Effects of Information Transmission by High Frequency Traders on Market Quality

Ahmad Peivandi* and Mohammad Abbas Rezaei†

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Abstract

We develop a two period model of trade where an insider, a noise trader, a high frequency trader (HFT) and a market maker trade a divisible asset that has a common value in two parallel markets. The market makers set competitive prices in both markets. We analyze the effects of the high frequency trader, who can gain from observing prices across markets, on market quality. Even though informed traders can transfer information across markets that is potentially useful in setting a more accurate price for the asset, we show that due to strategic interactions between the insider and the HFT, price discovery does not improve. Also, we show that HFT has a negative effect on market liquidity.

*Robinson College of Business, Georgia State University. Email: apeivandi@gsu.edu

†Haas School of Business, University of California, Berkeley. Email: Rezaei@berkeley.edu

1 Introduction

It is argued that fragmentation of equity markets forces exchanges to narrow their bid-ask spreads, which improves the overall market efficiency (Pagano (1989)). Regulations in the 80s and 90s trammeled the market power of centralized trading exchanges, which in turn promoted the equity markets to become fragmented. Nowadays, the same stock is traded in multiple exchanges. This has provided incentives to develop technologies to transfer market information across trading venues in a timely manner to facilitate some traders (referred to as high frequency traders (HFT)) to use these newly developed tools to employ trading strategies that depend on the information in other markets.[1] We develop a theoretical model to study the effects of information transmission by high frequency traders on market quality. We show despite the fact that high frequency traders reduce the market opaqueness by incorporating the information of other markets in their orders, because of the strategic response of insiders, price discovery and liquidity are harmed.

To conduct our analysis, we design a two period model where an indivisible asset with common value is traded in two markets. The agents in our model are: i) a market maker who, at the end of each period in each market, observes and absorbs the total order and sets the price of the asset equal to the expected value of the asset conditional on his observation, ii) an insider who knows the value of the asset and can submit orders in both markets, iii) noise traders who submit random orders to buy or sell the asset in both markets and both periods because of liquidity shocks or some other exogenous reasons, and iv) an informed trader who has access to a fast information transmission technology which enables him to observe the period 1 price and period 1 total order for both markets before submitting his order in period 2. Access to the fast information transmission between markets is costly. The informed trader is the only agent in the model who has such privilege. We call our model, the HFT model. In the simplest form, by eliminating the access of the informed trader to the information transmission technology, our model is reduced to two independent markets that operate similar to a two period model of Kyle (1985). We also consider a hypothetical model with full information transmission, where all agents have access to the information transmission technology. In other words, in the second period the price and total orders in both markets are common knowledge. We call this model the *transparent* model which is equivalent to a limiting case of "Informational integration with no integration" in Cespa and Colla (2017). The transparent model has the highest information integration and the Kyle model has no information integration.

Similar to the result of Kyle (1985), we show that in both the transparent model and the high frequency trading model a pure strategy sub-game perfect Nash equilibrium with linear strategies exists. Moreover, we show that the proposed equilibrium is the unique linear equilibrium. We analyze the effects of the informed

[1] Laughlin et al (2014) estimates the total cost of reducing the delay between Chicago and New York by 3 millisecond to be at least 300 million dollars.

trader on price discovery by comparing the equilibrium of our model with the equilibrium of the two period Kyle (1985) model and the transparent model.

The insider exploits his private information by submitting orders in both markets. In the HFT model, the insider does not have access to the information transmission technology to observe the period 1 prices in the other market. However, the insider strategically takes into account the fact that the informed trader observes the total order and the price at period 1. This strategic consideration is more severe in the transparent model, since market makers who set the period 2 price observe the prices at period 1. In period 1, the insider, in the presence of information transmission, submits a smaller order to disguise the value of the asset. Hence, as markets become more informationally integrated, the insider in period 1 becomes less sensitive to his private information. Therefore, the market liquidity, as measured by the volume of trade, declines in period 1. In period 2, even though the transparent model has the highest information integration, the insider is least sensitive to his private information in the HFT model. The reason is that the insider only observes the period 1 price in the same market, hence, he has no information about the quality of the signal that the informed trader receives by observing the other market's total order in period 1. Moreover, we show that the market liquidity, as measured by volume of trader, is higher in both periods when markets are opaque (Kyle model), compared to the HFT model. We show that in equilibrium, the informed trader submits an order that is increasing in the total period 1 order of the other market and decreasing in period 1 order in the same market. At period 1, the informativeness of the period 1 total order declines as markets become more informationally integrated. However, at period 2 the presence of the informed trader increases the informativeness of the period 2 total order. At period 1 the price discovery declines as markets become more informationally integrated. At period 2, however, the price discovery is best in the transparent model, this is because market makers observe the period 1 prices before they set the period 2 prices, they can set a more accurate price. However, in the HFT model the price discovery is similar to the price discovery in the Kyle model.

2 Related Literature

The concept of extracting information from order flow has been around for a while, to the best of our knowledge, this is the first work that applies this framework in settings with parallel markets. There is a strand of papers that consider the order flow information. Based on the celebrated work of Kyle (1985). Cespay and Collaz (2017) study a model in which traders trade in two parallel markets. They consider a setting where in the second period insiders can observe the period 1 price and total order in the other market. They consider a policy that mandates insiders to disclose their total orders in both markets before period 2 trade takes place. They show that this policy does not affect the price discovery. However, it reduces the co movement of price in response to common fundamental shocks. We complement their work by considering the strategic interactions between a separate agent who has access to the information transmission and insiders. In contrast to their finding, we show the strategic interaction harms the price discovery. Based on Madrigal (1996), Yang and Zhu (2015) considers a two period trading model where an agent (called a back runner) observes a noisy signal of the informed trader's order in period 1. They show that because of the strategic interactions of the inside trader with the back runner price discovery is harmed. We consider a setting were the informed trader's information comes from observing the total order and price in a parallel market. We extend their work by showing that despite the fact that the informed trader could possibly enhance the price discovery by bribing useful information from another market, the market quality is harmed. In contrast to their model, we prove existence and uniqueness of a pure linear equilibrium. Li (2014) considers a model of high frequency trading in which there are multiple HFTs with various speeds. In their setting HFTs observe the aggregate order flow before it reaches the market makers and make their orders on top of them. They show that the market quality is harmed when the HFTs have different speeds.

The benefits of existence of multiple markets where the same asset is traded have been studied in several papers such as Foucault and Menkveld (2008), O'Hara and Ye (2011), and Degryse et al (2015). However, researches disagree on the effects of HFT, who potentially benefit from market fragmentation, on the market efficiency. While Brogaard et al (2014) claims that high frequency trading facilitate price efficiency, Zhang (2010) presents evidence that HFT can be harmful. We argue that information transmission by HFTs across markets could harm the market quality.

Some earlier models also study the information of liquidity-driven order flows. Bernhardt and Taub (2008) studies the front-running problem in a two period model. In their model some agents (called speculators) observe noise traders orders in both periods before other agents and submit their orders. Attari, Mello, and Ruckes (2005) point out that arbitrageurs insufficient capitalization make their trades predictable. Other traders exploit this information and trade against them. They show that these trading strategies may significantly distort prices. Brunnermeier and Pedersen (2005) shows that predatory trading against distressed traders harms market liquidity. Carlin et al (2007) characterize conditions under which repeated interactions among traders result in predating each other or providing liquidity to each other. Our paper is also related to a strand of literature that study multi-market trading, Chowdhry and Nanda (1991), Pagano (1989), Baruch et al (2007), Pasquariello and Vega (2009). Non of these papers study the effects of information transmission by high frequency traders on market quality.

3 Model

We propose a model of trade in parallel markets in which a single asset is traded against a numeraire in two markets. We call our model the High Frequency Trading (HFT) model. The traded asset is perfectly divisible, and value of the asset, denoted by v, is normally distributed with mean p_0 and variance σ^2. Without loss of generality assume all agents start with zero endowment of money and zero endowment of the asset. The model has three dates $t = 0,\ 1,\ 2$ and 2 markets $m = A,\ B$. Trade takes place at periods $t = 1,\ 2$ in both markets and the value of the asset is realized at the end of date 2. Also, at the end of date 2, the final payoff of all agents are realized. This includes both the realized payoff from holding the asset and the payoff from monetary transfers. All agents are risk neutral and maximize the expected final payoff in each trading time. There are four types of agents in the model, an insider, a market maker for each market, a high frequency (HF) trader and noise traders.

At date 0 the insider learns the value v of the asset. The insider submits orders in two markets at periods 1 and 2. Since markets are physically separated, the insider submits orders thorough an agent in each market. Noise traders submit a total order of u_m^t in market $m \in A, B$ at time t which we normalize to have mean 0 and variance 1. We assume total orders by noise traders are independent across time periods and markets, and are also independent of v. The timing of the model is as follows:

1. Date 1: At the beginning of time1 the insider, who knows v, submits an order x_m^1 in markets $m \in \{A, B\}$. The market maker in market $m \in \{A, B\}$ observes the total order, comprising of the noise traders total order and the insider order in market m, and sets a price for the asset. Since Market makers are competitive, Market makers in both markets set competitive prices and their expected profit is zero. Hence, the market maker in each market sets the price equal to the expected value of the asset conditional on the total order.

2. Date 1.5: Between time 1 and 2, HF trader observes prices and quantities traded in both markets.

3. Date 2: At the beginning of $t = 2$, the insider, the noise trader and the HF trader submit orders in both markets. The insider's agent in market $m \in \{A, B\}$ observes the period one price and period one total order in market $m \in \{A, B\}$, and submits his order based on this information. We denote the insider's order and the HF trader's order in market m at period 2 by x_m^2 and y_m, respectively. The HF trader, who observed the total orders and prices of period 1 in both markets, submits orders in both market. The market marker in market $m \in \{A, B\}$ observes the total order in period 1, $x_m^1 + u_m^1$, and period 2, $x_m^2 + y_m + u_m^2$, and absorb the orders in period 2. Similar to period 1, the market maker sets the period 2 price equal to the expected value of the asset conditional on the information obtained

by observing the total order. Finally at the end of $t = 2$ the asset liquidates and agents receive the payout.

It is worth emphasizing that the HF trader is the only agent who has access to the fast information transmission to observe the period 1 prices and total orders in both markets. If the insider was the agent who had access to the fast information transmission technology, since the insider knows the value of the asset, in equilibrium he would ignore the information. Also, since the market makers make zero expected profit, the costly technology would not be beneficial to market makers.

We discuss two basic models which we use as reference points to compare our model with.

3.1 Two Period Kyle Model

Recall the two period Kyle's economy, Kyle (1985). Eliminating the HF trader in the HFT model reduces it to two separate markets that operate the same as the two period Kyle model. Since in absence of information transmission across markets, markets are symmetric, completely disjoint and operate independently, we consider only market A. Suppose the insider submits x_1 and x_2 in periods 1 and 2. Let p_1 be the price in period one and p_2 be the price in period two. The total orders in periods 1 and 2 are $\omega_2 = x_2 + u_2$ and $\omega_1 = x_1 + u_1$, respectively. Note that the insider is the only strategic agent in this model.

3.1.1 Equilibrium of the Two Period Kyle Model

Theorem (2) in Kyle (1985) for two periods can be written as the following proposition.

Proposition 1 *There exists an equilibrium of the two period Kyle model where the insiders submit the following orders:*

$$x_1 = \xi_1^{kyle}(v - p_0)$$
$$x_2 = \xi_2^{kyle}(v - p_1)$$

Prices follow:

$$p_1 = p_0 + \alpha_1^{kyle}\omega_1$$
$$p_2 = p_1 + \beta_2^{kyle}\omega_2$$

ξ_1^{kyle}, ξ_2^{kyle}, α_1^{kyle} and β_2^{kyle} are approximately as follows:

$$\xi_1^{kyle} = \frac{0.667}{\sigma}, \xi_2^{kyle} = \frac{1.202}{\sigma}$$
$$\alpha_1^{kyle} = 0.461\sigma, \beta_2^{kyle} = 0.416\sigma$$

3.2 Two Period Transparent Model

We modify the HFT model as follows: every trader observes the period one price in both markets. Note that in this case, since the market makers observe the period 1 price in both markets, the HF trader does not have any valuable information and cannot make a profit. For this reason, we assume the HF trader does not exist. We call the modified model, the *Transparent Model.* The equilibrium takes the following form.

Proposition 2 *The following is the unique Nash equilibrium with linear strategies of the transparent model::.*

$$x_1 = \xi_1^{trans}(v - p_0),$$
$$x_2 = \xi_2^{trans}(v - \hat{p}_1).$$

where $\hat{p}_1 = E[v|p_A^1, p_B^1]$ is the common belief of the market makers about the value of the asset after time 1. The prices can be written as:

$$p_1 = p_0 + \alpha_1^{trans}\omega_1$$
$$p_2 = \hat{p}_1 + \beta_2^{trans}\omega_2$$
$$\hat{p}_1 = p_0 + \alpha_2^{trans}(\omega_A^1 + \omega_B^1)$$

The parameters ξ_1^{trans}, ξ_2^{trans}, α_1^{trans}, β_2^{trans} and α_2^{trans} are approximately as follows:

$$\xi_1^{trans} = \frac{0.545}{\sigma}, \xi_2^{trans} = \frac{1.261}{\sigma}$$
$$\alpha_1^{trans} = 0.421, \alpha_2^{trans} = 0.342\sigma$$
$$\beta_2^{trans} = 0.397\sigma$$

See proof of Proposition 2 in Cespa and Colla (2017).

4 Equilibrium

Note that in our model the insider and the HF trader are both strategic players. We consider the Sub-game Perfect Nash Equilibrium of the HFT model. We conjecture that there exists a linear Nash Equilibrium, where the insider and HF trader submit orders as follows:

$$
\begin{aligned}
x_B^1 &= \theta_B^1 + \xi_B^1 v \\
x_B^2 &= \theta_B^2 + \xi_B^2 v + \gamma_B^2 \omega_B^1 \\
x_A^1 &= \theta_A^1 + \xi_A^1 v \\
x_A^2 &= \theta_A^2 + \xi_A^2 v + \gamma_A^2 \omega_A^1 \\
y_B &= y_B^0 + y_B^1 \omega_A^1 + \mu_B \omega_B^1 \\
y_A &= y_A^0 + y_A^1 \omega_B^1 + \mu_A \omega_A^1
\end{aligned} \tag{1}
$$

Moreover the prices are set by the market maker as the following:

$$
\begin{aligned}
p_B^1 &= \pi_B^1 + \alpha_B^1 \omega_B^1 \\
p_B^2 &= \pi_B^2 + \alpha_B^2 \omega_B^1 + \beta_B^2 \omega_B^2 \\
p_A^1 &= \pi_A^1 + \alpha_A^1 \omega_A^1 \\
p_A^2 &= \pi_A^2 + \alpha_A^2 \omega_A^1 + \beta_A^2 \omega_A^2
\end{aligned} \tag{2}
$$

We make the following intuitive conjecture:

- The equilibrium is symmetric between markets A and B.
- All orders that the insider and the HF trader set have zero expectation.
- From an ex-ante perspective, prior to learning any price and market order, the expected value of the price equals to the value of the asset.

In other words, there exists α_1, α_2, β_2, θ_1, θ_2, ξ_1, ξ_2, y_1, γ_2 and μ such that:

$$
\begin{aligned}
&\pi_A^1 = \pi_B^1 = p_0,\ \alpha_A^1 = \alpha_B^1 = \alpha_1 \\
&\pi_B^2 = \pi_A^2 = p_0,\ \alpha_B^2 = \alpha_A^2 = \alpha_2,\ \beta_A^2 = \beta_B^2 = \beta_2 \\
&\theta_A^1 = \theta_B^1 = \theta_1,\ \xi_A^1 = \xi_B^1 = \xi_1 \\
&\theta_A^2 = \theta_B^2 = \theta_2,\ \xi_A^2 = \xi_B^2 = \xi_2,\ \gamma_A^2 = \gamma_B^2 = \gamma_2 \\
&y_A^0 = y_B^0 = 0,\ y_A^1 = y_B^1 = y_1,\ \mu_A = \mu_B = \mu \\
&\theta_1 + \xi_1 p_0 = \theta_2 + \xi_2 p_0 = 0 \qquad (3)
\end{aligned}
$$

4.1 Period Two

Because of symmetric nature of the model, we only solve the model in one market, namely A. In period 2 market makers observe the period 1 total order ω_A^1 and period 2 total order ω_A^2 and set the price equal to the expected value of the asset conditional on their observation. The normality assumption and linearity of ω_A^1 and ω_A^2 implies:

$$
\begin{aligned}
p_A^2 &= E[v|\omega_A^1, \omega_A^2] \\
&= E[v] + \begin{bmatrix} Cov(v, \omega_A^1) & Cov(v, \omega_A^2) \end{bmatrix} \begin{bmatrix} Var(\omega_A^1) & Cov(\omega_A^1, \omega_A^2) \\ Cov(\omega_A^1, \omega_A^2) & Var(\omega_A^2) \end{bmatrix}^{-1} \begin{bmatrix} \omega_A^1 - E[\omega_A^1] \\ \omega_A^2 - E[\omega_A^2] \end{bmatrix} \qquad (4)
\end{aligned}
$$

Equation (4) gives us the parameters α_2 and β_2 in terms of other parameters.

The insider in market A in period 2 observes the value of the asset and period 1 total order and maximize his profit. The problem is the following:

$$
\max_{x_A^2} E[x_A^2(v - p_A^2)|v, \omega_A^1] \qquad (5)
$$

Program (5) gives us the ξ_2 and γ_2 in terms of other parameters.

The HF trader maximizes his expected profit conditional on observing the period 1 orders in markets A and B.

$$
\max_{y_A} E[y_A(v - p_A^2)|\omega_A^1, \omega_B^1] \qquad (6)
$$

Program (6) gives us the y_1 and μ in terms of other parameters.

4.2 Period One

At period one, the market maker sets the price as follows:

$$\begin{aligned} p_A^1 &= E[v|\omega_A^1] \\ &= E[v] + \frac{Cov(v,\omega_A^1)}{Var(\omega_A^1)}(\omega_A) \end{aligned} \tag{7}$$

And the insider solves the following problem at period one:

$$\max_{x_A^1} E[x_A^1(v-p_A^1)|v] + E[x_A^2(v-p_A^2)|v] + E[x_B^2(v-p_B^2)|v] \tag{8}$$

Note that the insider's choice of order in period one, x_A^1, affects his expected payoff in three ways:

1. It affects the expected profit in period one in market A. since the quantity of the asset x_A^1 and the price p_A^1 change.

2. Since the market maker in period two in market A sets the price based on the total order in period one and period two in market A, the price of the asset in market A at period two is affected by x_A^1. Also in period two, since the insider knows how the the optimal choice of order, x_A^2, is affected by the period two price. Hence, x_A^1 affects the expected profit from trade in period two by changing both x_A^2 and p_A^2.

3. Choice of x_A^1 affects the HF trader's information and the HF trader's order in market B in period two, which in turn affects the profit of the insider in market B in period two.

Note that equation (7) and programs (8) gives us α_1 and ξ_1 in terms of other parameters.

Theorem 1 *There exists a unique linear Nash Equilibrium of the trading game with linear prices according to equations (1), (2) and (3) where the parameters are approximately as follows:*

$$\begin{aligned} \alpha_2^{HFT} &= 0.546\sigma,\ \beta_2^{HFT} = 0.417\sigma \\ \gamma_2^{HFT} &= -0.584,\ \xi_1^{HFT} = \frac{0.606}{\sigma} \\ y_1^{HFT} &= 0.221, \quad \alpha_1^{HFT} = 0.443\sigma \\ \mu^{HFT} &= -0.142,\ \xi_2^{HFT} = \frac{1.132}{\sigma} \end{aligned}$$

See the appendix for a proof. Note that insider's order in period 1 can be written as $x_1^A = \xi_1^{HFT}(v-p_0)$.

5 Analysis

5.1 Equilibrium Behavior of Traders in the HFT Model

The insider acquires more assets if the final value is higher, hence, ξ_1^{HFT}, $\xi_2^{HFT} > 0$. In the second period the insider can take advantage of his private information, v, more when the public information, ω_1 sends the opposite (wrong) signal about the value; therefore, $\gamma_2^{HFT} < 0$. Note that σ is the ratio of asset value variance over the variance of the noise traders total order. Higher σ, implies that the insider's private information is more valuable. Hence the market maker assigns more weight to the observed total order in both periods and both markets when he sets the price; and the insider bids less aggressively to leak less information about the value of the asset to the market maker. Therefore, the insider's order sensitivity to the value is decreasing in σ.

The HF trader in period two observes the total order in the previous period in the same market (public information) and the other market (private information). Since the insider order is increasing in v, the HF trader rationally associates a higher order in the other market with a higher valuation of the asset, v. Hence, the HF trader submits a higher order when the total order in the other market is higher, $y_1^{HFT} > 0$. Similar to the insider, the HF trader takes advantage of his private information more when the private signal about the value of the asset is the opposite of the public information in the second period, hence $\mu^{HFT} < 0$. The HF trader observes the price and total order across markets. Higher σ has two opposite effects on the HF trader's order: (i) the insider's information in both markets is more valuable, hence the HF trader should react more aggressively to changes in the total order (ii) the market maker partially observes the HF trader's information through observing the total order, therefore, the HF trader should react less aggressively to leak less information about the order in the other market. It turns out that these two effects cancel out each other, hence the HF trader submits an order that is the sum of constant multipliers of total orders in period 1; in other words, y_1^{HFT} and μ^{HFT} are independent of σ.

5.2 Comparison with Base Models

We compare the equilibrium of the HFT model with the Kyle and Transparent models.

5.2.1 Strategy, Liquidity and Profit

We define the sensitivity of the insider to his private information v to be the coefficient of the v in the insider's order; in the HFT model these coefficients are ξ_1^{HFT} and ξ_2^{HFT}, respectively. The following is a corollary of theorem (1), propositions (1) and (2).

Corollary 1 *In period 1, the insider is least sensitive to his private information in the Transparent model and most sensitive in the Kyle model.*

In period 2 the insider is least sensitive to his private information in the HFT model and most sensitive in the Transparent model.

In period 1, as transparency between markets goes up, the insider submits an smaller order to leak less information about the value of the asset. Therefore, in the Transparent model, where agents can observe period 1 total orders in both markets, the insider is least sensitive to his private information.On the other hand, in the Kyle model, as markets are opaque, they submit their most aggressive order.

In contrast to the period 1 order, in the Transparent model, the insider is most sensitive to his private information at period 2. This is because he disguises his private information from the market makers more in period 1 by submitting a less informative order and make up for the difference by being more aggressive at period 2. In other words, in the Transparent model, the insider can set the stage at time one to be more aggressive at time two. However, in the HFT model there is a different scenario, where the insider does not know the period 1 price in the other market at period 2. Hence, he has partial information regarding the quality of the information of the HF trader. Therefore, he is more conservative about his order and submits an even less aggressive order compared to the Kyle model.

Note that the insider orders in the second period in the Transparent model and the Kyle model are a proportion of $v - \hat{p}_1$ and $v - p_1$, respectively. In the HFT model the insider's strategy in the second period can be rewritten as

$$x_A^2 = \xi_2^{HFT}(v - p_1^A) + (\xi_2^{HFT}\alpha_1^{HFT} + \gamma_2^{HFT})\omega_A^1$$

here $\xi_2^{HFT}\alpha_1^{HFT} + \gamma_2^{HFT} < 0$. In the second period of the HFT model, all agents in market A observe p_1^A. Also, in period 2 of market A the HF trader submits an order that is positively correlated with ω_A^1. Hence, the insider submits a larger order when the the HF trader is misguided by his signal, in other words, when period 1 price residual,$v - p_1^A$, and period 1 market order, ω_A^1, have opposite signs.

Note that insider's period 1 profit in all the models can be written as:

$$\pi_1 = -E[(v - p_1)u_1|v] = \alpha_1.$$

where α_1 is the sensitivity of the price to the total order at period 1.

Insider's profit in the HFT model in period 2 is:

$$\pi_2^{HFT} = E[(v - p_2^A)x_A^2] = \xi_2^{HFT} E[(v - p_0)(v - p_2^A)].$$

Note that the price follows a Markov property, $E[p_A^2|\omega_A^1] = E[p_A^2|p_A^1] = p_A^1$, hence the profit is equal to:

$$\xi_2^{HFT}\sigma^2(1 + \alpha_2^{HFT}\xi_1^{HFT} + \beta_2^{HFT}(\xi_2^{HFT} + \gamma_2^{HFT}\xi_1^{HFT} + y_1^{HFT}\xi_1^{HFT} + \mu^{HFT}\xi_1^{HFT})).$$

Corollary 2 *The insider's profit in both periods 1 and 2 is smallest in the transparent model and largest in the Kyle model.*

As pointed out earlier, the insider leaks less information as transparency goes up. Because of this, market makers put less weight on the total order to determine the price at period 1. We have the same order of profits in period 2 as in period 1.

We measure the market liquidity by total order. However, because markets may have different size of noise traders and prices, we calculate the expected value of the total order conditional on the value v. The liquidity in period 1 of the HFT model is equal to $x_A^1 = \xi_1^{HFT}(v - p_0)$ and in period 2 it is equal to

$$E[y_A + x_A^2|v] = (y_1\xi_1^{HFT} + \mu\xi_1^{HFT} + \xi_2^{HFT} + \gamma_2^{HFT}\xi_1^{HFT})(v - p_0)$$

Corollary 3 *In period 1, the liquidity is worst in the Transparent model and best in the Kyle model.*

In period 2 the liquidity of the HFT model is worse than the Kyle model.

5.2.2 Prices

The following is a corollary of theorem (1) and propositions (1) and (2).

Corollary 4 *At period 1, the market maker assigns the lowest weight on the period 1 total order in period 1 price in the transparent model and the highest weight in the Kyle model.*

At period 2, the market maker assigns the lowest weight on the period 1 total order in the period 2 price in the transparent model and the highest weight in the HFT model.

As we explained in the previous section, the insider's sensitivity to their private information decreases as the model becomes more transparent. The market maker rationally anticipates this and assigns less weight on the period 1 order when setting the period 1 price in the more transparent environment.

Since HF trader's order in period 2 is negatively related to the total order in the first period (in the same market), the total order in period 2 is negatively correlated with the period 1 total order. Hence, to keep the Markov property of the price the inequality $\alpha_2^{HFT} > \alpha_1^{HFT}$ must hold. Note that in the second period in the transparent model, the period 1 total orders in both markets enters the price via $\hat{p}_1^A = \alpha_2^{trans}(\omega_A^1 + \omega_B^1)$ instead of $p_1^A = \alpha_2^{kyle}\omega_A^1$ in the Kyle model. Since the total period 1 orders in both markets, ω_A^1 and ω_B^1, are positively correlated, the market maker sets $\alpha_1^{trans} > \alpha_2^{trans}$.

Note that total order in period 2 has almost the same weight in period 2 price compared with the Kyle model. This shows that all agents use their private information to extend possible at round two as in Kyle model and HFT model . However since in the transparent model the market maker has more accurate information about the price at the end of period one, the total order in period 2 is less valuable for the market maker

The price discovery in periods 1 and 2 is measured by $Var(v|p_1)$ and $Var(v|p_1, p_2)$, respectively. If the variance is higher, it shows that prices are a more noisy signal of the value, and price discovery is low. The following is a corollary of theorem (1) and propositions (1) and (2).

Proposition 3 *At period 1 the price discovery is best in the Kyle model and worst in the transparent model. At time two the price discovery is best in the transparent model and worst in the HFT model.*

At period 1, since the insider assigns a smaller weight on his private information in a more transparent model, the price discovery declines when the model becomes more transparent. However in period 2 the order changes. Our result shows that when markets are transparent for everyone, informativeness of the price goes up. However once it is shared by only a handful of agents who gain from it, the price discovery can worsen. In fact, the existence of the HF trader who observes both markets has two opposite effects on the price discovery compared to the Kyle model. The positive effect is that the HF trader reveals information about the other market's total order, which reveals a signal about the value of the asset. The negative effect comes from the fact that the HF trader competes with the insider; which gives the incentive to the insider to submit orders less aggressively. Proposition (3) states that the negative outweighs the positive effect.

6 Appendix: Proof of Theorem 1

We calculate the variance and covariance of the total orders with the asset value as follows:

$$Cov(v, \omega_B^1) = Cov(v, \omega_A^1) = Cov(v, \theta_1 + \xi_1 v + u_A^1) = \xi_1 \sigma^2$$

$$Var(\omega_A^1) = Var(\omega_B^1) = \xi_1^2 \sigma^2 + 1$$

$$Cov(\omega_A^1, \omega_B^1) = \xi_1^2 \sigma^2$$

$$Cov(\omega_A^2, \omega_B^2) = Cov(x_A^2 + u_A^2 + y_A, x_B^2 + u_B^2 + y_B) =$$

$$Cov(x_A^2 + y_A, x_B^2 + y_B) = (\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1)^2 \sigma^2 + 2y_1(\gamma_2 + \mu)$$

$$Var(\omega_A^2) = Var(\omega_B^2) = (\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1)^2 \sigma^2 + (\gamma_2 + \mu)^2 + y_1^2 + 1$$

$$Cov(v, \omega_A^2) = Cov(v, \omega_B^2) = (\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1) \sigma^2$$

$$Cov(\omega_A^1, \omega_A^2) = Cov(\omega_B^1, \omega_B^2) = \xi_1(\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1) \sigma^2 + (\gamma_2 + \mu)$$

$$Cov(v, \omega_A^1) Var(\omega_A^2) - Cov(v, \omega_A^2) Cov(\omega_A^1, \omega_A^2) =$$

$$\xi_1 \sigma^2 [y_1^2 + \sigma_2^2] - [(\gamma_2 + \mu)](\xi_2 + y_1 \xi_1) \sigma^2$$

$$Cov(v, \omega_A^2) Var(\omega_A^1) - Cov(v, \omega_A^1) Cov(\omega_A^1, \omega_A^2) = (\xi_2 + y_1 \xi_1) \sigma^2$$

$$Var(\omega_A^1) Var(\omega_A^2) - Cov(\omega_A^1, \omega_A^2)^2 =$$

$$(\xi_1^2 \sigma^2 + 1)[(\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1)^2 \sigma^2 + (\gamma_2 + \mu)^2 + y_1^2 + 1]$$

$$- [\xi_1(\xi_2 + \gamma_2 \xi_1 + y_1 \xi_1 + \mu \xi_1) \sigma^2 + (\gamma_2 + \mu)]^2$$

$$= \xi_1^2 \sigma^2 y_1^2 + \xi_1^2 \sigma^2 + \sigma^2 \xi_2^2 + 2\sigma^2 \xi_1 \xi_2 y_1 + \sigma^2 \xi_1^2 y_1^2 + y_1^2 + 1$$

$$Cov(v, \omega_A^1) Var(\omega_B^1) - Cov(v, \omega_B^1) Cov(\omega_A^1, \omega_B^1) =$$

$$\xi_1 \sigma^2 [(\xi_1^2 \sigma^2 + 1) - \xi_1^2 \sigma^2] = \xi_1 \sigma^2$$

$$Cov(v, \omega_B^1) Var(\omega_A^1) - Cov(v, \omega_A^1) Cov(\omega_A^1, \omega_B^1) = \xi_1 \sigma^2$$

$$Var(\omega_A^1) Var(\omega_B^1) - Cov(\omega_A^1, \omega_B^1)^2 = (\xi_1^2 \sigma^2 + \sigma_1^2)^2 - (\xi_1^2 \sigma^2)^2 = 2\xi_1^2 \sigma^2 + 1$$

Once we have these relations we first solve the problem at time 2 as outlined in the paper and then we go to time 1.

6.1 Period 2

At period two, Market makers set $p_A^2 = E[v|\omega_A^1, \omega_A^2]$, the normality assumption implies:

$$\begin{aligned}
p_A^2 &= E[v|\omega_A^1, \omega_A^2] \qquad (9)\\
&= E[v] + \begin{bmatrix} Cov(v,\omega_A^1) & Cov(v,\omega_A^2) \end{bmatrix} \begin{bmatrix} Var(\omega_A^1) & Cov(\omega_A^1,\omega_A^2) \\ Cov(\omega_A^1,\omega_A^2) & Var(\omega_A^2) \end{bmatrix}^{-1} \begin{bmatrix} \omega_A^1 - E[\omega_A^1] \\ \omega_A^2 - E[\omega_A^2] \end{bmatrix}\\
&= p_0 + \frac{[Cov(v,\omega_A^1)Var(\omega_A^2) - Cov(v,\omega_A^2)Cov(\omega_A^1,\omega_A^2)](\omega_A^1 - E[\omega_A^1])}{Var(\omega_A^1)Var(\omega_A^2) - Cov(\omega_A^1,\omega_A^2)^2}\\
&+ \frac{[Cov(v,\omega_A^2)Var(\omega_A^1) - Cov(v,\omega_A^1)Cov(\omega_A^1,\omega_A^2)](\omega_A^2 - E[\omega_A^2])}{Var(\omega_A^1)Var(\omega_A^2) - Cov(\omega_A^1,\omega_A^2)^2}\\
&= p_0 + \frac{\sigma^2[\xi_1(y_1+1) - (\gamma_2+\mu)(\xi_2 + y_1\xi_1)]\omega_A^1 + \sigma^2(\xi_2 + y_1\xi_1)\omega_A^2}{\xi_1^2\sigma^2 y_1^2 + \xi_1^2\sigma^2 + \sigma^2\xi_2^2 + 2\sigma^2\xi_1\xi_2 y_1 + \sigma^2\xi_1^2 y_1^2 + y_1^2 + 1}
\end{aligned}$$

More precisely, the computation above is followed from the fact that prices and total orders are a linear sum of independent normal random variables. We use this frequently from now on without mentioning the reason. Also, note that this confirms $\pi_2 = p_0$.

Since insider in market A know v and observes p_1^A at the beginning of period 2, his problem at time 2 is

$$\max_{x_A^2} E[x_A^2(v - p_A^2)|v, \omega_A^1]$$

To compute this note that:

$$\begin{aligned}
&E[y_A|v,\omega_A^1] =\\
&E[y_A^0 + y_A^1\omega_B^1 + \mu_A\omega_A^1|v,\omega_A^1] =\\
&E[y_A^0 + y_A^1(x_B^1 + u_B^1) + \mu_A\omega_A^1|v,\omega_A^1]\\
&= y_A^0 + y_A^1 x_B^1 + \mu_A\omega_A^1
\end{aligned}$$

So we get

$$
\begin{aligned}
&E[x_A^2(v - p_A^2)|v, \omega_A^1] \\
&= E[x_A^2(v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2\omega_A^2)|v, \omega_A^1] \\
&= E[x_A^2(v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2(u_A^2 + y_A + x_A^2))|v, \omega_A^1] \\
&= E[x_A^2(v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2(y_A + x_A^2))|v, \omega_A^1] \\
&= x_A^2[v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2(y_A^0 + y_A^1(\theta_B^1 + \xi_B^1 v) + \mu_A\omega_A^1 + x_A^2)]
\end{aligned}
$$

The first order condition is:

$$
-\beta_A^2 x_A^2 + [v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2(y_A^0 + y_A^1(\theta_B^1 + \xi_B^1 v) + \mu_A\omega_A^1 + x_A^2)] = 0
$$

The second order condition is $\beta_A^2 > 0$.

The first order condition implies:

$$
x_A^2 = -\frac{p_0 + \beta_2 y_1 \theta_1}{2\beta_2} - \frac{\alpha_2 + \beta_2\mu}{2\beta_2}\omega_A^1 + \frac{1 - \beta_2 y_1 \xi_1}{2\beta_2} v \tag{10}
$$

If we assume $\theta_1 + \xi_1 p_0 = 0$, which we verify once we write equations for period 1, the equation 10, gives us $\theta_2 + \xi_2 p_0 = 0$.

Informed trader observes p_1^A and p_1^B before making decision at time 2, so he solves:

$$
\begin{aligned}
&\max_{y_A} E[y_A(v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2\omega_A^2)|\omega_A^1, \omega_B^1] = \\
&\max_{y_A} E[y_A(v - \pi_A^2 - \alpha_A^2\omega_A^1 - \beta_A^2(y_A + \theta_A^2 + \xi_A^2 v + \gamma_A^2\omega_A^1))|\omega_A^1, \omega_B^1]
\end{aligned}
$$

Let $\hat{v} + p_0 = E[v|\omega_A^1, \omega_B^1]$ be the expected value of the asset conditioned informed trader's information. Note that

$$
\begin{aligned}
&E[v|\omega_A^1, \omega_B^1] = \\
&E[v] + \begin{bmatrix} Cov(v, \omega_A^1) & Cov(v, \omega_B^1) \end{bmatrix} \begin{bmatrix} Var(\omega_A^1) & Cov(\omega_A^1, \omega_B^1) \\ Cov(\omega_A^1, \omega_B^1) & Var(\omega_B^1) \end{bmatrix}^{-1} \begin{bmatrix} \omega_A^1 - E[\omega_A^1] \\ \omega_B^1 - E[\omega_B^1] \end{bmatrix} = \\
&p_0 + \frac{[Cov(v, \omega_A^1)Var(\omega_B^1) - Cov(v, \omega_B^1)Cov(\omega_A^1, \omega_B^1)](\omega_A^1 - E[\omega_A^1])}{Var(\omega_A^1)Var(\omega_B^1) - Cov(\omega_A^1, \omega_B^1)^2} \\
&+ \frac{[Cov(v, \omega_B^1)Var(\omega_A^1) - Cov(v, \omega_A^1)Cov(\omega_A^1, \omega_B^1)](\omega_B^1 - E[\omega_B^1])}{Var(\omega_A^1)Var(\omega_B^1) - Cov(\omega_A^1, \omega_B^1)^2} \\
&= p_0 + \frac{\xi_1 \sigma^2}{2\xi_1^2 \sigma^2 + \sigma_1^2}(\omega_A^1 + \omega_B^1)
\end{aligned}
$$

Hence, $\hat{v} = \frac{\xi_1 \sigma^2}{2\xi_1^2 \sigma^2 + \sigma_1^2}(\omega_A^1 + \omega_B^1)$. Assuming this, the FOC for informed trader's problem is:

$$
\begin{aligned}
&- \beta_A^2 y_A + (\hat{v} - \alpha_A^2 \omega_A^1 - \beta_A^2(y_A + \xi_A^2 \hat{v} + \gamma_A^2 \omega_A^1)) = 0 \\
&\Rightarrow y_A = -\frac{\alpha_2 + \beta_2 \gamma_2}{2\beta_2}\omega_A^1 + \frac{1 - \beta_2 \xi_2}{2\beta_2}\left(\frac{(\xi_1 \omega_A^1 + \xi_1 \omega_B^1)\sigma^2}{\sigma_1^2 + 2\xi_1^2 \sigma^2}\right) \\
&= \frac{\xi_1(1 - \beta_2 \xi_2)\sigma^2 - (\alpha_2 + \beta_2 \gamma_2)(\sigma_1^2 + 2\xi_1^2 \sigma^2)}{2\beta_2(\sigma_1^2 + 2\xi_1^2 \sigma^2)}\omega_A^1 + \frac{\xi_1(1 - \beta_2 \xi_2)\sigma^2}{2\beta_2(\sigma_1^2 + 2\xi_1^2 \sigma^2)}\omega_B^1
\end{aligned}
$$

This gives us

$$
\begin{aligned}
y_A &= -\frac{\alpha_2 + \beta_2 \gamma_2}{2\beta_2}\omega_A^1 + \frac{1 - \beta_2 \xi_2}{2\beta_2}\left(\frac{(\xi_1 \omega_A^1 + \xi_1 \omega_B^1)\sigma^2}{\sigma_1^2 + 2\xi_1^2 \sigma^2}\right) \\
&= \frac{\xi_1(1 - \beta_2 \xi_2)\sigma^2 - (\alpha_2 + \beta_2 \gamma_2)(\sigma_1^2 + 2\xi_1^2 \sigma^2)}{2\beta_2(\sigma_1^2 + 2\xi_1^2 \sigma^2)}\omega_A^1 + \frac{\xi_1(1 - \beta_2 \xi_2)\sigma^2}{2\beta_2(\sigma_1^2 + 2\xi_1^2 \sigma^2)}\omega_B^1
\end{aligned}
\tag{11}
$$

The second order condition for the HF trader is

$$\beta_2 > 0.$$

This conclues all the equations that we need for period 2.

6.2 Period 1

Market makers set the following price at period 1:

$$
\begin{aligned}
p_A^1 &= E[v|\omega_A^1] \qquad (12)\\
&= p_0 + \frac{\xi_1\sigma^2}{\xi_1^2\sigma^2+\sigma_1^2}\omega_A^1
\end{aligned}
$$

This gives us $\pi_1 = p_0$ as claimed before.

The insider's problem in period 1 is:

$$\max_{x_A^1} E[x_A^1(v-p_A^1)|v] + E[x_B^2(v-p_B^2)|v] + E[x_A^2(v-p_A^2)|v]$$

To simplify this, first we compute prices at time 2 given the information set of the insider. This is

$$
\begin{aligned}
&E[p_A^2|v] = E[p_B^2|v] = E[\pi_A^2 + \alpha_A^2\omega_A^1 + \beta_A^2\omega_A^2|v] = \\
&p_0 + \alpha_2(\theta_1+\xi_1 v) + \beta_2(\theta_2+\xi_2 v+\gamma_2(\theta_1+\xi_1 v))
\end{aligned}
$$

Hence:

$$E[v-p_A^2|v] = (v-p_0)(1-\alpha_2\xi_1-\beta_2(\xi_2+\gamma_2\xi_1))$$

Then we compute conditional value of x_A^2 at time 1. Since $x_A^2 = \theta_2+\xi_2 v+\gamma_2\omega_A^1$, we get

$$
\begin{aligned}
&E[x_A^2|v] = E[x_B^2|v] = \\
&\theta_2+\xi_2 v+\gamma_2(\theta_1+\xi_1 v) \\
&= (\xi_2+\gamma_2\xi_1)(v-p_0)
\end{aligned}
$$

Once we have these two equations, the first order condition is:

$$
\begin{aligned}
&-\alpha_1 x_A^1 + (v-p_0-\alpha_1 x_A^1) - \beta_2 y_1 E[x_B^2|v] - (\alpha_2+\beta_2\mu)E[x_A^2|v] + \gamma_2 E[(v-p_A^2)|v] - \gamma_2\beta_2 E[x_A^2|v] = 0 \Rightarrow \\
&-\alpha_A^1 x_A^1 + (v-p_0-\alpha_1 x_A^1) - (\beta_2 y_1+\alpha_2+\gamma_2\beta_2+\beta_2\mu)(\xi_2+\gamma_2\xi_1)(v-p_0) \\
&+\gamma_2(v-p_0)(1-\alpha_2\xi_1-\beta_2(\xi_2+\gamma_2\xi_1)) = 0
\end{aligned}
$$

The FOC implies:

$$x_A^1 = \frac{1}{2\alpha_1}[1 - (\beta_2 y_1 + \alpha_2 + \gamma_2\beta_2 + \beta_2\mu)(\xi_2 + \gamma_2\xi_1) + \gamma_2(1 - \alpha_2\xi_1 - \beta_2(\xi_2 + \gamma_2\xi_1))](v - p_0). \tag{13}$$

This also verifies $\theta_1 + \xi_1 p_0 = 0$.

The second order condition is: $-\alpha_1 < 0$ and $-2\alpha_1 + 2(-\alpha_2)(-\gamma_2) < 0$.

6.3 Solution

Once we have these formulas we get the following set of equations:

$$\begin{aligned}
&\sigma^2[\xi_1(y_1+1) - (\gamma_2+\mu)(\xi_2 + y_1\xi_1)] \\
&= \alpha_2[\xi_1^2\sigma^2 y_1^2 + \xi_1^2\sigma^2 + \sigma^2\xi_2^2 + 2\sigma^2\xi_1\xi_2 y_1 + \sigma^2\xi_1^2 y_1^2 + y_1^2 + 1] \\
&\sigma^2(\xi_2 + y_1\xi_1) = \beta_2[\xi_1^2\sigma^2 y_1^2 + \xi_1^2\sigma^2 + \sigma^2\xi_2^2 + 2\sigma^2\xi_1\xi_2 y_1 + \sigma^2\xi_1^2 y_1^2 + y_1^2 + 1] \\
&2\beta_2\xi_2 = 1 - \beta_2 y_1\xi_1 \\
&2\beta_2\gamma_2 = -\alpha_2 - \beta_2\mu \\
&2\beta_2\mu + \alpha_2 + \beta_2\gamma_2 = 2\beta_2 y_1 \\
&2\beta_2(\sigma_1^2 + 2\xi_1^2\sigma^2)y_1 = \xi_1(1 - \beta_2\xi_2)\sigma^2 \\
&\alpha_1(\xi_1^2\sigma^2 + \sigma_1^2) = \xi_1\sigma^2 \\
&2\alpha_1\xi_1 = 1 - (\beta_2 y_1 + \alpha_2 + 2\gamma_2\beta_2 + \beta_2\mu)(\xi_2 + \gamma_2\xi_1) + \gamma_2 - \gamma_2\alpha_2\xi_1
\end{aligned}$$

The first two equations come from the equation of the price at time 2 using (9). The next two equations are from the insider's problem at time 2 which is given by equation (10). The fifth and sixth equations are coming from insider's problem formulated by equation 11. Finally the last two equations are from the price and insider strategy at time 1, respectively given by equations (12) and (13). We can simplify this one more step and get

$$\beta_2[\xi_1(y_1+1)-(\gamma_2+\mu)(\xi_2+y_1\xi_1)]=\alpha_2(\xi_2+y_1\xi_1)$$

$$\sigma^2(\xi_2+y_1\xi_1)=\beta_2[\xi_1^2\sigma^2y_1^2+\xi_1^2\sigma^2+\sigma^2\xi_2^2+2\sigma^2\xi_1\xi_2y_1+\sigma^2\xi_1^2y_1^2+y_1^2+1]$$

$$2\beta_2\xi_2=1-\beta_2y_1\xi_1$$

$$2\beta_2\gamma_2=-\alpha_2-\beta_2\mu$$

$$2\beta_2\mu+\alpha_2+\beta_2\gamma_2=2\beta_2y_1$$

$$2\beta_2(\sigma_1^2+2\xi_1^2\sigma^2)y_1=\xi_1(1-\beta_2\xi_2)\sigma^2$$

$$\alpha_1(\xi_1^2\sigma^2+\sigma_1^2)=\xi_1\sigma^2$$

$$2\alpha_1\xi_1=1-(\beta_2y_1+\alpha_2+2\gamma_2\beta_2+\beta_2\mu)(\xi_2+\gamma_2\xi_1)+\gamma_2-\gamma_2\alpha_2\xi_1$$

In addition we have three second order conditions which are given by

$$-2\alpha_1+2(-\alpha_2)(-\gamma_2)<0$$

$$\beta_2>0$$

$$\alpha_1>0$$

Note that

Now we guess the following: $\xi_1=\frac{k}{\sigma}$, $\xi_2=\frac{l}{\sigma}$, $\beta_2=\beta\sigma$, $\alpha_2=\rho\sigma$ and $\alpha_1=\alpha\sigma$. The above equations are equivalent to:

$$\beta[k(y_1+1)-(\gamma_2+\mu)(l+ky_1)]=\rho(l+ky_1) \tag{14}$$

$$l+ky_1=\beta(2k^2y_1^2+l^2+k^2+2kly_1+y_1^2+1) \tag{15}$$

$$2\beta l=1-\beta y_1k \tag{16}$$

$$2\beta\gamma_2=-\rho-\beta\mu \tag{17}$$

$$2\beta\mu+\rho+\beta\gamma_2=2\beta y_1 \tag{18}$$

$$2\beta(1+2k^2)y_1=k(1-\beta l) \tag{19}$$

$$\alpha(k^2+1)=k \tag{20}$$

$$2\alpha k=1-(\beta y_1+\rho+2\beta\gamma_2+\beta\mu)(l+k\gamma_2)+\gamma_2-k\rho\gamma_2 \tag{21}$$

As we can see all of these equations are independent of σ. The second order conditions are equivalent to to:

$$\beta > 0$$

$$\alpha > \rho\gamma_2$$

$$\alpha > 0$$

The unique solution to equations (14) ... (21) that satisfy the second order conditions are:

$$\beta_2 = 0.416838\sigma$$
$$\xi_1 = \frac{0.606108}{\sigma}$$
$$\alpha_1 = 0.443266\sigma$$
$$\xi_2 = \frac{1.132449}{\sigma}$$
$$\alpha_2 = 0.546379\sigma$$
$$\gamma_2 = -0.584433$$
$$y_1 = 0.221265$$
$$\mu = -0.141903$$

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