Why are Bank Balance Sheets Exposed to Monetary Policy?

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Abstract

We propose an explanation for banks' exposure to movements in interest rates. Bank deposits provide liquidity, so an increase in interest rates, which raises the cost of liquidity, allows banks to earn higher spreads on deposits. If risk aversion is higher than one, banks' optimal dynamic hedging strategy is to take losses when interest rates rise, because they expect higher spreads looking forward. This can be achieved by a traditional maturity-mismatched balance sheet. The mechanism is quantitatively important and can match the level and time pattern of banks' maturity mismatch, regardless of whether interest rates are driven by monetary or real shocks.

Keywords: Monetary shocks, bank deposits, interest rate risk

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1 Introduction

The banking system is highly exposed to monetary policy. An increase in nominal interest rates creates large financial losses for banks, which typically hold long-duration nominal assets (like fixed-rate mortgages) and short-duration nominal liabilities (like deposits). This leaves banks with weakened balance sheets, which hinders their functioning and propagates shocks. But why do banks choose such a large exposure to movements in interest rates? One could conjecture that a maturity-mismatched balance sheet is inherent to the banking business and the resulting interest rate risk is an inevitable side effect. However, there exist deep and liquid markets for interest rate derivatives that banks can use to hedge their interest rate risk. Begenau et al. (2015) show that banks hold positions in these derivatives, but they use them to *amplify* their exposure. In this paper we argue that banks choose to bear interest rate risk as part of a dynamic hedging strategy.

We study a flexible-price monetary economy where the only source of shocks is monetary policy. The economy is populated by bankers and households. The distinguishing feature of bankers is that they are able to provide liquidity by issuing deposits that are close substitutes to currency, up to a leverage limit. Importantly, because markets are complete, bankers are able to choose their exposure to risk independently of their liquidity provision. In particular, we don't make any assumptions about what kind of securities bankers hold.

We show that if relative risk aversion is high (larger than one) bankers optimally choose to sustain losses when interest rates rise. As a result, the endogenous response of banks' balance sheets amplifies the effects of monetary policy shocks on the cost of liquidity. This exposure to risk can be achieved with a portfolio of long-duration nominal assets and short-duration nominal liabilities, as in a traditional bank balance sheet.

The mechanism works as follows. Because deposits provide liquidity services, bankers earn the spread between the nominal interest rate on illiquid bonds and the lower interest rate on deposits. If nominal interest rates rise, the opportunity cost of holding currency goes up, so agents substitute towards deposits. This drives up the equilibrium spread between the nominal interest rate and the interest rate on deposits, increasing bankers' return on wealth. Because risk aversion is higher than one, bankers want to transfer wealth from states of the world with high return on wealth to states of the world with low return on wealth. They are willing to take capital losses when interest rates rise because spreads going forward will be high.

We calibrate the model to match the observed behavior of interest rates, deposit spreads, bank leverage and other macroeconomic variables. We find that an increase in the short-term interest rate of 100 basis points produces losses of around 30% of banks' net worth. This exposure can be implemented with an average maturity mismatch between assets and liabilities of 3.2 years, which is close to the 3.9 years reported by English et al. (2012). Furthermore, the model reproduces the time pattern in the data, with the maturity mismatch rising during periods of low interest rates, such as 2002-2005. The correlation between the model and the data is 0.56.

The baseline model with only monetary shocks is intended as a benchmark to examine the mechanisms at play. We also study an arguably more realistic setting where the central bank follows an inflation targeting policy. The economy is hit by real shocks that move the equilibrium real interest rate and force the central bank to adjust the nominal interest rate in order to hit its inflation target. The quantitative results are similar to the benchmark model, with an average maturity mismatch of 3.6 years, and a correlation with the data of 0.57.

One possible interpretation of banks' observed exposure to interest rate risk is that it is evidence of risk-seeking behavior, which regulators should be concerned about. Our findings suggest an alternative, more benign, interpretation. Our model provides a quantitative benchmark to assess whether banks are engaging in risk-seeking. Large deviations from this benchmark *in either direction* would be indicative of risk-seeking. In particular, if banks did not expose their balance sheet to interest rates at all (for instance by having no maturity mismatch) they would in fact be taking on a large amount of risk due to the sensitivity of deposit spreads to interest rates. Our quantitative results show no evidence of risk-seeking: the size of banks' exposure to interest rate risk is consistent with a dynamic hedging strategy by highly risk averse agents.[1]

More generally, our theory provides a lens to understand banks' risk exposures beyond interest rate risk. It predicts that banks will choose exposure to risks that are correlated with their investment opportunities. While in this paper we focus on banks' role as providers of liquidity, banks are also involved in the origination and collection of loans and earn the spread between risky and safe bonds. The same logic implies that they should be willing to take losses when this spread goes up because they expect a higher return on wealth looking forward. In fact, Begenau et al. (2015) report that banks are highly exposed to credit risk: they face large losses when the spread between BBB and safe bonds rises. In contrast, when we add TFP shocks to the model, we find that these are shared proportionally by banks and

[1] Of course, banks may very well be engaging in risk-seeking behavior on other dimensions, just not on interest rate risk.

households. Our model therefore provides a theory not only of how much, but also what type of risk banks take.

Our paper fits into the literature that studies the role of the financial sector in the propagation and amplification of aggregate shocks (Brunnermeier and Sannikov (2014), He and Krishnamurthy (2011), He and Krishnamurthy (2012), Di Tella (2016), Gertler and Kiyotaki (2015)). Relative to this literature, the main innovation in our paper is that we model banks as providers of liquidity through deposits. This provides a lens to study the role of the banking sector in the transmission of monetary policy. An important question in this literature is why the financial sector is so exposed to certain aggregate shocks. Our approach has in common with Di Tella (2016) that we allow complete markets; the equilibrium allocation of aggregate risk reflects agents' dynamic hedging of investment opportunities. The economics, however, are very different. Explicitly modeling the banking business allows us to understand banks' dynamic hedging incentives, which are different from other financial institutions, and to assess them quantitatively.

An important feature of the mechanism is that the equilibrium spread between illiquid bonds and deposits is increasing in the nominal interest rate. We find this stylized fact is borne out by the empirical evidence. In our data, a 100 bp increase in interest rates is associated with a 66 bp increase in the deposit spread. This has been observed before. Hannan and Berger (1991) and Driscoll and Judson (2013) attribute it to a form of price stickiness; Drechsler et al. (2014) attribute it to imperfect competition among bank branches; Yankov (2014) attributes it to search costs. Nagel (2014) makes a related observation: the premium on other near-money assets (besides banks deposits) also co-moves with interest rates. He attributes this, as we do, to the substitutability between money and other liquid assets. Krishnamurthy and Vissing-Jorgensen (2015) document a negative correlation between the supply of publicly issued liquid assets and the supply of liquid bank liabilities, also consistent with their being substitutes. We choose the simplest possible specification to capture this: substitution between physical currency and deposits, but this literature suggests that the phenomenon is broader.[2] Relative to this literature, the contribution of our work is to derive the implications for equilibrium risk management in a model where the underlying risk in modeled explicitly.

Other studies have looked at different aspects of banks' interest rate risk exposure.

[2]There is also a large theoretical literature studying the nature of bank deposits (Diamond and Dybvig (1983), Diamond and Rajan (2001), etc.) and money (Kiyotaki and Wright (1989), Lagos and Wright (2005), etc.). We make no contribution to this literature, and simply assume that currency and deposits are substitutes in the utility function.

Rampini et al. (2015) provide an alternative explanation for why banks fail to hedge the exposure to interest rate risk that arises from their traditional business. They argue that collateral-constrained banks are willing to give up hedging to increase investment, and provide empirical evidence showing that banks who suffer financial losses consequently reduce their hedging. Our model explicitly abstracts from these considerations in the sense that all banks are equally constrained and never face a tradeoff between hedging and investment. Landier et al. (2013) show cross-sectional evidence that exposure to interest rate risk has consequences for bank lending. Haddad and Sraer (2015) propose a measure of banks' exposure to interest rate risk and find that it is positively correlated with the term premium. English et al. (2012) use high-frequency data around FOMC announcements to study how bank stock prices react to unexpected changes in the level and slope of the yield curve, and find that bank stocks fall after interest rate increases.

2 The Model

Preferences and technology. Time is continuous. There is a fixed capital stock k which produces a constant flow of consumption goods $y_t = ak$. There are two types of agents: households and bankers, a continuum of each. Both have identical Epstein-Zin preferences with intertemporal elasticity of substitution equal to 1, risk aversion γ and discount rate ρ:

$$U_t = \mathbb{E}_t \left[\int_t^\infty f(x_s, U_s)\, ds \right]$$

with

$$f(x, U) = \rho (1-\gamma) U \left(\log(x) - \frac{1}{1-\gamma} \log((1-\gamma) U) \right)$$

The good x is a Cobb-Douglas composite of consumption c and liquidity services from money holdings m:

$$x(c, m) = c^\beta m^{1-\beta} \tag{1}$$

Money itself is a CES composite of real currency holdings h (provided by the government) and real bank deposits d, with elasticity of substitution ϵ:[3]

$$m(h, d) = \left(\alpha^{\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)^{\frac{1}{\epsilon}} d^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

[3] Throughout, uppercase letters denote nominal variables and their corresponding lowercase letter are real variables. Hence $h \equiv \frac{H}{p}$ and $d \equiv \frac{D}{p}$ where p is the price of consumption goods in terms of currency, which we take as the numeraire.

Formulation (2) captures the idea that both currency and deposits are used in transactions, so they both provide liquidity services. Substitution between these types of money will determine the behavior of deposit interest rates.

Currency and deposits. The government supplies nominal currency H, following an exogenous stochastic process

$$\frac{dH_t}{H_t} = \mu_{H,t}dt + \sigma_{H,t}dB_t$$

where B is a standard Brownian motion. The process B drives equilibrium dynamics. The government distributes or withdraws currency to and from agents through lump-sum transfers or taxes.

Deposits are issued by banks. This is in fact the only difference between bankers and households. Deposits pay an equilibrium nominal interest rate i^d and also enter the utility function according to equation (2). The amount of deposits bankers can issue is subject to a leverage limit. A banker whose individual wealth is n can issue deposits d^S up to

$$d^S \leq \phi n \tag{3}$$

where ϕ is an exogenous parameter. Constraint (3) may be interpreted as either a regulatory constraint or a level of capitalization required for deposits to actually have the liquidity properties implied by (2). This constraint prevents banks from issuing an infinite amount of deposits, and makes banks' balance sheets important for the economy.

Monetary policy. The government chooses a path for currency supply H to implement the following stochastic process for the nominal interest rate i on short-term, safe but illiquid bonds:

$$di_t = \mu_i(i_t)\,dt + \sigma_i(i_t)\,dB_t \tag{4}$$

where the drift $\mu_i(\cdot)$ and volatility $\sigma_i(\cdot)$ are functions of i. Shocks B are our representation of monetary shocks, and they are the only source of risk in the economy.

There is more than one stochastic process H that will result in (4). Let

$$\frac{dp_t}{p_t} = \mu_{p,t}dt + \sigma_{p,t}dB_t$$

be the stochastic process for the price level (which is endogenous). We assume that the government implements the unique process H such that in equilibrium (4) holds and $\sigma_{p,t} = 0$.

Informally, this means that monetary shocks affect the rate of inflation μ_p but the price level moves smoothly.

Markets. There are complete markets where bankers and households can trade capital and contingent claims. We denote the real price of capital by q, the nominal interest rate by i, the real interest rate by r, and the price of risk by π (so an asset with exposure σ to the process B will pay an excess return $\sigma\pi$). All these processes are contingent on the history of shocks B.

The total real wealth of private agents in the economy includes the value of the capital stock qk, the real value of outstanding currency h and the net present value of future government transfers and taxes, which we denote by g. Total wealth is denoted by ω:

$$\omega = qk + h + g$$

Total household wealth is denoted by w and total bankers' wealth is denoted by n, so

$$n + w = \omega \tag{5}$$

and we denote by $z \equiv \frac{n}{\omega}$ the share of the aggregate wealth that is owned by bankers.

Discussion of assumptions. The assumption that bankers and households are separate agents deserves some discussion. After all, many banks are publicly held and their shares are owned by diversified agents. Bankers in this model represent bank insiders - managers or large investors - who have large undiversified stakes in their banks through either share ownership or incentive contracts. The risk aversion of bankers in the model is meant to represent the attitude to the risk embedded in these undiversified claims. We purposefully assume that bankers and households have the same preferences; the mechanisms that govern risk exposure in the model do not depend on differential attitudes towards risk.

We model money in a highly stylized way, with a simple "currency and deposits in the utility function" specification. In addition, we assume the market for deposits is perfectly competitive, but bankers are limited in their ability to supply deposits by the leverage constraint. This prevents them from competing away deposit spreads, effectively acting like market power for bankers as a whole. Our objective is not to develop a theory of money nor to account for all features of deposit contracts or the deposit market, but rather to write down the simplest framework where banks provide liquidity and deposit spreads increase with the nominal interest rate.

In this model there is no real reason for monetary policy to do anything other than follow the Friedman rule.[4] The choice to model random monetary policy as the only source of risk in the economy is obviously not driven by realism but by theoretical clarity. In Section 5 we instead look at a variant of the model where monetary policy follows an inflation targeting rule and only responds to real shocks that affect the equilibrium real interest rate, and show that our results also hold in this more realistic monetary policy regime.

The assumption of complete markets is theoretically important. We want to avoid mechanically assuming the result that banks are exposed to interest rate risk.[5] In our model, banks are perfectly able to issue deposits without any exposure to interest rate risk, for example by investing only in short term or adjustable assets, or by trading interest rate swaps. More generally, banks are completely free to take any risk exposure, independently of their deposit supply. Relatedly, while we specify deposit contracts in nominal terms, this is without loss of generality because banks could trade inflation swaps. One possible concern is that in practice households may not be able to trade interest rate swaps or other derivatives that allow them to share interest rare risk with bankers. However, households can share interest rate risk with bankers by adjusting the maturity of their assets and liabilities, or using adjustable rate debt.

We also don't make any assumptions on the kind of assets banks hold: both banks and households can hold capital. In our model banks are not particularly good at holding long term fixed rate nominal loans, or any other security. Finally, with complete markets it is not necessary to specify who receives government transfers when the supply of currency changes: all those transfers are priced in and included in the definition of wealth. Notice also that while banks can go bankrupt (if their net worth reaches zero), this never happens in equilibrium. Continuous trading allows them to scale down as their net worth falls and always avoid bankruptcy.

[4]The CES formulation (2) implies that currency demand is unbounded at $i = 0$ but the Friedman rule is optimal in a limiting sense.

[5]Even though markets are complete, there is no claim that the competitive allocation is efficient. Bankers' ability to produce deposits is limited by their wealth, which involves prices. A social planner would want to manipulate these prices to relax the constraint.

3 Equilibrium

Households' problem. Starting with some initial nominal wealth W_0, each household solves a standard portfolio problem:

$$\max_{W,x,c,h,d,\sigma_W} U(x)$$

subject to the budget constraint:

$$\begin{aligned} \frac{dW_t}{W_t} &= \left(i_t + \sigma_{W,t}\pi_t - \hat{c}_t - \hat{h}_t i_t - \hat{d}_t\left(i_t - i_t^d\right)\right) dt + \sigma_{W,t} dB_t \\ W_t &\geq 0 \end{aligned} \tag{6}$$

and equations (1) and (2). A hat denotes the variable is normalized by wealth, i.e. $\hat{c} = \frac{pc}{W} = \frac{c}{w}$. The household obtains a nominal return i_t on its wealth. It incurs an opportunity cost i_t on its holdings of currency. It also incurs an opportunity cost $\left(i_t - i_t^d\right)$ on its holdings of deposits. Let $s_t = i_t - i_t^d$ denote the spread between the deposit rate and the market interest rate. Furthermore, the household chooses its exposure σ_W to the monetary shock and obtains the risk premium $\pi\sigma_W$ in return.

Constraint (6) can be rewritten in real terms as

$$\frac{dw_t}{w_t} = \left(r_t + \sigma_{w,t}\pi_t - \hat{c}_t - \hat{h}_t i_t - \hat{d}_t s_t\right) dt + \sigma_{w,t} dB_t \tag{7}$$

where $r_t = i_t - \mu_{p,t}$ is the real interest rate.

Bankers' problem. Bankers are like households, except that they can issue deposits (denoted d^S) up to the leverage limit and earn the spread s_t on these. The banker's problem, expressed in real terms, is:

$$\max_{n,x,c,h,d,d^S,\sigma_n} U(x)$$

subject to:

$$\begin{aligned} \frac{dn_t}{n_t} &= \left(r_t + \sigma_{n,t}\pi_t - \hat{c}_t - \hat{h}_t i_t + \left(\hat{d}_t^S - \hat{d}_t\right) s_t\right) dt + \sigma_{n,t} dB_t \\ \hat{d}_t^S &\leq \phi \\ n_t &\geq 0 \end{aligned} \tag{8}$$

and equations (1) and (2).

Equilibrium definition Given an initial distribution of wealth between households and bankers z_0 and an interest rate process i, a competitive equilibrium is

1. a process for the supply of currency H
2. processes for prices p, i^d, q, g, r,π
3. a plan for the household: w, x^h, c^h, m^h, h^h, d^h, σ_w
4. a plan for the banker: n, x^b, c^b, m^b, h^b, d^b, d^S, σ_n

such that

1. Households and bankers optimize taking prices as given and $w_0 = (1 - z_0)(q_0 k + h_0 + g_0)$ and $n_0 = z_0 (q_0 k + h_0 + g_0)$
2. The goods, deposit and currency markets clear:
$$c_t^h + c_t^b = ak$$
$$d_t^h + d_t^b = d_t^S$$
$$h_t^h + h_t^b = h_t$$
3. Wealth holdings add up to total wealth:
$$w_t + n_t = q_t k + h_t + g_t$$
4. Capital and government transfers are priced by arbitrage:
$$q_t = \mathbb{E}_t^Q \left[a \int_t^\infty \exp\left(- \int_t^s r_u du \right) ds \right] \tag{9}$$
$$g_t = \mathbb{E}_t^Q \left[\int_t^\infty \exp\left(- \int_t^s r_u du \right) \frac{dH_s}{p_s} \right] \tag{10}$$
where Q is the equivalent martingale measure implied by r and π.
5. Monetary policy is consistent
$$i_t = r_t + \mu_{p,t}$$
$$\sigma_{p,t} = 0$$

Aggregate state variables. We look for a recursive equilibrium in terms of two state variables: the interest rate i (exogenous), and bankers' share of aggregate wealth z (endogenous) which is important because it affects bankers' ability to issue deposits and provide liquidity. Using the definition of $z = \frac{n}{n+w}$, we obtain a law of motion for z from Ito's lemma and the budget constraints:

$$\frac{dz_t}{z_t} = \underbrace{\left((1-z_t)\left((\sigma_{n,t}-\sigma_{w,t})\pi_t + \phi s_t - (\hat{x}_t^b - \hat{x}_t^h)\chi_t + \sigma_{w,t}(\sigma_{w,t}-\sigma_{n,t})\right) - \frac{z_t}{1-z_t}\sigma_{z,t}^2\right)}_{\equiv \mu_{z,t}} dt$$

$$+ \underbrace{(1-z_t)(\sigma_{n,t}-\sigma_{w,t})}_{\equiv \sigma_{z,t}} dB_t \tag{11}$$

while the law of motion of i is given by (4). All other equilibrium objects will be functions of i and z.

Static Decisions and Hamilton-Jacobi-Bellman equations. We study the banker's problem first. It can be separated into a static problem (choosing c, m, h and d given x) and a dynamic problem (choosing x and σ_n).

Consider the static problem first. Given the form of the aggregators (1) and (2), we immediately get that the minimized cost of one unit of money m is given by ι:

$$\iota(i,s) = \left(\alpha i^{1-\epsilon} + (1-\alpha)\, s^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{12}$$

the minimized cost of one unit of the good x is given by χ:

$$\chi(i,s) \;=\; \beta^{-\beta}\left(\frac{\iota}{1-\beta}\right)^{1-\beta} \tag{13}$$

and the static choices of c, m, h and d are given by:

$$\frac{c}{x} = \beta\chi \tag{14}$$
$$\frac{m}{x} = (1-\beta)\frac{\chi}{\iota} \tag{15}$$
$$\frac{h}{m} = \alpha\left(\frac{\iota}{i}\right)^{\epsilon} \tag{16}$$
$$\frac{d}{m} = (1-\alpha)\left(\frac{\iota}{s}\right)^{\epsilon} \tag{17}$$

Turn now to the dynamic problem. In equilibrium it will be the case that $i^d < i$ so bankers' leverage constraint will always bind. This means that (8) reduces to

$$\frac{dn_t}{n_t} = \underbrace{\left(r_t + \sigma_{n,t}\pi_t - \chi\left(i_i, s_t\right)\hat{x}_t + \phi s_t\right)}_{\equiv \mu_{n,t}} dt + \sigma_{n,t} dB_t \tag{18}$$

Given the homotheticity of preferences and the linearity of budget constraints the problem of the banker has a value function of the form:

$$V_t^b(n) = \frac{(\xi_t n)^{1-\gamma}}{1-\gamma}$$

ξ_t captures the value of the banker's investment opportunities, i.e. his ability to convert units of wealth into units of lifetime utility, and follows the law of motion

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{\xi,t} dB_t$$

where $\mu_{\xi,t}$ and $\sigma_{\xi,t}$ are equilibrium objects.

The associated Hamilton-Jacobi-Bellman equation is

$$0 = \max_{x,\sigma_n,\mu_n} f\left(x, V_t^b\right) + \mathbb{E}_t\left[dV_t^b\right]$$

Using Ito's lemma and simplifying, we obtain:

$$\begin{aligned}
0 = \max_{\hat{x},\sigma_n,\mu_n} \rho(1-\gamma)\frac{(\xi_t n_t)^{1-\gamma}}{1-\gamma}&\left[\log(\hat{x}n_t) - \frac{1}{1-\gamma}\log\left((\xi_t n_t)^{1-\gamma}\right)\right] \\
+ \xi_t^{1-\gamma} n_t^{1-\gamma}&\left(\mu_n + \mu_{\xi_t} - \frac{\gamma}{2}\sigma_n^2 - \frac{\gamma}{2}\sigma_{\xi_t}^2 + (1-\gamma)\sigma_{\xi_t}\sigma_n\right) \\
s.t. \mu_n = r_t + \sigma_n \pi_t + \phi s_t - \hat{x}\chi_t&
\end{aligned}$$

The household's problem is similar. The only difference is that the term ϕs_t is absent from the budget constraint. The value function has the form

$$V_t^h(w) = \frac{(\zeta_t w)^{1-\gamma}}{1-\gamma}$$

where

$$\frac{d\zeta_t}{\zeta_t} = \mu_{\zeta,t}dt + \sigma_{\zeta,t}dB_t$$

and the HJB equation is

$$\begin{aligned} 0 = & \max_{\hat{x},\sigma_w,\mu_w} \rho(1-\gamma)\frac{(\zeta_t w_t)^{1-\gamma}}{1-\gamma}\left[\log(\hat{x}w_t) - \frac{1}{1-\gamma}\log\left((\zeta_t w_t)^{1-\gamma}\right)\right] \\ & + \zeta_t^{1-\gamma} w_t^{1-\gamma}\left(\mu_w + \mu_{\zeta_t} - \frac{\gamma}{2}\sigma_w^2 - \frac{\gamma}{2}\sigma_{\zeta_t}^2 + (1-\gamma)\sigma_{\zeta_t}\sigma_w\right) \\ s.t. & \mu_w = r_t + \sigma_w \pi_t - \hat{x}\chi_t \end{aligned}$$

Total wealth, spreads and currency holdings. The first order conditions for $\hat{x}$ in the banker and household problem are both given by:

$$\hat{x}_t = \frac{\rho}{\chi_t} \tag{19}$$

Since the intertemporal elasticity of substitution is 1, both bankers and households spend their wealth at a constant rate ρ independent of prices.

Using (19) and the goods market clearing condition we can solve for total wealth:

$$\omega = \frac{ak}{\beta\rho} \tag{20}$$

Hence in this economy total wealth will be constant. This follows because the Cobb-Douglas form of the x aggregator implies that consumption is a constant share of spending (the rest is liquidity services), the rate of spending out of wealth is constant and total consumption is constant and equal to ak.

Using (15) and (17), the fact that deposit supply is ϕn and (19), the deposit market clearing condition can be written as:

$$\rho(1-\alpha)(1-\beta)\iota^{\epsilon-1}s^{-\epsilon} = \phi z \tag{21}$$

Solving (21) for s implicitly defines bank spreads $s(i, z)$ as a function of i and z. It's easy to show from (21) that the spread is increasing in i as long as $\epsilon > 1$. If currency and deposits are close substitutes, an increase in i, which increases the opportunity cost of holding currency, increases the demand for deposits, so the spread must rise to clear the deposit market. Likewise, (21) implies that the spread is decreasing in z. If bankers have a larger fraction of total wealth, they can supply more deposits so the spread must fall to clear the deposit market.

Finally, using (15), (16), (19) and (20), the currency market clearing condition simplifies to:

$$h = \frac{ak}{\beta}\alpha(1-\beta)\iota^{\epsilon-1}i^{-\epsilon} \tag{22}$$

Having solved for s, (22) immediately gives the level of real currency holdings $h(i, z)$.

Risk sharing. The first order conditions for bankers' choice of σ_n and households' choice of σ_w are, respectively:

$$\sigma_{n,t} = \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma}\sigma_{\xi,t} \tag{23}$$

$$\sigma_{w,t} = \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma}\sigma_{\zeta,t} \tag{24}$$

The first term in each of (23) and (24) relates exposure to B to the risk premium π; this is the myopic motive for choosing risk exposure: a higher premium will induce higher exposure. The second term captures the dynamic hedging motive, which depends on an income and a substitution effect. If the agent is sufficiently risk averse ($\gamma > 1$), then the income effect dominates. The agent will want to have more wealth when his investment opportunities (captured by ξ and ζ respectively) are worse.

From (23) and (24) we obtain the following expression for σ_z:[6]

$$\sigma_{z,t} = (1-z_t)\frac{1-\gamma}{\gamma}\left(\sigma_{\xi,t} - \sigma_{\zeta,t}\right) \tag{25}$$

The object σ_z measures how the bankers' share of wealth responds to the aggregate shock. The term $\sigma_{\xi,t} - \sigma_{\zeta,t}$ in (25) captures the relative sensitivity of bankers' and households'

[6] At this level of generality, this condition for aggregate risk sharing is analogous to the one in Di Tella (2016). However, the economic mechanism underlying the response of relative investment opportunities to aggregate shocks is specific to each setting.

investment opportunities to the aggregate shock. How this differential sensitivity feeds into changes in the wealth share depends on income and substitution effects. If agents are highly risk averse ($\gamma > 1$) they will shift aggregate wealth towards bankers after shocks that worsen their investment opportunities relative to households', i.e. $\frac{\xi}{\zeta}$ goes down.

We can use Ito's lemma to obtain an expression for $\sigma_\xi - \sigma_\zeta$:

$$\sigma_\xi - \sigma_\zeta = \left(\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta_z}\right)\sigma_z z + \left(\frac{\xi_i}{\xi} - \frac{\zeta_i}{\zeta}\right)\sigma_i \tag{26}$$

Notice that σ_z enters the expression for $\sigma_\xi - \sigma_\zeta$: the response of relative investment opportunities to aggregate shocks depends in part on aggregate risk sharing decisions as captured by σ_z. This is because in equilibrium investment opportunities depend on the distribution of wealth z, so we must look for a fixed point. Replacing (26) into (25) and solving for σ_z:

$$\sigma_z = \frac{(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_i}{\xi} - \frac{\zeta_i}{\zeta}\right)}{1 - z(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta}\right)}\sigma_i \tag{27}$$

Implementation. With complete markets, there is more than one way to attain the exposure dictated by equations (23) and (24). We are interested in seeing whether one possible way to do this is for banks to have a "traditional" balance sheet: long-term nominal assets, deposits as the only liability and no derivatives. To be concrete, we'll imagine a banker's balance sheet with net worth n, ϕn deposits and $(1+\phi)\,n$ nominal zero-coupon bonds that mature in T years.

As long as $T > 0$, this balance sheet will result in negative exposure: long term bond prices will fall when interest rates rise. The magnitude of the exposure will be a function of T, the level of bank leverage and the persistence of interest rates. We'll then ask whether there is a value of T that delivers the equilibrium level of desired exposure and how that compares with empirical measures of bank asset maturities.

4 Numerical Results

We make two minor changes to the baseline model to obtain quantitative results. First, we let productivity follow a geometric Brownian motion:

$$\frac{da_t}{a_t} = \mu_a dt + \tilde{\sigma}_a d\tilde{B}_t$$

where $\tilde{B}_t$ is a standard Brownian motion, independent of B_t.[7] The economy scales with a so this change does not introduce a new state variable. The main effect of this change is to lower the equilibrium real interest rate. Second, in order to obtain a stationary wealth distribution we add tax on bankers' wealth at a rate τ that is redistributed to households as a wealth subsidy.

We solve for the recursive equilibrium by mapping it into a system of partial differential equations for the equilibrium objects and solve them numerically using a finite difference scheme. Appendix A explains the modifications to the model and the numerical procedure in detail.

Parameter values. Table 1 summarizes the parameter values we use. We set the risk aversion parameter $\gamma = 10$, consistent with the asset pricing literature (see for instance Bansal and Yaron (2004)). We also perform a sensitivity analysis with different values of γ. EIS is 1 in our setting, in the interest of theoretical clarity and tractability, as explained above. It is also close to values used in the asset pricing literature. We choose the rest of the parameter values so that the model economy matches some key features of the US economy. The details of the data we use are in Appendix B.

Parameter	Meaning	Value
γ	Risk aversion	10
$\bar{i}$	Mean interest rate	3.5%
σ	Volatility of i	0.044
λ	Mean reversion of i	0.056
ρ	Discount rate	0.055
ϕ	Leverage	8.77
α	CES weight on currency	0.95
β	Cobb-Douglas weight on consumption	0.93
ϵ	Elasticity of substitution between currency and deposits	6.6
μ_a	Average growth rate of TFP	0.01
$\tilde{\sigma}_a$	Volatility of TFP	0.073
τ	Tax on bank equity	0.195

Table 1: Parameter values

We assume interest rates follow the Cox et al. (1985) stochastic process, so that $\mu_i(i) = -\lambda(i - \bar{i})$ and $\sigma_i(i) = \sigma\sqrt{i}$. The concept of i in the model corresponds to a short term

[7] We assume that monetary policy is carried out so that the price level is also not sensitive to $\tilde{B}$, i.e. $\tilde{\sigma}_p = 0$.

rate on an instrument that does not have the liquidity properties of bank deposits. We take the empirical counterpart to this to be the 6-month LIBOR rate in US dollars. We choose $\bar{i} = 3.5\%$ to match the average LIBOR rate between 1990 and 2014. Estimating the persistence parameter λ in a short sample has well known econometric difficulties (Phillips and Yu 2009). This parameter is very important in the model, for two related reasons. First, more persistence means that a change in interest rates has a long-lasting effect on bank spreads, which drive bankers' relative desire to hedge. Second, more persistence means that a change in interest rates will have a large effect on the prices of long-term bonds, so the maturity T needed to implement any desired σ_n shortens. We set $\lambda = 0.056$ and $\sigma = 0.044$ to match the standard deviation of the LIBOR rate (2.4%) and 10-year Treasury yields (1.8%) for the period 1990-2014.

We use equation (20) to choose a value for the discount rate ρ. The Flow of Funds reports a measure of aggregate wealth. To be consistent with our model which has no labor, we adjust this measure by dividing by 0.35 (the approximate capital share of GDP) in order to obtain a measure of wealth that capitalizes labor income. We then compute an average consumption-to-adjusted-wealth ratio between 1990 and 2014, taking consumption as consumption of nondurables and services from NIPA data. This results in $\frac{ak}{\omega} = 5.1\%$, which, given the value of β set below, leads to $\rho = 0.055$.

We use data on bank balance sheets from the Flow of Funds to set a value of the leverage parameter ϕ. In the model there is only one kind of liquid bank liability ("deposits") whereas in reality banks have many type of liabilities of varying degrees of liquidity, so any sharp line between "deposits" and "not deposits" involves a certain degree of arbitrariness. We choose the sum of checking and savings deposits as the empirical counterpart of the model's deposits, leaving out time deposits since these are less liquid and the spreads that banks obtain on them are much lower. We set $\phi = 8.77$ to match the average ratio of deposits to bank net worth between 1990 and 2014.

We construct a time series for z using data on banking sector net worth and total wealth from the Flow of Funds (total wealth is divided by 0.35 as before to account for labor income). The Flow of Funds data uses book values which is the right empirical counterpart for n in the model (market value of banks' equity includes the value of investment opportunities which is not part of n). We then use the data from Drechsler et al. (2014) on interest rates paid on checking and savings deposits and weight them by their relative volumes from the Flow of Funds to obtain a time series for the average interest rate paid on deposits.[8] We

[8]We thank Philipp Schnabl for kindly sharing this data with us.

subtract this from LIBOR to obtain a measure of spreads. We set β (the Cobb-Douglas weight on consumption as opposed to money), α (the CES weight on currency as opposed to deposits), and ϵ (the elasticity of substitution between currency and deposits) jointly to minimize the sum of squared distances between the spreads predicted by equation (21), given the measured time series for i and z, and the measured spreads. The data seem to prefer very high values of α so we, somewhat arbitrarily, fix $\alpha = 0.95$ (letting α take even higher values does not improve the fit very much). Minimizing over β and ϵ leads to $\beta = 0.93$ and $\epsilon = 6.6$.

We set the growth rate of productivity $\mu_a = 0.01$ and its volatility $\tilde{\sigma}_a = 0.073$ for the model to match the average real interest rate between 1990 and 2014, which was 1%. This value of $\tilde{\sigma}_a$ is close to that used by He and Krishnamurthy (2012), who use $\tilde{\sigma}_a = 0.09$.

Finally, we set the tax rate of bank capital to $\tau = 0.195$ for the average value of z in the model to match the average value in the data between 1990 and 2014, which is 0.56%.

Spreads. Figure 1 shows the spread as a function of i and z for our parameter values. As we know from equation (21), it is increasing in i and decreasing in z. Furthermore, it is concave in i. When i is high, agents are already holding very little currency, so further increases in i do not generate as much substitution into deposits and therefore don't lead to large increases in spreads.

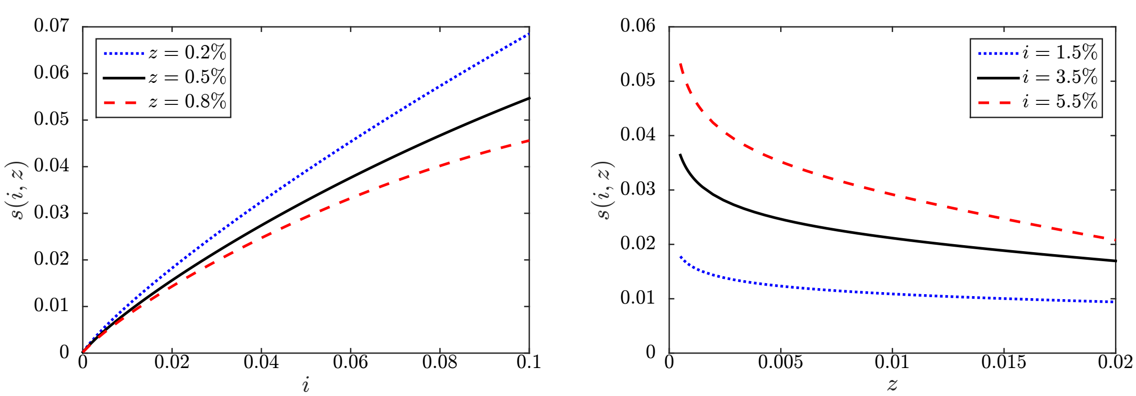

Figure 1: Spreads in the model as a function of i and z.

These properties of $s(i, z)$ are consistent with the data. Table 2 shows the results of regressing spreads on interest rates and banks' share of total wealth:

The first column, without a quadratic term, shows that a one percentage point increase in LIBOR is associated with a 66 basis points increase in bank spreads, while a one percentage point increase in banks' share of total wealth is associated with a 99 basis points fall in bank spreads. The second column, including a quadratic term, shows that there is indeed evidence that bank spreads flatten out as i increases.

	(1)	(2)
Constant	0.3%	−0.3%
	(0.22%)	(0.44%)
i	0.66	0.98
	(0.028)	(0.17)
i^2	–	−4.07
		(2.02)
z	−0.99	−0.71
	(0.25)	(0.32)
R^2	0.89	0.90
N	430	430

Note: The dependent variable is the spread. Newey and West (1987) standard errors are in parentheses.

Table 2: Spreads, interest rates and banks' share of aggregate wealth.

The model is able to match the time series behavior of spreads quite closely. Figure 2 compares the time series for $s(i, z)$ produced by the model with the time series of measured spreads from the Drechsler et al. (2014) data. However, it is worth noting that in order to do this, the model requires a high value of α (i.e. a strong preference for currency) and a high value of ϵ (i.e. a high elasticity of substitution between currency and deposits). This results in a high and variable currency-to-deposit ratio, which is not what we observe. Still, the calibration does mechanically yield an average deposits-to-GDP ratio that matches the data, which is what matters for the mechanism. Overall, we conclude that our microeconomic model of bank spreads is probably too simplistic and the observed co-movement of interest rates and bank spreads is also driven by imperfect competition between banks (Drechsler et al. 2014), stickiness in deposit rates (Hannan and Berger 1991, Driscoll and Judson 2013), search costs (Yankov 2014), or other factors. However, for the purposes of bank risk management, the exact microeconomic mechanism that drives spreads is not so essential. What matters is how these co-move with interest rates and banks' share of wealth.

Aggregate risk sharing. Figure 3 shows aggregate risk sharing. The top panels show bankers' exposure to interest rate risk. If the nominal interest rate rises by 100 basis points, bankers' net worth changes by $\frac{\sigma_n}{\sigma_i}$ percent. It is always negative, so banks face financial losses after an increase in nominal interest rates. Quantitatively, the effect is found to be quite large. At the mean levels of i and z, if interest rates rise by 100 basis points, banks lose about 30% percent of their net worth.

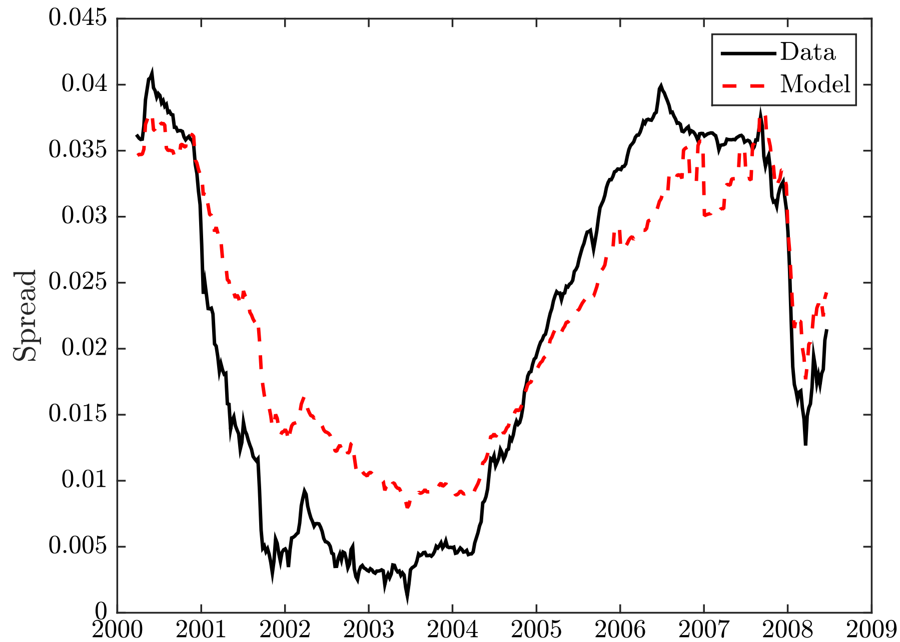

Figure 2: Spreads in the data compared to spreads implied by the $s(i, z)$ function given our parameter values and the measured time series of i and z.

To understand the mechanism, note that because aggregate wealth is insensitive to B, $\sigma_n = \sigma_z$ so movements in bankers' net worth and in their share of total net worth are equivalent. We know from (21) that an increase in the nominal interest rate raises the spread s. Since bankers earn this spread and households don't, bankers' relative investment opportunities $\frac{\xi}{\zeta}$ improve when the interest rate i rises, as shown in the middle-left panel of Figure 3. Equation (25) implies that z must fall in response, which further raises the spread s, amplifying the effect of monetary shocks on the cost of liquidity. As a result, bankers' relative investment opportunities $\frac{\xi}{\zeta}$ improve even more, as shown in the middle-right panel of Figure 3, which amplifies bankers' incentives to choose a negative σ_n (this is the reason the denominator in equation (27) is less than one).

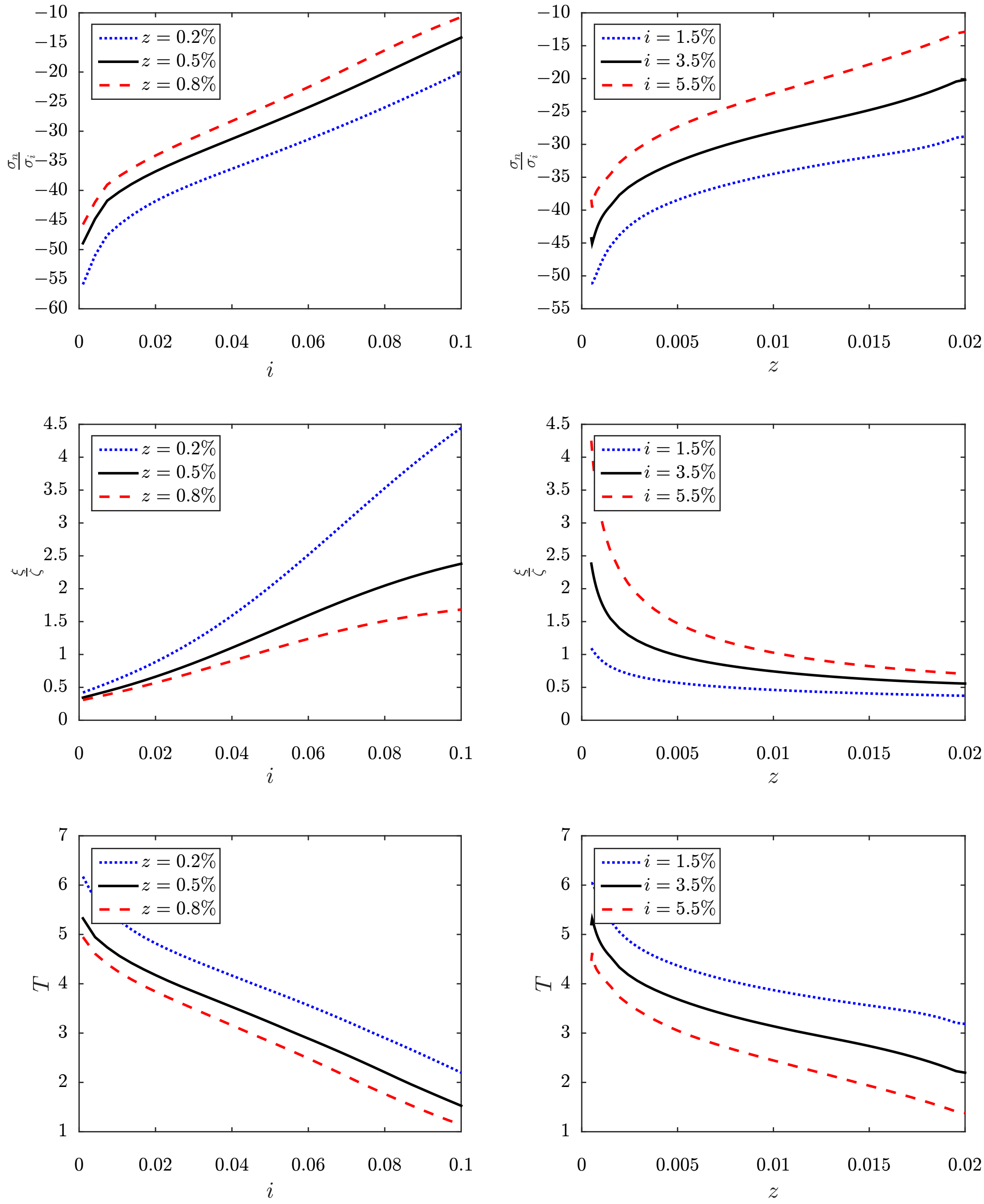

Figure 3: Aggregate risk sharing.

The hedging motive weakens at higher levels of i and z; $\frac{\sigma_n}{\sigma_i}$ is greater (in absolute value) for low i and z. This reflects the behavior of spreads. As shown in Figure 1, the spread flattens out for higher i and z. As a result, relative investment opportunities are less sensitive to i when i or z are high, so bankers choose lower exposure. To see the link between flattening spreads and lower exposure, we re-solved the banker's problem replacing the equilibrium $s(i,z)$ by the *linear* form $s(i,z) = 0.3\% + 0.66i - 0.99z$, which is the best linear approximation to the data as shown on Table 2. Since the sensitivity of spreads to i is constant in this experiment, the banker's exposure $\frac{\sigma_n}{\sigma_i}$ is almost constant as a function of i and z.

The fact that σ_n is always negative means that the desired exposure can be implemented with a "traditional" banking structure made up of deposits and long-maturity nominal bonds. In the model, the price $p^B(i,z;T)$ of a zero-coupon nominal bond of maturity T obeys the following partial differential equation:

$$\underbrace{\frac{p_i^B \mu_i + p_z^B \mu_z z + \frac{1}{2}\left[p_{ii}^B \sigma_i^2 + p_{zz}^B \sigma_z^2 z^2 + 2p_{iz}^B \sigma_i \sigma_z z\right]}{p^B} - \frac{p_T^B}{p^B}}_{\text{Nominal Capital gain}} - i = \underbrace{\pi \frac{p_i^B \sigma_i + p_z^B \sigma_z z}{p^B}}_{\text{Risk Premium}} \tag{28}$$

with boundary condition $p^B(i,z,0) = 1$ for all i, z. We use equation (28) to price bonds of all maturities at every point in the state space. The exposure to B of a traditional bank whose assets have maturity T is

$$\begin{aligned} \sigma_n &= (1+\phi)\,\sigma_{p^B} \\ &= (1+\phi)\,\frac{p_i^B(i,z;T)\,\sigma_i + p_z^B(i,z;T)\,\sigma_z z}{p^B(i,z;T)} \end{aligned} \tag{29}$$

We then find $T(i,z)$ for each point in the state space by solving (29) for T, taking σ_n from the equilibrium of the model. This is shown on the third row of Figure 3.

The maturity mismatch T is decreasing in both i and z. This reflects the higher desired exposure $\frac{\sigma_n}{\sigma_i}$ when i and z are low, which in turn results from the higher sensitivity of spreads to i in this region.[9] This basic correlation is borne out by the data. English et al. (2012) report banks' average, asset-weighted, maturity gap, which corresponds quite closely to our measure of T.[10] The maturity gap is constructed by recording the contractual maturity (in case of fixed-rate contracts) or repricing maturity (in case of floating-rate contracts) for each

[9] T depends on both the desired exposure σ_n and the sensitivity of bond prices σ_{p^B} for each maturity; the latter does vary with i but not by much, so the movement in T reflects mostly the movement in $\frac{\sigma_n}{\sigma_i}$.

[10] We thank Skander Van den Heuvel for kindly sharing this data with us.

line of the balance sheet, taking weighted averages of assets and liabilities and subtracting.[11] Table 3 shows the results of an OLS regression of banks' maturity mismatch T on i and z. A 100 bp increase in i is associated with decrease in T of 0.2 years; a 100 bp increase in z is associated with a decrease in T of 0.034 years.

	(1)
Constant	3.9
	(0.13)
i	−20.3
	(5.3)
z	−3.4
	(1.2)
R^2	0.45
N	41

Note: The dependent variable is the maturity mismatch T. Newey and West (1987) standard errors are in parentheses. i and z are demeaned.

Table 3: Maturity mismatch of banks, interest rates and banks' share of aggregate wealth.

Figure 4 compares the time series for T predicted by the model with the English et al. (2012) data. For the model values, we simply plug in the measured time series of i and z into the function $T(i, z)$ produced by the model. The model matches the behavior of T quite closely. The average T in the data is 3.9 years; in the model, it's 3.2 years. Furthermore, the model reproduces the time pattern in the data: the bank chooses higher T at times of low interest rates, as in 2003-2004. The model is less successful in the run-up to the financial crisis in 2006-2007, where it underpredicts T. Overall, the correlation between the model and the data is 0.56.

The model also produces a term premium. We can compute the excess return on a long term nominal bond simply as

$$ER = \sigma_{p^B} \pi$$

Piazzesi and Schneider (2007) report an average excess return on 5-year treasuries of 99 basis points. In the model, this excess return is 22 basis points, so the forces in the model explain about a fifth of the term premium. The reason a term premium emerges in the model is

[11] This measures the on-balance-sheet exposure, not the exposure through derivatives. English et al. (2012) show that for the majority of banks, this makes no difference since they do not trade derivatives. However, the evidence in Begenau et al. (2015) indicates that, especially for the largest banks, derivatives amplify interest rate exposure, so just measuring the on-balance-sheet positions underestimates the maturity mismatch. On the other hand, the option to refinance fixed-rate mortgages lowers their effective maturity. Using the contractual maturity therefore overestimates the maturity mismatch.

because χ (the cost of a unit of the composite good x) is increasing in the interest rate. In other words, high interest rates make liquidity expensive. Since liquidity is part of agents' composite consumption bundle, agents value wealth more in high-interest-rate states of the world. Therefore agents demand a premium to hold long term nominal bonds, which fall in value when interest rates rise.

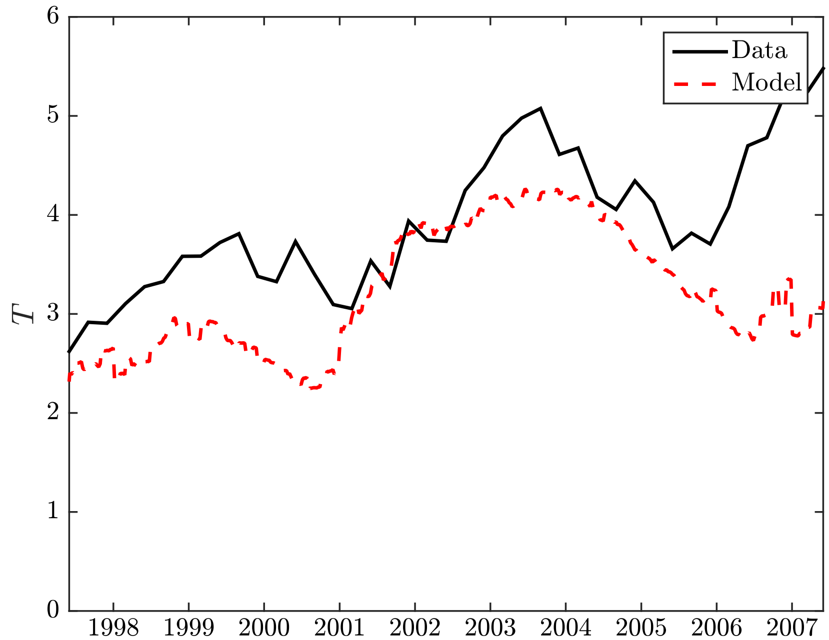

Figure 4: Maturity mismatch of banks in the English et al. (2012) data and in the model.

Notice however that the term premium does not play a role in equilibrium risk exposure. The premium provides equal incentives for households and bankers to take interest rate risk, as seen in the FOCs (23) and (24). As a result, π drops out of equation (25); only the relative hedging motive matters. In a richer model that could quantitatively account for the term premium, this basic force would remain unchanged.

Finally, it is important to note that while banks choose a large exposure to interest rate risk, TFP shocks $\tilde{B}$ are shared proportionally by both banks and households: $\tilde{\sigma}_n = \tilde{\sigma}_w = \tilde{\sigma}_a$. The reason for this is that these TFP shocks don't affect the investment opportunities of banks relative to households, so there is no relative hedging motive as in equation (26). Our theory therefore provides not only an explanation for why banks are exposed to risk in general, but also why they are exposed to interest rate risk in particular. A similar line of argument indicates that if banks also earn a credit spread, dynamic hedging motives would explain why they choose to be exposed to changes in this spread, as documented by Begenau et al. (2015).

Dynamics. Agents' endogenous exposure to interest rate risk leads to interesting equilibrium dynamics, shown in Figure 5. The upper panels show the drift of bankers' share of aggregate wealth z and the bottom panels its sensitivity to B. The drift of z is positive for small z and high i, because in this region the spread is high.

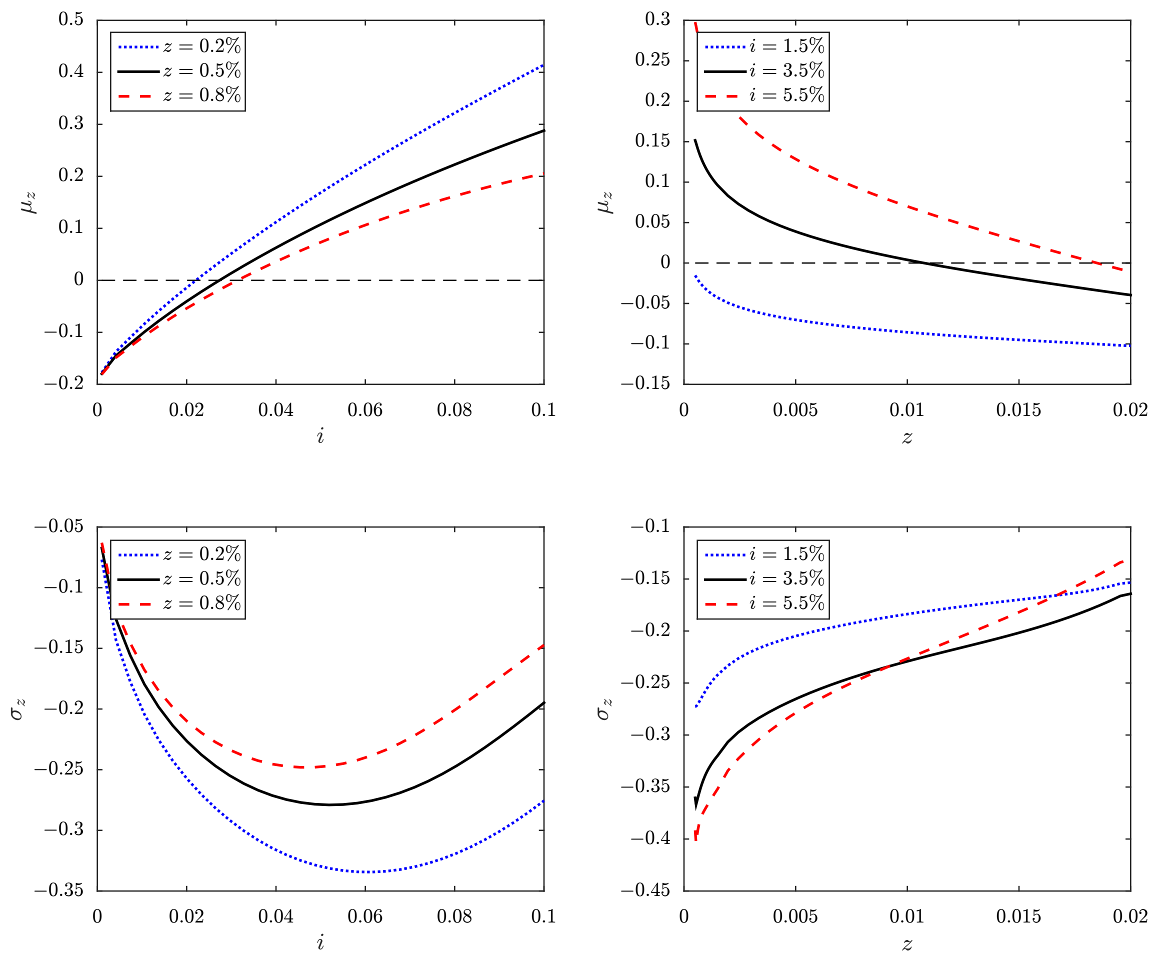

Figure 5: The drift of z, μ_z (upper panels) and its volatility σ_z (lower panels).

On impact, banks take losses when interest rates rise. Since total wealth ω is fixed, their share of aggregate wealth z fall. This is reflected in the bottom panels of Figure 5, where σ_z is negative. Over time, higher interest rates mean higher spreads and bank balance sheets strengthen. The resulting stationary distribution is shown in Figure 6.

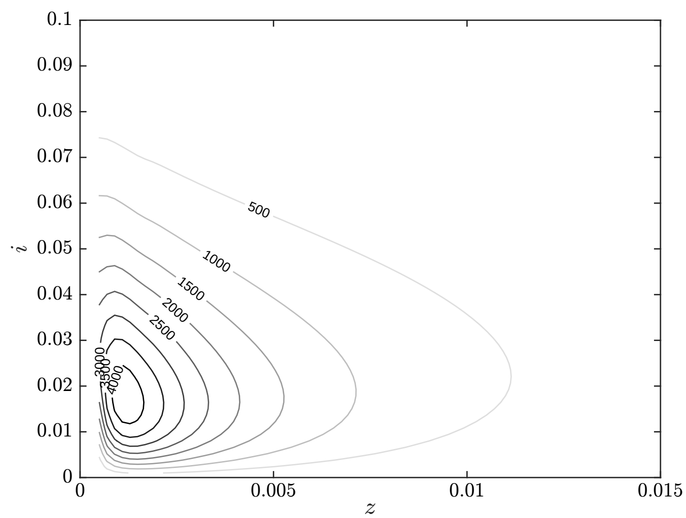

Figure 6: Stationary distribution over (i, z).

The role of risk aversion. We set the relative risk aversion parameter $\gamma = 10$. There is no consensus in the literature on the appropriate value for this parameter; lower values are more typical in macroeconomic models. Since the mechanism in this model is related to dynamic hedging, and more broadly to asset pricing, we use a value for γ in the range that has been found useful in matching asset pricing data, as in Bansal and Yaron (2004) or more recently Bansal et al. (2009).

We also perform a sensitivity analysis with $\gamma = 3$, $\gamma = 6$ and $\gamma = 20$. In each case, we set the rest of the parameter values to match the same targets and re-compute the time series for T predicted by the model. The results are shown in Table 4. Even with $\gamma = 3$, we get a significant maturity mismatch $T = 2$. Note that the maturity mismatch *increases* with risk aversion: banks take interest rate risk to insure against their stochastic investment opportunities. The effect is therefore stronger the higher risk aversion is. This can be seen in equation (25) (in particular, with $\gamma = 1$ we would get $T = 0$).

It is worth stressing that we cannot understand banks' risk taking behavior in isolation. Some other agent needs to take the other side (households in our model), so what matters is how monetary shocks affect their investment opportunities *relative* to households, as equation (25) shows. In other words, it is perfectly possible that neither banks nor households prefer losses after interest rates increases (liquidity is scarcer and the economic environment therefore worse for all agents), but banks dislike this less than households.

γ	3	6	10	20	English et al. (2012) data
Average T	2.0	2.9	3.2	3.6	3.9

Table 4: Average maturity mismatch for different values of risk aversion

5 Real Shocks under Inflation Targeting

Up to this point we have assumed that the only source of changes in interest rates is monetary policy shocks. This benchmark was useful to examine the mechanisms at play without confounding factors. In this section we look at the opposite case, where monetary policy follows an inflation targeting rule and changes in interest rates are the result of real shocks. There are many possible real shocks that could have an effect on equilibrium real interest rates. We will focus on a simple case, where the only shock is a change in the expected growth rate of TFP.

In particular, we assume that the growth rate of productivity μ_a is stochastic and follows:

$$\frac{d\mu_{a,t}}{\mu_{a,t}} = -\lambda\left(\mu_{a,t} - \bar{\mu}_a\right)dt + \sigma\sqrt{\mu_{a,t} - \mu_a^{min}}dB_t$$

This is a Cox et al. (1985) stochastic process for the growth $\mu_{a,t} - \mu_a^{min}$. $\bar{\mu}_a$ is the mean growth rate and μ_a^{min} is a, possibly negative, lower bound.

Monetary policy consists of targeting a constant rate of inflation $\bar{\mu}_p$ (keeping $\sigma_p = \tilde{\sigma}_p = 0$ as before). Therefore the nominal interest rate is just:

$$i_t = r_t + \bar{\mu}_p$$

where r_t is the endogenous real interest rate.

Instead of shocks to monetary policy, changes in the interest rate reflect the central bank's endogenous response to changes in the equilibrium real interest rate, which is driven by shocks to the expected growth rate. During booms when growth rates are high, equilibrium real interest rates must be high to clear the goods market. In order to maintain constant inflation, the central bank raises nominal interest rates.

The model can be solved along the same lines as the baseline model. The main difference is that the state variables are now μ_a (exogenous) and z (endogenous). Appendix B shows the details of the solution method.

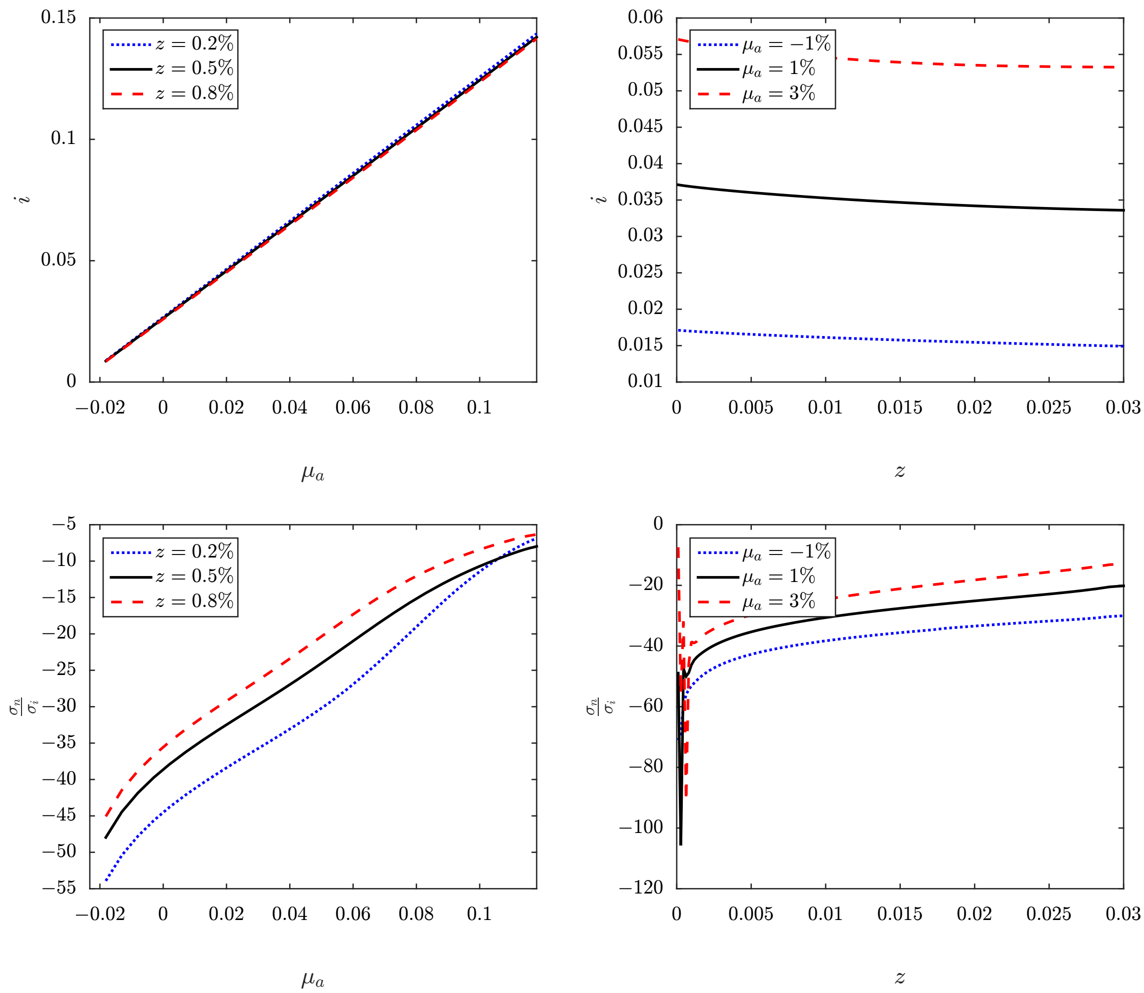

Figure 7: Aggregate risk sharing under inflation targeting.

Parameter values. We maintain most of the parameter values from the baseline model. In particular, we keep the same values for γ, ρ, ϕ, α, β and ϵ. We again set $\tilde{\sigma}_a$ to match average real interest rates, which results in $\tilde{\sigma}_a = 0.073$ and we set $\tau = 0.146$ to match the average level of z. We set the inflation target $\bar{\mu}_p = 2.53\%$ to match average inflation for 1990-2014. We set $\bar{\mu}_a = 0.01$ as in the baseline model and set $\lambda = 0.013$ and $\sigma = 0.024$ to match the standard deviation of LIBOR and the 10-year bond yield. This implies a very persistent and volatile process for the expected growth of the economy, much more so than the data. The goal is to match movements in both short and long interest rates that we observe, and which are central to the mechanism; a full theory of why equilibrium real interest rates move so much is beyond the scope of this exercise. The lower bound $\mu_a^{min} = \gamma\tilde{\sigma}_a^2 - \beta\rho - \bar{\mu}_p = -0.023$ is set to ensure that it's always possible to attain the inflation target.[12]

Results. Figure 7 shows the mechanisms at play. The top panels show how the nominal interest rate depends on μ_a and z. Lower growth rates lead to lower equilibrium real rates and, since inflation is constant, to lower nominal rates. Since holding currency is always an option, the nominal interest rate is always positive.

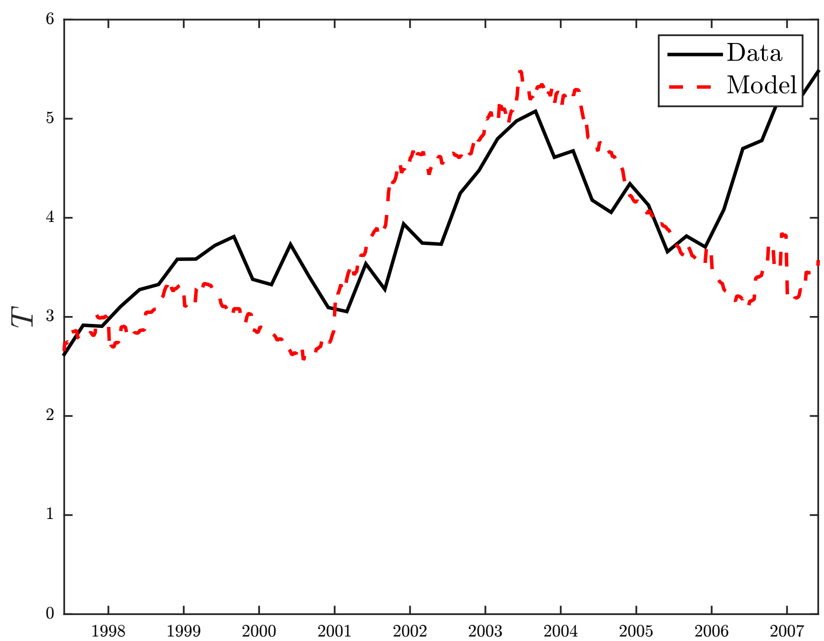

Figure 8: Maturity mismatch of banks in the English et al. (2012) data and in the model, under inflation targeting.

The bottom panels show how bankers and households share risk. Banks' exposure, again, is always negative and quite large. At the average values of μ_a and z, a change in the growth rate that induces a 100 basis point rise in the nominal interest rate results in banks losing

[12] Equilibrium requires a positive nominal interest rate $i = r + \bar{\mu}_p$. If the expected growth rate μ_a is very negative, the required equilibrium real interest rate could be too negative, $r < -\bar{\mu}_p$ for some (i, z), which would force the central bank to miss its inflation target.

about 35% of their net worth. The underlying mechanism is the same as in the baseline model and the magnitude of the effect is similar.

Figure 8 compares the time series for T predicted by the model with the measurements of bank asset-liability maturity mismatch from English et al. (2012). For the model values, we take the measured time series for i and z and back out the level of μ_a in the model that would generate the observed i given the observed z. We then plug in the imputed μ_a and the measured z into the function $T(\mu_a, z)$ produced by the model. Again, the model matches the behavior of T quite closely, both in terms of average levels and in terms of time pattern. The average T in the data is 3.9 years; in the model, it's 3.6 years. The correlation between the model and the data is 0.57.

We conclude from this that the explanatory power of the mechanism does not depend on random monetary policy being the driver of interest rates. Real shocks under an inflation targeting regime have approximately the same effect, as long as they imply similar movement in nominal interest rates.

6 Conclusion

We propose an explanation for banks' exposure to interest rate risk based on their role as providers of liquidity. Since the spread between (liquid) deposits and (illiquid) bonds rises after the interest rate increases, their exposure to interest rate risk can be seen as part of a dynamic hedging strategy. Banks are willing to take large losses after interest rates increase because they expect better investment opportunities looking forward (relative to households). This risk exposure can be achieved with a traditional banking balance sheet with a maturity mismatch between assets and liabilities.

When we calibrate the model to US data, we find an average maturity mismatch of 3.2 years, compared to 3.9 years in the data. Furthermore, the model reproduces the time pattern in the data, with a larger maturity mismatch during periods of low interest rates. This is true both when interest rates are driven by monetary policy shocks and when they are driven by real shocks under an inflation targeting regime. Seen through the lens of our model, banks' exposure to interest rate risk does not constitute risk seeking, bur rather a form of insurance, and *increases* with risk aversion.

More generally, our theory has implications for banks' risk exposure beyond interest rate risk. Banks will choose exposure to risks that are correlated with their investment opportunities. The approach in this paper can therefore be useful in studying not only how much, but also what type of risks banks take.

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Appendix A: Modified Model and Solution Method

Modified model with taxes and stochastic productivity. Let $\tilde{\sigma}$ denote exposure to the productivity shock $\tilde{B}_t$ and let $\tilde{\pi}_t$ denote the risk premium for exposure to this shock. Since the model scales linearly with the level of a we redefine ω as total wealth divided by a and likewise for h, g, k and q.

If τ is the tax rate on bankers' wealth, the government budget implies that $\tau \frac{z_t}{1-z_t}$ is the subsidy rate on households' wealth. The budget constraints thus become, respectively:

$$\begin{aligned}
\frac{dn_t}{n_t} &= \left(r_t - \tau + \sigma_{n,t}\pi_t + \tilde{\sigma}_{n,t}\tilde{\pi}_t - \chi_t \hat{x}_t + \phi s_t\right) dt + \sigma_{n,t} dB_t + \tilde{\sigma}_{n,t} d\tilde{B}_t \\
\frac{dw_t}{w_t} &= \left(r_t + \tau \frac{z_t}{1-z_t} + \sigma_{n,t}\pi_t + \tilde{\sigma}_{n,t}\tilde{\pi}_t - \chi_t \hat{x}_t + \phi s_t\right) dt + \sigma_{n,t} dB_t + \tilde{\sigma}_{n,t} d\tilde{B}_t
\end{aligned}$$

and the HJB equations are, respectively:

$$\begin{aligned}
0 = &\max_{\hat{x},\sigma_n,\tilde{\sigma}_n,\mu_n} \rho\,(1-\gamma)\,\frac{(\xi_t n_t)^{1-\gamma}}{1-\gamma}\left[\log(\hat{x} n_t) - \frac{1}{1-\gamma}\log\left((\xi_t n_t)^{1-\gamma}\right)\right] \\
&+ \xi_t^{1-\gamma} n_t^{1-\gamma}\left(\mu_n + \mu_{\xi,t} - \frac{\gamma}{2}\sigma_n^2 - \frac{\gamma}{2}\sigma_{\xi,t}^2 + (1-\gamma)\sigma_{\xi,t}\sigma_n - \frac{\gamma}{2}\tilde{\sigma}_n^2 - \frac{\gamma}{2}\tilde{\sigma}_{\xi,t}^2 + (1-\gamma)\tilde{\sigma}_{\xi,t}\tilde{\sigma}_n\right) \\
s.t.\quad & \mu_n = r_t - \tau + \sigma_n \pi_t + \tilde{\sigma}_n \tilde{\pi}_t + \phi s_t - \hat{x}\chi_t
\end{aligned}$$

and:

$$\begin{aligned}
0 = &\max_{\hat{x},\sigma_w,\tilde{\sigma}_w,\mu_w} \rho\,(1-\gamma)\,\frac{(\zeta_t w_t)^{1-\gamma}}{1-\gamma}\left[\log(\hat{x} w_t) - \frac{1}{1-\gamma}\log\left((\zeta_t w_t)^{1-\gamma}\right)\right] \\
&+ \zeta_t^{1-\gamma} w_t^{1-\gamma}\left(\mu_w + \mu_{\zeta,t} - \frac{\gamma}{2}\sigma_w^2 - \frac{\gamma}{2}\sigma_{\zeta,t}^2 + (1-\gamma)\sigma_{\zeta,t}\sigma_w - \frac{\gamma}{2}\tilde{\sigma}_w^2 - \frac{\gamma}{2}\tilde{\sigma}_{\zeta,t}^2 + (1-\gamma)\tilde{\sigma}_{\zeta,t}\tilde{\sigma}_w\right) \\
s.t.\quad & \mu_w = r_t + \tau\frac{z_t}{1-z_t} + \sigma_w \pi_t + \tilde{\sigma}_w \tilde{\pi}_t - \hat{x}\chi_t
\end{aligned}$$

The first order conditions (19), (23) and (24) are unaffected so formula (27) still applies. The first order conditions for $\tilde{\sigma}_n$ and $\tilde{\sigma}_w$ are:

$$\begin{aligned}
\tilde{\sigma}_n &= \frac{\tilde{\pi}_t}{\gamma} + \frac{1-\gamma}{\gamma}\tilde{\sigma}_{\xi,t} \\
\tilde{\sigma}_w &= \frac{\tilde{\pi}_t}{\gamma} + \frac{1-\gamma}{\gamma}\tilde{\sigma}_{\zeta,t}
\end{aligned}$$

The same steps that lead to (27) imply:

$$\tilde{\sigma}_z = \frac{(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_i}{\xi} - \frac{\zeta_i}{\zeta}\right)}{1 - z(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta}\right)}\tilde{\sigma}_i$$

and since, by definition, $\tilde{\sigma}_i = 0$, this implies $\tilde{\sigma}_z = 0$.

Its easy to see from the market clearing conditions that

$$\omega = \frac{k}{\beta\rho} \tag{30}$$

$$h = \frac{k}{\beta}\alpha(1-\beta)\iota^{\epsilon-1}i^{-\epsilon} \tag{31}$$

and condition (21) still applies.

Using Ito's lemma,

$$\tilde{\sigma}_\xi = \frac{\xi_i}{\xi}\tilde{\sigma}_i + \frac{\xi_z}{\xi}z\tilde{\sigma}_z$$

This implies that $\tilde{\sigma}_\xi = 0$ and similarly $\tilde{\sigma}_\zeta = 0$, so

$$\tilde{\sigma}_n = \tilde{\sigma}_w = \frac{\tilde{\pi}_t}{\gamma}$$

And since $n = za\omega$, then $\tilde{\sigma}_n = \tilde{\sigma}_z + \tilde{\sigma}_a + \tilde{\sigma}_\omega = \tilde{\sigma}_a$. Therefore:

$$\tilde{\pi}_t = \gamma\tilde{\sigma}_a \tag{32}$$

Replacing the first order conditions in the HJB equations and simplifying, these reduce to:

$$\rho\log(\xi_t) = \rho\log\left(\frac{\rho}{\chi_t}\right) + r_t - \tau - \rho + \phi s_t + \mu_{\xi,t} - \frac{\gamma}{2}\sigma_{\xi,t}^2 + \frac{\gamma}{2}\sigma_{n,t}^2 + \frac{\gamma}{2}\tilde{\sigma}_a^2 \tag{33}$$

$$\rho\log(\zeta_t) = \rho\log\left(\frac{\rho}{\chi_t}\right) + r_t + \tau\frac{z_t}{1-z_t} - \rho + \mu_{\zeta,t} - \frac{\gamma}{2}\sigma_{\zeta,t}^2 + \frac{\gamma}{2}\sigma_{w,t}^2 + \frac{\gamma}{2}\tilde{\sigma}_a^2 \tag{34}$$

The price of capital q follows the stochastic process:

$$\frac{dq_t}{q_t} = \mu_{q,t}dt + \sigma_{q,t}dB_t + \tilde{\sigma}_{q,t}d\tilde{B}_t \tag{35}$$

but $\tilde{\sigma}_{q,t} = 0$, because the TFP shock affects neither i nor z. Likewise for g and ψ below, we have $\tilde{\sigma}_{g,t} = 0$ and $\tilde{\sigma}_{\psi,t} = 0$.

Arbitrage pricing implies:

$$1 + \mu_a q_t + \mu_{q,t} q_t - r_t q_t = \pi_t \sigma_{q,t} q_t + \tilde{\pi}_t \tilde{\sigma}_a q_t \tag{36}$$

Similarly, the value of government transfers g follows the stochastic process:

$$dg_t = \mu_{g,t} dt + \sigma_{g,t} dB_t + \tilde{\sigma}_{g,t} d\tilde{B}_t \tag{37}$$

The real flow of transfers is $\frac{dH_t}{p_t}$ and since $h_t \equiv \frac{H_t}{a_t p_t}$ and $\sigma_p = \tilde{\sigma}_p = 0$, arbitrage pricing of g implies:

$$h_t \left(\mu_{h,t} + \mu_a + i_t - r_t\right) + \left(\mu_{g,t} + g_t \mu_a\right) - g_t r_t = \pi_t \left(\sigma_{h,t} h_t + \sigma_{g,t}\right) + \tilde{\pi}_t \left(h_t + g_t\right) \tilde{\sigma}_a \tag{38}$$

where

$$\frac{dh_t}{h_t} = \mu_{h,t} dt + \sigma_{h,t} dB_t + \tilde{\sigma}_{h,t} d\tilde{B}_t \tag{39}$$

is the stochastic process followed by h.

Let $\psi \equiv q + g$ follow the stochastic process:

$$d\psi_t = \mu_{\psi,t} dt + \sigma_{\psi,t} dB_t + \tilde{\sigma}_{\psi,t} d\tilde{B}_t \tag{40}$$

Adding (36) and (38) and rearranging:[13]

$$\left[1 + h_t \left(\mu_{h,t} + \mu_a + i_t - r_t\right)\right] + \left[\mu_{\psi,t} + \psi_t \mu_a\right] - r_t \psi_t \quad = \quad \pi_t \left[\sigma_{\psi,t} + \sigma_{h,t} h_t\right] + \tilde{\pi}_t \tilde{\sigma}_a \omega \tag{41}$$

Solution procedure. The solution method finds endogenous objects as functions of state variables. We divide the equilibrium objects into three groups. The first are the objects that we can find statically before knowing the value functions: s, h, r and ψ. The second group consists of π, σ_n and σ_w. These variables can be solved statically if we know ξ and ζ. The last group consists of the two value functions ξ and ζ. We'll express these as a system of differential equations and solve it backwards.

[13] Note that in expressions (36), (38) and (41), the stochastic processes for, g and ψ are expressed in absolute terms, as set out by (37) and (40). The stochastic processes for q and h are expressed in geometric terms, as set out by (35) and (39).

Objects solved statically. $s(i,z)$ comes from (21). $h(i,z)$ comes from (31). By definition, $\psi = q + g = \omega - h$, so $\psi(i,z)$ follows from subtracting $h(i,z)$ from (30). Finally, rearranging (41), using $\psi = \omega - h$ and using (32) to replace $\tilde{\pi}$ we obtain r:

$$r = \frac{1+hi}{\omega} + \mu_a - \gamma\tilde{\sigma}_a^2 \tag{42}$$

Solving for π, σ_n and σ_w given ξ, ζ, s, h, r and ψ Suppose we had found any function $X(i,z)$ that is a function of i and z. By Ito's Lemma it follows that the law of motion of X is:

$$dX(i,z) = \mu_X(i,z)\,dt + \sigma_X(i,z)\,dB \tag{43}$$

where the drift and volatility are

$$\begin{aligned}
\mu_X(i,z) &= X_z(i,z)\,\mu_z(i,z) + X_i(i,z)\,\mu_i(i) \\
&\quad + \frac{1}{2}\left[X_{zz}(i,z)\,\sigma_z^2(i,z)\,z^2 + X_{ii}(i,z)\,\sigma_i^2(i) + 2X_{zi}(i,z)\,\sigma_i(i)\,z\sigma_z(i,z)\right] \\
\sigma_X &= X_z(i,z)\,\sigma_z(i,z)\,z + X_i(i,z)\,\sigma_i(i)
\end{aligned}$$

or, in geometric form:

$$\frac{dX(i,z)}{X(i,z)} = \mu_X(i,z)\,dt + \sigma_X(i,z)\,dB \tag{44}$$

where the drift and volatility are

$$\begin{aligned}
\mu_X(i,z) &= \frac{X_z(i,z)}{X(i,z)}\mu_z(i,z) + \frac{X_i(i,z)}{X(i,z)}\mu_i(i) \\
&\quad + \frac{1}{2}\left[\frac{X_{zz}(i,z)}{X(i,z)}\sigma_z^2(i,z)\,z^2 + \frac{X_{ii}(i,z)}{X(i,z)}\sigma_i^2(i) + 2\frac{X_{zi}(i,z)}{X(i,z)}\sigma_i(i)\,z\sigma_z(i,z)\right] \\
\sigma_X &= \frac{X_z(i,z)}{X(i,z)}\sigma_z(i,z)\,z + \frac{X_i(i,z)}{X(i,z)}\sigma_i(i)
\end{aligned}$$

Hence if we know $\mu_z(i,z)$ and $\sigma_z(i,z)$ and we know the functions ξ, ζ, s, h and ψ and their derivatives, we know their drifts and volatilities at every point of the state space. Numerically, we approximate the derivatives with finite-difference matrices D_i, D_z, D_{ii} and D_{zz} such that for any set of values of ξ on a grid, the values of the derivatives on the grid are:

$$\begin{aligned}
\xi_i &\approx D_i \xi \\
\xi_z &\approx \xi D_z \\
\xi_{ii} &\approx D_{ii} \xi \\
\xi_{zz} &\approx \xi D_{zz} \\
\xi_{iz} &\approx D_i \xi D_z
\end{aligned}$$

The variables π, σ_n and σ_w can be found as follows. First, in order to apply formulas (43) or (44) we need to know $\mu_z(i,z)$ and $\sigma_z(i,z)$. We get $\sigma_z(i,z)$ from equation (27). Since $n = za\omega$ and ω is a constant and $\sigma_a = 0$, we have that $\sigma_n = \sigma_z$. Using the FOC (23), we can solve for

$$\pi = \gamma \sigma_n - (1-\gamma)\,\sigma_\xi$$

and using the FOC (24) we can solve for σ_w. Now, to obtain μ_z, note that

$$\frac{z}{1-z} = \frac{n}{w}$$

and therefore

$$\mu_z = (1-z)\left[\mu_n - \mu_w + \sigma_w(\sigma_w - \sigma_n)\right] - \frac{z}{(1-z)}\sigma_z^2$$

which, using the FOCs, reduces to

$$\mu_z = (1-z)\left[(\sigma_n - \sigma_w)\,\pi + \phi s - \frac{\tau}{1-z} + \sigma_w(\sigma_w - \sigma_n)\right] - \frac{z}{(1-z)}\sigma_z^2$$

Solving for ξ and ζ. We need to solve (33) and (34). To do so, we define time derivatives such that the equations hold exactly:

$$\dot{\xi} = -\left[\rho \log\left(\frac{\rho}{\chi}\right) + r - \tau - \rho + \phi s + \mu_\xi - \frac{\gamma}{2}\sigma_\xi^2 + \frac{\gamma}{2}\sigma_n^2 + \frac{\gamma}{2}\tilde{\sigma}_a^2 - \rho \log(\xi)\right]\xi \tag{45}$$

$$\dot{\zeta} = -\left[\rho \log\left(\frac{\rho}{\chi}\right) + r + \tau\frac{z}{1-z} - \rho + \mu_\zeta - \frac{\gamma}{2}\sigma_\zeta^2 + \frac{\gamma}{2}\sigma_w^2 + \frac{\gamma}{2}\tilde{\sigma}_a^2 - \rho \log(\zeta)\right]\zeta \tag{46}$$

The algorithm for finding ξ and ζ is as follows.

1. Guess values for ξ and ζ at every point in the state space

2. Compute the derivatives with respect to i and z by a finite difference approximation

3. Compute values for π, σ_n and σ_w at every point in the state space given the guess for ξ and ζ.

4. Compute $\dot{\xi}$ and $\dot{\zeta}$ at every point in the state space using (45) and (46)

5. Take a time-step backwards to define a new guess for ξ and ζ. We use a Runge-Kutta 4 procedure.

6. Repeat steps 1-5 until $\dot{\xi} \approx \dot{\zeta} \approx 0$.

The condition $\dot{\xi} \approx \dot{\zeta} \approx 0$ is equivalent to saying that equilibrium conditions hold.

Finding the stationary distribution. Once we solve for the equilibrium, this defines drifts and volatilities for the two state variables: $\mu_i(i,z)$, $\sigma_i(i,z)$, $\mu_z(i,z)$, $\sigma(i,z)$. The density $f(i,z)$ of the steady state distribution is the solution to the stationary Kolmogorov Forward Equation:

$$
\begin{aligned}
0 = & -\frac{\partial}{\partial i}\left[\mu_i(i,z) f(i,z)\right] - \frac{\partial}{\partial z}\left[\mu_z(i,z) f(i,z)\right] \\
& + \frac{1}{2}\left(\frac{\partial^2}{\partial i^2}\left[\sigma_i(i,z)^2 f(i,z)\right] + \frac{\partial^2}{\partial z^2}\left[\sigma_z(i,z)^2 f(i,z)\right] + 2\frac{\partial^2}{\partial i \partial z}\left[\sigma_i(i,z)\sigma_z(i,z) f(i,z)\right]\right)
\end{aligned} \tag{47}
$$

We solve this equation by rewriting it in matrix form.[14] The first step is to discretize the state space into a grid of $N_i \times N_z$ points and then convert it to a $N_i N_z \times 1$ vector. Let $vec(\cdot)$ be the operator that does this conversion. We then convert the differentiation matrices so that they are properly applied to vectors:

$$
\begin{aligned}
D_i^{vec} & \equiv I_{N_i} \otimes D_i \\
D_{ii}^{vec} & \equiv I_{N_i} \otimes D_{ii} \\
D_z^{vec} & \equiv M'\left(I_{N_z} \otimes D_z\right) M \\
D_{zz}^{vec} & \equiv M'\left(I_{N_z} \otimes D_{zz}\right) M \\
D_{iz}^{vec} & \equiv D_i^{vec} D_z^{vec}
\end{aligned}
$$

where $\otimes$ denotes the Kroenecker product and M is the vectorized transpose matrix such that $Mvec(A) = vec(A')$.

[14] See Achdou et al. (2014) for details on this procedure.

Now rewrite (47):

$$\begin{array}{rcl} -D_i^{vec}\cdot\left(diag\left(vec\left(\mu_i\right)\right)vec\left(f\right)\right)-D_z^{vec}\left(diag\left(vec\left(\mu_z\right)\right)vec\left(f\right)\right) & & \\ +\frac{1}{2}\left[\begin{array}{c} D_{ii}^{vec}\cdot\left(diag\left(vec\left(\sigma_i^2\right)\right)vec\left(f\right)\right)+D_{zz}^{vec}\left(diag\left(vec\left(\sigma_z^2\right)\right)vec\left(f\right)\right) \\ +2D_{iz}^{vec}\left(diag\left(vec\left(\sigma_i\right)\right)diag\left(vec\left(\sigma_z\right)\right)vec\left(f\right)\right) \end{array}\right] & = & 0 \end{array}$$

and therefore

$$Avec\left(f\right)=0 \tag{48}$$

where

$$\begin{aligned} A = & -D_i^{vec}\cdot diag\left(vec\left(\mu_i\right)\right)-D_z^{vec}\cdot diag\left(vec\left(\mu_z\right)\right) \\ & +\frac{1}{2}\left[D_{ii}^{vec}\cdot diag\left(vec\left(\sigma_i^2\right)\right)+D_{zz}^{vec}\cdot diag\left(vec\left(\sigma_z^2\right)\right)+2D_{iz}^{vec}\cdot\left(diag\left(vec\left(\sigma_i\right)\right)diag\left(vec\left(\sigma_z\right)\right)\right)\right] \end{aligned}$$

Equation (48) defines an eigenvalue problem. We solve it by imposing the additional condition that f integrates to 1.

Appendix B: Solution of the Model with Shocks to the Growth Rate

The steps that lead to equations (30) -(41) are unchanged, except for two differences. First, all objects are functions of μ_a and z instead of i and z. For instance:

$$\sigma_z=\frac{(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_{\mu_a}}{\xi}-\frac{\zeta_{\mu_a}}{\zeta}\right)}{1-z(1-z)\frac{1-\gamma}{\gamma}\left(\frac{\xi_z}{\xi}-\frac{\zeta_z}{\zeta}\right)}\sigma$$

Second, since i is not a state variable, we need to solve for it. However, it can simply be replaced by $i=r+\mu_p$.

The solution procedure is also to divide the equilibrium objects into three groups. The objects we can solve statically are: s, h, ψ and r. We can then find π, σ_n and σ_w if we know ξ and ζ. We find ξ and ζ by solving a system of differential equations.

We use the market clearing condition for deposits (21), the market clearing condition for currency (31), equation (42) for r and the definition $\omega=\psi+h$ to solve for h, s, r and ψ simultaneously. π, σ_n and σ_w are obtained in the same way as in the baseline model and the differential equations (45) and (46) still apply and can be solved the same way.

Appendix C: Data Sources

- We take the weekly 6-month LIBOR rate (series USD6MTD156N) and 10-year bond yields (series DGS10) from FRED.
- The wealth measure is "All sectors; U.S. wealth" (series Z1/Z1/FL892090005.Q) from the quarterly Flow of Funds.
- Consumption is "Personal Consumption Expenditures, Nondurable Goods" plus "Personal Consumption Expenditures, Services" from NIPA.
- Total checking deposits and total savings deposits are, respectively "Private depository institutions; checkable deposits; liability" (series Z1/Z1/FL703127005.Q) and "Private depository institutions; small time and savings deposits; liability" (series Z1/Z1/FL703131005.Q) from the quarterly Flow of Funds.
- Total net worth of banks is the difference between "Private depository institutions; total liabilities and equity" (series Z1/Z1/FL704194005.Q) and and "Private depository institutions; total liabilities" (series Z1/Z1/FL704190005.Q) from the quarterly Flow of Funds.
- The interest rates on checking and savings deposits are, respectively, the average rates on "Interest checking 0k" and "Money market deposit 10k" reported by Drechsler et al. (2014).
- Inflation, used to construct real interest rates, is the CPI inflation from the BLS