Meeting Technologies in Decentralized Asset Markets

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Abstract

We study decentralized trading networks where agents differ in both their time-varying taste for an asset and the constant frequency at which they meet others. We demonstrate that fast agents endogenously arise as intermediators whose net valuation of the asset gets moderated through their exposure to others. We show that allocating meetings in an ex-ante asymmetric fashion across agents generates higher welfare then a homogeneous distribution of meeting frequencies, only if some agents intermediate. We also characterize properties of the market equilibrium in which ex-ante identical agents choose their meeting rates, and show that an equilibrium with symmetric meeting rates does not exist. Finally we compare the properties of equilibrium outcome with the planner allocation.

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1 Introduction

A key feature of many asset markets is that agents trade in bilateral meetings and that the search for a counterparty is frictional. Another key feature of many asset markets is the vast heterogeneity across participants in terms of trading frequencies, trading volume, and number of trading partners. A natural candidate explanation is that the this heterogeneity reflects heterogeneous meeting technologies. For instance, one might think of some agents having invested in faster communication technologies, better visibility through location choices or advertisement, or relationships with more counterparties.

In this paper, we explore the consequences of such heterogeneity for equilibrium outcomes in a frictional asset market and offer an explanation for the coexistence of heterogeneous market participants. We further show that, from a normative viewpoint, such heterogeneity is in fact desirable. To that end, we model an over-the-counter asset market where individuals trade in stochastic bilateral meetings as in Rubinstein and Wolinsky (1987). As is standard in the literature following Duffie et al. (2005), we focus on an environment where the underlying reason for the trade of an asset are differences in the current flow utility agents receive from holding the asset. As in Vayanos and Wang (2007), Weill (2008), Hugonnier et al. (2014), Shen and Yan (2014), and Neklyudov (2013) time-varying tastes for the asset capture differential liquidity needs or hedging motives of individuals. Yet in our framework, market participant differ along a second dimension, namely in terms of how often they meet others. Specifically, individuals are endowed with search efficiencies that allow them to randomly and stochastically meet others at certain pace. Since agents meet bilaterally, this also implies that a given agent is more likely to draw a meeting partner with a high search efficiency.

The positive part of the paper characterizes equilibrium features under heterogeneous search efficiencies and then shows that heterogeneity is a natural market feature if agents can ex-ante invest into a search technology. We first show that faster individuals endogenously emerge as intermediaries in equilibrium. In particular, faster agents are more willing to take on mismatched asset positions that do not align with their idiosyncratic flow valuation. A fast agent with little taste for an asset is nonetheless willing to buy it from a slow agent with similar taste simply because she is more able to find a buyer with a taste for the asset. In turn, faster agents with a taste for an asset sell it to slow agents with similar taste because they are better at finding sellers. In order words, faster market participants have a larger option value of buying and selling which endogenously buffers the impact of their individual taste for the asset on the net valuation they ascribe to it. As a consequence of their moderate valuations these individuals emerge as middlemen at the center of the intermediation chain while slower individuals stay at its fringe.[1] Another way of putting this is that the equilibrium displays a core-periphery structure where the identity of the market participants at the core remains stable over time.

We then endogenize the equilibrium distribution of trading efficiencies by letting agents buy an expected meeting frequency ex ante. We show that not only a symmetric, degenerate distribution is not an equilibrium outcome if the search cost function is continuous and differentiable, but also the equilibrium distribution does not have any mass points. The intuition is that individuals positioned right above a mass point conduct substantially different types of trades than individuals right below a mass point, which introduces a convex kink into the profit function. In other words, the difference in the nature of trading opportunities right above and below a mass point implies a discontinuity in marginal benefit to speed. We proceed by characterizing the features of the distribution of search efficiencies that is the outcome of the ex ante investment stage.

We then evaluate the normative consequences of heterogeneity. In particular, we characterize both the optimal trading pattern and the optimal distribution of search efficiencies and contrast those with the equilibrium. We let a planner dictate a time-invariant set of trading rules so as to maximize steady state welfare. We find that the efficient trading pattern coincides with the equilibrium trading pattern given a distribution of search efficiencies. The planner dictates that faster agents take on mismatched positions, that is to intermediate, because of their superior ability to locate other mismatched individuals.

Next, we study the optimal allocation of search efficiencies across the population. Specifically, we endow the economy with an aggregate meeting technology that fixes the overall measure of bilateral meetings in the economy and then distribute search efficiencies across the population so as to maximize steady state welfare, with the optimal trading pattern. Our preliminary theoretical results establish that the planner prefers an intermediate level of heterogeneity over extreme solutions. To show this, we first contrast an economy with homogeneous speed with an extremely asymmetric economy where a subset of agents are in autarky and the remainder have accordingly higher meetings rates, and show that the symmetric economy has higher welfare. Yet we show next that the symmetric economy is dominated in terms of welfare by an asymmetric perturbation where a subset of agents are slightly faster than the fixed average meeting rate and the remainder accordingly slower. Intuitively, in a homogeneous environment, the asset solely flows from individuals with low to individuals with high taste. In an environment with heterogeneous search efficiencies,

[1] The result that speed moderates valuation is most easily understood by considering an agent whose meeting rate is very high. The value such an agent associates with holding the asset is close to independent of her current flow valuation because she finds another trading partner at very high pace.

this still happens yet there is additional trades that enhance welfare, namely trades where faster agents take on mismatch and intermediate. To show that it is the intermediation activity which enhances welfare we also prove that a heterogeneous environment is dominated by a homogeneous one if we do not allow trades across agents with common flow valuation. Finally, our simulation results suggest that the optimal distribution of speed across agents is continuous, that is the planners solution does not feature any mass points either. The logic behind the result is similar to the result that rules out a symmetric solution: One can "split" any mass of agents with homogeneous speed into two groups with marginally higher and lower speed leaving their interactions with the rest of the economy unchanged yet yielding new gainful trading opportunities across the two subgroups.

The last section offers several numerical exercises which characterize socially optimal and equilibrium distribution of search efficiency, and discusses how they compare to each other. We show that equilibrium distribution is too concentrated compared to the optimal distribution.There are too few very slow and too few very fast agents.

There are two distinct sources of inefficiency which leads to this difference: the first one is a classic bargaining inefficiency which leads to under-investment in search efficiency, as each individual does not take into account the fraction of surplus from trade which accrues to his counterparty. The second type of inefficiency is more novel and specific to our model: an individual does not internalize that a high search efficiency improves the overall allocation and decreases the (overall) gains to speed, which leads to over-investment. Our numerical results suggest that with linear cost function, the under-investment ineffciency dominates.

Literature Review: Our paper is closely related to a growing body of work on dynamic trading with search frictions, initiated by Duffie et al. (2005), and followed by Lagos and Rocheteau (2009), Weill (2008), and Hugonnier et al. (2014) among many others. However, in these setups agents meeting rates are either homogeneous, or there is an exogenously given market making sector which facilitates trade, as in Duffie et al. (2005) and Neklyudov (2013). We add to this literature by showing how market makers arise endogenously in an environment with heterogeneous meeting technologies and why such a market feature is a natural consequence of technology choice.

Hugonnier et al. (2014) show that intermediation chains emerge when agents can have a wide set of different flow valuation rather than just two. In particular, individuals with moderate current tastes act as intermediators, buying and selling to individuals with currently more extreme taste. In contrast to their setup, ours offers a theory where the identity of the individuals at the center of the intermediation chain remains stable over time which is a key empirical feature of many decentralized asset markets (see, for instance Bech and

Atalay (2010) for the federal funds market).

Chang and Zhang (2015) provide an alternative model of intermediation where agents differ in how volatile their taste for an asset is. As in our framework agents with more moderate types act as intermediators. The key difference is that we explicitly model the underlying reason for heterogeneous volatility and show why it is natural equilibrium outcome for ex-ante identical agents. Üslü (2015) offers a setup that allows for rich heterogeneity focusing on heterogeneity in pricing and inventories. Pagnotta and Philippon (2015) also considers the effect of differential speed on efficiency of financial markets. Unlike our model, they focus on a centralized model of trade so bargaining and search frictions play no role in their model.

The rest of the paper is organized as follows: Section 2 lays out the model, section 3 characterizes the equilibrium trading pattern and endogenizes the equilibrium distribution of search efficiencies. Section 4 discusses properties of the welfare maximizing trading pattern and distribution of search efficiencies, and section 5 provides several numerical examples. Section 6 concludes.

2 Model

Time is continuous and there is a perfectly divisible endowment of a homogeneous asset of measure $M = \frac{1}{2}$. The economy is populated by a measure one of infinitely lived agents who have time-varying taste for the asset and discount the future at rate ρ. Following Duffie et al. (2005), we assume that agents have current taste $s \in \{l, h\}$ which switches stochastically at rate γ. An agent with taste s receives flow payoff δ_s from holding the asset where $\delta_h > \delta_l$. We restrict agents' asset holdings to $j \in \{0, 1\}$. We call agents *mismatched* if they hold the asset in state l and if they do not hold the asset in state h.

Agents meet each other in bilateral meetings but the search process for a meeting partner is frictional. Specifically, an individual of type λ meets another individual at rate λ. An individual's type λ is time-invariant and distributed according to $G(\lambda)$. We use search efficiency, meeting rate, or speed to refer to λ, interchangeably. Search is random and the matching function is such that the probability of meeting any other individual λ' is proportional to their type λ', and independent of their current preference or asset position. Let $\Lambda = \int_0^\infty x dG(x)$ be the average meeting rate in the economy. Then, conditional on a meeting, the meeting partner has type $\lambda' \leq z$ with probability $\frac{\int_0^z x dG(x)}{\Lambda}$. Let $\mu_{s,j}(\lambda)$ denote the fraction of agent with type λ with taste s and asset holdings j, $\sum_{s,j} \mu_{s,j}(\lambda) = 1$, $\forall \lambda$. Finally, note that at any point half of the agents own an asset which implies that half of all meetings are between asset holders and non-holders.

If one agent in a meeting holds the asset and the other does not, the agents trade the asset when there is strictly positive surplus from doing so. In this event, the price is set via symmetric Nash bargaining as in Rubinstein and Wolinsky (1987) and allocates half of the joint surplus to either party.

We then endogenize the latter by studying the investment decision of ex-ante identical agents who can acquire a search technology. Specifically, agents commit ex-ante to permanently rent a search technology λ at flow cost $C(\lambda)$ so as to maximize their expected present value taking the equilibrium trading pattern as given. We assume $C(\lambda)$ to be strictly increasing, continuous, and differentiable. For expositional purposes we first characterize the equilibrium trading pattern given $G(\lambda)$ and then study agents' investment choices.

Value Functions

Let $V_{s,j}(\lambda)$ be the value of an individual with meeting technology λ, current taste s and current asset holdings j. Further, let $S_s(\lambda) \equiv V_{s,1} - V_{s,0}$ capture the net value of asset ownership. Denoting by $\tilde{s}$ the opposite taste of s, we can write the values of owning and not owning as

$$\begin{aligned} \rho V_{s,1}(\lambda) &= \delta_s + \gamma\left(V_{\tilde{s},1} - V_{s,1}\right) + \frac{\lambda}{2}\sum_{s'}\int \frac{\lambda'}{\Lambda}\max\left\{S_{s'}(\lambda') - S_s(\lambda), 0\right\}\mu_{s',0}(\lambda')\,dG(\lambda') \\ \rho V_{s,0}(\lambda) &= \gamma\left(V_{\tilde{s},0} - V_{s,0}\right) + \frac{\lambda}{2}\sum_{s'}\int \frac{\lambda'}{\Lambda}\max\left\{S_s(\lambda) - S_{s'}(\lambda'), 0\right\}\mu_{s',1}(\lambda')\,dG(\lambda'). \end{aligned}$$

Asset owners receive flow payoff δ_s, switch taste at rate γ, and meet others at rate λ. They draw meeting partners proportional to their speed λ' and, if they meet a non-owner who has a higher net valuation, they sell the asset and receive half of the gains from trade. Non-owners buy when they meet an owner with lower net valuation.

Using these expressions we can write the net value of asset ownership as

$$\rho S_s(\lambda) = \delta_s + \gamma\left(S_{\tilde{s}}(\lambda) - S_s(\lambda)\right) + \frac{\lambda}{2}O(\lambda) \tag{1}$$

where

$$O(\lambda) = \left(\sum_{s'} \int \frac{\lambda'}{\Lambda} \max\left\{S_{s'}(\lambda') - S_s(\lambda), 0\right\} \mu_{s',0}(\lambda')\, dG(\lambda')\right) - \left(\sum_{s'} \int \frac{\lambda'}{\Lambda} \max\left\{S_s(\lambda) - S_{s'}(\lambda'), 0\right\} \mu_{s',1}(\lambda')\, dG(\lambda')\right)$$

captures the option value of finding a buyer with a higher valuation net of the foregone option value of finding a seller with a lower valuation.

$S_s(\lambda)$ consists of a term capturing the flow utility and an option value term that is weighted by λ. $O(\lambda)$ has a dampening effect on an individuals net valuation: An agent who values the asset relatively highly takes into account that it is unlikely she will be able to profitably sell it and her net option value $O(\lambda)$ will thus be low. An agent who associates little value with the asset takes into account that she will likely be to profitably sell it and her net option value $O(\lambda)$ will thus be high. Jointly with the observation that $O(\lambda)$ is weighted by λ this heuristically suggests that search efficiency moderates the impact of taste on an agent valuation of an asset.

Law of Motion

Let $N(s, \lambda | s', \lambda')$ denote the endogenous probability that agent (s, λ) holds the asset after a meeting with (s', λ').[2] With this definition, we can write the law of motion for μ, suppressing the time dependence as

$$\begin{aligned} \dot{\mu}_{s,0}(\lambda) &= \gamma \mu_{\tilde{s},0}(\lambda) + \lambda \mu_{s,1}(\lambda) \int \frac{\lambda'}{\Lambda} \left(\sum_{s'} \mu_{s',0}(\lambda') N(s', \lambda' | s, \lambda)\right) dG(\lambda') \\ &\quad - \gamma \mu_{s,0}(\lambda) - \lambda \mu_{s,0}(\lambda) \int \frac{\lambda'}{\Lambda} \left(\sum_{s'} \mu_{s',1}(\lambda') N(s, \lambda | s', \lambda')\right) dG(\lambda') \\ \dot{\mu}_{s,1}(\lambda) &= \gamma \mu_{\tilde{s},1}(\lambda) + \lambda \mu_{s,0}(\lambda) \int \frac{\lambda'}{\Lambda} \left(\sum_{s'} \mu_{s',1}(\lambda') N(s, \lambda | s', \lambda')\right) dG(\lambda') \\ &\quad - \gamma \mu_{s,1}(\lambda) - \lambda \mu_{s,1}(\lambda) \int \frac{\lambda'}{\Lambda} \left(\sum_{s'} \mu_{s',0}(\lambda') N(s', \lambda' | s, \lambda)\right) dG(\lambda') \end{aligned} \tag{2}$$

[2] The object $N(s, \lambda | s', \lambda')$ is not strictly needed for the equilibrium characterization since it turns out that $N(s, \lambda | s', \lambda') = 1$ if $S_s(\lambda) > S_{s'}(\lambda)$and $N(s, \lambda | s', \lambda') = 0$ and $S_s(\lambda) < S_{s'}(\lambda)$ while any non-trivial interior cases will be ruled out. However, it is a useful object for the normative analysis which is why we introduce it here.

3 Equilibrium Characterization

The key equilibrium objects are the endogeneous distribution of meeting rates $G(\lambda)$, the distribution of asset holdings across agents which can be represented by $\mu_{s,j}(\lambda)$ given $G(\lambda)$, and the pattern at which agents trade, $N(s,\lambda|s',\lambda')$. We next restrict attention to a particular class of equilibria before characterizing, in that order, $\mu_{s,j}(\lambda)$, $N(s,\lambda|s',\lambda')$, and $G(\lambda)$.

Definition 1. [Stationary Symmetric Equilibrium]. We impose two restrictions. First, we impose stationarity, characterized by $\dot{\mu}_{s,j}(\lambda) = 0,\ \forall\ \lambda, s, j$ and time-invariant trading patterns. Second, we impose a particular, but natural form of symmetry: Specifically, we restrict attention to equilibria where, if $N(h,\lambda|h,\lambda') = 1$, then $N(l,\lambda'|l,\lambda) = 1$. In words, we impose that if a type-λ' asset owner in state h sells to a type-λ non-owner also in state h and thus mismatched, then it must be that a type-λ' non-owner buys in state l from a type-λ owner also in state l and thus mismatched. That is, mismatch gets swapped in the same direction.[3]Given these restrictions, no agent can profit by deviating to a different search efficiency or choosing a different set of trading partners in equilibrium.

3.1 Equilibrium Mismatch

The constant rate of taste change γ implies that in equilibrium, $\mu_{h,0}(\lambda)+\mu_{h,1}(\lambda) = \mu_{l,0}(\lambda)+\mu_{l,1}(\lambda) = \frac{1}{2}$. A symmetric equilibrium further implies that mismatch is symmetric $\mu_{h,0}(\lambda) = \mu_{l,1}(\lambda)$, and $\mu_{h,1}(\lambda) = \mu_{l,0}(\lambda)$.[4] It follows that $\mu_{h,0}(\lambda) + \mu_{l,0}(\lambda) = \mu_{h,1}(\lambda) + \mu_{l,1}(\lambda) = \frac{1}{2}$ which implies that agents of any search technology spend, in expectation, half their time in possession of the asset.

Let $\mu(\lambda) = \mu_{h,0}(\lambda) = \mu_{l,1}(\lambda)$, so that $\frac{1}{2} - \mu(\lambda) = \mu_{h,1}(\lambda) = \mu_{l,0}(\lambda)$. Moreover, let $m(\lambda) = 2\mu(\lambda) = \mu_{h,0}(\lambda)+\mu_{l,1}(\lambda)$ denote the fraction of agents of type who are *mismatched*, i.e. either hold the asset when in state l or not hold the asset when in state h. Note that exactly half of the mismatched (matched) agents of each speed are are in each preference state.

[3]To see why this is a natural restriction, note that our setup is isomorphic to the following two-asset economy: There is a measure one of agents. There are two assets, A and B, each in supply $\frac{1}{2}$ so that, at any point in time, each agent holds exactly one unit of either asset. Each agent either likes asset A or asset B, and her desire changes at rate γ. When two agents with different assets meet and there are gains from trade, they swap assets. In this isomorphic environment, the restriction to symmetric equilibria is natural: since everything is symmetric, when two agents who both currently value A trade in one direction we expect them to trade in the same direction when both currently value B.

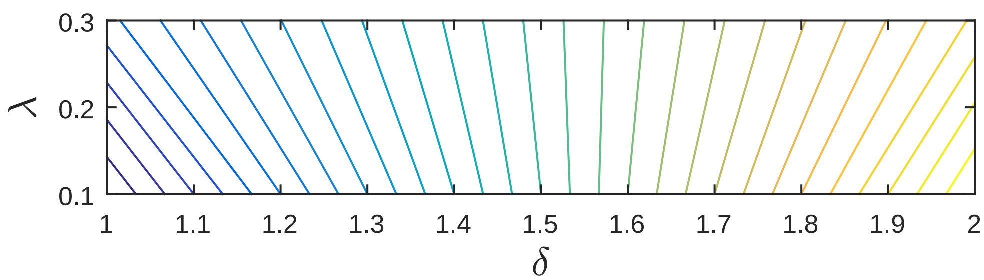
[4]It is straightforward to verify this using the inflow-outflow relations under symmetry.

3.2 The Equilibrium Trading Pattern

Illustration

Before we turn to a more formal treatment of the equilibrium trading pattern we numerically solve the model and illustrate the equilibrium trading pattern in figure 1. For illustrative purposes, we solve a version of the model where agents flow valuation δ can take a large number of different values. A line lies on combinations (δ, λ) that deliver the same net value as defined in equation (1) in equilibrium.[5] Lines are ordered such that lines further to the right correspond to a higher net valuation.

Figure 1: Iso-Net-Value Curves



Notes: We set $G(\lambda)$ to be uniform on $[.1, .3]$. Agents draw a new flow valuation δ from a uniform distribution on $[1, 2]$ at rate $\gamma = .03$ and discount with $\rho = .03$.

The main takeaway from figure 1 is that in this numerical example a high λ indeed moderates the impact of the flow valuation δ on the net value an individual ascribes to holding an asset: In meetings between individuals with low flow value δ the asset flows towards the faster agent. The opposite is true for meetings between individuals with high flow value. It follows that fast agents emerge endogenously as intermediators. They take on the asset from mismatched low-δ individuals even when they also have a low flow valuation and sell the asset to mismatched high-δ individuals even when they also have a high flow valuation. We next formally characterize key features of the equilibrium.

Formal Charaterization

Focusing on symmetric equilibria, let $V_0(\lambda)$ denote the average value of currently mismatched type λ individuals. Since $\mu_{h,0} = \mu_{l,1} \forall \lambda$, we have that $V_0(\lambda) = \frac{1}{2}\big(V_{h,0}(\lambda) + V_{l,1}(\lambda)\big)$. In turn, $V_1(\lambda) = \frac{1}{2}\big(V_{h,1}(\lambda) + V_{l,0}(\lambda)\big)$ denotes the analogous for matched type-λ individuals. Let $S(\lambda) \equiv V_1(\lambda) - V_0(\lambda)$ denote the difference. Further, normalize $\delta_h = 1$ and $\delta_l = 0$. Then,

[5] Given $G(\lambda)$, the equilibrium object $S_s(\lambda)$ determines the pattern of trade.

$$\rho V_0(\lambda) = \gamma S(\lambda) + \frac{\lambda}{4}\int_0^\infty \big(S(\lambda)+S(\lambda')\big)\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')$$
$$+\frac{\lambda}{4}\int_0^\infty \max\{S(\lambda)-S(\lambda'),0\}\frac{\lambda'}{\Lambda}(1-m(\lambda'))dG(\lambda') - C(\lambda)$$

and

$$\rho V_1(\lambda) = \frac{1}{2} - \gamma S(\lambda) + \frac{\lambda}{4}\int_0^\infty \max\{S(\lambda')-S(\lambda),0\}\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda') - C(\lambda).$$

A mismatched individual switches states at rate γ, becoming matched. She meets a mismatched person of type λ' with the opposite asset position at rate $\lambda\lambda' m(\lambda')dG(\lambda')/2\Lambda$ in which case they swap assets, both becoming matched, and share the gain from trade $S(\lambda)+S(\lambda')$ equally. She meets a matched person of type λ' with the opposite asset position at rate $\lambda\lambda'(1-m(\lambda'))dG(\lambda')/2\Lambda$, in which case they swap asset positions and share the gain from trade $S(\lambda)-S(\lambda')$ equally, if there is joint gains from doing so. A matched person gets on average utility $\frac{1}{2}$, switches states at rate γ, meets a mismatched type λ individual with the opposite asset position at rate $\lambda\lambda' m(\lambda')dG(\lambda')/2\Lambda$, in which case they swap asset positions and share the gain from trade $S(\lambda')-S(\lambda)$ equally, if there is joint gains from doing so. Taking the difference, we get

$$(\rho+2\gamma)S(\lambda) = \frac{1}{2} - \frac{\lambda}{4}\int_0^\infty \big(S(\lambda)+S(\lambda')\big)\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')$$
$$+\frac{\lambda}{4}\int_0^\infty \max\{S(\lambda')-S(\lambda),0\}\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda') \tag{3}$$
$$-\frac{\lambda}{4}\int_0^\infty \max\{S(\lambda)-S(\lambda'),0\}\frac{\lambda'}{\Lambda}(1-m(\lambda'))dG(\lambda').$$

We use this expression to establish the following proposition characterizing the equilibrium trading pattern.[6]

Proposition 1. *Given any $G(\lambda)$, the unique symmetric equilibrium trading pattern is such that 1) two mismatched agents always trade, 2) two matched agents never trade, and 3) a matched and a mismatched agents trade iff the matched agent has the better search technology.*

[6] Note that two agents can of course only trade with each other if one of them holds the asset and the other one does not.

Proof. See Appendix. □

The proof shows that net surplus function (3) is strictly decreasing in search efficiency λ. Proposition 1 establishes formally that faster agents *intermediate*: They buy the asset from slower low agents when both have taste δ_l and sell it to them when both have taste δ_h. In other words, they take positions *against* their own taste preference. In doing so, they align slower types asset holdings with their preferences and are compensated by bid-ask spreads. This also implies that faster agents not only meet other agents more frequently but also trade more frequently conditional on a meeting because they take on the mismatch from individuals with lower search efficiency.

3.3 Equilibrium Distribution of λ

Thus far we discussed how given a non-degenerate distribution $G(\lambda)$ faster agents act as intermediaries. However, it is unclear whether heterogeneous meeting technologies are a natural outcome when ex-ante identical agents invest into search efficiency. This subsection shows that this is indeed the case.

To do so, rewrite $S(\lambda)$ using proposition 1

(4)
$$\left(4\rho + 8\gamma + 2\lambda\int_0^\infty \frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')\right)S(\lambda) = 2 + \lambda\int_\lambda^\infty (S(\lambda') - S(\lambda))\frac{\lambda'}{\Lambda}(1-2m(\lambda'))dG(\lambda').$$

The equilibrium surplus function can then be used to solve uniquely for V_0 and V_1. Recall that we assume that ex-ante homogeneous indvididuals commit to permanently pay a strictly increasing flow cost $C(\lambda)$ for the search technology λ. We assume $C(\lambda)$ to be continuous and differentiable. Let $v(\lambda) = \lim_{\rho\to 0}\rho V_0(\lambda) - C(\lambda) = \lim_{\rho\to 0}\rho V_1(\lambda) - C(\lambda)$ denote the average flow value of an agent choosing meeting rate λ as we take discounting to zero.[7] In the Appendix, we derive the following explicit expressions for S(λ) and $v(\lambda)$,

(5)
$$S(\lambda) = \frac{1}{2\rho + 4\gamma}(1 - e^{-\int_\lambda^\infty \phi(\lambda')d\lambda'})$$

(6)
$$v(\lambda) = \frac{1}{4} + \frac{\gamma}{4}e^{-\int_\lambda^\infty \phi(\lambda')d\lambda'} + \frac{\lambda}{4}\int_0^\lambda \left(e^{-\int_\lambda^\infty \phi(\lambda'')d\lambda''} - e^{-\int_{\lambda'}^\infty \phi(\lambda'')d\lambda''}\right)\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda') - C(\lambda)$$

[7] In doing so we can ignore the initial allocation of the asset.

where

$$\phi(\lambda) \equiv \frac{4\rho + 8\gamma}{\lambda\left(4\rho + 8\gamma + 2\lambda \int_0^{\lambda} \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') + \lambda \int_{\lambda}^{\infty} \frac{\lambda'}{\Lambda} dG(\lambda')\right)}$$

and the lower bound of the support, $\underline{\lambda}$ is implicitly determined by

$$\frac{1}{2\rho + 4\gamma} e^{-\int_{\underline{\lambda}}^{\infty} \phi(\lambda') d\lambda'} = C'(\underline{\lambda})\underline{\lambda}\left(8\gamma + \underline{\lambda}\right)/(8\gamma^2). \tag{7}$$

The next proposition applies proposition 1 and equations (5) and (6) to establish that a degenerate distribution $G(\lambda)$ is never an equilibrium and that indeed any equilibrium distribution has continuous density:

Proposition 2. *The equilibrium distribution of search efficiency $G(\lambda)$ has no mass points.*

Proof. See Appendix. □

The above proposition implies that although all agents are ex-ante identical, there is no symmetric equilibrium in which all agents choose identical actions. Even stronger, no positive measure of agents takes the same action.

The distinct nature of the asset market in our model is that it is intermediated: the nature and frequency of an individual's trades depend starkly on her search technology relative to the ones in the population of market participants. We show that no agent chooses to be at a mass point because the trading opportunities in the vicinity of (hypothetical) mass point change in a way that makes the mass point a strictly inferior choice. The logic behind the result is best understood in the context of a single mass point λ^*: An agent choosing $\lambda^* + \epsilon$ intermediates all the agent at the mass point and collects bid-ask spreads proportional to the meeting frequency. As a result her gross profits are proportional to the meeting rate. An agent choosing $\lambda^* - \varepsilon$ is intermediated by all other market participans, so her gross profits are purely allocation driven. The improvement in allocation is concave in the meeting rate, which leads to a *convex kink* at any mass point. With any continuous differentiable cost function, a convex kink in gross profit function can never be an equilibrium.

We highlight that this feature is fundamentally different from the one driving heterogeneity in posted wage offers in the seminal work of Burdett and Mortensen (1998). In that environment the equilibrium wage offer distribution has no mass point either because the value function of a firm is discontinuous at any potential mass point, which leads to a profitable deviation. The same mechanism is applied in recent work on decentralized asset markets by Duffie et al. (2015) where dealers post prices to attract traders, just like firms post wages to attract workers in Burdett and Mortensen (1998). In these frameworks a buyer can discreetly increase the number of profitable transactions by incurring marginally higher cost. In our framework the value functions are *continuous* at mass points and the reason behind the kink is that an agents trading position with respect to a discreet number of events reverses on either side of the mass point.

We conclude this section by characterizing some properties of the equilibrium distribution when the cost function is linear, $C(\lambda) = c\lambda$. First note that in equilibrium, all agents make the same profit, $v(\lambda) = \bar{v}$. Second, if an agent is in autarky, i.e. chooses $\lambda = 0$, his expected value would be a constant equal to the probability that he owns the asset multipled by the expected value of holding the asset, $v_0 = \frac{1}{4}$. We call v_0 the value of being in autarky, and focus on equilibria in which agents make profits strictly higher than autarky,$\bar{v} > v_0$.

Proposition 3. *In any equilibrium in which equilibrium value is strictly larger than autarky, $\bar{v} > v_0$, the equilibrium distribution of search efficiencies $G(\lambda)$ has an open right tail and a strictly positive lower bound $\underline{\lambda} > 0$.*

Proof. See Appendix. □

The above proposition shows than with constant marginal cost, in an equilibrium with strictly positive profits no agent is in autarky. Moreover, there is no level of search efficiency $\bar{\lambda}$ where all agents choose $\lambda < \bar{\lambda}$. The equilibrium distribution is unbounded, i.e. there are agents who are infinitely fast. In other words, although all the agents are ex-ante perfectly identical, ex-post some of them choose to become *market makers*, who are in constant contact with the market. They intermediate every other agent and get compensated by bid-ask spreads.[8][9] Finally, we characterize the net profit that agents make in equilibrium.

Corollary 1. *In equilibrium, all agents make the same profit, $v(\lambda) = \bar{v}$ and $v'(\lambda) = 0$, $\forall\lambda$. Equilibrium profit is given by $\bar{v} = \frac{1}{4} + \gamma k - C(\underline{\lambda})$, where $k = \frac{1}{2\rho+4\gamma} e^{-\int_{\underline{\lambda}}^{\infty} \phi(\lambda')d\lambda'}$ and $\underline{\lambda}$ is defined by equation (7) .*

4 Normative Analysis

In this section we study the efficient distribution of meeting rate among agents, as well as the efficient trading pattern. Before formally defining the planner's problem, we start with an illustrative example.

[8]The only exception is the case where every agent makes profits equal to autarky in equilibrium, in which case a bounded support is also possible.

[9]If the cost function is convex enough, all the agent choose to be in autarky, $G(0) = 1$.

Example 1. Normalize $\gamma = 1$ and assume the meeting rate λ is identical across all individuals in the economy. The fraction of mismatched agents in the economy is

$$\mu(\lambda) = \mu_{h,0}(\lambda) = \mu_{l,1}(\lambda) = \frac{\sqrt{2+\lambda} - \sqrt{2}}{\sqrt{2}\lambda}$$

A notion of inefficiency is the size mismatch $\mu(\lambda)$, the fraction of individuals who value the asset but do not hold it. For large values of λ, this is well-approximated by $1/\sqrt{2\lambda}$, and in particular the mismatch rate declines with the square root of the matching efficiency.

We now consider how intermediation may improve efficiency in this economy. Holding the average search efficiency fixed at λ fixed we redistribute search efficiency unevenly across the population. In particular, we give a fraction α of the total flow of meetings λ to a fraction $1-\varepsilon$ of the population and the remaining $1-\alpha$ to a fraction ε of the population, called intermediaries. We also assume, as their name suggests, that intermediaries intermediate trade when they meet a regular trader.

Now let λ_1 and λ_2 denote the trading speed of regular traders and intermediaries, with $(1-\varepsilon)\lambda_1 = \alpha$ and $\varepsilon\lambda_2 = 1-\alpha$. Inefficiency is again given by the measure of individuals in the high preference state without the asset, $\mu(\lambda_1)(1-\varepsilon) + \mu(\lambda_2)\varepsilon$.

Using the inflow-outflow equations (2), we can solve explicitely for the fraction of mismatched agents in each type. The resulting solution is cumbersome, and so we focus here just on the limiting behavior as the number of intermediaries ε converges to zero. Optimizing over α, it turns out to always be optimal to set α slightly larger than $\frac{1}{2}$, but converging to $\frac{1}{2}$ as λ gets big. The optimal value of α is

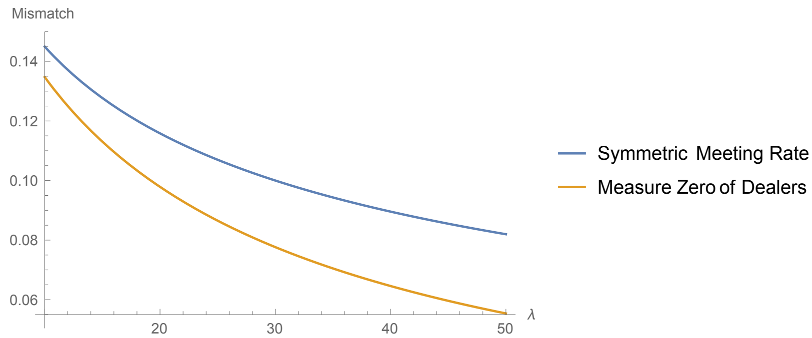
$$\sqrt{\frac{4}{\lambda^2} + \frac{3}{\lambda} + \frac{1}{16}} - \frac{2}{\lambda} + \frac{1}{4}.$$

At this value of α, the value of $\mu(\lambda_1)$ (which measures inefficiency as $\varepsilon \to 0$) is given by

$$\mu(\lambda_1) = \frac{1}{64}\left(-\sqrt{\lambda^2 + 48\lambda + 64} + \lambda + 24\right).$$

which converges to zero at a higher rate compared to rate of converge in the economy without intermediation, as $\lambda \to \infty$ as plotted in Figure 2. In other words, intermediation increases efficiency and the pace at which the market converges to its frictionless limit.

Figure 2: Rate of Convergence to Frictionless Limit



Why does intermediation improve the allocation for a given aggregate search technology. A key observation is that intermediaries are less likely to hold their desired asset position than are regular traders. The fact that they have many more meetings is outweighed by the fact that they frequently trade away from their desired asset position. Intermediation is useful because it makes it easier for the regular traders to obtain their desired asset holdings, not because intermediaries themselves have preferences well-aligned with their asset holdings.

4.1 Planner Problem

We focus on the problem of a social planner who is endowed with a fixed number of meetings, Λ. The social planner seeks to maximize the discounted sum of utilities, subject to the resource constraint of total number of meetings, by choosing the meeting distribution as well as trading pattern. The constraint planner problem is the dual of the problem of an unconstraint planner who faces a linear cost function $C(\lambda) = c\lambda$.

$$\max_{\{G(\lambda), N(s,\lambda|s',\lambda')(t)\}} \int e^{-\rho t} \left(\sum_s \delta_s \left(\int \mu_{s,1}(\lambda)(t) dG(\lambda) \right) \right) dt$$

$$\text{s.t.} \quad \int_0^\infty \lambda dG(\lambda) = \Lambda,$$

where $\mu_{s,j}(\lambda)(t)$ evolves according to equations (2) conditional on the planner's choice of trading pattern. We next restrict the planner to choose a time invariant trading pattern $N(s, \lambda|s', \lambda')$ and the limit $\rho \to 0$. Since $\sum_s \left(\int \mu_{s,1}(\lambda)(t) dG(\lambda) \right) = M$ we can rewrite the planner's problem as

$$\min_{\{G(\lambda),N(s,\lambda|s',\lambda')\}} \int \mu_{h,0}(\lambda)dG(\lambda)$$
$$\text{s.t.} \qquad \int_0^\infty \lambda dG(\lambda) = \Lambda,$$

where steady state mismatch $\mu_{h,0}$ follows from the $\dot{\mu}_{s,j} = 0 \forall s, j$ in the inflow-outflow equations (2). The social planner thus minimizes the steady state mismatch rate, subject to a constraint on aggregate search efficiency across all agents.

To formalize the intuition demonstrated in the above example, we proceed in two steps. First, we show that if there are only two different levels of search efficiency $\lambda_S < \lambda_F$ and the asset is in moderate supply, then the optimal trading pattern chosen by the planner resembles the intermediation pattern described above. The main idea is that when the asset supply is neither to high not too low both the option value the planner prefers a fast agent to be mismatched.[10] The next lemma summarizes this result.

Lemma 1. *If there are only two levels of search efficiency with $\lambda_S < \lambda_F$ and M equals the fraction of individuals in the high preference state then the socially optimal pattern of trade is for the fast agents to intermediate: $N(s, \lambda|s', \lambda') = 1$ iff $s = h$ and $s' = l$ or $s = s' = l$ and $\lambda > \lambda'$ or $s = s' = h$ and $\lambda < \lambda'$.*

Lemma 1 establishes that the planner requires the fast agents to take on a role as intermediaries. If both meeting partners are in state l the faster one has a better chance of finding a non-owner with taste h and thus the planner requires the fast agent to receive the asset. In turn, if both individuals are in state h the faster one has a better chance of finding an owner in state l and the agents thus requires the slow agent to receive the asset.

More generally, consider a general meeting rate distribution $G(\lambda)$ and the impose the intermediated trading pattern described in proposition 1 and note that mismatch is given by $\mu_{h,0}(\lambda) = \mu_{l,1}(\lambda) = \mu(\lambda)$. Manipulating the inflow outflow equations (2) yields the following steady-state relationship between inflows and outflows of mismatch rate

$$\frac{1}{2} - 2\int_{\underline{\lambda}}^\infty \mu(\lambda')dG(\lambda') = \frac{1}{\gamma\Lambda}\left(\int_{\underline{\lambda}}^\infty \lambda\mu(\lambda)dG(\lambda)\right)^2$$

[10]Note the role of asset scarcity in the optimal trading pattern. If the asset is sufficiently scarce then the planner requires the asset to flow toward the fast agent even even if both agents are in the high preference state. In this case the option value of replacing the asset is low because it is hard to find. It is thus optimal to keep it circulating among the those with a high search efficiency. The reverse is true if the asset is sufficiently abundant. Since we assume $M = \frac{1}{2}$ and the fraction of agents in the high preference state is $\frac{1}{2}$ these cases can be ruled out.

This equation conveys an important intuition: The net flow into mismatch is linearly increasing in the measure of matched individuals because of the taste shock γ. To offset this inflow mismatched agents need to meet each other. Thus, the larger the average search efficiency among the mismatched the better the allocation and shuffling mismatch to faster agents reduces mismatch. This is the main intuition why the intermediation trading pattern is optimal: mismatch is transferred from slow agents to fast agents who are more efficient at finding other mismatched with opposite taste.

Next we use this Lemma 1 to argue that the social planner, if restricted to two different search efficiencies λ_i, chooses an intermediate level of heterogeneity. In particular, the social planner does not choose a distribution with *extreme* dispersion: she does not leave any agent in autarky nor does she choose a single mass point.

Proposition 4. *Given a constant aggregate search technology Λ and two groups of agents with speed λ_i, the optimal distribution of search efficiency requires an intermediate degree of heterogeneity: No agent is in autarky, $\lambda_i > 0 \forall i$ nor do all agents have the same meeting rate, $\lambda_1 \neq \lambda_2$.*

Proof. See Appendix. □

Importantly, this result rules out that a single, homogeneous level of search efficiency is socially optimal. That is, without altering the aggregate search technology the allocation can be improved upon by distributing meetings in a (mildly) asymmetric way. In proving the latter result, we contrast mismatch under homogeneity with mismatch when half the population has seach efficiency $\Lambda+\epsilon$ and the other half has $\Lambda-\epsilon$. Using lemma 1 the optimal pattern of trade is the one where faster agents intermediate. So we impose this pattern, and show that the local "split" of the mass point improves the allocation, $\frac{1}{2}\big(\mu_{h,0}(\Lambda+\epsilon)+\mu_{h,0}(\Lambda+\epsilon)\big) < \mu_{h,0}(\Lambda)$. The reason is that with homogeneity and random search many meetings are among individuals with identical taste and are hence not gainful. In turn, whenever the two individuals in a meeting differ there is room for gains from trade. The economy takes advantage of a much larger fraction of meetings leading to first order improvements in the allocation. We suspect, although have not proven, that this is the logic behind our numerical findings below which suggest that the optimal distribution of search efficiencies does not display any mass points, allowing for any meeting to involve two differing individuals.

Finally observe that we do not let the planner chose the level of Λ. However, we point out that when considering the social planner's optimal level of total search efficiency at (linear) cost $c\Lambda$ there are diminishing returns to aggregate search efficiency. As the overall allocation in the economy improves the aggregate mismatch rate falls. However, there is a constant outflow from being matched due to type change. As all agents become perfectly matched, it becomes more difficult for the newly mismatched agents to find a trading partner, which creates the diminishing gain.

4.2 No Trade Within the Same Preference

We have shown that in our framework, some degree of heterogeneity and intermediation is welfare improving relative to a homogeneous benchmark. We next provide an important result that highlights the role of intermediation. It also illustrates that the gains associated with heterogeneity do not arise mechanically from the matching process.

To do so, assume the socially optimal trading pattern is such that there is no trade within agents of the same preference, even if they have different meeting rates. That is, just like in the case with a degenerate distribution of search efficiency, agents in the same preference state never trade and hence there is no intermediation. Assets get solely traded from taste l to taste h individual. The inflow-outflow equations (2) can then be written as

$$\left(\gamma + \lambda \int \frac{\lambda'}{\Lambda} \mu_{l,1}(\lambda'; G) dG(\lambda')\right) \mu_{h,0}(\lambda; G) = \gamma \mu_{l,0}(\lambda; G). \tag{8}$$

It is straightforward to verify that with this trading pattern, $\mu_{l,1}(\lambda) = \mu_{h,0}(\lambda) = \mu(\lambda; G)$ and $\mu_{l,0}(\lambda) = \mu_{h,1}(\lambda) = \frac{1}{2} - \mu(\lambda; G)$, where we write μ explicitly as a function of G to emphasize the dependence. Solving equation (8) for $\mu(\lambda, G)$ yields

$$\mu(\lambda; G) = \frac{\gamma}{2(2\gamma + \lambda X(G))} \tag{9}$$

where $X(G) \equiv \frac{\int \lambda \mu(\lambda; G) dG(\lambda)}{\int \lambda dG(\lambda)}$ denotes the speed-weighted average fraction of the population that can trade. The next proposition establishes that without intermediation, the socially optimal distribution of meeting rates is a mass point.

Proposition 5. *Consider a planner with constant aggregate search efficiency Λ. Assume that individuals trade iff the asset owner is in state l and the non-owner is in state h. Then the social planner endows all agents with the same search efficiency Λ.*

The above proposition shows that the gains from heterogeneity are due to intermediation. Without intermediation it turns out that heterogeneity is actually welfare decreasing reflecting the global concavity of welfare in search efficiency Λ. Endowing agents with heterogeneous meeting rates involves a cost since the slower agents lose more (relative to the homogeneous benchmark) than the fast agents win absent intermediation. This cost can only be covered by the first order gains from intermediation. If there is no intermediation among agents, endowing all the agents with the same search efficiency minimizes the mismatch rate.

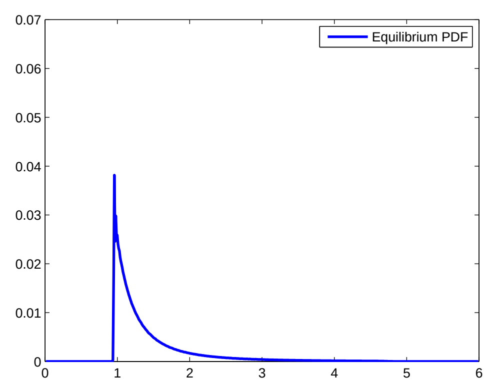
5 Numerical Examples

In this section, we numerically first compute the endogenous equilibrium distribution of search efficiencies under a linear cost function $C(\lambda) = c\lambda$. We then endow the planner with the equilibrium measure of matches Λ but let the planner freely redistribute those matches in the population to maximize steady state welfare.

5.1 Equilibrium Distribution

Using equations (5) and (6) along with Corollary 1, we search numerically for a distribution $G(\lambda)$ such that a measure one of ex ante homogeneous agents are indifferent across all λ with strictly positive density in equilibrium. Specifically, we search for weights $\tilde{g}(\lambda_i)$ on $N = 600$ uniformly distributed grid points on $[0, 6]$ such that $\sum_i^N \tilde{g}(\lambda_i) = 1$ and $v'(\lambda_i) = 0$ for all λ_i with $\tilde{g}(\lambda_i) > 0$.[11] Figure 3 plots the results.

Figure 3: Equilibrium Distribution of Search Efficiencies



In line with our theoretical findings in section 3.3, we find that $\exists \underline{\lambda} > 0$ such that $\tilde{g}(\lambda_i) = 0$ for $\lambda_i < \underline{\lambda}$. Further, the approximated equilibrium density is strictly declining above $\underline{\lambda}$ and displays an open right tail.[12]

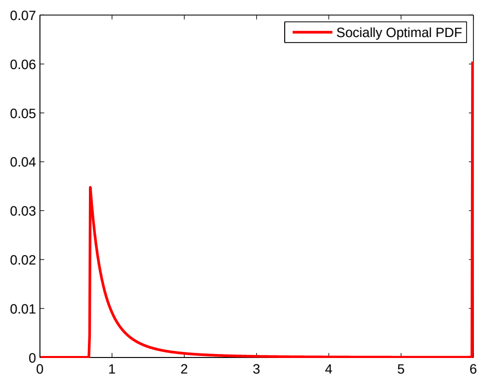
[11] Recall that $v(\lambda_i)$ is the value function net of the flow cost of meetings, so marginal flow cost c is in $v'(\lambda_i)$.

[12] Clearly, our numerical exercise assumes a finite support. However, we have experimented with large values for the upper bound of the gridspace and found that $\tilde{g}(\lambda_i) > 0$ in the right tail, independently of

5.2 Optimal Distribution

We next search for weights $\tilde{g}^P(\lambda_i)$ that maximize steady state welfare. To enable comparison with the equilibrium, we set Λ equal to the endogenous aggregate search efficiency in equilibrium. We then, directly and nonparametrically, search for the distribution of search efficiencies that minimize mismatch in steady state subject to the aggregate constraints $\sum_i^N \lambda \tilde{g}^P(\lambda_i) = \Lambda$ and $\sum_i^N \tilde{g}^P(\lambda_i) = 1$. Figure 4 plots the resulting density.

Figure 4: Optimal Distribution of Search Efficiencies



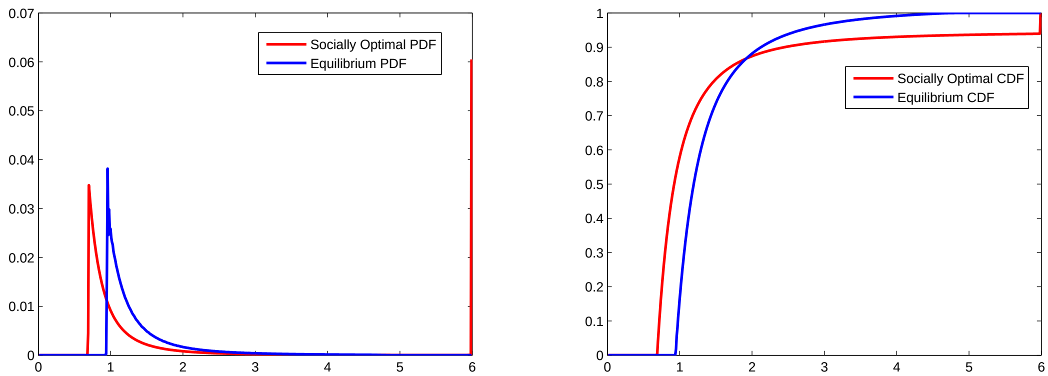
The optimal density has the same qualitative features as the equilibrium counterpart. It features a strictly positive lower bound and is decreasing yet strictly positive everywhere above. This confirms the theoretical observation that the optimal distribution does not feature any mass points. Note the "jump" in the density at the highest grid point. This reflects that we cut off the support, so the planner is prevented from choosing very fast agents.

5.3 Contrasting Equilibrium and Planner

In this section, we contrast several features of the equilibrium with their efficient counterpart. Figure 5 plots the distribution and density of search efficiency in the population in equilibrium against the planner's solution.

where we cut it off numerically.

Figure 5: Search Efficiencies - Equilibrium versus Planner

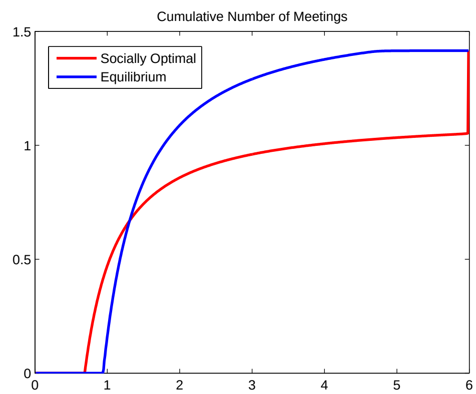


The difference between the two distributions sheds light on the main forces in the model. The two distributions are qualitatively similar, however, the equilibrium distribution is *more concentrated* than the efficient distribution. The intuition for this observation is that fast agents improve the allocation of slower agents, but this force is strongest when there are sufficiently many relatively slow agents who benefit from being intermediated. So the planner endows agents with more unequal meeting rates.

More specifically, fast agents derive most of their equilibrium profits from trade and not from having their asset holdings aligned with preferences. For exactly the same reason a planner values an individual with high λ primarily for her intermediation activity. This is not fully internalized and individuals hence *under-invest* in search efficiency at high levels of λ. In turn, since there is too few intermediators in equilibrium and, for that reason, bid-ask spreads are high agents *over-invest* at the low end of the meeting rate distribution. In other words, intermediation leads to a form of strategic substitutability and the inefficient investment decisions at the top of the distribution generate inefficient investment at the bottom. As a result, the equilibrium distribution is more concentrated than the socially optimal distribution.

Figure 6 plots the cumulative allocation of meetings over the support, i.e., how much aggregate search efficiency is accounted for by agents below each level of search efficiency. It clearly shows that the social planner allocates more meetings to very low speed agents, and many meetings to a very small fraction of very fast agents, which corresponds to the sharp spike in the last point of the socially optimal curve. Note that this figure plots the *cumulative* number of meetings allocated across different search efficiencies, so a spike (or discontinuity) in the planner curve indicates that there are a very small measure of very fast agents, who are responsible for a non-zero measure of total meetings. In fact our numerical experiments suggest that regardless of the support, the planner chooses to have a mass zero of infinitely fast agents who are responsible for strictly positive fraction of total search efficiency.[13] Note that the two curves coincide at the right boundary of the support because the social planner is endowed with the total search efficiency that arises in equilibrium. In other words, what planner does is to reallocate the same total number of meeting across agents to improve (decrease) the total mismatch rate.

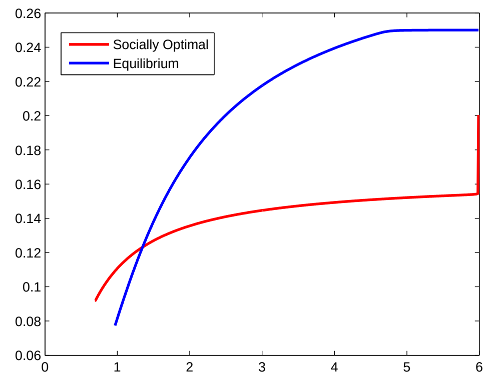
Figure 6: Cumulative Allocation of Meetings, Optimal versus Equilibrium



Finally, figure 7 plots mismatch against λ. Importantly agents with higher search efficiency have higher mismatch rate both in equilibrium and under the planner's solution. This is consistent with intermediation: intermediaries trade against their desired position and in so doing improve the allocation among those with lower search efficiency. In equilibrium intermediaries are compensated through bid-ask spreads. The reason the two curves cross is that at the lower end of distribution agents are better allocated in equilibrium exactly since they meet more frequently with faster indviduals which improves their allocation.

[13] We take care of this case separately by optimizing the fraction of total search efficiency that the planner can choose to allocate to a measure zero agents with are infinitely fast.

Figure 7: Mismatch Rate, Optimal versus Equilibrium



One can also interpret the difference between the two distributions in terms of two distinct sources of inefficieny: the first one is the classic bargaining inefficiency, also present in a large body of labor search models. Agents under-invest in their search technology because they do not internalize the share of surplus captured by their counterparties. The second source of inefficiency is rather unique to our model: when an individual agent chooses meeting rate he does not internalize that a high meeting rate improves the overall allocation and decreases the (overall) gains to search efficiency, which leads to over-investment. Our numerical results suggest that with linear cost function, both sources of ineffciency are sizable, but the under-investment inefficiency dominates.

6 Conclusions

We study a model of over-the-counter trading in asset markets in which ex-ante identical agents invest in trading technology and participate in bilateral trade. We show that when traders have heterogeneous search efficiencies, the fast agents intermediate: they trade against their desired position and take on misallocation from slower agents. Moreover, we characterize how starting with exante homogeneous agents, the distribution of search efficiency is determined endogenously in equilibrium, and how it compares with the corresponding socially optimal distribution. We argue that an economy with homogeneous meeting rate is nor an equilibrium, neither desirable from a social perspective. We also characterize properties of equilibrium and socially optimal meeting rate distribution and show how they compare to each other.

7 Appendix

Proof. **[Proposition 1]** Distributing the first integral on the right hand side of equation (3) between the two last integrals, and doing some further manipulation, equation (3) can be written as

$$S(\lambda) = \frac{2 + \lambda \int_0^\infty \min\{S(\lambda'), S(\lambda)\} \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')}{4\rho + 8\gamma + \lambda}.$$

View this as a mapping $S = T(S)$. We claim that for any cumulative distribution function G and mismatch function m with range $[0, 1/2]$, T is a contraction, mapping continuous functions on $[0, 1/(2\rho + 4\gamma)]$ into the same set of functions. Continuity is immediate. Similarly, if S is nonnegative, $T(S)$ is nonnegative. If $S \leq 1/(2\rho + 4\gamma)$,

$$T(S)(\lambda) \leq \frac{1}{2\rho + 4\gamma} \left(\frac{4(\rho + 2\gamma) + \lambda \int_0^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')}{4\rho + 8\gamma + \lambda} \right) < \frac{1}{2\rho + 4\gamma},$$

where the last inequality uses $m(\lambda) > 0$.

Finally, we prove T is a contraction. If $|S_1(\lambda) - S_2(\lambda)| \leq \varepsilon$ for all λ,

$$|T(S_1)(\lambda) - T(S_2)(\lambda)| \leq \frac{\lambda\varepsilon \int_0^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')}{4\rho + 8\gamma + \lambda} \leq \varepsilon \int_0^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda').$$

Note that the second inequality uses the fact that the fraction is increasing in λ and hence evaluates it at the limit as λ converges to infinity. Since $\int_0^\infty \lambda'(1 - 2m(\lambda'))dG(\lambda') < \int_0^\infty \lambda' dG(\lambda') = \Lambda$, this proves that T is a contraction in the sup-norm, with modulus $\int_0^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')$.

Next we prove that the mapping T takes nonincreasing functions S and maps them into decreasing functions. This implies that the equilibrium surplus function is decreasing. To prove this, let

$$I(\lambda) \equiv \int_0^\infty \min\{S(\lambda'), S(\lambda)\} \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda').$$

Note that if $S(\lambda) \in [0, 1/(2\rho + 4\gamma)]$ and is nonincreasing, $I(\lambda) \in [0, 1/(2\rho + 4\gamma))$ and is nonincreasing. Now take $\lambda_1 < \lambda_2$. Then

$$\begin{aligned} T(S)(\lambda_1) - T(S)(\lambda_2) &= \frac{2 + \lambda_1 I(\lambda_1)}{4\rho + 8\gamma + \lambda_1} - \frac{2 + \lambda_2 I(\lambda_2)}{4\rho + 8\gamma + \lambda_2} \\ &\geq \frac{2 + \lambda_1 I(\lambda_1)}{4\rho + 8\gamma + \lambda_1} - \frac{2 + \lambda_2 I(\lambda_1)}{4\rho + 8\gamma + \lambda_2} \\ &= \frac{2\big(1 - I(\lambda_1)(2\rho + 4\gamma)\big)(\lambda_2 - \lambda_1)}{(4\rho + 8\gamma + \lambda_1)(4\rho + 8\gamma + \lambda_2)} > 0. \end{aligned}$$

The first equality is the definition of T. The first inequality uses $I(\lambda_2) \leq I(\lambda_1)$. The second equality groups the two fractions over a common denominator. And the second equality uses $I(\lambda) < 1/(2\rho + 4\gamma)$. This proves the result. It follows that the equilibrium surplus function is decreasing. □

Proof. **[Equations (5) and (6)]** For all finite λ, solve equation (4) explicitly for $S(\lambda)$.

$$S(\lambda) = \frac{2 + \lambda \int_\lambda^\infty S(\lambda') \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')}{4\rho + 8\gamma + \lambda \left(1 - \int_0^\lambda \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')\right)}$$

First note that $S(0) = 1/(2\rho + 4\gamma)$. Next, note that for $\lambda = \infty$ we have

$$(\rho + 2\gamma)S(\lambda) = \frac{1}{2} + \frac{\lambda}{4}\Big[\int_0^\infty S(\lambda')\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda') - \int_0^\infty S(\lambda')\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda') - 2S(\lambda)(1-\alpha)\Big].$$

$$\lim_{\lambda\to\infty} S(\lambda) = \lim_{\lambda\to\infty} \frac{2}{2\rho + 4\gamma + \lambda(1-\alpha)m(\lambda)} = 0$$

where $(1 - \alpha)$ is the fraction of all meetings that agents with $\lambda = \infty$ are responsible for. With finite equilibrium Λ, mass of agents at with $\lambda = \infty$ goes to zero, but they can still be responsible for non-zero fraction of meetings because they have infinitely high meeting rate. This potentially non-zero fraction is captured by $1 - \alpha$.

Next, differentiate with respect to λ to get

$$\begin{aligned} &\left(4\rho + 8\gamma + 2\lambda\int_0^\infty \frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')\right) S'(\lambda) + 2\int_0^\infty \frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')S(\lambda) \\ &\quad = \int_\lambda^\infty (S(\lambda') - S(\lambda))\frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda') - \lambda S'(\lambda)\int_\lambda^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda') \\ &\quad = \frac{\left(4\rho + 8\gamma + 2\lambda\int_0^\infty \frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')\right)S(\lambda) - 2}{\lambda} - \lambda S'(\lambda)\int_\lambda^\infty \frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda'), \end{aligned}$$

where the second equation eliminates $\int_\lambda^\infty (S(\lambda) - S(\lambda'))\frac{\lambda'}{\Lambda}(1 - 2m(\lambda'))dG(\lambda')$ using equation (4). Now solve this expression for $S'(\lambda)$:

$$S'(\lambda) = \frac{(4\rho + 8\gamma)\, S(\lambda) - 2}{\lambda \left(4\rho + 8\gamma + 2\lambda \int_0^\lambda \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') + \lambda \int_\lambda^\infty \frac{\lambda'}{\Lambda} dG(\lambda')\right)} = \phi(\lambda) \left(S(\lambda) - \frac{1}{2\rho + 4\gamma}\right),$$

where

$$\phi(\lambda) \equiv \frac{4\rho + 8\gamma}{\lambda \left(4\rho + 8\gamma + 2\lambda \int_0^\lambda \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') + \lambda \int_\lambda^\infty \frac{\lambda'}{\Lambda} dG(\lambda')\right)}.$$

The general solution to this differential equation is

$$S(\lambda) = \frac{1}{2\rho + 4\gamma} - k e^{\int_{\underline{\lambda}}^{\lambda} \phi(\lambda') d\lambda'}, \tag{10}$$

and

$$S'(\lambda) = -k \phi(\lambda) e^{\int_{\underline{\lambda}}^{\lambda} \phi(\lambda') d\lambda'}. \tag{11}$$

As $S(\infty) = 0$, this gives us the constant of integration

$$k = \frac{1}{2\rho + 4\gamma} e^{-\int_{\underline{\lambda}}^{\infty} \phi(\lambda') d\lambda'},$$

This implies

$$S(\lambda) = \frac{1}{2\rho + 4\gamma} \left(1 - e^{-\int_{\lambda}^{\infty} \phi(\lambda') d\lambda'}\right).$$

Next, we can use the surplus function to solve uniquely for V_0 and V_1. In particular, recall $v(\lambda) = \lim_{\rho \to 0} \rho V_0(\lambda) - C(\lambda) = \lim_{\rho \to 0} \rho V_1(\lambda) - C(\lambda)$. This satisfies

$$\begin{aligned} v(\lambda) &= \frac{1}{2} - \gamma S(\lambda) - \frac{\lambda}{4} \int_0^\lambda (S(\lambda) - S(\lambda')) \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') - C(\lambda) \\ &= \frac{1}{4} + \gamma k e^{\int_{\underline{\lambda}}^{\lambda} \phi(\lambda') d\lambda'} + \frac{\lambda k}{4} \int_0^\lambda \left(e^{\int_{\underline{\lambda}}^{\lambda} \phi(\lambda'') d\lambda''} - e^{\int_1^{\lambda'} \phi(\lambda'') d\lambda''}\right) \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') - C(\lambda), \end{aligned}$$

which can be solved explicitly by substituting for k from above to get the desired result.

To get the implicit equation which defines the lower bound of the support, let $u(\lambda) = v(\lambda) + C(\lambda)$ denote the gross equilibrium profit of an agent with search efficiency λ. Differentiate u to get[14]

$$u'(\lambda) = -\left(\gamma + \frac{\lambda}{4}\int_0^{\lambda}\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')\right)S'(\lambda) - \frac{1}{4}\int_0^{\lambda}(S(\lambda)-S(\lambda'))\frac{\lambda'}{\Lambda}m(\lambda')dG(\lambda')$$

$$= \frac{u(\lambda)-\frac{1}{4}}{\lambda} - \frac{k\lambda\phi(\lambda)e^{\int_1^{\lambda}\phi(\lambda')d\lambda'}\int_{\lambda}^{\infty}\frac{\lambda'}{\Lambda}dG(\lambda')}{8} \tag{12}$$

$$= \frac{u(\lambda)-\frac{1}{4}}{\lambda} + \frac{S'(\lambda)\int_{\lambda}^{\infty}\frac{\lambda'}{\Lambda}dG(\lambda')}{8}$$

Moreover, note that

$$\phi(\underline{\lambda}) \equiv \frac{4\rho+8\gamma}{\underline{\lambda}\,(4\rho+8\gamma+\underline{\lambda})}$$

so using equation (12), and noting that $\rho \to 0$, we have

$$v'(\underline{\lambda}) = -\gamma S'(\underline{\lambda}) - C'(\underline{\lambda}) = +\gamma k\phi(\underline{\lambda})e^{\int_{\underline{\lambda}}^{\underline{\lambda}}\phi(\lambda')d\lambda'} - C'(\underline{\lambda}) = \gamma k\frac{8\gamma}{\underline{\lambda}\,(8\gamma+\underline{\lambda})} - C'(\underline{\lambda})$$

We know that in equilibrium, $v(\lambda)$ is constant over the support, so $v'(\lambda) = 0$. Evaluating this final equality at $\underline{\lambda}$ and substituting for k gives the implicit equation that defines the lower bound of the support. □

Proof. **[Proposition 2]** Note that if the type distribution G has a mass point at λ, denominator of ϕ jumps down at λ as well, since $2m(\lambda') < 1$. As a result ϕ jumps up. The expression for S implies S is continuous, but the differential equation for S' implies that the derivative jumps down discretely at λ (given the expression for S, S' equals ϕ multiplied by a negative number). That is, S has a concave kink at any mass point λ.

As before, let $u(\lambda) = v(\lambda) + C(\lambda)$ denote the gross profit of agents. As shown in that proof

$$u'(\lambda) = \frac{u(\lambda)-\frac{1}{4}}{\lambda} + \frac{S'(\lambda)\int_{\lambda}^{\infty}\frac{\lambda'}{\Lambda}dG(\lambda')}{8}$$

This implies that $u(\lambda)$ has a convex kink whenever S has a concave kink, i.e. at any mass point. Basically, the first term on RHS is continuous. In the second term, $S'(\lambda) < 0$ and it jumps down, and $\int_{\lambda}^{\infty}\frac{\lambda'}{\Lambda}dG(\lambda')$ is positive but also jumps down.

Finally, suppose that individuals choose λ to maximize $v(\lambda)$, and $C(\lambda)$ is a differentiable cost function. If there is a mass point in G at some λ, the gross benefit $u(\lambda)$ has a convex

[14] Note that when integrating under $dG(\lambda)$ distribution, seting the lower bound of integral to 0 gives the same result as setting the lowerbound to $\underline{\lambda}$ as by definition, there is no density on $0 < \lambda < \underline{\lambda}$.

kink at λ, and λ is a local minimum of the gross profit function. This proves that there are no mass points in the equilibrium speed distribution. □

Proof. **[Proposition 3]** Assume there is an upper bound on the support of equilibrium distribution, $\bar{\lambda}$. Let $u(\lambda) = v(\lambda) + C(\lambda)$ again denote the gross profit of agents. Then

$$u'(\lambda) = \frac{u'(\lambda) - \frac{1}{4}}{\lambda} + \frac{S'(\lambda) \int_{\lambda}^{\infty} \frac{\lambda'}{\Lambda} dG(\lambda')}{8} \tag{13}$$

Then $\forall\ \lambda > \bar{\lambda}$, $\lambda u'(\lambda) = u(\lambda) - \frac{1}{4}$ as there is no agent above $\lambda > \bar{\lambda}$. As a result

$$u(\lambda) = \frac{u(\bar{\lambda}) - \frac{1}{4}}{\bar{\lambda}} \lambda + \frac{1}{4} \qquad \forall\ \lambda \geq \bar{\lambda}$$

In order for no agent to be profitable to deviate to $\lambda > \bar{\lambda}$, we must have $v'(\lambda) < 0\ \ \forall\ \ \lambda > \bar{\lambda}$, or equivalently $c \geq \frac{u(\bar{\lambda}) - \frac{1}{4}}{\bar{\lambda}}$.

However, we know that every agent in equilibrium makes at least net $\frac{1}{4}$ profits, which is the value of being in autarky, $\lim_{\lambda \to 0} v(\lambda) = \lim_{\lambda \to 0} u(\lambda) = \frac{1}{4}$. So the same must hold for the agent with $\lambda = \bar{\lambda}$, who has at least paid $c\bar{\lambda}$ in cost. So we must have $u(\bar{\lambda}) \geq \frac{1}{4} + c\bar{\lambda}$, or $c \leq \frac{u(\bar{\lambda}) - \frac{1}{4}}{\bar{\lambda}}$.

The above two conditions can simultaneously be satisfied only if $c = \frac{u(\bar{\lambda}) - \frac{1}{4}}{\bar{\lambda}}$. As a result, either everyone in equilibrium makes zero profit (beyond the $\frac{1}{4}$ in autarky), in which case support can be bounded [or if the marginal cost is too high then all the mass at zero is an equilibrium]; or all the agents make profits strictly more than autarky, in which case support is unbounded. This also implies that $\lambda = 0$ can only be part of a solution in which everyone's profit is exactly $\frac{1}{4}$: in any equilibrium everyone must make the same profit, and in equilibria with unbounded support everyone makes strictly more profit than autarky. As a result, any equilibrium with positive profits and unbounded support has a strictly positive lower bound $\lambda = \underline{\lambda}$. □

Proof. **[Corollary 1]** Since all agents are ex-ante identical, they must all make the same profit in equilibrium, otherwise there will be a profitable deviation for any agent who makes a lower profit. This equilibrium profit can be evaluated at the lower bound from equation (6)

$$v(\underline{\lambda}) = \frac{q}{4} + \gamma k - C(\underline{\lambda})$$

where k and $\underline{\lambda}$ are computed when deriving equations (5) and (6). □

Proof. **[Lemma 1]** We need to introduce some more notation to prove this statement.[15] Let $\tilde{x}$ be the object in social planner problem parallel to object x in equilibrium.

The net social value, $\tilde{S}(\lambda)$, analogous to equilibrium object $S(\lambda)$ as defined in equation 1 is given by □

$$\rho\tilde{S}_s(\lambda) = \delta_s + \gamma\left(\tilde{S}_{\tilde{s}}(\lambda) - \tilde{S}_s(\lambda)\right) + \lambda\tilde{O}(\lambda) \tag{14}$$

where $\tilde{O}(\lambda)$ is the option value of search in the social planner problem defined analogous to $O(\lambda)$ in individual problem.

We represent the different trading rules that might emerge as optimal as chains of inequalities that determine $N(\delta,\lambda|\delta',\lambda')$. Note that agents of identical speed always trade towards the agent with higher flow utility. For brevity, we write $S(F)$ for $\lambda_S(\lambda_F)$.

1. Intermediation

$$\tilde{S}_l(S) < \tilde{S}_l(F) < \tilde{S}_h(F) < \tilde{S}_h(S) \tag{15}$$

   This trading pattern corresponds to the one where the fast agents act as intermediariers.

2. Trade towards the fast agents, which spans 2 cases:

   (a)

$$\tilde{S}_l(S) < \tilde{S}_l(F) < \tilde{S}_h(S) < \tilde{S}_h(F) \tag{16}$$

   At same flow value asset flows toward fast agent, but low fast sells to high slow.

   (b)

$$\tilde{S}_l(S) < \tilde{S}_h(S) < \tilde{S}_l(F) < \tilde{S}_h(F) \tag{17}$$

   Asset flows toward fast agent even when a high slow owner meets a low fast non-owner: in this case fast agents are *"absorbing"* the asset and the asset is either held solely by the fast agents or all the fast agents hold an asset which they never trade and the remaining assets are traded between the slow types.

[15] We assume γ is constant across the two types ($\gamma_{hl} = \gamma_{lh} = \gamma$), which implies excatly half of the agents are high and half are low. The generalization is straightforward.

3. Trade towards the slow agents, and again spans 2 cases:

   (a)

$$\tilde{S}_l(F) < \tilde{S}_l(S) < \tilde{S}_h(F) < \tilde{S}_h(S)$$

   At same flow value asset flows toward slow agent, but low slow sells to high fast.

   (b)

$$\tilde{S}_l(F) < \tilde{S}_h(F) < \tilde{S}_l(S) < \tilde{S}_h(S)$$

   Asset flows toward slow agent even when a high fast owner meets a low slow non-owner: in this case slow agents are *"absorbing"* the asset.

The proof works by ruling out all the cases except the first case, i.e. intermediation.

Proof. **[Proposition 4]** This proof proceeds in two steps. First we show that the planner prefers a symmetric distribution to an extremely asymmetric one where one group of agents is in autarky. Then we show that a perturbation of symmetric speed distribution improves welfare, which along with the first part completes the proof.

Step 1 Consider an average speed level λ and construct the following family of speed distributions. Choose a fraction x of the population and give it speed $\frac{\lambda}{x}$. Give the remaining $(1-x)$ fraction of population speed zero, so that average speed in population remains λ. This family of speed distributions is features "extreme asymmetry" in the sense that given (λ, x), the two groups of agents have speeds as different as possible.

Next, find the division of total amount of asset between the fast and slow (zero speed) sub-populations which maximizes total welfare, $x\mu_{h,1}(\frac{\lambda}{x}, m_1) + (1-x)\mu_{h,1}(0, m_2)$. In this division, m_1 denotes the per capita amount of asset own by non-zero speed agents, so $xm_1 + (1-x)m_2 = M$. Note that $x = 1$ corresponds to the symmetric speed distribution where every agent has speed λ.

Next, we show that among all members of this family (indexed by x), the symmetric distribution maximizes welfare (at the corresponding optimal division of asset). In other words

$$\hat{x} = \text{argmax}_x \max_{m1} \{x\mu_{h,1}(\frac{\lambda}{x}, m_1) + (1-x)\mu_{h,1}(0, m_2)\}. \tag{18}$$

$$\text{s.t. } xm_1 + (1-x)m_2 = M \tag{19}$$

We want to show that $\hat{x} = 1$. This is sufficient condition for the social planner not to choose an "extreme-asymmetric" speed distribution.

To do so, we solve the *unconstrained* version of problem 18 and show that $x = 1$ maximizes it. The relevant constraints are $m_1 > 0$, $m_1 \leq 1$ and $m_1 \leq \frac{M}{x}$ (i.e. $m_2 > 0$). Note that at $x = 1$ (symmetric equilibrium) none of these constraints bind, while for other x's they can potentially bind. So if $x = 1$ is the argmax of the unconstrained problem, it is also the argmax of the constrained problem. This completes the first step of the proof.

Step Two The next step is to show that a small pertubation to the the symmetric speed distribution, which makes some agents faster and some slower, holding the aggregate speed constant, is welfare-improving.

We use the following approach: Conjecture a perturbation in which half of agents are fast and half are slow, $g = \{g_S, g_F\} = \{\frac{1}{2}, \frac{1}{2}\}$. Next consider the flow equations for μ's which characterize an "intermediation" trading rule as defined above. Impose $\lambda_F = \lambda_S = \lambda$ and compute the equilibrium stocks. In other words, endow all agents with same speed λ, but label half of them S and the other half F, and then force the S and F agents to trade in a fan trading pattern.

With some abuse of notation, let $\mu_{s,m}(F)$ ($\mu_{s,m}(F)$) denote the fraction of agents labeled F (S) who are in state s with ownership status m, while both groups have the same speed λ. The first thing to note is that

$$\frac{1}{2}\mu_{s,m}(S) + \frac{1}{2}\mu_{s,m}(F) = \mu_{s,m}(\lambda), \quad s \in \{l, h\},\ m \in \{0, 1\} \tag{20}$$

where $\mu_{s,m}(\lambda)$ is the corresponding fraction in the symmetric equilibrium.

Now to show that a intermediation trading pattern with asymmetric speed improves welfare, we need to show that

$$\left(\frac{\partial \mu_{h,1}(\lambda - \epsilon)}{\partial \epsilon} + \frac{\partial \mu_{h,1}(\lambda + \epsilon)}{\partial \epsilon}\right)\Big|_{\epsilon=0} > 0. \tag{21}$$

To compute the above partial derivatives totally differential the system of equations that characterize the steady state stocks μ and evaluate it at $\epsilon = 0$. Equation 21 boils down to

$$\mu_{l,1}(F)\mu_{h,0}(F) > \mu_{l,1}(S)\mu_{h,0}(S) \tag{22}$$

as defined above. To show that this equation always holds, consider the following: assume the above exercise, except that after labeling agents as S and F we force all low agents to sell to all high agents and to no-one else (irrespective of their label). So the S and F populations will be identical, $\mu_{s,m}(F) = \mu_{s,m}(S) \quad \forall\ s,\ m$; which means the above inequality will hold with equality. Now enforce that low S owners sell to low F non-owners and high F owners sell to high S non-owners. As equation 20 holds as long as the two labeled populations have the same trading speed, we will have $\mu_{l,1}(F) > \mu_{l,1}(S)$. A symmetric argument for the high types implies $\mu_{h1}(F) < \mu_{h1}(S)$. Note that because of our choice of perturbation ($\frac{1}{2}$ of agents being each F and S), $\mu_{h,1}(F) + \mu_{h,0}(F) = \mu_{h,1}(S) + \mu_{h,0}(S) = \frac{1}{2}f_h$. This implies $\mu_{h,0}(F) > \mu_{h,0}(S)$ which along $\mu_{l,1}(F) > \mu_{l,1}(S)$ implies equation 22 and completes the proof.

The above two together show that social planner chooses some intermediate asymmetric distribution. So the planner does not want all the market participants to have the same speed, but he does not want some very fast and some extremely slow agents either. □

Proof. **[Proposition 5]** Observe that $\mu(\lambda; G)$ is a convex function of λ (given G) and $\lambda\mu(\lambda; G)$ is a concave function of λ (given G).

Start from a distribution function G that is degenerate at Λ and consider any other distribution $\tilde{G}$ with the same mean, $\int \lambda d\tilde{G}(\lambda) = \Lambda$. Our goal is to prove that

$$\int \mu(\lambda; G)dG(\lambda) < \int \mu(\lambda; \tilde{G})d\tilde{G}(\lambda).$$

We do this in two steps.

First, we prove $X(G) > X(\tilde{G})$. We prove this by contradiction, assuming instead that $X(G) \leq X(\tilde{G})$. Equation (9) then implies $\mu(\lambda; G) \geq \mu(\lambda; \tilde{G})$ for all λ. Then

$$\int \lambda\mu(\lambda; G)dG(\lambda) \geq \int \lambda\mu(\lambda; \tilde{G})dG(\lambda) > \int \lambda\mu(\lambda; \tilde{G})d\tilde{G}(\lambda),$$

where the first inequality uses $\mu(\lambda; G) \geq \mu(\lambda; \tilde{G})$ and the second uses Jensen's inequality: $\lambda\mu(\lambda; \tilde{G})$ is concave in λ and $\tilde{G}$ is a mean-preserving spread of G. Since G and $\tilde{G}$ have the same mean, this implies $X(G) > X(\tilde{G})$, a contradiction which proves $X(G) > X(\tilde{G})$.

Next, $X(G) > X(\tilde{G})$ implies $\mu(\lambda; G) < \mu(\lambda; \tilde{G})$ for all λ by equation (9). Finally,

$$\int \mu(\lambda; G)dG(\lambda) < \int \mu(\lambda; \tilde{G})dG(\lambda) < \int \mu(\lambda; \tilde{G})d\tilde{G}(\lambda),$$

□

References

Bech, Morten L. and Enghin Atalay, "The topology of the federal funds market," *Physica A: Statistical Mechanics and its Applications*, 2010, *389* (22), 5223–5246.

Burdett, Kenneth and Dale T Mortensen, "Wage differentials, employer size, and unemployment," *International Economic Review*, 1998, pp. 257–273.

Chang, Briana and Shengxing Zhang, "Endogenous Market Making and Network Formation," Technical Report, LSE 2015.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, "Over-the-Counter Markets," *Econometrica*, 2005, *73* (6), 1815–1847.

_ **, Piotr Dworczak, and Haoxiang Zhu**, "Benchmarks in search markets," Technical Report, National Bureau of Economic Research 2015.

Hugonnier, Julien, Benjamin Lester, and Pierre-Olivier Weill, "Heterogeneity in decentralized asset markets," Technical Report, National Bureau of Economic Research 2014.

Lagos, Ricardo and Guillaume Rocheteau, "Liquidity in asset markets with search frictions," *Econometrica*, 2009, *77*, 403–426.

Neklyudov, Artem V, "Bid-ask spreads and the decentralized interdealer markets: Core and peripheral dealers," Technical Report, Working Paper, University of Lausanne 2013.

Pagnotta, Emiliano and Thomas Philippon, "Competing on speed," Technical Report, National Bureau of Economic Research 2015.

Rubinstein, Ariel and Asher Wolinsky, "Middlemen," *The Quarterly Journal of Economics*, 1987, pp. 581–594.

Shen, Ji and Hongjun Yan, "A search model of the aggregate demand for safe and liquid assets," *Available at SSRN 2384895*, 2014.

Üslü, Semih, "Pricing and Liquidity in Decentralized Asset Markets," Technical Report, UCLA 2015.

Vayanos, Dimitri and Tan Wang, "Search and endogenous concentration of liquidity in asset markets," *Journal of Economic Theory*, 2007, *136* (1), 66–104.

Weill, Pierre-Olivier, "Liquidity premia in dynamic bargaining markets," *Journal of Economic Theory*, 2008, *140* (1), 66–96.