# Equilibrium Labor Force Participation and Wage Dispersion 

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#### Abstract

An important gap in the literature on labor market search is that the majority of studies focus on within labor-market transitions, while ignoring workers' decisions to participate in or leave the labor force. This paper develops a model of equilibrium search which simultaneously determines workers' labor market participation/exit decisions, and job acceptance rules. Workers revise their labor force entry and exit strategies through occasional shocks to the utility of non-work options. The model is applied to the labor market experience of black and white high school graduate women from the NLSY79 to discern the sources of female black-white labor market inequality in wages and participation.


## 1 Introduction

Equilibrium models of labor market search have become very useful for understanding the effect of frictions on labor market phenomena such as worker transition behavior, labor market policies (e.g. unemployment compensation); and the nature of the wage offer distribution. Albrecht and Axell (1984) and Burdett and Mortensen (1998) provided the theoretical impetus for this line of research. The initial focus of these papers was on ob-
taining endogenous dispersion in wage offers, perhaps due to the well-known difficulty in generating non-degenerate wage offer distributions in models of sequential search ${ }^{1}$. The subsequent empirical literature, however, has extended and applied their framework to a wide variety of questions: inter alia, the transition from school to work of young graduates (see, for example Eckstein and Wolpin (1990), and Bowlus, Kiefer and Newmann (2001)); the sources of wage inequality among workers of different observational types (race) (Bowlus and Eckstein (1999)); the examination of intra and inter-industry wage differentials (Van den Berg and Ridder (1998)) and the relative effects of worker and firm heterogeneity (Robin and Postel-Vinay (2002)).

A significant gap in this literature concerns the treatment of labor force participation. In much of this literature, transitions into and out of the labor force (which in this paper may be referred to as "participation flows") have received comparatively less attention than on transitions between employment and unemployment (correspondingly referred to as "intra labor force flows" ${ }^{2}$ ). One flaw frequently encounted in this literature is that inadequate steps are taken to address potential sample selection bias. All the papers cited above, for example, simply drop non labor-force participants from the sample at the estimation stage. Van den Berg and Ridder (1998), for example, argue that
"the main features of the [Burdett-Mortensen] model are insensitive to the inclusion of such a [nonparticipant] state. Moreover, transitions to and from nonparticipation are rare in the data." (pp. 1194, Van den Berg and Ridder (1998)).

Data and research suggest otherwise, and in fact, it is not hard to show that nonparticipation, even at a glance, appears to be more significant than Van den Berg and Ridder assert. Firstly, data consistently shows participation flows to be of a similar order of magnitude to job flows.

## Table1here

[^0]Panel A of table 1 is constructed from the 1979 National Longitudinal Survey of Youth ${ }^{3}$ (NLSY79) and shows the average monthly flows over the period 1978 to 1996. The monthly flow from nonparticipation to employment $(n \rightarrow e)$ is $3.0 \%$ of the population, larger than than the flow from unemployment to employment ( $u \rightarrow e, 1.12 \%$ ) . Likewise, the rate of flow from jobs into nonparticipation $(e \rightarrow n)$ is larger than the rate of flow to unemployment $(e \rightarrow u, 2.93$ vs 1.0). This is consistent with evidence from the Current Population Survey, taken from Kim (2001) and shown in Panel B of table 1. There, the $n \rightarrow e$ flow is $0.9 \%$, only slightly smaller than the $u \rightarrow e$ flow of $1.1 \%$. The $e \rightarrow u$ and $e \rightarrow n$ flows are of similar magnitude as well. ${ }^{4}$ Therefore regardless of whether we look at individuals over their life cycles, or at a cross section of the population, flows into and out of the labor force are at least as large as flows within the labor force.

Second, research by Flinn and Heckman (1983) and Gonul (1992), suggest that the unemployed (U), and the out-of-labor force (OLF) are indeed behaviorally different. Flinn and Heckman (1983) find OLF and U to be distinct states among white male high school graduates, in that the OLF exit to employment at a lower rate than the unemployed. Gonul (1992) on the other hand finds that there is no distinguishable difference between OLF and U among white high school graduate males, but does find a difference among white high school graduate females. Unlike Flinn and Heckman, Gonul finds evidence of this behavioral difference not among males but among females; that is, a white OLF female is less likely to find work compared to an unemployed female. One interpretation of these differentials is that labor force participation is a choice. Gonul's findings also suggest that women appear to be somewhat "more conscious" in the choice of labor force particpation choice than men are. These different findings by these two papers suggest that this question is far from settled.

Third, related to the second point above, there is a natural setting where participation flows are of potentially significant importance: that of the participation behavior of females in the labor market. Life cycle events such as marriage, divorce, childbirth affect the

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Figure 1: Labor Force Participation from 1990 Census.
labor supply of women more significantly than men. Figure (1) depicts the life cycle labor supply of women from the 1990 census, by age cohort, who have completed high school or have a GED. Notice that the typical black female has a "hump-shaped" life cycle labor force participation profile, whereas white women begin to leave the labor force in their midtwenties, and re-enter the labor force again roughly ten to fifteen years later, thus generating a slight non-concave profile. This suggests that changes in the value of non-work activities during these years might be significant determinants of wage and transition behavior among women.

As mentioned above, most models applied to consider the effects of labor market policy characterize the labor market in two states-employment and unemployment, neglecting considering effects on the participation margin. Such an omission may result in misleading policy predictions because these policies affect not only individuals' reservation wages but also their desire to remain in the labor force.

From the above, it is clear that a two-state model will not be satisfactory in explaining the movements into and out of the labor force and wage structure of females. What is
needed is a model which allows the simultaneous determination of entry and exit from the labor force, as well as the determination of wage acceptance strategies. The goal of this paper is to develop such a model. In this model it is also neccessary to endow individuals with some form of heterogeneity, in order to generate participation flows. This paper considers the role of non-work options which may change from time to time. In order to obtain an endogenous wage distribution, firms are assumed to make profit-maximizing wage offers. This allows us to examine the wage structure of workers, in addition to workers' transition behavior.

This paper aims to do the following. First, to develop a model of labor market search where agents make both participation and reservation strategies, and which generates wage dispersion based upon the model's fundamental parameters (in this case both friction parameters and heterogeneity in work and non-work options). I then estimate such a model to determine how important is the distinction between the states of OLF and U among females. Second and more interestingly, I assess the empirical validity of the model by applying it to a contemporary policy-relevant issue: accounting for the wage-inequality among young black and white females. To establish this second goal, I estimate this model on two samples of females-one of black women, another of white women-which are observationally the same along the dimensions of education and age and differ only in race. The estimates will show the extent to which (1) search impediments; (2) labor market and (3) non-labor market options differ between white and black women. Using this mode, I can conduct a number of counter factual experiments to discern which are the more significant sources of labor market inequality among white and black women.

There is a large literature researching the extent of labor market inequality among white and black females in the US. I shall only relate my paper to a small subset of this literature. Several studies (Chandra (2000) and Heckman, Lyons and Todd (2000)) have observed that black-white wage gaps in reported wages may be much smaller than the corresponding wage gaps in estimated potential wages. Furthermore, the observed-wage gap has been found to be smaller for women than for men. These findings suggest that wage inequality between black and white females have been understated, and that conventional wisdom presents a misleading picture of racial differences in labor market opportunities. In a similar vein,

Derek Neal (2003) observed that significant black-white differences in marriage market opportunity and household structure could account for labor market differences. Using a wage imputation method accounting for marital opportunities, Neal finds the black-white gap in potential wages to be about 60 percent larger than the gap in reported wages. I find that the current paper is able to speak to this debate. The various attempts at imputing the unobserved "potential" wages, are analogous to obtaining the wage offer distribution. Furthermore, this paper goes on further to suggest where the sources of these differences may lie: whether from differences in search impediments, non-work options, or the types of jobs available to each group. Beyond wage inequality, the search model also generates flows and steady state spell lengths, allowing us to compare other dimensions of labor market inequality.

In the model by Albrecht and Axell, heterogeneity in non-work alternatives was the driving force that generated wage dispersion. In the model by Burdett and Mortensen, wage dispersion was generated by workers who engage in on-the-job search. This paper blends the insights of the models of the two models in that the wage offer distribution can be explained by frictions in the job (and on-the-job) search process and the distribution of opportunity cost of work. However unlike the current literature, heterogeneity in workers' opportunity costs of work generates participation flows as well. Bontemps et. al. (1999/2000) found that heterogeneity in workers' opportunity costs did not play an important role in the wage distribution. However Bontemps et. al. consider only two states, and in effect ignore the possibility that workers' opportunity costs play a role in generating flows, even if it may not explain wage dispersion.

Preliminary estimates from the model find relatively large differences in the transition parameters between black and white females. As one would expect, the arrival rates of jobs are lower for black females than for white females. Interestingly, the parameter estimates of the distribution of non-work options suggest that black women in the NLSY79 face more frequent and on average better shocks to utility of their non-work option, and hence this appears to be an important factor accounting for black-white wage inequality. I further find that the implied mean wage offer to black females, is actually slightly higher than that for white females in the NLSY79. On the other hand, black and white women are found to
have very similar reservation wage functions, suggesting that it is not that black women are more selective in accepting jobs, but that the effect of lower arrival rates just about offsets the higher non-work utility options, and average wage offers that they face.

Finally, it should be mentioned that within the vast literature on equilibrium search, Kim (2001) is closest to the current work. He presents an version of the Mortensen and Pissarides (1994) search model with the participation decision. In his paper, transitions are generated by shocks to workers' productivity leading to layoffs and exits from the labor force. Unlike the present paper, there is no on-the-job search and no quits. He calibrates the model to match various macroeconomic aggregates and job flow data whereas the present paper represents to my knowledge the first attempt at estimating a three-state equilibrium model from individual transitions and wage data.

The paper is organized as follows. The next section presents the model, the third section discusses the data, solution and estimation of the mode. Section 4 considers some policy experiments, and the last section concludes.

## 2 The Model.

In the model I consider, workers not attached to a job make two choices: whether to participate in the labor force, and what wage to accept, if a participant. Workers attached to a job decide whether or not to separate from the job, and if separated, whether or not to drop out of the labor force all together. What motivates these decisions is that workers face an occasional shock to the utility of their non-work activity, sometimes referred to as leisure. Firms differ in their endowed marginal product of labor and make profit-maximizing wage offers. There are two sources of heterogeneity: in the worker's valuation of the non-work option, and in the firms' marginal product of labor. In the equilibrium, the wage offer depend on the shapes of the distribution of non-work options as well as job options available to the workers, in addition to the transition parameters. The model thus permits us to ask to what extent do the availability and quality of non-work and work options affect wage offers, and influence labor market transitions.

### 2.1 The Worker's Problem.

There is a unit measure of workers. Each worker takes as given the distribution of wage offers $F(w)$, and distribution of the opportunity cost of work or equivalently utility of leisure, $H(b)$. There are three states that the worker can find herself in: employment at wage $w$, with associated value function $V(w)$, unemployed with utility of leisure $b$, denoted by $U(b)$, or nonparticipation with utility of leisure $b$, denoted by $N(b)$. While unemployed, the worker is actively searching for a job, at utility cost $s$, and receives a job offer from $F(w)$ at rate $\lambda$. While a nonparticipant, the worker is assumed to not expend search effort $s$ but nonetheless may receive a wage offer from $F(w)$ at rate a lower arrival rate $\lambda_{0}<\lambda$. Think of this as passive search. There are no other differences between workers.

On the job search is allowed, so workers employed at current wage $w$ may sample a take it or leave it offer $w^{\prime}$ from another firm. To simplify the subsequent analysis, we assume this happens with rate $\lambda$, equal to the arrival rate for the unemployed, and that job offers arrive without the worker having expended effort in job search while on the job. In doing so I do not consider job-search choice while on the job.

Regardless of the state they are in, all agents sample a new draw of utility of leisure $b^{\prime}$ at rate $\gamma$. However workers in different states will respond to this draw differently. Workers currently employed at $w$ will have to decide whether to stay on the job, or quit and take the "offer" of new utility of leisure. If they quit, they will have to decide whether to quit to unemployment or to nonparticipation. Unemployed and nonparticipant individuals, however, do not have the option of rejecting the new draw of $b^{\prime}$. As they are already enjoying $b$, the arrival of a new $b^{\prime}$ is treated as a shock to utility of leisure. The only decision they make is whether to change their participation status.

The three flow parameters, $\left\{\gamma, \lambda, \lambda_{0}\right\}$ generate the six transitions across all the three states ${ }^{5}$ observed in the labor market (seven, if one includes job-to-job transitions). The driving force of these transitions is simply the idea that opportunity costs to labor market

[^2]employment may change from time to time in unanticipated ways, and cause participation and quit decisions. This is a parsimonious theory of labor force participation that interprets a number of significant life-cycle changes as a markov process. For instance, the arrival of a positive shock to $b$ can be thought of as an increase in spousal wealth, the arrival of a marital suitor, or a child-bearing opportunity. Likewise a negative shock may be the corresponding departure of that wealthy spouse or similar adverse change in family size.

Bellman Equations In a time interval $\Delta$, the worker employed at wage $w$ can encounter two possible events: a wage offer $w^{\prime}$ with probability $\Delta \lambda$, or experience a shock to utility of leisure $b^{\prime}$ with probability $\Delta \gamma$. Both events are assumed not to happen at the same time ${ }^{6}$. Let

$$
P(b)=\max [U(b), N(b)]
$$

The Bellman equation of employment at wage $w$ is

$$
\begin{aligned}
(1+r \Delta) V(w)= & \Delta w+\Delta \lambda E \max \left[V\left(w^{\prime}\right), V(w)\right]+\Delta \gamma E \max \left[P\left(b^{\prime}\right), V(w)\right] \\
& +(1-\Delta \gamma-\Delta \lambda) V(w)
\end{aligned}
$$

Similarly we can derive the Bellman equations for unemployment and nonparticipation.

$$
(1+r \Delta) U(b)=\Delta(b-s)+\Delta \lambda E \max \left[V\left(w^{\prime}\right), U(b)\right]+\Delta \gamma P\left(b^{\prime}\right)+(1-\Delta \gamma-\Delta \lambda) U(b)
$$

and

$$
(1+r \Delta) N(b)=\Delta b+\Delta \lambda_{0} E \max \left[V\left(w^{\prime}\right), N(b)\right]+\Delta \gamma P\left(b^{\prime}\right)+(1-\Delta \gamma-\Delta \lambda) N(b)
$$

Dividing through by $\Delta$ and letting $\Delta \rightarrow 0$ yields the continuous time limits

$$
\begin{align*}
r V(w) & =w+\lambda E \max \left[V\left(w^{\prime}\right)-V(w), 0\right]+\gamma E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]  \tag{1}\\
r U(b) & =b-s+\lambda E \max [V(w)-U(b), 0]+\gamma E\left[P\left(b^{\prime}\right)-U(b)\right]  \tag{2}\\
r N(b) & =b+\lambda_{0} E \max [V(w)-N(b), 0]+\gamma E\left[P\left(b^{\prime}\right)-N(b)\right]  \tag{3}\\
P(b) & =\max [U(b), N(b)] \tag{4}
\end{align*}
$$

[^3]
### 2.1.1 Individual Decision Rules.

The workers' decision rules regarding reservation wages and participation choices can be established via a series of lemmas. First, observe that the value function in each labor force state is increasing only in their respective state variables. Next this implies that for each state not attached to a job there is a unique reservation wage. Thirdly, I show that there is a unique cutoff level of leisure $b^{*}$ above which all individuals not attached to a job will prefer to exit the labor force, and below which they will enter the labor market and engage in active search. The arguments are straightforward and proofs can be found in the appendix.

Lemma $1 V^{\prime}(w), U^{\prime}(b)$, and $N^{\prime}(b)$ are strictly increasing. In particular, $V^{\prime}(w)=$ $\frac{1}{r+\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{j}(w)\right)\right]}, j \in\{u, n\}$ where $\phi()=.R^{-1}($.$) .$

Lemma 2 Given $F(w)$ and $H(b)$, there exist unique reservation wage strategies $R_{u}(b)$ and $R_{n}(b)$ for the unemployed and nonparticipant workers respectively such that

1. An unemployed worker accepts all wage offers $w \geq R_{u}(b)$;
2. A nonparticipating worker accepts all wage offers $w \geq R_{n}(b)$;

Furthermore, $R_{u}(b)$ and $R_{n}(b)$ are strictly increasing in $b$.

Proposition 1 Assume that $s<B(\bar{b}-\underline{b})$ where $B=\frac{\lambda-\lambda_{0}}{r+\gamma+\lambda_{0}}$.. Then given $F(w)$ and $H(b)$, there exists a unique $b^{*} \in(\underline{b}, \bar{b})$ such that $U\left(b^{*}\right)=N\left(b^{*}\right)$. Furthermore, $U(b) \gtrless$ $N(b) \Longleftrightarrow b^{*} \gtrless b$.

That the value functions depend only on their respective state variables is important for uniqueness. An interpretation of this is that there is no "recall" of the previous state: once an individual accepts a wage offer $w$, or a utility draw $b^{\prime}$, she immediately "forgets" the utility of leisure $b$ or wage $w$ she used to enjoy ${ }^{7}$. Given unique reservation wages, it

[^4]

Figure 2: Summary of Individual Worker Strategies and Labor Market Choices.
is easy to show that $R_{n}(b)$ is steeper than and intersects only once with $R_{u}(b)$, implying that $b^{*}$ uniquely divides the population into two groups (proposition 1). Workers who have an opportunity cost of work $b<b^{*}$ will prefer unemployment to nonparticipation, while workers with $b>b^{*}$ will prefer to be nonparticipants. Workers with exactly $b^{*}$ are indifferent. Thus with no loss of generality, $R_{u}(b)$ specifies the reservation strategy for $b<b^{*}$ and $R_{n}(b)$ specifies the reservation strategy for $b>b^{*}$. Setting $V(w)=U(b)$ and $V(w)=N(b)$ over these respective two regions, the reservation strategies are

$$
\begin{array}{lc}
R_{u}(b)=b-s & b<b^{*} \\
R_{n}(b)=b-\left(\lambda-\lambda_{0}\right) \varphi\left[R_{n}(b)\right] & b>b^{*} \tag{6}
\end{array}
$$

where

$$
\varphi(w)=\int_{w}^{\bar{w}} \frac{1-F(x)}{r+\lambda[1-F(x)]+\gamma\left[1-H\left(\phi_{j}(x)\right)\right]} d x
$$

The diagram below summarizes the participation choices of workers in $(b, w)$ space.

The lines labeled $R_{u}(b)$ and $R_{n}(b)$ denote the reservation wages of an unemployed and nonparticipant individual respectively. They are also respectively the loci of indifference between being employed and unemployed, and being employed and OLF. The upper envelope of both functions is indicated in bold. The regions marked $E, U$, and $N$ mark the preferred state, given any selected $(b, w)$ pair. All individuals to the left of $b^{*}$ prefer unemployment, and all individuals to the right of $b^{*}$ choose nonparticipation. Individuals in the shaded region choose employment, and require values of $b^{\prime}$ to the right of the bold envelope to leave their jobs. Proposition 1 also obtains a sufficient condition for the existence of $b^{*}$ : that the cost of search is not too large relative to the support of the opportunity cost of leisure, i.e. $\frac{s}{\bar{b}-\underline{b}}<\frac{\lambda-\lambda_{0}}{r+\gamma+\lambda_{0}}$. Otherwise no workers would choose to be unemployed (i.e. to engage in active job search).

From figure (2) it is also particularly easy to characterize the job-to-job movements, and quits. A worker currently employed at $w$ will encounter other job offers at rate $\lambda$ and will accept any offer $w^{\prime}>w$. Workers' quit strategies depend on where they are on the wage distribution. At low wages, in particular $w<b^{*}-s$, a worker may decide to exit the labor force altogether, or quit to unemployment, continue searching in hope of a better job. At high wages, i.e. above $w>b^{*}-s$, a worker quits only when she samples a very high value of leisure. The following lemma summarizes workers' quit strategies given their current wage $w$ :

Lemma 3 Workers quit when they receive a new draw $b^{\prime}$ such that $R_{j}\left(b^{\prime}\right)>w$, where $j \in$ $\{u, n\}$. (1) Workers earning $w>b^{*}-s$ quit to nonparticipation only (when $\left.R_{n}\left(b^{\prime}\right)>w\right)$. (2) Workers earning $w<b^{*}-s$ may quit to either nonparticipation (when $w<R_{u}\left(b^{\prime}\right)<w^{*}$ ) or unemployment (when $R_{n}\left(b^{\prime}\right)>w^{*}$ ).

To summarize, given $H(b)$ and $F(w)$, we are able to obtain a complete characterization of the workers' strategies, which are summarized in the following proposition

Proposition 2 Given $H(b)$ and $F(w)$ each individual adopts the following optimal strategies

1. Reservation Strategies. If unemployed, accept any wage offer greater than

$$
R_{u}(b)=b-s \quad \text { when } b<b^{*}
$$

Whereas if out of the labor force, accept any wage offer greater than

$$
R_{n}(b)=b-\left(\lambda-\lambda_{0}\right) \varphi\left[R_{n}(b)\right] \quad \text { when } b>b^{*}
$$

where

$$
\varphi(w)=\int_{w}^{\bar{w}} \frac{1-F(x)}{r+\lambda[1-F(x)]+\gamma\left[1-H\left(R_{n}^{-1}(x)\right)\right]} d x
$$

2. Participation Strategies. Choose $U$ if $b<b^{*}$ and choose $N$ if $b \geq b^{*}$, where $b^{*}$ is given by the solution to

$$
s=\left(\lambda-\lambda_{0}\right) \varphi\left(b^{*}-s\right)
$$

3. Quit and On-the-job search Strategies. If currently employed at wage w,
(a) Accept any wage offer $w^{\prime}>w$.
(b) If $w>b^{*}-s$ quit to nonparticipation only when receive $a$ draw $b^{\prime}$ such that $R_{n}\left(b^{\prime}\right)>w$.
(c) If $w<b^{*}-s$ and $w<R_{u}\left(b^{\prime}\right)<b^{*}-s$, quit to nonparticipation. But if $R_{n}\left(b^{\prime}\right)>b^{*}-s$ quit to unemployment.

Having provided a full characterization of the workers' individual problem, we can now move on to consider transitions in steady state.

### 2.1.2 Steady State Aggregates

We now derive the steady state distributions of the model. Let $G(w)$ be the distribution of paid wages less than or equal to $w$. Clearly, $G(w)$ differs from $F(w)$ since not all wage offers are accepted. Let $J_{u}(b)$ be the distribution of currently unemployed workers with opportunity cost of work less than or equal to $b$, and $J_{n}(b)$ be the distribution of nonparticipants with opportunity cost of work less than or equal to $b$. Just as $G(w)$ differs from $F(w), J_{u}(b)$ and $J_{n}(b)$ differ from $H(b)$ because not all workers receiving a shock to the utility of leisure will quit their job ${ }^{8} . G(w), J_{u}(b)$ and $J_{n}(b)$ have the property that $\int_{\underline{w}}^{\bar{w}} d G(w)=1, \int_{\underline{b}}^{b^{*}} d J_{u}(b)=1$ and $\int_{b^{*}}^{\bar{b}} d J_{n}(b)=1$ respectively. Let $e, u, n$ be the steady

[^5]state proportions of employed, unemployed and nonparticipant individuals in the economy respectively. Naturally $e+u+n=1$. The steady state flows from state $i$ to $j, T_{i \rightarrow j}$ (.). can be written as
\[

$$
\begin{aligned}
T_{u \rightarrow e}(b) & =\lambda \int_{b_{l}}^{b}\left[1-F\left(R_{u}(x)\right)\right] d J_{u}(x) \\
T_{u \rightarrow n} & =\gamma\left[1-H\left(b^{*}\right)\right] \forall b \in\left[\underline{b}, b^{*}\right] \\
T_{n \rightarrow e}(b) & =\lambda_{0} \int_{b^{*}}^{b}\left[1-F\left(R_{n}(x)\right)\right] d J_{n}(x) \\
T_{n \rightarrow u} & =\gamma H\left(b^{*}\right) \forall b \in\left[b^{*}, \bar{b}\right] \\
T_{e \rightarrow u}\left(R_{u}(b)\right) & =\gamma \int_{\underline{w}}^{R_{u}(b)}\left[H(b)-H\left(\phi_{u}(x)\right)\right] d G(x) \\
T_{e \rightarrow n}\left(R_{n}(b)\right) & =\gamma\left\{\left[H(b)-H\left(b^{*}\right)\right] G\left(R\left(b^{*}\right)\right)+\int_{R\left(b^{*}\right)}^{R_{n}(b)}\left[H(b)-H\left(\phi_{n}(x)\right)\right] d G(x)\right\}
\end{aligned}
$$
\]

$T_{u \rightarrow e}(b)$ is derived as follows. Each unemployed worker given $b$, receives wage offers at rate $\lambda$ and accepts wages higher than $R_{u}(b)$. Thus this probability is $\lambda\left[1-F\left(R_{u}(b)\right)\right]$. This must be integrated over $J_{u}(b)$, the distribution of workers' leisure utilities. All the other transitions are derived in the same fashion. The Steady state conditions for $u, e$, and $n$ must satsify the following three equations:

$$
\begin{aligned}
u\left[T_{u \rightarrow e}\left(b^{*}\right)+T_{u \rightarrow n}\right] & =n T_{n \rightarrow u}+e T_{e \rightarrow u}\left(R_{u}\left(b^{*}\right)\right) \\
n\left[T_{n \rightarrow e}\left(b_{u}\right)+T_{n \rightarrow u}\right] & =u T_{u \rightarrow n}+e T_{e \rightarrow n}\left(R_{n}(\bar{b})\right) \\
e+u+n & =1
\end{aligned}
$$

To preserve readability of the paper, the expressions of $e, u$ and $n$ can be found in the appendix.

Likewise, applying an analogous steady state condition, $J_{u}(b)$ and $J_{n}(b)$ must respectively satisfy:

$$
\begin{equation*}
u\left\{T_{u \rightarrow e}(b)+T_{u \rightarrow n} J_{u}(b)\right\}=n T_{n \rightarrow u}+e T_{e \rightarrow u}\left(R_{u}(b)\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
n\left\{T_{n \rightarrow e}(b)+\left[T_{n \rightarrow u}+\gamma[1-H(b)]\right] J_{n}(b)\right\}=u\left\{\gamma[1-H(b)]+T_{u \rightarrow n}\right\}+e T_{e \rightarrow n}\left(R_{n}(b)\right) \tag{8}
\end{equation*}
$$

There are, unfortunately, no closed form expressions for $J_{u}(b)$ and $J_{n}(b)$. Nevertheless, it is possible to show that $J_{u}(b)$ and $J_{n}(b)$ are unique expressions. The proof, found in the appendix, shows that the densities of equations (7) and (8), $J_{u}^{\prime}(b)$ and $J_{n}^{\prime}(b)$, satisfy the Lipschitz conditions for a first order ordinary differential equation, and are thus unique solutions for a given initial condition .

Proposition 3 Given $F(w)$ and $H(b)$, there exists a unique $J_{u}(b)$ satisfying (7) and a unique $J_{n}(b)$ satisfying (8).

Steady state wage offers $F(w)$ can be derived in like manner. However there are two cases to consider due to the presence of unemployed and nonparticipants: one where $w<b^{*}-s$ and another where $w \geq b^{*}-s$. We consider each one in turn.

Case 1: $w<b^{*}-s$. Entrants into $G(w)$ come only from $U$ because nobody in $N$ would accept wage offers less than $b^{*}-s$. Similarly, exits from $G(w)$ consist of quits and job-tojob movements. Furthermore there is a lower threshold $b_{l}$ where every worker with $b<b_{l}$ accepts every job offer. Thus equating inflows and outflows, $G(w)$ must satisfy

$$
\begin{align*}
& u \lambda\left\{F(w) J_{u}\left(b_{l}\right)+\int_{b_{l}}^{\phi_{u}(w)}\left[F(w)-F\left(R_{u}(b)\right)\right] d J_{u}(b)\right\}  \tag{9}\\
= & e\left\{\lambda[1-F(w)] G(w)+\gamma \int_{\underline{w}}^{w}\left[1-H\left(\phi_{u}(x)\right)\right] d G(x)\right\}
\end{align*}
$$

Case 2: $w \geq b^{*}-s$. In this case entrants into $G(w)$ come from both $U$ and $N$. Thus transitions into/out of $G(w)$ must satisfy

$$
\begin{aligned}
& \left.u \lambda\left\{J_{u}\left(b_{l}\right) F(w)+\int_{b_{l}}^{b^{*}}\left[F(w)-F\left(R_{u}(b)\right)\right] d J_{u}(b)\right\}+n \lambda_{0} \int_{b^{*}}^{\phi_{n}(w)}\left[F(w)-F\left(R_{n}(b)\right)\right] d J_{n}((\hat{b}))\right) \\
= & e\left\{\lambda[1-F(w)] G(w)+\gamma \int_{\underline{w}}^{R\left(b^{*}\right)}\left[1-H\left(\phi_{u}(x)\right)\right] d G(x)+\gamma \int_{R\left(b^{*}\right)}^{w}\left[1-H\left(\phi_{n}(x)\right)\right] d G(x)\right\}
\end{aligned}
$$

Differentiating (9) and (10) we obtain an expression for the "labor supply function" $\frac{G^{\prime}(w)}{F^{\prime}(w)}$. We get, respectively,

$$
\begin{equation*}
\frac{G^{\prime}(w)}{F^{\prime}(w)}=\frac{\lambda}{e} \frac{u J_{u}\left(\phi_{u}(w)\right)+e G(w)}{\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{u}(w)\right)\right]}, w<b^{*}-s \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{G^{\prime}(w)}{F^{\prime}(w)}=\frac{1}{e} \frac{\lambda[u+e G(w)]+n \lambda_{0} J_{n}\left(\phi_{n}(w)\right)}{\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{n}(w)\right)\right]}, w \geq b^{*}-s \tag{12}
\end{equation*}
$$

Having provided a complete characterization of the worker's problem and the labor side of the market, we can now turn to the firm's problem.

### 2.2 The Firm's Problem and Equilibrium

There is a unit mass of firms, each of which has a linear production technology with marginal product of labor $p$. Firms are heterogeneous in $p$. Assume that $p \sim \Gamma(p)$ continuous on $[\underline{p}, \bar{p}]$ Firms takes as given workers' strategies on participation, quit and acceptance, and in turn sets the wage. Each firm maximizes steady state profit per worker by choosing its wage offer, $w$. It's objective function is

$$
\Pi(w)=\max _{w}(p-w) \frac{G^{\prime}(w)}{F^{\prime}(w)}
$$

Thus taking $F(w), R_{u}(b), R_{n}(b), b^{*}$ as given, the firm's profit-maximizing wage offer, $w=\kappa(p)$, satisfies

$$
\kappa(p) \in \arg \max _{x}(p-w) \frac{G^{\prime}(w)}{F^{\prime}(w)}
$$

We are now able to define an equilibrium

Definition $1 A$ steady state equilibrium in this economy is $\left\{R_{n}(b), R_{u}(b), \kappa(p), b^{*}\right.$ and $\left.F(w)\right\}$ such that

1. Given $H(b)$ and $F(w)$, workers' reservation wages are determined according to

$$
R_{u}(b)=b-s
$$

and

$$
R_{n}(b)=b-\left(\lambda-\lambda_{0}\right) \int_{R_{n}(b)}^{\bar{w}} \frac{1-F(w)}{r+\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{n}(w)\right)\right]} d w
$$

2. The labor force participation rate is determined by $b^{*}$ satisfying

$$
s=\left(\lambda-\lambda_{0}\right) \int_{b^{*}-s}^{\bar{w}} \frac{1-F(w)}{r+\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{n}(w)\right)\right]} d w
$$

3. $\kappa(p) \in \arg \max _{x}(p-w) \frac{G^{\prime}(w)}{F^{\prime}(w)}$.

Strictly speaking, we do not need heterogeneity in the marginal product of labor in order to generate wage dispersion. If all firms had the same production technology, the equilibrium is obtained by an equal maximum profit condition rather than a profit-maximizing condition. While this would be sufficient to achieve wage dispersion, it may result in a wage offer distribution that is not of an "admissible" shape when compared with the data. To overcome this problem, heterogeneity in firm technology is introduced. A discussion of these issues are beyond the scope of this paper and may be found in Bontemps et. al. (2000).

Applying the usual concavity conditions on the firm's objective function, it can be easily shown that existence and uniqueness of the equilibrium is guaranteed if $l(w)=\frac{G^{\prime}(w)}{F^{\prime}(w)}$ is $\log$ concave. Furthermore the profit maximizing wage offer $\tilde{w}$ as a function of marginal product of labor satisfies

$$
p=\tilde{w}+\frac{l(\tilde{w})}{l^{\prime}(\tilde{w})}
$$

Unfortunately, because expressions such as $J_{u}(b)$ and $J_{n}(b)$ do not have closed forms, it is difficult to establish analytically that $l(w)$ is log-concave. However it turns out from simulations that for common distributions and realistic parameter values, log-concavity is satisfied. Let $w=\kappa(p)$ be the solution to the firm's profit maximization problem. As long as $l(w)$ is $\log$ concave, it is easy to establish that wage offers are increasing in $p$.

Lemma 4 If $l(w)$ is log concave, then $\kappa^{\prime}(p)>0$.

This then implies the upper and lower support of the wage distribution are obtained from the productivities of the most productive and least productive firms, i, e that $\underline{w}=\kappa(\underline{p})$ and $\bar{w}=\kappa(\bar{p})$. Also, note that all we need to guarantee uniqueness of $\kappa(p)$, is that $\frac{1}{(\underline{p}-\underline{w})^{2}}$ is large enough compared to $l(\underline{w})$.

Finally, we can obtain the distribution of wage offers.
Lemma 5 Since $\kappa^{\prime}>0$, the proportion of wage offers less than $w$ is $\Gamma\left(\kappa^{-1}(w)\right)$, so

$$
\begin{equation*}
F(w)=\Gamma\left(w+\frac{l(w)}{l^{\prime}(w)}\right) \tag{13}
\end{equation*}
$$

where $l(w)=\frac{G^{\prime}(w)}{F^{\prime}(w)}$.

## 3 Solution, Data and Estimation

### 3.1 Solution of the Model

The lack of closed forms for the endogenous distributions $G(w), J_{u}(b)$ and $J_{n}(b)$ imply that the solution must be obtained numerically. The procedure is as follows:

1. obtain $G(w)$ using a kernel estimate and observed $\underline{w}$ and $\bar{w}$ are observed from the data.
2. Using an initial guess of $F($.$) , solve for the workers' decision rules R_{u}($.$) and R_{n}($. and $b^{*}$. $R_{n}(n)$ has no closed form, but can easily be solved either by guessing an intial $R_{n}^{0}(b)$ and iterating on the reservation wage equation (6) until convergence, or numerically solving (6) using differential equation techniques.
3. Next solve for $J_{u}(b)$ and $J_{n}(b)$, and $u, e, n$. By guessing $J_{u}^{(0)}($.$) and J_{n}^{(0)}($.$) , we can$ obtain $u, e$, and $n$, which are then used to provide updates of $J_{u}^{(1)}$ and $J_{n}^{(1)}$ using the steady state conditions(7) and (8). This step is repeated until convergence of $J_{u}$ (.) and $J_{n}($.$) .$
4. With updated $J_{u}(b)$ and $J_{n}(b)$ update $F^{(1)}($.$) using the steady state expressions for$ $l(w) .(11),(12)$ and (13). Repeat steps (2) to (4) until convergence of $F(w)$.

### 3.2 Data

The model is estimated using data from the 1979 Youth Cohort of the National Longituditudinal Survey (NLSY79) sponsored by the Bureau of Labor Statistics. The NLSY79 is a nationally representative panel of 12,686 young men and women who were 14-22 years old in 1979. Our sample is drawn from the cross-sectional sample and the supplemental sample of the NLSY79. The cross sectional sample consists of 6,111 respondents designed to be representative of the noninstitutionalized civilian segment of young people living in the United States. The supplemental sample consists of 5,295 respondents designed to oversample civilian Hispanic, black, and economically disadvantaged non-black/non-Hispanic youth living in the United States.

The main criteria in choosing a sample are that individuals should have the same education and are unlikely to attain more schooling, are at the same age and stage in their life cycle, and are relatively likely to make a transition motivated by changes in utility of leisure. To this end, I select two samples of white and black women who have completed high school, and did not complete another year of schooling for at least five years after that. This results in 963 white women, and 633 black women. In this sample, only a small minority (around 5 percent) attain further education after a five year absence. I begin observing these women at the start of the second year after high school graduation (i.e. from June of the year following high school graduation), until the first transition observed after that period, up to a maximum of eight years ${ }^{9}$. This period corresponds to the age range where the black-white differences in labor force participation appear to be largest, as depicted in figure 1. According to the NLSY79 data, about 95 percent of these women completed high school at ages 18 or 19. By the age of $24,51 \%$ of white women and $75 \%$ of black women in this sample would have had their first birth. At that same point in their lives, $73 \%$ of white women and $47 \%$ of black women would have been married. This suggests that the chosen period indeed contains the arrival of significant events that may induce respondents to alter their labor force status. This way of selecting the sample and period reflects a balance between the realities of the life cycle changes with a model that delivers transitions in a steady state.

An alternative way would be to use the event of the first birth or mariage as a proxy for the event signifying an arrival of a new value of leisure. However I do not do so because the date of labor market transition may vary greatly from the date of birth of the child.

Table 2 presents summary statistics of the sample
table2here

From the first two columns of table 2, observe that black females have a much higher unemployment (about three times) and nonparticipation rate than white females. Their mean accepted wage and LFP rate are about $90 \%$ of white women, and their average

[^6]unemployment spells about $18 \%$ longer. The data does not show a significant difference in the mean lengths of employed and OLF spells. Turning to the flows, observe that the transitions between $u$ and $n$ for black women are about three times as large as for white women. Job to job transitions are about $30 \%$ smaller for black women compared with white women. Quits from jobs are about the same, with more blacks quitting to unemployment, but more whites quitting out of the labor force. From these data, we can guess that black women may receive more frequent shocks to their non-work utility, and that their arrival rate of jobs is lower while in the labor force.

### 3.3 Estimation

The model is estimated via maximum likelihood. I assume that the distribution of leisure utility and the distribution of marginal product of labor are distributed as log-normals, that is, $b \sim L N\left(\mu_{H}, \sigma_{H}^{2}\right)$ and $p \sim L N\left(\mu_{P}, \sigma_{P}^{2}\right)$. Along with the transition parameters $\left\{\lambda, \lambda_{0}, \gamma\right\}$, the search cost $s$, the rate of discount $r$, the model has nine parameters.

The seven types of transitions imply seven types of contributions to the likelihood function. Below I write down the likelihood contributions assuming that $b$ are unobserved, but durations and wages are observed without error. In any given labor force state, the escape rate follows an exponential distribution given $b$ or $w$ as per the following

$$
\begin{aligned}
\text { from unemployment } & : \lambda\left[1-F\left(R_{u}(b)\right)\right]+\gamma\left[1-H\left(b^{*}\right)\right] \\
\text { from nonparticipation } & : \lambda_{0}\left[1-F\left(R_{n}(b)\right)\right]+\gamma H\left(b^{*}\right) \\
\text { from current employment } & : \lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{j}(w)\right)\right], \text { where } \phi(.)=R^{-1}(.)
\end{aligned}
$$

The escape rate from each state is the sum of the escape rates into the other two states. For instance, unemployed individuals can transit into employment or to nonparticipation. Hence given $b$ her probability of leaving unemployment is the sum of the hazard rates into employment $\lambda\left[1-F\left(R_{u}(b)\right)\right]$ and nonparticipation $\gamma\left[1-H\left(b^{*}\right)\right]$. The other two states can be derived in the same way.

Let $L\left(\theta ; t_{i},\left\{w_{i}, w_{i}^{\prime}\right\} \mid j, k\right)$ be the likelihood contribution of individual $i$ who sojourns in state $j$ for a duration of $t_{i}$ and transits to state $k . w_{i}$ and $w_{i}^{\prime}$ are optional arguments depending on the type of transition, representing the old wage and new wage (if any). Let
$l()=.\ln L($.$) . The log-likelihood function is thus$

$$
l(\theta)=\sum_{i=1}^{N} l\left(\theta ; t_{i}, w \mid j, k\right)
$$

$U \rightarrow E$ Transition In this transition we observe unemployed duration $t_{u}$ and accepted wage $w$. The contribution of this type of transition to the log-likelihood is
$l\left(\theta ; t_{u}, w \mid u, e\right)=\ln \lambda-\left\{\lambda+\gamma\left[1-H\left(b^{*}\right)\right]\right\} t_{u}+\ln f(w)+\ln \left(\int_{\underline{b}}^{b^{*}} \exp \left\{\lambda F(b-s) t_{u}\right\} d J_{u}(b)\right)$
$U \rightarrow N$ Transition This transition requires we observe only duration $t_{u}$. The loglikelihood is
$l\left(\theta ; t_{u} \mid u, n\right)=\ln \gamma+\ln \left[1-H\left(b^{*}\right)\right]-\left\{\lambda+\gamma\left[1-H\left(b^{*}\right)\right]\right\} t_{u}+\ln \left(\int_{\underline{b}}^{b^{*}} \exp \left\{\lambda F(b-s) t_{u}\right\} d J_{u}(b)\right)$
$N \rightarrow E$ Transition In this type of transition I should observe $t_{n}$ and the accepted wage $w$
$l\left(\theta ; t_{n}, w \mid n, e\right)=\ln \lambda_{0}-\left\{\lambda_{0}+\gamma H\left(b^{*}\right)\right\} t_{n}+\ln f(w)+\ln \left(\int_{b^{*}}^{\bar{b}} \exp \left\{\lambda_{0} F\left[R_{n}(b)\right] t_{n}\right\} d J_{n}(b)\right)$
$N \rightarrow U$ Transition This transition requires we observe only observe $t_{n}$

$$
\begin{equation*}
l\left(\theta, t_{n} \mid n, u\right)=\ln \gamma-\left[\lambda_{0}+\gamma H\left(b^{*}\right)\right] t_{u}+\ln H\left(b^{*}\right)+\ln \left(\int_{b^{*}}^{\bar{b}} \exp \left\{\lambda_{0} F\left[R_{n}(b)\right] t_{n}\right\} d J_{n}(b)\right) \tag{17}
\end{equation*}
$$

$E \rightarrow E$ Transition I should observe duration of current job $t_{e}$, wage at current job $w$, and the new wage $w^{\prime}$ that I get. Likelihood would be

$$
\begin{equation*}
l\left(\theta ; t_{e}, w, w^{\prime} \mid e, e\right)=\ln \lambda-\left\{\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{j}(w)\right)\right]\right\} t_{e}+\ln f\left(w^{\prime}\right) \tag{18}
\end{equation*}
$$

$E \rightarrow U$ transitions These transitions only happen when current accepted wage is low enough, i.e. when $\underline{w}<w<b^{*}-s$, but the new draw of $b$ is not too high. I should observe $w$ and $t_{e}$. Likelihood is

$$
\begin{equation*}
l\left(\theta, t_{e}, w \mid e, u\right)=\ln \gamma+\ln \left[H\left(b^{*}\right)-H(w+s)\right]-\{\lambda[1-F(w)]+\gamma[1-H(w+s)]\} t_{e} \tag{19}
\end{equation*}
$$

$E \rightarrow N$ transitions There are two possibilities: (1) when $w<b^{*}-s$. The likelihood of a transition to $N$ is

$$
\begin{equation*}
l\left(\theta ; t_{e}, w \mid e, n, w<b^{*}-s\right)=\ln \gamma+\ln \left[1-H\left(b^{*}\right)\right]-\{\lambda[1-F(w)]+\gamma[1-H(w+s)]\} t_{e} \tag{20}
\end{equation*}
$$

When (2) $w \geq b^{*}-s$, a worker only quits to nonparticipation the contribution to likelihood is
$l\left(\theta ; t_{e}, w \mid e, n, w>b^{*}-s\right)=\ln \gamma+\ln \left\{1-H\left[\phi_{n}(w)\right]\right\}-\left\{\lambda[1-F(w)]+\gamma\left[1-H\left(\phi_{n}(w)\right)\right]\right\} t_{e}$

Treatment of Missing Observations An empirical issue is that for job to job transitions, it is sometimes observed that $w^{\prime}<w$. This is not admissible under the model as it generates a likelihood value of zero. To get around this problem, one either incorporates measurement error, or disregards $w^{\prime}$. In the version without measurement error, I take the latter approach.

### 3.4 Measurement Error (TO BE ADDED)

Assume that wages are measured with multiplicative error. If $w$ is the true wage and $\tilde{w}$ is the observed wage, then the relationship between true and observed wage is

$$
w=\tilde{w} \varepsilon
$$

Assume that $\varepsilon \sim \Phi(\varepsilon)$.

### 3.5 Estimation Results

Table 3 presents the parameter estimates of the maximum likelihood estimation assuming that wages are measured without error. Standard errors are calculated from the empirical hessian of the maximized likelihood.

## table3here

That $\lambda>\lambda_{0}>0$ and $\gamma>0$ are key results of the paper. The arrival rate of jobs when OLF is lower than when employed as expected, but significantly different from zero. Also that $\gamma>0$ imples that shocks to non-work options do drive females' labor force participation choices. The results indicate that white females experience a higher arrival rate of jobs while employed and unemployed ( 0.0670 vs 0.041 )and OLF ( 0.0256 vs 0.0161 )compared with black females. Both flow parameters are $59 \%$ and $63 \%$ higher respectively. On the other hand, the arrival rate of shocks to utility of leisure, $\gamma$, is $18 \%$ higher among black women than white women ( 0.0432 vs 0.0367 ). The mean of the distribution of leisure values from the black sample is also higher than for the white sample, while the variance is slightly lower. Since this is assumed to be lognormal, after converting to 1994 US dollars, this implies a mean non-work value of $\$ 287.54$ vs $\$ 262.27$ per week for black and white females respectively. Put another way, the estimates imply that black women in the sample have about a $10 \%$ higher mean value of non-work, and slightly less variation in their non-work utility than the white sample. In addition, they encounter these shocks more frequently. Search costs turn out to be lower for the black sample than for the white sample, but only slightly so. The weekly rates of discount are about equal $r^{b}=0.0026$ for black sample and $r^{w}=0.0024$ for the white sample. These correspond an annual discount rates of 0.874 for black women, and 0.883 for white women. ${ }^{10}$

Turning to the distribution parameters of labor productivity, we find that black women have a $12 \%$ higher mean value of labor productivity. The distribution of productivity, also a lognormal, implies a mean of about $\$ 309.99$ and $\$ 277.01$ per week for the black and white

[^7]sample respectively. Care should be taken to interpret the productivity parameters. These are productivity parameters of the firm, not the worker, so in this particular set of estimates, black females are employed at firms with lower average productivity than white females. In addition, the estimates should be thought of as a measure of "aggregate" productivity and are not industry specific. This is largely because the of the relatively small sample chosen from the NLSY79. With a larger sample, one can stratify the data by industry and perform a more finely divided analysis that accounts for inter and intra industry differentials.

The overall picture emerging from these results is that the various sources of labor market differentials between black and white women that are captured by the model appear to work in opposite directions. Lower arrival rates of job offers faced by black females compared with white females would decrease reservation wages, but this effect is offset by higher mean values of leisure and of productivity, which increase reservation wages.

We now turn to table 4 which reports steady state statistics from the model

## table4here

Panel A reports the predicted steady state proportion in each of unemployment, employment and OLF. As much as $47 \%$ of black females are not attached to a job, as opposed to $31 \%$ of white females. In panel B, the mean wage offers are $\$ 151.52$ and $\$ 137.53$ for black and white women respectively: much lower than the mean accepted wages, and implying that the black-white gap in offered wages is in the opposite direction than that of accepted wages. Looking at the mean sojourn spells, we see that all spells among black females are shorter than for white females. The higher frequency of shocks to utility of leisure faced by black females as compared to whites reduces their overall length of time spent in each state.

Turning to table 5, we compare the average hazards obtained from the sample, with the hazards implied by the model. For the most part, the model and data hazards are of the same order of magnitude. It is interesting to note that the model significantly underpredicts the $n \rightarrow e$ average hazard rate. The model's average $u \rightarrow e$ hazard is about 40 times as large as the average $n \rightarrow e$ hazard. This underscores the behavioral difference between the unemployed and the OLF. The hazards governing movement between $u$ and $n$ are as of the


Figure 3: Wage Offer and Accepted Densities for Black and White Females.
same order of magnitude as the hazards into employment, and as insignificant, as suggested by Van Den Berg and Ridder (1998).

We now turn to the distributions implied by the model. Figure 3 displays the distributions of offered wages $F(w)$ and accepted wages $G(w)$. Indeed, looking at accepted wages, would give a different picture than offered wages. Offered wages are not a sufficient picture of racial labor market differences either. One could argue that offered wages are higher for blacks, but these are offset by the fact that frictions in job search are also higher. Figure 4 displays the reservation wages implied by the model. The two reservation wage functions are quite close together, suggesting that no group is more "picky" about jobs than the other. In sum, the estimated model seems to suggest a mixed picture of black-white labor market inequality

The differences in labor market frictions and shocks to non-work options are quite large and are in favor of white females, the differences in work (productivity) and non-work (leisure) options are more moderate, and favor the black females, In all, these two effects appear to cancel each other, resulting in very similar reservation wage functions. Also, the


Figure 4: Reservation Wages Implied by the Model.
higher rate of shocks to leisure does shorten the average labor market attachment of the black women compared with white women.

## 4 Counterfactual Experiments

## TO BE ADDED

## 5 Conclusion

This paper presents the first attempt at formulating and estimating an equilibrium search model in more than two states. The model developed here incorporates the participation decision as well as the reservation strategy. Its main driving force in generating participation is the fact that individuals receive changes to the utility of their non-work option, and respond by reassessing their labor market choices. They may choose to quit their jobs, and if so, whether to engage in active search (unemployed) at a utility cost, or passive search (OLF) at no cost. This formulation is meant to capture, in a parsimonious way, potentially
long term life cycle changes that are reflected by a change in the valuation of non-work alternatives.

The estimated model allowed us to examine labor force inequality along several dimensions, namely offered wages, unemployment and participation rates, and spell lengths. In summary, the estimated model suggests that black labor market spells are shorted as a result of more frequent changes in non-work options. The parameters also suggest that labor force inequality arises from (1) more search frictions encountered by black women than by white women (as evidenced by the lower arrival rate of jobs to black women while unemployed). However this is somewhat countered by (2) the higher valuation of, and higher frequency of shocks to non-work alternatives by black women.

The model suggests that putting more structure on search frictions parameters, such as incorporating endogenous search, would be a fruitful direction of further research. Likewise, further work to understand the nature of the non-work alternatives would also be fruitful.

TO BE CONTINUED...

## 6 Appendix

### 6.1 Proofs

Derivation of $R_{u}(b)$ and $R_{n}(b)$.. Rewrite the value functions characterizing the model

$$
\begin{align*}
r V(w) & =w+\lambda \int_{w}^{\bar{w}}\left[V\left(w^{\prime}\right)-V(w)\right] d F\left(w^{\prime}\right)+\gamma E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]  \tag{22}\\
r U(b) & =b-s+\lambda \int_{R_{u}(b)}^{\bar{w}}\left[V\left(w^{\prime}\right)-V\left(R_{u}(b)\right)\right] d F\left(w^{\prime}\right)+\gamma E\left[P\left(b^{\prime}\right)-U(b)\right]  \tag{23}\\
r N(b) & =b+\lambda_{0} \int_{R_{n}(b)}^{\bar{w}}\left[V\left(w^{\prime}\right)-V\left(R_{n}(b)\right)\right] d F\left(w^{\prime}\right)+\gamma E\left[P\left(b^{\prime}\right)-N(b)\right]  \tag{24}\\
P(b) & =\max [U(b), N(b)] \tag{25}
\end{align*}
$$

We begin by solving for the reservation wage for the unemployed. Setting $U(b)=V\left[R_{u}(b)\right]$ we obtain the expression for the reservation wage of an unemployed worker with $b$ :

$$
\begin{equation*}
R_{u}(b)=b-s \quad \text { if } E P\left(b^{\prime}\right) \geq U(b) \tag{26}
\end{equation*}
$$

Similarly, setting $N(b)=V\left[R_{n}(b)\right]$. This implies

$$
\begin{equation*}
R_{n}(b)=b-\left(\lambda-\lambda_{0}\right) \varphi\left[R_{n}(b)\right] \quad \text { if } E P\left(b^{\prime}\right) \geq N(b) \tag{27}
\end{equation*}
$$

where by integrating by parts,
$\varphi\left[R_{n}(b)\right]=\int_{R_{n}(b)}^{\bar{w}}\left[V\left(w^{\prime}\right)-V\left(R_{n}(b)\right)\right] d F\left(w^{\prime}\right)=\int_{R_{n}(b)}^{\bar{w}}[1-F(w)] V^{\prime}(w) d w$.
To summarize, the workers adopt the following reservation strategy: unemployed and nonparticipating workers with utility $b$ accept wage offers greater than the reservation wage equations defined by (26) and (27) respectively.

Proof of Lemma 1.. $U^{\prime}(b), N^{\prime}(b), V^{\prime}(w)>0$.
Step 1: Recall that

$$
r V(w)=w+\lambda \int_{w}^{\bar{w}}[1-F(z)] V^{\prime}(z) d z+\gamma E \max \left\{\max \left[N\left(b^{\prime}\right), U\left(b^{\prime}\right)\right]-V(w)\right\}
$$

where we need to evaluate $P\left(b^{\prime}\right)$. Suppose the worker receives an opportunity to draw a new $b^{\prime}$. We conjecture that there exists a $b^{*}$ such that the worker is indifferent between $U\left(b^{*}\right)$ and $N\left(b^{*}\right)$. If the worker draws $b^{\prime}>b^{*}$, she would prefer nonparticipation, because there the arrival rate of job offers is low, allowing her to enjoy a high $b^{\prime}$ for a longer period. If the worker draws $b^{\prime}<b^{*}$, she would prefer to choose unemployment, engage in active job search, and suffer a short term utility loss so as to exit unemployment as soon as possible. This suggests that

$$
P\left(b^{\prime}\right)=\left\{\begin{array}{l}
N\left(b^{\prime}\right) \text { if } b^{\prime} \geq b^{*} \\
U\left(b^{\prime}\right) \text { if } b^{\prime}<b^{*}
\end{array}\right.
$$

Since there is a $b^{*}$ such that $U\left(b^{*}\right)=N\left(b^{*}\right)$, it must also be true that $V\left[R_{u}\left(b^{*}\right)\right]=$ $V\left[R_{n}\left(b^{*}\right)\right]$. And in order that $V^{\prime}()>$.0 , it should be true that $R_{u}\left(b^{*}\right)=R_{n}\left(b^{*}\right)=R\left(b^{*}\right)$ also. Therefore we consider 2 cases.

Case 1: $w<R\left(b^{*}\right):$
In this case workers have the option to quit to both nonparticipation and to unemployment. Workers would quit to unemployment only if the currently received low wage $\left(w<R\left(b^{*}\right)\right)$ is exceeded by a new sample of $b^{\prime}$ such that $U\left(b^{\prime}\right)>V(w)$, and $U\left(b^{\prime}\right)>N\left(b^{\prime}\right)$. Thus $R_{u}\left(b^{\prime}\right)>w$. (i.e. the wage she received was so low, that she'd rather quit to unemployment if the opportunity came up). On the other hand if she samples a $b^{\prime} \geq b^{*}$, she would prefer to quit to nonparticipation. This discussion implies that the worker would (1) quit to unemployment when $w$ is low enough and the worker samples $b^{\prime} \in\left[R_{u}^{-1}(w), b^{*}\right)$,
or (2) quit to nonparticipation if samples a $b^{\prime} \in\left[b^{*}, \bar{b}\right]$. Therefore when $w<R\left(b^{*}\right)$

$$
\begin{align*}
E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]= & \int_{R_{u}^{-1}(w)}^{b^{*}}\left[V\left(R_{u}\left(b^{\prime}\right)\right)-V(w)\right] d H\left(b^{\prime}\right)  \tag{28}\\
& +\int_{b^{*}}^{\bar{b}}\left\{V\left[R_{n}\left(b^{\prime}\right)\right]-V(w)\right\} d H\left(b^{\prime}\right) \tag{29}
\end{align*}
$$

Case 2: $w \geq R\left(b^{*}\right)$.
In this case, the worker quits only to nonparticipation. This would happen if she draws a $b^{\prime}$ such that $b^{\prime} \geq b^{*}$ and is moreover sufficiently high compared to his current wage that she is induced to quit. Thus $R_{n}\left(b^{\prime}\right)>w$. In other words she must sample $b^{\prime} \in\left[R_{n}^{-1}(w), \bar{b}\right]$. This discussion implies that when $w \geq R\left(b^{*}\right)$,

$$
\begin{equation*}
E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]=\int_{R_{n}^{-1}(w)}^{\bar{b}}\left\{V\left[R_{n}\left(b^{\prime}\right)\right]-V(w)\right\} d H\left(b^{\prime}\right) \tag{30}
\end{equation*}
$$

Combining the two expressions (28) and (30), we now have

$$
\begin{align*}
& E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]  \tag{31}\\
= & \left\{\begin{array}{c}
\int_{R_{u}^{-1}(w)}^{b^{*}}\left[V\left(R_{u}\left(b^{\prime}\right)\right)-V(w)\right] d H\left(b^{\prime}\right)+\int_{b^{*}}^{\bar{b}}\left\{V\left[R_{n}\left(b^{\prime}\right)\right]-V(w)\right\} d H\left(b^{\prime}\right) \text { if } w<R\left(b^{*}\right) \\
\int_{R_{n}^{-1}(w)}^{\bar{b}}\left\{V\left[R_{n}\left(b^{\prime}\right)\right]-V(w)\right\} d H\left(b^{\prime}\right) \text { if } w \geq R\left(b^{*}\right)
\end{array}\right.
\end{align*}
$$

Differentiating equation (31) using Leibniz's rule:

$$
\frac{\partial E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]}{\partial w}=\left\{\begin{array}{c}
-\left[1-H\left[\phi_{u}(w)\right]\right] V^{\prime}(w) \text { if } w<R\left(b^{*}\right)  \tag{32}\\
-\left\{1-H\left[\eta_{n}(w)\right]\right\} V^{\prime}(w) \text { if } w \geq R\left(b^{*}\right)
\end{array}\right.
$$

Step 2: Differentiating equation (22) by Leibniz's rule, we have :

$$
\begin{equation*}
r V^{\prime}(w)=1-\lambda[1-F(w)] V^{\prime}(w)+\gamma \frac{\partial E \max \left[P\left(b^{\prime}\right)-V(w), 0\right]}{\partial w} \tag{33}
\end{equation*}
$$

Substituting (32) into (33) and letting

$$
1-H\left[\phi_{j}(w)\right]=\left\{\begin{array}{c}
1-H\left[\phi_{u}(w)\right] \text { if } w<R\left(b^{*}\right) \\
1-H\left[\phi_{n}(w)\right] \text { if } \quad w \geq R\left(b^{*}\right)
\end{array}\right.
$$

allow us to obtain $V^{\prime}(w)$

$$
V^{\prime}(w)=\frac{1}{r+\lambda[1-F(w)]+\gamma\left\{1-H\left[\phi_{j}(w)\right]\right\}}>0
$$

Step 3: Differentiating the Bellman Equations via Leibniz's rule and rearranging, we have

$$
\begin{aligned}
U^{\prime}(b) & =\frac{1}{r+\gamma+\lambda\left[1-F\left(R_{u}(b)\right)\right]} \\
N^{\prime}(b) & =\frac{1}{r+\gamma+\lambda_{0}\left[1-F\left(R_{n}(b)\right)\right]}
\end{aligned}
$$

which are both strictly positive.
Proof of Lemma 2.. Consider an unemployed worker with $b$. Since $U(b)$ is independent of $w$ and $V^{\prime}(w)>0$, a reservation strategy $R_{u}(b)$ exists and is unique. Conversely since $U^{\prime}(b)>0$ and $V(w)$ is independent of $b, R_{u}(b)$ is strictly increasing in $b$. The same arguments apply for $R_{n}(b)^{11}$.

Proof of Proposition 1.. We first show existence and uniqueness. Setting $U\left(b^{*}\right)=$ $N\left(b^{*}\right)$ implies $b^{*}$ must solve

$$
\frac{s}{\lambda-\lambda_{0}}=\varphi\left(b^{*}-s\right)
$$

where $\varphi(w)=\int_{w}^{\bar{w}} \frac{1-F(x)}{r+\lambda[1-F(x)]+\gamma\left[1-H\left(\phi_{j}(x)\right)\right]} d x$. It is straightforward to show that $\varphi(\underline{b}-s)$ is monotonically declining and is bounded above by the rectangular area $\frac{\bar{b}-b}{r+\gamma+\lambda}$. Furthermore, $\varphi(\bar{b})=0$. Continuity and monotonicity guarantee the existence of $b^{*}$. Observing further that $\frac{\bar{b}-b}{r+\gamma+\lambda}>\frac{s}{\lambda-\lambda_{0}}$, we obtain the bound $B=\frac{\lambda-\lambda_{0}}{r+\gamma+\lambda_{0}}$.

Next, consider $U(b)-N(b)$. Using a first order Taylor expansion around $b^{*}-s$,

$$
\begin{aligned}
U(b)-N(b) & \gtrless 0 \Longleftrightarrow-s+\left(\lambda-\lambda_{0}\right) \varphi(b-s) \gtrless 0 \\
& \Longleftrightarrow \underbrace{-s+\left(\lambda-\lambda_{0}\right) \varphi\left(b^{*}-s\right)}_{=0}+\left(\lambda-\lambda_{0}\right) \varphi^{\prime}\left(b^{*}-s\right)\left(b-b^{*}\right) \gtrless 0 \\
& \Longleftrightarrow \varphi^{\prime}\left(b^{*}-s\right)\left(b-b^{*}\right) \gtrless 0
\end{aligned}
$$

where since $\varphi^{\prime}<0$, the inequality can be flipped to obtain the result.

[^8]
### 6.2 Solving the Endogenous Transitions and Distributions

### 6.2.1 Aggregate Transitions and Stocks in Steady State

The model generates six transitions: $u \rightarrow e, u \rightarrow n, n \rightarrow e, n \rightarrow u, e \rightarrow u, e \rightarrow n$.
Aggregate transitions in steady states therefore must satisfy the following flow conditions. Inflows to and outflows from unemployment must satisfy:

$$
\begin{align*}
& u\left\{\lambda\left[J_{u}\left(b_{l}\right)+\int_{b_{l}}^{b^{*}}\left[1-F\left(R_{u}(b)\right)\right] d J_{u}(b)\right]+\gamma\left[1-H\left(b^{*}\right)\right]\right\}  \tag{34}\\
= & n \gamma H\left(b^{*}\right)+e \gamma \int_{\underline{w}}^{R\left(b^{*}\right)}\left[H\left(b^{*}\right)-H\left(\phi_{u}(w)\right)\right] d G(w) \tag{35}
\end{align*}
$$

which simplifies to

$$
u\left\{\lambda[1-A]+\gamma\left[1-H\left(b^{*}\right)\right]\right\}=\gamma\left\{H\left(b^{*}\right)\left[n+e G\left(R\left(b^{*}\right)\right)\right]-e B\right\}
$$

where

$$
\begin{aligned}
A & =\int_{b_{l}}^{b^{*}} F\left(R_{u}(b)\right) d J_{u}(b) \\
B & =\int_{\underline{w}}^{R\left(b^{*}\right)} H\left(\phi_{u}(w)\right) d G(w)
\end{aligned}
$$

Inflows to and outflows from nonparticipation have to satisfy:

$$
\begin{align*}
& n\left\{\lambda_{0} \int_{b^{*}}^{b_{u}}\left[1-F\left(R_{n}(b)\right)\right] d J_{n}(b)+\gamma H\left(b^{*}\right)\right\}  \tag{36}\\
= & u \gamma\left[1-H\left(b^{*}\right)\right]+e \gamma\left\{\left[1-H\left(b^{*}\right)\right] G\left(R\left(b^{*}\right)\right)+\int_{R\left(b^{*}\right)}^{\bar{w}}\left[1-H\left(\phi_{n}(w)\right)\right] d G(w)\right\}
\end{align*}
$$

which simplifies to

$$
n\left\{\lambda_{0}\left[J_{n}\left(b_{u}\right)-C\right]+\gamma H\left(b^{*}\right)\right\}=\gamma\left\{u\left[1-H\left(b^{*}\right)\right]+e\left[1-H\left(b^{*}\right) G\left(R\left(b^{*}\right)\right)-D\right]\right\}
$$

where

$$
\begin{aligned}
C & =\int_{b^{*}}^{b_{u}} F\left(R_{n}(b)\right) d J_{n}(b) \\
D & =\int_{R\left(b^{*}\right)}^{\bar{w}} H\left(\phi_{n}(w)\right) d G(w)
\end{aligned}
$$

And finally, imposing the requirement that $e+n+u=1$, we have 3 equations in 3 unknowns for which we solve for steady state ergodic distribution of all workers. The closed form solutions to the steady state stocks of $u, e, n$ are

$$
\begin{align*}
u & =\frac{\gamma^{2} H(1-B-D)+\lambda_{0} \gamma(G H-B)(J-C)}{\Delta}  \tag{37}\\
e & =\frac{\lambda_{0}(J-C)[\gamma(1-H)+\lambda(1-A)]+\gamma \lambda(1-A) H}{\Delta}  \tag{38}\\
n & =\frac{\gamma^{2}(1-H)(1-B-D)+\gamma \lambda(1-A)(1-D-H G)}{\Delta} \tag{39}
\end{align*}
$$

where

$$
\begin{aligned}
\Delta= & \gamma^{2}(1-B-D)+\lambda(1-A)\left[\gamma(1-D+H-G H)+\lambda_{0}(J-C)\right] \\
& +\lambda_{0} \gamma(J-C)(1-B+H-G H)
\end{aligned}
$$

### 6.2.2 $J_{u}(b)$ and $J_{n}(b)$.

The support of $b$ must be split into four regions: $\left[\underline{b}, b_{l}\right]$, where workers will accept any job because their utility of leisure is very low, $\left[b_{l}, b^{*}\right]$, and $\left[b^{*}, b_{u}\right]$, where workers accept some jobs but reject others, and $\left[b_{u}, \bar{b}\right]$ where workers reject all jobs because they enjoy leisure too much.

These specify four cases:

Case 1: $\underline{b}<b<b_{l} \quad$ By the same logic of equating inflows and outflows, $J_{u}(b)$ must satisfy

$$
u\{\lambda+\gamma[1-H(b)]\} J_{u}(b)=\gamma n H(b)
$$

implying

$$
\begin{equation*}
J_{u}(b)=\frac{n}{u} \frac{\gamma H(b)}{\lambda+\gamma[1-H(b)]} \tag{40}
\end{equation*}
$$

Case 2: $b_{l}<b<b^{*} \quad J_{u}(b)$ must respectively satisfy:

$$
\begin{align*}
& u\left\{\lambda J_{u}\left(b_{l}\right)+\int_{b_{l}}^{b} \lambda\left[1-F\left(R_{u}(x)\right)\right] d J_{u}(x)+\gamma[1-H(b)] J_{u}(b)\right\}  \tag{41}\\
= & \gamma\left[n H(b)+e \int_{\underline{w}}^{R_{u}(b)}\left[H(b)-H\left(\phi_{u}(x)\right)\right] d G(x)\right]
\end{align*}
$$

Case 3: $b^{*}<b<b_{u} \quad J_{n}(b)$ in this case would be

$$
\begin{align*}
& n\left\{\int_{b^{*}}^{b} \lambda_{0}\left[1-F\left(R_{n}(x)\right)\right] d J_{n}(x)+\gamma\left[1+H\left(b^{*}\right)-H(b)\right] J_{n}(b)\right\}  \tag{42}\\
= & \gamma\left\{u\left[H(b)-H\left(b^{*}\right)\right]+e\left[G\left(R\left(b^{*}\right)\right)\left[H(b)-H\left(b^{*}\right)\right]+\int_{R\left(b^{*}\right)}^{R_{n}(b)}\left[H(b)-H\left(\phi_{j}(x)\right)\right] d G(x)\right]\right\}
\end{align*}
$$

Case 4: $b_{u}<b<\bar{b}$

$$
\begin{align*}
& n\left\{\int_{b^{*}}^{b_{u}} \lambda_{0}\left[1-F\left(R_{n}(x)\right)\right] d J_{n}(x)+\gamma\left[1+H\left(b^{*}\right)-H(b)\right] J_{n}(b)\right\}  \tag{43}\\
= & \gamma\left\{u\left[H(b)-H\left(b^{*}\right)\right]+e \int_{R\left(b^{*}\right)}^{R_{n}\left(b_{u}\right)}\left[H\left(b_{u}\right)-H\left(\phi_{u}(x)\right)\right] d G(x)+e\left[H(b)-H\left(b_{u}\right)\right]\right\}
\end{align*}
$$

Proposition 4 Given $F(w)$ and $H(b)$, there exists a unique $J_{u}(b)$ satisfying (41) and a unique $J_{n}(b)$ satisfying (42) and (43)

Proof. We will prove by differentiating (41) to (43) with respect to $b$ and showing that the resulting differential equations satisfy the Lipschitz condition. Differentiating expression (41) and rearranging we get

$$
J_{u}^{\prime}(b)=\frac{\gamma}{u} \frac{\left[n+e G\left(R_{u}(b)\right)+u J_{u}(b)\right] H^{\prime}(b)}{\lambda\left[1-F\left(R_{u}(b)\right)\right]+\gamma[1-H(b)]}
$$

Define the RHS as $K(b, x)=\frac{\left[n+e G\left(R_{u}(b)\right)+u x\right] H^{\prime}(b)}{\lambda\left[1-F\left(R_{u}(b)\right)\right]+\gamma[1-H(b)]}$. Observe that $K(b, x)$ is continuous for all $x$ and for all $b$ satisfying $\lambda\left[1-F\left(R_{u}(b)\right)\right]+\gamma[1-H(b)]>0$ which is true as long as $b^{*}<\bar{b}$. Next observe that

$$
\left|K\left(b, x_{2}\right)-K\left(b, x_{1}\right)\right|=\left|\frac{u H^{\prime}(b)\left(x_{2}-x_{1}\right)}{\lambda\left[1-F\left(R_{u}(b)\right)\right]+\gamma[1-H(b)]}\right|
$$

which satisfies the Lipschitz condition because $b^{*}<\bar{b}$. From this together with our initial condition that $J_{u}(\underline{b})=0$, we conclude that $J_{u}(b)$ exists and is unique. The proof for $J_{n}(b)$ is identical, with the analogous expressions for the differential equation

$$
\begin{aligned}
J_{n}^{\prime}(b) & =\frac{\gamma}{n} \frac{\left\{n J_{n}(b)+u+e G\left(R_{n}(b)\right)\right\} H^{\prime}(b)}{\lambda_{0}\left[1-F\left(R_{n}(b)\right)\right]+\gamma\left[1+H\left(b^{*}\right)-H(b)\right]} \\
b & \in\left[b^{*}, b_{u}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
J_{n}^{\prime}(b) & =\frac{1}{n} \frac{H^{\prime}(b)\left[1-n+n J_{n}(b)\right]}{\left[1+H\left(b^{*}\right)-H(b)\right]} \\
b & \in\left[b_{u}, \bar{b}\right]
\end{aligned}
$$

and the initial condition $J_{n}\left(b^{*}\right)=0$. Note now that the denominator in this case is always $>0$.

### 6.3 Computation of the Model

### 6.3.1 Computation of $J_{u}(b)$ and $J_{n}(b)$.

Given a solution to $\{u, e, n\}$ we need to update the $J_{u}(b)$ and $J_{n}(b)$ expressions. Using the steady state conditions $(40),(41),(42)$ and (43), we can write updating expressions as follows:

Case 1: $\underline{b}<b<b_{l}$

$$
\begin{equation*}
J_{u}(b)=\frac{n}{u} \frac{\gamma H(b)}{\lambda+\gamma[1-H(b)]} \tag{44}
\end{equation*}
$$

Case 2: $b_{l}<b<b^{*}$

$$
\begin{equation*}
J_{u}^{(t+1)}(b)=\frac{u \lambda A(b)+\gamma H(b)[n+e G(b-s)]-e \gamma B(b-s)}{u\{\lambda+\gamma[1-H(b)]\}} \tag{45}
\end{equation*}
$$

where

$$
\begin{aligned}
A(b) & =\int_{\underline{w}+s}^{b} F^{(t)}(b-s) J_{u}^{\prime(t)}(b) d b \\
B(b-s) & =\int_{\underline{w}}^{b-s} H(w+s) G^{\prime}(w) d w
\end{aligned}
$$

Case 3: $b^{*}<b<b_{u}$
$J_{n}^{(t+1)}(b)=\frac{n \lambda_{0} C(b)+u \gamma\left[H(b)-H\left(b^{*}\right)\right]+e \gamma H(b)\left[G\left(R_{n}(b)\right)-G\left(b^{*}-s\right)\right]-e \gamma D\left(R_{n}(b)\right)}{n\left\{\lambda_{0}+\gamma\left[1+H\left(b^{*}\right)-H(b)\right]\right\}}$
where

$$
\begin{aligned}
C(b) & =\int_{b^{*}}^{b} F^{(t)}\left[R_{n}(b)\right] J_{n}^{\prime(t)}(b) d b \\
D\left(R_{n}(b)\right) & =\int_{b^{*}-s}^{R_{n}(b)} H\left[\phi_{n}(w)\right] G^{\prime}(w) d w
\end{aligned}
$$

Case 4: $b_{u}<b<\bar{b}$ and

$$
\begin{aligned}
& J_{n}^{(t+1)}(b)= \\
& \frac{-n \lambda_{0}\left[J_{n}^{(t)}\left(b_{u}\right)-C\left(b_{u}\right)\right]+u \gamma\left[H(b)-H\left(b^{*}\right)\right]+e \gamma H\left(b_{u}\right)\left[G\left(R_{n}\left(b_{u}\right)\right)-G\left(b^{*}-s\right)\right]-e \gamma D\left(R_{n}\left(b_{u}\right)\right)}{n \gamma\left[1+H\left(b^{*}\right)-H(b)\right]}
\end{aligned}
$$

where noting that $R_{n}\left(b_{u}\right)=\bar{w}$,
$J_{n}^{(t+1)}(b)=\frac{-n \lambda_{0}\left[J_{n}^{(t)}\left(b_{u}\right)-C\left(b_{u}\right)\right]+u \gamma\left[H(b)-H\left(b^{*}\right)\right]+e \gamma H\left(b_{u}\right)\left[1-G\left(b^{*}-s\right)\right]-e \gamma D(\bar{w})}{n \gamma\left[1+H\left(b^{*}\right)-H(b)\right]}$
where

$$
\begin{aligned}
C\left(b_{u}\right) & =\int_{b^{*}}^{b_{u}} F^{(t)}\left[R_{n}(b)\right] J_{n}^{\prime(t)}(b) d b \\
D(\bar{w}) & =\int_{b^{*}-s}^{\bar{w}} H\left[\phi_{n}(w)\right] G^{\prime}(w) d w
\end{aligned}
$$

### 6.3.2 Computation of $F(w)$.

Once we have converged on $J_{n}(b)$ and $J_{u}(b)$ we are in a position to update $F(w)$. Using equations (9) and (10) we can write the updating equations as

$$
\begin{align*}
F^{(t+1)}(w) & =\frac{(\gamma+\lambda) e G(w)+u \lambda A(w+s)-e \gamma B(w)}{u \lambda J_{u}(w+s)+e \lambda G(w)}  \tag{47}\\
\text { where } w & <b^{*}-s
\end{align*}
$$

where

$$
\begin{aligned}
A(w+s) & =\int_{\underline{w}+s}^{w+s} F^{(t)}(b-s) J_{u}^{\prime(t)}(b) d b \\
B(w) & =\int_{\underline{w}}^{w} H(w+s) G^{\prime}(w) d w
\end{aligned}
$$

and

$$
F^{(t+1)}(w)=\frac{(\gamma+\lambda) e G(w)+u \lambda A\left(b^{*}\right)+n \lambda_{0} C\left[\phi_{n}(w)\right]-e \gamma\left[B\left(b^{*}-s\right)+D(w)\right]}{\lambda[u+e G(w)]+n \lambda_{0} J_{n}\left[\phi_{n}(w)\right]}(4 \varepsilon
$$

$$
\begin{equation*}
\text { where } w \geq b^{*}-s \tag{49}
\end{equation*}
$$

where

$$
\begin{aligned}
A\left(b^{*}\right) & =\int_{\underline{w}+s}^{b^{*}} F^{(t)}(b-s) J_{u}^{\prime(t)}(b) d b \\
B\left(b^{*}-s\right) & =\int_{\underline{w}}^{b^{*}-s} H(w+s) G^{\prime}(w) d w \\
C\left[\phi_{n}(w)\right] & =\int_{b^{*}(t)}^{\phi_{n}(w)} F^{(t)}\left[R_{n}(b)\right] J_{n}^{\prime(t)}(b) d b \\
D(w) & =\int_{b^{*}-s}^{w} H\left[\phi_{n}(x)\right] G^{\prime}(x) d x
\end{aligned}
$$

Hence Update Rules for $F(w)$ are (47) and (48).

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Table 1 : Average Monthly Worker Transitions as a Percentage of Sample

|  | \% Sample |
| :--- | ---: |
| Panel A : NLSY79 Cross Section Sample |  |
| (N=6111) averaged over Jan 1978 - Dec 1996 |  |
| $\mathrm{n} \rightarrow \mathrm{u}$ | 1.07 |
| $\mathrm{n} \rightarrow \mathrm{e}$ | 3.00 |
| $\mathrm{u} \rightarrow \mathrm{n}$ | 0.96 |
| $\mathrm{u} \rightarrow \mathrm{e}$ | 1.12 |
| $\mathrm{e} \rightarrow \mathrm{u}$ | 1.00 |
| $\mathrm{e} \rightarrow \mathrm{n}$ | 2.93 |
| $\mathrm{e} \rightarrow \mathrm{e}$ | 1.89 |

Panel B : CPS Jan 1968 to May 1986
$\mathrm{n} \rightarrow \mathrm{u} \quad 0.66$
$\mathrm{n} \rightarrow \mathrm{e} \quad 0.90$
$\mathrm{u} \rightarrow \mathrm{n} \quad 0.55$
$\mathrm{u} \rightarrow \mathrm{e} \quad 1.01$
$\begin{array}{ll}\mathrm{e} \rightarrow \mathrm{u} & 0.80\end{array}$
$\mathrm{e} \rightarrow \mathrm{n} \quad 1.02$
$\mathrm{e} \rightarrow \mathrm{e}$
Notes: Panel B taken from Kim (2001).

Table 2 : Summary Statistics of the Data

|  | Black | White | B/W Ratio |
| :---: | :---: | :---: | :---: |
| Unemployment | 0.122 | 0.046 | 2.646 |
| Employment | 0.580 | 0.733 | 0.791 |
| OLF | 0.299 | 0.221 | 1.353 |
| Unemployment Rate | 0.173 | 0.059 | 2.932 |
| LFP Rate | 0.701 | 0.779 | 0.900 |
| Mean Accepted Wage/Week* | 294.9 | 336.98 | 0.875 |
| Std dev. Wage | 149.18 | 160.49 |  |

Spell Length Before Transition (week)

| Mean Unemployed Spell | 14.7 | 12.4 | 1.185 |
| :--- | :--- | :--- | :--- |
| Mean Employed Spell | 35.1 | 34.9 | 1.006 |
| Mean OLF Spell | 23.4 | 23.8 | 0.984 |

## Weekly Worker Flows (\%)

| $\mathrm{n} \rightarrow \mathrm{u}$ | 0.425 | 0.132 | 3.218 |
| :--- | :--- | :--- | :--- |
| $\mathrm{n} \rightarrow \mathrm{e}$ | 0.447 | 0.485 | 0.921 |
| $\mathrm{u} \rightarrow \mathrm{n}$ | 0.429 | 0.133 | 3.227 |
| $\mathrm{u} \rightarrow \mathrm{e}$ | 0.329 | 0.264 | 1.245 |
| $\mathrm{e} \rightarrow \mathrm{u}$ | 0.283 | 0.233 | 1.212 |
| $\mathrm{e} \rightarrow \mathrm{n}$ | 0.448 | 0.502 | 0.893 |
| $\mathrm{e} \rightarrow \mathrm{e}$ | 0.980 | 1.292 | 0.759 |

## Weekly Hazards(\%)

| $\mathrm{n} \rightarrow \mathrm{u}$ | 1.423 | 0.598 | 2.378 |
| :--- | :--- | :--- | :--- |
| $\mathrm{n} \rightarrow \mathrm{e}$ | 1.495 | 2.196 | 0.681 |
| $\mathrm{u} \rightarrow \mathrm{n}$ | 3.519 | 2.886 | 1.219 |
| $\mathrm{u} \rightarrow \mathrm{e}$ | 2.699 | 5.736 | 0.471 |
| $\mathrm{e} \rightarrow \mathrm{u}$ | 0.487 | 0.318 | 1.532 |
| $\mathrm{e} \rightarrow \mathrm{n}$ | 0.773 | 0.685 | 1.129 |
| $\mathrm{e} \rightarrow \mathrm{e}$ | 1.689 | 1.762 | 0.959 |

[^9]Table 3 : Maximum Likelihood Estimation Results

|  | Black | White |
| :---: | :---: | :---: |
| $\lambda$ | $\begin{array}{r} 0.0410 \\ (0.0006) \end{array}$ | $\begin{array}{r} 0.0670 \\ (0.0005) \end{array}$ |
| $\lambda_{0}$ | $\begin{array}{r} 0.0161 \\ (0.0008) \end{array}$ | $\begin{array}{r} 0.0256 \\ (0.0006) \end{array}$ |
| $\gamma$ | $\begin{array}{r} 0.0432 \\ (0.0006) \end{array}$ | $\begin{array}{r} 0.0367 \\ (0.0006) \end{array}$ |
| $s$ | $\begin{array}{r} 25.59 \\ (0.0006) \end{array}$ | $\begin{array}{r} 27.20 \\ (0.0006) \end{array}$ |
| $r$ | $\begin{array}{r} 0.0026 \\ (0.0011) \end{array}$ | $\begin{array}{r} 0.0024 \\ (0.0011) \end{array}$ |
| $\mu_{H}$ | $\begin{array}{r} 5.0333 \\ (0.0003) \end{array}$ | $\begin{array}{r} 4.9248 \\ (0.0003) \end{array}$ |
| $\sigma_{H}$ | $\begin{array}{r} 1.1207 \\ (0.0002) \end{array}$ | $\begin{array}{r} 1.1354 \\ (0.0002) \end{array}$ |
| $\mu_{P}$ | $\begin{array}{r} 5.5144 \\ (0.0003) \end{array}$ | $\begin{array}{r} 5.3100 \\ (0.0003) \end{array}$ |
| $\sigma_{P}$ | $\begin{array}{r} 0.6665 \\ (0.0002) \end{array}$ | $\begin{array}{r} 0.7925 \\ (0.0002) \end{array}$ |
| Negative of log-likelihood | 2908.6 | 5892.933 |
| Number of Observations | 462 | 827 |
| Mean Value of Leisure | 287.54 | 262.27 |
| S.d. Value of Leisure | 455.68 | 425.28 |
| Mean Marginal Product of Labor | 309.99 | 277.01 |
| S.d. Marginal Product of Labor | 231.85 | 258.97 |

Table 4: Statistics Implied by the Estimated Model

|  | Black | White | B/W Ratio |
| :--- | ---: | ---: | ---: |
| Panel A: \% in State. |  |  |  |
| Unemployment | 0.240 | 0.133 | 1.798 |
| Employment | 0.531 | 0.687 | 0.772 |
| OLF | 0.230 | 0.179 | 1.280 |
| Panel B: Labor Market Statistics |  |  |  |
| Unemployment Rate (U/LF) | 0.311 | 0.163 | 1.915 |
| LFP Rate (LF/POP) | 0.770 | 0.821 | 0.939 |
| Mean Wage Offer | 151.52 | 137.53 | 1.102 |
| S.d. Wage Offer | 170.00 | 156.07 |  |
| Mean Accepted Wage From Data | 286.26 | 320.7 | 0.893 |
| b* | 330.61 | 353.96 |  |
| Mean Unemployed Spell (wk) | 34.81 | 40.6 | 0.857 |
| Mean Employed Spell (wk) | 60.96 | 77.3 | 0.789 |
| Mean OLF Spell (wk) | 25.63 | 28.8 | 0.891 |

Table 5: Comparison of Hazards and Flows from Data and Model

| Weekly Average Hazard (\%) | Black |  | White |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Data | Model | Data | Model |
| $\mathrm{n} \rightarrow \mathrm{u}$ | 1.423 | 3.251 | 0.598 | 2.924 |
| $\mathrm{n} \rightarrow \mathrm{e}$ | 1.495 | 0.065 | 2.196 | 0.119 |
| $\mathrm{u} \rightarrow \mathrm{n}$ | 3.519 | 1.065 | 2.886 | 0.744 |
| $\mathrm{u} \rightarrow \mathrm{e}$ | 2.699 | 2.798 | 5.736 | 4.529 |
| $\mathrm{e} \rightarrow \mathrm{u}$ | 0.487 | 0.340 | 0.318 | 0.261 |
| $\mathrm{e} \rightarrow \mathrm{n}$ | 0.773 | 0.953 | 0.685 | 0.650 |
| $\mathrm{e} \rightarrow \mathrm{e}$ | 1.689 | 1.374 | 1.762 | 1.681 |
| Weekly Worker Flows (\%) | Data | Model | Data | Model |
| $\mathrm{n} \rightarrow \mathrm{u}$ | 0.425 | 0.746 | 0.132 | 0.524 |
| $\mathrm{n} \rightarrow \mathrm{e}$ | 0.447 | 0.015 | 0.485 | 0.021 |
| $\mathrm{u} \rightarrow \mathrm{n}$ | 0.429 | 0.255 | 0.133 | 0.099 |
| $\mathrm{u} \rightarrow \mathrm{e}$ | 0.329 | 0.671 | 0.264 | 0.604 |
| $\mathrm{e} \rightarrow \mathrm{u}$ | 0.283 | 0.180 | 0.233 | 0.179 |
| $\mathrm{e} \rightarrow \mathrm{n}$ | 0.448 | 0.506 | 0.502 | 0.447 |
| $\mathrm{e} \rightarrow \mathrm{e}$ | 0.980 | 0.729 | 1.292 | 1.155 |


[^0]:    ${ }^{1}$ See Diamond (1971).
    2 "Participation" flows refer to transitions between a labor force participation state, unemployment or employment, and nonparticipation. "Intra labor force" flows refer to transitions between employment and unemployment.

[^1]:    ${ }^{3}$ Panel A is constructed from the cross sectional sample of the NLSY79 ( $\mathrm{N}=6111$ ).
    ${ }^{4}$ The figures from the NLSY79 and the CPS are not directly comparable. The flows from the NLSY79 are averaged over the life cycle of a fairly homogeneous age cohort, while the flows calculated from the CPS are for a cross section of the population.

[^2]:    ${ }^{5}$ It should be clear that $\gamma=0$ implies that one's endowment of non-work utility is permanent and the model will not produce $u \leftrightarrow e$ transitions. This, coupled with $\lambda_{0}=0$, effectively reduces the model to the heterogeneous worker case in Burdett-Mortensen (1998). If there were furthermore no on-the-job search, it corresponds to the Albrecht-Axell (1984) model.

[^3]:    ${ }^{6}$ This is not a strong assumption, because in the continuous time limit, no two events will happen at a point in time.

[^4]:    ${ }^{7}$ We can do so because regardless of the state the worker is in $V(w), U(b)$ or $N(b)$, she only has to track one state variable, either $b$ or $w$. This would not be the case, for example, if $V(w, b)$, such as if employed workers can recall their $b$ when laid off. This becomes more complicated because we can no longer guarantee the monotonicity of the reservation wage strategies, which is needed for uniqueness of reservation wages.

[^5]:    ${ }^{8}$ As can be inferred from figure 2, they quit their job if their offer of leisure utility is far enough to the right of the bold envelope.

[^6]:    ${ }^{9}$ In a minority of cases who graduated earlier than 1977 , I began following them from the start of the survey. A period of one to eight years after graduation yields very few truncated spells, so censoring is not an issue.

[^7]:    ${ }^{10}$ There can be several explanations as to why these appear to be rather low annual discount rates. Firstly, it may be because the sample consists of relatively young women at ages 20 to 28 ; secondly it may also be because the rate of discount also incorporates the probability of exogenous "death" of a job match.

[^8]:    ${ }^{11} \mathrm{We}$ can do so because regardless of the state the worker is in $V(w), U(b)$ or $N(b)$, she only has to track one state variable, either $b$ or $w$. This would not be the case, for example, if $V(w, b)$, such as if employed workers can recall their $b$ when laid off. This becomes more complicated because we can no longer guarantee the monotonicity of the reservation wage strategies, which is needed for uniqueness of reservation wages.

[^9]:    *Wages are in 1994 dollars.

