Accountability and representation in repeated elections with uncertain policy constraints

Adam Meirowitz[1]
ameirowi@princeton.edu
Princeton University
September 25, 2003

We consider repeated elections in which two long lived parties have fixed preferences over a two dimensional policy space. In each period the government is constrained in its policy selection decisions. The public never observes this constraint making control of the government quite difficult. In pure strategies the voter's preferred equilibria have the following features: (i) parties favored by the current constraint enact the voter's constrained optimum and are reelected while parties not favored by the current constraint distort policy in their preferred direction, (ii) reelection may follow the enactment of policies that seem biased in favor of the governing party (iii) the sets of polices that will result in reelection differ across parties. Voters prefer equilibria involving partial control to repeatedly selecting the best ex-ante party. In addition, if the voters can randomize (as in mixed strategies) then full control is possible.

[1] I thank Chris Achen, Jim Alt, David Austen-Smith, Larry Bartels, Joshua Clinton, John Duggan, Keith Krehbiel, Nolan McCarty, Roger Myerson, Tom Palfrey, Tom Romer, Ken Shotts, Jim Snyder, Jean Tirole and Alan Wiseman for helpful comments and discussions. I thank Natasha Zharinova for her careful assistance.

1 Introduction

In February 2002, newly elected New York City Mayor Michael Bloomberg faced a public challenge. Financial strain on city resources and a faltering economy exposed the city to a dramatic budgetary problem. Aside from the policy challenge of managing a city under tight conditions Bloomberg faced a difficult but subtle political problem in choosing a budget. Voters could not fully comprehend the complex trade-offs associated with running a city under changing economic conditions. Specifically, a centrist voter would have a hard time discerning if Bloomberg's policy represented a desirable compromise given the complex trade-offs, or a policy that inappropriately favored Bloomberg's agenda. The former might merit reelection while the latter would not. In response Bloomberg offered what the New York Times labeled "a budget that hurts everyone" (Cooper 2002). This description of policy making and politics differs from extant models of representation and accountability.

This paper examines the extent to which voters can control government officials in the presence of this type of informational asymmetry. Extant models of accountability draw from agency theory and consider the choice of effort by government officials when voters (the principals) imperfectly observe effort. A central question is whether elections serve as a means to *control* governmental actions or *select* the best available governments. Fearon (1999) presents a clear review of the formal literature on accountability and forwards a *general* conclusion. He argues that viewing *elections as a means to select high quality candidates* is more appropriate than viewing *elections as a means to induce accountability by sanctioning poor performance.* Fearon defends this claim by describing conditions under which the selection explanation is more plausible than the control explanation,

> (i) repeated elections do not work well as a mechanism of accountability, because [voters] believe that their ability to observe what politicians do and to interpret whether it is in the public interest is so negligible; and (ii) there actually is relevant variation in the types of candidates for political office, and these can be distinguished to some extent,... (p. 68)

While this conjecture is reasonable in the context of the models reviewed, it needs further investigation. In potential contrast, Besley and Case (1995a, 1995b) find evidence consistent with the *sanctioning* hypothesis: (1) voters reward governors that outperform those of neighboring states and (2) term limited governors tend to shirk in fiscal policy making relative to non-limited governors. Analyzing gubernatorial and legislative elections Lowry, Alt and Ferree conclude that "electoral accountability for fiscal policy outcomes is strong but highly contingent on a complex configuration of party labels, partisan control, expectations, and institutions." (p. 759) The tension between Fearon's conclusion and these empirical findings, as well as the fact that existing theories of elections do not adequately capture the two conditions that Fearon highlights as important motivate analysis of a different type of model. We capture the monitoring problem through hidden knowledge

– the party in office observes the feasibility constraint prior to policy selection and the public and out of office party never observe this information. Variation in candidates is captured by two identifiable infinitely lived parties that each desire to maximize a different policy issue. Contrary to the selection explanation, we find that, in the public's most preferred equilibrium, parties of quite different types (with one ex-ante preferred to the other) are controlled in a manner that makes voters unconcerned with the selection problem.[2] Additionally, (as discussed in the concluding section) the equilibrium analysis offers some intuition/motivation for the empirical regularities found by Lowry, Alt and Ferree.

The model differs from existing models in three ways. First, agents have monotone preferences over two dimensions of primitive goods that the government can provide (a classic example is defense spending-guns and social spending-butter, a more timely example is privacy and security). Preference heterogeneity surfaces in the form of differing marginal rates of substitution (while we all want privacy and security we differ on how much of one we are willing to sacrifice for the other).[3] Second, the government faces a feasibility constraint (increasing security requires that privacy is sacrificed). The party in office knows the constraint, but the public and out party do not. This assumption is consistent with the claim that typical voters devote little effort to information acquisition about the subtleties of policy-making. In contrast to existing formal studies of elections, which start with induced Euclidean preferences over policy or treat the government as choosing a level of costly effort, preference divergence is over the willingness to substitute between issues that are viewed as goods. Here, induced preferences over policies and information asymmetries are the product of an explicitly modeled feasibility constraint. This approach represents an alternative starting point for modeling elections. Elections serve as a process for aggregating primitive preferences over goods (like security and privacy) into policy decisions that satisfy certain feasibility requirements. Third, while our focus is on control of the government, the notion of shirking differs from most applications of agency theory. Here, shirking describes the selection of policy which the governing party prefers to the voters' constrained-optimum.

The analysis leads to six conclusions.

- Using non random voting rules, the public can at best partially control the government, so that in

[2]Standard models in agency theory emphasize the concepts of moral hazard and/or adverse selection. Strictly speaking the current model does not involve adverse selection or moral hazard. Instead the model involves hidden knowledge which limits the set of control schemes available to the public. The extent to which one party may have preferences that are more aligned with the voters' suggests that one agent might be ex-ante more desirable and thus the phrase "solving the selection problem" has a natural meaning. Similarly, while the choice variable of the parties is not simply effort, the extent to which incentives might induce the party to act as if she were trying to maximize the public's utility renders the term "control" meaningful. Accordingly, we will use the terms *control* and *selection* as adjectives describing equilibria. These concepts have similarities with the terms *moral hazard* and *adverse selection* which tend to describe models.

[3]This approach is motivated by work in political behavior. With overwhelming regularity scholars of public opinion report that voters "want to have their cake and eat it too" (Zaller 1998).

equilibrium the incumbent shirks for some realizations of the random constraint.

- With mixed strategies, in the form of a probabilistic reelection function, perfect control is possible as the government is made to internalize the public's preferences.
- Voters will treat the parties differently–the reelection functions differ across parties. This is true in either mixed or pure strategy equilibria.
- Adoption of policies that are very desirable to the in party does not necessarily result in removal, as some such polices can be credibly justified as optimal for the voters given some constraint.
- With a monitoring problem of this form, the best pure strategy equilibria (in terms of voter preferences) do not involve always electing and retaining the ex-ante best party, thus simply solving the selection problem without addressing the control problem is suboptimal.
- If there are random valence shocks, in periods in which the government has a valence advantage it is less controllable and will shirk more.

The first two points are descriptions of the equilibrium analysis. The pure strategy equilibria speak to the types of speeches that may credibly be made by incumbents when justifying their actions to the constituency (Fenno 1978). From this perspective *trust in the government* is a phenomena partially dependent on uncontrollable features of the political landscape (specifically stochastic constraints) and partially dependent on the choices made by the party in office. The sharp difference between mixed and pure strategy results demonstrates that control becomes easier when the set of actions available to the public is enlarged. The third and fourth points describe pure strategy equilibria and speak to the types of voting behavior we might observe (in say U.S. presidential or gubernatorial contests). The model offers an explanation for the finding that voters treat the parties differently (Lowry, Alt and Ferree 1998) . Policies (or more generally outcomes) that are considered acceptable if implemented by one party may be viewed as unacceptable when the other party is in office. The fifth point speaks to the question of whether elections should be viewed as devices for selection or control and counters Fearon's conjecture. The sixth point is in conflict with Fiorina's (1973) *marginality hypothesis* -electoraly weak incumbents will tend to moderate more than strong incumbents. While empirical support for the hypothesis is mixed, recent formal work involving single period elections with valence and policy commitment (e.g, Groseclose 2001) finds support for the hypothesis while the current model challenges it.

In section 2 we present a brief review of related models. In section 3 we begin by formulating a model involving just uncertainty about the relative trade-offs between each policy coordinate. After describing the model and equilibrium concepts we show that perfect monitoring is impossible in pure strategies. We then characterize simple pure strategy equilibria that exhibit the maximal amount of control possible. We also address Fearon's conjecture and characterize the voter's most preferred pure strategy equilibria. In section 4 we consider the case where there is uncertainty about both the relative price of each issue and the total amount of resources available. We show that no degree of control is possible in pure strategy equilibria. We then establish the existence of mixed strategy equilibria exhibiting perfect control if the value to office is sufficiently high. In Section 5 we extend the analysis to allow for the possibility that there is a policy independent (valence) shock to voter preferences in each period. Section 6 raises a few natural extensions to the basic model and discusses the robustness of the findings to these variations. In section 7 we conclude with a discussion. Appendix A provides the proof of one purely technical result used in the paper and Appendix B provides a characterization of the voter's optimal pure strategy equilibria.

2 Previous Literature

Existing principal-agent models of elections focus on the problem of creating incentives for the government to undertake the optimal amount of costly but publicly desirable effort. Barro (1973), Austen-Smith and Banks (1989), and Ferejohn (1986) consider the moral hazard aspects of this problem, and Banks and Sundaram (1993, 1998) and Ashworth (2001) consider the problem of both moral hazard and adverse selection. While focused on the control of shirking the current paper is conceptually quite distinct. Whereas the above moral hazard models deal with the creation of incentives for the government to not shirk in its effort choice (a variable upon which the principal and agent have diametrically opposed preferences), we deal with the creation of incentives for the government to not shirk in its policy selection (a variable upon which preferences may be aligned but not identical). Aside from this critical departure, the current model is similar to the two party model that Ferejohn considers. Both involve informational asymmetries, an infinite horizon, and a pool of two long lived parties. In both models voting serves to constrain governmental action and the voter must be indifferent between having either party in office. The equilibrium results and intuition differ in most other respects.

Banks and Duggan (2001) analyze a repeated election citizen candidate model in which preelection commitment is not possible.[4] Their model involves a large population of ex ante indistinguishable candidates, and uncertainty only about the preferences of candidates. Banks and Duggan bridge the gap between social choice theory and the restrictive Downsian/Hotelling world while the current model addresses representation and accountability in two party elections with complex or changing political environments. While Banks and Duggan predict convergence to a particular government and policy for reasonable parameterizations we predict stochastic oscillation between the two parties and non-convergence of policy.[5] Caines-Wrone, Heron and Shotts (2001) present a two-period model which explains shirking, in the sense of policy choice that

[4]See also Duggan's (2000) unidimensional model.

[5]We do not necessarily predict that each party is in office with equal probability.

differs from the public optimum, even when preferences are perfectly aligned. The explanation hinges on the executive's incentive to convince the voter that it is competent and should thus be retained. Given the preference alignment the question is not why do candidates act on the behalf of the voter. Rather the puzzle is why do they sometimes shirk. In contrast, the presence of non-shirking behavior needs an explanation in the current paper as the incumbent has an ideological/preference motivation to not select policies desirable to the electorate. Rogoff and Sibert (1988) analyze a dynamic model of macroeconomic policy in which the government has temporarily private information about its fitness and find that this short-term informational asymmetry can generate political business cycles. While there are similarities in terms of the number of agents and the sequence of play, in Rogoff and Sibert the uncertainty pertains to the competency of the government, and policy is unidimensional–the provision of a good. These differences result in a starkly different equilibrium intuition.

3 The basic model

3.1 Players and preferences

We consider a model that is quite distinct from existing theories of representation and accountability. To make the incentives clear we focus on just two parties and a representative voter, each infinitely lived and concerned with discounted streams of per period payoffs. The policy space is two-dimensional, with parties each seeking to maximize one of the two issues. Voters have well-behaved preferences in which each issue is a "normal good". Feasibility constraints are represented by linear constraints, and information asymmetry about the constraint is captured by assuming that initially the slope (and then in a later section the resource level also) are observed only by the in-government party. Since the analysis can rely on spatial intuition as opposed to algebraic manipulation we avoid specifying specific functional forms for utility functions and stochastic distributions. Instead a few key assumptions impose the relevant structure on the problem. Since the structure of the model is distinct from existing agency and election theories, in several places we have foregone generalizations which do not alter the qualitative properties of the results but do complicate the exposition/notation. (For example there are ways to: allow parties to care about both issues; relax the assumption that the constraint surface is linear; and consider an arbitrary number (odd) of voters. These points are taken up in section 6.)

We consider three players interacting in an infinite number of periods. A set of two parties $P = \{l, r\}$ compete for office in each period, and the representative voter m selects between the parties. We sometimes denote parties witht the subscripts p and $-p$. We consider only the case of a single voter to emphasize the difficulties of controlling parties. In section 6 we show how the model can be generalized to multiple voters. The policy space is $X = \mathbb{R}^2_+$, the positive orthant of two dimensional Euclidean space. We use bold letters to denote a policy which is a vector, and non bold typeface with subscripts 1 or 2 to denote the coordinates of a policy. Thus $\mathbf{x} = (x_1, x_2)$. We assume that each party cares about the policy enacted and values holding office. If policy $\mathbf{x}^t$ is chosen in period t and party p is in office during period t the period t payoff to party p is

$$u_p(\mathbf{x}^t) + \eta_p. \tag{1}$$

The party specific term $\eta_p \geq 0$ measures the non-policy rents associated with holding office. In contrast if party p is not in office but policy $\mathbf{x}^t$ is chosen the t period payoff to party p is $u_p(\mathbf{x}^t)$. We assume that the policy-specific utility function $u_p(\mathbf{x}) : X \to \mathbb{R}^1$ is twice differentiable.

[Figure 1 about here]

Voter m cares only about policy and has twice differentiable utility function $u_m(\mathbf{x})$. Players l, r, m are assumed to have globally non-satiated preferences. We are interested in the case where party l is a high demander of dimension 2, party r is a high demander of dimension 1 and m likes more of each dimension. This approach represents a dramatic departure from standard models of politics which usually assume that agents have ideal points. These "bliss-point" or spatial preferences are generally motivated as stemming from required trade-offs and traditional economic (non-satiated) preferences over goods (like consumption, safety, privacy, ...). In standard models both of these features are unmodeled and the analysis starts with spatial preferences over policy. Here, the feasibility constraint is explicitly modeled, so the right starting point is preferences over primitive issues (like privacy and safety). Endogenous to the model is how policy should and will balance competing desires.

Formally, we define the marginal rate of substitution at $\mathbf{x}$ for player i as

$$MRS_i(\mathbf{x}) = \frac{\frac{\partial u_i(x)}{\partial x_1}}{\frac{\partial u_i(x)}{\partial x_2}} \tag{2}$$

and assume that for any $\mathbf{x} \in X$ $\;MRS_l(\mathbf{x}) < MRS_m(\mathbf{x}) < MRS_r(\mathbf{x})$. For simplicity we take the extreme case where $MRS_l(\mathbf{x}) = 0$ and $MRS_r(\mathbf{x}) = \infty$ for every $\mathbf{x} \in X$. This holds when $u_l(\mathbf{x}) = h_l(x_2)$ and $u_r(\mathbf{x}) = h_r(x_1)$ with $h_l(\cdot)$ and $h_r(\cdot)$ strictly increasing functions.[6] We assume that the voter, m, has strictly convex preferences. Figure 1 depicts a generic representation of policy specific indifference curves for the three agents.

We consider an infinite sequence of elections. In period t nature, a non strategic player, randomly selects a constraint set $B^t \subset X$ and the government, $g^t \in P$ after observing B^t, selects a policy point $\mathbf{x}^t \in B^t$. The voter knowing $\mathbf{x}^t$ but not B^t then casts a ballot $v^t \in \{0, 1\}$ where a vote of 1 is a vote to keep the incumbent

[6] In section 6 we discuss how this assumption can be relaxed.

and a vote of 0 is a vote to replace the incumbent with party $P\backslash g^t$. In period $t+1$ a new constraint B^{t+1} is realized, a new policy $\mathbf{x}^{t+1} \in B^{t+1}$ is selected by g^{t+1} and a new election occurs. Without loss of generality we assume that period 1 involves selection of $\mathbf{x}^1 \in B^1$ by government $g^1 = l$. This game form necessitates that we extend the period utility functions to preferences over an infinite horizon. Accordingly, for a sequence $\{\mathbf{x}^t, g^t\} = \{(\mathbf{x}^1, g^1), ..., (\mathbf{x}^t, g^t), ...\}$ party p's utility is

$$U_p(\{\mathbf{x}^t, g^t\}) = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} \left[u_p(\mathbf{x}^t) + \eta_p 1_p(g^t) \right]. \tag{3}$$

where $\delta \in (0,1)$ is a common discount rate[7] and $1_p(g^t)$ is an indicator taking the value 1 if $g^t = p$ and 0 otherwise. Similarly, the voter's utility over such a sequence is

$$U_m(\{\mathbf{x}^t, g^t\}) = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} u_m(\mathbf{x}^t). \tag{4}$$

[Figure 2 about here]

We assume that any feasible constraint is of the form $B^t = \{\mathbf{x} \in X : b^t x_1 + (1-b^t)x_2 \leq 1\}$ where $b^t \in [\gamma, 1-\gamma]$. In other words a constraint is given by a relative price b^t. The constant $0 < \gamma < \frac{1}{2}$ is a measure of the spread of the support on the random relative price. To make clear the dependence of the constraint on the parameters we sometimes denote a constraint by $B(b)$. The set of possible constraints is isomorphic to $\mathbb{B} = [\gamma, 1-\gamma]$. Figure 2 depicts the set of policies which are feasible for some constraint, $\beta = \cup_{b \in [\gamma, 1-\gamma]} B(b)$. By $\widehat{\beta}$ we denote the efficient boundary of β. This is the set of points that satisfy $b^t x_1 + (1-b^t)x_2 = 1$ for some $b \in [\gamma, 1-\gamma]$. In section 3 we introduce uncertainty not just about the relative price but also about the total size of the pie. In this case the constraint is $\{\mathbf{x} \in X : b^t x_1 + (1-b^t)x_2 \leq c^t\}$ and both b^t and c^t are treated as random variables.[8]

We assume that the common belief is that for every t the parameter b^t is given by an independent draw from the continuous and strictly increasing distribution function $F_b(\cdot)$ on support $[\gamma, 1-\gamma]$.[9] The parties l and r only observe the value b^t if they are in office at period t. The parameter b^t is never revealed to players other than g^t. The game, thus, involves hidden knowledge about the period state variable b^t.

We introduce a convenient partial ordering on X. For two policies $\mathbf{x}, \mathbf{y} \in X$ we say $\mathbf{x} \nwarrow \mathbf{y}$ if $x_2 > y_2$ and $x_1 < y_1$. Intuitively $\mathbf{x} \nwarrow \mathbf{y}$ means that $\mathbf{x}$ is to the northwest of $\mathbf{y}$. Compactness and convexity of the

[7] The assumption of common discount rates is made purely to simplify the notation.

[8] In section 6 we discuss convex constraints with nonlinear boundaries.

[9] The assumption that b is generated by iid draws is unnecessary. In section 6 we discuss the extension to non independent b^t's.

constraint and continuity and strict convexity of the preferences ensure that the set of induced ideal policies for agent $i \in \{m, l, r\}$ for any given constraint b

$$\mathbf{x}_i^*(b) = \arg\max_{\mathbf{x} \in B(b)} u_i(\mathbf{x}) \tag{5}$$

contains exactly one point. Moreover, by the theorem of the maximum (Berge 1963) the function mapping constraints into induced ideal points is continuous. We impose two additional assumptions on the preferences of m.

Assumption 1: For some $b^* \in (\gamma, 1-\gamma)$

$$\mathbf{x}_m^*(b^*) = (1, 1). \tag{6}$$

This condition states that for some feasible b^*, $m's$ optimal policy subject to the constraint $B(b^*)$ corresponds to the point $(1, 1)$. Note that this is the unique point that lies on the boundary of every feasible constraint. This assumption is satisfied if

$$\frac{\gamma}{1-\gamma} < MRS_m((1,1)) < \frac{1-\gamma}{\gamma}. \tag{7}$$

Assumption 2: If $b < b'$ then $\mathbf{x}_m^*(b') \nwarrow \mathbf{x}_m^*(b)$.

This assumption requires that m respond to increases in the relative price of issue 1 by selecting less of issue 1 and more of issue 2. One example satisfying these conditions is, the commonly studied case of Cobb-Douglas utility functions, $u_m(\mathbf{x}) = x_1^\alpha x_2^{1-\alpha}$. Here the solution is $\mathbf{x}_m(b) = (\frac{\alpha}{b}, \frac{1-\alpha}{1-b})$. So $\mathbf{x}_m(\alpha) = (1, 1)$ satisfying Assumption 1. The derivatives are $\frac{dx_1}{db} < 0$, $\frac{dx_2}{db} > 0$, $\frac{dx_1}{d(1-b)} > 0$, $\frac{dx_2}{d(1-b)} < 0$, satisfying Assumption 2.

3.2 Interpretations

One stylized interpretation of the model is to think about issue 1 as defense spending and issue 2 as welfare or redistributive spending. In this interpretation c^t represents the available revenue (from taxing and deficit spending). Party l is then the Democrat party and r is the Republican party.[10] An alternative interpretation is closer in spirit to the public finance literature. Define $x_1 = 1 - \tau$ were τ is the tax rate, and let x_2 denote the amount of government redistribution. The constraint is $b(1-\tau) + (1-b)x_2 \leq c$, and the production function on redistribution is $x_2 = \frac{c - b(1-\tau)}{(1-b)}$ which is stochastic with random parameters b and c. Party l seeks the maximization of welfare spending, x_2, and party r seeks the minimization of taxation. The representative voter seeks to balance the marginal cost and benefit of redistribution when she has strictly convex preferences over $(1-\tau)$ and x_2. The model may also be applicable to areas of regulatory

[10] In this case, one might interpret $\frac{b}{1-b}$ as the relative price of missile defense systems in terms of subsidized health insurance.

politics in which the executive or congress can select from a set of feasible agencies in defining discretion. Additionally, it is possible that internal agency decision making, may be described in this manner with a principal choosing between different departments each staffed with agents that have certain policy biases.

3.3 Strategies and equilibria

We focus on stationary perfect Bayesian equilibria (SPBE). A SPBE consists of a government policy function $\psi_p(b) : [\gamma, 1-\gamma] \rightarrow B(b)$ for each $p \in P$, a ballot function $\upsilon(\mathbf{x}, g) : \beta \times P \rightarrow \{0, 1\}$ for the voter, m, and a voter belief $\pi(b \mid \mathbf{x}, g)$ about the constraint faced by the government conditional on the policy chosen and the government identity. This belief mapping is a distribution function on $\mathbb{B}$ conditional on a policy $\mathbf{x} \in \beta$ and identity $p \in P$. The policy function of p needs to be optimal given the ballot function and the policy function of $-p$, the ballot function needs to be sequentially rational relative to the belief mapping and the policy functions, and the belief mapping needs to satisfy Bayes' rule when it is defined. The assumption of stationarity is satisfied by this description as we have required strategies to hinge only on the state variable $(\mathbf{x}^t, g^t)$.

The assumption of stationarity is tenable as voting or policy selection strategies that hinge on a long history of past elections seem peculiar when these past elections provide no payoff relevant information. Moreover stationary equilibria exhibit retrospective voting –a feature that surfaces in the voting literature.[11] Subsequently, we discuss how the equilibrium set enlarges when the stationarity restriction is relaxed. Since this is a model of incomplete information we focus on perfect Bayesian equilibria to ensure that the equilibria do not hinge on unreasonable voter beliefs. In this game the only relevant uncertainty is faced by the voter, m, when she must decide whether to retain or remove the incumbent. Accordingly, SPBE require that beliefs about b^t be consistent with the observation $\mathbf{x}^t$ and the strategy $\psi_g(\cdot)$. The beliefs are not very important to the analysis here, which distinguishes this model from many others with imperfect information. Under a fixed profile of stationary strategies $\psi_l(b), \psi_r(b)$ the voter's preference for retaining or removing g is not dependent on b^t. Accordingly, the extent to which a ballot strategy $\upsilon(\mathbf{x}, g)$ is sequentially rational does not depend on the beliefs. Intuitively, sequential rationality would constrain prospective behavior (in this case voting), but the information observable to m is of no value in predicting future play under a given pair of stationary policy functions. Given this observation we suppress the beliefs about b^t from subsequent statements of and arguments about equilibria. We characterize strategy profiles as *supportable as SPBE* when there exists a belief mapping for which the strategy profile and belief would constitute a SPBE. Sequential rationality imposes the following constraint on the ballot function $\upsilon(\mathbf{x}, g)$.

[11] Our usage of the phrase retrospective voting differs from that of Fiorina (1981). By retrospective voting we mean behavior in which voting decisions over tomorrow's government are based only on the policy choice of today's government. In contrast to typical applications of the concept, we do not require that retrospective voting specifies a utility cut-off rule or aspiration level, just that voting strategies are dependent only on the last observed policy.

Condition 1 *Given $\psi_l(b), \psi_r(b)$, the mapping $\upsilon(\mathbf{x}, g)$ is sequentially rational iff*

$$\int u_m(\psi_g(b))dF_b(b) > (<) \int u_m(\psi_{-g}(b))dF_b(b)$$

implies $\upsilon(\mathbf{x}, g) = 1(0)$.

Given this condition in any SPBE in which one party is not always in office we must have

$$\int u_m(\psi_l(b))dF_b(b) = \int u_m(\psi_r(b))dF_b(b). \tag{8}$$

This means that in any non-trivial equilibrium, the ballot function will not be prospective in nature. In fact given the desire of m to create incentives for $\psi_g(b)$ to be close to $\mathbf{x}_m^*(b^t)$ it is natural to think about the game as a mechanism design problem, where m selects a ballot function satisfying Condition 1 to maximize the left and right hand side of (8) and the policy mappings $\psi_p(\cdot)$ are mutual best responses to the ballot function. Our analysis will serve to characterizes the relevant incentive compatibility constraints on ballot functions.

Definition 1 *We say a SPBE exhibits **perfect control** if for every $p \in P$, $\psi_p(b) = \mathbf{x}_m^*(b)$ for almost every b. We say a SPBE exhibits **partial control** if for every $p \in P$, $\psi_p(b) = \mathbf{x}_m^*(b)$ for $b \in D_p$ with D_p a subset of $[\gamma, 1-\gamma]$ having positive Lebesgue measure.*

Intuitively, under SPBE with perfect control the in-government party (almost) always adopts the voter's most preferred feasible policy. In an SPBE exhibiting partial control, the in-government party sometimes adopts its most preferred feasible policy that results in reelection (which sometimes coincides with $\mathbf{x}_m^*(b)$) and sometimes this party shirks adopting its most preferred feasible policy. Given a pure strategy ballot function the set of polices which will result in an incumbent victory is given by the acceptance sets

$$\begin{aligned} A_l &= \{\mathbf{x} : \upsilon(\mathbf{x}, l) = 1\} \\ A_r &= \{\mathbf{x} : \upsilon(\mathbf{x}, r) = 1\}. \end{aligned} \tag{9}$$

The compliments of these sets result in loss of office. By choosing the acceptance sets m influences the policies that governments will enact. If she makes A_p to small or restrictive then when party p is in office it will select a policy to maximize its current utility over policy and not retain office next period. Conversely, if A_p is too large or unrestrictive when party p is in office it will be able to retain office while selecting a policy that is far from the voter's optimum (subject to B^t). Agent m would like to select A_p so that when party p is in office she selects $\mathbf{x}_m^*(b^t)$, the voter's most preferred policy in $B(b^t)$. Perfect control by the voter is difficult because she never learns b^t and thus may not be able to discern if a chosen (and observed) policy was indeed in her best interest.

3.4 The impossibility of perfect control in pure strategies

We now show that in pure strategies there are no SPBE in which the voter perfectly controls the candidates. This finding is of more than just technical interest. It indicates that the monitoring problem faced by the public is severe and insolvable with simple voting strategies that either retain or remove the in-government party deterministically. Here, the institutional controls open to the public (a binary action) are insufficient to create the *right* incentives for the government. The problem that the voter faces is that when the price b is low (high) party l (r) can choose an inefficient policy $(x_1 b + (1-b)x_2 < 1)$ which is optimal for m under some other higher (lower) price $b' > (<) b$. This inefficient policy will be more desirable to the party than m's most preferred policy given b. If m conjectures that parties are always choosing $\mathbf{x}^*_m(b)$ she will not be able to discern a deviation of the form just described since she does not know b.

Given any constraint $B(b)$ the incumbent party $p \in P$ must decide whether to select a policy that results in reelection if such a policy is feasible (i.e., $A_p \cap B(b) \neq \emptyset$). On purely policy grounds p's most preferred feasible policy is given by

$$\begin{aligned} \mathbf{x}^*_l(b) &= (0, \frac{1}{1-b}) \\ \mathbf{x}^*_r(b) &= (\frac{1}{b}, 0). \end{aligned} \tag{10}$$

Alternatively, given A_p and the constraint $B(b)$ the optimal policy that will result in reelection is given by

$$\mathbf{x}^w_p(b) = \arg \max_{\mathbf{x} \in B(b) \cap A_p} u_p(\mathbf{x}). \tag{11}$$

Throughout, the ballot functions we consider will induce sets $B(b) \cap A_p$ that are compact. Given the continuity of the preferences this implies that for any p, b the set $\mathbf{x}^w_p(b)$ is non-empty. Moreover, this set will turn out to be a singleton if $B(b) \cap A_p$ is non-empty.

[Figure 3 about here]

Figure 3 exhibits the intuition. In any SPBE with perfect control in which party p is elected with positive probability, for almost every b it must be the case that $\mathbf{x}^w_p(b) = \mathbf{x}^*_m(b)$. By assumption 1 there exists a b^* s.t. $\mathbf{x}^*_m(b^*) = (1,1)$. We will repeatedly refer to this special slope b^*. Now consider $b^* > b > b'$. Prices b and b' both induce a constraint with a boundary that is flatter than the boundary of $B(b^*)$. By assumption 2, $\mathbf{x}^*_m(b^*) \nwarrow \mathbf{x}^*_m(b) \nwarrow \mathbf{x}^*_m(b')$. This and $b > b'$ imply that $\mathbf{x}^*_m(b) \in B(b')$ and $u_l(\mathbf{x}^*_m(b')) < u_l(\mathbf{x}^*_m(b))$. Accordingly party l facing a constraint $b' < b^*$ will strictly prefer to enact a policy $\mathbf{x}^*_m(b)$ which is optimal for the voter under some other constraint and feasible under the constraint $B(b')$. The following proposition states the conclusion we have just demonstrated.

Proposition 1 *In pure strategies there are no SPBE that exhibit perfect control.*

The problem that prevents us from constructing SPBE with perfect control is slightly peculiar. When m observes l, a high demander of x_2, choose a low value of x_2 she cannot tell if the government shirked (choosing a policy that l prefers to $\mathbf{x}_m^*(b)$) . In contrast when l chooses high values of x_2 the voter can be certain that no shirking occurred. This is true because when $b < b^*$ a deviation from $\mathbf{x}_m^*(b)$ that increases x_2 is not feasible (it is outside $B(b)$). Thus, m is unable to determine if l is selecting the right policy, not when the enacted policy involves a high quantity of the government's preferred coordinate, but rather when the enacted policy involves a low quantity of the government's preferred coordinate and a very low quantity of the other coordinate. The intuition being, when the constraint favors party r, party l has an incentive to select an inefficient policy that makes it look like the constraint favors party l by a little less. There are always regions of the set $\mathbb{B}$ where party l can get away with this. It should be noted that even in non stationary strategies perfect control is not possible. The barrier to control is not the size of the stick and carrot, but rather the inability of the voter to determine when to use the stick and when to use the carrot.

Why does proposition 1 require the condition *in pure strategies*? If following $\mathbf{x}_l$ player m retains l with probability $\lambda(\mathbf{x}, l)$ and this function is decreasing in the first coordinate of $\mathbf{x}$ it might be possible for l to prefer selection of $\mathbf{x}_m^*(b)$. In a subsequent section we explore this possibility and derive a mixed strategy SPBE in which the voter's mixed ballot function creates the *right* incentives for the in government party. The existence of such an equilibrium hinges on the slopes of the party utility functions (over policy) not being too steep relative to the term $\delta\eta_p$.

3.5 Imperfect control in pure strategies

While perfect control cannot occur in a pure strategy SPBE, sometimes there are pure strategy SPBE in which the voter can exert partial control on the parties. To develop the basic intuition we first consider cases with a fair amount of symmetry making it easy to satisfy condition 1.

Definition 2 *We say* ***symmetry*** *is satisfied if the voter is indifferent between the following two lotteries*

$$\psi_l(b) = \begin{cases} \mathbf{x}_m^*(b) \ \textit{if } b > b^* \\ \mathbf{x}_l^*(b) \ \textit{otherwise} \end{cases}$$

$$\psi_r(b) = \begin{cases} \mathbf{x}_m^*(b) \ \textit{if } b < b^* \\ \mathbf{x}_r^*(b) \ \textit{otherwise} \end{cases}.$$

Recall that since b is a random variable in each period these policy mappings induce lotteries over policy. An example satisfying symmetry involves $F_b(\cdot)$ uniform and $u_m(\mathbf{x}) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. This is not the only parameterization that satisfies the condition. Symmetry is a joint restriction on the distribution $F_b(\cdot)$ and the voter's preferences $u_m(\cdot)$. When symmetry is satisfied we can characterize strategy profiles that are supportable as SPBE in which each party selects the voter's constrained optimum when b is on its desirable side of b^* and it selects the party constrained optimum when b is on its undesirable side of b^*. The construction uses the fact that when $(1,1) \nwarrow \mathbf{x}^t$ and $\mathbf{x}^t$ solves the voter's problem for some constraint the public can trust that r has not shirked and when $\mathbf{x}^t \nwarrow (1,1)$ and and $\mathbf{x}^t$ solves the voter's problem for some constraint the public can trust that l has not shirked. In the converse cases it is not possible to infer that the parties are not shirking. By $\mathbf{x}_m^{*-1}(\mathbf{x})$ we denote the inverse of $\mathbf{x}_m^*(b)$. Thus $\mathbf{x}_m^{*-1}(\mathbf{x}) = \{b : \mathbf{x}_m^*(b) = \mathbf{x}\}$. In addition to symmetry two additional conditions are needed for pure strategy SPBE with partial control.

Proposition 2 *If symmetry is satisfied the following profile is supportable as a SPBE with partial control*

$$
\begin{aligned}
A_l &= \{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) > b^*\} \\
A_r &= \{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) < b^*\} \\
\psi_l(b) &= \begin{cases} \mathbf{x}_m^*(b) \ \textit{if } b > b^* \\ \mathbf{x}_l^*(b) \ \textit{otherwise} \end{cases} \\
\psi_r(b) &= \begin{cases} \mathbf{x}_m^*(b) \ \textit{if } b < b^* \\ \mathbf{x}_r^*(b) \ \textit{otherwise} \end{cases}
\end{aligned}
$$

if the following conditions are satisfied

$$\frac{\max_{b \in [b^*, 1-\gamma]} [u_l(\mathbf{x}_l^*(b)) - u_l(\mathbf{x}_m^*(b))]}{\eta_l + \int [u_l(\psi_l(b')) - u_l(\psi_r(b'))] \, dF_b(b')} \leq \delta \tag{C1}$$

$$\frac{\max_{b \in [\gamma, b^*]} [u_r(\mathbf{x}_r^*(b)) - u_r(\mathbf{x}_m^*(b))]}{\eta_r + \int [u_r(\psi_r(b')) - u_r(\psi_l(b'))] \, dF_b(b')} \leq \delta. \tag{C2}$$

Proof: Given symmetry is satisfied each party's policy function induces a lottery over policy with the same expected utility for m and thus Condition 1 is satisfied, so the ballot function is sequentially rational. It remains only to verify that the policy mappings are mutual best responses.

-Consider party l : Assume that $\psi_r(b)$ and A_l, A_r are given by the proposition. It is sufficient to show that no unilateral single-period deviation from $\psi_l(b)$ is desirable. If $b > b^*$ then selection of $\mathbf{x}_m^*(b)$ involves reelection and selection of any other feasible policy involves either loss of office or less of x_2. We let $v_l^l(b)$ denote the continuation value to l from being in office with constraint parameter b and $v_l^r(b)$ be the continuation value to l for having r in office with constraint parameter b. We define

$$Ev_l = [F_b(1-\gamma) - F_b(b^*)] \int v_l^l(b')dF_b(b') + [F_b(b^*) - F_b(\gamma)] \int v_l^r(b')dF_b(b'). \tag{12}$$

The continuation value to l from selecting $\mathbf{x}_m^*(b)$ (with $b > b^*$) and staying in office is

$$v_l^l(b) = u_l(\mathbf{x}_m^*(b)) + (1+\delta)\eta_l + \delta \int u_l(\psi_l(b'))dF_b(b') + \delta^2 Ev_l. \tag{13}$$

The continuation value to l from selecting $\mathbf{x}_l^*(b)$ and losing office is

$$v_l^r(b) = u_l(\mathbf{x}_l^*(b)) + \eta_l + \delta \int u_l(\psi_r(b'))dF_b(b') + \delta^2 Ev_l. \tag{14}$$

Subtracting and rearranging demonstrates the deviation is not desirable for any $b > b^*$ if (C1) is satisfied. Now if $b < b^*$ the strategy profile $\psi_l(b)$ is clearly optimal as no policy that would attain reelection is in $B(b)$ and thus selection of $\mathbf{x}_l^*(b)$ is a best response. Interchanging l and r and the appropriate ranges of b in the argument yields the result for party r.■

This SPBE involves successful control over governments that receive constraints which they find relatively desirable, and no control over governments that receive constraints that are not desirable. In the latter case the government shirks, giving itself as desirable a policy as possible and then leaves office. In the event of a desirable constraint the government forgoes the opportunity to shirk because it values the prospect of retaining office. The value to office consists of the exogenous term η_p and the endogenous term

$$\int \left[u_p(\psi_p(b')) - u_p(\psi_{-p}(b'))\right] dF_b(b'). \tag{15}$$

Note that because the equilibrium is stationary the punishment to shirking derives only from 1 period of play. It is important to note that δ and η_p affect whether this SPBE exists but they do not affect observable behavior in such a SPBE. In other words if the SPBE exists under the triple (δ, η_l, η_r) then a triple $(\delta', \eta_l', \eta_r')$ which is bigger in each coordinate induces an observationally equivalent partial control SPBE. It should also be noted that this equilibrium involves purely retrospective voting in the sense that voters base voting decisions on what the incumbent has done for them lately.[12] The voter's action serves to control shirking by parties, since the voter will punish any shirking that is observable. This feature of voting is in contrast to the Banks and Duggan model. The key distinction is that the stochastic element of the current model is not correlated with the actions of parties. Accordingly there is no room for the voter to learn about the future from past play. Thus, voting here is best characterized by the heuristic "I will vote to keep you in office only if I trust that you did not shirk in your last policy choice". Finally, note that with the assumption of symmetry there is no selection problem since neither party is ex ante "better" for the voter. Figure 4 exhibits the shape of a generic acceptance set A_l.

[12]Recall, that our notion of retrospective voting differs from Fiorina's, in that the voter does not use a cutpoint rule.

[Figure 4 about here]

Given the argument preceding proposition 1 we see that it is impossible (in a pure strategy SPBE) to control l (r) for $b < (>) b^*$. This means that this SPBE (if it exists) involves the maximal amount of control. Exactly half of the possible (b, g) pairs can be controlled.[13]

Corollary 1 *If the SPBE in proposition 2 exists no other pure strategy SPBE exhibits control on any more pairs (b, g).*

When symmetry is satisfied but C1 or C2 fail, it may be possible to attain SPBE with control for a smaller subset of the possible (b, g) pairs. We do not consider this extension as no additional intuition is gained, and C1 and C2 are satisfied as long as (η_l, η_r, δ) are big enough. Symmetry on the other hand involves a knife edged condition and we want to understand what happens when the condition does not hold. If symmetry is violated and the parties use the policy functions defined in proposition 2, punishment of one of the parties is no longer a best response for the voter as condition 1 would require that the one party is always reelected and the other party is never reelected. The voter strictly prefers having one party in office and that party will not find the threat of punishment credible following a single period deviation. In this case the selection problem seems to make credible solution of the control problem impossible. However, we can modify this SPBE to accommodate cases where symmetry is not satisfied. One modification involves reducing the cases where the advantaged or favored party is controlled.

Definition 3 *We say that party l is advantaged if given the two policy mappings*

$$\psi_l(b) = \begin{cases} \mathbf{x}_m^*(b) \text{ if } b > b^* \\ \mathbf{x}_l^*(b) \text{ otherwise} \end{cases}$$

$$\psi_r(b) = \begin{cases} \mathbf{x}_m^*(b) \text{ if } b < b^* \\ \mathbf{x}_r^*(b) \text{ otherwise} \end{cases} .$$

we have $\int u_m(\psi_l(b))dF_b(b) > \int u_m(\psi_r(b))dF_b(b)$.

Specifically, if l is the advantaged party then for some $b^{\#} > b^*$ whenever $b > b^{\#}$, l will select $\mathbf{x}_l^*(b)$ and m will not punish l. This makes the value of having l in office decrease. Accordingly, in constructing a SPBE with partial control when symmetry fails we will use a $b^{\#}$ which is chosen to equate the expected utility to m of having each party in office. The advantaged party will then shirk for some values of b on the desirable side of b^* (namely the extreme ones with $b > b^{\#}$). It is obvious that analysis with r advantaged would be completely analogous. We now formalize this extension.

[13] Of course control of a particular party may happen more or less than half the time in the SPBE, as the distribution $F_b(b)$ may assign any probability to the set $[\gamma, b^*]$.

We first define the critical point $b^{\#}$ by the equation

$$\int_{\gamma}^{b^*} u_m(\mathbf{x}_l^*(b))dF(b) + \int_{b^*}^{b^{\#}} u_m(\mathbf{x}_m^*(b))dF(b) + \int_{b^{\#}}^{1-\gamma} u_m(\mathbf{x}_l^*(b))dF(b) = \int u_m(\psi_r(b))dF_b(b) \tag{16}$$

where the mapping $\psi_r(b)$ is identical to that in definition 2. Using the intermediate value theorem we can establish the existence and uniqueness of the point $b^{\#}$ when l is advantaged. The proof of the following lemma appears in the appendix.

Lemma 1 *If l is advantaged and*

$$\int u_m(\psi_r(b))dF_b(b) > \int u_m(\mathbf{x}_l^*(b))dF_b(b) \tag{wa}$$

then exactly one point $b^{\#} \in (b^*, 1-\gamma)$ exists that solves (16).

Condition (wa) states that l is not so advantaged that the voter would prefer having l select her optimum in every state to having r use the partial control strategy. The analogue to proposition 2 when l is advantaged (but not by too much) can now be stated and proven.

Proposition 3 *If l is advantaged and condition (wa) holds then the following profile is supportable as a SPBE with partial control*

$$\begin{aligned}
A_l &= \{\mathbf{x} : b^{\#} > \mathbf{x}_m^{*-1}(\mathbf{x}) > b^*\} \cup \{\mathbf{x} \in \beta : \ x_2 \geq \frac{1}{b^{\#}} \ \} \\
A_r &= \{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) < b^*\} \\
\psi_l'(b) &= \begin{cases} \mathbf{x}_m^*(b) \ \text{if } b \in (b^*, b^{\#}) \\ \mathbf{x}_l^*(b) \ \text{otherwise} \end{cases} \\
\psi_r'(b) = \psi_r(b) &= \begin{cases} \mathbf{x}_m^*(b) \ \text{if } b < b^* \\ \mathbf{x}_r^*(b) \ \text{otherwise} \end{cases}
\end{aligned}$$

if the following conditions are satisfied

$$\frac{\max_{b\in[b^*,b^{\#}]}[u_l(\mathbf{x}_l^*(b)) - u_l(\mathbf{x}_m^*(b))]}{\eta_l + \int [u_l(\psi_l'(b')) - u_l(\psi_r'(b'))]\, dF_b(b')} \leq \delta \tag{C1'}$$

$$\frac{\max_{b\in[\gamma,b^*]}[u_r(\mathbf{x}_r^*(b)) - u_r(\mathbf{x}_m^*(b))]}{\eta_r + \int [u_r(\psi_r'(b')) - u_r(\psi_l'(b'))]\, dF_b(b')} \leq \delta. \tag{C2'}$$

Proof: By construction $b^{\#}$ is chosen so that Condition 1 is satisfied by $\psi_l'(b)$ and $\psi_r'(b)$. It remains only to show that the policy functions are mutual best responses.

-Consider party l : Assume that $\psi_r'(b)$ and A_l, A_r are given by the proposition. It is sufficient to show that no unilateral single-period deviation from $\psi_l'(b)$ is desirable. If $b > b^{\#}$ then selection of $\mathbf{x}_l^*(b)$ is clearly optimal as it results in reelection and is the optimal feasible policy for l. Thus no deviation from $\psi_l'(b)$ is desirable in this case. If $b \in (b^*, b^{\#})$, no policy in $\{\mathbf{x} \in \beta : x_2 \geq \frac{1}{b^{\#}}\ \}$ is feasible and thus l faces exactly the choice she did under the ballot function in proposition 2. Thus the proof of the optimallity of $\psi_l'(b)$ for $b \in (b^*, b^{\#})$ is the same as that for the optimallity of $\psi_l(b)$ for $b \in [b^*, 1-\gamma]$ in the proof of proposition 2 and the associated condition C1' attains. Similarly the optimallity of $\psi_{rl}'(b)$ follows from a similar argument.■

When l is advantaged it is not possible to support SPBE in which l chooses $\mathbf{x}_l^*(b)$ for values of b that are moderately higher than b^* and l chooses $\mathbf{x}_m^*(b)$ for values of b that are substantially higher than b^* . A ballot function like this is not incentive compatible for values of $b > b^*$. If l is supposed to select $\mathbf{x}_m^*(b)$ for a moderately high b but not a very high b, following a very high b she could always select a policy $\mathbf{x}$ which is feasible and coincides with $\mathbf{x}_m^*(b')$ for a moderately high b' but which l prefers to $\mathbf{x}_m^*(b)$. The voter would not be able to determine if shirking had occurred. That is, as b goes from b^* to $1-\gamma$ incentive compatibility requires that we only transition from requirements of $\mathbf{x}_m^*(b)$ to $\mathbf{x}_l^*(b)$. Even though $b^{\#}$ is unique the SPBE in proposition 3 may not maximize the amount of control. It may be possible to satisfy condition 1 by making the disadvantaged party, r, select a policy that is more desirable for m (than $\mathbf{x}_r^*(b)$) for some values of $b > b^*$. In the next section we address this issue in characterizing the optimal pure strategy SPBE. We can however conclude the following.

Corollary 2 *If the SPBE in proposition 3 exists no other pure strategy SPBE exhibits control on any more pairs* (b, r).

Corollaries 1 and 2 have an alternative interpretation. Instead of considering the set of pairs (b, g) for which control occurs, we can consider the probability that the government enacts the voter's constrained optimum $\mathbf{x}_m^*(b)$. The nature of the equilibrium in proposition 3 and the argument proceeding corollary 2 imply the following result.

Corollary 3 *In the equilibrium of proposition 3, (1) a government of the right party selects the voter's constrained optimum with probability* $F(b^*)$, *(2) a government of the left party selects the voter's constrained optimum with probability* $F(b^{\#}) - F(b^*)$, *and (3) no pure strategy equilibrium involves control of the disadvantaged party with higher probability.*

3.6 Voter welfare analysis

In principle it may be possible for m to prefer always having party l in office and uncontrolled. The single period expected utility to m of this arrangement is

$$Eu_m^l = \int u_m(0, \frac{1}{b})dF_b(b). \tag{17}$$

Since m is indifferent between having either party in office in the equilibrium of proposition 3, the single period expected utility to m from the equilibrium in proposition 3 is

$$\int_{\gamma}^{b^*} u_m(0, \frac{1}{b})dF_b(b) + \int_{b^*}^{b^\#} u_m(\mathbf{x}_m^*(b))dF_b(b) + \int_{b^\#}^{1-\gamma} u_m(0, \frac{1}{b})dF_b(b). \tag{18}$$

By inspection, we see that the latter is clearly higher than the former as $u_m(\mathbf{x}_m^*(b)) \geq u_m(0, \frac{1}{b})$ for every b.

[Figure 5 about here]

Figure 5 depicts (for a generic example) the set of policies that can be enacted in the equilibrium of proposition 3 and those that can be enacted under the selection equilibrium of always reelecting party l. Inspection of the two sets of feasible pictures demonstrates the algebraic argument. The conclusion is a strong *contradiction of Fearon's conjecture*, "Introduce any variation in politician's attributes or propensities relevant to their performance in office, and it makes sense for the electorate to focus completely on choosing the best type when it comes to vote" (p. 77). Instead, we have just established the following result.

Proposition 4 *If the equilibrium in proposition 3 exists then the public,* m, *would rather solve the control problem by using the SPBE in proposition 3, then the selection problem of always deferring to the advantaged party,* l.

While we have shown that solving the control problem is preferred to solving the selection problem, we have said nothing about the optimal pure strategy equilibria (in terms of the voter's utility). Optimal pure strategy SPBE are similar to the equilibrium in proposition 3. For desirable constraints the government will be controlled. The difference lies in what happens when the constraint is undesirable. In the above equilibrium a government facing an undesirable constraint cannot retain office, and so its best response is to shirk (enacting $\mathbf{x}_p^*(b)$ instead of $\mathbf{x}_m^*(b)$). This is costly for m. One possibility is to allow the point $(1, 1)$ to result in reelection. In this case any government can always select a policy that results in reelection. Furthermore if δ is large enough this will be desirable. Accordingly, for some parameterizations there is a pure strategy SPBE in which on the equilibrium path the same party is always retained and the maximal amount of partial control occurs. In Appendix B we characterize the optimal pure strategy equilibria.

4 Uncertainty about the price and level of the constraint

We now consider the case where there is more uncertainty about the form of the feasibility constraint. Let $B(b,c) = \{\mathbf{x} \in X : bx_1 + (1-b)x_2 \leq c\}$. We assume that in each period b^t and c^t are generated by independent draws from the continuous and strictly monotone joint distribution function $F_{bc}(b,c)$. The support is assumed to be $[\gamma, 1-\gamma]\times$ $[\varsigma, 1+\varsigma]$ with $0 < \varsigma < 1$. We redefine $\mathbb{B}$ in the natural manner. We define $\mathbf{x}^*_m(b,c)$ and $\mathbf{x}^*_p(b,c)$ and $\psi_p(b,c)$ in the natural manner. Extension of the concepts partial and full control is natural.

Definition 4 *We say a SPBE exhibits* ***perfect control*** *if for every* $p \in P$ $\psi_p(b,c) = \mathbf{x}^*_m(b,c)$ *for almost every* b,c. *We say a SPBE exhibits* ***partial control*** *if for every* $p \in P$, $\psi_p(b,c) = \mathbf{x}^*_m(b,c)$ *for* $b,c \in D_p$ *with* D_p *a subset of* $[\gamma, 1-\gamma] \times [\varsigma, 1+\varsigma]$ *having positive measure.*

Uncertainty about the resource level c has dramatic implications for the possibility of monitoring and control. If m expects l to select $\mathbf{x}^*_m(b,c)$ for any pair (b,c) with $c > \varsigma$ then there is always a deviation to a feasible policy $\mathbf{x}' = \mathbf{x}^*_m(b',c') \in B(b,c)$ with $c' < c$ and $b' > b$ that is desirable for l. Similarly, r has an incentive to deviate from $\mathbf{x}^*_m(b,c)$ to a policy $\mathbf{x}'' = \mathbf{x}^*_m(b'',c'') \in B(b,c)$ with $c'' < c$ and $b'' < b$.

[Figure 6 about here]

Accordingly, now it is not possible to attain control on any subset of the space $\mathbb{B}$ in pure strategies. Figure 6 illustrates this point by plotting two constraints $B(b,c)$ and $B(b',c')$ and two possible points $\mathbf{x}^*_m(b,c)$ and $\mathbf{x}^*_m(b',c')$. Upon observing $\mathbf{x}^*_m(b',c')$ the voter cannot determine if b,c have attained and l has shirked or if b',c' have attained and l has not shirked. With the addition of uncertainty about c, equilibria of the type in proposition 3 cannot be constructed. Here the monitoring problem is almost always severe. This leads us to the following conclusion.

Proposition 5 *With uncertainty about* b *and* c *there are no pure strategy SPBE in which partial control occurs.*

However, if the slope of $h_p(\cdot)$ is not too steep there are mixed strategy SPBE in which perfect control occurs. The construction hinges on creating mixed ballot functions that induce each party to choose $\mathbf{x}^*_m(b,c)$. We let $\lambda(\mathbf{x},p)$ denote the probability that p is retained if she selects policy $\mathbf{x}$. Using the single deviation principle we can derive the incentive compatibility condition that a mixed ballot function must satisfy. Suppose both parties will select $\mathbf{x}^*_m(b,c)$ whenever they are in office (except possibly for party l this period). Given this, a ballot function $\lambda(\mathbf{x},l)$ and a constraint $B(b,c)$, party l must solve the problem

$$\arg\max_{\mathbf{x}\in B(b,c)} h_l(x_2) + \lambda(\mathbf{x},l)\delta\eta_l + k \tag{19}$$

where k is a constant with respect to the choice variable $\mathbf{x}$. By definition we have

$$\mathbf{x}_m^*(b,c) = \arg \max_{\mathbf{x} \in B(b,c)} u_m(\mathbf{x}). \tag{20}$$

Since (19) and (20) involve the same constraint, if $\lambda(\mathbf{x}, l)$ is chosen so that the first order conditions from problem (19) are equated with the first order conditions from (20), party l will have an incentive to choose $\mathbf{x}_m^*(b,c)$. This requires

$$\begin{aligned} \frac{\partial h_l(x_2)}{\partial x_2} + \delta\eta_l \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_2} &= \frac{\partial u_m(\mathbf{x})}{\partial x_2} \\ \delta\eta_l \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_1} &= \frac{\partial u_m(\mathbf{x})}{\partial x_1} \end{aligned} \tag{21}$$

Rearranging yields

$$\begin{aligned} \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_2} &= \frac{1}{\delta\eta_l}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_2} - \frac{\partial h_l(x_2)}{\partial x_2}\right) \\ \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_1} &= \frac{1}{\delta\eta_l}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_1}\right). \end{aligned} \tag{22}$$

A function that satisfies this condition is

$$\lambda((x_1,x_2),l) = \frac{1}{\delta\eta_l}\left(u_m(\mathbf{x}) - h_l(x_2)\right) + q \tag{23}$$

where q is a scalar. It remains only to verify that it is possible to construct a mapping $\lambda(\mathbf{x}, l)$ with image $[0,1]$ that satisfies these conditions. This requires that

$$\max_{\mathbf{x} \in \beta} \lambda((x_1,x_2),l) - \min_{\mathbf{x} \in \beta} \lambda((x_1,x_2),l) < 1. \tag{24}$$

This difference is bounded by

$$\frac{1}{\delta\eta_l}\left(h_l(\frac{1+\varsigma}{\gamma}) - h_l(0)\right). \tag{25}$$

Thus, if

$$h_l(\frac{1+\varsigma}{\gamma}) - h_l(0) \leq \delta\eta_l \tag{C7}$$

an incentive compatible mixed ballot function can be constructed. Similar logic yields the conditions

$$\begin{aligned} \frac{\partial \lambda(\mathbf{x}, r)}{\partial x_1} &= \frac{1}{\delta\eta_r}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_1} - \frac{\partial h_r(x_1)}{\partial x_1}\right) \\ \frac{\partial \lambda(\mathbf{x}, r)}{\partial x_2} &= \frac{1}{\delta\eta_r}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_2}\right) \end{aligned} \tag{26}$$

$$h_r(\frac{1+\varsigma}{\gamma}) - h_r(0) \leq \delta\eta_r. \tag{C8}$$

We are then left with the following conclusion.[14]

Proposition 6 *With uncertainty about b and c (or just b), if conditions C7 and C8 are satisfied there exist scalars q_l and q_r such that full control is supportable in a mixed strategy SPBE with the following ballot functions:*

$$\lambda(\mathbf{x}, l) = \frac{1}{\delta\eta_l}\left(u_m(\mathbf{x}) - h_l(x_2)\right) + q_l$$
$$\lambda(\mathbf{x}, r) = \frac{1}{\delta\eta_r}\left(u_m(\mathbf{x}) - h_r(x_1)\right) + q_r.$$

The mixed strategy equilibria characterized above are first-best for the public. Since the stationarity assumption only limits the size of the carrot and stick, when C7 and C8 are satisfied (and thus the carrot and stick are big enough), the restriction to stationary strategies does not limit the public's ability to control the government in mixed strategies, as there are no equilibria that do better than these stationary mixed strategy equilibria. When conditions C7 and C8 are not satisfied allowing punishment to last for multiple periods can make full control in mixed strategies possible.

More generally, relaxing the restriction of stationary strategies has no effect on the amount of control when $\delta\eta_p$ are high enough for the equilibria in propositions 2,3 and 6 to exist. However, when the value of retaining office is too small, multi-period punishments and rewards may enlarge the set of parameterizations for which partial (and in the case of mixed strategies full) control is possible.

5 Extension - varying valence advantages

While we consider the case of violations of the symmetry condition in propositions 3,4,5,7 there are alternative potential sources of asymmetry. One possibility is that the l party has a valence advantage so that when the l party is in office and enacts policy $\mathbf{x}$ the voter's per period payoff is $u_m(\mathbf{x}) + \varepsilon$ where $\varepsilon > 0$ is the net valence advantage of party l. When party r is in office and enacts policy $\mathbf{x}$ the voter's per period payoff is $u_m(\mathbf{x})$. As long is v is not too large propositions 3 and 4 attain in this model. The analysis is virtually unchanged as the valence advantage just makes party l advantaged. In this case as v increases $b^{\#}$ decreases. When $\varepsilon > 0$ the mixed strategy analysis is affected. Since the mixed strategy equilibrium that supports proposition 7 involves perfect control, $\varepsilon > 0$ means that m is no longer indifferent between each party. As a consequence if the parties are controlled then mixing is not a best response for m. If ε is not too big it is possible to characterize a mixed strategy equilibrium in which party r always selects $x_m^*(b, c)$ and party l shirks a bit. Qualitatively the equilibrium is quite similar to the one when $\varepsilon = 0$.

[14] The mixed strategy equilibriua can also be constructed in the simpler model where there is no uncertainty about c.

A more interesting possibility is that the valence advantage varies over time. A simple extension that captures this possibility involves ε_t independently drawn from a distribution $H(\cdot)$ on $[\underline{k}, \overline{k}]$. We assume that the realization of ε_{t+1} is revealed after the choice of policy in period t but before the selection of the government for period t. Once realized, the value is public information. The shock, ε_{t+1}, combines with policy utility, $u_m(\mathbf{x}_{t+1})$, to give the period $t+1$ payoff to the voter. One plausible interpretation is that the incumbent receives accurate polling data measuring partisan or personality-based preferences (independent of policy). Intuitively, if l is in office for period t and a positive value of ε_t is realized then the voter will be predisposed to keep l in office for period $t+1$. As a consequence, the shock ε_t results in variation in the identity of the advantaged candidate and the magnitude of the advantage. If the parties are using strategies that yield the same expected policy utility then the shock ε_t will result in a strict preference over the parties. This strict preference has important implications for control. Given the logic behind condition 1, some degree of control requires that the new shock dependent strategies $\psi_c(b, c, \varepsilon)$ involve shock compensation. Partial (or full) control requires that

Condition 2 *Given the mappings $\psi_l(b, c, \varepsilon), \psi_r(b, c, \varepsilon)$, the mapping $v(\mathbf{x}, g, \varepsilon)$ is sequentially rational iff*

$$\int u_m(\psi_l(b, c, \varepsilon))dF_{bc}(b, c) + \varepsilon > (<) \int u_m(\psi_r(b, c, \varepsilon))dF_{bc}(b, c)$$

implies $v(\mathbf{x}, l, \varepsilon) = 1(0)$ and $v(\mathbf{x}, r, \varepsilon) = 0(1)$.

We now characterize an optimal (for the voter) mixed strategy equilibrium. Optimallity requires that the less desirable party enact the voter's most preferred policy. Suppose when $\varepsilon < 0$, l selects

$$\overline{\psi}_l(b, c, \varepsilon) = \mathbf{x}^*_m(b, c) \tag{27}$$

Given this and Condition 2, indifference by the voter requires that when $\varepsilon < 0$ party r's strategy satisfy the condition

$$\int u_m(\mathbf{x}^*_m(b, c))dF_{bc}(b, c) + \varepsilon = \int u_m(\psi_r(b, c, \varepsilon))dF_{bc}(b, c). \tag{28}$$

For a fixed $\varepsilon < 0$ let $C_r(\varepsilon)$ denote the set of functions $\psi_r(b, c, \varepsilon)$ that satisfy this constraint. This set is non-empty as long as the lower bound $\underline{k}$ on the support of ε is greater than

$$k^- := \int u_m((\frac{c}{b}, 0))dF_{bc}(b, c) - \int u_m(\mathbf{x}^*_m(b, c))dF_{bc}(b, c). \tag{29}$$

Assume that $\underline{k} \geq k^-$ and let $\overline{\psi}_r(b, c, \varepsilon)$ be a selection from the set

$$\arg\max\{u_r(x(b, c)) \text{ s.t. } x(b, c) \in C_r(\varepsilon)\}. \tag{30}$$

Suppose when $\varepsilon > 0$ r selects

$$\overline{\psi}_r(b,c,\varepsilon) = \mathbf{x}_m^*(b,c). \tag{31}$$

Given this and Condition 2 indifference by the voter requires that when $\varepsilon > 0$ party l's strategy satisfy the condition

$$\int u_m(\mathbf{x}_m^*(b,c))dF_{bc}(b,c) = \int u_m(\psi_l(b,c,\varepsilon))dF_{bc}(b,c) + \varepsilon. \tag{32}$$

For a fixed $\varepsilon > 0$ let $C_l(\varepsilon)$ denote the set of functions $\psi_l(b,c,\varepsilon)$ that satisfy this constraint. This set is non-empty as long as the upper bound $\overline{k}$ on the support of ε is less than

$$k^+ := \int u_m(\mathbf{x}_m^*(b,c))dF_{bc}(b,c) - \int u_m((0, \frac{c}{1-b}))dF_{bc}(b,c). \tag{33}$$

Assume that $\overline{k} \leq k^+$ and let $\overline{\psi}_l(b,c,\varepsilon)$ be a selection from the set

$$\arg\max\{u_l(x(b,c)) \text{ s.t. } x(b,c) \in C_l(\varepsilon)\}. \tag{34}$$

Thus, given ε_t the voter is indifferent between the parties if they use the following ε_t dependent strategies

$$\psi_l(b,c,\varepsilon) = \begin{cases} \mathbf{x}_m^*(b,c) \text{ if } \varepsilon_t \leq 0 \\ \overline{\psi}_l(b,c,\varepsilon) \text{ otherwise} \end{cases} \tag{35}$$

$$\psi_r(b,c,\varepsilon) = \begin{cases} \mathbf{x}_m^*(b,c) \text{ if } \varepsilon_t \geq 0 \\ \overline{\psi}_r(b,c,\varepsilon) \text{ otherwise} \end{cases}. \tag{36}$$

It remains only to characterize party and ε_t dependent reelection mixtures that make these strategies best responses for the parties. Conditional on $\varepsilon_t \leq (\geq) 0$ the function $\lambda(\mathbf{x}, l)$ ($\lambda(\mathbf{x}, r)$) characterized above works. For the remaining cases it is sufficient for the voter to use functions $\overline{\lambda}(\mathbf{x}, l, \varepsilon), \overline{\lambda}(\mathbf{x}, r, \varepsilon)$ that satisfy the condition: for every $\varepsilon > 0$

$$\overline{\psi}_r(b,c,\varepsilon) \in \arg\max_{\mathbf{x}\in B(b,c)} h_l(x_2) + \overline{\lambda}(\mathbf{x}, l, \varepsilon)\delta\eta_l \tag{37}$$

and for every $\varepsilon < 0$

$$\overline{\psi}_l(b,c,\varepsilon) \in \arg\max_{\mathbf{x}\in B(b,c)} h_l(x_2) + \overline{\lambda}(\mathbf{x}, r, \varepsilon)\delta\eta_r. \tag{38}$$

Since $u_c(\overline{\psi}_c(b,c,\varepsilon)) \geq u_c(\mathbf{x}_m^*(b,c))$ the bounds (C7) and (C8) are sufficient to ensure that functions $\overline{\lambda}(\mathbf{x}, l, \varepsilon), \overline{\lambda}(\mathbf{x}, r, \varepsilon)$ satisfying the above condition exist. We are thus left with the result.

Proposition 7 *With uncertainty about b_t,c_t and ε_t drawn from the support $[k^-, k^+]$ if conditions C7 and C8 are satisfied the following strategies are supportable in a mixed SPBE:*

$$\psi_l(b,c,\varepsilon) = \begin{cases} \mathbf{x}_m^*(b,c) \ \textit{if } \varepsilon_t \leq 0 \\ \overline{\psi}_l(b,c,\varepsilon) \ \textit{otherwise} \end{cases}$$

$$\psi_r(b,c,\varepsilon) = \begin{cases} \mathbf{x}_m^*(b,c) \ \textit{if } \varepsilon_t \geq 0 \\ \overline{\psi}_r(b,c,\varepsilon) \ \textit{otherwise} \end{cases} .$$

$$\lambda(\mathbf{x},l,\varepsilon) = \begin{cases} \overline{\lambda}(\mathbf{x},l,\varepsilon) \ \textit{if } \varepsilon > 0 \\ \lambda(\mathbf{x},l) \ \textit{otherwise} \end{cases}$$

$$\lambda(\mathbf{x},r,\varepsilon) = \begin{cases} \overline{\lambda}(\mathbf{x},r,\varepsilon) \ \textit{if } \varepsilon < 0 \\ \lambda(\mathbf{x},r) \ \textit{otherwise.} \end{cases}$$

Furthermore, no other SPBE yields a higher payoff to the voter.

Party preferences can be interpreted as rents that the advantaged party gets to extract by selecting policies that it prefers to $\mathbf{x}_m^*(b,c)$. In other words the equilibrium offers a simple insight.

Corollary 4 *The extent to which party p will shirk in period t, $s_p(\varepsilon) := \int \|\psi_p(b,c,\varepsilon) - \mathbf{x}_m^*(b,c)\| \, dF_{bc}(b,c)$ has the following relationship with $|\varepsilon_t|$:*

(1) If $|\varepsilon_t| > 0$ then $s_l(\varepsilon_t)$ is increasing ($s_r(\varepsilon_t)$ is constant) in $|\varepsilon_t|$.

(2) If $|\varepsilon_t| < 0$ then $s_r(\varepsilon_t)$ is increasing ($s_l(\varepsilon_t)$ is constant) in $|\varepsilon_t|$.

Non-policy preferences distort the control relationship by causing the valence advantaged party to shirk-selecting policies closer to her ideal. In the one shot Downsian setting with commitment, Groseclose (2001) finds that the opposite holds. When candidates have policy and office motivation and face uncertainty about the location of the median voter the candidate with a valence advantage will select policies closer to the center of the policy space (and thus further from her ideal policy).

6 Other extensions

In this section we discuss how the results are affected by a few extensions. With respect to the number of voters the model is more general than it seems. Since voter decisions involve choices over lotteries on $\mathbb{R}_+^2$ there will be a representative voter, if the voter preferences over lotteries on $\mathbb{R}_+^2$ are representative. Banks and Duggan (2001) have shown that if preferences are quadratic and a core exists (in an arbitrary dimensional space) then the voter with the ideal point corresponding to the core is decisive over lotteries. Following this result we can construct a model with an odd population of voters having quadratic preferences: (1) with ideal points that are in the first quadrant, (2) with ideal points that have a large enough magnitude so that preferences are monotone on the set $\mathbb{B}$, and (3) such that the agent ideal points are colinear so that the core in $\mathbb{R}^2_+$ is non-empty. In this problem Banks and Duggan's result implies that there is a voter who's preferences over lotteries in $\mathbb{B}$ are decisive for majority rule. This voter would be called m.

A more direct model with n (odd) voters involves agent Von Neumann-Morgenstern utility functions on $\mathbb{R}^2_+$ of the log-Cobb Douglass form,

$$u_i(x_1, x_2) = \alpha_i \ln x_1 + (1 - \alpha_i) \ln x_2.$$

The expected utility extension of these preferences to lotteries on $\mathbb{R}^2_+$ yields the representation of the expected utility of a lottery λ on $\mathbb{R}^2_+$

$$\int u_i(x_1, x_2) d\lambda(x_1, x_2) = \alpha_i \int \ln x_1 d\lambda(x_1) + (1 - \alpha_i) \int \ln x_2 d\lambda(x_2),$$

where the right-hand side is attained from the linearity of the expectation operator. Accordingly, lottery λ is weakly preferred to lottery φ iff

$$\begin{gathered}\alpha_i \int \ln x_1 d\lambda(x_1) + (1 - \alpha_i) \int \ln x_2 d\lambda(x_2) \geq \\ \alpha_i \int \ln x_1 d\varphi(x_1) + (1 - \alpha_i) \int \ln x_2 d\varphi(x_2).\end{gathered}$$

This holds iff

$$\frac{\alpha_i}{1 - \alpha_i} \geq \frac{\int \ln x_2 d\varphi(x_2) - \int \ln x_2 d\lambda(x_2)}{\int \ln x_1 d\lambda(x_1) - \int \ln x_1 d\varphi(x_1)}.$$

Thus, we have shown that n agents have order restricted preferences over the set of lotteries on $\mathbb{R}^2_+$ and they are ordered by α_i. Since order restricted preferences are representative, the agent with the median value of α_i is decisive and we can call this voter m.

A slight relaxation of the assumption that parties care only about one issue has no effect on the results. This assumption dramatically simplifies the notation without altering the general incentives of the problem. As long as $MRS_l(\mathbf{x})$ is sufficiently low and $MRS_r(\mathbf{x})$ is sufficiently high for every $\mathbf{x}$ the analysis presented here holds. In this sense the results of the current model are not knife-edged with respect to this assumption. More extreme departures that still satisfy the condition $MRS_l(\mathbf{x}) < MRS_m(\mathbf{x}) < MRS_h(\mathbf{x})$ are not problematic, as the analysis can be applied to the portion of the feasibility constraint on which party and voter preferences are not aligned. If this ordering is not satisfied then the incentives may be quite different, and the results will not generally hold.

The standard model assumes that only the in-party government knows the parameters of the constraint. We can relax this assumption by considering an extension in which the out-party observes b^t, c^t and can make an announcement $a_t(b^t, c^t, \mathbf{x}^t; p) \in \{0, 1\}$ after policy $\mathbf{x}^t$ is enacted but before voting for the $t + 1$

period government. One interesting question is: is it possible to support more control that that occurring in proposition 3 in pure strategy SPBE? While monitoring by the out-party seems plausible, this type of behavior cannot be supported in stationary equilibria. The reason is simple. If voters react to the message a_t then since the out-party wants to be reelected, it will have an incentive to select a_t which results in the removal of the incumbent (or at least increases the probability of this). Thus the cheap talk nature of the out-party's communication renders it meaningless in this setting where the out party always wants to remove the incumbent. The only way that such dishonesty can be controlled is for the parties to somehow punish each other for such behavior. This type of behavior is not possible in stationary strategies.

Relaxing the assumption that the feasibility constraint has a linear boundary is quite simple. A natural extension is to assume that every constraint is compact and convex and that the intersection of all possible constraints is a singleton $\mathbf{x}^*$. This point then corresponds to the point $(1, 1)$ in the model in which $c = 1$. If the family of constraint boundaries represent rotations about this common intersection point results similar to propositions 1-5 can be attained in a model of this form.[15] More precisely stated, if $\{B_\rho\}_\rho$ is the collection of feasible constraint boundaries then propositions 1-5 would extend if $\{\cap_\rho B_\rho\} = \mathbf{x}^*$ is a singleton and for every $\rho \neq \rho'$, $B_\rho \cap B_{\rho'} = \mathbf{x}^*$. The logic behind these results breaks down if there are pairs of constraints that have some intersections that are to the northwest and others that are to the southeast of the point $\mathbf{x}^*$. More generally, as long as the public constrained optima and the parties constrained optima are well defined the construction in proposition 6 can be used to establish the existence of mixed strategy equilibria with full control when agents value office enough.

A simplifying assumption of the model is that the constraint parameters b^t or (b^t, c^t) are independently and identically distributed over time. If only the identical part of the assumption is relaxed the pure strategy analysis extends as long as time dependent versions of condition 1 and C1' and C2' are satisfied. In mixed strategies. the equilibrium characterized in proposition 6 will exist unchanged since the voter is indifferent between having either party in office when both parties select $\mathbf{x}_m^*(b)$. If it is assumed that b^t or (b^t, c^t) are not independent over time then again the mixed strategy equilibrium with full control still exists unchanged for precisely the same reason – the voter is indifferent between either party enacting $\mathbf{x}_m^*(b)$ regardless of the distribution on b^t and c^t. The implications for the pure strategy analysis are more serious. If b^t and b^{t-1} are correlated a more severe selection problem surfaces. The voter will want to select l (r) if b^{t+1} is likely to be high (low). Formally, non independence requires that a history dependent symmetry condition hold in order for m to be willing to remove a particular party. Additionally, the possibility that x^t can affect m's beliefs about b^{t+1} introduces potential incentives for p to manipulate m's beliefs. We could also allow b^t, c^t to be correlated with x^{t-1}. When the policy area involves control of economic factors this extension might be quite defensible. These extensions are beyond the scope of the current paper.

[15]See Meirowitz (2001) for analysis of social choice on convex constraint sets.

7 Discussion

The model generates some novel predictions about representation: policy differences over time, governments that are sometimes retained and sometimes thrown out (even in the long run). Both of these findings are in contrast to the repeated election models and obviously can't be compared with the static election models in the Downsian tradition. In the pure strategy equilibria with partial control we find a non-monotonic relationship between reelection and the quantity of a party's preferred issue. Most striking is the prediction that governments that enact policies that have a very high quality of the issue they care about will be retained. This last fact is a pattern that the analyst would observe from election data, but it is not a statement about the causality. When policies that contain a high degree of the party's preferred issue are feasible, the monitoring problem is solvable and thus these polices are also those that the public would enact under the constraint.

In the pure strategy partial control equilibria voters treat incumbents from different parties differently. This prediction does not surface in either the quality-based moral hazard and adverse selection models (Barro, Ferejohn, Austen-Smith and Banks, Banks and Sundaram, Ashworth) or the spatial representation models (Duggan, Banks and Duggan). While many scholars have focused on the role of party in elections, there is a paucity of empirical specifications that test the interaction of incumbent party and how voters assess the incumbents performance on different policy issues. In their analysis of state elections Lowry, Alt and Ferree find evidence that governors are disproportionately rewarded for increases in scale and balance.

> We find evidence that voters dislike both budget deficits and surpluses. Moreover, voters hold politicians accountable for changes in fiscal scale in a partisan way: They punish Republican incumbents and reward Democrats for unexpected increases in fiscal scale, but they reward Republicans and penalize Democrats for unexpected cuts. (p. 759)

The theory presented here offers a rough justification for this finding. Republicans prefer less scale and Democrats do not. Republicans are punished for increases in government scale because the voters cannot determine if the Republican increased scale enough. In contrast. Democrats are rewarded for such action because the voter can determine that the Democrat did not increase scale by too much. While far from being clear empirical evidence in favor of the theory, this stylized fact suggests that subsequent empirical analysis can shed some light on the validity of the equilibrium predictions.[16] Estimating acceptance sets for Democrat and Republican incumbents relative to budget allocations across typically Democrat and Republican issues from Gubernatorial elections might yield a direct empirical test of one of the models predictions.

The model offers a perspective on the types of rhetoric governments might use when justifying their actions. Fenno defends the importance of this feature of legislator-constituent relationships, and it is

[16] A more thorough analysis might involve comparison of the estimated probability of reelection function and the mixed strategy reelection function.

reasonable to imagine that executives also must justify their record to the voters. While the model does not involve explicit communication between voters and parties, the equilibrium intuition is easy to interpret in this manner. Following a constraint and policy choice which is supposed to result in reelection the incumbent can make speeches espousing her policy selection as in the public's interest given the constraint. There is no compelling retort that the challenger can give. In contrast, following either an out of equilibrium policy or any policy that is feasible given a constraint that does not favor the incumbent, the challenger will be able to persuade voters that the incumbent (might have) shirked. The analysis formalizes how the public can adjudicate this type of debate without knowing the constraint.

In terms of the principal agent perspective the theory is primarily one of control and not selection. While the model offers the potential for a non-trivial selection problem as one party might be ex-ante preferred, we find that in the desirable equilibrium the public is indifferent between the pool of available agents/parties and thus the selection problem is resolved. The control problem is persistent and we find only partially solvable in pure strategies. In contrast, with mixed strategies the problem is solvable. This demonstrates that a finer control device (choice of mixtures as opposed to a binary decision) allows the principal to exert increased control.

In Fearon's comparison of selection and control explanations he notes that "if politicians vary in policy preferences, even a little, then voters are no longer generically indifferent between the incumbent and possible replacements." (p.75). This conclusion does not hold in the current model. Indifference over the ideal policies of the parties is not necessary for indifference between an incumbent or challenger in equilibrium. The relevant condition is more endogenous. In equilibrium the voter must be indifferent between the lotteries induced by having either party use its equilibrium strategy in office. The intuition hinges on the analysis when symmetry is not satisfied. In this case, one party (l) is more desirable than the other in a clear policy sense. Even in this case the voter is indifferent between having either party in office. Moreover, Fearon's conclusion that the gain to voters from solving the selection problem is higher than the gain from solving the control problem is not valid in the current model. With only uncertainty about b^t, Proposition 4 indicates that the public would rather partially solve the control problem than fully solve the selection problem. In the model with uncertainty about b^t and c^t the mixed strategy SPBE are first best for the public as they induce the policy $x^*_m(b^t, c^t)$ in every period.[17] Our conclusion that control is a relevant feature of the voter-government relationship is consistent with the cited empirical findings. From an institutional choice perspective the possibility of control and its reliance on the future horizon and rents to holding office has important implications for debates about term limits and office compensation.

[17] In one case Fearon's conjecture is supported. With uncertainty about both the slope and resource level if only pure strategy equilibria are considered even partial control is infeasible.

8 Appendix A:

Proof of Lemma 1: Let the left hand side of (16) be denoted by the function $\phi(b^{\#}) : (b^*, 1-\gamma) \to \mathbb{R}^1$. Note that by continuity

$$\lim_{b^{\#} \to b^*} \phi(b^{\#}) = \int u_m(\mathbf{x}_l^*(b)) dF_b(b). \tag{39}$$

By (wa) the right hand side of the above is less than $\int u_m(\psi_r(b)) dF_b(b)$, thus

$$\lim_{b^{\#} \to b^*} \phi(b^{\#}) < \int u_m(\psi_r(b)) dF_b(b). \tag{40}$$

On the other hand

$$\lim_{b^{\#} \to 1-\gamma} \phi(b^{\#}) = \int_{\gamma}^{b^*} u_m(\mathbf{x}_l^*(b)) dF_b(b) + \int_{b^*}^{1-\gamma} u_m(\mathbf{x}_m^*(b)) dF_b(b). \tag{41}$$

The right hand side of this is greater than $\int u_m(\psi_r(b)) dF_b(b)$ because l is advantaged. Since $\phi(\cdot)$ is continuous there is a value $b^{\#}$ solving (16) by the intermediate value theorem. Since $\phi(\cdot)$ is strictly monotone this value is unique.■

9 Appendix B: Optimal pure strategy equilibria

Given this observation the optimal equilibrium can be constructed by determining when it is that the in-government party can stay in office without enacting $\mathbf{x}_m^*(b)$. Since it is not possible to create incentives for selection of $\mathbf{x}_m^*(b)$ when the constraint is disadvantageous (i.e. $b \in D_l := \{b : b < b^*\}$ for l and $b \in D_r = \{b : b > b^*\}$ for r), the best that m can do is create incentives for the government to select a relatively good policy on these sets. Recall that β is the union of the constraints and $\mathbb{B}$ is the set of possible prices. By $\widehat{B}(b) = \{\mathbf{x} : px_1 + (1-p)x_2 = 1\}$ we denote the efficient boundary of constraint $B(b)$. An efficient policy function is a mapping $\mathbf{x}_p^+(b) : \mathbb{B} \to \widehat{B}(b)$. Given such a policy function the reelection set

$$A_p^+ = \bigcup_{b \in \mathbb{B} \setminus \{b : \mathbf{x}_p^+(b) = \mathbf{x}_p^*(b) \ \& \ b \in D_p\}} \mathbf{x}_p^+(b) \tag{42}$$

involves retention of p following any policy in $\cup_{b \in \mathbb{B} \setminus D_p} \mathbf{x}_p^+(b)$ and any policy in $\cup_{b \in D_p} \mathbf{x}_p^+(b)$ that is not optimal for p under some constraint.

Characterization of the optimal acceptance sets involves first determining the set of possible efficient mappings $x_p^+(b) : \mathbb{B} \to \widehat{B}(b)$ that can be implemented in the following sense. We say a function $\mathbf{x}_p^+(b) : \mathbb{B} \to$

$\widehat{B}(b) \cup \varnothing$ is *potentially implementable* if given the set $A_p^+ = \cup_{b \in D_p} \mathbf{x}_p^+(b)$, for any $b \in D_p$ it is the case that there is no other policy $\mathbf{x}' \in B(b) \cup A_p^+$ with $u_p(\mathbf{x}') > u_p(\mathbf{x}_p^+(b))$. The statement $\mathbf{x}_p^+(b) = \varnothing$ means that at b, p cannot select any feasible policy that results in reelection. We use the term potentially implementable to describe such a mapping because for sufficiently high $\delta\eta_p$ a best response for party p to the set A_p^+ and price b will be to select policy $x_p^+(b)$. We know that on $\mathbb{B} - D_p$, $\mathbf{x}_p^+(b) = \mathbf{x}_m^*(b)$ is potentially implementable and optimal for m so we now focus on the problematic sets D_p. We show that any function which satisfies the following condition is potentially implementable:

$$\text{for every } b, b' \in D_r \text{ with } \mathbf{x}_r^+(b) \neq \varnothing \text{ and } \mathbf{x}_r^+(b') \neq \varnothing \text{ if } b < b' \text{ then } x_r^+(b')_1 \geq x_r^+(b)_1 \text{ and } x_r^+(b')_2 \geq x_r^+(b)_2 \tag{IC-R}$$

$$\text{for every } b, b' \in D_l \text{ with } \mathbf{x}_l^+(b) \neq \varnothing \text{ and } \mathbf{x}_l^+(b') \neq \varnothing \text{ if } b < b' \text{ then } x_l^+(b')_1 \leq x_l^+(b)_1 \text{ and } x_l^+(b')_2 \leq x_l^+(b)_2 \tag{IC-L}$$

Condition (IC) ensures that for a price $b \in D_p$ there is not an alternative policy $\mathbf{x}' \in B(b)$ s.t. $\mathbf{x}' = \mathbf{x}_p^+(b')$ for some value $b' \in D_p$ (implying $\mathbf{x}' \in A_p^+$) and $u_p(\mathbf{x}') > u_p(\mathbf{x}_p^+(b))$. To see why condition (IC) captures this constraint notice that for party l facing a constraint $b \in D_l$ the set of policies that are preferred to $\mathbf{x}_l^+(b)$ are those that contain more of dimension 2 and since $\mathbf{x}_l^+(b) \in \widehat{B}(b)$ (it is efficient) any feasible policy with more of dimension 2 must involve less of dimension 1. In other words if $\mathbf{x}'$ is a desirable deviation from $\mathbf{x}_l^+(b)$ then $x_1' < x_l^+(b)_1$ and $x_2' > x_l^+(b)_2$. Condition (IC) requires that there is no b' for which $\mathbf{x}' = \mathbf{x}_l^+(b')$. Informally, the set A_l^+ is a curve with positive slope so that if $\mathbf{x}_l^+(b) \in A_l^+$ no point to the northwest ($\mathbf{x}' \nwarrow \mathbf{x}_l^+(b)$) is on the curve. A similar argument holds for r. Replicating arguments in proposition 2 to attain a bound on δ yields the following result

Proposition 8 *If the mappings $\mathbf{x}_l^+(b)$ and $\mathbf{x}_r^+(b)$ are potentially implementable and the following conditions hold*

$$\int u_m(\mathbf{x}_l^+(b'))dF_b(b') = \int u_m(\mathbf{x}_r^+(b'))dF_b(b') \tag{43}$$

$$\frac{\max_{b \in \mathbb{B}}[u_l(\mathbf{x}_l^*(b)) - u_l(\mathbf{x}_l^+(b))]}{\eta_l + \int \left[u_l(\mathbf{x}_l^+(b')) - u_l(\mathbf{x}_r^+(b'))\right] dF_b(b')} \leq \delta$$

$$\frac{\max_{b \in \mathbb{B}}[u_r(\mathbf{x}_r^*(b)) - u_r(\mathbf{x}_r^+(b))]}{\eta_r + \int \left[u_r(\mathbf{x}_r^+(b')) - u_r(\mathbf{x}_l^+(b'))\right] dF_b(b')} \leq \delta$$

there is a SPBE in which the acceptance sets are A_p^+ and the policy selection mappings are $\mathbf{x}_p^+(b)$.

Proof: If the mappings $\mathbf{x}_p^+(b)$ and $\mathbf{x}_p^+(b)$ are potentially implementable then as long as δ satisfies the relevant constraint, $\mathbf{x}_p^+(b)$ is a best response for candidate p when $-p$ uses $\mathbf{x}_{-p}^+(b)$ and m uses the acceptance set A_p^+. If the first constraint is satisfied then Condition 1 is satisfied so m is willing to vote based on the acceptance sets A_l^+ and A_r^+.■

The optimal equilibrium for the voter can be chosen by choosing $\mathbf{x}_l^+(b)$ and $\mathbf{x}_r^+(b)$ to maximize $\int u_m(\mathbf{x}_l^+(b'))dF_b(b')$ subject to the constraints (IC) and (43). By assumption 2, if $\mathbf{x}_l^+(b)$ satisfies IC-L then $\mathbf{x}_m(b)$ and $\mathbf{x}_l^+(b)$ respond to changes in $b \in D_l$ in the opposite direction. This means that the constraint $x_l^+(b')_1 \leq x_l^+(b)_1$ in IC-L binds. Accordingly, the maximal expected utility to m from controlling l (if condition 1 were not imposed) can be found by selecting a point $\overline{\mathbf{x}} \in \cup_{b\in D_l}\mathbf{x}_m^*(b)$ to maximize the payoff to m from the potentially implementable mapping

$$\mathbf{x}_l^+(b;\overline{\mathbf{x}}) = \begin{cases} \mathbf{x}_m^*(b) \text{ if } b \in B - D_l \\ \mathbf{x}_l^*(b) \text{ if } b \in (\mathbf{x}_m^{-1}(\overline{\mathbf{x}}), b^*) \\ (\frac{1-(1-b)\overline{x}_2}{b}, \overline{x}_2) \text{ otherwise.} \end{cases}$$

This mapping is a best response for l to the acceptance set

$$\overline{A}_l = \left(\cup_{b\in B\setminus D_l}\mathbf{x}_m^*(b)\right) \cup \{x : x_1 \geq \overline{x}_1 \ \&\ x_2 = \overline{x}_2\}.$$

Similarly, the maximal utility that m can get from having r in office is found by selecting a point $\widetilde{\mathbf{x}} \in \cup_{b\in D_r}\mathbf{x}_m^*(b)$ to maximize the payoff to m from the potentially implementable mapping

$$\mathbf{x}_r^+(b;\widetilde{\mathbf{x}}) = \begin{cases} \mathbf{x}_m^*(b) \text{ if } b \in B - D_r \\ \mathbf{x}_l^*(b) \text{ if } b \in (b^*, \mathbf{x}_m^{-1}(\widetilde{\mathbf{x}})) \\ (\widetilde{x}_1 \frac{1-b\widetilde{x}_1}{1-b}) \text{ otherwise.} \end{cases}$$

This mapping is a best response for l to the acceptance set

$$\widetilde{A}_r = \left(\cup_{b\in B\setminus D_r}\mathbf{x}_m^*(b)\right) \cup \{x : x_1 = \widetilde{x}_1 \ \&\ x_2 \geq \widetilde{x}_2\}.$$

Let $\overline{v}_m = \arg\max_{\overline{\mathbf{x}}} \int u_m(\mathbf{x}_l^+(b;\overline{\mathbf{x}}))dF_b(b)$ and $\widetilde{v}_m = \arg\max_{\widetilde{\mathbf{x}}} \int u_m(\mathbf{x}_r^+(b;\widetilde{\mathbf{x}}))dF_b(b)$ denote the maximal values from these types of potentially implementable mapping. Proposition 8 yields the following corollary.

Corollary 5 *If $\delta\eta_p$ are sufficiently high then there exists a pure strategy SPBE in which the expected utility to m is $\frac{\min\{\overline{v}_m,\widetilde{v}_m)}{1-\delta}$ and no pure strategy SPBE yields a higher value to m.*

References

[1] Austen-Smith, David, and Jeffrey Banks. 1989. "Electoral Accountability and Incumbency." in Ordeshook, ed. *Models of Strategic Choices in Politics.* University of Michigan Press.

[2] Banks, Jeffrey, and Rangarazan Sundaram. 1993. "Moral Hazard and Adverse Selection in a Model of Repeated Elections." in Barnett et al., eds. *Political Economy: Institutions, Information, Competition, and Representation.* Cambridge University Press.

[3] Banks, Jeffrey, and Rangarazan Sundaram. 1998. "Optimal Retention in Agency Problems" *Journal of Economic Theory* 82:293-323..

[4] Banks, Jeffrey, and John Duggan. 2000. "A Multidimensional Model of Repeated Elections", *California Institute of Technology Typescript.*

[5] Banks, Jeffrey, and John Duggan. 2001. "A social choice lemma on voting over lotteries with applications to a class of dynamic games", *California Institute of Technology Typescript.*

[6] Barro, Robert. 1973. "The control of politicians: an economic model." *Public Choice* 14:19-42.

[7] Baron, David, and Ehud Kalai. 1993. "The simplest equilibrium of a majority rule game." *Journal of Economic Theory* 61:290-301.

[8] Berge, Claude. 1963. *Topological Spaces.* Translated by E.M. Patterson, New York: Macmillan.

[9] Besley, Timothy, and Anne Case 1995. "Incumbent Behavior: Vote-Seeking, Tax-Setting, and Yardstick Competition." *The American Economic Review* 85:25-45.

[10] Besley, Timothy, and Anne Case 1995. "Does Electoral Accountability Affect Economic Policy Choices? Evidence from Gubernatorial Term Limits". *The Quarterly Journal of Economics* 110:769-798.

[11] Black, Duncan, 1958. *The Theory of Committees and Elections.* Cambridge: Cambridge university Press.

[12] Caines-Wrone, Brandice, Michael Herron, and Kenneth Shotts. 2001. "Leadership and Pandering: A Theory of Executive Policymaking." *American Journal of Political Science.* 45:532-550.

[13] Calvert, Randal. 1985. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence. *American Journal of Political Science*, 28(1):127-146.

[14] Citrin, Jack. 1979. "Do People Want Something for Nothing: Public Opinion on Taxes and Government Spending." *National Tax Journal* 32:113-30.

[15] Cooper, Michael. 2002. "The Mayor's Budget Proposal" *New York Times*, Late Edition, February 14, Section A, Page 1.

[16] Davis, Otto, and Melvin Hinich. 1970. "An Expository Development of a Mathematical Model of the Electoral Process." American Political Science Review, 64(2):426-448.

[17] Downs, Anthony 1957. *An Economic Theory of Democracy.* Harper Collins. New York. N.Y.

[18] Duggan, John. 2000. "Repeated Elections with Asymmetric Information", *Economics and Politics* 12:109-136.

[19] Fearon, James. 1999. "Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance." in Przeworski et al., eds. *Democracy, accountability, and representation.* Cambridge University Press.

[20] Fenno, Richard F. Jr. 1978. *Home Style: House Members in Their Districts.* Boston: Little, Brown.

[21] Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50:5-26.

[22] Fiorina, Morris. 1973. "Electoral Margins, Constituency Influence, and Policy Moderation: A Critical Assesment." American Politics Quarterly 1:479-498.

[23] Fiorina, Morris. 1981. *Retrospective Voting in American National Elections.* New Haven:Yale University Press.

[24] Groseclose, Tim. 2001. "A Model of Candidate Location When One Candidate Has a Valence Advantage", *American Journal of Political Science*, 45(4):862-886.

[25] Hansen, John. 1998. "Individuals, Institutions, and Public Preferences over Public Finance." *American Political Science Review* 92:513-31.

[26] Ledyard, John. 1984. "The Pure Theory of Large Two-Candidate Elections." *Public Choice.* 44:7-41

[27] Lowry, Robert, C., James Alt, and Karen E. Ferree. 1998. "Fiscal Policy Outcomes and Electoral Accountability in American States." *American Political Science Review* 92(4):759-774.

[28] McKelvey, Richard, and Peter C. Ordeshook. 1985. "Elections with Limited Information: A Fulfilled Expectations Model using Contemporaneous Poll and Endorsement Data as Information Sources." *Journal of Economic Theory*, 36:55-85.

[29] McKelvey, Richard, and Peter C. Ordeshook. 1986a. "Sequential Elections with Limited Information." *American Journal of Political Science* 29(4):480-512.

[30] McKelvey, Richard, and Peter C. Ordeshook. 1986b. "Information, Electoral Equilibria, and the Democratic Ideal." *Journal of Politics* 48(4):909-937.

[31] Meirowitz, Adam. 2001. "The core of social choice problems with monotone preferences and a feasibility constraint". *Princeton University Typescript.*

[32] Modigliani, Andre, and Franco Modigliani. 1987. "The Growth of the Federal Deficit and the Role of Public Attitudes." *Public Opinion Quarterly* 51:459-80.

[33] Rogoff, Kenneth, and Anne Sibert. 1988. "Elections and Macroeconomic Policy Cycles". *The Review of Economic Studies* 55:1-16.

[34] Wittman, Donald. 1977. "Candidates with Policy Preferences: A dynamic model." *Journal of Economic Theory*, 14:180-89.

[35] Wittman, Donald. 1983. "Candidate motivation: A synthesis of alternatives." *American Political Science Review*, 77:142-57.

[36] Zaller, J., 1998, "Coming to grips with V.O. Key's concept of latent opinion". *U.C.L.A. typescript.*

Figure 1 Indifference curves

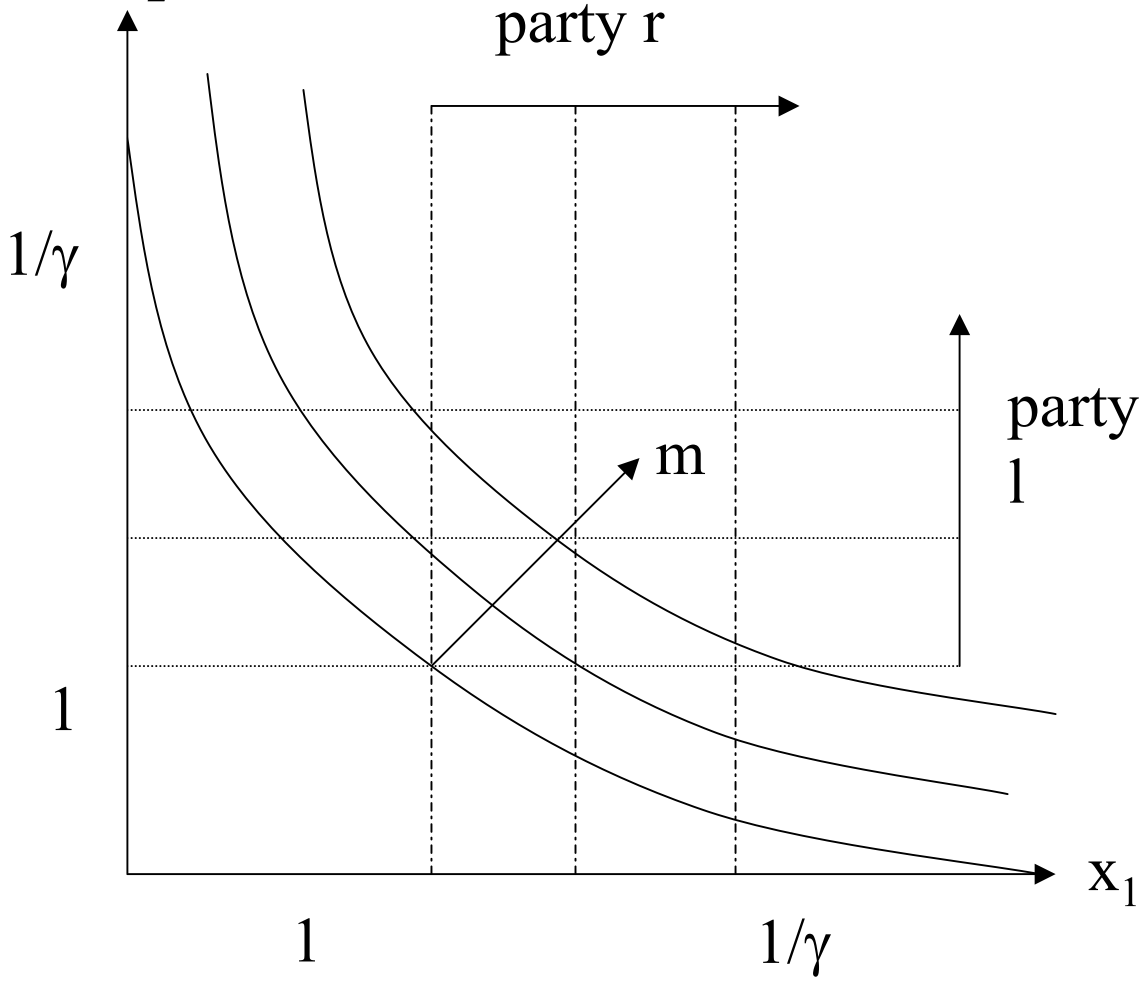


Figure 2 The set

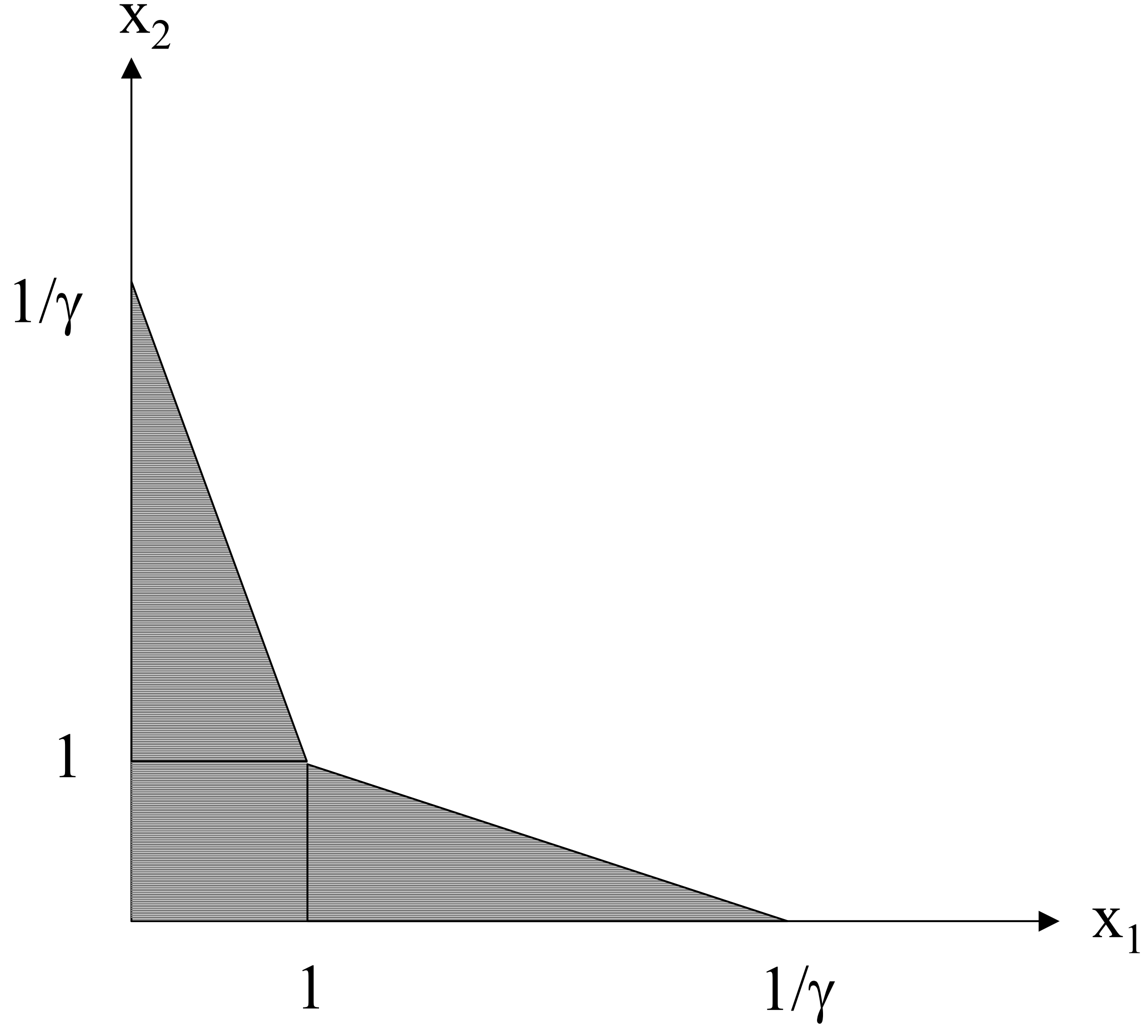


Figure 3 Monitoring problem for uncertain b

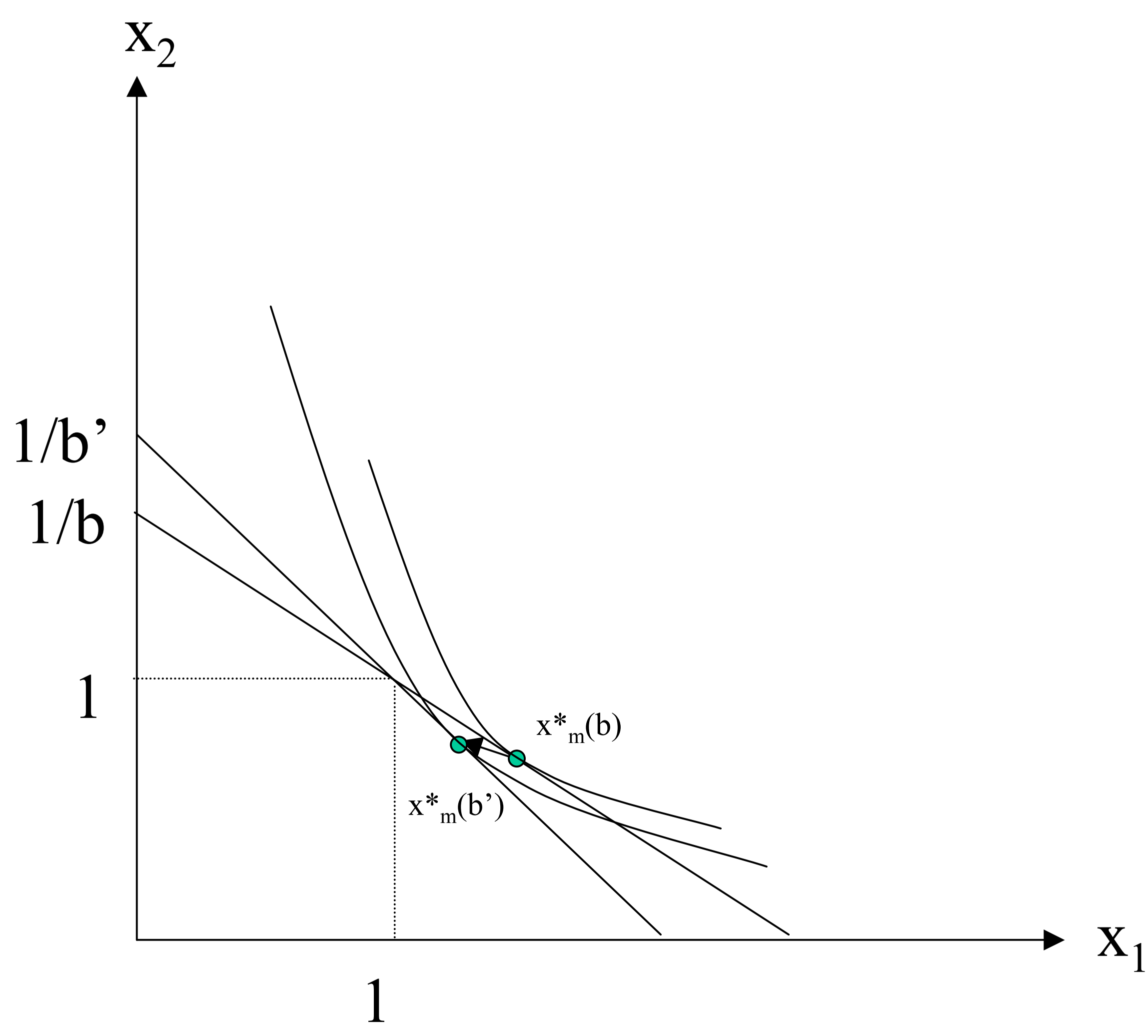


Figure 4 The set A_1

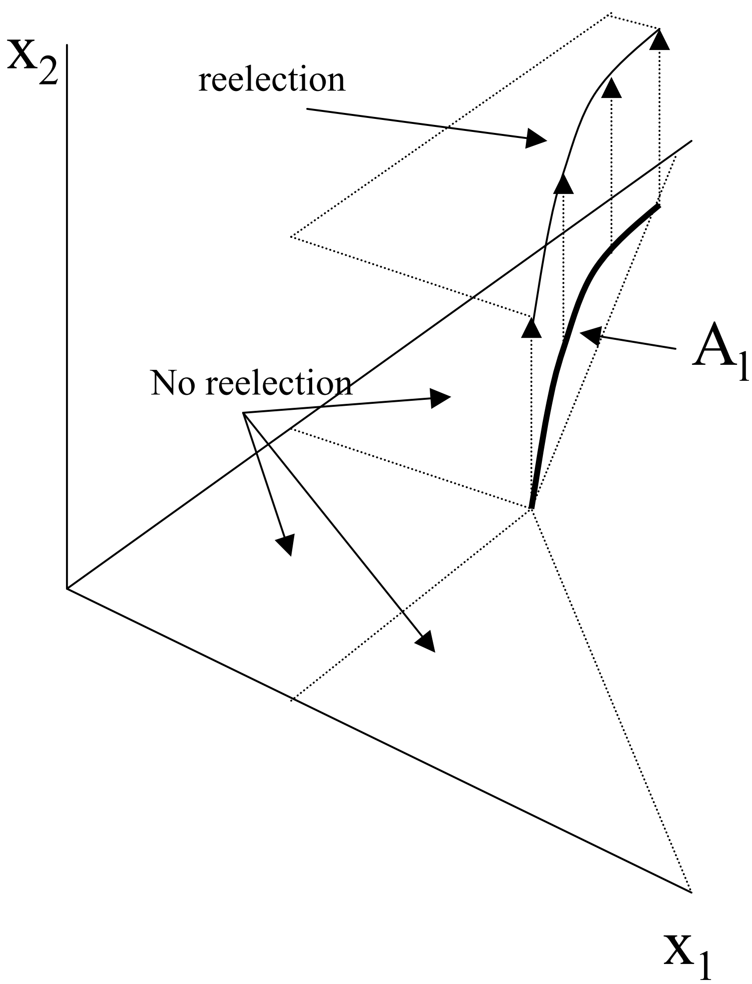
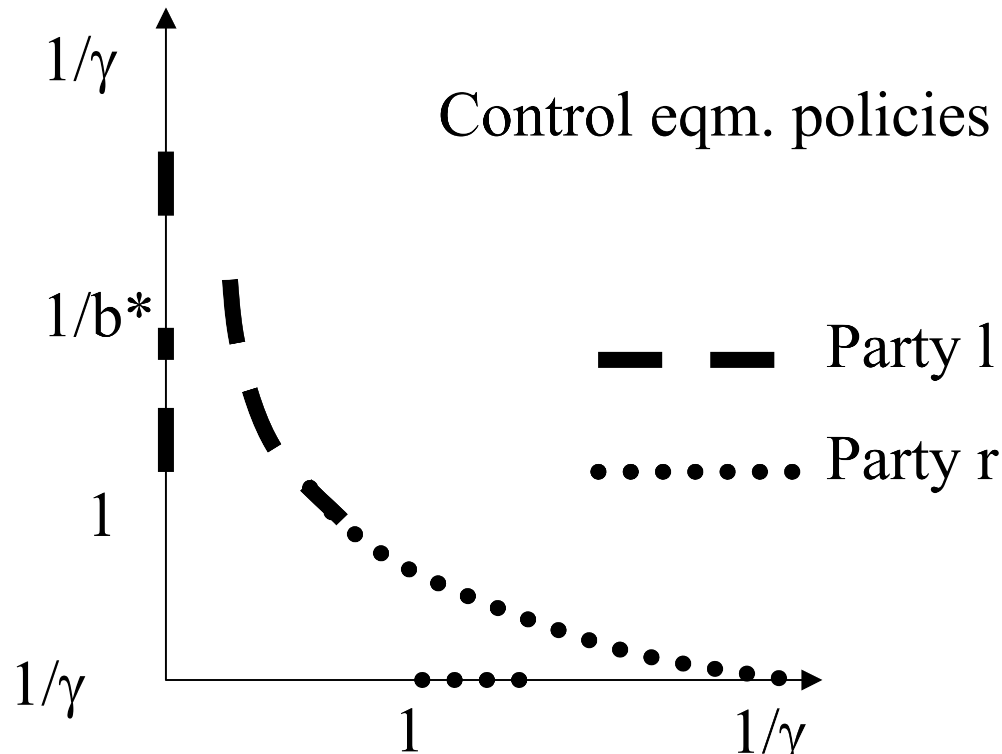


Figure 5 Control vs. selection



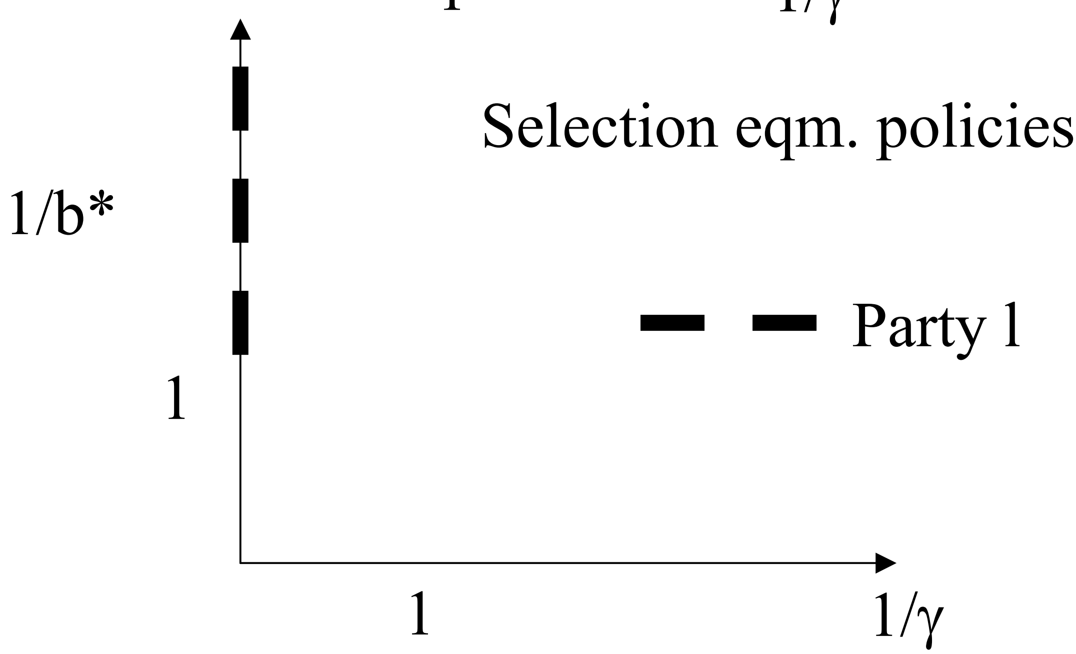


Figure 6 Monitoring problem for uncertain b,c