

Federal Reserve Bank of Minneapolis
Research Department

September 16, 2013

On the Social Usefulness of Fractional Reserve Banking *

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ABSTRACT

This paper presents a simple means-of-payment-in-advance model where households can purchase the single consumption good with either deposits from a price taking representative bank or cash. We show that even if facilitating trade through deposits costs more than using cash, such trade can occur in equilibrium if banks face a low enough reserve ratio. Thus it is possible that fractional reserve banking occurs in the unique equilibrium, but is strictly dominated by the equilibrium associate with a 100% reserve ratio, even when bank runs are explicitly ruled out. We further show that if households can, with some probability, access all of their wealth just prior to making purchases, the social benefits of fractional reserve banking converge to zero as this probability approaches one.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. PRELIMINARY AND INCOMPLETE. DO NOT CIRCULATE.

1. Introduction:

Fractional reserve banking is, to put it mildly, problematic. Banks with fractional reserves have been historically subject to runs and panics with arguably enormous external effects. Further, banks are costly. Even abstracting from runs and panics (as we do in this paper), the banking sector uses up real resources — labor and capital — which could be put to alternative uses. But fractional reserve banking is also ubiquitous. It appears to occur throughout history, with or without bailouts, making it difficult to argue that this ubiquity is due simply to bailouts or government subsidies. Usually, the ubiquity of an economic arrangement itself argues that this arrangement serves a valuable social purpose. Historically, one such purpose is that banks have allowed individuals and firms to pay for goods and services through their provision of bank checks and other widely accepted claims. Therefore, those individuals and firms haven't had to resort to costly barter or non-interest bearing specie trade.

In this paper, we raise two possibilities: First, that this facilitation of trade using interest bearing deposits serves a *privately* useful function, while providing lower (and perhaps negative) *social* benefits. In one simple example we provide, banks are worse than useless in that they use up real resources but provide no societal benefit, and this is true even when we explicitly rule out significant societal costs such as bank runs. The more general point we wish to make is that the private benefits from creating private payments systems may exceed the social benefits. Second, we argue that as technology changes to allow households to more easily (or less expensively) access all of their wealth when needed, the benefits associated with fractional reserve banking decrease, and thus now, or in the near future, the benefits of fractional reserve banking may no longer justify its associated risk to society.

Our paper is related to a large literature in money and banking. Our paper has in common with Aiyagari, Braun, and Eckstein (1998) that private money creation uses real resources so that

inflation stimulates the demand for private money and has real costs. They focus on the welfare costs of inflation while our focus is on examining the benefits and costs of changes in reserve requirements. Our paper is also related to Monnet and Sanches (2011) who show, among other results, that 100% reserve requirements may be undesirable. Their result derives from lack of commitment of bankers to repay depositors. We also show that 100% reserve requirements may be undesirable, but our results derive from comparing the costs of private money creating versus the insurance benefits associated with private money. He and Wright (2005) develop a search model of fractional reserve banking, where, like ours, fractional reserve banking occurs in equilibrium, but do not focus on welfare implications.

Other papers, see for example Gu et.al. (2012) have developed theories of banking using mechanism design approaches. Almost by construction, such theories imply that allocations are efficient. While we believe such an approach is very useful, in our paper we focus on inefficiencies that could arise when private and social interests do not coincide.

Finally, our paper is related to an extensive literature on so called “pecuniary externalities” that arise when prices enter consumption or production sets. See, for instance, Kehoe and Levine (1993), Kiyotaki and Moore (1997), Lorenzoni (2008), and Hart and Zingales (2011). Perhaps the most closely related of these is Hart and Zingales (2011) who develop a four-period model in which they argue that private liquidity provision can be inefficiently high due to a pecuniary externality. They do not analyze the welfare effects of fractional reserve banking.

2. The Model:

The model is a simple, infinitely repeated, means-of-payment-in-advance economy. There exists a unit continuum of identical households, a single price-taking representative bank, and a monetary authority whose sole function is to (possibly) make lump sum monetary transfers to households. Time is discrete and is denoted $t \in \{0, 1, 2, \dots\}$.

The commodities in the economy are a single, divisible, non-storable consumption good, of which each household receives a constant endowment y , and a commodity which we refer to as “gold”. Gold can take three forms: coins (m), which can be used to facilitate trade, gold bars (g^b), which can be used by the bank as reserves, and jewelry (g^h), which brings direct enjoyment to the owner. At the beginning of each period, gold can be linearly and costlessly transformed across its three forms. Thus for each date $t \geq 0$,

$$m_t + g_t^h + g_t^b = G_t, \tag{1}$$

where G_t is the aggregate supply of gold at date t . We assume $G_0 = 1$ and $G_{t+1} = (1 + \eta)G_t$.

At each date t , the bank is assumed to be able to freely create deposits, d_t , subject to a constraint that $\alpha d_t \leq g_t^b$, where g_t^b is the bank’s holdings of gold, and α is a reserve ratio.

Households cannot eat their own endowments but must instead trade with other households. To facilitate this trade, households must use coins, deposits, or, with some probability $\sigma(z = 1)$, may purchase goods facing no means-of-payment in advance constraint, instead purchasing the consumption good using all the wealth available to the household. If such trade occurs using coins, (the selling household receives coins in exchange for the consumption good), we assume it occurs with a physical cost ϕ^m in terms of the consumption good. Specifically, if a household wishes to consume c units of the consumption good and purchase these goods with coins, it must actually purchase $(1 + \phi^m)c$ units of the consumption good, where $\phi^m c$ units of the consumption are used up facilitating the trade. If such trade occurs using bank deposits, (the selling household receives bank deposits in exchange for the consumption good), we assume it occurs with a physical cost $\phi^d > \phi^m$ in terms of the consumption good. If trade occurs using neither coins nor deposits, it occurs with physical cost $\phi^b > \phi^d$. These costs (ϕ^m, ϕ^d, ϕ^b) stand in for the real resources (labor and capital)

of running various systems for facilitating trade and the assumption that $\phi^b > \phi^d > \phi^m$ reflects the idea that a more sophisticated means-of-payment system uses more real resources. Formally, they are simply iceberg costs.

Households care about streams of consumption and (possibly) jewelry holdings according to the per-period utility function

$$u(c_t, \theta_t) + v(g_t^h), \tag{2}$$

where c_t is consumption, $\theta_t \in \Theta$ is a publicly observed idiosyncratic i.i.d. preference shock, and g^h is gold held by the household in non-monetary form (jewelry). (Assume $\lim_{c \rightarrow 0} u_c(c, \theta) = \infty$ for all θ .) If $v(g^h) = 0$ for all g^h , then gold is interpreted as a fiat currency. So that a stationary equilibrium exists we assume

Assumption 1. *Either $v(g^h) = 0$ for all g^h (fiat currency) or $G_t = 1$ for all t ($\eta = 0$ or gold is in fixed supply).*

A. Markets:

At the beginning of each period, t , households can freely transform gold between coins and jewelry, and there exists a market among households and the bank with trade in gold, deposits, and promises to deliver gold at the beginning of the next period contingent on the (yet to be realized) taste shock $\theta_t \in \Theta$ and liquidity shock $z_t \in \{0, 1\}$. Let $q_t(\theta_t, z_t)$ be the price of claim to a unit of (θ_t, z_t) -contingent gold next period and let $x_t(\theta_t, z_t)$ denote the quantity of (θ_t, z_t) -contingent gold a household purchases. (If $x_t(\theta_t, z_t) < 0$, then the household has sold a promise to deliver gold if shocks turn out to be (θ_t, z_t) .) Also assume households are free at this point to obtain gold directly from the bank, one for one, by presenting deposits to the bank. That is, a deposit is considered to be a legal right to trade deposits for gold with the bank, one-for-one. Households may also give gold to the bank in return for deposits, again, one-for-one, if the bank chooses to create the deposit.

We further assume, as part of this market structure, that a deposit entails a legal requirement on the bank to pay a market determined rate of interest, r_t , to the depositor, one period later. These assumptions deliver the household budget constraint in this market as

$$m_t + d_t + g_t^h + \sum_{\theta_t, z_t} q_t(\theta_t, \psi_t) x_t(\theta_t, z_t) \leq w_t, \quad (3)$$

where w_t is the household's beginning of period wealth in terms of coins, deposits and jewelry.

After the financial markets close, each household receives its endowment of y units of the consumption good and realizes its i.i.d. preference shock $\theta \in \Theta$ and access-to-its-wealth shock $z \in \{0, 1\}$. (Let $\pi(\theta_t)$ denote the fraction of households with preference shock θ_t and $\sigma(z_t)$ denote the fraction of households with liquidity shock z_t .) Households then split into shopper-seller pairs.

Sellers inelastically supply y units to market. Shoppers pay p_t^c for the price of consumption good regardless of the form of payment (since sellers are indifferent). Let $c_t^m(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) using coins, $c_t^d(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) household using deposits, and $c_t^b(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) household using the household's general wealth. Shoppers face means-of-payment-in-advance constraints such that for all (θ_t, z_t)

$$p_t^c(1 + \phi^m)c_t^m(\theta_t, z_t) \leq m_t, \quad (4)$$

$$p_t^c(1 + \phi^d)c_t^d(\theta_t) \leq d_t, \quad (5)$$

and for all θ_t ,

$$c_t^b(\theta_t, 0) = 0. \quad (6)$$

The latter restriction implies that if $z_t = 0$, household consumption must be purchased using either

coins or deposits.¹ Note that the timing does not allow the household's allocation of its wealth between coins, deposits, and jewelry to depend on its preference shock θ or access shock z . After trade, households eat and receive utility $u(c_t^m(\theta_t, z_t) + c_t^d(\theta_t, z_t) + c_t^b(\theta_t, z_t), \theta_t) + v(g_t^h)$.

At the end of period t , households receive transfers $x_t(\theta_t, z_t)$, interest on deposits $r_t d_t$ and a lump-sum gold transfer from the monetary authority η . (Note that interest is paid on deposits held *before* the market for the consumption good. This ensures that sellers equally value coins and deposits.) Thus a household starts period $t + 1$ with wealth (per unit of date $t + 1$ gold)

$$w_{t+1}(\theta_t, z_t) = \frac{1}{1 + \eta} \left(\eta + (1 + r_t)d_t + g_t^h + m_t + x_t(\theta_t, z_t) + p_t^c \left(y - (1 + \phi^m)c_t^m(\theta_t, z_t) - (1 + \phi^d)c_t^d(\theta_t, z_t) - (1 + \phi^b)c_t^b(\theta_t, z_t) \right) \right). \quad (7)$$

3. Allocations:

Let $h^t = (\theta_0, z_0, \dots, \theta_t, z_t)$. An allocation (with prices) is a sequence

$$\{c_t^m(h^t), c_t^d(h^t), c_t^b(h^t), m_t(h^{t-1}), d_t(h^{t-1}), g_t^h(h^{t-1}), x_t(\theta_t, z_t)(h^{t-1}), g_t^b, q_t(\theta, z), p_t^c, r_t\}_{t=0}^\infty. \quad (8)$$

An allocation is resource feasible if for all $t \geq 0$

$$\sum_{h^t} \pi(\theta_0) \dots \pi(\theta_t) \sigma(z_0) \dots \sigma(z_t) ((1 + \phi^m)c_t^m(h^t) + (1 + \phi^d)c_t^d(h^t) + (1 + \phi^b)c_t^b(h^t)) = y. \quad (9)$$

and

$$\sum_{h^{t-1}} \pi(\theta_0) \dots \pi(\theta_{t-1}) \sigma(z_0) \dots \sigma(z_t) (m_t(h^{t-1}) + g_t^h(h^{t-1})) + g_t^b = 1. \quad (10)$$

¹Although not modeled explicitly, one can interpret the event $z = 1$ as the household's buyer having the ability to trade general wealth for either cash or bank deposits immediately prior to a transaction and the household's seller being able to immediately trade the proceeds from a transaction for other assets.

An allocation is a competitive equilibrium if it is resource feasible and solves the household and bank problems outlined below. To simplify somewhat, we only consider *stationary* allocations where prices p_t^c , r_t , and $q_t(\theta, z)$ are all constants.

A. Household Problem:

Given constant prices, the household problem is recursive (in beginning of period wealth relative to the aggregate gold supply) and can be expressed as

$$V(w) \equiv \max_{c^m(\theta, z), c^d(\theta, z), c^b(\theta, z), d, m, g^h, x(\theta, z)} \sum_{\theta} \pi(\theta) \sigma(z) [u(c^m(\theta, z) + c^d(\theta, z) + c^b(\theta, z), \theta) + v(g^h) + \beta V(w'(\theta, z))] \quad (11)$$

subject to

$$m + d + g^h + \sum_{\theta} q(\theta, z) x(\theta, z) \leq w, \quad (12)$$

$$p_c(1 + \phi^m) c^m(\theta, z) \leq m, \text{ for all } (\theta, z) \quad (13)$$

$$p_c(1 + \phi^d) c^d(\theta, z) \leq d, \text{ for all } (\theta, z) \quad (14)$$

$$w'(\theta, z) = \frac{1}{1 + \eta} \left(\eta + (1 + r)d + g^h + m + x(\theta, z) + p^c \left(y - (1 + \phi^m) c^m(\theta, z) - (1 + \phi^d) c^d(\theta, z) - (1 + \phi^b) c^b(\theta, z) \right) \right) \quad (15)$$

as well as $c^b(\theta, 0) = 0$ for all θ and that all choice variables other than $x(\theta)$ be non-negative. Let μ be the Lagrange multiplier on the budget constraint (12) and $\pi(\theta) \sigma(z) \gamma^m(\theta, z)$ and $\pi(\theta) \sigma(z) \gamma^d(\theta, z)$ be the Lagrange multipliers on the means of payment in advance constraints (13) and (14).

B. Bank Problem:

At all dates, the representative bank is free to create deposits, d_t , subject to the constraint that $\alpha d_t \leq g_t^b$, where g_t^b is the banks reserve holdings of gold. Further, the creation of a deposit d_t obligates the bank to pay rd_t to the depositor at all future dates starting at date $t + 1$ (assuming, without loss, that the deposit is never destroyed.) Thus a choice of (d_t, g_t^b) must solve

$$\max_{d_t} d_t - g_t^b - \frac{q}{1 - q} d_t \quad (16)$$

subject to $\alpha d_t \leq g_t^b$, where $q \equiv \sum_{\theta, z} q(\theta, z)$, or the unconditional price at date t to a unit of gold at date $t + 1$.

4. Stationary Equilibria:

To characterize stationary equilibria (equilibria with constant prices), first consider the first order condition with respect to $x(\theta, z)$ in the household problem, or

$$\beta\pi(\theta)\sigma(z)V'(w'(\theta, z))\frac{1}{1 + \eta} = q(\theta, z)\mu. \quad (17)$$

If $q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta)$, this becomes

$$V'(w'(\theta)) = \mu, \quad (18)$$

for all θ . Thus given $q(\theta) = \beta\pi(\theta)\sigma(z)/(1 + \eta)$, $w'(\theta)$ is independent of θ . Further, standard envelope arguments imply $V'(w) = \mu$, thus

$$V'(w'(\theta, z)) = V'(w), \quad (19)$$

for all (θ, z) , which implies $w'(\theta, z) = w$ for all (θ, z) . In words, when $q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta)$, a condition of household optimization is that household wealth is constant over all dates and all histories of preference and access-to-the-bank shocks. That

$$q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta) \quad (20)$$

is from now on imposed.

Next, consider the implications of profit maximization by the representative bank. If one substitutes that bank's reserve requirement, $\alpha d_t \leq g_t^b$, at equality into its objective function (16) it becomes

$$d\left[1 - \alpha - \frac{q}{1 - q}r\right], \quad (21)$$

which is linear in d . Thus a necessary condition for maximization (which also implies zero profits) is that the bracketed expression in (21) equal zero, or

$$r = (1 - \alpha)\left(\frac{1}{q} - 1\right) = (1 - \alpha)\left(\frac{1 + \eta}{\beta} - 1\right), \quad (22)$$

or that the interest rate on deposits equals $(1 - \alpha)$ times the nominal interest rate.

A. Further Characterization

Proposition 1. *There exists a cutoff interest rate on deposits $r^* = \frac{(1+\eta)}{\beta} \frac{(\phi^d - \phi^m)}{(1+\phi^d)}$ such that if $r > r^*$, household optimization implies $m = 0$ and $c^m(\theta, z) = 0$ for all (θ, z) and if $r < r^*$, $d = 0$ and $c^d(\theta, z) = 0$ for all (θ, z) .*

Proof. Consider the household problem. The first order condition with respect to d (ignoring non-

negativity and imposing (18)) delivers

$$(1 - \beta \frac{1+r}{1+\eta})\mu = \sum_{\theta, z} \pi(\theta)\sigma(z)\gamma^d(\theta, z). \quad (23)$$

Let $c(\theta, z) = c^m(\theta, z) + c^d(\theta, z) + c^b(\theta, z)$. The first order condition with respect to $c^d(\theta, z)$ and imposing (18) delivers

$$u_c(c(\theta, z), \theta) = p_c(1 + \phi^d)(\frac{\beta\mu}{1+\eta} + \gamma^d(\theta, z)), \quad (24)$$

which implies

$$\sum_{\theta, z} \pi(\theta)\sigma(z)u_c(c(\theta, z), \theta) = p_c(1 + \phi^d)(\frac{\beta\mu}{1+\eta} + \sum_{\theta} \pi(\theta)\sigma(z)\gamma^d(\theta, z)), \quad (25)$$

Together, (23) and (25) imply

$$\sum_{\theta, z} \pi(\theta)\sigma(z)u_c(c(\theta, z), \theta) = p_c(1 + \phi^d)\mu(1 - \frac{\beta r}{1+\eta}). \quad (26)$$

Next, the first order condition with respect to m (again ignoring non-negativity and imposing (18)) delivers

$$(1 - \frac{\beta}{1+\eta})\mu = \sum_{\theta, z} \pi(\theta)\sigma(z)\gamma^m(\theta, z), \quad (27)$$

and the first order condition with respect to $c^m(\theta, z)$ and imposing (18) delivers

$$u_c(c(\theta, z), \theta) = p_c(1 + \phi^m)(\frac{\beta\mu}{1+\eta} + \gamma^m(\theta, z)), \quad (28)$$

which implies

$$\sum_{\theta,z} \pi(\theta)\sigma(z)u_c(c(\theta,z),\theta) = p_c(1+\phi^m)\left(\frac{\beta\mu}{1+\eta} + \sum_{\theta,z} \pi(\theta)\sigma(z)\gamma^m(\theta,z)\right), \quad (29)$$

Together, (27) and (29) imply

$$\sum_{\theta,z} \pi(\theta)\sigma(z)u_c(c(\theta,z),\theta) = p_c(1+\phi^m)\mu. \quad (30)$$

Note that the left hand sides of equation (26) and (30) are identical. Thus m and d can both be positive only if the right hand sides are equal or

$$(1+\phi^d)\left(1 - \frac{\beta r}{1+\eta}\right) = 1 + \phi^m. \quad (31)$$

Let r^* solve (31) or

$$r^* = \frac{(1+\eta)(\phi^d - \phi^m)}{\beta(1+\phi^d)}. \quad (32)$$

The result follows immediately. \square

An equilibrium with $d = 0$ and $m > 0$ is referred to as a *cash equilibrium*. An equilibrium with $d > 0$ and $m = 0$ is referred to as a *banking equilibrium*.

Corollary 1. *There exists a cutoff reserve ratio $\alpha^* \equiv \frac{1+\eta-\beta(1+\phi^d)-(1+\eta)\phi^m}{(1+\eta-\beta)(1+\phi^d)}$ such that if $\alpha > \alpha^*$, a banking equilibrium cannot exist, and if $\alpha < \alpha^*$, a cash equilibrium cannot exist.*

Proof. Since r is a function of the reserve ratio α , one can solve for α^* - the cutoff reserve ratio by equating the right hand sides of (22) and (32) to derive

$$\alpha^* \equiv \frac{1+\eta-\beta(1+\phi^d)-(1+\eta)\phi^m}{(1+\eta-\beta)(1+\phi^d)}. \quad (33)$$

□

Corollary 2. *If $\frac{1+\phi^d}{1+\phi^m} > \frac{1+\eta}{\beta}$ (or the relative price of consumption purchased using deposits versus cash exceeds the gross nominal interest rate), then a banking equilibrium cannot exist. (In particular, if $\frac{1+\eta}{\beta} = 1$, or the net nominal interest rate equals zero, then a banking equilibrium cannot exist.) If $\frac{1+\phi^d}{1+\phi^m} < \frac{1+\eta}{\beta}$ then there exists a sufficiently low reserve ratio $\alpha > 0$ such that a monetary equilibrium does not exist.*

Proof. For households to be willing to transact using deposits, one needs $r \geq \frac{(1+\eta)}{\beta} \frac{(\phi^d - \phi^m)}{(1+\phi^d)}$. For the representative bank to be willing to create deposits, it needs $r \leq (1 - \alpha)(\frac{1+\eta}{\beta} - 1)$. Thus one needs

$$\frac{(1 + \eta) (\phi^d - \phi^m)}{\beta (1 + \phi^d)} \leq (1 - \alpha) \left(\frac{1 + \eta}{\beta} - 1 \right). \quad (34)$$

Since the right hand side of (34) is decreasing in α , setting $\alpha = 0$ relaxes (34) as much as possible. Thus if $\frac{(1+\eta)}{\beta} \frac{(\phi^d - \phi^m)}{(1+\phi^d)} > \frac{1+\eta}{\beta} - 1$, which solves for $\frac{1+\phi^d}{1+\phi^m} > \frac{1+\eta}{\beta}$, banking is incompatible with either household optimization or non-negative profits for the representative bank for all $\alpha > 0$. Likewise, if $\frac{1+\phi^d}{1+\phi^m} < \frac{1+\eta}{\beta}$, then α can be set low enough (but still positive) such that (34) can be satisfied as a strict inequality. □

Corollary 3. *If $\sigma(0) > 0$ and $\alpha \neq \alpha^*$, then either $m > 0$ and $d = 0$ or $d > 0$ and $m = 0$.*

Proof. For any given θ , that $\sigma(0) > 0$ implies $u(c(\theta, 0), \theta)$ has positive weight in the household objective function. That $\lim_{c \rightarrow 0} u_c(c, \theta) = \infty$ implies $c(\theta, 0) > 0$. That $c(\theta, 0) = c^m(\theta, 0) + c^d(\theta, 0)$ (from $c^b(\theta, 0) = 0$) then implies either $c^m(\theta, 0) > 0$ or $c^d(\theta, 0) > 0$. Proposition 1 and the means-of-payment in advance constraints (13) and (14) then imply the result. □

5. Welfare in the $\sigma(0) = 1$ Economy:

To see the welfare effects of deposits substituting for currency, first consider the case where $\sigma(0) = 1$, or households must pay for all consumption with either coins or deposits. If $v(g^h) = 0$ for all g^h (fiat money) and Ω is a singleton (no preference shocks), then characterization and welfare analysis is greatly simplified, thus we consider this case first.

As noted in the previous section, if $\alpha > \alpha^*$, $d = 0$. Bank optimization then implies $g^b = 0$ (banks hold no reserves). That $v(g^h) \equiv 0$ implies $g^h = 0$, and thus market clearing for gold (equation (1)) implies all gold is held as coins, or $m = 1$, where the gold supply is normalized to unity each period. Goods market clearing implies $c = y/(1 + \phi^m)$ and the cash-in-advance constraint implies $p^c = \frac{1}{y}$ (where p^c is in terms of current period gold). That $m_0 = 1$, $m_{t+1} = (1+\eta)m_t$, $x_{t+1} = (1+\eta)x_t$, $q = \beta/(1 + \eta)$ and $m_{t+1} + qx_{t+1} = m_t + \eta + x_t$ then implies $x_t = 0$ for all $t \geq 0$.

Thus the stationary monetary equilibrium with fiat money, no ability to purchase consumption without currency and no preference shocks is quite simple. Each period, a household enters with wealth $w = 1$ (where the current aggregate supply of gold is the numeraire) in the form of one gold coin. It makes no trades in the initial financial market. The household's shopper goes off and purchases y units of the consumption good while the household's seller sells the household's endowment, y , for one gold coin. The household is then handed η gold coins by the monetary authority. At the beginning of the next period, the household holds $(1 + \eta)$ coins in terms of the previous period's numeraire, or one gold coin in terms of the current numeraire.

Next, suppose $\alpha < \alpha^*$ and thus $m = 0$. As before, that $v(g^h) \equiv 0$ implies $g^h = 0$, and thus market clearing for gold (equation (1)) implies all gold is held as bank reserves, or $g^b = 1$ and $d = 1/\alpha$. Goods market clearing implies $(1 + \phi^d)c = y$, and the cash-in-advance constraint, $p^c(1 + \phi^d)c = d$ implies $p^c = \frac{1}{\alpha y}$. That $d_0 = 1/\alpha$, $d_{t+1} = (1 + \eta)d_t$, $x_{t+1} = (1 + \eta)x_t$, $q = \beta/(1 + \eta)$,

$r = (1 - \alpha)(1/q - 1)$ and $d_{t+1} + qx_{t+1} = d_t(1 + r) + \eta + x_t$ then implies

$$x = \frac{-(1 - \alpha)}{\alpha} \frac{1 + \eta}{\beta} < 0. \quad (35)$$

Thus the stationary banking equilibrium can be described as follows: Each period, a household again enters the period with wealth $w = 1$ in terms of the current gold supply. At the initial financial market, the household converts this wealth into a unit of deposits and borrows an additional $-qx$ from the bank, leaving the financial market with $d = 1/\alpha$ deposits and an obligation to pay the bank $-x$ at the end of the period. ($d = 1 - qx = 1/\alpha$ from the derived equilibrium values for q and x). The household's shopper then goes off and purchases y units of the consumption good (of which $y/(1 + \phi^d)$ can be used for consumption) while the household's seller sells the household's endowment, y , for $1/\alpha$ deposits. The household is then handed η gold coins by the monetary authority, receives r/α interest on its deposits, and pays $-x$ to the bank, leaving it with wealth, in terms of the new gold supply of $w = (1/(1 + \eta))(\eta + (1/\alpha)(1 + r) + x) = 1$.

Proposition 2. *Suppose $v(g^h) = 0$ for all g^h (fiat money), Ω is a singleton (no preference shocks), and $\sigma(0) = 1$. Then the cash equilibrium dominates the banking equilibrium (or the optimal policy is $\alpha = 1$).*

Proof. In the cash equilibrium, steady state consumption is equal to $y/(1 + \phi^m)$. In the banking equilibrium, steady state consumption is equal to $y/(1 + \phi^d)$. Given $\phi^d > \phi^m$, the result follows. \square

Note that the cash equilibrium can be enforced here either by setting $\alpha = 1$ (actually $\alpha > \alpha^*$) or by setting $\eta = \beta - 1$ (deflating according to the Friedman rule).

A useful question is why, given $\phi^m < \phi^d$, a household doesn't deviate and use cash in the banking equilibrium, since its utility in the cash equilibrium is higher than in the banking equilibrium. The simple answer is that it can't afford to. Steady state wealth is the same ($w = 1$) in

both equilibria. But the price level is $1/\alpha$ times higher in the banking equilibrium, since the supply of “money” (assets which can be used for trade) is $1/\alpha$ times higher in the banking equilibrium. Thus a household which used only cash and didn’t borrow in the banking equilibrium could only afford $1/p^c = \alpha y/(1 + \phi^m)$ units of consumption in the first period. (In subsequent periods, the household could afford to purchase $y/(1 + \phi^m)$ units of consumption each period.) Consuming $y/(1 + \phi^d)$ units of the consumption good each period is preferable to consuming $\alpha y/(1 + \phi^m)$ units in the first period, and $y/(1 + \phi^m)$ units thereafter. Now this is not the only deviation where only cash is used. The deviating household could borrow from the bank, for instance, to smooth consumption and nevertheless only use cash. However, the point remains that that using cash in every period to purchase $y/(1 + \phi^m)$ units of consumption as in the cash equilibrium is not in a household’s constraint set when $p^c = 1/\alpha$. Further, the effect on the price level of the increase in the money supply induced by a household’s decision to use deposits is external to that household. This externality, through the means-of-payment-in-advance constraints, is the source of the competitive equilibrium inefficiency.

Next, consider when $v(g^h) \neq 0$ or when Ω is not a singleton, still assuming $\sigma(0) = 1$. (As before, assume if $v(g^h) \neq 0$, then $\eta = 0$, so that a steady state equilibrium exists).

Proposition 3. *If $\theta \in \{L, H\}$ and $\alpha > \alpha^*$, then steady state expected utility is independent of α .*

Proof. Equation (28) and $\gamma^m(L) = 0$ with (30) implies

$$\frac{\beta}{1 + \eta} \sum_{\theta} \pi(\theta) u_c(c(\theta), \theta) = u_c(c(L), L), \quad (36)$$

or,

$$\frac{\beta}{1 + \eta} \pi(H) u_c(c(H), H) = \left(1 - \frac{\beta}{1 + \eta} \pi(L)\right) u_c(c(L), L), \quad (37)$$

This, along with

$$\pi(L)c(L) + \pi(H)c(H) = y/(1 + \phi^m), \quad (38)$$

solve for $c(L)$ and $c(H)$ independently of α .

To solve for g^h (jewelry), note that the cash-in-advance constraint holding with equality for $\theta = H$ implies the real value of money, or

$$p_c c(H) = m. \quad (39)$$

The first order condition with respect to g^h (jewelry) is

$$v'(g^h) = \left(1 - \frac{\beta}{1 + \eta}\right)\mu. \quad (40)$$

Finally, since $d = 0$, $g^b = 0$. Equations (30), (39), (40) and

$$g^h + m = 1 \quad (41)$$

then solve for m , g^h , p_c and μ as functions of $c(L)$ and $c(H)$ (which, again, do not depend on α).

(If $v(g^h) \equiv 0$ for all g^h (fiat money), then $g^h = 0$ replaces equation (40)). \square

Proposition 4. *If $\theta \in \{L, H\}$ and $\alpha < \alpha^*$, then steady state expected utility from consumption $(\sum_{\theta} \pi(\theta)u(c(\theta), \theta))$ is decreasing in α . In particular, as $\alpha \rightarrow 0$, $c(\theta)$ approaches full insurance $(u_c(c(L), L) = u_c(c(H), H))$ with $\pi(L)c(L) + \pi(H)c(H) = y/(1 + \phi^d)$. If $v(g^h) \equiv 0$ for all g^h (fiat money), then welfare is decreasing in α for all $\alpha < \alpha^*$.*

Proof. Equation (24) and $\gamma^d(L) = 0$ with (26) implies

$$\frac{\beta}{1+\eta} \sum_{\theta} \pi(\theta) u_c(c(\theta), \theta) = u_c(c(L), L) \left(1 - \frac{\beta r}{1+\eta}\right), \quad (42)$$

or,

$$\frac{\beta}{1+\eta} \pi(H) u_c(c(H), H) = \left(1 - \frac{\beta}{1+\eta} r - \frac{\beta}{1+\eta} \pi(L)\right) u_c(c(L), L), \quad (43)$$

This, along with

$$\pi(L)c(L) + \pi(H)c(H) = \frac{y}{1+\phi^d}, \quad (44)$$

solves for $c(L)$ and $c(H)$ as functions of r (which is a decreasing function of α). Note that if $\alpha = 0$, $r = \left(\frac{1+\eta}{\beta} - 1\right)$ and (43) implies $u_c(c(H), H) = u_c(c(L), L)$. In particular, as α increases (and r decreases), aggregate consumption is unchanged but $u_c(c(L), L)$ decreases relative to $u_c(c(H), H)$, decreasing expected utility from consumption. \square

In words, the first of the previous two propositions states that if $\alpha > \alpha^*$ (or that the stationary equilibrium involves only cash), then the actual value of α is irrelevant. Intuitively, if the reserve ratio is high enough to shut down banking, increasing it further has no effects. The second states that if $\alpha < \alpha^*$ (or that the stationary equilibrium involves only deposits), then increasing α *decreases* welfare. Intuitively, if the reserve ratio is low enough to allow banking, decreasing it further increases welfare (or at least that part of welfare which comes from consumption and not jewelry). This occurs because decreasing the reserve ratio increases the equilibrium interest rate on deposits. This allows better insurance across θ shocks. In particular, if $\alpha = 0$ (or, more carefully, as $\alpha \rightarrow 0$ from above to avoid an infinite price level) the equilibrium allocation has the interest rate on deposits equal the nominal interest rate in the credit market and thus mimics the non-cash-in-advance constrained allocation for the economy where each household's per period endowment is $y/(1+\phi^d)$. For higher

values of α (but still low enough to allow banking), each household conserves on deposits (since there is an opportunity cost to holding wealth in the form of deposits), distorting the household's consumption choice $c^d(\theta)$.

This logic then allows a further result: If $\eta = \beta - 1$, (or money growth is negative at the deflationary Friedman rule rate) then full insurance, $u_c(c(L), L) = u_c(c(H), H)$ with $\pi(L)c(L) + \pi(H)c(H) = y/(1 + \phi^m)$, is achieved in the cash equilibrium, which is guaranteed to occur since under this money growth rate $\alpha^* < 0$. If $\eta > \beta - 1$, or $v(\cdot) \neq 0$ (jewelry is valued), no welfare comparison can, in general, be made between the monetary equilibrium ($\alpha > \alpha^*$) and the various banking equilibria (one for each $\alpha < \alpha^*$). If one considers, as a thought experiment, moving α from just above α^* to just below, several countervailing forces on welfare occur. First, consumption falls from $y/(1 + \phi^m)$ on average per household to $y/(1 + \phi)^d$ on average per household. This hurts welfare. But second, the interest rate on the equilibrium means-of-payment rises from zero (the interest rate on cash) to $r = (1 - \alpha^*)((1 + \eta)/\beta - 1) > 0$ – a discontinuous jump as a function of α . This jump in the interest rate causes a discontinuous and welfare improving change in $c(\theta)$ for the reasons outlined in the previous paragraph. Finally, in the switch from the monetary equilibrium to the banking equilibrium, there is a discontinuous change (again as a function of α) in the price of gold relative to consumption. If jewelry is valued ($v(\cdot) \neq 0$), this causes a discontinuous change in the steady state level of gold held as jewelry. Intuitively, fractional reserve banking allows fewer units of gold to be used facilitating trade and more units to be used in a form which directly increases utility.

A. Access to the bank possible

Next consider when $\sigma(1) > 0$, or it is possible that a shopper has access to all her wealth after discovering the household's preference shock. The assumption that $\phi^b > \phi^d > \phi^m$ then delivers that households will spend the same amount of cash or deposits regardless of whether it can access its

wealth. If, for a particular θ realization, the household's means-of-payment in advance constraint is slack, it makes no transaction even when given the chance to do so ($c^b(\theta, 1) = 0$) and if instead the means-of-payment in advance constraint binds, the household "tops off" consumption by accessing its wealth.

Proposition 5. *In a cash equilibrium ($m > 0, d = 0$), for all θ , $c^m(\theta, 0) = c^m(\theta, 1)$. Further, if $p_c(1 + \phi^m)c^m(\theta, 1) < m$, then $c^b(\theta, 1) = 0$. Likewise, in a banking equilibrium, ($m = 0, d > 0$), for all θ , $c^d(\theta, 0) = c^d(\theta, 1)$ and if $p_c(1 + \phi^d)c^m(\theta, 1) < d$, then $c^b(\theta, 1) = 0$.*

Proof. First suppose a cash equilibrium and $p_c(1 + \phi^m)c^m(\theta, 1) < m$ which implies $\gamma^m(\theta, 1) = 0$. This contradicts the first order condition of the household problem with respect to $c^m(\theta, 1)$ (28) and the first order condition for $c^b(\theta, 1)$ assuming $c^b(\theta, 1) > 0$,

$$u_c(c(\theta, 1), \theta) = p_c(1 + \phi^b) \frac{\beta\mu}{1 + \eta}, \quad (45)$$

since $\phi^b > \phi^m$. Thus if $p_c(1 + \phi^m)c^m(\theta, 1) < m$ in a cash equilibrium, $c^b(\theta, 1) = 0$.

Still assuming a cash equilibrium, if $c^b(\theta, 1) > 0$, (28) and (45) imply $\gamma^m(\theta, 1) > 0$ and thus $c^m(\theta, 1) = \frac{m}{p_c(1 + \phi^m)}$ (from (13)). That $c^m(\theta, 0) \leq c^m(\theta, 1)$ (again from (13)), the first-order-conditions with respect to $c^m(\theta, 0)$, $c^m(\theta, 1)$ and the concavity of u imply $\gamma^m(\theta, 0) > \gamma^m(\theta, 1) > 0$. Thus $c^m(\theta, 0) = \frac{m}{p_c(1 + \phi^m)} = c^m(\theta, 1)$.

If $c^b(\theta, 1) = 0$, then condition (28) for both $z = 1$ and $z = 0$ implies if $\gamma^m(\theta, 0) = 0$ and $\gamma^m(\theta, 1) = 0$, then $c^m(\theta, 0) = c^m(\theta, 1)$. If $\gamma^m(\theta, 0) > 0$ and $\gamma^m(\theta, 1) > 0$, then (13) implies $c^m(\theta, 0) = c^m(\theta, 1) = \frac{m}{p_c(1 + \phi^m)}$. If $\gamma^m(\theta, 0) > 0$ and $\gamma^m(\theta, 1) = 0$ (and likewise if $\gamma^m(\theta, 0) = 0$ and $\gamma^m(\theta, 1) > 0$) then condition (28) for both $z = 1$ and $z = 0$ and the concavity of u imply $c^m(\theta, 0) < c^m(\theta, 1)$ (or likewise $c^m(\theta, 0) > c^m(\theta, 1)$) which, given the lesser one equals $\frac{m}{p_c(1 + \phi^m)}$ violates (13) and is thus a contradiction. Thus $c^m(\theta, 0) = c^m(\theta, 1)$ for all θ in all cash equilibria.

This exact argument, using the first order conditions of the household problem for $c^d(\theta, z)$ instead of $c^m(\theta, z)$ and using the fact that $\phi^d < \phi^b$ establishes that in any banking equilibrium, if $p_c(1 + \phi^d)c^m(\theta, 1) < d$, then $c^b(\theta, 1) = 0$, $c^d(\theta, 0) = c^d(\theta, 1)$ for all θ . \square

Next, we show that if ϕ^b is not too large and $\sigma(1) \rightarrow 1$, then the equilibrium approaches full insurance with an endowment of $y/(1 + \phi^b)$.

Proposition 6. *Consider a sequence of economies where $\sigma(1) \rightarrow 1$ and $\frac{\phi^b - \phi^m}{1 + \phi^m} < (\frac{1 + \eta}{\beta} - 1)$. Then, $c(L, 1) \rightarrow c^*(L)$ and $c(H, 1) \rightarrow c^*(H)$ where $u_c(c^*(L), L) = u_c(c^*(H), H)$ and $\pi(L)c^*(L) + \pi(H)c^*(H) = y/(1 + \phi^b)$.*

Proof. To be completed. \square

An implication of Proposition 6 is that if $v(\cdot) = 0$ (fiat money), ϕ^b is sufficiently close to ϕ^d , and $\sigma(1)$ is sufficiently close to zero, then welfare given $\alpha = 1$, or 100% reserves under a cash equilibrium approaches welfare given $\alpha = 0$ under a deposit equilibrium, even when runs on a deposit equilibrium are explicitly ruled out. It is this result which we interpret as implying that while fractional reserve banking may have once been desirable, if bank runs have significant costs, now, or in the near future, the benefits of fractional reserve banking may no longer justify its associated risk to society.

6. Conclusion:

We have developed a model in which privately produced money provides a socially useful insurance role and a privately useful but socially costly medium of exchange. We have shown that if reserve requirements are sufficiently low then private money drives out specie as a medium of exchange. We have further shown that if the assets used to back private money have no direct value in preferences or technology and the social value of insurance is sufficiently small, equilibria with fractional reserve banking are necessarily inefficient. Finally, we have shown that if the assets

used to back private money have no direct value in preferences or technology, then it is desirable to set reserve requirements either to zero or 100%. And if the technology for households allows more and more of a household's wealth to be accessible for transactions, the benefits to fractional reserve banking decrease to zero.

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