Housing and Liquidity*

Chao He
Hanqing Advanced Institute, Renmin University of China

Randall Wright
University of Wisconsin-Madison, FRB Minneapolis and NBER

Yu Zhu
University of Wisconsin-Madison

September 17, 2012

Abstract

In addition to providing utility, and possibly capital gains, housing facilitates credit transactions when home equity serves as collateral. We document big increases in home-equity loans coinciding with the US house-price boom, and suggest a connection. When it is used as collateral, housing bears a liquidity premium. Since liquidity is endogenous, and depends to some extent on beleifs, even when fundamentals are deterministic and time invariant equilibrium house prices can display complicated patterns, including cyclic, chaotic and stochastic trajectories. Some equilibrium price paths look a lot like bubbles. The framework is tractable, yet captures several salient features of housing markets qualitatively, and to some extent quantitatively. We examine various mechanisms for determining the terms of trade, and different ways of specifying credit restrictions. We also study the impact of monetary policy on housing markets,

JEL Classification: E44, G21, R21, R31
Keywords: Housing, Liquidity, Collateral, Dynamics, Bubbles

*We thank many friends and colleagues for input, especially Charles Engle, Guillaume Rocheteau, Chao Gu, Derek Stacey, Dean Corbae, Ed Glaeser and Gadi Barley. We also thank participants in presentations at Wisconsin, the Chicago Fed, the University of Colorado, the SED in Cyprus and the NBER. Wright acknowledges support from the NSF and the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business. The usual discliamer applies.

1 Introduction

Housing plays three roles. First, it provides direct utility as shelter. Second, like other assets houses may generate capital gains or losses. Third, housing can facilitate intertemporal transactions when credit markets are imperfect: in the presence of limited commitment, it can be difficult for consumers to get unsecured loans, and this generates a role for home equity as collateral. This implies that equilibrium house prices can bear a *liquidity premium* – i.e., one may be willing to pay more than the fundamental value (defined below) because home ownership conveys security in the event that one needs a loan. Since liquidity is at least partly endogenous and depends on beliefs, even when fundamentals are deterministic and time invariant equilibrium house prices can display complicated patterns, including cyclic, chaotic and stochastic trajectories. Some of these resemble *bubbles*.[1]

Our goal is to make these ideas precise and study their implications for the aggregate US housing market experience since 2000. It is commonly heard there was a price bubble during this period, which eventually burst, leading to all kinds of problems. It has also been noted that there was a big increase in home-equity loans. Reinhart and Rogoff (2009) contend financial development[2] allowed consumers "to turn their previously illiquid housing assets into ATM machines." Ferguson (2008) also says households began to "treat their homes as cash machines," and reports that between 1997 and 2006 US consumers withdrew an estimated $9 trillion from home equity. Greenspan and Kennedy (2007) find home equity withdrawal financed about 3% of personal consumption between 2001 and 2005, while Disney and Gathergood (2011) find a fifth of the recent growth in household debt is explained by house prices. Mian and Sufi (2011) report that homeowners extracted 25 cents for every dollar increase in home equity, adding $1.25 trillion to debt from 2002 to 2008. They also

[1] Heuristically, one might say housing has a certain *moneyness*, in that it ameliorates intertemporal trading frictions. We argue below, however, that houses are different from currency. By way of preview, on the supply side, in contrast to currency houses are produced by profit-maximizers. And on the demand side, since houses yield utility, there are no equilibria where the price is 0 or where it goes to 0, which makes it more challenging to construct interesting dynamics in housing models than in monetary models.

[2] The development they have in mind is securitization. As Holmstrom and Tirole (2011) say, "In the runup to the subprime crisis, securitization of mortgages played a major role ... by making [previously] nontradable mortgages tradable, led to a dramatic growth in the US volume of mortgages, home equity loans, and mortgage-backed securities in 2000 to 2008."

report the loans were used for consumption, rather than, e.g., paying off credit card debt or buying financial assets, and were used more by those who are younger and have lower credit scores. This is all consistent with the theory presented below.

Figure 1 shows data for the US over the relevant period (exact data definitions and sources are given in Appendix D; all figures are at the end of the paper). First, there are house prices, deflated in two ways. One divides by the CPI to correct for the purely nominal effect of inflation. The other divides by an index of rental rates, to correct for inflation plus changes in the demand for shelter relative to other goods and services. These data illustrate what people mean by a bubble: a dramatic price run up, followed by collapse. We also show housing investment, normalized by GDP, to indicate what was happening with supply. And we show home-equity loans, this time normalized three ways. The first again uses the CPI to control for purely nominal effects. The second divides by nominal GDP, to show an increase in home-equity loans relative to general economic growth. The third divides by a measure of home equity, to make it clear that loans as a fraction of collateral increased, consistent with financial innovation. While the exact number depends on which of the series one uses, home-equity loans increased a lot, from a stable value normalized to 0.3 in the 1990s, to somewhere between 0.7 and 1.0 at their peak.

The message we take away from this is the following: coinciding with the start of the price boom, there began a sizable increase in the real value of home-equity loans, and an increase in housing investment; later prices drop fast, and investment falls, while home-equity lending stays up, at least for a while. Understanding home-equity lending is relevant for understanding these observations. If one considers a house only as a consumer durable, the rent-price ratio should be roughly the sum of the discount and depreciation rates. There can be other costs and benefits of owning, including tax implications, but while these may affect the level, as long as they are approximately constant the rent-price ratio should not look like the series in Figure 1.[3] Our position is that financial innovation led to a bigger role

[3] Others have considered this. Harding et al. (2007) estimate a depreciation rate around 2.5 percent, so for a reasonable discount rate, the rent-price ratio should be around 5. In the Campbell et al. (2009) data, from 1975 to 1995 the ratio is close to 5, but declines to 3.7 in 2007, consistent with our theory.

for home equity in credit markets, and this fueled housing demand. Taking the liquidity role of home equity into account, we show how to generate equilibria that are qualitatively, and to some extent quantitatively, consistent with experience. Again, some of our equilibria display trajectories that look a lot like bubbles.

As Case and Shiller (2003) say, "The term 'bubble' is widely used but rarely clearly defined. We believe ... the term refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated." Shiller (2011) more recently says "bubbles are social epidemics, fostered by a sort of interpersonal contagion. A bubble forms when the contagion rate goes up for ideas that support a bubble. But contagion rates depend on patterns of thinking, which are difficult to judge." Although phenomena like "excessive public expectations, social epidemics and interpersonal contagion" seem fascinating, we instead emphasize liquidity. Precisely, a bubble here is an asset price different from its fundamental value, the present value of holding the asset for its returns, which can arise due to liquidity considerations. This is consistent with, e.g., Stiglitz (1990), who says "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists." We do not want to get bogged down in semantics, however, and would not mind if others prefer to use *liquidity premium* instead of *bubble*.

In emphasizing credit frictions, we follow a large literature summarized in Gertler and Kiyotaki (2010) or Holmstrőm and Tirole (2011), with a direct antecedent being Kiyotaki and Moore (1997,2005). But there is a key difference: those papers put borrowing restrictions on *firms*; we put the restrictions on *households*.[4] There is a huge literature on asset pricing, generally, but we argue that housing is different from many other assets – e.g., typical assets generate returns that enter your budget equation, while houses enter your

[4] In a sense, this makes the approach similar at least is spirit to the monetary model in Kiyotaki and Wright (1989), although except for an extension in Section 6, there is no currency here. The literaure on modern versions of the related monetary theory is discussed in the recent surveys by Williamson and Wright (2010) and Nosal and Rocheteau (2011). The defining characteristic of the approach is that it tries to be explicit about the process of exchange, by going into detail concerning how agents trade (bilateral, intermediated, etc.), using which instruments (money, unsecured credit, secured credit, etc.) and at what terms (as determined by various mechanisms, including price taking, posting, bargaining, etc.).

budget equation and your utility function. This changes the circumstances under which bubbles can exist from one of low supply, for financial assets, to one of either low or high supply depending on the utility function, for housing. Relatedly, welfare may decrease with an increase in the housing stock, which typically does not happen with other assets.

Farhi and Tirole (2011) provide a recent review of research on bubbles generally. As for housing, several papers study the precautionary or collateral function of home equity, and at the risk of neglecting some important contributions we can only cite a few examples that influenced our thinking.[5] A difference from some recent work (e.g., the novel approach in Burnside et al. 2011) is that we focus on fully rational agents, with homogenous beliefs. Indeed, we generate bubble-like equilibria under perfect foresight. We also consider both a fixed and an endogenous housing supply, which is relevant because some suggest "The housing-price boom of the 2000's was little more than a construction-supply bottleneck, an inability to satisfy investment demand fast enough, and was (or in some places will be) eliminated with massive increases in supply" (Shiller 2011; see also Glaeser et al. 2008). We consider both exogenous and endogenous debt limits, and various mechanisms for determining the terms of trade. Finally, we study the effects of monetary policy.

Section 2 presents the economic environment. Sections 3 and 4 discuss steady states and dynamics with a fixed supply. Section 5 endogenizes supply. Section 6 studies a monetary version of the model and the effects of inflation. Section 7 concludes.

2 The Basic Model

In each period of discrete time, agents interact in two distinct markets. One is a frictionless centralized market, labeled AD for Arrow-Debreu. The other is a decentralized market, with frictions detailed below, labeled KM for Kiyotaki-Moore. At each date t, in addition

[5] Related work includes Carroll et al. (2003), Hurst and Stafford (2004), Campbell and Hercowitz (2005), Arce and Lopez-Salido (2011), Brady and Stimel (2011), Coulson and Fisher (2009), Ngai and Tenreyro (2009), Novy-Marx (2009), Piazzesi and Schneider (2009), Jaccard (2011) and Aruoba et al. (2011). Liu et al. (2011) also assume real estate is used as collateral, but by producers, not consumers. See also Miao and Wang (2011) and Liu and Wang (2011). Caballero and Krishnamurthy (2006) and Fostel and Geanakoplos (2008) are somewhat related. Also, in terms of the literature, this paper is *not* about imperfect housing markets: houses here are traded in frictionless markets, like capital in standard growth theory. A recent paper by Head and Lloyd-Ellis (2012) provides references to work on frictional housing markets.

to labor ℓ_t, there are two nonstorable consumption goods x_t and y_t, plus housing h_t. We assume ℓ_t, x_t and h_t are traded in the AD market, and y_t in the KM market. The utility of a household is given by

$$\lim_{T\to\infty} \mathbb{E} \sum_{t=0}^{T} \beta^t \left[\mathcal{U}(x_t, y_t, h_t) - A\ell_t \right], \tag{1}$$

where $\beta \in (0,1)$ and $A > 0$.[6] Although it is not crucial, to ease the presentation, let $\mathcal{U}(x_t, y_t, h_t) = U(x_t, h_t) + u(y_t)$, where $U(\cdot)$ and $u(\cdot)$ satisfy all the usual assumptions, as well as $U(0,0) = u(0) = 0$.

For now there is a fixed stock of housing H. In terms of AD goods, ℓ_t can be converted one-for-one into x_t (it is easy to use more general technologies). In terms of KM goods, some agents can produce y_t using a technology summarized by cost function $v(y_t)$. In related papers, sometimes households buy y_t from each other and $v(y_t)$ is interpreted as a disutility cost; in other papers, households buy from firms or retailers (see the Nosal-Rocheteau or Williamson-Wright surveys mentioned above). Although it does not matter for our results, we follow the latter approach, with households buying y_t from KM retailers. The retail technology works as follows: by investing at $t-1$ a fixed amount, normalized to 1, of the AD numeraire x_{t-1}, a retailer can at t generate any $y_t \leq 1$ of the KM good plus $x_t = F(1-y_t)$ of the AD good. The profit from this activity, conditional on selling y_t in the KM market at t for revenue R_t, measured in period t numeraire, is $R_t + F(1-y_t) - (1+r)$, given the initial investment is repaid at interest rate $1 + r = 1/\beta$.

Not all retailers earn the same payoff, since not all trade in the KM market. Let α_f be the probability a retailer trades in KM, and α_h the probability a household trades. Also, assume $y \leq 1$ is not binding, as is the case, e.g., if $F'(0) = \infty$. Then expected profit is

$$\begin{aligned} \Pi_t &= \alpha_f \left[R_t + F(1-y_t) \right] + (1-\alpha_f) F(1) - (1+r) \\ &= \alpha_f \left[R_t - v(y_t) \right] + F(1) - (1+r), \end{aligned} \tag{2}$$

where $v(y_t) \equiv F(1) - F(1-y_t)$ is the opportunity cost of selling y_t in the KM market. As in standard search theory (Pissarides 2000), if there is a $[0,1]$ continuum of households

[6] We assume here the limit in (1) exists; if not, one can use more advanced optimization techniques (see the discussion and citations in Rocheteau and Wright 2010).

and a $[0, N]$ continuum of retailers, the KM trading probabilities can be endogenized by $\alpha_h = \alpha(n)$ and $\alpha_f = \alpha(n)/n$, where $\alpha(\cdot)$ comes from a standard matching technology and $n \leq N$ is the measure of firms participating in KM. Firms may have to pay an entry cost, in addition to their initial investment in x_t, and n can be determined by a standard free-entry condition. To make our main points, however, we assume this cost is small and $1+r < F(1)$, so that $n = N$. This implies $\alpha_h = \alpha$ and $\alpha_f = \alpha/N$ are fixed constants.

It is useful to have a frictional KM market.[7] One can model this using search theory, but it is not necessary to invoke search *per se*. An alternative is to assume households sometimes realize a demand for y_t due to preference or opportunity shocks. Nice examples include the possibility that one has occasion to throw a party, or to buy a boat at a good price; not-so-nice examples include the possibility that one has a medical emergency, or one's boat sinks. In any case, to show our results are robust, we consider various options for KM pricing, which as discussed below can be interpreted as either bilateral or multilateral trading. More significantly, in the KM market households have to buy on credit, since they have nothing to offer retailers by way of *quid pro quo* for now (in Section 6 they can pay with cash) So y_t is acquired in exchange for debt d_t to be retired in the next AD market. Note that with quasi-linear utility, households are indifferent between short- and long-term loans if they are not credit constrained, and actually prefer short-term debt if they are constrained, assuming an interior solution for ℓ.

Credit is limited by lack of commitment: households are free to default, albeit at the risk of punishment. At one extreme, punishment is so severe that credit is effectively perfect. At the other extreme, there is no punishment, not even exclusion from future credit, maybe because borrowers are anonymous, so unsecured credit is impossible. In general, we impose a debt limit $d \leq D = D(e_t)$, with $e_t = \psi_t h_t$, where ψ_t is the price of h_t in terms of x_t. Often we focus on $D(e_t) = D_0 + D_1 e_t$, with $D_0 \geq 0 \leq D_1 \leq 1$, but only to ease the presentation. It makes sense to consider $D_1 < 1$ if a creditor can seize only some assets after a default

[7] The reason (we learned from Peralta-Alva et al. 2011) is the following. Our claim is that home-equity loans are important in the recent housing episode. How can this be when consumption did not rise all that much? The answer is that households have a *precautionary demand* for home-equity loans: these loans can be highly valued even if they get used with probability $\alpha < 1$.

– e.g., he gets the house while the debtor absconds with the appliances. But $D(e_t) > e_t$ is also possible when we have punishments beyond confiscating collateral. For now $D(e_t)$ is exogenous, but in Appendix C we show how to endogenize it as in Kehoe-Levine (1993) or Alvarez-Jermann (2000), both for the case where punishment for default involves taking away unsecured credit, and for the case where it involves taking way all future credit.

Let $W_t(d_t, h_t)$ be a household's value function entering the AD market with debt d_t and housing h_t. Since d_t is paid off each period in AD, households start debt free in KM, where $V_{t+1}(h_{t+1})$ is the value function, next period. The household's AD problem is

$$W_t(d_t, h_t) = \max_{x_t, \ell_t, h_{t+1}} \{U(x_t, h_t) - A\ell_t + \beta V_{t+1}(h_{t+1})\} \tag{3}$$
$$\text{st } x_t + \psi_t h_{t+1} = \ell_t + \psi_t h_t + T_t - d_t \text{ and } \ell_t \in [0, \bar{\ell}]$$

where T_t denotes all other income, including transfers minus taxes, dividends from firm ownership, etc. With quasi-linear utility, T_t affects nothing except leisure, so we do not need to specify other income explicitly to derive the main results. We do need to assume that wealth other than home equity – e.g., future wage or profit income – cannot be used to secure loans, perhaps because it is hard to seize (in the language of Holmstrom and Tirole 2011, it is not *pledgeable*).

Before solving (3), we digress briefly to show how our approach is consistent with much micro and macro research on household production (Aruoba et al. 2012 provide a recent example and references). Suppose households value goods acquired on the market x_t^1, and those produced at home x_t^2. They also engage in market work ℓ_t^1, and home work ℓ_t^2. If $x_t^2 = G_t\left(\ell_t^2, h_t\right)$ is the home production function, the generalization of (3) is

$$W_t(d_t, h_t) = \max_{x_t^1, x_t^2, \ell_t^1, \ell_t^2, h_{t+1}} \left\{U(x_t^1, x_t^2) - A_1\ell_t^1 - A_2\ell_t^2 + \beta V_{t+1}(h_{t+1})\right\} \tag{4}$$
$$\text{st } x_t^1 + \psi_t h_{t+1} = \ell_t^1 + \psi_t h_t + T_t - d_t,\ x_t^2 = G_t\left(\ell_t^2, h_t\right) \text{ and } \ell_t^1 + \ell_t^2 \in [0, \bar{\ell}].$$

Now h_t does not enter $U(\cdot)$ directly, but indirectly as an input to $G_t(\cdot)$ (one could have both, disaggregating home capital into, say, vacuum cleaners and plasma TV's). As is well known, we can substitute out $x_t^2 = G_t\left(\ell_t^2, h_t\right)$ and maximize out ℓ_t^2, to derive a reduced-form that depends only on market variables. Given this, although sometimes there are reasons for being more explicit about home production, we assume h_t enters $U(\cdot)$ directly.

Returning to (3), assuming interiority $0 < \ell_t < \bar{\ell}$, we eliminate ℓ_t and write

$$W_t(d_t, h_t) = \psi_t h_t + T_t - d_t + \max_{x_t} \{U(x_t, h_t) - x_t\} + \max_{h_{t+1}} \{\beta V_{t+1}(h_{t+1}) - \psi_t h_{t+1}\} \quad (5)$$

where we normalize the disutility of work to $A = 1$. Immediately this implies choices at t, and in particular h_{t+1}, are independent of (d_t, h_t), which simplifies the analysis because we do not have to keep track of the wealth distribution.[8] The FOC's from (5) are

$$U_1(x_t, h_t) = 1 \text{ and } \psi_t = \beta \frac{\partial V_{t+1}}{\partial h_{t+1}}. \quad (6)$$

The envelope conditions are

$$\frac{\partial W_t}{\partial d_t} = -1 \text{ and } \frac{\partial W_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t, \quad (7)$$

so that W is linear in debt – more generally, net worth – but not housing since h_t affects $U(\cdot)$ directly as well as indirectly through the budget constraint.

In the KM market, retailers (or sellers) with trading opportunities produce and households (or buyers) with trading opportunities consume y_t, in return for which the latter issue debt $d_t \leq D(e_t)$ coming due in the next AD market. To determine (y_t, d_t), we begin with Walrasian pricing To motivate this, one can imagine KM trade is multilateral as in a Lucas-Prescott search model, e.g., as opposed to bilateral as in a Mortenson-Pissarides model. In any case, the trade surplus for buyers is $S_{bt} = u(y_t) + W(d_t, h_t) - W(h_t) = u(y_t) - d_t$, since $W(\cdot)$ is linear in d_t. Buyers maximize S_{bt} subject to $d_t = p_t y_t \leq D_t$, taking as given p_t and D_t. Sellers maximize $S_{st} = p_t y_t - v(y_t)$. To reduce notation, assume the measures of buyers and sellers are equal. Then, if we ignore the credit constraint, for a moment, equilibrium is $y_t = y^*$, where $u'(y^*) = v'(y^*)$, and $p^* = v'(y^*)$. If $d^* = p^* y^* \leq D_t$ the actual equilibrium entails $(y_t, d_t) = (y^*, d^*)$. But if $d^* > D_t$ then the equilibrium entails $p_t = v'(y_t)$ from

[8] To be clear, this presumes the constraint $\ell_t \in [0, \bar{\ell}]$ is slack. More generally, people with very low or high net worth may be unable to set ℓ_t high or low enough to settle all debt d_t and get to their preferred h_{t+1} in one period using their labor income; but if we start with a distribution of (h_t, d_t) that is not too disperse relative to $[0, \bar{\ell}]$, that is not a problem.

the sellers' FOC, and $y_t = D_t/p_t$ from the buyers' constraint. Thus, if $d^* > D_t$ we have a debt-constrained equilibrium, where $d_t = D_t$ and y_t solves $v'(y_t)\, y_t = D_t$.

For future reference let $g(y_t) \equiv v'(y_t)\, y_t$, and note that $g'(y_t) > 0$, so that when $d^* > D_t$ we can write $y_t = g^{-1}(D_t) < y^*$. We formalize these results as follows:

Proposition 1 *Let (y^*, d^*) be the equilibrium ignoring $d_t \leq D_t$, and let $d^* = g(y^*)$. As shown in Figure 2, KM equilibrium at t is given by:*

$$y_t = \begin{cases} g^{-1}(D_t) & \text{if } D_t < d^* \\ y^* & \text{if } D_t \geq d^* \end{cases} \quad \text{and } d_t = \begin{cases} D_t & \text{if } D_t < d^* \\ d^* & \text{if } D_t \geq d^* \end{cases} \tag{8}$$

Instead of Walrasian pricing, suppose we pair off buyers and sellers and let them bargain bilaterally, as in a Mortenson-Pissarides model. Consider generalized Nash bargaining,

$$(y_t, d_t) = \arg\max S_{bt}^{\theta} S_{st}^{1-\theta} \text{ st } d_t \leq D_t.$$

One can show this gives the same outcome as (8) except instead of $g(y) = v'(y)\, y$ we have

$$g(y) = \frac{\theta v(y)\, u'(y) + (1-\theta)\, u(y)\, v'(y)}{\theta u'(y) + (1-\theta)\, v'(y)}. \tag{9}$$

Thus, Nash says $d^* = g(y^*) = \theta v(y^*) + (1-\theta)\, u(y^*)$, while Walras says $d^* = v'(y^*)\, y^*$, but either way Proposition 1 holds. Another option is Kalai bargaining. As one can show, this also satisfies (8), but with $g(y) = \theta v(y) + (1-\theta)\, u(y)$, which differs from Nash iff $d_t \leq D_t$ binds. As final option, we present an explicit strategic bargaining game in Appendix A that also satisfies (8). We consider all these options to show the results are robust.[9]

In each case, the equilibrium outcome is given by (8) for a particular $g(\cdot)$, and we can write the KM value function as

$$V_t(h_t) = W(0, h_t) + \alpha \left\{ u\left[y(\psi_t h_t)\right] - d(\psi_t h_t) \right\}, \tag{10}$$

where it is understood that $y(\psi_t h_t)$ and $d(\psi_t h_t)$ are given by (8) with $D_t = D(\psi_t h_t)$. Using (7)-(8) we derive

$$\begin{aligned} \frac{\partial V_t}{\partial h_t} &= U_2(x_t, h_t) + \psi_t + \alpha \left[u'(y)\, y'(\psi_t h_t) - d'(\psi_t h_t) \right] \psi_t \\ &= U_2(x_t, h_t) + \psi_t + \alpha D_1 \mathcal{L}\left[y(\psi_t h_t) \right] \psi_t \end{aligned} \tag{11}$$

[9] Strategic bargaining is also relevant because one may well question the use of axiomatic solutions in nonstationary models (e.g., Coles and Muthoo 2003).

where we define the *liquidity premium*:

$$\mathcal{L}(y) \equiv \begin{cases} u'(y)/g'(y) - 1 & \text{if } y < y^* \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Inserting these results into the FOC for h_{t+1} in (6), we get the Euler equation

$$r\psi_t = U_2(x_{t+1}, h_{t+1}) + \left(\psi_{t+1} - \psi_t\right) + \alpha D_1 \mathcal{L}\left[y\left(\psi_t h_t\right)\right]\psi_{t+1}. \tag{13}$$

The three terms on the RHS of (13) describe the three effects of a having bigger house mentioned in the Introduction: 1) it provides more utility; 2) it generates more capital gains or losses; and 3) it secures more credit. Setting $h_t = H$ and using the FOC $U_1(x_t, H) = 1$ to define $x_t = X(H)$, (13) gives us a univariate difference equation in the price of housing, $\psi_t = \Psi\left(\psi_{t+1}\right)$. An equilibrium is any nonnegative and bounded sequence $\{\psi_t\}$ solving $\psi_t = \Psi\left(\psi_{t+1}\right)$.[10] From ψ_t, we easily recover $e_t = \psi_t H$, $D_t = D\left(e_t\right)$, $y_t = y\left(e_t\right)$, etc.

3 Steady State

A steady state solves $\psi = \Psi\left(\psi\right)$, so there are no capital gains, and (13) becomes

$$r\psi = U_2\left[X(h), h\right] + \alpha D_1 \mathcal{L}\left[y\left(e\right)\right]\psi. \tag{14}$$

One can interpret this as the long-run demand for housing, with slope

$$\frac{\partial h}{\partial \psi} = \frac{U_{11}\left(U_2 - \alpha\psi^2 h \mathcal{L}' y'\right)}{\left(U_{11}U_{22} - U_{12}^2 + U_{11}\alpha\psi^2 \mathcal{L}' y'\right)\psi}. \tag{15}$$

Obviously $\partial h/\partial\psi < 0$ if $\mathcal{L}'\left(y\right) < 0$. One can prove $\mathcal{L}'\left(y\right) < 0$ for Walras and Kalai pricing, but not Nash (without additional assumptions, like θ close to 1). Even without $\mathcal{L}'\left(y\right) < 0$ one can prove $\partial h/\partial\psi < 0$, as in Wright (2010), but to avoid these technicalities let us suppose $\mathcal{L}'\left(y\right) < 0$. Then we have:

Proposition 2 *With fixed H, there exists a unique steady state $\psi = \psi^s$.*

If $e = \psi H > e^*$ then $\mathcal{L}(e) = 0$ and (14) implies $\psi^s = \psi^* \equiv U_2\left[X(H), H\right]/r$, where we define the fundamental price ψ^* by the discounted marginal utility of living in house

[10] Boundedness is required to satisfy a standard transversality condition (Rocheteau and Wright 2010).

H forever. But if $e < e^*$, then $\mathcal{L}(y) > 0$ and (14) implies $\psi^s > \psi^*$. In this case, houses bear a premium that we call a bubble, although we prefer not to debate semantics at this stage and instead focus on economics. The simple economic idea is this: when credit constraints bind, agents are willing to pay a premium for assets that relax them. One example is cash, but there are related results for similar models with equity and neoclassical capital (Lagos and Wright 2005; Geromichalos et al. 2007; Lagos and Rocheteau 2008). Housing is different. Consider cash. In monetary models, whenever there is a steady state where cash is valued there is another where it is not; our housing model has a unique steady state.

Now consider a financial asset in fixed supply, say equity in a "Lucas tree" paying a dividend ξ in terms of numeraire, with fundamental price ξ/r. One can show there is a liquidity premium iff S is low in this kind of model (see, e.g., Lester et al. 2012). With housing there is also a liquidity premium iff e is low, but e can be low either when H is low or when H is high, depending on the elasticity of demand: $\partial e/\partial H$ takes the same sign as $-H\left(U_{22}U_{11} - U_{21}^2\right) - U_2 U_{11}$, which is ambiguous. For $U(x,h) = \tilde{U}(x) + h^{1-\sigma}/(1-\sigma)$, e.g., $\sigma < 1$ implies housing bears a premium when H is low, and $\sigma > 1$ implies it bears a premium when H is high. Because of this, welfare W can actually fall as H increases, while W is always increasing in the supply of "tress" in this kind of model. When H increases AD utility must rise, but KM utility can fall, and in examples it can fall by enough for a net decrease in W. Intuitively, if demand is elastic we may want H to be scarce, since this makes it valuable and that relaxes credit restrictions.

Relatedly, while ψ^s always increase with α, it is ambiguous how it changes with D_1. Higher D_1 makes home equity more useful as collateral, but also means a given amount of collateral goes further, which makes it less valuable at the margin. We summarize the results about steady state as follows:

Proposition 3 *If $e > e^*$ then $\psi^s = \psi^*$; if $e < e^*$ then $\psi^s > \psi^*$. We can have $e < e^*$ either when H is low or when H is high, depending on $U(\cdot)$. Also, $\partial\psi^s/\partial\alpha > 0$, but $\partial\psi^s/\partial D_1$ is ambiguous, as is $\partial W/\partial H$.*

4 Dynamics

Consider first deterministic equilibria, given by nonnegative and bounded solutions to

$$\psi_t = \Psi\left(\psi_{t+1}\right) = \frac{U_2\left[X\left(H\right), H\right] + \psi_{t+1} + \alpha D_1 \mathcal{L}\left[y(\psi_{t+1}H)\right]\psi_{t+1}}{1+r}. \tag{16}$$

The first observation is that any interesting dynamics must emerge from liquidity considerations, which show up in the nonlinear term $\mathcal{L}\left[y(\psi_{t+1}H)\right]$. To see this, set $\alpha = 0$ or $D_1 = 0$. Then (16) is a linear difference equation that can be rewritten $\psi_{t+1} = -U_2\left[X\left(H\right), H\right] + \left(1+r\right)\psi_t$. This has a unique steady state at the fundamental price ψ^*, which is also the unique equilibrium, since any solution other than $\psi_t = \psi^*\ \forall t$ has ψ_t unbounded. There are no interesting dynamics when the liquidity motive is inoperative.

When $\alpha D_1 > 0$, however, as long as $H\psi_{t+1} < e^*$ we have $\mathcal{L}\left[y(\psi_{t+1}H)\right] > 0$, and the nonlinearity kicks in. We analyze this in $\left(\psi_{t+1}, \psi_t\right)$ space, where it is natural to think of ψ_t as a function of ψ_{t+1}, because given ψ_{t+1} demand for h_t is single-valued, and so market clearing uniquely pins down ψ_t. However, as usual, there can be multiple values of ψ_{t+1} for which this mapping yields the same ψ_t, so the inverse $\psi_{t+1} = \Psi^{-1}\left(\psi_t\right)$ can be a correspondence. Of course Ψ and Ψ^{-1} cross on the 45^o line at the unique steady state. Textbook methods (e.g., Azariadis 1993) tell us that whenever Ψ^{-1} and Ψ cross off the 45^o line there is a cycle of period 2, i.e., a solution $\left(\psi^1, \psi^2\right)$ to $\psi^2 = \Psi\left(\psi^1\right)$ and $\psi^1 = \Psi\left(\psi^2\right)$, or a fixed point of Ψ^2, that is nondegenerate in the sense that $\psi^1 \neq \psi^2$. This happens whenever Ψ has a slope less than -1 on the 45^o line. In a 2-cycle ψ oscillates between ψ^1 and ψ^2 as a self-fulfilling prophecy.

Before discussing the economics, consider more generally n-period cycles, which are nondegenerate solutions to $\psi = \Psi^n\left(\psi\right)$. We show that n-cycles exist for different n by way of example, since there is no claim that exotic equilibria always obtain, only that they can. Consider $H = 1$, $v\left(y\right) = y$, $D\left(e\right) = e$ and[11]

$$U\left(x, h\right) = \tilde{U}\left(x\right) + \kappa\frac{h^{1-\sigma}}{1-\sigma} \text{ and } u\left(y\right) = \eta\frac{\left(y+\varepsilon\right)^{1-\gamma} - \varepsilon^{1-\gamma}}{1-\gamma}.$$

[11] The functional form of $\tilde{U}$ is irrelevant, as is σ for some results, in which case we report n/a in Table 1. The role of ε is simply to force $u\left(0\right) = 0$.

Parameter values for different examples are given in Table 1 for various KM pricing mechanisms, including Walrasian pricing and axiomatic or strategic bargaining. We emphasize this because in some models one gets cycles for certain mechanisms and not others – e.g., Gu et al. (2012) can construct cycles in a Kehoe-Levine model using generalized Nash, but not Kalai or buyer-take-all bargaining. Nonlinearity here comes from the nature of liquidity, not from the nonmonotonicity of generalized Nash bargaining.

	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6
Mechanism	Walras	Walras	Kalai	Game	Walras	Walras
α	0.5	0.5	0.9	0.5	0.99	0.99
β	0.8	0.6	0.6	0.8	0.95	0.95
θ	n/a	n/a	0.9	0.6	n/a	n/a
κ	0.125	0.3333	0.1	0.125	0.1	0.1
σ	n/a	n/a	n/a	n/a	4	4
η	1.5125	3.2479	0.5882	3.0368	1.027	1.028
γ	2	7	9	8	8	16
ε	0.1	0.1	0.5	0.1	0.0001	0.0001

Table 1: Parameter Values for the Examples

In Example 1, with Walrasian pricing, the steady state is the unique equilibrium (Figure 3a). In Example 2, with Walrasian pricing but different parameters, there is a 2-cycle in addition to the steady state (Figure 3b). In this case, $y^* = 1.0833$, the constraint binds iff $\psi < y^*$, given we have $D(e) = e$ and $H = 1$, and this happens in alternate periods. Examples 3 and 4 display 2-cycles with Kalai and strategic bargaining, resp. Example 2 also has a 3-cycle, with $\psi^1 = 0.8680 < y^*$, $\psi^2 = 1.5223 > y^*$ and $\psi^3 = 1.1134 > y^*$. If a 3-cycle exists, there exist n-cycles for all n by the Sarkovskii theorem, as well as chaotic dynamics by the Li-Yorke theorem (again see Azariadis 1993). Chaos is observationally equivalent to a stochastic process for house prices, and obtains even though we have time-invariant and deterministic fundamentals. We also have fully rational agents – indeed, for now they have perfect foresight. House prices fluctuate because housing conveys liquidity, which can involve self-fulfilling prophecies, as we now explain.

Consider a 2-cycle. Suppose at t agents expect ψ_{t+1} will be high. Then home equity and hence liquidity will be relatively plentiful at $t+1$, and this lowers the amount people are willing to pay for it at t. Thus, low ψ_t is consistent with market clearing given high ψ_{t+1}. By the same (i.e., opposite) logic high ψ_{t+1} is consistent with low ψ_{t+2}, and so on. Agents are willing to pay more for H when they know the price is about to drop, precisely because the price drop means liquidity will soon be scarce, which makes it currently dear. Hence, the prices of liquid assets like housing have an inherent tendency to oscillate, i.e., there can be equilibria where they oscillate, even though there also always exists a steady state equilibrium with $\psi_t = \psi^s \; \forall t$. Cycles of order $n > 2$ and chaos are more complicated self-fulfilling prophecies, but the idea is similar.

While these examples make a point, ψ_t tends to go up and down rather too regularly, compared the stereotypical bubble pattern of a prolonged run up followed by collapse. So, Example 5 displays an equilibrium with just such a pattern. Figure 4 depicts in blue the 45^o line and in red the curve giving ψ_{t+1} as a correspondence of ψ_t, zooming in around the steady state. As one can see, ψ^s corresponds to two values of ψ_{t+1}. One equilibrium is $\psi_t = \psi^s \; \forall t$. But there is another, where at some completely arbitrary date we jump to $\psi_{t+1} = \psi_1 > \psi^s$, then set off on the increasing trajectory shown by the dashed lines. Along this path ψ_t keeps increasing, but this cannot go on forever (without violating transversality). In the equilibrium under construction, after 5 periods of increases, the price plummets from ψ_2 to $\psi_3 < \psi^s$, before recovering in an oscillatory path back to the original ψ^s. This example clearly *does* display the stereotypical pattern. Moreover, it uses a very reasonable $\beta = 0.95$, although risk aversion is somewhat high with $\gamma = 8$, and lowering γ requires lowering β to get an equilibrium with a similar pattern.[12]

Rather than trying to fine tune the parameters, at this stage, we instead provide a negative result, or at least describe something one cannot do with perfect foresight (suggested by conversations with Charles Engle). First, rearrange (16) as

$$\frac{\psi_{t+1} - \psi_t}{\psi_t} = r - \frac{U_2\left[X\left(H\right), H\right]}{\psi_t} - \frac{\alpha \psi_{t+1} D_1 \mathcal{L}\left[y(\psi_{t+1} H)\right]}{\psi_t}. \tag{17}$$

The capital gain on the LHS is bounded above by r, since the RHS is r minus two positive

[12] Note that γ is risk aversion for KM utility $u(y)$ (approximately, with the approximation exact when $\varepsilon \to 0$). Our results are independent of risk aversion or any other property of AD utility $\tilde{U}(x)$.

terms, the service flow and the liquidity premium. Hence, there is no way for perfect-foresight dynamics to generate capital gains exceeding r, intuitively, because that would open up opportunities for arbitrage profits. To the extent that one observes episodes with capital gains above what one considers reasonable values for r, as is arguably the case for the data in Figure 1, this is inconsistent with perfect-foresight equilibrium.

Based on this, we consider *sunspot equilibria*, where ψ_t fluctuates stochastically, even though fundamentals are deterministic. As a simple example, suppose that when the price is ψ^1 it jumps to $\psi^2 > \psi^1$ with probability λ^1 and stays put with probability $1 - \lambda^1$, while when it is ψ^2 it falls back to ψ^1 with probability λ^2 (and of course agents have rational expectations: they know λ^1 and λ^2). Using standard methods, one can show:[13]

Proposition 4 *For some parameters sunspot equilibria as well as deterministic cycles exist.*

The intuition for stochastic fluctuations is similar to what we gave for deterministic cycles, but quantitatively there is a difference. Example 6 displays an equilibrium where prior to $t = 4$, $\psi_t = \psi^s = 0.5255$. From $t = 4$ to 8, every period there is a probability of jumping to a deterministic path transitioning between $\psi = 0.5350$ and ψ^s. These probabilities change each period, and agents know this. After $t = 8$, everything is deterministic again. One realization of this stochastic process has ψ increasing at 9% per year for 5 years, then collapsing, and oscillating back to ψ^s. The example uses $\gamma = 16$, which we do not claim is realistic, but it makes the important point that in principle sunspots overcome the bound on capital gains given in (17). If we want big capital gains, naturally, they must occur with a low probability, so a realization where ψ_t increases by a lot several periods in a row is a rare event. In the example it can be expected to happen about once a century. This actually seems about right: even if bubbles are recurrent events, as one learns from Reinhart and Rogoff (2009), they do not happen all the time.

[13] Again see Azariadis (1993). One method is to note that in the limit when $\lambda^1 = \lambda^2 = 1$, the sunspot equilibrium described in the text reduces to a 2-cycle, and then appeal to continuity. This guarantees sunspot equilibria exist for the same parameters that generate a 2-cycle, and thus constitutes a proof of the Proposition. Such equilibria, however, are quuatitatively similar to 2-cycles, and not much like the data. We construct the example below directly, not using the existence argument and continuity. Hence, this example looks less like a 2-cycle, and more like the data.

To close Section 4, we acknowledge that some of our results are reminiscent of monetary economics, which makes sense, given that H and M are both assets that facilitate intertemporal transactions. There are also notable differences. Models of fiat money always have multiple steady states, including $\psi = 0$. Here we have a unique steady state $\psi^s > 0$. This makes it more difficult to construct nonstationary equilibria, e.g., one cannot focus on paths that transit from one steady state to another, or oscillate between points close to two steady states. We mention this because part of our motivation came from an example of Narayana Kocherlakota, with something he calls housing, where in equilibrium price jumps at a random date from $\psi > 0$ to $\psi = 0$. This is a one-time event: after ψ goes down it can never come back. In his example H is a fiat object, better labeled M, because it has zero fundamental value (provides no utility). Our H has intrinsic value, ruling out equilibria where $\psi = 0$, and those where $\psi_t \to 0$, either stochastically or deterministically.

Still we can generate equilibria with recurrent cycles, as well as equilibria with booming-and-bursting prices. We still give credit to Kocherlakota for the example, and for noticing that bubbles are not necessarily bad – a point to which we return below.

5 Construction

Another way H differs from M is that the former can be produced by profit-maximizing private agents, which is relevant because the supply side is an interesting part of the recent housing-market experience. We introduce a technology for home building, where producing Δh_t units requires an input of $c(\Delta h_t)$ in AD numeraire. The construction business, like other AD activity, is perfectly competitive. Therefore, in equilibrium

$$\psi_t = c' \left[h_{t+1} - (1-\delta) h_t \right], \tag{18}$$

where δ is a depreciation rate. The households' AD problem is unchanged, except now $e_t = \psi_t (1-\delta) h_t$, and the Euler equation becomes

$$(1+r) \psi_t = U_2 \left[X (h_{t+1}), h_{t+1} \right] + \psi_{t+1} (1-\delta) + \alpha D_1 (1-\delta) \mathcal{L} \left[y \left(h_{t+1} \psi_{t+1} \right) \right] \psi_{t+1}. \tag{19}$$

In steady state, (18)-(19) can be written

$$(r+\delta)\,\psi = U_2\left[X\left(h\right),h\right] + \alpha\left(1-\delta\right)D_1\mathcal{L}\left[y\left(e\right)\right]\psi \tag{20}$$

$$\psi = c'\left(\delta h\right), \tag{21}$$

where (20) is a straightforward generalization of long-run demand (14), while (21) is a long-run supply relation. Combining (20) and (21), we get

$$r+\delta = \frac{U_2\left[x\left(h\right),h\right]}{c'\left(\delta h\right)} + \alpha\left(1-\delta\right)D_1\mathcal{L}\left[\left(1-\delta\right)c'\left(\delta h\right)h\right].$$

The RHS goes to ∞ as h goes to 0, and vice-versa, and it is strictly decreasing. Hence, there exists a steady state (h^s, ψ^s) and it is unique. As before, we can get a liquidity premium when the (now endogenous) supply is high or when it is low, depending on elasticities. As a special case, the previous model is recovered by making supply vertical. At the other extreme, if supply is horizontal the price is pinned down by the constant marginal cost. But except for the case of horizontal supply, all results derived earlier survive.

Proposition 5 *Propositions 2-4 hold with H endogenous and $c'(\Delta h_t) > 0$.*

Moving beyond steady states, an equilibrium is defined by a nonnegative and bounded path for $\{h_t, \psi_t\}$ satisfying (18)-(19). One should anticipate the existence of complicated equilibria in this bivariate system, given the univariate results. Instead of an exhaustive analysis of dynamics, we instead use the model to organize a particular narrative concerning recent events. As the story goes, at the start of the episode in question, financial innovation gave households easier access to home-equity loans – this is what it means to say they started using previously illiquid housing assets as ATM's. This stimulated demand, and hence prices, followed by a downturn as supply eventually caught up. We now show how this can happen in two distinct ways. One is to specify parameters that yield a multiplicity of dynamic equilibria, and select one that resembles the data. The other relies on financial innovation occurring gradually over time. Heuristically, for this exercise, it is useful to recall from Proposition 3 that ψ is not necessarily monotonic in D_1.

For the first experiment, we start in steady state where $D(e) = D_1 e$ with D_1 small, and consider an unexpected, once-and-for-all, increase in D_1. There is one predetermined variable (quantity) and one jump variable (price), and for some parameter values we get saddle-path stability. In this case, there is a unique equilibrium, where ψ jumps at $t = 2$ and monotonically declines to the new (h^s, ψ^s) as construction gradually catches up with demand. For other parameters, the system displays a classic indeterminacy where (h^s, ψ^s) is a sink. In this case, there are many perfect-foresight paths leading to the new steady state. This means ψ can jump after the change in D_1 to any value in some interval before beginning the transition, giving us some freedom to pick a path that looks something like the episode in question. One such equilibrium is shown in Figure 5, constructed under Walrasian pricing, using parameters such that $y \leq D(e)$ is binding, and verifying numerically that both eigenvalues are real and less than 1 at (h^s, ψ^s). We claim Figure 5 looks like Figure 1.

By this we do not mean, of course, that they look *exactly* the same, but the paths are similar in the following sense: the price first soars then tumbles, whether we measure it by the price relative to numeraire or the price-rent ratio; home-equity loans go up, and stay up, as households take advantage of the new financial opportunities; and construction rises, then drops, as we approach the new steady state. Note that home-equity lending rises quickly, even though h_t takes time to adjust due to increasing marginal construction costs, because the price jump makes $e_t = \psi_t h_t$ rise before h_t. Also note from Figure 5 that welfare W increases over the period. Financial development, our increase in D_1, is good for W because it relaxes credit constraints, even though it can lead to a transition that resembles a bubble, complete with collapse. Now, in the real world, some agents had the bad fortune of buying high and selling low, but this paper is not about redistributive effects (on that, see Kiyotaki et al. 2011). Financial innovation is beneficial for the representative agent, even if some people get hurt.

The above example uses a low $\beta = 0.6$, which is required to make (h^s, ψ^s) a sink. Also, as with any multiple equilibrium story, it is difficult to say just what theory predicts – we presented one of many equilibria, and others look different. And it is a stretch to think that there was a once-and-or-all innovation that is accurately modeled as an unanticipated D_1 shock. More likely, financial developments occurred gradually, with some anticipated and others more of a surprise. For the next experiment, we pick parameters so that equilibrium is unique, and try to capture the data by having D_1 change slowly over time. Again, for this it is useful to recall that D_1 may increase or decrease ψ. The eventual price decline in the following example is due to h_t ultimately becoming so good as collateral that the liquidity premium goes down. Results for this kind of experiment hinge on what agents know and when they know it, so we have to specify expectations.

To produce an empirical series for D_1, first divide home-equity loans by the value of residential fixed assets, 1996-2010; then divide this by $\alpha = 0.25$, the approximate proportion of home owners with such loans. This series constructed in the way has the following properties: D_1 increases gradually from 0.17 in 1996 to 0.23 in 2005; then jumps to 0.26 in 2006; and jumps again to 0.35 in 2008. Suppose in 1996 households perfectly predict D_1 up to 2005, then predict it stays constant (i.e., they think financial innovation will be complete by then). But in 2004, households update their expectations and realize D_1 will jump to 0.30 in 2006. Again they expect it to stay put. But in 2008, it unexpectedly increases to 0.35. After that, households predict D_1 will go to 0.55 after 2010 and then stay put. Figure 6 shows the paths for prices, loans and residential investment. Notice ψ_t peaks in 2007 around 8% above the 1996 price, which is only about 1/5 of the price change in the data. So, this story can account for some, but not the majority, of the price run up, although it captures the episode fairly well qualitatively, and for some series it does better quantitatively (e.g., home equity loans double).

Our examples are meant to be just that – examples. More work can be done to fine tune the estimates of expectations and parameters. The goal here was to illustrate how dynamic models can be used to think about the episode in a general way. Of course, we run into selection issues when we appeal to multiplicity, and when equilibrium is unique we need to take a stand on what people knew and when they knew it. Still, it is interesting to know these models can generate outcomes that look somewhat like the data.

6 Money and Banking

To this point, households put up home equity to secure stylized consumption loans directly from retailers. In reality, typically, households use home equity to borrow cash from banks, then use cash to buy goods. We now model this explicitly, not only for the sake of realism, but because once we introduce money and banking we can study monetary policy. There is a literature on the effects of inflation (or nominal interest rates) on the housing market, with Aruoba et al. (2012) providing a recent example plus references to earlier work. That paper documents that in the data there is a clear relationship between the real value of the aggregate housing stock and inflation (or nominal interest rates), using various data sets, for the US and other countries. The approach presented here provides a new way to think about those facts.

Here we assume supply is fixed at H and $\delta = 0$. To ease the presentation, each period households want to consume the retail good y_t with probability α, but conditional on wanting to consume they trade with probability 1. We assume for this discussion that money is the only means of payment in the decentralized retail market, and hence change its label from KM to KW (Kiyotaki-Wright replacing Kiyotaki-Moore). This can be formalized, following Kocherlakota (1998), by assuming that decentralized trade is anonymous. Heuristically, if a retailer who does not know the identity of a customer offers him a loan secured by a claim on a house, the customer could come up with a claim on a nonexistent house, or one belonging to someone else, or one that is under water. However, bankers have the ability to check the authenticity and/or desirability of real estate, making home equity valid collateral for bank loans, if not retail loans.

At the start of each period, before KW opens, households have access to cash brought in from the previous AD market, and can also access a new market called the DD market, for Diamond and Dybvig (1983).[14] This works as follows. Households that want to consume in

[14] Our DD market is in the spirit of Diamond-Dybvig models, generally, in the sense that banks provide liquidity insurance. One difference is that our bankers deal in cash, not goods, making it closer to the model in Berentsen et al. (2007). We also mention Li and Li (2010), who study a related model where real assets are used to secure cash loans, and give references to other papers.

the retail market (borrowers) can withdraw cash from a bank, while those that do not want to consume (savers) do not withdraw. Settlement occurs in the next AD market. While DD is competitive, we still assume limited commitment: households can renege on bank loans, so home equity is required as collateral. It is not important, and cannot be determined, who carries money out of AD and into the next period, since it can always be reallocated in DD. Hence, we assume all cash is deposited in banks at the end of the AD market. Then, in the DD market, those that want to consume in KW withdraw, generally more than their deposits, and those that do not want to consume in KW leave their deposits alone.

The generalized AD value function is $W_t(d_t, h_t, m_t)$, where a portfolio now consists of debt, housing and money in the bank. The DD value function $J_{t+1}(h_{t+1}, m_{t+1})$ depends on housing and cash, only, since debt is settled in AD. Then

$$W_t(d_t, h_t, m_t) = \max_{x_t, \ell_t, h_{t+1}, m_{t+1}} \{U(x_t, h_t) - \ell_t + \beta J_{t+1}(h_{t+1}, m_{t+1})\}$$
$$\text{st } x_t + \psi_t h_{t+1} + \phi_t m_{t+1} = \ell_t + \psi_t h_t + T_t + \phi_t m_t - d_t$$

where ϕ_t is the value of a dollar in terms of numeraire x_t. Eliminating ℓ_t and taking FOC's, we get

$$U_1(x_t, h_t) = 1,\ \psi_t = \beta \frac{\partial J_{t+1}}{\partial h_{t+1}} \text{ and } \phi_t = \beta \frac{\partial J_{t+1}}{\partial m_{t+1}}. \tag{22}$$

Generalizing the baseline model, now (x_t, h_{t+1}, m_{t+1}) is independent of (d_t, h_t, m_t) and W_t is linear in wealth. The DD value function satisfies

$$J_t(h_t, m_t) = \alpha \max_{\hat{m}_t} V_t[(1+\rho_t)(\hat{m}_t - m_t)\phi_t, h_t, \hat{m}_t] + (1-\alpha) W_t[-(1+\rho_t) m_t \phi_t, h_t, 0]$$
$$\text{st } (1+\rho_t)(\hat{m}_t - m_t)\phi_t \le D(\psi_t h_t),$$

where ρ_t is the interest rate and $D(\psi_t h_t)$ the limit on debt owed to the banking system.

Thus, with probability α households increase m_t to $\hat{m}_t$, spend it in KW, and incur a real obligation of $(1+\rho_t)(\hat{m}_t - m_t)\phi_t$. And with probability $1-\alpha$ they leave their money in the bank, skip the KW market, and enter the next AD market with an obligation $-(1+\rho_t) m_t \phi_t$.[15] Here V_t is the KW value function conditional on wanting to consume,

$$V_t(d_t, h_t, m_t) = u(y_t) - \phi_t \hat{m}_t + W_t(d_t, h_t, m_t) \tag{23}$$

[15] A positive bank balance is a negative liability.

where, again, some trading mechanism (price taking or bargaining) determines $g(y_t) = \phi_t \hat{m}_t$. Generalizing the baseline model, the outcome depends on whether the DD debt limit, $(1+\rho_t)(\hat{m}_t - m_t)\phi_t \leq D(\psi_t h_t)$, binds.

In Case 1, it does not bind, and the FOC for $\hat{m}$ is

$$-(1+\rho_t)\phi_t + \frac{\partial V_t}{\partial \hat{m}_t} = 0. \tag{24}$$

Using (23), this reduces to $\mathcal{L}(y_t) = \rho_t$, which equates the liquidity premium and the loan rate. Then we have the Euler equation for m_{t+1},

$$(1+r)\phi_t = \left(1+\rho_{t+1}\right)\phi_{t+1}. \tag{25}$$

Let i_t be the nominal interest rate that makes agents willing to give up a dollar in AD at t and get back $1+i_t$ dollars in AD at $t+1$. Then the Fisher Equation is $1+i_t = \phi_t/\beta\phi_{t+1}$, since $\phi_t/\phi_{t+1} = 1+\pi_t$ is inflation and $1/\beta = 1+r_t$ is the real interest rate. In this case (25) says $\rho_{t+1} = i$ and the Euler equation for h_{t+1} is

$$(1+r)\psi_t = U_2(x_{t+1}, h_{t+1}) + \psi_{t+1}. \tag{26}$$

Therefore, when the debt constraint is slack housing is priced fundamentally.

In Case 2 the debt constraint binds, so borrowers in DD go to their limit and

$$J_t(h_t, m_t) = \alpha V_t\left[D(\psi_t h_t), h_t, m_t + \frac{D(\psi_t h_t)}{(1+\rho_t)\phi_t}\right] + (1-\alpha) W_t\left[-(1+\rho_t)\phi_t m_t, h_t, 0\right]$$

In this case, the Euler equation for cash is

$$(1+r)\phi_t = \alpha\left[\mathcal{L}(y_{t+1}) + 1\right]\phi_{t+1} + (1-\alpha)\rho_{t+1}\phi_{t+1}, \tag{27}$$

and the Euler equation for housing is

$$(1+r)\psi_t = U_2(x_{t+1}, h_{t+1}) + \psi_{t+1} + \frac{\alpha D_1 \psi_{t+1}}{1+\rho_{t+1}}\left[\mathcal{L}(y_{t+1}) - \rho_{t+1}\right], \tag{28}$$

where compared to (26) the liquidity premium now appears in the last term.

There are two subcases in Case 2. In Case 2a, bank lending exhausts deposits, so there is no idle vault cash. In this case we need $\rho_{t+1} > 0$ to clear the market. This yields

$g(y_{t+1}) = D(\psi_{t+1}h_{t+1}) / (1-\alpha)$. In Case 2b, when all borrowers go to the limit they do not exhaust deposits, so there is idle vault cash and $\rho_{t+1} = 0$. Then the Euler equation for housing is

$$(1+r)\psi_t = \psi_{t+1} + U_2(x_{t+1}, h_{t+1}) + D_1\psi_{t+1}i.$$

Two conditions determine which case obtains. One is the individual debt limit: can individuals borrow from the bank as much as they want? If $D(e)$ is low, they are constrained and housing bears a premium. The other is an aggregate condition: do deposits satiate loan demand? If so, there is idle cash and $\rho = 0$. So, there are three logical possibilities: 1) the aggregate and individual limits are slack; 2a) the individual limit binds but the aggregate limit is slack; and 2b) both bind (a fourth possibility, where the aggregate limit binds but the individual limit is slack, can easily be ruled out). Given $D(e_t) = D_0 + D_1e_t$, Figure 7 partitions (D_0, D_1) space into regions where each case obtains, separated by two downward sloping curves, $D_1 = B_1(D_0)$ and $D_1 = B_2(D_0)$, derived in Appendix B. For large D_0 and D_1, we get Case 1 and debt limits do not bind. As D_0 and D_1 decrease, we move to Case 2a, with $\rho \in (0, i)$. As D_0 and D_1 decrease further we move to Case 2b, with $\rho = 0$.

This machinery allows us to study the impact of monetary policy, in terms of the inflation rate π or, equivalently, by the Fisher Equation, the nominal interest rate i. In Case 1, $\rho = i$ and houses are priced fundamentally at ψ^*. So as long as we are in Case 1 monetary policy does not affect real home prices. In Case 2a, $\rho \in (0, i)$ and house prices may go up or down with i, depending on a condition given in Proposition 6 below. In Case 2b, $\rho = 0$ and real house prices must rise with i. See Appendix B. The result we want to emphasize is that, although monetary policy generally has ambiguous effects on the housing market, one can derive precise conditions under which $\partial\psi/\partial i$ is positive, negative or zero, depending on the tightness of debt limits. The model can generate a positive relationship between inflation and home values, at least when credit conditions are relatively tight, consistent with the empirical findings in Aruoba et al. (212), but for completely different reasons.[16]

[16] The channel discussed in that paper is this: as a tax on market activity, inflation encourages agents to switch at the margin into more home production, which can increase all home inputs, including time and capital. Since houses are part of home capital, inflation increases the demand for housing.

Figure 8 presents an example, where ψ first increases, then decreases, and eventually becomes independent of i. The interest rate on bank loans and deposits ρ is also shown, as is KW consumption y (one can show analytically that y is decreasing in i). While more can be done, the goal was primarily to illustrate how a theory of the housing market that incorporates liquidity considerations allows us to think about the effects of monetary policy in a new light. We summarize as follows:

Proposition 6 *There is a unique steady state and it implies:*

1) if $B_2(D_0) < D_1$ *then* $\rho = i$, $\psi = U_2/r$ *and* $\partial\psi/\partial i = 0$*;*

2a) if $B_1(D_0) < D_1 < B_2(D_0)$ *then* $\rho \in (0, i)$ *and* $\partial\psi/\partial i \lesseqgtr 0$ *as* $\frac{\partial}{\partial y}[1 + \mathcal{L}(y)]\, g(y) \gtreqless 0$*;*

2b) if $B_1(D_0) > D_1$ *then* $\rho = 0$, $\psi = U_2/(r - iD_1)$ *and* $\partial\psi/\partial i > 0$*.*

7 Conclusion

If housing can be used to collateralize loans, either from retailers or bankers, house prices can bear a liquidity premium. We document the use of home-equity lending increased significantly since 2000, as consumers began treating their houses "as cash machines" (Ferguson 2008; Reinhart and Rogoff 2009). This is consistent with empirical work, e.g., Mian and Sufi (2011), that finds: homeowners extracted 25 cents for every dollar increase in home equity, adding \$1.25 trillion to debt; the loans were used for consumption, not paying off credit card debt or buying assets; and they were used more by young people and those with low credit scores (captured in the model by lower D_0). Once it is acknowledged that housing conveys liquidity, it follows that prices can exhibit complicated dynamics. First, we showed there is an tendency for prices to oscillate in anything from a 2-cycle to chaos. We also provided an example of a perfect-foresight equilibrium with price increases lasting several periods before collapsing. However, in any perfect-foresight equilibrium capital gains are bounded by r. So we produced a sunspot equilibria, where expected capital gains are bounded by r, but with some probability there can be large price increases several periods in a row.

The main results are robust to having different mechanisms, including not only price taking, but axiomatic or strategic bargaining. This is perhaps a contribution in itself (think of it as integrating Kiyotaki-Moore and Mortensen-Pissarides). They are also robust to having exogenous or endogenous debt limits (Appendix C). We also endogenized H, and in this version we described two experiments. In one, after financial innovation there are many equilibria, and we tried to select one that looks something like the data. In the other, there is a unique equilibrium, and we used the model to measure how much of the observed price boom can be accounted for by gradual financial innovation. The finding is: some, but not the majority. How much of the remainder is due to self-fulfilling prophecies and sunspots of the type analyzed here? We don't have a definitive answer, yet, but future work might profitably pursue this question. We also studied monetary policy. This is relevant because there is in the data a relationship between housing and inflation (Aruoba et al. 2012), and this approach provides a new way to think about those facts. More generally, all of this is intended to illustrate how one can think productively about interactions between housing and liquidity.

We regard the paper as an exercise in theory, designed to show how one can generate rational expectations equilibria with interesting or at least complicated qualitative patterns in housing market variables. Again, studying in more detail the extent to which one can do well quantitatively with these models is left to future work. Another extension that may be relevant is to consider multiple liquid assets, but there should be no presumption that the results, at least the qualitative results, would change. If consumers can hold portfolios $(h, m, b, ...)$ of houses, money, bonds, etc., they will choose these based on liquidity as well as returns, risk, etc. When it becomes easier to use home equity as collateral they will increase their demand for h at the margin. This is true when housing is, as in this paper, the only asset available to secure loans above some baseline level D_0. It can continue to be true in generalized models with alternative liquid assets. As long as conditions make it difficult to get as much credit as one might sometimes need or want, assets that relax the relevant constraints, as home equity surely does, will bear a liquidity premium. In this situation, our general approach will apply.

Appendix A: Consider the following bargaining game:

Stage 1: The seller offers (y_t, d_t).

Stage 2: The buyer responds by accepting or rejecting, where:

- accept implies trade at these terms;
- reject implies they go to stage 3.

Stage 3: There is a coin toss, such that:

- with probability θ, the buyer makes a take-it-or-leave-it offer;
- with probability $1-\theta$, the seller makes a take-it-or-leave-it offer.

Any offer must satisfy $d_t \leq D_t$. We claim there is a unique SPE, characterized by acceptance of the initial Stage 1 offer, given by

$$(y_t, d_t) = \arg\max S_{st} \text{ st } S_{bt} = \theta \left[u(\bar{y}_t) - \bar{d}_t \right] \text{ and } d \leq D_t, \tag{29}$$

where $\left(\bar{y}_t, \bar{d}_t\right)$ is the offer a buyer would make if (off the equilibrium path) at Stage 3.

The first observation is that, off the equilibrium path, if bargaining were to go to Stage 3 and the buyer got to make a final offer, he would offer $\left(\bar{y}, \bar{d}\right)$ where:

$$\bar{y} = \begin{cases} v^{-1}(D_t) & \text{if } D_t < v(y^*) \\ y^* & \text{if } D_t \geq v(y^*) \end{cases} \text{ and } \bar{d} = \begin{cases} D_t & \text{if } D_t < v(y^*) \\ v(y^*) & \text{if } D_t \geq v(y^*) \end{cases}$$

There are four logical possibilities: 1) the constraint $d \leq D$ is slack at the initial and final offer stage; 2) it binds in the initial but not the final offer stage; 3) it binds in both; and 4) it binds in the final but not the initial offer stage. It is easy to check that case 4 cannot arise, so we are left with three.

Case 1: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y,d} \{u(y) - d\} \text{ st } d = v(y),$$

with solution $y = y^*$ and $d = v(y^*)$. If the seller proposes the buyer gets no surplus, so the buyer's expected surplus before the coin flip is $\theta [u(y^*) - v(y^*)]$. Therefore, in the initial offer stage, the seller's problem is

$$\max_{y,d} \{d - v(y)\} \text{ st } u(y) - d = \theta [u(y^*) - v(y^*)],$$

with solution $y = y^*$ and $d = d^* = (1-\theta)\, u(y^*) + \theta v(y^*)$. Since $d^* > v(y^*)$, this case occurs iff $D > d^*$.

Case 2: The buyer's expected payoff before the coin flip is again $\theta\,[u(y^*) - v(y^*)]$, but at the initial offer stage the constraint binds, so the seller solves

$$\max_y \{D - v(y)\} \text{ st } u(y) - D = \theta\,[u(y^*) - v(y^*)]\,.$$

This implies $u(y) = D + \theta\,[u(y^*) - v(y^*)]$ and $d = D$. This case occurs iff $v(y^*) < D < d^*$.

Case 3: In the final offer stage, if the buyer proposes, his problem is

$$\max_y \{u(y) - D\} \text{ st } D = v(y)\,.$$

This implies $y = v^{-1}(D)$, and his expected surplus before the coin flip is $\theta\left[u \circ v^{-1}(D) - D\right]$. At the initial offer stage, the seller's problem is

$$\max_y \{D - v(y)\} \text{ st } u(y) - D = \theta\left[u \circ v^{-1}(D) - D\right].$$

The solution satisfies $u(y) = \theta u \circ v^{-1}(D) + (1-\theta)\, D$ and $d = D$. This case occurs iff $D < v(y^*)$ and $D < u(y^*) - \theta u \circ v^{-1}(D) + \theta D$, the last inequality coming from the observation that, at the first stage, if the constraint is slack, the buyer pays $u(y^*) - \theta u \circ v^{-1}(D) + \theta D$ to get y^*. This last inequality is equivalent to $(1-\theta)\, D < u(y^*) - \theta u \circ v^{-1}(D)$, which always holds if $D < v(y^*)$.

To sum up, $d = D$ if $D < d^*$ and $d = d^*$ if $d^* \leq D$; and y is given by

$$y = \begin{cases} u^{-1}\left[\theta u \circ v^{-1}(D) + (1-\theta)\, D\right] & \text{if } D < v(y^*) \\ u^{-1}\left[D + \theta\,[u(y^*) - v(y^*)]\right] & \text{if } v(y^*) < D < d^* \\ y^* & \text{if } D > d^* \end{cases}.$$

It is easy to check $y = g^{-1}(D)$ is differentiable and strictly increasing for $D < d^*$. ■

Appendix B: Here we verify the results in Proposition 6, and derive

$$\begin{aligned} B_1(D_0) &= \begin{cases} \dfrac{r\,[g(\tilde{y})(1-\alpha) - D_0]}{ig(\tilde{y})(1-\alpha) - iD_0 + HU_2} & \text{if } D_0 < g(\tilde{y})(1-\alpha) \\ 0 & \text{if } D_0 > g(\tilde{y})(1-\alpha) \end{cases} \\ B_2(D_0) &= \max\left\{\frac{r\,[g(\bar{y})(1-\alpha)(1+i) - D_0]}{HU_2}, 0\right\} \end{aligned}$$

where $\tilde{y}$ and $\bar{y}$ satisfy $\mathcal{L}(\tilde{y}) = i/\alpha$ and $\mathcal{L}(\bar{y}) = i$.

Case 1: The debt constraint is not binding. In steady state, we have

$$\begin{aligned} i &= \mathcal{L}(y)\,,\ \rho = i \\ r\psi &= U_2\,[X(H), H] \\ g(y) &< \frac{D_0 + D_1 \psi H}{(1+\rho)(1-\alpha)}. \end{aligned}$$

The last condition comes from two observations: when $\rho > 0$, to clear the market we must have $g(y) = \phi_t M_t / \alpha$; and when the debt constraint is slack, $(1-\alpha)\phi_t M_t < \alpha (D_0 + D_1 \psi H)/(1+\rho)$. This equilibrium exists iff

$$g(\bar{y}) < \frac{D_0 + D_1 \psi H}{(1-\alpha)(1+\rho)}$$

with $\psi = U_2 / r$, or $D_1 > g(\bar{y})(1+i)(1-\alpha) - D_0$. Uniqueness follows immediately. Furthermore, $\partial \psi / \partial i = 0$ and $\partial y / \partial i < 0$.

Case 2: The debt constraint is binding. In steady state,

$$\begin{aligned} i &= \alpha \mathcal{L}(y) + (1-\alpha)\rho && (30)\\ r\psi &= \alpha [\mathcal{L}(y) - \rho] \frac{\psi D_1}{1+\rho} + U_2 [X(H), H] && (31)\\ g(y) &= \phi_t M_t + \frac{D_0 + D_1 \psi H}{1+\rho}. && (32) \end{aligned}$$

We now consider the subcases separately.

Case 2a: If $\rho > 0$, market clearing and a binding debt constraint imply

$$\phi_t M_t = \frac{\alpha (D_0 + D_1 \psi h)}{(1-\alpha)(1+\rho)}. \tag{33}$$

Using (30), we get $\rho = (i - \alpha \mathcal{L})/(1-\alpha)$. This, (33) and (32) yield

$$\psi = \frac{g(y)[1 + i - 1 - \alpha \mathcal{L}(y)] - D_0}{D_1 H}.$$

Substituting these into (31), we get

$$\frac{r}{D_1} = \frac{\alpha \mathcal{L}(y) - i}{1 + i - \pi [1 + \alpha \mathcal{L}(y)]} + \frac{H U_2 [X(H), H]}{g(y)[1 + i - \pi - \pi \alpha \mathcal{L}(y)] - D_0} \equiv \Phi(y). \tag{34}$$

The RHS is decreasing in y, so there is at most one solution. Note in this subcase $\rho < \mathcal{L}(y)$, implying $0 < \rho < i$. This and (30) imply $\alpha i < \mathcal{L}(y) < i$. Consequently, this equilibrium exists iff (34) has a solution in $(\tilde{y}, \bar{y})$, where $\mathcal{L}(\tilde{y}) = i/\alpha$ and $\mathcal{L}(\bar{y}) = i$. This requires $\Phi(\tilde{y}) > r/D_1$ and $\Phi(\bar{y}) < r/D_1$, or $B_1(D_0) < D_1 < B_2(D_0)$. One can derive

$$\begin{aligned} \frac{\partial y}{\partial i} &\approx -\frac{g D_1 (\mathcal{L}+1)\alpha}{\psi (1+\rho)^2} - \frac{g U_h}{(1+\rho)\psi^2} < 0\\ \frac{\partial \rho}{\partial i} &\approx -\frac{\alpha \mathcal{L}' D_1 g}{(1+\rho)\psi} + \frac{U_2 g'}{\psi^2} > 0\\ \frac{\partial \psi}{\partial i} &\approx -D_1 \alpha \left[g \mathcal{L}' + g(\mathcal{L}+1) \right] \approx -\frac{\partial}{\partial y} [\mathcal{L}(y) + 1] g(y), \end{aligned}$$

where $A \approx B$ means A and B have the same sign. Therefore, $\partial \psi / \partial i < 0$ iff $[\mathcal{L}(y) + 1] g(y)$ is increasing.

Case 2b: If $\rho = 0$, steady state is characterized by

$$\begin{aligned} i/\alpha &= \mathcal{L}(y) && (35) \\ r\psi &= i\psi D_1 + U_2[X(H), H] && (36) \\ g(y) &> \frac{D(\psi H)}{1-\alpha}. && (37) \end{aligned}$$

Now (35) determines y and (36) determines ψ. This equilibrium exists iff (37) holds, given y and ψ, which leads to $B_1(D_0) > D_1$. It is obvious in this case that $\partial y/\partial i < 0$ and $\partial \psi/\partial i > 0$. ■

Appendix C: Suppose D_1 is exogenous, and consider making D_0 endogenous. Debtors always repay secured loans, given $D_1 \leq 1$, but may renege on unsecured loans. We consider two types of punishment: 1) take away a defaulter's unsecured credit; 2) take away all future credit. To simplify the exposition, let $U(x,h) = U(x) + \upsilon(h)$ and $\alpha = 1$. Then AD consumption is fixed, and we normalize $U(X) - X = 0$, with no loss of generality.

The KM value function along the equilibrium path and after a default are

$$\begin{aligned} V^E &= \frac{1}{1-\beta}\left\{u\left(y^E\right) + \upsilon\left(h^E\right) - d^E\right\} \\ V^D &= \frac{1}{1-\beta}\left\{u\left(y^D\right) + \upsilon\left(h^D\right) - d^D\right\}, \end{aligned}$$

where $\left(y^E, h^E, d^E\right)$ and $\left(y^D, h^D, d^D\right)$ both depend on D_0, and on the punishment. The repayment constraint is

$$\upsilon\left(h^E\right) - d_0 + \beta V^E \geq \upsilon\left(h^E\right) + \psi\left(h^E - h^D\right) + \beta V^D.$$

Combining these, the unsecured debt limit $\tilde{D}_0$ is

$$\tilde{D}_0 = T(D_0) \equiv \frac{1}{r}\left\{u\left(y^E\right) + \upsilon\left(h^E\right) - d^E\right\} - \frac{1}{r}\left\{u\left(y^D\right) + \upsilon\left(h^D\right) - d^D\right\} - \psi\left(h^E - h^D\right),$$

where

$$\begin{aligned} y^E &= \min\left\{g^{-1}\left(D_0 + D_1\psi H\right), y^*\right\} \\ d^E &= \min\left\{g\left(y^*\right), D_0 + D_1\psi H\right\} \\ r\psi &= \upsilon'(H) + \psi D_1 \mathcal{L}\left(y^E\right). \end{aligned}$$

Exactly as in Alvarez and Jermann (2000), the equilibrium debt limit is a fixed point of $T(D_0)$.

Punishment 1: Given ψ, y^D, h^D and d^D we solve

$$\begin{aligned} y^D &= \min\left\{g^{-1}\left(D_1\psi h^D\right), y^*\right\} \\ d^D &= \min\left\{g\left(y^*\right), D_1\psi h^D\right\} \\ r\psi &= \upsilon'\left(h^D\right) + \psi D_1 \mathcal{L}\left(y^D\right). \end{aligned}$$

Obviously $T(0) = 0$. Moreover, as $D_1 \to 0$, $T'(0) \to \mathcal{L}(0)/r$. If D_1 is sufficiently small and $\mathcal{L}(0) > r$, there exists an equilibrium with unsecured credit. If D_1 is too big, no unsecured credit can exist.

Punishment 2: Now $y^D = 0$, $h^D = \upsilon'^{-1}(r\psi) \leq h^E = H$, $d^D = 0$ and

$$T(D_0) = \frac{1}{r}\left[u\left(y^E\right) + \upsilon\left(h^E\right) - d^E - \upsilon\left(h^D\right)\right] - \psi\left(h^E - h^D\right).$$

Notice $T(0) > 0$ and, if D_0 is big enough that nondefaulters are not constrained, $T(D_0)$ is constant. So a strictly positive endogenous unsecured debt limit always exists. Also, T is increasing in D_1. To see this, if nondefaulters are constrained,

$$\frac{\partial T(D_0)}{\partial D_1} = \frac{1}{r}\mathcal{L}\left(y^E\right) H \frac{dD_1\psi}{dD_1} - \frac{d\psi}{dD_1}\left(H - h^D\right). \tag{38}$$

As $d\psi/dD_1 = \psi\left(\mathcal{L} + \psi D_1 H \mathcal{L}' y'\right) / \left(r - D_1\mathcal{L} - D_1^2\psi\mathcal{L}'y'H\right)$, we have

$$\frac{\partial T(D_0)}{\partial D_1} = \frac{\psi h^D \mathcal{L} - \psi^2 D_1 H \mathcal{L}' y'\left(H - h^D\right)}{r - D_1\mathcal{L} - D_1^2\psi\mathcal{L}'y'H} > 0.$$

If nondefaulters are not constrained, $\partial T(D_0)/\partial D_1 = 0$. So $\partial T(D_0)/\partial D_1 \geq 0$. Since T is increasing in D_1, the equilibrium debt limit is, too. In this case, secured credit helps unsecured credit. ■

Appendix D: Data in Figure 1. Home Prices are given by the FHFA Purchase Only price index. To turn it into a real variable, we divide by the CPI, or by the Rent index from BLS, with the real series normalized to 1 in 1993 to fit on the chart. Loan data are from the Federal Reserve Flow of Funds Accounts. Home-equity loans (HEL) are divided by CPI, by nominal GDP, and by Home Equity with the resulting series all normalized to 0.3 in 1993. Home Equity data is obtained by first subtracting home-equity loans from mortgage loans to get closed-end mortgages, and then subtracting that from the Market Value of Homes, which includes the value of land, as constructed by Davis and Heathcote (2007). The Rensidential Fixed Investment data are from BEA, divided by nominal GDP, normalized to 0.5 in 1993.

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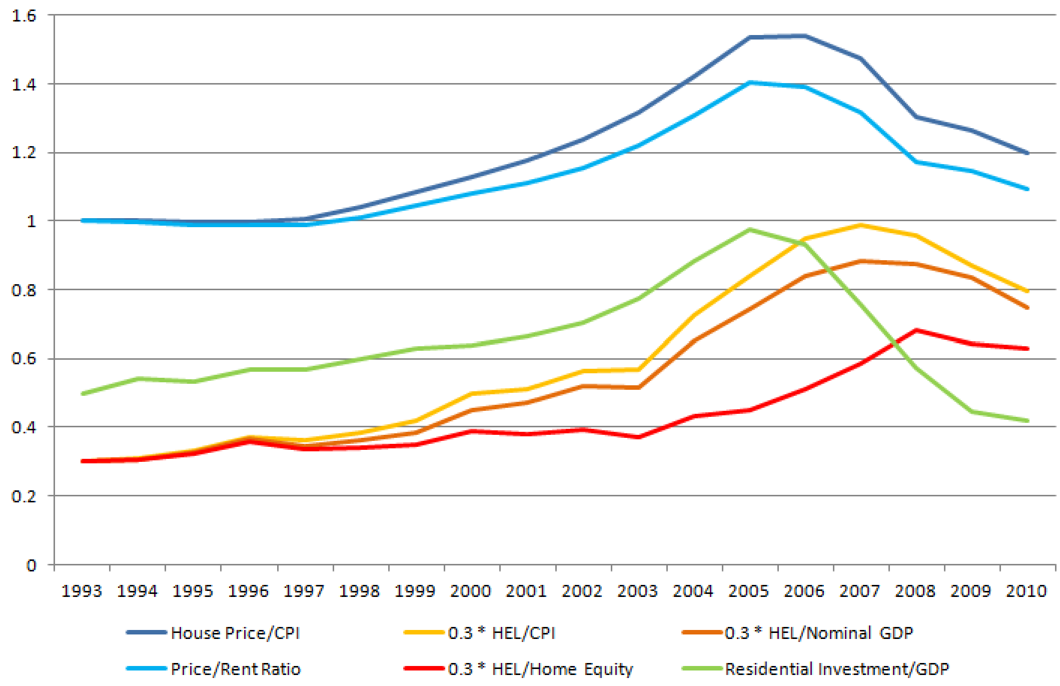


Figure 1: Housing Sector and Home Equity Loans

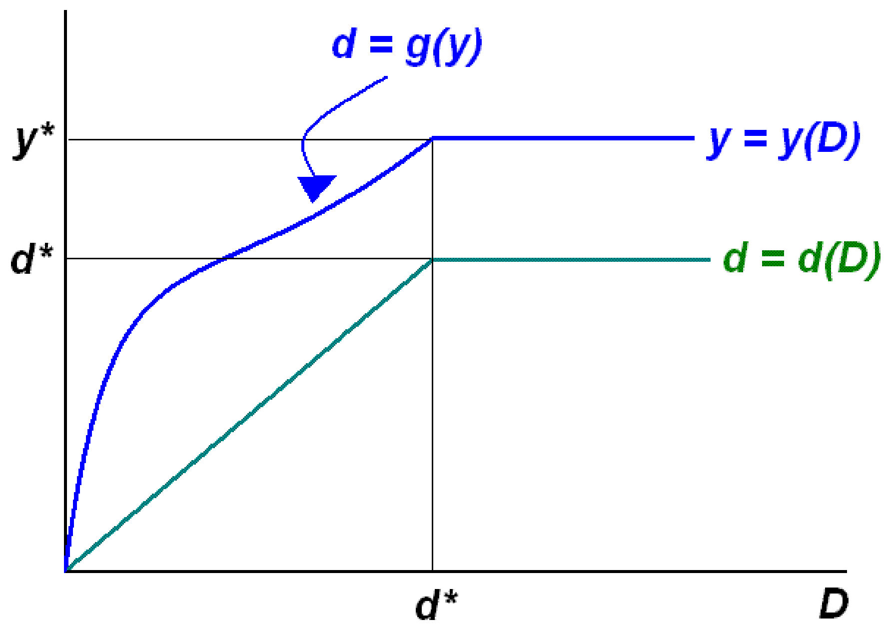
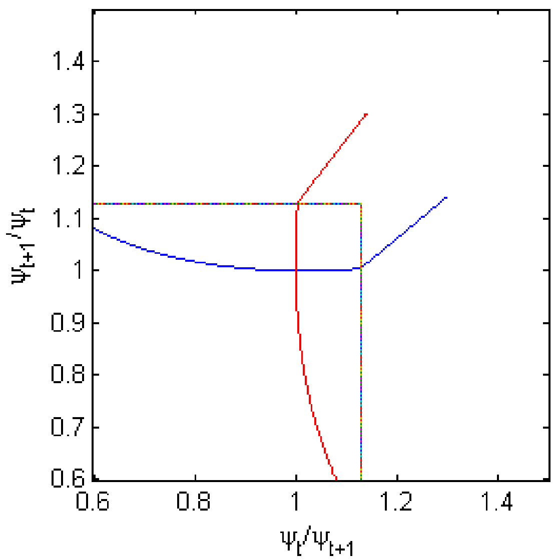
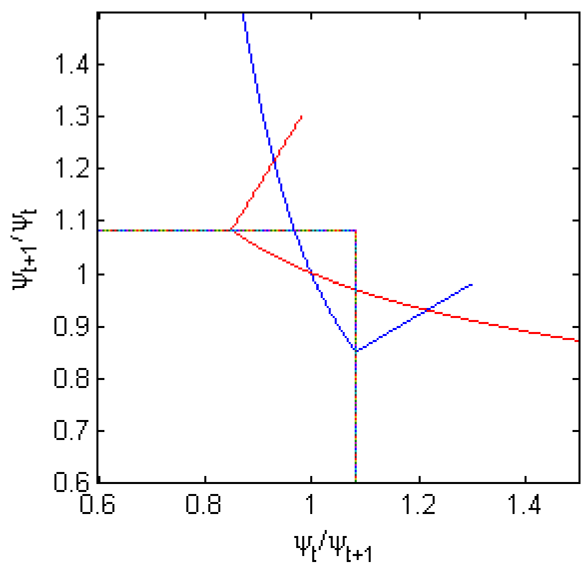


Figure 2: Trading Mechanism



(a) Example 1



(b) Example 2

Figure 3: Housing Price Dynamics

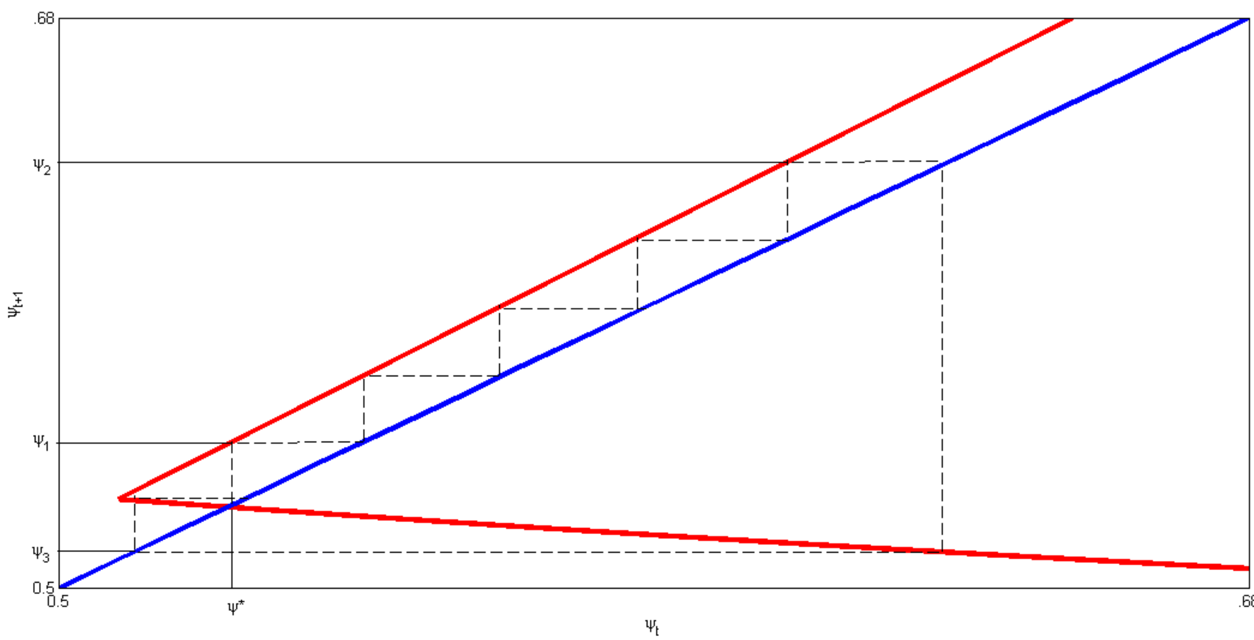


Figure 4: Price Dynamics

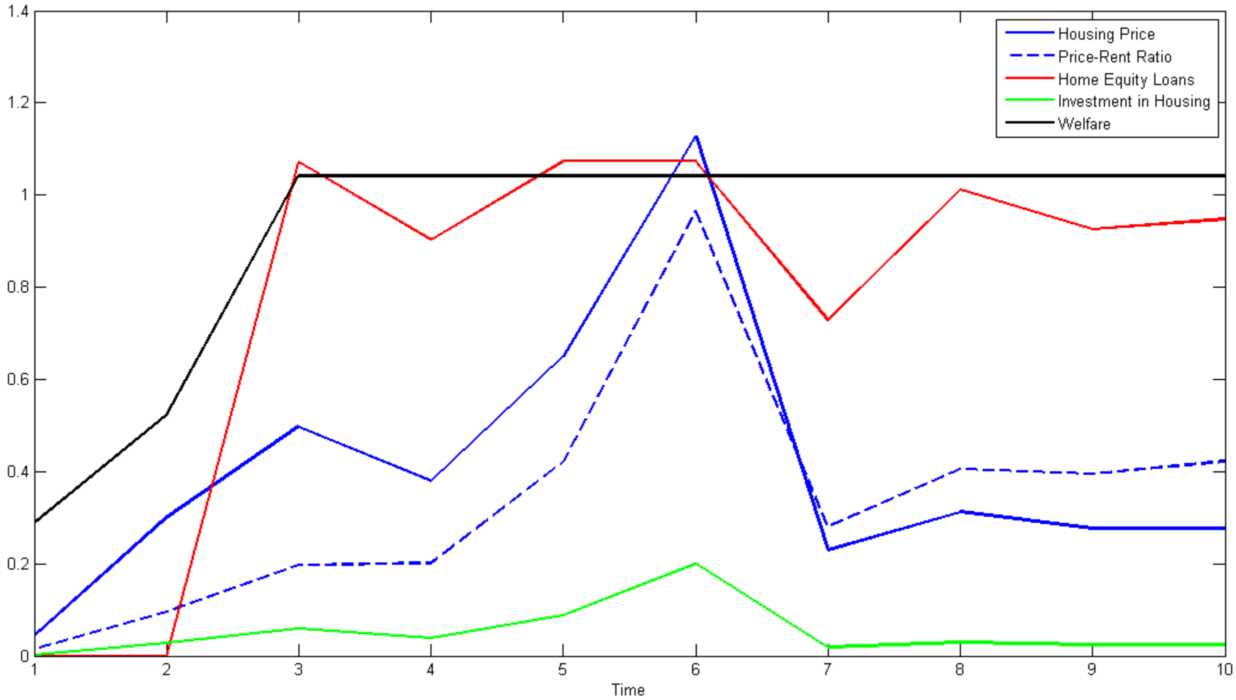


Figure 5: Housing Price Dynamics: One Period Unexpected Change in Financial Market

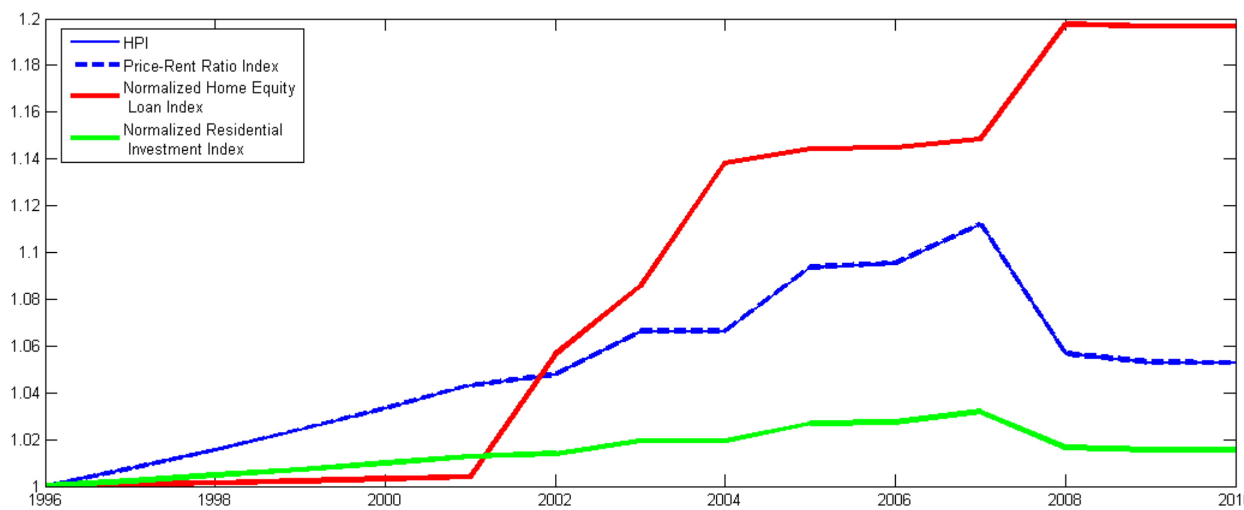


Figure 6: Housing Price Dynamics: Transition Path Trigered by Gradual Change in Financial Market

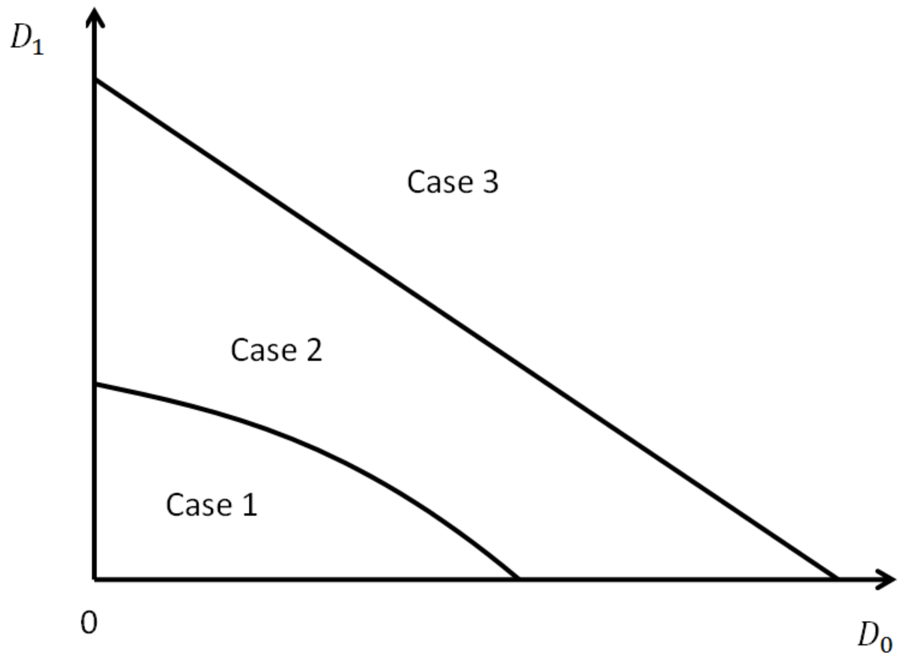


Figure 7

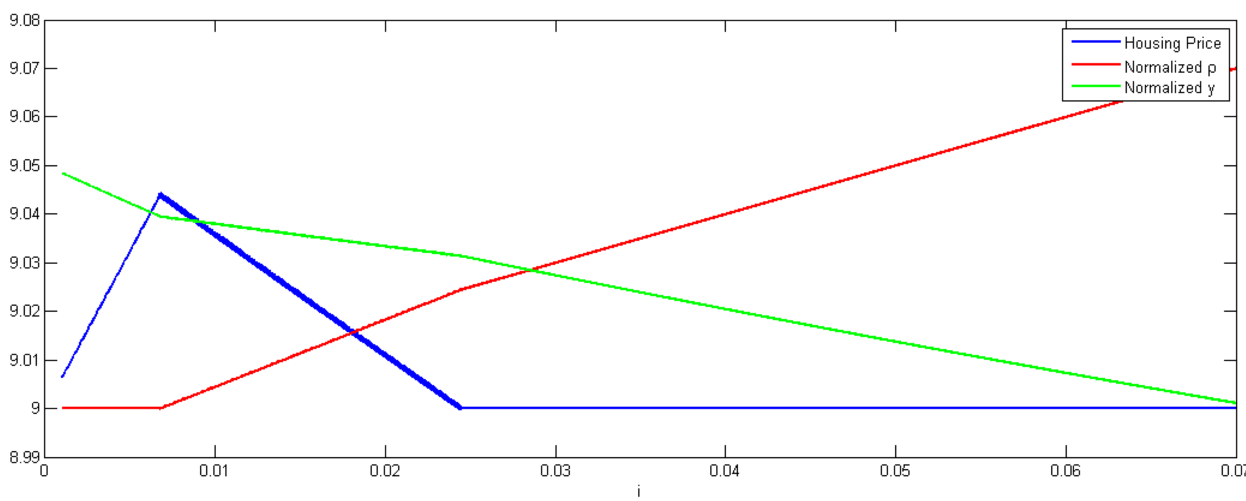


Figure 8