

Detecting Large-Scale Collusion in Procurement Auctions*

Kei Kawai[†] Jun Nakabayashi[‡]

June 5, 2014

Abstract

This paper documents evidence of widespread collusion among construction firms using a novel dataset covering most of the construction projects procured by the Japanese national government from 2003 to 2006. By examining rebids that occur for auctions when all (initial) bids fail to meet the reserve price, we identify collusion using ideas similar to regression discontinuity. We identify about 1,000 firms whose conduct is inconsistent with competitive behavior. These bidders were awarded about 7,600 projects, or close to one fifth of the total number of construction projects in our sample. The value of these projects totals about \$8.6 billion.

Key words: Collusion, Procurement Auctions, Antitrust

JEL classification: D44, H57, K21, L12

*We thank John Asker, Robin Lee, Margaret Levenstein, and Larry White. This research has been supported by JSPS KAKENHI Grant Number 26285052.

[†]Department of Economics, NYU Stern, Henry Kaufman Management Center Rm 7-160, 44 West 4th Street, New York, NY, 10012. Email:kkawai@stern.nyu.edu.

[‡]Department of Economics, Tohoku University, Kawauchi 27-1, Aoba, Sendai, 980-8576, Japan. Email: nakabayashi@econ.tohoku.ac.jp.

1 Introduction

One of the central themes of competition policy is to deter, detect, and punish collusion. While there is almost universal agreement among economists that collusion among firms is socially undesirable, firms often have private incentives to engage in collusive behavior absent regulatory sanctions. Therefore, it is crucial to ensure that the antitrust agencies have the authority and the resources to detect and punish collusion in order to promote competition among firms. To the extent that collusive activities remain undetected or unpunished, collusion may become the norm rather than the exception, with potentially large detrimental effects on the economy.

In this paper, we document widespread collusion among Japanese construction firms using bidding data from government procurement auctions. Our novel dataset, which covers April 2003 through December 2006, accounts for most of the construction projects procured by Japan's national government during this period. Our data contain more than 40,000 auctions worth more than \$42 billion in total. On an annual basis, the total size of the award amount is close to \$14 billion, or about 3% of the national tax revenue. Using this large dataset, we provide evidence of widespread collusion among bidders. We find patterns of collusion that persist across regions, across types of construction projects and across time.

While the antitrust authorities (JFTC) brought only four collusion cases against construction firms in connection with the procurement projects in our sample, there is wide spread speculation that many firms were engaging in bid rigging. For example, the Japanese Bar Association issued a study in 2001 which concluded that bid rigging among construction firms is wide spread in Japan with extremely high probability based, in part, on the testimony of the defendants in five criminal collusion cases (JFBA (2001)). Another example is a widely publicized incident in 1997 in which Sakae Hirashima, a former corporate executive of Obayashi Corporation – who was the leader of a bidding ring in the Kansai area and was often referred to as the “don” or the “emperor” of the construction industry – filed a report to the JFTC that implicated more than 150 construction firms. Hirashima claimed that he was involved in allocating among the ring members, more than \$50 billion worth

of construction projects in 1996 alone.¹ In fact, collusion among construction firms was deemed so pervasive that it became one of the sticking points during talks over U.S. - Japan trade frictions.²

Despite many news reports and articles that document evidence of possible collusion in Japanese public procurement auctions, however, the allegations are based largely on isolated incidents or anecdotes. As far as we are aware, there has not been any concrete evidence regarding the pervasiveness of collusion. By examining most of the construction projects procured by the Japanese national government during 2003 to 2006, this paper provides a systematic account of collusion among construction firms in procurement auctions. In particular, using an idea similar to regression discontinuity, we identify more than 1,000 firms whose conduct is inconsistent with competitive behavior. The number of projects awarded to these bidders during our sample period is close to 7,600, and the value of these auctions totals about \$8.6 billion. The detection method we propose in this paper is very simple and requires only bid data. Moreover, it does not rely on the specifics of the model such as independent signals, private values, risk neutrality, etc. Our method can thus be useful for law enforcement agencies in identifying bidding rings.

In principle, bidding rings can be organized in a variety of ways, depending on whether or not members engage in side-payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that ring members pick a predetermined winner beforehand and reduce competitive pressure in the actual procurement auction. Hence, all the ring members, except for the predetermined winner, submit non-serious high bids, and the sole serious bid is submitted by the predetermined winner. Of course, even the serious bid is inflated relative to the competitive bid, ensuring that the ring extracts surplus from the buyer. Almost all of the existing evidence indicates that bidding rings in the Japanese construction industry are or-

¹The projects he claimed to allocate include those procured by the national government, as well as by local governments. The latter accounts for about ten times the value of the former. The JFTC did not press charges against Hirashima or any of the firms he implicated in his report.

²See Japan Structural Impediments Initiative Joint Report (1990) – in particular, the Report by the Japanese delegation, Section IV (Exclusionary Business Practices) II (Measures to be Taken) -(7) (Effective Deterrence against Bidrigging).

ganized in this manner.³

The auction mechanism used in Japanese public construction projects is a variant of the first-price sealed-bid (FPSB) mechanism with a secret reserve price.⁴ In fact, the auction mechanism is exactly the same as the FPSB auction as long as the lowest bid is below the secret reserve price, in which case the lowest bidder becomes the winner and the auction ends. If none of the bids is below the reserve price, however, the buyer reveals the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are *not* revealed). The second round bidding takes place typically 30 minutes after the initial round, with the same set of bidders and the same (secret) reserve price. If the lowest bid in the second round is still higher than the secret reserve price, there is a third round of bidding.⁵ Approximately 20% of all auctions advance to the second round and about 3% advance to the third round in our data.⁶

In order to identify collusion, we use an idea that is similar to regression discontinuity design. In particular, we look for patterns in the data where the identity of the lowest bidder is very persistent across rounds in an auction – consistent with designating a predetermined winner among the ring members – beyond what competitive behavior can explain. To be more concrete, let $i(1)$ and $i(2)$ be the lowest and the second-lowest bidders, respectively, in the first round. We then examine the second-round bids of $i(1)$ and $i(2)$ for the set of auctions that go to the second round and where the first-round bids of $i(1)$ and $i(2)$ are only ε apart. Note that conditional on the first-round bids being very close to each other, the bidders that turn out to be the lowest/second-lowest in the first round are as good as random un-

³All of the criminal collusion cases cited in the previously mentioned Bar Association study (JFBA (2001)), as well as the four bidding rings that were prosecuted by the JFTC during our sample period, were organized in this manner.

⁴Towards the end of our sample (starting around mid 2006), the government started to introduce scoring auctions for a substantial fraction of procurement auctions. By the end of 2007, most procurement auctions were awarded through scoring auctions.

⁵After the third round, a bilateral negotiation takes place between the buyer and the lowest third-round bidder. The same secret reserve price is used in all three rounds.

⁶Ji and Li (2008) examine procurement auctions of the Department of Transportation in Indiana, which have almost the same auction format. In their sample, they find that about 12.5% of auctions proceed to the second round.

der competition. Hence, the two bidders can be thought of as symmetric, in terms of costs, risk aversion, beliefs over the distribution of the reserve price, etc. Thus, absent information asymmetry that exists between $i(1)$ and $i(2)$ given that only the first-round lowest bid is revealed to the participants, the likelihood that $i(2)$ outbids $i(1)$ in the second round should be close to 50% as long as ε is small enough. It turns out that when we factor in the information asymmetry, it makes it even more likely that $i(2)$ outbids $i(1)$ in the second round under competitive behavior.⁷

To summarize, as long as the first-round bids of $i(1)$ and $i(2)$ are sufficiently close, we would expect $i(2)$ to win at least as often as $i(1)$ in the second round under competitive bidding.⁸ However, we find that $i(2)$ rarely outbids $i(1)$ in the second round in the actual data. For example, when we set ε to be 1% of the reserve price, $i(2)$ outbids $i(1)$ only about 2.6% of the time (56 out of 2,160 auctions).⁹ The probability that $i(1)$ remains the lowest bidder in the second round is around 96.4%.¹⁰

Of course, it is possible that our findings are driven by inherent cost differences among the firms; i.e., the bandwidth we use (e.g., $\varepsilon = 1\%$ of the reserve price) is not small enough to adequately control for differences in costs, etc., among the bidders. In order to rule out this possibility, we compare the second-round bids of $i(2)$ and $i(3)$ (the second- and the third-lowest bidders in the first round). In contrast to the case of $i(1)$ and $i(2)$, we find that $i(3)$ outbids $i(2)$ in the second round close to 50% of the time. For example, when we examine the second-round bids of $i(2)$

⁷The buyer reveals the lowest bid, but none of the other bids. Hence, $i(1)$ knows only that its bid was the lowest among the first-round bids, while $i(2)$ gains knowledge of two bids – its own bid and the lowest bid. This means that conditional on the two lowest bids being very close to each other, $i(2)$ has an information advantage in the second round: While $i(1)$ does not know that there is another bidder who bid just above its bid, $i(2)$ knows that it was outbid by a small margin.

⁸More precisely, we expect $i(2)$ to do no worse than $i(1)$. This does not logically imply that $i(2)$ should win more often if profit margins are very thin. We address this concern by examining auctions that go to the third round and in which $i(2)$ submits substantially lower bids in the third round. For this set of auctions, we have an upper bound on $i(2)$'s costs or, equivalently, a lower bound on $i(2)$'s profit margin. Even for these auctions, we find that $i(2)$ almost never outbids $i(1)$ in the second round.

⁹This includes 7 ties in which $i(1)$ and $i(2)$ bid exactly the same amount in the second round.

¹⁰The reason why this probability is lower than 97.4% ($= 100\% - 2.6\%$) is because, occasionally, the third-lowest bidder in the first round (or the fourth-lowest, etc.) becomes the lowest bidder in the second round.

and $i(3)$ for the set of auctions where the bid difference between $i(2)$ and $i(3)$ in the first round is less than 1% of the reserve price, we find that $i(3)$ outbids $i(2)$ in about 49.0% of the cases (2,555 out of 5,218 auctions).¹¹ This gives assurance that the bandwidth we choose for ε is sufficiently small for purging much of the inherent differences among the bidders. Our results, thus, suggest that there is much more persistence – across multiple rounds within the same auction – in the identity of the lowest bidder than competition can explain.

In order to provide more evidence on collusion, we further examine the shape of the distribution of Δ_{12} (i.e., the difference between the second-round bids of $i(1)$ and $i(2)$, normalized by the reserve price).¹² In the left panel of Figure 1, we plot the distribution of Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 1%. First, we find that the distribution of Δ_{12} lies mostly to the right of zero, confirming our previous finding that $i(2)$ almost never outbids $i(1)$ in the second round. In contrast, when we examine the distribution of Δ_{23} (i.e., the difference between the second-round bids of $i(2)$ and $i(3)$, normalized by the reserve price), we find that the distribution of Δ_{23} is symmetric around zero.¹³ The right panel of Figure 1 plots the distribution of Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 1%. The fact that the distribution of Δ_{23} is symmetric around zero means that $i(3)$ outbids $i(2)$ almost 50% of the time in the second round.

The second finding – perhaps more-conclusive evidence of collusion – is that there appears to be a discontinuity at exactly zero for the distribution of Δ_{12} . That is, when we focus on a small band around zero, we find hundreds of auctions in which Δ_{12} falls just to the right of zero (i.e., $\Delta_{12} \in (0, t)$ for some small positive t), whereas we find very few auctions in which Δ_{12} falls just to the left of zero (i.e., $\Delta_{12} \in (-t, 0)$). This implies that there are many auctions in which $i(2)$ loses to $i(1)$ in the second round by a tiny margin, but almost no auctions in which $i(2)$ wins by a tiny margin. The distribution for Δ_{23} , on the other hand, is continuous

¹¹This includes 478 ties in which $i(2)$ and $i(3)$ bid exactly the same amount in the second round.

¹²We define Δ_{12} by subtracting the second-round bid of $i(1)$ from the second-round bid of $i(2)$ and dividing by the reserve price.

¹³We define Δ_{23} by subtracting the second-round bid of $i(2)$ from the second-round bid of $i(3)$ and dividing by the reserve price.

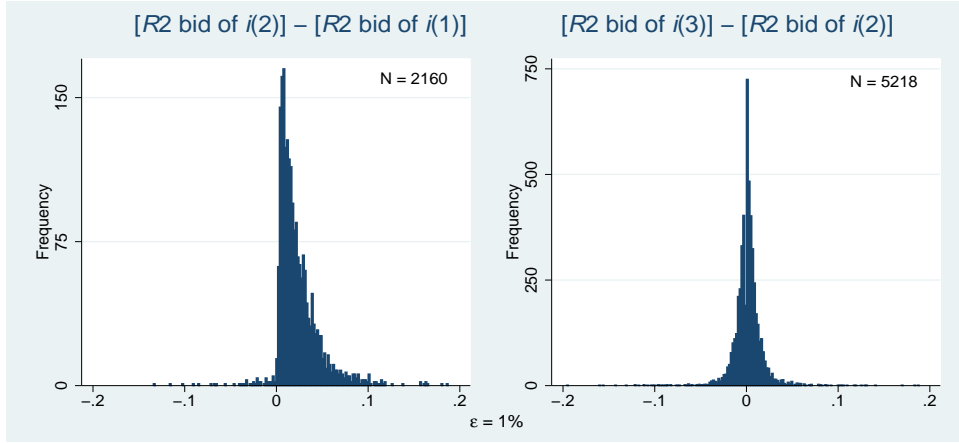


Figure 1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel). The left panel plots Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 1%. The right panel plots Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 1%.

and symmetric around zero with a fair amount of variance.

The discontinuity exhibited in the distribution of Δ_{12} at zero strongly suggests that the bidders know how each other will bid in the second round and, moreover, that auction participants designate a predetermined winner in advance. To see this, suppose the contrary: If $i(1)$ and $i(2)$ were uncertain as to how each other will bid in the second round, then one should observe a similar number of auctions in which $i(2)$ outbids $i(1)$ by a tiny margin as auctions in which $i(1)$ outbids $i(2)$ by a tiny margin. Hence, the fact that the distribution of Δ_{12} seems discontinuous at zero suggests that the bidders are aware of how each other will bid. But if this is the case, why else would $i(2)$ *lose* by a small margin (rather than *win* by a small margin) other than to yield to the predetermined winner?

The discontinuity in the distribution of Δ_{12} at zero persists even when we condition on auctions in which $i(2)$ must have had much to gain by outbidding $i(1)$ in the second round. That is, we reexamine the distribution of Δ_{12} and Δ_{23} for the set of auctions that proceeded to the third round, and $i(2)$ bid substantially less in the third round than it did in the second round. To the extent that the third-round bid of $i(2)$ gives an upper bound on $i(2)$'s cost, $i(2)$ must have had a lot to gain

in the second round by outbidding $i(1)$ for this set of auctions. However, we still find a sharp discontinuity in Δ_{12} at zero. We take this as further evidence of bidder collusion.

The bidding pattern that we identify as suggestive of collusion also holds for the bids of known bidding ring members. The JFTC prosecuted about 90 firms in four collusion cases during the sample period. When we examine the distributions Δ_{12} and Δ_{23} of these firms, we find that Δ_{12} is asymmetric and discontinuous around zero while Δ_{23} is symmetric around zero.

Overall, the bidding pattern that we identify as evidence of collusion is prevalent across regions, time, and types of project. We find that the shape of the distribution of Δ_{12} and Δ_{23} looks consistent regardless of how we condition on observables, suggesting that collusion is widespread.

Lastly, we develop a test statistic of collusive behavior that formalizes the idea that Δ_{12} should not be discontinuous at zero under competitive behavior. Our test statistic is composed of two parts: a measure of how sharply the distribution of Δ_{12} changes around zero, and the variance of Δ_{23} . Under the null of competitive bidding, it can be shown that the variance of Δ_{23} puts a bound on how sharply the distribution of Δ_{12} can change around zero. Thus, our test statistic compares the change in the distribution of Δ_{12} around zero with the variance of Δ_{23} . We then apply this test to each firm in our dataset by computing the test statistic for each firm using just the sample of auctions in which the firm participated.

In our baseline result, we find about 1,000 construction firms for whom we reject the null hypothesis of competitive behavior at the 95% confidence level. The number of auctions these firms won totals 7,600, or close to one fifth of the total number of auctions in our sample. These auctions range from small scale projects such as painting and paving to large and complex projects such as building tunnels and bridges. The total award amount of these auctions is about \$8.6 billion. We estimate that, absent collusion by these firms, taxpayers could have saved about \$721 million. While this is already a large number, it is worth mentioning that a large fraction of firms that we identify as uncompetitive are also active in other public procurement projects, such as municipal and prefectural projects. Given that the total value of these public projects is close to ten times the size of our dataset,

the overall impact of collusion on taxpayers can be staggering.

1.1 Related Literature

This paper is most closely related to the empirical literature on the detection of collusion.¹⁴ Existing empirical studies of collusion tend to take advantage of known episodes of cartel activity, e.g., paving in highway construction in Nassau and Suffolk counties (Porter and Zona 1993); school milk in Ohio (Porter and Zona 1999); school milk in Florida and Texas (Pesendorfer 2000); and collectible stamps in North America (Asker 2010). While none of our analysis requires information on known bidding rings, it is still useful to study the bidding behavior of known cartels for validation purposes. We do this in Section 5 for the four known bidding cartels that were prosecuted by the JFTC.

There is another strand of literature that tests for collusion in the absence of any prior knowledge of bidder conduct. Examples include bidding in seal coat contracts in three states in the U.S. Midwest (Bajari and Ye 1999); U.S. Forest Service timber sales (Baldwin, Marshall and Richard 1997; Athey, Levin and Seira 2011); Offshore gas and oil lease (Hendricks and Porter 1988; Haile, Hendricks, Porter and Onuma 2013); roadwork contracts in Italy (Conley and Decaloris, 2013); and public-works consulting in Japan (Ishii, 2009). Ishii (2009) studies 175 auctions for design consultant contracts in Naha, Okinawa and analyzes how the winner of the auctions can be explained by exchange of favors. While her identification is based on bid patterns across auctions, our identification strategy focuses on how bidders bid within a given auction. Our study also looks at most of the construction projects procured by the national government, whereas she studies a specific local market.¹⁵

Lastly, there is a small literature on multiple-round bidding in procurement auctions. Ji and Li (2008) study procurement auctions let by the Indiana Department

¹⁴For a brief survey, see Asker (2010a). For a more comprehensive study, see, e.g., Marshall and Marx (2012).

¹⁵For a more general overview of bidding rings among procurement firms in Japan, see McMillan (1991). See, also, Ohashi (2009), who discusses how the change in auction design in Mie Prefecture affected collusion.

of Transportation. They structurally estimate a multiple-round auction model with competitive bidding.

2 Institutional Background

Auction Mechanism The auction mechanism used in our sample is a variant of the first-price sealed bid (FPSB) auction with a secret reserve price. In fact, the auction mechanism is exactly the same as the FPSB auction as long as the lowest bid is below the secret reserve price, in which case, the lowest bidder becomes the winner with price equal to the lowest bid, and the auction ends. If none of the bids is below the reserve price, however, the buyer reveals the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are *not* revealed). The second round bidding takes place typically 30 minutes after the initial round, with the same set of bidders and the same (secret) reserve price.¹⁶ This means that when bidding in the second round, the bidders know that the secret reserve price is lower than the lowest first-round bid.

The second round proceeds in the same manner as the initial round; if the lowest bid is below the reserve price, the auction ends, and the lowest bidder wins. Otherwise, the buyer reveals the lowest second-round bid to the bidders, and the auction goes to the third round. The third round is the final round. If no bid meets the reserve price in the third round, bilateral negotiation takes place between the buyer and lowest third-round bidder. The same secret reserve price is used in all three rounds.¹⁷

Bidder Participation As is the case in many countries, participation in procurement auctions in Japan is not fully open. A contractor that wishes to participate must first go through screening to be pre-qualified. Because pre-qualification occurs at the regional level, a contractor needs to be pre-qualified for each region in

¹⁶See, e.g., Bidding Guidelines of Ministry of Land, Infrastructure, and Transportation, Chugoku Regional Developing Bureau.

¹⁷The reserve price, the identity of the bidders, and all the bids in each round are made public after the auction ends.

which it wishes to bid on projects.¹⁸

In addition to pre-qualification, there may be additional restrictions on participation: Depending on how restrictive they are, the auctions can be divided into four categories: The first and the second categories are the most restrictive, with participation by government invitation. In these two categories, the government typically invites ten bidders from the pool of pre-qualified contractors. The difference between the two categories is that in the first category, the invited bidders are chosen randomly from the pool, while in the second category, the government chooses bidders based on contractors' preferences over project type, project location, etc., submitted by the contractors in advance.¹⁹

The third and fourth categories are less restrictive. The set of potential bidders is still restricted to the pool of pre-qualified contractors, but any pre-qualified contractor can participate in the bidding. The difference between the third and fourth categories is that in the third category, the government reserves the right to exclude potential bidders from participating in the auction under certain conditions.

Collusive Behavior In principle, bidding rings can be organized in a variety of ways, depending on whether or not members engage in side-payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that the ring picks a predetermined winner in advance and that the rest of the ring members help the predetermined winner win. Almost all of the existing evidence indicates that bidding rings in the construction industry in Japan are organized in this manner.

There are also two other documented features of prosecuted bidding rings in the construction industry that are worth mentioning: The first feature is that, typically, the designated winner alone incurs the cost of estimating the project cost.²⁰ Estimating the project cost can be quite expensive, and the non-designated bidders typically

¹⁸Our data set is divided into nine regions.

¹⁹Each pre-qualified contractor submits a form to the government to express its preferences over the type and location of projects it wishes to bid on.

²⁰See, e.g., the criminal bid-rigging case regarding the construction of a sewage system in Hisai city (Tsu District Court, No. 165 (Wa), 1997), the bid-rigging case regarding the construction of a waste incineration plant in Nagoya city (Nagoya District Court, No. 1903 (Wa), 1995), etc.

avoid incurring this cost.²¹ Note that this makes it risky for a non-designated bidder to accidentally win the auction. The second feature is that the designated winner of a bidding ring would often communicate to other members how it would bid in each of the three rounds (as opposed to communicating how it would bid just in the first round).²²

3 Data

We use a novel dataset of auctions for public construction projects obtained from the Ministry of Land, Infrastructure and Transportation, the largest single procurement buyer in Japan. The dataset spans April 2003 through December 2006 and covers most of the construction works auctioned by the Japanese national government during this period. After dropping scoring auctions, unit-price auctions, and those with missing or mistakenly recorded entries, we are left with 42,561 auctions with a total award amount of more than \$42 billion.²³ The award amount is close to \$14 billion annually, accounting for about 3% of the national government tax revenue.

The data include information on all bids, bidder identity, the (secret) reserve price, auction date, auction category (which corresponds to how restrictive bidder participation is), location of the construction site, and the type of project.²⁴ The data also contain information on whether the auction proceeded to the second round or the third round, as well as all the bids in each round. Table 1 provides summary statistics of the data. In the table, we report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics

²¹Estimating the project cost involves understanding the specifications of the project, assessing the quantity and quality of materials required, negotiating prices for construction material and arranging for available subcontractors. These costs are often quite substantial.

²²See Japan Federation of Bar Associations (2001), p19 and JFTC Ruling #27 (2010), pp.10-11.

²³Samples with missing or mistakenly recorded entries account for 1.3% and 1.3% of the entire dataset, respectively. The scoring auction data account for 15.8%.

²⁴Construction projects are divided into 21 types of construction work, such as civil engineering, architecture, bridges, paving, dredging, painting, etc.

Concluding Round	(R)eserve	(W)inbid	(W)/(R)	Lowest bid / Reserve			#	N
	Yen M.	Yen M.		Round 1	Round 2	Round 3	Bidders	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	103.459 (246.55)	97.011 (234.01)	0.927 (0.085)	0.927 (0.085)	-	-	9.86 (2.60)	34,104 80.1%
2	81.033 (177.85)	78.526 (173.40)	0.964 (0.033)	1.056 (0.075)	0.964 (0.033)	-	9.90 (2.40)	7,207 16.9%
3	62.375 (166.45)	60.050 (157.39)	0.962 (0.035)	1.143 (0.113)	1.071 (0.089)	0.962 (0.035)	9.42 (2.25)	1,250 2.9%
All	98.455 (234.49)	92.795 (223.11)	0.934 (0.079)	0.955 (0.102)	0.980 (0.059)	0.962 (0.035)	9.86 (2.56)	42,561 100.0%

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations except for the last column, where we report the fraction of auctions that ended in the first, second, and third rounds. First and second columns are in millions of yen.

Table 1: Sample Statistics.

are reported separately by whether the auction concluded in Round 1, Round 2, or Round 3.

In the first and second columns of the table, we find that the average reserve price of the auctions is about 98 million yen and the average winning bid is about 93 million yen. In the third column, we find that the winning bid ranges between 92% and 97% of the reserve price. In the next three columns, we report the lowest bid in each round as a fraction of the reserve price. Note that for auctions that conclude in the first round, Column (4) is equal to Column (3). For auctions that conclude in the second or third round, the numbers reported in Column (4) are higher than unity by construction. Column (7) reports the average number of bidders, and Column (8) reports the sample size. We find that 16.9% of the auctions go to the second round, and 2.9% advance to the third round.

		Round 2				
		1	2	3	4	5+
Round 1	1	96.70%	1.61%	0.62%	0.26%	0.82%
	2	1.59%	26.62%	18.63%	13.50%	39.66%
	3	0.53%	18.81%	18.65%	13.89%	48.11%
	4	0.37%	14.24%	15.94%	15.36%	54.10%
	5+	0.13%	6.75%	9.21%	10.34%	73.56%

Note: The (i, j) element of the matrix denotes the probability that a bidder submits the j -th lowest bid in the second round conditional on submitting the i -th lowest bid in the first round. When there are ties, multiple bidders are assigned to the same rank. The number of auctions is 8,089.

Table 2: Rank of the Second-Round Bid by Rank of the First-Round Bid

4 Analysis

4.1 Persistence of the Identity of the Lowest Bidder

Persistence in the Second Round We begin our analysis by studying the extent to which the lowest bidder in the first round is also the lowest bidder in later rounds for a given auction. Recall that a typical feature of bidding rings is that there is a designated winner and that ring members other than the designated winner submit bids in such a way as to ensure that the designated bidder is the lowest bidder. Because, in the setting we study, the reserve price is unknown from the perspective of the bidding ring, the ring members must make sure that the designated bidder is the lowest bidder in each successive round if the auction takes multiple rounds. This is especially important if the designated bidder is the only one that has estimated the project cost. This implies that we should observe persistence in the identity of the lowest bidder across rounds under bidder collusion.

In Table 2, we report how the rank of the bidders changes from the first round to the second round for all auctions that proceed to the second round with five or more participants ($N = 8,089$). The (i, j) element of the matrix corresponds to the probability that a bidder submits the j -th lowest bid in the second round, conditional on submitting the i -th lowest bid in the first round; i.e., $\Pr(j\text{-th lowest} | i\text{-th lowest})$.

th lowest). Thus, the diagonal elements correspond to the probability that a given bidder remains in the same rank in both rounds. Note that the horizontal sum of the probabilities is one.

What is striking about this table is the probability in the $(1, 1)$ cell. We find that in 96.70% of cases, the lowest bidder in the first round is still the lowest bidder in the second round. The flip side of this is that if a bidder is not the lowest bidder in the first round, the bidder is almost never the lowest bidder in the second round. For example, the conditional probability that a second-lowest bidder in Round 1 becomes the lowest bidder in Round 2 is only 1.59%. Note, also, that the diagonal elements other than the $(1, 1)$ element are much smaller: the probability that the second-lowest bidder in the first round remains the second-lowest bidder is just 26.62%. There is very strong persistence in the identity of the lowest bidder, but not necessarily for other positions.

In order to illustrate this point further, we examine more closely how the three lowest bidders in the first round behave in the second round. In what follows, we let $i(k)$ denote the identity of the bidder who submits the k -th lowest bid in Round 1. We also denote the (normalized) bid of bidder $i(k)$ in round t by $b_{i(k)}^t$. Because there is considerable variation in project size, we work with the normalized bids by dividing the actual bids by the reserve price of the auction. Hence, $b_{i(1)}^2$, for example, denotes the second-round bid of the first-round lowest bidder as a percentage of the reserve price.

In the top left panel of Figure 2, we plot the histogram of $\Delta_{12}^2 \equiv b_{i(2)}^2 - b_{i(1)}^2$ for the set of auctions that go to the second round. That is, we plot the difference in the (normalized) second-round bids of $i(1)$ and $i(2)$.²⁵ Note that almost all of the mass lies to the right of zero, which confirms what we report in Table 2: A flip in the ordering between the lowest and the second-lowest bidders almost never happens across rounds. In the top right panel of Figure 2, we plot the histogram of $\Delta_{23}^2 \equiv b_{i(3)}^2 - b_{i(2)}^2$, i.e., the difference in the normalized rebids of $i(2)$ and $i(3)$, for the set of auctions that go to the second round. In stark contrast to the left panel, the shape of the histogram for Δ_{23}^2 is quite symmetric around zero. This implies

²⁵The sample sizes are different between the top left and the top right panels because in some auctions, $i(1)$ or $i(3)$ does not bid in the second round.

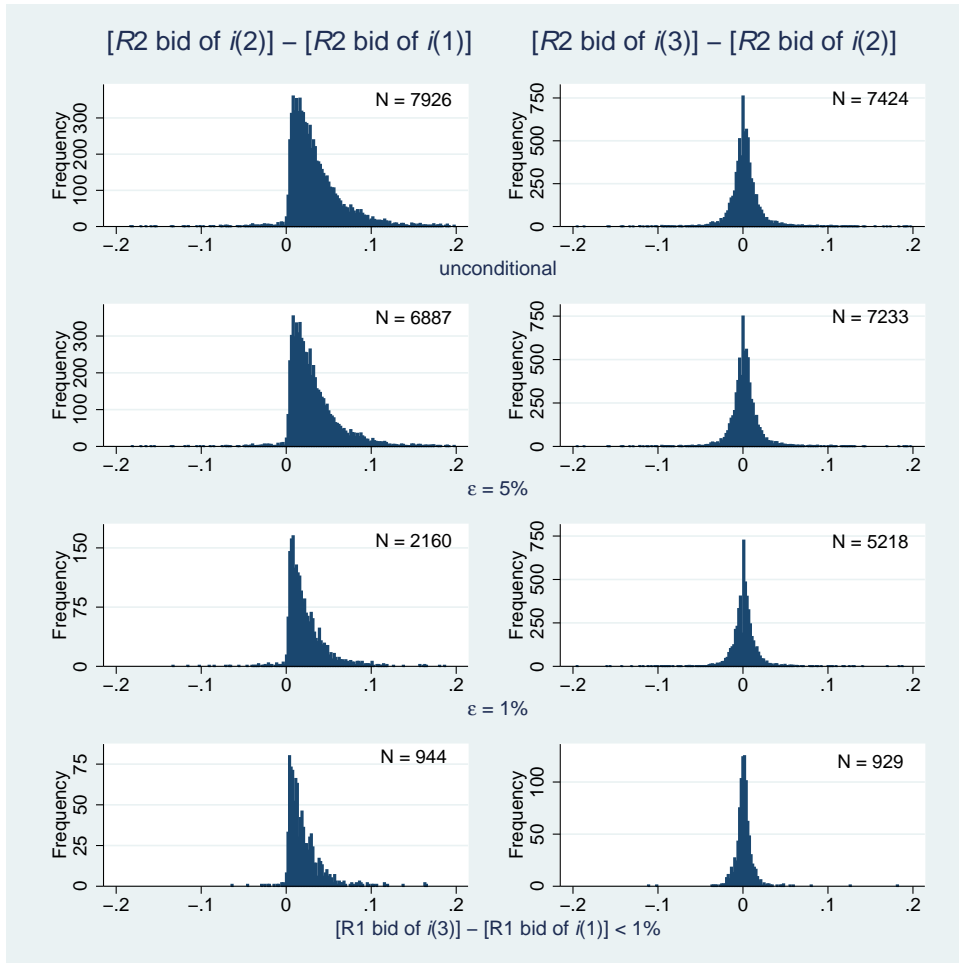


Figure 2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row is the histogram for the set of auctions that reach the second stage; and $i(1)$ and $i(2)$ (or $i(2)$ and $i(3)$) submit valid bids in the second round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

that the ranking between $i(2)$ and $i(3)$ flips in the second round with almost 50% probability. This also seems consistent with our previous finding that there is much less persistence in the ranking for the second and third places.

So far, the results that we have presented correspond to all of the auctions that proceeded to the second round. However, it is possible that our results are driven by

inherent differences among firms such as costs, risk attitude, beliefs over the reserve price, etc. For instance, if there are significant cost differences between the lowest bidder and all of the other bidders, our results may be generated by competitive bidding. In order to rule out this possibility, we perform the same analysis by conditioning on the set of auctions in which the first-round bids are close to each other. The idea is that if, for example, the first-round bids of $i(1)$ and $i(2)$ are sufficiently close (i.e., $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$ for some small ε), there should be little inherent differences among them, on average. In fact, if ε is small enough, which bidder turns out to be the lowest/second-lowest bidder in the first round is as good as random. Hence, $i(1)$ and $i(2)$ should be interchangeable, in terms of costs, risk attitude, beliefs over the reserve price, etc.

In the second row of Figure 2, we plot Δ_{12}^2 and Δ_{23}^2 for the subset of auctions for which the bids in the first round are within 5% of each other.²⁶ In particular, we plot the histogram of Δ_{12}^2 for the set of auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$ in the left panel and the histogram of Δ_{23}^2 for the set of auctions with $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$ in the right panel. Note that the shape of the distribution of Δ_{12}^2 in the left panel is still very skewed and asymmetric around zero, while the distribution of Δ_{23}^2 in the right panel remains symmetric around zero. The fact that the distribution of Δ_{23}^2 is symmetric around zero and very similar to the top panel suggests that cost differences between bidders do not seem to play a large role: If cost differences were driving the skewed bid pattern for Δ_{12}^2 in the left panel, we should also expect to see a distribution of Δ_{23}^2 that is skewed to the right of zero. The third row plots the distribution of Δ_{12}^2 and Δ_{23}^2 , but now conditioning on auctions with $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$ and $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$, respectively. Lastly, the bottom row shows the distribution of Δ_{12}^2 and Δ_{23}^2 conditional on the event that the three lowest bids in the first round are *all* within 1% of each other, $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$.²⁷ Taken together, Figure 2 suggests that it is not differences in costs, etc. that are driving the persistence in the identity

²⁶The sample sizes are different between the two panels because there are more auctions in which $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$ than auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$. Similarly for the two panels in the third row. The difference in the sample sizes in the fourth row is due to the fact that in some auctions, $i(1)$ or $i(3)$ does not bid in the second round.

²⁷Note that $b_{i(3)}^1 \geq b_{i(2)}^1 \geq b_{i(1)}^1$, by construction. Hence, $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$ implies $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$ and $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$.

of the lowest bidder.

In the Online Appendix, we explore whether the distributions of Δ_{12}^2 and Δ_{23}^2 exhibit similar patterns when we condition the sample by various auction characteristics, such as region, auction category, project type, and year. We find that the distributions of Δ_{12}^2 and Δ_{23}^2 often look very similar to those shown in Figure 2: The distribution of Δ_{12}^2 is skewed to the right and displays what appears to be a discontinuity at zero, while the distribution of Δ_{23}^2 is symmetric around zero. In the Online Appendix, we also plot the second-round bid differences of $i(1)$ and $i(2)$ and $i(2)$ and $i(3)$ without normalizing the bids by the reserve price. The graphs also appear similar to Figure 2.

Information Advantage of $i(2)$ Recall from Section 2 that the lowest bid is announced in each round, but none of the other bids are. This means that while $i(1)$ only gains knowledge that it was the lowest bidder in the first round, $i(2)$ learns exactly what the lowest bidder bid in the first round in addition to what it bid itself. This implies that conditional on the two lowest bids being very close to each other, $i(2)$ has an information advantage over $i(1)$ in the second round. To see this, consider the case in which $i(1)$ and $i(2)$ bid almost exactly the same amount, say $\$Z$. The information revealed to $i(1)$ at the end of the first round is that $\$Z$ is the lowest bid and that it bid the lowest. The information revealed to $i(2)$, on the other hand, is that $\$Z$ is the lowest bid and that (at least) one other firm beside itself bid $\$Z$. Clearly, $i(2)$ has a bigger information set at the end of the first round.

So far, we have documented that the ordering between $i(1)$ and $i(2)$ is very persistent across rounds, while the ordering between $i(2)$ and $i(3)$ is not. Given $i(2)$'s information advantage, however, the fact that the ordering between $i(1)$ and $i(2)$ does not change is even more surprising. Once we condition on auctions in which $i(1)$ and $i(2)$ bid close to each other in Round 1, $i(2)$ should be aware that by bidding a little more aggressively, it can beat $i(1)$ in the next round with high probability. Hence, given the informational advantage of $i(2)$, we would normally expect the order of $i(1)$ and $i(2)$ to flip more, and not less, frequently than 50% under competitive behavior. Hence, the persistence in the identity of the lowest bidder seems at odds with competitive behavior.

Persistence in the Third Round For the subset of auctions that go to the third round, we can further examine whether a similar pattern continues to hold in the third round. In the top two panels of Figure 3, we plot the difference in the third-round bids of $i(1)$ and $i(2)$, i.e., $\Delta_{12}^3 \equiv b_{i(2)}^3 - b_{i(1)}^3$ (left panel), and the difference in the third-round bids of $i(2)$ and $i(3)$, i.e., $\Delta_{23}^3 \equiv b_{i(3)}^3 - b_{i(2)}^3$ (right panel) for all auctions that advance to the third round. In rows two to four of Figure 3, we plot the histogram conditioning on the set of auctions in which the first-round bids were sufficiently close. Focusing on the left panels, the second row plots Δ_{12}^3 for the set of auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$; the third row plots Δ_{12}^3 for which $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$; and the last row plots Δ_{12}^3 for which $b_{i(3)}^1 - b_{i(1)}^1 < 0.03$. Similarly, the second through the fourth panels in the right column plot Δ_{23}^3 for the set of auctions in which $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$, $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$ and $b_{i(3)}^1 - b_{i(1)}^1 < 0.03$, respectively.

4.2 Discontinuity of Δ_{12}^2 at Zero

One striking feature of the distribution of Δ_{12}^2 (and Δ_{12}^3) is that there is what appears to be a discontinuous jump at exactly zero. This is in stark contrast to the distribution of Δ_{23}^2 , which is symmetric and continuous around zero. We argue that this pattern of bidding is also inconsistent with competitive behavior.

Consider, first, the distribution of Δ_{23}^2 in the right panels of Figure 2. Note that, even among bidders that submit almost identical first-round bids, there is a certain amount of variance in Δ_{23}^2 . To the extent that these bids are generated under competitive behavior, this seems to indicate that for many auctions, there is a reasonable amount of idiosyncrasy among the bidders with regard to the beliefs over the distribution of the reserve price, risk preference, etc., inducing variance in the second-round bids. In other words, idiosyncratic reasons seem to induce at least a certain amount of uncertainty in the second-round bidding for many auctions even among bidders that submit almost identical first-round bids.

Now consider the distribution of Δ_{12}^2 in the left panels of Figure 2. As long as there exists a reasonable amount of idiosyncrasy among the bidders, $i(2)$ should outbid $i(1)$ in the second round by a narrow margin just as often as $i(1)$ outbids

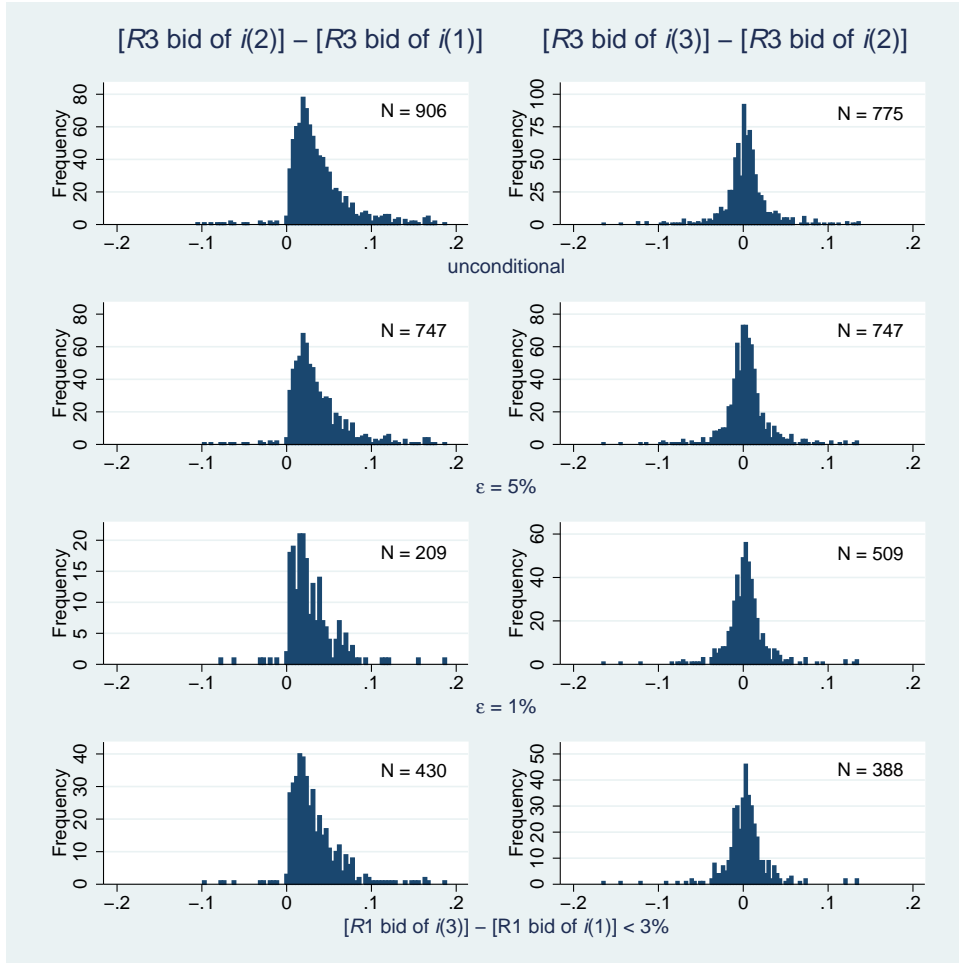


Figure 3: Difference in the Third-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Third-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row corresponds to all auctions that reached the third round and $i(1)$ and $i(2)$ (in the case of the left panel) or $i(2)$ and $i(3)$ (in the case of the right panel) submitted valid bids in the third round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

$i(2)$ by a narrow margin. That is, there should be a similar number of observations in which $\Delta_{12}^2 \in [-t, 0]$ and $\Delta_{12}^2 \in [0, t]$ for small values of t – a feature which we clearly do not see in any of the histograms of the left panels of Figure 2. This is inconsistent with competitive behavior.

In fact, the discreteness exhibited in the histogram of Δ_{12}^2 at zero suggests that

the bidders know exactly how the other bidders will bid in the second round. If, on the contrary, $i(1)$ and $i(2)$ were both uncertain about each other's bid, there should be just as many cases where $i(2)$ won by a tiny margin as cases where $i(2)$ lost by a tiny margin. Hence, the discontinuity of Δ_{12}^2 suggests that the bidders have prior knowledge about how each other will bid and that $i(2)$ is deliberately losing by submitting a slightly higher bid than $i(1)$ (rather than winning by slightly underbidding $i(1)$).

Regarding whether ring members can achieve such coordination without communication, it seems unlikely. There is large heterogeneity in project size, specification, etc., between auctions. This makes it hard for bidders to predict a particular price that could serve as an obvious anchor of tacit (i.e., no communication) collusion, in general. Therefore, the observed bid pattern seems to indicate communication.²⁸

Note that our findings also suggest that bidding rings communicate beforehand how each ring member should bid in the second round – not just how to bid in the first round. This is natural given that a substantial fraction of auctions go to the second round and that there are only 30 minutes between rounds. In fact, this is consistent with the feature of bidding rings documented in court rulings (See, e.g., Nagoya District Court, No. 1903 (Wa), 1995).

4.3 Optimality of $i(2)$'s Second-Round Bid

We now explore the persistence in the identity of the lowest bidder and the discontinuity of Δ_{12}^2 from the perspective of the optimality of $i(2)$'s second-round bid. Recall that there are many cases in which $i(2)$ could have outbid $i(1)$ in the second round by shading its second-round bid by a tiny margin. For example, focusing on the left panel of the second row in Figure 2, we find that about 4.44%, 15.75%, and 38.67% of the distribution lies within $[0, 0.005]$, $[0, 0.01]$, and $[0, 0.02]$, respectively. On the other hand, the probability that the distribution lies to the left of zero is only 1.73%. This suggests that $i(2)$ can increase the probability of outbidding

²⁸But see Section 5 for an example of a bidding ring which used the first-round lowest bid as an anchor.

$i(1)$ substantially by shading its bid only slightly, raising the question of whether $i(2)$'s second-round bid is optimal.

Of course, outbidding $i(1)$ is not the same as winning the auction because one must outbid all of the other bidders as well as the secret reserve price in order to win the auction.²⁹ To take this into consideration, we shade the second-round bid of $i(2)$ in every auction by 0.5%, 1%, and 2% and count the number of instances in which the shaded bid is lower than the secret reserve price and all of the other bids. We find that $i(2)$ would win the auction 3.75%, 11.46% and 30.55% of the time, respectively. In contrast, the actual fraction of auctions in which $i(2)$ won (either in the second round or the third round) was a mere 1.41%. This means, for example, that $i(2)$ could have increased the probability of winning the auction by about 270%, from around 1.41% to 3.75%, by lowering its bid by merely 0.5%. Unless the profit margin of $i(2)$ is very thin – in fact, less than 0.80% of its second-round bid – $i(2)$ could have increased its profits by lowering its second-round bid by 0.5%, i.e., the observed bid of $i(2)$ is not optimal.³⁰

While a profit margin of only around 0.80% of the second-round bid seems too small to be reasonable, it is difficult to obtain direct cost measures that would allow us to test this claim. What we do, instead, is consider a subset of auctions: 1) that go to the third round; and 2) in which $i(2)$ bids substantially less in the third round than in the second round. For these auctions, the third-round bid of $i(2)$ gives us a lower bound on $i(2)$'s profit margin. That is, if $b_{i(2)}^3 < (b_{i(2)}^2 \times x\%)$, then we know that $i(2)$ was willing to win the auction at $x\%$ of its second-round bid, implying a profit margin of at least $(100 - x)\%$.

In the top left panel of Figure 4, we plot $\Delta_{12}^2 \equiv b_{i(2)}^2 - b_{i(1)}^2$ for the set of

²⁹Given that $i(2)$ does not know that it came in second at the time of rebidding (it only learns that it came close to being first), our test of optimality may be too strict. In order to address this point, we also consider the optimality of the second-round bid of any bidder that came close to $i(1)$ in Round 1, regardless of whether or not it came in second. We find that when we shade by 0.5% (1%, 2%), the second-round bids of any bidder who bid within 5% of $i(1)$, the probability of winning increases from 0.43% to 2.15% (8.57%, 27.20%), or about a five-fold increase.

³⁰ $i(2)$ is better off lowering its second-round bid by 0.5% unless its cost is higher than 99.20% of its second-round bid:

$$(b_{i(2)}^2 - c) \times 1.41\% \leq (0.995 \times b_{i(2)}^2 - c) \times 3.75\%.$$

Solving for c gives about $c \leq 99.20\% \times b_{i(2)}^2$.

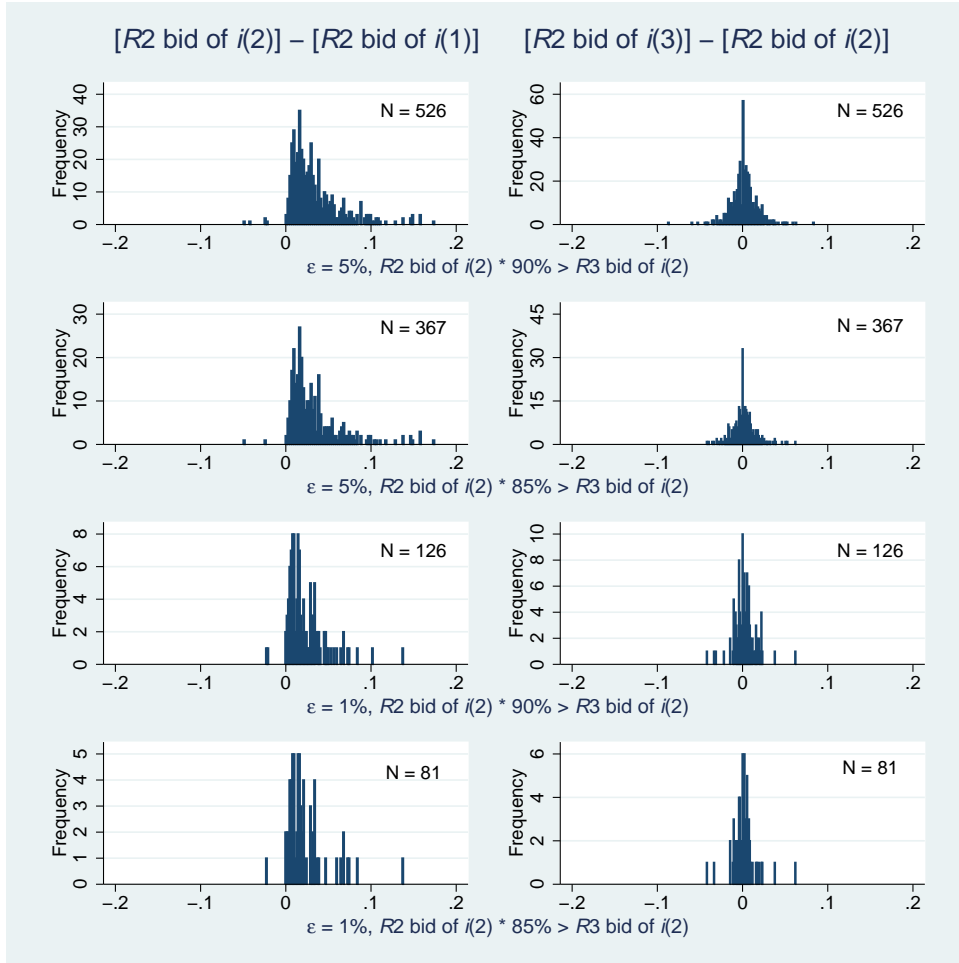


Figure 4: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) for Auctions with Large Profit Margin. The figure plots the histogram for the set of auctions that eventually reach the third round; and $i(2)$'s third-round bid is less than 90% (first two rows) or 85% (last two rows) of its second-round bid.

auctions: 1) that proceed to the third round; 2) in which $b_{i(2)}^3$ is at least 10% lower than $b_{i(2)}^2$; and 3) in which the first-round bids of $i(1)$ and $i(2)$ are within 5% (i.e., $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$). The first two conditions ensure that we are examining only the set of auctions in which the profit margin of $i(2)$ is sufficiently high in the second round; and the third condition ensures that the differences between $i(1)$ and $i(2)$ are relatively modest.

Note that the shape of Δ_{12}^2 in the top left panel of Figure 4 remains more or less the same compared to the distribution of Δ_{12}^2 plotted in the left panels of Figure 2, i.e., there is a substantial mass just to the right of zero, but almost none to the left of zero. This suggests that low profit margins cannot explain the reluctance of $i(2)$ to outbid $i(1)$ in the second round. For comparison, the top right panel of Figure 4 plots the distribution of Δ_{23}^2 for the same set of auctions as the top left panel. By and large, the distribution of Δ_{23}^2 is symmetric around zero, as before. The two panels in the second row plot the histograms of Δ_{12}^2 and Δ_{23}^2 when we further condition the sample to the set of auctions in which $i(2)$ bids at least 15% less in the third round ($b_{i(2)}^3 < 0.85 \times b_{i(2)}^2$). Again, we see a similar pattern as before. Lastly, the panels in the third and fourth rows plot Δ_{12}^2 and Δ_{23}^2 for auctions with $b_{i(2)}^3 < 0.9 \times b_{i(2)}^2$ and $b_{i(2)}^3 < 0.85 \times b_{i(2)}^2$, respectively, but now, only for the subset of auctions with $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$. The panels in the third and fourth rows appear similar to panels in the first two rows.

To sum, we find that the discontinuity in the distribution of Δ_{12}^2 at zero remains even for the set of auctions where $i(2)$ must have had a lot to gain by outbidding $i(1)$ in the second round. This suggests that $i(2)$'s second-round bidding is inconsistent with profit-maximizing behavior.

Discussion: Equilibrium Play

So far, we have abstracted from discussing equilibrium play of the auction. One reason for this is that characterization of the equilibria requires assumptions on the correlation structure of bidder values and signals, risk attitude of the bidders, etc., which we have not needed for our analysis. Another reason, however, is that a full characterization of the equilibria is very hard, even in a simplified model of two bidders and two rounds. This is because the first-round bidding has a signaling aspect, given that the lowest bid is revealed conditional on proceeding to the second round.

Below, we offer a (very limited) characterization of competitive bidding behavior in the second round of a two-round auction, where we take as given that bidders play monotone strategies in the first round. While there is no guarantee that bidders play monotone strategies in the first round, this seems like a natural benchmark.

The theoretical exercise is relevant for our empirical analysis because it gives predictions as to how Δ_{12}^2 should be distributed under competitive bidding. Consistent with the actual auction, we assume that the lowest bid from the first round is revealed upon proceeding to the second round.

Proposition 1 *Consider a symmetric IPV procurement auction with two bidders, a secret reserve price and, at most, two rounds. Assume that bidders play symmetric and strictly monotone pure strategies in the first round. Then, there exists a pair of best responses in the second round in which $i(1)$ plays a mixed strategy over some support $[\underline{b}, \bar{b}]$; and $i(2)$ plays a strictly monotone pure strategy in which the type with the lowest cost bids \underline{b} . Moreover, for any pair of best responses in the second round, the strategies of $i(1)$ and $i(2)$ have the following properties: 1) $i(1)$ plays a mixed strategy in which there exists no mass at the lower bound, \underline{b} , of the support; 2) $i(2)$'s strategy is weakly increasing in its costs; 3) for any $\epsilon > 0$, there exists a strictly positive mass of $i(2)$ types that bid less than $\underline{b} + \epsilon$.*

The proof of Proposition 1 is found in the Online Appendix. Note that if bidders play strictly monotone pure strategies in the first round, the cost of $i(1)$ is revealed to $i(2)$ upon proceeding to the second round. This induces $i(1)$ to mix in the second round.

What is more relevant for our analysis is given by the following corollary:

Corollary 1 *Suppose, again, that both bidders play symmetric and strictly monotone pure strategies in the first round. Then, if we consider auctions that proceed to the second round and in which $b_{i(2)}^1 - b_{i(1)}^1 < \epsilon$, the probability that $i(2)$ outbids $i(1)$ in the second round approaches 1 as ϵ goes to zero.*

This corollary is an immediate consequence of properties 1) through 3) of Proposition 1. The corollary claims that if we take auctions in which the first-round bids of $i(1)$ and $i(2)$ are very close to each other, we should observe $i(2)$ outbidding $i(1)$ with close to 100% probability. This suggests that, if anything, competitive behavior should result in the distribution of Δ_{12}^2 to lie *to the left of zero*, rather than almost entirely to the right of zero.

5 Case Study

In this section, we analyze four collusion cases that were implicated by the JFTC during our sample period. The four cases that we examine are the bidding ring of (A) prestressed concrete providers; (B) firms installing traffic signs; (C) builders of bridge upper structure; and (D) floodgate builders.³¹ In all of these cases, firms were found to have engaged in activities such as deciding on a predetermined winner for each project and communicating among the members how each bidder will bid.³² All of the implicated firms in cases (B), (C) and (D) admitted wrongdoing soon after the start of the investigation, but none of the firms implicated in case (A) admitted any wrongdoing initially, and the case went to trial.³³

Before we analyze these four cases, we point out one interesting feature of the bidding ring in case (A): According to the ruling in case (A), an internal rule existed among the subset of the ring members operating in the Kansai region, which prescribed that 1) the predetermined winner should aim to bid below the reserve price in the first round; 2) if the predetermined winner did not bid below the reserve price in the first round, the predetermined winner should submit a second-round bid that is less than some prespecified fraction (e.g., 97%) of its first-round bid (e.g., $b_{i(1)}^2 < 0.97 \times b_{i(1)}^1$); and 3) the rest of the ring members should submit second-round bids that are higher than the prespecified fraction of the *predetermined winner's* first-round bid (e.g., $b_{i(k)}^2 > 0.97 \times b_{i(1)}^1$ for $k \geq 2$). The prespecified fraction used in the ring was 96% for auctions with an expected value less than 100 million yen, 97% for auctions with an expected value between 100 million yen and 500 million yen, and 97.5% for auctions expected to worth more than 500 million yen.³⁴ One

³¹See JFTC Recommendation #27-28 (2004) and Ruling #26-27 (2010) for case (A); JFTC Recommendation and Ruling #5-8 (2005) for case (B); JFTC Recommendation and Ruling #12 (2005) for case (C); and JFTC Cease and Desist Order #2-5 (2007) for case (D).

³²The ring members took turns being the predetermined winner. The determination of who would be the predetermined winner depended on factors such as whether a given firm has an existing project that is closely related to the auction in question and the number of auctions a given firm has won in the past.

³³Out of 20 firms that were initially implicated in Case (A), one firm was acquired by another firm, one was acquitted, and the rest of the firms eventually settled with the JFTC after going to trial.

³⁴There is evidence that ring members actively communicated with each other on what the prespecified fraction should be. For example, a memo which was obtained by the JFTC from one of the ring members records a discussion among the members over the prespecified fraction. According to

consequence of this internal rule is that we would observe the same lowest bidder in Round 1 and Round 2.

In Figure 5, we plot the winning bid (lowest bid of the concluding round as a percentage of the reserve price) against the calendar date for all auctions in which the winner is a member of one of the implicated bidding rings. We have also drawn a vertical line that corresponds to the “end date” of collusion. The “end date” is the date in the JFTC’s ruling after which the ring members were deemed to have stopped colluding. Note that in panels (B) and (C) of Figure 5, there exist periods after the collusion end date during which no ring member wins an auction. This reflects the fact that implicated ring members in cases (B) and (C) were banned from participating in public procurement projects for a period of up to 18 months.³⁵

We see that for cases (B), (C), and (D), there is a general drop in the winning bid of about 8.3%, 19.5%, and 5.3%, respectively, after the collusion end date. However, there is almost no change in the winning bid for case (A) before and after the end date. Also, it is worth mentioning that, even for cases (B), (C), and (D), there are some auctions in which the winning bid is extremely high after the end date. In fact, about 24.4% of auctions after the end date have a winning bid higher than 95% for cases (B), (C) and (D). While the investigation and the ruling of the JFTC seemed to have made collusion harder, it is far from clear whether the prices after the end date are truly at competitive levels. Hence, the price drops that we see in Figure 5 may be a conservative estimate of the effect of collusion. We discuss this point more below.

We now examine the second-round bids of $i(1)$, $i(2)$, and $i(3)$ during the period in which the firms were colluding. If the distinctive shapes of the distribution of Δ_{12}^2 and Δ_{23}^2 that we found in Section 4 are indeed evidence of collusion, we should expect to see the same pattern among the second-round bids of these colluding firms. Figure 6 plots the histogram of Δ_{12}^2 and Δ_{23}^2 before the collusion end date for each of the four bidding rings. The samples used for the figure correspond to the set of auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ for the left column and $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$

the memo, one of the members wanted to use different fractions for auctions worth, for example, 10 million yen and 99 million yen (JFTC Ruling #27 (2010)).

³⁵The ring members involved in cases (A) and (D) were banned from bidding in procurement auctions for certain periods in 2010 and 2007, respectively.

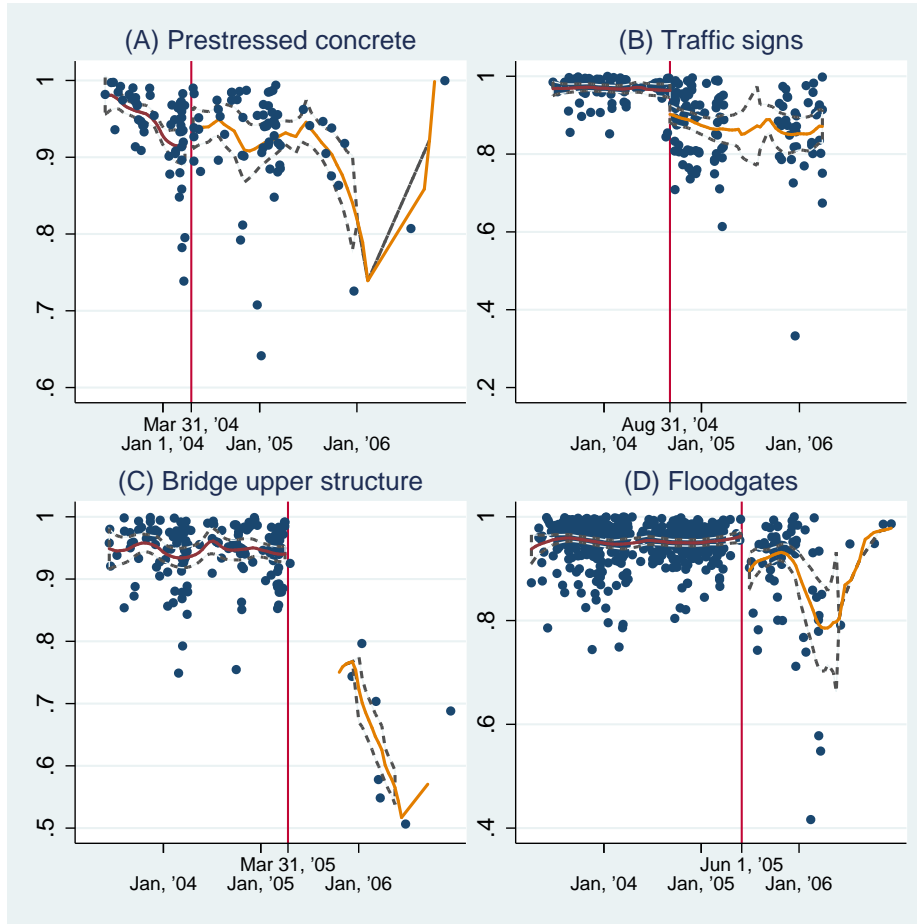


Figure 5: Winning Bid of Auctions in Which the Winner Was Involved in One of the Four Bidding Rings. The horizontal axis corresponds to the calendar date from the beginning of our sample (i.e., April 1, 2003), and the vertical axis corresponds to the winning bid as a percentage of the reserve price. The vertical line in each of the four panels corresponds to the collusion “end date.”

for the right column, i.e., $\varepsilon = 0.05$. We see that for all four bidding rings, Δ_{12}^2 is asymmetric around zero, while Δ_{23}^2 is symmetric around zero, as before. Thus, Figure 6 suggests that the distinctive shapes of the distributions of Δ_{12}^2 and Δ_{23}^2 are a hallmark of collusive bidding.

We next examine the second-round bids of the ring members, but for auctions occurring after the collusion “end date.” To the extent that ring members stopped colluding after the “end date,” we should expect to see the distribution of Δ_{12}^2 dis-

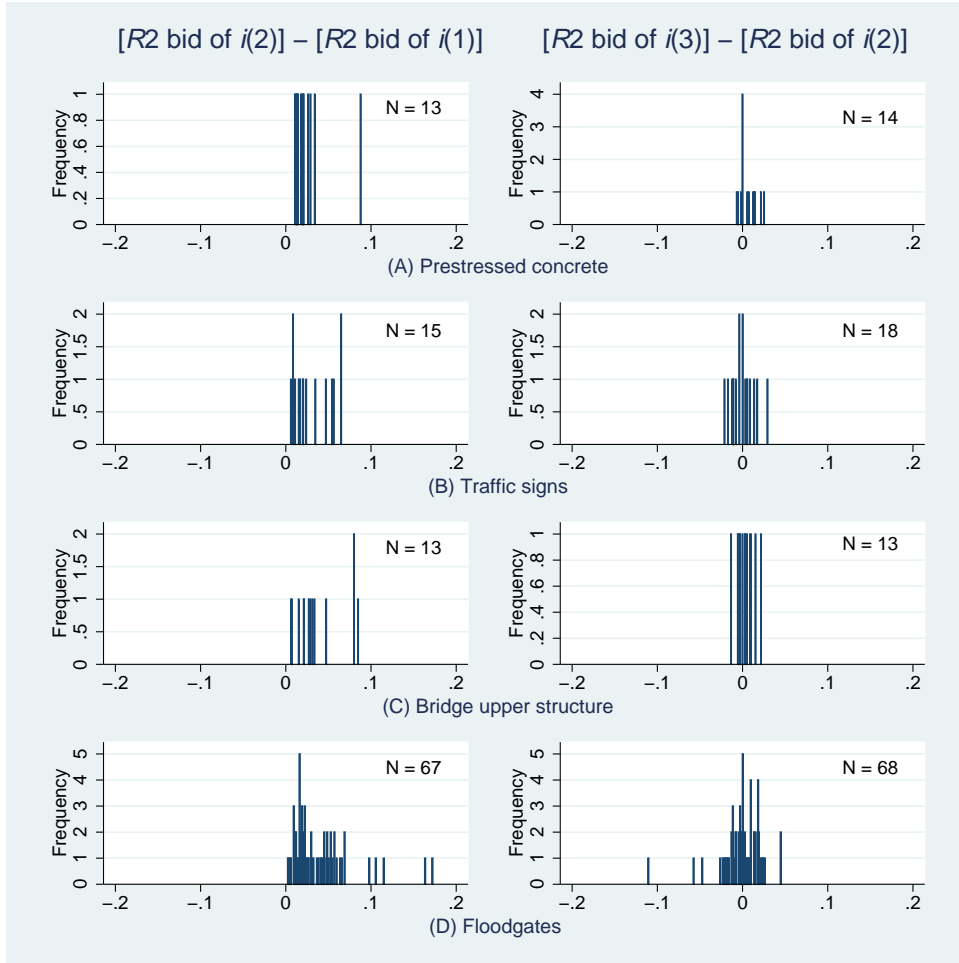


Figure 6: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) Before the Collusion End Date. We use $\varepsilon = 0.05$; hence, the differences in the first-round bids are relatively small.

tributed to the left of zero. Figure 7 plots the histogram of Δ_{12}^2 and Δ_{23}^2 for each of the four bidding rings with $\varepsilon = 5\%$. Although the sample size is very small, the distribution of Δ_{12}^2 and Δ_{23}^2 in Figure 7 are similar to those in Figure 6. That is, Δ_{12}^2 is distributed to the right of zero while Δ_{23}^2 is distributed symmetrically around zero. This may seem to cast doubt on our analysis – why do the distinctive patterns in the distribution of Δ_{12}^2 and Δ_{23}^2 persist even after the collusion end date, when firms presumably started behaving competitively?

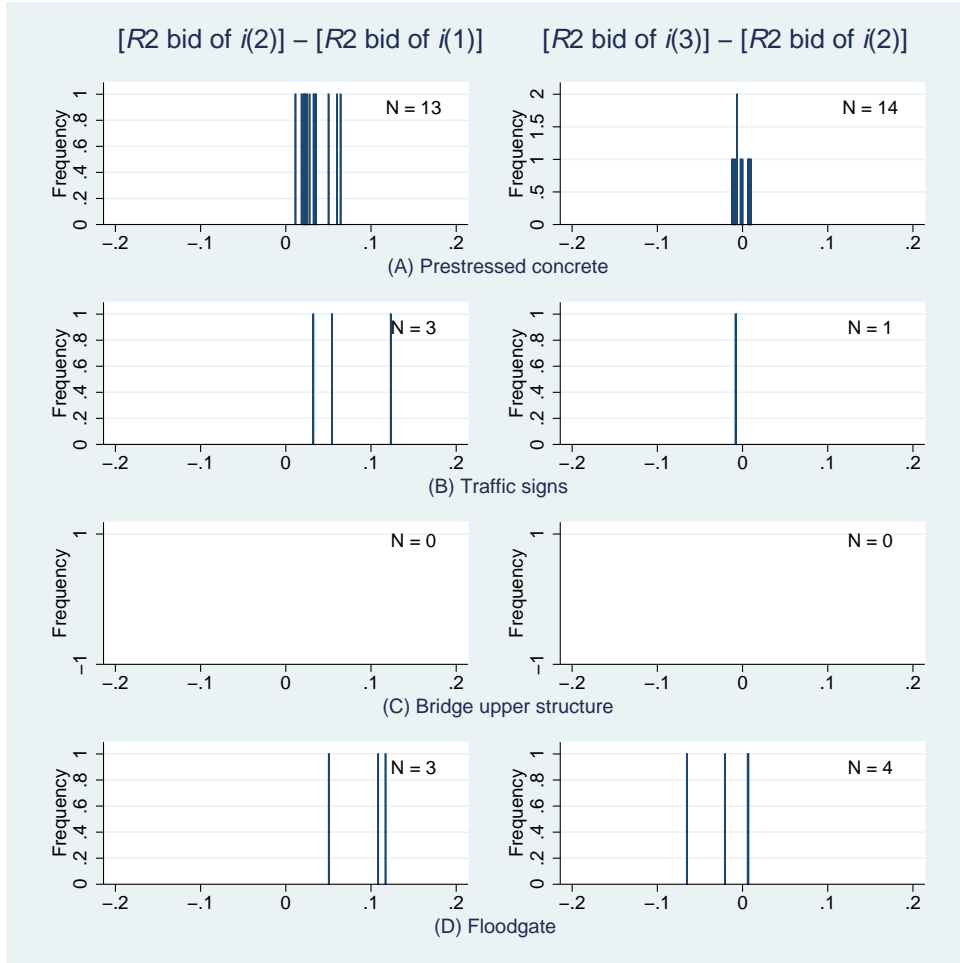


Figure 7: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) After the Collusion End Date. We use $\varepsilon = 0.05$; hence, the differences in the first-round bids are relatively small.

Our view is that asymmetry in the distribution of Δ_{12}^2 should be taken as evidence that firms may have been able to continue colluding at least on some auctions even after the “end date.” While the bidding rings seem to have changed their behavior around the time of the “end date,” – as the drop in the winning bid suggests in Figure 5 – this does not necessarily mean that the firms completely ceased to collude. For example, in the ruling on case (A) issued in 2010, more than five years after the start of the investigation, the judges ordered the ring members, among

other things, to take various measures to prevent collusion from recurring.³⁶ This is because the judges determined that there were still circumstances conducive to collusion even after the “end date” and that ring members needed to take steps to ensure that they do not collude.³⁷ Moreover, many firms that were implicated in these cases are repeat offenders. For example, one firm involved in case (A) had been found guilty in four previous collusion cases.³⁸ A number of firms implicated in case (C) were also subsequently charged and found guilty of collusion in a separate case by the JFTC. It seems that being implicated by the JFTC is no guarantee that a firm will behave competitively thereafter; firms may have been able to continue colluding well beyond the “end date,” at least for some auctions.

With respect to case (A), there is additional evidence that the ring members continued to collude beyond the end date, by following the formula for rebids that we described earlier. Recall that a subset of the prestressed concrete ring members in the Kansai region had a prespecified discount (96% for auctions valued at less than 100 million yen, 97% for auctions valued between 100 million yen and 500 million yen, and 97.5% for auctions valued at more than 500 million yen.) that they used when rebidding in the second round. Figure 8 plots the second-round bids of the ring members in the Kansai region as a fraction of the lowest first-round bid. The top panel corresponds to auctions with a reserve price below 100 million yen; the middle corresponds to those with a reserve price between 100 and 500 million yen; and the last panel corresponds to those with a reserve price of more than 500 million yen. The horizontal axis in the figure corresponds to the calendar date. The vertical line in each panel corresponds to the collusion end date. Thus, auctions that took place before the end date appear to the left of this line. The circles represent $b_{i(1)}^2/b_{i(1)}^1$, and the Xs represent $b_{i(k)}^2/b_{i(1)}^1$ for $k \geq 2$. We have drawn a horizontal line at 96% (top panel), 97% (middle panel), and 97.5% (bottom panel).

While the top and the bottom panels are not very informative, note that all of $i(1)$'s second-round bids in the middle panel of Figure 8 are below 97% of $i(1)$'s

³⁶JFTC Ruling #26-27 (2010). In the ruling, the firms were ordered to take preventative measures such as periodic auditing by a legal officer, etc.

³⁷See JFTC Rulings #26 (2010) pp.66-68 and #27 (2010) pp.51-54.

³⁸JFTC Rulings issued on January 10, 1975; February 25, 1977; July 12, 1977; and June 16, 2000.

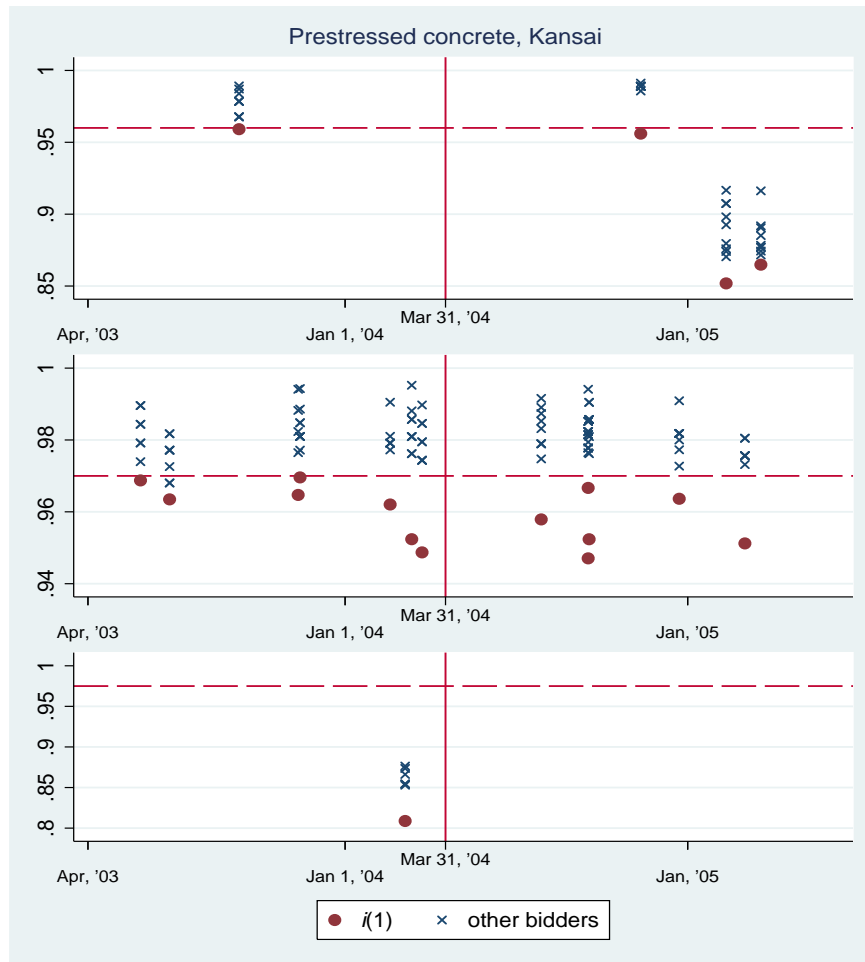


Figure 8: Second-Round Bids of the Ring Members of Kansai Region as a Fraction of the Lowest First-Round Bid. The top panel corresponds to auctions with reserve price less than 100 million yen; the second panel corresponds to auctions with reserve price between 100 million and 500 million yen; and the last panel corresponds to reserve price above 500 million yen. The horizontal axis corresponds to calendar date, starting from April 1, 2003.

first-round bid. Moreover, the bids of all of the others are above 97% of $i(1)$'s first-round bid, except for one auction. If we focus on auctions after the collusion end date, the second-round bids of $i(k)$ ($k \geq 2$) are all above 97%. The bidding pattern in Figure 8 suggests that bidders continued to use the prespecified discount as the threshold value for submitting second-round bids. It seems quite likely that the ring

members were able to maintain collusion even after the “end date.”

6 Detection of Collusive Bidders

In this section, we develop a formal statistical test of collusive behavior based on the idea we discussed in Section 4.2, namely, the distribution of Δ_{12}^2 should not be discontinuous at zero under competitive bidding. We then apply our test to each firm in order to examine whether or not its bidding behavior is consistent with competitive bidding.

Test Statistic Recall from Section 4.2 that there is a reasonable amount of variance in Δ_{23}^2 even among bidders that submit almost identical first-round bids. To the extent that bids are generated by competitive behavior, this means that there is a reasonable amount of bidder-specific idiosyncrasy with regard to the beliefs over the distribution of the reserve price, risk preference, etc., that induce variance in the second-round bids. This, in turn, implies that $i(1)$ cannot be outbidding $i(2)$ in the second round by a small margin all the time under competitive bidding. If $i(1)$ wins some, it has to lose some. Thus, the amount of idiosyncrasy measured by the variance of Δ_{23}^2 puts a bound on how sharply the distribution of Δ_{12}^2 can change around zero. The test statistic that we propose below formalizes this idea by looking for violations of this bound.

We begin by specifying the second-round bids of $i(2)$ and $i(3)$ as follows:

$$\begin{aligned} b_{i(2)}^2 &= X + u_2 \\ b_{i(3)}^2 &= X + u_3, \end{aligned}$$

where X is a common component, and u_2, u_3 are bidder-specific idiosyncratic shocks distributed independently and identically according to F_u . As long as we condition on auctions in which the first-round bids of $i(2)$ and $i(3)$ are close enough, this specification seems natural: Both $i(2)$ and $i(3)$ should have similar cost structures and similar information, which is captured in the common component, X . Note that X is a random variable whose distribution can arbitrarily depend on the

object being auctioned, information revealed in the first round, etc. Basically, X captures all observed and unobserved common factors between $i(2)$ and $i(3)$. The error terms, u_2 and u_3 , are independent bidder-specific idiosyncrasies that result from differences in the bidders' beliefs over the secret reserve price, heterogeneity in the bidders' risk preferences, etc. We assume that u_2 and u_3 are independent of X . Now, given that Δ_{23}^2 is just the difference between $b_{i(3)}^2$ and $b_{i(2)}^2$, we have

$$\begin{aligned}\Delta_{23}^2 &\equiv b_{i(3)}^2 - b_{i(2)}^2 \\ &= u_3 - u_2.\end{aligned}$$

Given our i.i.d. assumptions on (u_2, u_3) , we can recover F_u from realizations of Δ_{23}^2 .

We now consider putting bounds on the distribution of Δ_{12}^2 using F_u . Let us denote by Y the second-round bid of $i(1)$:

$$b_{i(1)}^2 = Y.^{39}$$

Given that $i(1)$ has a different information set than all of the other bidders (as well as, perhaps, having different costs), we do not impose any restrictions on the distribution of Y other than independence with respect to (u_2, u_3) ; i.e., $Y \perp (u_2, u_3)$. In particular, Y can have arbitrary correlation with respect to X .

Note that $\Delta_{12}^2 = X + u_2 - Y$, given that $\Delta_{12}^2 = b_{i(2)}^2 - b_{i(1)}^2$. Now, we define $d(t)$ ($t \in \mathbb{R}^{++}$), a measure of how discontinuous the distribution of Δ_{12}^2 is around zero:

$$d(t) = \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]).$$

$\Pr(\Delta_{12}^2 \in [-t, 0])$ is just the probability that Δ_{12}^2 falls within $[-t, 0]$, and $\Pr(\Delta_{12}^2 \in [0, t])$ is the probability that Δ_{12}^2 falls within $[0, t]$. Hence, $d(t)$ is the difference between the probability that Δ_{12}^2 falls just to the right of zero and the probability that Δ_{12}^2 falls just to the left of zero.

³⁹Note that our formulation incorporates specifications such as $b_{i(1)}^2 = Y + u_1$.

We can derive a simple bound on $d(t)$ using F_u after some algebra,

$$\begin{aligned}
d(t) &= \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]) \\
&= \int 1_{\{X+u_2-Y \in [0, t]\}} dF_{X,Y}(X, Y) dF_u(u_2) \\
&\quad - \int 1_{\{X+u_2-Y \in [-t, 0]\}} dF_{X,Y}(X, Y) dF_u(u_2) \\
&= \int F_u(Y - X + t) - F_u(Y - X) dF_{X,Y}(X, Y) \\
&\quad - \int F_u(Y - X) - F_u(Y - X - t) dF_{X,Y}(X, Y) \\
&= \int F_u(Y - X + t) + F_u(Y - X - t) - 2F_u(Y - X) dF_{X,Y}(X, Y) \\
&\leq \sup_x \|F_u(x + t) + F_u(x - t) - 2F_u(x)\|,
\end{aligned}$$

where the second line uses independence of u_2 with respect to X and Y and $F_{X,Y}$ is the joint cumulative distribution function of X and Y .

Our test statistic simply compares $d(t)$ with the bound derived from F_u . Define $\tau(t)$ as

$$\tau(t) \equiv \sup_x \|F_u(x + t) + F_u(x - t) - 2F_u(x)\| - d(t).$$

Given that we can estimate F_u and $d(t)$, we can estimate $\tau(t)$. Under the null hypothesis of competitive behavior, $\tau(t)$ should be nonnegative.

Detecting Collusive Bidders We now apply this test to each firm that we observe in the data. In particular, for a given firm, we collect all auctions in which the firm participated. We then estimate $d(t)$ and F_u parametrically, for each firm, using realizations of Δ_{12}^2 and Δ_{23}^2 from a subset of these auctions where 1) the auction proceeded to the second round; and 2) the first-round bids of $i(2)$ and $i(3)$ were sufficiently close to each other, i.e., $b_{i(3)}^1 - b_{i(2)}^1 < \varepsilon$.⁴⁰ We use a frequency estimator for $d(t)$ and a maximum likelihood estimator for F_u by specifying F_u to

⁴⁰Note that we condition on the set of auctions where the second- and third-lowest bids in the first round are within ε , given the assumptions on u_2 and u_3 . Note, also, that we drop auctions if Δ_{23}^2 is bigger than 30% to make sure that we exclude misrecordings, etc. This biases against finding collusion.

be a mean-zero Normal distribution with parameter σ_u ($u \sim N(0, \sigma_u^2)$). While our test statistic can easily accommodate a nonparametric estimate of F_u , we impose functional form assumptions on F_u because the number of auctions per firm is not very large. In practice, we estimate $\tau(t)$ for every firm that participated in at least five auctions that meet the two criteria mentioned above.⁴¹ Given our parametric assumption on F_u , $\tau(t)$ has an asymptotically Normal distribution.

In the top left panel of Figure 9, we plot the estimates of $\tau(t)$ for each firm for $t = 1\%$ and $\varepsilon = 5\%$. As shown in the panel, the estimated distribution of $\tau(t)$ lies somewhat to the right of zero, but there is also a substantial mass below zero. Under the null hypothesis of competitive bidding, the value of $\tau(t)$ should be positive; thus, a negative estimate of $\tau(t)$ raises concerns about possible collusive behavior. In the top right panel, we plot the t -statistic for each firm. Again, we find that the estimated t -statistic is negative for a substantial fraction of firms. In particular, there are 674 firms (out of 3,998 firms) whose t -statistic is less than -1.65 , which is the one-sided critical value for rejecting the null hypothesis of competitive behavior at the 95% confidence level. The set of 674 firms includes 21 firms (out of a total of 92 firms) that were implicated in one of the four bid-rigging cases. In the second row of Figure 9, we plot our estimate of $\tau(t)$ and the t -statistic for $t = 2\%$ and $\varepsilon = 5\%$. The results are qualitatively similar. For this case, we find that 578 firms have a t -statistic less than -1.65 .

In the bottom two panels of Figure 9, we repeat the same exercise with $\varepsilon = 1\%$. The panels in the third row correspond to $t = 1\%$, $\varepsilon = 1\%$, and the bottom panels correspond to $t = 2\%$, $\varepsilon = 1\%$. In the third row, there are 403 firms (out of 3,073 firms) whose estimated t -statistic is less than -1.65 , and in the fourth row, we find that 314 firms have an estimated t -statistic less than -1.65 .

It should be clear from the construction of the test statistic that the value of $\tau(t)$ should be nonnegative for all values of t under competitive bidding. Hence, we next conduct a joint hypothesis test. In particular, we pick $t = 1\%$ and $t = 2\%$ and test whether $(\tau(1\%), \tau(2\%))$ is jointly nonnegative. Under the joint hypothesis test, we

⁴¹A total of 21,622 construction firms are observed in our analysis, among which 3,998 (3,073) firms participated in at least five auctions that proceeded to the second round with $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ ($b_{i(3)}^1 - b_{i(2)}^1 < 1\%$).

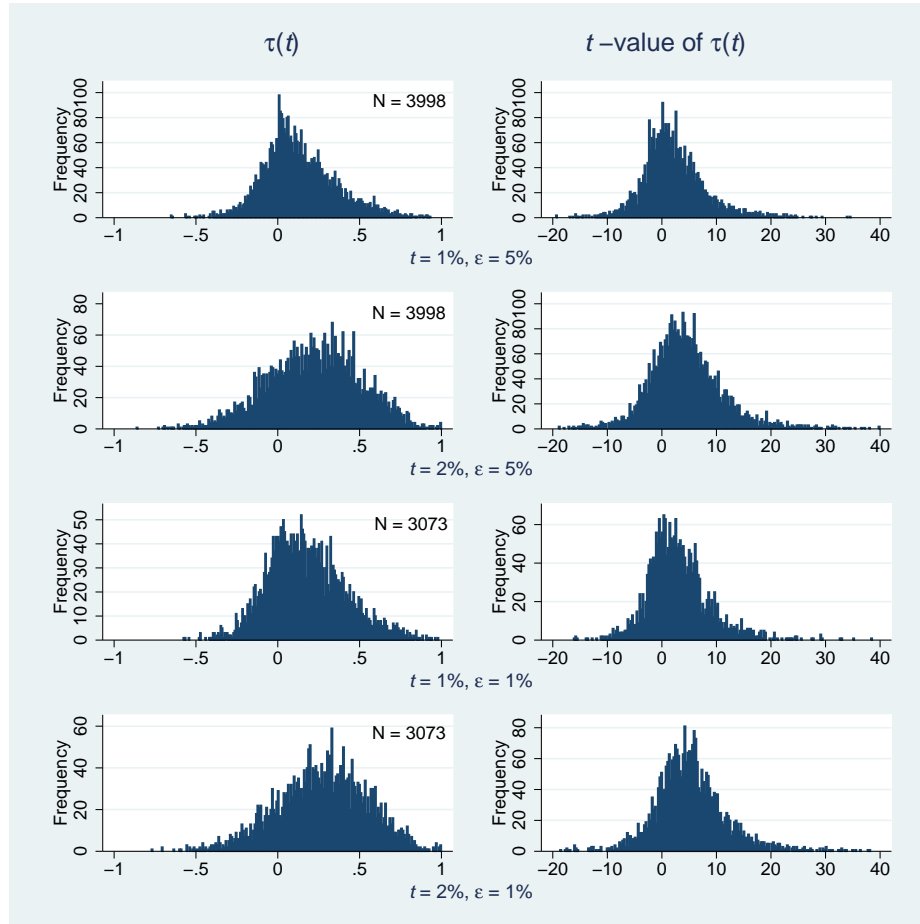


Figure 9: Estimate of $\tau(t)$ (Left Panel) and t -Statistic (Right Panel). We estimated $\tau(t)$ for each firm using only the subset of auctions in which it participated. Top two panels plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 5\%$. Bottom two panels plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 1\%$.

find that we can reject the null for 1,008 firms for $\varepsilon = 5\%$ (586 firms for $\varepsilon = 1\%$).⁴² The joint hypothesis test for $\varepsilon = 5\%$ picks out 25 firms out of 92 firms (27 firms for $\varepsilon = 1\%$) that were implicated in one of the four bid-rigging cases.

To get a sense of the magnitude of our findings, note that the total number of auctions awarded to the 1,008 “suspicious” firms that we identify (in the joint hy-

⁴²In practice, we estimate the joint (2-dimensional) distribution of $(\tau(1\%), \tau(2\%))$. We then simulate 500 draws of (2×1) random vectors according to the estimated joint distribution. We test whether there are more than 25 (= 5% of 500) draws whose elements are both positive.

pothesis test for $\varepsilon = 5\%$) is about 7,600, or close to one fifth of the total number of auctions in our sample. The total award amount of these auctions equals about \$8.6 billion. Given that the four case studies show about a 8.4% average drop in the winning bid after the bidding rings were implicated, our results suggest that taxpayers could have saved about \$721 million in the absence of collusion.⁴³ Moreover, if we consider the fact that the total award amount of municipal and prefectural construction projects in Japan is close to ten times the total value of the auctions in our dataset, the impact of collusion can even be bigger as a whole. There is also ample reason to believe that collusion is just as rampant among municipal and prefectural construction projects, given that some of the same construction firms in our dataset participate in these auctions, as well.

7 Conclusion

In this paper, we document large-scale collusion among construction firms in Japan using bidding data from government procurement projects. We find evidence of collusion across regions, types of construction projects and time. We then test, for each firm, whether its bidding behavior is consistent with competitive behavior. Our test identifies about 1,000 “suspicious” firms that won a total of about 7,600 auctions, or about one fifth of the total number of auctions during our sample period.

The detection method we propose in this paper is very simple and requires only bid data. While our test is not a definitive proof of collusion, we believe that our method can be useful for law enforcement agencies in identifying possible cases of bid rigging.

References

- [1] Asker, John, “Bidding Rings,” *New Palgrave Dictionary of Economics Online*, Eds. Steven Durlauf and Lawrence Blume, Palgrave Macmillan 2010a.

⁴³The drop in the winning bid after the “collusion end date” was 0.6% for (A) prestressed concrete; 8.3% for (B) traffic signs; 19.5% for (C) bridge upper structure; and 5.3% for (D) floodgates. A simple average of the four numbers yields 8.4%.

- [2] Asker, John, "A Study of the Internal Organization of a Bidding Cartel," *American Economic Review*, Vol. 100, No. 3, 2010b, pp. 724-762
- [3] Athey, Susan, Jonathan Levin and Enrique Seira, "Comparing open and Sealed Bid Auctions: Evidence from Timber Auctions," *Quarterly Journal of Economics* 126 (1), 2011, pp.207-257.
- [4] Bajari, Patrick and Lixin Ye, "Deciding Between Competition and Collusion," *Review of Economics and Statistics*, Vol. 85, No. 4, 2003, pp. 971-989
- [5] Baldwin, Laura, Robert Marshall, and Jean-Francois Richard, "Bidder Collusion at Forest Service Timber Sales," *Journal of Political Economy*, Vol. 105, No. 4, 1997, pp. 657-699
- [6] Bidding Guidelines of Ministry of Land, Infrastructure, and Transportation, Chugoku Regional Developing Bureau.
- [7] Conley, Timothy and Francesco Decarolis, "Detecting Bidders Groups in Collusive Auctions," working paper, Boston University
- [8] Haile, Philip, Kenneth Hendricks, Robert Porter, and Toshi Onuma, "Testing Competition in U.S. Offshore Oil and Gas Lease," 2012, working paper, Yale University
- [9] Hendricks, Kenneth and Robert Porter, "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review*, Vol. 78 Issue 5, 1988, pp. 865-83.
- [10] Ishii, Rieko, "Favor Exchange in Collusion: Empirical Study of Repeated Procurement Auctions in Japan," *International Journal of Industrial Organization*, Vol. 27, Issue 2, 2009, pp. 137-144
- [11] Japan Fair Trade Commission (JFTC) Ruling H16#26-27, Sept 21, 2010; JFTC Recommendation and Ruling S49#45, January 10, 1975; S52#1, February 25, 1977; S52#15 July 12, 1977; H12#7, June 16, 2000; H17#5-8, Apr 27, 2005; H17#12, Nov 11, 2005; JFTC Cease and Desist Order H19#2-5 Mar 8, 2007.

- [12] Japan Federation Bar Association (JFBA), “Recommendations on procurement auction reform and a report on bidding behavior (Nyusatsu seido kaikaku ni kansuru teigen to nyusatsu jittai chousa houkoku),” 2001, pp.1-22. (in Japanese).
- [13] Japan Structural Impediments Initiative Joint Report, June 28, 1990.
- [14] Ji, Lu and Tong Li, “Multi-Round Procurement Auctions with Secret Reserve Prices: Theory and Evidence,” *Journal of Applied Econometrics*, vol. 23(7), 2008, pp.897-923.
- [15] Judicial District Court Records: Tsu District Court, No. 165 (Wa), 1997, Nagoya District Court, No. 1903 (Wa), 1995
- [16] Marshall, Robert, and Leslie Marx, “The Economics of Collusion,” The MIT Press, 2012
- [17] McMillan, John, “Dango: Japan’s Price-Fixing Conspiracies,” *Economics and Politics*, vol. 3(3), 1991, pp.201-218.
- [18] Ohashi, Hiroshi, “Effects of Transparency in Procurement Practices on Bidding Behavior: The Experience of Municipal Public Works,” *Review of Industrial Organization*, 34(3), 2009: 267-85
- [19] Pesendorfer, Martin, “A Study of Collusion in First-Price Auctions,” *Review of Economic Studies* Vol. 67 (3), 2000, pp.381-411.
- [20] Porter, Robert and J. Douglas Zona, “Detection of Bid Rigging in Procurement Auctions,” *Journal of Political Economy*, Vol. 101, No. 3 1993, pp. 518-538
- [21] Porter, Robert and J. Douglas Zona, “Ohio School Milk Markets: An Analysis of Bidding,” *RAND Journal of Economics*, Vol. 30, No. 2 1999, pp. 263-288

For Online Publication

Analysis of Collusive Behavior by Region, Auction Category, Project Type, and Time

In this Appendix, we show that the shape of the distributions of Δ_{12}^2 and Δ_{23}^2 in Figure 1 is robust to conditioning on region, auction category, project type, and year. We also show that the shape of the distribution is robust to whether or not we normalize the bids by the reserve price. Note that for all of the figures in this section (Figures A.1 - A.5), we set ε equal to 5%, i.e., the figures plot auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ (left panels) or $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ (right panels).

By Region

Figure A.1 plots the histogram of Δ_{12}^2 and Δ_{23}^2 for four of the nine regions of Japan with the largest number of auctions. The regions that we show are Hokkaido, Kanto, Kansai and Chubu, in decreasing order of number of total auctions.

By Auction Category

Figure A.2 plots the histogram of Δ_{12}^2 and Δ_{23}^2 for each of the four auction categories that we discussed in Section 2. Category 1 corresponds to auctions with the most restrictions on participation, and category 4 corresponds to auctions with the least restrictions.

By Project Type

In Figure A.3, we plot the histogram of Δ_{12}^2 and Δ_{23}^2 for the four types of projects with the largest number of auctions. The four types of projects are civil engineering, repair and maintenance, paving, and communication equipment, in decreasing order of number of total auctions.

By Year

In Figure A.4, we plot the histogram of Δ_{12}^2 and Δ_{23}^2 , by year.

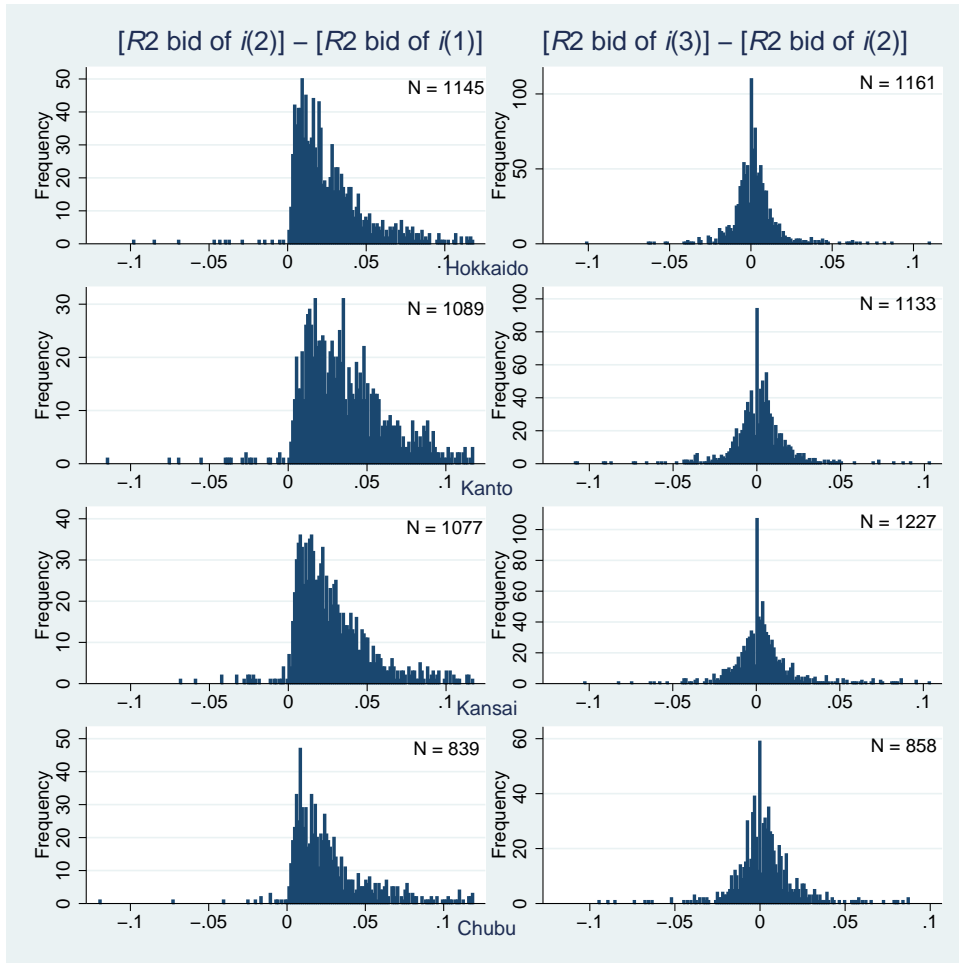


Figure A.1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Region. The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Raw Bids

Finally, in Figure A.5, we plot the raw difference in the second-round bids without normalizing by the reserve price. The left panels plot the second-round bid differences of $i(1)$ and $i(2)$. The right panels plot the second-round bid differences of $i(2)$ and $i(3)$. The top panels correspond to auctions whose reserve price is between 20-22 million yen. The middle and bottom panels correspond to auctions with a re-

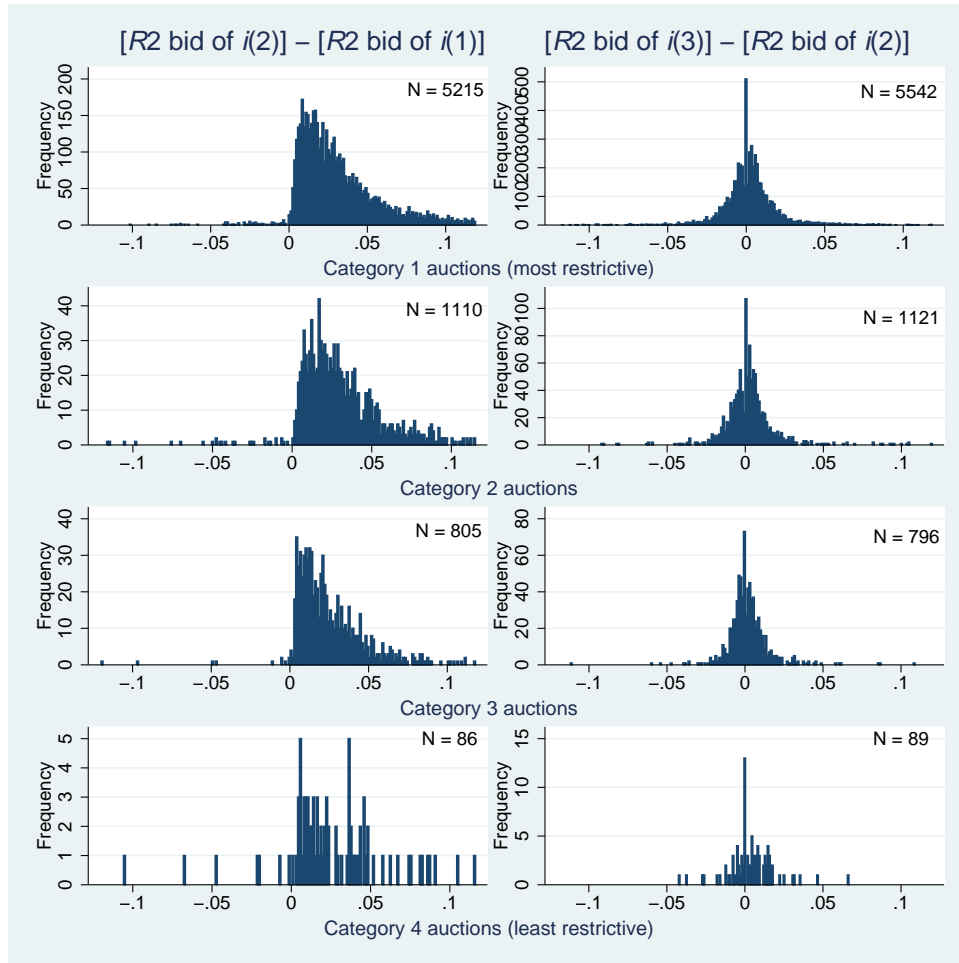


Figure A.2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Auction Category. The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

serve price between 60-66 million yen and 90-99 million yen, respectively.⁴⁴ The auctions in each row roughly correspond to the 25%, 50% and 75% quantiles in terms of project size.

⁴⁴The length of the bandwidth we use (i.e., 2 million, 6 million, and 9 million yen, respectively) is roughly 10% of the reserve price in each row.

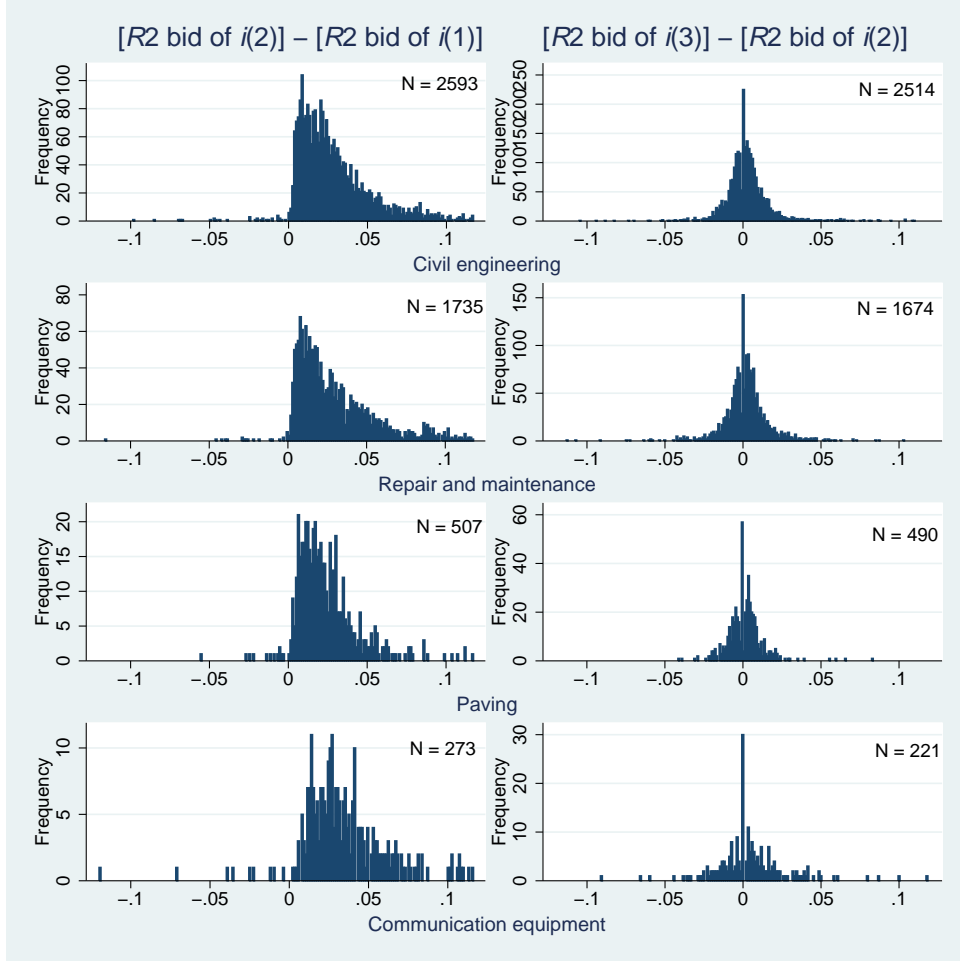


Figure A.3: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Project Type. The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Proof of Proposition 1

In this Appendix, we give a proof of Proposition 1. Suppose that there are two risk-neutral bidders with independently and identically distributed costs, following a distribution F on support $C = [\underline{c}, \bar{c}]$ with $F(\underline{c}) = 0$, $F(\bar{c}) = 1$ with density $f(\cdot) > 0$. Let $c_1, c_2 \in C$ be the costs of $i(1)$ and $i(2)$ with $c_1 \leq c_2$. Since $i(1)$'s first-round bid is revealed at the end of the first round, and the first-round bidding

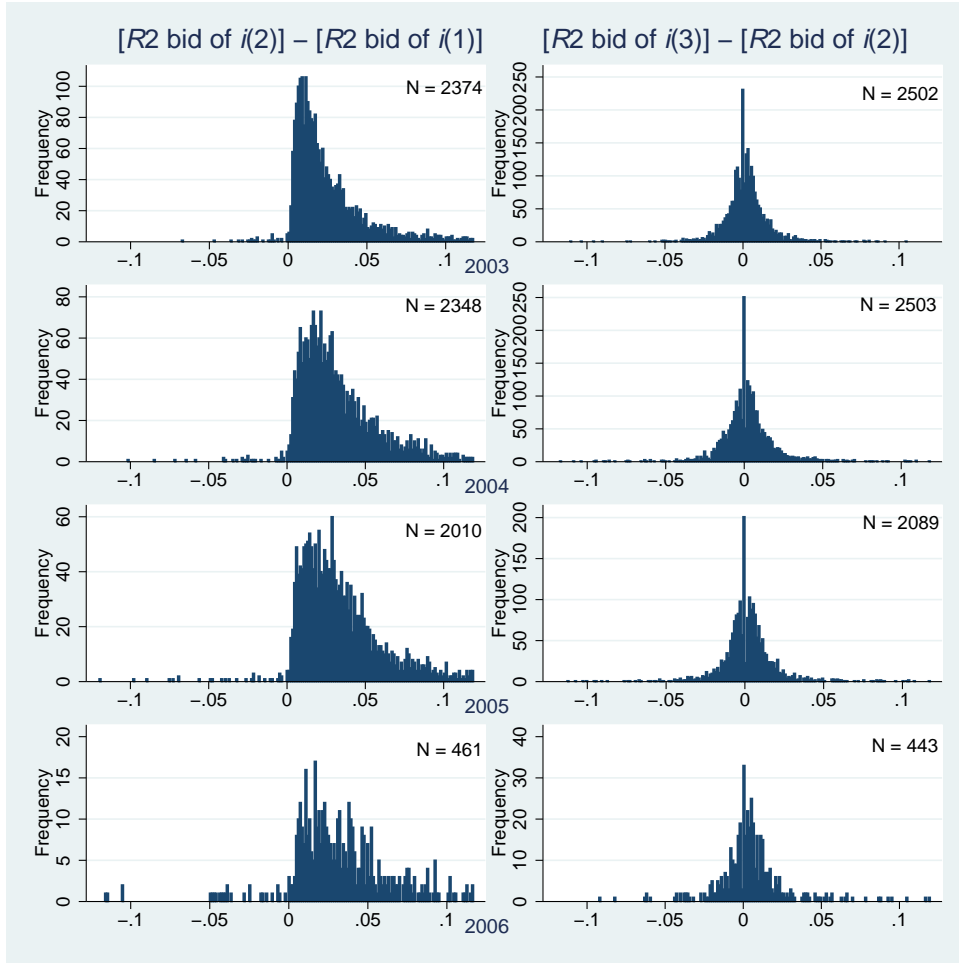


Figure A.4: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Year. The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

strategy is symmetric and strictly monotone, $i(2)$ knows c_1 , while $i(1)$ knows only that c_2 is no less than c_1 . Two bidders have identical beliefs over the distribution of the secret reserve price after the first round. Let $R(\cdot)$ denote the distribution of bidders' beliefs over the reserve price with density $r(\cdot)$ and $R(c_1) < 1$. We assume that the hazard rate of $R(\cdot)$:

$$\frac{r(x)}{1 - R(x)}$$

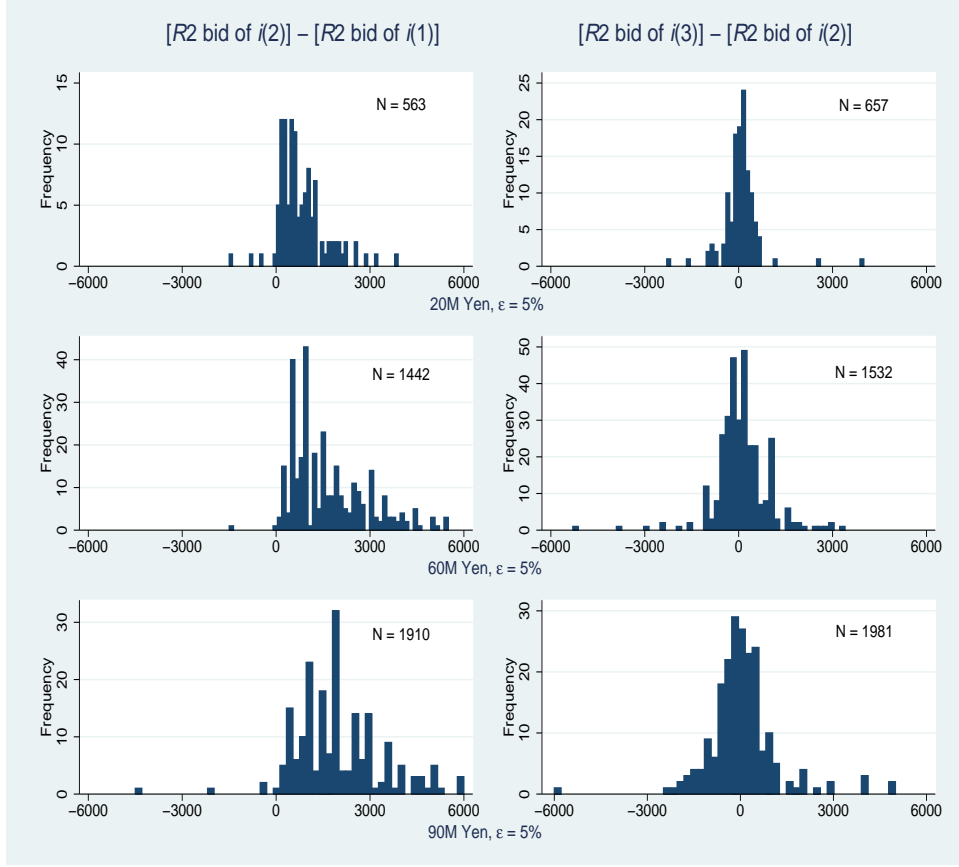


Figure A.5: Raw Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Raw Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The left panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5% of the reserve price. The right panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5% of the reserve price.

is strictly increasing in x . Define two functions $H(\cdot)$ and $\beta(\cdot)$

$$H(b) = \begin{cases} 1 & \text{if } b > \bar{b} \\ 1 - \exp\left(\int_{\underline{b}}^b \frac{-1}{x - F^{-1}\left(1 - \frac{y}{x - c_1} \cdot \frac{1}{1 - R(x)}\right)} dx - \ln \frac{1 - R(b)}{1 - R(\underline{b})}\right) & \text{if } b \in [\underline{b}, \bar{b}] \\ 0 & \text{if } b < \underline{b} \end{cases} \quad (1)$$

$$\beta^{-1}(b) = \begin{cases} F^{-1}\left(1 - \frac{y}{b - c_1} \cdot \frac{1}{1 - R(b)}\right) & \text{if } b \in [\underline{b}, \bar{b}] \\ b & \text{if } b \in [\bar{b}, \bar{c}], \end{cases} \quad (2)$$

where \bar{b} , \underline{b} , and y are defined as follows:

$$\begin{aligned}\pi_1(b) &= (b - c_1) \frac{1 - F(b)}{1 - F(c_1)} [1 - R(b)], \\ \bar{b} &= \min_{\hat{b}} \left\{ \hat{b} \mid \hat{b} = \arg \max_b \pi_1(b) \right\}, \\ \underline{b} &= \{b \mid (b - c_1) [1 - R(b)] = \pi_1(\bar{b}) \wedge b \leq \bar{b}\}, \\ y &= \pi_1(\bar{b}) [1 - F(c_1)].\end{aligned}$$

We first show that $H(\cdot)$ and $\beta(\cdot)$ are well-defined, strictly increasing and continuously differentiable in (\underline{b}, \bar{b}) (Lemma 1). We then show that $i(1)$'s strategy of mixing its bid on support $B = [\underline{b}, \bar{b}]$ following distribution $H(\cdot)$ and $i(2)$'s pure monotone strategy given by $\beta(\cdot)$ are mutual best responses in the second round (taking as given that bidders play symmetric pure monotone strategies in the first round). Finally, we show that, for any pair of best responses in the second round, the strategies of $i(1)$ and $i(2)$ have the following properties: 1) $i(1)$ plays a mixed strategy in which there exists no mass at the lower bound, \underline{b}' , of the support; 2) $i(2)$'s strategy is weakly increasing in its costs; 3) for any $\epsilon > 0$, there exists a strictly positive mass of $i(2)$ types that bid less than \underline{b}' .

Lemma 1 *Both $\beta^{-1}(\cdot)$ and $H(\cdot)$ are well-defined. That is, there exists a unique value \underline{b} defined in the above expression, the argument of $F^{-1}(\cdot)$ in (2) is in $[0, 1]$, and $H(\cdot)$ is a proper distribution function. Moreover, $\beta^{-1}(\cdot)$ and $H(\cdot)$ are strictly increasing and continuously differentiable in $b \in (\underline{b}, \bar{b})$.*

Lemma 1 guarantees that $\beta^{-1}(\cdot)$ and $H(\cdot)$ are well-defined. The following lemmas are convenient for showing that $H(\cdot)$ and $\beta(\cdot)$ are best responses.

Lemma 2 *\underline{b} is strictly greater than c_1 .*

Lemma 3 *If $R(\cdot)$ has a strictly increasing hazard rate, then $(b - c_1)[1 - R(b)]$ is strictly increasing in $b \leq \bar{b}$.*

The proofs of Lemma 1-3 are given at the end.

We now show that $\beta(\cdot)$ is $i(2)$'s best response given $H(\cdot)$. If $c_2 \geq \bar{b}$, $i(2)$ has no chance of winning. It follows that bidding its cost is $i(2)$'s best response if $c_2 \geq \bar{b}$.

Now, consider the case that $c_2 < \bar{b}$.⁴⁵ Given $H(\cdot)$, $i(2)$'s expected profit, $\pi_2(b, c_2)$, is given by

$$\pi_2(b, c_2) = (b - c_2)[1 - H(b)][1 - R(b)].$$

Let b_2^* be $i(2)$'s optimal bid. Then, b_2^* satisfies the first-order condition for maximizing $\pi_2(b, c_2)$ as

$$\frac{1}{b_2^* - c_2} = \frac{h(b_2^*)}{1 - H(b_2^*)} + \frac{r(b_2^*)}{1 - R(b_2^*)}. \quad (3)$$

If we substitute out $h(\cdot)[1 - H(\cdot)]^{-1}$ using (1), Equation (3) becomes

$$\frac{1}{b_2^* - c_2} = \frac{1}{b_2^* - \beta^{-1}(b_2^*)}.$$

This implies that the first-order condition for $i(2)$'s problem is satisfied at $c_2 = \beta^{-1}(b_2^*)$. Moreover, this argument holds for all $c_2 \in [c_1, \bar{b}]$.

To see that $\beta(\cdot)$ is indeed $i(2)$'s best response, we show that $\beta(\cdot)$ satisfies the sufficient condition for $i(2)$'s maximization problem; from (3),

$$\frac{1}{\beta(\tilde{c}) - c_2} \begin{matrix} \geq \\ < \end{matrix} \frac{h(\beta(\tilde{c}))}{1 - H(\beta(\tilde{c}))} + \frac{r(\beta(\tilde{c}))}{1 - R(\beta(\tilde{c}))},$$

if and only if $\tilde{c} \begin{matrix} \geq \\ < \end{matrix} c_2$ for all $\tilde{c}, c_2 \in [c_1, \bar{b}]$ because the left-hand side is strictly increasing in c_2 . Define \tilde{b} such that $\beta^{-1}(\tilde{b}) = \tilde{c}$. Given that $\beta(\cdot)$ is continuous and strictly increasing, $\tilde{b} \begin{matrix} \geq \\ < \end{matrix} \beta(c_2)$ if and only if $\tilde{c} \begin{matrix} \geq \\ < \end{matrix} c_2$. Hence, we have

$$\frac{1}{\tilde{b} - c_2} \begin{matrix} \geq \\ < \end{matrix} \frac{h(\tilde{b})}{1 - H(\tilde{b})} + \frac{r(\tilde{b})}{1 - R(\tilde{b})},$$

if and only if $\tilde{b} \begin{matrix} \geq \\ < \end{matrix} \beta(c_2)$. Thus, $\beta(\cdot)$ gives a unique global maximum of $\pi_2(\cdot, c_2)$ for

⁴⁵This event occurs with positive probability since Lemma 2 implies that $c_1 < \bar{b}$.

any c_2 . Hence, $\beta(\cdot)$ is $i(2)$'s best response.

We next show that $H(\cdot)$ is $i(1)$'s best response given $\beta(\cdot)$. Given that $\beta^{-1}(\cdot)$ is strictly increasing, the probability that $i(2)$'s bid is above b is given by

$$\Pr\{\beta(c_2) \geq b\} = \Pr\{c_2 \geq \beta^{-1}(b)\} = \frac{1 - F(\beta^{-1}(b))}{1 - F(c_1)}.$$

Therefore, $i(1)$'s expected payoff from bidding an arbitrary value b is characterized as

$$(b - c_1) \frac{1 - F(\beta^{-1}(b))}{1 - F(c_1)} [1 - R(b)].$$

It is easy to see that if we substitute out $\beta^{-1}(\cdot)$ using expression (2), this equals $\pi_1(\bar{b})$ for all $b \in B$. Hence, any bid between \underline{b} and \bar{b} gives $i(1)$ exactly the same payoffs as long as $i(2)$ plays $\beta(\cdot)$.

To see that $H(\cdot)$ is indeed $i(1)$'s best response, it is sufficient to show that $i(1)$'s expected payoff is maximized at \bar{b} . Given that $i(2)$ bids its cost if the cost is above \bar{b} , $\pi_1(\bar{b})$ is $i(1)$'s expected payoff from bidding \bar{b} . Recall that \bar{b} is a maximizer of $\pi_1(b)$ by definition and that $i(1)$'s expected payoff is constant for all $b \in B$. Moreover, by Lemma 3, $i(1)$'s expected payoff from bidding $b < \underline{b}$ is strictly lower than $\pi_1(\underline{b})$ ($= \pi_1(\bar{b})$). Hence, $H(\cdot)$ is $i(1)$'s best response.

We have shown, so far, that there exists a pair of best responses in which $i(1)$ mixes its bid on B and $i(2)$ bids a strictly pure monotone strategy. We now show that, for any pair of best responses in the second round, the strategies of $i(1)$ and $i(2)$ have the following properties: 1) $i(1)$ plays a mixed strategy in which there exists no mass at the lower bound, \underline{b}' , of the support; 2) $i(2)$'s strategy is weakly increasing in its costs; 3) for any $\epsilon > 0$, there exists a strictly positive mass of $i(2)$ types that bid less than $\underline{b}' + \epsilon$. Here, we denote the lower bound of $i(1)$'s strategy as \underline{b}' , which may not be equal to \underline{b} defined at the beginning of this proof.

First, we note that \underline{b}' is not "too high" or "too low," that is,

$$\begin{aligned} \underline{b}' &\in (c_1, \tilde{b}), \\ \tilde{b} &= \arg \max_b (b - c_1)[1 - R(b)]. \end{aligned}$$

The expression $(b - c_1)[1 - R(b)]$ is $i(1)$'s profit from bidding b when $i(2)$ does not bid. Note that there is a unique maximand of this expression; hence, \tilde{b} is well-defined.⁴⁶ It is easy to see \underline{b}' must be strictly higher than c_1 , so we focus on showing $\underline{b}' < \tilde{b}$. Observe that bidding \tilde{b} dominates bidding above \tilde{b} . That is, regardless of $i(2)$'s strategy, $i(1)$'s profit from bidding \tilde{b} is strictly higher than bidding above \tilde{b} . Hence, \underline{b}' must be weakly below \tilde{b} . To see that \underline{b}' is strictly below \tilde{b} , suppose to the contrary. Given that bidding above \tilde{b} is dominated by bidding \tilde{b} , $i(1)$ must bid \tilde{b} with probability one. However, this cannot be part of a best response, because a positive mass of $i(2)$ types would want to undercut $i(1)$ by an infinitesimal amount.

We now show 2). To demonstrate that $i(2)$'s strategy is weakly increasing, it is sufficient to show that $i(2)$'s expected payoff function,

$$(b - c_2)[1 - \tilde{H}(b)][1 - R(b)],$$

satisfies the single-crossing condition, where $\tilde{H}(\cdot) \in [0, 1]$ is the cumulative distribution function of $i(1)$'s bid. The derivative with respect to c_2 is given as

$$-[1 - \tilde{H}(b)][1 - R(b)],$$

which is increasing in b . Hence, $i(2)$'s expected payoff function satisfies the single crossing condition.

We next show 3). Suppose, to the contrary, that there is not a strictly positive mass of $i(2)$ types that bid less than $\underline{b}' + \epsilon$. In this case, $i(1)$'s expected payoff from bidding $b = \underline{b}'$ is

$$(\underline{b}' - c_1)[1 - R(\underline{b}')], \tag{4}$$

and the expected payoff from bidding $\underline{b}' + \epsilon/2$

$$(\underline{b}' + \epsilon/2 - c_1)[1 - R(\underline{b}' + \epsilon/2)]. \tag{5}$$

As long as $\underline{b}' < \tilde{b}$, (5) is strictly greater than (4) because $R(b)$ has a monotone

⁴⁶It is easy to see that the derivative of the expression with respect to b is strictly decreasing.

hazard rate. Hence, bidding $b = \underline{b}'$ is strictly worse than bidding $b = \underline{b}' + \epsilon/2$ for $i(1)$. This is a contradiction. Thus, there exists a strictly positive mass of $i(2)$ types that bid less than $\underline{b}' + \epsilon$. Note that we can apply the same argument to the lower bound of $i(2)$. That is, if we let \underline{b}'' denote the lower bound of $i(2)$'s bids, $i(1)$ bids between \underline{b}'' and $\underline{b}'' + \epsilon$ with positive probability. This implies that $\underline{b}' = \underline{b}''$.

We finally show 1). Suppose, to the contrary, that $i(1)$'s bid has a point-mass at \underline{b}' . Then, there exists a positive mass of $i(2)$ types that would gain by undercutting \underline{b}' by an infinitesimal amount. Hence, $i(1)$'s bid cannot have a point-mass at \underline{b}' .

Proof of Lemmas 1, 2 and 3

We show Lemma 1 after proving Lemmas 2 and 3.

Proof of Lemma 2 Since $c_2 > c_1$ with a positive probability, it cannot be $i(2)$'s best response to bid c_1 or less with probability equal to 1. Therefore, $i(1)$ obtains a strictly positive expected gain if and only if it bids strictly above c_1 . Hence, \underline{b} is strictly greater than c_1 .

Q.E.D.

Proof of Lemma 3 Since \bar{b} maximizes $\pi_1(b)$, it necessarily satisfies the first-order condition as

$$\frac{1}{\bar{b} - c_1} - \frac{r(\bar{b})}{1 - R(\bar{b})} = \frac{f(\bar{b})}{1 - F(\bar{b})}.$$

The right-hand side is strictly positive for all $\bar{b} \in C$. Hence,

$$\frac{1}{b - c_1} - \frac{r(b)}{1 - R(b)} \tag{6}$$

is strictly positive at $b = \bar{b}$. Furthermore, it is easy to see that (6) is strictly decreasing in b if $R(b)$ has a strictly increasing hazard rate. Hence, (6) is strictly positive for all $b \leq \bar{b}$. Note that (6) is equivalent to the derivative of $\ln(b - c_1)[1 - R(b)]$ with respect to b . Therefore, $(b - c_1)[1 - R(b)]$ is strictly increasing in $b \leq \bar{b}$.

Q.E.D.

Proof for Lemma 1 We first show that $\beta^{-1}(b)$ is strictly increasing and continuously differentiable for all $b \in (\underline{b}, \bar{b})$. To demonstrate this, we show that \underline{b} is uniquely given with $\underline{b} > c_1$; recall that \underline{b} is defined as

$$\underline{b} = \{b \mid (b - c_1) [1 - R(b)] = \pi_1(\bar{b}) \wedge b \leq \bar{b}\}.$$

Since $\pi_1(\bar{b})$ is constant, and $(b - c_1) [1 - R(b)]$ is continuous and strictly increasing in b for all $b < \bar{b}$ by Lemma 3, there is a unique value of b less than \bar{b} that satisfies

$$(b - c_1) [1 - R(b)] = \pi_1(\bar{b}). \quad (7)$$

Hence, \underline{b} exists and, moreover, is unique.

We now show that $\beta^{-1}(\cdot)$ is strictly increasing and continuously differentiable for all $b \in (\underline{b}, \bar{b})$. By Lemma 2, $(b - c_1) [1 - R(\underline{b})] > 0$. In addition, Lemma 3 implies that $(b - c_1) [1 - R(b)]$ is strictly increasing in b for all $b < \bar{b}$. Hence, $(b - c_1) [1 - R(b)]$ is strictly positive for all $b \in B$.

Then, it is easy to see that the right-hand side in (2),

$$F^{-1} \left(1 - \frac{y}{b - c_1} \cdot \frac{1}{1 - R(b)} \right) \equiv F^{-1} \left(1 - \frac{\bar{b} - c_1}{b - c_1} [1 - F(\bar{b})] \frac{1 - R(\bar{b})}{1 - R(b)} \right),$$

is well-defined because the object inside $F^{-1}(\cdot)$ in the above expression is 1) strictly increasing and continuously differentiable with respect to b and; 2) contained within $[F(c_1), F(\bar{b})] \subset [0, 1]$ for all $b \in (\underline{b}, \bar{b})$. Hence, $\beta^{-1}(\cdot)$ is strictly increasing and continuously differentiable in $b \in (\underline{b}, \bar{b})$.

We next show that $H(\cdot)$ is strictly increasing and continuously differentiable for all $b \in (\underline{b}, \bar{b})$. From (1), the derivative of $-\ln(1 - H(b))$ is given as

$$-\frac{d}{db} \ln(1 - H(b)) = \frac{1}{b - \beta^{-1}(b)} - \frac{r(b)}{1 - R(b)}, \quad (8)$$

as long as $b \neq \beta^{-1}(b)$, where $F^{-1}(\cdot)$ is replaced with β^{-1} . Hence, $H(\cdot)$ is continuously differentiable in $b \in (\underline{b}, \bar{b})$ as long as we can show that $b > \beta^{-1}(b)$ holds for

any $b \in (\underline{b}, \bar{b})$. To see this, note that

$$1 - F(\beta^{-1}(b)) = \frac{\bar{b} - c_1}{b - c_1} [1 - F(\bar{b})] \frac{1 - R(\bar{b})}{1 - R(b)} \text{ for all } b \in B,$$

by (2). Multiplying $(b - c_1)[1 - R(b)][1 - F(c_1)]^{-1}$ on both sides gives

$$\begin{aligned} (b - c_1) \frac{1 - F(\beta^{-1}(b))}{1 - F(c_1)} [1 - R(b)] &= (\bar{b} - c_1) \frac{1 - F(\bar{b})}{1 - F(c_1)} [1 - R(\bar{b})] \\ &= \max_{\hat{b}} \pi_1(\hat{b}). \end{aligned}$$

If $b \leq \beta^{-1}(b)$ were to occur for some $b \in (\underline{b}, \bar{b})$, we would have

$$(b - c_1) \frac{1 - F(b)}{1 - F(c_1)} [1 - R(b)] \geq (b - c_1) \frac{1 - F(\beta^{-1}(b))}{1 - F(c_1)} [1 - R(b)],$$

at b . Recall that the left-hand side of the above inequality is $\pi_1(b)$. It follows that there exists $b < \bar{b}$ such that $\pi_1(b) \geq \max_{\hat{b}} \pi_1(\hat{b})$. However, this violates the definition of $\bar{b} = \min \arg \max_{\hat{b}} \pi_1(\hat{b})$. Hence, $b > \beta^{-1}(b)$, and $H(\cdot)$ is continuously differentiable in $b \in (\underline{b}, \bar{b})$.

We now show that $H(\cdot)$ is strictly increasing. Note first that

$$\frac{1}{b - c_1} - \frac{r(b)}{1 - R(b)} \leq \frac{1}{b - \beta^{-1}(b)} - \frac{r(b)}{1 - R(b)}$$

for any $\beta^{-1}(b) \in [c_1, \bar{b}]$ and that the equality holds if and only if $b = \underline{b}$. The left hand side of this inequality is strictly positive for all $b \in (\underline{b}, \bar{b})$ because it is the derivative of $\ln(b - c_1)(1 - R(b))$, which is strictly increasing by Lemma 3. This means that the right-hand side in (8) is strictly positive for all $b \in (\underline{b}, \bar{b})$, or equivalently, $H(b)$ is strictly increasing.

Q.E.D.