Evolution and Walrasian Behavior in Market Games*

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Abstract

We revisit the question of price formation in general equilibrium theory. We explore whether evolutionary forces lead to Walrasian equilibrium in the context of a market game, introduced by Shubik (1972). Market games have Pareto inferior (strict) Nash equilibria, in which some, and possibly all, markets are closed. We introduce a strong version of evolutionary stable strategies (*SESS*) for finite populations. Our concept requires stability against deviations by coalitions of agents. We show that a small coalition of trading agents is sufficient for Pareto improving trade to be generated. In addition, provided that agents lack market power, Nash equilibria corresponding to approximate Walrasian equilibria constitute the only approximate SESS.
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1 Introduction

Walrasian equilibrium is a cornerstone of modern economics. It is, therefore, not surprising that the question of price formation has received considerable attention in general equilibrium theory. The tâtonnement process has been used extensively in this context.[1] The study of tâtonnement, however, has produced largely negative results, and this has led some researchers to conclude that decentralized information about prices alone is not sufficient to bring the economy to the Walrasian equilibrium. In addition, and perhaps more importantly, the tâtonnement has been criticized for lacking micro foundations since the price adjustment process is not the outcome of the individual optimization.

Even if we put the traditional stability question aside, Walrasian equilibrium may be challenged on the basis of complexity considerations. Can "unsophisticated" agents learn to behave in such a way that an outside observer of the economy will see a Walrasian equilibrium allocation? Evolutionary game theory provides an appropriate framework to formulate this question. After all, competitive outcomes are often justified by appealing to the natural selection of behavior that is more "fit."[2] In this paper we explore whether evolutionary forces can lead to Walrasian equilibrium in the context of a market game, introduced by Shubik (1972).[3] Our story is not explicitly dynamic. Rather, we demonstrate that certain outcomes can be disturbed by the introduction of a small number of "noise-traders," who can become better off in relative terms by choosing different trading patterns.

Market games are one of the structures that give rise to competitive outcomes when agents lack market power. Thus, it has served as a non-cooperative foundation for the Walrasian equilibrium. Even in large economies, however,

[1]See Arrow and Hurwicz (1959) for a classic reference.

[2]See Alchian (1950) for one of the first attempts to formalize this argument. See, for example, Weibull (1995) or Samuelson (1997) for a review of evolutionary models.

[3]There is extensive literature on market games. Standard references include Shapley (1977), Shapley and Shubik (1977), Dubey and Shubik (1977), and Mas-Colell (1982).

in addition to approximately Walrasian outcomes, market games obtain Pareto inferior (strict) Nash equilibria, in which some, and possibly all, markets are closed due to a coordination failure. Our study concerns a pure exchange economy with a finite number of agents and a finite number of goods. We study the case formalized by Postlewaite and Schmeidler (**PS,** 1978), where the number of agents is large. We introduce a strong version of evolutionary stable strategies (SESS) for asymmetric, finite games. Roughly speaking, SESS requires stability against all coalitions consisting of at most one agent per population.

We demonstrate that, in an approximate sense that we make precise, (partial) autarky outcomes are not SESS. A suitable small-size coalition can generate Pareto improving trade and open a market. Thus, evolutionary forces provide an avenue through which the economy can avoid situations where some markets are closed due to a coordination failure. In addition, feasible outcomes in which all markets are open, and in which non-Walrasian prices prevail in some markets, can also be disturbed through a small coalition. More precisely, we demonstrate that if the game is sufficiently large so that agents' market power is insignificant, Nash equilibria that support approximate Walrasian equilibria of the underlying economy are the only approximate SESS. Interestingly, the lack of market power is necessary for this to be true.[4] The reason for this is simple. As in Schaffer (1988), our concept emphasizes the role of relative performance: a successful strategy makes an agent better off relative to others of the same type. In a large economy agents lack market power and, therefore, their ability to affect the payoffs of others is small. In that case, the best strategy is to optimize by maximizing absolute performance.[5]

We can summarize the intuition behind our findings as follows. Since a single

[4]Our results are related to Dubey and Shubik (1978), who introduce an outside agency that ensures that arbitrarily small amounts of bids and asks are present in all markets. Our argument, however, does not rely on the existence of such an agency. In addition, we impose minimal rationality requirements on our agents, and we explicitly consider non-Nash outcomes.

[5]See Alchian (1950) for a discussion of the merits of maximizing relative performance by firms.

agent cannot create beneficial trade, Pareto inferior outcomes, in which some markets are closed, cannot be disturbed by one agent. On the other hand, the introduction of one trading agent *on each side of the market* is sufficient to open a market, thus leading the economy to a Pareto superior trading regime. All other non-Nash states that involve trade, but not individual optimization, can be disturbed by a single agent who chooses the best basket at the given prices. An important ingredient in our analysis is that the number of agents in the economy under study is much greater than the size of the deviating coalition. Consequently, while such coalitions can change certain agents' baskets, they only have a negligible effect on prices. As a result, no deviations by small-size coalitions can lead to improvements, in an approximate sense, if the economy is close to a Walrasian equilibrium. Thus, consistent with the traditionally held view, our findings provide support for the belief that evolutionary forces lead to competitive outcomes, but *only* when individual agents are of small size.[6]

The paper proceeds as follows. Section 2 reviews some concepts from evolutionary game theory, introduces our solution concept, and presents an example. In Section 3, we apply the solution concept to the market game and discuss the main result. A brief discussion section follows.

2 The Solution Concept

We start by stating two existing definitions of evolutionary stability in the context of an abstract normal form game. First, consider a single population consisting of a continuum of identical agents, and assume that N agents are selected to play a normal-form game $\Gamma = (N, S, U)$, where S is the set of available (pure) strategies, and U represents payoffs. The standard definition of an *Evolution-*

[6] It is worth noting that outcomes in which some markets are closed are not evolutionary stable even if the underlying economy is "small." However, it is only in sufficiently large economies that Walrasian outcomes are evolutionary stable. More precisely, the Nash equilibrium in which all markets are open may fail to be evolutionary stable if it does not correspond to an approximate Walrasian equilibrium. This is in contrast to some recent papers in the literature, notably Vega-Redondo (1997).

ary Stable Strategy (ESS) for $(N = 2)$-player symmetric games is as follows (see Weibull, 1995):

Definition 1 *A strategy $s \in \Delta$ is an* **ESS** *if, for every strategy $t \neq s$, there exists $\varepsilon_t > 0$ such that*

$$U\left(s, (1-\varepsilon) s + \varepsilon t\right) > U\left(t, (1-\varepsilon) s + \varepsilon t\right), \tag{1}$$

for all $\varepsilon \in (0, \varepsilon_t)$, where Δ is the set of all mixed strategies.

Next, consider any finite population of size N. The definition of ESS for N-player symmetric games is as follows (see Schaffer, 1988, 1989):

Definition 2 *A strategy $s \in S$ is an* **ESS** *if, for any strategy $t \neq s$,*

$$U\left(s, (t, \overline{s})\right) \geq U(t, (s, \overline{s})), \tag{2}$$

where $(t, \overline{s})$ and $(s, \overline{s})$ denote the strategies of the other $(n-1)$ players. In particular, $(s, \overline{s})$ indicates that all other players play strategy s, while $(t, \overline{s})$ indicates that one player plays strategy t, while all other players play s.

Note that, unlike Nash equilibrium, the ESS criterion refers to *relative*, as opposed to absolute, performance. We will amend Schaffer's (1988) definition in two ways. First, we extend the definition of an ESS from one finite population to multiple, distinct, finite populations. Second, we will require a strong version of evolutionary stability: one that requires stability against a simultaneous deviation by multiple agents from different populations.

We first present the concept in the context of an example. In the next section, we will apply it to a market game. Assume that there are $K > 1$ finite populations. Each population, i, contains $n_i \geq 2$ agents. We assume that agents play an N-player game, Γ, where $N = n_1 + ... + n_K$. The game is assumed to have the following symmetry property. All players from population i have the same set of strategies, X^i, and the same payoff function, U^i. In other words, if two players (from the same population) play the same strategy, they will obtain the same payoffs. Hence, we indicate the normal form game as

$$\Gamma = \left(\{n_1 + ... + n_K\}; \underbrace{S^1 \times ... \times S^1}_{n_1 \text{ times}} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K \text{ times}}; \left(U^1; ...; U^K\right) \right). \tag{3}$$

In what follows, we will need to consider the situation where one agent from population i plays strategy t^i, while every other agent from that population plays strategy s^i. More generally, in the case where at most one agent in each population plays a strategy, t, which is different from the one chosen by every other agent in his population, the payoff of the agent from population i who plays a different strategy than his peers can be written as:

$$U^i\left(t^i; (t^1, \overline{s^1}); ...; (t^i, \overline{s^i}); ...; (t^K, \overline{s^K})\right), \tag{4}$$

where, as before, $\left(t^i, \overline{s^i}\right)$ denotes that one agent from population i plays t^i, while all other agents from population i play s^i.

We are now ready to define our main concept.

Definition 3 *A symmetric strategy profile* $\overline{s} = \left(\underbrace{s^1, ..., s^1}_{n_1}; ...; \underbrace{s^K, ..., s^K}_{n_K}\right) \in \underbrace{S^1 \times ... \times S^1}_{n_1} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K}$ *is a* **Strong ESS (SESS)** *if, for all* i*,*

$$U^i\left(s^i; \gamma^1, ..., \gamma^K\right) \geq U^i\left(t^i; \gamma^1, ..., \gamma^K\right), \tag{5}$$

for any strategy $t^i \neq s^i$*, and for all* γ^j*, such that* $\gamma^j = \left(s^j, \overline{s^j}\right)$*, or* $\gamma^j = \left(t^j, \overline{s^j}\right)$.

In other words, a notable feature of the SESS is that it requires stability against up to K simultaneous deviations (one per population). Clearly, this is a stronger concept than Schaffer's ESS. Thus, SESS will not exist in general. An important feature of our concept is that, while it requires a symmetric outcome, it can be applied to asymmetric games. Below, we give an example of a four-player coordination-like game in which SESS uniquely selects the Pareto efficient Nash equilibrium even though there is another ESS.

Example: Suppose that there are two populations (I and II), each consisting of two players. Each player has two available actions (a and b). Let $\theta_{I(II)}$ stand for the number of a-players in population $I(II)$. Payoffs are defined as follows.

$$\begin{array}{cccc} \underline{U_I(a,\theta_I,\theta_{II})} & \underline{U_I(b,\theta_I,\theta_{II})} & \underline{U_{II}(a,\theta_I,\theta_{II})} & \underline{U_{II}(b,\theta_I,\theta_{II})} \\ U_I(a,1,0)=0 & U_I(b,0,0)=2 & U_{II}(a,0,1)=0 & U_{II}(b,0,0)=2 \\ U_I(a,2,0)=0 & U_I(b,1,0)=2 & U_{II}(a,0,2)=0 & U_{II}(b,0,1)=2 \\ U_I(a,1,1)=3 & U_I(b,0,1)=1 & U_{II}(a,1,1)=3 & U_{II}(b,1,0)=1 \\ U_I(a,2,1)=3 & U_I(b,1,1)=1 & U_{II}(a,1,2)=3 & U_{II}(b,1,1)=1 \\ U_I(a,1,2)=4 & U_I(b,0,2)=0 & U_{II}(a,2,1)=4 & U_{II}(b,2,0)=0 \\ U_I(a,2,2)=4 & U_I(b,1,2)=0 & U_{II}(a,2,2)=4 & U_{II}(b,2,1)=0 \end{array} \tag{6}$$

For example, $U_I(a,1,0)=0$ means that the payoff of the player in population I who plays action a, when all other players (one player in population I and two players in population II) play action b, is zero. Clearly, this coordination game obtains two symmetric strict Nash equilibria in which all agents play a and all play b, respectively. The a-equilibrium is an SESS. Notice, however, that the b-equilibrium is not an SESS since a coalition consisting of one agent per population (type) deviating to playing strategy a will result in a payoff of 3 for each of the two deviators (instead of 1 for the b-players).

Later, we shall make use of the following approximate notion of an SESS.

Definition 4 *A symmetric strategy profile* $\overline{s} = \left(\underbrace{s^1,...,s^1}_{n_1};...;\underbrace{s^K,...,s^K}_{n_K}\right) \in \underbrace{S^1 \times ... \times S^1}_{n_1} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K}$ *is an* ϵ-**SESS** *if, for all* i,

$$U^i\left(s^i;\gamma^1,...,\gamma^K\right) \geq U^i\left(t^i;\gamma^1,...,\gamma^K\right) - \epsilon, \tag{7}$$

for any $t^i \neq s^i$, *and for all* γ^j, *such that* $\gamma^j = \left(s^j, \overline{s^j}\right)$, *or* $\gamma^j = \left(t^j, \overline{s^j}\right)$.

Thus, an ϵ-SESS requires that no agent can be better off by more than a small amount, ϵ. In the next section we motivate and use ϵ-SESS in the context of our main topic of study, a strategic market game.

3 The Market Game

3.1 Preliminaries

We consider a finite, convex pure exchange economy with L consumption goods. The economy is described by $\mathcal{E} = \langle I, X^i, w^i, u^i \rangle_{i \in I}$, where $I = \{1, ..., N\}$ is a finite set of agents belonging to $K > 1$ different populations (or types); $X^i = \mathbb{R}^L_+$ denotes the consumption possibility set for agent i; $w^i \in \mathbb{R}^L_+$ is the endowment vector of agent i; and $u^i : \mathbb{R}^L_+ \to \mathbb{R}$ is the utility function of agent i. Agents belonging to the same type have identical preferences and endowments. We assume that u^i is continuous, strictly increasing in all its variables, and strictly quasi-concave.

Agents participate in a strategic market game related to the one in Shapley and Shubik (1977). We will follow **PS** in specifying the market game corresponding to $\mathcal{E}$.[7] An N-person market game in normal form is defined as follows. For each $i \in I$, let $S^i = \{s^i = (b^i, q^i) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ : q^i \leq w^i\}$ be the set of strategies of agent i. Given any (symmetric) N-list of strategies $(b^i, q^i)_{i \in I}$, the payoff to agent i is denoted by $U^i\left((b^1, q^1); ...; (b^i, q^i); ...(b^N, q^N)\right)$. Here, b^i denotes the vector of bids or "goods requested" by agent i, measured in abstract units of account (such as "dollars"), while q^i denotes the vector of goods offered by agent i. $U^i : S^1 \times .. \times S^N \to \mathbb{R}$ is the Von-Neuman and Morgenstern utility function of agent i.

Individual agents have to satisfy a *balance or bankruptcy condition,* which requires that the total value of an agent's bids has to be less than the total "receipts" from his sales of goods. More precisely, the individual balance condition is given by

$$\sum_{l \in L} b^i_l \leq \sum_{l \in L} \frac{q^i_l}{\sum_{j \in I} q^j_l} \sum_{j \in I} b^j_l. \tag{8}$$

[7] Our main argument will apply under alternative specifications of the market game provided that they allow for a Nash equilibrium of the game to be an approximately Walrasian equilibrium of the underlying economy.

One issue is what happens to agents who violate the balance condition. This is particularly important in our case for two reasons. First, unlike **PS**, we explicitly consider non-Nash states in which this constraint might be violated. Second, since agents in our model are concerned with *relative* performance, they might take an action that will make them worse off in absolute terms, if this will lead to other agents of their type becoming further worse off. This could occur, for example, if an action by a single agent would lead to other agents' becoming bankrupt. This possibility arises under the **PS** specification since they assume that agents who violate the balance condition have all their resources confiscated. With these considerations in mind, we impose the milder assumption that an agent whose total value of goods requested exceeds his total receipt value has his bid vector "shaved" by an amount that is proportional to his overbidding. More precisely, let

$$\alpha^i = \frac{\sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j}{\sum_{l \in L} b_l^i} \tag{9}$$

and let

$$\widetilde{b}_l^i = \begin{cases} \alpha^i b_l^i, & \text{if } \sum_{l \in L} b_l^i > \sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j \\ b_l^i, & \text{otherwise.} \end{cases} \tag{10}$$

The determination of the agents' resulting consumption baskets operates as follows. For all $i \in I$, and $l \in L$, let $c_l^i \in \mathbb{R}_+$ be the consumption of good l by agent i. This is determined by

$$c_l^i = w_l^i - q_l^i + \frac{\widetilde{b}_l^i}{\sum_{j \in I} \widetilde{b}_l^j} \sum_{j \in I} q_l^j. \tag{11}$$

As usual, a (symmetric) strategy profile $(\widehat{s}_1, ..., \widehat{s}_N)$ is a *Nash equilibrium* if for all $i \in I$ and all $s^i \in S^i$,

$$U^i(\widehat{s}_1, ..., \widehat{s}_i, ...\widehat{s}_N) \geq U^i(\widehat{s}_1, ..., s_i, ...\widehat{s}_N).$$

A Nash equilibrium is *full* if all markets are open; i.e., for all $l \in L$,

$$\sum_{i \in I} \widehat{b}_l^i > 0 \text{ and } \sum_{i \in I} \widehat{q}_l^i > 0.$$

We shall only consider economies in which a full Nash equilibrium exists, in which there is (sufficiently large) positive trade in all commodities.

Proceeding as in **PS**, for all l, and for a distinguished agent i, we can write $B_l = b_l^i + B_l^{-i} = b_l^i + \sum_{j\in I, j\neq i} b_l^j$, and $Q_l = q_l^i + Q_l^{-i} = q_l^i + \sum_{j\in I, j\neq i} q_l^j$. Let $p_l = B_l/Q_l$ denote the average price of commodity l (provided that the denominator of this expression is strictly positive). Define an allocation $\widehat{z}$ resulting from a full Nash equilibrium to be ϵ-Walrasian if all markets are open, and there exists $\widehat{p} = (\widehat{p}_1, ..., \widehat{p}_L)$ such that for all $i \in I$, $\widehat{p}\widehat{z}^i = \widehat{p}w^i$, and

$$\#\{i \in I : \forall z^i,\ u^i\left(z^i\right) > u^i\left(\widehat{z}^i\right) \Rightarrow \widehat{p}z^i > \widehat{p}(1-\epsilon)w^i\} > (1-\epsilon)\#I, \tag{12}$$

where, as stated above, prices correspond to ratios of aggregate bids. Next, we state the main result of **PS**. It establishes the connection between full Nash equilibria of the market game and approximate Walrasian equilibria of the underlying economy.

Proposition 1 *(***PS***):* *For any positive numbers α, β, and ϵ, any allocation resulting from a full Nash equilibrium in an economy $\mathcal{E} = \left\langle I, X^i, w^i, u^i \right\rangle_{i\in I}$ with $w^i < \beta(1, ...1)$ for all $i \in I$, $\sum_{i\in I} w^i > N\alpha(1, ..., 1)$, and $N > 16L\beta/\alpha\epsilon^2$ is ϵ-Walrasian.*

In what follows, we find it convenient to rewrite agent i's utility as a function of his own strategy and of the bid and offer profile by all other agents; i.e., $U^i = U^i\left(s^i; B^{-i}, Q^{-i}\right)$, where $B^{-i} = (B_1^{-i}, ..., B_L^{-i})$ and $Q^{-i} = (Q_1^{-i}, ..., Q_L^{-i})$. This completes the description of the market game. For a more detailed discussion of these concepts we refer the reader to **PS.** Henceforth, we will concentrate on the evolutionary stability properties of full Nash equilibria.

3.2 Evolutionary Stability

Before we analyze the market game from an evolutionary point of view, we introduce the main argument in an informal way. This will also serve as a motivating discussion for the concepts we introduced in the previous section. First, notice that no Nash equilibrium in which some markets are closed can be disturbed by a single deviating agent. This is because at least one agent *on each side of the market* is necessary for any trade. While the existence of such (partial) autarky Nash outcomes is plausible, it is also insightful to study under what conditions evolutionary forces will result in the "opening of markets," leading to a Pareto superior outcome. To our knowledge, ours is the first example to demonstrate that evolutionary pressure can lead to the opening of markets. The fact that this requires the simultaneous deviation from each side of the market is exactly what SESS is designed to capture.

A separate issue from whether all markets will be open is whether evolution will give rise to an efficient or, more restrictively, to a Walrasian outcome. Even if no state in which some or all markets are closed corresponds to an SESS, one might ask whether states that correspond to full Nash equilibria are SESS. Here, a difficulty arises. The fact that we deal with a finite game implies that each individual agent has some market power. Of course, this market power vanishes as the number of agents increases. This suggests that in the case where the economy is large enough, we can expect that the above question will be answered in the affirmative, but only in an approximate sense.

To see this, let us suppose that the economy is at a full Nash equilibrium. Suppose that an agent switches to a different bid/offer. Clearly, since the previous situation was a Nash equilibrium, the deviating agent will be worse off. However, this does *not* imply the evolutionary stability of full Nash equilibria. The reason is as follows. Since there is a finite number of agents, the deviation will result in slightly different prices for at least some agents. While the deviator is worse off under the new prices, it could be that other agents of his type are even more worse off or, in other words, the deviator could be better off *in relative terms*. Thus, the evolutionary stability of full Nash equilibria is not automatic. A continuity argument, however, guarantees that if the economy is large enough, a deviation by a small-size coalition cannot make the deviators better off by more than an arbitrarily small amount. Thus, a full Nash equilibrium of a large enough economy, which **PS** have shown to be approximately Walrasian, will also be an approximate SESS, provided that agents lack significant market power. Formalizing the details of this argument is the main purpose of this section. We begin by presenting a definition of ϵ-**SESS** in the context of a market game.

Definition 5 *A symmetric strategy profile* $\overline{s} = \left(\overline{s}^1, ..., \overline{s}^1; ..., \overline{s}^K, ..., \overline{s}^K\right) \in \underbrace{S^1 \times ... \times S^1}_{n_1} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K}$ *is an* ϵ-**SESS** *of the market game if, for all* $i \in I$ *and for all* $t^i \in S^i$,

$$U^i\left(\overline{s^i}; \widetilde{B}^{-i}, \widetilde{Q}^{-i}\right) \geq U^i\left(t^i; \widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i}\right) - \epsilon, \tag{13}$$

where $(\widetilde{B}^{-i}, \widetilde{Q}^{-i})$ *and* $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$ *are such that*

$$\begin{aligned}
\widetilde{B}_l^{-i} &= (n_i - 2)\, \overline{b}_l^i + \sum_{k=1,k\neq i}^{K} (n_k - 1)\, \overline{b}_l^k + \sum_{k=1}^{K} \widetilde{b}_l^k, \\
\widetilde{Q}_l^{-i} &= (n_i - 2)\, \overline{q}_l^i + \sum_{k=1,k\neq i}^{K} (n_k - 1)\, \overline{q}_l^k + \sum_{k=1}^{K} \widetilde{q}_l^k, \\
\widetilde{\widetilde{B}}^{-i} &= \sum_{k=1}^{K} (n_k - 1)\, \overline{b}_l^k + \sum_{k=1,k\neq i}^{K} \widetilde{b}_l^k, \\
\widetilde{\widetilde{Q}}^{-i} &= \sum_{k=1}^{K} (n_k - 1)\, \overline{q}_l^k + \sum_{k=1,k\neq i}^{K} \widetilde{q}_l^k.
\end{aligned} \tag{14}$$

As before, the above conditions require that a distinguished deviating agent is at most better off by ϵ relative to the other agents of his type when at most one agent per population deviates. The variables $(\widetilde{B}^{-i}, \widetilde{Q}^{-i})$ and $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$ give rise to the resulting prices before and after the deviation by our distinguished agent. The next result is stated analogously to the result in **PS**, but in the context of our evolutionary analysis.

Consider an economy $\mathcal{E}$ and suppose that $(\widehat{s}_1, ..., \widehat{s}_K)$ is a symmetric full Nash equilibrium profile. Let $\max_i \widehat{s}_j^i = \lambda_j > 0$ denote the largest bid/offer strategy in this equilibrium. We have the following.

Theorem 1 *Consider an economy $\mathcal{E}$ that, for any positive numbers ϵ, β, and λ, satisfies the following: (1) $(\lambda_1, ..., \lambda_L) > \lambda (1, ..., 1)$, (2) $w^i < \beta (1, ..., 1)$, and (3) there exists $\delta(\epsilon) > 0$ such that for any two populations i and j, $2(L-1) K \left(\frac{\beta^2}{\lambda_i \lambda_j}\right) \frac{N}{n_l n_j} < \delta(\epsilon)$. Then, the symmetric full Nash equilibrium profile $(\widehat{s}_1, ..., \widehat{s}_K)$ of the market game associated with $\mathcal{E}$ is ϵ-SESS.*

Before presenting the proof, we briefly discuss the nature of the conditions needed for the above Theorem. The first condition requires that the full Nash equilibrium involves a strictly positive amount of trade in all markets. The second condition is also used in **PS**. It assumes that individuals' endowments are "small." Finally, the third condition requires that the number of agents belonging to each type is sufficiently large.

Proof: Consider an economy $\mathcal{E}$. Fix $\epsilon > 0$, and suppose that $(\widehat{s}_1, ..., \widehat{s}_K)$ is a symmetric full Nash equilibrium profile of the associated market game. Consider a coalition of agents, $C \neq \emptyset$, consisting of at most one agent per type, who deviates from the full Nash equilibrium. Denote by t^i a deviation by an agent of type i. Let $(\widehat{B}^{-i}, \widehat{Q}^{-i})$ denote the bids and offers faced by an agent of type i in a full Nash equilibrium. Similarly, let $(\widetilde{B}^{-i}, \widetilde{Q}^{-i})$ and $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$ denote, respectively, the bids and offers faced by a non-deviant and by a deviant agent of type i after the deviation. Finally, $\widehat{s}^i$ and $\widetilde{\widetilde{s}}^i$ denote the best response for an agent of type i given prices $(\widehat{B}^{-i}, \widehat{Q}^{-i})$ (the bids and offers in the full Nash equilibrium) and $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$, respectively.

Since all markets are open, U^i is continuous in all arguments. Therefore, for all vectors (B^{-i}, Q^{-i}) such that $\left\|(B^{-i}, Q^{-i}) - (\widehat{B}^{-i}, \widehat{Q}^{-i})\right\| < \delta_1(\epsilon)$, we have that

$$\left|U^i\left(\widehat{s}^i; B^{-i}, Q^{-i}\right) - U^i\left(\widehat{s}^i; \widehat{B}^{-i}, \widehat{Q}^{-i}\right)\right| < \frac{\epsilon}{2}. \tag{15}$$

In addition, by the *Theorem of the Maximum* (see Berge (1997) p. 116), for all $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$ such that $\left\|(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i}) - (\widehat{B}^{-i}, \widehat{Q}^{-i})\right\| < \delta_2(\epsilon)$, we have that

$$\left|U^i\left(\widehat{s}^i; \widehat{B}^{-i}, \widehat{Q}^{-i}\right) - U^i(\widetilde{\widetilde{s}}^i; \widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})\right| < \frac{\epsilon}{2}. \tag{16}$$

Let $\delta(\epsilon) \leq \min\{\delta_1(\epsilon), \delta_2(\epsilon)\}$ be such that condition *(3)* of the Theorem holds. We want to show that the maximum possible influence that the deviating coalition will have on the terms of trade in the full Nash equilibrium is smaller than $\delta(\epsilon)$.

In this equilibrium, the "price" faced by agent i in the market for good j is

$$\widehat{p}_j^{\,i} = \frac{\widehat{B}_j^{-i}}{\widehat{Q}_j^{-i}}. \tag{17}$$

Thus, in the full Nash equilibrium, the deviation by coalition C can maximally increase this price by

$$\left(p_j^i\right)^+ = \frac{\widehat{B}_j^{-i-C} + B_j^C}{\widehat{Q}_j^{-i-C} + 0}, \tag{18}$$

where $B_j^C = \sum_{t\in C} B_j^t = \sum_{t\in C}\sum_{l\neq j} \frac{w_l^t}{\widehat{Q}_l^{-C} + \sum_{t\in C} w_l^t}\widehat{B}_l^{-C}$.

Similarly, at the full Nash equilibrium, the deviation by C can maximally decrease price $\widehat{p}_j^{\,i}$ by

$$\left(p_j^i\right)^- = \frac{\widehat{B}_j^{-i-C} + 0}{\widehat{Q}_j^{-i-C} + w_j^C}, \tag{19}$$

where $w_j^C = \sum_{t\in C} w_j^t$. We now have

$$\left|\left(p_j^i\right)^+ - \widehat{p}_j^i\right| = \left|\frac{\widehat{B}_j^{-i-C} + B_j^C}{\widehat{Q}_j^{-i-C}} - \frac{\widehat{B}_j^{-i-C} + \widehat{B}_j^C}{\widehat{Q}_j^{-i-C} + \widehat{Q}_j^C}\right| =$$

$$\left|\frac{\left(B_j^C - \widehat{B}_j^C\right)\widehat{Q}_j^{-i-C} + \left(\widehat{B}_j^{-i-C} + B_j^C\right)\widehat{Q}_j^C}{\widehat{Q}_j^{-i-C}\left(\widehat{Q}_j^{-i-C} + \widehat{Q}_j^C\right)}\right| =$$

$$\left|\frac{\left(B_j^C - \widehat{B}_j^C\right)}{\widehat{Q}_j^{-i}} + \frac{\widehat{Q}_j^C}{\widehat{Q}_j^{-i-C}}\frac{\left(\widehat{B}_j^{-i-C} + B_j^C\right)}{\left(\widehat{Q}_j^{-i-C} + \widehat{Q}_j^C\right)}\right| \leq$$

$$\left|\frac{\left(B_j^C - \widehat{B}_j^C\right)}{\widehat{Q}_j^{-i}}\right| + \frac{\widehat{Q}_j^C}{\widehat{Q}_j^{-i-C}}\left|\frac{\left(\widehat{B}_j^{-i-C} + B_j^C\right)}{\left(\widehat{Q}_j^{-i-C} + \widehat{Q}_j^C\right)}\right|. \tag{20}$$

Note that

$$\left|B_j^C - \widehat{B}_j^C\right| \leq \sum_{t\in C}\sum_{l\neq j} w_l^t \frac{N}{n_l}\frac{\beta}{\lambda_l}. \tag{21}$$

By condition *(1)* in the Theorem, we have

$$\widehat{Q}_j^{-i} \geq n_j\lambda_j. \tag{22}$$

In addition,

$$\widehat{Q}_j^C \leq Kw_j \leq K\beta. \tag{23}$$

Hence

$$\left|\frac{\left(B_j^C - \widehat{B}_j^C\right)}{\widehat{Q}_j^{-i}}\right| \leq \left|\frac{\sum_{t\in C}\sum_{l\neq j} w_l^t \frac{N}{n_l}\frac{\beta}{\lambda_l}}{n_j\lambda_j}\right| \leq$$

$$(L-1)\frac{\beta}{\lambda_l\lambda_j}\sum_{t\in C} w^t\left|\frac{N}{n_l n_j}\right| \leq \left[(L-1)K\frac{\beta^2}{\lambda_l\lambda_j}\right]\frac{N}{n_l n_j}.$$

By condition *(3)* in the Theorem,

$$\frac{1}{2}\delta(\epsilon) > (L-1)K\left(\frac{\beta}{\lambda_i\lambda_j}\right)^2\frac{N}{n_l n_j} \geq \left|\frac{\left(B_j^C - \widehat{B}_j^C\right)}{\widehat{Q}_j^{-i}}\right|. \tag{24}$$

Thus, we have

$$\frac{\widehat{Q}_j^C}{\widehat{Q}_j^{-i-C}}\left|\frac{\left(\widehat{B}_j^{-i-C} + B_j^C\right)}{\left(\widehat{Q}_j^{-i-C} + \widehat{Q}_j^C\right)}\right| \leq$$

$$\frac{\beta K}{(n_j-1)\lambda_j}\left|\frac{N\beta}{n_j\lambda_j}\right| = \left[K\frac{\beta^2}{\lambda_j\lambda_j}\right]\frac{N}{(n_j-1)\,n_j} \leq \frac{1}{2}\delta(\epsilon), \tag{25}$$

and we finally obtain

$$\left|\left(p_j^i\right)^+ - \widehat{p}_j^i\right| \leq \delta(\epsilon). \tag{26}$$

A similar argument establishes that

$$\left|\left(p_j^i\right)^- - \widehat{p}_j^i\right| \leq \delta(\epsilon). \tag{27}$$

Hence, we have demonstrated that the maximum possible influence that the deviating coalition can have on the terms of trade in the full Nash equilibrium is smaller than $\delta(\epsilon)$. Thus inequalities (15) and (16) hold, and we obtain

$$U^i\left(\widehat{s}^i;(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right) - U^i(t^i;(\widetilde{\widetilde{B}}^{-i},\widetilde{\widetilde{Q}}^{-i})) =$$

$$\left[U^i\left(\widehat{s}^i;(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right) - U^i\left(\widehat{s}^i;(\widehat{B}^{-i},\widehat{Q}^{-i})\right)\right] + \left[U^i\left(\widehat{s}^i;(\widehat{B}^{-i},\widehat{Q}^{-i})\right) - U^i(t^i;(\widetilde{\widetilde{B}}^{-i},\widetilde{\widetilde{Q}}^{-i}))\right]$$

$$\geq -\left|U^i\left(\widehat{s}^i;(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right) - U^i\left(\widehat{s}^i;(\widehat{B}^{-i},\widehat{Q}^{-i})\right)\right| +$$

$$\left[U^i\left(\widehat{s}^i;(\widehat{B}^{-i},\widehat{Q}^{-i})\right) - U^i(\widetilde{\widetilde{s}}^i;(\widetilde{\widetilde{B}}^{-i},\widetilde{\widetilde{Q}}^{-i}))\right] + \left[U^i(\widetilde{\widetilde{s}}^i;(\widetilde{\widetilde{B}}^{-i},\widetilde{\widetilde{Q}}^{-i})) - U^i(t^i;(\widetilde{\widetilde{B}}^{-i},\widetilde{\widetilde{Q}}^{-i}))\right]$$

$$\geq -\frac{\epsilon}{2} - \frac{\epsilon}{2} + 0 \geq -\epsilon, \tag{28}$$

where $t^i \in S^i$, for all i. In other words, a full Nash equilibrium profile is an ϵ-SESS. ■

A couple of remarks are in order. First, notice that the above proof uses the "large economy" assumption. We consider this to be an important feature of our model as it suggests that evolutionary arguments can be used as a foundation for Walrasian equilibria only when agents lack market power. We discuss this issue further below. We next provide a partial converse of Theorem 1. For any strategy profile $\left(t^1, ..., t^K\right)$, define $\max_i t^i_j = \lambda_j > 0$, for all goods j for which the corresponding market is open. We have the following.

Theorem 2 *Consider an economy $\mathcal{E}$ and any profile $\left(t^1, ..., t^K\right)$ that does not constitute a symmetric full Nash equilibrium of the underlying market game. Let β, λ, and δ_0 be positive numbers for which the following conditions hold: (1) $\lambda_j \geq \lambda$, for any good j for which the corresponding market is open, (2) $w^i \leq \beta\,(1, ..., 1)$, and (3) $2\,(L-1)\,K\left(\frac{\beta}{\lambda}\right)^2 \frac{N}{n_l n_j} < \delta_0$. Then, there exists $\epsilon_0 > 0$ such that the profile $\left(t^1, ..., t^K\right)$ is not an ϵ_0-SESS.*

Proof: Since the definition of ϵ-SESS involves symmetry, it is sufficient to consider only symmetric profiles. It is instructive to first demonstrate the result for the autarchy (no trade) Nash equilibrium. We will then move to the general case.

Consider the strategy profile $(0, ..., 0)$, associated with no trade. Since, by assumption, there exists a full Nash equilibrium, we can find a coalition, C, such that, by opening all markets, each member of the coalition obtains a payoff at least as great as that in the no-trade equilibrium, while at least one member of the coalition, say agent i, obtains a strictly higher payoff. Let $(B, Q) = (\widehat{0}, \widehat{0})$ denote the state where all agents follow a no-trade strategy, and $(\widetilde{B}, \widetilde{Q})$ denote the corresponding state after the deviation by C. We have argued that there exists $i \in C$ whose payoff when he follows trading strategy $\widetilde{s}^i$ satisfies

$$U^i(\widetilde{s}^i; \widetilde{B}^{-i}, \widetilde{Q}^{-i}) > U^i\left(0; \widehat{0}, \widehat{0}\right). \tag{29}$$

Let

$$\epsilon_0 = \frac{1}{2}\left[U^i(\widetilde{s}^i; \widetilde{B}^{-i}, \widetilde{Q}^{-i}) - U^i\left(0; \widehat{0}, \widehat{0}\right)\right] > 0. \tag{30}$$

Then

$$U^i(\widetilde{s}^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})-U^i\left(0;\widetilde{B}^{-i},\widetilde{Q}^{-i}\right)=$$

$$U^i(\widetilde{s}^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})-U^i\left(0;\widehat{0},\widehat{0}\right)>\epsilon_0. \tag{31}$$

This is because non-deviant agents do not trade even when coalition C opens all markets. Thus, the payoffs to all such agents remain the same as in no-trade equilibrium.

More generally, now, consider any profile $\left(t^1,...,t^K\right)\neq(0,...,0)$ that does not constitute a full Nash equilibrium. Then, there exists a coalition of agents (possibly a singleton), C, such that by opening a market or (if all markets open) by deviating, each member of C obtains a payoff at least as great as her original payoff, and there exists an agent i in C who obtains a strictly higher payoff than before. Let (B,Q) denote the original state and $(\widetilde{B},\widetilde{Q})$ denote the corresponding state after the deviation by C. The payoff to agent i that follows trading strategy s^i satisfies

$$U^i(s^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})>U^i\left(t^i;B^{-i},Q^{-i}\right). \tag{32}$$

Next, we need to demonstrate that payoffs to non-deviant agents can change only by a small amount.

Let

$$\epsilon_0=\frac{2}{3}\left[U^i(s^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})-U^i\left(t^i;B^{-i},Q^{-i}\right)\right]>0. \tag{33}$$

Clearly, the payoff to non-deviant agents can be affected only in those markets that are initially open. Suppose that there are $1\leq M\leq L$ such markets. Let $\|\bullet\|_M$ denote the norm of a vector of bids and offers restricted to the M initially open markets. The payoff function, U^i, is continuous in the bids and offers that take place in the M originally open markets. Therefore, for any player $i\notin C$, there exists a $\delta_0>0$ such that for all $(\widetilde{B}^{-i},\widetilde{Q}^{-i})$ such that

$\left\|(B^{-i},Q^{-i})-(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right\|_M<\delta_0$, we have

$$\left|U^i\left(t^i;B^{-i},Q^{-i}\right)-U^i\left(t^i;\widetilde{B}^{-i},\widetilde{Q}^{-i}\right)\right|<\frac{\epsilon_0}{2}. \tag{34}$$

Thus, for $\left\|(B^{-i},Q^{-i})-(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right\|_M<\delta_0$ we have

$$U^i\left(s^i;\widetilde{B}^{-i},\widetilde{Q}^{-i}\right)-U^i(t^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})=$$

$$U^i\left(s^i;\widetilde{B}^{-i},\widetilde{Q}^{-i}\right)-U^i(t^i;B^{-i},Q^{-i})+U^i(t^i;B^{-i},Q^{-i})-U^i(t^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})\geq$$

$$\left[U^i\left(s^i;\widetilde{B}^{-i},\widetilde{Q}^{-i}\right)-U^i(t^i;B^{-i},Q^{-i})\right]-\left|U^i(t^i;\widetilde{B}^{-i},\widetilde{Q}^{-i})-U^i(t^i;B^{-i},Q^{-i})\right|\geq$$

$$\frac{3}{2}\epsilon_0-\frac{1}{2}\epsilon_0>\epsilon_0. \tag{35}$$

In order to complete the proof, we need to bound the effects of a deviation by the coalition C. In other words, we need to demonstrate that indeed

$$\left\|(B^{-i},Q^{-i})-(\widetilde{B}^{-i},\widetilde{Q}^{-i})\right\|_M<\delta_0. \tag{36}$$

The argument proceeds in parallel to that in the proof of Theorem 1. First, consider all initially open markets. For such markets, j, define $\max_i t^i_j=\lambda_j>\lambda>0$. The "price" faced by agent i in market j is

$$p^i_j=\frac{B^{-i}_j}{Q^{-i}_j}. \tag{37}$$

At the original state $\left(t^1,...,t^K\right)$, the coalition C can maximally increase the price faced by agent i on market j by

$$\left(p^i_j\right)^+=\frac{B^{-i-C}_j+\widetilde{B}^C_j}{Q^{-i-C}_j+0}, \tag{38}$$

where $\widetilde{B}^C_j=\sum_{t\in C}\widetilde{B}^t_j=\sum_{t\in C}\sum_{l\neq j}\frac{w^t_l}{\widetilde{Q}^{-C}_l+\sum_{t\in C}w^t_l}B^{-C}_l$. Similarly, C can maximally decrease the above price by

$$\left(p^i_j\right)^-=\frac{B^{-i-C}_j+0}{Q^{-i-C}_j+w^C_j}, \tag{39}$$

where $w_j^C = \sum_{t\in C} w_j^t$. Thus,

$$\left|\left(p_j^i\right)^+ - p_j^i\right| = \left|\frac{B_j^{-i-C} + \widetilde{B}_j^C}{Q_j^{-i-C}} - \frac{B_j^{-i-C} + B_j^C}{Q_j^{-i-C} + Q_j^C}\right| \leq$$

$$\left|\frac{\left(\widetilde{B}_j^C - B_j^C\right)}{\left(Q_j^{-i-C} + Q_j^C\right)}\right| + \frac{Q_j^C}{Q_j^{-i-C}}\left|\frac{\left(B_j^{-i-C} + \widetilde{B}_j^C\right)}{\left(Q_j^{-i-C} + Q_j^C\right)}\right|. \tag{40}$$

Note that

$$\left|\widetilde{B}_j^C - B_j^C\right| \leq \sum_{t\in C}\sum_{l\neq j} w_l^t \frac{N}{n_l}\frac{\beta}{\lambda_l}.$$

By condition *(2)* in the statement of the Theorem,

$$Q_j^{-i} \geq n_j\lambda_j, \tag{41}$$

and since C involves at most one agent from each of the K types of consumers,

$$Q_j^C \leq Kw_j \leq K\beta. \tag{42}$$

Thus,

$$\left|\frac{\left(\widetilde{B}_j^C - B_j^C\right)}{\left(Q_j^{-i-C} + Q_j^C\right)}\right| \leq \left[(L-1)\,K\frac{\beta^2}{\lambda_l\lambda_j}\right]\frac{N}{n_l n_j} \leq \frac{1}{2}\delta_0.$$

Next, note that

$$\frac{Q_j^C}{Q_j^{-i-C}}\left|\frac{\left(B_j^{-i-C} + \widetilde{B}_j^C\right)}{\left(Q_j^{-i-C} + Q_j^C\right)}\right| \leq \left[K\frac{\beta^2}{\lambda_j\lambda_j}\right]\frac{N}{(n_j-1)\,n_j} \leq \frac{1}{2}\delta_0.$$

Hence,

$$\left|\left(p_j^i\right)^+ - \widehat{p}_j^i\right| \leq \delta_0. \tag{43}$$

By an analogous argument,

$$\left|\left(p_j^i\right)^- - \widehat{p}_j^i\right| \leq \delta_0. \tag{44}$$

Therefore, the profile $\left(t^1, ..., t^K\right)$ is not an ϵ_0-SESS.■

Finally, the next Corollary connects our solution concept to Walrasian equilibrium. It follows directly from our Theorem 2 when we invoke the main Proposition in **PS.**

Corollary 1 *Consider an economy $\mathcal{E}$ that, for any positive numbers α, ϵ, β, and λ satisfies the following: (1) $(\lambda_1, ..., \lambda_L) > \lambda\,(1, ..., 1)$, (2) $w^i < \beta\,(1, ..., 1)$, (3) there exists $\delta(\epsilon) > 0$ such that for any two populations i and j, $2\,(L-1)\,K\left(\frac{\beta^2}{\lambda_i\lambda_j}\right)\frac{N}{n_l n_j} < \delta(\epsilon)$, (4) $\sum_{i\in I} w^i > N\alpha(1, ..., 1)$, and (5) $N > 16L\beta/\alpha\epsilon^2$. Then any allocation resulting from a strategy profile that constitutes an ϵ-SESS is ϵ-Walrasian.*

Conditions *(1)-(3)* are needed for both Theorems 1 and 2. Conditions *(4)* and *(5)* are used in **PS** and *further* strengthen the first three. They jointly require that the number of agents in the economy are sufficiently large and that the aggregate endowment vectors are sufficiently small.

As we mentioned before, the above results will *not* hold in general if the economy is populated by a small number of agents. In that case, by having a non-negligible effect on prices, an agent deviating from the full Nash equilibrium allocation may be able to make himself better off relative to the other agents of his type. Therefore, full Nash equilibria may not correspond to ϵ-SESS if agents have significant market power. While this observation is consistent with the traditionally held view that competitive outcomes arise when individual agents are of insignificant size, it is distinct from Vega-Redondo (1997), in which a competitive outcome is shown to be evolutionary stable in the context of a Cournot oligopoly model where agents have significant market power. This suggests that whether a partial or a general equilibrium framework is assumed matters when determining the evolutionary stability of Walrasian outcomes.

4 Discussion

We studied the evolutionary stability of full Nash equilibria in the context of strategic market games. We introduced a strong version of evolutionary stable strategies, SESS, for asymmetric games played by finite populations. SESS requires stability against coalitions consisting of multiple agents. The introduction of a small number of "mutants" is sufficient for Pareto improving trade to be generated. Thus, Pareto inferior strict Nash equilibria where some or all markets are closed due to a coordination failure, as well as all non-Nash outcomes, do not constitute SESS. Provided that agents lack market power, full Nash equilibrium outcomes were shown to be the only ϵ-SESS. While our specification of the market game closely follows the one in **PS**, we believe that our analysis holds under alternative specifications. One extension that we are currently pursuing concerns studying *replicas* of an arbitrary pure exchange economy and investigating the relation between the corresponding ϵ-SESS and Walrasian equilibria.[8]

Throughout the paper we required stability against coalitions consisting of K agents (one per population). One could ask whether our results would be different if we required stability against *any* coalitions of size K (possibly several per population). Indeed, our results would hold under this more general specification, and we adopted the more restrictive notion for notational convenience. The reason is as follows. Clearly, any outcome that does not satisfy our notion of stability would not satisfy the more general notion. In addition, full Nash equilibria will satisfy the more general notion since, provided that the economy is sufficiently large, there is no coalition consisting of K agents that will have an appreciable effect on the price vector. Thus, the approximate evolutionary stability of full Nash equilibria will remain intact under the more

[8] The notion of a replica economy goes back to Edgeworth (1881) and Cournot (1897).

general specification.[9]

Peck and Shell (1990) and Ghosal and Morelli (2004) study variations of market games in which competitive outcomes prevail even when the number of traders is small. It would be interesting to study whether our evolutionary story can be embedded in their setups. An important extension of our analysis concerns the relation between our static SESS concept and the asymptotically stable points of a suitably defined dynamic system describing the learning process. Such a dynamic system must be able to distinguish between Walrasian outcomes and other strict Nash equilibria involving (partial) autarky outcomes. This extension is left to future research. Finally, an advantage of the proposed setup is that it is simple enough to be implemented in an experimental environment. In future work, we plan to study under what specifications of preferences and endowments human subjects will exhibit behavior consistent with SESS in a laboratory environment.

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[9]Clearly, the two notions will not be equivalent in all games. Our concept could be used in biological examples in which simultaneous mutations, say by a male and a female, might be needed in order to increase population fitness. See Noldeke and Samuelson (2003) and references therein for related examples in biology.

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