Enterprise Dynamics and Finance: Distinguishing Mechanism Design from Exogenously Incomplete Markets Models*

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Abstract

We formulate and solve a range of dynamic models of information-constrained credit markets that allow for moral hazard and unobservable investment. We compare them to the exogenously incomplete markets environments of autarky, saving only, and borrowing and lending in a single asset. We develop computational methods based on mechanism design theory, linear programming, and maximum likelihood techniques to structurally estimate, compare and statistically distinguish among the competing theoretical models of credit market imperfections. Our methods can be applied with both cross-sectional and panel data and allow for measurement error and unobserved heterogeneity in initial conditions. The models match major stylized facts from the empirical literature on firm dynamics as listed by Cooley and Quadrini (2001). Empirically, we find that using consumption, cashflow and investment data jointly or using dynamic data improves the researcher's ability to distinguish across the various model regimes relative to using consumption or investment only data, especially in the presence of high measurement error. We also estimate our models using data on Thai households running small businesses. We find that the borrowing and saving only frameworks provide the best fit when using joint data on consumption, cashflow and investment.

Keywords: financial constraints, mechanism design, structural estimation and testing

JEL Classifications: C61, D82

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1 Introduction

We study the enterprises run by households that are so common not only in emerging markets but also other countries. More specifically, we compute, estimate, and contrast the consumption and investment behavior of risk averse households running business under various financial market environments, including both exogenously incomplete ones (autarky, savings only, borrowing and lending) and endogenously information-constrained ones (moral hazard with observed or unobserved investment, both relative to full insurance). We compare the predictions of each of these financial regimes to stylized facts reported in the literature (e.g., Cooley and Quadrini, 2001) and discuss in what circumstances they might be distinguished in data. Indeed, we develop methods for empirical estimation of mechanism design models and test the various models against each other using both data generated from the models themselves and actual data on Thai households running small businesses.

The relatively small businesses that we study are only beginning to be recognized as a major economic factor in a variety of economies, both developing and developed. In addition, these household business typically “fall between the cracks” in the literature. To make clear the objects we study, we elaborate a bit at the outset on these two issues.

In Thailand’s high growth period from 1976 to 1996, occupation shifts from agriculture to wage work and enterprise account for 18-21% of the change in per capita income. These shifts, along with an increase followed by a decrease of the enterprise income premium, profits minus wages, account for 33-39% of the change in inequality. Occupation shifts alone account for 29% reduction in the poverty rate, the most important factor. Using a micro-founded structural model of credit constraints, Jeong and Townsend (2007) estimate that 73% of the change in Thailand’s total factor productivity can be explained by a combination of occupational choice and changes in the financial infrastructure. In India, own-account enterprises using at most unpaid family labor account for 68% of all non-agricultural enterprises and 36% of all employment.

Even in advanced OECD countries such as Spain, large fractions of some bank portfolios are loans to entrepreneurs, as distinct from mortgages, consumer durables, and loans to listed, officially registered corporations in various sectors. In the US, non-employers account for 70% of all establishments, though only 14% of employment. Using data from the Survey of Consumer Finances, Cagetti and De Nardi (2006) in their study of US inequality find that the 7.6% of self-employed business owners hold 33% of the wealth.

Yet, with few exceptions, the literature maintains a dichotomy embedded in the national accounts: households are consumers and suppliers of market inputs, whereas firms produce and hire labor and other factors. This gives rise, on the one hand, to a large development literature which studies household consumption smoothing. There is voluminous work estimating the permanent income model, the full risk sharing model, buffer stock models (Zeldes, 1989; Deaton and Laroque, 1996) and, lately, models with private information (Phelan 1994; Ligon, 1998; Werning, 2001) or limited commitment (Ligon, Thomas and Worrall, 2002) . On the other hand, the consumers-firms dichotomy gives rise to an equally large literature on investment. There is the adjustment costs approach of Abel and Blanchard (1983) and Bond and Meghir (1994), for example. In an industrial organization setting, Hopenhayn (1992) and Ericson and Pakes (1995) model the entry and exit of firms with Cobb-Douglas or CES production technologies where investment augments capital with a lag and output produced from capital, labor and other factors is subject to factor-neutral Markov technology shocks.

Mostly firms are modeled to be risk neutral maximizers of expected discounted profits or dividends to owners. There are models attempting to explain stylized facts for firm growth, with higher mean growth and variance in growth for small firms, e.g. Cooley and Quadrini (2001) among others. The more recent work by Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006) introduce either private information or limited commitment but maintain risk neutrality. Applied general equilibrium models feature both consumption and investment in the same context, as Rossi-Hansberg and Wright (2007), but there the complete markets hypothesis justifies, within the model, a separation of the decisions of households from the decisions of firms. Alem and Townsend (2008) provide an explicit derivation with full risk sharing with equilibrium stochastic discount factors rationalizing the apparent risk neutrality of households as firms making investment decisions.

The work that is closest to this paper, and complementary with it, does feature households as firms but largely assumes that certain markets are missing. For example, Cagetti and De Nardi (2006) follow Aiyagari (1994) in their study of inequality and assume labor income is stochastic and uninsured. In contrast, Angeletos and Calvet (2007) and Covas (2006) in their study of buffer stock motives and macro savings rates feature uninsured entrepreneurial risk. In the asset pricing vein, Heaton and Lucas (2000) feature entrepreneurial investment as a portfolio choice problem, assuming exogenously incomplete markets, as in the tradition of Geanakoplos and Polemarchakis (1986) and Zame (1993).

These papers beg the question of how good an approximation are the various assumptions on financial regimes, different across the different papers. That is, what would be a reasonable assumption for the financial regime if that part too were taken to the data? We take that view below to see how far we get. The adjustment costs investment literature may be picking up constraints implied by financing, not adjustment costs per se. The pecking order investment literature (Myers and Majuf, 1984) assumes that internally generated funds are least expensive, then debt, and finally equity, discussing wedges and distortions. Berger and Udell (2002) have a long discussion in this spirit of small vs. large firm finance. He points out that small firms would seem to be informally opaque yet have received funds from family, friends, angels, and venture capitalists. Bitler, Moskowitz and Vissing-Jorgensen (2005) argue likewise that agency considerations play a key role. The empirical work of Fazzari, Hubbard and Petersen (1988) picks up systematic distortions for small firms, but the nature of the credit market imperfection is not modeled, leading to criticisms of their interpretation of the tests (Kaplan and Zingales, 2000).

Our methods in this paper follow logically from Paulson, Townsend and Karaivanov (2006) where we estimated whether moral hazard or limited commitment is the key financial obstacle causing the observed positive monotonic relationship between initial wealth and subsequent decision to enter into business. Buera and Shin (2007) extend this to dynamics and endogenous savings decision in a model with limited borrowing. Here we abstract from occupational choice and focus much more on the dynamics, investment, and especially on a wide variety of financial regimes.

We also analyze the advantages of using a combination of data on consumption and the smoothing of income shocks, with data on firms size distribution and the smoothing of investment from cash flow fluctuations, in effect filling the gap between the dichotomies of the literature. In estimating both exogenously incomplete and endogenous information-constrained regimes we also break new ground. The only other effort of which we are aware is Meh and Quadrini (2006), who compare and contrast a bond economy to an economy in which unobserved diversion of capital creates an incentive constraint.

In this paper we focus on whether, and in what circumstances, it is possible to distinguish financial markets regimes, depending on the data used. To that end, we feature tests where we have full control, i.e., we know what the financial regime really is, using data generated from the model. Our paper is thus both a conceptual and methodological contribution. We show how all the regimes can be formulated as linear programming problems, often of large dimension, and how likelihood functions, naturally in the space of lotteries or histograms, can be estimated. We allow for i.i.d. measurement error, the need to estimate the underlying distribution of unobserved state variables, and related, the use of data from economies in transition, before they reach steady state.

When using model-generated data we find that, naturally, the ability to distinguish between the regimes depends on both the type of data used and the amount of measurement error. With low measurement error we are able to distinguish between virtually all regime pairs. As expected, however, higher levels of measurement error in the data reduce the power of our model comparison tests to such an extent that some of the regimes cannot be distinguished from the data generating baseline as well as from each other. This is especially pronounced for the case of firm size data under full depreciation. Using consumption/income data, we typically cannot distinguish between the moral hazard and full information regimes when there is high measurement error. In contrast, in virtually all cases we are able to distinguish between regime groups, that is the exogenously incomplete regimes versus the mechanism design moral hazard and full information regimes.

Using joint data on consumption, investment and cash flow markedly improves the ability to distinguish across the regimes including when there is high measurement error. In that case, if the hypothesized null regime is true, then we can distinguish the truth from other regimes. But if the researcher guesses incorrectly, and the null is a counterfactual regime, then there is sometimes less ability to distinguish the null from other regimes nearby. Thus researchers should be cautious when testing a given regime against an alternative when they fail to reject it in the data. Both the null and the alternatives may not be the true regime. We also incorporate intertemporal data from the model through either repeated cross-sections or panels. We find that doing so improves the ability to distinguish the regimes relative to when using single cross-sections.

Additionally, we do take the next step and apply our methods to our featured emerging market economy, Thailand, to make the point that our methods offer a feasible practical approach to real data when the researcher aims to provide insights as to the source and nature of financial constraints. We chose Thailand for two reasons. First, our data source (the Townsend Thai surveys) includes panel data on both consumption and enterprise size and this is rare. We can thus see if the combination of consumption and firm size data really helps in practice. Second, we also learn about potential next steps in modeling financial regimes. We know in particular from other work with these data that consumption smoothing is quite good, i.e., it is difficult to reject full insurance, in the sense that the coefficient on idiosyncratic income is small if significant (Chiappori, Shulhulfer-Wohl, Samphantharak, and Townsend, 2008). We also know that investment is sensitive to cash flow, especially for the poor, but on the other hand this is overcome by networks of family and friends (Samphantharak and Townsend, 2008). We are interested in how these same data look when viewed jointly though the lens of the various financial regimes modelled in this paper. We also want to be assured that our methods which rely on grid approximation, cell size, reasonable measurement error, estimation of unobserved distribution of utility promises, and transition dynamics are, as a practical matter, applicable to actual data.

We find that by and large out methods work, though sometimes the number of probability cells needed to use joint data in the estimation (and in some instances, panel data) exceeds our sample size. This also hurts some of the other bilateral comparison. Otherwise, we obtain results consistent with those using theory-generated data and some striking findings. There are some puzzles as well. We find that the Thai consumption and income data are most consistent with the moral hazard regime but full information is a close second and sometimes statistically tied, depending on the specification. Here, using repeated cross sections data helps pin down the dominance of moral hazard. However, a two-year panel separated by several years offers a very blurry picture, though this tends to happen as well in model-generated data. We also find that the investment and cash flow data are most consistent with the savings and/or borrowing lending regime, again with ties depending on the specification. Again, repeated cross sections help pin down the dominance of the borrowing regimes. Using firm size and investment data alone does better than using consumption and income data alone. The results using combined consumption and investment data also lend support to the savings/borrowing lending regime but now it is hard to distinguish borrowing from saving only. The dominance of different financial regimes depending on the data used is something that does not occur in the model-generated data. We explore this further in the conclusions.

2 Theory

2.1 Basic Setting

Consider an economy of agents (firms) heterogeneous in their initial endowments (assets), k_0 of a single good that can be used for both consumption and investment. The agents live T periods, where T can be infinity. They can interact with a financial intermediary, entering into saving, debt, or insurance contracts. We characterize the optimal dynamic financial contracts that arise between the agents and the intermediary in different information or credit access regimes.

Agents are risk averse and have time-separable preferences defined over consumption, c, and labor effort, z represented by $U(c, z)$ where $U_1 > 0$, $U_2 < 0$. They discount future utility using a discount factor, β where $\beta \in (0, 1)$. For computational reasons (see below) we assume that c and z belong to the finite discrete sets (grids) C, Z accordingly. The agents have access to a stochastic output technology, $P(q|z, k) : Q \times Z \times K \to [0, 1]$ giving the probability of obtaining output, q from effort level z and capital level k[1]. The sets Q and K are also finite and discrete[2]. In all information regimes we study, output is assumed to be observable and verifiable. However, one or both of the inputs, k and z may be unobservable to third parties, leading to moral hazard and/or adverse selection problems. Capital, k depreciates at a rate δ every period. Depending on the application we have in mind, the lowest capital level ($k = 0$) could be interpreted as a "worker" occupation (similar to PTK, 2006) or as a firm "exit" state.

The financial intermediary is risk neutral and has access to an outside credit market with opportunity cost of funds R. Using the linear programming approach of Prescott and Townsend (1984) and Phelan and Townsend (1991), we model the optimal financial contracts as probability distributions over assigned or implemented allocations of consumption, output, effort, and investment (see below for details). There are two possible ways to interpret this. First, one can think of the intermediary (the principal) contracting with a single agent/firm at a time, in which case the contracts specify mixed strategies over allocations. Alternatively, one can think of a principal contracting with a continuum of agents, so that the optimal contract specifies the fraction of agents of given type that receive a particular deterministic allocation. It is further assumed that there are no aggregate shocks, there are no technological links between the agents, and the agents cannot collude. Finally, the principal and the agents are assumed to be able to fully commit to the ex-ante optimal contract (although our methods allow us to relax this assumption).

[1]We can easily incorporate heterogeneity in entrepreneurial ability across agents as in Paulson et al. (2006), for instance by adding a talent parameter θ in the production function $P(q|z, k)$.

[2]This can be either a technological or computational assumption depending on the application.

2.2 Information and Credit Access Regimes

We write down a dynamic linear programming problem determining the (constrained) optimal contract in each regime. To characterize the contracts with incomplete information we invoke the revelation principle and study direct mechanisms in which the agents announce truthfully their type[3].

The information / credit access regimes we study can be classified into two groups. The first group are regimes with *exogenously incomplete* markets: *autarky (A)*, *savings only (S)*, and *borrowing and lending (B)*. In these models the feasible financial contracts take a specific, exogenously defined form (no access to financial markets, a deposit contract, or a debt contract). In contrast, in the second group of regimes the financial contracts are endogenously determined as solving a mechanism design problem, potentially subject to information and incentive constraints. We look at two such *endogenously incomplete* markets regimes – *moral hazard (MH),* in which agents' effort is unobserved but capital and investment are observed, and *moral hazard with unobserved capital (UC)*, in which both effort and investment are unobservable. All regimes are compared to the *full information (FI)* regime (the first best).

2.2.1 Exogenously Incomplete Markets Regimes

Autarky

In this regime the agent is assumed to have no access to financial intermediaries or storage. The timeline is as follows. The agent starts the current period with initial capital k which he invests into production. At this time the agent also decides on his effort z. At the end of the period output q is realized, the agent decides on the next period capital level, $k' \in K$, and consumes $c = (1-\delta)k + q - k'$. Capital, k is the state variable in the recursive formulation of the agent's optimization problem. This is a relatively simple problem and can be solved by standard non-linear dynamic programming techniques. To be consistent with the solution methods used for the endogenously incomplete regimes, where non-linear techniques may be inapplicable due to non-convexities introduced by the incentive and truth-telling constraints, we reformulate the problem as a linear program with respect to the joint probabilities of obtaining allocations (q, z, k') given k:

$$V(k) = \max_{\pi(q,z,k'|k)} \sum_{Q\times Z\times K} \pi(q,z,k'|k)[u((1-\delta)k + q - k', z) + \beta V(k')] \tag{1}$$

The maximization in (1) is subject to a set of constraints on the choice variables, π. First, for each $k \in K$, the $\pi' s$ have to be Bayes-consistent with the probability distribution over outputs, P:

$$\sum_K \pi(q,z,k'|k) = P(\bar{q}|\bar{z},k) \sum_{Q\times K} \pi(q,\bar{z},k'|k) \text{ for all } (\bar{q},\bar{z}) \in Q \times Z \tag{2}$$

Given that $\pi' s$ are allocation probabilities, we also must have $\pi(q,z,k'|k) \geq 0$ (non-negativity) for all $(q,z,k') \in Q \times Z \times K'$, as well as (adding-up):

$$\sum_{Q\times Z\times K} \pi(q,z,k'|k) = 1 \tag{3}$$

The policy variables $\pi(q,z,k'|k)$ that solve the maximization problem determine the optimal effort and investment level z and k' in autarky.

[3]The proofs that the optimal contracting problem can be written in a recursive form and that the revelation principle applies follow from Doepke and Townsend (2006) and hence are omitted.

Saving Only / Borrowing

In this financial environment the agent is able to either save (which we call the *saving only, (S)* regime) or borrow and save (called the *borrowing, (B)* regime) through a competitive financial intermediary. Thus, the agent can use debt/savings to smooth consumption or investment, on top of what he could do under autarky. More specifically, if the agent borrows (saves) an amount b, next period he has to repay (collect) an amount Rb, independently of the state of the world. As with all other variables, b is assumed to take values on the finite discrete set, B. By convention, a negative value of b represents savings, thus in the S regime the upper bound of the grid B is zero – the agent can only accumulate and run down a buffer stock. Default is ruled out by assuming that the principal refuses to lend to a borrower who is at risk of not repaying[4]. By shutting down all contingencies in the debt contracts we aim for better differentiation with the endogenously constrained state-contingent contracts described next.

The timeline is as follows: the agent starts the current period with capital k carried and uses it in production together with effort z. In the end of the period, output q is realized, the agent repays Rb, and borrows or saves $b' \in B$. He also puts aside (invests) next period's capital, k' and consumes $c = (1-\delta)k + q + b' - Rb - k'$. The two assets k and b can be freely converted into one another each period. The problem of an agent with current capital stock k and debt/savings level b can be written recursively as:

$$V(k,b) = \max_{\pi(q,z,k',b'|k,b)} \sum_{Q\times Z\times K\times B} \pi(q,z,k',b'|k,b)[U((1-\delta)k + q + b' - Rb - k', z) + \beta V(k',b')] \tag{4}$$

subject to the Bayes-consistency and adding-up constraints analogous to (2) and (3) and subject to $\pi(q,z,k',b'|k,b) \geq 0$ for all $(q,z,k',b') \in Q \times Z \times K' \times B'$.

2.2.2 Mechanism Design Regimes

Full Information

With full information the principal observes and can contract upon agent's effort and investment. We write the corresponding dynamic principal-agent problem as an extension of Phelan and Townsend (1991) with capital accumulation. As is standard in such settings (see Spear and Srivastava, 1987; Doepke and Townsend, 2006), to obtain a recursive formulation we need an additional state variable – *promised utility*, i.e., discounted future utility, w belonging to the discrete set[5] W. As in Phelan and Townsend (1991) the grid of promised utilities we use has a lower bound, $w_{\min}$ corresponding to the lowest possible consumption, $c_{\min}$ (obtained from the lowest possible $\tau \in T$ and the highest $k' \in K$) and the highest possible effort, $z_{\max}$ promised forever. The set's upper bound, $w_{\max}$ corresponds to promising the highest possible consumption, $c_{\max}$ and lowest possible effort forever, i.e.,

$$w^{FI}_{\min} = \frac{U(c_{\min}, z_{\max})}{1-\beta} \text{ and } w^{FI}_{\max} = \frac{U(c_{\max}, z_{\min})}{1-\beta}$$

The optimal full information (FI) contract for an agent with current promised utility w and capital k consists of effort and investment levels, z, k', next period's promised utility $w' \in W$, as well as a transfer, τ (belonging to the discrete set T) between the principal and the agent. A positive value of τ denotes a transfer from the principal to the agent. The timing of events is the same as before, with the addition that the transfer occurs after output is observed. The principal's

[4]Computationally, this is achieved by assigning a very low utility value for such borrower.

[5]In principle, some values in this set may be infeasible. The set of feasible values is determined along with iterating on the value function using the methods proposed by Abreu, Pearce and Stacchetti (1990).

objective function, $V(w,k)$ when contracting with an agent at state (w,k) maximizes the expected value of output net of transfers to the agent plus the discounted value of future outputs and transfers. We write the mechanism design problem as a linear program in the joint probabilities, $\pi(\tau,q,z,k',w'|w,k)$ over allocations (τ,q,z,k',w'):

$$V(w,k) = \max_{\{\pi(\tau,q,z,k',w'|w,k)\}} \sum_{T\times Q\times Z\times K\times W} \pi(\tau,q,z,k',w'|w,k)[q-\tau+(1/R)V(w',k')] \tag{5}$$

The maximization in (5) is subject to the familiar Bayes-consistency and adding-up constraints on the probabilities π:

$$\sum_{T\times K\times W} \pi(\tau,\bar{q},\bar{z},k',w'|w,k) = P(\bar{q}|\bar{z},k) \sum_{T\times Q\times K\times W} \pi(\tau,q,\bar{z},k',w'|w,k) \text{ for all } (\bar{q},\bar{z}) \in Q\times Z, \tag{6}$$

$$\sum_{T\times Q\times Z\times K\times W} \pi(\tau,q,z,k',w'|w,k) = 1, \tag{7}$$

as well as non-negativity: $\pi(\tau,q,z,k',w'|w,k) \geq 0$ for all $(\tau,q,z,k',w') \in T\times Q\times Z\times K\times W$.

Because of the extra state, w there is an additional constraint, the *promise keeping* constraint, which ensures that the agent's expected utility equals his current utility promise:

$$\sum_{T\times Q\times Z\times K\times W} \pi(\tau,q,z,k',w'|w,k)[U(\tau+(1-\delta)k-k',z)+\beta w'] = w \tag{8}$$

By varying the initial value of w one can trace the whole Pareto frontier of expected utilities for the principal and the agent (see the Appendix for computed examples). The optimal FI contract maximizes (5) subject to the constraints (6), (7) and (8). We solve the dynamic linear program numerically. The solution is a vector of probabilities $\pi^*(\tau,q,z,k',w'|w,k)$ representing the optimal contract between the bank and the agent.

Moral Hazard

In this regime the principal can still observe and control capital and investment (k and k') but he can no longer observe or verify agent's effort, z. With capital observed and controlled, it can be interpreted as endogenous collateral when output is low. The timing is the same as in the FI regime. The unobservability of effort implies a moral hazard problem, i.e., the principal must induce effort from the agent. This is achieved by requiring that the optimal contract, $\pi(\tau,q,z,k',w'|w,k)$ satisfy an incentive-compatibility constraint (ICC) in addition to (6)-(8). The ICC states that, given the agent's state (w,k), a recommended effort level, $\bar{z}$, and known and enforced capital level k' and transfer τ, the agent must not be able to achieve higher expected utility by deviating to an alternative effort level $\hat{z}$, i.e., for all $(\bar{z},\hat{z}) \in Z\times Z$:

$$\sum_{T\times Q\times W'\times K'} \pi(\tau,q,z,k',w'|w,k)[U(\tau+(1-\delta)k-k',\bar{z})+\beta w'] \geq$$

$$\geq \sum_{T\times Q\times W'\times K'} \pi(\tau,q,z,k',w'|w,k)\frac{P(q|\hat{z},k)}{P(q|\bar{z},k)}[U(\tau+(1-\delta)k-k',\hat{z})+\beta w'] \tag{9}$$

For details on the derivation of the ICC in the linear programming approach, see Prescott and Townsend (1984). The key term is the "likelihood ratio", $\frac{P(q|\hat{z},k)}{P(q|\bar{z},k)}$ reflecting the fact that, if the agent deviates, he changes the probability distribution of output and the probabilities π must be adjusted to preserve Bayes rule consistency.

Apart from the constraint (9), the moral hazard regime also differs from the full information regime in the set of feasible promised utilities, W. Specifically, the lowest possible promise under moral hazard can no longer be the value $w^{FI}_{\min}$. Indeed, if the agent is assigned minimum consumption forever he would not supply any effort level above the minimum possible. The range of promised utilities for the MH setting is instead:

$$w^{MH}_{\min} = \frac{U(c_{\min}, z_{\min})}{1-\beta} \text{ and } w^{MH}_{\max} = \frac{U(c_{\max}, z_{\min})}{1-\beta}$$

The derivation of the bound $w^{MH}_{\min}$ is as in Phelan and Townsend (1991). Intuitively, the principal cannot promise a slightly higher consumption in exchange for much higher effort so that agent's utility falls below $w^{MH}_{\min}$ since this is not incentive compatible. If the agent does not follow the principal's recommendations but deviates to $z_{\min}$ the worst punishment he can receive is $c_{\min}$ forever.

The (constrained) optimal contract in the moral hazard regime is the solution to the linear program defined by (5)–(9). The contract features incomplete consumption insurance and intertemporal tie-ins, i.e., it is not simply a repetition of the optimal one-period contracts (Townsend, 1982).

Moral Hazard with Unobserved Investment

Now suppose the effort exerted by the agent is still unobservable by the principal but, in addition, assume that the principal also cannot observe the agent's capital level, k and the level of capital investment planned for next period, k'. Thus, there is both a dynamic adverse selection problem about the agent's unobserved state, k, as well as the moral hazard problem of two unobserved actions, z and k'.

To model the mechanism design problem that arises, assume that the agent sends a message about his capital level k to the principal who offers a contract conditional on the agent's message consisting of transfer τ, recommended effort, z, investment, k' and future promised utility. Due to the physical linkage between time periods and the dynamic adverse selection problem in k, following Fernandes and Phelan (2000) the proper state variable for the problem's recursive representation is a promise function, $w(k)$ instead of the scalar w in the MH regime. The reason why utility promises cannot be independent of k, in general, is the different incentives of agents entering next period with different k' (see Kocherlakota, 2004). Thus, to induce incentive compatibility, the principal needs to offer an optimal promised utility schedule dependent on assets, $\mathbf{w} \equiv \{w(k_1), w(k_2), ..., w(k_{\#K})\} \in \mathbf{W}$ where k_1, k_2, etc. are the elements of the grid K[6]. The set $\mathbf{W}$ is endogenously determined and, computationally, must be iterated upon together with the value and policy functions (Abreu, Pierce and Stacchetti, 1990).

Within this financial regime, we study two different scenarios regarding investment, which affect the way we write the mechanism design problem. First, we study the case of capital depreciating fully during the production process, i.e., $\delta = 1$ (e.g. interpret k as materials). This case is easier to solve because the states are not technologically linked (as in Fernandes and Phelan, 2000 or Doepke and Townsend, 2006). Matters become more complicated when capital depreciates incompletely, i.e., $\delta < 1$ (e.g. k is "machines"), creating an additional inter-dependence between time periods. We manage to resolve these difficulties by judicious usage of extra state variables and utility bounds (see the Appendix).

[6] We use bold font to denote vector variables. The notation $\#X$ means "the number of elements of vector X".

The computational method we propose to solve for the optimal contract in this regimes require separability in consumption and leisure, $U(c,z) = u(c) - d(z)$ (this is not needed in the MH, FI or the exogenously incomplete regimes). The separability allows to split each period into two stages and use dynamic programming not only across but also within time periods. This helps us keep the curse of dimensionality in check since the resulting stage sub-problems are of lower dimensionality. The first stage includes: the announcement of k by the agent, the principal's effort recommendation, z, the agent's effort supply, and the realization of the output q. The second stage includes: the transfer τ, the announcement of the promised utility vector, $\mathbf{w}'$, the investment recommendation, k', and agent's consumption and investment. To tie the two sub-periods together, we introduce an extra variable that we call *interim utility*, a mathematical object representing the agent's expected utility from the end of sub-period 1 onwards.

Consider the full depreciation case first (see the Appendix for the incomplete depreciation case). The first sub-period problem for computing the optimal contract between the principal and an agent who has announced k and has been promised $\mathbf{w}$ is:

Program UC1:

$$V(\mathbf{w},k) = \max_{\{\pi(q,z,w_m|\mathbf{w},k)\}} \sum_{Q\times Z\times W_m} \pi(q,z,w_m|\mathbf{w},k)[q + V_m(w_m)] \tag{10}$$

The choice variables are the probabilities over allocations $(q,z,w_m) \in Q \times Z \times W_m$. The set of interim utilities, W_m is a discrete finite set with bounds consistent with those on W. The function $V_m(w_m)$ is defined in the second stage problem (see below). The maximization is subject to a number of constraints. First, the optimal contract must deliver the promised utility, $w(k)$:

$$\sum_{Q\times Z\times W_m} \pi(q,z,w_m|\mathbf{w},k)[-d(z) + w_m] = w(k) \tag{11}$$

The utility from consumption, as well as discounted future utility are incorporated in w_m. Second, as in the MH regime, the optimal contract must satisfy incentive compatibility. That is, $\forall(z,\hat{z}) \in Z \times Z$:

$$\sum_{Q\times W_m} \pi(q,z,w_m|\mathbf{w},k)[-d(z) + w_m] \geq \sum_{Q\times W_m} \pi(q,z,w_m|\mathbf{w},k)[-d(\hat{z}) + w_m]\frac{P(q|\hat{z},k)}{P(q|z,k)} \tag{12}$$

Third, the state k is private information, so the agents need incentives to reveal it truthfully. On top of that, agents can presumably consider joint deviations in their announcements, k and effort choices, z. To rule out joint deviations in k and z, truth-telling must hold regardless of whether the agent decides to follow the effort recommendation, z or considers a deviation to another effort level, $\delta(z)$ where $\delta(z)$ denotes all possible mappings from Z to Z. Such behavior is ruled out by imposing the following *truth-telling constraints,* which must hold for all $\hat{k} \neq k$ and $\delta(z) \in Z$:

$$w(\hat{k}) \geq \sum_{Q\times Z\times W_m} \pi(q,z,w_m|\mathbf{w},k)[-d(\delta(z)) + w_m]\frac{P(q|\delta(z),\hat{k})}{P(q|z,k)} \tag{13}$$

In words, an agent who actually has $\hat{k}$ but considers announcing k should find any such deviation unattractive. There are $(\#K-1)\#Z^{\#Z}$ such constraints in total. Finally, the contract must satisfy the familiar Bayes-consistency, adding-up, and non-negativity constraints for $\pi(q,z,w_m)$.

To solve Program UC1, we first need to compute the principal's interim value function $V_m(w_m)$. Thus, we compute, for each $w_m \in W_m$ the following:

Program UC2:

$$V_m(w_m) = \max_{\{\pi(\tau,k',\mathbf{w}'|w_m)\}} \sum_{T\times K\times \mathbf{W}} \pi(\tau,k',\mathbf{w}'|w_m)[-\tau + (1/R)V(k',\mathbf{w}')] \tag{14}$$

The maximization in (14) is subject to the following constraints. First, we impose the definition of interim utility:

$$w_m = \sum_{T\times K\times \mathbf{W}} \pi(\tau,k',\mathbf{w}'|w_m)[u(\tau-k') + \beta w'(k')] \tag{15}$$

Next, obedience in the investment decision must be ensured by providing incentives for the agent not to deviate from the recommendation, k' to some alternative value, $\hat{k}'$. Because of our timing, this has to hold for any transfer τ, i.e., we must have that for all $\tau \in T$, k', $\hat{k}' \in K'$, $\hat{k}' \neq k'$:

$$\sum_{\mathbf{W}} \pi(\tau,k',\mathbf{w}'|w_m)[u(\tau-k') + \beta w'(k')] \geq \sum_{\mathbf{W}} \pi(\tau,k',\mathbf{w}'|w_m)[u(\tau-\hat{k}') + \beta w'(\hat{k}')] \tag{16}$$

Finally, adding-up and non-negativity must hold for $\pi(\tau,k',\mathbf{w}'|w_m)$.

3 Computation

3.1 Techniques

We solve for the optimal financial contracts from the previous section numerically[7]. Specifically, we use the linear programming (LP) methods proposed by Prescott and Townsend (1984) and Phelan and Townsend (1991). An alternative to the LP methodology is the "first order approach" (Rogerson, 1985), used for instance by Abraham and Pavoni (2005). A potential problem with the latter method is the non-convexity introduced by the incentive and/or truth-telling constraints. In contrast, the LP approach is extremely general and can be applied for any possible preference and technology specifications, as it convexifies the problem by allowing for all possible lotteries over allocations. A potential downside is that the LP method may suffer from a "curse of dimensionality". However, as we show above, by judicious formulation of the linear programs this deficiency is minimized.

To speed-up computation, the dynamic problems for each regime are solved using policy function iteration (Judd, 1998). We start with a suitable initial guess for the value function and iterate until convergence on the Bellman operator in policy space. At each iteration step, we solve a linear program[8] in the policy variables π. In the unobserved capital (UC) regime the promised utilities set, $\mathbf{W}$ is endogenously determined (Abreu, Pierce and Stacchetti, 1990) and has to be solved for together with V. Using incentive compatibility, we restrict attention to non-decreasing promise vectors $\mathbf{w}(k)$. Specifically, we "discretize" the functional set $\mathbf{W}$ by starting with a broad dense set $\mathbf{W}_0$ consisting of linear functions $\mathbf{w}(k)$ with intercepts that take values from the grid $W = \{w_{\min}, w_2, ...w_{\max}\}$ defined above and a set of non-negative slopes. We initially iterate on the

[7] Given our primarily empirical objectives, we have chosen general functional forms that preclude analytical tractability. As is standard in computational work, we are aware of the fact that computed examples do not constitute proofs. We verify robustness by using multiple parameterizations and initial conditions. The full set of numerical computations is available on request.

[8] All coefficient matrices of the objective and the constraints were created in Matlab while all linear programs were solved using the comercial solver CPLEX. All computations were performed on a dual core, 2.2 Ghz, 2GB RAM, Windows XP machine.

UC dynamic programming problem using value function iteration (Judd, 1998), that is, iterate over the feasible promise set together with the value function by dropping all infeasible vectors $\mathbf{w}$ at each iteration. Once we have successively eliminated all vectors in $\mathbf{W}_0$ for which the linear programs have no feasible solution, i.e., we have converged to the feasible promises set $\mathbf{W}^*$, we switch to policy function iteration[9] and continue iterating on the Bellman equation until convergence in V.

3.2 Functional Forms, Grids, and Parameters

Below we describe the functional forms used in the numerical analysis. Agent preferences are of the CES type[10]:

$$u(c,z) = \frac{c^{1-\sigma}}{1-\sigma} - \xi z^{\theta}$$

The production function, P representing the probability of obtaining given output level, $q \in Q \equiv \{q_1, q_2, ..q_{\#Q}\}$, from effort z and capital, k is:

$$P(q = q_1|z,k) = 1 - (\eta k^{\rho} + (1-\eta)z^{\rho})^{1/\rho}$$
$$P(q = q_i|z,k) = (\frac{1-\lambda}{1-\lambda^{\#Q-1}})\lambda^{i-1}(\eta k^{\rho} + (1-\eta)z^{\rho})^{1/\rho} \text{ for } i = 2, ..\#Q$$

where the lowest output, q_1 is interpreted as "failure". The probability of obtaining any output level is bounded away from zero[11]. Our formulation allows for a wide range of production technologies. With $\rho = 1$ we have a perfect substitutes technology, with $\rho \to 0$ we obtain the Cobb-Douglas form, and with $\rho \to -\infty$ the technology is Leontief. The weight parameter $\lambda \in (0,1)$ ensures that the probabilities add up to 1.

The grids for each variable are defined in Table 1. For simplicity, and to allow easier interpretation of the results, we assume two output levels (low and high), q_0 and q_1 with $q_0 < q_1$. Effort takes three values[12]. In the simulations and empirical exercises below we use different parametrizations for the grid bounds in the cases of full and incomplete depreciation. The reason is to avoid corner solutions e.g., zero investment.

To get an idea of the computational difficulty of the dynamic contracting problems we compute, Table 2 reports the number of linear programs, variables and constraints that need to be solved at each iteration for the various regimes. The number of linear programs is closely related to the number of state variables while the number of variables and constraints is related to the product of the grid dimensions. The biggest computational difficulties arise from increasing $\#K$ and $\#Z$ because of the exponential increase in the number of variables or constraints to which this leads. That is why we keep these dimensions relatively low while, for example, increasing $\#T$ is relatively "cheap" computationally. In practice, the number of variables for which we solve is slightly lower than the numbers reported in the table above because we drop from the computation any allocations that result in negative consumption, i.e., we assign probability zero to their corresponding probabilities, π. Because of the huge dimensionality and computational time requirements for the

[9] We have also verified our results against proceeding with value function iteration all the way.

[10] Our LP numerical methodology does not require separable preferences but separability is commonly used in the literature and simplifies the analysis in the unobserved k case.

[11] To have well-defined likelihood ratios and satisfy the full support condition, the probabilities we actually use in the computation are constrained between .01 and .99. That is, if the formulae above imply some probability value P, we actually use $\tilde{P} = \min\{.99, \max\{.01, P\}\}$.

[12] The lowest value is set to be slightly higher than zero for technical reasons.

unobserved capital case with incomplete depreciation we only compute results for the full depreciation baseline. Finally, table 3 displays the baseline parameters used in the simulation, estimation and testing exercises[13].

3.3 Generating Cross-Sectional and Panel Data from the Model

Our numerical approach allows us to study the models from both static and dynamic angles using the policy functions, $\pi^*(.)$ solving the dynamic programming problems in section 2. Using the policy functions we first construct the Markov state transition matrices corresponding to each regime. Formally, denote by $s \in S$ the current state (k in the A regime, (k, b) in S/B, and (k, w) in the MH/FI regimes). The transition probability of going from s to some next period state, s' can be found using the π^*, integrating out all non-state variables. For example, in the MH regime we have:

$$\Pr(w', k'|w, k) = \sum_{T \times Q \times Z} \pi^*(\tau, q, z, k', w'|w, k)$$

Putting those together, we form the model's state transition matrix, $\mathbf{M}$ of dimension $\#S \times \#S$ with elements m_{ij} corresponding to the transition probability of going from state i to state j. This matrix completely characterizes the dynamics of the model. In particular, we can use $\mathbf{M}$ to compute the cross-sectional probability distribution over states at any time t, $\mathbf{D}_t(s) \equiv (d_t^1, .., d_t^{\#S})$, starting from an arbitrary initial state distribution, $\mathbf{D}_0(s)$:

$$\mathbf{D}_t(\mathbf{s}) = (\mathbf{M}')^t \mathbf{D}_0(s) \tag{17}$$

Setting $t = \infty$ gives the stationary state distribution (if one exists).

The state distribution (17) can be further used, in conjunction with the policy functions, π^*, to compute cross-sectional distributions (histograms), $\mathbf{H}_t(x)$ for any variable of interest, x (could be k, c, z, τ, q, etc.), or any combination thereof, at any time period. For example, in the MH regime, the probability distribution of next period's firm size, k' over the grid K with elements k_i', $i = 1, ..\#K$ at time t can be computed as:

$$\Pr{}_t(k' = k_i'|\mathbf{D}_0) = \sum_{j=1..\#S} d_t^j \sum_{T \times Q \times Z \times W'} \pi_t^*(\tau, q, z, k' = k_i', w'|s^j)$$

We can also use the state distribution, $\mathbf{D}_t(s)$ and the Markov matrix, $\mathbf{M}$ to compute transition probabilities, $\mathbf{P}_t(x, x')$ for any variable, x at any time period, t. Furthermore, the transition and the cross-sectional probabilities can be combined into joint probability distributions encompassing several periods at a time as in a panel. Model dynamics can be also studied by generating population-weighted time paths (empirical mean of a given variable over time), starting from any initial state distribution. For example, to compute the expected time path of next period's firm size, we sum over all possible grid points, k_j' in K weighted by the corresponding fraction of agents at grid point k_j' at time t, $\mathbf{H}_t(k_j')$:

$$E_t(k') = \sum_{j=1..\#K} k_j' \mathbf{H}_t(k_j') \tag{18}$$

[13]We also performed many robustness checks for the preference and technology parameter values (shown in parentheses).

3.4 Numerical Results and Examples

In this section we compute a baseline set of numerical results followed by a series of robustness checks for alternative parameters and initial conditions. Consistent with our objectives, we concentrate on investment, firm assets, and consumption smoothing. We also explore the model predictions with regards to the stylized facts from the firm dynamics literature - firm growth, cash flow sensitivity and growth variance.

3.4.1 Firm Size, Investment, and Cashflow

Cooley and Quadrini (2001) report the following empirical regularities about firm size, growth and investment[14]:

Fact 1 – Firm growth decreases with firm size

Fact 2 – The variability of firm growth decreases with firm size

Fact 3 – Small firms invest more.

Fact 4 – Small firms take on more debt.

Fact 5 – The investment of small firms is more sensitive to cash flows even after controlling for their future profitability.

To compare the predictions of our models with the above regularities[15], we first map the model variables into their empirical counterparts. We interpret k as firm size (assets). Firm growth is then given by the ratio k'/k, while investment is $i = k' - (1 - \delta)k$. Firm growth variability is measured by the variance, $Var(k'/k)$. Output, q is firm's cash flow. We measure the sensitivity of investment to cash flow by the difference between expected investment when cash flow is high and low, $E(i(q_2)) - E(i(q_1))$. Note that this is equivalent to looking at the difference $E(k'(q_2)) - E(k'(q_1))$. As k and z are fixed before output is realized, the randomness in q_2 vs. q_1 has nothing to do with productivity. Indeed, this difference is zero in the full information case. Also, note that firm growth k'/k as a function of firm size is the same (up to a constant) as relative investment i/k as a function of firm size since $i/k = k'/k - (1 - \delta)$, so Facts 1 and 3 can be analyzed jointly in our model.

Figure 1 displays, for each model regime, the expected values[16] of firm growth, sensitivity to cash flows, growth variance (as defined above), and, for the borrowing regime only, relative indebtedness, $E(b'/k)$, each plotted as function of firm size, k. The top panels show that virtually all regimes, from autarky to full information, match qualitatively facts 1 and 3. However, there are noticeable *quantitative* differences between the regimes that could be used in testing them against one other. Specifically, the autarky lines are farthest from the FI, followed by S, B, etc. The MH regime produces lines that are very close to those for the FI and UC regimes. While all regimes predict that $E(k'/k)$ is downward sloping, they differ in the slopes, with the autarky line flattest and the full information line steepest. Intuitively, households are trying to get to the firm size determined by productivity and the less constrained the regime, the higher the ability to adjust assets. These different slopes are a testable implication of our model.

The second row of panels on fig. 1 shows that all regimes exhibit decreasing variance of firm size growth as function of k, which matches qualitatively Fact 2. However, once again, there are

[14]We omit facts that our model does not currently allow to match or verify.

[15]Currently we explore only the facts regarding firm size and not firm age. In principle, we can account for firm age by interpreting $k = 0$ as "being out of business" as in PTK (2006). This would enable us to study firm entry, aging and exit. We save this for future applications.

[16]The expectation is taken over the initial normal distribution for w or b, in addition to over any lotteries in the optimal contract. Using a uniform distribution or putting all mass at a single state (done as robustness checks) produces qualitatively the same results.

significant quantitative differences e.g., the exogenously incomplete regimes display markedly higher growth variability at lower firm size. Intuitively, being more constrained, these regimes do worse smoothing out cash flow fluctuations and are more sensitive to variation in b compared to the mechanism design regimes (FI, UC and MH).

The empirical fact that produces qualitatively different results across the model regimes is Fact 5 (cash flow sensitivity of investment diminishing in firm size), exhibited on the third row of panels in fig. 1 We find this regularity to hold to some extent for the exogenously incomplete regimes with incomplete depreciation (see also fig. 4) but it does not obtain in the MH or FI environments and also when capital depreciates fully. In our setting, this finding is consistent with the hypothesis that, on average, firms in the Cooley and Quadrini data are more likely to behave as if facing constraints to smoothing investment as modelled by the saving and/or borrowing regimes. Comparing across the regimes, the level of cash flow sensitivity decreases as the exogenous or information constraints are relaxed (going from A to FI). This is another testable implication. Finally, the bottom row of panels of fig. 1 shows that Fact 4 is matched as well by the borrowing regime.

Overall, we find that the model matches successfully the firm growth and finance empirical regularities from the Cooley and Quadrini list. However, this success should be taken with some caution. The finding that most regularities are matched qualitatively by financial market environments filling the whole spectrum from no access to financial markets (A) to complete markets (FI) suggests the qualitative stylized facts in the list seem not to provide much insight for selecting across theoretical models of firm finance. However, we found significant quantitative differences across the regimes that could be used as a basis to statistically test and distinguish between the competing models. We explore this in the empirical part of the paper.

3.4.2 Consumption Smoothing

Now treat the agents in the model as consumers. Figures 2a-2b depict the expected values of c (integrated over b or w) across the regimes as function of assets, k. As expected, we see large variation in consumption across the high and low output states in the autarky regime, as the only channel to smooth consumption is investment. In the S regime the consumption differential is reduced in half, with the agent now able to use savings as a second smoothing instrument. Moving to the B regime, the consumption differential is reduced even further as the agent can also borrow. The UC regime provides even more smoothing, especially at low assets. Consumption smoothing is almost perfect in the MH regime, as in our specification most of incentive provision occurs through promised utility rather than consumption as function of income, q.

Overall, the consumption smoothing dimension of our model suggests that the different regimes provide households with different ability to smooth income shocks across states of the world and time. This offers another basis to empirically distinguish among the regimes in cross-sectional or panel data on consumption, c and income, q. We perform this in section 5.

3.4.3 Dynamics

The model regimes have different predictions about dynamics of consumption, firm size and investment. One way to compare is by looking at population-weighted expected time paths of c and k' using (18), starting from a uniform initial state distribution (the results are not sensitive to this). Figure 3 shows that in the most constrained regimes (A and S), consumption and investment converge relatively fast (around 10 and 50 periods, respectively) to non-degenerate stationary distributions. That is, while there may be a lot of mobility inside the distribution, the overall cross-sectional consumption distribution and hence its variance may be observed not to vary over time in the long-run. In contrast, the less constrained regimes (B and MH) do not converge in means within 100 periods. These results are important with regards to the literature on testing full vs. partial insurance (e.g., Blundell, Pistaferri and Preston, 2008). Observing time-varying variance in the consumption distribution is surely a sufficient condition to reject the full insurance hypothesis but, as our results show, the opposite is not true: observing zero time variation in the cross-sectional consumption variance is consistent with partial insurance.

The simulations suggest that the ability to distinguish the regimes may depend on the length of the time period from which the data are drawn. In section 5 we test this hypothesis by estimating the regimes on the basis of repeated cross-sections or panel data lying either one period apart (short run dynamics) or fifty periods apart (longer run dynamics).

4 Empirical Implementation

4.1 Maximum Likelihood Estimation

In Paulson, Townsend and Karaivanov (2006) we estimated via maximum likelihood a one-period model of occupational choice with financial constraints. We used only binary data on occupational choice and data on ex-ante wealth. The dynamic models presented here significantly expand this approach to fit a wide range of cross-sectional frequencies, panel data, or repeated cross-sections of consumption, investment, firm size and income/cash flow[17]. Both model-simulated and actual data are used in the estimation exercises (see next section for details).

Specifically, let the data in frequency form (e.g., the joint empirical distribution of c, q) be $\hat{m}_{ij}$ where $\sum_{j=1}^{J} \hat{m}_{ij} = 1$ for all $i = 1..I$. The subscript $j = 1..J$ refers to data frequencies in mutually exclusive cells (e.g., if there are $\#C$ grid points for c and $\#Q$ grid points for q, we have $J = \#C \times \#Q$) while the subscript i refers to different *sets* of data frequencies (e.g., $i = 1, 2$ could denote the frequency distributions of two cross-sections of c, q).

The structural model is parametrized by a vector, ϕ that we are interested in estimating. The elements of ϕ can include any of the preference or technology parameters, as well as distributional parameters (e.g., mean, variance) of unobserved state variables (promises, w or debt, b) or measurement error. Given ϕ, let the counterpart of the data $\hat{m}_{ij}$ in the model be the model-predicted frequencies, $m_{ij}(\phi)$, where once again, by construction, $\sum_{j=1}^{J} m_{ij}(\phi) = 1$ for all i.

The maximum likelihood (ML) estimator, ϕ^{MLE} is defined by:

$$\hat{\phi}^{MLE} = \arg\max_{\phi}\ n \sum_{i=1}^{I} \left[\sum_{j=1}^{J-1} \hat{m}_{ij} \ln m_{ij}(\phi) + (1 - \sum_{j=1}^{J-1} \hat{m}_{ij}) \ln(1 - \sum_{j=1}^{J-1} m_{ij}(\phi)) \right] \tag{19}$$

where n is the overall sample size. The maximization above can be done by any optimization algorithm robust to local maxima, e.g., pattern search, genetic algorithms, or simulated annealing (Goffe, Ferrier and Rogers, 1994).

The requirement we impose, that the data be in the form of frequencies over mutually exclusive cells[18], has two significant advantages. First, it allows us to write the log-likelihood function in

[17] In general, the MLE approach proposed here can be used on any data in the form of probabilities/ fractions – that is any single-period or repeated cross-sectional distibutions or panels of model variables or transitional probabilities. Any dataset can be put in this form by sorting the observations in "bins" and then using the bin frequencies to perform the estimation (see also Jappelli and Pistaferri, 2006).

[18] Note that given our MLE methodology, the need for grids in our LP algorithm turns out to be perfectly suited for our estimation approach.

explicit form. Second, it allows us to use a formal statistical test, Vuong's (1989) test, to compare across the competing model regimes. In theory, one could also employ a GMM or minimum-χ^2 techniques to estimate on the basis of any data moments instead of using frequency distributions over cells. Unfortunately, to our knowledge, no tractable way of forming and implementing a statistical test to compare across the models exists for this more general methodology (see Rivers and Vuong, 2002 for a theoretical discussion).

4.2 Testing and Model Selection

We use the results of Vuong (1989) to compute an asymptotic test statistic to distinguish between competing non-nested models. Vuong's test is based on the maximum likelihood method. An attractive feature of the test is that it does not require that either of the compared models be correctly specified. A further advantage, extremely useful for numerical work is that the computational method used to estimate the competing models need not optimize the model fit criterion as long as it is held constant across the regimes (see Rivers and Vuong, 2002).

Suppose the values of the estimation criterion function being minimized (i.e., minus the log-likelihood) for two non-nested[19] competing models are given by $L_n^1(\hat{\phi}^1)$ and $L_n^2(\hat{\phi}^2)$ where n is the common sample size and $\hat{\phi}^1$ and $\hat{\phi}^2$ are the parameter estimates for the two non-nested models. The null hypothesis, H_0 of the Vuong test is that the two models are "asymptotically equivalent" relative to the true data generating process. Define the "difference in lack-of fit" statistic:

$$T_n = n^{-1/2} \frac{L_n^1(\hat{\phi}^1) - L_n^2(\hat{\phi}^2)}{\hat{\sigma}_n}$$

where $\hat{\sigma}_n$ is a consistent estimate of the asymptotic variance[20], σ_n of $L_n^1(\hat{\phi}^1) - L_n^2(\hat{\phi}^2)$ (the likelihood ratio). The main result is:

Proposition (Vuong, 1989):

Under some regularity assumptions, if the compared models are non-nested, then the Vuong test-statistic, T_n is distributed $N(0,1)$ under the null.

4.3 Methodological Discussion

Initial Conditions: Steady States vs. Transitions

When estimating dynamic models, an important issue is initialization. In our setup the initial conditions are the $t = 0$ values of the state variables (k for A, (k,b) for S/B, and (k,w) for the mechanism design regimes), some of which are unobserved by the econometrician. For example, the initial promise, w being a purely mathematical object keeping track of output history, is clearly not observable. Thus, if we are interested in transitional dynamics (more on this below), w needs to be estimated to initialize the MH/FI models.

Indeed, all model regimes we study have implications for both transitional dynamics and long-run distributions. This has to be reflected in the estimation strategy. If the researcher believes that the data represents a steady state then one can estimate the model to match the simulated

[19] It is crucial to point out that the different models we study are statistically non-nested. Formally, for our purposes we say that model A nests model B, if, for any possible allocation that can arise in model B, there exist parameter values such that this is the allocation in model A (see Paulson et al., 2006 for more details).

[20] A consistent estimate of σ_n is given by the sample analogue of the variance of the LR statistic (see Vuong, 1989, p. 314).

stationary distribution with the actual empirical distribution disregarding transitional dynamics and initial conditions. If, however, one is dealing with data that is more likely to correspond to a transition (as might be natural to assume if the data are from a developing economy) then the model needs to be estimated using both cross-sectional and intertemporal data. We focus on the latter case which we find empirically more plausible given the applications we have in mind.

Identification

Due to the analytical complexity of our models, it is not possible to provide theoretical identification proofs. In fact, we are aware (Honore and Tamer, 2006) that point identification can sometimes fail in complex structural models like ours. To address this issue we use the following verification procedure, which is a form of numerical identification: Step 1 – take a baseline model regime parametrized by a vector of parameters, ϕ^{base}; Step 2 – generate simulated data from the baseline regime; Step 3 – estimate the baseline model using the data in Step 2 using maximum likelihood and obtain estimates, $\hat{\phi}^{base}$; and Step 4 – if the estimates from Step 3 are numerically close to the baseline ϕ^{base}, report success, otherwise report failure.

Sample Size and Number of Frequency Cells

As explained above, our ML estimation strategy is based on matching model-generated with actual data frequencies in mutually exclusive "frequency cells". Holding sample size, n constant, increasing the number of frequency cells can have two opposite effects on the precision of the estimates and on our ability to distinguish between the models. The first effect is positive, if the increase in the number of frequency cells being fitted is due to using richer data (e.g., data on c, q, k, k' instead of c, q only, or using panel data instead of a single cross-section). This, by itself, improves our ability to statistically distinguish across the regimes since more dimensions in which the models differ are utilized in the estimation. There is, however, a second effect, which goes in the opposite direction. Namely, holding n constant, increasing the number of cells means that, on average, the empirical probabilities in each cell are less precisely estimated. Thus, the relative magnitudes of the number of cells, n and the sample size, IJ are important for our ability to distinguish across regimes. We find evidence of this trade-off in the estimation results below.

5 Estimation and Model Selection Using Simulated Data

In this section we estimate and test across the model regimes using the following dimensions suggested by the theory and simulations in sections 2 and 3: (i) firm size, investment and cash flow; (ii) consumption smoothing, and (iii) dynamics.

To keep the environment under complete control when assessing our methodology, we first test across the regimes using simulated data from one of the regimes. In the next section, we show how the same methodology can be applied to real data. We adopt as a baseline the moral hazard model with observed investment (MH) and generate data from it to be used in the estimation. Regarding firm size, investment and cash flow, we generate data from the joint distribution (k, k', q) of firms' current and future assets[21], k and k' and cash flow, q. This distinguishes across the regimes based on the stylized facts from section 3.4 which involve firm size and growth (k, k') and cash flow. To evaluate consumption smoothing as a basis to differentiate between the regimes, we use the cross-sectional distribution of (c, q). We also explore whether using data on firm assets, investment and consumption *jointly* (the joint distribution of c, q, k, k') improves our ability to distinguish

[21] In the UC regime k and k' are unobserved to third parties in the model but assumed observed to the econometrician.

the regimes. Finally, section 3 suggests that the various regimes have different implications about consumption and investment dynamics. We test this dimension of the model using data from repeated cross-sections (e.g., c, q and c', q') or panel data (e.g., the joint distribution c, q, c', q').

5.1 Generating Data From the Model

We use the two baseline parametrizations in table 3 (for incomplete or full depreciation) to generate baseline data from the model. Because of the heavy computational requirements of the MLE procedure (the need to compute and iterate on the linear programs at each parameter vector during the grid search and estimation) we use the following grid sizes (smaller than those in table 1): $\#K = 5$, $\#T = 19$, $\#W = 5$, $\#B = 5$ (for S) and $\#B = 9$ (for B)[22]. Consumption, c generated from the models is gridded up[23] using $\#C = 10$ values on $[0, 1.2]$ or $[0, 3.5]$, respectively for the $\delta = .05$ and $\delta = 1$ cases.

To initialize the baseline model (MH) from which we draw the data used to estimate all regimes, we assume an initial distribution over states (k, w) with an equal number of data points for each gridpoint in K and normally distributed[24] in w, i.e., $w \tilde{} N(\mu_w, \gamma_w^2)$ for each $k \in K$. We pick μ_w equal to the average promise [25]. We then draw n random numbers from $N(\mu_w, \gamma_w^2)$ (that is, $n/\#K$ draws for each k) and generate the sample distribution over the initial state space. We then compute the baseline model at the baseline parameters (see table 3) and use the state transition matrix, $\mathbf{M}$ to generate n values for each c, q and k'.

As explained above, we allow for additive measurement error[26] in consumption c, and/or firm size k and k'. The measurement errors ε_i, $i = 1, ..n$ applied to each variable are drawn from the normal distribution $N(0, \tilde{\gamma}_{me}^2)$. We perform all estimation and testing exercises for two specifications of measurement error (m.e.): *low m.e.* i.e., $\tilde{\gamma}_{me}$ equal to 10% of the grid span of the respective variable and *high m.e.* i.e., $\tilde{\gamma}_{me}$ equal to 50% of the grid span. That is, the standard deviation of the measurement error is taken as proportional to the variable's grid span: $\tilde{\gamma}_{me} = \gamma_{me}(\textit{gridspan})$ where γ_{me} (=.1 or .5) is the proportionality parameter[27]. For example, if the true value of consumption (the value generated from the baseline model at the initial state) is c_i, the "observed" data value used in the estimation is $\tilde{c}_i = c_i + \varepsilon_i$[28]. Finally, we use $\tilde{c}$, $\tilde{k}$,etc. to create the actual data used in the estimation – e.g., $(\tilde{c}, q)$ cross-sectional data, $(\tilde{k}, \tilde{k}', q)$ repeated cross-sections, etc.

5.2 Baseline Results Using Model-Generated Data

The parameters we estimate are the three distributional parameters for promises and measurement error (μ_w, γ_w and γ_{me}) as well as three of the model's structural parameters: the preference parameters, σ and θ and the technology parameter, ρ. The rest of the parameters are set at their

[22]Our methods can handle much larger problems at the cost of additional computational time.

[23]In all models in section 2 consumption is a resultant variable, dependent on the grids for k, b, etc. It is fitted on an explicit grid here to use in the MLE procedure.

[24]Our methods allow using mixtures of normals to approximate any initial state distribution at the cost of more parameters to be estimated.

[25]This implies setting the baseline distributional parameters to $(\mu_w, \gamma_w) = (19.99, 8)$ for $\delta = .05$ and $(34.64, 15)$ for $\delta = 1$.

[26]Cash flow, q is assumed to be observed without error for computational reasons, as it takes only two values and moderate measurement error will not affect its distribution over the grid. In principle, our method allows for measurement error in q as well.

[27]By using a *relative* rather than absolute level of the measurement error, we able to keep the standard deviation commeasurable across variables with different grid spans.

[28]Unavoidably, allowing for measurement error could sometimes lead to "truncation" and information loss if the resulting value falls outside the grid (we then assign the grid end point).

benchmark values in table 3. In the S and B regimes, instead of the promise parameters, μ_w and γ_w we estimate the mean, μ_b and standard deviation, γ_b of the debt variable b (assumed unobserved and drawn from a normal distribution to be consistent with the MH/FI cases).

For each regime we follow the same procedure described above: draw initial states, generate simulated data, apply measurement error, and populate the necessary frequency cells corresponding to the dimension of the model being used. We then form the likelihood function, (19) to compute the criterion value and use an optimization routine[29] to solve for the estimates $\hat{\phi}$ maximizing the likelihood between the baseline data and the model regime being estimated. We also estimate the data-generating regime itself, to verify how well we recover the parameters used to generate the baseline data (see the "numerical identification" discussion above). Next, we use the pairwise Vuong test to find out whether we can distinguish statistically between the data-generating (MH) and the rest of the regimes, as well as between any possible regime pairs including counterfactual ones (e.g., B vs. S).

5.2.1 Firm Size, Investment, and Cash Flow

We first estimate and test the regimes based on their implications about firm size, investment and cash flow. That is, we generate data from the baseline MH regime on the joint distribution of (k, k', q). Tables 4 and 5 display, respectively, the estimation and model selection results[30]. Table 4 shows that the baseline parameters used to generate the data are recovered well in the estimation process (when estimating the baseline, MH) in the low measurement error specification although, not surprisingly, not so well in the high m.e. case. In terms of log-likelihood values the regimes follow a robust order: the data-generating MH regime naturally has the highest likelihood, followed by the other mechanism design regimes – FI (and UC), then the exogenously incomplete B, S regimes, and finally A, although with high measurement error the likelihood values are sometimes very close.

The parameter estimates differ across the estimated regimes, as the MLE procedure is trying to fit the data as well as possible, but the estimates are generally similar between the FI and MH regimes. The exogenously incomplete regimes (B, S, A) seem to require a higher value for the measurement error variance (in the range $.44 - .90$ for low m.e. and $.62 - .85$ for high m.e.) compared to the baseline (.1 and .5 respectively) to fit the data. Assuming larger measurement error decreases the likelihood values for all regimes when using (k, k', q) data although this does not always happen below, when using different data. The standard errors of the parameter estimates also tend to rise for the least likely regimes, at least in the incomplete depreciation case.

Turning to the results of the pairwise Vuong tests between the competing regimes (table 5), we find that in the low m.e. specification we are able to distinguish between the baseline MH regime and each of the other regimes almost perfectly (at less than 1% confidence level). Moreover, this result also holds (with two exceptions, at the 5% or 10% level), in the pairwise comparisons between the counterfactual (non-MH) regimes. That is, even if the researcher (incorrectly) believes that e.g., the data were generated from the FI regime, he can still distinguish it from the others.

In contrast, with high measurement error, the distinction between the regimes is blurred and we cannot differentiate statistically between the MH and FI regimes in the incomplete depreciation

[29] We first perform a detailed grid search over the parameter space to rule out local extrema and then use the Matlab routines `patternsearch` and `fminsearch` to maximize the likelihood.

[30] Due to computational reasons the UC regime is only estimated in the full depreciation ($\delta = 1$) specification. We are working on paralellizing our numerical algorithm which should eventually allow us to overcome the computational constraints.

case or between any of the MH, FI, B, UC and S models in the full depreciation specification. Assuming full depreciation breaks the link between firm size today and tomorrow, k and k', so less information is available to use in the Vuong test relative to the alternative specification $\delta = .05$. This suggests that more data dimensions are needed to be able to distinguish between the competing regimes in this case.

The UC regime achieves very similar likelihoods to the MH and FI regimes indicating that it might be difficult to distinguish between the endogenous information-constrained regimes. Table 5 shows that, with low m.e., we distinguish between UC and FI only at a 5% confidence level, while with high m.e. we cannot discern between MH, FI and UC. These findings suggest that, in terms of its firm size, growth and cash flow implications (i.e., regarding the Cooley and Quadrini, 1999 facts), the moral hazard regime produces similar data to the full information or several other regimes when there is high measurement error. In contrast, in all cases, including high m.e., all non-autarky regimes are statistically distinguishable at the 1% level from the autarky regime.

We visualize these results on Figure 4 which plots firm size growth, growth variance and cash flow sensitivity at the parameter estimates for the incomplete depreciation case for both the low and high m.e. parametrizations. We see that the MH and FI regimes are indeed very close in terms of their firm dynamics implications. The figure also shows clearly how higher measurement error blurs the distinction between the regimes, relative to Figure 1.

5.2.2 Consumption Smoothing

We estimate and test whether we can distinguish between the regimes based on the degree of consumption smoothing, as embedded in the consumption-income, (c, q) joint distribution. The results[31] are in table 6. As when using (k, k', q) data, the likelihood values we obtain are ordered MH, FI/UC, B, S and A from the highest to the lowest likelihood[32]. This indicates that the ranking of the regimes relative to the data-generating MH specification is robust and is not affected by the type of data used (with the exception of the UC/FI pair). In terms of ability to distinguish the regimes using (c, q) data, with low measurement error, both the baseline and the counterfactual regimes are once again distinguished with high accuracy from the alternatives. In contrast, in the high measurement error specification we are able to distinguish between the baseline (MH) and alternative regimes (B, FI or S with $\delta = .05$ and FI, B, UC with $\delta = 1$) only at lower confidence levels. As in the results using (k, k', q) data, some other regime pairs are also indistinguishable when γ_{me} is high. Unlike using (k, k', q) data, using (c, q) data makes it harder to distinguish the regimes in the incomplete depreciation case at high measurement error but the opposite is true in the complete depreciation case. Finally, as before, autarky is statistically distinguished from all alternative regimes (with a single exception), even if γ_{me} is high.

5.2.3 Using Joint Data on Consumption, Firm Size, Growth and Income / Cash Flow

A natural question is whether using *joint data* on consumption and investment can improve our ability to distinguish the competing regimes even with high measurement error. Theoretically, it is known that, with incomplete markets as all of our regimes but FI stipulate, the classical "separation" result between consumption and production/investment decisions fails. Thus, other things held constant, more information should be present in the joint data on c, k, k' and q than

[31]For lack of space we omit the parameter estimates (available upon request). The same disclaimer applies to the rest of the runs in this section. We find the same general patterns as in the (k, k', q) case.

[32]The UC and FI regimes flip positions depending on measurement error.

in consumption and investment data separately. However, note that using joint data implies a substantially larger number of frequency cells (500 for c, q, k, k' data compared to 20 for c, q data and 50 for k, k', q data). Thus, holding the sample size n constant, a less precise estimate of each cell would be possible, especially if n is small, which might work against the theoretical advantage of using joint consumption / investment data.

Table 7 reports the estimation and test results using the joint distribution (c, q, k, k'). The ability to distinguish across regimes remains very good under low measurement error. More importantly, we do observe a significant improvement in the ability to distinguish between the data-generating regime against each alternative (this occurs in all possible cases compared to tables 5 and 6 when using consumption or investment data separately. The ability to distinguish between pairs of counterfactual regimes in the high measurement error specification improves as well, although a few cases remain in which if the researcher guesses (incorrectly) the data-generating regime he would be unable to distinguish with some alternative, nearby regime.

More specifically, using (k, k', q) data only (see table 5) there are 10 "tie" cases (i.e., being unable to distinguish) among the compared regimes at $\delta = 1$, while using (c, q) data (see table 6) there are 4 ties (plus 2 weak rejections). In contrast, using joint data on (c, q, k, k'), all regime comparisons are significant at the 5% confidence level and all but two at the 1% level. The results are similar for the incomplete depreciation high measurement error case. In summary, even quite substantial measurement error (50% of the entire range of values consumption and investment can take) does not impede our ability to distinguish across the regimes once joint consumption, investment and cash flow data are used.

5.2.4 Using Intertemporal Data: Panel and Repeated Cross-sections

We also estimate and test the financial regimes based on their implications about short and longer run dynamics of consumption and income. Specifically, we use model-generated data on the joint distribution of consumption and income (c, q) in two different periods, $t = 0$, 1 (or $t = 0$, 50) as in a panel dataset. Table 8 contains the results from the pairwise Vuong tests. We report only the high measurement error case to investigate whether using intertemporal data improves our ability to distinguish across the regimes compared to using single cross-sections as above.

Compared to table 6, the results in Table 8 show that adding a time dimension to the data used in the estimation improves significantly our ability to distinguish the regimes, especially in the case of incomplete depreciation (the number of ties diminishes from 5 to 2 and, in general, the Vuong test statistics are larger in all cases). The improvement in ability to discern the regimes is however smaller compared to when joint data on consumption, firm growth and cash flow was used (Table 7 vs. Table 8). A second observation concerns the period length, one vs. fifty periods apart. We find that using data further apart improves somewhat our ability to distinguish across the regimes in the 100% depreciation specification (the number of ties falls from 3 to 2), but the opposite is true with incomplete depreciation.

An alternative way to use dynamic data is through repeated cross-sections instead of panel. The advantage is that using repeated cross-sections requires a lower number of probability cells (40 instead of 400 for c, q data) which increases precision if n is small. Table 9 reports model comparison results using two cross-sections of the joint (c, q) distribution at two time periods (as above, $t = 0$ and 1 or $t = 0$ and 50). Incorporating intertemporal data once again improves our ability to distinguish the regimes compared to when using single a cross-section as in table 6. But, the length of the period at which we take the cross-sections (1 or 50) does not seem to affect the results in any definite way as with panel data.

5.3 Robustness Runs Using Model-Generated Data

5.3.1 Using Data from the Savings Only Regime

We also performed several additional runs to study the robustness of our results. First, instead of generating the baseline data from the MH model, we generated it from the saving only regime (S). The data generation procedure is exactly as before, and measurement error is allowed once again. Table 10 presents the Vuong test results using data on firm size, investment and cash flow (k, k', q). We compare these results to table 5 where MH is the data-generating regime.

We see that when the data are generated by the S regime, the likelihood ranking changes accordingly, with the S regime producing the highest likelihood, followed by B, UC, FI or MH and A. Interestingly, the UC regime comes closer to matching the firm size and investment data produced by the saving only compared to the MH or FI regimes. This is consistent with the literature on hidden savings (e.g., Allen, 1985). The autarky regime is again furthest away from the data-generating (S) regime although now, in a few cases, it cannot be distinguished statistically from the MH and FI specifications. As in table 5 we find that larger measurement error reduces our ability to distinguish the regimes, especially in the full depreciation case, in which we cannot differentiate between S and B.

5.3.2 Additional Robustness Checks

Table 11 contains the results from additional robustness runs performed using (k, k', q) data generated from the baseline MH regime for incomplete depreciation. In the top panel we analyze the effect of allowing even larger measurement error ($\gamma_{me} = .7$). As expected, the Vuong test statistics worsen relative to those in table 5 and now, in addition to the MH/FI pair, we also cannot distinguish between the S and B regimes. The autarky regime remains statistically distinguishable from all the rest even with this high level of error.

In the middle two panels we study the effect of sample size, n on our ability to distinguish the regimes. We find that reducing the sample size from 1,000 to 200 observations significantly reduces the power of the Vuong test and so we cannot distinguish between any of the MH, FI and B regimes (also, we can distinguish them only at 5% significance level from the S regime). In contrast, increasing n to 5,000 significantly improves our ability to discern among the regimes relative to the table 5 baseline. Larger sample size is thus quite helpful.

Finally, the last panel of table 11 investigates the sensitivity of the results to grid size. Reducing the size of the consumption grid to $\#C = 5$ (from 10) does not affect the Vuong statistics significantly relative to the table 5 baseline reassuring for the robustness of our findings and for our computational methods which sometimes require relatively coarse grids.

5.4 Summary of Findings Using Model-Generated Data

We summarize the main findings from the model comparisons with data generated from the MH model (tables 4-9) in table 12. First, we find that some regime pairs are always distinguished whatever type of data are used: MH/A; FI/A; B/A; UC/A. Thus, autarky is almost always distinguished, even with high measurement error. The regime pairs MH/S and FI/S are also distinguishable in most cases (less than 10% exceptions). On the other end, the pairs that are most rarely distinguishable are B/UC; B/S; FI/UC and FI/B. Intuitively the borrowing and saving regime are harder to distinguish as they share very similar theoretical structure. We also see that, especially with high measurement error, the non-data-generating mechanism design regimes (FI, UC) might not be statistically distinguishable from the exogenously incomplete (B, S) regimes.

Comparing the incidence of Vuong test ties (see panel 2), *within* the groups of mechanism design regimes (MH, UC, FI) and exogenously incomplete regimes (B, S, A) vs. *across* those groups, we see that more ties occur within groups (25% and 15.3% of all pairs respectively) against 11.5% only for the across group comparisons (8.3% if exclude the UC regime). A similar picture emerges comparing the number of indistinguishable pairs involving the data-generating MH regime – 23.5% ties when compared to FI/UC vs. only 5.6% ties when MH is compared to B, S and A. The fact that there are more ties within the exogenous or mechanism design regime groups vs. across them exists for both specifications but is more pronounced with incomplete depreciation. The reason is that the UC regime is often tied with B, S in the full depreciation case.

6 An Application to Thai Data

This section demonstrates how our methodology can be applied to actual data from a developing country.

6.1 Data

The data we use come from the Townsend Thai monthly survey (Townsend, Paulson and Lee). The survey began in August 1997 with a comprehensive baseline questionnaire on an extensive set of topics, followed by interviews roughly every month. Initially consumption data were gathered weekly, then bi-weekly. The data we use here begins in January 1998, so that technique and questionnaire adjustments were essentially done. We use a panel of 531 households observed for seven consecutive years, 1998 to 2004. These data are gathered from 16 villages in four provinces, two in the relatively wealthy industrializing Central region near Bangkok, and two in the relatively poor semi-arid Northeast.

Consumption expenditures, c include owner-produced consumption (rice, fish, etc.). Income, q is measured on an accrual basis (see Samphantharak and Townsend, 2008). Wage labor income is excluded to try to capture income as return to assets, as in the model. Assets, k include business and farm equipment, but exclude livestock and household assets such as durable goods (the distinction is sometimes not so obvious). Assets other than land are depreciated, so investment includes gross change. All variables were added up to produce annual numbers. All variables are in nominal terms (but inflation was low over this period). The variables are not converted to per-capita terms, i.e., household size is not brought into consideration. Summary statistics for the Thai data are displayed in Table 13.

6.2 Data Preparation

The first step in our empirical strategy is to put the Thai data in the frequency form needed to perform the MLE estimation. To start, we convert the data into model units rather than currency. We do this by dividing all currency values in the data by the 90-th percentile of the assets distribution (1,742,557 baht). The normalized assets are then placed on the 5-point grid[33], $[0, .03, .2, .6, 1]$. The unequal spacing of the grid reflects the skewedness of the assets distribution in the data, with many small and few large firms. The data fractions (in the whole sample) at each of

[33] We use a standard histogram function based on distance to closest gridpoint (Matlab's command `hist`) to fit the data on the grids.

those grid points are respectively [28.3%, 24.3%, 17.4%, 13.8%, 16.1%]. Consumption is normalized by the same factor and fit on a 10-point linear grid on $[0, .1]$ while normalized income values are fitted, consistent with the model, on a two-point grid $[.005, .13]$. The grid bounds reflect the extreme values of c and q in the data relative to those for k. Having fitted the data on those grids for all years we can construct any joint distributions we need, e.g., (c, q), (k, k', q), etc. Admittedly, some information could be lost approximating the continuous data in this fashion. We perform various robustness checks to verify this does not affect our model comparison results.

We use the same procedure as in section 5.1 to draw the initial distribution of (unobservable) states assuming normal distributions for w, b with means and variances to be estimated. Additive measurement error with standard deviation proportional to grid span is allowed for c, k and k' as before.

6.3 Estimation Results

Given the normalized Thai data, we simulate each model regime and form the likelihood function between the model-generated frequencies and their counterpart in the data. As with model-generated data, for computational reasons, we estimate a subset of the structural parameters,[34] (σ, θ, ρ), together with the distributional parameters, μ_w (or μ_b), γ_w (or γ_b), and the standard error parameter, γ_{me}.

The results are summarized in tables 14-16. Table 14 contains the parameter estimates, bootstrap standard errors, and likelihoods when using cross-sectional investment and cash flow data from 1998-99. The actual Vuong test statistics are in table 15. In table 16 we report a summary of the regime ranking in terms of likelihood, the proportion of statistically insignificant regime comparisons ("ties"), and the fraction of those ties within the groups of mechanism design vs. exogenously incomplete markets regimes. As the top lines of Tables 14.1, 14.2 and 14.3 list the best fitting regimes, one can see that the parameter estimates are sensitive to the data used inclusive of bootstrap standard errors.

6.3.1 Business Assets, Investment and Cash Flow Data

We first estimate and test the implications of the models about firm size, investment and cash flow. That is, the data used are the joint distribution of (k, k', q). For robustness we estimate using either the first two ('98-99) or the last two ('03-04) years of the Thai data ($n = 531$) for each parametrization.

The main finding is that, when estimated from k, k', q cross-sectional data (see Table 16), the financial regimes rank (in decreasing order of likelihood) as: B, S, MH, FI, A, with the Vuong test unable to reject the hypothesis that the B and S regimes are equally close to the Thai '98-99 data. Furthermore, exactly as in our simulations with model-generated data, in all pairwise comparisons the autarky regime always does significantly worse than the others. Also, as before, whenever Vuong test ties are observed in table 15, they are within the groups of mechanism design regimes (MH, FI) or exogenously constrained regimes (B, S) and not across these groups. As in the model-generated data, the parameter estimates (see table 14.1) differ across the different regimes as the model needs to adjust to achieve best fit. The lowest likelihood full information and autarky regimes exhibit the highest estimated level of measurement error.

[34] Given with the data-determined grids, to generate interior solutions, we set $\eta = .8$ and $\xi = .1$. As a robustness check we estimated (η, ξ, ρ) instead of (σ, θ, ρ) with very similar results (available upon request). All runs with Thai data are computed for the incomplete depreciation case.

As a graphical illustration of the results, fig. 5 plots the (k, k', q) joint distribution histograms[35] in the Thai data vs. the histograms produced from the model at the MLE parameter estimates for 1998-99 data. Evidently, one dimension of the data that model regimes struggle to match is the high persistence in assets between periods (note the high frequencies at the $k = k'$ bars). This probably has to do with the low frequency of investment in the data.

6.3.2 Business Assets, Investment, Cash Flow and Consumption Data

The regime likelihood ranking we obtain when using joint data on consumption, investment and cash flow (see section 2 of tables 15-16) is the same we obtained in the investment only, (k, k', q) case but now we are unable to distinguish between the S and B regimes. Note, however, that the number of cells when using all available data is large (500 now vs. 50 before). We do not see a more significant improvement in the ability to distinguish the regimes relative to the (k, k', q) case when adding the consumption data, due to the low precision we have in each probability cell. This intuition is confirmed in section 6.3.5 below in which we perform a robustness run with model-generated data using the MLE parameters.

6.3.3 Consumption Smoothing Data

We test whether we can distinguish between the studied regimes based on the degree of consumption smoothing as embedded in the (c, q) cross-sectional distribution in the Thai data. Despite the fact that there are many fewer cells, we observe significantly more ties among the regime comparisons in Table 15 (7 ties and 3 marginally significant (at 10%) comparisons) relative to when using joint investment and consumption data (3 ties). This confirms our findings from section 5 that using joint investment-consumption data improves our ability to distinguish the regimes and shows that the general forces from the model-generated data section are at work. Using investment data only, (k, k', q) performs better than using consumption data only. Unlike model-generated data, the consumption Thai data alone seem to be unable to pin down precisely the best fitting regime.

The regime likelihoods using (c, q) data also rank differently compared to when using (k, k', q) or (c, k, k', q) data. As before, the autarky regime has the worst likelihood and is always rejected, but now the mechanism design regimes (MH/FI) achieve slightly higher likelihoods than the exogenously incomplete (B/S) regimes. However, the likelihood differences are usually statistically insignificant, and thus we cannot reject the hypothesis that the B/S regimes are as close to the data as the MH/FI regimes in most cases.

In general, the parameter estimates in table 14.3 differ relative to the (k, k', q) case as the MLE is adjusting the estimates to the different data. We illustrate the ability of the competing financial regimes to match the Thai consumption data on fig. 6. Note the bad performance of the autarky regime which puts too much weight on the incorrect (c, q) cells.

6.3.4 Dynamics: Using Repeated Cross-sections and Panel Data

We also estimate and test across the regimes based on repeated cross-sections and panel data targeting the model dynamics. Specifically, we use cross-sections of the joint distributions (c, q), (k, k', q) or (c, q, k, k') taken at two different time periods, one or five years apart (6 years for c, q)– see tables 15 and 16. Using repeated cross sections of (k, k', q) data improves significantly our ability to distinguish the regimes relative to when using a single (k, k', q) cross-section. Using

[35] On the horizontal axis (k') the first five columns of bars refer to the (k, k') values when $q = .005$ and the next five columns refer to $q = .13$.

repeated (c, q) cross-sections we also find, in general, a much better ability to distinguish compared to using a single cross-section. These results confirm our findings from the model-generated data section that using more data dimensions improves our ability to distinguish across regimes.

Next, we compare the regimes using data on the joint distribution of consumption and income in two different periods t and t', i.e., (c, q, c', q') data as in a panel[36] (section 5 of tables 15-16). The ordering of regimes in terms of likelihood from the single (c, q) cross-section case is preserved, with the MH/FI regimes coming slightly (often insignificantly) ahead and the A regime always rejected against any other. As in section 5, with incomplete depreciation our ability to distinguish the regimes does not improve with the time between surveys in the panel.

6.3.5 Model-Generated Data

Finally, we also ran the MLE and model comparison tests on model-generated data but for parameter values actually estimated from corresponding Thai data. The sample size and variable grids are also chosen to be the same as in the Thai data runs in tables 14-16. We report the Vuong statistics in table 17. For each data type (e.g., (c, q), (k, k', q),etc.) we pick as data-generating the MH or S regime that is found to fit the Thai data better, as indicated in the table.

The measurement error variance parameter, γ_{me} is estimated and turns out to be generally low, in the range 0.07-0.11 (the exception is 0.23 using c, q, k, k' data). This measurement error level corresponds closely to our "low m.e." specification in the runs from section 5. As in that section, we see that at this low level of measurement error, most regime pairs are statistically distinguishable from each other with the B/S pair being the hardest to discern as in our baseline parameter results. In terms of likelihoods, the ranking among the regimes remains robust. As before, the autarky regime is always distinguished at the 1% significance level from all alternatives.

The main finding from this exercise is that, when using model-generated data and controlling for parameter values, grids and sample size we obtain much better ability to distinguish among the regimes compared to when using actual Thai data (compare the counts of regime ties in tables 15 and 17).

6.3.6 Summary

The findings from all estimation exercises with Thai data are summarized in Table 16. We saw that the type of data used can affect our ability to pin down the model of financial constraints that matches the Thai data best or, in general, to distinguish across the regimes. The results using combined data on (c, q, k, k') where the likelihood ranking among the regimes is the same as that when using (k, k', q) data and where our ability to distinguish regimes is better than using consumption data only, seems to imply that the type of financial constraints represented by the B/S regimes are the important factor in shaping overall outcomes in the Thai data.

The results from this section should be put in perspective relative to our findings in PTK (2006) where we estimated a single-period model of occupational choice between starting a small business and subsistence farming. We found moral hazard (rather than limited liability) to be the predominant source of financial constraints for rural Thai entrepreneurs but the borrowing and saving regimes we study here were not in the set of compared models. On the other hand, Karaivanov (2007) finds that, in an occupational choice setting very similar to PTK's, one cannot distinguish statistically between a model of moral hazard vs. a model of borrowing with default.

[36] Unfortunately, using panel data on (k, k', q) or (c, q, k, k') is infeasible with our Thai dataset due to the very high number of cells required to be fitted (2,500 or 25,000) using only 531 observations.

7 Discussion and Conclusions

We formulated and solved numerically a wide range of multi-period financial market regimes with exogenous or endogenous asset structure that allow for moral hazard and unobservable capital/investment. We characterized the optimal allocations implied by the regimes from both cross-sectional and intertemporal perspectives. We developed methods based on mechanism design theory and linear programming and used them to structurally estimate, compare and distinguish between the different information regimes. We showed that such models can match stylized facts from the empirical firm dynamic literature as listed by Cooley and Quadrini (2001). The compared regimes were also demonstrated to differ significantly with respect to their qualitative and/or quantitative implications for investment, consumption, financial flows, and insurance in cross-section, transitions, and long-run outcomes. Joint consumption and investment data are particularly useful in pinning down the financial regime generating the data. Our methods can handle unobserved heterogeneity, grid approximations, transitional dynamics, and reasonable measurement error.

One striking finding is that we can readily distinguish exogenously incomplete financial regimes from endogenous incomplete ones, where the latter are solutions to mechanism design problems with unobserved effort and hidden state variables. As the literature we surveyed in the introduction typically takes one route, or the other, we believe this ability to distinguish will prove quite helpful in future research and the applications of others. We are also able to distinguish within these groups, though this depends on measurement error, the variables coming from a survey, and whether or not we have more than static cross-sections. Of course, we do not claim we have covered all possible models, only six typical prototypes. Obvious inclusions for future work would include observed effort and unobserved capital, unobserved output, and costly state verification. We could also easily incorporate limited enforcement/limited commitment in our formulation by requiring that the minimum possible promised utility be equal to the agent's discounted value under autarky or, if the agent's saving or borrowing cannot be controlled by the bank, his value under the saving only or borrowing regimes.

We are still somewhat limited on the computation side, though we are encouraged with recent advances we have been making. In this paper, we estimate only a subset of parameters at one time (though we conducted robustness check to verify that none of our conclusions depended on this). We also had some difficulty estimating the unobserved capital regime with incomplete depreciation. In recent collaboration with research scientists, we have been exploring the use of parallel processing and the Grid to speed up existing codes and allow more complexity. What we do thus far is, for want of better terminology, brute force. There may be further gains from more streamlined programs and more efficient search, i.e., where to refine the grids, when to use non-linear or mixed methods, the use of the dual, and so on.

We have established that our methods work on real data from villages in Thailand. We echo previous work which finds that full risk sharing is rejected, but not by much, and indeed find that the moral hazard regime is consistent with the income and consumption data. We also echo previous work which finds that investment is not smooth and may be sensitive to cash flow and indeed find that savings only, or borrowing and saving regimes seem to characterize the investment, firm size, and cash flow data. Of course, we recognize that these results are an anomaly: the regime which the Thai data fits best depends on which data are used, unlike the model-generated data comparisons.

We thought perhaps we might recover a more sophisticated contract theoretic regime if we restricted attention to family networks, but was not the case. So we suspect our results are instead due to the infrequent nature of investment in the data and the relatively large size of investment compared to capital when investment takes place. That is, the Thai data results may have more to do with the technology and not the financial regime. Indeed, recall that we dropped costly adjustment per se and tried to view the data though the lens of financial regimes alone. Evidently we have learned something from our approach, or at least know what would be one obvious next step. Fortunately this is doable. Other future steps include distinctions across different technologies (fish/shrimp, livestock, business, and so on), and the inclusion of aggregate shocks (shrimp disease, rainfall). We would also return to the issue of entrepreneurial talent, as in our earlier work (PTK, 2006), and allow for heterogeneity in returns. Other work (Pawasutipaisit and Townsend, 2007) shows that ROA varies considerably across households and is persistent. On the other hand, these data summaries have trouble finding consistent patterns with respect to finance, suggesting the data be viewed through the lens of revised models.

We have our eyes on other economies as well, in part because we get more entry and exit from business in other countries, and in part because we need large sample sizes for our methods to work. Unfortunately, we do not typically find both consumption and income data which is why we chose the Thai data to begin with. But preliminary work with enterprise data from Spain shows not only the stylized facts of firm growth, declining mean and variance with size and age, but also a salient financial life cycle, with newly born small firms relying on informal finance, then, conditional on survival, gaining credit from one formal financial intermediary, and finally, having access to multiple banks (Zambrano, Saurina and Townsend, 2008). The methods of this paper allow in principle for transitions across financial regimes, jointly with the growth facts. Fortunately, there are as many as 10,000 data points in each Spanish cohort.

Other complementary approaches include doing more on the supply side. It may be that lenders have rules for access and credit levels that are hard to mimic with the current regimes at hand. Hsieh and Klenow (2007) also find that the distribution of firm size may have to do with regulatory distortions. Assuncao, Mityakov, and Townsend (2008) are working in this direction.

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8 Appendix

8.1 Moral Hazard with Unobserved Investment and Incomplete Depreciation

We extend the moral hazard with unobserved investment model to include the possibility of incomplete depreciation, $\delta < 1$. This makes the interim principal's value, V_m dependent on k since capital does not expire in production. Furthermore, the interim utility must now take into account that the agent might have deviated in his announcement of k when entering the second stage, i.e., we need to define it as a vector $\mathbf{w}_m = \{w_m(k_1), w_m(k_2), ...\} \in \mathbf{W}_m$ similarly to $\mathbf{w}$. The set $\mathbf{W}_m$ is endogenously determined during the value function iteration, similarly to the set $\mathbf{W}$. The first sub-period problem is then (compare with program UC1):

Program UC3:

$$V(\mathbf{w}, k) = \max_{\{\pi(q,z,\mathbf{w}_m|\mathbf{w},k)\}} \sum_{Q\times Z\times \mathbf{W}_m} \pi(q, z, \mathbf{w}_m|\mathbf{w},k)[q + V_m(\mathbf{w}_m, k)] \tag{20}$$

The maximization is subject to the constraints (11) and (12), replacing w_m with $w_m(k)$ and $\pi(q, z, w_m|\mathbf{w},k)$ with $\pi(q, z, \mathbf{w}_m|\mathbf{w},k)$. Additionally, truth-telling needs to be induced, as in (13), only replacing w_m with $w_m(\hat{k})$ (the interim utility needs to be consistent with the agent's true type) and $\pi(q, z, w_m|\mathbf{w},k)$ with $\pi(q, z, \mathbf{w}_m|\mathbf{w},k)$. The rest of the constraints are the familiar Bayes-consistency, adding-up, and non-negativity.

The state variables in the second stage sub-problem are different from the $\delta = 1$ case, namely now they are the *vector* of interim utilities, $\mathbf{w}_m$ and the announcement k. This introduces extra truth-telling and obedience constraints in the second stage program The reason is that now we need to ensure that, when deciding on k', the agent cannot get more than his interim utility, $w_m(k)$ for *any* announcement k. Due to the higher dimensional state space, we also need to compute a much larger number of second stage linear programs – one for each possible $(k, \mathbf{w}_m) \in K \times \mathbf{W}_m$ given by:

Program UC4:

$$V_m(\mathbf{w}_m, k) = \max_{\{\pi(\tau,k',\mathbf{w}'|\mathbf{w}_m,k)\},\{v(k,\hat{k},k',\tau)\}} \sum_{T\times K'\times \mathbf{W}'} \pi(\tau, k', \mathbf{w}'|\mathbf{w}_m, k)[-\tau + (1/R)V(k', \mathbf{w}')] \tag{21}$$

In addition to the probabilities, $\pi(\tau, k', \mathbf{w}'|\mathbf{w}_m, k)$, we now add more choice variables, namely $v(k, \hat{k}, k', \tau)$. These variables, which we call *utility bounds*, (see Prescott, 2003 for details) specify the maximum expected utility that an agent of type $\hat{k}$ could obtain by not announcing truthfully (report k instead) and who receives transfer τ and an investment recommendation k'. This translates into the constraint:

$$\sum_{\mathbf{W}'} \pi(\tau, k', \mathbf{w}'|\mathbf{w}_m, k)[u(\tau + (1-\delta)\hat{k} - \hat{k}') + \beta w'(\hat{k}')] \le v(k, \hat{k}, k', \tau) \tag{22}$$

holding for all possible combinations $\tau, k', \hat{k}', \hat{k} \neq k$, and $\hat{k}' \neq k'$. To obtain the utility, $w_m(\hat{k})$ that an agent obtains in the second sub-period by reporting k when the true state is $\hat{k}$, we add up the bounds $v(k, \hat{k}, k', \tau)$ over all possible $\tau, k' \in T \times K'$ resulting in the constraint:

$$\sum_{T\times K'} v(k, \hat{k}, k', \tau) \le w_m(\hat{k}) \tag{23}$$

The two sets of constraints, (22) and (23) also rule out any joint deviations in the report k and the action k'. Further, by definition, the interim utility must satisfy:

$$w_m(k) = \sum_{T\times K'\times \mathbf{W}'} \pi(\tau, k', \mathbf{w}'|\mathbf{w}_m, k)[u(\tau + (1-\delta)k - k') + \beta w'(k')] \tag{24}$$

Finally, the π's must satisfy non-negativity and adding-up as usual.

8.2 Welfare

The analysis in section 2 implies that the information regimes we study can be unambiguously ranked in terms of efficiency, as measured by how far the Pareto frontier extends. The autarky regime (no access to markets) is characterized with lowest welfare. It is followed by the saving only and borrowing regimes which allow non state-contingent intertemporal consumption smoothing but no extra insurance across states apart from self-insurance. In turn, if $\beta = 1/R$ as in our baseline, the moral hazard regime with unobserved investment (UC) Pareto dominates the B regime since the optimal allocation achieved by B is incentive and truth-telling compatible in UC (it can be achieved by allowing the agent to borrow and lend through the principal). In general, however, UC can provide extra insurance relative to the borrowing regime[37]. The moral hazard regime with observed investment (MH) is in turn Pareto superior to the unobserved investment regime as the truth-telling constraints are relaxed. Finally, the full information regime dominates all other regimes since, by definition, it achieves the first best allocation.

The Pareto frontiers for all regimes, computed at the baseline parameters from table 3 are graphed on fig. A1-2. To plot the B and S frontiers we use that agent's savings determining his utility map into negative principal value and conversely about agent's debt. The autarky frontier is a single point. The increasing portion of the frontier for the MH and UC regimes is due to the fact that at very low promised utility the agent's incentives to exert effort and invest are diminished (see Phelan and Townsend, 1991 for discussion).

[37]The results of Allen (1985) and Cole and Kocherlakota (2001) for dynamic adverse selection problems which state that no additional insurance on top of self-insurance can be provided by the principal do not apply in our moral hazard setting because of the endogeneity of the stochastic process of output (see Abraham and Pavoni, 2005).

Table 1 - Baseline Grids

	Grid Size	Grid Range (full depr.)	Grid Range (inc. depr.)
Q	2	{.1, 3}	{.1, .5}
K	11	[0, 1]	[0, 1]
Z	3	[.01, 1]	[.01, 1]
B	21	S: [-5, 0], B: [-5, 5]	S: [-1, 0], B: [-1, 1]
T	31 (86 if $\delta < 1$)	[0, 3]	[0, 1]
W	21	[w_{min}, w_{max}]	[w_{min}, w_{max}]
$\mathbf{W_m}$	31 (630 if $\delta < 1$)	[w_{min}, $\beta w_{max}+u(t_{max})$]	[w_{min}, $\beta w_{max}+u(t_{max})$]

Table 2 - Dimensionality

	# linear programs	# LP variables (π)	# LP constraints
Autarky	11	66	7
Saving / Borrowing	231	1386	7
Full Information	231	42,966	8
Moral Hazard, full depr.	231	42,966	14
Moral Hazard, inc. depr.	231	119,196	14
Unobserved k, full depr. stage 1	6,930	186	284
Unobserved k, full depr. stage 2	21	410,130	188
Unobserved k, inc. depr. stage 1	6,930	3,780	284
Unobserved k, inc. depr. stage 2	6,930	1,137,780	95,548

Table 3 - Baseline Parameters

Parameter	Value(s)
depreciation rate, δ	1 (full depr.), 0.05 (incomplete depr.)
agent's discount factor, β	0.95
principal's discount factor, 1/R	0.95
risk aversion, σ	0.5 *(0, 2)*
effort curvature, θ	2
effort cost, ξ	1 *(0.1)*
technology parameter, ρ	0 *(-1, 1)*
capital share, η	0.5 *(0.8)*
probability scaling factor, λ	0.5

Table 4 - Parameter Estimates Using Model-Generated Data on Firm Size, Investment and Cashflow (k, k', q)
Data-Generating Model is Moral Hazard, n = 1000

Incomplete depreciation (δ = 5%)

Estimates for:	$\mu_{w/b}$	$\gamma_{w/b}$	γ_{me}	σ	θ	ρ	LL Value
Model	**Low Measurement Error (stdev = 0.1 * gridmax)**						
Moral Hazard (base) - MH	20.1875 *(.586)*	8.2500 *(.798)*	0.1000 *(.005)*	0.5000 *(.000)*	2.0313 *(.043)*	0.0000 *(.053)*	**-3165.1**
Full Information - FI	19.9766 *(.862)*	8.7188 *(1.06)*	0.1366 *(.011)*	0.5039 *(.008)*	1.9961 *(.058)*	-0.0547 *(.028)*	**-3221.4**
Borrowing & Lending - B	-0.1521 *(.142)*	0.2161 *(.232)*	0.5276 *(.044)*	0.6000 *(.199)*	1.3750 *(4.04)*	-0.9063 *(.161)*	**-3441.7**
Saving Only - S	-0.4000 *(.001)*	0.2000 *(.071)*	0.6374 *(.056)*	0.1313 *(.027)*	1.2000 *(.513)*	2.3984 *(.376)*	**-3480.3**
Autarky - A	n.a.	n.a.	0.7039 *(.043)*	0.1000 *(.473)*	8.1000 *(1.88)*	0.2188 *(6.10)*	**-3617.2**
baseline values	*19.9999*	*8*	*0.1*	*0.5*	*2*	*0*	
	High Measurement Error (stdev = 0.5 * gridmax)						
Moral Hazard (base) - MH	19.0000 *(6.40)*	9.0000 *(6.52)*	0.6414 *(.075)*	0.5000 *(.002)*	2.0156 *(.771)*	-0.0469 *(1.06)*	**-3581.3**
Full Information - FI	19.9688 *(7.22)*	8.0625 *(8.86)*	0.5684 *(.105)*	0.5000 *(.012)*	1.9844 *(.802)*	-0.0625 *(.721)*	**-3595.2**
Borrowing & Lending - B	0.0000 *(.250)*	0.1377 *(.108)*	0.6559 *(.066)*	0.5000 *(.492)*	2.0000 *(4.31)*	-3.8203 *(3.13)*	**-3630.3**
Saving Only - S	-0.1651 *(.081)*	0.0000 *(.286)*	0.6619 *(.031)*	0.4537 *(.071)*	2.0753 *(.718)*	-2.7498 *(1.78)*	**-3679.6**
Autarky - A	n.a.	n.a.	0.6845 *(.035)*	1.1125 *(.365)*	1.4917 *(1.38)*	-3.4563 *(4.00)*	**-3741.2**
baseline values	*19.9999*	*8*	*0.5*	*0.5*	*2*	*0*	

Complete depreciation (δ = 100%)

Estimates for:	$\mu_{w/b}$	$\gamma_{w/b}$	γ_{me}	σ	θ	ρ	LL Value
Model	**Low Measurement Error (stdev = 0.1 * gridmax)**						
Moral Hazard (base) - MH	36.7338 *(1.73)*	15.0000 *(1.42)*	0.1039 *(.003)*	0.5000 *(.006)*	2.0234 *(.027)*	-0.0313 *(.129)*	**-2976.3**
Unobserved k - UC	n.a.	n.a.	0.1020 *(.079)*	0.5469 *(.192)*	4.1375 *(.045)*	-0.2813 *(.608)*	**-3018.1**
Full Information - FI	34.1713 *(1.82)*	17.7266 *(2.47)*	0.1937 *(.018)*	0.5068 *(.004)*	2.5859 *(.451)*	-0.0313 *(.152)*	**-3059.4**
Borrowing & Lending - B	-1.0000 *(.003)*	0.4367 *(.016)*	0.5517 *(.013)*	0.9750 *(.352)*	4.0000 *(.589)*	0.0000 *(.164)*	**-3165.3**
Saving Only - S	-1.0000 *(.003)*	0.2873 *(.096)*	0.4402 *(.017)*	0.0375 *(.435)*	6.0000 *(.004)*	0.2402 *(.171)*	**-3184.6**
Autarky - A	n.a.	n.a.	0.9031 *(.083)*	0.1000 *(.002)*	10.0000 *(.000)*	0.7344 *(.002)*	**-3868.4**
baseline values	*34.64*	*15*	*0.1*	*0.5*	*2*	*0*	
	High Measurement Error (stdev = 0.5 * gridmax)						
Moral Hazard (base) - MH	34.6400 *(6.14)*	14.5000 *(2.13)*	0.5898 *(.035)*	0.5000 *(.026)*	2.2188 *(1.89)*	0.3125 *(.082)*	**-3514.9**
Full Information - FI	34.6400 *(.019)*	14.9688 *(7.42)*	0.6168 *(.088)*	0.5000 *(.008)*	2.4375 *(1.60)*	0.0000 *(.191)*	**-3522.9**
Borrowing & Lending - B	-1.0000 *(.107)*	0.2414 *(.195)*	0.6335 *(.044)*	0.2188 *(.059)*	10.0000 *(.000)*	-0.3750 *(.394)*	**-3528.4**
Unobserved k - UC	n.a.	n.a.	0.6224 *(.029)*	0.1352 *(.160)*	5.0156 *(3.04)*	-5.5938 *(1.83)*	**-3528.7**
Saving Only - S	-1.0000 *(.006)*	0.0250 *(.182)*	0.6550 *(.079)*	0.4750 *(.192)*	8.1000 *(.045)*	0.2344 *(.608)*	**-3530.4**
Autarky - A	n.a.	n.a.	0.8521 *(.056)*	0.1000 *(.002)*	10.0000 *(.000)*	0.7344 *(.006)*	**-3952.3**
baseline values	*34.64*	*15*	*0.5*	*0.5*	*2*	*0*	

Note: Bootstrap standard errors are in the parentheses next to the estimates.

Table 5 - Model Comparisons Using Data on Investment and Cash Flow (k,k',q)
Data-Generating Model is Moral Hazard, n=1000

Incomplete depreciation (δ = 5%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats					LL value[1]
	MH	FI	B	S	A	
MH	n.a.					-3165.1
FI	6.143***(MH)	n.a.				-3221.4
B	11.25***(MH)	9.233***(FI)	n.a.			-3441.7
S	11.22***(MH)	9.217***(FI)	1.720*(B)	n.a.		-3480.3
A	14.36***(MH)	12.33***(FI)	6.981***(B)	8.213***(S)	n.a.	-3617.2

Incomplete depreciation (δ = 5%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-3581.3
FI	1.177(tie)	n.a.				-3595.2
B	4.904***(MH)	2.376**(FI)	n.a.			-3630.3
S	6.302***(MH)	6.566***(FI)	3.171***(B)	n.a.		-3676.9
A	8.573***(MH)	6.588***(FI)	6.824***(B)	3.499***(S)	n.a.	-3741.2

Complete depreciation (δ = 100%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	UC	FI	B	S	A	
MH	n.a.						-2976.3
UC	3.576***(MH)	n.a.					-3018.1
FI	4.978***(MH)	2.25**(UC)	n.a.				-3059.4
B	10.53***(MH)	7.55***(UC)	7.336***(FI)	n.a.			-3165.3
S	9.598***(MH)	7.11***(UC)	8.364***(FI)	1.783*(B)	n.a.		-3184.6
A	21.16***(MH)	20.5***(UC)	20.32***(FI)	18.59***(B)	17.74***(S)	n.a.	-3868.4

Complete depreciation (δ = 100%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	FI	B	UC	S	A	
MH	n.a.						-3514.9
FI	1.379(tie)	n.a.					-3522.9
B	1.362(tie)	0.535(tie)	n.a.				-3528.4
UC	1.419(tie)	0.531(tie)	0.025(tie)	n.a.			-3528.7
S	1.285(tie)	0.715(tie)	0.145(tie)	0.12(tie)	n.a.		-3530.4
A	14.98***(MH)	14.12***(FI)	14.30***(B)	12.9***(UC)	13.54***(S)	n.a.	-3952.3

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 6 - Model Comparisons Using Data on Consumption and Income (c,q)
Data-Generating Model is Moral Hazard, n=1000

Incomplete depreciation (δ = 5%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-2561.3
FI	5.157***(MH)	n.a.				-2597.4
B	7.250***(MH)	3.963***(FI)	n.a.			-2646.4
S	8.750***(MH)	6.320***(FI)	4.059***(B)	n.a.		-2682.4
A	13.52***(MH)	12.11***(FI)	16.35***(B)	12.81***(S)	n.a.	-2793.6

Incomplete depreciation (δ = 5%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	B	FI	S	A	
MH	n.a.					-2715.8
B	1.633(tie)	n.a.				-2721.4
FI	2.105**(MH)	0.791(tie)	n.a.			-2724.7
S	2.240**(MH)	1.582(tie)	0.748(tie)	n.a.		-2729.7
A	3.087***(MH)	2.989***(B)	2.066**(FI)	1.151(tie)	n.a.	-2735.7

Complete depreciation (δ = 100%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	UC	FI	B	S	A	
MH	n.a.						-2729.6
UC	3.308***(MH)	n.a.					-2754.2
FI	4.593***(MH)	1.67*(UC)	n.a.				-2768.8
B	6.856***(MH)	6.09***(UC)	2.943***(FI)	n.a.			-2811.4
S	9.616***(MH)	8.40***(UC)	5.958***(FI)	5.636***(B)	n.a.		-2860.2
A	17.18***(MH)	16.5***(UC)	14.14***(FI)	17.11***(B)	19.18***(S)	n.a.	-3096.3

Complete depreciation (δ = 100%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	FI	UC	B	S	A	
MH	n.a.						-2790.6
FI	0.857(tie)	n.a.					-2793.8
UC	1.346(tie)	0.680(tie)	n.a.				-2797.5
B	2.009**(MH)	1.646*(FI)	0.77(tie)	n.a.			-2801.9
S	4.923***(MH)	4.560***(FI)	4.15***(UC)	4.793***(B)	n.a.		-2845.1
A	6.664***(MH)	6.113***(FI)	6.24***(UC)	7.100***(B)	5.077***(S)	n.a.	-2881.7

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 7 - Model Comparisons Using Data on Investment, Consumption and Cash Flow (c,q,k,k')
Data-Generating Model is Moral Hazard, n=1000

Incomplete depreciation (δ = 5%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-4655.7
FI	8.785***(MH)	n.a.				-5129.2
B	22.85***(MH)	12.11***(FI)	n.a.			-6282.7
S	25.78***(MH)	14.33***(FI)	1.467(tie)	n.a.		-6403.3
A	27.98***(MH)	16.92***(FI)	6.439***(B)	4.712***(S)	n.a.	-6829.3

Incomplete depreciation (δ = 5%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-5744.3
FI	3.754***(MH)	n.a.				-5939.0
B	7.217***(MH)	4.031***(FI)	n.a.			-6267.2
S	8.853***(MH)	6.288***(FI)	2.380**(B)	n.a.		-6455.7
A	11.23***(MH)	8.893***(FI)	5.375***(B)	3.277***(S)	n.a.	-6794.9

Complete depreciation (δ = 100%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	UC	FI	B	S	A	
MH	n.a.						-4688.8
UC	12.27***(MH)	n.a.					-5308.8
FI	10.33***(MH)	1.13(tie)	n.a.				-5404.7
B	13.69***(MH)	5.15***(UC)	4.457***(FI)	n.a.			-5766.6
S	20.90***(MH)	9.28***(UC)	7.102***(FI)	4.438***(B)	n.a.		-6068.3
A	30.58***(MH)	23.7***(UC)	20.25***(FI)	18.06***(B)	18.45***(S)	n.a.	-8262.4

Complete depreciation (δ = 100%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats						LL value
	MH	FI	UC	B	S	A	
MH	n.a.						-5715.2
FI	2.947***(MH)	n.a.					-5908.6
UC	5.324***(MH)	2.382**(FI)	n.a.				-6087.1
B	7.304***(MH)	4.548***(FI)	2.56**(UC)	n.a.			-6306.7
S	9.936***(MH)	7.197***(FI)	5.65***(UC)	3.216***(B)	n.a.		-6614.7
A	15.10***(MH)	12.13***(FI)	10.6***(UC)	7.734***(B)	13.54***(S)	n.a.	-7206.2

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 8 - Model Comparisons Using Panel Data on Consumption and Income (c,q,c',q')
Data-Generating Model is Moral Hazard, n=1000; high measurement error (γ_{me} = 0.5 * gridspan)

Incomplete depreciation (δ = 5%), using (c,q) data at t = 0 and t = 1 (joint distribution)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-5711.0
FI	3.520***(MH)	n.a.				-5940.9
B	6.378***(MH)	3.212***(FI)	n.a.			-6171.3
S	6.438***(MH)	3.307***(FI)	0.725(tie)	n.a.		-6229.2
A	10.86***(MH)	8.216***(FI)	6.162***(B)	5.166***(S)	n.a.	-6720.4

Complete depreciation (δ = 100%), using (c,q) data at t = 0 and t = 1 (joint distribution)

Model	Vuong Test Z-stats						LL value
	MH	FI	UC	B	S	A	
MH	n.a.						-5622.6
FI	1.862*(MH)	n.a.					-5730.3
UC	4.917***(MH)	3.606***(FI)	n.a.				-5954.0
B	5.045***(MH)	3.471***(FI)	0.600(tie)	n.a.			-5997.5
S	5.771***(MH)	4.304***(FI)	1.415(tie)	0.980(tie)	n.a.		-6069.0
A	9.678***(MH)	8.200***(FI)	5.91***(UC)	5.594***(B)	5.431***(S)	n.a.	-6501.0

Incomplete depreciation (δ = 5%), using (c,q) data at t = 0 and t = 50 (joint distribution)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-5707.3
FI	3.279***(MH)	n.a.				-5924.9
B	4.511***(MH)	1.545(tie)	n.a.			-6045.2
S	7.332***(MH)	4.360***(FI)	2.826***(B)	n.a.		-6282.0
A	8.397***(MH)	5.624***(FI)	4.438***(B)	1.420(tie)	n.a.	-6404.2

Complete depreciation (δ = 100%), using (c,q) data at t = 0 and t = 50 (joint distribution)

Model	Vuong Test Z-stats						LL value
	MH	FI	B	UC	S	A	
MH	n.a.						-5616.5
FI	4.246***(MH)	n.a.					-5831.8
B	5.304***(MH)	2.110**(FI)	n.a.				-5999.3
UC	6.094***(MH)	2.871***(FI)	0.574(tie)	n.a.			-6050.2
S	8.937***(MH)	5.874***(FI)	4.676***(B)	2.93***(UC)	n.a.		-6348.3
A	9.826***(MH)	6.928***(FI)	5.217***(B)	4.51***(UC)	1.606(tie)	n.a.	-6478.3

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 9 - Model Comparisons Using Repeated Cross-sections of Consumption and Income
Data-Generating Model is Moral Hazard; high measurement error

Incomplete depreciation (δ = 5%); using (c,q) data at t = 0 and t = 1 (two cross-sections)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-5557.6
FI	2.864***(MH)	n.a.				-5575.7
B	3.516***(MH)	0.662(tie)	n.a.			-5580.9
S	4.217***(MH)	2.198**(FI)	1.880*(B)	n.a.		-5594.5
A	7.458***(MH)	6.583***(FI)	6.483***(B)	6.997***(S)	n.a.	-5661.1

Complete depreciation (δ = 100%), using (c,q) data at t = 0 and t = 1 (two cross-sections)

Model	Vuong Test Z-stats						LL value
	MH	FI	B	UC	S	A	
MH	n.a.						-5558.3
FI	1.984**(MH)	n.a.					-5581.5
B	4.449***(MH)	3.270***(FI)	n.a.				-5654.5
UC	5.699***(MH)	3.950***(FI)	0.561(tie)	n.a.			-5671.6
S	5.760***(MH)	4.750***(FI)	3.141***(B)	1.155(tie)	n.a.		-5709.9
A	8.959***(MH)	8.229***(FI)	7.661***(B)	5.32***(UC)	7.985***(S)	n.a.	-5855.5

Incomplete depreciation (δ = 5%); using (c,q) data at t = 0 and t = 50 (two cross-sections)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-5549.6
FI	3.292***(MH)	n.a.				-5574.5
B	4.074***(MH)	1.034(tie)	n.a.			-5586.3
S	5.013***(MH)	2.062**(FI)	1.473(tie)	n.a.		-5599.3
A	6.899***(MH)	4.490***(FI)	5.100***(B)	4.496***(S)	n.a.	-5639.2

Complete depreciation (δ = 100%), using (c,q) data at t = 0 and t = 50 (two cross-sections)

Model	Vuong Test Z-stats						LL value
	MH	FI	B	UC	S	A	
MH	n.a.						-5572.4
FI	3.530***(MH)	n.a.					-5592.8
B	5.263***(MH)	2.696***(FI)	n.a.				-5626.3
UC	5.670***(MH)	3.440***(FI)	0.717(tie)	n.a.			-5637.3
S	9.054***(MH)	7.361***(FI)	9.563***(B)	4.33***(UC)	n.a.		-5729.4
A	9.678***(MH)	11.03***(FI)	13.30***(B)	9.83***(UC)	6.261***(S)	n.a.	-5850.3

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 10 - Robustness: Model Comparisons Using Data on Investment and Cash Flow (k,k',q)
Data-Generating Model is Saving Only, n=1000

Incomplete depreciation (δ = 5%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats					LL value
	S	B	FI	MH	A	
S (baseline)	n.a.					-2868.1
B	3.649***(S)	n.a.				-2896.2
FI	11.86***(S)	9.096***(B)	n.a.			-3063.5
MH	11.24***(S)	8.931***(B)	0.189(tie)	n.a.		-3065.1
A	13.15***(S)	11.83***(B)	4.158***(FI)	4.012***(MH)	n.a.	-3164.1

Incomplete depreciation (δ = 5%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats					LL value
	S	B	FI	MH	A	
S (baseline)	n.a.					-3547.7
B	0.335(tie)	n.a.				-3549.8
FI	2.248**(S)	2.253**(B)	n.a.			-3565.9
MH	4.733***(S)	4.455***(B)	3.086***(FI)	n.a.		-3600.4
A	5.774***(S)	5.052***(B)	4.334***(FI)	0.411(tie)	n.a.	-3606.8

Complete depreciation (δ = 100%), low measurement error (γ_{me} = 0.1 * gridspan)

Model	Vuong Test Z-stats						LL value
	S	B	UC	FI	A	MH	
S	n.a.						-2743.0
B	0.658(tie)	n.a.					-2743.7
UC	7.302***(S)	7.215***(B)	n.a.				-2846.3
FI	17.40***(S)	17.23***(B)	15.0***(UC)	n.a.			-3165.2
A	19.71***(S)	19.66***(B)	15.8***(UC)	1.230(tie)	n.a.		-3197.9
MH	17.71***(S)	17.54***(B)	15.7***(UC)	3.175***(FI)	0.136(tie)	n.a.	-3201.8

Complete depreciation (δ = 100%), high measurement error (γ_{me} = 0.5 * gridspan)

Model	Vuong Test Z-stats						LL value
	S	B	UC	MH	FI	A	
S	n.a.						-3366.9
B	0.163(tie)	n.a.					-3369.0
UC	1.004(tie)	0.667(tie)	n.a.				-3379.5
MH	3.862***(S)	3.904***(B)	3.79***(UC)	n.a.			-3432.4
FI	4.234***(S)	4.520***(B)	4.33***(UC)	1.053(tie)	n.a.		-3442.3
A	4.383***(S)	4.239***(B)	3.48***(UC)	0.585(tie)	0.120(tie)	n.a.	-3447.0

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 11 - Robustness Checks: More Regime Comparisons using (k,k',q) data
benchmark is Moral Hazard; incomplete depreciation (δ = 5%)

Larger measurement error (γ_{me} = 0.7 * gridspan)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-3566.0
FI	0.293(tie)	n.a.				-3569.0
B	2.951***(MH)	2.562**(FI)	n.a.			-3598.3
S	2.666***(MH)	3.567***(FI)	0.305(tie)	n.a.		-3601.5
A	7.507***(MH)	7.099***(FI)	5.688***(B)	6.484***(S)	n.a.	-3690.8

Smaller number of data points (n = 200)

Model	Vuong Test Z-stats					LL value
	FI	MH	B	S	A	
FI	n.a.					-3568.6
MH	0.032(tie)	n.a.				-3569.2
B	0.336(tie)	0.332(tie)	n.a.			-3579.5
S	2.427**(FI)	2.266**(MH)	2.105**(B)	n.a.		-3647.6
A	4.490***(FI)	4.074***(MH)	3.962***(B)	3.034***(S)	n.a.	-3830.1

Larger number of data points (n = 5000)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-3566.4
FI	3.256***(MH)	n.a.				-3572.0
B	4.989***(MH)	3.586***(FI)	n.a.			-3584.8
S	7.198***(MH)	5.815***(FI)	4.769***(B)	n.a.		-3596.7
A	25.07***(MH)	24.32***(FI)	23.76***(B)	25.49***(S)	n.a.	-3792.4

Smaller Grid Size (#C = 5)

Model	Vuong Test Z-stats					LL value
	MH	FI	B	S	A	
MH	n.a.					-3579.7
FI	1.113(tie)	n.a.				-3593.3
B	3.132***(MH)	1.758*(FI)	n.a.			-3622.8
S	5.865***(MH)	2.811***(FI)	1.133(tie)	n.a.		-3638.9
A	8.379***(MH)	8.184***(FI)	5.100***(B)	4.496***(S)	n.a.	-3737.8

NOTES:
1. The regimes in all tables are ordered in decreasing likelihood.
2. *** = 1%, ** = 5%, * = 10% two-sided significance level; the better fitting regime is in the parentheses;
3. "tie" denotes the tested regimes cannot be statistically distinguished from each other relative to the data.

Table 12 - Summary of Results Using Model-Generated Data from the MH regime

1. Ties Between Regime Pairs	% ties overall	% ties, δ = .05	% ties, δ = 1
MH v. FI	25%	29%	20%
MH v. B	13%	14%	10%
MH v. S	4%	0%	10%
MH v. A	0%	0%	0%
MH v. UC	20%	n.a.	20%
FI v. B	25%	36%	10%
FI v. S	8%	7%	10%
FI v. A	0%	0%	0%
FI v. UC	30%	n.a.	30%
B v. S	33%	43%	20%
B v. A	0%	0%	0%
B v. UC	60%	n.a.	60%
S v. A	13%	14%	10%
S v. UC	20%	n.a.	20%
A v. UC	0%	n.a.	0%

2. Ties Within v. Between Exogenous (B, S, A) and Mechanism Design (MH, FI, UC) Regimes	
% ties within the mechanism design regime group	25.0%
% ties within the exogenous regime group	15.3%
% ties across the exogenous and mechanism design groups	11.5%
% ties between MH/FI and any exogenous regime	8.3%
% ties between MH and any exogenous regime	5.6%
% ties between MH and any mechanism design regime	23.5%

3. Ties Within v. Between Exogenous (B, S, A) and Mechanism Design (MH, FI, UC) Regimes, high measurement error specification only	
% ties within the mechanism design regime group	31.3%
% ties within the exogenous regime group	18.5%
% ties across the exogenous and mechanism design groups	15.5%
% ties between MH/FI and any exogenous regime	11.1%
% ties between MH and any exogenous regime	7.4%
% ties between MH and any mechanism design regime	32.0%

Table 13 - Thai Data Summary Statistics

Variable	mean	median	std. dev.	min	max
Assets, k	963,580	164,250	3,744,100	0	97,006,000
Net Income from production, q	128,700	65,016	240,630	-1,473,100	5,211,800
Consumption expenditure, c	64,172	47,868	53,284	4,210	610,860

1. Sample size is 531 households observed over 7 years (1998-2004). Unit is Thai baht.
2. The summary statistics are computed for the overall (pooled) data.

Table 14 - Parameter Estimates and Likelihoods Using Thai Data

14.1. Estimates using investment and cashflow (k,k',q) data; years used: '98-99; δ = .05; n = 531

Model	$\mu_{w/b}$	$\gamma_{w/b}$	γ_{me}	σ	θ	ρ	LL Value[1]
Borrowing and Lending - B	**-0.1454**	**0.1528**	**0.1154**	**0.4057**	**1.3499**	**0.8955**	**-2.8836**
	0.0452	*0.0449*	*0.0197*	*0.0802*	*3.3505*	*0.0134*	
Saving Only - S	**-0.1540**	**0.1552**	**0.1101**	**0.2842**	**0.1908**	**0.9766**	**-2.9311**
	0.0221	*0.0106*	*0.0300*	*0.0200*	*0.0636*	*0.0956*	
Moral Hazard - MH	**10.1792**	**0.6016**	**0.2005**	**0.5057**	**2.2763**	**-2.2537**	**-3.1554**
	0.3942	*1.5149*	*0.0989*	*0.0164*	*0.1961*	*1.1570*	
Full Information - FI	**10.6670**	**0.0000**	**0.4728**	**0.5167**	**2.6916**	**0.8918**	**-3.1656**
	0.0745	*0.2221*	*0.0933*	*0.0135*	*1.5591*	*0.2308*	
Autarky - A	n.a.	n.a.	**0.4517**	**0.1000**	**2.2964**	**2.6028**	**-3.4679**
			0.1957	*0.2615*	*0.4178*	*1.2346*	

14.2. Estimates using investment, consumption and cashflow (c,q,k,k') data; years used: '98-99; δ = .05; n = 531

Model	$\mu_{w/b}$	$\gamma_{w/b}$	γ_{me}	σ	θ	ρ	LL Value
Saving Only - S	**-0.3333**	**0.0083**	**0.2208**	**1.0928**	**1.2007**	**-0.0282**	**-5.5033**
	0.0392	*0.0318*	*0.0407*	*0.1324*	*0.5732*	*0.1150*	
Borrowing and Lending - B	**-0.2924**	**0.0543**	**0.2040**	**1.2370**	**0.2471**	**0.0000**	**-5.5659**
	0.0158	*0.0526*	*0.0512*	*0.0839*	*1.7019*	*0.1307*	
Moral Hazard - MH	**6.3367**	**4.9798**	**0.1650**	**0.5000**	**2.8590**	**0.2398**	**-6.3853**
	0.8588	*1.8416*	*0.0751*	*0.0246*	*1.2363*	*0.2488*	
Full Information - FI	**5.8082**	**3.1694**	**0.3088**	**0.5000**	**1.7330**	**-0.8014**	**-6.7038**
	1.4404	*0.9605*	*0.0638*	*1.9415*	*0.5509*	*5.9247*	
Autarky - A	n.a.	n.a.	**0.3204**	**2.2940**	**0.1003**	**0.2838**	**-7.1196**
			0.0723	*0.7939*	*0.0339*	*0.2191*	

14.3. Estimates using consumption and cashflow (c,q) data; years used: '98; δ = .05; n = 531

Model	$\mu_{w/b}$	$\gamma_{w/b}$	γ_{me}	σ	θ	ρ	LL Value
Moral Hazard - MH	**6.3860**	**0.9362**	**0.1045**	**0.5000**	**1.9273**	**-12.2697**	**-2.4131**
	0.2752	*0.1251*	*0.0070*	*0.0010*	*1.5072*	*3.2229*	
Full Information - FI	**5.9436**	**0.7363**	**0.1045**	**0.4785**	**0.4632**	**0.4405**	**-2.4160**
	0.0961	*0.0861*	*0.0077*	*0.0024*	*0.0984*	*0.0590*	
Borrowing and Lending - B	**-0.1392**	**0.1368**	**0.0734**	**0.4057**	**0.2903**	**0.8955**	**-2.4622**
	0.0237	*0.0225*	*0.0088*	*0.0000*	*0.0000*	*0.0000*	
Saving Only - S	**-0.3989**	**0.3236**	**0.0119**	**0.0475**	**1.2364**	**3.0255**	**-2.4687**
	0.0363	*0.0539*	*0.0120*	*0.0023*	*0.1097*	*0.2340*	
Autarky - A	n.a.	n.a.	**0.3085**	**0.5000**	**0.1000**	**0.3237**	**-2.7792**
			0.0079	*0.0176*	*0.0341*	*0.3407*	

1. In all tables the regimes are ordered in decreasing order of likelihood to the Thai data.
2. Bootstrap standard errors are in italics below the estimates.

Table 15 - Model Comparisons Using Thai Data - Vuong Test Z-Statistics

Comparison	MH v FI	MH v B	MH v S	MH v A	FI v B	FI v S	FI v A	B v S	B v A	S v A
1. Using (k,k',q) data										
1.1. years: 98-99	0.29(tie)	-4.98***(B)	-3.42***(S)	5.15***(MH)	-6.73***(B)	-4.74***(S)	7.10***(FI)	1.36(tie)	13.78***(B)	9.77***(S)
1.2. years: 03-04	-0.44(tie)	-10.7***(B)	-11.8***(S)	3.43***(MH)	-8.76***(B)	-8.58***(S)	2.74***(FI)	2.26**(B)	13.15***(B)	13.56***(S)
2. Using (c,q,k,k') data										
2.1. years: 98-99	2.31**(MH)	-5.74***(B)	-6.54***(S)	5.49***(MH)	-7.09***(B)	-8.10***(S)	3.05***(FI)	-0.81(tie)	10.49***(B)	12.09***(S)
2.2. years: 03-04	0.97(tie)	-5.32***(B)	-5.00***(S)	3.61***(MH)	-5.59***(B)	-5.49***(S)	2.50**(FI)	0.21(tie)	8.93***(B)	8.43***(S)
3. Using (c,q) data										
3.1. year: 98	0.26(tie)	1.52(tie)	1.71*(MH)	8.50***(MH)	1.36(tie)	1.72*(FI)	9.39***(FI)	0.27(tie)	9.80***(B)	12.98***(S)
3.2. year: 04	4.92***(MH)	0.62(tie)	0.42(tie)	7.99***(MH)	-2.31**(B)	-1.92*(S)	4.76***(FI)	-0.17(tie)	10.28***(B)	9.01***(S)
<u>Dynamics</u>										
4. Repeated Cross-Sections										
4.1. (k,k',q), yrs: 98-9 & 99-0	7.86***(MH)	-7.12***(B)	-4.38***(S)	14.8***(MH)	-15.2***(B)	-9.71***(S)	7.89***(FI)	3.34***(B)	20.8***(B)	18.2***(S)
4.2. (k,k',q), yrs: 98-9 & 03-4	4.28***(MH)	-7.97***(B)	-8.37***(S)	8.55***(MH)	-12.5***(B)	-14.9***(S)	7.72***(FI)	0.54(tie)	18.5***(B)	21.3***(S)
4.3. (c,q,k,k'), 98-9 & 99-0	-0.09(tie)	-4.51***(B)	-5.52***(S)	4.51***(MH)	-4.61***(B)	-5.64***(S)	4.78***(FI)	-0.50(tie)	10.3***(B)	11.3***(S)
4.4. (c,q,k,k'), 98-9 & 03-4	0.15(tie)	-5.10***(B)	-5.01***(S)	3.20***(MH)	-5.04***(B)	-4.96***(S)	2.84***(FI)	0.95(tie)	8.18***(B)	8.01***(S)
4.5. (c,q), yrs: 98 & 99	10.5***(MH)	2.82***(MH)	1.84*(MH)	15.3***(MH)	-2.52**(B)	-3.64***(S)	11.2***(FI)	-2.56**(S)	14.5***(B)	15.7***(S)
4.6. (c,q), yrs: 98 & 04	10.8***(MH)	3.72***(MH)	3.78***(MH)	13.8***(MH)	-2.58***(B)	-3.54***(S)	6.51***(FI)	-0.86(tie)	12.8***(B)	15.6***(S)
5. Two-Year Panel										
5.1. (c,q), yrs: 98 and 99	4.01***(MH)	2.62***(MH)	3.40***(MH)	10.4***(MH)	0.32(tie)	1.41(tie)	8.45***(FI)	1.77*(B)	8.89***(B)	7.57***(S)
5.2. (c,q), yrs: 98 and 04	1.06(tie)	-0.07(tie)	-0.82(tie)	6.97***(MH)	-0.70(tie)	-1.55(tie)	6.53***(FI)	-1.02(tie)	8.61***(B)	9.89***(S)

<u>NOTES:</u>

*** = 1%, ** = 5%, * = 10% two-sided significance level, the "winning" regime is in the parentheses

Z-stat cutoffs and colors: 2.575 = *** 1.96 = ** 1.645 = * "tie"

Table 16 - Summary of Model Comparisons Using Thai Data

Data used	sample size, n	# frequency cells	LL Ranking	% ties	% within group ties of all ties
1. Model tests using (k,k',q) data					
1.1. using '98-99 data	531	50	B=S, MH=FI, A	20%	100%
1.2. using '03-04 data	531	50	B, S, MH=FI, A	10%	100%
2. Model tests using (c,q,k,k') data					
2.1. using '98-99 data	531	500	S=B, MH, FI, A	10%	100%
2.2. using '03-04 data	531	500	B=S, MH=FI, A	20%	100%
3. Model tests using (c,q) data					
3.1. using '98 data	531	20	MH=FI=B, S, A	40%	50%
3.2. using '04 data	531	20	MH=S=B, FI, A	30%	33%
4. Using Repeated Cross-Sections					
4.1. using (k,k',q) data, years: '98-99 and '99-00	1062	100	B, S, MH, FI, A	0%	n.a.
4.2. using (k,k',q) data, years: '98-99 and '03-04	1062	100	B=S, MH, FI, A	10%	100%
4.3. using (c,q,k,k') data, years: '98-99 and '99-00	1062	1000	S=B, FI=MH, A	20%	100%
4.4. using (c,q,k,k') data, years: '98-99 and '03-04	1062	1000	B=S, MH=FI, A	20%	100%
4.4. using (c,q) data, years: '98 and '99	1062	40	MH, S, B, FI, A	0%	n.a.
4.6. using (c,q) data, years: '98 and '04	1062	40	MH, S=B, FI, A	10%	100%
5. Using 2-Year Panel					
5.1. using (c,q) data, years: '98 and '99	531	400	MH, FI=B, S, A	20%	0%
5.2. using (c,q) data, years: '98 and '04	531	400	S=B=MH=FI, A	60%	33%

Table 17 - Comparisons Using Model-Generated Data at Best-Fit Thai Data Estimates - Vuong Test Z-Statistics[1, 2]

Comparison	MH v FI	MH v B	MH v S	MH v A	FI v B	FI v S	FI v A	B v S	B v A	S v A
1. Using (k,k',q) data, n=531										
DGM[3]: B; est. γ_{me}= 0.1154	2.90***(MH)	-10.7***(B)	-8.75***(S)	4.43***(MH)	-11.5***(B)	-10.2***(S)	2.88***(FI)	1.27(tie)	14.03***(B)	12.18***(S)
2. Using (c,q,k,k') data, n=531										
DGM: S; est. γ_{me}= 0.2208	1.59(tie)	-8.15***(B)	-11.9***(S)	3.22***(MH)	-9.57***(B)	12.9***(S)	1.69*(FI)	-4.55***(S)	11.23***(B)	13.92***(S)
3. Using (c,q) data, n=531										
DGM: MH; est. γ_{me}= 0.1045	2.63***(MH)	8.30***(MH)	7.70***(S)	19.4***(MH)	6.90***(FI)	6.59***(FI)	17.8***(FI)	-1.14(tie)	13.10***(B)	15.78***(S)
4. Using Repeated Cross-Sections, n=1062										
4.1. using (k,k',q) data										
DGM: B, 1-yr; est.γ_{me}= 0.0898	4.03***(MH)	-11.5***(B)	-9.02***(S)	4.72***(MH)	-16.6***(B)	-13.9***(S)	3.13***(FI)	0.64(tie)	9.40***(B)	9.88***(S)
DGM: B, 5-yr; est.γ_{me}= 0.0564	3.08***(MH)	-18.6***(B)	-6.91***(S)	1.88*(MH)	-18.2***(B)	-9.40***(S)	0.08(tie)	4.51***(B)	11.6***(B)	6.82***(S)
4.2. using (c,q,k,k') data										
DGM: S, 1-yr; est.γ_{me}= 0.2386	1.49(tie)	-8.06***(B)	-14.7***(S)	6.67***(MH)	-9.37***(B)	-16.9***(S)	5.44***(FI)	-6.16***(S)	14.34***(B)	19.78***(S)
DGM: B, 5-yr; est.γ_{me}= 0.3124	0.85(tie)	-13.9***(B)	-13.8***(S)	5.75***(MH)	-14.8***(B)	-14.7***(S)	5.35***(FI)	1.38(tie)	17.60***(B)	17.49***(S)
4.3. using (c,q) data										
DGM: MH, 1-yr; est.γ_{me}= 0.1059	0.59(tie)	13.7***(MH)	17.3***(MH)	35.9***(MH)	6.69***(FI)	7.15***(FI)	15.3***(FI)	0.58(tie)	17.10***(B)	20.86***(S)
DGM: MH, 6-yr; est.γ_{me}= 0.1147	3.86***(MH)	12.1***(MH)	11.8***(MH)	26.3***(MH)	11.4***(MH)	11.1***(FI)	24.6***(FI)	-3.29***(S)	28.35***(B)	31.43***(S)
5. Two-Year Panel of (c,q), n=531										
DGM: MH, 1-yr; est.γ_{me}=0.1012	3.16***(MH)	9.02***(MH)	9.42***(MH)	17.8***(MH)	6.99***(FI)	6.79***(FI)	15.2***(FI)	-0.59(tie)	8.54***(B)	10.20***(S)
DGM: S, 6-yr; est.γ_{me}=0.1872	3.78***(MH)	-1.20(tie)	-1.16(tie)	7.28***(MH)	-4.83***(B)	-4.80***(S)	4.49***(FI)	0.03(tie)	8.05***(B)	7.61***(S)

NOTES:

1. *** = 1%, ** = 5%, * = 10% two-sided significance level, the "winning" regime is in the parentheses
2. Z-stat cutoffs and colors: 2.575 = *** 1.96 = ** 1.645 = * "tie"
3. DGM: data-generating model

Figure 1 − Financial Indicators − baseline parameters

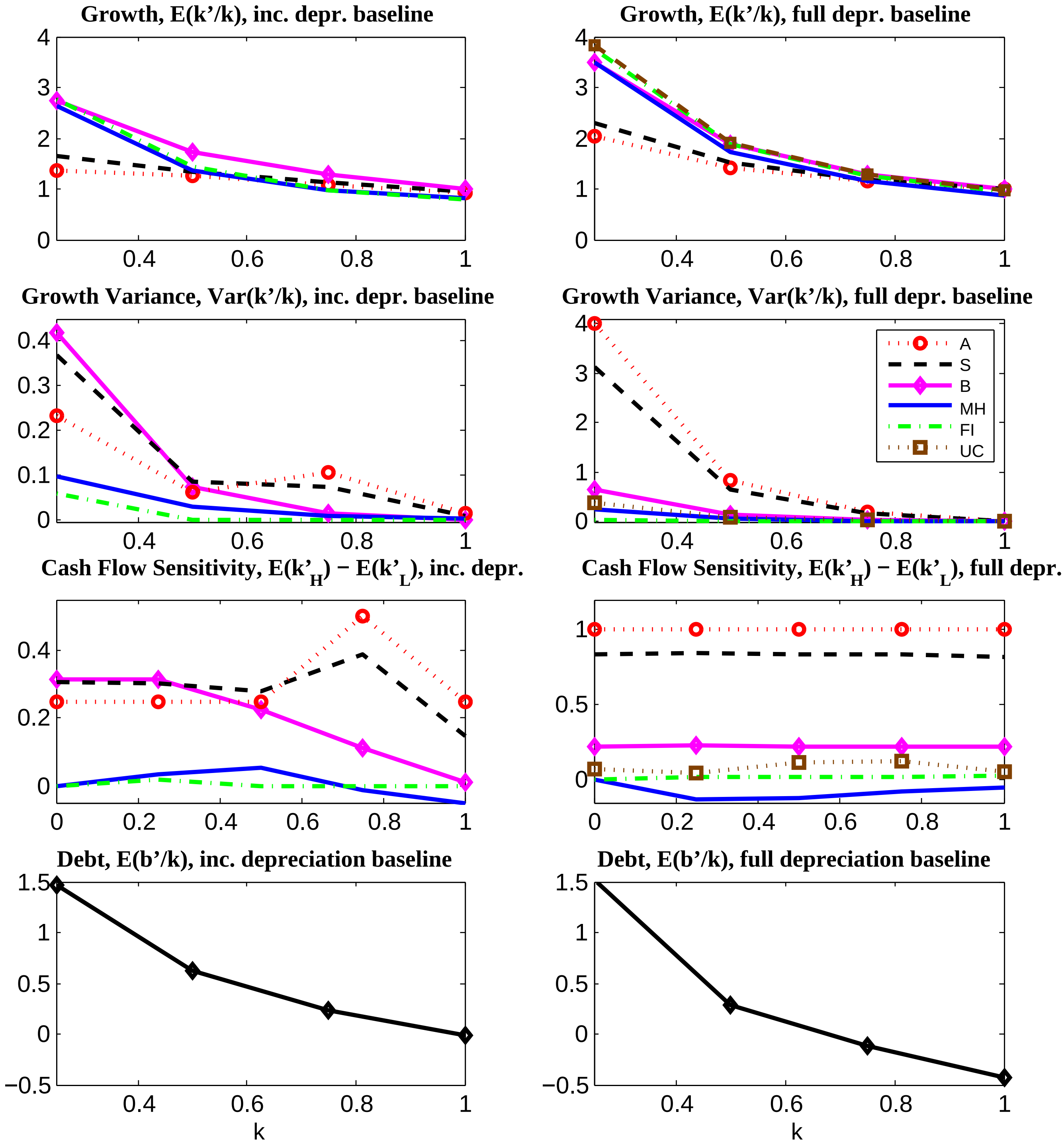

Fig. 2a − Consumption, full depreciation baseline

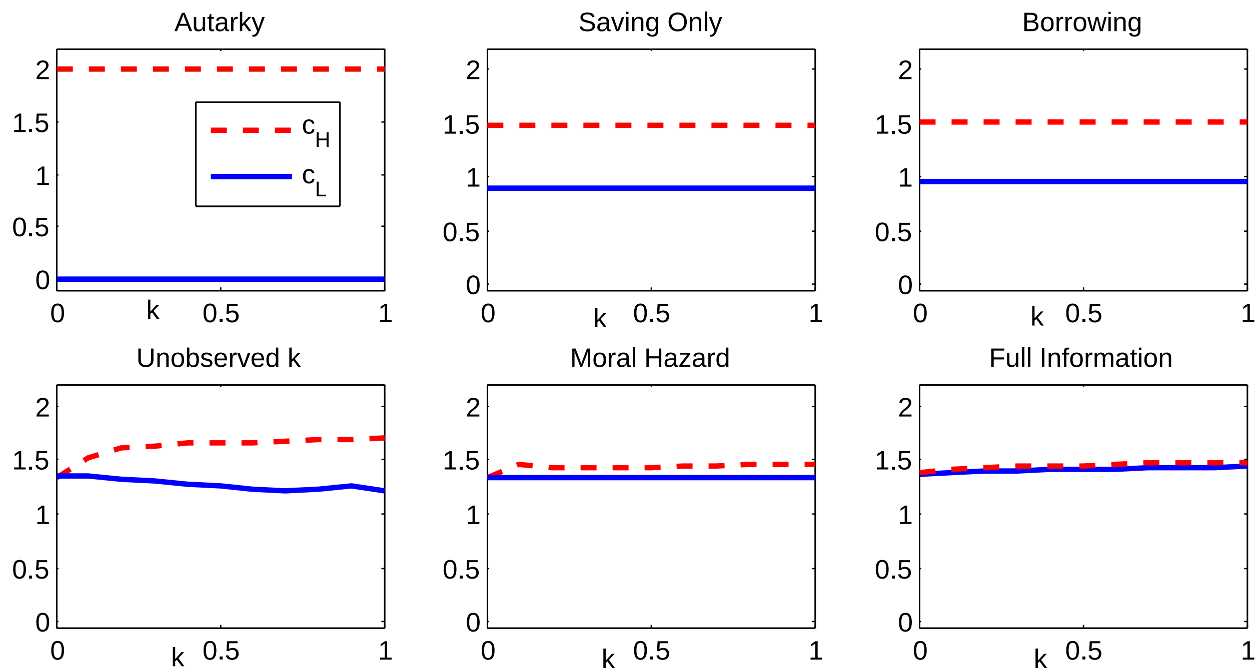

Fig. 2b − Consumption, incomplete depreciation baseline

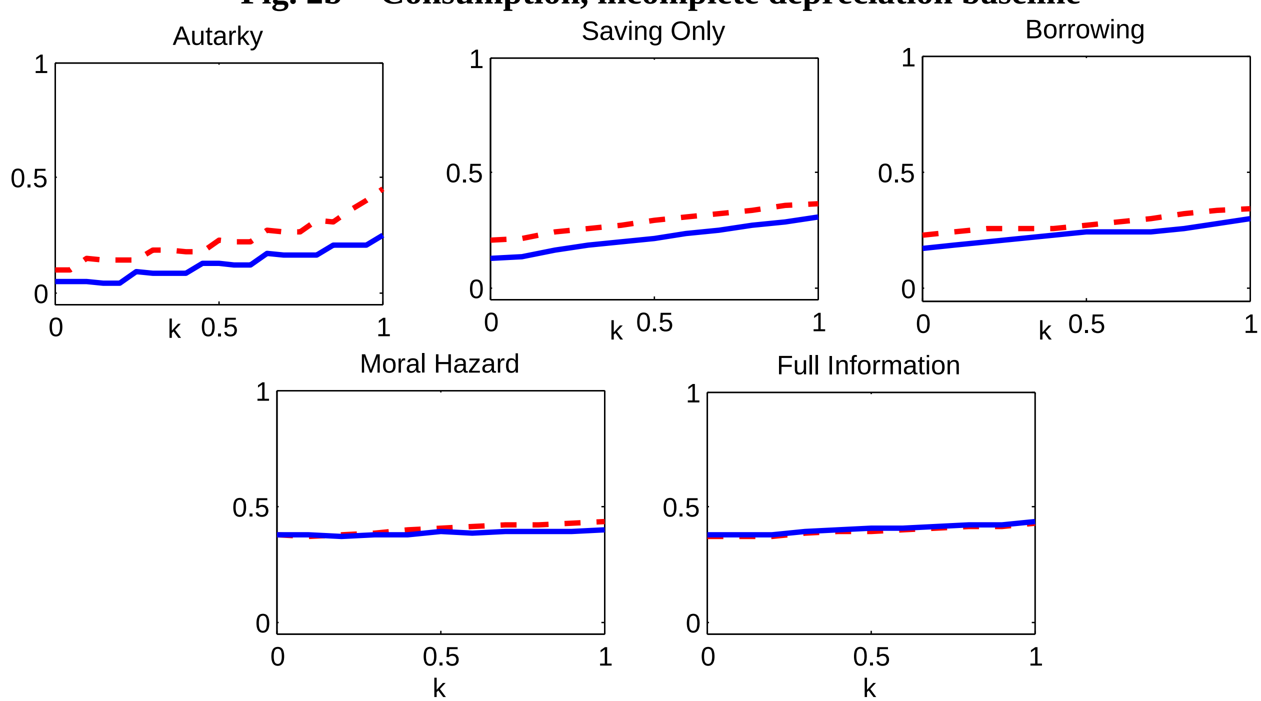

Fig. 3 − Time Paths − baseline parameters

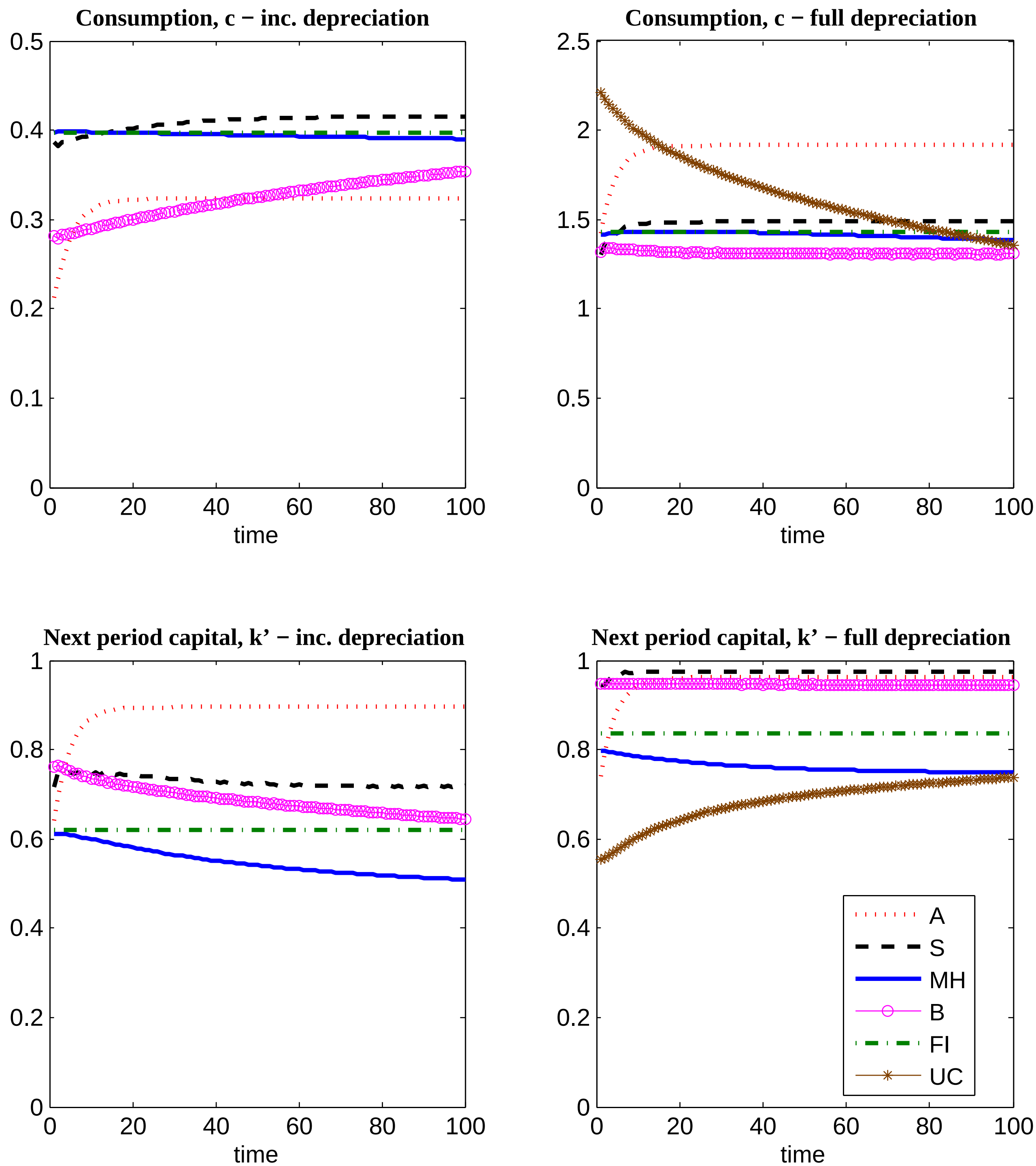

Fig. 4 − Financial Indicators, estimated parameters, δ=0.05

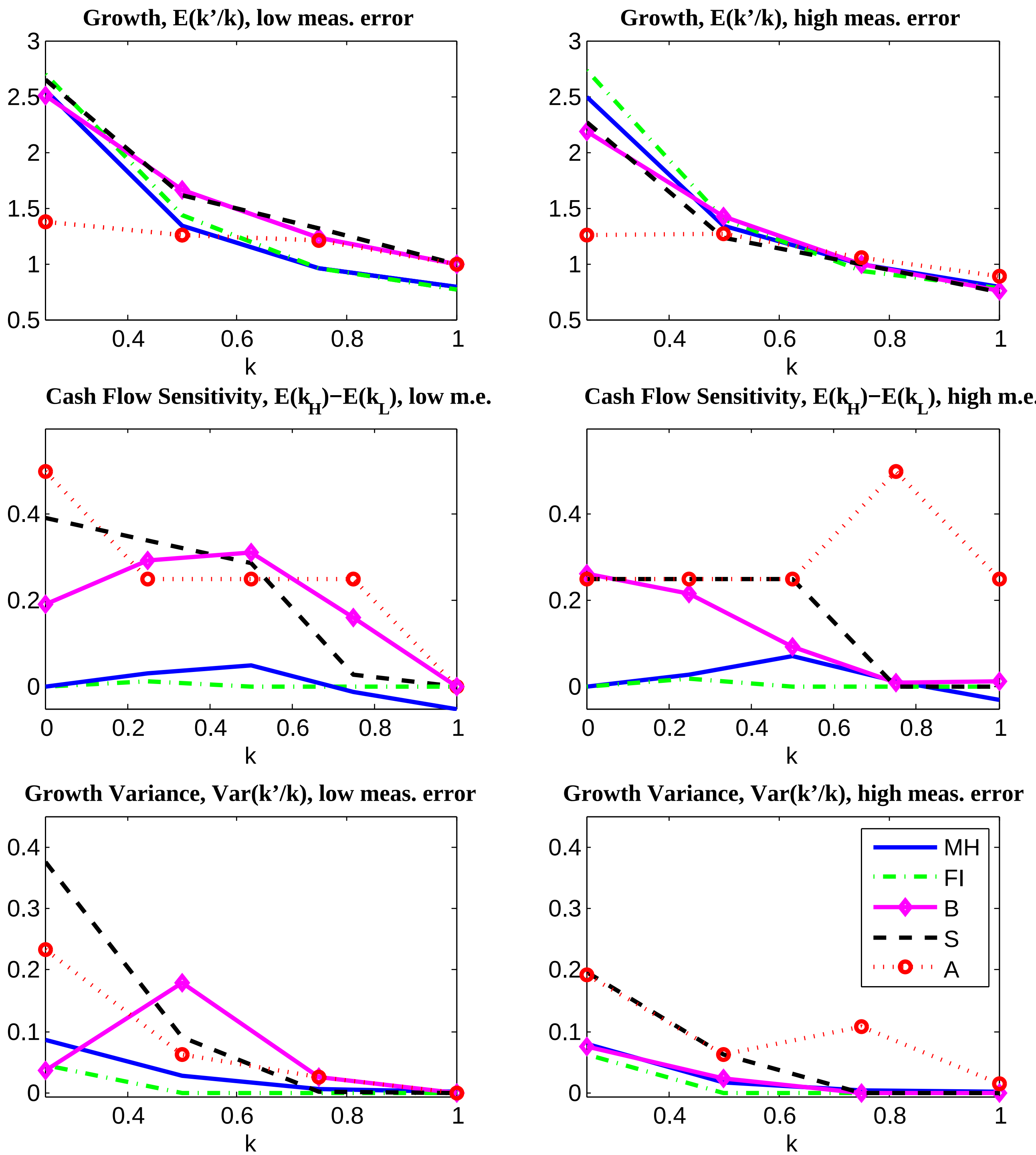

Fig. 5 − Data and Model Histograms using (k, k’, q) data

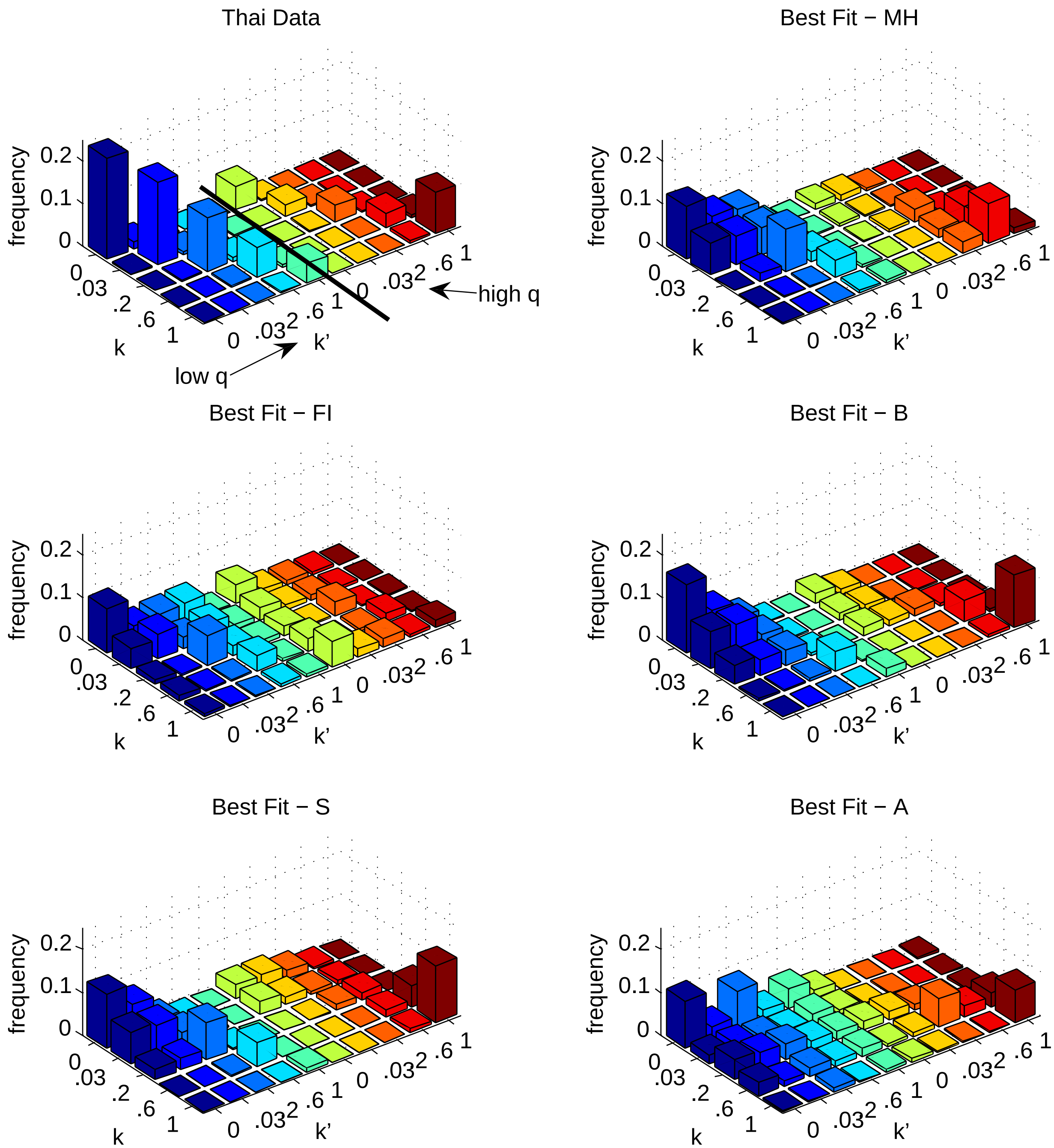

Fig. 6 − Data and Model Histograms using (c, q) data

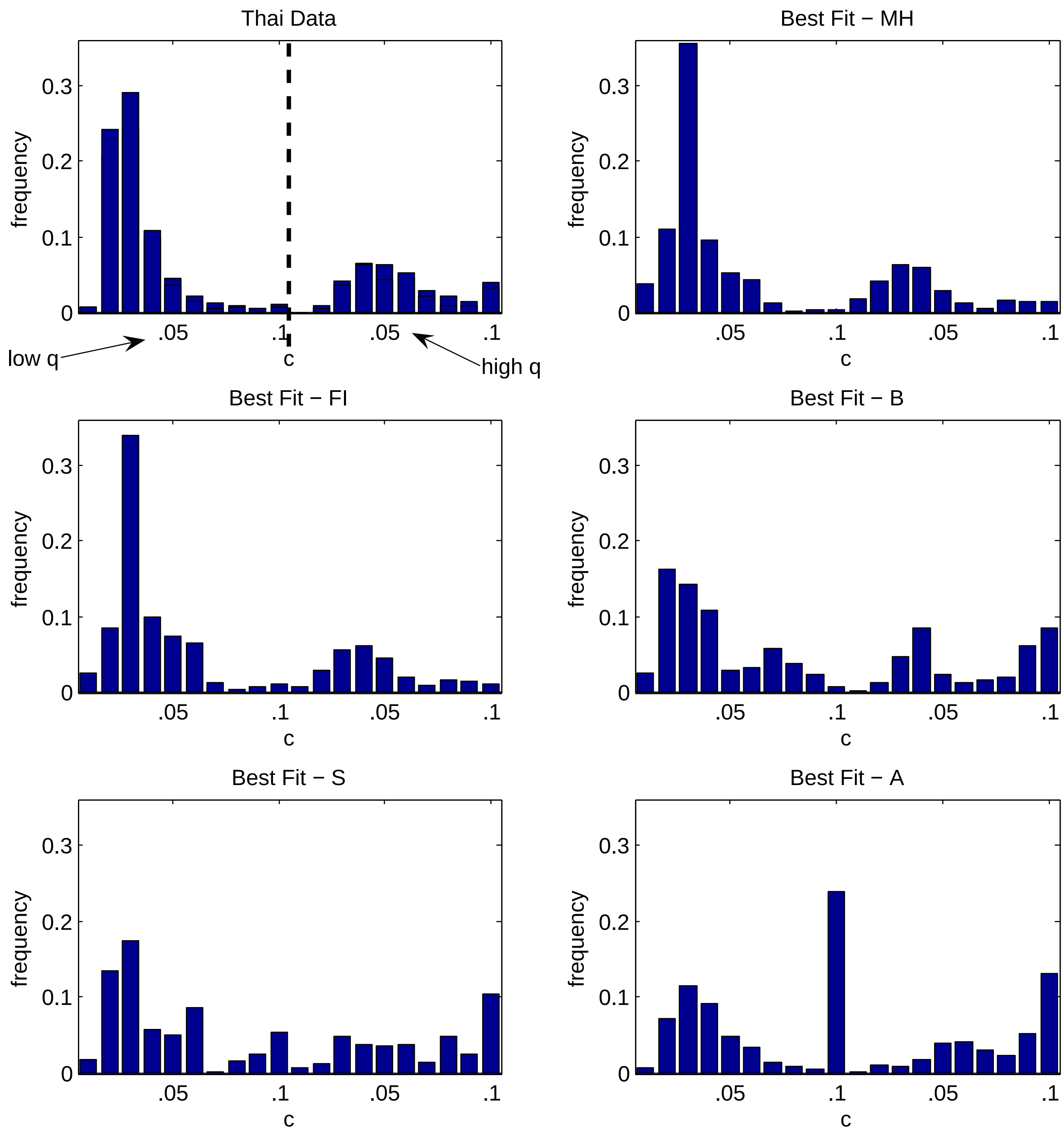

Fig. A1 − Pareto Frontiers, incomplete depreciation baseline

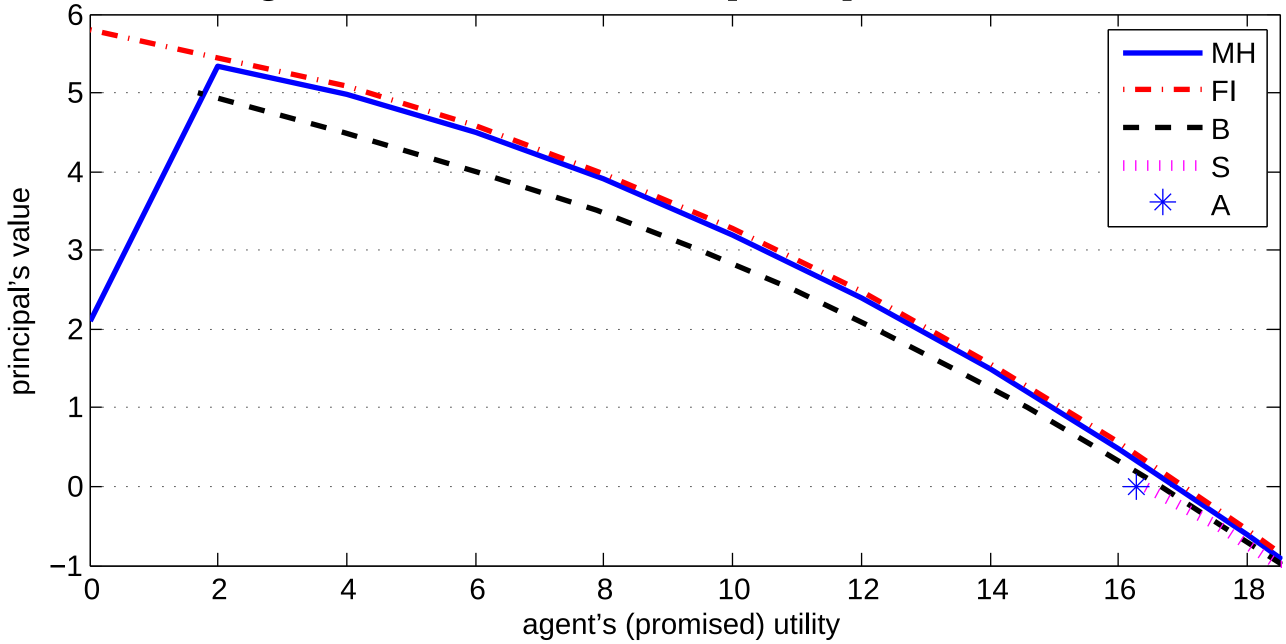

Fig. A2 − Pareto Frontiers, full depreciation baseline