# Product Choice and Price Discrimination in Markets with Search Costs*

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**Abstract**

In a seminal paper, Champsaur and Rochet (1989) showed that competing firms choose non-overlapping qualities so as to soften price competition, thus giving up profitable opportunities to price discriminate. In this paper we show that an arbitrarily small amount of search costs is enough to overturn this prediction. In markets with search costs, competing firms face the monopolist's incentive to price discriminate, which induces them to carry overlapping product lines and thus to compete head-to-head. Thus, search costs provide a foundation for the existence of multi-product retailers, one that is not based on consumers' preferences for one-stop shopping. Our analysis also provides predictions regarding pricing by multi-product firms in markets with search costs under various retail market structures. Products choices and pricing by online bookstores motivate our findings.

Keywords: second degree price discrimination, search, vertical differentiation, retail competition.

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# 1 Introduction

Since the classical work of Chamberlin (1933), a well known principle in economics is that firms differentiate their products in order to relax competition. Champsaur and Rochet (1989) (CR, thereafter) formalized this Chamberlinian incentive in a model in which quality choices are followed by price competition.[1] They showed that firms choose non-overlapping qualities because the incentives to soften price competition dominate over the incentives to better discriminate heterogeneous consumers. Yet, in many markets, competing firms often carry overlapping qualities even when this creates fierce competition among them.

In this paper, we argue that CR's prediction fails in practice because it omits a fundamental ingredient: consumers' search is costly.[2] When consumers are not perfectly informed about firms' prices and qualities, they cannot choose their preferred option unless they incur search costs to learn and compare all options. Since the seminal work of Diamond (1971), the search literature has shown that the introduction of search frictions can have substantial effects on competition, no matter how search costs are modeled.[3] However, this literature has broadly neglected the possibility that firms engage in price discrimination through quality choice.[4] Understanding the interaction between search costs and price discrimination, and their effects on product choice and pricing by competing firms, is the main goal of this paper.

By introducing search costs *à la* Varian (1980) in a simplified version of CR, we show that an arbitrarily small amount of search costs is all it takes to induce firms to offer overlapping product lines, and through this, to restore Bertrand competition. More generally, in the presence of search costs, the opposite as in CR's model holds: the incentives to better discriminate heterogeneous consumers dominate over the incentives to soften price competition. Hence, regardless of whether search costs are high or low, the equilibrium involves full product overlap, with all consumers buying their preferred products at lower prices than if firms could coordinate on CR's equilibrium. This does not necessarily imply that search costs make consumers better off: while an arbitrarily

[1]Shaked and Sutton (1982) formalized the same idea in a model similar to Champsaur and Rochet (1989), with the difference that firms are allowed to offer one quality only. Thus, in Shaked and Sutton (1982), firms cannot discriminate consumers through quality.

[2]There is a large empirical literature analyzing pricing in markets that fit well into CR's theoretical framework and in which search costs are relevant. See Section 2 for more on this.

[3]Search models can essentially be classified as models of either simultaneous search (Burdett and Judd, 1983) or sequential search (Stahl, 1989). De los Santos *et al.* (2012) test which of the two processes best represents actual search for online books, and conclude in favor of the simultaneous search model, which is the approach we adopt in this paper.

[4]Unlike the current paper, in which we model second-degree price discrimination, Fabra and Reguant (2017) allow for third-degree price discrimination in markets with search costs.

small amount of search costs reduces prices through the effect on product choice,[5] further increases in search costs relax competition, eventually leading to prices that exceed those in frictionless markets.

Beyond investigating the effects of search costs on firms' product lines, we also aim at understanding their effects on equilibrium pricing in general, even in markets where firms cannot reach the subgame perfect equilibrium product choices (e.g. if there are substantial fixed costs of carrying a product, or any exogenous obstacle that constrains firms' product choices). We show that the incentive compatibility constraints faced by multi-product firms introduce an important departure from Varian (1980): the prices for the various goods sold within a store cannot be chosen independently from each other. In particular, as it is standard in games with asymmetric information, the price of the high quality good has to be reduced to discourage those consumers with high quality preferences from buying the low quality good, just as the price of the low quality good has to be reduced to discourage those consumers with low quality preferences from buying the high quality one. Yet, when competition becomes particularly intense, multi-product firms reduce the price difference between the two goods below the level that is necessary to induce the high types to buy the high quality good. In other words, during periods of sales *a la* Varian, the incentives to compete may dominate over the incentives to minimize information rents. Additionally, incentive compatibility considerations imply that on average multi-product firms tend to charge lower prices than single-product firms.

Our paper is related to two strands of the literature: (*i*) papers that analyze competition with search costs, and (*ii*) papers that characterize quality choices under imperfect competition. The vast part of the search literature assumes that consumers search for one unit of an homogenous good, with two exceptions. Some search models allow for product differentiation across firms but, unlike ours, assume that each firm carries a single product.[6] Other search models allow firms to carry several products but, unlike ours, typically assume that such goods are complements and that consumers search for more than one ('multi-product search'). In these models, consumers differ in their preference

[5]In general, search costs are thought to relax competition, thus leading to higher prices, although not as intensively as the Diamond paradox would have anticipated (Diamond, 1971). There are some exceptions to this general prediction. Some recent papers have shown that search costs can lead to lower prices, particularly so when search costs affect the types of consumers who search. For instance, see Moraga-González *et al.* (2017) and Fabra and Reguant (2017).

[6]See for instance Anderson and Renault (1999) for horizontally differentiated products, and Wildenbeest (2011) for vertically differentiated products. Unlike us, the latter assumes that all consumers have the same preference for quality. He finds that all firms use the same symmetric mixed strategy in utility space, which means that firms use asymmetric price distributions depending on the quality of their product. In contrast, we find that firms use different pricing strategies for the same product, with this asymmetry arising because of price discrimination within the store.

for buying all goods in the same store ('one-stop shopping') rather than on their preferences for quality. The distinctions between these two types of search models and ours are important. In the first case, the single-product firm assumption leaves no scope for price discrimination within the firm. Hence, pricing is solely driven by competitive forces. In the second case, the multi-product search assumption implies that discrimination is based on heterogeneity in consumers' shopping costs, which become the main determinant of firms' product line decisions (Klemperer, 1992).

Within the 'multi-product search' literature, two papers deserve special attention. In line with our results, Zhou (2014) finds that multi-product firms tend to charge lower prices than single-product firms. This is not driven by the interaction between competition and price discrimination, as in our paper, but rather by a 'joint search' effect, i.e., multi-product firms charge less because they gain more by discouraging consumers from searching competitors. In Rhodes and Zhou (2016), increases in search costs imply that consumers value one-stop shopping more, thus making it more likely that the equilibrium involves multi-product firms. When search costs are small, Rhodes and Zhou (2016) predict asymmetric market structures, with single-product and multi-product firms co-existing.[7] The driving force underlying our prediction is different: since in our model consumers buy a single good, the multi-product firm equilibrium is not driven by one-stop shopping considerations but rather by firms' incentives to price discriminate consumers with heterogenous quality preferences. Despite these differences, our paper has one common element with both Rhodes (2014) and Rhodes and Zhou (2016), which is the fact that search costs can give rise to lower prices through their effect on endogenous product choices.

As far as we are aware of, Garret *et al.* (2016) is the only paper that, like ours, introduces frictions in a model of price competition in which firms can carry more than one product but in which consumers buy only one. However, there are two important distinctions between the their analysis and ours. First, in Garret *et al.* (2016), firms decide qualities and prices simultaneously, rather than sequentially. The simultaneous timing is appropriate in settings where firms can change product design rather quickly, or alternatively, when firms commit to prices for long periods of time; for example, under long term contracts. In contrast, the sequential timing is better suited to capture the notion that in many markets prices vary at will, even daily, but a change in product line decisions happens less often as it usually involves changes in the production and/or retail facilities (Brander and Eaton, 1984). This distinction is relevant as in simultaneous settings firms cannot affect competition by pre-committing to quality, which is a fundamental driving force of our results. Furthermore, as it is common in simultaneous choice settings,

[7]One-stop shopping considerations are also the driving force behind the evidence of price dispersion across stores documented by Kaplan *et al.* (2016).

Garret *et al.* (2016) focus on symmetric equilibria, giving rise to overlapping product lines. However, they do not explore whether a non-overlapping product equilibrium *à la* Champsaur and Rochet would arise if one allowed for asymmetric configurations. Still, given that in our set-up firms are endogenously symmetric, our analysis shares common predictions with Garret *et al.*'s concerning the comparative statics of prices and relative prices. We further explore the comparative statics of prices for given (asymmetric) product choices, as these may turn to be meaningful in cases in which exogenously given obstacles stop firms from reaching the symmetric subgame perfect equilibrium.

Last, our paper also relates to the literature that analyzes quality choices *followed* by imperfect competition, either quantity competition (Gal-Or, 1983; Johnson and Myatt 2003 and 2015) or price competition with horizontal differentiation (Stole, 1995). As already noted by CR (p. 535), one of the main consequences of less competitive pricing is to induce wider and, very likely, overlapping product lines. While one may view search costs as an alternative way to allow for less competitive pricing, there is a fundamental difference in the way search costs affect firms' product choices as compared to other forms of imperfect competition. To see this, consider a model of horizontal product differentiation with no search costs (i.e., all consumers are shoppers). A firm selling a high quality good that decides to deviate from CR's non-overlapping equilibrium by also carrying a low quality one must weigh two countervailing effects. One the one hand, due to horizontal differentiation, the firm will be able to capture rents through the sales of the low quality good. On the other hand, it will also have to discount the price of the high quality one to avoid that consumers who value high quality end up buying the low quality good instead. If there is little horizontal differentiation, competition will drive the price of the low quality good almost down to marginal costs, thus leading to low profits on both goods. Hence, the profit loss on the high quality good dominates, and the firm prefers not to add the low quality product to its product line, just as CR had predicted. As a consequence, CR's prediction is robust to allowing for some degree of horizontal differentiation, as long as it is not too strong. In contrast, the impact of search costs is fundamentally different: even infinitesimally small search costs destroy CR's prediction. The reason is that search costs restore firms' monopoly power over those consumers who do not search, even when competition for those who search is very fierce. Through the non-shoppers, firms can make rents on the low quality good that exceed the required discount on the high quality one, no matter whether the mass of non-shoppers is large or small.

The rest of the paper is organized as follows. Section 2 describes an illustrative example that conveys the main intuition of the model while providing supportive empirical evidence. Section 3 describes the model. Section 4 shows that in the absence of search costs firms escape the Bertrand paradox by carrying non-overlapping product lines. In contrast, Section 5 shows that an arbitrarily small amount of search frictions is enough to overturn this prediction, leading to overlapping product choices and prices close to marginal costs. Section 6 characterizes equilibrium pricing for all potential product choice configurations, as well as the Subgame Perfect Equilibrium product choices for all levels of the search costs. Section 7 concludes, and most proofs and robustness checks are postponed to the Appendix.

# 2 An Illustrative Example

Price discrimination is pervasive in a wide range of markets in which search costs matter. In gasoline markets, consumers have the choice of paying for full-service or self-service gasoline at the same station, or of searching for competing stations (Shepard, 1991). In the airline industry, travellers can choose whether to fly in business or in economy class within the same flight, or to search for alternative airlines offering the same route. Other examples in which price discrimination, competition and search coexist include coffee shops (McManus, 2000), cereals (Nevo and Wolfram, 2002), theaters (Leslie, 2004), Yellow Pages advertising (Busse and Rysman, 2005), mobile telephony (Miravete and Röller, 2004), cable TV (Crawford and Shum, 2007), or several markets in which competing firms offer advanced-purchase discounts (Möller and Watanabe, 2016; Nocke *et al.*, 2011), among others.

To build intuition on the main forces underlying our model, we focus on a market that fits well our modeling framework: the market for online books. While previous empirical papers have analyzed search in these markets (De Los Santos *et al.*, 2012; Hong and Shum, 2006), their focus has been on estimating buyers' search behaviour for given product choices and prices. Rather, our focus here is simply to motivate and illustrate the predictions of the model by exploring firms' product choices and prices given consumers' search behaviour. For this purpose, we have collected daily book prices at Amazon and Barnes & Noble, the two leading online booksellers from December 2016 to March 2017, for each of the 2012-2016 #1 New York Times fiction and non-fiction best-sellers.

## 2.1 Theoretical intuition

Let us think of two online stores, Amazon (A) and Barnes and Noble (B), competing to sell books to consumers with heterogenous preferences for quality. Before choosing prices, booksellers must decide whether to offer both the hardcopy and the paperback versions of each book, or just one of the two, if any. Since the hardcover version is generally thought of being of better quality than the paperback, we will sometimes refer to the two as the high and low quality goods, respectively. In turn, we will refer to those consumers who are willing to pay the extra cost of producing the hardcover as the high types, and the remaining consumers as the low types.

In the absence of search costs, CR's prediction states that one store will offer the hardcover, and the other one the paperback. If one of the bookstores deviated from this equilibrium, competition would drive the price of the overlapping version down to marginal costs, making such a deviation unprofitable. Furthermore, the store would have to discount the non-overlapping version as all consumers would be tempted to buy the version that is priced at marginal costs.

Alternatively, suppose that there is an arbitrary small amount of consumers (so-called 'non-shoppers') who visit one of two sites at random without searching any further (since these consumers can be thought of having very large search costs, the mass of non-shoppers is a proxy for the level of search costs).[8] Once in the site, the non-shoppers buy the version of the book that offers them higher utility (if positive), given their quality preferences and the prices of the two versions. Since each store is a monopolist over its non-shoppers, the duopolists face similar incentives as the ones that induce a monopolist to offer both goods. Indeed, the presence of non-shoppers rules out CR's non-overlapping equilibrium. Instead, a new equilibrium arises in which there is complete overlap and firms compete in mixed strategies.

Why does the non-overlapping equilibrium unravel? Under such equilibrium, the bookstore selling the paperback can profitably deviate to also sell the hardcover. In particular, the deviating bookstore could leave the price of the paperback unchanged (so that all the non-shopper low types would still buy the paperback at the same price) and sell the hardcover at the maximum price that the non-shoppers high types would pay for the hardcover, given the price of the paperback. Since the latter are willing to pay more for the hardcover than for the paperback, the bookstore's profits increase. Of course, the deviating bookstore could sometimes do better than this, e.g. if there are many high types searching for the hardcover, the bookstore might be better off reducing the hardcover's price to attract them. In any event, if this were the case, this would further strengthen the bookstore' incentives to deviate from CR's equilibrium. This result holds true whenever the mass of non-shoppers is strictly positive, no matter how small.

Anticipating that the rival bookstore will carry both versions, the bookstore carrying the hardcover cannot refrain from also offering the paperback. This would allow the store to sell the paperback to the non-shoppers low types, for whom the hardcover was

[8] In the context of online books, De los Santos *et al.* (2012) show that, within a 7 days window, 76% of consumers only visit one store. They also report the presence of loyal consumers: 24% of consumers engage in multiple transactions but only buy from one store, even if it exhibits a higher price, thus suggesting the presence of specific store preferences independent of price.

too expensive. As a consequence, in the presence of non-shoppers, both booksellers end up offering the two versions of the book, despite being worse-off than under CR's non-overlapping equilibrium. If the bookstores could coordinate on CR's equilibrium, some non-shopper low types would not buy any book at all, whereas other high type consumers would end up buying the paperback instead (i.e., discrimination is incomplete). However, despite losing sales and the possibility to fully separate consumers, the bookstores' profits under the CR's equilibrium would be higher because of the softening of competition effect. In this sense, search costs take the stores into a prisoner's dilemma as they remove the stores' ability to coordinate on the most profitable equilibrium.

## 2.2 Evidence in the data

The patterns observed in the data are in stark contrast with the predictions of the CR model, but can be rationalized when accounting for search costs. First, and not surprisingly in the online bookstore context, we find that the two stores sell both the hardcover and the paperback version whenever these versions exist, i.e., they offer overlapping product lines.[9] Furthermore, even though the two stores offer identical product lines of homogeneous goods, pricing does not reflect what would be predicted by simple economic theory: prices equal to marginal cost if consumers perceive no horizontal differentiation between the stores, or they add a constant markup over marginal costs. In either case, prices would remain rather stable - at least during short periods of time when other relevant variables remain unchanged (e.g., market size, consumer tastes, and costs). Instead, we observe substantial price dispersion across stores and over time.[10]

Figure 1 summarizes price dispersion patterns in the online bookstore market. In Panel 1a, one can see that prices fluctuate substantially for each title, even after partialling out book and day-of-sample means.[11,12] Consistent with our model, the presence of non-shoppers is a force that can generate price dispersion, as tension between attracting the shoppers versus exploiting the non-shoppers implies that firms price according to

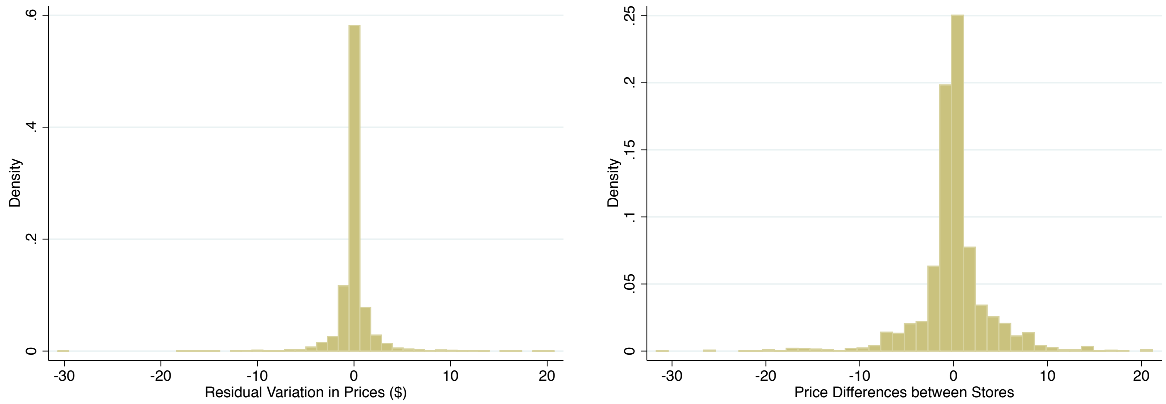
[9] Out of the 200 books that we consider, some of them are only available in paperback or hardcover. For those available in both versions, all of them are served by Amazon and Barnes and Noble at the same time, except in one instance in which Amazon does no longer offer the hardcover version (which is only sold by other sellers). Given the almost complete overlap, we focus our analysis in cases in which both stores sell both versions.

[10] Borenstein and Rose (1994) explore sources of price dispersion in the airline industry, but they do not consider second-degree price discrimination, as in this paper, as they focus on pricing for economy class seats.

[11] Average prices for hardcover and paperback are roughly \$20 and \$10, respectively. Therefore, variation of a few dollars can imply substantial variation in prices.

[12] The figure shows prices for both Amazon and B&N, hardcover and paper back. Separate figures for each book format and store exhibit similar distributions and are relegated to the appendix.

Figure 1: Patterns in prices for online bookstores




(a) Residual price dispersion

(b) Residual price differences between stores

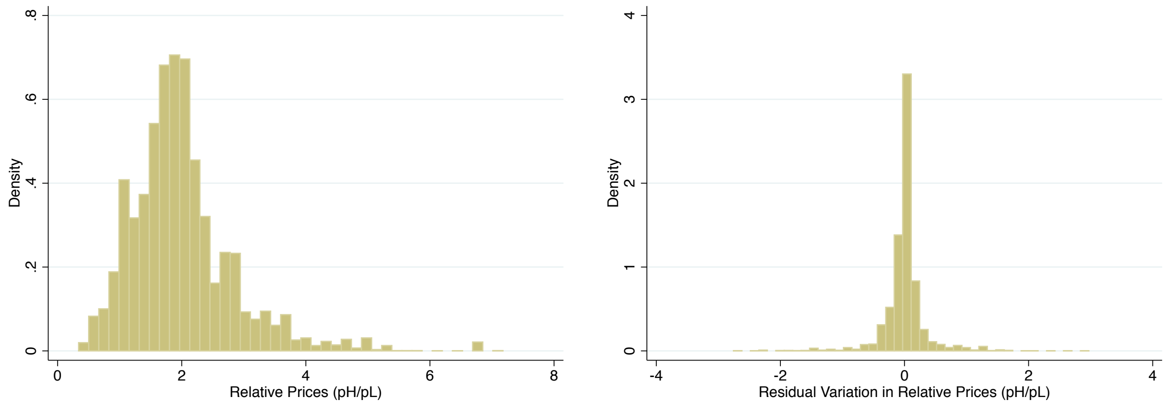
*Notes:* This figure shows patterns in online book prices. Panel (a) shows price dispersion after partialling out book-store-format means. Panel (b) takes the difference in prices at a given date between stores for the same book-format.

mixed strategies. Panel 1b shows that this dispersion is not just due to common fluctuations, e.g., fluctuations for particular books over time that are common across stores. The figure shows price differences between stores, which also fluctuate substantially even after taking out constant mean differences by book.

The interaction between price discrimination and search costs can also explain another stylized fact that we observe in the data, namely, the dispersion in the relative prices of the two goods. Whereas existing search models cannot capture fluctuations in relative prices because they do not allow for price discrimination, our model predicts that search costs not only lead to price differences across stores, but also to price differences across different versions of the book sold within a store and over time. Figure 2 summarize patterns in relative prices. Panel 2a shows the distribution of relative prices between the hardcover and the paperback of a given title. One can see that there is substantial variation in relative prices, partly due to differences across different titles, and partly due to variation in such relative prices over time. Panel 2b shows the variation in relative prices after partialling out book-store means. One can see that relative prices between the hardcover and the paperback versions also move over time, and that such variation is not just due to variation across books, but also to variation within book titles.

In sum, search costs in the market for online books seem to play an important role in shaping product choices and price patterns, in line with the predictions of our model. First, the norm is that all booksellers offer the hardcopy and the paperback versions of the same book, whenever available, even if when triggers intense competition for

Figure 2: Patterns in relative prices for online bookstores




(a) Relative prices

(b) Price dispersion in relative prices

*Notes:* This figure shows patterns in relative prices. Panel (a) shows the distribution of relative prices, with a peak around 2, i.e., the hardcover version of a book is about twice as expensive as the paperback version on average. Panel (b) shows residual variation in relative prices after partialling out book-store means.

almost identical goods (up to the horizontal differences that consumers may perceive across stores). Additionally, book prices fluctuate substantially, both at the book-store level but, more importantly, also across stores, thus making search meaningful. Relative prices between book versions also exhibit substantial dispersion, indicating that this is another dimension that firms use when sorting out consumers and attracting them from rivals.

# 3 The Model

## 3.1 Model Description

Consider a market served by two competing retailers. Retailers carry either one or the two goods that are available for sale: one with high quality $q^H$ and high costs $c^H$, and another one with lower quality $q^L$ and lower costs $c^L$;[13] we use $\Delta q \equiv q^H - q^L > 0$ and $\Delta c \equiv c^H - c^L > 0$ to denote the quality and cost differences across goods.[14]

[13]Without substantial effort, our model could be interpreted as one of quantity discounts, with firms offering the different quantities of the same product to consumers with either low or high demands. Results would go through as long as costs are not linear in the quality; for instance, if bigger bundles require costly product design features, such as packaging.

[14]We can think of these costs as the wholesale prices at which retailers buy the products from either competitive manufacturers, or from a monopoly manufacturer. Endogenizing the qualities of the

There is a unit mass of consumers who buy at most one good. Consumers differ in their preferences over quality. A fraction $\lambda > 0$ of them have a low valuation for quality $\theta^L$, while the remaining $1 - \lambda$ fraction of consumers have a high quality valuation $\theta^H$, with $\Delta\theta \equiv \theta^H - \theta^L > 0$. As in Mussa and Rosen (1978), a consumer of type $i = L, H$ who purchases good $j = L, H$ at price $p^j$ obtains net utility $u^i = \theta^i q^j - p^j$. We assume that the gross utility of a low type (high type) from consuming the low (high) quality product always exceeds the costs of producing it, i.e., $c^i < \theta^i q^i$ for $i = L, H$.

The timing of the game is as follows. First, retailers simultaneously decide which product(s) they offer for sale (or "product line"). Carrying a product entails an arbitrarily small fixed cost for the retailer.[15] Once chosen, firms observe the product line of the rival but consumers don't. Second, retailers simultaneously choose the prices for the product(s) they carry and consumers visit the stores in order to learn firms' product choices and their respective prices.

Following Varian (1980), we assume that there is a fraction $\mu \leq 1$ of consumers who always visit the two stores (the *shoppers*), and hence know where to find the cheapest product of each quality type.[16] Since the remaining $1 - \mu$ fraction of consumers only visit one store (the *non-shoppers*),[17] they can compare the prices of the goods sold *within* the store they have visited, but not *across* stores. We assume that the non-shoppers visit one of the two stores with equal probability.[18] Once consumers have visited the store(s), they buy the product that gives them higher utility, provided it is non-negative. In case of indifference, low (high) type consumers buy the low (high) quality product.

products or the costs faced by the retailers is out of the scope of this paper.

[15]This cost does not play a major role in the analysis. It is only used as an equilibrium selection device in the case of no search costs (Section 3).

[16]There is a fundamental distinction between introducing search costs which are equal across consumers, versus introducing a fraction of uninformed consumers. It is well known that in a (single-product, homogeneous good) Bertrand model, the former gives rise to the Diamond Paradox, such that all firms charge the monopoly price and consumers do not engage in search. A similar outcome would arise in our set-up. Varian's approach, which we adopt here, avoids the Diamond Paradox. Furthermore, as already noted, the empirical evidence reports that a large fraction of consumers are uninformed (De los Santos *et al.*, 2012).

[17]An implicit assumption is that the fractions $\mu$ and $\lambda$ are uncorrelated. As we show in Appendix B, our main results do not change if there was some correlation between $\mu$ and $\lambda$.

[18]Since in some settings it may be reasonable to assume that non-shoppers observe product lines but not their prices, in Appendix B we consider the case in which non-shoppers visit the store that carries their preferred product (and split randomly between the two stores in case they both carry it). The main results of the paper, in particular Propositions 1 and 2, remain unchanged.

## 3.2 Preliminaries

We start by characterizing the benchmark solutions of monopoly and marginal-cost pricing. This will serve to introduce some concepts and assumptions to be used in the rest of the analysis.

**The monopoly solution** A monopolist carrying both products that is able to perfectly discriminate consumer types would extract all their surplus by charging the (unconstrained) monopoly prices $p^i = \theta^i q^i$, for a per unit profit of $\pi^i = \theta^i q^i - c^i$, $i \in \{L, H\}$. This holds true regardless of the number of consumers of each type, and regardless of the relative profitability of serving one type or another. This is no longer true however, as we move to the more relevant case of a multi-product monopolist that cannot perfectly discriminate across consumers.

Our first assumption is that a monopolist carrying both goods finds it optimal to sort consumers out. At the optimal solution, the following incentive compatibility constraints must hold

$$\theta^i q^i - p^i \geq \theta^i q^j - p^j, \tag{$IC^i$}$$

for $i, j \in \{L, H\}$ and $i \neq j$, which can also be re-written as

$$p^i \leq \theta^i q^i - \left(\theta^i q^j - p^j\right).$$

The second term on the right-hand side of the inequality represents consumers' *information rents*, i.e., the minimum surplus a type $i \in \{L, H\}$ consumer needs to obtain to be willing to buy good $i$ instead of good $j \neq i \in \{L, H\}$. For the multi-product monopolist, the incentive compatible (i.e., constrained monopoly) prices are thus

$$\begin{aligned} p^L &= \theta^L q^L \text{ and} \\ p^H &= \theta^H q^H - \Delta\theta q^L \\ &= \theta^L q^L + \Delta\theta q^H. \end{aligned}$$

The alternative for the monopolist is to only sell good $H$ to the high types at the (unconstrained) monopoly price $\theta^H q^H$,[19] thus avoiding to leave information rents to the high types. To guarantee that this alternative is indeed less profitable than selling the two goods requires that the profit from selling good $L$ to the low types be enough to compensate for the information rents that must be left with the high types:[20]

[19]Note that this alternative implicitly assumes that serving the high types with product $H$ is more profitable than serving all consumers with product $H$ at price $\theta^L q^H$. Relaxing this would only add more cases to the analysis without altering any fundamental result.

[20]In turn, *(A1)* also guarantees that the optimal price for a monopolist that only carries good $L$ is $\theta^L q^L$. In particular, serving all consumers at this price is more profitable than just serving the high types at $\theta^H q^L$.

$(A1)$ $\lambda\left(\theta^L q^L - c^L\right) \geq (1-\lambda) q^L \Delta\theta.$

Note that $(A1)$ does not rule out the possibility that a firm that sells only one good, say $i \in \{L, H\}$, at the (unconstrained) monopoly price finds it unprofitable to carry the two goods if the price of good $j \neq i \in \{L, H\}$ is below the monopoly price. To see this, consider a firm selling good $H$ that is deciding whether to also carry good $L$. Adding product $L$ would yield extra profits $\lambda\left(p^L - c^L\right)$, but would also create information rents on good $H$, $(1-\lambda)\left(\theta^H q^L - p^L\right)$. Assumption $(A1)$ does not prevent the latter to exceed the former if $p^L$ is sufficiently below $\theta^L q^L$.

Similarly, consider a firm selling good $L$ at the monopoly price that is deciding whether to also carry good $H$. Adding product $H$ would allow the firm to separate the two types, yielding extra profits $(1-\lambda)\left(p^H - \theta^L q^L - \Delta c\right)$, but would also create information rents on good $L$, $\lambda\left(\theta^L q^H - p^H\right)$, which are positive whenever $p^H < \theta^L q^H$. Again, assumption $(A1)$ does not prevent the latter to exceed the former if $p^H$ is sufficiently below $\theta^L q^H$. In sum, being able to capture positive rents on good $j$ is not sufficient for a firm to be willing to carry it unless these exceed the information rents that the firm has to give up on good $i$.

In both cases, adding good $j \in \{L, H\}$ is relatively less appealing the lower its own price: not only a lower $p^j$ implies lower profits on good $j$, but it also increases the information rents on good $i \neq j \in \{L, H\}$. To the extent that competition drives prices down, this creates a potential trade-off between competition and firms' incentives to discriminate through quality choices. This trade-off will play an important role in the analysis that follows.

**The competitive solution** Our second assumption rules out 'bunching' at the competitive solution. This requires marginal-cost pricing to be incentive compatible, which is equivalent to assuming that the high types are willing to pay for the extra cost of high quality, whereas the low types are not:

$(A2)$ $\Delta c \in \left(\theta^L \Delta q, \theta^H \Delta q\right).$

Note that implicit in $(A2)$ is the standard property (see, for example, CR) that the cost of providing quality is strictly convex in quality, i.e., $c^H/q^H > c^L/q^L$; otherwise, either type would buy the high quality product or nothing at all.[21]

The price difference at the competitive solution equals $\Delta c$ while it equals $\theta^H \Delta q$ at the monopoly solution, indicating that the price difference between the high and the low quality products is wider under monopoly than under perfect competition.

[21]Note also that $c^H/q^H > c^L/q^L$ ensures that there is a non-empty region of values of $\lambda$ for which $(A1)$ is valid.

In turn, since the (constrained) monopoly profits of selling the high quality product can be written as $\pi^H - q^L\Delta\theta = \pi^L + \varphi > \pi^L$, $(A2)$ implies that the monopolist always finds it more profitable to sell the high quality product than the low quality one even though the monopolist is not able to extract the high types' full surplus. The parameter $\varphi$ thus measures the monopolist's extra profitability from selling the high quality good, and will play an important role in the analysis that follows.

We are now ready to solve the game. We start by analyzing the case in which all consumers are shoppers, $\mu = 1$ (i.e., no search costs), then move on to introducing an arbitrarily small fraction of non-shoppers, $\mu \to 1$, and finish by providing a full equilibrium characterization for all parameter values, $\mu \in [0,1]$. Appendix B contains robustness checks of our main results.

# 4 Escaping the Bertrand Paradox

In this section we characterize the Subgame Perfect Equilibrium (SPE) product choices, and subsequent pricing decisions given those choices, in the absence of search costs. Specifically, we assume that all consumers are shoppers, i.e., $\mu = 1$. We write $(\phi_1, \phi_2)$ to denote firms' product choices, with $\phi_i \in \{\varnothing, L, H, LH\}$, $i = 1, 2$.

First, it is simple to show that there cannot exist a SPE involving overlapping product choices. The reason is that, with no search costs, Bertrand competition drives prices down to marginal cost for each overlapping product. Hence, firms lose no profits when dropping such products, but save on the fixed cost $\epsilon \to 0$ of carrying a product.

The remaining candidate equilibria are of two types: either $(L, H)$, such that firms specialize in selling distinct products ("specialization"), and/or $(LH, \varnothing)$, $(L, \varnothing)$, and $(H, \varnothing)$, such that one firm monopolizes the market while the other remains inactive ("monopolization").

Analyzing whether $(L, H)$ constitutes a SPE requires characterizing profits at the subgames to which firms could deviate, i.e., $(L, LH)$ and $(LH, H)$. At subgame $(L, LH)$, good $L$ is priced at marginal cost $c^L$ while good $H$ is sold at the highest price that satisfies the high types' incentive compatibility constraint, i.e., $c^L + \theta^H\Delta q$.[22] Firm $L$ makes zero profits while firm $LH$ makes minimax profits $(1-\lambda)\left(\theta^H\Delta q - \Delta c\right)$, which by $(A2)$ are strictly positive. Similarly, at subgame $(LH, H)$, good $H$ is priced at marginal cost $c^H$ while good $L$ is sold at the highest price that satisfies the low types' participation and incentive compatibility constraints, i.e., $\max\left\{\theta^L q^L, c^H - \theta^L\Delta q\right\}$.[23] Firm $H$ makes zero

[22]Note that at this price, the high types' participation constraint is always satisfied, as $\theta^H q^L - c^L > \theta^L q^L - c^L > 0$.

[23]If $c^H \leq \theta^L q^H$, the binding constraint is the incentive compatibility constraint; or the participation

profits while firm $LH$ makes minimax profits $\lambda \max\left\{\pi^L, \Delta c - \theta^L \Delta q\right\}$, which again are strictly positive by $(A2)$.

In contrast, at $(L, H)$, a pure strategy equilibrium does not exist. This stems from an important result: at any pure strategy equilibrium, firms' prices must satisfy incentive compatibility. Otherwise, the firm selling good $H$ would sell nothing and would thus be better off reducing its price until incentive compatibility is achieved. However, if the high types' incentive compatibility constraint is binding, the firm carrying good $L$ could in turn attract all customers by slightly reducing its own price. Since these opposing forces destroy any candidate pure strategy, the equilibrium has to be in mixed strategies. In turn, since firms cannot obtain profits below their minmax, and these are strictly positive, it follows that all prices in the support of the mixed strategies must exceed marginal costs.

This has important implications for equilibrium product choices. First, since at $(L, H)$ product $L$ is priced *above* marginal costs, the firm carrying good $H$ has to give up lower information rents at $(L, H)$ than at $(L, LH)$. In turn, since under $(L, LH)$ product $L$ yields no profits, the firm that carries product $H$ is strictly better off at $(L, H)$ than at $(L, LH)$.

Similarly, at $(L, H)$, the firm that carries good $L$ does not have to discount the price of good $L$ (weakly) as much under $(L, H)$ as under $(LH, H)$, given that the information rents of the low types, if positive, are lower when good $H$ is priced above marginal costs. Again, since the profits on product $H$ at $(LH, H)$ are zero, and the profits on good $L$ are (weakly) higher at $(L, H)$ than at $(LH, H)$, it follows that the firm that carries product $L$ is (weakly) better off at $(L, H)$ than at $(LH, H)$. In case the low types' participation constraint is binding, its profits at these two subgames are equal, but the per-product fixed cost $\epsilon$ breaks the indifference in favour of $(L, H)$. In sum, since neither firm can gain by deviating from $(L, H)$, the "specialization equilibrium" constitutes a SPE of the game with no search costs.

Last, we also need to consider the subgames in which one firm chooses not to carry any product. Among these, the only relevant one is $(LH, \varnothing)$, as $(A1)$ implies that a monopolist is better off carrying the two products. In line with the analysis of entry games followed by Bertrand competition, it is straightforward to see that $(LH, \varnothing)$ constitutes a SPE of the game with no search costs: entry would drive prices down to marginal costs, not allowing the firm to recover $\epsilon$, even for $\epsilon$ arbitrarily small.

The following Proposition summarizes our main result of this section:

**Proposition 1** *Assume $\mu = 1$. All Subgame Perfect Equilibria (SPE) involve non-overlapping product choices. In particular, there exist two (pure-strategy) SPE: the "spe-*

constraint, otherwise.

*cialization equilibrium" $(L,H)$, and the "monopolization equilibrium" $(LH,\varnothing)$. In equilibrium, all prices are strictly above marginal costs.*

**Proof.** See the Appendix. ■

The above result is fully in line with CR who were the first to show that quality choices followed by price competition lead firms to choose non-overlapping product lines. Firms give up profitable opportunities to discriminate consumers because doing so would come at the cost of intensifying competition. In the next section we assess whether this prediction is robust to introducing search frictions.

# 5 Back to the Bertrand Paradox

Before solving the game for $\mu \in [0,1]$, in this section we show that the "specialization" and "monopolization" equilibria of Proposition 1 are not robust to introducing a small amount of search costs.[24] In a nutshell: search frictions restore the monopolist's incentives to discriminate; these induce both firms to carry the whole product line, and thus to compete head-to-head when setting prices. A small amount of search costs is thus enough to restore the Bertrand paradox.

To explore this in more detail, let us revisit the equilibrium predictions of Proposition 1 when the mass of shoppers $\mu$ falls slightly below 1. Consider first the "specialization" equilibrium, $(L,H)$. Now, the firm carrying product $L$ is strictly better off adding product $H$, given that under $(LH,H)$ it can now price discriminate the non-shoppers. Indeed, the firm would be able to increase its profits by $(1-\mu)(1-\lambda)\left(\pi^H - q^L\Delta\theta - \pi^L\right)/2 > 0$ from selling the high rather than the low quality product to the non-shopper high types, without affecting its profits from selling the low quality product to the low types.[25] Similarly, the presence of non-shoppers breaks the "monopolization" equilibria as the inactive firm can now make strictly positive profits by selling either one or the two products to the non-shoppers.

If non-overlapping product lines cannot be sustained in equilibrium, what do equilibrium product lines look like then? To study this, consider first the subgame with symmetric multi-product firms, $(LH,LH)$. Search costs, no matter how small, imply that marginal cost pricing is no longer an equilibrium as firms could make positive profits out of the non-shoppers. More generally, search costs rule out any equilibrium in pure strategies, as firms face a trade off between serving the non-shoppers at monopoly prices versus charging lower prices to also attract the shoppers. Since firms must be

[24] A formal treatment of this case is deferred to the next section.

[25] As it will become clear in the next section, the equilibrium in this case is in mixed strategies, with the multi-product firm obtaining the same profits as if it sold the two products independently.

indifferent between charging any price in the support, expected equilibrium profits can be computed by characterizing profits at the upper bound, where firms serve their share of non-shoppers at (constrained) monopoly prices,

$$\Pi(LH, LH) = \frac{1-\mu}{2}\left[\lambda\pi^L + (1-\lambda)(\pi^H - \Delta\theta q^L)\right]. \tag{1}$$

Importantly, each firm's profits are a fraction $(1-\mu)/2$ of the multi-product monopoly profits, precisely because firms only make profits out of the non-shoppers. This is true in expectation only, as for prices below the upper bound firms make profits out of the shoppers too, as these pay prices below the monopoly level but above marginal cost. As $\mu$ approaches 1 and all customers become shoppers, the equilibrium price distributions concentrate around marginal costs, and firms' profits are driven down to (almost) zero. Thus, the Bertrand outcome is restored.

Could firms escape from the Bertrand paradox by having one of them drop one product, either $L$ or $H$?[26] Let us first analyze the incentives of moving from $(LH, LH)$ to $(H, LH)$. Since a pure strategy equilibrium does not exist, and firms have to be indifferent across all prices in the support, expected profits equal those of serving the non-shoppers at the upper bound. Since firm $H$ is not constrained by incentive compatibility, its optimal price is the (unconstrained) monopoly price, and its expected profits become

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H. \tag{2}$$

Since firm $H$'s profits are a fraction $(1-\mu)/2$ of monopoly profits, comparing (1) and (2) is equivalent to assessing the monopolist's incentives to carry the high quality good only versus the two goods. Assumption $(A1)$ guarantees that (1) exceeds (2) as the losses from not selling the low quality product, $(1-\mu)\lambda\pi^L/2$, exceed the information rents left to the high types, $(1-\mu)(1-\lambda)\Delta\theta q^L/2$.

The alternative is for one of the two firms to drop product $H$, thus moving from $(LH, LH)$ to $(L, LH)$. Now, the expected profits of firm $L$ must be equal to the profits of serving all the non-shoppers at the unconstrained monopoly price,[27]

$$\Pi(L, LH) = \frac{1-\mu}{2}\pi^L.$$

Again, this payoff is strictly less than (1) since the firm gives up the extra profit that firm $L$ could make by selling the high quality good to the non-shopper high types.

In sum, the presence of non-shoppers restores firms' monopoly power over the non-shoppers, while competition for the shoppers implies that firms make no expected profits

[26] No firm has incentives to drop both products altogether as they both make positive profits at $(LH, LH)$.

[27] Note that in this case the firm would serve both the low and the high-types, since the latter are also willing to buy the low-quality product at the unconstrained monopoly price.

out of them. Hence, firms' incentives to price discriminate through product choice mimic those of the monopolist. Consequently, we are left with a unique equilibrium prediction: in the presence of arbitrarily small search costs, firms choose overlapping product lines $(LH, LH)$, in stark contrast with CR's prediction.

**Proposition 2** *Assume $\mu \to 1$. The unique SPE involves full product overlap, $(LH, LH)$. In equilibrium, all prices are arbitrarily close to marginal costs.*

**Proof.** See the discussion above. A formal derivation can be found as a particular case of the proof to Proposition 7. ■

Propositions 1 and 2 form a remarkable result: an arbitrarily small amount of search costs is all it takes to restore Bertrand competition. Search costs are generally thought to help firms relax competition, but here they do the exact opposite by altering firms' product choices. This is a clear example of a search externality, as the presence of non-shoppers improves the deals offered to the shoppers (Armstrong, 2015). As will become clear in what follows, the prediction of overlapping product lines is robust regardless of the level of search costs. However, the result that search costs reduce prices is not generally true, as further increases in search costs soften competition (i.e., when the share of shoppers $\mu$ goes well below 1), leading to price increases beyond those characterized in the absence of search costs (Proposition 1).

Last, it is worth pointing out that our main prediction is not driven by the rents created by search frictions. Indeed, as this section has demonstrated, when search costs are arbitrarily small and such rents are close to zero, firms' product lines fully overlap. In contrast, in previous papers analyzing quality choices followed by imperfect competition (Gal-Or, 1983; Johnson and Myatt 2003 and 2015; Stole, 1995), the rents created by imperfect competition have to be high enough to overturn CR's prediction (e.g., few firms have to compete *à la* Cournot,[28] or their products be sufficiently differentiated under price competition). In those papers, just as in CR, there is a tension between competition and price discrimination: competition reduces the rents on the overlapping products at the same time as it as it enlarges consumers' information rents, thus reducing the gains from price discrimination. In the presence of search costs, such a tension is present when firms determine prices, but does not drive product choices because (in expectation) firms only care about the profits made out of the non-shoppers. Accordingly, all the search models in which a fraction of consumers, no matter how small, search only once would deliver similar predictions.

[28]In fact, Gal-Or (1983) has already shown that a symmetric (i.e., overlapping) equilibrium fails to exist in a Cournot environment with many firms.

# 6 Equilibrium Product and Price Choices

We have shown that a small amount of search costs induce firms to carry both products. It is evident that this multi-product configuration carries through for very high search costs (i.e., $\mu \to 0$), when retailers charge the (constrained) monopoly prices for the two goods. In this case, rent extraction motives rather than competition induces firms to carry both products. It is less evident, however, that the multi-product configuration also holds for intermediate levels of search costs, when there is again a real trade-off between rent extraction and competition. To see why, we need to characterize the equilibria for all values of $\mu < 1$. We again proceed by backwards induction by first analyzing equilibrium pricing behavior and then product line choices.

## 6.1 Pricing Behavior

We first provide an important property of pricing behavior by multi-product retailers.

**Lemma 1** *In equilibrium, multi-product firms choose incentive compatible prices for their products, i.e.,* $\Delta p \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$.

**Proof.** Argue by contradiction and suppose that the firm chooses $\Delta p > \theta^H \Delta q$. Hence, all buyers visiting the store buy product $L$, and the firm makes a profit margin equal to $\left(p^L - c^L\right)$. If the firm reduced $p^H$ so that $\Delta p = \theta^H \Delta q$, it would still sell product $L$ to the low types at the same price, but would now sell product $H$ to the high types with a higher profit margin $\left(p^H - c^H\right) = \left(p^L + \theta^H \Delta q - c^H\right) > \left(p^L - c^L\right)$, where the inequality follows from $(A2)$. A similar reasoning applies to rule out $\Delta p < \theta^L \Delta q$. ■

Lemma above shows that it is always optimal for a multi-product retailer to choose prices that satisfy incentive compatibility. The intuition is simple. If the price of the high quality product is too high so that all consumers buy the low quality product, it is profitable for the firm to reduce $p^H$, while leaving $p^L$ unchanged, so as to attract the high types and obtain a larger profit margin.[29] Similarly, if the price of the high quality product is too low so that all consumers buy it, it is profitable for the firm to increase $p^H$, while leaving $p^L$ unchanged, so as to extract more surplus from the high types as these are willing to pay more for higher quality. This result constitutes an important departure from Varian (1980), as it implies that the price of one product cannot be picked independently from the price of another product *within* the same store.[30]

We are now ready to characterize equilibrium pricing at every possible subgame.

[29]While these incentives are analogous to those in the monopoly case, they are stronger under oligopoly, given that the reduction in $p^H$ needed to achieve incentive compatibility might possibly attract consumers that would otherwise have bought from the rival firm.

[30]This is in contrast to Johnson and Myatt (2015) prediction. In a model of quality choice followed by

**Full product overlap** We start by considering subgames with full product overlap: $(LH, LH)$, $(L, L)$, and $(H, H)$. The last two are similar to Varian's. Since single-product firms selling the same product are not constrained by incentive compatibility, they play a mixed strategy equilibrium with an upper bound equal to the (unconstrained) monopoly price. Under $(L, L)$ all consumers are served, but under $(H, H)$ the low types are left out of the market. We thus focus here on the case of full product overlap among multi-product retailers, $(LH, LH)$.

**Proposition 3** *Given product choices $(LH, LH)$, there does not exist a pure strategy equilibrium. There exist a continuum of mixed strategy equilibria satisfying the following properties:*

*(i) At the upper bound of the price support, firms choose the (constrained) monopoly prices, $\overline{p}^H = \theta^H q^H - \theta^L \Delta q$ and $\overline{p}^L = \theta^L q^L$. Thus, at the upper bound, the high types' incentive compatibility constraint is binding, $\Delta\overline{p} \equiv \overline{p}^H - \overline{p}^L = \theta^H \Delta q$.*

*(ii) At the lower bound of the price support, firms choose prices that are strictly above marginal costs, $\underline{p}^i > c^i$ for $i = L, H$, and such that the high types' incentive compatibility constraint is not binding, $\Delta\underline{p} \in [\Delta c, \theta^H \Delta q)$.*

*(iii) In equilibrium, both firms obtain their minmax profits. In particular, each firm makes the same profits as it is was a monopolist over the non-shoppers.*

**Proof.** See the Appendix. ■

The non-existence of pure strategy equilibria is shared with most simultaneous search cost models, starting with Varian (1980). It stems from firms' countervailing incentives, as on the one hand they want to reduce prices to attract the shoppers, but on the other, they want to extract all rents from the non-shoppers.

Despite this similarity, our analysis shows that equilibrium pricing by multi-product firms has a distinctive feature: it is constrained by incentive compatibility (Lemma 1). This comes up clearly when characterizing the upper bound of the price support: firms are not able to extract all the surplus from the non-shopper high types because firms have to give up information rents $\Delta\theta q^L$.

Since firms make strictly positive profits at the upper bound, prices at the lower bound must be strictly above marginal costs. The reduction in prices from the upper to the lower bound is more pronounced for the high quality product: competition for the high types is fiercer because the profitability of selling the high quality product is higher (recall from $(A4)$ that $\varphi > 0$). In turn, this implies that at the lower bound, the incentive compatibility constraint for the high types is not binding, so that the price

Cournot competition, they find conditions under which the equilibrium prices chosen my multi-product oligopolists are close to the single-product prices.

wedge between the two products is lower than at the upper bound. We can conclude that high quality products are relatively cheaper during periods of "sales" *à la* Varian, when both goods are priced at the lower bound. However, since firms need not necessarily price the two goods simultaneously at the lower bound, the relative price differences can be as large as $\theta^H \Delta q$ or as small as $\Delta c$- in any case, lower than at the monopoly solution.

Since firms have to be indifferent between charging any price in the support, including the upper bounds, expected equilibrium profits are unambiguously given by[31]

$$\begin{aligned}\Pi\left(LH, LH\right) &= \frac{1-\mu}{2}\left[\lambda\pi^L + (1-\lambda)(\pi^H - \Delta\theta q^L)\right] \\ &= \frac{1-\mu}{2}\left[\pi^L + (1-\lambda)\varphi\right].\end{aligned} \tag{3}$$

At the lower bound, each firm attracts all the shoppers plus its share of the non-shoppers of each type. Hence, expected profits can also be expressed as a function of the lower bounds,

$$\Pi\left(LH, LH\right) = \frac{1+\mu}{2}\left[\lambda(\underline{p}^L - c^L) + (1-\lambda)(\underline{p}^H - c^H)\right]. \tag{4}$$

Since there are two goods, and only one profit level, as defined in equations (3) and (4), the problem has an extra degree of freedom: there is a continuum of price pairs $\underline{p}^L$ and $\underline{p}^H$ satisfying $\Delta\underline{p} \in [\Delta c, \theta^H \Delta q)$ while simultaneously yielding the same equilibrium profits. This implies that, even though equilibrium profits are unique and well defined, there might be multiplicity of mixed strategy equilibria.

Given that the challenge is to discourage high type consumers from buying the low quality product, a natural equilibrium to consider is one in which firms keep on pricing the low quality product as if they were just selling that product, but adjust their pricing for the high quality one. The following Lemma characterizes such equilibrium:

**Lemma 2** *Given product choices $(LH, LH)$, there exists a mixed-strategy equilibrium in which firms choose $p^L$ in $\left[\underline{p}^L, \overline{p}^L\right]$ according to*

$$F^L(p^L) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^L - c^L)}{(p^L - c^L)}$$

**Proposition 4** *and such that, for given $p^L$, the price $p^H$ is chosen in $\left[\underline{p}^H, \overline{p}^H\right]$ to satisfy*

$$\frac{p^H - c^H}{p^L - c^L} = \frac{\overline{p}^H - c^H}{\overline{p}^L - c^L} \tag{5}$$

*where, for $i = L, H$,*

$$\underline{p}^i = c^i + \frac{1-\mu}{1+\mu}(\overline{p}^i - c^i) > c^i.$$

[31] Note that firms' profits are decreasing in $\mu$.

**Proof.** See the Appendix. ■

The proposed equilibrium has several appealing features. While firms price the low quality product as if they were just selling that product (as in Varian's model), on average they choose lower prices for the high quality product than when they only sell that product. This is a direct implication of the fact that $\overline{p}^H < \theta^H q^H$ because of the information rents left to the high types. Indeed, the distribution of $p^H$,

$$F^H(p^H) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^H - c^L)}{(p^H - c^L)}$$

puts higher weight on lower prices all along the support than in the independent products case.

Under this equilibrium, the choice of $p^L$ results in a unique choice of $p^H$ such that the relative profit margin of the two products remains constant along the whole support; see equation (5). In particular, the relative markups of the two products are the same as under monopoly. That is, competition affects the *price levels* but not the *price structure* within the firm.

The price difference that is embodied in this price structure can be expressed as

$$\Delta p = \alpha\theta^H \Delta q + (1-\alpha)\,\Delta c,$$

Consistently with Lemma 1, the price difference is a weighted average between $\theta^H \Delta q$ (i.e., the monopoly separation) and $\Delta c$ (i.e., the competitive separation), where the weight $\alpha = \left(p^L - c^L\right) / \left(\overline{p}^L - c^L\right)$ represents the distance to the upper bound. Thus, the higher (lower) $p^L$, the closer is the price difference to the monopoly separation (competitive separation). At the upper bound, when the incentive compatibility constraint of the high types is binding, the price difference is maximal, $\Delta\overline{p} = \theta^H \Delta q$. As we move down the support, the incentive compatibility constraint is satisfied with slack and the price difference narrows down. The difference is minimal at the lower bound, when $\alpha = (1-\mu)/(1+\mu)$. Importantly, as $\mu$ approaches one, the prices at the lower bound converge to marginal cost (in line with Proposition 1), and the price gap approaches the competitive separation. On the other extreme, as $\mu$ approaches zero, the prices at the lower bound converge to monopoly prices so that the price gap approaches the monopoly separation. The equilibrium would thus collapse to the monopoly solution.

**Partial product overlap** Let us now characterize equilibrium pricing in the subgames with partial overlap: $(H, LH)$, $(L, LH)$. Interestingly, we will see that even though the single-product firm does not face an incentive compatibility constraint *within* its store, its pricing is nevertheless affected by incentive compatibility considerations through the effect of competition.

**Proposition 5** *Given product choices $(H, LH)$, there exists $\widehat{\mu} \in (0,1)$ such that:*

*(i) For $\mu \leq \widehat{\mu}$, there exists a unique pure strategy equilibrium. At this equilibrium, firm $H$ chooses the (unconstrained) monopoly price $p^H = \theta^H q^H$, and firm $LH$ chooses the (constrained) monopoly prices, $p^H = \theta^H q^H - \Delta\theta q^L$ and $p^L = \theta^L q^L$.*

*(ii) For $\mu > \widehat{\mu}$, there does not exist a pure strategy equilibrium.*

*(iii) In equilibrium, for all $\mu$, firm $H$ obtains its minmax profits whereas firm $LH$'s profits (strictly) exceed its minmax. In particular, firm $H$ makes the same profits as it is was a monopolist over the non-shoppers.*

**Proof.** See the Appendix. ■

There now exists a pure strategy equilibrium as long as the fraction of shoppers $\mu$ is small enough. At this equilibrium, the multi-product firm charges the (constrained) monopoly prices, while the single-product firm charges either the (unconstrained) monopoly price for the high quality product or the highest price that the non-shopper high types are willing to pay the for the high quality good. The single-product firm does not want to fight for the shoppers as it is better off just serving the non-shoppers but extracting all of their surplus (or as much as possible), than fighting for the shoppers but having to leave informational rents to the non-shoppers.

The above is no longer an equilibrium when the fraction of shoppers is higher, as it now pays the single-product firm to fight for them. In this case, the equilibrium must be in mixed strategies.[32],[33] The precise shape of the mixed strategy equilibrium depends on whether it pays firm $H$ to serve the low types or not. The Appendix contains details on the characterization of the mixed strategy equilibria for all $\mu$.

For the case in which it never pays firm $H$ to serve the low types because the costs of high quality exceed their willingness to pay for it, $c^H \geq \theta^L q^H$, the two firms compete for the shopper high types by randomly choosing their prices for the high quality product. The low quality product is still priced at the monopoly level, $\theta^L q^L$, as the multi-product firm competes for the shopper high types by simply lowering the price of the high quality good. It follows that the incentive compatibility constraint of the multi-product firm is not binding, so its profits are the same as if the two products were sold independently.

In contrast, when $c^H < \theta^L q^H$, the low types might be tempted to buy the high quality good when its price is sufficiently low, e.g. at the lower bound of the support when competition is particularly intense, i.e., for high $\mu$. In this case, the price of

[32]Interestingly, there is continuity between the pure and the mixed-strategy equilibrium. The two firms charge the upper bounds of their price supports, $\theta^H q^H - \Delta\theta q^L$ and $\theta^H q^H$, with positive and identical mass. This mass fades away as $\mu$ grows larger—from one, when $\mu \to \widehat{\mu}$ towards zero, when $\mu \to 1$.

[33]Unlike in subgame $(LH, LH)$, the equilibrium is now unique: since one firm only has one product, there are no longer two degrees of freedom as in the symmetric two product case.

the low quality good is strictly below the monopoly price, and such that the low types are indifferent between the low and high quality goods. Thus, at the lower bound of the support, the price difference between the two goods is narrower than their cost differences, $\Delta \underline{p} = \theta^L \Delta q < \Delta c$ . In sum, even though the multi-product firm is a monopolist over the low quality good, competition with the rival's high quality good forces the firm to reduce the prices of both goods.

Regarding the single-product firm, since $\theta^H q^H - \Delta\theta q^L$ is the highest price that the multi-product firm would ever charge for the high quality good, the firm will play either the (unconstrained) monopoly price, $\theta^H q^H$, or something less than the (constrained) monopoly price, $\theta^H q^H - \Delta\theta q^L$. Any price in between is unprofitable, either because it doesn't extract enough from the non-shopper high types or because it doesn't attract the shoppers when the multi-product firm happens to price the good at or below $\theta^H q^H - \Delta\theta q^L$. In either case, profits remain as in the pure strategy equilibrium, either because $\theta^H q^H$ is in the support, or when it is not, because the multi-product firm puts sufficiently high mass at its upper bound so that the single-product firm is indifferent between charging $\theta^H q^H$ or any price in the support lower than $\theta^H q^H - \Delta\theta q^L$.

Comparing the pricing behavior of the two firms, the equilibrium price distribution used by the multi-product firm (weakly) first-order stochastically dominates that of the single-product firm. It follows that, on average, the price charged by the single-product firm for the high quality product exceeds the one charged by the multi-product firm.[34]

The existence of a pure strategy equilibrium for some values of $\mu$ does not extend to the $(L, LH)$ subgame , i.e., when the single-product firm sells the low quality product. Since firms are not constrained by incentive compatibility when selling the low quality product, there is no price wedge between the prices that the single- and the multi-product firms are willing to charge for the low quality product. Thus, nothing stops them from undercutting each other to attract the shopper low types, and a pure strategy equilibrium fails to exist. The following Proposition characterizes the unique mixed-strategy equilibrium at this subgame.

**Proposition 6** *Given product choices $(L, LH)$:*

*(i) A pure strategy equilibrium does not exist.*

*(ii) At the unique mixed-strategy equilibrium, firm LH charges $p^H = p^L + \theta^H \Delta q$, and both firms choose $p^L$ in $\left[\underline{p}^L, \theta^L q^L\right]$, with firm L putting a probability mass at the upper bound.*

*(iii) In equilibrium, for all $\mu$, firm L obtains its minmax profits whereas firm LH's profits (strictly) exceed its minmax. In particular, firm L makes the same profits as it is*

[34]Note that the non-shoppers cannot benefit from these price differences given that they do not observe product lines before deciding which store to visit.

*was a monopolist over the non-shoppers.*

**Proof.** See the Appendix. ■

In equilibrium, the two firms choose random prices for the low quality product over a common support. In turn, given its price choice for the low quality good, the multi-product firm prices the high quality product to just comply with incentive compatibility. Hence, the price difference between the two products remains constant at $\theta^H \Delta q$ over the whole support, and the density of prices for the high quality product is the same as that for the low quality product (just shifted out to the right by $\theta^H \Delta q$). It follows that, whenever the multi-product firm has the low price for the low quality product, all the shoppers (both the low or the high types) buy from it. Otherwise, the single-product firm serves all the shoppers, including the the low and the high types.

As in the previous subgame, the multi-product firm charges lower prices on average as compared to the single-product firm. The reason is that, when it has the low price, its ability to discriminate between the low and the high types allows the firm to make extra profits $\mu(1-\lambda)\varphi$ out of the shopper high types. Hence, the multi-product firm has stronger incentives to undercut its rival's price. As a consequence, the single product firm has to put at probability mass at the upper-bound, which implies that the single-product firm's profits equal its minmax while those of the multi-product firm exceed that level.

**Non-overlap** Let us now move to characterizing equilibrium pricing in the subgames with no product overlap: $(\varnothing, L)$, $(\varnothing, H)$, $(\varnothing, LH)$ and $(L, H)$. The first three correspond to the monopoly solution already characterized in Section 2.2 above. Hence, here we turn our attention to the more interesting subgame with asymmetric single-product firms, $(L, H)$ - recall from Section 3 that this was the SPE of the game with $\mu = 1$.

**Proposition 7** *Given product choices $(L, H)$, there exists $\widetilde{\mu} \in (\widehat{\mu}, 1)$ such that:*

*(i) For $\mu \leq \widehat{\mu}$, there exists a unique pure strategy price equilibrium: firms charge the (unconstrained) monopoly prices $p^H = \theta^H q^H$ and $p^L = \theta^L q^L$.*

*(ii) For $\mu > \widehat{\mu}$ there does not exist a pure strategy equilibrium. At the unique mixed-strategy equilibrium, firm L chooses prices $p^L$ in $\left[\underline{p}^L, \theta^L q^L\right]$ with a mass on $\theta^L q^L$. If $\mu \in (\widehat{\mu}, \widetilde{\mu})$ firm H chooses prices $p^H$ in $\left\{\left[\underline{p}^H, \theta^H q^H - \Delta\theta q^L\right], \theta^H q^H\right\}$ with a mass on $\theta^H q^H$ that falls to zero as $\mu \to \widetilde{\mu}$; if $\mu \geq \widetilde{\mu}$, $\theta^H q^H$ is not part of firm H's support.*

**Proof.** See the Appendix. ■

Equilibrium pricing at subgames $(L, H)$ and $(H, LH)$ share some similarities. In particular, just as in Proposition 5, if the mass of shoppers $\mu$ is small enough, there exists a pure strategy equilibrium as the firm selling the high quality product is better off serving the non-shopper high types than competing for the shopper high types.[35] Furthermore, there is continuity between the pure and the mixed strategy equilibrium in that the probability mass that the high quality firm puts on the (unconstrained) monopoly price fades away as $\mu$ grows larger.

The main difference between the two subgames is that, under $(L, H)$, the high quality firm chooses not to include the unconstrained monopoly price in the support when $\mu$ is very large. The reason is that the profits from serving a small fraction of non-shoppers are always lower than the profits from fighting for the shoppers. To illustrate this, note that the former converge to zero as $\mu \rightarrow 1$. However, the high quality firm's profits cannot be lower than its minmax, which is strictly positive as the firm can always make a profit margin of at least $\varphi$ when selling the high quality product to the high types.

We are now ready to analyze product line decisions.

## 6.2 Product Choices

In this section we analyze product choice decisions given the continuation equilibria characterized above. In the previous section we showed that in the absence of search costs ($\mu = 1$), the SPE has single-product retailers with non-overlapping products. We also showed that this equilibrium prediction breaks down as soon as we add an arbitrarily small amount of non-shoppers, in which case the unique equilibrium has fully overlapping product lines. The next Proposition shows that this remains true for any value of $\mu < 1$.

**Proposition 8** *Assume $\mu < 1$. The unique SPE involves full product overlap, $(LH, LH)$. In equilibrium, firms choose prices according to Proposition 3.*

**Proof.** See the Appendix. ■

We close this section by pointing out that, given the SPE product choices, (expected) prices are monotonically decreasing in $\mu \in [0, 1)$, i.e., a reduction in search costs leads to lower prices. Hence, as concluded from Propositions 1 and 2, the only non-monotonicity arises in the limit as all consumers become shoppers, i.e., when search costs vanish out at $\mu = 1$ and firms manage to mitigate competition by choosing non-overlapping product lines.

[35] Note that the threshold for the existence of a pure-strategy equilibrium is the same under both subgames.

# 7 Conclusions

In this paper we have analyzed a model of product choice followed by price competition in markets with search costs. We have found that an arbitrarily small amount of search costs is enough to overturn the prediction that firms soften competition by carrying non-overlapping product lines, as in Champsaur and Rochet (1989). Through product choice, search costs thus have important implications for market outcomes beyond their well studied price effects. Furthermore, we have shown that analyzing the price effects of search costs without endogenizing product choices can sometimes lead to overestimating the anticompetitive effects of search costs. In particular, a small amount of search costs can create head-to-head competition by inducing firms to carry overlapping products. Our results are robust to a number of assumptions, including the possibility that search costs and quality tastes are positively or negatively correlated, or that consumers know firms' product lines without incurring search costs.

The multi-product nature of firms also adds important twists to the analysis of competition in the presence of search costs. In line with Varian (1980), we show that search costs give rise to price dispersion when the two competing firms are multi-product- a possibility not considered by Varian (1980). However, if one of the firms specializes in selling the high quality product, the dispersion prediction might no longer hold. In particular, the market might be segmented between the non-shoppers who visit the single-product high quality store, and the remaining consumers who pay lower prices at the multi-product store.

We further show that multi-product retailing adds an important departure from Varian (1980), as goods within a store cannot be priced independently from each other. In particular, the incentives to separate both consumer types impose an upper bound on the highest price that can be charged for a high quality good, given the price of the low quality one. This holds true even for a single-product firm competing with a multi-product one, as price discrimination within the latter spreads to the former through the effect of competition.

Throughout the analysis, we have assumed that firms incur an arbitrarily small fixed cost of carrying a product. We have not allowed for higher fixed costs so as to highlight the strategic motives underlying product choice. Admittedly, there are several motives other than the ones studied in this paper that shape firms' product choices. For instance, higher fixed costs of carrying a product (which could arguably be higher for high quality products),[36] could induce firms to offer fewer and possibly non-overlapping products.

[36]In some cases, such costs can be substantial, e.g. firms have to advertise that they are carrying an additional product, or the transaction costs of dealing with an additional provider can sometimes be high. The marketing literature has analyzed several factors explaining the limited number of products

Our prediction is not that competitors should always carry overlapping product lines. Rather, our analysis suggests that if their product lines do not overlap, it must be for reasons other than the strategic ones- at least in markets where search frictions prevail.

# Appendix A: Proofs

**Proof of Proposition 1 [SPE under** $\mu = 1$] First, at subgames $(LH, LH)$, $(L, L)$ and $(H, H)$, both firms make zero profits. Second, at subgame $(L, LH)$ the low quality product is priced at marginal cost $c^L$ while the high quality product is sold at the highest price that satisfies the high types' incentive compatibility constraint, i.e., $c^L + \theta^H \Delta q$. Firm $L$ makes zero profits while firm $LH$ gets a payoff of $(1 - \lambda)(\theta^H \Delta q - \Delta c)$, which equals its minimax. Third, at subgame $(H, LH)$, the high quality product is priced at marginal cost $c^H$ while the low quality product is sold at the highest price that satisfies the low types' incentive compatibility constraint and participation constraints, i.e., $\min \left\{c^H - \theta^L \Delta q, \theta^L q^L\right\}$. Firm $H$ makes zero profits while firm $LH$ makes profits $\lambda \pi^L$ if $c^H > \theta^L q^H$ or $\lambda \left(\Delta c - \theta^L \Delta q\right)$ otherwise. Finally, at subgame $(L, H)$ the equilibrium is in mixed strategies. For the purposes of this proof, it suffices to put bounds on equilibrium profits. Minmax profits for each firm are computed by characterizing the firm's best response to the rival pricing its good at marginal cost. Following our previous analysis, the minmax profits for the $H$ firm are $(1-\lambda)(\theta^H \Delta q - \Delta c) > 0$, while the minmax profits for the $L$ firm are $\lambda \pi^L > 0$ if $c^H > \theta^L q^H$ or $\lambda \left(\Delta c - \theta^L \Delta q\right) > 0$ otherwise. Since at the mixed strategy equilibrium firms always price above marginal costs (otherwise they would have zero profits, but this cannot be since their minmax profits are positive), equilibrium profits are *strictly above* the minimax whenever the participation constraint is not binding. The only case where above marginal cost pricing does not necessarily imply that firm $L$'s profits are strictly above its minmax is when $c^H > \theta^L q^H$, as in this case firm $L$'s best response is the the same regardless of whether firm $H$ prices at $c^H$ or above.[37] Indeed, for the case $c^H > \theta^L q^H$, we can show that equilibrium profits are exactly

sold per firm. For instance, Villas-Boas (2004) analyzes product line decisions when firms face costs of communicating about the different products they carry to their customers. They show that costly advertising can induce firms to carry fewer products as well as to charge lower prices for their high-quality goods.

[37] It is straightforward to see that in a mixed strategy equilibrium we must have $\underline{p}^H > c^H$ and $\underline{p}^L > c^L$; otherwise, each firm's profits would be zero, but this leads to a contradiction since profits cannot be below the minimax. Hence, firm $H$ would never like to price lower than $\underline{p}^L + \theta^H \Delta q > c^L + \theta^H \Delta q > c^H$. Since at a price $\underline{p}^L + \theta^H \Delta q$ firm $H$ would at least be serving the high types, its profits must be strictly greater than its minmax $(1 - \lambda)(\theta^H \Delta q - \Delta c)$. Similarly, if $\underline{p}^H > \theta^L q^H$, firm $L$ would be a monopolist over the low-types, so it could always secure profits of at least $\lambda \pi^L$. If $\underline{p}^H < \theta^L q^H$, firm $L$ would never like to charge prices lower than $\underline{p}^H - \theta^L \Delta q > c^H - \theta^L \Delta q$. Since at a price $\underline{p}^H - \theta^L \Delta q$ firm $L$ would at

equal to the minmax $\lambda\pi^L$. To see this, note that at the $MSE$ the upper bounds of firms' price supports are the constrained monopoly prices. Furthermore, firm $L$ has to play a probability mass at its upper bound. Otherwise, firm $H$ would make zero profits at its upper bound (as all consumers would strictly prefer to buy from firm $L$), but this cannot be the case since its minmax is strictly positive. Last, the two firms cannot put positive mass at their upper bounds as firm $L$ would be better off putting all its mass slightly below its upper bound (so as to attract all consumers whenever the rival plays the mass at the upper bound). It thus follows that when firm $L$ plays its upper bound, the rival is pricing below its upper bound with probability one. Hence, at the upper bound firm $L$ only serves the low types, thus making profits that exactly equal its minmax, $\lambda\pi^L$.

We are now ready to show that $(L, H)$ is the unique $SPE$ product choice. Starting at $(L, H)$, firm $H$ does not want to carry good $L$ as at $(L, LH)$ its profits are equal to the minmax, while they are strictly above that level at $(L, H)$. Similarly, firm $L$ does not want to carry good $H$ as at $(LH, H)$ its profits are equal to the minmax, while at $(L, H)$ its profits are (weakly) greater than its minmax. When the comparison is weak because firms' profits are $\lambda\pi^L$ at both subgames (i.e., when $c^H > \theta^L q^H$), the fixed cost $\epsilon \to 0$ of carrying a product breaks the indifference in favour of $(L, H)$.

Last, we note that $c^H > \theta^L q^H$ implies that there does not exist a $MSE$ in which firms mix between $L$ and $H$. In particular, we show that a firm would be better off deviating to carrying both products. Suppose that the rival chooses $L$ with probability $\alpha$ and $H$ with probability $(1 - \alpha)$. If the firm chooses $L$, its profits are zero with probability $\alpha$ and equal profits at $(L, H)$ with probability $(1 - \alpha)$. Instead, suppose that a firm deviates to $LH$. With probability $\alpha$ firms would be at the subgame $(LH, L)$ instead of $(L, L)$, so its profits would increase from zero to $(1 - \lambda)(\theta^H \Delta q - \Delta c)$; with probability $(1 - \alpha)$ firms would be at the subgame $(LH, H)$ instead of $(L, H)$. If $c^H > \theta^L q^H$, the profits at $(L, H)$ as well as at $(LH, H)$ are $\lambda\pi^L$. Hence, the firm is strictly better off deviating to $LH$ so a $MSE$ in which firms mix between $L$ and $H$ does not exist when $c^H > \theta^L q^H$. When $c^H < \theta^L q^H$ we only have a lower bound for profits at $(L, H)$ so we cannot assure that firms will always deviate from the $MSE$ in which firms mix between carrying either $L$ or $H$. But even if such a mixed strategy equilibrium equilibrium exists, it would still give rise to non-overlapping product lines with strictly positive (expected) payoffs for both firms. $Q.E.D.$

**Proof of Proposition 3 [pricing at subgame** $(LH, LH)$**]** The non-existence of a pure strategy equilibrium follows from standard arguments. Firms cannot tie in prices as a slight reduction in the price would allow a firm to attract all the shoppers. Firms cannot

least be serving the low types, its profits must be strictly greater than its minmax $\lambda\left(\Delta c - \theta^L \Delta q\right)$.

charge different prices either as the high-priced firm would only serve the non-shoppers and would thus be better off by either undercutting the rival's price or by charging the (constrained) monopoly prices to maximize profits out of the non-shoppers; in turn, if the high-priced firm priced as the (constrained) monopolist, the other firm would find it profitable to slightly price below that level, thus not making it profitable any more for the rival to charge the (constrained) monopoly prices. Thus, the equilibrium must be in mixed-strategies. Since firms are symmetric, we focus on characterizing the symmetric mixed strategy equilibria. Standard arguments imply that there are no holes in the support and that firms play no mass point at any price of the support, including the upper bound (see, for instance, Narasimhan, 1998). (i) At the upper bound, firms serve the non-shoppers only. Since profits are increasing in prices subject to the $\left(IC^H\right)$, the optimal prices at the upper are $\overline{p}^H = \theta^H q^H - q^L \Delta\theta$ and $\overline{p}^L = \theta^L q^L$, so that $\Delta\overline{p} = \theta^H \Delta q$.

We now demonstrate (ii), i.e., that at the lower bound $\Delta \underline{p} \in [\Delta c, \theta^H \Delta q)$. We will first demonstrate that $\Delta \underline{p} < \theta^H \Delta q$. Suppose otherwise that the price gap $p^H - p^L$ is constant and equal to $\theta^H \Delta q$ at and in the neighborhood of the lower bound (or throughout the entire price support for that matter). When a firm plays $\underline{\mathbf{p}} = (\underline{p}^H, \underline{p}^L)$ it obtains

$$\Pi(LH, LH; \underline{\mathbf{p}}) = \left(\mu + \frac{1-\mu}{2}\right)(1-\lambda)(\underline{p}^H - c^H) + \left(\mu + \frac{1-\mu}{2}\right)\lambda(\underline{p}^L - c^L).$$

Using $\Pi(\cdot; \underline{\mathbf{p}}) = \bar{\pi} \equiv (1-\mu)\left[\pi^L + (1-\lambda)\varphi\right]/2$, the payoff at the upper bound, and the assumption that $\underline{p}^H - \underline{p}^L = \theta^H \Delta q$ we obtain

$$\underline{p}^H - c^H = \frac{1-\mu}{1+\mu}(\bar{p}^H - c^H) + \lambda \frac{2\mu}{1+\mu}\varphi \tag{6}$$

and

$$\underline{p}^L - c^L = \frac{1-\mu}{1+\mu}(\bar{p}^L - c^L) - (1-\lambda)\frac{2\mu}{1+\mu}\varphi \tag{7}$$

where $\varphi \equiv \theta^H \Delta q - \Delta c > 0$. We now now compute the cdf $F(p^H)$ firms use in equilibrium to randomize prices. First, notice that if one firm plays something in the support, the other firm never wants to deviate and serve just the high type with a price $\theta^H q^H$, because according to $(A2)$ the payoff of doing so is strictly lower than $\bar{\pi}$. Thus, to obtain the cdf $F(p^H)$ around the lower bound, notice that playing any pair $p^H$ and $p^L = p^H - \theta^H \Delta q$ around the lower bound yields an expected payoff of

$$\begin{aligned} \Pi(\cdot; p^H, p^L) \quad = \quad & (1-\lambda)(p^H - c^H)\left[\frac{1-\mu}{2} + \mu(1 - F(p^H))\right] + \\ & \lambda(p^L - c^L)\left[\frac{1-\mu}{2} + \mu(1 - F(p^H))\right] \end{aligned}$$

where $1 - F(p^H)$ is the probability to attract both high and low type shoppers. Rearranging terms and using $\Pi(p^H, p^L = p^H - \theta^H \Delta q) = \bar{\pi}$ leads to

$$(\bar{p}^H - p^H)\frac{1-\mu}{2} = \left[1 - F(p^H)\right]\left[\mu(p^H - c^H) - \lambda\mu\varphi\right]. \tag{8}$$

From this expression we obtain

$$f(\bar{p}^H) = \frac{1-\mu}{2[\mu(\bar{p}^H - c^H) - \lambda\mu\varphi]} > 0$$

and

$$f(\underline{p}^H) = \frac{1+\mu}{2[\mu(\underline{p}^H - c^H) - \lambda\mu\varphi]} > 0. \tag{9}$$

Since $\underline{p}^H - c^H = \underline{p}^L - c^L + \varphi$, from (9) we also obtain that

$$f(\underline{p}^L) = \frac{1+\mu}{2[\mu(\underline{p}^L - c^L) + (1-\lambda)\mu\varphi]} > 0. \tag{10}$$

If the lower bound $\underline{\mathbf{p}}$ is indeed part of the equilibrium support, firms would not want to deviate from it. There are four possible (local) deviations to consider: (i) $\underline{p}^L$ and $p^H = \underline{p}^H - \varepsilon$, (ii) $\underline{p}^L$ and $p^H = \underline{p}^H + \varepsilon$, (iii) $\underline{p}^H$ and $p^L = \underline{p}^L - \varepsilon$, and (iv) $\underline{p}^H$ and $p^L = \underline{p}^L + \varepsilon$, where $\varepsilon \to 0$. The first deviation is clearly not profitable. If the firm plays $p^H = \underline{p}^H - \varepsilon$, it sells the same amount but at a lower price. Playing (ii) $p^H = \underline{p}^H + \varepsilon$ is also unprofitable. It violates the IC for high type consumers; so the firm would end up selling only low quality products to both, all shoppers and the non-shoppers coming to the store. Deviation (iii) is also unprofitable for the same reason (ii) is. We are left with deviation (iv). Notice first that playing $\underline{p}^H$ and $p^L = \underline{p}^L + \varepsilon$ only affects profits from low type consumers. The change in profit is

$$\Delta\Pi = \lambda\frac{1-\mu}{2}\varepsilon + \lambda\mu\left[(1 - F(\underline{p}^L + \varepsilon))(\underline{p}^L + \varepsilon - c^L) - (\underline{p}^L - c^L)\right].$$

The first term captures the gain from non-shoppers and the term in brackets captures the trade-off of losing the shoppers and charging them a bit more. We now take the derivative of $\Delta\Pi$ with respect to $\varepsilon$ and evaluate it at $\varepsilon = 0$ to obtain

$$\left.\frac{\partial\Delta\Pi}{\partial\varepsilon}\right|_{\varepsilon=0} = \frac{\lambda(1-\mu)}{2} + \lambda\mu[1 - f(\underline{p}^L)(\underline{p}^L - c^L)]. \tag{11}$$

Replacing $f(\underline{p}^L)$ that follows from (10) into (11) we obtain $\partial\Delta\Pi/\partial\varepsilon|_{\varepsilon=0} > 0$, which contradicts that playing $\underline{\mathbf{p}}$ was an equilibrium.

We now demonstrate that we cannot rule out $\Delta\underline{p} = \Delta c$ at and around the lower bound. Suppose then that $\underline{p}^H - \underline{p}^L = \Delta c$. Proceeding as before we can obtain the lower bounds

$$\underline{p}^H - c^H = \frac{1-\mu}{1+\mu}(\bar{p}^H - c^H) - \frac{\lambda(1-\mu)}{1+\mu}\varphi \tag{12}$$

and

$$\underline{p}^L - c^L = \frac{1-\mu}{1+\mu}(\bar{p}^L - c^L) + \frac{(1-\lambda)(1-\mu)\mu}{1+\mu}\varphi. \tag{13}$$

We now evaluate whether a deviation at the lower bound is profitable; in particular, playing $\underline{p}^L$ and $p^H = \underline{p}^H + \varepsilon$ (this is the only relevant deviation). If $F(p^H)$ is the cdf, then such deviation reports an extra gain/loss equal to

$$\Delta\Pi = \frac{1-\mu}{2}(1-\lambda)\varepsilon + \mu(1-\lambda)\left[(1-F(\underline{p}^H+\varepsilon))(\underline{p}^H+\varepsilon-c^H) - (\underline{p}^H-c^H)\right].$$

Take the derivative with respect to $\varepsilon$ and evaluate it at $\varepsilon = 0$ to obtain

$$\left.\frac{\partial\Delta\Pi}{\partial\varepsilon}\right|_{\varepsilon=0} = \frac{1-\mu}{2}(1-\lambda) + \mu(1-\lambda)\left[1-f(\underline{p}^H)(\underline{p}^H-c^H)\right] \tag{14}$$

To continue with the proof we need $f(\underline{p}^H)$, which requires to obtain $F(p^H)$ first. For this latter notice that playing any pair $p^H$ and $p^L = p^H - \Delta c$ in the neighborhood of $\underline{\mathbf{p}}$ (where we are assuming that $p^H - p^L = \Delta c$) reports $\bar{\pi}$, so

$$\begin{aligned}\bar{\pi} \;\; = \;\; & (1-\lambda)(p^H-c^H)\left[\frac{1-\mu}{2}+\mu(1-F(p^H))\right] + \\ & \lambda(p^L-c^L)\left[\frac{1-\mu}{2}+\mu(1-F(p^H))\right]\end{aligned}$$

Rearranging terms and using $p^L = p^H - \Delta c$ leads to

$$\bar{\pi} = (p^H-c^H)\left[\frac{1-\mu}{2}+\mu(1-F(p^H))\right]$$

from which we obtain

$$\mu f(p^H)(p^H-c^H) = \frac{1-\mu}{2}+\mu(1-F(p^H))$$

and

$$f(\underline{p}^H) = \frac{1+\mu}{2\mu(\underline{p}^H-c^H)} \tag{15}$$

Plugging (15) into (14) yields

$$\left.\frac{\partial\Delta\Pi}{\partial\varepsilon}\right|_{\varepsilon=0} = 0$$

which concludes this part of the proof. Proceeding like this we can also rule out $\Delta\underline{p} < \Delta c$. Just assume that $p^H - p^L = \Delta c - \xi$ holds, with $\xi$ positive but small, and show that $\partial\Delta\Pi/\partial\varepsilon|_{\varepsilon=0} > 0$. *Q.E.D.*

**Proof of Lemma 2 [pricing at subgame** $(LH, LH)$**]** We want to show that the equilibrium in the statement of the Proposition is indeed an equilibrium. First, firms could deviate by playing the price pairs in the support with different probabilities, while still choosing price pairs that satisfy incentive compatibility. However, this is unprofitable given that all price-pairs in the support give equal expected profits. Indeed, the equilibrium has been constructed so that

$$(p^L - c^L)\left[\frac{1-\mu}{2} + \mu(1 - F^L(p^L))\right] = \frac{1-\mu}{2}(\overline{p}^L - c^L) = \frac{1+\mu}{2}(\underline{p}^L - c^L)$$

and

$$(p^H - c^H)\left[\frac{1-\mu}{2} + \mu(1 - F^H(p^H))\right] = \frac{1-\mu}{2}(\overline{p}^H - c^H) = \frac{1+\mu}{2}(\underline{p}^H - c^H)$$

with the ratio (5) derived in order for the price pair $\left(p^H, p^L\right)$ to satisfy $F^H(p^H) = F^L(p^L)$, i.e., the choice of $p^L$ results in a choice of $p^H$, so that the prices satisfying that ratio are played with equal probability. Therefore, expected profits at the proposed equilibrium are as in (3).

Second, firms could deviate by choosing $p^L$ and $p^H$ not satisfying equation 5 while still satisfying incentive compatibility. Again, these deviations are not profitable since all the prices in the support give equal profits. Deviating to prices that do not satisfy incentive compatibility is unprofitable because of Lemma 1.

Last, firms could deviate by playing price pairs outside the support. Choosing any prices above $\left(\overline{p}^L, \overline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is only selling to the non-shoppers and $\left(\overline{p}^L, \overline{p}^H\right)$ are the optimal monopoly prices. Choosing any prices below $\left(\underline{p}^L, \underline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is inelastically selling to all consumers with probability one and would thus gain by rasing the price up to $\left(\underline{p}^L, \underline{p}^H\right)$.

**Proof of Proposition 5 [pricing at subgame** $(H, LH)$**]** At the PSE candidate $p^H = \theta^H q^H$ for firm $H$ and $p^L = \theta^L q^L$ and $p^H = \theta^H q^H - q^L \Delta\theta$ for firm $LH$, firm $H$ only sells to the non-shopper high types,[38] while firm $LH$ sells to all the rest.

Firms' profits at the candidate PSE are

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H \tag{16}$$

$$\Pi(LH, H) = \left(\frac{1-\mu}{2} + \mu\right)\left(\lambda\pi^L + (1-\lambda)(\pi^H - q^L\Delta\theta)\right). \tag{17}$$

For this to be an equilibrium, it must be the case that neither firm wants to deviate from it. Firm $L$ cannot profitably deviate. In turn, firm $H$ could charge slightly less

[38] Recall that we are assuming that the firm prefers to leave demand from the low types unserved rather than having to reduce prices so that the low types also buy. Allowing for this possibility would duplicate the number of cases we need to consider without adding any qualitatively different results.

than $\theta^H q^H - q^L \Delta\theta$ to also attract the high type shoppers (note that at this price there are still unserved low types). If it does, it would sell to all the high types except to the non-shoppers who buy from firm $LH$. It would thus make profits

$$\Pi'(H, LH) = \left(1 - \frac{1-\mu}{2}\right)(1-\lambda)\left(\pi^H - \Delta\theta q^L\right) \tag{18}$$

Comparing the profit expressions for firm $H$, this deviation is unprofitable if

$$\mu \leq \widehat{\mu} \equiv \frac{q^L \Delta\theta}{\pi^H + (\pi^H - q^L \Delta\theta)} = \frac{\pi^H - \left(\pi^L + \varphi\right)}{\pi^H + (\pi^L + \varphi)}. \tag{19}$$

Hence, the PSE exists if and only if $\mu \leq \widehat{\mu}$, otherwise the equilibrium must be in mixed strategies.

We now characterize the $MSE$. The upper bounds for firm $LM$ must be $\theta^L q^L$ and $\theta^H q^H - q^L \Delta\theta$. For firm $H$, the upper bound could either be the unconstrained monopoly price $\theta^H q^H$ or the constrained monopoly price $\theta^H q^H - q^L \Delta\theta$. In either case, firm $H$ never plays a price in between these two prices. Before considering these two possibilities, we first note that firm $L$ has to play a mass at its upper bound. Otherwise, when firm $H$ charged $\theta^H q^H - q^L \Delta\theta$, since firm $L$ would be charging prices below it with probability one, firm $H$ would make strictly lower profits than if it charged $\theta^H q^H$, as it would serve the same set of consumers at a lower price. Accordingly, suppose firm $LM$ plays a mass point at the upper bound, denoted $\omega^l > 0$. Similarly, let $\omega^h$ denote the mass point that firm $H$ puts on its upper bound.

As for the lower bounds, we need to consider two possibilities, either $\underline{p}^H \geq \theta^L q^H$ o $\underline{p}^H < \theta^L q^H$.

**Case $\underline{p}^H \geq \theta^L q^H$** In this case, the low types never want to buy good $H$, so their $(IC)$ is never binding. Thus, firm $H$ never serves the low types and firm $LM$ can always price good $L$ at $\theta^L q^L$ since the low types will never buy good $H$ even when it is priced at the lower bound.

**Case 1: $\underline{p}^H \geq \theta^L q^H$ and the upper bound for firm $H$ is $\theta^H q^H$** Firm $H$'s profits at the lower and upper bounds are

$$\Pi\left(H, LH; \bar{\mathbf{p}}\right) = \frac{1-\mu}{2}(1-\lambda)\pi^H. \tag{20}$$

$$\Pi(H, LH; \underline{\mathbf{p}}) = \left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)\left(\underline{p}^H - c^H\right).$$

which implies that

$$\underline{p}^H = c^H + \frac{1-\mu}{1+\mu}\pi^H. \tag{21}$$

Firm $H$ must also be indifferent between playing $\theta^H q^H$ and $\theta^H q^H - \Delta\theta q^L - \epsilon$ (with $\epsilon \to 0$)

$$\left(\frac{1-\mu}{2} + \mu\omega^{lh}\right)(1-\lambda)(\theta^H q^H - \Delta\theta q^L - c^H) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

which implies that

$$\omega^{lh} = \frac{(1-\mu)\Delta\theta q^L}{2\mu(\pi^H - \Delta\theta q^L)}.$$

(Note that the mass is decreasing in $\mu$: for $\mu \to 1$, firm $LH$ puts no mass at the upper bound, $\omega^{lh} = 0$, whereas for $\mu \to \Delta\theta q^L/(2\pi^H - \Delta\theta q^L)$, they put all the mass on the upper bounds, $\omega^{lh} = 1$. Hence, there is continuity between the PSE and the MSE). On the other hand, if firm $LH$ plays $p^L = \theta^L q^L$ and $p^H = \theta^H q^H - \Delta\theta q^L$, it obtains

$$\left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L + (1-\lambda)\left[\omega^h\left(\frac{1-\mu}{2} + \mu\right) + (1-\omega^h)\frac{1-\mu}{2}\right](\pi^H - \Delta\theta q^L)$$

which must be equal to what it gets by playing $\underline{p}^H$, which is

$$\left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L + \left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)(\underline{p}^H - c^H).$$

Equalizing these two latter expressions yields

$$\omega^h = \frac{(1-\mu)\Delta\theta q^L}{2\mu(\pi^H - \Delta\theta q^L)},$$

so the two firms play the same mass at the upper bounds.

We need to check that, as we had assumed,

$$\underline{p}^H = c^H + \frac{1-\mu}{1+\mu}\pi^H \geq \theta^L q^H$$

This requires that $\mu$ is sufficiently low,

$$\mu < \mu^* \equiv \frac{\pi^H - \left(\theta^L q^H - c^H\right)}{\pi^H + \left(\theta^L q^H - c^H\right)}$$

If $c^H \geq \theta^L q^H$ then $\mu^* \geq 1$ so this condition is always satisfied. In contrast, if $c^H < \theta^L q^H$, then $\mu^* < 1$. Hence, the condition characterized above is only valid for $\mu \in (\widehat{\mu}, \mu^*)$. [Note that $\widehat{\mu} < \mu^*$ since $\left(\pi^L + \varphi\right) > \left(\theta^L q^H - c^H\right)$].

We also need to make sure that firm $H$ does not want to deviate outside of the support to serve both the high as well as the low types. A sufficient condition to guarantee this is

$$\left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)\left(\underline{p}^H - c^H\right) > \left(\frac{1-\mu}{2} + \mu\right)\left(\theta^L q^H - c^H\right)$$

given that the low types will never buy good $H$ unless it gives them positive utility. Rearranging, a sufficient condition is

$$\mu < \mu^{**} \equiv \frac{(1-\lambda)\pi^H - \left(\theta^L q^H - c^H\right)}{(1-\lambda)\pi^H + (\theta^L q^H - c^H)}$$

It is easy to see that $\mu^{**} < \mu^*$. Hence, for $\mu < \mu^{**}$ this equilibrium is guaranteed to exist, even though it could possibly also exist for some higher $\mu < \mu^*$. [later on we will see that for $\mu > \mu^{**}$ another equilibrium is also guaranteed to exist, so equilibrium existence is not at stake]

Furthermore note that at $\mu = \mu^{**}$, $\underline{p}^H = \theta^L q^H$ [as we will see later on, for $\mu \geq \mu^{**}$, the equilibrium also has $\underline{p}^H = \theta^L q^H$ so there is continuity in the lower bounds, and therefore in profits, even if there is a change in the equilibrium expressions]

Equilibrium profits are, for $\mu \in (\widehat{\mu}, \mu^*)$ :

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H \tag{22}$$

$$\Pi(LH, H) = \left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L + \frac{1-\mu}{2}(1-\lambda)\pi^H. \tag{23}$$

**Case 2: $\underline{p}^H \geq \theta^L q^H$ and the upper bound for firm $H$ is $\theta^H q^H - q^L \Delta\theta$** Firm $H$ cannot play a mass point at the upper bound $\theta^H q^H - q^L\Delta\theta$. If it did, firm $L$ could make more profits at prices slightly below the monopoly price than at the monopoly price. Since the monopoly price must be in the support of firm $L$, it follows that $\omega^h = 0$. Thus, equilibrium profits for the $LM$ firm at the upper bound are[39]

$$\Pi(LH, H) = \left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L + (1-\lambda)\frac{1-\mu}{2}(\pi^H - \Delta\theta q^L)$$

Equalizing profits for firm $LM$ at the lower and upper bounds,

$$\frac{1-\mu}{2}(1-\lambda)(\pi^H - \Delta\theta q^L) = \left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)(\underline{p}^H - c^H)$$

shows that

$$\underline{p}^H = c^H + \frac{1-\mu}{1+\mu}(\pi^H - \Delta\theta q^L)$$

Since firm $H$ must also be indifferent between playing $\underline{p}^H$ and $\theta^H q^H - \Delta\theta q^L - \epsilon$ (with $\epsilon \to 0$)

$$\left(\frac{1-\mu}{2} + \mu\omega^{lh}\right)(1-\lambda)(\pi^H - \Delta\theta q^L) = \frac{1-\mu}{2}(1-\lambda)(\pi^H - \Delta\theta q^L)$$

from which ti follows that we should also have $\omega^{lh} = 0$. However, firm $H$ would then rather deviate to charging the unconstrained monopoly price to obtain profits $\frac{1-\mu}{2}(1-\lambda)\pi^H$. It follows that this case can never be an equilibrium: if $\underline{p}^H > \theta^L q^H$, firm $H$ must have the upper bound at the unconstrained monopoly price $\theta^H q^H$.

[39]Note that profits are positive regardless of the share of informed consumers.

**Case $\underline{p}^H < \theta^L q^H$** Note that this case can only arise under the assumption $c^H < \theta^L q^H$.

In this case, we have to consider the possibility that the $(IC)$ constraint of the low types is binding. In other words, firm $LH$ cannot always price good $L$ at $\theta^L q^L$ given that for $p^H < \theta^L q^H$ the low types would buy good $H$. Hence, firm $LH$ plays $p^L = \min\left\{\theta^L q^L, p^H - \theta^L \Delta q\right\}$ and both firms choose $p^H$ randomly. Note that for $p^H > \theta^L q^H$ this results in $p^L = \theta^L q^L$ with profits on good $L$ equal to $\pi^L$; whereas for $p^H < \theta^L q^H$ this results in $p^L = p^H - \theta^L \Delta q$ with profits on good $L$ equal to $\left(p^H - c^H\right) + \left(\Delta c - \theta^L \Delta q\right)$.

We first note that when playing $\underline{p}^H$, firm $H$ attracts all the shopper, both high as well as the low types, with probability one. The reason why the low types buy good $H$ from firm $H$ is simple: when firm $LH$ is pricing good $H$ above $\theta^L q^H$, the price for $L$ is $\theta^L q^L$ so the low types' utility from choosing $L$ is zero and hence, they would rather buy $H$ at $\underline{p}^H < \theta^L q^H$; if firm $LH$ is pricing $H$ below $\theta^L q^H$ but above $\underline{p}^H$, the low types are indifferent between choosing $H$ or $L$ at firm $LH$, since at the lower bound firm $H$ prices good $H$ at a price lower than firm $LH$, it follows that the shopper low types also buy from firm $H$.

We again need to consider two cases for firm $H$'s upper bound.

**Case 3: $\underline{p}^H < \theta^L q^H$ and the upper bound for firm $H$ is $\theta^H q^H$** Firm $H$ has to be indifferent between playing the lower bound or $\theta^H q^H$ it must hold

$$\left(\frac{1-\mu}{2} + \mu\right)(\underline{p}^H - c^H) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

Leading to [note that the second term is now multiplied by $(1-\lambda)$ so the lower bound is now lower than in the previous case]

$$\underline{p}^H = c^H + \frac{1-\mu}{1+\mu}(1-\lambda)\pi^H$$

Firm $H$ must also be indifferent between playing $\theta^H q^H$ and $\theta^H q^H - \Delta\theta q^L - \epsilon$ (with $\epsilon \to 0$), so again

$$\left(\frac{1-\mu}{2} + \mu\omega^{lh}\right)(1-\lambda)(\theta^H q^H - \Delta\theta q^L - c^H) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

which implies that

$$\omega^{lh} = \frac{(1-\mu)\Delta\theta q^L}{2\mu(\pi^H - \Delta\theta q^L)}.$$

On the other hand, if firm $LH$ plays $p^L = \theta^L q^L$ and $p^H = \theta^H q^H - \Delta\theta q^L$, it obtains

$$\left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L + (1-\lambda)\left[\omega^h\left(\frac{1-\mu}{2} + \mu\right) + (1-\omega^h)\frac{1-\mu}{2}\right](\pi^H - \Delta\theta q^L)$$

which must be equal to what it gets by playing $\underline{p}^H$, which is [note that the $(IC)$ is now biding at the lower bound for the low types]

$$\left(\frac{1-\mu}{2}+\mu\right)\lambda\left((\underline{p}^H-c^H)+\left(\Delta c-\theta^L\Delta q\right)\right)+\left(\frac{1-\mu}{2}+\mu\right)(1-\lambda)(\underline{p}^H-c^H)$$

which equals [note that at the lower bound firm $LH$ makes more profits than firm $H$ since it can discriminate]

$$\frac{1-\mu}{2}(1-\lambda)\pi^H+\left(\frac{1-\mu}{2}+\mu\right)\lambda\left(\Delta c-\theta^L\Delta q\right)$$

Equalizing these two latter expressions yields

$$\omega^h=\frac{1-\mu}{2\mu}\left(\frac{\Delta\theta q^L}{\pi^H-\Delta\theta q^L}\right)-\frac{1+\mu}{2\mu}\frac{\lambda}{1-\lambda}\frac{\left(\theta^L q^H-c^H\right)}{\pi^H-\Delta\theta q^L}$$

which is lower than the one in the previous case (the difference is the second term which is negative since $c^H<\theta^L q^H$).

Since $\omega^h\geq 0$, we require

$$\mu<\mu'=\frac{\frac{1-\lambda}{\lambda}\Delta\theta q^L-\left(\theta^L q^H-c^H\right)}{\frac{1-\lambda}{\lambda}\Delta\theta q^L+\left(\theta^L q^H-c^H\right)}<1.$$

Hence, for $\mu>\mu'$, we must have $\omega^h=0$. For $\mu\geq\mu'$, firm $LH$'s profits would fall below its minmax. Since this cannot be the case, this equilibrium cannot exist for $\mu>\mu'$ (we elaborate on this further below).Note that $\mu'<\mu^*$ since $\frac{1-\lambda}{\lambda}\Delta\theta q^L<\pi^L<\pi^H$.

Furthermore, we also have to check that

$$\underline{p}^H=c^H+\frac{1-\mu}{1+\mu}(1-\lambda)\pi^H<\theta^L q^H$$

which requires

$$\mu>\mu^{**}\equiv\frac{(1-\lambda)\pi^H-\left(\theta^L q^H-c^H\right)}{(1-\lambda)\pi^H+\left(\theta^L q^H-c^H\right)}$$

If we assume $(1-\lambda)\pi^H>\pi^L$ we can also rank all critical values: $\mu'<\mu^{**}<\mu^*$. Under this ranking, this equilibrium can never exist.

Note we do not need to check firms' incentives to deviate outside of the support since at the lower bound firms are already serving both types of customers.

**Case 4: $\underline{p}^H<\theta^L q^H$ and the upper bound for firm $H$ is $\theta^H q^H-q^L\Delta\theta$** Firm $H$ cannot play a mass point at the upper bound $\theta^H q^H-q^L\Delta\theta$. If it did, firm $LH$ could make more profits at prices slightly below the monopoly price than at the monopoly price. Since the monopoly price must be in the support of firm $LH$, it follows that $\omega^h = 0$. Thus, equilibrium profits for the $LH$ firm at the upper bound are[40]

$$\Pi(LH, H) = \left(\frac{1-\mu}{2} + \mu\left(1 - \underline{w}^h\right)\right)\lambda\pi^L + (1-\lambda)\frac{1-\mu}{2}(\pi^H - \Delta\theta q^L)$$

where we now denote $\underline{w}^h = \Pr\left(p^H < \theta^L q^H\right) = F\left(\theta^L q^H\right)$.

Equalizing profits for firm $LH$ at the lower and upper bounds,

$$\begin{aligned} & \left(\frac{1-\mu}{2} + \mu\right)\lambda(\underline{p}^H - \theta^L\Delta q - c^L) + \left(\frac{1-\mu}{2} + \mu\right)(1-\lambda)(\underline{p}^H - c^H) \\ = & \left(\frac{1-\mu}{2} + \mu\left(1 - \underline{w}^h\right)\right)\lambda\pi^L + (1-\lambda)\frac{1-\mu}{2}(\pi^H - \Delta\theta q^L) \end{aligned}$$

Since firm $H$ must be indifferent between playing $\underline{p}^H$ and $\theta^H q^H - \Delta\theta q^L - \epsilon$ (with $\epsilon \to 0$)

$$\left(\frac{1-\mu}{2} + \mu\omega^{lh}\right)(1-\lambda)(\pi^H - \Delta\theta q^L) = \left(\frac{1-\mu}{2} + \mu\right)(\underline{p}^H - c^H)$$

My conjecture is that at least one firm must get its minmax [this needs to be argued but I believe it must be true].

Suppose firm $H$ gets its minmax:

$$\frac{1-\mu}{2}(1-\lambda)\pi^H = \left(\frac{1-\mu}{2} + \mu\right)(\underline{p}^H - c^H)$$

so that

$$\underline{p}^H = c^H + \frac{1-\mu}{1+\mu}(1-\lambda)\pi^H$$

From the analysis above we know that $\underline{p}^H < \theta^L q^H$ iff $\mu > \mu^{**}$.

Then firm $LH$'s profits are

$$\Pi(LH, H) = \left(\frac{1-\mu}{2} + \mu\right)\lambda\left(\Delta c - \theta^L\Delta q\right) + \frac{1-\mu}{2}(1-\lambda)\pi^H$$

For $\mu \to 1$, we must have $\underline{p}^H = c^H$: firm $H$'s profits converge to zero and firm $LH$'s profits converge to $\lambda\left(\Delta c - \theta^L\Delta q\right)$.

We can also compute the probability $\underline{w}^h$ using the indifference condition of firm $LH$

$$\left(\frac{1-\mu}{2} + \mu\left(1 - \underline{w}^h\right)\right)\lambda\pi^L + (1-\lambda)\frac{1-\mu}{2}(\pi^H - \Delta\theta q^L) = \left(\frac{1-\mu}{2} + \mu\right)\lambda\left(\Delta c - \theta^L\Delta q\right) + \frac{1-\mu}{2}(1-\lambda)\pi^H$$

[40] Note that profits are positive regardless of the share of informed consumers.

Clearly, we must have $\underline{w}^h > 0$. By $(A1)$, if $\underline{w}^h = 0$, we would reach a contradiction as

$$\left(\frac{1-\mu}{2}+\mu\right)\lambda\left(\pi^L-\left(\Delta c-\theta^L\Delta q\right)\right)+(1-\lambda)\frac{1-\mu}{2}\Delta\theta q^L > 0$$

i.e., the LHS would exceed the RHS, and since the LHS is decreasing in $\underline{w}^h$ we must have $\underline{w}^h > 0$ to equalize LHS and RHS. This mass should be $\underline{w}^h = 0$ at $\mu^{**}$ and increasing in $\mu$. Note that

$$\lim_{\mu\to 1}\underline{w}^h = \frac{\pi^L-\left(\Delta c-\theta^L\Delta q\right)}{\pi^L} = \frac{\theta^L q^H - c^H}{\pi^L} > 0$$

and $\underline{w}^h \to 1$ only when $\Delta c \to \theta^L \Delta q$.

We can also compute the mass of firm $LM$ at the upper bound, which is the same as before. Note that it is always decreasing it it reaches zero when $\mu \to 1$.

$$\omega^{lh} = \frac{(1-\mu)\Delta\theta q^L}{2\mu(\pi^H-\Delta\theta q^L)}.$$

Suppose now that firm $LM$ gets its minmax:

Minmax for firm $LH$: If firm $H$ prices at $c^H$, the BR of firm $LM$ is either (1) $p^L = c^H - \theta^L\Delta q$ and $p^H = p^L + \theta^H\Delta q = c^H + \Delta\theta\Delta q$ so as to attract all low types and separate the low from the high types non shoppers, and get profits

$$\frac{1-\mu}{2}(1-\lambda)\Delta\theta\Delta q+\left(\frac{1-\mu}{2}+\mu\right)\lambda\left(\Delta c-\theta^L\Delta q\right)$$

or (2) set the constrained monopoly prices and only serve the non shoppers, and get profits

$$\frac{1-\mu}{2}(1-\lambda)(\pi^H-\Delta\theta q^L)+\frac{1-\mu}{2}\lambda\pi^L$$

So the minmax is the greatest between the two.

If $\mu$ is very close to 1 the first option is more profitable [we would need to check in this $\mu$ region this is always the case that the minmax is the first equation].

Hence, [if it is true that firm $LH$ gets its minmax] for $\mu$ close to 1

$$\begin{aligned}&\frac{1-\mu}{2}(1-\lambda)\Delta\theta\Delta q+\left(\frac{1-\mu}{2}+\mu\right)\lambda\left(\Delta c-\theta^L\Delta q\right)\\ =\;&\left(\frac{1-\mu}{2}+\mu\right)\lambda(\underline{p}^H-\theta^L\Delta q-c^L)+\left(\frac{1-\mu}{2}+\mu\right)(1-\lambda)(\underline{p}^H-c^H)\end{aligned}$$

so that

$$\underline{p}^H = c^H+\frac{1-\mu}{1+\mu}(1-\lambda)\Delta\theta\Delta q$$

Firm $H$'s profits are

$$\begin{aligned}\Pi(H, LH) &= \left(\frac{1-\mu}{2}+\mu\right)\left(\underline{p}^H - c^H\right) \\ &= \frac{1+\mu}{2}\frac{1-\mu}{1+\mu}(1-\lambda)\Delta\theta\Delta q \\ &= \frac{1-\mu}{2}(1-\lambda)\Delta\theta\Delta q\end{aligned}$$

But this cannot be the case since these profits are below the minmax $\frac{1-\mu}{2}(1-\lambda)\pi^H$, a contradiction.

**Wrapping up** For $c^H > \theta^L q^H$ :

1. If $\mu < \widehat{\mu}$ : PSE with firm $H$ charging the unconstrained monopoly prices, and firm $LH$ charging the constrained ones.

2. If $\mu \in (\widehat{\mu}, 1)$ : MSE with firm $H$ charging the unconstrained monopoly price at the upper bound

$$\Pi(LH, H) = \left(\frac{1-\mu}{2}+\mu\right)\lambda\pi^L + \frac{1-\mu}{2}(1-\lambda)\pi^H$$

For $c^H > \theta^L q^H$ :

1. If $\mu < \widehat{\mu}$ : PSE with firm $H$ charging the unconstrained monopoly prices, and firm $LH$ charging the constrained ones.

2. If $\mu \in (\widehat{\mu}, \mu^*)$ : MSE with firm $H$ charging the unconstrained monopoly price at the upper bound

$$\Pi(LH, H) = \left(\frac{1-\mu}{2}+\mu\right)\lambda\pi^L + \frac{1-\mu}{2}(1-\lambda)\pi^H$$

3. If $\mu \in (\mu^{**}, 1)$ : MSE with firm $H$ charging the constrained monopoly price at the upper bound

$$\Pi(LH, H) = \left(\frac{1-\mu}{2}+\mu\right)\lambda\left(\Delta c - \theta^L \Delta q\right) + \frac{1-\mu}{2}(1-\lambda)\pi^H$$

Note that for some $\mu \in (\mu^{**}, \mu^*)$ there are two possible equilibria. In both, firm $H$ makes equal profits; the Pareto dominant equilibrium is 2. This multiplicity does not affect the profits of firm $H$ which always remain at

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

(these are the equilibria we are mostly interested in for Proposition 8).

Firm $LM$ makes profits strictly above its minmax, while firm $H$'s profits are always equal to its minmax.

*Q.E.D.*

**Proof of Proposition 6 [pricing at subgame** $(L, LH)$**]** It is easy to see that the equilibrium must be in mixed strategies. Since both firms are competing to attract the shoppers, any $PSE$ candidate can be ruled out by either firm's incentives to slightly undercut its rival. Note that by $(A1)$ it does not pay firm $LH$ to only serve the high types. Both firms choose $p^L$ in $\left[\underline{p}^L, \theta^L q^L\right]$ and incentive compatibility restricts firm $LH$ to price the high quality product at $p^H = p^L + \theta^H \Delta q$.

We now show, by contradiction, that one of the two firms must be placing an atom at the upper bound. Suppose not, in which cases upper-bound payoffs are given by[41]

$$\Pi\left(L, LH; \bar{\mathbf{p}}\right) = \frac{1-\mu}{2}\pi^L \tag{24}$$

and

$$\Pi\left(LH, L; \bar{\mathbf{p}}\right) = \frac{1-\mu}{2}\left(\pi^L + (1-\lambda)\varphi\right) \tag{25}$$

respectively. In the absence of atoms, when firm $L$ prices at the upper bound, the shoppers low and all high types buy from the rival because with probability one firm $LH$ would be pricing both goods at a lower price (after controlling for quality), whereas when $LH$ prices at the upper bound, both shoppers' types prefer to buy the rival's low quality product. With these hypothetical "upper-bound" payoffs we can now find the lower bound of the price support $\underline{p}^L$, which must be atomless to rule out any deviation. If we equalize (24) to what firm $L$ would get by pricing at the lower bound and attracting all shoppers and half of the non-shopper low and high types, i.e.,

$$\Pi\left(L, LH; \underline{p}^L\right) = \left(\frac{1-\mu}{2} + \mu\right)(\underline{p}^L - c^L).$$

We obtain

$$\underline{p}_l^L - c^L = \frac{1-\mu}{1+\mu}\pi^L \tag{26}$$

where the subindex $l$ indicates that $\underline{p}_l^L$ is obtained using $L$'s indifference condition. Similarly, if we equalize (25) to what firm $LH$ would get by pricing at the lower bound and attracting all shoppers as well as non-shopper high types, i.e.,

$$\begin{aligned}\Pi\left(LH, L; \underline{p}^L, \underline{p}^H\right) &= \left(\frac{1-\mu}{2} + \mu\right)\left(\lambda(\underline{p}^L - c^L) + (1-\lambda)\left(\underline{p}^L + \theta^H \Delta q - c^H\right)\right) \\ &= \left(\frac{1-\mu}{2} + \mu\right)\left((\underline{p}^L - c^L) + (1-\lambda)\,\varphi\right).\end{aligned}$$

[41] Note that firm $L$ serves the non-shoppers, both the low and the high types at $\theta^L q^L$.

We obtain

$$\underline{p}_{lh}^{L}-c^{L}=\frac{1-\mu}{1+\mu}\pi^{L}-\frac{2\mu}{1+\mu}(1-\lambda)\varphi \tag{27}$$

where the subindex $lh$ indicates that $\underline{p}_{lh}^{L}$ is obtained using $LH$'s indifference condition. Importantly, note that $\underline{p}_{lh}^{L}-c^{L}$ is decreasing in $\varphi$. Simple inspection of (26) and (27) shows that $\underline{p}_{l}^{L}\neq\underline{p}_{lh}^{L}$, except for $\mu=0$, which contradicts the initial assumption that no firm places an atom at the upper bound for any $\mu>0$. Furthermore, we have $\underline{p}_{l}^{L}>\underline{p}_{lh}^{L}$ for all $\mu>0$. This implies that $\underline{p}_{l}^{L}$ is the actual lower bound and that firm $L$ must be playing the upper bound with positive probability mass, while firm $LH$ plays no mass at any price. The size of that mass $\omega^{l}$, can be obtained by invoking $LH$'s indifference condition[42]

$$\left(\frac{1-\mu}{2}+\mu\right)\left(\left(\underline{p}^{L}-c^{L}\right)+\left(1-\lambda\right)\varphi\right)=\left(\frac{1-\mu}{2}+\mu\omega^{l}\right)\left(\pi^{L}+(1-\lambda)\varphi\right)\right)$$

and after replacing $\underline{p}_{l}^{L}-c^{L}$ from (26),

$$\omega^{l}=\frac{1+\mu}{2\mu}\frac{\frac{1-\mu}{1+\mu}\pi^{L}+\left(1-\lambda\right)\varphi}{\pi^{L}+(1-\lambda)\varphi}-\frac{1-\mu}{2\mu}.$$

Equilibrium profits at this $MSE$ are

$$\begin{aligned}
\Pi\left(L,LH\right) &= \frac{1-\mu}{2}\pi^{L}\\
\Pi\left(LH,L\right) &= \left(\frac{1-\mu}{2}+\mu\right)\left(\frac{1-\mu}{1+\mu}\pi^{L}+\left(1-\lambda\right)\varphi\right).
\end{aligned}$$

Hence, firm $L$ obtains minmax profits while firm $LH$ obtains profits above its minmax. For this reason, the profits of firm $LH$ are no longer proportional to its mass of non-shoppers.

The characterization of the $MSE$ ends with the probability distribution $F(p^{L})$ used by the firms to set prices in the range $[\underline{p}^{L},\theta^{L}q^{L})$. Since both firms play according to the same distribution, one can derive such distribution from either firm's indifference condition. $Q.E.D.$

**Proof of Proposition 7 [pricing at subgame $(L,H)$]** [We need to recharacterize this equilibrium under the single crossing condition]

At the PSE candidate $p^{H}=\theta^{H}q^{H}$ for firm $H$ and $p^{L}=\theta^{L}q^{L}$ for firm $L$, firm $H$ only sells to the non-shopper high types, while firm $L$ sells to all the rest (except for the

[42]Notice that when firm $L$ plays the atom $\theta^{L}q^{L}$, which happens with probability $\omega^{l}$, firm $LH$'s upper bound for $p^{L}$ is not $\theta^{L}q^{L}$ but $\theta^{L}q^{L}-\varepsilon$ with $\varepsilon\rightarrow 0$. The reason is that in equilibrium there cannot be a tie on a price with positive probability.

non-shoppers that visit firm $H$). Thus, firms' profits are

$$\begin{aligned} \Pi(H,L) &= \frac{1-\mu}{2}(1-\lambda)\pi^H \\ \Pi(L,H) &= \frac{1+\mu}{2}\pi^L. \end{aligned} \tag{28}$$

For this to be an equilibrium, it must be the case that neither firm wants to deviate from it. In particular, firm $H$ could charge $\theta^H q^H - q^L \Delta\theta$ to also attract the high type shoppers. It would thus sell to all the high types (except for the non-shoppers who visit firm $L$) and would make profits

$$\Pi'(H,L) = \frac{1+\mu}{2}(1-\lambda)(\pi^H - q^L\Delta\theta). \tag{29}$$

Comparing the profit expressions for firm $H$, the PSE equilibrium exists if and only if $\mu \leq \widehat{\mu}$ as defined in (19). Otherwise, the equilibrium must be in mixed strategies. Note we we do not need to check that firm does not want to charge $\theta^L q^H - c^H$ so as to serve all the non-shoppers, including the low types since we have assumed $(1-\lambda)\pi^H > \theta^L q^H - c^H$.

In order to characterize the $MSE$, we start by characterizing the lower bounds of the price supports. For given $\underline{p}^L$, it never pays firm $H$ to charge less than $\underline{p}^L + \theta^H \Delta q$ since at this price it attracts all the shopper high types. Hence, we must have $\underline{p}^H \geq \underline{p}^L + \theta^H \Delta q$. Second, for given $\underline{p}^H$, it never pays firm $L$ to charge less than $\underline{p}^H - \theta^H \Delta q$ since at this price it attracts all the shoppers. Hence, we must have $\underline{p}^L \geq \underline{p}^H - \theta^H \Delta q$. Putting these two conditions together, it follows that we must have $\underline{p}^H = \underline{p}^L + \theta^H \Delta q$. Therefore, when firm $H$ charges $\underline{p}^H$, with probability one firm $L$ is charging prices above $\underline{p}^H - \theta^H \Delta q$. Hence, all the high types buy from firm $H$ (except for the non-shoppers that visit $L$) implying that firm $H$'s profits in equilibrium must satisfy

$$\begin{aligned} \Pi(H,L) &= \left(1 - \frac{1-\mu}{2}\right)(1-\lambda)\left(\underline{p}^H - c^H\right) \\ &= \frac{1+\mu}{2}(1-\lambda)\left(\underline{p}^L - c^L + \varphi\right). \end{aligned} \tag{30}$$

Similarly, when firm $L$ charges $\underline{p}^L$, with probability one firm $H$ is charging prices above $\underline{p}^L + \theta^H \Delta q$. Hence, the firm serves its non-shoppers plus all the shoppers, both high and low types, implying the firm $L$'s profits in equilibrium must satisfy

$$\Pi(L,H) = \frac{1+\mu}{2}\left(\underline{p}^L - c^L\right). \tag{31}$$

Let us now characterize the upper bounds of the price supports. The upper bound for firm $L$ must be $\theta^L q^L$, as for the low quality product the constrained and unconstrained monopoly prices coincide. Similar arguments as above imply that the upper bound for firm $H$ could either be the unconstrained monopoly price $\theta^H q^H$ or the constrained monopoly price $\theta^H q^H - q^L \Delta\theta$. In either case, firm $H$ never plays a price in between these two prices. Before considering these two possibilities, we first note that firm $L$ has to play a mass at its upper bound $\theta^L q^L$. Otherwise, when firm $H$ charged $\theta^H q^H - q^L \Delta\theta$, since firm $L$ would be charging prices below $\theta^L q^L$ with probability one, firm $H$ would make strictly lower profits than if it charged $\theta^H q^H$, as it would serve the same set of consumers at a lower price. Accordingly, suppose firm $L$ plays a mass point at $\overline{p}^L = \theta^L q^L$, denoted $\omega^l > 0$. Similarly, let $\omega^h$ denote the mass point that firm $H$ puts on its upper bound.

**Case 1: the upper bound for firm H is $\theta^H q^H$**

Suppose that firm $H$'s upper bound is $\theta^H q^H$. Since firm $L$ is pricing below $\theta^H q^H$ with probability one, firm $H$'s profits in this case are

$$\Pi\left(H, L\right) = \frac{1-\mu}{2}(1-\lambda)\pi^H. \tag{32}$$

Since firm $H$ must be indifferent between playing $\theta^H q^H$ and $\theta^H q^H - \Delta\theta q^L$, it follows that the mass that firm $L$ puts on the monopoly price in Case 1, $\omega_1^l$, must satisfy

$$\left(\frac{1-\mu}{2} + \mu\omega_1^l\right)(1-\lambda)\left(\pi^H - q^L\Delta\theta\right) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

which implies

$$\omega_1^l = \frac{1}{2}\frac{q^L\Delta\theta}{\left(\pi^H - q^L\Delta\theta\right)}\left(\frac{1}{\mu} - 1\right). \tag{33}$$

The mass $\omega_1^l$ is decreasing in $\mu$, for $\mu \to 1$, $\omega_1^l = 0$ and for $\mu \to \widehat{\mu}$, $\omega_1^l = 1$. Hence, there is continuity between the pure strategy equilibrium and this mixed-strategy equilibrium.

Firm $H$ must also be indifferent between playing $\theta^H q^H$ and $\underline{p}^H$. Equating profits,

$$\frac{1+\mu}{2}(1-\lambda)\left(\underline{p}^H - c^H\right) = \frac{1-\mu}{2}(1-\lambda)\pi^H$$

which implies[43]

$$\underline{p}^H - c^H = \frac{1-\mu}{1+\mu}\pi^H.$$

Since we must have $\underline{p}^H = \underline{p}^L + \theta^H\Delta q$, then

$$\underline{p}^L - c^L = \frac{1-\mu}{1+\mu}\pi^H - \varphi.$$

Using equation (31), equilibrium profits for firm $L$ are thus

$$\Pi\left(L, H\right) = \frac{1+\mu}{2}\left(\frac{1-\mu}{1+\mu}\pi^H - \varphi\right).$$

[43]Note that as $\mu \to 1$ the firm would be making zero profits, which cannot be the case since that is below its minmax. Hence, at some point his equilibrium must cease to exist, as we will see below.

When the firm prices at the upper bound $\theta^L q^L$, it makes profits

$$\Pi(L,H) = \left(\frac{1+\mu}{2}\lambda + \left(\frac{1-\mu}{2} + \mu\omega^h\right)(1-\lambda)\right)\pi^L. \tag{34}$$

Equalizing these two expressions and rearranging terms yields

$$\omega^h = \frac{1-\mu}{2}\frac{\left(\pi^H - \pi^L - \varphi\right)}{\mu(1-\lambda)\pi^L} - \frac{\lambda\pi^L + \varphi}{(1-\lambda)\pi^L}. \tag{35}$$

Note that for $\mu \to \widehat{\mu}$, $\omega^h \to 1$. Hence, there is continuity between the pure strategy equilibrium and this mixed-strategy equilibrium.

For

$$\mu = \widetilde{\mu} \equiv \frac{\pi^H - \left(\pi^L + \varphi\right)}{\pi^H + (\pi^L + \varphi) - 2(1-\lambda)\pi^L} \tag{36}$$

we have $\omega^h = 0$, so that firm $L$ 's profits in equation (34) are equal to the minmax $\left(\frac{1+\mu}{2}\lambda + \frac{1-\mu}{2}(1-\lambda)\right)\pi^L$. Note that $\widetilde{\mu} \in (\widehat{\mu}, 1)$. For $\mu > \widetilde{\mu}$, firm $L$ 's profits would fall below its minmax. Since this cannot be the case, this equilibrium cannot exist for $\mu > \widetilde{\mu}$ (we elaborate on this further below).

**Case 2: the upper bound for firm H is $\theta^H q^H - q^L\Delta\theta$**

Firm $H$ cannot play a mass point at the upper bound $\theta^H q^H - q^L\Delta\theta$. If it did, firm $L$ could make more profits at prices slightly below the monopoly price than at the monopoly price. Since the monopoly price must be in the support of firm $L$, it follows that $\omega^h = 0$. Thus, equilibrium profits for the $L$ firm at the upper bound are[44]

$$\Pi(L,H) = \left(\frac{1+\mu}{2}\lambda + \frac{1-\mu}{2}(1-\lambda)\right)\pi^L.$$

Equalizing profits for firm $L$ at the lower and upper bounds,

$$\left(\frac{1+\mu}{2}\lambda + \frac{1-\mu}{2}(1-\lambda)\right)\pi^L = \frac{1+\mu}{2}\left(\underline{p}^L - c^L\right)$$

shows that

$$\underline{p}^L - c^L = \left(\lambda + \frac{1-\mu}{1+\mu}(1-\lambda)\right)\pi^L.$$

So that

$$\underline{p}^H - c^H = \left(\lambda + \frac{1-\mu}{1+\mu}(1-\lambda)\right)\pi^L + \varphi.$$

Using equation (30), equilibrium profits for the $H$ firm are thus[45]

$$\Pi(H,L) = \frac{1+\mu}{2}(1-\lambda)\left(\left(\lambda + \frac{1-\mu}{1+\mu}(1-\lambda)\right)\pi^L + \varphi\right). \tag{37}$$

[44] Note that profits are positive regardless of the share of informed consumers.

[45] Again, profits are positive regardless of the share of informed consumers. The higher $\mu$, the lower the profits.

Since firm $H$ must be indifferent between playing $\theta^H q^H - \Delta\theta q^L$ and $\underline{p}^H$, it follows that the mass that firm $L$ puts at the upper bound in case 2, $\omega_2^l$, must satisfy

$$\left(\frac{1-\mu}{2}+\mu\omega_2^l\right)(1-\lambda)\left(\pi^L+\varphi\right)=\frac{1+\mu}{2}(1-\lambda)\left(\frac{1-\mu+2\lambda\mu}{1+\mu}\pi^L+\varphi\right).$$

After some algebra,

$$\omega_2^l=\frac{\varphi+\lambda\pi^L}{\varphi+\pi^L}. \tag{38}$$

equations (33) and (38) cross at a single value of $\mu$, call it $\mu^*$. Since $\omega_1^l$ is decreasing in $\mu$ and $\omega_2^l$ is flat, $\omega_1^l \geq \omega_2^l$ if and only if $\mu \leq \mu^*$. At $\mu^*$, since $\omega_1^l = \omega_2^l$, the profits made by firm $H$ in cases 1 and 2 must coincide, i.e., expressions (32) and (37) must be equal. At $\mu^*$, using expressions (32) and (37),

$$\frac{1-\mu}{2}(1-\lambda)\pi^H=\frac{1+\mu}{2}(1-\lambda)\left(\frac{1-\mu+2\lambda\mu}{1+\mu}\pi^L+\varphi\right)$$

and rearranging,

$$\frac{1+\mu}{2}\left(\frac{1-\mu}{1+\mu}\pi^H-\varphi\right)=\left(\frac{1+\mu}{2}\lambda+\left(\frac{1-\mu}{2}\right)(1-\lambda)\right)\pi^L.$$

This equation is satisfied at $\mu=\widetilde{\mu}$ (from equation (35), recall how we defined $\widetilde{\mu}$). Hence, we must have $\mu^*=\widetilde{\mu}$.

This has important implications for equilibrium existence and uniqueness. First, for $\mu<\widetilde{\mu}$, $\omega_1^l>\omega_2^l$ implies that firm $H$ makes more profits at the unconstrained monopoly price than at the constrained monopoly price. Hence, the equilibrium characterized in case 2 cannot exist: the unique equilibrium is the one characterized in case 1. Second, for $\mu\geq\widetilde{\mu}$, profits for firm $L$ in case 1 fall below its minmax, so that the unique equilibrium is the one characterized in case 2. Last note that the equilibrium is continuous in $\mu$. In particular, the masses with which firms play their upper bounds are continuous in $\mu$ at $\widehat{\mu}$ as we move form the PSE to the MSE in case 1, and at $\widetilde{\mu}$ when we move to the MSE in cases 1 to 2.

To sum up, equilibrium profits are as follows, for $\widehat{\mu}$ and $\widetilde{\mu}$ defined in (19) and (36).

For $\mu\in[0,\widehat{\mu}]$ :

$$\begin{aligned}\Pi(H,L) &= \frac{1-\mu}{2}(1-\lambda)\pi^H\\ \Pi(L,H) &= \frac{1+\mu}{2}\pi^L.\end{aligned} \tag{39}$$

For $\mu\in[\widehat{\mu},\widetilde{\mu}]$ :

$$\begin{aligned}\Pi(H,L) &= \frac{1-\mu}{2}(1-\lambda)\pi^H\\ \Pi(L,H) &= \frac{1+\mu}{2}\left(\frac{1-\mu}{1+\mu}\pi^H-\varphi\right).\end{aligned} \tag{40}$$

For $\mu \in [\widetilde{\mu}, 1]$ :

$$\begin{aligned}\Pi(H,L) &= \frac{1+\mu}{2}(1-\lambda)\left(\left(\lambda+\frac{1-\mu}{1+\mu}(1-\lambda)\right)\pi^L+\varphi\right)\\ \Pi(L,H) &= \left(\frac{1+\mu}{2}\lambda+\frac{1-\mu}{2}(1-\lambda)\right)\pi^L. \end{aligned} \tag{41}$$

*Q.E.D.*

**Proof of Proposition 8 [product choices]** We have to solve a 4x4 game as each firms has four potential choices $\{\varnothing, L, H, LH\}$. To prove that $(LH, H)$ is the unique SPE of the game we proceed as follows: first, we show that $L$ and $\varnothing$ are dominated by $LH$; second, we show that in the reduced 2x2 game (once $L$ and $\varnothing$ have been eliminated), $LH$ is a dominant strategy.

It is trivial to show that $\varnothing$ is dominated by $LH$ since profits at $LH$ are strictly positive. To show that $L$ is dominated by $LH$, we compare the profit expressions in the paper,

$$\begin{aligned}\Pi(L,\varnothing)-\Pi(LH,\varnothing) &= -(1-\lambda)\varphi<0\\ \Pi(L,L)-\Pi(LH,L) &= -\frac{1+\mu}{2}(1-\lambda)\varphi<0\\ \Pi(L,LH)-\Pi(LH,LH) &= -\frac{1-\mu}{2}(1-\lambda)\varphi<0.\end{aligned}$$

There remains to compare the incentives to deviate from $(L, H)$ to $(LH, H)$. Even though we do not have a closed-form expression for $\Pi(L, H)$, it is not difficult to see that it must be lower than $\Pi(LH, H)$ since by adding product $H$, firm $L$ can *at least* increase its profits by selling $H$ to the non-shoppers at the (constrained) monopoly price, and thus make extra profits $(1-\mu)(1-\lambda)\varphi/2$, while maintaining the exact same expected profits on the low types. Formally, we need to consider three regions for $\mu$: from the proofs of Propositions 5 and 7 we have that, for $\mu \in [\widetilde{\mu}, 1)$, where $\widetilde{\mu} > \widehat{\mu}$, profits for the firm carrying product $L$ are (41) at $(L, H)$ and (23) at $(LH, H)$. Substracting yields

$$\Pi(L,H)-\Pi(LH,H) = -\frac{1-\mu}{2}(1-\lambda)\left(\pi^H-\pi^L\right)<0,$$

which shows that $L$ would deviate to also carry good $H$ whenever $\mu \in [\widetilde{\mu}, 1)$.

From the same two proofs we also have that, for $\mu \in [\widehat{\mu}, \widetilde{\mu})$, profits for the firm carrying product $L$ are (40) at $(L, H)$ and (23) at $(LH, H)$. The difference is

$$\Pi(L,H)-\Pi(LH,H) = \frac{1}{2}\lambda\left(\pi^H(1-\mu)-\pi^L(1+\mu)\right)-\frac{1+\mu}{2}\varphi.$$

Since the above expression is decreasing in $\mu$, it suffices to check its sign at $\mu = \widehat{\mu}$,

$$\Pi(L,H) - \Pi(LH,H) = -(1-\lambda)\frac{\pi^H \varphi}{\varphi + \pi^H + \pi^L} < 0.$$

Since it is negative, it again shows that firm $L$ would deviate to also carry good $H$ whenever $\mu \in [\widehat{\mu}, \widetilde{\mu})$.

Last, for $\mu \in (0, \widehat{\mu})$, profits for the firm carrying product $L$ are (39) at $(L,H)$ and (17) at $(LH,H)$. The difference is

$$\Pi(L,H) - \Pi(LH,H) = -\frac{1+\mu}{2}(1-\lambda)\varphi < 0,$$

which completes the proof that $L$ is a dominated strategy for all $\mu < 1$.

Second, in the reduced 2x2 game, we show that $H$ is dominated by $LH$. For the comparison of $(H,H)$ and $(LH,H)$ we again need to consider two cases: for $\mu \in [0, \widehat{\mu}]$, for $\mu \in (\widehat{\mu}, 1)$ if $c^H > \theta^L q^H$ or for $\mu \in (\widehat{\mu}, \mu^*)$ if $c^H > \theta^L q^H$ :

$$\Pi(H,H) - \Pi(LH,H) = -\left(\frac{1-\mu}{2} + \mu\right)\lambda\pi^L < 0,$$

For $\mu \in (\mu^*, 1)$ if $c^H < \theta^L q^H$

$$\Pi(H,H) - \Pi(LH,H) = -\left(\frac{1-\mu}{2} + \mu\right)\lambda\left(\Delta c - \theta^L \Delta q\right) < 0,$$

Last, comparison of $(H,LH)$ and $(LH,LH)$ shows that,

$$\Pi(H,LH) - \Pi(LH,LH) = -\frac{1-\mu}{2}\left(\lambda\pi^L - (1-\lambda)q^L\Delta\theta\right)\right) < 0.$$

where the last inequality follows from $(A1)$.

This establishes that the unique equilibrium is for both firms to carry both products and price them according to Proposition 3. *Q.E.D.*

# Appendix B: Robustness Checks

## 7.1 Correlation between search costs and quality types

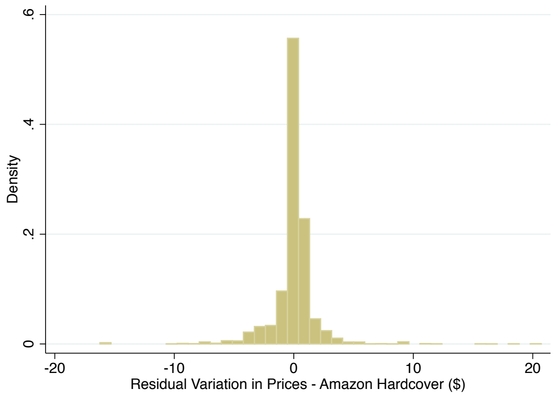
So far, we have assumed that shoppers and non-shoppers are equally likely to be either high or low types. This may not hold in practice. For instance, if low types are lower income consumers with more time to search, then non-shoppers are more likely to be high types. Alternatively, if high types enjoy shopping for their preferred (high quality) product, then non-shoppers are more likely to be low types. Ultimately, this is an empirical question whose answer may vary depending on the type of product. The objective here is to understand the consequences for our main results (Propositions 1 and 2) of introducing correlation between search costs and quality types.

Proposition 1 remains intact since all consumers are shoppers by definition. As for the implications for Proposition 2, consider first the case in which the non-shoppers are more likely to be high types relatively to the shoppers. One can formalize this by assuming $\lambda^S > \lambda^{NS}$, where $\lambda^S$ and $\lambda^{NS}$ are the fraction of low types among the shoppers and non-shoppers, respectively (this assumption introduces positive correlation between search costs and quality types). Since profits on good $H$ are proportional to $\left(1-\lambda^{NS}\right)$, firm $L$ would have stronger incentives now to also carry $H$ if it anticipates that its rival is only carrying $H$. The same is true if it anticipates that its rival is carrying both products: not carrying $H$ would entail an even higher profit loss of $(1-\mu)(1-\lambda^{NS})\varphi/2$. Just as in the case of no correlation, this destroys the "specialization equilibrium".

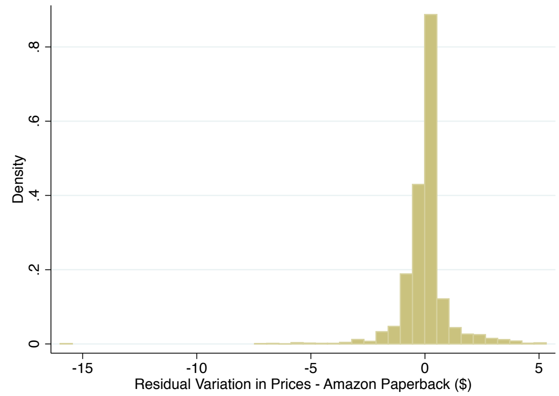
As long as there exists a positive fraction, no matter how small, of non-shoppers high types, our main predictions also prevail under the alternative case in which the non-shoppers are more likely to be low types (i.e., $\lambda^S < \lambda^{NS} < 1$). Firm $L$'s incentives to deviate from the "specialization equilibrium" would disappear only in the unlikely case that all the non-shoppers are low types ($\lambda^{NS} = 1$). Anticipating this, the firm carrying good $H$ would also refrain from carrying $L$ since the gains would more than offset the losses from intensifying competition. Thus, the "specialization" equilibrium of Proposition 1 would be reestablished. This is not a surprising result, given that the case with $\lambda^{NS} = 1$ is similar to the case with no search costs.

# Appendix C: Additional Figures

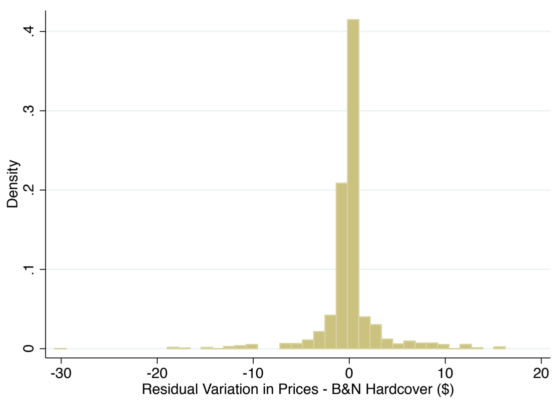
Figure 3: Residual Dispersion in Prices



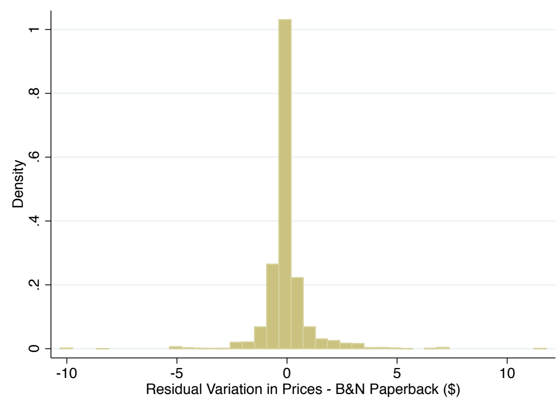

(a) Amazon — Hardcover




(b) Amazon — Paperback

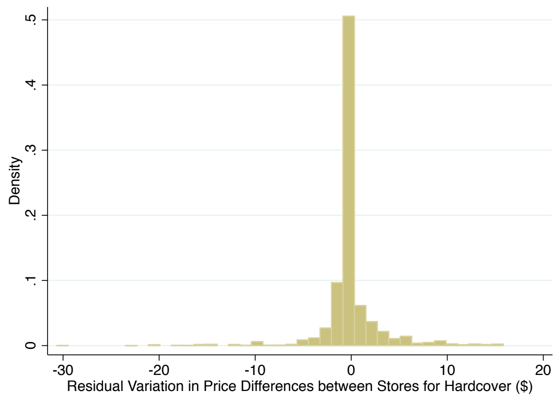



(c) B&N — Hardcover




(d) B&N — Paperback

Figure 4: Residual Dispersion in Price Differences between Stores

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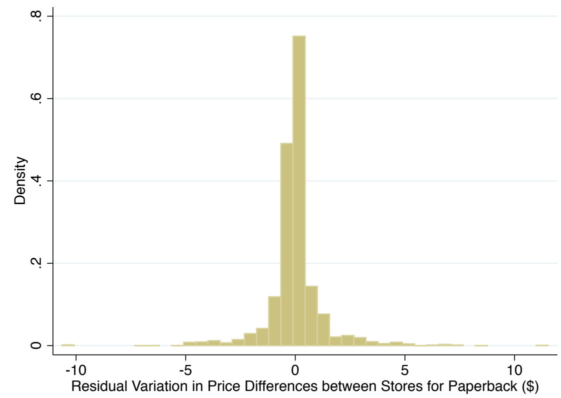
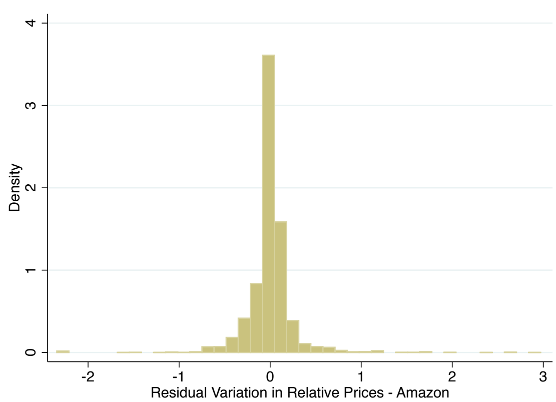
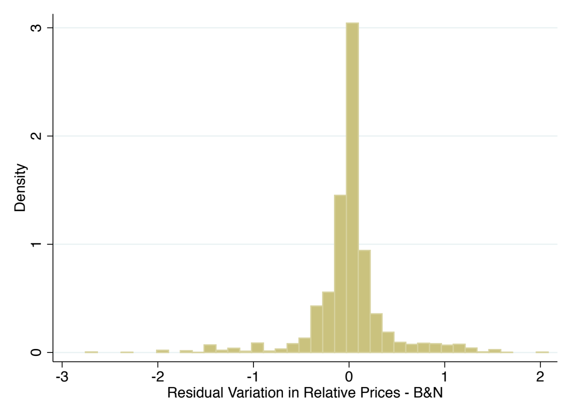



(b) Paperback

Figure 5: Residual Dispersion in Relative Prices




(a) Hardcover




(b) Paperback

# References


[1] Armstrong, M. (2015), Search and Rip-offs Externalities, *Review of Industrial Organization* 47(3), 273–302.

[2] Armstrong, M. and Vickers, J. (2002), Competitive Price Discrimination, *RAND Journal of Economics* 32(4), 1-27.

[3] Anderson, S. and Renault, R. (1999), Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model, *RAND Journal of Economics* 30(4), 719-735.

[4] Borenstein, S. and Rose, N.L. (1994), Competition and Price Dispersion in the U.S. Airline Industry, *Journal of Political Economy*, 102, 653-68.

[5] Brander, J. and Eaton, J. (1984), Product-line Rivalry, *American Economic Review* 74, 323-334.

[6] Burdett, K. and Judd, K.L. (1983), Equilibrium Price Dispersion, *Econometrica* 51(4), 955-969.

[7] Busse, M. and Rysman, M. (2005), Competition and Price Discrimination in Yellow Pages Advertising, *RAND Journal of Economics* 36(2), 378-390.

[8] Chamberlin, E. H. (1933), The Theory of Monopolistic Competition. Cambridge, MA: Harvard University Press.

[9] Champsaur, P. and Rochet, J.-C. (1989), Multiproduct Duopolists, *Econometrica* 57, 533-557.

[10] Crawford, G. and Shum, M. (2007), Monopoly Quality Degradation in Cable Television, *Journal of Law and Economics* 50.

[11] Diamond, P. (1971), A Model of Price Adjustment, *Journal of Economic Theory* 3, 156-168.

[12] De Los Santos, B., Hortaçsu, A. and Wildenbeest, M. (2012), Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior, *American Economic Review* 102(6), 2955-80.

[13] Fabra, N. and Reguant, M. (2017), A Model of Search with Price Discrimination, mimeo, Northwestern University.

[14] Gal-Or, E. (1983), Quality and Quantity Competition, *Bell Journal of Economics* 14, 590-600.

[15] Garret, D., Gomes, R., and Maestric, L. (2016), Competitive Screening under Heterogeneous Information, mimeo, Toulouse School of Economics.

[16] Hong, H., and Shum, M. (2006), Using Price Distributions to Estimate Search Costs, *RAND Journal of Economics* 37(2), 257-75.

[17] Iyer, G., and Seetharaman, P.B. (2003), To Price Discriminate or Not: Product Choice and the Selection Bias Problem, *Quantitative Marketing and Economics* 1, 155-178.

[18] Johnson, J.P., and Myatt, D.P. (2003), Multiproduct Quality Competition: Fighting Brands and Product Line Pruning, *American Economic Review*, 749-774.

[19] Johnson, J.P., and Myatt, D.P. (2015), The Properties of Product Line Prices, *International Journal of Industrial Organization* 43, 182-188.

[20] Kaplan, G., Menzio, G., Rudanko, L. and Thachter, N. (2016), Relative Price Dispersion: Evidence and Theory, NBER Working Paper No. 21931.

[21] Klemperer, P. (1992), Equilibrium Product Lines: Competing Head-to-Head May be Less Competitive, *American Economic Review* 82(4), 740-55.

[22] Leslie, P. (2004), Price Discrimination in Broadway Theater, *RAND Journal of Economics* 35, 520-541.

[23] Mc Manus, B. (2007), Nonlinear Pricing in an Oligopoly Market: The Case of Specialty Coffee, *RAND Journal of Economics* 38, 513-533.

[24] Miravete, E. and Röller, L. (2004), Estimating Markups under Nonlinear Pricing Competition, *Journal of the European Economic Association* 2(2/3), 526-535.

[25] Möller, M. and Watanabe, M. (2016), Competition in the Presence of Individual Demand Uncertainty, *RAND* Journal of Economics, 47(2), 273-292.

[26] Moraga González, J.L., Sandor, Z. and Wildenbeest, M. (2017), Prices and Heterogeneous Search Costs, *RAND Journal of Economics* 48(1), 125-146.

[27] Mussa, M. and Rosen, S. (1978), Monopoly and Product Quality, *Journal of Economic Theory* 18, 301-317.

[28] Nevo, A. and Wolfram, C. (2002), Why Do Manufacturers Issue Coupons? An Empirical Analysis of Breakfast Cereals?, *RAND Journal of Economics* 33, 319-339.

[29] Nocke, V., Peitz, M. and Rosar, F. (2011), Advance-Purchase Discounts as a Price Discrimination Device, *Journal of Economic Theory* 146(1), 141-162.

[30] Pesendorfer, M. (2002), Retail Sales: A Study of Pricing Behavior in Supermarkets, *Journal of Business* 75(1), 33-66.

[31] Rhodes, A. (2015), Multiproduct Retailing, *Review of Economic Studies* 82(1), 360-390.

[32] Rhodes, A., and Zhou, J. (2016), Consumer Search and Retail Market Structure, mimeo, Toulouse School of Economics.

[33] Rochet, J.-C. and Stole, L. (2002), Nonlinear Pricing with Random Participation, *Review of Economic Studies* 69, 277-311.

[34] Seim K. and Sinkinson, M. (2016), Mixed Pricing in Online Marketplaces, *Quantitative Marketing and Economics* 14 (2), 129-155.

[35] Shaked, A. and Sutton, J. (1982), Relaxing Price Competition Through Product Differentiation, *The Review of Economic Studies* 49(1), 3-13.

[36] Shepard, A. (1991), Price Discrimination and Retail Configuration, *Journal of Political Economy* 99(1), 30-53.

[37] Stahl, D.O. (1989), Oligopolistic Pricing with Sequential Consumer Search, *American Economic Review* 79(4), 700-712.

[38] Stole, L. (1995), Nonlinear Pricing and Oligopoly, *Journal of Economics & Management Strategy* 4, 529-562.

[39] Stole, L. (2007), Price Discrimination and Competition, *Handbook of Industrial Organization*, 3 (eds. M. Armstrong and R. Porter), North-Holland.

[40] Tirole, J. (1988), The Theory of Industrial Organization, Cambridge, MA: MIT Press.

[41] Villas-Boas, M. (2004), Communication Strategies and Product Line Design, *Marketing Science* 23, 304-316.

[42] Varian, H. (1980), A Model of Sales, *American Economic Review* 70(4), 651-659.

[43] Wernerfelt, B. (1988), Umbrella Branding as a Signal of New Product Quality: an example of signalling by posting a bond, *RAND Journal of Economics* 19, 458-466.

[44] Wildenbeest, M. R. (2011), An Empirical Model of Search with Vertically Differentiated Products, *RAND Journal of Economics* 42(4), 729-757.

[45] Zhou, J. (2014), Multiproduct Search and the Joint Search Effect, *American Economic Review*, 104(9), 2918-2939.