Defending A Policy Regime Against Speculative Attacks

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Abstract

Many policies, such as currency pegs and government's bailouts, are subject to speculative attacks, which may result in financial and economic crises. Because the main cause of these speculative attacks is the government's limited commitment, the government needs to resolve the limited commitment problem when defending against speculative attacks. This paper analyzes a dynamic regime change game, where a policy maker may obtain the commitment power by building a reputation and a continuum of speculators may learn the policy maker's type. I show that if speculators' learning speed is slow (speculators individually learn), the model has a unique equilibrium in which no speculator attacks and the policy maker sustains the status quo forever. This is because the incentive of building a reputation brings the policy maker the commitment power. If learning is fast, multiple equilibria with attacks exist. In any equilibrium with attacks, the first attacking period depends on the entire learning process, the time interval between two consecutive attacking periods is uniformly bounded, and the weak policy maker abandons the policy regime almost surely.

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Contents

1 Introduction

Speculative attacks on currency pegs have occurred in many countries. The main cause of such attacks is the central bank's limited commitment to sustain the currency peg. Despite various instruments developed to defend against currency attacks (see Drazen (2000)), sustaining the currency peg when facing attacks is costly. If the speculative attacks become sufficiently aggressive, the cost incurred in sustaining the currency peg may outweigh the benefit, leading the central bank to abandon it and so making the attacks profitable. Therefore, whether attacks occur depends on whether speculators believe the central bank will abandon the currency peg.

Speculators' beliefs that the central bank will abandon the currency peg are affected by the central bank's past behaviors: if the currency peg survives past attacks, the central bank is believed to have strong interests in sustaining the currency peg in the future. Hence, by defending against speculative attacks, the central bank can build a "reputation", which may deter future attacks. Following previous works on the reputation effect, such as Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenburg and Levine (1989), I assume the central bank has private information about the value it derives from the currency peg. If this value is sufficiently high, the central bank would like to pay any cost of defending to sustain the currency peg. Then in case of a low value, by defending against speculative attacks, the central bank makes speculators believe that it is more likely to have a high value and thus more likely to sustain the currency peg in the future.

Although the benefit is unknown by all speculators, they may exogenously learn about it. Specifically, each speculator collects one private signal about the benefit in every period, and the accumulated signals become infinitely precise in the limit. Therefore, if the benefit is low, speculative attacks may happen because the central bank is believed not to have commitment to sustain the currency peg. As the possibility of future attacks increases, the incentive of the central bank to build a reputation is weakened.

In this paper, I shed light on the issue of defending a policy regime against speculative attacks by analyzing the interaction between a policy maker's reputation building and speculators' learning in a dynamic regime change game. While the individual learning (which leads to the common learning in my model) is usually sufficient for attacks in some equilibrium (for example, see Angeletos, Hellwig, and Pavan (2007)), I show that when the learning speed is relatively slow, my model has a unique equilibrium, in which no speculator attacks and the central bank sustains the status quo forever. The equilibrium uniqueness implies that the central bank's lowest equilibrium average discounted payoff is its highest feasible payoff. Therefore when the learning speed is relatively slow, the central bank effectively defends the status quo against speculative attacks.

Turning to the details of my model, the policy maker decides to sustain the status quo or abandon it in every period. If she abandons the status quo, she receives a zero flow payoff, and the game ends. If she sustains the status quo, she obtains a period benefit but incurs a defending cost, and then the game enters the next period. The period benefit could be either high or low. It is fixed and is the policy maker's private information. There is a continuum of speculators, each exogenously observing one piece of private information in every period. Each speculator then uses his own private signals to decide whether to attack the status quo or not. Assume each speculator individually learns the policy maker's benefit.

When the learning speed is slow, there is no equilibrium with attacks in this framework. Why does the learning speed in the limit matter for the equilibrium characterization? First, in an equilibrium with attacks, speculators attack infinitely often. Otherwise, there is a period such that no speculator attacks ever again if the status quo is in place after that period. Then a sufficiently patient policy maker sustains the status quo for sure in the last "attacking" period, which leads to a contradiction that no attack happens in an attacking period. Second, the time interval between any two consecutive attacking periods is uniformly bounded by some integer $\bar{K}$. If speculators do not attack within $\bar{K}$ periods after the former attacking period, the policy maker can profitably deviate to sustain the status quo in the former attacking period. These two facts, together with a bounded (away from 1) probability that the weak policy maker defends, establish a lower bound of the reputation building speed. On the other hand, a common belief is a necessary condition for attacks in any particular period, because of the coordination motives among speculators. So if the learning speed is slow, a common belief may not be formed within a bounded number of periods, which implies that infinitely many attacks cannot happen in an equilibrium. Therefore, the strategy profile in which no speculator attacks and the policy maker sustains the status quo forever is the unique equilibrium.

The equilibrium uniqueness stems from the commitment power brought by the policy maker's incentive to build a reputation. When the learning speed in the limit is slow, the weak policy maker's average discounted payoff from sustaining the status quo is positive in any equilibrium. So sustaining the status quo forever is dominating abandoning the status quo in some period in any equilibrium. Since the policy maker will sustain the status quo no matter how aggressive the attacks are, the attacking cost deters any individual speculator from attacking.

There are fast learning speeds with which infinitely many equilibria with attacks exist. The weak policy maker will abandon the status quo almost surely in finite periods in these equilibria. Though the equilibrium regime change outcome is the same as that in some equilibria in a dynamic regime change game without a defender, the dynamics of attacking are significantly different. First, the earliest possible first attacking period may not be the first time speculators form a common belief. Though speculators have a common belief that the policy maker is weak in some period, if their learning speed in the subsequent periods is slow, they cannot form common beliefs fast enough to guarantee subsequent attacks. Then the weak policy maker has strict incentives to sustain the status quo and build a reputation. Hence, the earliest possible first attacking period depends on the entire learning process. Second, as analyzed in the slow learning case, the time interval between two attacking periods is bounded. Therefore, only if the learning speed is sufficiently fast such that a common belief be formed within any $\bar{K}$ periods, an equilibrium with attacks exists. The existence of an equilibrium with attacks implies the existence of infinitely many equilibria with attacks. As the policy maker becomes arbitrarily patient, her lowest equilibrium payoff is close to 0, which means the policy maker cannot effectively defend against speculative attacks.

My model implies that the pre-established institutions will determine whether a policy maker can effectively defend the policy regime against speculative attacks. Most importantly, while there are a lot of researches supporting the transparency of the policy maker in facilitating her communications with the market, my model suggests that the transparency increases the difficulties in defending against speculative attacks. In the model, the speculators' learning process captures the transparency of the policy maker. So the results of the model imply that if the policy maker is required to release her information eventually, it is better for her to release it slowly when defending against speculative attacks.

Currency pegs are not the only target of speculators. Speculators have attacked the price ceiling supported by buffer stock sales in the gold market (Henderson and Salant, 1978; Salant, 1983), the unallocated cumulative catch quotas in fisheries (Gaudet, Moreaux, and Salant, 2002), and the unallocated "stock quotas' on autos and H1B visas (Gaudet and Salant, 2003). Recently, people have been concerned about the potential speculative attacks on the European central bank's bailouts package to help several countries in the eurozone with their sovereign debt crises. If such attacks occur, the European central bank will find it hard to maintain the bailouts package, which will increase the sovereign default risk. Then not only the European financial market is affected, but the US equity market will also be hurt (Jeanneret, 2011). My model could be directly applied to these issues.

1.1 Previous Works on Currency Attacks

Speculative attacks induced by policies have drawn economists' attentions. Following the canonical crisis model developed by Henderson and Salant (1978), economists analyze the issue of defending against speculative attacks extensively in the foreign exchange market, where speculators attack the pegged currency. Krugman (1979), Flood and Garber (1984), and Broner (2008) study the "first generation" models of currency attacks, which treats currency attacks as a run on the central bank's foreign reserves. Besides arguments against the dependence of currency attacks on reserves (Drazen, 2000), I show in Subsection 6.1 that under a reasonable borrowing constraint, the issue of defending against speculative attacks is independent of foreign reserves.

The "second generation" models of currency attacks emphasize the limited commitment of the central bank to sustain the fixed exchange rate. In these models, speculators attack the currency because they expect other speculators to attack and the central bank to abandon the currency peg. Obstfeld (1996) analyzes a complete information model, in which multiple equilibria exist because of the coordination motive among speculators. Morris and Shin (1998) apply global games (introduced by Carlsson and van Damme (1993)) to currency attacks, in which speculators observe the relevant fundamentals with small noises, and show that there exists a unique equilibrium as the noise diminishes. The central bank in these two models plays a passive role as in the one-shot game in my model, because in a static environment, the central bank abandons the fixed exchange rate once the defending cost is greater than the benefit.

Angeletos, Hellwig, and Pavan (2006), and Angeletos and Pavan (2011) demonstrate that the signaling effects of preemptive instruments lead to multiple equilibria. In my model, the reputation may preempt future attacks. In addition, when speculators' learning speed is slow, the incentive of building a reputation will preempt all speculative attacks, which leads to the equilibrium uniqueness in my model. Goldstein, Ozdenoren, and Yuan (2011) uncover the informational complementarity among speculators, because the central bank is uncertain about its benefit from the fixed exchange rate and thus learn from the market. In my model, since it is common knowledge that the lowest period benefit is positive, the policy maker may still have incentive to build a reputation that her signals favoring a high benefit. All these works are in a static environment. But defending the policy regime against speculative attacks is an intrinsically dynamic process, because speculators have the option to attack the status quo repeatedly. Additionally, a potentially infinite horizon model provides a better framework to analyze the interaction between the reputation effect and the learning effect.

Economists have focused on dynamic global games recently. Angeletos, Hellwig, and Pavan (2007) show the equilibrium multiplicity in a dynamic regime change game with learning. Because the regime change rule is exogenously given, their model is an analogue of the induced game in my model. I will discuss the difference between results in their model and those in my induced game in Subsection 6.3. Dasgupta, Steiner, and Stewart (2010) study a dynamic global game with private learning in which the asynchronous coordination is allowed. Since the asynchronous coordination requires asynchronous common belief, they draw very different conclusions from those in my model, where synchronous coordination among speculators is a necessary condition for the policy maker to abandon the status quo.

Huang (2011) analyzes the interaction between coordination and social learning in a dynamic regime change game. In all these studies, the regime change rule is exogenously given, which is not a realistic assumption in the issue of defending against speculative attacks.

1.2 Other Related Literature

The policy maker's reputation in my model is not just a particular equilibrium (Barro and Gordon (1983) and Ljungqvist and Sargent (2002)), but is defined as the public belief of speculators. Wiseman (2009) studies the reputation bound of an informed player with uninformed players exogenously learning in a repeated chain store game. He establishes a lower bound of the chain store's equilibrium payoff when the precision of exogenous signals is small, which is strictly smaller than the stackelberg payoff. Though he dose not show the lower bound is tight, many equilibria with the informed player's payoff lower than the stackelberg payoff can be constructed. My model features coordination motives among speculators and the stopping property which result in significantly different conclusions.

Because of the coordination motive among speculators, a common belief among speculators is necessary for attacks. The common belief concept is introduced by Monderer and Samet (1989) and generalized by Morris and Shin (2007). Since speculators learn to form common beliefs, this paper is related to the learning literature. Cripps, Ely, Mailath, and Samuelson (2008) provide general conditions for common learning. All these theoretical works focus on economies with a finite number of players. Complementing these works, I define and apply the common belief and the common learning among a continuum of speculators.

The remainder of the paper is organized as follows. In Section 2, I present the model of defending against speculative attacks. I then first analyze the equilibrium behaviors of the policy maker in Section 3. Given candidate equilibrium actions of the policy maker, speculators play a dynamic regime change game with an exogenous regime change rule. Section 4 is devoted to the analysis of such "induced" game. In Section 5, I characterize the equilibrium of the model of defending against speculative attacks and show how the interaction between the reputation and the learning determines the outcome of the model. In Section 6, I discuss some related issues and how my model differs from closely related papers. Section 7 concludes. All omitted proofs are presented in the Appendix.

2 Defending A Regime Against Attacks

Time is discrete and is indexed by $t \in \{1, 2, \dots\}$. The game starts with the status quo in place. There is a continuum of long-lived speculators of measure 1, indexed by i and uniformly distributed over $[0, 1]$. In any period t, speculator i ($i \in [0, 1]$) chooses between attacking the status quo or not. Denote by $a_{it} = 1$ that speculator i attacks in period t and by $a_{it} = 0$ otherwise. The size of attacks in period t is defined as the measure of speculators attacking. Let A_t denote the size of attacks in period t, then $A_t = \int_0^1 a_{it} di$. In every period, after observing the size of attacks in that period, a policy maker decides whether to sustain the status quo or abandon it. The game continues as long as the status quo is in place and ends once the policy maker abandon the status quo.

2.1 Payoffs

In period t, any speculator's flow payoff depends on his own action and the regime change outcome in that period. The flow payoff from not attacking is normalized to be 0, no matter the status quo is abandoned or not. If speculator i attacks in period t, he receives $1-c$ if the status quo is abandoned in period t and $-c$ otherwise. Here, $c \in (0,1)$ is the opportunity cost of attacking.

The policy maker receives a period benefit from maintaining the status quo, θ. But in order to sustain the status quo in period t, the policy maker needs to pay a cost A_t, which is just the size of attacks in period t.[1] So the net period t payoff of the policy maker from maintaining the status quo is $\theta - A_t$. If the policy maker abandons the status quo in period t, her period t payoff is 0.

Assume all agents in this model share a common discount factor $\delta \in (0,1)$. Then the average discounted payoff of a speculator i is:

$$v_i = \begin{cases} (1-\delta)\left[\sum_{t=1}^{T-1} \delta^{t-1}(-c)a_{it} + \delta^{T-1}(1-c)a_{iT}\right], & \text{if the regime changes in period } T; \\ (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} a_{it}(-c), & \text{if the regime never changes,} \end{cases}$$

and the average discounted payoff of the policy maker is:

$$u = \begin{cases} (1-\delta)\sum_{t=1}^{T-1} \delta^{t-1}(\theta - A_t), & \text{if the regime changes in period } T; \\ (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}(\theta - A_t), & \text{if the regime never changes.} \end{cases}$$

2.2 Information

The policy maker's period benefit from maintaining the status quo, θ, is drawn from the set $\Theta \equiv \{L, H\}$ at the beginning of the game, where $0 < L < 1 < H$. All agents share a common prior belief about $\theta = L$, denoted by $\mu_1 = \Pr(\theta = L)$. Once picked, θ is fixed. The policy maker knows the picked θ as her private information. No speculator knows θ.

[1] Generalizing the defending cost to be a function of the size of attacks $C(A_t)$ with $0 \leq C(A_t) \leq C(1) < H$ would not change results in this paper.

In any period t, before making his decision, speculator i observes a private signal $z_{it} = \theta + \xi_{it}$ about θ. Assume $\xi_{it} \sim \mathcal{N}(0, 1/\eta_t)$ is independent of θ, independent and identically distributed across i, and serially uncorrelated. That is, speculators observe conditionally independent signals in every period. Let z_i^t denote speculator i's private signals up to period t. Then in period t, all speculators simultaneously make decisions after observing their own private signals up to period t. The policy maker, observing the size of attacks A_t, then chooses to maintain the status quo or abandon it. Neither individual nor aggregate actions are observed by speculators; hence the only public information at the beginning of period t is that the status quo is still in place.

2.3 Equilibrium

The policy maker decides whether to maintain the status quo in period t based on past and current sizes of attacks and her type. Hence the policy maker's strategy is a mapping from her type and the attacking history to a real number in $[0, 1]$. So $s_2(\theta, \{A_\tau\}_{\tau=1}^t)$ is the probability that the type θ policy maker maintains the status quo in period t, given the attacking history $\{A_\tau\}_{\tau=1}^t$.

The private history of a speculator i consists of his own private signals and past actions. Denote a typical history that any speculator observes before he makes the decision in period t by $h^t \in \mathbb{R}^t \times \{0, 1\}^{t-1}$ ($h^1 \in \mathbb{R}$ is just a speculator's private signal in the first period). Let $H = \cup_{t=1}^{\infty} h^t$ be the set of all relevant histories. Then any speculator i's strategy is defined as $s_i : H \to [0, 1]$, that is, $s_i(h^t)$ is the probability that speculator i attacks in period t, given his private history h^t.

The solution concept of this game is Perfect Bayesian equilibrium (PBE). Some special features of this game simplify the definition of a PBE. So it is helpful to first analyze these features to get a simplified definition of a PBE for this game. First, because there is a continuum of speculators, any individual speculator is so "small" that his action cannot affect the current and future sizes of attacks. Hence, given the policy maker's strategy, any individual speculator's action does not affect the time when the regime changes. As a result, a strategy of speculator i is part of a PBE, if and only if it prescribes an action after any history h_i^t to maximize his period t flow payoff. That is, in a PBE, any speculator behaves "myopically".

Second, define speculator i's *private belief* about L in period t to be the belief formed after the history h_i^t. Besides the private history h_i^t, speculator i also makes inference from the fact that the status quo is in place at the beginning of period t. In particular, since there is a continuum of speculators, fix a strategy profile, conditional on θ, the size of attacks in any period t is a deterministic number, $A_t(\theta)$. Then based on the policy maker's strategy, speculators update their beliefs. Since speculators share a common prior, in a PBE, their updated beliefs just based on the public history must be same. Call this belief the *public belief*, and denote the period t public belief about $\theta = L$ by μ_t. Then in a PBE, any speculator i forms the private belief in two steps. (i) From the fact that the status quo is in place at the beginning of period t, speculator i forms the public belief about $\theta = L$. (ii) Speculator i then employ Bayes' rule to form his private belief about $\theta = L$ based on the public belief μ_t and his private history h_i^t. Denote this private belief about $\theta = L$ by $\rho^{\mu_t}(h_i^t)$. Because speculators behave myopically in a PBE, their period t equilibrium actions depend only on the public belief about L in period t and their own private history up to period t.

Third, the policy maker is sequentially rational in a PBE. That is, after any attacking history $\{A_\tau\}_{\tau=1}^t$, the policy maker's equilibrium action has to maximizes her continuation average discounted payoff. But given a public belief μ_t, the past attacks $\{A_\tau\}_{\tau=1}^{t-1}$ do not affect future plays. Hence, the policy maker's equilibrium continuation strategy in any period t only depends on θ (her type), μ_t (the public belief), and A_t (the size of attacks in period t).

Finally in a PBE, given the associated public belief μ_t, no regime change in period t is always on the equilibrium path unless the policy maker chooses to abandon the status quo in period t for all θ. But $H > 1$ and largest possible cost incurred in sustaining the status quo is 1 (because the total measure of speculators is 1), so always maintaining the status quo is the unique dominant strategy of the policy maker with $\theta = H$. This implies that abandoning the status quo for all θ in period t is not a part of a PBE. As a result, in a PBE no speculator has information sets off the equilibrium path.

Definition 1 *A strategy profile $s = (s_i)_{i\in[0,1]\cup\{2\}}$ and a public belief system $\{\mu_t\}_t$ constitute a Perfect Bayesian equilibrium if*

1. *given $(s_i)_{i\in[0,1]}$ and $\{\mu_t\}_t$, $s_2(\theta)$ prescribes a strategy after any attacking history with associated (μ_t, A_t) to maximize the type θ policy maker's continuation average discounted payoff, $\forall\theta \in \Theta$;*

2. *Given s_2 and other speculators' strategies, in any period t with associated μ_t, $s_i(h_i^t)$ solves the following maximization problem for any h_i^t:*

$$\max_{a\in[0,1]} \left\{ \left[(1 - s_2(L, \mu_t, A_t(L)))\rho^{\mu_t}(h_i^t) + (1 - s_2(H, \mu_t, A_t(H)))(1 - \rho^{\mu_t}(h_i^t)) - c\right] a \right\};$$

3. *Given s, $\{\mu_t\}_t$ is calculated by Bayes' rule on the path of play.*

3 The Policy Maker's Reputation

Because always maintaining the status quo is the unique dominant strategy for the policy maker when $\theta = H$, the model is like a reputation model in which $\theta = H$ is the commitment type. Therefore, we say that the policy maker is of a "strong" type if $\theta = H$ and is of a "weak" type if $\theta = L$. In an equilibrium, the strong policy maker always defends the status quo against any speculative attacks. Then how about the weak policy maker? In any equilibrium, if the size of attacks is smaller than L in any period t, the weak policy maker would like to maintain the status quo. Because by sustaining the status quo, the weak policy maker receives a positive flow payoff in period t (since $L > A_t(L)$) and non-negative continuation payoffs (since she can always abandon the status quo in period $t+1$), sustaining the status quo in period t with $A_t(L) < L$ dominates abandoning it.

The interesting case is when $A_t(L) \geq L$. Since the cost incurred in sustaining the status quo outweighs the benefit from the status quo, it is optimal for a myopic weak policy maker to abandon the status quo. But the policy maker would take into account her future payoffs when making the current decision. The following lemma shows that in any equilibrium, if $A_t(L) \geq L$ on the equilibrium path, the weak policy maker will randomize, provided that she is sufficiently patient.

Lemma 1 *Fix any $\delta \in (1 - L, 1)$. In any equilibrium, the weak policy maker sustains the status quo with probability $q_t \in (0, 1 - c)$ in period t after $A_t(L) \geq L$ on the equilibrium path.*

The intuition about the randomization of the weak policy maker when $A_t(L) \geq L$ in any equilibrium (provided that δ is sufficiently large) follows the argument in the reputation literature (Fudenberg and Levine, 1989). On one hand, in an equilibrium, if the probability that the weak policy maker maintains the status quo is high, the expected payoff of any speculator from attacking would be less than the attacking cost. Therefore, the size of attacks is 0 (smaller than L). On the other hand, if the weak policy maker abandons the status quo for sure when $A_t(L) \geq L$, by deviating to maintain the status quo, she can quickly signal herself as a strong policy maker, so that she can deter all future attacks. When the policy maker is patient enough ($\delta > 1 - L$), this deviation is profitable.

When $A_t(L) \geq L$, Lemma 1 not only describes the weak policy maker's behavior on the equilibrium path, but also helps to pin down the continuation payoff of the weak policy maker in period t. Because abandoning the status quo brings the weak policy maker 0 average discounted payoff, the fact that the weak policy maker randomizes on the equilibrium path when $A_t(L) \geq L$ implies her equilibrium continuation payoff in period t is 0. Then fix the continuation strategy profile, after any $A'_t > A_t(L)$, the sequential rationality requires the weak policy maker to abandon the status quo in period t, because she will receive a negative average discounted payoff by sustaining the status quo. Similarly, for all $A'_t < A_t(L)$, the weak policy maker will sustain the status quo for sure.[2] In another case of $A_t(L) < L$ on the

[2]The size of attacks $A'_t \neq A_t$ cannot be reached by any individual speculator's unilateral deviation, but the weak policy maker needs to play optimally after A'_t, given the continuation strategy profile. Since the

equilibrium path, the weak policy maker sustains the status quo for sure in the equilibrium. Her action after any off-equilibrium size of attacks A'_t is also pinned down by the continuation strategy profile. In this case, the policy maker's decision rule in period t is in one of the following two forms: (i) if sustaining the status quo brings a positive average discounted payoff when the size of attacks is 1, the policy maker will sustain the status quo for sure for any A'_t; (ii) if there is $\hat{A}_t$ such that sustaining the status quo after $\hat{A}_t$ brings a zero average discounted payoff, the weak policy maker sustains the status quo for sure for all $A'_t < \hat{A}_t$, sustains the status quo with probability $q_t \in [0,1]$ when $A'_t = \hat{A}_t$, and abandons the status quo for sure for all $A'_t > \hat{A}_t$.

To sum up, fix the continuation strategy profile, the weak policy maker's equilibrium strategy is in the following form: assume the weak policy maker's average discounted payoff from sustaining the status quo after $\hat{A}_t(L)$ is 0, then

$$s_2(L, \mu_t, A'_t) = \begin{cases} 1, & \text{if} \quad A'_t < \hat{A}_t(L); \\ q_t, & \text{if} \quad A'_t = \hat{A}_t(L); \\ 0, & \text{if} \quad A'_t > \hat{A}_t(L). \end{cases}$$

In addition, if $\hat{A}_t$ is the equilibrium size of attacks in period t, $q_t \in (0, 1-c)$ after $A'_t = \hat{A}_t$. Otherwise, $q_t \in [0,1]$.

Lemma 1 also implies that if the weak policy maker sustains the status quo in period t when facing attacks with size $A_t(L) \geq L$, the public belief updates according to the Bayes' rule:

$$\mu_{t+1} = \frac{\mu_t q_t}{\mu_t q_t + (1-\mu_t)}.$$

Since $q_t < 1-c$, $\mu_{t+1} < \mu_t$. Define the policy maker's *reputation* as the public belief about $\theta = H$, then the weak policy maker can build her reputation by defending the status quo against attacks. The higher the policy maker's reputation is, the less likely speculators attack in the future. As a result, the weak policy maker has an incentive to mimic the strong policy maker, which provides the weak policy maker some commitment power.

4 Common Learning among Speculators

In this section, I analyze speculators' equilibrium behaviors. Suppose a strategy profile of speculators is part of an equilibrium, and A_t is the equilibrium size of attacks in period t. Then the corresponding equilibrium action of the weak policy maker is

$$s_2(L, \mu_t, A_t) = \begin{cases} 1, & \text{if} \quad A_t < L; \\ q_t \in (0, 1-c), & \text{if} \quad A_t \geq L. \end{cases}$$

aggregate actions of speculators are not observable to speculators, they will play as if the size of attacks in period t is A_t. Therefore, the continuation payoff from sustaining the status quo is strictly positive if $A'_t < A_t$ and is strictly negative if $A'_t > A_t$.

That is, given a relevant candidate equilibrium strategy of the policy maker, speculators know that if the size of attacks is less than the benefit θ, the regime does not change; if the size of attacks is greater than or equal to the benefit (this is true only if $\theta = L$), the regime changes with probability q_t. Because if $A_t = 0$ in period t, q_t could be any number in $[0, 1]$ for a possible size of attacks greater than or equal to L, let's first consider a general sequence $\{q_t\}_t$, with $q_t \in (0, 1]$ for all t. Then the regime change rule induced by the candidate equilibrium action of the policy maker is as follows (denote by $\hat{\tau}$ the time the regime changes):

$$\Pr(\hat{\tau} = t | \hat{\tau} \geq t) = \begin{cases} 0, & \text{if} \quad A_t < \theta; \\ 1 - q_t, & \text{if} \quad A_t \geq \theta. \end{cases}$$

Take this regime change rule as exogenously given, the game played by speculators is called the "induced" game.

Consider the strategy profile in which no speculator attacks, no matter what the private history is. Because $A_t = 0$ for all t on the path of play, according to the regime change rule, the status quo will be in place forever. Hence, it is the best for any speculator not to attack. Therefore, the strategy profile without attacks is an equilibrium (call it *no attack equilibrium*). This pure coordination failure equilibrium directly follows from the continuum speculators assumption. Then, is there any equilibrium with attacks? If so, when do attacks happen? How about the regime change outcome in such an equilibrium?

4.1 Conditions of Attacking

Any speculator i's equilibrium choice in any period t depends on both his private signals and his past actions. This dependence on private histories makes equilibrium strategies rather complicated. However, the following lemma 2 shows that in any equilibrium, speculators' strategies are in a simple form. Let $\beta_1 = \eta_1$, and recursively define $\beta_{t+1} = \beta_t + \eta_{t+1}$. Then define a new sequence $\{x_{it}\}_t$ by letting $x_{i1} = z_{i1}$ and $x_{it+1} = \frac{\beta_t}{\beta_{t+1}} x_{it} + \frac{\eta_{t+1}}{\beta_{t+1}} z_{it+1}$. From the standard Gaussian updating formula, $x_{it} \sim \mathcal{N}(\theta, 1/\beta_t)$ is the sufficient statistic of z_i^t about θ.

Lemma 2 *In any equilibrium, speculators employ symmetric cutoff rules in every period. In particular, any equilibrium is characterized by a sequence $\{x_t^*\}_t^\infty$ with $x_t^* \in \mathbb{R} \cup \{-\infty\}$, and any speculator attacks in period t if and only if $x_{it} \leq x_t^*$.*

So in any equilibrium, any speculator i's decision only depends on the sufficient statistic of his private signals. This is so both because private actions are not informative about θ and because the sufficient statistic leads to the same private belief as private signals do. Hence, in any equilibrium with the associated public belief system $\{\mu_t\}_t$, $\rho^{\mu_t}(h_i^t) = \rho^{\mu_t}(x_{it})$ ($\forall i$ and $\forall t$). In the no attack equilibrium, $x_t^* = -\infty$ for all t. Therefore, the question whether there is an equilibrium with attacks can be formulated as the problem whether there is a sequence $\{x_t^*\}_t$ such that $x_t^* \in \mathbb{R}$ for some t and x_t^* is speculators' equilibrium threshold point in period t for all t.

Define $\tilde{x}_t$ as the sufficient statistic in period t such that

$$\Pr(x_t \leq \tilde{x}_t | \theta = L) = \Phi(\sqrt{\beta_t}(\tilde{x}_t - L)) = L,$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution. So conditional on $\theta = L$, in period t, the measure of speculators who have the sufficient statistic lower than or equal to $\tilde{x}_t$ is exactly L.

Lemma 3 *If the induced game has an equilibrium in which the associated public belief in period t is μ_t and conditional on $\hat{\tau} \geq t$, a positive measure of speculators attack in period t, then*

$$\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q_t}. \tag{1}$$

The intuition of Lemma 3 is illustrated in the following figure. Let

$$g(x, \mu_t) = \chi(\Pr(x_t \leq x | \theta = L) \geq L)(1 - q_t)\rho^{\mu_t}(x) - c,$$

where $\chi(\cdot)$ is the indicator function. So $g(x, \mu_t)$ is a speculator's expected payoff from attacking in period t, when his own private sufficient statistic is x, all other speculators are using the cutoff rule with the threshold point x, and the public belief in period t is μ_t. Then in an equilibrium with μ_t and $A_t > 0$, $g(x, \mu_t) = 0$ must have a solution. In the figure, if $x < \tilde{x}_t$, $\chi(\Pr(x_t \leq x|\theta = L) \geq L) = 0$ which in turn implies that $g(x, \mu_t) = -c < 0$. For $x \geq \tilde{x}_t$, $\chi(\Pr(x_t \leq x|\theta = L) \geq L)(1 - q_t)\rho^{\mu_t}(x) - c = (1 - q_t)\rho^{\mu_t}(x) - c$. Because $\rho^{\mu_t}(x)$ is continuous and strictly decreasing in x, and $\lim_{x\to\infty} \rho^{\mu_t}(x) = 0$, the necessary condition for the existence of a solution to $g(x, \mu_t) = 0$ is $\max_{x\geq\tilde{x}_t} \rho^{\mu_t}(x) = \rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q_t}$.

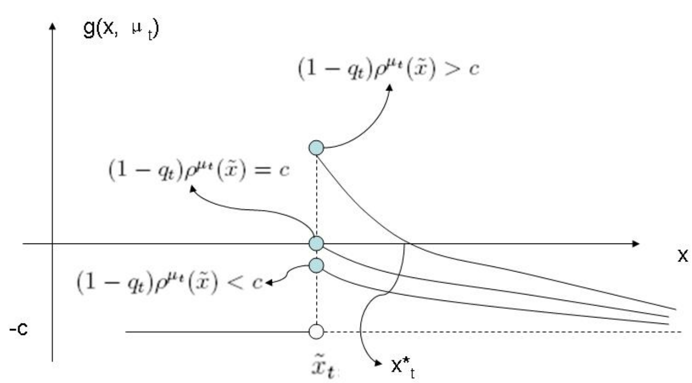

Figure 1: Function $g(x, \mu_t)$.

Figure 1 also suggests a sufficient condition for the existence of an equilibrium in which a positive measure of speculators attack in period t.

Lemma 4 *If $\rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1-q_t}$, the induced game has an equilibrium in which a positive measure of speculators attack in period t.*

I prove this lemma in the appendix by construction. The key point of the construction is that if no speculator chooses to attack in any period τ, the regime dose not change for sure, no matter $\theta = L$ or $\theta = H$. Therefore, the public belief does not change. So if speculators do not attack until period t, the public belief $\mu_t = \mu_1$. Then the condition of this lemma guarantees the possibility of attacks in period t.

4.2 Common Beliefs and Common Learning

Given the exogenous regime change rule, the game played by speculators features coordination motives in every period. However, because the regime does not change when $\theta = H$, speculators are uncertain about the coordination result. As a result, if a speculator attacks in some period t, he must believe $\theta = L$ with probability at least $\frac{c}{1-q_t}$. But because of the coordination motive, this speculator needs to form a belief about other speculators' beliefs, form a belief about other speculators' beliefs about other speculators' beliefs, and so on. This infinite hierarchy of beliefs is called *common belief* (Monderer and Samet, 1989). In this subsection, I follow Morris and Shin (2007) to define a version of common belief in the model and apply this concept to the analysis of speculators' behaviors.

Consider the conditions for speculator i to attack. Because speculator i behaves "myopically", he attacks if and only if he believes that the regime changes with probability at least c. If $\theta = H$, the regime does not change. And conditional on $\theta = L$, the regime changes only if the size of attacks is greater than or equal to L. Conditional on the joint event $\theta = L$ and $A_t(L) \geq L$, the regime changes with probability $1 - q_t$. Therefore, if speculator i attacks, his private belief about the joint event $\theta = L$ and $A_t(L) \geq L$ is at least $\frac{c}{1-q_t}$. Because $A_t(L) \geq L$ only if at least L measure speculators $\frac{c}{1-q_t}$-believe the joint event, at least L measure speculators $\frac{c}{1-q_t}$-believe that at least L measure speculators $\frac{c}{1-q_t}$-believe the joint event, and so on. Therefore, speculator i attacks only if he $\frac{c}{1-q_t}$-believes the entire list of following events:[3]

1. $\theta = L$;
2. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe $\theta = L$;
3. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe statement (2);
4. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe statement (3);

[3]As in Monderer and Samet (1989), a player p-believes an event if his posterior belief about the event is at least p.

5. ...

If when $\theta = L$, there are at least L measure speculators $\frac{c}{1-q_t}$-believe the above entire list of events, there is a *common* $(L, \frac{c}{1-q_t})$*-belief about* L among speculators.

Proposition 1 shows that if attacks happen in period t in an equilibrium, then there is a common $(L, \frac{c}{1-q_t})$-belief among speculators.

Proposition 1 *If the induced game has an equilibrium in which the associated public belief in period* t *is* μ_t *and conditional on* $\hat{\tau} \geq t$*, a positive measure of speculators attack in period* t*, then there is a common* $(L, \frac{c}{1-q_t})$*-belief about* $\theta = L$ *among speculators in period* t*.*

Proof. Because there is a continuum of speculators, a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$ is equivalent to that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$. To see this, consider any speculator i's private belief in period t about the whole list of events:

$$\begin{aligned} & \Pr(\theta = L, \text{statement 2, statement 3}, \ldots | x_{it}) \\ = \; & \Pr(\text{statement 3}, \ldots | \text{statement 2}, \theta = L, x_{it}) \Pr(\text{statement 2} | \theta = L, x_{it}) \Pr(\theta = L | x_{it}). \end{aligned}$$

Obviously, $\Pr(\text{statement 3}, \ldots | \text{statement 2}, \theta = L, x_{it}) = 1$. Since sufficient statistics are conditionally independent, if there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$, $\Pr(\text{statement 2} | \theta = L, x_{it}) = \Pr(\text{statement 2} | \theta = L) = 1$. So if there are L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$, there is a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$. Conversely, if there are less than L measure of speculators $\frac{c}{1-q_t}$-believing $\theta = L$, any speculator i's private belief in period t about the whole list of events above is 0. So there is not a common $(L, \frac{c}{1-q_t})$-belief about L.

Then, from Lemma 3, I only need to show that the inequality (1) is equivalent to that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$. First, suppose $\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q}$. Because $\rho^{\mu_t}(x_t)$ is strictly decreasing in x_t, any speculator i with private sufficient statistic $x_{it} \leq \tilde{x}_t$ has a posterior belief greater than or equal to $\frac{c}{1-q_t}$. Then the definition of $\tilde{x}_t$ implies that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$.

Now, suppose $\rho^{\mu_t}(\tilde{x}_t) < \frac{c}{1-q_t}$. Since $\rho^{\mu_t}(x_t)$ is continuous and strictly decreasing in x_t, $\exists\ \epsilon > 0$ such that $\rho^{\mu_t}(\tilde{x}_t - \epsilon) < \frac{c}{1-q_t}$. In addition, conditional on $\theta = L$, only speculators with private sufficient statistic less than $\tilde{x}_t - \epsilon$ have private beliefs about $\theta = L$ at least $\frac{c}{1-q_t}$. Then since $\Pr(x_t < \tilde{x}_t - \epsilon | \theta = L) < \Pr(x_t < \tilde{x}_t | \theta = L) = L$, a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ cannot be formed. ■

Since a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$ is necessary for attacks in period t in the equilibrium, it is natural to ask how $\theta = L$ is commonly $(L, \frac{c}{1-q_t})$-believed. The answer to this question is "by learning". Each speculator collects one private signal in every period, hence his sufficient statistic is increasingly accurate. Given $p \in (0,1)$ and $\mu_1 \in (0,1)$, define

$B_{it}^{p}(\theta) = \{x_{it} : \Pr^{\mu_1}(\theta|x_{it}) \geq p\}$ for any period t and any speculator i. Then $B_{it}^{p}(\theta)$ is the event that speculator i p-believes θ. Note the public belief which is used by any speculator i to calculate his private beliefs in all periods is μ_1. That is, to form the private belief $\Pr^{\mu_1}(\theta|x_{it})$ about θ, speculator i ignores the information revealed from the public history $\hat{\tau} \geq t$.

Definition 2 *Speculator i learns $\theta \in \Theta$ individually, if for each $p \in (0,1)$, there is T such that for all $t > T$, $\Pr(B_{it}^{p}(\theta)|\theta) \geq p$. Speculator i learns Θ if he learns each $\theta \in \Theta$.*

The definition of *individual learning* is the same as that in Cripps, Ely, Mailath and Samuelson (2008). The following Lemma 5 provides a sufficient and necessary condition for any speculator to learn Θ.

Lemma 5 *Any speculator learns Θ individually, if and only if*

$$\lim_{t\to\infty} \beta_t = +\infty.$$

But individual learning about Θ may not be sufficient for attacks in an equilibrium, because the coordination motive requires common $(L, \frac{c}{1-q})$-belief about $\theta = L$. Intuitively, the notion of common learning in an economy consisting of a continuum of agents of measure 1 should be that after some period T, there is a common (p,p)-belief among speculators in every period for any $p \in (0,1)$. However, this notion is too strong to be necessary for attacks in an equilibrium, because as shown in Lemma 4 that attacks happens in period t in an equilibrium if $\theta = L$ is common $(L, \frac{c}{1-q})$-belief in period t. So the common learning concept which is defined as follows is weaker than that in Cripps, Ely, Mailath and Samuelson (2008).

Definition 3 *Speculators $(L, \frac{c}{1-q_t})$-commonly learns $\theta = L$ in period t, if fix the public belief at μ_1, there is a common $(L, \frac{c}{1-q_t})$-belief among speculators in period t.*

If speculators cannot individually learn Θ, that is, the sequence $\{\beta_t\}_t$ is bounded above by $\bar{\beta} < +\infty$, fix $q_t = q$ sufficiently close to $1-c$, speculators cannot $(L, \frac{c}{1-q})$-commonly learn L in any period t. Then no attack equilibrium will be the unique equilibrium. So for the possibility of attacks in some equilibrium, I assume that speculators individually learn Θ, that is, $\lim_{t\to\infty} \beta_t = +\infty$. Lemma 6 below, together with Lemma 4, shows that if there is a subsequence of $\{q_t\}_t$ bounded above by some $\tilde{q} < 1-c$, and speculators individually learn Θ, then there is an equilibrium in which attacks happen in some period T.

Lemma 6 *If there is $\tilde{q} < 1-c$ such that $\{q_t\}_t$ has a subsequence bounded above by $\tilde{q}$, then individual learning implies common learning in some period T.*

4.3 Equilibrium of The Induced Game

The no attack equilibrium always exists in the induced game. According to the exogenous regime change rule, the regime does not change in the no attack equilibrium. Then what are conditions for the existence of an equilibrium with attacks? Suppose an equilibrium with attacks exists, what are the dynamics of attacks? Will the regime change when $\theta = L$?

Because of the flexibility of the sequence $\{q_t\}_t$, it is hard to get interesting conclusions in the induced game. Therefore, I focus on the case that the sequence $\{q_t\}_t$ has a subsequence bounded above by $\tilde{q} < 1 - c$.

Proposition 2 *Fix any $\mu_1 \in (0, 1)$, any sequence $\{q_t\}_t$ with a subsequence bounded above by $\tilde{q} < 1 - c$, and any strictly increasing and unbounded sequence $\{\beta_t\}_t$. With the exogenous regime change rule, multiple equilibria exist in the induced game:*

1. *no attack equilibrium exists;*

2. *there exists an equilibrium with attacks in which there is T such that no attacks happen after period T;*

3. *there exists an equilibrium with attacks in which there is $t > T$ such that attacks happen in period t, for any T.*

The proof of Proposition 2 is illustrated in Figure 2 below. Suppose $q_t = \tilde{q} < 1 - c$, and β_1 is sufficiently large so that $\tilde{x}_1 \leq \frac{H+L}{2}$. The x-axis is the public belief about L, and the y-axis is the variance of the private sufficient statistic. This "public belief – variance" space describes the condition for attacks in any period. When $\mu \geq \frac{c}{1-\tilde{q}}$, attacks are possible for all $\beta \geq \beta_1$. When $\mu < \frac{c}{1-\tilde{q}}$, there is $\tilde{\beta} > \beta_1$ such that attacks are possible if and only if $\beta \geq \tilde{\beta}$. $\tilde{\beta}$ is a strictly increasing function of μ. Therefore, in all graphs in Figure 2, the "public belief – variance" space is divided into two parts by the function $\tilde{\beta}(\mu)$: in the lower right part, attacks are possible because there is a common $(L, \frac{c}{1-\tilde{q}})$-belief about L; and in the upper left part, there is no common $(L, \frac{c}{1-\tilde{q}})$-belief about L, so no attack can happen. Fix an equilibrium, arrows indicate directions to which points move.

The left graph of Figure 2 shows the no attack equilibrium. Since speculators do not attack, the public belief does not change. And as they accumulate private signals, the variance of the sufficient statistic goes to 0. Therefore, all arrows are going down in this graph. The middle graph of Figure 2 illustrates an equilibrium, in which speculators attack once and if the policy maker sustains the status quo when facing attacks, no speculator attacks ever again. Note attacks happen when the point $(\mu, 1/\beta)$ is in the lower right part. And if attacks happen at some point $(\mu, 1/\beta)$, the arrows point to the southwest, because the policy maker's reputation increases (μ decreases) and speculators keep learning. The

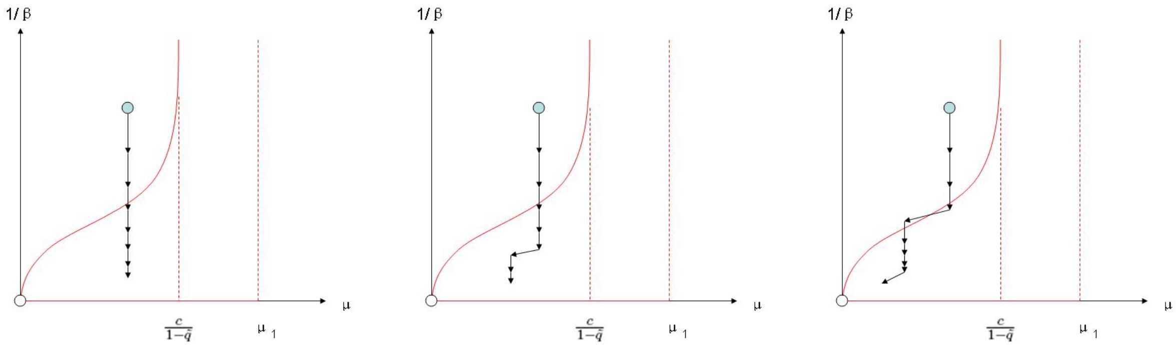

Figure 2: Multiple equilibria in the induced game

right graph of Figure 2 illustrates an equilibrium in which speculators attack infinitely often. The key point here is the individual learning. If the initial point is in the upper left part, speculators cannot attack, so that the public belief does not change. Then individual learning leads the path to cross the line $\tilde{\beta}(\mu)$, so attacks become possible.

In the no attack equilibrium and in any equilibrium with attacks in at most finitely many periods, speculators will learn the true state eventually. Hence, if $\theta = L$, they $(L, \frac{c}{1-\tilde{q}})$-commonly learn $\theta = L$ infinitely often. So no attack after some period is just due to the pure coordination failure. In these equilibria, even if $\theta = L$, with positive probability the regime does not change. Therefore, it is more interesting to analyze equilibria in which attacks happen infinitely often.

I first summarize three straightforward properties of an equilibrium in which attacks happen infinitely often. First, the exogenous regime change rule implies that conditional on $\theta = L$, if attacks happen in period t, $A_t(L) \geq L$. Otherwise, no speculator will choose to attack, since the regime changes with probability 0. Second, because attacks happen infinitely often, if $\theta = L$, the regime changes with probability 1. Third, even if $\theta = H$, attacks happen infinitely often. This is so because fix the public belief, speculators $(L, \frac{c}{1-\tilde{q}})$-commonly learn $\theta = L$ infinitely often even though the true state is H. In the following, I investigate two more equilibrium properties, which are significantly different from those of the model with the regime change outcomes endogenously determined by the policy maker.

Let $T_1(s)$ be the first period in which attacks happen in the equilibrium s. Then $\min_s T_1(s)$ is pinned down by the first period in which there is a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ among speculators.

Corollary 1 *Suppose* $\lim_{t\to\infty} \beta_t = +\infty$, *then*

$$\min_s T_1(s) = \min_t \{t : \rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1-q_t}\}.$$

Note for a fixed sequence $\{\beta_t\}_t$, Corollary 1 implies that $\min_s T_1(s)$ does not depend on how speculators learn after the period in which a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ forms in the first time.

Fix an equilibrium s. Define $\mathcal{Q}(s) \subset \mathbb{N}$ such that in s, $A_t > 0$ if and only if $t \in \mathcal{Q}(s)$. That is, in the equilibrium s, $\mathcal{Q}(s)$ is the set of periods in which attacks happen.

Corollary 2 *Given any integer $K \in \mathbb{N}$, there is an equilibrium s' such that $|T' - T| > K$ for any $T, T' \in \mathcal{Q}(s')$.*

Suppose T and T' are two consecutive periods in which attacks happen. Then Corollary 2 implies that the number of periods between T and T' may be unbounded in an equilibrium. Call the periods between T and T' the *common learning phase*, since speculators only collect private signals and the public belief does not change. This is actually the phase of tranquility in Angeletos, Hellwig, and Pavan (2007). Note for a fixed equilibrium, in some periods in the common learning phase, there is a common $(L, \frac{c}{1-q})$-belief about $\theta = L$, but speculators choose not to attack. An implication of this corollary is that the sequence of sizes of attacks is not monotone in some equilibrium s'. For any three consecutive periods T, T', and T'' in $\mathcal{Q}(s')$, if $T' - T$ is sufficiently large and $T'' - T'$ is relative small, it is possible that $A_{T'} > A_T$ and $A_{T'} > A_{T''}$.

5 Reputation Versus Common Learning

Let's go back to the model where the regime change outcome is endogenously chosen by the policy maker. It is straightforward that the strategy profile in which the policy maker always maintains the status quo and no speculator attacks is an equilibrium. Call this equilibrium the *no attack equilibrium*. In the "no attack equilibrium", the type θ policy maker's average discounted payoff is θ, her largest feasible payoff (or the "stackelberg payoff" in the reputation literature). Then are there equilibria with attacks? If so, what is the lowest equilibrium payoff (the reputation bound) of the policy maker?

The analysis in section 3 shows that in any equilibrium with attacks, the policy maker's strategy must be in the following form: assume the weak policy maker's average discounted payoff from sustaining the status quo after $\hat{A}_t(L)$ is 0. Then

$$s_2(L, \mu_t, A'_t) = \begin{cases} 1, & \text{if} \quad A'_t < \hat{A}_t(L); \\ q_t, & \text{if} \quad A'_t = \hat{A}_t(L); \\ 0, & \text{if} \quad A'_t > \hat{A}_t(L). \end{cases}$$

In addition, if $\hat{A}_t$ is the equilibrium size of attacks in period t, $q_t \in (0, 1-c)$ after $A'_t = \hat{A}_t$. Otherwise, $q_t \in [0, 1]$. Lemma 2 shows that in any equilibrium given the regime change rule induced by the policy maker's equilibrium actions, speculators employ cutoff rule in every period and make decisions only based on the public belief and the private sufficient statistics. Furthermore, a slightly modified version of Proposition 1 shows that if attacks happen in period t in some equilibrium, then given the public belief μ_t in period t, there must be a common (L, c)-belief about $\theta = L$ in period t. The requirement of a common (L, c)-belief about $\theta = L$ is due to the freedom of choosing $q_t \in (0, 1-c)$. It seems that simply putting these two parts together, we can characterize all equilibria with attacks. Therefore, the equilibrium characterization should be very similar to Proposition 2, when speculators are assumed to be able to learn Θ individually. Is this generally true?

Let's first consider a strategy profile specifying (1) attacks happen, and (2) if the policy maker sustains the status quo in some period, no speculator attacks ever again.

Lemma 7 *Fix any $\delta \in (1-L, 1)$. Consider a strategy profile with attacks. Suppose there is T such that speculators refrain from attacking even again after period T, if the policy maker sustains the status quo at the end of period T. Then the strategy profile cannot be an equilibrium.*

Proof. Suppose there is an equilibrium s in which attacks happen and conditional on $\hat{\tau} > T$, no speculator attacks after period T. Without losing generalization, let $T = \max \mathcal{Q}(s)$, then T is the last period in which attacks happen. Because $A_t > 0$, $A_t(L) \geq L$. Therefore, the probability that the weak policy maker maintains the status quo in period T is $q_T < 1 - c$. Therefore, because abandoning the status quo brings the weak policy maker 0 average discounted payoff in period T, the weak policy maker's average discounted payoff in period T is 0 on the equilibrium path.

Now consider the deviation of the policy maker in period T to $q_T = 1$. That is, the weak policy maker maintains the status quo for sure. By this deviation, $\hat{\tau} > T$. Since the deviation is not observable, no speculator chooses not to attack after period T. Then the weak policy maker's average discounted payoff in period T from this deviation is:

$$\begin{aligned} & (1-\delta)[(L - A_t) + L \sum_{\tau=1}^{\infty} \delta^{\tau}] \\ > \ & (1-\delta)(L-1) + \delta L \\ > \ & 0. \end{aligned}$$

Hence, this deviation is profitable. ■

Lemma 7 implies that in any equilibrium, once attacks happen, speculators cannot terminate attacking. This is different from the second part of Proposition 2. In Proposition 2, the regime change rule is exogenous, so speculators do not attack after some period in some equilibrium. However, the regime change rule is endogenously chosen by the policy maker in the model. If speculators do not attack after some period T, the weak policy maker, who is sufficiently patient, will maintain the status quo for sure when facing attacks.

So whether there is an equilibrium with attacks is equivalent to whether there is an equilibrium in which speculators attack infinitely often. In any period t, if $A_t(L) \geq L$ and the policy maker sustains the status quo,

$$\mu_{t+1} = \frac{\mu_t q_t}{\mu_t q_t + (1 - \mu_t)}.$$

Since in any equilibrium with attacks, $q_t \in (0, 1-c)$, so $\mu_{t+1} < \mu_t$. That is, if attacks happen in period t, and the status quo is in place at the end of period t, speculators believe that the policy maker is more likely to be strong. So by defending against attacks, the weak policy maker builds her reputation. On the other hand, however, speculators get more accurate information about the policy maker's type over time. This weakens the policy maker's incentive to build her reputation. An implicit assumption for this argument is that the common learning phase can be arbitrarily long. However, the following Lemma 8 shows that the number of periods in any common learning phase is uniformly bounded. Fix $\delta \in (1-L, 1)$. Let $\bar{K}$ be the smallest integer such that

$$(L-1) + L \sum_{\tau=1}^{\bar{K}} \delta^{\tau} \geq 0.$$[4]

Lemma 8 *Fix any $\delta \in (1-L, 1)$. Suppose there is an equilibrium s in which attacks happen in period t if and only if $t \in \mathcal{Q}(s)$. Then for any two consecutive periods T_n and T_{n+1} in $\mathcal{Q}(s)$, $T_{n+1} - T_n \leq \bar{K}$.*

Proof. Because attacks happen in period T_n and period T_{n+1} in the equilibrium s, the weak policy maker must randomize in these two periods. Further more, from Lemma 1, $q_{T_n} \in (0, 1-c)$ and $q_{T_{n+1}} \in (0, 1-c)$. Therefore, because abandoning the status quo always brings the policy maker 0 average discounted payoff, the weak policy maker's average discounted payoff is 0 in both period T_n and period T_{n+1}.

Now let's calculate the weak policy maker's average discounted payoff in period T_n from maintaining the status quo:

$$(1-\delta)[(L - A_{T_n}(L)) + L \sum_{\tau=1}^{T_{n+1}-T_n-1} \delta^{\tau}] + \delta^{T_{n+1}-T_n} 0 = 0.$$

[4]To see the existence of $\bar{K}$, note that $L - 1 < 0$ and that $(L-1) + \frac{\delta L}{1-\delta} > 0$. So $\bar{K} > 0$.

Therefore,

$$\begin{aligned}0 &= (L - A_{T_n}(L)) + L \sum_{\tau=1}^{T_{n+1}-T_n-1} \delta^\tau \\ &> (L-1) + L \sum_{\tau=1}^{T_{n+1}-T_n-1} \delta^\tau.\end{aligned}$$

So $\sum_{\tau=1}^{T_{n+1}-T_n-1} \delta^\tau < \sum_{\tau=1}^{\bar{K}} \delta^\tau$ which implies that $T_{n+1} - T_n - 1 < \bar{K}$. Therefore, $T_{n+1} - T_n \leq \bar{K}$. Since this is true for all n, the claim is true. ■

The fact that in any equilibrium common learning phases cannot be arbitrarily long is due to the weak policy maker's indifference between maintaining and abandoning the status quo when facing attacks. When speculators are in a common learning phase, while they acquire more accurate information about the policy maker's type, the policy maker is accumulating flow payoffs. Therefore, in order to make the policy maker indifferent at the beginning of a common learning phase, speculators have to attack again before the policy maker collects too many flow payoffs. This is different from Corollary 2, in which the regime change rule is exogenously given, so speculators do not need to make a policy maker randomize when she is facing attacks.

In any equilibrium with speculators attacking infinitely often, failing attacks decrease speculators' public belief about $\theta = L$. So consider the public history, the formed common (L, c)-belief about $\theta = L$ may be ruined. This happens especially when the incremental accuracy of private information cannot offset the discrete drop of the public belief due to the failing attacks. Therefore, speculators have to learn to form a common (L, c)-belief about $\theta = L$ again within a fixed number of periods. This suggests that the equilibrium characterization is determined by the comparison between the speed in which the public belief decreases and the speed in which speculators commonly learn, that is, the comparison between the policy maker's reputation building and speculators' common learning.

5.1 Slow Common Learning

The comparison between the reputation and the common learning is determined by three factors. First, attacks provide the policy maker chances to build her reputation. In addition, the more frequently attacks happen, the quicker the reputation is built. Once attacks begin, speculators have to attack again within $\bar{K}$ period. So the reputation building speed is bounded below by $(1-c)^{\frac{t-T_1}{\bar{K}}}$, because the probability of maintaining the status quo in any "attacking" period is bounded above by $(1-c)$. Second, the accuracy of speculators' private sufficient statistics are strictly increasing. In a common learning phase, the policy maker cannot build the reputation, but speculators knows more and more about her type. The learning speed is captured by the increasing rate of β, that is, the accuracy of new private signals. Third, β_{T_1} is the "stock" accuracy of speculators' private information, while the increments of β's are the "flow" accuracy. Though speculators can freely choose the "stock" accuracy before attacks happen (by coordinating not to attack until some T_1), the "flow" accuracy is given exogenously.

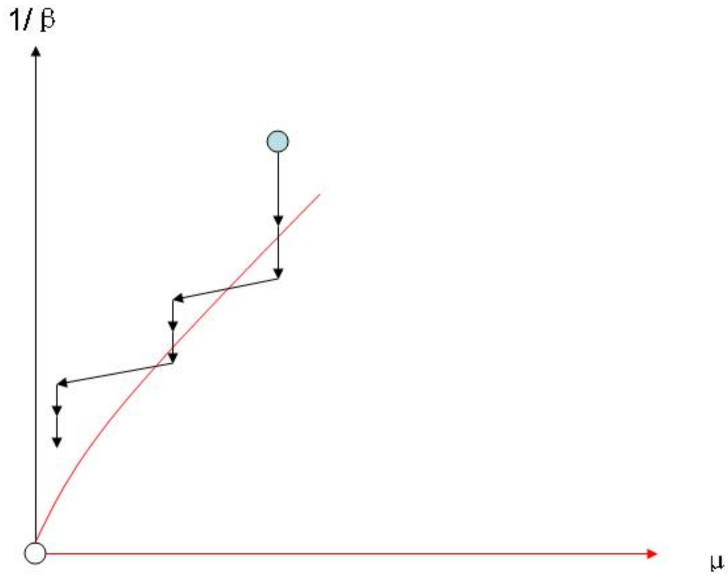

Figure 3: Reputation v.s. Common Learning.

Figure 3 shows the possibility that if speculators' learning speed is relatively slow, a strategy profile specifying attacks infinitely often cannot be an equilibrium. No matter what the "stock" accuracy speculators choose (the initial point in the graph), it is possible that the path will cross the line $\tilde{\beta}(\mu)$. Then the slow learning speed implies that within $\bar{K}$ periods, the path cannot cross the line $\tilde{\beta}(\mu)$ from above. So the policy maker will deviate to sustain the status quo for sure.

Proposition 3 below formalizes this argument and provides a sufficient condition for the uniqueness of the equilibrium. In particular, the condition (2) below captures the above three factors: the numerator is the lower bound of the policy maker's reputation building speed, the denominator is speculators' common learning speed, and it is independent of the first time in which attacks happen.

Proposition 3 *Fix any $\delta \in (1 - L, 1)$. Suppose*

$$\lim_{t \to \infty} \frac{(1-c)^{\frac{t}{\bar{K}}}}{\phi[\Phi^{-1}(L) - (H - L)\sqrt{\beta_t}]} = 0. \tag{2}$$

There is no equilibrium in which attacks happen. In this case, the type θ policy maker's lowest equilibrium average discounted payoff is θ.

Proof. Lemma 7 implies that if attacks happen in an equilibrium s, $\mathcal{Q}(s)$ is unbounded. Let $T_1 \in \mathcal{Q}(s)$ be the first period in which attacks happen. Recall that $\Pr(x_t \leq \tilde{x}_t | \theta = L) = \Phi[\sqrt{\beta_t}(\tilde{x}_t - L)] = L$. So

$$\tilde{x}_t = \frac{\Phi^{-1}(L)}{\sqrt{\beta_t}} + L.$$

Define $\tilde{\mu}_t$ such that when the public belief is $\tilde{\mu}_t$, the speculator who has the private sufficient statistic $\tilde{x}_t$ forms the posterior belief $\rho^{\tilde{\mu}_t}(\tilde{x}_t) = c$. Then

$$\begin{aligned}
& \rho^{\tilde{\mu}_t}(\tilde{x}_t) \\
= & \frac{\tilde{\mu}_t \phi[\sqrt{\beta_t}(\tilde{x}_t - L)]}{\tilde{\mu}_t \phi[\sqrt{\beta_t}(\tilde{x}_t - L)] + (1 - \tilde{\mu}_t)\phi[\sqrt{\beta_t}(\tilde{x}_t - H)]} \\
= & \frac{\tilde{\mu}_t \phi[\Phi^{-1}(L)]}{\tilde{\mu}_t \phi[\Phi^{-1}(L)] + (1 - \tilde{\mu}_t)\phi[\Phi^{-1}(L) - (H - L)\sqrt{\beta_t}]} \\
= & c.
\end{aligned}$$

Therefore, by rearranging terms, we have

$$\begin{aligned}
\tilde{\kappa}_t & \equiv \frac{\tilde{\mu}_t}{1 - \tilde{\mu}_t} \\
& = \frac{c}{(1 - c)\phi[\Phi^{-1}(L)]}\phi[\Phi^{-1}(L) - (H - L)\sqrt{\beta_t}].
\end{aligned}$$

Now define $\kappa_t \equiv \frac{\mu_t}{1-\mu_t}$, then $\mu_t \geq \tilde{\mu}_t$ if and only if $\kappa_t \geq \tilde{\kappa}_t$. For $t \leq T_1$, $\kappa_t = \kappa_1$. But by the beginning of any period $t > T_1$, Lemma 8 implies that attacks have happened at least Q times. Here Q is the smallest integer which is larger than or equal to $\frac{t-T_1}{\bar{K}}$. Therefore,

$$\begin{aligned}
\kappa_t & = \frac{\mu_t}{1 - \mu_t} \\
& = \left(\frac{\mu_1}{1 - \mu_1}\right) \prod_{\tau \in \mathcal{Q}(s) \cap \{\tau : \tau < t\}} q_\tau \\
& < \left(\frac{\mu_1}{1 - \mu_1}\right) (1 - c)^Q \\
& \leq \left(\frac{\mu_1}{1 - \mu_1}\right) (1 - c)^{\frac{t-T_1}{\bar{K}}}.
\end{aligned}$$

Suppose condition (2) holds. Then for any $\epsilon > 0$, there exists T such that for all $t > T$,

$$\frac{(1 - c)^{\frac{t}{\bar{K}}}}{\phi[\Phi^{-1}(L) - (H - L)\sqrt{\beta_t}]} < \epsilon.$$

Take

$$\epsilon < \frac{c(1 - c)^{\frac{T_1}{\bar{K}} - 1}}{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]},$$

then for all $t > T$,

$$
\begin{aligned}
& \frac{\kappa_t}{\tilde{\kappa}_t} \\
< & \left[\frac{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]}{c(1-c)^{\frac{T_1}{K}-1}}\right]\frac{(1-c)^{\frac{t}{K}}}{\phi[\Phi^{-1}(L)-(H-L)\sqrt{\beta_t}]} \\
< & \left[\frac{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]}{c(1-c)^{\frac{T_1}{K}-1}}\right]\epsilon \\
< & \ 1.
\end{aligned}
$$

This inequality holds because the term in the square bracket is independent of t. Though T_1 depends on the specific equilibrium candidate (so I have to take different ϵ for different equilibrium candidate), the fact that we can find T is independent of the equilibrium candidate.

Therefore, for the strategy profile s, $\kappa_t < \tilde{\kappa}_t$ is equivalent to $\mu_t < \tilde{\mu}_t$. So for all $t > T$, $\rho^{\mu_t}(\tilde{x}_t) < c$ which implies that there is no common (L, c)-belief in period t. If s is an equilibrium in which attacks happen infinitely often, then Proposition 1 says that for any T, there is $t > T$ such that there is a common (L, c)-belief about $\theta = L$ among speculators. These lead to the contradiction. ■

Three remarks about Proposition 3 are worth emphasizing. First, the equilibrium uniqueness is because of the commitment power brought by the policy maker's incentive to build the reputation. Because the strong policy maker behaves as a commitment type who always maintains the status quo, the weak policy maker wants to mimic so that she builds the reputation to be the strong type. This reputation incentive provides the weak policy maker a commitment power. Since if the status quo is in place, any speculator attacking will get negative payoff, no speculator want to attack. Second, for any T, how speculators learn in the first T periods does not affect the equilibrium characterization. Because speculators will learn slowly after period T, the weak policy maker will want to build her reputation in the tail. The reputation incentive in the tail results in no attacks after some period T. Because once attacks happen speculators cannot terminate attacking, speculators will never start attacking. Third, the policy maker does not need to be very patient. As long as $\delta > 1 - L$, the weak policy maker has the reputation incentive (that is, it is valuable for her to build the reputation to deter future attacks). Because $\bar{K}$ is non increasing in δ, the reputation building speed is nondecreasing in δ.[5]

There are two intuitive comparative static analyses. First, when the attacking cost c becomes large, the numerator converges to 0 faster. So equation (2) is easier to hold. That

[5]Because $\bar{K}$ is defined to be an integer, there is $\epsilon > 0$ such that for all $\delta \in (1-\epsilon, 1)$, $\bar{K}$ reaches its minimum, so the reputation building speed reaches its maximum.

is, the higher the attacking cost, the less likely the status quo is attacked. Second, if the benefit L increases, the policy maker needs fewer periods to collect flow payoffs to make her average discounted payoff 0. That is, $\bar{K}$ is an increasing function of L. Hence, the larger the flow payoff is, the less likely speculators attack. Note, an increase in H does not have any effect on the equilibrium characterization, because no matter how large H is, the strong policy maker's equilibrium behavior does not change.

Compare Proposition 3 with Proposition 2, it is easy to see the role of the policy maker's reputation. In Proposition 2, there is a sequence $\{q_t\}_t$ such that for all possible prior beliefs and all strictly increasing and unbounded sequences of $\{\beta\}_t$, there are infinitely many equilibria with attacks. This conclusion is under the assumption that the regime change rule is exogenously given. But in Proposition 3, the policy maker decides whether to sustain or abandon the status quo, so she may deviate to sustain it, which leads to the difference between Proposition 3 and Proposition 2.

5.2 Fast Common Learning

When the common learning is fast, there may exist equilibria with attacks. A sequence of $\{\beta_t\}_t$ for an equilibrium with attacks can be identified by the method of "reverse engineering". Suppose we want an equilibrium in which attacks happen in period t if and only if $t \in \mathcal{Q} \subset \mathbb{N}$. Then a strictly increasing sequence of $\{\beta_t\}_t$ can be found by the following algorithm:

1. Arrange elements in $\mathcal{Q}$ to be $\{T_1, T_2, \dots\}$ such that $T_{n+1} > T_n$;
2. Find $A_{T_1}(L) = L \sum_{\tau=1}^{T_2-T_1-1} \delta^\tau$;
3. Choose β_{T_1} such that
$$\frac{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))] + (1-\mu_{T_1})\phi[\Phi^{-1}(A_{T_1}(L)) - (H-L)\sqrt{\beta_{T_1}}]} > c;$$
4. Given β_{T_1}, calculate $x^*_{T_1}$ and q_{T_1} such that
$$\Phi[\sqrt{\beta_{T_1}}(x^*_{T_1} - L)] = A_{T_1}(L)$$
and
$$(1-q_{T_1})\frac{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))] + (1-\mu_{T_1})\phi[\Phi^{-1}(A_{T_1}(L)) - (H-L)\sqrt{\beta_{T_1}}]} = c;$$
5. From Bayes' rule, calculate
$$\mu_{T_2} = \frac{\mu_{T_1}q_{T_1}}{\mu_{T_1}q_{T_1} + (1-\mu_{T_1})};$$

6. In any $T_n \in \mathcal{Q}$ with μ_{T_n}, determine $A_{T_n}(L)$. Then similar to the case in T_1, calculate β_{T_n}, $x^*_{T_n}$, q_{T_n} and $\mu_{T_{n+1}}$. Note, $\beta_{T_{n+1}} > \beta_{T_n}$ for all $T_n, T_{n+1} \in \mathcal{Q}$;

7. Given $T_n, T_{n+1} \in \mathcal{Q}$, in any t such that $T_{n+1} > t > T_n$, pick any $\beta_t \in (\beta_{T_n}, \beta_{T_{n+1}})$ and $\beta_{t+1} > \beta_t$ if $t+1 < T_{n+1}$. Then in any $t \notin \mathcal{Q}$, set $q_t = 1$ and $x^*_t = -\infty$.

Proposition 4 *Fix any $\delta \in (1-L, 1)$. The sequence $\{\beta_t\}_t$ constructed in the above algorithm exists and leads to an equilibrium in which speculators attack in period $t \in \mathcal{Q}$ if and only if $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$.*

Suppose given the sequence of $\{\beta_t\}_t$, there is an equilibrium with attacks. Then, it is straightforward to show that there are infinitely many equilibria with attacks. So it is interesting to compare the properties of these equilibria with those of equilibria in the induced game. First, given any equilibrium with attacks, speculators never terminate attacking (see Lemma 7). This is due to the policy maker's incentive to maintain the status quo for future payoffs. But with exogenous regime change rule, there are equilibria in which attacks only happen in finitely many periods.

Second, recall that $T_1(s)$ is the first period in which attacks happen in the equilibrium s. Then the first possible "attacking" period $\min_s T_1(s)$ depends on the entire sequence $\{\beta_t\}_t$. The reason why there may be no attack in period T by which there is a common (L, c)-belief among speculators is due to the policy maker's incentive to build her reputation. For example, speculators (L, c)-commonly learn $\theta = L$ by period T, but after period T, the accuracy of private sufficient statistics increases very slowly for a long time and then increases fast. Because the "stock" of the accuracy of the private sufficient statistic in period T is bounded, if the slow increasing phase is very long, a common (L, c)-belief about $\theta = L$ cannot be form at the end of such phase. Then the policy maker has a strict incentive to maintain the status quo in period T no matter how large the attack is. So no speculator chooses to attack in period T in any equilibrium. This is different from the conclusion in Corollary 1, where the weak status quo is abandoned exogenously with positive probability if the size of attacks is larger than or equal to L.

6 Discussion

6.1 Borrowing Constraints

Traditional models, such as the first generation models of currency attacks, explain speculative attacks as a run on the capacity of the policy maker to sustain the policy regime. I show in this subsection that imposing a reasonable borrowing constraint to the policy maker will not change the result of the model. Consider the currency attacks example. Suppose the central bank holds a credit line in an outside borrower, so that it can borrow at most 1 unit of the foreign reserve. Therefore, the central bank can borrow again if and only if its outstanding balance has been fully repaid.

Let T_n be the nth attacking period in an equilibrium. Because the borrowing constraint is not binding at the beginning of period T_1, as the same argument in Lemma 1, the policy maker sustains the status quo with probability $q_{T_1} \in (0, 1-c)$ if $A_{T_1}(L) \geq L$ in the equilibrium. So the policy maker's average discounted payoff from sustaining the status quo in period T_1 is 0. If the policy maker cannot repay her outstanding balance by period T_2, then the discounted benefits she collects until period T_2 cannot cover the defending cost in period T_1. Since she has to abandon the status quo in period T_2, her average discounted payoff in period T_2 is 0. This implies that the policy maker's average discounted payoff in period T_1 is negative, which leads to the contradiction. Therefore, the borrowing constraint is not binding at the beginning of period T_2. Then by induction, it can be shown that imposing the borrowing constraint does not change the results of this paper.

6.2 The Myopic Policy Maker

When $\delta = 0$, the policy maker is myopic. So the equilibrium of the model prescribes an equilibrium in the one-shot game in every period with associated public belief μ_t. Since the policy maker does not value future payoffs, in period t, if $A_t(L) > L$, the policy maker will abandon the status quo for sure. When $A_t(L) = L$, the policy maker may randomize. Given a candidate equilibrium strategy of the policy maker, the induced game in period t could be solved by Figure 1. Suppose when $A_t(L) \geq L$, the policy maker abandons the status quo for sure. If and only if $\rho^{\mu_t}(\tilde{x}_t) \geq c$, the equation $g(x, \mu_t) = 0$ has a solution $x_t^* \in \mathbb{R}$. Therefore, there exists an equilibrium with attacks in the one-shot game in period t if and only if there is a common (L, c)-belief about $\theta = L$. In such an equilibrium, the policy maker sustains the status quo if and only if $A_t(L) < L$, and speculator i attacks if and only if $x_{it} \leq x_t^*$. Given this equilibrium in period t, if the status quo is in place at the beginning of period $t+1$, $\mu_{t+1} = 0$.

Given the policy maker's equilibrium strategy, the induced game among speculators in the one-shot game in any period differs from the model in Morris and Shin (1998). Because the policy maker sustains the status quo for sure if $A_t(L) < L$, attacking is not a dominant strategy for any private signal. Hence, this induced game is not a global game, and it has either a unique equilibrium in which no speculator attacks or multiple equilibria.

6.3 Continuum State Space

Angeletos, Hellwig, and Pavan (2007) (AHP) analyze a dynamic regime change game, in which θ is drawn from the real line, and in period t the regime change if and only if $A_t(\theta) \geq \theta$. Since the second period, their model is very similar to the induced model in my paper, provided that $q_t = \tilde{q} < 1 - c$ for all t. However, the outcome of the induced model is different from the model in AHP. In the induced game, given any prior belief $\mu_1 \in (0,1)$, individual learning results in infinitely many equilibria. In AHP, denote the infimum of the state surviving the attacks in the first period by $\underline{\theta}$. If $\underline{\theta}$ is sufficiently close to 1 (due to the extremely aggressive attacks in the first period), there exists a unique equilibrium, in which no attack can happen ever again.

This difference relies on the different common belief requirements. In the induced game, as long as there is a common $(L, \frac{c}{1-\tilde{q}})$-belief in any period t, there is an equilibrium in which some speculators attack in period t. Here, L and $\tilde{q}$ is fixed. So the learning effects will overturn any public belief. But in AHP, given a $\theta' < 1$, attacks happen only if there is a common $(\theta', \frac{c}{\Pr(\theta \leq \theta' | \theta > \underline{\theta})})$-belief about $\theta \leq \theta'$. But this common belief cannot be formed as $\underline{\theta}$ is sufficiently close to 1.

6.4 Exogenous Public Information

The reputation bound of an informed player when uninformed short-lived players are learning about the informed player's type has been studied by Wiseman (2009) in a repeated chain store game. Though Wiseman (2009) does not show the tightness of the established reputation bound, one can construct an equilibrium with the chain store's payoff strictly lower than the stackelberg payoff, no matter how slow the learning speed is. This is different from Proposition 3. And two assumptions of my model lead to this difference. First, speculators' private information is idiosyncratic. Because of the coordination feature, speculators put more weight on the public information when making decisions. The public information in my model is the policy maker's reputation, hence, the policy maker has stronger incentive to build the reputation. Second, the game ends once the policy maker abandons the status quo. Because the continuation payoff from abandoning the status quo is the policy maker's minmax value, the policy maker also has stronger incentive to sustain the status quo.

Now suppose besides private signals, in every period t speculators observe an exogenous public signal $y_t = \theta + \vartheta_t$, where $\vartheta \sim \mathcal{N}(0, 1/\alpha_t)$. Assume $\sum_{t=1}^{\infty} \alpha_t = +\infty$. Then conditional on $\theta = L$, in the limit, the sufficient statistic of the public signal is extremely precise. So the public signals will overturn the public belief formed from the fact that the status quo is in place. That is, the reputation built is easily ruined by the public signals. Therefore, a common (L, c)-belief can be formed frequently. Hence, an equilibrium with attacks exist.

7 Conclusion

I analyze a model where the policy maker is building a reputation by taking advantage of her private information and speculators are learning the policy maker's private information. The interaction between the reputation and the learning determines the equilibrium characterization and the outcome of defending against speculative attacks. In particular, when speculators learning speed is slow in the limit, the reputation effect will dominate the learning effect. As a result, the unique equilibrium of the model is the no attack equilibrium, in which no speculator attacks and the policy maker sustains the policy regime forever. Therefore, when the learning speed is slow in the limit, the policy maker effectively defends speculative attacks, because the incentive of building a reputation brings the policy maker the commitment power. In case of a fast learning speed, equilibria with attacks may exist. In any equilibrium with attacks, the first attacking period depends on the whole learning process, the time interval between two consecutive attacking periods is bounded, and the weak policy maker abandons the status quo almost surely.

From a theoretical perspective, I show that the reputation bound in a stopping game equals to the stackelberg payoff, when the uninformed players' learning speed is slow in the limit. This reputation bound does not require the informed player to be extremely patient. Besides, I complement the common belief and common learning literature, by defining and applying the common belief and the common learning in an economy consisting of a continuum of players.

From an applied perspective, I demonstrate that the transparency of a government has significant effects on the outcome of defending the policy against speculative attacks. The more transparent the government is, the faster the speculators learn the government's private information. Consequently, the policy established by the government is more likely to be attacked.

A Omitted Proofs

This section includes proofs of Propositions and Lemmas, which are stated in the text but not proved.

Proof of Lemma 1:

Suppose first, in the equilibrium, $A_t(L) \geq L$ implies $q_t \geq 1 - c$. Then any speculator i's payoff from attacking in period t is

$$(1 - q_t)\rho^{\mu_t}(h_i^t) - c < c - c = 0.$$

The strict inequality is due to the common support assumption of private signals with respect to θ. Therefore, any speculator who is attacking would like to deviate to not attack. This implies $A_t = 0$, which leads to a contradiction.

Now suppose $q_t = 0$, that is, the weak policy maker abandons the status quo for sure when the defending cost is larger than or equal to the flow payoff. Consider a deviation to maintain the status quo for sure in all periods $\tau \geq t$. Because the strong policy maker defends against any attack, no regime change in period t implies $\mu_{t+1} = 0$. That is, since this deviation is not observable by speculators, the public belief about $\theta = H$ shifts to 1. Then no speculator wants to attack in any period $\tau > t$. Therefore, the weak policy maker's average discounted payoff in period t is (note if the public belief about $\theta = H$ is 1, no speculator wants to attack ever again):

$$\begin{aligned}
& (1-\delta)[(L - A_t(L)) + \sum_{\tau=1}^{\infty} \delta^\tau L] \\
> \; & (1-\delta)[\frac{L}{1-\delta} - 1] \\
= \; & L - (1-\delta) \\
> \; & 0.
\end{aligned}$$

So this deviation is profitable, which implies that $q_t = 0$ when $A_t(L) \geq L$ is not a part of an equilibrium.

Q.E.D.

Proof of Lemma 2:

Let's first show that in any equilibrium, speculators employ symmetric strategies. In any period t with the public belief μ_t. Given all other speculators' strategies, conditional on θ, the total measure of attack in period t is a deterministic number. Therefore, the probability of the regime change is $\chi(A_t(L) \geq L)(1 - q)$ which is exogenously given to all speculators. If $\chi(A_t(L) \geq L)(1-q) \leq c$, no matter what the private history is, a speculator will choose not to attack. If $\chi(A_t(L) \geq L)(1-q) > c$, any speculator i chooses to attack if and only if $\rho^{\mu_t}(h_i^t) \geq \frac{c}{1-q}$. Since two speculators with the same private history will form the same posterior belief, they will make the same choice. As a result, in any equilibrium, speculators employ symmetric strategies.

Now, in an equilibrium, because any individual actions cannot publicly observed, a speculator's past actions are not informative about θ. Hence, a speculator forms his posterior belief only based on his private signals. By the standard Gaussian updating formula, for any speculator i, in any period t given μ_t, z_i^t and x_{it} lead to the same posterior belief. Therefore, $\rho^{\mu_t}(h_i^t) = \rho^{\mu_t}(x_{it})$. Because

$$\rho^{\mu_1}(x_{i1}) = \frac{\mu_1 \phi(\sqrt{\beta_1}(x_{i1} - L))}{\mu_1 \phi(\sqrt{\beta_1}(x_{i1} - L)) + (1-\mu_1)\phi(\sqrt{\beta_1}(x_{i1} - H))},$$

the monotone likelihood ratio property implies that $\rho^{\mu_t}(x_{it})$ is strictly decreasing in x_{it}. As a result, if $\chi(A_t(L) \geq L)(1-q) \leq c$ which is equivalent to $\chi(A_t(L) < L)$ because $1-q > c$, all speculators will choose not to attack. That is, any speculators attack if and only if $x_{it} \leq x_t^* = -\infty$. If $\chi(A_t(L) \geq L)(1-q) > c$, there is an $x_t^* \in \mathbb{R}$ such that any speculator i attacks if and only $x_{it} \leq x_t^*$.

Q.E.D.

Proof of Lemma 3:

Suppose there is an equilibrium in which the public belief in period t is μ_t and conditional on $\hat{\tau} \geq t$, $A_t > 0$. According to the exogenous regime change rule, conditional on θ, if $A_t(\theta) < \theta$, $\Pr(\hat{\tau} = t | \hat{\tau} \geq t) = 0$. Hence, any speculator i's problem in period t is

$$\max_{a \in [0,1]} \quad [\chi(A_t(L) \geq L)(1-q_t)\rho^{\mu_t}(x_{it}) - c]a.$$

If $A_t(L) < L$, speculator i will choose not to attack, no matter what his private sufficient statistic is. Therefore, if $A_t(L) < L$, $A_t(L) = 0$. Equivalently, $A_t(L) > 0$ implies $A_t(L) \geq L$. From Lemma 2, any speculator i attacks in period t if and only if $x_{it} \leq x_t^*$. Hence, the speculator with private sufficient statistic x_t^* will receive 0 expected payoff from attacking. That is, $\rho^{\mu_t}(x_t^*) = \frac{c}{1-q_t}$. So for $A_t(L) \geq L$, $\Pr(x_t \leq x_t^* | \theta = L) \geq L = \Pr(x_t \leq \tilde{x}_t | \theta = L)$. So $x_t^* \geq \tilde{x}_t$. Because $\rho^{\mu_t}(x_t)$ is a strictly decreasing function of x_t, $\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q_t}$.

Q.E.D.

Proof of Lemma 4:

I prove this lemma by construction. Let's consider the strategy profile in which no speculator chooses to attack until period t. Because $A_\tau(L) = A_\tau(H) = 0$ for all $\tau < t$, $\hat{\tau} \geq t$

no matter $\theta = H$ or $\theta = L$. Then $\mu_t = \mu_1$. So $\rho^{\mu_t}(\tilde{x}_t) = \rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1-q_t}$. Since $\rho^{\mu_t}(x)$ is continuous in x and $\lim\limits_{x\to\infty} \rho^{\mu_t}(x) = 0$, $\exists x_t^* \in [\tilde{x}_t, \infty)$ such that $\rho^{\mu_t}(x_t^*) = \frac{c}{1-q_t}$. Then in period t, any speculator i attacks if and only if $x_{it} \leq \tilde{x}_t$. If attacks in period t fail, no speculator attacks ever again.

Let's verify the constructed strategy profile is an equilibrium. In all periods $\tau \neq t$, since $A_\tau = 0$, $\Pr(\hat{\tau} = t|\hat{\tau} \geq t) = 0$. Therefore, refraining from attacking is the best response of any speculator. In period t, consider any speculator i. Since other speculators use the cutoff rule with the threshold point $x_t^* \in [\tilde{x}_t, \infty)$, $\Pr(x_t \leq x_t^*|\theta = L) \geq \Pr(x_t \leq \tilde{x}_t|\theta = L) = L$. So $\chi(A_t(L) \geq L) = 1$. In addition, $\rho^{\mu_t}(x_{it}) \geq \rho^{\mu_t}(x_t^*) = \frac{c}{1-q}$ if and only if $x_{it} \leq x_t^*$. Hence, speculator i attacks if and only if $x_{it} \leq x_t^*$. Therefore, the constructed strategy profile is an equilibrium.

Q.E.D.

Proof of Lemma 5:

Without losing any generalization, I prove that $\theta = L$ is individually learned if and only if $\lim\limits_{t\to\infty} \beta_t = +\infty$. First suppose $\lim\limits_{t\to\infty} \beta_t = \bar{\beta} < +\infty$. Note $\rho^{\mu_1}(x_{it}) \geq p$ is equivalent to

$$x_{it} \leq \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}.$$

Fix any p sufficiently close to 1,

$$\begin{aligned}
& \Pr(B_{it}^p(L)|L) \\
= \; & \Pr(\{x_{it} : \rho^{\mu_1}(x_{it}) \geq p\}|L) \\
= \; & \Pr\left(\left\{x_{it} : \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}\right\}|L\right) \\
= \; & \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right] \\
< \; & \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\bar{\beta}}(H-L)} + \sqrt{\bar{\beta}}\frac{H-L}{2}\right] \\
< \; & p.
\end{aligned}$$

That is, for p sufficiently close to 1, $\Pr(B_{it}^p(L)|L) < p$ for all t. As a result, no speculator can individually learn $\theta = L$. Put differently, if speculators can individually learn $\theta = L$, $\lim\limits_{t\to\infty} \beta_t = +\infty$.

Now suppose $\lim_{t\to\infty} \beta_t = +\infty$. Fix any $p \in (0,1)$,

$$\begin{aligned} & \Pr(B_{it}^p(L)|L) \\ = \; & \Pr(\{x_{it} : \rho^{\mu_1}(x_{it}) \geq p\}|L) \\ = \; & \Pr\left(\left\{x_{it} : \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}\right\}|L\right) \\ = \; & \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right]. \end{aligned}$$

Because $\lim_{t\to\infty}\left[\frac{\ln[(1-p)\mu_1]-\ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right] = +\infty$, there is T such that

$$\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_T}(H-L)} + \sqrt{\beta_T}\frac{H-L}{2}\right] > p.$$

. Since $\Phi\left[\frac{\ln[(1-p)\mu_1]-\ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right]$ is increasing in β_t, for all $t > T$,

$$\begin{aligned} & \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right] \\ > \; & \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_T}(H-L)} + \sqrt{\beta_T}\frac{H-L}{2}\right] \\ > \; & p. \end{aligned}$$

So if $\lim_{t\to\infty} \beta_t = +\infty$, speculators individually learn $\theta = L$.

Q.E.D.

Proof of Lemma 6:

In Proposition 1, I show that a common $(L, \frac{c}{1-q_T})$-belief among speculators in period T is equivalent to the inequality $(1-q_T)\rho^{\mu_T}(\tilde{x}_T) \geq c$. Therefore, fix public belief $\mu_t = \mu_1$ for all t, then

$$\begin{aligned} & (1-q_t)\rho^{\mu_1}(\tilde{x}_t) \\ = \; & (1-q_t)\frac{\mu_1\phi[\Phi^{-1}(L)]}{\mu_1\phi[\Phi^{-1}(L)] + (1-\mu_1)\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}]}. \end{aligned}$$

Because speculators individually learn Θ, $\lim_{t\to\infty} \beta_t = +\infty$, which implies that $\lim_{t\to\infty} \rho^{\mu_1}(\tilde{x}_t) = 1$. Since there is a subsequence of $\{q_t\}_t$ which is bounded above by $\tilde{q} < 1-c$, there is T such that $(1-q_T)\rho^{\mu_1}(\tilde{x}_T) \geq c$.

Q.E.D.

Proof of Proposition 2:

The first part is trivial and directly follows from the continuum speculators assumption and the exogenous regime change rule.

For the second part, Lemma 6 shows that individual learning is sufficient for common learning in some period t. Therefore, there is an equilibrium with attacks. But after some period T, speculators just choose the pure "not attack" strategy, which leads to no attack after period T.

For the third part, because individual learning implies common learning in some period for any prior belief $\mu_1 \in (0,1)$, there is an equilibrium in which attacks happen. If attacks fail in some period T, $\mu_T < \mu_1$, but $\mu_T \in (0,1)$. Therefore, speculators (L,c)-commonly learn $\theta = L$ by some period $t > T$. Therefore, attacks can happen again in or after period t.

Q.E.D.

Proof of Proposition 4:

Suppose a sequence of $\{\beta_t\}_t$ is constructed according to the algorithm and leads to an equilibrium consisting of sequences $\{\mu_t\}_t$, $\{x_t^*\}$, and $\{q_t\}_t$. Then Lemma 8 implies the necessity of $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$.

Now suppose that $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$, we want to show that the constructed sequence of $\{\beta_t\}_t$ exists, and that the associated sequences $\{\mu_t\}_t$, $\{x_t^*\}$, and $\{q_t\}_t$ constitute an equilibrium. Since $T_2 - T_1 \leq \bar{K}$ and $\delta \in (1-L, 1)$, A_{T_1} is well defined. Because $\mu_{T_1} = \mu_1 \in (0,1)$, and

$$\lim_{\beta \to \infty} \frac{\mu_{T_1} \phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1} \phi[\Phi^{-1}(A_{T_1}(L))] + (1-\mu_{T_1}) \phi[\Phi^{-1}(A_{T_1}(L)) - (H-L)\sqrt{\beta}]} = 1 > c,$$

β_{T_1} is well defined. Then $x_{T_1}^* \in \mathbb{R}$ and $q_{T_1} \in (0, 1-c)$ are uniquely determined. Therefore, μ_{T_2} can be calculated from Bayes' rule. The rest of the proof follows from the induction.

Q.E.D.

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