Household Intertemporal Behavior: a Collective Characterization and Empirical Tests

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Abstract

In this paper, each household is characterized as a group of agents making joint decisions. Under the assumption of full efficiency, it is shown that households with several adults can be characterized by a unique utility function if and only if individual members have identical discount factors, HARA utility functions with identical curvature parameter and identical beliefs. If this conditions are violated, the household objective function and consequently Euler equations depend on the decision power of each spouse. If the assumption of full efficiency is relaxed, I find that standard Euler equations of couples are replaced by inequalities, even if the restrictions are satisfied. However, the Euler equations of singles should be fulfilled with and without efficiency. Using the PSID as well as the CEX, these theoretical implications are tested. I find that the standard model of household intertemporal behavior is consistent with the consumption pattern of singles, but not with the consumption behavior of couples. Finally it is shown that full efficiency has different predictions for household Euler equations from the limited commitment version of the model. These predictions will be tested using both the PSID and the CEX.

1 Introduction

In this paper, it is shown that the standard model of household intertemporal behavior is consistent with the consumption pattern of US households with only one adult. However, the standard framework is rejected for US couples. These results and the approach employed in the paper are new in several respects. First, it is explicitly taken into consideration that households are composed by several individual members with different preferences. In most of the consumption literature, each household is characterized as if a single agent were making the decisions. Specifically, a unique utility function is maximized under a budget constraint. Using the alternative approach, it is shown that the testable implications of the standard intertemporal maximization model should be satisfied by households composed by only one adult, but rejected for household with several adults. Second, I consider a full commitment as well as a limited commitment version of intertemporal behavior and find that the two approaches have different implications for household Euler equations. Third, the theoretical restrictions of the intertemporal optimization models are tested using the Panel Study of Income Dynamics (PSID) as well as the Consumer Expenditure Survey (CEX). I find that the assignment of individual preferences to household members and a proper treatment of their interactions can be important theoretically as well as empirically.

The PSID and CEX are the most frequently used data sets to test the theoretical restrictions of intertemporal optimization. In both data sets, the majority of the households are married or cohabiting couples. Consequently, the outcome of any test is driven by the pattern of consumption of households with several adults. Therefore the theoretical and empirical results of this paper represent a possible structural explanation of the rejections of the theory reported in the past 20 years. Hall and Mishkin (1982), for instance, find that food consumption is excessively sensitive to lagged labor income. Hayashi (1985) finds excess sensitivity using seven different consumption goods. Flavin (1981) and Campbell and Mankiw (1989) strongly reject the Euler equations using aggregate time-series data.

In this paper, the intertemporal allocation of resources is modelled as the joint decision of household members. To that end, I characterize the household as a group of agents, each of them being represented by individual preferences. By means of this framework, the following results are shown.

a) Under the assumption of full efficiency, households with several adults can be characterized by a unique utility function if and only if strong restrictions on preferences are satisfied. Specifically, if and only if all household members have identical discount factors, Harmonic Absolute Risk Aversion utility functions with identical curvature parameter and identical beliefs. For instance, under the standard assumption of Constant Relative Risk Aversion utilities, these restrictions are satisfied if and only if individual members have identical risk aversion, hypothesis that is strongly rejected by Barsky, Juster, Kimball and Shapiro (1997). In all other cases, the only objective function that can be assigned to the household depends on the decision power of individual members and on all variables having an effect on it. Consequently, Euler equations of couples should depend on all the factors affecting the intra-household allocation of resources, such as individual income, differences in age and family composition.

b) If the assumption of full efficiency is relaxed to allow for limited commitment, the results are even stronger. Euler equations of couples are replaced by supermartingales, even if the restrictions on preferences are satisfied. Therefore, tests of intertemporal optimization based on Euler equations should reject the classical model even if liquidity constraints are not binding.

c) The full efficiency version of the model proposed in this paper has different implications for household Euler equations from the limited commitment version. Under full commitment, the factors affecting individual decision power should enter household Euler equations only as interaction terms with household consumption. If the assumption of full commitment is not satisfied, those factors should enter household Euler equations both directly and as interaction terms with household consumption.

d) The presence of those factors in household Euler equations generates additional testable implications: the corresponding coefficients must satisfy proportionality conditions that are very unlikely to be fulfilled unless the model at stake is correct.

e) Using the PSID and the CEX, I find that consumption of couples is excessively sensitive to household income. However, I do not find excess sensitivity for households with only one adult. Therefore I can reject the model that assigns a unique utility function to the household independently of the family structure. But I cannot reject the alternative specification introduced in this paper. The empirical results are consistent with the findings of Attanasio and Weber (1995). In their insightful paper, Attanasio and Weber (1995) show that the intertemporal model of optimizing behavior for consumption is consistent with US micro data if household preferences are modelled so as to take into consideration changes in family composition and labor supply behavior over the life cycle.

The model of household consumption introduced in this paper extends the collective model developed by Chiappori (1988, 1992) in two directions. The collective model is first generalized to a multi-period framework to analyze household intertemporal optimization. The extension to a multi-period framework raises a commitment issue. In the paper I consider two possible approaches to household commitment. In the first model, household members can commit in their first period to an allocation of resources for the future. In the second framework household members can only commit to an allocation of resources for the current period but not for the future. This distinction allows me to determine the impact of limited efficiency on Euler equations. Lundberg, Startz and Stillman (forthcoming) use a three-period collective model with no uncertainty to explore the retirement-consumption puzzle. The topic analyzed in this paper is very different. The emphasis is on the relation between household intertemporal behavior and family structure. Moreover, the model that I propose is more general in several dimensions. Uncertainty is a crucial component of household behavior as emphasized by Mazzocco (2001) and modelled in this paper. I take into consideration the additive form of altruism. Finally, I allow for an arbitrary number of periods. Lundberg and Pollack (2001) use a non-stationary multi-stage game to analyze theoretically the location decision of a married couple. They show that marital decisions involving the future are in general not efficient. These results are consistent with the finding of this paper that Euler equations are replaced by inequalities when the assumption of full efficiency is relaxed. The approach of splitting the sample in two groups based on theoretical ground has been extensively used in the literature. For example, Bernanke (1984), Hayashi (1985), Zeldes (1989) and more recently Browning and Chiappori (1998) use this strategy. This paper differs from the previous literature as it allows for self selection. Moreover, this is the first paper that estimates Euler equations separately for singles and couples.

The paper is organized as follows. In section 2, the full efficiency Intertemporal Collective Model (ICM) is introduced. Afterwards, the assumption of full efficiency is relaxed and the case of limited commitment is analyzed. It is shown that the full efficiency collective model is equivalent to a two-stage framework in which first the household allocates optimally total resources across individual members. Second, given the allocation of resources, each spouse maximizes her own utility subject to individual budget constraints. This alternative formulation is much more tractable theoretically as well as empirically. Section 3 establishes the conditions on preferences and efficiency under which the standard Euler equations are satisfied. Moreover, it discusses the consequences for the estimation of Euler equations if these restrictions are violated. In section 4, tests to evaluate the standard unitary model against the ICM are outlined. Section 5 describes the data. Section 6 discusses some econometric issues. Section 7 reports the estimation results. Some concluding remarks are presented in the final section.

2 The Intertemporal Collective Model

To answer several policy questions, precise estimates of the intertemporal and intratemporal rate of substitutions are required. In the past twenty years, economists have estimated these coefficients using household Euler equations. With rare exceptions, all these works are based on the crucial assumption that household intertemporal behavior can be characterized as if a single agent were making the decisions. In reality, the majority of households are composed by several agents making joint decisions. In the CEX, about 66 percent of the interviewed households are married or cohabiting couples. A similar pattern is displayed in the PSID. It is therefore important to determine the impact of this assumption on the estimation of Euler equations.

I will consider a household living for T periods and composed by n agents. Agent i is characterized by individual preferences over a private composite good, c^i, and a public composite good, Q, an individual discount factor, β_i, and individual beliefs. Individual preferences are assumed to be separable over time and to be represented by a von Neumann-Morgenstern utility function, $u^i\left(c^i, Q^i\right)$. A probability measure $F^i(\omega)$ describes the individual beliefs, where $\omega \in \Omega$ is a potential state of the world. In each period $t \in \{0, ..., T\}$ and state of the economy $\omega \in \Omega$, member i is endowed with an exogenous stochastic income stream $\left\{y^i(t, \omega)\right\}_{t \in \mathbf{T}, \omega \in \Omega}$. For any given (t, ω), the household can either consume or save. One risk-free asset is traded in the economy. $s(t, \omega)$ and $R(t)$ denote respectively the amount of wealth invested in the risk-free asset and the gross return on the risk-free asset. Total household income and total household consumption are denoted with $Y(t, \omega) = \sum_{i=1}^n y^i(t, \omega)$, $C(t, \omega) = \sum_{i=1}^n c^i(t, \omega)$. Let $Z = \{z_1, ..., z_m\}$, be the set of variables affecting the individual decision power. Throughout the paper, utility functions are assumed to be continuous, continuously differentiable, have positive first derivative and negative second derivative. All functions are assumed to be measurable with respect to the probability measure F^i.

2.1 The Unitary Model

Consumption is generally measured only at the household level. To overcome this obstacle, most of the literature on intertemporal optimization assumes that household preferences can be represented by a unique von Neumann-Morgenstern utility function, $U(C, Q)$. Hence, the household allocates resources over time and across states of the world according to the following program:

$$\max_{\{C_t, Q_t, s_t\}_{t \in \mathbf{T}, \omega \in \Omega}} E\left[\sum_{t=0}^{T} \beta^t U(C_t, Q_t)\right] \tag{1}$$

$$s.t.\ C_t + P_t Q_t + s_t \leq \sum_{i=1}^{n} y_t^i + R_t s_{t-1} \ \ \forall t, \omega$$

$$s_T \geq 0 \ \ \forall \omega.$$

This approach overlooks that the household is composed by several agents with different preferences, interacting in a complex way. In particular, in this framework the distribution of decision power between members is not explicitly modelled and only full efficiency can be considered.[1]

[1] Becker (1991) offers a justification for this approach.

2.2 The Full-Efficiency Intertemporal Collective Model

To take into consideration that households are composed by several agents making joint decisions, each individual member is represented by individual preferences. Building on the seminal work of Chiappori (1988, 1992), it is assumed that household decisions are efficient, i.e. the corresponding allocation of resources is always on the Pareto frontier. In an intertemporal framework, household efficiency requires two assumption. First, members must be able to commit at $t = 0$ to an allocation of resources for any future time, t, and state of nature, ω. Second, household members must have symmetric information. Under these assumptions, household intertemporal decisions can be represented by means of the following Pareto problem:

$$\max_{\{c_t^i, Q_t, s_t\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\ldots,n}} E^1\left[\sum_{t=0}^{T}\beta_1^t u^1(c_t^1, Q_t)\right] \tag{2}$$

$$s.t.\ \mu_i : E^i\left[\sum_{t=0}^{T}\beta_i^t u^i(c_t^i, Q_t)\right] \geq \underline{u}_i\,(Z) \quad i = 2, ..., n$$

$$\sum_{i=1}^{n} c^i + P_t Q_t + s_t \leq \sum_{i=1}^{n} y_t^i + R_t s_{t-1} \ \ \forall t, \omega$$

$$s_T \geq 0 \ \ \forall \omega,$$

for some set of reservation utilities $\{\underline{u}_i\}_{i=2,\ldots,n}$. In general, each $\underline{u}_i$ is a function of all the variables affecting the individual decision power, Z. For a given Z, the set of efficient outcomes obtains as $\{\underline{u}_i\}_{i=2,\ldots,n}$ varies within its domain. The full commitment assumption is implicit in the participation constraints of problem (2). Indeed, the household commits to an allocation at $t = 0$ and sticks to it for the next T periods even if the relative power of the n members changes over time.[2]

The full efficiency ICM can be formulated using an alternative two-stage framework. In the first stage, the household decides how much to consume of the public good and the optimal distribution of total income between the n agents for each pair (t, ω). In the second stage, each

[2] It is useful to note that with this framework it is possible to take into consideration the additive form of altruism,

$$U^i\left(c^1, .., c^i, .., c^n, Q\right) = u^i(c^i, Q) + \sum_{j\neq i}\delta_{ji}u^j(c^j, Q).$$

agent chooses individual private consumption and savings, given her share of total income and the chosen quantity of the public good for each (t,ω). The two-stage approach is appealing for two reasons. It is mathematically and empirically more tractable. Moreover, the first stage gives a complete description of the efficient distribution of available resources between the n members. Formally, let $\left\{\rho^i\left(t,\omega\right)\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$ be an arbitrary distribution of available non labor income among the n members for each pair (t,ω). Let $\left\{Q\left(t,\omega\right)\right\}_{t\in\mathbf{T},\omega\in\Omega}$ be an arbitrary consumption of the public good for each pair (t,ω). Then, given $\left\{\rho^i\left(t,\omega\right)\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$ and $\left\{Q\left(t,\omega\right)\right\}_{t\in\mathbf{T},\omega\in\Omega}$, in the second stage agent i solves the following program:

$$V^i(\left\{\rho_{t,\omega}^i, Q_{t,\omega}\right\}_{t\in\mathbf{T},\omega\in\Omega}) = \max_{\left\{c_t^i,s_t^i\right\}_{t\in\mathbf{T},\omega\in\Omega}} E^i\left[\sum_{t=0}^{T}\beta_i^t u^i(c_t^i,Q_t)\right] \tag{4}$$

$$s.t.\ c_t^i + s_t^i \leq \rho_t^i + R_t s_{t-1}^i \ \ \forall\left(t,\omega\right)$$

$$s_T \geq 0 \ \ \forall\omega.$$

In the first stage, the household determines the optimal allocation of total income between the n members, solving the following program:

To see this note that (2) can be written using the Kuhn-Tucker multipliers associated with the participation constraints:

$$\max_{\left\{c^i,Q,s\right\}_{i=1,\dots,n}} \bar{\mu}^1 E_0\left[\sum_{t=0}^{T}\beta_1^t u^1(c_t^1,Q)\right] + .. + \bar{\mu}^n E_0\left[\sum_{t=0}^{T}\beta_n^t u^n(c_t^n,Q)\right] \tag{3}$$

$$\sum_{i=1}^{n} c_t^i + P_t Q_t + s_t \leq \sum_{i=1}^{n} y_t^i + R_t s_{t-1} \ \ \forall\left(t,\omega\right)$$

$$s_T \geq 0 \ \ \forall\omega, \in\Omega$$

This formulation is equivalent to a model with additive altruism in which $\bar{\mu}^1,\dots,\bar{\mu}^n$ are the sum of Pareto weights and altruism parameters.

$$\max_{\{\rho_t^i, Q_t\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}} V^1(\{\rho_{t,\omega}^1, Q_{t,\omega}\}_{t\in\mathbf{T},\omega\in\Omega}) \tag{5}$$

$$s.t.\ V^i(\{\rho_{t,\omega}^i, Q_{t,\omega}\}_{t\in\mathbf{T},\omega\in\Omega}) \geq \underline{u}_i \quad i = 2, ..., n$$

$$\sum_{i=1}^{n} \rho_t^i + P_t Q_t \leq \sum_{i=1}^{n} y_t^i \quad \forall (t, \omega) .$$

The following proposition states that the two-stage interpretation is equivalent to the full efficiency intertemporal collective model.

Proposition 1 *Let* $\{c_{t,\omega}^i, Q_{t,\omega}, s_{t,\omega}\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$ *be a solution of the two-stage model. Let* $s_{t,\omega} = \sum_{i=1}^n s_{t,\omega}^i$ *for every* (t, ω)*. Then* $\{c_{t,\omega}^i, Q_{t,\omega}, s_{t,\omega},\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$ *is a solution of the ICM, (2). Let* $\{c_{t,\omega}^i, Q_{t,\omega}, s_{t,\omega}\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$ *be a solution of the ICM, (2). Then there exist a sharing rule* $\{\rho_{t,\omega}^i\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$*, consumption of the public good* $\{Q_{t,\omega}\}_{t\in\mathbf{T},\omega\in\Omega}$ *and individual savings* $\{s_{t,\omega}^i\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$*, with* $s_{t,\omega} = \sum_{i=1}^n s_{t,\omega}^i$ *for every* (t, ω)*, such that* $\{\rho_{t,\omega}^i, c_{t,\omega}^i, s_{t,\omega}^i\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,\dots,n}$*, is a solution of the two-stage model.*

Proof. First and Second Welfare Theorem with production. ■

2.3 The Limited-Commitment ICM

In the full efficiency ICM, it is assumed that individual members have symmetric information and can commit to an allocation of resources for the future. The assumption of symmetric information is likely to be satisfied for most households. The assumption on commitment is much stronger. To verify the effect of limited commitment on household intertemporal behavior, in this section I will consider a framework in which household members can commit within periods but not on future plans, i.e. the intertemporal allocation of resources must satisfy individual participation constraints for each period and state of the world. Formally, the household behaves according to the following program:

$$\max_{\left\{c_t^i,Q_t,s_t\right\}_{t\in\mathbf{T},\omega\in\Omega}} E_0^1\left[\sum_{t=0}^{T}\beta_1^t\mu_1 u^1(c_t^1,Q_t)+\right]...+E_0^2\left[\sum_{t=0}^{T}\beta_n^t\mu_n u^n(c_t^n,Q_t)\right] \tag{6}$$

$$s.t.\ \mu_i\left(\tau,\omega\right):E_\tau^i\left[\sum_{t=0}^{T-\tau}\beta_i^t u^i(c_{t+\tau}^i,Q_{t+\tau}^i)\right]\geq \underline{u}_{i,\tau,\omega}\left(s_{\tau-1}\right)\ \ \forall\omega,\tau>0,i=1,...,n$$

$$\sum_{i=1}^{T}c_t^i+P_tQ_t+s_t\leq\sum_{i=1}^{T}y_t^i+R_ts_{t-1}\ \ \forall\left(t,\omega\right)$$

$$b_T+s_T\geq 0\ \ \forall\omega.$$

for some set of reservation utilities $\left\{\underline{u}_{i,t,\omega}\left(s_{t-1}\right)\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,...,n}$, which may depend on household savings and more generally on Z.

The main difference between the full efficiency and limited commitment ICM is summarized by the multipliers $\{\mu_i\}_{i=1,..n}$ in (2) and the set of multipliers $\{\mu_i\left(t,\omega\right)\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,...,n}$ in (3). μ_i can be interpreted as the decision power of member i at $t=0$ in the collective decision process. For any pair (t,ω), $\mu_i\left(t,\omega\right)$ is the decision power of member i at (t,ω). Thus the solution of (2) is only affected by the distribution of decision power between the n members at $t=0$. Whereas the optimal allocation in (3) is a function of the distribution of power between the n members at each pair (s,ω).

3 Household Euler Equations

In the past twenty years, Euler equations have been derived and estimated using the unitary model. Specifically, household intertemporal behavior has been tested by means of the following equation:

$$U_C\left(C_t,Q_t\right)=\beta E_t\left[U_C\left(C_{t+1},Q_{t+1}\right)R_{t+1}\right]. \tag{7}$$

Household Euler equations can also be derived using the ICM. Under the assumption of full efficiency and separability over time and across states of nature, it is always possible to construct household preferences solving the following representative agent problem:

$$V\left(C,Q,\mu(Z)\right)=\max_{\left\{c^{i},Q\right\}_{i=1,\ldots,n}}\beta_{1}F^{1}(\omega)\mu_{1}\left(Z\right)u^{1}\left(c^{1},Q\right)+\ldots+\beta_{n}F^{n}(\omega)\mu_{n}\left(Z\right)u^{n}\left(c^{n},Q\right)$$

$$s.t.\sum_{i=1}^{n}c^{i}+PQ=C$$

where $\mu(Z)=\left\{\mu_{i}\left(Z\right)\right\}_{i=1,\ldots,n}$ are the individual Pareto weights. It is then straightforward to derive the following household Euler equation using standard arguments:

$$V_{c}\left(C_{t},Q_{t},\mu(Z)\right)=\beta E_{t}\left[V_{c}\left(C_{t+1},Q_{t},\mu(Z)\right)R_{t+1}\right]. \tag{8}$$

If the assumption of full commitment is not satisfied, it is still possible to derive household preferences and the corresponding Euler equations using the approach developed in Marcet and Marimon (1992) and Marcet and Marimon (1998). It is possible to show that the limited commitment ICM can be rewritten using the following formulation:

$$\max_{\left\{c_{t}^{i},Q_{t},s_{t}\right\}}\sum_{t=0}^{T}E_{0}^{1}\left[\beta_{1}^{t}M_{1,t}u^{1}(c_{t}^{1},Q_{t})-\mu_{1,n}\underline{u}_{1,t}\right]+\ldots+\sum_{t=0}^{T}E_{0}^{n}\left[\beta_{n}^{t}M_{n,t}u^{n}(c_{t}^{n},Q_{t})-\mu_{n,t}\underline{u}_{n,t}\right]$$

$$\sum_{i=1}^{T}c_{t}^{i}+P_{t}Q_{t}+s_{t}\leq\sum_{i=1}^{T}y_{t}^{i}+R_{t}s_{t-1}\ \ \forall\left(t,\omega\right)$$

$$s_{T}\geq 0\ \forall\omega,$$

where $M_{i,t}=\sum_{\tau=0}^{t}\mu_{i,t}$, $\mu_{i,t}$ is the Kuhn-Tucker multiplier corresponding to the participation constraint of member i at time t and $\mu_{i,0}$ is the initial Pareto weight of agent i. Under the assumption that $\underline{u}_{i,t}$ is independent of savings and other decision variables, it is possible to exploit the separability of preferences over time and across states of the world to determine household preferences by solving a representative agent problem at each t:

$$V\left(C,Q,M_{t}(Z)\right)=\max_{\left\{c^{i},Q\right\}_{i=1,\ldots,n}}\beta_{1}F^{1}(\omega)M_{1,t}\left(Z\right)u^{1}\left(c^{1},Q\right)+\ldots+\beta_{n}F^{n}(\omega)M_{n,t}\left(Z\right)u^{n}\left(c^{n},Q\right)$$

$$s.t.\sum_{i=1}^{n}c^{i}+PQ=C.$$

Consequently, if the assumption of limited commitment is not satisfied, household Euler equations can be written in the form,

$$V_c\left(C_t, Q_t, M_t(Z)\right) = \beta E_t\left[V_c\left(C_{t+1}, Q_t, M_t(Z)\right) R_{t+1}\right], \tag{9}$$

i.e. Euler equations still depend on individual decision power, but the distribution of decision power can change over time if the individual outside options vary.

Given the extensive use of the unitary approach to estimate Euler equations it is important to determine under which restrictions individual preferences can be aggregated in a unique utility function so that,

$$V_c\left(C_t, Q_t, \mu(Z)\right) = U_C\left(C_t, Q_t\right) \tag{10}$$

or alternatively,

$$V_c\left(C_t, Q_t, M_t(Z)\right) = U_C\left(C_t, Q_t\right) \tag{11}$$

Moreover, if these conditions are fulfilled, it is crucial to determine which additional assumptions are required for household Euler equations to be satisfied.

Definition 1 *For any given level of Q, define $u_Q^i\left(c^i\right) = u^i\left(c^i, Q\right)$. An ISHARA household is a household satisfying the following conditions: (i) all members have identical discount factor β and beliefs $F(\omega)$; (ii) for $i = 1, ..., n$,*

$$u_Q^{i\prime}\left(c^i\right) = \left(a_i + \frac{c^i}{\gamma_i}\right)^{-\gamma_i}$$

with $\gamma_1 = ... = \gamma_n$, i.e. individual preferences conditional on the public good are Harmonic Absolute Risk Aversion with identical curvature parameter.[3]

It is worth noting that the assumption of HARA preferences with identical shape parameter is very restrictive. For instance, if all agents have CRRA preferences, the household is ISHARA

[3] Note that in the conditional utility function $u_Q^i\left(c^i\right)$, a_i and γ_i will be in general complex functions of Q. Therefore preferences will not be in general separable between Q and c^i even under the assumption that the household belongs to the ISHARA class.

if and only if all members have identical preferences. The following result is a generalization of Gorman aggregation to an intertemporal framework and states that an ISHARA household is a sufficient and necessary condition for a unique utility function to exist.

Assumption 1 *Each agent has strictly positive Pareto weights, i.e.* $\{\mu_i\}_{i=1,...,n}$.

Assumption 2 *For each* $i = 1, ..., n$, $\lim_{x \to 0} u^{i\prime}(x) = \infty$.

Assumption 3 *For each* $i = 1, ..., n$, *the probability measure* $F_i(\omega)$ *has a density* $f_i(\omega)$.

Theorem 1 *The household can be represented using a unique utility function that does not depend on the Pareto weights if and only if the household belongs to the ISHARA class.*

Proof. In the appendix. ■

To understand theorems 1, observe that the unitary and collective frameworks are equivalent if and only if the optimal allocation of total income between members has no effect on the aggregate behavior of the household. Intuitively, for this to happen two conditions must be satisfied. First, each agent's income expansion path must be linear. If not, exploiting the nonlinearity, it is always possible to find two households with identical aggregate incomes, but different allocation of resources and hence different intertemporal behavior. This has an important consequence: even if all agents have identical utility functions, the standard and collective framework are not necessarily equivalent. Second, the slope of the linear income expansion path must be identical across agents. The strong implication is that household members must have the same sensitivity to risk and uncertainty and this is not supported by empirical evidence. For instance, Barsky, Juster, Kimball and Shapiro (1997) find that the correlation of relative risk tolerance across household members is only 0.12. Only ISHARA households satisfy both conditions.

The following result is a simple corollary of theorem 1 and it claims that under the assumptions of full efficiency and ISHARA households the standard Euler equation (7) is satisfied.

Corollary 1 *Let* $\left\{c^i(t,\omega), Q(t,\omega)\right\}_{t \in \mathbf{T}, \omega \in \Omega}^{i=1,...,n}$ *be the solution of the full efficiency ICM. Set* $C(t,\omega) = \sum_{i=1}^{n} c^i(t,\omega)$ *for every* (t,ω). *Assume that the household belongs to the ISHARA class. Then, the* *following household Euler equation is satisfied:*

$$U_C\left(C\left(t,\omega\right),Q\left(t,\omega\right)\right)=\beta E_t\left[U_C\left(C\left(t+1,\omega\right),Q\left(t+1,\omega\right)\right)R_{t+1}\right].$$

Proof. In the appendix. ■

This result suggest that, for non-ISHARA households, the only aggregator that can be used to characterize household preferences will depend on the distribution of decision power and on the set of variables having an effect on it.

It is important to establish whether this result still applies if full efficiency is replaced by limited commitment. In particular, suppose that households belong to the ISHARA class, but individual members cannot commit for the future. The following result states that household Euler equations are replaced by supermartingales, even if the conditions for Gorman aggregations are satisfied.

Assumption 4 *The reservation utility of each member, in each period and state of the world is and increasing function of household savings, i.e.*

$$\frac{\partial \underline{u}_{i,t+1}(\omega)}{\partial s_t\left(\omega\right)}\geq 0\, for\, any\, i,\ t,\ \omega. \tag{12}$$

Theorem 2 *Consumption Euler equations are replace by consumption supermartingales,*[4]

$$U_C\left(C\left(t,\omega\right),Q\left(t,\omega\right)\right)>\beta E_t\left[U_C\left(C\left(t+1,\omega\right),Q\left(t+1,\omega\right)\right)R_{t+1}\right], \tag{13}$$

Two remarks are in order. First of all, condition (13) resembles in part the results obtained in the literature on limited commitment.[5]. In this literature, the inequality applies to each agent, but not necessarily to the group. Theorem 2 suggests that a similar inequality applies to the household, if it belongs to the ISHARA class. Second, condition (13) is isomorphic to the findings of the literature on liquidity constraints. Consequently, a test designed to detect liquidity constraints using this inequality has no power against the alternative of limited commitment.

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[4]Intuitively savings should have a positive effect on individual outside options. If savings affect negatively the reservation utility of at least one agent, then consumption Euler equations are still replaced by inequalities, but the direction of the inequality depends on the relative effect of savings on individual outside options

[5]See for example Ligon, Thomas and Worral (1990), Kocherlakota (1996) and Attanasio and Rios-Rull (2000)

In the next section, these results will be used to set up three tests to verify the empirical relevance of the ICM.

4 Testing Household Intertemporal Behavior

The unitary model and the ICM generate different household Euler equations. Since household Euler equations are generally highly non-linear it is difficult to determine how these differences affect household Euler equations. A possible solution is to linearize the Euler equations using a Taylor expansion.

In this section, I will restrict the analysis to households with 2 decision makers. At the same time, I will generalize the analyzes to consider also household Euler equations for the public good. In the following theorem, log-linearized household Euler equations are derived using the full efficiency ICM. Let ϕ_1 and ϕ_2 be defined as follows:

$$\phi_1\left(\hat{C},\hat{Q},\hat{Z}\right)=\ln\left\{V_C\left(\exp\left\{\ln\left(\hat{C}E[C]\right)\right\},\exp\left\{\ln\left(\hat{Q}E[Q]\right)\right\},\kappa\left(\exp\left\{\ln\left(\hat{Z}E[Z]\right)\right\}\right)\right)\right\}$$
$$\phi_2\left(\hat{C},\hat{Q},\hat{Z}\right)=\ln\left\{V_Q\left(\exp\left\{\ln\left(\hat{C}E[C]\right)\right\},\exp\left\{\ln\left(\hat{Q}E[Q]\right)\right\},\kappa\left(\exp\left\{\ln\left(\hat{Z}E[Z]\right)\right\}\right)\right)\right\}$$

where $\hat{C}=\ln\frac{C}{E[C]}$, $\hat{Q}=\ln\frac{Q}{E[Q]}$, $\hat{Z}=\ln\frac{Z}{E[Z]}$ and κ is equal to μ if the full efficiency ICM is considered and equal to M if the limited commitment ICM is considered.

Assumption 5 $\phi_i\left(\hat{C},\hat{Q},Z\right):\Theta_i\rightarrow\mathbb{R}$, *where* $\phi_i\in C^3$, $\Theta_i\in\mathbb{R}^2\times\mathbb{R}^m$ *is open, convex and* $0\in\Theta_i$, *for i=1,2.*

Theorem 3 *Under assumption 5, the household Euler equations for the full efficiency ICM can be written as follows:*

$$\ln\frac{C_{t+1}}{C_t}=\alpha_0+\alpha_1\ln R_{t+1}+\alpha_2\ln\frac{Q_{t+1}}{Q_t}+\sum_{i=1}^{m}\alpha_{i,3}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Ci}{\hat{C}_i}+\sum_{i=1}^{m}\alpha_{i,4}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Q_t}{\hat{Q}}$$
$$+\alpha_5\left[\left(\ln\frac{C_{t+1}}{\hat{C}}\right)^2-\left(\ln\frac{C_t}{\hat{C}}\right)^2\right]+\alpha_6\left[\left(\ln\frac{Q_{t+1}}{\hat{Q}}\right)^2-\left(\ln\frac{Q_t}{\hat{Q}}\right)^2\right]$$
$$+\alpha_7\left[\ln\frac{C_{t+1}}{\hat{C}}\ln\frac{Q_{t+1}}{\hat{Q}}-\ln\frac{C_t}{\hat{C}}\ln\frac{Q_t}{\hat{Q}}\right]+R_C\left(\hat{C},\hat{Q},\hat{Z}\right)+\ln\left(1+e_{t+1,C}\right),$$

$$
\begin{aligned}
\ln\frac{Q_{t+1}}{Q_t} &= \delta_0+\delta_1\ln R_{t+1}+\delta_2\ln\frac{C_{t+1}}{C_t}+\sum_{i=1}^{m}\delta_{i,3}\ln\frac{z_i}{\hat{z}_i}\ln\frac{C_{t+1}}{\hat{C}_t}+\sum_{i=1}^{m}\delta_{i,4}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Q_{t+1}}{Q_t} \\
&+\delta_5\left[\left(\ln\frac{C_{t+1}}{\hat{C}}\right)^2-\left(\ln\frac{C_t}{\hat{C}}\right)^2\right]+\delta_6\left[\left(\ln\frac{Q_{t+1}}{\hat{Q}}\right)^2-\left(\ln\frac{Q_t}{\hat{Q}}\right)^2\right] \\
&+\delta_7\left[\ln\frac{C_{t+1}}{\hat{C}}\ln\frac{Q_{t+1}}{\hat{Q}}-\ln\frac{C_t}{\hat{C}}\ln\frac{Q_t}{\hat{Q}}\right]+R_Q\left(\hat{C},\hat{Q},\hat{Z}\right)+\ln\left(1+e_{t+1,Q}\right),
\end{aligned}
$$

with

$$
\begin{aligned}
\alpha_{i,3} &= \frac{V_C V_{CC\mu}-V_{CC}V_{C\mu}}{V_{CC}V_C}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{C}\bar{z}_i, \quad \alpha_{i,4}=\frac{V_C V_{CQ\mu}-V_{CQ}V_{C\mu}}{V_{CC}V_C}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{Q}\bar{z}_i, \\
\delta_{i,3} &= \frac{V_Q V_{QC\mu}-V_{QC}V_{Q\mu}}{V_{QQ}V_Q}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{C}\bar{z}_i, \quad \delta_{i,4}=\frac{V_Q V_{QQ\mu}-V_{QQ}V_{Q\mu}}{V_{QQ}V_Q}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{Q}\bar{z}_i,
\end{aligned}
$$

where $\bar{C}=E[C]$, $\bar{Q}=E[Q]$, $\bar{z}_i=E[z_i]$, R_C and R_Q are the remainders of the Taylor's formula and e_C and e_Q are the expectation errors.

Proof. In the appendix. ■

It is interesting to note that using the full efficiency ICM, the factors determining individual decision power enter the household Euler equations only as interaction terms with the private and public good. To have some insight note that under the assumption of separability over time and across states of nature, the household intertemporal optimization can be analyzed by considering a specific period and state of nature at a time. Under the assumption of full efficiency only the relative power of household members at time $t=0$ is relevant in the decision process, i.e. any change in individual outside options has no effect on the allocation of resources. Hence, each period and state of nature is characterized by an identical function $\mu(Z)$. Consequently, a change in one of the factors, z_i, has a different effect on (t',ω') relative to (t'',ω'') only because optimal consumption at (t',ω') differs from the optimal consumption at (t'',ω''), i.e. for each (t,ω) the change in $\mu(Z)$ is identical but it occurs at a different point of the (t,ω)-Pareto frontier. Since the Taylor expansion is calculate at the average value of C, Q and Z for each pair (t,ω), the first order term in z_i will be equal to zero. All the effect is captured by the interaction terms, which evaluate how the average value of optimal consumption differ from the actual value.

To be able to distinguish between the full efficiency ICM and the limited commitment ICM, it is important to derive a log-linearized version of the household Euler equation with limited commitment. The following theorem establishes that the factors affecting the individual decision power enter the intertemporal optimization condition, not only as interaction terms with private and public consumption as in the full efficiency case, but also directly.

Theorem 4 *Under assumption 5, the household Euler equation for the limited commitment ICM can be written as follows:*

$$\ln\frac{C_{t+1}}{C_t}=\alpha_0+\alpha_1\ln R_{t+1}+\alpha_2\ln\frac{Q_{t+1}}{Q_t}+\sum_{i=1}^{m}\alpha_3\ln\frac{z_i}{\hat{z}_i}+\sum_{i=1}^{m}\alpha_{i,4}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Ci}{\hat{C}_i}+\sum_{i=1}^{m}\alpha_{i,5}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Q_t}{\hat{Q}}$$
$$+\sum_{i=1}^{m}\alpha_6\left(\ln\frac{z_i}{\hat{z}_i}\right)^2+\alpha_7\left[\left(\ln\frac{C_{t+1}}{\hat{C}}\right)^2-\left(\ln\frac{C_t}{\hat{C}}\right)^2\right]+\alpha_8\left[\left(\ln\frac{Q_{t+1}}{\hat{Q}}\right)^2-\left(\ln\frac{Q_t}{\hat{Q}}\right)^2\right]$$
$$+\alpha_9\left[\ln\frac{C_{t+1}}{\hat{C}}\ln\frac{Q_{t+1}}{\hat{Q}}-\ln\frac{C_t}{\hat{C}}\ln\frac{Q_t}{\hat{Q}}\right]+R_C\left(\hat{C},\hat{Q},\hat{Z}\right)+\ln\left(1+e_{t+1,C}\right),$$

$$\ln\frac{Q_{t+1}}{Q_t}=\delta_0+\delta_1\ln R_{t+1}+\delta_2\ln\frac{C_{t+1}}{C_t}+\sum_{i=1}^{m}\delta_3\ln\frac{z_i}{\hat{z}_i}+\sum_{i=1}^{m}\delta_{i,4}\ln\frac{z_i}{\hat{z}_i}\ln\frac{C_{t+1}}{\hat{C}_t}+\sum_{i=1}^{m}\delta_{i,5}\ln\frac{z_i}{\hat{z}_i}\ln\frac{Q_{t+1}}{Q_t}$$
$$+\sum_{i=1}^{m}\delta_6\left(\ln\frac{z_i}{\hat{z}_i}\right)^2+\delta_7\left[\left(\ln\frac{C_{t+1}}{\hat{C}}\right)^2-\left(\ln\frac{C_t}{\hat{C}}\right)^2\right]+\delta_8\left[\left(\ln\frac{Q_{t+1}}{\hat{Q}}\right)^2-\left(\ln\frac{Q_t}{\hat{Q}}\right)^2\right]$$
$$+\delta_9\left[\ln\frac{C_{t+1}}{\hat{C}}\ln\frac{Q_{t+1}}{\hat{Q}}-\ln\frac{C_t}{\hat{C}}\ln\frac{Q_t}{\hat{Q}}\right]+R_Q\left(\hat{C},\hat{Q},\hat{Z}\right)+\ln\left(1+e_{t+1,Q}\right),$$

with

$$\alpha_{i,3}=\frac{V_{CM}}{V_{CC}}\frac{\partial\mu_t\left(\bar{Z}\right)}{\partial z_i}\bar{z}_i,\qquad\alpha_{i,4}=\frac{V_CV_{CC\mu}-V_{CC}V_{C\mu}}{V_{CC}V_C}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{C}\bar{z}_i,$$
$$\alpha_{i,5}=\frac{V_CV_{CQ\mu}-V_{CQ}V_{C\mu}}{V_{CC}V_C}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{Q}\bar{z}_i,\qquad\delta_{i,3}=\frac{V_{QM}}{V_{QQ}}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{z}_i,$$
$$\delta_{i,4}=\frac{V_QV_{QC\mu}-V_{QC}V_{Q\mu}}{V_{QQ}V_Q}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{C}\bar{z}_i,\qquad\delta_{i,5}=\frac{V_QV_{QQ\mu}-V_{QQ}V_{Q\mu}}{V_{QQ}V_Q}\frac{\partial\mu\left(\bar{Z}\right)}{\partial z_i}\bar{Q}\bar{z}_i,$$

where $\bar{C}=E[C]$, $\bar{Q}=E[Q]$, $\bar{z}_i=E[z_i]$, R_C *and* R_Q *are the remainders of the Taylor's formula,* e_C *and* e_Q *are the expectation errors,* $M_t=\sum_{\tau=0}^{t}\mu_\tau$ *and* μ_t *is the Kuhn-Tucker multiplier at time* t.

Proof. In the appendix. ■

To understand why the factors Z enter directly the Euler equation in the limited commitment ICM, note that in this version of the model, the relative power of each household member can vary over time through changes in the individual outside options. Consequently, a variation in the factor z_i will affect differently the distribution of decision power at time t, M_t, relative to the distribution of decision power at $t+1$, $M_{t+1} = M_t + \mu_{t+1}$.

The following result is a simple corollary of theorems 3 and 4.

Corollary 2 *If the household is composed by one decision maker, household consumption Euler equations simplify to the following standard form:*

$$\ln \frac{C_{t+1}}{C_t} = \alpha_0 + \alpha_1 \ln R_{t+1} + \alpha_2 \ln \frac{Q_{t+1}}{Q_t} + \alpha_5 \left[\left(\ln \frac{C_{t+1}}{\hat{C}} \right)^2 - \left(\ln \frac{C_t}{\hat{C}} \right)^2 \right]$$
$$+\alpha_6 \left[\left(\ln \frac{Q_{t+1}}{\hat{Q}} \right)^2 - \left(\ln \frac{Q_t}{\hat{Q}} \right)^2 \right] + R_C \left(\hat{C}, \hat{Q}, \hat{Z} \right) + \ln (1 + e_{t+1,C}),$$
$$\ln \frac{Q_{t+1}}{Q_t} = \delta_0 + \delta_1 \ln R_{t+1} + \delta_2 \ln \frac{C_{t+1}}{C_t} + \delta_5 \left[\left(\ln \frac{C_{t+1}}{\hat{C}} \right)^2 - \left(\ln \frac{C_t}{\hat{C}} \right)^2 \right]$$
$$+\delta_6 \left[\left(\ln \frac{Q_{t+1}}{\hat{Q}} \right)^2 - \left(\ln \frac{Q_t}{\hat{Q}} \right)^2 \right] + R_Q \left(\hat{C}, \hat{Q}, \hat{Z} \right) + \ln (1 + e_{t+1,Q}),$$

By means of theorems 3, 4 and corollary 2 it is possible to set up a first test to evaluate the ability of the unitary and collective framework to approximate household intertemporal behavior.

TEST 1. *Under the assumption of rational expectations, Euler equations of households with only one decision maker should not exhibit excess sensitivity to income or other variables known at time t. However, income and other factors known at time t should enter significantly household Euler equations, if these variables affect the individual decision power. This test can be implemented estimating separately Euler equations for single and couples after controlling for self selection.*

The approach of splitting the sample in two groups based on theoretical ground has been extensively used in the literature. For example, Bernanke (1984), Hayashi (1985), Zeldes (1989) and more recently Browning and Chiappori (1998) make use of this strategy. Theorems 3 and 4 make clear that test 1 evaluates the unitary model against the ICM, but it is not possible to distinguish between the full-efficiency and the limited commitment version of the model. However, if test 1 rejects the unitary model, by means of theorems 3 and 4 it is possible to construct an additional test to discern between the two versions of the ICM.

TEST 2. *Under the assumption of full commitment, the factors Z should not enter the household Euler equations directly, but only as interaction terms with private and public consumption. If limited commitment is a correct specification of household intertemporal behavior, the factors Z should enter both directly and as interaction terms in the household Euler equations.*

If test 1 rejects the unitary model, test 2 enables one to distinguish between the two versions of the ICM. However, other competing models may be consistent with the outcome of tests 1 and 2. The next corollary to theorems 3 and 4 contains the ingredients to derive a proportionality condition that is unlikely to be fulfilled if the ICM is not a correct characterization of household behavior.

Corollary 3 *If the full efficiency ICM is a correct representation of household intertemporal behavior, then,*

$$\frac{\alpha_{i,3}}{\alpha_{j,3}} = \frac{\delta_{i,3}}{\delta_{j,3}} = \Psi_{i,j}, \qquad \frac{\alpha_{i,4}}{\alpha_{j,4}} = \frac{\delta_{i,4}}{\delta_{j,4}} = \Phi_{i,j},$$

for any pair of factors z_i, z_j. If the limited commitment ICM is a correct representation of household intertemporal behavior, then,

$$\frac{\alpha_{i,3}}{\alpha_{j,3}} = \frac{\delta_{i,3}}{\delta_{j,3}} = \psi_{i,j}, \qquad \frac{\alpha_{i,4}}{\alpha_{j,4}} = \frac{\delta_{i,4}}{\delta_{j,4}} = \phi_{i,j}, \qquad \frac{\alpha_{i,5}}{\alpha_{j,5}} = \frac{\delta_{i,5}}{\delta_{j,5}} = \upsilon_{i,j},$$

for any pair of factors z_i, z_j.

A test to determine the empirical validity of the full efficiency ICM or limited commitment ICM can now be constructed.

TEST 3. *Euler equations for private consumption and the public good should be estimated simultaneously. Then the proportionality condition can be tested.*

To perform this test at least two consumption goods and two factors are required.

5 Data

In the estimation of household Euler equations two different data sets are employed: the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). Both data sets have advantages and disadvantages. The PSID is a true panel, but unfortunately it contains only data on food consumption. The CEX is a survey eliciting detailed information on almost any category of consumption, but it is a rotating panel. In particular, households are followed for only 4 quarters and then dropped from the sample. The next two subsections describe the two data sets.

5.1 PSID

The PSID has gathered data on income and consumption annually each spring from 1968, following the same households and their split-offs over time. In the estimation, I use the data collected from 1975 to 1987. I do not use the data from 1968 to 1974 because the marginal tax rate was not calculated before 1974.[6] I do not use data from 1988 to 1993, the last available wave, because the food consumption questions were not asked in 1988 and 1989. Two variables are crucial in the tests derived in the previous section: household total consumption and income.

Consumption. The PSID does not contain data on total consumption. The only measure of consumption reported in the survey is annual household expenditure on food. There are advantages and disadvantages in utilizing the *PSID*. The use of food consumption in the estimation of the Euler equation is justified only if food is separable from other consumption components.[7] It is also well documented the existence of measurement errors in the data contained in the PSID.[8] However, the estimation of the Euler equation requires the service flows of consumption goods for a given period. Food expenditure is less likely to contain durable parts. Finally, a legitimate question is whether the empirical relevance of the ICM can be determined using only food

[6]The marginal tax rate can be computed using the NBER's TAXSIM model. However, this is likely to introduce additional measurement errors.

[7]There is mixed evidence regarding the separability between food and other consumption goods (see for example Deaton and Muellbauer (1980) and Mankiw, Rotenberg and Summer (1985))

[8]See for example Altonji and Siow (1987) and Runkle (1991).

consumption. Intuitively any type of consumption should be appropriate. The PSID collects information on three separate components of food expenditure: the cost of food consumed at home; the amount spent in restaurants; the value of food purchased with food stamps. Any empirical work utilizing consumption data from the PSID struggles with the timing of the question concerning food consumption. In fact, it is not clear what period the survey question refers to. The food consumption question in the PSID questionnaire is, " How much do you spend on the food that you use at home in an average week?" From 1977 this question has been asked following the food stamp question "Did you receive or buy food stamps, *last month*?". If the answer was affirmative, the following question was asked, "In addition to what you spent on food stamps, did you spend any money on food that you use at home?" and, if yes "How much?". Therefore, it seems evident that the surveyors are trying to extract information about food consumption for the period that immediately precedes the interview. Hence, given that the interviews take place in March or April, I interpret the answers as concerning food consumption for the first quarter of the interview year and choose accordingly the timing of the interest rate and of the inflation rates. I also experiment with the alternative interpretation that the consumption question refers to the prior calendar year and obtain similar results[9]. To have total food consumption, I deflate food consumed at home and away from home by their respective consumer price indexes (CPI) and sum the two real components.

Disposable income. Disposable income is computed as total household income, minus taxes, plus transfers.

The real after-tax interest rate. The interest rate is the first quarter average of the 1-year Treasury bill rate. I use the marginal tax rate on earned income for husband and wife for year $t+1$ as estimated by the Survey Research Center of the University of Michigan. The measure of the inflation rate is the growth of overall food CPI between the first quarter of year t and year $t+1$.

[9]Zeldes (1989) and Runkle (1991) give the same interpretation of the question on food consumption, whereas Hall and Mishkin (1982) and Lawrance (1991) use the alternative interpretation.

5.2 CEX

Since 1980, the CEX survey has been collecting data on household consumption, income and different types of demographics. The survey is a rotating panel organized by the Bureau of Labor Statistics (BLS). Each quarter about 4500 households, representative of the US population, are interviewed: 80% are reinterviewed the following quarter, while the remaining 20% are replaced by a new randomly selected group. Each household is interviewed at most for four quarters and detailed information are elicited in regard to expenditures for each of the three months preceding the interview, and in regard to income and demographics for the quarter preceding the interview.[10] The data used in the estimation cover the period 1980-1995.

Consumption. The CEX data set contains monthly data on consumption. Total consumption is computed as the sum of food at home, food out, tobacco, alcohol, other nondurable goods and services such as heating fuel, public and private transportation, personal care and semidurable goods which include clothing and shoes. In particular, from the definition of total consumption I exclude consumer durables, housing, education and health expenditure. Total consumption is deflated using the Consumer Price Indices published by the BLS. Specifically, the price index for the composite good is calculated as a weighted average of individual price indices, with weights equal to the expenditure share for the particular consumption good.

Disposable income. Disposable income is computed as total household income, minus taxes.

The real after-tax interest rate. The interest rate is the 3-month Treasury bill rate. The marginal tax rate on earned income is computed using the NBER's TAXSIM model. The measure of the inflation rate is the household specific price index.

Rather than employing the short panel dimension of the CEX, I construct synthetic panels. The synthetic panels are constructed using two variables: the year of birth of the head of the household and a dummy equal to 1 if the head is married and 0 otherwise. All households are assigned to one of these cells. I then average the variables of interest over all the households belonging to a given cohort observed in a given quarter. To avoid unnecessary overlapping between quarters, for each

[10] Each household is interviewed for 5 quarter, but the first interview is used to make contact and no information is publicly available.

Table 1: cohort definition for couples and singles

cohort	Year of Birth	Age in 1980	Average Cell Size Couples	Average Cell Size Singles	Used in Estimation
1	1905-1909	71-77	61	71	no
2	1910-1914	66-72	118	107	no
3	1915-1919	61-67	153	113	yes
4	1920-1924	56-62	178	119	yes
5	1925-1929	51-57	193	118	yes
6	1930-1934	46-52	196	102	yes
7	1935-1939	41-47	210	103	yes
8	1940-1944	36-42	248	128	yes
9	1945-1949	31-37	312	170	yes
10	1950-1954	26-32	322	205	yes
11	1955-1959	21-27	319	238	yes
12	1960-1964	16-20	263	241	yes
13	1965-1969	11-15	193	205	no

household in each quarter, I use only the consumption data for the month preceding the interview and drop the data for the previous two months. Table 5.2 contains a description of the cohort.

5.3 Selection of the PSID Sample

The PSID data set contains a subsample in which low-income households are overrepresented. I eliminate this group to have a representative sample of the US economy. I include split-off families as separate families, but eliminate from the sample the observations associated to their first year as a new family to reduce the measurement errors. I eliminate outliers from the sample. Specifically, an observation is considered an outlier if $ln\left(c_{i,t+1}/c_{it}\right) > 1.1$, i.e. if the level of food consumption rose or fell by more than a factor of 3 in a year. I only include in the sample households whose head is between the ages of 19 and 75. Finally, I do not use an observation if consumption or income data were estimated by the Survey Research Center.

5.4 Selection of the CEX Sample

I exclude from the sample rural households, households living in student housing, household in which the head is younger than 19 and older than 75 and households with incomplete income responses. I drop households experiencing a change in marital status. Finally a cohort is dropped if the cell size is lower than 100.

6 Econometric Issues

The household Euler equations derived in section 4 contain the logarithm of the expectation error. By assumption, the expectation error term $e_{i,t+1}$ has mean zero and is orthogonal to the information set at time t. However, it could be correlated across households because of macro shocks. To solve this problem, I will assume, as in Zeldes (1989), that $(1+e_{i,t+1})$ can be decomposed in the product of two orthogonal components with mean zero: an aggregate component that is common to all households, $\left(1+e^{a}_{t+1}\right)$, and an idiosyncratic component, $(1+\bar{e}_{i,t+1})$. By construction, $ln\,(1+\bar{e}_{i,t+1})$ does not necessarily have zero expectation, implying that the error term of the log-linearized Euler equations is not, in general, a mean zero variable. To solve this problem, notice that the expected value of the second order Taylor expansion of $ln\,(1+\bar{e}_{it+1})$ is given by,

$$E_t\left[ln\,(1+\bar{e}_{i,t+1})\right] = -\frac{1}{2}\sigma^2_{i,t+1},$$

where $\sigma^2_{i,t+1} = Var_t\left[\bar{e}_{i,t+1}\right]$. As a consequence $\eta_{i,t+1} = ln\,(1+\bar{e}_{i,t+1}) - \frac{1}{2}\sigma^2_{i,t+1}$ has mean zero. Taking into account all this and assuming that $Var_t\left[\bar{e}_{i,t+1}\right]$ is constant over time and across families, the log-linearized Euler equations can be estimated consistently including $Var_t\left[\bar{e}_{i,t+1}\right]$ in the constant. The likely presence of aggregate shocks implies that Euler equations can be consistently estimated only if households are observed over a long period of time. Indeed, if this condition is satisfied, aggregate expectation errors are averaged out. In the PSID, most households are observed for the entire sample period. Therefore aggregate errors should be averaged out.[11] In regard to the CEX, since I construct synthetic cohorts and these cohorts are followed for the whole sample period, the aggregate error should not affect the estimation results. This solves the small T problem discussed by Chamberlain (1984).

In the theoretical part of the paper, a public good is always included in the Euler equations for two reasons. Some of the tests derived in section 4 requires the joint estimation of Euler equations for two different goods. More important, there is at least one public good that is likely to affect household intertemporal behavior: consumption of children. In most households, children are not decision makers. Consequently, they do not enter the household decision process directly. However, their consumption should be in the utility functions of their parents. Unfortunately, the consumption of children is not measured in the PSID and CEX. As a partial solution, I use the number of children as a proxy for consumption of children.

In the estimation of Euler equations with the CEX, two demographic variables are always included: the age of the head of the household and family size growth. When the PSID data set is used, family size growth is replaced by annual food needs growth, since only food consumption is observed.

I will assume that the number of children, age and family size at time $t+1$ are known to the household at time t, implying that they will be uncorrelated with the error term. In general the after-tax interest rate will not be known at time t and will probably be correlated with the expectation error because of the correlation between consumption and the marginal tax rate at $t+1$. To solve this problem, I instrument the after-tax interest rate. Since total income may be affected by measurement errors, this variable is also instrumented. Under the assumption of rational expectations, variables known at time t should be valid instruments. Measurement errors may introduce dependence between variables known at time t and future variables, even under rational expectations. To avoid this problem I only use variables known at $t-1$. The following instruments are used in the estimations: the second, third and fourth lag of the nominal interest rate; the second, third and fourth lag of consumption growth; the second, third and fourth lag of

[11] The PSID sample is not a balanced panel, implying that different households are in the survey for different periods.

family size; the second, third and fourth lag of children.[12]

In the derivation of household Euler equations, labor supply decisions are never considered. The implicit assumption is that consumption and leisure are separable. The PSID contains only food consumption. Consequently, I will assume separability between food and other goods. When Euler equations are estimated using the CEX data set, all consumption goods are aggregated in a composite good. Hence, I will assume that all the conditions of the composite good theorem are satisfied.

To perform the tests, I divide the sample in two groups using marital status at time t. The first group is composed by singles, widows, divorcees and separated couples. The second group contains married and cohabiting couples. If the Euler equations are estimated using OLS, the estimates are likely to be affected by self selection bias. Therefore, I employ a Heckman two-step estimator adding the inverse Mill's ratio to the Euler equations derived in this section.

The main implication of the theoretical part is that the family structure is extremely important for the determination of the Euler equation, as it affects the household decision process. While it is beyond the scope of this paper to model theoretically how households choose between different family structures, from an econometric point of view it is crucial to take it into consideration.[13]

Let $D^m_{j,i,t}$ be a dummy variable equal to 1 if agent j is married or cohabiting in household i. Let $E\left[V^s_{j,i}(X,t)\,|\mathfrak{F}_t\right]$ and $E\left[V^m_{j,i}(X,t)\,|\mathfrak{F}_t\right]$ be, respectively, the expected value of being single and married today for agent j in household i, where X is the set of variables affecting $V^s_{j,i}$ and $V^m_{j,i}$ and $\mathfrak{F}_t$ is the information known at time t. I will make the simplifying assumption that $E\left[V^s_{j,i}(X,t)\,|\mathfrak{F}_t\right]$ and $E\left[V^m_{j,i}(X,t)\,|\mathfrak{F}_t\right]$ are linear functions of current variables, X_t.

$$V^s_{j,i}(X,t) = X_{j,i,t}\beta^s_t + e^s_{j,i,t}$$

$$V^m_{j,i}(X,t) = X_{j,i,t}\beta^m_t + e^m_{j,i,t}$$

with $E\left[e^h_{j,i,t}\,|\mathfrak{F}_t\right] = 0$, $h = s, m$. Denote with $\hat{V}^m_{j,i}(X,t)$ the best feasible match available to

[12]In the estimations with the quarterly CEX data, three seasonal dummies are also included.

[13]For a discussion on the variables affecting the marriage decision see for instance Lillard et al. (1995) and Rindfuss and VandenHeuvel (1990).

agent j, i.e.

$$\hat{V}^m_{j,i}(X,t) = X_{j,i,t}\hat{\beta}^m_t + \hat{e}^m_{j,i,t} = \max_{k\in K} V^m_{j,k}(X,t)$$

where K is the set of feasible households available to agent j. Then the selection decision can be written as follows,

$$D^m_{j,i,t} = 1 \;\; if \;\; \left(X_{j,i,t}\hat{\beta}^m_t + \hat{e}^m_{j,i,t}\right) - \left(X_{j,i,t}\beta^s_t + e^s_{j,i,t}\right) \geq 0 \qquad (14)$$
$$D^m_{j,i,t} = 0 \;\; otherwise.$$

Given the definition of $D^m_{j,i,t}$, the Euler equation for household i can be written as follows:

$$GC^s_{i,t+1} = \delta^s + Y_{i,t}\gamma^s + \eta^s_{i,t+1} \;\; if \;\; e_{j,i,t} < -X_{j,i,t}\beta_t \qquad (15)$$
$$GC^m_{i,t+1} = \delta^m + Y_{i,t}\gamma^m + \eta^m_{i,t+1} \;\; otherwise \qquad (16)$$

where $e_{it} = \hat{e}^m_{j,i,t} - e^s_{j,i,t}$, $\beta_t = \hat{\beta}^m_t - \beta^s_t$ and $Y_{i,t}$ is the set of variables characterizing the Euler equation. Under the assumption that for each t and for $h = s, m$, (η^h_i, e_i) is a bivariate normal distribution with mean vector 0 and covariance matrix,

$$\begin{bmatrix} \sigma^2_{\eta,h} & \rho_{\eta,h} \\ & 1 \end{bmatrix}$$

the household Euler equations can be consistently estimated applying a standard argument to equations (15) and (16) to derive,[14]

$$GC^s_{i,t+1} = \delta^s + Y_{i,t}\gamma^s + \rho_{\eta,s}\sigma_{\eta,s}\lambda^s_{i,t} + \epsilon^s_{i,t+1} \qquad (17)$$
$$GC^m_{i,t+1} = \delta^m + Y_{i,t}\gamma^m + \rho_{\eta,m}\sigma_{\eta,m}\lambda^m_{i,t} + \epsilon^m_{i,t+1}. \qquad (18)$$

where $\rho_{\eta,s}\sigma_{\eta,s}\lambda^s_{i,t} = E\left[\eta^s_{i,t+1}\,|D^m_{it} = 0, Y_{i,t}\right]$, $\rho_{\eta,m}\sigma_{\eta,m}\lambda^m_{i,t} = E\left[\eta^m_{i,t+1}\,|D^m_{it} = 1, Y_{i,t}\right]$, $\lambda^h_{i,t}$ is the inverse Mill's ratio and $\epsilon^h_{i,t+1}$ are the new residuals, with zero conditional mean. The two-stage estimation procedure can now be spelled out. First, estimates of β_t are obtained by estimating T

[14] For a thorough discussion of the Heckman two-steps estimator see Heckman (1976, 1979), Lee (1978).

cross-sectional probit specifications by maximum likelihood. [15] I will then estimate equations (17) and (18) by OLS, substituting the estimated $\hat{\lambda}^s_{i,t}$ and $\hat{\lambda}^m_{i,t}$ for $\lambda^s_{i,t}$ and $\lambda^m_{i,t}$.

To calculate the asymptotic covariance matrix for the parameter estimates, I follow Chamberlain (1984) and MaCurdy (1982). Let $\psi^h = \rho_{\eta h}\sigma^2_{\eta h}$. Moreover, let θ, Λ, and Z be defined as follows:

$$
\begin{aligned}
\theta &= (\beta_1, ..., \beta_T) \\
\xi^k &= \left(\gamma^k, \psi^h\right), \quad k = s, m \\
Z &= \left(Y, \hat{\lambda}^h\right) \\
\Lambda^k &= \left[\frac{\partial \hat{\lambda}^k_1}{\partial \theta}, \frac{\partial \hat{\lambda}^k_2}{\partial \theta}, ..., \frac{\partial \hat{\lambda}^k_{NT}}{\partial \theta}\right],
\end{aligned}
$$

where NT is the total number of observations. Then the asymptotic covariance matrix for the parameter estimates of equations (17) and (18) is,

$$
Var\left(\hat{\xi}^k\right) = \left(Z'Z\right)^{-1} Z' \left[Var\left(\eta^k\right) + \sigma^2_{\eta k}\rho^2_{\eta k}\Lambda^k Var\left(\hat{\theta}\right)\Lambda^{k\prime}\right] Z \left(Z'Z\right)^{-1}
$$

where $Var\left(\eta^k\right)$ is the covariance matrix of the residual of the standard Euler equations. Since the first step estimates are obtained using T cross-sectional probits, I follow Chamberlain (1984) and MaCurdy (1982) to determine $Var\left(\hat{\theta}\right)$. Specifically, let $Q\left(\theta\right)$ be the likelihood function of the probit. Let ψ, Δ and J be defined as follows:

$$
\psi\left(X, \theta\right) = \frac{\partial Q}{\partial \theta}
$$

$$
\Delta = E\left[\psi\left(X, \theta\right)\psi\left(X, \theta\right)'\right]
$$

$$
J = E\left[\frac{\partial \psi\left(X, \theta\right)}{\partial \theta'}\right].
$$

[15] The estimation of T cross-sectional probit specifications by maximum likelihood enables one to allow for a general structure of the variance-covariance matrix without computing multiple integrals. Since the inverse Mill's ratio is computed using a panel, it is important to allow for this generalization. The cost is a loss in efficiency. When the CEX data are used in the estimation, the first step estimates are computed by means of a single probit, because I do not have enough observations each period to compute T cross-sectional probits.

Then[16],

$$Var\left(\hat{\theta}\right) = J^{-1}\Delta J^{-1}. \tag{19}$$

Finally, to allow each family to have a different and unrestricted covariance structure, I follow White (1984) and Keane and Runkle (1992) and estimate $Var\left(\eta^k\right)$ as follows,

$$\widehat{Var}\left(\eta^k\right) = I_N \otimes \frac{1}{N}\sum_{i=1}^{N} u_i u_i', \tag{20}$$

where u_i is the vector of estimated Euler equation residuals for family i.

Depending on the data set employed in the estimation, two different exclusion restrictions are used. With the PSID, it is assumed that the number of siblings of the head of the household and race dummies are correlated with the marriage decision, but not with the consumption decision. The CEX does not report the number of siblings of the head. Dummies indicating the origin or ancestry of the head are used to proxy for the number of siblings and the race dummies.

7 Results

Tables 4 and 5 in the appendix contain a summary of means for the main variables. Tables 2 and 3 contain the estimates for singles and couples when the log of household total income is included in the Euler equations as a proxy for the factors affecting individual decision power.

7.1 Test 1

Group 1. The ICM has one clear prediction: the household Euler equation should not be violated for families with one adult. Tables 2 and 3 suggest that this hypothesis is not rejected using both the PSID and the CEX. The coefficient on the log of available income at time t is not significant and small in magnitude. The coefficient on the interest rate is positive. The coefficient on age is negative. The coefficient on family size growth is positive as expected and strongly significant.

[16] Since $\hat{\theta}$ is estimated by means of T cross-sectional probits, J will be a block diagonal matrix with the generic t-block equivalent to the information matrix at t.

The coefficient on the public good, number of children, is negative. The self selection correction term is not significant.

Group 2. Euler equations for couples exhibit excess sensitivity to income using the PSID as well as the CEX. Specifically, the coefficient on the log of disposable income is significant and, more important, its magnitude is two and one half times the magnitude of the coefficient obtained for group 1 in the CEX, and almost ten times larger in the PSID. The coefficient on age is negative. The coefficient on family growth is positive and strongly significant. Finally the coefficient on the interest rate is positive as expected. As for groups 1 the sample selection coefficient is not significant.

7.2 Test 2

To be added.

7.3 Test 3

To be added.

Table 2: PSID data set. Group 1: singles, divorcees, separated and widows. Group 2: married and cohabiting couples (standard errors in brackets).

Euler Equation Estimates for Two Subgroup		
Independent Variable	Group 1	Group 2
age of head	-0.001	-0.0011
	[0.00023]	[0.00043]
growth in annual food needs	0.39	0.27
	[0.064]	[0.056]
growth in n. of children	-0.0026	0.0008
	[0.0049]	[0.0021]
real after-tax interest rate	0.53	0.48
	[0.31]	[0.14]
log of disposable income	-0.004	-0.039
	[0.021]	[0.014]
inverse Mill's ration	0.0088	-0.012
	[0.012]	[0.028]
number of observations	3727	15258
number of families	797	2178

Table 3: CEX data set. Group 1: cohorts composed by singles, divorcees, separated and widows. Group 2: cohorts composed by married and cohabiting couples (standard errors in brackets).

Euler Equation Estimates for Two Subgroup		
Independent Variable	Group 1	Group 2
age of head	-0.0003	-5.5e-06
	[0.0003]	[0.0002]
growth family size	0.28	0.84
	[0.13]	[0.27]
growth in n. of children	-0.021	-0.041
	[0.026]	[0.018]
real after-tax interest rate	0.63	0.87
	[0.19]	[0.20]
log of disposable income	0.015	0.037
	[0.015]	[0.013]
inverse Mill's ration	-0.013	0.003
	[0.018]	[0.015]
number of observations	464	544
number of cohorts	10	10

8 Conclusions

In the paper, each household is represented as a group of agents making joint decisions. By means of this framework, it is shown that, under the assumption of full efficiency, the household can be represented by means of a unique utility function if and only if all members have identical discount factors and HARA preferences with identical shape parameter. If these conditions are not satisfied, the intertemporal allocation of resources will depend on the distribution of decision power. In particular, the traditional Euler equation will not be satisfied. For the limited commitment case, it is shown that, even if these restrictions are fulfilled, the household Euler equation is replaced by an inequality. It is also shown that the full efficiency ICM has different predictions for Euler equations from the limited commitment ICM: under full commitment, the factors affecting individual decision power should enter household Euler equations only as interaction terms with household consumption; if the assumption of full commitment is not satisfied, those factors should enter household Euler equations both directly and as interaction terms with household consumption.

The standard unitary framework as well as the ICM predict that the traditional Euler equation should not be violated for households with one decision maker. The two frameworks have opposite implications for households with several decision makers, however. The unitary model predicts that the standard Euler equation should not be violated, whereas the ICM predicts that it should be rejected. These predictions are tested using the PSID as well as the CEX. I find that Euler equations are not violated for households with one adult. However, Euler equations are strongly rejected for couples with both members in the labor force. The theoretical and empirical results indicate that it is crucial to model households with several decision makers as groups of agents with heterogeneous preferences and different decision power.

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A Appendix

A.1 Proof of theorem 1

(Sufficiency) Rubinstein (1974).

(Necessity) The second part of Theorem 1 will be proved in four steps.

First, the intertemporal decision process can be divided in two steps. In the first step, for an arbitrary sequence of public consumption, $\{Q_{t,\omega}\}$, the household decides the optimal sequence of private consumption and savings, $\left\{c^i_{t,\omega}, s_{t,\omega}\right\}^{i=1,\ldots,n}_{\omega\in\Omega,t\in T}$. In the second step, given the optimal sequence of private consumption and savings for any given sequence of the public good, the household chooses the optimal sequence of the public good.

The remaining parts of the theorem deal with the first step of the intertemporal decision process. Specifically, all the results should be interpreted conditional on the public good. To simplify the notation I will suppress the dependence on the public good.

Lemma 1 *Let $\left\{c^{i*}(t,\omega), s^{i*}(t,\omega)\right\}^{\omega\in\Omega,t\in T}_{i=1,\ldots n}$ be the optimal household allocation. Assume $\{\mu_i\}_{i=1,\ldots,n} \gg 0$ and $\lim\limits_{x\to 0} u^{i\prime}(x) = \infty$. Moreover suppose that u^i is continuous, strictly increasing, strictly concave and twice continuously differentiable. Then there exist prices $\{p(t,\omega)\}_{\omega\in\Omega,t\in T}$ and transfers $\left\{W^i\right\}_{i=1,\ldots n}$ such that $\{p(t,\omega)\}_{\omega\in\Omega,t\in T}$, $\left\{W^i\right\}_{i=1,\ldots n}$ and $\left\{c^{i*}(t,\omega), s^{i*}(t,\omega)\right\}^{\omega\in\Omega,t\in T}_{i=1,\ldots n}$ is an Arrow-Debreu equilibrium with transfers, or equivalently,*

(i) for each $i = 1, \ldots n$, $\left\{c^{i*}(t,\omega), s^{i*}(t,\omega)\right\}^{\omega\in\Omega,t\in T}_{i=1,\ldots n}$ solves

$$\max_{\{c^{i*},s^{i*}\}_{\omega\in\Omega,t\in T}} \sum_{t=0}^{T} E^i\left[\beta_i^t u_i\left(c^i(t,\omega)\right)\right]$$
$$s.t. \quad \sum_{t=0}^{T} \int_\Omega p(t,\omega)\, c^i(t,\omega)\, d\omega = W^i.$$

(ii) for each t, ω,

$$\sum_{i=1}^{n} c^{i*}(t,\omega) = Y(t,\omega) + R(t)\, s(t-1,\omega) - s(t,\omega)\,;$$

(ii) for each s, t, ω,

$$p(t,\omega)\,R(t)\,s^*(t-1,\omega) - p(t-1,\omega)\,s^*(t-1,\omega) \geq$$
$$p(t,\omega)\,R(t)\,s(t-1,\omega) - p(t-1,\omega)\,s(t-1,\omega)\,.$$

Proof. The technology $y = f(x) = Rx$ is convex. The utility functions are strictly concave and increasing. Let $X^i = \left\{c^i \in \Re^{n_\Omega} \times \Re^{T+1} : c^i \geq 0\right\}$ be the consumption set of agent i, where n_Ω denotes the cardinality of Ω. Then X^i is convex and $0 \in X^i$. Preferences are continuous by continuity of u^i for $i = 1, ..., n$. Finally, $\{\mu_i\}_{i=1,...,n} \gg 0$ and $\lim_{x\to 0} u^{i\prime}(x) = \infty$ imply that $\left(W^1, ..., W^n\right) \gg 0$. Hence the second welfare theorem with production can be applied and the result follows. ■

Let $W = \sum_{i=1}^n W^i$ and $c^i\left(t,\omega;W^i\right)$ be optimal consumption at (t,ω) if individual wealth is equal to W^i. Then household exact aggregation is defined as follows:

$$\sum_{i=1}^n c^i\left(t,\omega;W^i\right) = C(t,\omega;W) \quad \forall t,\omega,$$

i.e. for each time and state of the world, total household consumption depends only on total resources W, but not on its allocation between household members.

Lemma 2 *Household exact aggregation is satisfied if and only if for each pair (t,ω) individual Engel curves are linear with identical slope, i.e.*

$$c^i\left(t,\omega;W^i\right) = a^i(t,\omega) + b(t,\omega)\,W^i \quad \forall i,t,\omega.$$

Proof. Lemma 1 implies that the household problem can be written as a static problem. By Gorman (1951), in a static framework, exact aggregation is satisfied if and only if for each consumption good individual Engel curves are linear with identical slope. In this framework this is equivalent to

$$c^i\left(t,\omega;W^i\right) = a^i(t,\omega) + b(t,\omega)\,W^i \quad \forall i,t,\omega.$$

■

The next lemma establishes conditions on preferences for the Engel curves to be linear with identical slope and hence for exact aggregation to apply.

Lemma 3 *Individual Engel curves are linear with identical slope if and only if household members have identical discount factors, individual utility functions are HARA with identical curvature parameter.*

Proof. By lemma 1, the solution of the household problem can be determined solving for $i = 1, ..., n$ the following program:

$$\max_{\{c^{i*}, s^{i*}\}_{\omega \in \Omega, t \in T}} \sum_{t=0}^{T} \int_{\Omega} \beta_i^t u_i \left(c^i (t, \omega) \right) F_i (d\omega) \tag{21}$$
$$s.t. \quad \sum_{t=0}^{T} \int_{\Omega} p(t, \omega) c^i (t, \omega) d\omega = W^i.$$

By assumption, for each $i = 1, ..., n$, the probability measure $F_i(\omega)$ has a density $f_i(\omega)$. Therefore the program (21) can be written in the form,

$$\max_{\{c^{i*}, s^{i*}\}_{\omega \in \Omega, t \in T}} \sum_{t=0}^{T} \int_{\Omega} \beta_i^t u_i \left(c^i (t, \omega) \right) f_i (\omega) d\omega$$
$$s.t. \quad \sum_{t=0}^{T} \int_{\Omega} p(t, \omega) c^i (t, \omega) d\omega = W^i.$$

The corresponding Lagrangian is,

$$\mathcal{L} = \sum_{t=0}^{T} \int_{\Omega} \left[\beta_i^t u_i \left(c^i (t, \omega) \right) f_i (\omega) - \lambda_i p(t, \omega) c^i (t, \omega) \right] d\omega,$$

where λ_i is the Lagrangian multiplier. By the calculus of variations, the following equations are necessary conditions for $\left\{ c^{i*} (t, \omega) \right\}_{\omega \in \Omega, t \in T}$ to be a solution of the household problem:

$$\beta_i^t f_i (\omega) u_i' \left(c^i (t, \omega) \right) = \lambda_i p(t, \omega) \quad \forall t, \omega.$$

Hence,

$$\frac{\hat{f}_i (t, \omega) u_i' \left(c^i (t, \omega) \right)}{\hat{f}_i (t + k, \omega') u_i' \left(c^i (t + k, \omega') \right)} = \frac{p(t, \omega)}{p(t + k, \omega')} \quad \forall t, k, \omega, \omega',$$

where $\hat{f}_i (t, \omega) = \beta_i^t f_i (\omega)$. It is now possible to apply theorems 2 and 3 in Brennan and Kraus (1978) to conclude that individual Engel curves are parallel straight lines if and only if the following to conditions are jointly satisfied:

(i) all household members have HARA utility functions with identical curvature parameter;

(ii) $\hat{f}_i(t,\omega) = \hat{f}_j(t,\omega)$ for any i, j, t, ω.

Since $\hat{f}_i(t,\omega) = \beta_i^t f_i(\omega)$, condition (ii) is satisfied if and only if all agents have identical discount factors and identical probability measure over Ω. ■

A.2 Proof of corollary 1

In order to prove corollary 1 and theorems 2 the following results are required. The following lemma is theorem 198 in Hardy, Littlewood and Polya (1934).

Lemma 4 *Let x_1 and x_2 be nonnegative random variables defined on $(\Omega, \mathfrak{F})$ and finite almost everywhere. Set $x = x_1 + x_2$. If $\gamma \in \mathbb{R}$ and $\gamma < 0$, then the function $\left(\int x^\gamma dP\right)^{\frac{1}{\gamma}}$ is concave in x or equivalently (given homogeneity of degree 1),*

$$\left(\int x^\gamma dP\right)^{\frac{1}{\gamma}} \geq \left(\int x_1^\gamma dP\right)^{\frac{1}{\gamma}} + \left(\int x_2^\gamma dP\right)^{\frac{1}{\gamma}}.$$

Let A and B be two constants. If $P\{\omega \in \Omega : Ax_1(\omega) = Bx_2(\omega)\} < 1$, and $P\{\omega \in \Omega : x_1(\omega) = x_2(\omega) = 0\} = 0$, then

$$\left(\int x^\gamma dP\right)^{\frac{1}{\gamma}} > \left(\int x_1^\gamma dP\right)^{\frac{1}{\gamma}} + \left(\int x_2^\gamma dP\right)^{\frac{1}{\gamma}}.$$

Proof. See Hardy, Littlewood and Polya (1934). ■

Following Ash (1972), let a random object X_t be a mapping $X_t : (\Omega, \mathfrak{F}) \to (\Omega', \mathfrak{F}')$, for some measurable spaces $(\Omega, \mathfrak{F})$ and $(\Omega', \mathfrak{F}')$. Define $\sigma\langle X_t\rangle$ to be the σ-field generated by X_t, i.e. the minimum σ-field that makes X_t measurable between $\mathfrak{F}$ and $\mathfrak{F}'$. The following lemma gives a characterization of the filtration $\{\mathfrak{F}_t\}_{t\in T}$ in terms of random variables.

Lemma 5 *Let $\{\mathfrak{F}_t\}_{t\in T}$ be the filtration associated with $\mathfrak{F}$. Then for every $t \in \mathbf{T}$ there exists a random object $X_t : (\Omega, \mathfrak{F}) \to (\Omega', \mathfrak{F}')$ such that $\sigma\langle X_t\rangle = \mathfrak{F}_t$.*

Proof. Define X_t as follows,

(i) $X_t : (\Omega, \mathfrak{F}) \to (\Omega, \mathfrak{F}_t)$;

(ii) X_t is the identity map, i.e. $X_t(\omega) = \omega \; \forall \omega \in \Omega$.

$\forall A \in \mathfrak{F}_t$, $X_t^{-1}(A) = \{\omega \in \Omega : X_t(\omega) \in A\} = A$ by construction, which implies $X_t^{-1}(\mathfrak{F}_t) = \mathfrak{F}_t$.

Then by theorem 6.4.2 in Ash (1972), $\sigma \langle X_t \rangle = X_t^{-1}(\mathfrak{F}_t) = \mathfrak{F}_t$. ■

The following lemma claims that a conditional expectation given a σ-field can be written as a function of a well-chosen random object.

Lemma 6 *Let* $h(\omega) = E[Y | \mathfrak{F}_t]$ *and* $g(x) = E[Y | X_t = x]$. *Then there exists a random object* $X_t : (\Omega, \mathfrak{F}) \to (\Omega, \mathfrak{F}_t)$ *such that,*

$$h(\omega) = (g \circ X_t)(\omega) \; \forall \omega \in \Omega.$$

Proof. By Lemma 5, there exists a random object $X_t : (\Omega, \mathfrak{F}) \to (\Omega, \mathfrak{F}_t)$ such that $\sigma \langle X_t \rangle = X_t^{-1}(\mathfrak{F}_t) = \mathfrak{F}_t$.

Let $g(x) = E[Y | X_t = x]$. Then by theorem 6.4.3 in Ash (1972),

$$g(X_t(\omega)) = E[Y | \sigma \langle X_t \rangle] = E[Y | \mathfrak{F}_t] = h(\omega) \; \forall \omega \in \Omega.$$

■

Let X_t be the random object defined in lemma 5 and x a realization of X_t.

Lemma 7 *Assume* $Y : (\Omega, \mathfrak{F}) \to (\mathbb{R}, \mathfrak{R})$ *is a nonnegative random variable. For every* x *and* $B \in \mathfrak{R}$, *let* $P(x, B)$ *be a probability measure in* B *for each fixed* x *and a Borel measurable function of* x *for each fixed* B, *i.e.* $P(x, B)$ *is the conditional distribution of* Y *given* $X_t = x$. *Let* $\gamma \in \mathbb{R}$. *Then*

$$E[Y^\gamma | X_t = x] = \int_{\mathbb{R}} Y^\gamma P(x, dy).$$

Proof. The argument of the proof follows closely Ash (1972) section 6.3.5 part (d).

By construction, $Y : (\Omega, \mathfrak{F}) \to (\mathbb{R}, \mathfrak{R})$ and $X_t : (\Omega, \mathfrak{F}) \to (\Omega, \mathfrak{F}_t)$.

Let $\Omega' = \mathbb{R} \times \nleq$, $\mathfrak{F}' = \mathfrak{R} \times \mathfrak{F}$, $\pi_1(x,y) = x$ and $\pi_2(x,y) = y$.

$\forall A \in \mathfrak{F}_t$ let $P_{X_t}[A] = P[\{\omega \in \Omega : X_t(\omega) \in A\}]$, i.e. P_{X_t} is the probability on $(\Omega, \mathfrak{F}_t)$ induced from P by X_t.

Define P_{xy} as follows,

$$P_{xy}[A,B] = \int\limits_A P(x,B)\, dP_{X_t}(x).$$

By the product measure theorem, P_{xy} is the unique probability measure on $\mathfrak{F}'$ determined by P_{X_t} and $P(x,\cdot)$.

Consider $(\pi_2)^\gamma = y^\gamma : (\mathbb{R}, \mathfrak{R}) \to (\mathbb{R}, \mathfrak{R})$. $\int_{\mathbb{R}} y^\gamma dP(x,dy)$ exists by $Y(\omega) \geq 0\ \forall \omega \in \Omega$. Then, by Fubini's theorem and definition of P_{xy},

$$\begin{aligned}
&\int\limits_{\{X_t \in A\}} \pi_2^\gamma dP_{xy} = \int\limits_{\Omega'} \pi_2^\gamma I_{\{X_t \in A\}} dP_{xy} = \\
&= \int\limits_{\Omega} \int\limits_{\mathbb{R}} \pi_2(x,y)^\gamma I_A(x)\, P(x,dy)\, dP_{X_t}(x) = \\
&= \int\limits_A \left[\int\limits_{\mathbb{R}} y^\gamma P(x,dy) \right] dP_{X_t}(x).
\end{aligned}$$

By definition of conditional expectation, this implies,

$$E[\pi_2^\gamma | X_t = x] = E[Y^\gamma | X_t = x] = \int\limits_{\mathbb{R}} Y^\gamma P(x,dy).$$

■

Finally, for (4) to be well-defined we have to assign a conditional probability governing the transition between t and $t+1$. Let X_t be the random object defined in lemma 5 and x a particular realization of X_t. For every x and $B \in \mathfrak{F}$, let $P(x,B)$ be a probability measure in B for each fixed x, and a Borel measurable function of x for each fixed B. Consider a random variable $Y : (\Omega, \mathfrak{F}) \to (\mathbb{R}, \mathfrak{R})$. For every x and $A \in \mathfrak{R}$, let $P_Y(x,A) = P_Y\left(x, Y^{-1}(A)\right) = P_Y(x, \{\omega \in \Omega : Y(\omega) \in A\})$, i.e. $P_Y(x,A)$ is the conditional distribution induced from P of a random variable $Y : (\Omega, \mathfrak{F}) \to (\mathbb{R}, \mathfrak{R})$, given $X_t = x$.

It is now possible to prove corollary 1.

Proof. I will prove the theorems for $n = 2$. The proof for an arbitrary n can be obtained iterating $n-1$ times the same argument. Given the assumption of ISHARA households, without loss of generality I will assume $\beta R_{t+1} = 1$. By proposition 1 the solution of the ICM (2) is equivalent to the solution of the two-stage budgeting model. The assumption of HARA preferences ensures that second stage (4) has an interior solution. The first order conditions for the second stage (4) imply,

$$c_t^i = \left(u^{i\prime}\right)^{-1}\left(E\left[u^{i\prime}\left(c_{t+1}^i\right)|\mathfrak{F}_t\right]\right)\ for\ i = 1, 2.$$

Given assumption 2,

$$\left(u^{i\prime}\right)^{-1}\left(E\left[u^{i\prime}\left(c_{t+1}^i\right)|\mathfrak{F}_t\right]\right) = \gamma\left(\left(E\left[\left(a_i + \frac{c_{t+1}^i}{\gamma}\right)^{-\gamma}|\mathfrak{F}_t\right]\right)^{-\frac{1}{\gamma}} - a_i\right).$$

Fix an $\omega \in \Omega$. Define $z_{t+1}^i = a_i + \dfrac{c_{t+1}^i}{\gamma}$. By Lemma 7,

$$h^i(\omega) = E\left[\left(z_{t+1}^i\right)^{-\gamma}|\mathfrak{F}_t\right](\omega) = E\left[\left(z_{t+1}^i\right)^{-\gamma}|X_t(\omega) = x\right].$$

By definition of permissible income process, $z_{t+1}^i = a_i + \dfrac{c_{t+1}^i}{\gamma} \geq 0\ \forall\omega, i = 1, 2$. Let $P_{z^i}(x, B) = P_i^x(B)$. For any $B \in \mathfrak{R}$,

$$P_i^x(B) = P^x\left(z^i \in B\right) = P^x\left(\left\{\omega \in \Omega : z^i(\omega) \in B\right\}\right),$$

i.e. P_i^x is a probability measure induced from P^x by z^i. By Lemma 7 and the change of variable theorem,

$$E\left[\left(z_{t+1}^i\right)^{-\gamma}|X_t(\omega) = x\right] = \int_{\mathbb{R}}\left(z_{t+1}^i\right)^{-\gamma} dP_i^x = \int_{\nleq}\left(z_{t+1}^i(\omega)\right)^{-\gamma} dP^x(\omega). \qquad \text{(A.1)}$$

The first order conditions of the first stage (5) imply,

$$P\left[\left\{\omega \in \Omega : Az_t^1(\omega) = Bz_t^2(\omega)\right\}\right] = 1\ \forall(t, \omega), \qquad \text{(A.2)}$$

for some constant A and B. This condition is simply the Borsh rule. Lemma 4, equations (A.1) and (A.2) imply,

$$
\begin{aligned}
c_t^1 + c_t^2 &= \gamma \left(\left(\int_{\nleq} \left(z_{t+1}^1 (\omega) \right)^{-\gamma} dP^x (\omega) \right)^{-\frac{1}{\gamma}} - a_1 \right) + \gamma \left(\left(\int_{\nleq} \left(z_{t+1}^2 (\omega) \right)^{-\gamma} dP^x (\omega) \right)^{-\frac{1}{\gamma}} - a_2 \right) = \\
&= \gamma \left(\left(\int_{\nleq} \left(z_{t+1}^1 \right)^{-\gamma} dP^x \right)^{-\frac{1}{\gamma}} + \left(\int_{\nleq} \left(z_{t+1}^2 \right)^{-\gamma} dP^x \right)^{-\frac{1}{\gamma}} - (a_1 + a_2) \right) = \\
&= \gamma \left(\left(\int_{\nleq} \left(z_{t+1}^1 + z_{t+1}^2 \right)^{-\gamma} dP^x \right)^{-\frac{1}{\gamma}} - (a_1 + a_2) \right) = \\
&= \gamma \left(\left(\int_{\nleq} \left(a_1 + a_2 + \frac{c_{t+1}^1 (\omega) + c_{t+1}^2 (\omega)}{\gamma} \right)^{-\gamma} dP^x \right)^{-\frac{1}{\gamma}} - (a_1 + a_2) \right) = \\
&= \left(U' \right)^{-1} \left(E \left[U' \left(c_{t+1}^1 + c_{t+1}^2 \right) | \mathfrak{F}_t \right] \right).
\end{aligned}
$$

Applying U' to both sides,

$$
U' (c_t) = E \left[U' (c_{t+1}) | \mathfrak{F}_t \right].
$$

■

A.3 Proof of theorem 2

Proof. The FOC's for the limited commitment ICM imply the following:

$$
\begin{aligned}
u_1'(c_t^1) &= E_t \left[\left(1 + \frac{\hat{k}_{t+1,\omega}^1}{\hat{K}_t^1} \right) u_1'(c_{t+1}^1) + \frac{\hat{k}_{t+1,\omega}^1}{\hat{K}_t^1} \frac{\partial \underline{u}_{1,t+1,\omega}}{\partial s_t} + \frac{\hat{k}_{t+1,\omega}^2}{\hat{K}_t^1} \frac{\partial \underline{u}_{2,t+1,\omega}}{\partial s_t} \right] \qquad (22) \\
u_2'(c_t^2) &= E_t \left[\left(1 + \frac{\hat{k}_{t+1,\omega}^2}{\hat{K}_t^2} \right) u_2'(c_{t+1}^2) + \frac{\hat{k}_{t+1,\omega}^1}{\hat{K}_t^2} \frac{\partial \underline{u}_{1,t+1,\omega}}{\partial s_t} + \frac{\hat{k}_{t+1,\omega}^2}{K_t^2} \frac{\partial \underline{u}_{2,t+1,\omega}}{\partial s_t} \right],
\end{aligned}
$$

where $\hat{k}_{t+1,\omega}^1 \geq 0$ is the Kuhn-Tucker multiplier associated with the participation constraint adjusted for the discount factor and the probability function and $\hat{K}^i (t, \omega) = \sum_{\tau=0}^t \hat{k}^i (\tau, \omega) \geq 0$.

Assume $\dfrac{\partial \underline{u}_{i,t+1,\omega}}{\partial s_t(\omega)} = 0$ for every i. Then (22) simplifies to,

$$u^{1\prime}\left(c_t^1\right) = E\left[\left(1+\frac{\hat{k}^1_{t+1,\omega}}{\hat{K}^1_t}\right)u^{1\prime}\left(c^1_{t+1}\right)|\mathfrak{F}_t\right]$$
$$u^{2\prime}\left(c_t^2\right) = E\left[\left(1+\frac{\hat{k}^2_{t+1,\omega}}{\hat{K}^2_t}\right)u^{2\prime}\left(c^2_{t+1}\right)|\mathfrak{F}_t\right],$$

which implies,

$$c_t^1 \le \left(u^{1\prime}\right)^{-1}\left(E\left[u^{1\prime}\left(c^1_{t+1}\right)|\mathfrak{F}_t\right]\right)$$
$$c_t^2 \le \left(u^{2\prime}\right)^{-1}\left(E\left[u^{2\prime}\left(c^2_{t+1}\right)|\mathfrak{F}_t\right]\right)$$

Given the assumption of HARA preferences,

$$\left(u^{i\prime}\right)^{-1}\left(E\left[u^{i\prime}\left(c^i_{t+1}\right)|\mathfrak{F}_t\right]\right) = \gamma\left(\left(E\left[\left(a_i+\frac{c^i_{t+1}}{\gamma}\right)^{-\gamma}|\mathfrak{F}_t\right]\right)^{-\frac{1}{\gamma}} - a_i\right).$$

Fix an $\omega \in \Omega$. Define $z^i_{t+1} = a_i + \dfrac{c^i_{t+1}}{\gamma}$. The first order conditions of (3) imply,

$$P\left[\left\{\omega\in\Omega : Az_1(\omega) = Bz_2(\omega)\right\}\right] < 1. \tag{A.2'}$$

Lemma 4, equations (A.1) and (A.2'), and the assumption on preferences imply,

$$c_t^1 + c_t^2 \le \gamma\left(\left(\int_{\nleq}\left(z^1_{t+1}(\omega)\right)^{-\gamma} dP^x(\omega)\right)^{-\frac{1}{\gamma}} - a_1\right) + \gamma\left(\left(\int_{\nleq}\left(z^2_{t+1}(\omega)\right)^{-\gamma} dP^x(\omega)\right)^{-\frac{1}{\gamma}} - a_2\right)$$
$$= \gamma\left(\left(\int_{\nleq}\left(z^1_{t+1}\right)^{-\gamma} dP^x\right)^{-\frac{1}{\gamma}} + \left(\int_{\nleq}\left(z^2_{t+1}\right)^{-\gamma} dP^x\right)^{-\frac{1}{\gamma}} - (a_1+a_2)\right)$$
$$< \gamma\left(\left(\int_{\nleq}\left(z^1_{t+1}+z^2_{t+1}\right)^{-\gamma} dP^x\right)^{-\frac{1}{\gamma}} - (a_1+a_2)\right)$$
$$= \gamma\left(\left(\int_{\nleq}\left(a_1+a_2+\frac{c^1_{t+1}(\omega)+c^2_{t+1}(\omega)}{\gamma}\right)^{-\gamma} dP^x\right)^{-\frac{1}{\gamma}} - (a_1+a_2)\right)$$

$$= \left(U'\right)^{-1}\left(E\left[U'\left(c^1_{t+1}+c^2_{t+1}\right)|\mathfrak{F}_t\right]\right).$$

Applying U' to both sides and using the fact that U' is decreasing,

$$U'\left(c_t\right) > E\left[U'\left(c_{t+1}\right)|\mathfrak{F}_t\right].$$

Assume $\sum_{i=1}^{n}\hat{k}^i_{t+1,\omega}\frac{\partial \underline{u}_{i,t+1,\omega}}{\partial s_t\left(\omega\right)} \geq 0$. Then by (22),

$$u_1^{'}(c^1_t) \geq E_t\left[\left(1+\frac{\hat{k}^1_{t+1,\omega}}{\hat{K}^1_t}\right)u_1^{'}(c^1_{t+1})\right] \geq E_t\left[u_1^{'}(c^1_{t+1})\right]$$
$$u_2^{'}(c^2_t) \geq E_t\left[\left(1+\frac{\hat{k}^2_{t+1,\omega}}{\hat{K}^2_t}\right)u_2^{'}(c^2_{t+1})\right] \geq E_t\left[u_2^{'}(c^2_{t+1})\right],$$

which implies,

$$c^1_t \leq \left(u^{1\prime}\right)^{-1}\left(E\left[u^{1\prime}\left(c^1_{t+1}\right)|\mathfrak{F}_t\right]\right)$$
$$c^2_t \leq \left(u^{2\prime}\right)^{-1}\left(E\left[u^{2\prime}\left(c^2_{t+1}\right)|\mathfrak{F}_t\right]\right).$$

Hence, applying the same argument as for $\frac{\partial \underline{u}_{i,t+1,\omega}}{\partial s_t\left(\omega\right)} = 0$ for every i, it follows,

$$U'\left(c_t\right) > E\left[U'\left(c_{t+1}\right)|\mathfrak{F}_t\right].$$

■

A.4 Proof of theorem 3 and 4

Proof. The one-variable functions $\vartheta_1 : I_1 \to \mathbb{R}$ and $\vartheta_2 : I_2 \to \mathbb{R}$ are defined as follows:

$$\vartheta_1(t) = \phi_1(t\hat{C}, t\hat{Q}, t\hat{Z})$$
$$\vartheta_2(t) = \phi_2(t\hat{C}, t\hat{Q}, t\hat{Z})$$

where $I_1 = (-a, a)$ and $I_2 = (-b, b)$. Applying the one-variable Taylor's formula with remainder,

$$\vartheta_i(t) = \vartheta_i(0) + \vartheta_i'(0)t + \vartheta_i''(0)t^2 + r_i(t), \quad for\, i = 1, 2, \tag{23}$$

with

$$r_i(t) = \frac{1}{3!}\int_0^t (t-s)^3\vartheta_i'''(s)ds.$$

Applying the chain rule, we have,

$$\vartheta_i'(t) = \frac{\partial \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{C}}\hat{C} + \frac{\partial \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{Q}}\hat{Q} + \sum_j \frac{\partial \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{z}_i}\hat{z}_i$$

$$\vartheta_i''(t) = \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{C}^2}\hat{C}^2 + \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{Q}^2}\hat{Q}^2 + \sum_j \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{z_i}^2}\hat{z_i}^2$$

$$+\sum_j \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{Q}\partial \hat{C}}\hat{Q}\hat{C} + +\sum_j \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{z}_i\partial \hat{C}}\hat{z}_i\hat{C} + \sum_j \frac{\partial^2 \phi_i(t\hat{C}, t\hat{Q}, t\hat{Z})}{\partial \hat{z}_i\partial \hat{Q}}\hat{z}_i\hat{Q}$$

Hence, from (23), with $t = 1$,

$$\phi_i(\hat{C}, \hat{Q}, \hat{Z}) = \phi_i(0) + \frac{\partial \phi_i(0)}{\partial \hat{C}}\hat{C} + \frac{\partial \phi_i(0)}{\partial \hat{Q}}\hat{Q} + \sum_j \frac{\partial \phi_i(0)}{\partial \hat{z}_i}\hat{z}_i + \frac{\partial^2 \phi_i(0)}{\partial \hat{C}^2}\hat{C}^2 + \frac{\partial^2 \phi_i(0)}{\partial \hat{Q}^2}\hat{Q}^2$$

$$+\sum_j \frac{\partial^2 \phi_i(0)}{\partial \hat{z}_i^2}\hat{z}_i^2 + \sum_j \frac{\partial^2 \phi_i(0)}{\partial \hat{C}\partial \hat{Q}}\hat{C}\hat{Q} + \sum_j \frac{\partial^2 \phi_i(0)}{\partial \hat{C}\partial \hat{z}_i}\hat{C}\hat{z}^i + \sum_j \frac{\partial^2 \phi_i(0)}{\partial \hat{Q}\partial \hat{z}_i}\hat{Q}\hat{z}^i + R_i(\hat{C}, \hat{Q}, \hat{Z}) \quad (24)$$

Finally by definition of $\phi_i(\hat{C}, \hat{Q}, \hat{Z})$, we have,

$$\frac{\partial \phi_1}{\partial \hat{z}_i} = \frac{V_{C\kappa}}{V_C}\frac{\partial \kappa}{\partial z_i}z_i, \quad \frac{\partial^2 \phi_1}{\partial \hat{C}\partial \hat{z}_i} = \frac{V_C V_{CC\kappa} - V_{C\kappa}V_{CC}}{V_C^2}\frac{\partial \kappa}{\partial z_i}Cz_i, \quad (25)$$

$$\frac{\partial^2 \phi_1}{\partial \hat{Q}\partial \hat{z}_i} = \frac{V_C V_{CQ\kappa} - V_{C\kappa}V_{CQ}}{V_C^2}\frac{\partial \kappa}{\partial z_i}Qz_i, \quad \frac{\partial \phi_2}{\partial \hat{z}_i} = \frac{V_{Q\kappa}}{V_Q}\frac{\partial \kappa}{\partial z_i}z_i, \quad (26)$$

$$\frac{\partial^2 \phi_2}{\partial \hat{C}\partial \hat{z}_i} = \frac{V_C V_{QC\kappa} - V_{Q\kappa}V_{QC}}{V_Q^2}\frac{\partial \kappa}{\partial z_i}Cz_i, \quad \frac{\partial^2 \phi_2}{\partial \hat{Q}\partial \hat{z}_i} = \frac{V_Q V_{QQ\kappa} - V_{Q\kappa}V_{QQ}}{V_Q^2}\frac{\partial \kappa}{\partial z_i}Qz_i, \quad (27)$$

where $\kappa = \mu$ if the full efficiency ICM is considered and $\kappa = M$ if the limited commitment ICM is considered.

Under the assumption of rational expectations, the household Euler equations can be written in the form,

$$\frac{V_C\left(C_{t+1}, Q_{t+1}, \kappa(Z)\right)\beta R_{t+1}}{V_C\left(C_t, Q_t, \kappa(Z)\right)} = 1 + e_{t+1}$$

where e_{t+1} is the expectation error. Taking logs and using $\phi_1 = \ln V_C$ and $\phi_2 = \ln V_Q$, we have,

$$\phi_i(\hat{C}_{t+1}, \hat{Q}_{t+1}, \hat{Z}_{t+1}) - \phi_i(\hat{C}_t, \hat{Q}_t, \hat{Z}_t) = -\ln\beta - \ln R_{t+1} + ln(1 + e_{t+1}) \;\; for\ i = 1, 2.$$

Table 4: PSID summary statistics

Variable	Mean for Singles	Mean for Couples
consumption growth	-0.014	-0.0005
age of head	42.6	41.8
growth in annual food needs	-0.0029	-0.0039
annual real consumption	2999.2	4832.7
annual after tax income	16806	30237
number of observations	3205	15366
number of families	716	2115

Consider first the full efficiency ICM. The relative Pareto weight μ is constant over time. Moreover the vector of factors Z is also constant over time. Hence, from (24), (25), (26), (27), $\hat{C} = \ln \frac{C}{E[C]}$ and $\hat{Q} = \ln \frac{Q}{E[Q]}$, the result follows.

Consider the limited commitment ICM. Note that,

$$M_{t+1} = M_t + \mu_{t+1} \quad for\ any\ (t, \omega). \tag{28}$$

Consequently,

$$\frac{\partial M_{t+1}}{\partial z_i} = \frac{\partial M_t}{\partial z_i} + \frac{\partial \mu_{t+1}}{\partial z_i} \quad for\ any\ (t, \omega). \tag{29}$$

Hence, from (24), (25), (26), (27), $\hat{C} = \ln \frac{C}{E[C]}$ and $\hat{Q} = \ln \frac{Q}{E[Q]}$, the result follows. ■

A.5 Summary statistics

Tables 4 and 5 contains sample means of the main variables used in the estimation.

Table 5: CEX summary statistics

Variable	Mean for Singles	Mean for Couples
consumption growth	0.0004	-0.0015
age of head	46.6	46.7
growth in family size	-0.0009	-0.0031
monthly real consumption	552.1	882.1
annual after tax income	15959	29063
number of observations	908	906