Foreign Debt and Ricardian Equivalence

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Abstract

This paper establishes a connection between Bulow and Rogoff's "no sovereign lending" result and Ricardian equivalence. When a government strictly prefers debt financing to tax financing, an endogenous cost of default arises, prompting the government to repay. More precisely, a government which does not have enough tools to reach the first best (in which Ricardian equivalence holds) through taxes and transfers, is also unable to redistribute precisely the gains from defaulting, and therefore domestic net losses appear in the economy, making foreign debt repayment sustainable.

Keywords: Sovereign debt, Ricardian equivalence, bubbles.
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I Introduction

Countries have to raise taxes or cut expenditures to repay their debts. Thus, a country might be tempted to default *ex post* on its commitment abroad so as to avoid the associated taxes or cuts in expenditures. When the country's debt is also held domestically, the decision to repay or default on outstanding debt also affects the distribution of wealth among domestic agents. Therefore, *ex ante*, debt sustainability hinges on agents' anticipation of the country's future fiscal policies. By focusing on a representative agent formulation, the literature on sovereign debt and default (cf. Eaton and Gersovitz, 1981, Bulow and Rogoff, 1989, among others) has mostly abstracted from the connection between external default incentives and domestic fiscal policies..

In this paper, I consider a general setting with heterogenous agents where a country's debt is traded by both domestic and foreign agents. In the absence of external sanctions or reputation costs, a country may repay its debt to avoid the redistributive effects of defaults on domestic bond holdings. I show that, when the domestic economy is Ricardian (as in Barro, 1974), the country is be better off defaulting on its commitment, and no external debt level is sustainable. Conversely, when the country's debt is net wealth for domestic residents, the government may be better off honoring its commitment.

I consider (Section III) a small open endowment economy where a government can finance expenditures either by taxing lump-sum domestic residents facing idiosyncratic shocks or by borrowing from them and from foreign investors. If the government defaults, the whole country is only excluded from future international *borrowing* but it can still lend abroad as in Bulow and Rogoff (1989). Finally, I assume that the country's preferences are increasing in each domestic residents' consumption, while keeping the other residents' consumption constant.

I show (Section IV) that Bulow and Rogoff (1989)'s no-borrowing result extends to every economy where Ricardian equivalence holds (Theorem 1). With a sufficiently large set of tax instruments (or sufficiently available insurance contracts), the country is able to perfectly redistribute the gains from defaulting. As a result, in the absence of external costs of default, the country has then no incentives to repay. Conversely, I show that credible repayment on strictly positive foreign-owned government's debt is feasible if debt funding is strictly preferred by the government over tax funding. I call these economies *debt-oriented non-Ricardian economies*. In the absence of external costs of default, a preference for debt is sufficient to explain foreign creditors' repayment (Theorem 2)[1].

In Section V, I show that a sufficient condition for obtaining a preference for debt is that

[1] *A fortiori*, economies preferring tax-based funding are unable to sustain foreign borrowing.

the domestic economy be able to sustain unbacked public debt. The resulting connection between bubbles and international borrowing relies not only on the *possibility* of bubbles but also on their *desirability* in the sense of Diamond (1965) or Tirole (1985). By constrast with Hellwig and Lorenzoni (2009), the connection with bubbles considered in this paper emphasizes bubbles *inside* the country and not *outside*, in international capital markets. However, inside and outside bubbles share the same emergence conditions so that external costs of default emerge concomitantly with internal ones. To illustrate these results, I provide two examples of *debt-oriented non-Ricardian economies*. The first example is the overlapping generation model with public debt as in the seminal contribution of Diamond (1965). The second example is the Bewley-Aiyagari model, where domestic households can save in public bonds[2]. Conversely, models with distortionary taxationfinst are not debt-oriented non-Ricardian as public debt is not net wealth in these models.

In the end, the connection between Ricardian equivalence and the internal cost of default's theory suggests that the quantitative assessment of these costs should not only rely on sectoral approaches[3] but should look at the aggregate Ricardian properties of an economy. In terms of theoretical contribution, the connection between sovereign borrowing and Ricardian equivalence suggests that the trade-offs experienced in sovereign defaults should be studied from a global perspective with other elements of fiscal policy design (e.g. distortionary versus lump-sum taxes as in Werning, 2007).

The rest of the paper is organized as follows: Section II provides a two-period example illustrating how ability to tax affects the country's willingness to repay. Section III introduces the general environnement and shows how to map the domestic allocations on government's choices using a preference relation. Section IV states the two main results on internal costs of default and Section V extends the approach to general equilibrium and gives examples of debt-oriented non-Ricardian economies.

Related literature Several papers challenged Bulow and Rogoff's result by introducing features which temper saving incentives. These features alter the basic assumptions of Bulow and Rogoff's result: inability to commit to save (Gul and Pesendorfer, 2004, Amador, 2008), foreign lenders (Cole and Kehoe, 1995, Hellwig and Lorenzoni, 2009) or reputation spillovers (Cole and Kehoe, 1998, among others). The internal cost of default theory has been theo-

[2] As in Aiyagari and McGrattan (1998) for example. Further examples may include economies with liquidity needs *à la* Woodford (1990) or Holmstrom and Tirole (1998) where future possible reinvestment requires transferring wealth.

[3] As in Brutti (2011) for firms' liquidity needs or in Gennaioli et al. (2011) for banks.

retically investigated by Kremer and Mehta (2000), Guembel and Sussman (2009), Broner et al. (2010) or Mengus (2013). In the latter, I introduce an internal cost of default theory where Ricardian equivalence breaks down because of domestic agents' inability to pledge future investments' revenues.

Ricardian equivalence was formally introduced by Barro (1974) in overlapping generation models with dynastic altruism. Bernheim and Bagwell (1988) have extended the equivalence to more complex altruistic interlinkages among agents (cf. Seater, 1993, for a detailed discussion of the theoretical and empirical aspect of Ricardian equivalence).

Kumhof and Tanner (2005) or Krishnamurthy and Vissing-Jorgensen (2012), among others, have documented that public debt is preferred to any other asset that is privately issued and that there may not be enough public debt. As a result, bubbles on privately-issued assets appear only on top of unbacked public debt. Kraay and Ventura (2007) analyze the crowding out of the Dot-Com bubble by issuance of public debt and Krishnamurthy and Vissing-Jorgensen (2013) show that government debt is a substitute for private short-term debt.

My result shares similarities with the literature on bubbles. Sovereign debt and bubbles have been connected by Hellwig and Lorenzoni (2009) through interest rates: low interest rates that allow the emergence of bubbles also make debt repayment affordable. Here I emphasize another channel, through welfare. The positive welfare impact of bubbles has been studied by Scheinkman and Weiss (1986) or Santos and Woodford (1997) in the incomplete market model, by Farhi and Tirole (2012) in Woodford-style models, by Tirole (1985) in OLG models.

Fiscal policies in no commitment models have received much attention when taxes are distortionary (e.g). In this paper, I focus on lump-sum taxes that may also suffer from time-inconsistency as in Calvo and Obstfeld (1988).

This paper is also connected with the issue of public versus private international borrowing as studied by Jeske (2006) and Wright (2006), but this paper's focus is on public international borrowing. Notice that both Jeske (2006) and Wright (2006) use Ricardian models.

II A two-period two-generation example

This section illustrates the connection between tax instruments and willingness to repay in a two-period two-generation example, where, depending on the set of assumptions on government's transfers, Ricardian equivalence may hold or not. I show that debt cannot be sustained when the set of assumptions allows Ricardian equivalence to hold and when tax instruments are constrained, the country is able to sustain positive foreign-owned debt.

II.1 Setting

Consider a small open economy populated by a government and domestic households that face competitive and risk-neutral foreign investors. There is no uncertainty and time is discrete and indexed by $t \in \{0, 1\}$.

Domestic agents consist of two generations of households. The first one lives in period 0 and 1 and the second one lives only in period 1.

The first generation receives an endowment of y in period 0 and 0 afterwards and the second generation receives an endowment of y in period 1.

Foreign investors' pricing kernel is q^*.

Assets There are two assets traded in period 0. Domestic households and foreign investors can purchase a risk-less foreign store of value at a price q^* that pays 1 in period 1 with probability 1. Alternatively, they can purchase domestic public debt at a price q for a promised repayment of 1 in period 1. This debt repayment endogenous probability is denoted by $\delta \in \{0, 1\}$, where $\delta = 0$ denotes default and $\delta = 1$ denotes repayment.

The foreign investors' exposures to domestic public debt is denoted by B^*.

Domestic households Domestic households in the first generation have preferences over consumption sequences following:

$$U^O = u(c_0^O) + \beta u(c_1^O),$$

where u increasing and strictly concave.

They pay taxes and invest in domestic public bonds. I denote by T_0^O the taxes that they pay in period 0 and by T_1^O those paid in period 1. Finally, B^O denotes the amount of domestic bonds that they purchase in period 0.

The second generation has preferences over consumption sequences as follows:

$$U^Y = \beta u(c_1^Y),$$

and this generation pays taxes T_1^Y in period 1.

Programs The first generation consumption-saving problem is then: given expected taxes T_0^O and T_1^O:

$$\max_{B^O, c_0^O, c_1^O} u(c_0^O) + \beta u(c_1^O)$$
$$\text{s.t. } c_0^O + T_0^O + qB^O = y \text{ and } c_1^O + T_1^O = \delta B^O$$

The second generation's problem is: given taxes T_1^Y:

$$\max_{c_1^Y} \beta u(c_1^Y) \text{ s.t. } c_1^Y + T_1^Y = y$$

Government The government has to finance exogenous public expenditures: g_t in period t. It can do so either by imposing lump sum taxes on domestic residents: T_t^i, $i \in \{O, Y\}$ and $t \in \{0, 1\}$ or by borrowing both from domestic households and foreign investors.

However, the government funding tools are potentially constrained.

First, the government is unable to commit to repay its debt. I assume that it cannot default selectively on foreign-owned debt[4]. I also assume that there are no sanctions or any international enforcement tools.

Second, there are potential restrictions on tax instruments. I consider two cases in this example.

1. Full availability: taxes can be contingent on agents' types. This means that T_1^Y can be contingent on Y, T_0^O and T_1^O on O.

2. Restricted availability: taxes cannot be non-contingent on agents' types, implying that: $T_1^O = T_1^Y$.

Finally, government's preferences are increasing in each domestic household's consumption level.

Equilibrium An equilibrium is this economy is consumption levels (c_0^O, c_1^O and c_1^Y), domestic bond holdings (B^O) and foreign bond holdings (B^*), taxes (T_0^O, T_1^O and T_1^Y) and a repayment decision (δ) solving households' problems, the goverment's problem and so that markets clear.

[4]This assumes that the government is unable to discriminate among bondholders as, for example, in Guembel and Sussman (2009).

II.2 Government ex post repayment incentives

A default implies an internal redistribution within the domestic economy. The first generation losses due to bond holdings but gains from adjustments of taxes: $B^O - T_1^O + T_1^{d,O}$. Conversely, the second generation is unaffected except through taxes: $T_1^Y - T_1^{d,Y}$.

By adjusting taxes, the government can potentially compensate these two generations of households. This depends on its available tax instruments.

Ricardian case When the government has access to a full set of tax instruments, it can at least perfectly compensate all its domestic residents, e.g. by implementing:

$$T_1^{d,O} = T_1^O - B^O \text{ and } T_1^{d,Y} = T_1^Y.$$

When defaulting, whatever the level of foreign-owned debt, each domestic agent is always at least better off.

As a result, the government is better off always defaulting, and so, no sovereign debt is sustainable.

Non-Ricardian case Conversely, when taxes are restricted, each domestic agent is better off when the first generation of agents holding bonds is better off:

$$T_1^Y = T_1^{d,O} \leq T_1^O - B^O.$$

By plugging, the government's budget constraint in this inequality, I obtain that each generation is better off as soon as the levels of domestically-owned debt and foreign-owned debt satisfy:

$$B^O \leq B^*/2.$$

In the end, sovereign debt can be sustained and debt repayment incentives derive from *non-compensable* positive domestic holdings of debt. This contrasts with the insights of Guembel and Sussman (2009) or Broner et al. (2010), among others, who abstract from the possibility of domestic compensation through taxes.

II.3 Summary

This section illustrates that two elements are necessary and sufficient for sovereign debt repayment incentives. First, there is a need of domestic bondholdings associated with a lack of domestic private insurance. In this example, this lack of insurance derives from agents' inabilty to insure against the risk to be born in one generation. Second, tax instruments have to be restricted. Otherwise, the government can compensate its domestic residents. In what follows, I generalize these insights to a more general setting.

III The environment

In this section, I introduce a model featuring a government and domestic and foreign investors. The government raises taxes from domestic residents and issues debt to finance exogenous expenditures. However, it cannot commit in advance and will repay its debt only if it is in the country's interest to do so. The key element of the model is the measurability restrictions of taxes and asset payoffs with respect to aggregate and idiosyncratic shocks, which defines an economy as Ricardian or non-Ricardian.

III.1 Model

Consider an economy populated by a government, a continuum of domestic private agents normalized to 1: $\mathfrak{D} = [0,1]$, and foreign investors. Time is discrete and indexed by $t \in \{0, 1, ..., \bar{t}\}$, with $\bar{t} \leq \infty$.

Uncertainty For any date t, the economy can be affected by both aggregate shocks, denoted by z_t, and idiosyncratic shocks to agents, denoted by $h_t = \{h_{i,t}\}_{i \in \mathfrak{D}}$[5]. The vector $s_t = (z_t, h_t)$ summarizes these two components. The entire history of shocks at time t is denoted by: $s^t = \{s_0, s_1, ..., s_t\}$. A state s^τ with $\tau > t$ is said to follow s^t if $s^\tau = \{s^t, s_{t+1}, ...s_\tau\}$ and this is denoted by $s^\tau | s^t$. I define similarly h^t and z^t and so $s^t = (z^t, h^t)$.

The unconditional probability of state s^t is $\pi(s^t) > 0$ and $\pi(s_t|s^{t-1})$ is the conditional probability of state s_t knowing the realization of state s^{t-1}. I assume that $\pi(s_t|s^{t-1})$ can be decomposed as $\pi(s_{t+1}|s^t) = \lambda(z^{t+1}|z^t)\phi(h_{t+1}|z^{t+1}, h^t)$. The law of large number holds, so that $\pi(z^t, h_i^t)/\pi(z^t)$ stands for the fraction of agents $i \in \mathfrak{D}$ in aggregate state z^t that have drawn an history h_i^t.

Domestic households Each agent $i \in \mathfrak{D}$ receives a stream of endowments $\{y_i(s^\tau)\}_{s^\tau|s^0}$ and chooses a stream of consumption $\{c_i(s^\tau)\}_{s^\tau}$ so as to maximize utility

$$U_{s^t}\left(\{c_i(s^\tau)\}_{s^\tau|s^t}\right),$$

[5] This exogenous source of heterogeneity prevents using this paper's results on endogenous forms of heterogeneity as the one I consider in Mengus (2013).

where U_{s^t} is increasing in each $c_i(s^\tau)$. I assume that U_{s^t} depends only on current and future stream of consumption in possible states and that these preferences are time-consistent. More precisely, following Johnsen and Donaldson (1985), for all $i \in \mathfrak{D}$, there exists a continuous and monotone function g_i:

$$U_{s^t}(\{c_i(s^\tau)\}_{s^\tau|s^t}) = g_i\left(\{c_i(s^t)\}_{s^t}, \left[U_{s^{t+1}}\left(\{c_i(s^\tau)\}_{s^\tau|s^{t+1}}\right)\right]_{s^{t+1}|s^t}\right)$$

Remark. This formulation encompasses standard forms of utility function such as recursive utility functions. I do not provide further structure for agents to keep the approach as general as possible. In particular, the type of agents may correspond to *ex ante* heterogeneity (e.g. differences in endowment processes) or *ex post* heterogeneity (e.g. because of different histories of idiosyncratic shocks as in Aiyagari (1994)).

Assets and foreign investors There are two types assets available in the economy: one-period foreign assets and one-period domestic government bonds.

Agents face trading frictions that allow them to only purchase assets contingent on $\Xi(s^t)$ where Ξ is a projection of the realized state on what the agents can trade. For example, when $\Xi(s^t) = z^t$, agents can trade only assets contingent on aggregate states. Conversely, when $\Xi(s^t) = s^t$, the agents can trade assets contingent on both aggregate and idiosyncratic states, so that they can perfectly insured.

I denote by $q^*(s^t)$ the price of a foreign asset that pays 1 in state s^t and $q(s^t)$ the price of the domestic bonds that pays 1 in state s^t.

The date-t-1 price of the basket of foreign asset that pays 1 in aggregate state z^t is denoted by $q^*(z^t) = \sum \pi(h^t, z^t)/\pi(z^t)q^*(h^t, z^t)$ and, similarly, the basket of government bonds that pays 1 in aggregate state z^t is traded at $t-1$ at price $q(z^t) = \sum \pi(h^t, z^t)/\pi(z^t)q(h^t, z^t)$.

$B_i(s^t)$ denotes the government's promised repayment to domestic agent $i \in \mathfrak{D}$ in state s^t. $B_i(s^t)$ is measurable on $\{s^{t-1}, \Xi(s^t)\}$.

Foreign investors' aggregate holdings are denoted by: $B^*(z^t)$. The superscript $*$ refers to foreign agents in the rest of the paper. The whole stock of government's repayment promises in state z^t is $B(z^t) = B^*(z^t) + \int_i B_i(s^t)di$.

I restrict attention to centralized borrowing arrangements (cf. Jeske, 2006) where only foreign agents and the government can access international capital markets[6]. This leads to the following assumption:

[6]This paper's results are robust to assuming that private agents can access international markets as well, provided that they are excluded from international borrowing after the country's default and by focusing on equilibria where they use domestic debt to smooth consumption.

Assumption 1. *Foreign investors can invest either in domestic bonds or in foreign assets and domestic agents can only purchase domestic government contingent bonds.*

Government The government faces a stream of exogenous expenditures $\{g(z^t)\}_{z^t}$. To finance these expenditures, the government can raise taxes from domestic agents or it can borrow.

The government is benevolent, I denote by V its objective function. At each date t, V is increasing in domestic agents' consumption $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}$. I also assume that there exists a continuous and monotone function f:

$$V_{z^t}(\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}) = f\left(\{c_i(s^t)\}_{i\in\mathfrak{D},s^t}, \left[V_{z^{t+1}}\left(\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^{t+1}}\right)\right]_{z^{t+1}|z^t}\right).$$

This make government's preferences time-consistent.

Taxes Domestic agents ($i \in \mathfrak{D}$) pay lump-sum taxes to and receive lump-sum transfers from the government. Let $T_i(s^t)$ denotes the deterministic net lump-sum tax paid by agent i in state s^t. I impose the following measurability constraint:

$$\forall s^t, \forall i \in \mathfrak{D},\ T_i(s^t) = T_i(\Gamma(z^t, h^t)). \tag{1}$$

The function Γ is then a projection of realized states on what the government can observe (and tax). The Ricardian properties of the economy depend on the form of Γ. For example, $\Gamma(z^t, h^t) = z^t$ means that the government can condition individual taxes only on the aggregate state and $\Gamma(z^t, h^t) = \{z^t, h^t\}$ means that the government can condition individual taxes on both aggregate and idiosyncratic states.

I assume that the government cannot tax more than agents' total endowment: $\sum_i T_i(s^t) \leq \sum_i y_i(s^t)$, and so, taxes are bounded.

Commitment assumptions The government cannot commit to honor its debt. I assume that the decision to default is measurable on aggregate state z^t and I denote by $\delta(z^t) \in \{0, 1\}$ the discrete decision variable associated with the repayment decision in state z^t for z^t-contingent securities. This decision variable equals 1 when the government decides to honor its debt and 0 otherwise.

I assume that the government can only default wholesale and so cannot selectively default on foreign-owned debt.

In addition, I assume that the government cannot commit on future taxes. $\{T_i(s^\tau)\}_{s^\tau|s^t}$ then denotes the *anticipated* flow of future net taxes after state s^t.

Remark. These two commitment assumptions are complementary. If the government were able to commit on taxes, it could rule out future defaults by committing to some ”crazy” tax paths after default, ensuring no default.

Tax schedules and budget constraints I denote by $\Theta(s^t)$ the set of bounded and time-consistent $\{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}$ satisfying the measurability constraint (1). Any tax vector $\{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}} \in \Theta(s^t)$ is said to be *admissible*.

The government budget constraint is then:

$$g(z^t) = \sum_{i\in\mathfrak{D}} T_i(s^t) + \sum_{z^{t+1}>z^t} q(z^{t+1})B(z^{t+1}) - \delta(z^t)B(z^t), \tag{2}$$

where $B(z^t)$ is the stock of debt satisfying:

$$\forall z^t,\ B(z^t) = \sum_{i\in\mathfrak{D}} B_i(z^t) + B^*(z^t).$$

Punishment scheme I follow Bulow and Rogoff (1989) in assuming that a defaulting country is excluded from future borrowing but not from future lending and that foreign investors cannot seize the country's assets abroad. This results in the following constraint:

Assumption 2. *When defaulting in state z^t, the foreign-owned debt satisfies:*

$$\forall \tau \geq t, \forall z^\tau|z^t, B^*(z^\tau) \leq 0. \tag{3}$$

In state z^t, if the government has already defaulted in a previous period or if it defaults in period t, I denote its objective function by $V^D(z^t)$. Otherwise, I denote government's objective by $V^R(z^t)$.

Conversely, I assume that domestic residents cannot punish their government after a default, and so, no restriction affects domestic holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D}}$ for states after default took place.

Finite endowment Finally, I make throughout the paper the following assumption on the country's endowment:

Assumption 3. *The economy's endowment is finite, i.e.*

$$\sum_{z^\tau|z^t} \pi(z^\tau|z^t)\frac{q^*(z^\tau)}{q^*(z^t)}\int_i y_i(s^t)di < \infty$$

Indeed, Hellwig and Lorenzoni (2009) show that, under further conditions, if interest rates are low enough, the discounted value becomes infinite, allowing for endogenous external costs of default[7].

By contrast, Assumptions 2 and 3 imply that the country faces no external cost of default and is therefore potentially willing to default on its debt.

Equilibrium An equilibrium in this economy is defined by a stream of domestic bond holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$ and domestic consumption $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$, a stream of foreign bond holdings $\{B^*(z^\tau)\}_{z^\tau|z^0}$, a stream of taxes $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$ and repayment decisions $\{\delta(z^\tau)\}_{z^\tau|z^0}$ solving the domestic households problem and the government problem at each state s^t and so that markets clear.

Summary of the timing Figure 1 summarizes the timing of the economy:

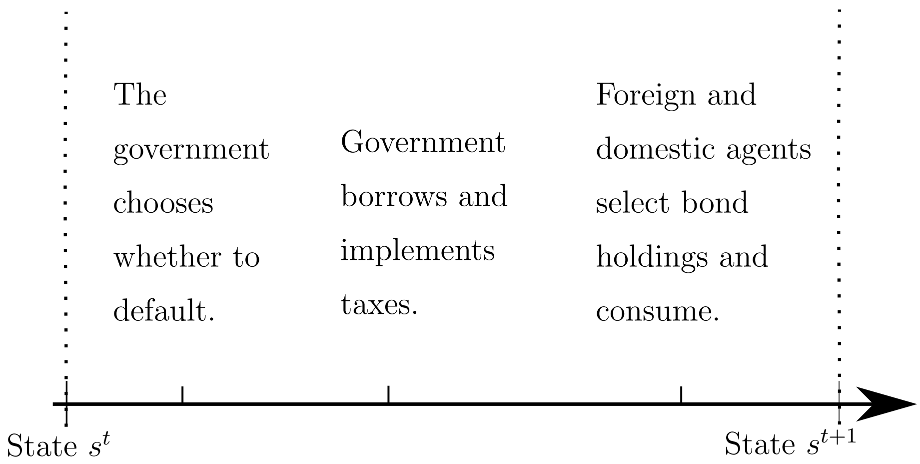

Figure 1: Date-t timing

III.2 Borrowing limits and default-free equilibria

This subsection defines borrowing limits.

Lemma 1 (Borrowing limit). *In each state z^t, there exists $\overline{B}^{*,R}(z^t) \geq 0$ such that for all $B^*(z^t) \leq \overline{B}^{*,R}(z^t)$, the country repays ($\delta(z^t) = 1$) and for $B^*(z^t) > \overline{B}^{*,R}(z^t)$, the country defaults ($\delta(z^t) = 0$). After defaulting, the borrowing limit is $\overline{B}^{*,D}(z^t) = 0$.*

Proof. See appendix. □

[7]Notice that this assumption resembles Cass (1972)'s criterion for dynamically efficient economies, which can be written, in this paper's framework as: an economy is dynamically inefficient if and only if $\sum_{z^\tau|z^t} \frac{\pi(z^\tau|z^t)q(z^t)}{q(z^\tau)} < \infty$.

Indeed, the set of $B^*(z^t)$ such that $\delta(z^t) = 1$ is not empty and contains at least 0. It thus admits an upper bound, which I denote $\overline{B}^{*,R}(z^t)$. The monotonicity property derives from the assumption that government's objective functions are increasing in consumption levels.

Similarly, Assumption 2 induces the borrowing limit for the defaulting country:

$$\forall z^t, \overline{B}^{*,D}(z^t) = 0.$$

In what follows, I focus on default-free equilibria (i.e. where, for all state z^t, $\delta(z^t) = 1$, and so, where $B^*(z^t) \leq \overline{B}^{*,R}(z^t)$)[8]. I build debt limits by backward induction: I determine current the debt limit, given future debt limits.

In those default-free equilibria, the domestic and the foreign asset prices are equal with each other: $q(z^t) = q^*(z^t)$ for all state z^t. Indeed, *before default* foreign investors arbitrate between foreign and domestic assets:

$$q(z^t) = q^*(z^t)\delta(z^t), \text{ for all } z^t.$$

As a result, when the government is expected to honor its debt, we obtain that $q(z^t) = q^*(z^t)$, for all z^t.

III.3 The preference relation

To characterize the government's choices, this subsection introduces a preference relation over these choices. More precisely, I map government's preferences that are on allocations into a policy preference relation that encompasses the level of foreign-owned debt, the distribution of domestic bond holdings and the path of expected taxes. This preference relation then allows me to analyze the Ricardian properties of the domestic economy as well as the willingness to default.

Domestic households In this paragraph, I show that domestic households' stream of consumption only depends on current debt holdings and tax schedules.

The program solved by agent $i \in \mathfrak{D}$ in state s^t is:

Problem 1 (Domestic agents)**.** *Given anticipated taxes $\{T_i(s^\tau)\}_{s^\tau|s^t}$ and initial portfolio $B_i(s^t)$,*

$$\max_{\{B_i(z^\tau)\}_{s^\tau|s^t}} U_i\left(\{c_i(s^\tau)\}_{s^\tau|s^t}, s^t\right)$$
$$s.t\ \forall s^\tau|s^t, c_i(s^\tau) = y_i(s^\tau) + B_i(z^\tau) - \sum_{z^{\tau+1}|z^\tau} q(z^{\tau+1})B_i(z^{\tau+1}) - T_i(s^\tau)$$

[8] I show in the appendix that there is no loss of generality to consider only default-free equilibria.

The solution to this program yields a function Ψ_i such that

$$\Psi_i \left[B_i(s^t), \{T_i(s^\tau)\}_{s^\tau|s^t}\right] = \{c_i(s^\tau)\}_{s^\tau|s^t}.$$

Considering all agents in $\mathfrak{D}$ yields a function $\Psi = \times_{i\in\mathfrak{D}}\Psi_i$ such that

$$\Psi \left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right] = \{c_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}.$$

The government The government's objective function is $V\left(\{c_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right)$. I now look at whether we can map consumption streams to fiscal variables.

First, using the function Ψ, we can define an indirect objective function W:

$$\begin{aligned} V\left(\{c_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right) &= V\left(\Psi\left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right]\right) \\ &\equiv W\left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right] \end{aligned}$$

Second, given initial foreign holdings $B^*(z^t)$, for any tax schedules and initial holdings, we can back out a sequence of future levels of external debt $\{B^*(z^\tau)\}_{z^\tau|z^t}$ so that government's budget constraints from state z^t onwards can are satisfied. Indeed, a tax schedule and initial holdings yield a path of future domestic holdings. Using the government's budget constraints, we can obtain the sequence of foreign debt as a residual:

$$\begin{aligned} &\forall z^\tau, \tau > t, B^*(z^\tau) - \sum_{z^{\tau+1}|z^\tau} q^*(z^{\tau+1})B^*(z^{\tau+1}) = \\ &\sum_{z^{\tau+1}|z^\tau} q^*(z^{\tau+1}) \sum_{i\in\mathfrak{D}} B_i(z^{\tau+1}) - \sum_{i\in\mathfrak{D}} B_i(z^\tau) + \sum_{i\in\mathfrak{D}} T_i(s^\tau) - g(z^\tau) \end{aligned}$$

The required values for the sequence of B^* do not necessarily satisfy future debt limits:

Definition 1 (Self-enforceability). *The triplet* $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right]$ *is self-enforceable when the sequence of required future borrowing satisfies future debt limits:*

$$\forall z^\tau|z^t, B^*(z^\tau) \leq \overline{B}^*(z^\tau).$$

Finally, for any given level of foreign debt and for any given distribution of domestic bond holdings $(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}})$, I denote by $\Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ the set of feasible tax schedules $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Theta(s^t)$ such that the triplet $\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right)$ is self-enforceable. This defines a correspondence Λ.

In the end:

Lemma 2. *V^R and V^D are functions of foreign and domestic holdings of debt and tax schedules:*

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}, s^t\right],$$

where

- $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right]$ *is sustainable:*

$$\{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right).$$

- $V^R\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right] = W\left(\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right).$

and similarly for V^D.

The preference relation Using these elements, I can map government's choices into a preference relation on fiscal variables as, according to Lemma 2, for each of these fiscal variables

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right],$$

there exists a real number $V^R\left(\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right]\right)$.

Using the usual order on $\mathbb{R}$, I can introduce a preference relation:

Definition 2. *Let* $\succeq$ *be a preference relation such that*

$$\left[B_1^*(z^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^T)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right] \succeq \left[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right],$$

the government weakly prefers the left-hand term to the right-hand term. $\succ$ *indicates strict preference and* $\approx$ *indifference.*

General properties of the relation As the preference relation is related to V^R, it inherits the properties of the usual order on $\mathbb{R}$, i.e. completeness, reflexivity, antisymmetry and transitivity (cf. Appendix for the definition of these properties). Thus $\succeq$ is a complete order on fiscal variables.

III.4 Default decisions

When defaulting, the country's value function is V^D and debt limits satisfy Bulow and Rogoff (1989)'s punishment scheme. Furthermore, foreign debt holdings is reset at 0 and domestic holdings as well ($\{0\}$). In other words, a default is a combination of a selective default on foreign-owned debt and a selective default on domestically-owned debt.

Lemma 3. *Given a level of external debt* $B^*(z^t)$ *and a distribution of domestic holdings* $\{B_i(s^\tau)\}_{i\in\mathfrak{D}, s^\tau|s^t}$*, for all* $\{T_i(s^\tau)\}_{i\in\mathfrak{D}, s^\tau|s^t} \in \Lambda\left(B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$*, there exists a vector of taxes* $\{T_i'(s^t)\}_{i\in\mathfrak{D}, s^\tau|s^t} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right)$ *such that*

$$\left[0, 0, \{T_i'(s^t)\}_{i\in\mathfrak{D}, s^\tau|s^t}\right] \succeq \left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \tag{4}$$

if and only if the country is better off defaulting.

Proof. See Appendix □

Thus, for expressing the willingness to repay or to default, it is sufficient to compare V^R evaluated at some positive levels of domestic and foreign debt with V^R evaluated when the country has no current debts. For example, the country prefers to repay when:

Selective default No domestic cost arises when the government can selectively default on its foreign-owned debt[9]. Then sovereign repayment hinges on the presence of external costs of default, i.e. costs deriving from the foreign investors' punishment. However, when the present value of country's endowment is finite, such external costs of default do not emerge, leading to the following restatement of Bulow and Rogoff's no-sovereign-borrowing result:

Proposition 1 (Selective default)**.** *For all level of external debt $B^*(z^t) \geq 0$, for all distribution of domestic holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}$ and for all anticipated tax schedule $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Lambda\left(B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$, there exists a vector of anticipated taxes $\{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Lambda\left(0, \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ such that*

$$V^D\left[0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t}\right] \geq V^R\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \tag{5}$$

with equality if and only if $B^*(z^t) = 0$.

Proof. See Appendix. □

The proof closely follows Bulow and Rogoff's arbitrage argument, as I only need to show that when debt is positive, the government can default and engage in a sequence of investments abroad at price $q^*(s^t)$ and satisfying Assumption 2 on the punishment's scheme. A key assumption for this result is that domestic behaviors are unaffected by the lower taxes resulting from government's savings.

III.5 Ricardian and non-Ricardian economies

In this subsection, I define a Ricardian economy in this paper's context and I also define some deviations from Ricardian equivalence.

Ricardian economies In this paragraph, I introduce definitions of Ricardian economies. Informally, a Ricardian economy is an economy where the government can alter the path of taxes without any constraint (cf. Barro, 1974, Seater, 1993). We may want to extend Barro's

[9]In the absence of external costs, if a country were able to do so, it would always default selectively on its foreign-owned debt, and so no domestic default would ever occur.

definition to account for transfers (deriving from insurance or redistribution motives), and so, in this case, a Ricardian economy is an economy where, after transfers, the government is indifferent among changing tax paths. Formally:

Definition 3 (Ricardian economy)**.** *For any tax schedule* $\{T^1(z^\tau)\}_{z^\tau|z^t}$ *and* $\{T^2(z^\tau)\}_{z^\tau|z^t}$*, an economy is Ricardian if there exists a feasible tax schedule* $\{T_i(s^\tau)\}_{s^\tau,i\in\mathfrak{D}}$ *satisfying*

$$\forall z^\tau, \int_i T_i'(s^\tau)di = 0$$

such that the government is indifferent between $\{T_i(s^\tau)+T^1(z^\tau)\}_{s^\tau,i\in\mathfrak{D}}$ *and* $\{T_i(s^\tau)+T^2(z^\tau)\}_{s^\tau,i\in\mathfrak{D}}$*.*

This definition allows to consider multiple agents. In a representative agent economy, the vector $\{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}$ equals $\{0\}$ and the government is indifferent between any tax schedule $\{T(z^\tau)\}_{z^\tau}$, and so we recover Barro's definition.

In particular, notice that if an economy is Ricardian, there exists $\{T_i'(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}$ satisfying

$$\forall z^\tau, \int_i T_i'(s^\tau)di = 0.$$

Indeed, $T_i'(s^\tau = T_i(s^\tau) - \int_i T_i'(s^\tau)di$. By construction, we have that $\forall z^\tau, \int_i T_i'(s^\tau)di = 0$. In other words, the government is indifferent between any schedules of aggregate taxes as soon as there exists a vector of taxes and transfers redistributing resources among agents.

Ex post Ricardian economy As the government's repayment decision takes place *after* private portfolios have been selected, I need to define a form of *ex post* Ricardian equivalence implying taxes but also current debt holdings. When changing portfolios and tax schedules, government's indifference between taxes and debt held by the domestic sector can be written formally as:

Definition 4 (Ex post Ricardian economy)**.** *An economy is Ex post Ricardian if and only if for any given level of foreign debt* $B^*(z^t)$*, any level of domestic debt* $\{B_i(s^t)\}$*, for any change in debt* $\{\Delta B_i\}$*, for all tax schedules* $\{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ *and* $\{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t) - \Delta B_i(s^t)\}_{i\in\mathfrak{D}}\right)$

$$\left[B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \approx \left[B^*(s^t), \{B_i(s^t) - \Delta B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D}}\right] \quad (6)$$

A Ricardian economy is also ex post Ricardian. When the government is *ex ante* indifferent between tax schedules, it is also indifferent *ex post*. But more generally, ex post Ricardian economies also include economies where the government has enough (fiscal) tools to offset the frictions preventing Ricardian equivalence to hold.

Ex post non-Ricardian economies I also define a specific set of *ex post* non-Ricardian economies, i.e. economies where the government is not indifferent between issuing debt and raising taxes. Here I focus on a particular subclass of non-Ricardian economies, those where debt is preferred to taxes:

Definition 5 (Debt-oriented non-Ricardian economy). *Given external debt $B^*(z^t)$, a debt-oriented non-Ricardian economy is such that, for any debt level $\{B_i\}_{i\in\mathfrak{D}}$,*

$$\forall\{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall\{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(s^t), \{0\}_{i\in\mathfrak{D}}\right), \quad (7)$$

$$\left[B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(s^t), \{0\}_{i\in\mathfrak{D}}, \left(\{T_i'(s^t)\}_{i\in\mathfrak{D}}\right)\right] \quad (8)$$

with strict inequality ($\succ$) at least for some positive value of $\{B_i(s^t)\}_{i\in\mathfrak{D}}$.

A debt-oriented non-Ricardian economy is an economy where debt is weakly preferred to taxes and strictly preferred for some values. Other non-Ricardian economies could be defined as tax-oriented economies or even any mixture between tax-oriented and debt-oriented non-Ricardian economies (i.e. when taxes and debt are alternatively preferred to each others).

Remark. So far, I have not considered distortionary taxes. It is a well-known result that, when taxes are distortionary, debt is used to smooth distortions over time (cf. Lucas and Stokey, 1983). Nevertheless, the tax smoothing motives for debt issuance does not prevent defaulting, as a default reduces the amount of tax to be raised and the corresponding distortionary cost (cf. Chari and Kehoe, 1993).

Measurability conditions and Ricardian properties Finally, the following lemma connects the Ricardian properties with the model assumption:

Lemma 4. *When one of these two conditions is satisfied:*

1. *Taxes can be perfectly targeted: $\Gamma(z^t, h^t) = (z^t, h^t)$,*

2. *Asset markets are complete: $\Xi(z^t, h^t) = (z^t, h^t)$.*

the economy is Ricardian.

When taxes and asset markets are measurable only on aggregate states

$$\Xi(z^t, h^t) = (z^t) \text{ and } \Gamma(z^t, h^t) = (z^t),$$

the economy is debt-oriented non-Ricardian.

When the government is perfectly able to observe agents' shocks it can perfectly compensate them. When asset markets are sufficiently rich, domestic agents can smooth perfectly future outcomes.

IV Sovereign debt and internal cost of default

In this section, I present the two main results of the paper: the extension of Bulow and Rogoff's result to domestic Ricardian economies and the characterization of the deviations from Ricardian equivalence required for foreign borrowing.

IV.1 Ricardian economies

The first theorem extends Bulow and Rogoff (1989)'s result in Ricardian economies:

Theorem 1 (Bulow and Rogoff)**.** *If an economy is Ex Post Ricardian, defaulting is weakly preferred, with strict preference if and only if $B^*(z^t) > 0$. In any state z^t, the debt limit satisfies $\overline{B}^*(z^t) = 0$.*

In particular, this holds for an economy where Ricardian equivalence is satisfied.

Proof. Suppose that the economy is Ex post Ricardian and suppose that $B^*(z^t) > 0$. The Ex post Ricardian property allows to write:

$$\forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),$$
$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \approx \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}}\right]$$

Besides, a selective default is always weakly preferred (cf. Proposition 1):

$$\exists \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right), \forall \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),$$
$$\left[0, 0, \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}}\right]$$

with equality if and only if $B^*(z^t) = 0$. Then:

$$\exists \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}, \forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right),$$
$$\left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right]$$

with equality if and only if $B^*(z^t) = 0$. □

When an economy is Ricardian, no internal frictions prevent the government from reducing domestic debt in exchange of lower taxes, making default non-costly. In the absence of external costs of default, the country is better off defaulting as soon as the level of external debt is strictly positive.

The general intuition behind this result is that the gains from default, i.e., here, the tax cuts due to the default, may be offset by the losses due to the default resulting from domestic holdings. At the level of an individual agent, the losses are exactly the difference between the direct losses through debt holdings and the gains of lower future taxes, or in other words, the net worth associated with government's bond holdings. Thus, the government's choice depends on this net worth, and, hence, on the Ricardian properties of the economy.

This result holds for a larger set of economies, i.e. all economies that are Ex post Ricardian. The Theorem's result does not require that the government has to be indifferent between *any* tax schedule. Indeed, it is sufficient that the government can offset domestic losses by transfers or tax cuts. Notice that, in this case, if the government has the power to smooth losses *ex post*, it can also implement transfers *ex ante*.

IV.2 Non-Ricardian economies

When does a country prefer to repay its debt? I establish now a necessary and sufficient condition in terms of deviation of Ricardian equivalence under which foreign-owned debt is honored.

First, let me make the following further assumption on the preference relation:

Assumption 4 (Local non-satiation). *For any $B^*(z^t) \geq 0$ such that*

$$\left[B^*(z^t), \{B_i^1(s^t)\}_{i\in\mathfrak{D}}, \{T_i^1(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[B^*(z^t), \{B_i^2(s^t)\}_{i\in\mathfrak{D}}, \{T_i^2(s^t)\}_{i\in\mathfrak{D}}\right] \tag{9}$$

there exists $\Delta B^* > 0$ such that:

$$\begin{aligned} &\forall B^* \in \left(B^*(z^t) - \Delta B^*(z^t), B^*(z^t) + \Delta B^*(z^t)\right), \\ &\left[B^*(z^t), \{B_i^1(s^t)\}_{i\in\mathfrak{D}}, \{T_i^1(s^t)\}_{i\in\mathfrak{D}}\right] > \left[B^*(z^t), \{B_i^2(s^t)\}_{i\in\mathfrak{D}}, \{T_i^2(s^t)\}_{i\in\mathfrak{D}}\right] \end{aligned} \tag{10}$$

A sufficient condition for Assumption 4 to hold is that V^R be a continuous function of each of its variable $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^T)\}_{s^T>s^t, i\in\mathfrak{D}}\right]$.

The following theorem establishes how Theorem 1's result evolves when an economy deviates from Ricardian equivalence:

Theorem 2 (Non-Ricardian economies). *When Assumption 4 is satisfied, a government can borrow against state z^t if and only if its economy is debt-oriented non-Ricardian.*

In this case, when domestic debt is strictly positive ($\{B_i(s^t)\}$ is strictly positive for some $i \in \mathfrak{D}$) there exists a strictly positive level of foreign-owned debt ($\overline{B}^*(z^t) > 0$) such that the government prefers to honor its debt for any lower level of debt ($B^*(z^t) \leq \overline{B}^*(z^t)$).

Proof. See Appendix. □

This Theorem's result derives from a continuity argument. In the absence of foreign-owned debt, some domestically-held debt forces the government not to default in the case of a debt-oriented Ricardian economy. When the level of foreign debt is positive, the gains of default, i.e., here, the reduction in future domestic taxes, become positive and might offset the costs of default due to domestic holdings. However, Assumption (4) ensures that the gains of default remain low as long as foreign-owned debt also remains low.

In the absence of external costs of default, a preference for debt is even sufficient to sustain external debt. This result does not hold anymore when external costs of default are present, for example, by relaxing Assumption 3. In this case, any economy, Ricardian or not, can sustain foreign-owned debt.

In the end, the costs of internal default and redistribution due to positive domestic holdings of debt makes external debt sustainable. Indeed, a default makes some domestic agents strictly worse off when they hold positive amounts of domestic debt and when they cannot be perfectly compensated with tax instruments (implying that government's bonds are net wealth). When the gain of defaulting, i.e. when the level of external debt is sufficiently low, the government is better off repaying. I find a similar conclusion in Mengus (2013) where redistribution motives derives from endogenous portfolio allocation.

V Examples

This section illustrates the two results of Theorems 1 and 2. More specifically, I provide examples of debt-oriented non-Ricardian economies in which external debt is sustainable.

V.1 Low interest rates and debt-oriented non-Ricardian economies

In this subsection, I establish a connection between domestic unbacked public debt with internal costs of defaults.

Without loss of generality, I assume that there is no public spending: $g(z^\tau) = 0$ for every z^τ. As a result, there is no need to tax at any point of time. In the absence of external debt, the government's budget constraint is:

$$\sum_{s^{t+1} > s^t} q(z^{t+1}) B(z^{t+1}) = \delta(z^t) B(z^t) \tag{11}$$

Suppose that there exists unbacked public debt domestically, i.e. there exists strictly positive portfolios $\{B_i(z^t\}_{i \in \mathfrak{D}}$ and 0-value external debt $(B^*(z^t))$ such that (11) is satisfied. In terms of portfolio allocation, this corresponds to $\{0, \{B_i(s^t)\}_{i \in \mathfrak{D}}, 0\}$. When defaulting, the government has to redistribute

$$B(z^t) = \int_{i \in \mathfrak{D}} B_i(s^t) di$$

to domestic agents. As a result the allocation after the default is $\{0, 0, \{-B(z^t)\}\}$.

When there is no outside debt, two situations may arise:

$$\text{Case (i)}: \left[0, \{B_i(s^t)\}_{i \in \mathfrak{D}}, 0\right\} \succ \{0, \{0\}_{i \in \mathfrak{D}}, \{-B_i(s^t)\}_{i \in \mathfrak{D}}\right]$$

$$\text{Case (ii)}: \left[0, \{0\}_{i \in \mathfrak{D}}, \{-B_i(s^t)\}\right\} \succeq \{0, \{B_i(s^t)\}_{i \in \mathfrak{D}}, 0\right]$$

When taxing is weakly better than issuing debt (case (ii)), the government could have *ex ante* limited the inefficiencies that makes unbacked public debt desirable. This requires $\{-B_i(s^t)\}_{i \in \mathfrak{D}}$ to be element of $\Theta(s^t)$: the government can exactly offset domestic losses due to the default. Ex ante, the government could have implemented these transfers and done at least as well as with debt. In contrast, when there are sufficient restrictions on the government's ability to transfer ($\{-B_i(s^t)\}_{i \in \mathfrak{D}} \notin \Theta(s^t)$), case (i) may arise.

This leads to the following proposition:

Proposition 2. *If unbacked public debt is sustainable, an economy is debt-oriented non-Ricardian.*

Unbacked public debt exists if Assumption 3 does not hold.

The conditions under which unbacked public debt is sustained in an economy are well-known (see Tirole (1982, 1985) and Santos and Woodford (1997)). This theorem gives also a very simple mapping with Hellwig and Lorenzoni (2009)'s result. They have shown that international borrowing necessitates "low" interest rates. These "low" interest rates reduce the cost of future borrowing as well as the gains from saving, so that countries are not tempted to default and save, as in the Bulow and Rogoff (1989)'s argument. The presence of such low interest rates is equivalent to the existence of a bubble on international capital markets and they require that Assumption 3 does not hold.

Remark. Here, unbacked public debt is assumed to be the only bubble sustained in the economy. Allowing for private bubbles may crowd out public debt. This will result in a lower ability to borrow abroad as the domestic cost of default is also lowered. Possibly, when private bubbles are stochastic, and when domestic agents are risk-adverse, public debt can be preferred to private bubbles. This holds obviously as long as public debt is sufficiently safe.

V.2 Overlapping generation models

This subsection provides a first example of a debt-oriented non-Ricardian economy: the overlapping generation *à la* Diamond (1965).

In this first example, the domestic private sector consists of two overlapping generations of households who live two periods (young and old)[10]. For simplicity, I do not consider aggregate shocks and so there are only idiosyncratic states: $h_t^i \in \{Y, O\}$, when $i \in G_t \cup G_{t-1}$, with G_t denotes the set of agents who were born in period t.

In any period t the young households' endowment $(y(Y))$ is greater than the old households' endowment $(y(O))$: $y(Y) > y(O)$.

Each household $i \in G_t$ chooses consumption so as to maximizes its lifetime utility function:

$$U_i = u(c_i(Y)) + \beta u(c_i(O)),$$

where $u(.)$ is increasing, twice-differentiable and concave.

I denote taxes net of transfers by $T_i(h^t)$. I assume that taxes and transfers are non-contingent on household's types h^t. This induces the following measurability condition on $T_i(h^t)$:

$$\forall i, \ T_i(h^t) = T_i. \tag{12}$$

The program of one household is then:

$$\max u(c_i(Y)) + \beta u_i(c(O))$$
$$\text{s.t.} c(Y) = y^Y - qB - T^Y$$
$$c^O = y^O + \delta B - T^O$$

The solution of this problem is a non-zero demand for bonds $(B > 0)$ as long as $y^Y - T^Y > y^O - T^O$.

The measurability constraint (12) implies that net taxes T_i do not depend on types. When defaulting, the government differentially impacts generations. In particular, the generation which becomes old at the time of the default is potentially a net loser. Indeed agents in that generation receive only T and lose B. When $T < B$, the old generation loses.

No other generation is negatively affected. Generations born and dead before the default are not affected at all. Generations after the default are positively affected as they gain the difference between saving and borrowing as in Bulow and Rogoff (1989).

In comparison, if taxes were contingent to types, the government would be able to replicate the revenues of public bonds by giving at least 0 to young households and at least B to old households. In turn, contingent taxes would imply that, in normal times, the government can also redistribute from the young households to the old ones, shrinking down heterogeneity and thus the net demand for public bonds.

[10] Finite horizon guarantees that Assumption 3 is satisfied.

Government default decision Turning to government decisions, a key parameter is the reaction of the government's objective to the welfare of the old generation at the time of the default: as long as this parameter is large enough, the losses suffered by this generation cannot be to compensated by the gains of every future generation.

The government's problem is:

$$\max_{\delta,T} \Phi^O U^O + \Phi^Y U^Y$$
$$\text{s.t. } \forall i \in [0,1], c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - qB_i(h^t)$$
$$\delta\left(\int_i B_i + B^*\right) + g = q\left(\int_i B_i + B^*\right) + \int_i T_i(h^t)di$$

Without defaulting, T balances the government budget constraint: $(1-q)(B+B^*)+g = T$. When defaulting, the governement decrease taxes from T to $T - \Delta T$ to balance its budget constraint: $-qB + g = T - \Delta T$ and so $\Delta T = B + B^*(1-q)$.

The net outcome for the old generation is: $-B/2 + B^*(1-q)/2$ while it is at least $\Delta T/2$. As long as $\Phi^O > 0$, when B^* is sufficiently low, the old generation loses from the default, and so, defaulting is not Pareto improving. In particular, there exist political weights Φ^Y and Φ^O such that when $B^* = 0$ the government is better off repaying its debt (for example, $\Phi^O = 1$ and $\Phi^Y = 0$).

This gives rise to the following proposition:

Proposition 3. *The economy is debt oriented non-Ricardian if and only if*

1. *Political weights Φ^Y and Φ^O are such that when $B^* = 0$ the government is better off repaying.*
2. *When T_i satisfies the measurability constraint (12).*

When these conditions are satisfied, there exists a strictly positive level of foreign-owned debt ($\overline{B}^*$) such that for any lower level B^*, the government honors its debt ($\delta = 1$).

The two conditions are both sufficient. When T_i is contingent on types, the government defaults for any strictly positive level of foreign-owned debt $B^* > 0$, as the governement can at least replicate the flows of defaulted debt for the old generation and the the young generation as well.

Rather than using the standard OLG model with unequal endowment for young and old households, several other demand for stores of value by generations of agents can be introduced: Blanchard (1985)'s finite horizon model, a political economy model as in Guembel and Sussman (2009), a demand by entrepreneurs, either because of a mistiming of investment as in Woodford (1990) or Farhi and Tirole (2012)[11], or due to the expectation of reinvestment shocks as in Holmstrom and Tirole (1998). Brutti (2011) has already considered this latter demand for stores of value as a source of international borrowing.

V.3 Uninsurable idiosyncratic risk economy

This subsection provides a second example of debt-oriented non-Ricardian economies: Bewley-Aiyagari economies.

To this purpose, let me consider a stylized Bewley-Aiyagari economy where agents face an uninsurable idiosyncratic risk. The domestic private sector is a continuum of mass one of infinitively-lived households. They choose consumption so as to maximize:

$$\max \sum_{t,h^t} \beta^t \pi(h^t) u(c_i(h^t))$$

Each of them receives a endowment $y + \epsilon_i(h^t)$ where $\epsilon_i(h^t)$ is an zero-mean i.i.d. idiosyncratic risk. For simplicity, we assume that $\epsilon_i(h^t)$ can take only two values: $+\epsilon$ or $-\epsilon$ with $0 < \epsilon < \min(y(h^t))$.

As in the previous subsection on overlapping generations models, I assume that the government cannot observe nor elicit types, and so taxes and transfers are non-contingent to types. This induces the following measurability constraint:

$$T_i(h^t) = T_i. \tag{13}$$

The only asset that households use to smooth consumption is public debt. Their holdings is denoted by $B_i(h^t)$ as previously. Households cannot short public debt imposing $B_i(h^t) \geq 0$ (cf. Aiyagari, 1994).

Consequently the problem of household i is:

$$\begin{aligned} U_i = \max & \sum_{t,h^t} \pi(h^t) u(c_i(h^t)) \\ & \text{s.t. } c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - q B_i(h^t) \\ & B_i(h^t) \geq 0 \end{aligned}$$

This problem leads as well to a non-zero demand for bonds. More precisely, following the results by Aiyagari (1994), there exist N holdings levels: $\{0, B^1, ..., B^N\}$.

The measurability constraint (13) implies that the government implements a uniform tax or transfer T to households.

[11] In the online appendix, I completely describe an OLG model along Farhi and Tirole (2012)'s lines.

In case of default, there exists $i \in \{0, 1..N\}$ such that $T \in [B_i, B_{i+1}]$. Consequently, each household holding B_j, with $j \geq i+1$ faces losses equal to $T - B_j$.

The government problem when deciding whether to default is:

$$\max_{\delta,T} \int_0^1 \Phi_j U_j dj$$
$$\text{s.t. } \forall i \in [0,1], c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - qB_i(h^t)$$
$$\delta(\int_i B_i(h^t)di + B^*) + g = q(\int_i B_i + B^*) + \int_i T_i(h^t)di$$

Without default, $T = g + (1-q)(\int_i B_i di + B^*)$. With default, the level of tax decrease by ΔT satisfying $T - \Delta T = g + q' \int_i B_i' di$. As a result, $\Delta T = (1-q)(\int_i B_i di + B^*) + q' \int_i B_i' di \geq \int_i B_i di + (1-q)B^*$.

The default is thus not Pareto improving as it leaves some agents strictly worse off. There exists weights $\Phi^j, j \in [0,1]$ so that, when $B^* = 0$ the government is better off not defaulting. This leads to the following Proposition:

Proposition 4 (Aiyagari economy)**.** *The economy is debt-oriented non-Ricardian when* $\Phi^j, j \in [0,1]$ *are so that, when* $B^* = 0$*, the government is better off not defaulting and when* T_i *satisfies the measurability constraint (13).*

As a result, there exists a strictly positive foreign-owned debt level ($\overline{B}^*$) *so that, for any lower level of debt* $B^* \leq \overline{B}^*$*, the government strictly prefers to repay* ($\delta = 1$)*.*

For these two examples, the general intuition is that when the government is unable to condition taxes net of transfers on agents' report, one can check that the cost due to second best restrictions may prevent the government to redistribute the gains of default and, hence, from defaulting itself. Presumably, the more the government is able to elicit information by having greater flexibility in tax schemes, the less the default is costly.

Other examples of debt-oriented non-Ricardian economies include economies suffering from political economy frictions as in Amador (2008) or Gul and Pesendorfer (2004) and, similarly, the strategic use of debt in the switching-government environment of Persson and Svensson (1989) or of Alesina and Tabellini (1990).

V.4 Distortionary taxes

This subsection looks at distortionary taxes. They are a well-known deviation from Ricardian equivalence as they favor the issuance of debt to smooth the costs involved by tax distortions (cf. Lucas and Stokey, 1983). However, Chari and Kehoe (1993) show that the government is always better off defaulting to reduce the cost of future taxes: debt is used to mimic lump-sum taxes.

Following the Ramsey taxation literature, I consider one representative household who consumes, provides labor and invest in domestic debt and in capital.

The household's preferences on consumption and labor are:

$$\sum_{z^\tau|z^t} \pi(z^\tau)\beta^t u\left(c(z^\tau), l(z^\tau)\right)$$

with $\beta \in (0,1)$ the discount factor and u is a concave function, increasing in consumption but decreasing in labor. I assume that u satisfies the standard Inada conditions. The household's budget constraint reads:

$$\begin{aligned} c(z^t) =& B(z^t) - \sum_{z^{t+1}>z^t} q(z^{t+1})B(z^{t+1}) \\ &+ (1-\tau^l(z^t))w(z^t)l(z^t) + (1-\tau^k(z^t))\left(F(k(z^{t-1}), l_(z^t)) - w(z^t)l(z^t)\right) - k_i(z^t) + T_i(z^t) \end{aligned} \tag{14}$$

In equilibrium, the household's first order conditions are:

$$q(z^{t+1})u_C(c(z^t), l(z^t)) = \pi(z^{t+1}|z^t)u_C(c(z^{t+1}), l(z^{t+1})) \tag{15}$$

$$u_C(c(z^t), l(z^t)) = \sum_{z^{t+1}>z^t} \beta\pi(z^{t+1}|z^t)u_C(c(z^{t+1}), l(z^{t+1}))\left(1 + (1-\tau(z^{t+1})F_k(z^{t+1})\right)$$

$$\tau^l(z^t) = 1 + \frac{u_C(c(z^t), l(z^t))}{u_C(c(z^t), l(z^t))w(z^t)} \tag{16}$$

$$w(z^t) = F_l(z^t)$$

The government's budget constraint is:

$$g(z^t) = -B(z^t) + \sum_{z^{t+1}>z^t} q(z^{t+1})B(z^{t+1}) + \tau^l(z^t)w(z^t)l(z^t) + \tau^k(z^t)\left(F(k(z^{t-1}), l_(z^t)) - w(z^t)l(z^t)\right) \tag{17}$$

Definition 6 (A Ramsey problem)**.** max U, *s.t. (17), (14), (15) and (16).*

Lump sum taxes When the government has this ability to raise lump sum taxes (i.e. when (15) and (16) do not bind), it is well-known that Ricardian equivalence holds. In such case, the domestic private sector decisions and allocation depend only on the net present value of futures taxes. Indeed, summing the government's budget constraint over all future periods, we have:

$$B(z^0) \leq \sum_{z^t} q(z^t)/q(z^0)\left(T(z^t) - g(z^t)\right)$$

Only the net present value of taxes matter.

Distortionary taxes To make the problem simple, I make two assumptions. First the preferences satisfy Zhu (1992)'s condition: u is separable in consumption and labor and is CRRA with respect to consumption. Under this condition, the government will tax only labor as in Judd (1985) or Chamley (1986). Second the utility is convex with respect to labor and the relative curvature is constant. This makes the tax rate on labor constant across states. Finally, the utility function is of the form:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1-\xi}}{1-\xi}$$

with $\sigma, \xi > 1$.

In that context, the budget constraint of the government writes as:

$$B(z^t) + B^*(z^t) + \sum_{z^T > z^t} q(z^t) g(z^T) = \tau_w \sum_{z^T > z^t} q(z^t) w(z^t)$$

By defaulting on the whole stock of debt $B(z^t) + B^*(z^t)$, the tax rate on labor after the default τ_w^D is such that:

$$\sum_{z^T > z^t} q(z^t) g(z^T) = \tau_w^D \sum_{z^T > z^t} q(z^t) w(z^t)$$

and $\tau_w^D < \tau_w$.

This change in tax rate affects domestic welfare in two dimensions: through the amount disposable resources for households and through the change in the distortions.

For the former effect, using the household's budget constraint, the decrease in taxes is beneficial for the domestic household as he benefits from a net tax cut:

$$\tau_w^D \sum_{z^T > z^t} q(z^t) w(z^t) \leq \tau_w \sum_{z^T > z^t} q(z^t) w(z^t) - B(z^t)$$

with equality if and only if $B^*(z^t) = 0$?

For the distortionary effect, the tax cut correspond also to a net gain in terms of utility.

The following proposition sums up these results:

Proposition 5 (Distortionary taxes)**.** *When taxes are distortionary, a default is always strictly preferred for* $B^*(z^t) > 0$.

This result sheds some light on debt-oriented non-Ricardian economies. The preference for debt is such economies is not only *ex ante*, when issuing debt, but also *ex post*, when debt has to be repaid. With distortionary taxes, debt is desirable *ex ante* as this reduces the welfare cost of distortionary taxes, but not *ex post* as debt repayment implies distortions in the future.

VI Conclusion

This paper identifies a link between Ricardian equivalence and the existence of sovereign debt. As long as an economy is Ricardian, *per se* or because the government has enough tools to replicate the first best allocation, no sovereign lending is possible. However, when the economy has a preference for debt-financed expenditures, sovereign lending becomes sustainable up to some state-contingent upper bound. Furthermore, I show that such a preference for debt appears when unbacked public debt can be domestically sustained and I provide examples of debt-oriented non-Ricardian economies: the overlapping generation model and the Bewley-Aiyagari idiosyncratic risk model. Yet, when public debt is used to smooth taxes over time, as in the case of distortionary taxes, an economy is not debt-oriented non-Ricardian.

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A Additional elements

A.1 Default-free equilibria

Lemma 5. *There is no loss of generality in considering only default-free allocations.*

Proof. Suppose that there exists an equilibrium path $[\{T_I(s^\tau), B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}, B^*(z^\tau), \delta(z^t)]_{z^\tau}$ guaranteeing $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}$ such that there exists z^t in which the government defaults $\delta(z^t) = 0$. Then, by borrowing $B^*(z^t) = 0$ and by setting $\delta(z^t) = 1$, the equilibrium path guaranteeing $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}$ is still implementable. Indeed, in each state z^t, the government faces looser borrowing constraints after state z^t.

What I need to check is that in each state $z^\tau|z^t, \tau > t$, the government is not better off consuming its savings abroad and borrow. This is guaranteed when preferences are time-consistent. The rest follows from the proof of Proposition 1: saving rather than borrowing involves a lower cost at each period compared with borrowing (notice that this does not require selective defaults but relies only from the fact that, when external debt is 0, saving is preferred to borrowing). □

To show that there is no loss of generality to consider default-free allocation, I show that any equilibrium allocation featuring default can be implemented without default. This is not a foregone conclusion. Indeed, intuitively, the time-consistent policy under tighter borrowing constraints (those resulting from defaulting) is not necessarily time-consistent when considering looser borrowing constraints. Tighter borrowing constraints may end up being a commitment tool to restrict the set of time-consistent tax paths to more desirable ones. Neverthless, in the proof, I show that the tax path with default is also consistent without default, as with time-consistent preferences, the government is always better off following the saving strategy regardless of the possibility of borrowing. Indeed, following Bulow and Rogoff (1989)'s argument, savings is always cheaper, when defaulting or, as in Lemma 5, when external debt is set at 0.

Yet, Lemma 5 does not require to ensure that the commitment path of taxes is implementable. In particular, when the horizon is finite and taxes are measurable only on aggregate states, commitment tax schedules may be time-inconsistent (cf. Calvo and Obstfeld, 1988).

Remark. This result is not robust to assuming time-inconsistent preferences as in Gul and Pesendorfer (2004) or Amador (2008), as the government might be willing to deviate and consume its savings abroad.

A.2 The preference relation's properties

The preference relation's properties are:

$$\forall \left[B_1^*(z^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}\right], \forall \left[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}\right]$$
$$\text{and } \forall \left[B_3^*(z^t), \{B_{3i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{3i}(s^t)\}_{i\in\mathfrak{D}}\right],$$

(i) *Completeness*: either $[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$ or $[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$.

(ii) *Reflexivity*: $[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_1^*(z^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$.

(iii) *Antisymmetry*: if $[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$ and

$[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$, then:

$$\left[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}\right] \approx \left[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}\right].$$

(iv) *Transitivity*: if $[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$

and $[B_2^*(s^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_3^*(z^t), \{B_{3i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{3i}(s^t)\}_{i\in\mathfrak{D}}]$,

then: $[B_1^*(s^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^t)\}_{i\in\mathfrak{D}}] \succeq [B_3^*(z^t), \{B_{3i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{3i}(s^t)\}_{i\in\mathfrak{D}}]$.

B Proofs

B.1 Proof of Lemma 1.

The set of $B^*(z^t)$ such that $\delta(z^t) = 1$ is not empty as it contains at least 0.

Let me denote by $B^{*,R}(z^t)$ the upper bound of this set. Suppose that there exists $B^*(z^t) \leq B^{*,R}(z^t)$ such that the government is better off defaulting. Then, $V^R(z^t)$ evaluated at $B^*(z^t)$ is larger than $V^R(z^t)$ evaluated at $B^{*,R}(z^t)$, involving that the country should default when the external debt equals $B^{*,R}(z^t)$.

B.2 Proof of Proposition 1.

Default and saving Suppose that the discounted value of agents' future endowments is finite. Then the discounted value of government's taxes is bounded as well. We can then replicate Bulow and Rogoff (1989)'s proof, as they only use an arbitrage argument.

I denote by $W(z^t) = \sum_{\tau,z^\tau} q^*(z^\tau)/q^*(z^t) \int_{i\in\mathfrak{D}} y_i(s^\tau)di$. In particular, what the government actually taxes is bounded:

$$\sum_{\tau,z^\tau} q^*(z^\tau)/q^*(z^t) \int_{i\in\mathfrak{D}} T_i(s^\tau)di \leq W(z^t)$$

. I denote by $y(z^t) = \int_{i\in\mathfrak{D}} y_i(s^\tau)di$.

In state z^t, the net payment is $P(z^t) = B^*(z^t) - \sum_{z^\tau|z^t} q^*(z^\tau)B^*(z^\tau)$. The total country debt $D^*(z^t) = \sum q^*(z^\tau)/q(z^t)P(z^\tau)$ and $D^*(z^t) \leq kW(z^t)$ with $k \leq 1$. We have all the elements of Bulow and Rogoff's proof.

Suppose that $B^*(z^t) \geq k(W(z^t) - y(z^t))$. Then the government can default and engage in saving as follows:

$$\text{It purchases in period t} A(z^t) = P(z^t) - D^*(z^t) + k(W(z^t) - y(z^t)).$$

$$\text{It purchases in period } \tau A(z^\tau) = G_\tau(z^\tau) + P(z^\tau) - ky(z^\tau).$$

$$\text{It obtains in period } \tau G(z^\tau) = kW(z^\tau) - D^*(z^\tau).$$

I need to check that:

$$A(z^\tau) = \sum_{z^{\tau+1} > z^\tau} q^*(z^{\tau+1})/q^*(z^\tau)G(z^\tau).$$

This holds as

$$W(z^\tau) - y(z^\tau) = \sum_{z^{\tau+1} > z^\tau} q^*(z^{\tau+1})/q^*(z^\tau)W(z^{\tau+1}),$$

and

$$P(z^t) - D^*(z^t) = -\sum_{z^{t+1} > z^t} q^*(z^{t+1})/q^*(z^t) D^*(\tau + 1).$$

Finally, we can notice that $A(z^t) \leq P(z^t)$ by assumption and $P(z^\tau) - ky(z^t) \leq P(z\tau)$ for all $z^\tau | z^t$. Then, as in Bulow and Rogoff (1989), this implies that $k = 0$ and so D^* and B^* equal 0.

Tax path Is there a tax path consistent with the default and saving option? Suppose that the anticipated tax schedule before the default $\{T_i(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t}$ is time-consistent. This tax schedule is consistent with borrowing constraints and budget constraints after default, as the default and saving path only involves paying lower taxes. In particular, each individual net tax is lower.

B.3 Proof of Lemma 3

For selective defaults on foreign-owned debt, Proposition 1 implies that:

$$\forall \{T_i^1(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda(0, \{0\}), \exists \{T_i^2(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda^D(0, \{0\}), \tag{18}$$

$$V^R\left(0, 0, \{T_i(s^t)\}_{i \in \mathfrak{D}}\right) = V^D\left(0, 0, \{T_i'(s^t\}_{i \in \mathfrak{D}}\right) \tag{19}$$

As a result, if there exists a vector $\{T_i'(s^t)\}_{i \in \mathfrak{D}, s^\tau | s^t}$ satisfying the lemma's condition, there exists there a vector $\{T_i''(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda^D(0, \{0\})$, such that

$$V^R\left(0, 0, \{T_i'(s^t)\}_{i \in \mathfrak{D}}\right) = V^D\left(0, 0, \{T_i''(s^t\}_{i \in \mathfrak{D}}\right),$$

and so the country is better off defaulting.

B.4 Proof of Theorem 2.

Suppose that an economy strictly prefers not to default: there exists $B^*(z^t) > 0$ so that, $\forall \{T_i(s^\tau)\}_{i \in \mathfrak{D}}, s^\tau | s^t\} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i \in \mathfrak{D}}\right)$ and $\forall \{T_i''(s^\tau)\}_{i \in \mathfrak{D},, s^\tau | s^t} \in \Lambda\left(0, \{0\}_{i \in \mathfrak{D}}\right)$,

$$\left[B^*(z^t), \{B_i(s^t)\}_{i \in \mathfrak{D}}, \{T_i(s^\tau)\}_{i \in \mathfrak{D},, s^\tau | s^t} \succ \{0, \{0\}_{i \in \mathfrak{D}}, \{T_i''(s^\tau)\}_{i \in \mathfrak{D},, s^\tau | s^t}\right].$$

Suppose that the economy is Ricardian: $\forall \{T_i'(s^\tau)\}_{i \in \mathfrak{D},, s^\tau | s^t} \in \Lambda\left(B^*(z^t), \{0\}_{i \in \mathfrak{D}}\right)$,

$$\left[B^*(z^t), \{B_i(s^t)\}_{i \in \mathfrak{D}}, \{T_i(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t}\right] \approx \left[B^*(z^t), \{0\}_{i \in \mathfrak{D}}, \{T_i'(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t}\right]$$

$$\text{Then: } \left[B^*(z^t), \{0\}_{i \in \mathfrak{D}}, \{T_i'(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t}\right] \succ \left[0, \{0\}_{i \in \mathfrak{D}}, \{T_i''(s^t)\}_{i \in \mathfrak{D}, s^\tau | s^t}\right]$$

which contradicts Proposition 1 on selective defaults.

Reciprocally: Suppose that the economy is debt oriented non-Ricardian: in particular, there exists some $\{B_i(s^t)\}$ such that:

$$\forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall \{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),$$
$$\left[0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i(s^t) - B_i(s^t)\}_{i\in\mathfrak{D}}\right]$$

Using Assumption 4, there exists $dB^*(s^t) > 0$, such that:

$$\left[dB^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[dB^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D}}\right]$$

and then, using Proposition 1, $\forall \{T_i''(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right)$,

$$\left[dB^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i''(s^t)\}_{i\in\mathfrak{D}}\right]$$