A New Explanation for
European Unemployment based on Rational Institutions

Rafael Di Tella*
Harvard Business School

and

Robert MacCulloch**
London School of Economics

This draft: May 19, 2001

Abstract
We show that European-style hysteresis can arise in a normative model where labor market institutions are determined optimally. We focus on the government's decision to set unemployment benefits in response to an unemployment shock. The government balances insurance considerations with the tax burden of benefits and the possibility that they introduce adverse “incentive effects” whereby benefits increase the unemployment rate. It is found that when the shock occurs, benefits should be increased in those economies where the adverse incentive effects of benefits are largest. Adjustment costs of changing benefits can introduce hysteresis in benefit setting and unemployment. A good temporary shock can permanently reduce unemployment by making it optimal to have a cut in unemployment benefits. Desirable features of the model are that we obtain an asymmetry out of a symmetric environment and that the mechanism yielding hysteresis is both simple (requires the third derivative of the utility function to be non-negative) and self-correcting.

JEL Classification: J6
Keywords: Optimal unemployment benefits, hysteresis, natural rate of unemployment.

* Morgan Hall, Soldier Field Rd, Boston, MA 02163, USA. E-mail: rditella@hbs.edu. ** STICERD, Houghton Street, London WC2A 2AE, UK. E-mail: robertmacculloch@compuserve.com. We thank Fernando Alvarez for generous ideas and suggestions as well as Tim Besley, Alan Manning, Juan Pablo Nicolini, Jorn-Steefen Pischke, Julio Rotemberg and Justin Wolfers as well as seminar participants at London School of Economics, MIT (macro), Harvard (labor), UTDT, Stanford (Political Economy) and the Milan (2000) conference of the European/American Association of Labor Economists.
I. Introduction

An important challenge in economics is to build a theoretical model that can explain European unemployment. The ideal model of hysteresis should explain two kinds of asymmetries. The first is an asymmetry over time: it should explain how unemployment could increase after an adverse shock and then remain up for a very long period of time. The second is an asymmetry across countries: ideally the theory should also be able to explain why, once the shock disappears, unemployment drops in some countries but not in others. In this paper we aim to provide one such theory. It is based on the idea that labor market institutions are determined optimally.

The contrasting labor market performances of Europe and the US have been the subject of much research. The standard explanation is based on institutions. It is argued that generous unemployment benefits and strict employment protection drive up European unemployment.1 Of course this, even if true, would not explain either of the two asymmetries. The first papers to focus on the problem of hysteresis were Blanchard and Summers (1986) and Lindbeck and Snower (1988). They argued that when wages are set unilaterally by “insiders”, wage (rather than employment) gains would follow the withdrawal of a temporary bad shock.2 Another literature has made the case for plausible asymmetries in the nature of morale and skill decay following joblessness (see Layard and Nickell (1987)). When these “duration” effects are not so severe as to induce withdrawal from the labor force they are a potential source of unemployment persistence.3 In this paper we take a different approach. We argue that before assessing the performance of positive theories of hysteresis, we must ask if this phenomenon can be explained as the outcome in a normative model.

We present a simple model where the government sets the level of taxes on employed workers to pay out unemployment benefits to the unemployed. The economic environment implies that the current rate of unemployment depends on the generosity of unemployment benefits and a shock. A key feature of our model is that, for some simple cases, we can evaluate the effects of an increase in the

1 See Bentolila and Bertola (1990), Lazear (1990), Alvarez and Veracierto (1999), Caballero and Hammour (1998), Ljungqvist and Sargent (1998), inter alia. See Gregg and Manning (1996) for a review.
2 The debate concerning the “insider-outsider” model of wage determination used in these models includes Hall (1986), Lindbeck and Snower (1990), Fehr (1990), Rotemberg (1999), inter alia.
3 Blanchard and Wolfers (2000) have examined the way shocks and exogenous institutions interact to yield unemployment persistence. See also Bertola (1990), Ljungqvist and Sargent (1998) and Lemieux and McLeod (1998).
level of risk in the economy. Since unemployment benefits are supposed to provide insurance, the level of risk is a key parameter in the formulation of the problem. A large literature in public economics examines the optimal provision of unemployment insurance. In general, however, this literature does not look at the problem of providing unemployment insurance when the level of risk in the environment changes. Changing these models to address this question is not always feasible. For example, the problem studied by Hopenhayn and Nicolini (1997) is how to achieve a certain level of insurance at minimum cost, so that changing some risk parameters in the problem will not answer the questions we are after.

In an important review article, Blanchard and Katz (1997) have suggested that if unemployment shocks lead to increases in unemployment benefits, then we may have a way to explain the high persistence of European unemployment. Using OECD data for the 1970's and 80's, Di Tella and MacCulloch (1995) presents evidence consistent with the idea that unemployment benefits tend to fall with the unemployment rate (a tax effect) and increase when there are positive changes in the unemployment rate (an insurance effect). In the present paper we formalize Blanchard and Katz’s (1997) intuition and show how such “endogenous institutions” lead to hysteresis, even in a normative model of unemployment benefits. A number of authors have explained how differences in the welfare state, such as those that characterize Europe and the US, can arise out of a set of common fundamentals (see, for example, Piketty (1995), Benabou (2000) and Benabou and Tirole (2002)), although this literature has not emphasized differences over time or the role of shocks.

Our first task in the paper is to formalize the meaning of policy generated hysteresis. We focus

---

4 Important papers include Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) on how unemployment insurance (UI) ought to be paid over time, Feldstein (1978) and Topel (1983) on the effect of UI (and UI-financing) on layoff and quit behavior and Mortensen (1977) on the effect on job search.

5 Hansen and Imrohoroglu (1992) present a model showing how costly it is to set the wrong (non-optimal) level of unemployment benefits in a general equilibrium model where there are liquidity constraints and moral hazard. We experimented with a (much) simpler version of that model to see if it could be used to study the determination of unemployment benefits at different levels of risk. The fundamental problem encountered is that the parameters that determine the unemployment rate and that could be used to capture the risk in the environment also affect the degree of risk aversion that individuals have. Thus, it is impossible to disentangle in that model what is happening because individuals have become more risk-averse and what occurs because the environment is more risky.

6 Positive theories include the voting model of Wright (1986) and also Atkinson (1990). Neither of these models, however, considers the role of “incentive effects”. These can be thought of as the coefficient on benefits in an unemployment regression. Saint Paul (1996) presents a good review of positive models, and discusses other institutions, such as job security provisions.
on an objective function, $S(b, \varepsilon)$, where $b$ is a choice variable (i.e. a policy) and $\varepsilon$ is a stationary random variable whose value is known when $b$ is set. Changes in the value of $\varepsilon$ (for example from 0 to $\varepsilon_1$) correspond to shocks. Put simply, hysteresis can exist when the value of adjusting the choice variable, $b$, once the shock occurs, $\Delta S^{\varepsilon_1}$, is different than the value of adjusting back once the shock has disappeared, $\Delta S^0$. This would be true if there is some “adjustment cost” that lies strictly between these two values. Note that unless strong restrictions are placed on the functional form of $S(b, \varepsilon)$ to guarantee the occurrence of the special case in which $\Delta S^0 = \Delta S^{\varepsilon_1}$, hysteresis as we define it here will be a pervasive feature of the world. We first show that a sufficient condition for hysteresis to exist is that the degree of concavity at the maximum of $S(.)$ changes with the size of the shock.

This may be helpful in putting more structure to the definition of hysteresis, but as such, says little about European unemployment. For all we know, a formulation where economic variables are used to construct $S(.)$, converting the objective function in our problem into a utilitarian social welfare function, could lead to all the wrong correlations. For instance, it could be that a shock that increases unemployment leads to lower unemployment benefits. Or it could be that hysteresis occurs only for shocks that increase social welfare. This would hardly be descriptive of the European experience after the oil shocks of the 1970’s. The challenge for the second part of the paper is to show that, when $S(.)$ is a reasonable social welfare function such as the weighted sum of the utility of the employed and the unemployed, a shock that increases unemployment can reduce social welfare and lead to permanently higher levels of benefits and unemployment, even after the shock has disappeared. We show that this will happen if two key conditions are satisfied. First, the degree of concavity of $S(.)$ at the point where it reaches its maximum increases once the adverse unemployment shock occurs. And second, the shock leads to a higher optimal level of unemployment benefits.

A key condition for the degree of concavity of $S(.)$ to increase with the shock is that the individual utility function has a non-negative third derivative. In other words, we require that individuals do not become more risk averse at higher income, a condition that is satisfied by most utility functions commonly used (for example, logarithmic, CRRA, CARA). The reason why this leads to hysteresis is because all of the effects of the shock on the concavity of $S(.)$ have the same sign for a given level of unemployment benefits. When an unemployment shock takes place, the social welfare function now incorporates some more people on benefits and loses an equal number of people on wages. As long as the replacement rate is less than one, this change will incorporate people who are on
a more concave part of their utility function. A second effect is that higher benefit payments to the unemployed mean a higher tax burden. This means lower net wages, so that now the employed are also on a more concave part of the utility function.7

The second condition for European-style hysteresis to exist, namely that unemployment benefits ought to be increased following an unemployment shock, is that the adverse incentive effects of benefits are large. The larger these incentive effects are, the more likely it is that the optimal response to a shock is to raise benefits. The intuition is simple once we note that benefits are set optimally at all times, including the moment just before the shock takes place. If incentive effects are large, benefits ought to be set low prior to the shock to minimize unemployment problems generated by the welfare state. In the limit, we can imagine a situation where unemployment is zero if benefits are zero. Then it is clear that the optimal level of benefits prior to the shock must be zero. But after the shock takes place, the marginal gain from an extra unit of insurance is particularly large.

In contrast to previous models in the literature, the mechanism that yields hysteresis is no longer relevant when unemployment becomes high because tax considerations yield a self-correcting mechanism (see Hall (1986)).8 Furthermore, it is simple (requires that the third derivative of the utility function is non-negative) and symmetric in the sense that the same mechanism is at play in the presence of negative and positive shocks. In particular, it does not assume any behavioral asymmetry, between insiders or outsiders or between the long-term and the short-term unemployed. The only ad-hoc feature is that it requires the existence of “adjustment costs” that are not modeled. We conjecture that an in-depth look at the extended benefit system in the US may provide some empirical clues as to what may give rise to these costs.

Although the paper deals with unemployment insurance, the results seem to have a more general application to other situations where the objective function depends on an individual’s utility function. Two key features of this paper - the fact that a shock increases the concavity of the objective function and there are some adjustment costs - are present in the rational design of other institutions such as job security provisions or minimum wages. Section II provides a definition and an outline of the general structure of rational hysteresis. Section III presents the general problem in a simple

---

7 Another effect with the same sign is described after Proposition 4.
8 Dixit and Pindyck (1994) have shown how hysteresis can exist in the context of irreversible investment decisions made under uncertainty. In contrast our results are not related to the option value of waiting. A more closely related paper, Hamermesh (1995), shows hysteresis can depend on the history of labor market policies.
economic model of optimal benefit setting and solves the simplest case when there is full discounting and no adjustment costs to develop the basic intuition. Section IV includes the effect of an adjustment cost of changing the benefit level and derives the conditions required for hysteresis to occur. Section V discusses the results and empirical implications, while Section VI concludes.

II. Formal Definition of Hysteresis

Define an objective function $S(b, \varepsilon)$ where $b$ is a choice variable, $\varepsilon$ is a shock and $\frac{\partial^2 S(b, \varepsilon)}{\partial b^2} < 0$. Assume that there is a fixed cost, $m$, of changing $b$. Each period, $b$ is set to maximize the current value of the objective function $S(b, \varepsilon)$ minus the adjustment cost, after observing the value of the shock. Call $b^0 = \text{argmax}_b S(b, 0)$ and $b^{\varepsilon} = \text{argmax}_b S(b, \varepsilon)$. Without loss of generality, let $b^0 < b^{\varepsilon}$ and $\varepsilon > 0$. Figure 1 illustrates. The gain obtained by adjusting from $b^0$ to $b^{\varepsilon}$ equals $\Delta S^{\varepsilon} - m = S(b^{\varepsilon}, \varepsilon) - S(b^0, \varepsilon) - m$ and the gain obtained by adjusting from $b^{\varepsilon}$ to $b^0$ equals $\Delta S^0 - m = S(b^0, 0) - S(b^{\varepsilon}, 0) - m$. Consider a shock of absolute size $|\varepsilon_1|$ that occurs but subsequently disappears. The question is whether the system returns to its initial state (no hysteresis).

**Proposition 1:**

a. If $\min(\Delta S^0, \Delta S^{\varepsilon}) < m < \max(\Delta S^0, \Delta S^{\varepsilon})$ and $\frac{\partial^2 S(b^{\varepsilon} - x, \varepsilon)}{\partial b^2} < \frac{\partial^2 S(b^0 + x, 0)}{\partial b^2} < 0$ $\forall x \in (0, b^{\varepsilon} - b^0)$ then hysteresis exists for a shock of size, $\varepsilon_1$.

b. If $\min(\Delta S^0, \Delta S^{\varepsilon}) < m < \max(\Delta S^0, \Delta S^{\varepsilon})$ and $0 > \frac{\partial^2 S(b^{\varepsilon} - x, \varepsilon)}{\partial b^2} > \frac{\partial^2 S(b^0 + x, 0)}{\partial b^2}$ $\forall x \in (0, b^{\varepsilon} - b^0)$ then hysteresis exists for a shock of size, $-\varepsilon_1$.

c. If $\frac{\partial^2 S(b^{\varepsilon} - x, \varepsilon)}{\partial b^2} = \frac{\partial^2 S(b^0 + x, 0)}{\partial b^2}$ $\forall x \in (0, b^{\varepsilon} - b^0)$ then hysteresis cannot exist for any size of adjustment cost.

**Proof:** See Appendix I. #

Proposition 1 argues, rather trivially, that in the presence of a shock concavity at the maximum of the $S(.)$ function will increase, decrease or remain unchanged. It then shows that the first two cases yield hysteresis. Part (a) of the Proposition refers to the first case and proves that there is hysteresis for positive values of the shock. Part (b) refers to the second case, where the degree of concavity of the
objective function falls due to the shock, and proves that hysteresis will exist for negative values of the shock. In both cases adjustment costs must lie in the specified range. Note that, because the condition for hysteresis involves comparing concavity of two functions at two different values of the choice variable, a condition on the change in concavity at a given point is enough only in the cases where \(|b^{cl} - b^0|\) is sufficiently small. What is required in the general case is a comparison of concavity of the original function around its maximum and concavity of the new function around the new maximum. This is what the terms \(b^0 + x\) and \(b^{cl} - x\) refer to. Less formally, hysteresis exists when the objective function becomes more sharply "peaked" in the presence of the shock. Only in the rare case that there is no change in concavity (for example the objective function has the form, \(Q(b) + \varepsilon b\), where \(Q(b)\) is quadratic) can there be no hysteresis.

The conditions required in Proposition 1 for hysteresis to occur could be satisfied by a large number of functional forms. However we still must check that it can hold for a social welfare function: \(S(b, \varepsilon) = SW(b, u, T, \varepsilon)\) where \(b\) is the level of unemployment benefits, \(u=f(b, \varepsilon)\) is the unemployment rate and \(T=g(b, u)\) is the level of taxes. More importantly, once economic relationships are considered, nothing leads us to expect that hysteresis will occur with the correct co-variation in the variables, in the sense that they are compatible with the European experience. For example, none of the cases covered by part (b) of Proposition 1 will do because European unemployment is not associated with a shock that reduced unemployment. Furthermore, a shock that increases unemployment could easily lead to a lower level of optimal benefits. In other words, nothing precludes that in Figure 1 the function with the shock, \(S'\), has a maximum to the left of \(b^0\). Put differently, we ask if the predictions in our model are compatible with European unemployment, where we ask that an adverse shock that hits the economy and then disappears leaves benefits and the unemployment rate at a higher level and social welfare at a lower level than those prevailing prior to the shock. This is the challenge for the rest of the paper.

III. A Simple Model of Unemployment Benefit Determination

III. A. Individual Preferences
Assume an economy populated with identical risk-averse individuals with strictly concave utility defined over income, \(U(i)\) (where \(U'(i) > 0\) and \(U''(i) < 0\)). Individuals cannot save or insure themselves
in private insurance markets.\textsuperscript{9} The unemployment benefit program pays $b_t$ to the unemployed, funding it with a tax equal to $T_t$ levied on employed individuals at time, $t$.

\textbf{III. B. Labor Market}

At any point in time we denote the equilibrium unemployment rate, $u_t=f(b_t, \varepsilon_t)$, where $\varepsilon_t$ is a random, stationary shock defined on $[\varepsilon^L, \varepsilon^H]$. It has mean zero and $\partial u_t/\partial \varepsilon_t > 0$.\textsuperscript{10} Unemployment is also affected by the generosity of the unemployment benefit program, $b_t$, where $\partial u_t/\partial b_t > 0$.\textsuperscript{11}

\textbf{III. C. The Government’s Problem}

We assume that the shock occurring at time $t$ is random but known when benefits are set at time $t$.\textsuperscript{12} There is an adjustment cost, $m_t$, to changing the level of the policy variable, unemployment benefits. This is defined in utils and could be due to several factors, including administrative costs and the costs of coordination that are incurred if political support for such changes is required. The government must pay the same cost both when it wants to increase benefits and when it wishes to cut them. Clearly, allowing for differences in adjustment costs could make it more likely that hysteresis obtains.

After observing the shock, the government’s problem is to set benefits to maximize the present discounted value of expected welfare, conditional on information at time $t$, subject to the budget constraint, the possibility that higher benefits may cause higher unemployment and the adjustment

\textsuperscript{9} Chiu and Karni (1998) explain the role of private information in the failure of private insurance markets.

\textsuperscript{10} All the arguments in the paper can be developed using perfect foresight as we simply seek to show that hysteresis can arise in a normative model for some plausible environment.

\textsuperscript{11} This can be derived from a variety of standard models of equilibrium unemployment, including an efficiency wage model, a union bargaining model or a search model. The following example illustrates. Assume that firms pay workers the gross real wage, $W^g_t$, and competition ensures zero profits: $\pi(W^g_t, \varepsilon_t)=0$ (where $\partial \pi/\partial W^g_t < 0$, $\partial \pi/\partial \varepsilon_t < 0$). Assume workers can shirk on their job (in which case their work effort equals 0) but if caught, they are fired. The expected income from being fired equals the probability of staying unemployed (equal to $a(U_t)$ where $U_t$ is the unemployment rate and $\partial a/\partial U_t > 0$) multiplied by the level of benefits, plus the probability of finding a new job (equal to $1-a(U_t)$) multiplied by the wage net of taxes and effort costs. This formulation implicitly assumes that newly hired workers who have been caught shirking once are not able to shirk again. The “No-Shirking-Condition” equates the value from exerting effort on the job to the value of shirking: $C(W^g_t, b_t, U_t)=0$ (where $\partial C/\partial W^g_t > 0$, $\partial C/\partial b_t < 0$, $\partial C/\partial U_t > 0$). Equilibrium unemployment, $u_t$, can now be expressed as a function of both the level of benefits and the shock ($u_t=f(b_t, \varepsilon_t)$ where $\partial f/\partial b_t > 0$, $\partial f/\partial \varepsilon_t > 0$). The equilibrium gross wage, $w^g_t$, can be expressed solely as a function of the shock ($w^g_t=w^g(\varepsilon_t)$ where $\partial w^g/\partial \varepsilon_t < 0$).

\textsuperscript{12} This assumption about timing ensures that, for the cases we study, the level of unemployment is the relevant measure of “risk” in the economy. Other timings would require us to look at the distribution of $\varepsilon$. 
costs. If the social rate of time preference equals $\theta$, the government’s problem as of time zero is:

$$
\max_{b_t, \epsilon_t, \ldots} \sum_{t=0}^{\infty} \mathbb{E} \left[ \frac{SW_t - M_t}{(1 + \theta)^t} \mid t = 0 \right]
$$

subject to

1. $u_t = f(b_t, \epsilon_t)$ \hspace{1cm} Incentive Constraint \hspace{1cm} (2)
2. $T_t = \frac{u_t b_t}{1 - u_t}$ \hspace{1cm} Budget Constraint \hspace{1cm} (3)
3. $m_t \geq 0 \text{ if } |b_t - b_{t-1}| \neq 0$
4. $M_t = \begin{cases} 0 & \text{if } |b_t - b_{t-1}| = 0 \end{cases}$ \hspace{1cm} Adjustment Costs \hspace{1cm} (4)

where $SW_t = (1 - u_t)U(w_t) + u_t U(b_t)$ and $w_t = w^g(\epsilon_t) - T_t$ is the net wage. Substituting in $SW_t$ for constraints (2) and (3) yields $S(b_t, \epsilon_t)$. This formulation implies the simplest assumption regarding transitional dynamics: each period the government ignores the employment history. Thus, a situation where a person is unemployed for two periods is identical to the situation where that person is unemployed for one period and another is unemployed the next. If we define the value function as:

$$
V(b_{t-1}, \epsilon_t) = \max_{b_t, b_{t+1}, \ldots} \mathbb{E} \left[ \sum_{t=0}^{\infty} \frac{S(b_t, \epsilon_t) - M_t}{(1 + \theta)^{t-1}} \mid t \right]
$$

then the solution to the government’s problem satisfies:

$$
V(b_{t-1}, \epsilon_t) = \max_b \{S(b_t, \epsilon_t) - M_t + (1 + \theta)^{-1} \mathbb{E}[V(b_{t+1}, \epsilon_{t+1}) \mid t] \}
$$

This Bellman equation fully characterizes the solution to the government’s unemployment benefit problem. More intuition can be gained, however, by examining the government’s problem in extreme cases, such as when there is full discounting or when adjustment costs are zero.

13 Alternatively, the same social welfare function (divided by the discount rate) is obtained if we consider the lifetime expected utility of employed and unemployed workers used in Shapiro and Stiglitz (1984). Transitional dynamics are analyzed in Kimball (1994).
III. D. Basic Results with Full Discounting and No Adjustment Costs

As is standard in this type of problem, it is useful to start by assuming that only current period welfare is valued and there are no adjustment costs. The problem reduces to:

$$ \max_b \quad SW(b, u, T, \varepsilon) = (1-u)U(w^*(\varepsilon) - T) + uU(b) \quad (7) $$

subject to

$$ u = f(b, \varepsilon) \quad \text{Incentive Constraint} \quad (8) $$

$$ T = \frac{ub}{1-u} \quad \text{Budget Constraint} \quad (9) $$

The First Order Condition (FOC) is:

$$ - (1-u)U'(w)[\frac{u}{1-u} + \frac{b}{(1-u)^2} \frac{\partial u}{\partial b}] + uU'(b) - \frac{\partial u}{\partial b} [U(w) - U(b)] = 0 \quad (10) $$

When the second order condition holds, the FOC implicitly defines optimal benefits as a function of the magnitude of incentive effects, $\frac{\partial u}{\partial b}$. Clearly if there are no adverse incentive effects of benefits, marginal utility must be equalized across states and there is simply full insurance. Inspection of the FOC above suggests that incentive effects would sometimes tend to reduce the optimal level of benefits. For simplicity, assume that incentive effects are linear and the shock is additive.\footnote{\text{A sufficient condition for the Second Order Condition to hold under these conditions is } \alpha \leq u^f. \text{ It is possible to derive some of the results below for other cases as well, available on request.}} At each point in time, unemployment is given by $u = u^f + \alpha b + \varepsilon$.\footnote{\text{This makes two simplifying assumptions. First, a linear approximation has been used. Second, it is implicitly assumed that the shock does not directly affect labor market "incentives" (i.e. } \frac{\partial f}{\partial \varepsilon} \alpha = 0). \text{ The reason is tractability. For details, as well as empirical evidence on the determination of benefits, see Di Tella and MacCulloch (1995).}} This equals the sum of frictional unemployment, $u^f$, unemployment arising from the adverse incentive effects of the benefit system, $\alpha b$, and from random shocks, $\varepsilon$.

**Proposition 2:** The government should set benefits low when incentive effects are large.
Proof: Compute $db/d\alpha<0$, using the implicit function rule on the FOC (10).

The intuition for this result is simple. At the optimum, the government balances insurance against the tax costs to fund the program as well as the adverse incentive effects that unemployment benefits introduce (which increase unemployment). When incentive effects are large the government will try to restrict benefits because, for a given level of insurance, benefits now have a bigger effect on the unemployment rate and the tax burden of the employed. We can now study what happens to the optimal level of benefits when there is an exogenous shock to the unemployment rate.

**Proposition 3:**

a. When incentive effects are small, the government should reduce benefits following the occurrence of an adverse shock.

b. When incentive effects are large, the government should increase benefits following the occurrence of an adverse shock.

Proof: See Appendix I.

If there are only small incentive effects of benefits on unemployment, benefits should decrease due to exogenous adverse shocks to unemployment. The reason is that benefits should initially be set at relatively generous levels (the replacement ratio close to 1) when $\alpha$ is small so that the main impact of the shock is to raise taxes (via the budget constraint) and reduce the affordable level of benefits.

Perhaps the more interesting case is when incentive effects are large. Initially, unemployment benefits are set at relatively low levels and the optimal response to an adverse unemployment shock may be to increase, rather than reduce, the generosity of benefits. In this case increases in insurance have large positive marginal effects on social welfare in the presence of an adverse shock. Consider an example where utility is logarithmic. If $U(x) = \log x$ then pre-shock social welfare is $S(b,0) = u\log b + (1-u)\log [w^\delta - ub/(1-u)]$ where $u = u^f + \alpha b$. This can be re-expressed as $S(b,0) = \log w^\delta + u\log(b/w^\delta) + (1-u)\log[1-u(b/w^\delta)/(1-u)]$. For simplicity, let $\partial w^\delta/\partial \varepsilon = 0$. When benefits are set low so that $b << w^\delta$, taxes are low and consequently $S(b,0) \approx \log w^\delta + u(\log(b/w^\delta)-b/w^\delta)$ (since $\log(1+x) \approx x$ for small $x$). In the presence of a shock that adds $\varepsilon l$ to the unemployment rate, $S(b,\varepsilon l) \approx S(b,0) + \varepsilon l(\log(b/w^\delta)-b/w^\delta)$. The second term has
a positive derivative with respect to \( b \), equal to \( \varepsilon l(l/b-1/w^2) \). Hence if benefits were being set optimally before the shock occurred, well below the wage due to the large incentive effects, there now exists a positive marginal welfare gain from more insurance. The smaller is the initial level of benefits, the larger is the gain from adjusting.

Figure 2 illustrates how optimal benefits vary with the unemployment shock for different levels of the incentive effects. The top curve shows the case when incentives are small, and in most situations tax effects dominate so that shocks lead to lower benefits. The lowest curve shows the case for large incentive effects where shocks lead to increases in benefits. Clearly if \( \varepsilon + u^f = 0 \) then \( b^e = 0 \). As the size of the adverse shock increases, \( b^e \) rises before ultimately falling as tax effects begin to dominate benefit setting. Again, it is clear that if \( \varepsilon + u^f = 1 \) then benefits must be zero. Note that movements along the top curve lead to relatively large cuts in benefits, whereas movements along the bottom curve (denoting large incentive effects) involve relatively small increases in benefits.

A fundamental aspect of this problem is that the effect of an adverse shock on the objective function (social welfare) is to increase its degree of concavity for a given value of benefits. In other words, the second derivative of the welfare function, with respect to benefits, becomes more negative in the presence of the shock.

**Proposition 4:** Provided \( U''''(w) \geq 0 \) then

\[
\frac{\partial^2 S(b_1, \varepsilon 1)}{\partial b^2} < \frac{\partial^2 S(b, 0)}{\partial b^2} \quad \forall b, \ \forall \varepsilon 1 > 0
\]  

**Proof:** See Appendix I.  

There are several effects that give rise to this result. First, an adverse shock shifts a proportion of workers from employment to unemployment. Once unemployed they find themselves on a lower part of their utility function (where \( U'' \) is more negative) since they are now only earning the benefit (which is lower than the wage). Second, an adverse shock cuts the level of net wages by lowering the gross wage that workers are paid and by increasing the level of taxes due to the greater numbers of unemployed. Hence even those workers who stay employed are pushed onto a lower part of their utility function (where \( U'' \) is more negative). Third, the greater numbers of unemployed due to the shock
mean that higher benefits have increasingly more severe effects on taxes, which also makes the second derivative of the welfare function more negative.16

In most cases, the result in Proposition 4 of concavity increasing at a given $b$ will be enough to guarantee an increase in the degree of concavity at the top of the $S(.)$ function once the shock occurs. In some cases, however, the shock will induce too large a change in benefits. It is theoretically conceivable that a large change in benefits could affect the degree of concavity at the top of $S(.)$ by adding a term with the wrong sign (e.g. if the shock induces an increase in benefits that moves the unemployed to a less concave part of their utility function). It is to avoid these pathological cases that we will later impose a restriction "for a small change in benefits".

For the logarithmic utility example where incentive effects are large so benefits are initially set low, the expression for $\frac{\partial^3 S}{\partial \epsilon \partial b^2}$ is dominated by the negative term, $-1/b^2$. This term captures the degree of concavity of the utility function, $U(b)=\log b$, of the workers who are made unemployed due to the shock. More generally, Figure 1 in Appendix II shows the case when incentive effects are large. Social welfare varies with benefits along the curve $SS$ in the absence of a shock. The optimal level of benefits is set relatively low at $b^0$. Social welfare is $S(b^0,0)$ at point A. This figure also shows the impact of a shock to unemployment, $\epsilon_1 > 0$. Social welfare now varies with benefits along the curve $SS'$. From Proposition 3(b) we know that the optimal level of benefits rises to $b^{\epsilon_1}$ and social welfare equals $S(b^{\epsilon_1},\epsilon_1)$ at point C. From Proposition 4 we know that, for a given $b$, the degree of concavity of the post-shock social welfare function, $SS'$, is greater than the degree of concavity of the pre-shock function, $SS$.

### III. E. Results Without Full Discounting and No Adjustment Costs

Assume that the government positively weights welfare in future periods and the adjustment cost is zero. The solution to problem (1) remains the same as in sub-section III.D since benefits should be set each period at the level that maximizes $S(b_t,\epsilon_t)$.

---

16 This last effect means that even when $U'''(w)=0$ the social welfare function would become more concave after the shock. To see this note that in the quadratic utility example, $U(w)=cw-dw^2$ where $w=w^*-ub/(1-u)$, the second derivative with respect to $b$ becomes more negative when the unemployment rate is higher.
IV. Optimal Benefits with Positive Adjustment Costs Yield Hysteresis

In sub-sections IV.A and IV.B we assume that there exists a positive fixed cost of adjusting benefits. In other words, \( m_t = m > 0 \) if \( |b_t - b_{t-1}| \neq 0 \) and is zero otherwise.

IV. A. The Case with Positive Fixed Costs of Adjustment

If the fixed cost of adjusting benefits is very large so benefits must be set initially at a single level that cannot be changed in future periods, then the optimal level of benefits solves the problem: 

\[
\max_b S(b, \varepsilon_0) + E[\sum_{t=1}^{\infty} S(b, \varepsilon_t)/(1+\theta)^t | t=1].
\]

For intermediate levels of adjustment costs, hysteresis in benefit setting can arise. The easiest way to see this is to start from an equilibrium where \( \varepsilon = 0 \) and benefits are being set to maximize social welfare in the current period. Let \( b^0 = \arg\max_b S(b, 0) \), \( b^{\varepsilon_1} = \arg\max_b S(b, \varepsilon_1) \), \( \Delta S^0 = S(b^0, 0) - S(b^{\varepsilon_1}, 0) \) and \( \Delta S^{\varepsilon_1} = S(b^{\varepsilon_1}, \varepsilon_1) - S(b^0, \varepsilon_1) \).

**Proposition 5 (Hysteresis):** Consider the effect of an adverse shock to unemployment of size, \( \varepsilon_1 > 0 \). For sufficiently high rates of time preference, hysteresis can exist for a range of adjustment costs.

*Proof:* See Appendix I. 

The intuition is that social welfare, drawn as a function of benefits, becomes more sharply “peaked” in the presence of an adverse shock to unemployment. Consequently there exists an asymmetry between the welfare gain from adjusting benefits in the presence of the shock and the welfare gain from adjusting benefits once the shock has gone. Note that our results do not depend on \( \theta \to \infty \). Although the general problem is quite complex, we can gain some understanding about the behavior of the problem by considering an extreme case. Assume that a bad shock hits the economy and it is known, ex-ante, that the shock will disappear after one period and that it will never return. It is easy to see that unemployment benefits should be adjusted provided \( \Delta S^{\varepsilon_1} - m > \Delta S^0/\theta \). After benefits have been changed and the shock has disappeared forever (i.e. \( \varepsilon = 0 \) for all current and future \( t \)) the government may still want to keep benefits at their old level. By not changing the government saves on adjustment costs today, but loses the social welfare gain of having the “correct” level of benefits in the
future. This means that there will be hysteresis provided \( m > \Delta S^0 / \theta \). Thus, even if the shock takes such an extreme form and the future is not completely discounted, there will be hysteresis as long as:

\[
\Delta S^{\epsilon_1} - \Delta S^0 / \theta > m > \Delta S^0 (1 + \theta) / \theta
\]

(12)

Note, however, that as the rate of time preference becomes small there can be no hysteresis. When there does exist hysteresis in benefit-setting, there exists a corresponding hysteresis effect in unemployment. The reason is that if unemployment benefits are increased in the presence of a shock but not subsequently reduced once it disappears then the unemployment rate will also increase but not subsequently return to its pre-shock level. The extent of the rise in unemployment depends on the size of the incentive effects of benefits.

Figure 1 illustrates. Benefits are set at the pre-shock level, \( b^0 \), where social welfare equals \( S(b^0, 0) \) at point A. In the presence of an adverse shock to unemployment, \( \epsilon I \), social welfare now varies with benefits along the curve \( S' S' \). If benefits are kept at their pre-shock level then welfare drops to \( S(b^0, \epsilon I) \) at point B. However if benefits are increased to \( b^{\epsilon_1} \), which maximizes \( S' S' \), then social welfare can be increased to \( S(b^{\epsilon_1}, \epsilon I) \) at point C. In other words, before paying adjustment costs, increasing benefits after the shock raises social welfare by \( \Delta S^{\epsilon_1} \). After the shock disappears, welfare equals \( S(b^{\epsilon_1}, 0) \) at point D. The gain from reducing benefits from \( b^{\epsilon_1} \) to the optimal value \( b^0 \) (their initial value) equals \( \Delta S^0 \) before paying the adjustment cost. The welfare gain from increasing benefits in the presence of the shock, \( \Delta S^{\epsilon_1} \), is larger than the size of the welfare gain from reducing benefits after it has gone, \( \Delta S^0 \). If adjustment costs are zero (or small) then along the optimal path benefits should be increased from \( b^0 \) to \( b^{\epsilon_1} \) and then subsequently reduced back to \( b^0 \). However if adjustment costs are larger, satisfying \( \Delta S^0 < m < \Delta S^{\epsilon_1} \), and the government’s rate of time preference is high so that it only values current period welfare, then it is worthwhile for benefits to be increased to \( b^{\epsilon_1} \) in the period that the shock occurs but not reduced once it disappears. The unemployment rate also does not return to its initial value, due to the higher level of benefits.
IV. B. Characterization of the Degree of Hysteresis

The purpose of this section is to characterize the amount of hysteresis in the economy.

**Definition 1:** If $\rho$ and $\varepsilon^*$ are two shocks such that:

$$S(b(-\rho),-\rho) - S(b(0),-\rho) = m = S(b(\varepsilon^*),\varepsilon^*) - S(b(0),\varepsilon^*)$$

then the degree of hysteresis in the economy, $\eta$, can be characterized by:

$$\eta = \frac{b(-\rho) - b(0)}{b(\varepsilon^*) - b(0)}$$

Given the uncertainty structure, this measure best captures the asymmetric range of inaction of the government when it sets benefits. When $\eta$ is larger than one, it reflects the asymmetry resulting from the increase in the degree of concavity of the social welfare function in the presence of an unemployment shock. The more the degree of concavity rises, the larger $\eta$ becomes. The smaller the change in concavity, the closer $\eta$ is to one.

The nature of this measure can be seen in Figures 3 and 4. They are drawn for the case of large incentive effects and when only current period welfare is valued due to a high rate of time preference. Figure 3 shows two functions, both of which define social welfare (in the absence of adjustment costs) as a function of the size of the unemployment shock, $\varepsilon$. The function, $S(b(\varepsilon),\varepsilon)$, which is depicted by the thick line, shows how social welfare varies with $\varepsilon$ when benefits, $b(\varepsilon)=b(\varepsilon)$, vary optimally so as to maximize $S(.)$ for each level of $\varepsilon$. In other words, if $b(\varepsilon)=\text{argmax}_b S(b,\varepsilon)$ then:

$$\frac{dS(b(\varepsilon),\varepsilon)}{d\varepsilon} = \frac{\partial S(b,\varepsilon)}{\partial b} \frac{\partial b}{\partial \varepsilon} + \frac{\partial S(b,\varepsilon)}{\partial \varepsilon} \frac{\partial S(b,\varepsilon)}{\partial \varepsilon}$$

by the Envelope Theorem. The function, $S(b(0),\varepsilon)$, depicted by the thin line, shows how $S(.)$ varies with $\varepsilon$ when benefits are fixed at the level, $b^0(0)=b(0)$, where $b(0)=\text{argmax}_b S(b,0)$. These two functions are

---

17 We thank Fernando Alvarez for ideas and help with this section. Errors are our own.
tangential when $\varepsilon=0$. For other values of the shock, $S(b(0),\varepsilon)<S(b(\varepsilon),\varepsilon)$. If benefits are set initially at $b^0$, then the increase in welfare that can be obtained from changing the level of benefits when there is an adjustment cost of size, $m$, equals $S(b(\varepsilon),\varepsilon)-S(b(0),\varepsilon)-m$. This is the motivation for the proposed definition of $\eta$.

Figure 4 draws the same problem, but in $(\varepsilon,b)$ space. The thick line, $b(\varepsilon)$, describes how benefits vary optimally so as to maximize $S(.)$ for each level of the shock, $\varepsilon$. It is upward sloping since we are focusing on the case where incentive effects of benefits are large (see Proposition 3(b)). The thin lines depict the limits of the regions of inaction. In the absence of a shock, benefits are set optimally at $b^0=b(0)$. In the presence of a shock that increases unemployment, but is smaller than $\varepsilon^*$, benefits should not be changed due to the cost, $m$, of doing so. In the presence of a shock that reduces unemployment, benefits should also not be changed provided that $\varepsilon>-\rho$. In Figure 4, $\eta$ is the vertical distance between points D and F divided by the vertical distance between points A and B. If there are no changes in the degree of concavity of $S(b,\varepsilon)$ between points F and B then these two distances must be the same.

Consider the example of a shock that is marginally larger than $\varepsilon^*$. Benefits should be increased from $b(0)$ to $b(\varepsilon^*)$ (from point A to point B). Once the shock has disappeared, benefits should be kept at $b(\varepsilon^*)$. Only if a shock reduces unemployment by more than the level measured by the horizontal distance between points O and C should benefits be cut. Figures 3 and 4 show that the nature of the solution to problem (1) follows an $(S,s)$ rule: when the size of the shock deviates sufficiently from its previous value at which benefits were being set optimally, benefits should be adjusted so that they become optimal for the size of the new shock.\footnote{There is a related literature on $(S,s)$ pricing rules where the shock is usually assumed to be non-stationary.}

Further intuition can be gained by expressing the degree of hysteresis in terms of the concavity of the objective function. Provided $|b(\varepsilon^*)-b(-\rho)|$ is a small number, then $m=\frac{1}{2}\frac{\partial^2 S(b(-\rho),\varepsilon^*)}{\partial b^2}(b(-\rho)-b(0))^2$ and $m=\frac{1}{2}\frac{\partial^2 S(b(0),\varepsilon^*)}{\partial b^2}(b(\varepsilon^*)-b(0))^2$ (using second order Taylor approximations). Hence:

$$\eta = \left| \frac{b(-\rho)-b(0)}{b(\varepsilon^*)-b(0)} \right| \approx \sqrt{\frac{\partial^2 S(b(\varepsilon^*),\varepsilon^*)}{\partial b^2}/\frac{\partial^2 S(b(-\rho),-\rho)}{\partial b^2}} = \sqrt{1+\Delta P} \quad (16)$$
where $\Delta P$ is the percentage change in concavity. Thus, our hysteresis measure suggests that the ratio of the ranges of inaction is proportional to the square root of one plus the percentage change in concavity.

V. Discussion

V. A. Good Shocks

As in previous models of hysteresis, temporary good shocks may now have permanent effects on unemployment. Consider the effect of a good shock that temporarily lowers unemployment risk in the economy. When this occurs, there is less demand for insurance and a relatively large welfare gain to be captured by cutting benefits. Provided this gain now exceeds the adjustment cost of changing benefits the government can increase social welfare by reducing benefits and keeping them low in future periods (in the absence of further shocks).

In Figure 5 benefits are initially set equal to $b^0$ and welfare is at point A. Assume that the government’s rate of time preference is high and that there exists an adjustment cost, $m=\Delta S^0+c<\Delta S^{el}$ (where $c$ is small). In the presence of an adverse shock to unemployment, benefits are increased to $b^{el}$ and social welfare (before paying the adjustment cost) equals $S(b^{el},\varepsilon 1)$ at point C. Once the shock disappears, and in the absence of further shocks, benefits remain at $b^{el}$ and social welfare equals $S(b^{el},0)$ at point D. Now consider the effect of a good shock that lowers the rate of unemployment by $q$. The direct effect of the shock is to increase social welfare to $S(b^{el},-q)$ at point E on the curve, $S’S’’$. In traditional models, with exogenous benefits fixed at $b^{el}$, the good shock could only lead to a temporary reduction in unemployment. However if benefits are set optimally they should be cut to $b^{q}$ in the presence of the good shock so that welfare can be further increased by $\Delta S^{q}=S(b^{q},-q)-S(b^{el},-q)-m$ (from point E to point F). After the shock has disappeared (and in the absence of further ones) the level of benefits remains low at $b^{q}$ and consequently the unemployment rate is also lower in future periods compared with the case of exogenous benefits.19

V. B. A Succession of Bad Shocks

A different implication of the model from the previous literature concerns the effect of a succession of

19 It is possible to argue that the good shock could be generated by some government action.
adverse shocks. Most (positive) models of hysteresis lack a self-correcting mechanism so it is possible
that the unemployment rate could increase monotonically with the occurrence of negative shocks (see
Hall (1986)). This does not occur in the present model. Two cases are worth analyzing. In the first case,
insurance effects continue to dominate benefit setting, whereas in the second case tax effects dominate.

The first case occurs when an economy gets stuck at a high level of unemployment because
benefits get stuck at a high level after a shock. This is the standard case depicted by point D in Figure
1. Now imagine that a second, even bigger shock hits the economy. Social welfare falls below $S'\bar{S}$ and,
if insurance considerations prevail, the new maximum lies to the right of $b^\epsilon 1$. Assume that it is optimal
to adjust benefits up in the presence of this big shock. It is also true that the gain from adjusting
benefits down will now be bigger than $\Delta S^0$. Thus, a bigger shock may actually make it optimal to have
an adjustment down of benefits.

A second case concerns bad shocks of such large magnitude that tax effects, rather than
insurance effects, begin to dominate the government’s benefit setting problem. First, assume that a bad
shock has driven up optimal benefits and, because of institutional adjustment costs, it is optimal not to
reduce them once the shock disappears. Now assume that a further bad shock hits the economy that
leaves unemployed a large proportion of the labor force. In the traditional hysteresis models there are
no self-correcting mechanisms. In our model, there always comes a point where the shock is
sufficiently large that benefits must be reduced by the government directly due to its budget constraint.

**Proposition 6:** As the unemployment rate tends to 1, the benefit level that maximizes current period
social welfare decreases in the presence of an adverse shock.

*Proof:* See Appendix I. 

Appendix III contains a numerical simulation of the model.

**V. C. The Natural Rate of Unemployment**

Work on the natural rate of unemployment defines it independently of aggregate demand conditions
and the current rate of unemployment (Friedman (1968), Phelps (1968, 1994)). Previous work on hysteresis has pointed out that this distinction may be overstated. A similar point can be made in this model. Only if institutions (in our case benefits) are set exogenously can we define a natural rate of unemployment, \( u^n \), independently of the random shocks affecting the economy: 
\[
\begin{align*}
  u_t &= a_b t + \epsilon_t \\
  u^n &= E(u_t) = a b_t \\
  E(\epsilon_t) &= 0.
\end{align*}
\]
On the other hand, if benefits are set optimally, then the “natural rate” will in general depend on the history of shocks to unemployment, via the effect of these shocks on the level of benefits: 
\[
  u^n = E(u_t) = a b(\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \ldots).
\]

V. D. General Discussion and Empirical Implications

In seeking to provide the simplest environment where hysteresis will emerge out of a normative process, we have made a number of simplifying assumptions. In our view, most of them are licit in the sense that there is no reason to think why a more sophisticated model would show such assumptions to be implausible. An example is the assumption that the shock does not affect incentive effects that is implicit in the linear approximation to labor market equilibrium. Whereas a more detailed model of the labor market may certainly have the property that the effect of the shock on \( \partial u / \partial b \) is non-zero, there is no reason to believe that it must be so.

There is one exception however. The model is built around the assumption of a fixed value of the adjustment cost, in the sense that it does not change with the shock. More formally, the adjustment cost, \( m \), is denominated in utils. An alternative model would put \( m \) inside the utility function, so that paying this cost in bad times costs more in utils. A couple of issues are worth noting, however. First, such an approach will reduce the set of shocks for which there is hysteresis, but it does not eliminate them. Second, once this is done, we really need a theory of what causes these adjustment costs. For example, one may wonder why we should not explicitly make \( m \) a function of the shock. A plausible assumption is that political coordination costs are smaller in bad times. This would mean that the original assumption of \( m \) being fixed in utils could again become correct to a first approximation. A limitation of our approach is that we do not have a theory of the behavior of these adjustment costs. All we do is characterize the behavior of the economy for the case when these costs are fixed in utils.

---

20 Friedman (1968) defines it as "the level which would be ground out by the Walrasian system of general equilibrium equations, provided that there is embedded in them the actual structural characteristics of the labor
Our results suggest that European unemployment, in some limited sense, may be optimal. If there were high costs of changing institutions in Europe, the optimal course of action after the oil shocks in the 1970's could well have been to increase benefits and not lower them after the oil price came down. Another way of putting it would be to note that the period of high unemployment that follows a shock is not too costly in terms of welfare, particularly since it is associated with high unemployment benefits.

The empirical prediction of our theory is that unemployment benefits in Europe should have been raised after the oil shocks in the 70's and then kept high during the 80's. It also predicts that unemployment should have risen with the oil shock as well as with the increase in generosity of the unemployment benefit program during the 70's and then should have dropped partially during the 80's once the oil shock disappeared. This is consistent with the evidence (see Di Tella and MacCulloch (1995)). Figures 6 and 7 illustrate with the case of Spain and compare it with the US. The differential performance between the US and Europe could be explained if the cost of changing institutions is lower in the US. The implication is that the coordination, legislative and political costs of changing the level of benefits are higher in Europe.

A particularly simple way of reducing these costs would be to specify explicitly in advance a rule or formula defining how benefits are going to be adjusted in the presence of a shock. An example would be a contingent rule such as unemployment benefits are $x$ if the unemployment rate is less than $y$ and $z$ if it rises above $y$. The U.S. has such a rule in the Extended Compensation Act. Japan and Canada also have variants of these laws. These countries have laws stating that benefits depend on aggregate unemployment conditions. In the U.S., the Federal/State Extended Compensation Act of 1970 established a second layer of benefits for claimants who exhaust their regular entitlement during periods of relatively high unemployment in a State. This program provides for up to 13 extra weeks of benefits at the claimant's usual weekly benefit amount. The benefits are triggered on “if the State's insured unemployment rate for the past 13-week period is 20 percent higher than the rate for the corresponding period in the past two years and the rate is at least 5 percent.” Hence in the U.S.

and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the cost of mobility, and so on.”

21 The OECD measure of benefits describes the parameters of the unemployment benefit system. It is calculated as the pre-tax average of the unemployment benefit replacement ratios for two earnings levels, three family situations and three durations of unemployment (see the OECD Jobs Study (1994) for details).
adverse shocks that increase the unemployment rate also increase benefit generosity, by law. If this type of legislation lowers the adjustment costs of changing benefits we may expect to observe less evidence of hysteresis: benefits would be more likely to be increased in the presence of an adverse shock but returned to their initial value once the shock has gone. In fact, the U.S. Federal/State Extended Compensation Act does specify that “extended benefits cease to become available when the insured unemployment rate does not meet either the 20 percent requirement or the 5 percent requirement”. In other words, the primitive in our model is differences in how easy it is change unemployment benefits rather than differences in the generosity or otherwise of these programs across countries. In some sense this is closer to the way the word “institution” is often used by authors such as North (1990), where attention is given to the rules of the game, rather than the outcomes.

From a positive point of view, an empirical prediction of the model is that, other things equal, differences in the government’s rate of time preference should explain changes in the unemployment benefit system. A possible way of capturing differences in impatience is to focus on political color, as it is sometimes argued that left-wing parties discount the future more than right-wing parties. It is also possible that the length of the electoral cycle influences the government’s discount rate. Then we may expect to see less evidence of benefit hysteresis in countries that have longer periods between elections.

VI. Conclusions

A number of economists have blamed European unemployment on labor market institutions. Since institutions are primitives in these models, a lot of the dynamics have been left unexplained. Consider for example, the time path of unemployment benefits. Figures 6 and 7 show them increasing sharply in the US and Spain in the years immediately after 1973 and 1979. A similar pattern is present in the data for other OECD countries. If we believe institutions are exogenous, we must also believe that these countries were incredibly unlucky. Just when they got hit by an oil shock, politicians decided to increase benefits, worsening their unemployment problems. Only the US turned out to be lucky in the 1980’s when benefits returned to their pre-shock levels. A less ad-hoc story involves developing a theory where institutions are rational. In such a theory, unemployment benefits can certainly increase the unemployment rate, but it should also allow us to understand what drives movements in benefits. This is the objective of our paper.
We present a model where the government sets unemployment benefits to maximize social welfare in response to an unemployment shock, subject to a budget constraint and the possibility that unemployment benefits may introduce incentive problems that increase the unemployment rate. The following results can be established:

1. In the absence of incentive effects (whereby higher benefits increase the unemployment rate) there should be full insurance. Unemployment benefits, on the other hand, should be set lowest (highest) when the adverse incentive effects of benefits are largest (smallest).

2. In response to a shock that increases unemployment, benefits should be increased in those economies where the adverse incentive effects are most severe. The intuition for this result stems from the fact that benefits are set optimally at all times, including the moment just before the shock occurs. Thus, large incentive effects imply a low initial level of benefits and large welfare gains derived from better insurance when there is an unemployment shock.

3. In the presence of an adjustment cost of changing the level of benefits there may exist hysteresis in benefit setting and unemployment. In other words, the level of benefits (and unemployment) may rise in the presence of an adverse shock and remain higher than the initial value once the shock has disappeared.

4. The reason for the asymmetry is that a shock increases the degree of concavity of the objective function (social welfare). This occurs because the shock incorporates into the objective function a group of people who are on a more concave part of their utility function. This suggests that the key assumption driving hysteresis is that the individual utility function has a positive third derivative (people do not become more risk averse as they become richer). Contrary to previous models, we do not require any behavioral asymmetries between "insiders" and "outsiders" or between the short-term unemployed and the long-term unemployed. And when unemployment tends to one, tax considerations prevail so the mechanism is self-correcting.

5. As in previous models of hysteresis, temporary good shocks may now have permanent effects on unemployment. The reason is that lower unemployment reduces "risk" in the economy and may make lower benefits optimal.
Appendix I

Proof of Proposition 1

a. Consider the outcome when $\varepsilon$ changes from 0 to $\varepsilon l$ (i.e. the shock is of size $\varepsilon l$). Assume as an initial condition that $b=b^0$. Define $f(x)=S(b^0+x,0)$ and $g(x)=S(b^{el}-x,\varepsilon l)$ where $g''(x)<f''(x)<0 \ \forall x \in (0,b^{el}-b^0)$ by assumption. Integrating both sides gives $\int_0^x g''(x)dx < \int_0^x f''(x)dx \ \forall z \in (0,b^{el}-b^0)$. Hence $g'(z)-f'(0)<f'(z)-f'(0)$ since $g'(0)=f'(0)=0$. Integrating both sides again gives $\int_0^z (g''(x))dx < \int_0^z (f''(x))dx$. Hence $g(b^{el}-b^0)-g(0)<f(b^{el}-b^0)-f(0)$ implies $\Delta S^{el}<\Delta S^0$. Provided $\Delta S^{el}<\Delta S^0$, then in the presence of the shock, $b$ changes from $b^0$ to $b^{el}$ (the gain is $\Delta S^{el}-m>0$) but not back again once the shock has gone (the loss would be $\Delta S^0-m<0$). Consequently there exists hysteresis.

b. Consider the outcome when $\varepsilon$ changes from $\varepsilon l$ to 0 to $\varepsilon l$ (i.e. the shock is of size $-\varepsilon l$). Assume as an initial condition that $b=b^{el}$. Again define $f(x)=S(b^0+x,0)$ and $g(x)=S(b^{el}-x,\varepsilon l)$. Since in this case $f''(x)<g''(x)<0 \ \forall x \in (0,b^{el}-b^0)$ by assumption, then $\Delta S^{el}<\Delta S^0$ using a similar logic as in part (a). Provided $\Delta S^{el}<\Delta S^0$ then in the presence of the shock, $b$ changes from $b^{el}$ to $b^0$ (the gain is $\Delta S^{el}-m>0$) but not back again once the shock has gone (the loss would be $\Delta S^0-m<0$). Consequently there again exists hysteresis.

c. Define $f(x)=S(b^0+x,0)$ and $g(x)=S(b^{el}-x,\varepsilon l)$. Since in this case $f''(x)=g''(x) \ \forall x \in (0,b^{el}-b^0)$ by assumption, then $\Delta S^{el}=\Delta S^0$ using a similar logic as in part (a). Provided $m<\Delta S^0=\Delta S^{el}$ then in the presence of the shock, $b$ changes from $b^0$ to $b^{el}$ (the gain is $\Delta S^{el}-m>0$) and hence it must also pay to change back again once the shock has gone (the gain is $\Delta S^{el}-m<0$). If $m>\Delta S^0=\Delta S^{el}$ then it does not pay to change $b$ at all. Consequently there cannot exist hysteresis.

Proof of Proposition 3

Substituting in $SW(b,u,T,\varepsilon)$, for constraints (8) and (9) yields $S(b,\varepsilon)$. The effect of a shock on the marginal gain from increasing benefits is:

\[
\frac{\partial^2 S}{\partial \varepsilon \partial b} = U'(b) - U'(w) - \frac{\partial \omega^{eq}}{\partial \varepsilon} U'(w)[\alpha - r (u + \frac{ab}{1-u})(1 - \frac{b}{(\partial \omega^{eq}/\partial \varepsilon)(1-u)^2})] \quad (A1)
\]

where $\partial \omega^{eq}/\partial \varepsilon<0$ and $r=-U''(w)/U'(w)$ is the coefficient of absolute risk aversion.

a. As $\alpha \to 0$, the FOC (10) implies that $U'(w) \to U'(b)$ and from (A1):

\[
\frac{\partial^2 S}{\partial \varepsilon \partial b} \to U'(w) r (u + \frac{ab}{1-u})(\frac{\partial \omega^{eq}}{\partial \varepsilon} - \frac{b}{(1-u)^2}) \quad (A2)
\]

which is negative. Hence using the implicit function theorem, benefits should be cut following the occurrence of an adverse shock when incentive effects are small.
b. If incentive effects are large so $b$ is small then:

$$\frac{\partial^2 S}{\partial \alpha \partial b} \rightarrow U'(b) - U'(w) - \frac{\partial w}{\partial \alpha} U'(w) [\alpha - r(u + \frac{ab}{1-u})]$$  \hspace{1cm} (A3)$$

which is positive provided that the utility function is strictly concave and the coefficient of absolute risk aversion, $r$, has an upper bound. Hence using the implicit function theorem, benefits should be increased following the occurrence of an adverse shock when incentive effects are large. #

**Proof of Proposition 4**

The second derivative of the social welfare function is:

$$\frac{\partial^2 S}{\partial b^2} = u U''(b) + 2\alpha[U'(b) - \frac{\partial w}{\partial b} U'(w)] + (1-u)[\frac{\partial w}{\partial b} U''(w) + \frac{\partial^2 w}{\partial b^2} U'(w)]$$  \hspace{1cm} (A4)$$

The effect of a shock on the concavity of the welfare function for a given value of $b$ is:

$$\frac{\partial}{\partial \varepsilon} \left( \frac{\partial^2 S}{\partial b^2} \right) = U''(b) + \Phi U''(w) - \psi U''(w) \frac{(u + ab)^2}{1-u}$$  \hspace{1cm} (A5)$$

where $\Phi=\alpha b[4+3ab/(1-u)]/(1-u)^3 + u(2-u)/(1-u)^2 - 2\alpha(\partial w/\partial \varepsilon)$ and $\psi=-\partial w/\partial \varepsilon + b/(1-u)^2$. Since both $\Phi$ and $\psi$ are positive, $U''(b)<0$ and $U''(w)<0$, a sufficient condition for (A5) to be negative is that $U''(w)>0$ (which is the case for all quadratic, Constant Absolute and Constant Relative Risk Aversion functions). Hence $\partial^2 S/\partial b^2$ is a monotonically decreasing function of $\varepsilon$ so $\partial^2 S(b, \varepsilon I)/\partial b^2 < \partial^2 S(b, 0)/\partial b^2 \forall b, \forall \varepsilon I>0$. #

**Proof of Proposition 5**

It is simple, but not necessary, to let $\theta \to \infty$ so that only current period welfare is valued by the government. In the presence of the shock, welfare changes by $\Delta S^{\varepsilon I}-m$ if benefits are adjusted from $b^0$ to $b^{\varepsilon I}$. After the shock has gone, welfare changes by $\Delta S^0-m$ if benefits are adjusted back from $b^{\varepsilon I}$ to $b^0$. Proposition 4 states that the degree of concavity of the welfare function increases in the presence of shocks, $\varepsilon I>0$: $\Delta S^{\varepsilon I}-\Delta S^0<0$. For a small change in benefits (i.e. when $|b^{\varepsilon I}-b^0|$ is small) the condition in Proposition 1(a) is satisfied and hence there exists hysteresis whenever adjustment costs lie in the range, $\Delta S^0<m<\Delta S^{\varepsilon I}$. An estimate of the difference, $\Delta S^{\varepsilon I}-\Delta S^0$, is:

$$-\frac{1}{2} \frac{\partial^2 S(b^{\varepsilon I})}{\partial b^2} - \frac{\partial^2 S(b^0)}{\partial b^2} |b^{\varepsilon I}-b^0|^2 > 0 \quad \text{(using 2nd order Taylor approximations)}.$$  \hspace{1cm} #

**Proof of Proposition 6**

As $u \to 1$ in equation (A1), $\partial^2 S/\partial \varepsilon \partial b$ becomes negative (for given $\alpha$). Hence using the implicit function theorem, $db/\partial \varepsilon < 0$. #
Figure 1: The objective function, $S(b, \varepsilon)$, versus the choice variable, $b$, before a shock ($SS$) and during an adverse shock ($S'S'$).
Figure 2: The benefit curve, drawn for different levels of “incentive effects”.

Optimal Benefits

Frictional Unemployment + Shock

$(u^f + \varepsilon)$

Increasing $\alpha$

Increasing $\varepsilon$

Low $\alpha$

High $\alpha$

A

B

C

D
Figure 3: Social Welfare versus the Unemployment Shock. $S(b(\varepsilon), \varepsilon)$ is the envelope over which benefits are changed optimally depending on the size of the shock. When the adjustment cost is $m$, the corresponding region of inaction is $(-\rho, \varepsilon^*)$. 
Figure 4: The Optimal Unemployment Benefit Setting Rule. Segment DA denotes the (asymmetric) region of inaction.
Figure 5: Social Welfare versus Unemployment Benefits before a shock (SS), during an adverse shock to unemployment (S'S') and during a positive shock that reduces unemployment (S'S').
Figure 6: Spain’s Unemployment Benefits and Real Commodity Prices from 1969 to 1993.

Figure 7: The US’s Unemployment Benefits and Real Commodity Prices from 1969 to 1993.
Appendix III
A Numerical Simulation
Let utility be logarithmic, $w^g=1$, $u^f=0.04$, $\alpha=0.2$ and the government’s rate of time preference be high so that it only values current period welfare. Assume there are no adjustment costs. The optimal level of benefits, $b^0$, equals 0.13, and the unemployment rate equals 0.07. Social welfare, $S(b^0,0)=-0.14$. Consider the effect of an adverse unemployment shock, $\varepsilon_1$, of size 0.04. For simplicity, let the gross wage be unaffected by the shock. If benefits remain at their pre-shock level then welfare drops to $S(b^0,\varepsilon_1)=-0.23$. However if they are increased to $b^{\varepsilon_1}=0.32$, which maximizes $S(b,\varepsilon_1)$, then welfare would be $S(b^{\varepsilon_1},\varepsilon_1)=-0.21$. In other words, welfare can be increased by $\Delta S^{\varepsilon_1}=S(b^{\varepsilon_1},\varepsilon_1)-S(b^0,\varepsilon_1)=0.02$ by increasing benefits. After the shock has gone, the gain from reducing benefits from $b^{\varepsilon_1}$ back to their initial value, $b^0$, equals $\Delta S^0=S(b^0,0)-S(b^{\varepsilon_1},0)=0.01$. Hence the welfare gain from increasing benefits in the presence of the shock is twice the gain from reducing them after the shock has gone.

Optimal Benefits with Adjustment Costs
Now assume that adjustment costs satisfy $0.01<m<0.02$. In such a case, it is worthwhile for the government to raise benefits in the period that the temporary shock occurs but not to reduce them once $\varepsilon=0$. If $m=0.015$ then in the period of the shock, social welfare can be increased by 0.005 ($\Delta S^{\varepsilon_1}-m=0.02-0.015$) by raising benefits from $b^0=0.13$ to $b^{\varepsilon_1}=0.32$. In the period that the shock disappears, benefits should not be cut since this would result in a welfare loss equal to $-0.005$ ($\Delta S^0-m=0.01-0.015$). Unemployment does not return to its initial value, due to the higher level of benefits. After the shock has gone, the unemployment rate equal 0.10, which is 0.03 higher than its pre-shock value.

Good Shocks
Start from the position where benefits have been optimally increased to $b^{\varepsilon_1}=0.32$ following the adverse shock of size, $\varepsilon_1=0.04$, and have become stuck at this level due to the adjustment cost, $m=0.015$. Social welfare, $S(b^{\varepsilon_1},0)=-0.15$. Now consider the effect of a good shock that temporarily reduces unemployment by $-q=-0.02$. Social welfare rises by 0.02 as a direct consequence ($=S(b^{\varepsilon_1},-q)-S(b^{\varepsilon_1},0)=-0.13-(-0.15)$). By cutting benefits to $b^{q}=0.07$, welfare can be further increased by 0.01 ($=\Delta S^q-m=S(b^{q},-q)-S(b^{\varepsilon_1},-q)-m=-0.105-(-0.13)-0.015$). In the absence of further shocks, benefits will permanently remain at this lower level. Unemployment also remains low, equal to 0.05, compared to its pre-shock value of 0.10.

A Succession of Bad Shocks
Start from the position in which benefits have been optimally increased from $b^0=0.13$ to $b^{\varepsilon_1}=0.32$ following the first adverse shock of size, $\varepsilon_1=0.04$, and have become stuck at this level due to the adjustment cost, $m=0.015$. Social welfare, $S(b^{\varepsilon_1},0)=-0.15$. Now consider the effect of another bad, but larger shock to unemployment of size, $\varepsilon_2=0.16$. If benefits are not changed then welfare drops to $S(b^{\varepsilon_1},\varepsilon_2)=-0.45$. However if benefits are increased further to $b^{\varepsilon_2}=0.46$, which maximizes $S(b,\varepsilon_2)$, then welfare can be increased above this level by 0.005 ($S(b^{\varepsilon_2},\varepsilon_2)-S(b^{\varepsilon_1},\varepsilon_2)-m=-0.43-(-0.45)-0.015$). After the second shock has gone, it is worthwhile to cut benefits back to their initial value, $b^0=0.13$, since the gain from doing so, even after paying the adjustment cost, is positive. It equals 0.015 ($=S(b^0,0)-S(b^{\varepsilon_2},0)-m=-0.14-(-0.17)-0.015=0.015$). Unemployment returns to its initial value of 0.07.
References


Insurance System”, NBER 6732.