Wage Bargaining with On-the-job Search:  
A Structural Econometric Model\textsuperscript{1}

Pierre Cahuc  
Université de Paris I, Paris,  
CREST-INSEE, Malakoff,  
CEPR, London

Fabien Postel-Vinay\textsuperscript{2}  
INRA-Paris Jourdan, Paris,  
CREST-INSEE, Malakoff,  
CEPR, London

Jean-Marc Robin  
Université de Paris I, Paris,  
CREST-INSEE, Malakoff,  
INRA Paris-Jourdan, Paris,  
CEPR, London on

November 12, 2002

\textsuperscript{1} Conversations with Bruno Crépon, Zvi Eckstein, Francis Kramarz and Barbara Petrongolo were helpful for the preparation of this paper. The authors also wish to thank participants in the following conferences and workshops: the CEPR DAEUP meeting in Paris (May 2002), the CEPR/IZA ESSLE meeting in Buch an Amersee (Sept. 2002), ERC conference in Chicago (Oct. 2002). The customary disclaimer applies.

\textsuperscript{2} Corresponding author: INRA Paris-Jourdan, Ecole Normale Superieure, 48 boulevard Jourdan 75014 Paris, France. Email: fpostel@delta.ens.fr
Abstract. We write and estimate an equilibrium model with strategic wage bargaining and on-the-job search and use it to take another look at the determinants of wages in France. There are three essential determinants of wages in our model: productivity, competition between employers resulting from on-the-job search, and the workers’ bargaining power. We find that between-firm competition matters a lot in the determination of wages. In particular, we detect no significant bargaining power for unskilled workers. However, inter-industry differentials are mainly due to differences in productivity, and bargaining power, in the case of skilled workers.
1 Introduction

The empirical studies that estimate the impact of observable workers characteristics on wages typically explain no more than 50% of the variation in compensation across individuals. On the other hand, many empirical studies have shown that wage differentials are influenced by differences in pay policies across firms (Abowd et al., 1999). These findings, which suggest that similar workers employed in different firms can be paid differently, have fuelled a strand of literature that stresses the importance of labor market frictions for understanding wage determination (Mortensen, 2002, provides a recent survey). It is well known that by preventing workers from bringing employers into Bertrand wage competition, search frictions give strictly positive value to formed firm-worker pairs, and they also give some local monopsony power to the firms. In this context, that is a rent sharing mechanism that explains why similar workers are paid differently.

Models with search frictions have highlighted two reasons why workers can get some part of the rents. The so-called matching model (Pissarides, 2000, Mortensen and Pissarides, 1999) focuses on bilateral bargaining between firms and workers as the prevailing rent sharing mechanism. In this model, wages are continuously renegotiated by Nash bargaining over the expected match value. Even when on-the-job-search is allowed it is assumed that the worker who is being offered an alternative to his or her current job takes the best outcome of two separate Nash bargaining games, where the worker’s outside option is supposed to be unemployment, so that on-the-job search does not give rise to between-firm competition. In the wage posting/equilibrium search models à la Burdett and Mortensen (1998), in which employers make take-it-or-leave-it offers, on-the-job search leads employers to leave a part of the rents to their employees in order to reduce labor turnover.

The principal argument of our paper is that the Bertrand competition paradigm, which is at the root of the competitive model of wage formation, may have been too quickly abandoned.¹

¹A certain schizophrenia nonetheless persists in the profession as the competitive paradigm seems at least
There is little doubt that workers are unable to instantaneously or costlessly bring employers into Bertrand wage competition, the outcome of which would be marginal productivity payment. Search frictions limit competition and imperfect information about the location of available employment opportunities, their scarcity, are definite sources of rent appropriation by firms. Yet, limited competition does not mean absent competition.

In previous work, Postel-Vinay and Robin (2002a, 2002b) have explored this idea in a formal equilibrium search model sharing many features with the models of Mortensen (1990) and Burdett and Mortensen (1998), except that Postel-Vinay and Robin did not assume wage posting. Wage posting models assume that firms make take-it-or-leave-it offers to workers and rule out any possibility for employers to counter the attempts by competitors to poach their employees. Postel-Vinay’s and Robin’s model also assumes that firms make take-it-or-leave-it offers to unemployed workers. The situation is different when an employee is contacted by another employer because raising an outside offer is the means by which an employee can activate the between-employer Bertrand competition. The incumbent employer and the poacher are forced, by a well understood interest, to compete for the worker, this competition resulting either in a wage raise or in a job mobility if the poacher’s reservation value (the match productivity) is higher than the incumbent’s.

The present paper extends the equilibrium search model with sequential auctions of Postel-Vinay and Robin by replacing Bertrand competition by a more general wage bargaining scheme involving workers and all the incumbent and other would-be employers that are brought into contact. In this perspective, we construct simple strategic bargaining games reflecting in a stylized way the most prominent features of the negotiation and renegotiation of employment contracts. Specifically, we make a sharp distinction between negotiation on new matches and renegotiation on continuing jobs. The former always leads to severance in case of disagreement.

as widely accepted in other empirical segments of labor economics. Mincerian wage equations, supposed to exclusively reflect productivity differences, are indeed endorsed without restriction whatsoever by many major empirical contributions ranging from Heckman’s various estimates of the Roy model, to dynamic discrete choice models, of which Keane and Wolpin (1994) is a perfect example.
The latter, started by employees receiving an outside offer, involves between-firm competition and allows the parties to continue under the terms of the current contract in case of disagreement. Our model thus explains why and when renegotiations occur and suggests that negotiation and renegotiation put the parties in different situations.

This approach allows us to evaluate the separate contributions of between-firm competition and the bargaining power on wages. This issue is important for many purposes. For instance, according to the so-called Hosios-Pissarides condition, the labor market is efficient if the surplus share accruing to workers takes a certain value, hinging on properties of the matching function (Hosios, 1990, Pissarides, 2000). From this perspective, estimating the bargaining power is a first step towards a proper evaluation of labor market efficiency. Also, reducing the workers’ bargaining power can be thought of as a policy to cut unemployment. A biased measure of this bargaining power, that does not disentangle between-firm competition from wage bargaining effects can cause the implementation of policies aiming at reducing the workers’ bargaining power when it is not needed.

**Related literature.** Probably the paper most closely related to ours is Dey and Flinn (2000). They represent the negotiation process by the Nash bargaining solution in the presence of between-firm competition for workers. Their idea is that a worker who is currently employed at a wage $w_0$ and receives an outside offer bargains with the poaching firm on the basis that if the negotiation fails the current wage contract $w_0$ will prevail. Let $w_1$ be the wage thus negotiated with the poacher. In a second step, the worker uses this offer $w_1$ as an outside option to negotiate a new wage, $w_2$, with his/her current employer. A sequence of bilateral Nash bargaining games is played instantaneously until one of the firms can no longer bid up, that is when the sequence of wage offers has reached the smallest firm reservation value or match productivity. Our contribution provides an explicit non-cooperative bargaining game that relies on a precise definition of the strategic interactions at work in the wage renegotiation. Moreover, Dey and Flinn consider a more complex framework, with multidimensional employment contracts
stipulating wages and health insurance provisions, that does not yield wage equations allowing the estimation of the worker’s bargaining power. In our paper, we do estimate the bargaining power using matched worker-firm panel data from France.\footnote{An earlier, also closely related paper assumes that workers may not search sequentially. Burdett and Judd (1982) show that if unemployed workers have a probability strictly between 0 and 1 of receiving more than one wage offer at a time, then the equilibrium wage distribution is necessarily dispersed even if all firms and workers are alike and even without on-the-job search.}

The reader who is familiar with the early contribution of Eckstein and Wolpin (1995) will understand that our work can also be viewed as an extension of theirs. Their paper is the first paper that we know of that has proceeded to the estimation of a fully specified general equilibrium model of wage bargaining on micro data. Eckstein and Wolpin adapted the standard Pissarides matching model to fit the micro data of the NLSY. One is then entitled to ask how we shall be able to identify workers’ bargaining power when Eckstein and Wolpin claimed that it was not possible to separately identify the location parameter of the distribution of match productivities from the bargaining power parameter. This is because they used worker data and did not observe match productivity separately from wages. We do not face this identification problem as we use both worker and firm data, which will allow us to estimate a system of two equations: a production function and a wage equation, allowing for a separate identification of the parameters of the production function and the wage equation.

Outline and main results. In the following section of this paper, we develop formal non-cooperative negotiation and renegotiation games which allow us to express wages as functions of worker ability, firm productivity, matching frictions and the bargaining power of workers. Our first contribution is to provide closed-form expressions for wages and wage distributions that hinge on these four elements in a unified theoretical model.

The empirical implementation of the model is presented in section 3. We use the framework of section 2 to estimate the influence of productivity, between-firm competition and the bargaining power of workers on wages. Because it is particularly important to separately iden-
tify the different sources of wage dispersion, we choose to implement a multi-stage estimation procedure. We first estimate the friction parameters (arrival rate of job offers, job destruction rate) from worker data on employment spell durations. We then use a firm panel of data on value-added and employment differentiated by skill to estimate labor productivity at the firm level. Lastly, we relate mean wages per firm to productivity to estimate bargaining powers.

Section 4 presents some empirical applications.

First, it is shown that ignoring on-the-job search causes substantial upwards biases in the bargaining power estimates. In particular, we find that “unskilled” workers (workers with no managerial tasks) have zero bargaining power in most industries whereas “skilled” workers (supervisors, managers of all ranks and educated engineers) generally have positive bargaining power, the extent of which varies across industries. This result suggests that most previous empirical studies overestimated the bargaining power. These studies (a non exhaustive list of the papers includes Abowd and Lemieux, 1993, Blanchflower et al., 1996, Van Reenen, 1996, Margolis and Salvanes, 2001, Kramarz, 2002), based on simple static models where some bargaining process leads to splitting the job surplus, typically defined as the difference between productivity and an outside wage that depends on worker characteristics and selected labor market variables such as the (local) unemployment rate and the industry- or economy-wide mean wage.

Then, we use our model to take another look at the determinants of inter-industry wage differentials. There are three essential determinants of inter-industry wage differentials in our model: productivity, between-firm competition, and the bargaining power. We find that, even though taking account of job-to-job mobility matters in the determination of wages, inter-industry differentials are mainly due to differences in productivity and bargaining power. It is worth stressing that these results do not mean that between-firm competition does not influence wages. Actually, our empirical results suggest that it does play a crucial role. However, it turns out that the differences in the intensity of between-firm competition across industries, although
significant, explain a very small share of inter-industry wage differentials. In this section, we also analyze inter-skill wage differentials and find, with no surprise, that skill wage differentials essentially reflects productivity differences.

2 Theory

We first describe the characteristics and objectives of workers and firms. The matching process and the negotiation game that workers and firms play to determine wages is then explained. In the third and last subsection, the steady-state general equilibrium of this economy is characterized.

2.1 Workers and firms

We consider a labor market in which a measure $M$ of atomistic workers face a continuum of competitive firms, with a mass normalized to 1, that produce one unique multi-purpose good. Time is continuous, workers and firms live forever. The market unemployment rate is denoted as $u$. The pool of unemployed workers is steadily fueled by layoffs that occur at the exogenous rate $\delta$.

Workers have different professional skills. A given worker’s ability is measured by the amount $\varepsilon$ of efficiency units of labor she/he supplies per unit time. The distribution of ability values in the population of workers is exogenous, with cdf $H$ over the interval $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$. We only consider continuous ability distributions and further designate the corresponding density by $h$.

Summing over all employee ability values for a given firm defines the efficient firm size. The marginal productivity of efficient labor is denoted as $p$. Firms differ in the technologies that they operate, meaning that parameter $p$ is distributed across firms with a cdf $\Gamma$ over the support $[p_{\text{min}}, p_{\text{max}}]$. This distribution is assumed continuous with density $\gamma$. The marginal productivity of the match $(\varepsilon, p)$ of a worker with ability $\varepsilon$ and a firm with technology $p$ is $\varepsilon p$.

A type-$\varepsilon$ unemployed worker receives an income flow of $\varepsilon b$, with $b$ a positive constant,
which he has to forgo from the moment he finds a job. Being unemployed is thus equivalent to working at a “virtual” firm with labor productivity equal to $b$ that would operate on a frictionless competitive labor market, therefore paying each employee their marginal productivity, $\varepsilon b$.

Workers discount the future at an exogenous and constant rate $\rho > 0$ and seek to maximize the expected discounted sum of future utility flows. The instantaneous utility flow enjoyed from a flow of income $x$ is $U(x) = x^\gamma$. Firms maximize profits.

2.2 Matching and wage bargaining

Firms and workers are brought together pairwise through a sequential, random and time-consuming search process. Specifically, unemployed workers sample job offers sequentially at a Poisson rate $\lambda_0$. As in the original Burdett and Mortensen (1998) paper, employees may also search for a better job while employed. The arrival rate of offers to on-the-job searchers is $\lambda_1$. The type $p$ of the firm from which a given offer originates is assumed to be randomly selected in $[p_{\min}, p_{\max}]$ according to a sampling distribution with cdf $F$ (and $\bar{F} \equiv 1 - F$) and density $f$. The sampling distribution is the same for all workers irrespective of their ability or employment status. Note that we a priori assume no connection between the probability density of sampling a firm of given type $p$, $f(p)$, and the density $\gamma(p)$ of such types in the population of firms. When a match is formed, the wage contract is negotiated between the different parties according to the following rules.

2.2.1 Assumptions

Wages are bargained over by workers and employers in a complete information context. In particular, all agents that are brought to interact by the random matching process are perfectly aware of one another’s types. All wage and job offers are also perfectly observed and verifiable. Specifically, we make the following three assumptions about wage strategies and wage contracts:

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3 This is merely for simplicity. The theoretical model is tractable with an arbitrary utility function (provided that intertemporal transfers are ruled out), and the empirical analysis can in principle be conducted for any CRRA specification (see Postel-Vinay and Robin, 2002).
Assumption A1 Wage contracts stipulate a fixed wage that can be renegotiated by mutual agreement only.

Assumption A1 implies that renegotiations occur only if one party can credibly threaten the other to leave the match for good if the latter refuses to renegotiate. In our framework, renegotiations can be triggered only when employees receive outside offers. The assumption of renegotiation by mutual agreement captures an important and often neglected feature of employment contracts (see the enlightening survey by Malcomson, 1999).

The following two assumptions describe the structure of the negotiation game that is played by an unemployed worker and an employer (Assumption A2), and that of the renegotiation game that is played by a currently employed worker, his/her current employer and a poaching employer (Assumption A3).

Assumption A2 When an unemployed worker meets a firm, the wage is determined according to the following bargaining game:

1. The firm makes a wage offer;
2. The worker either accepts the offer and signs the contract, or s/he rejects it;
3. In case of rejection at step 2, some time elapses. Then:
   - With probability $\beta$, the worker makes a wage offer;
   - With probability $1 - \beta$, the firm makes a wage offer;
4. The player who has received the offer at step 3 either accepts it and signs the contract, or rejects it. In case of rejection the match ends and the worker remains unemployed.

Assumption A3 An employed worker who receives an outside job offer renegotiates his/her wage according to the following game:

1. The firms make simultaneous noncooperative wage offers;
2. The worker either chooses one wage offer and signs a new contract or keeps the pre-existing contract;

3. If the worker has chosen one wage offer at step 2, some time elapses. Then the players can participate in the following game:

   - With probability $\beta$, the worker makes separate wage offers to both employers;
   - With probability $1 - \beta$, the firms make simultaneous non-cooperative wage offers.

4. Any player who has received an offer at step 3 either accepts or rejects it. In case of disagreement at step 4, the worker’s decision at step 2 prevails. In case of agreement between the worker and either firm, a new contract is signed. The worker chooses among the firms if both accept the offer s/he made at step 3.

Assumptions A2 and A3 describe two very simple strategic negotiation games adapted from Osborne and Rubinstein (1990). The seminal contributions of Binmore, Rubinstein and Wolinsky (1986) and Osborne and Rubinstein (1990) have shown that the Nash sharing rule can be derived from strategic bargaining games that are very useful to properly define the threat payoffs. Obviously, any strategic bargaining game is necessarily peculiar. Our game has been designed to provide a simple and tractable tool to understand the renegotiation process in the presence of between-firm competition for workers.

The negotiation game that is played between two firms and an initially employed worker looks like the game between a firm and an unemployed worker except that the former has three players instead of two. Steps 1 and 2 have been modified to enable the worker to maneuver in order to get himself an optimal credible threat point in the renegotiation subgame (steps 3 to 4). Namely, if the worker accepts the offer of the poaching firm at step 2, he quits the incumbent firm and this offer becomes his threat point in the renegotiation. Conversely, his threat point is the offer of the incumbent employer if that offer is accepted at step 2. This game can appear somewhat unrealistic at first glance, as it gives the employee the option to
momentarily quit the incumbent employer to eventually come back with a new contract at the end of the renegotiation. Such back-and-forth worker movements don’t happen in the real world. Neither do they in our game, as we wish to emphasize, since temporarily quitting to a less attractive employer is only a threat available for the worker to use, which is never implemented in equilibrium.

It is also worth insisting on the fact that whenever the worker receives an outside offer, the pre-existing contract with the incumbent employer prevails if no agreement is reached (at step 2). This is an important difference with the negotiation on new matches—between unemployed workers and firms—that are dissolved in case of disagreement. We view this assumption as more in accordance with actual labor market institutions than the usual one according to which matches always break up in case of renegotiation failure (Pissarides, 2000, Mortensen and Pissarides, 1999). It is indeed legally considered in most OECD countries that an offer to modify the terms of a contract does not constitute a repudiation. Accordingly, a rejection of the offer by either party leaves the pre-existing terms in place, which means that the job continues under those terms if the renegotiation fails (Malcomson, 1999, p. 2,321).

2.2.2 Wage contracts and job mobility

We now exploit the preceding series of assumptions to derive the precise values of wages and the job mobility patterns.

The subgame perfect equilibria of the two bargaining games described above are characterized in Appendix A.1. In both games the worker receives a share $\beta$ of the match rent. Let $V_0(\varepsilon)$ denote the lifetime utility of an unemployed worker of type $\varepsilon$ and $V(\varepsilon, w, p)$ that of the same worker when employed at a firm of type $p$ and paid a wage $w$. The rent of a match between a type-$\varepsilon$ unemployed worker and a type-$p$ job amounts to $V(\varepsilon, \varepsilon p, p) - V_0(\varepsilon)$. It is shown in the Appendix that the wage bargained on a match between a type-$\varepsilon$ unemployed worker and a
type-\(p\) firm, denoted as \(\phi_0(\varepsilon,p)\), solves:

\[
V(\varepsilon, \phi_0(\varepsilon,p), p) = V_0(\varepsilon) + \beta [V(\varepsilon, \varepsilon p, p) - V_0(\varepsilon)].
\] (1)

This equation merely states that a type-\(\varepsilon\) unemployed worker matched with a type-\(p\) firm gets his reservation utility, \(V_0(\varepsilon)\), plus a share \(\beta\) of the rent accruing to the job.

The assumption of long term contracts, renegotiated by mutual agreement only, implies that wages can be renegotiated only if employees receive new job offers. Moreover, an employee paid a wage \(w\) in a type-\(p\) firm and who receives an outside offer from a type-\(p'\) firm is willing to trigger a renegotiation only if firm \(p'\) is competitive enough:

If \(p' \leq p\), the worker stays at the type-\(p\) firm, because the match with the type-\(p'\) firm is associated with a lower rent. However, the employee can get wage increases if \(p'\) is sufficiently high in regard of his/her current wage, \(w\). If the employee triggers a renegotiation (by accepting the poacher’s first offer at step 2), he eventually stays at his initial firm (the type \(p\) firm) with a new wage \(\phi(\varepsilon, p', p)\) as defined by:

\[
V(\varepsilon, \phi(\varepsilon, p', p), p) = V(\varepsilon, \varepsilon p', p') + \beta \left[ V(\varepsilon, \varepsilon p, p) - V(\varepsilon, \varepsilon p', p') \right].
\] (2)

Obviously, the employee decides to trigger a renegotiation only if it is a way to get a wage increase, i.e. if the productivity parameter of the new match, \(p'\), exceeds a threshold value \(q(\varepsilon, w, p)\), that satisfies:

\[
\phi(\varepsilon, q(\varepsilon, w, p), p) = w.
\] (3)

Let us insist a bit on the role played by the game structure at this point. Note that

\[
V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)) = V(\varepsilon, w, p) - \frac{\beta}{1 - \beta} [V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p)]
\]

\[
\leq V(\varepsilon, w, p)
\]

(with strict inequality if \(w < p\)). The observant reader will thus have noticed that an outside offer from a type \(p'\) firm can result in a wage increase even when \(V(\varepsilon, \varepsilon p', p') < V(\varepsilon, w, p)\), i.e. even when the poacher’s productivity is so low that it can’t even afford to compensate the
worker for his/her pre-existing value $V(\varepsilon, w, p)$. This results from the existence of steps 3 and 4, which ensure that the worker can credibly threaten to accept the weaker firm’s offer at step 2, even in cases where that offer is lower than what s/he would have gotten at status quo. In other words, in order to force his/her incumbent employer to renegotiate, the worker is willing to “take the chance” of accepting a very unattractive offer from the poacher because s/he knows that it is then in the interest of her incumbent employer to attract him/her back with a wage increase at later stages of the renegotiation game.

If $p' > p$, the outside offer creates a (private) rent supplement equal to $V(\varepsilon, \varepsilon p', p') - V(\varepsilon, \varepsilon p, p)$. The renegotiation game thus implies that the worker moves to the type-$p'$ job, where s/he gets a wage $\phi(\varepsilon, p, p')$ that solves:

$$V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p) + \beta \left[ V(\varepsilon, \varepsilon p', p') - V(\varepsilon, \varepsilon p, p) \right].$$

(4)

It can be seen that an employee who moves from a type-$p$ to a type-$p'$ firm gets a value equal to the maximum that s/he could get from staying at the type-$p$ firm, plus a share $\beta$ of the new match rent. Note that the wage $\phi(\varepsilon, p, p')$ obtained in the new firm can be smaller than the wage $w$ paid in the previous job, because the worker expects larger wage raises in firms with higher productivity.

To sum up, one of the following three situations may arise when a type-$\varepsilon$ worker, paid a wage $w$ by a type-$p$ firm, receives a type-$p'$ job offer:

(i) $p' \leq q(\varepsilon, w, p)$, and nothing changes.

(ii) $p \geq p' > q(\varepsilon, w, p)$, and the worker obtains a wage raise $\phi(\varepsilon, p', p) - w > 0$ from his/her current employer.

(iii) $p' > p$, and the worker moves to firm $p'$ for a wage $\phi(\varepsilon, p, p')$ that may be greater or smaller than $w$.

Before we go any further, we should note that Dey and Flinn (2000) have reached similar sharing rules to those just derived in a similar framework by applying the Nash bargaining
solution. Our contribution shows that this result can be derived from a precisely defined strategic bargaining game compatible with job continuation when renegotiations fail.4

The precise form of wages can be obtained from the expressions of lifetime utilities (see Appendix A.2 for the corresponding algebra). The wage \( \phi(\varepsilon, p', p) \) of a type-\( \varepsilon \) worker, currently working at a type-\( p \) firm and whose last job offer was made by a type-\( p_0 \) firm, is defined by:

\[
\phi(\varepsilon, p', p) = \varepsilon \cdot \left( p - (1 - \beta) \int_{p'}^{p} \frac{\rho + \delta + \lambda_1 F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx \right). 
\]

This expression shows that the returns to on-the-job search depend on the bargaining power parameter \( \beta \). It can be seen that outside offers trigger wage increases within the firm only if employers have some bargaining power. In the limiting case where \( \beta = 1 \), the worker appropriates all the surplus up-front and gets a wage equal to \( \varepsilon p \), whether or not s/he searches on the job. In the opposite extreme case, where \( \beta = 0 \), the wage increases as outside offers come since all offers from firms of type \( p' \in (q(\varepsilon, w, p), p) \) provoke within-firm wage raises.

The wage \( \phi_0(\varepsilon, p) \), obtained by a type-\( \varepsilon \) unemployed workers when matched with a type-\( p \) firm, writes as:

\[
\phi_0(\varepsilon, p) = \varepsilon \cdot \left( p_{\text{inf}} - (1 - \beta) \int_{p_{\text{inf}}}^{p} \frac{\rho + \delta + \lambda_1 F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx \right),
\]

where \( p_{\text{inf}} \) is the lowest viable marginal productivity of labor. The latter is defined as the productivity value that is just sufficient to compensate an unemployed worker for his forgone value of unemployment, given that he would be paid his marginal productivity, thus letting the firm with zero profits. Analytically:

\[
V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon) \quad \Uparrow
\]

\[
p_{\text{inf}} = b + \beta(\lambda_0 - \lambda_1) \int_{p_{\text{inf}}}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx
\]

\footnote{Moreover, Dey and Flinn focus on the renegotiation issue in a more complex framework with multidimensional employment contracts stipulating wages and health insurance provisions. Due to this added complexity, they are unable to come up with closed-form expression for wages and wage distributions.}
It appears that \( p_{\text{inf}} \) differs from the unemployment income if workers have positive bargaining power. For instance, \( \varepsilon p_{\text{inf}} \) is greater than the unemployment income flow \( \varepsilon b \) if the arrival rate of job offers to unemployed workers \( \lambda_0 \) is larger than the arrival rate to employees, \( \lambda_1 \). In that case, accepting a job reduces the efficiency of future job search. The worker needs to be compensated for this loss through a wage higher than his unemployment income. Operating firms thus have to be able to afford wages at least equal to \( p_{\text{inf}} \), which imposes the obvious condition that they be at least as productive as \( p_{\text{inf}} \). It is worth noting that the lower support of observed marginal productivities, that we denote by \( p_{\text{min}} \), can be strictly above the lower support of viable productivities \( p_{\text{inf}} \), for instance if free entry is not guaranteed on the search market.

The definition (6) of \( \phi_0(\varepsilon, p) \) together with the definition (7) of \( p_{\text{inf}} \) shows that entry wages, received by individuals who exit from unemployment, are not necessarily higher than the unemployment income. It actually appears that those wages are always smaller than the unemployment income if workers have no bargaining power, because accepting a job is a means to obtain future wage raises. Entry wages obviously increase with the bargaining power parameter \( \beta \).

We end this Section by some comments on comparative statics. The wage function \( \phi(\varepsilon, p, p') \) decreases with \( \lambda_1 \) and \( F \) (with respect to first-order stochastic ordering), and increases with \( \delta \). These properties reflect an option value effect: workers are willing to pay for higher future prospects. Of course \( \phi(\varepsilon, p, p') \) increases with bargaining power, \( \beta \). The wage function is an increasing function of worker ability \( \varepsilon \) and the type \( p \) of the less competitive employer, as both Bertrand competition and Nash-bargaining work in tandem to push wages up. However, we note an ambiguous effect of the type \( p' \) of the employer winning the auction: \( \phi(\varepsilon, p, p') \) decreases with \( p' \) if \( \beta \) is small enough for the option value effect to dominate. A high \( p' \) means that the upper bound put on future renegotiated wages is more remote (as it is equal to \( p' \)) and the worker is thus willing to trade lower present wages for a promise of higher future wages.

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However, $\phi(\varepsilon, p, p')$ increases with $p'$ if $\beta$ is large enough for the bargaining power effect on rent sharing to take over the option value effect.

2.3 Steady-state equilibrium

We know from what precedes that a type $\varepsilon$ employee of a type $p$ firm is currently paid a wage $w$ that is either equal to $\phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{inf}}, p)$, if $w$ is the first wage after unemployment, or is equal to $\phi(\varepsilon, q, p)$, with $p_{\text{inf}} \leq p_{\text{min}} < q \leq p$, if the last wage mobility is the outcome of a bargain between the worker, the incumbent employer and another firm of type $q$. The cross-sectional distribution of wages therefore has three components: a worker fixed effect ($\varepsilon$), an employer fixed effect ($p$) and a random effect ($q$) that characterizes the most recent wage mobility. In this section we determine the joint distribution of these three components.

In a steady state a fraction $u$ of workers is unemployed and a density $\ell(\varepsilon, p)$ of type-$\varepsilon$ workers is employed at type-$p$ firms. Let $\ell(p) = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \ell(\varepsilon, p) \, d\varepsilon$ be the density of employees working at type-$p$ firms. The average size of a firm of type $p$ is then equal to $M \ell(p)/\gamma(p)$. We designate the corresponding cdfs with capital letters $L(\varepsilon, p)$ and $L(p)$, and we let $G(w | \varepsilon, p)$ represent the cdf of the (not absolutely continuous, as we shall see) conditional distribution of wages within the set of workers of ability $\varepsilon$ within type-$p$ firms.

We now proceed to the derivation of these different distributional parameters by increasing order of complexity. The steady state assumption implies that inflows must balance outflows for all stocks of workers defined by a status (unemployed or employed), a personal type $\varepsilon$, a wage $w$, an employer type $p$. The relevant flow-balance equations are spelled out in Appendix A.3. They lead to the following series of definitions/results:

- **Unemployment rate:**

\[
\frac{\delta}{\delta + \lambda_0}. \tag{8}
\]

- **Distribution of firm types across employed workers:** The fraction of workers employed at
a firm with mpl less than $p$ is

$$
L(p) = \frac{F(p)}{1 + \kappa_1 F(p)},
$$

(9)

with $\kappa_1 = \frac{\lambda_1}{F'}$, and the density of workers in firms of type $p$ follows from differentiation as

$$
\ell(p) = \frac{1 + \kappa_1}{[1 + \kappa_1 F(p)]^2} f(p).
$$

(10)

- **Distribution of matches:** The density of matches $(\varepsilon, p)$ is

$$
\ell(\varepsilon, p) = h(\varepsilon) \ell(p).
$$

(11)

- **Within-firm distribution of wages:** The fraction of employees of ability $\varepsilon$ in firms with mpl $p$ is

$$
G(w|\varepsilon, p) = \left( \frac{1 + \kappa_1 F(p)}{1 + \kappa_1 F[q(\varepsilon, w, p)]} \right)^2 = \left( \frac{1 + \kappa_1 L[q(\varepsilon, w, p)]}{1 + \kappa_1 L(p)} \right)^2.
$$

(12)

where $q(\varepsilon, w, p)$, defined equation (3), stands for the threshold value of the productivity of new matches above which a type-$\varepsilon$ employee with a current wage $w$ can get a wage increase.

Equation (8) is standard in equilibrium search models (see Burdett and Mortensen, 1998) and merely relates the unemployment rate to unemployment in- and outflows.

Equation (9) is a particularly important empirical relationship as it will allow us to back out the sampling distribution $F$ from its empirical counterpart $L$.\footnote{It is exactly the same equilibrium relationship as between the distribution of wage offers and the distribution of earnings in the Burdett and Mortensen model.}

Equation (11) implies that, under the model’s assumptions, the within-firm distribution of individual heterogeneity is independent of firm types. Nothing thus prevents the formation of highly dissimilar pairs (low $\varepsilon$, high $p$, or low $p$, high $\varepsilon$) if profitable to both the firm and the worker. This results from the assumptions of constant returns to scale, scalar heterogeneity and undirected search.

Finally, equation (12) expresses the conditional cdf of wages in the population of type $\varepsilon$ workers hired by a type $p$ firm. What the pair of equations (11,12) shows is that a random
draw from the steady-state equilibrium distribution of wages is a value \( \phi(\varepsilon, q, p) \) where \( (\varepsilon, p, q) \)
are three random variables such that

(i) \( \varepsilon \) is independent of \( (p, q) \),

(ii) the cdf of the marginal distribution of \( \varepsilon \) is \( H \) over \([\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]\),

(iii) the cdf of the marginal distribution of \( p \) is \( L \) over \([p_{\text{min}}, p_{\text{max}}]\), and

(iv) the cdf of the conditional distribution of \( q \) given \( p \) is \( \tilde{G}(\cdot|p) \) over \( \{p_{\text{inf}}\} \cup [p_{\text{min}}, p] \) such that

\[
\tilde{G}(q|p) = G(\phi(\varepsilon, q, p)|\varepsilon, p) = \frac{[1 + \kappa_1 F(p)]^2}{[1 + \kappa_1 F(q)]^2}
\]

for all \( q \in \{p_{\text{inf}}\} \cup [p_{\text{min}}, p] \). The latter distribution has a mass point at \( p_{\text{inf}} \) and is otherwise continuous over the interval \([p_{\text{min}}, p]\).

3 Estimation

In this section, we describe the data and the estimation procedure and discuss the results.

3.1 Data

We use a dataset constructed by Crépon and Desplat (2002). This dataset covers the period 1993-1997 and contains various accounting informations drawn from the BRN firm-data source ("Bénéfices Réels Normaux"), collected by the French National Statistical Institute (INSEE) : total compensation costs, value added, current operating surplus, gross productive assets, etc., plus an Auerbach-type measure of the user cost of capital computed by Crépon and Desplat using data on the age of capital. The BRN data are supposedly exhaustive of all private enterprises (not establishments) with a sales turnover of more than 3.5 million FRF (about 530,000 Euros) and liable to profit tax.\(^6\) Note that the necessary “cleaning” of this administrative data source (mainly outlier detection and construction of the capital cost variable) let them retain

\(^6\)The BRN is a subset of a larger firm sample, the BIC, "Bénéfices Industriels et Commerciaux".
only about 30% of all the firms present in the original sample (87,371 firms). In addition, Crépon and Desplatz used the DADS worker data source ("Déclarations Annuelles de Données Sociales") to compute labor costs and employment, at the enterprise level, for different worker categories (skill, age, sex). The DADS data are based on mandatory employer (establishments) reports of the earnings of each salaried employee of the private sector subject to French payroll taxes over one given year. This very large dataset was thus “collapsed” by enterprise and skill category and then merged with the BRN dataset.\(^7\)

Aggregating worker wages and labor into two skill categories (“skilled” and “unskilled”)\(^8\), we have formed four panels of firm data on value-added, employment by skill and average wage by skill, covering the period 1993-1997 and corresponding to the following thirteen distinct industries:

1. Intermediate goods  
2. Investment goods  
3. Consumption goods  
4. Electrical & electronic equipment  
5. Construction  
6. Transportation  
7. Wholesale, food  
8. Wholesale, nonfood  
9. Retail, food  
10. Retail, nonfood  
11. Automobile repair & trade  
12. Hotels & restaurants  
13. Personal services

\(\{\text{Manufacturing}\}
\)

\(\{\text{Trade}\}
\)

\(\{\text{Services}\}
\)

Table 1 contains some descriptive statistics for the selected variables. We note that a large majority of workers in every industry are unskilled, the proportion of unskilled workers ranging from 57\% (Wholesale, nonfood) to 81.8\% (Construction). Moreover, the skilled-unskilled average annual compensation cost ratio ranges between 1.5 (Hotel & Restaurants) and 2 (Intermediate goods, Wholesale-food).

\(^7\)For more information on these datasets, we refer to the paper by Crépon and Desplatz and to Abowd, Kramarz and Margolis (1999) for another very precise description of the same data sources and others.

\(^8\)The unskilled category comprises unskilled manual workers and trade employees. The skilled category comprises skilled manual workers, administrative employees (secretaries, ...), engineers, and all employees with some managerial function in the firm.
Finally, estimating the model requires data on worker mobility. We use the French Labor Force Survey ("Enquête Emploi") which is a three-year rotating panel of individual professional trajectories similar to the American CPS ("Current Population Survey"). We prefer to use the LFS panel instead of the larger DADS panel as the latter is known to be affected by large attrition biases. Moreover, the LFS is precisely designed to study unemployment and worker mobility.

3.2 Productivity

The values and distribution of firms’ marginal productivity values $p$ are crucial determinants of wages in the structural model. Since these values are not directly observed in the data, their construction is a key step in the estimation procedure. A central principle that we want to stick with in the design of this procedure is that the productivity parameters $p$ should not be constructed to a priori fit the wage data, but should rather be identified from value-added data alone. This, we believe, is the only way to get credible estimates of the bargaining power $\beta$, which in turn will be identified by the connection that exists in the data between wages and productivity.\(^9\)

The construction of the $p$'s and their distributions for each labor category from value-added data requires some additional structure on the production technology.

**Further assumptions on the production technology.** The population of workers is clustered into two statistical categories called “skilled” and “unskilled”. We assume that each category of workers faces the same transition rate values denoted as $\delta_s, \lambda_{0s}, \lambda_{1s}$ (resp. $\delta_u, \lambda_{0u}, \lambda_{1u}$) for, respectively, skilled and unskilled workers. Idem for the values of non market time $b_s$ and $b_u$.

\(^9\)It is therefore essential that our dataset contain information on both individual wages and value-added. In the absence of the latter, Postel-Vinay and Robin (2002b) had to rely on the sole wage data to construct the $p$’s using the structural relationship implied by the model between $p$ and the conditional mean wage $E(w/p)$.
Moreover, the observed skill type does not necessarily capture all the productive heterogeneity of workers. Specifically, there are $M_s$ skilled workers and $M_u$ unskilled workers in the economy, with corresponding densities of professional ability $h_s(\varepsilon)$ and $h_u(\varepsilon)$ respectively.\textsuperscript{10}

Firms’ labor input is an aggregate of skilled and unskilled labor constructed as follows. Let $M_j = M_{sj} + M_{uj}$ be the size of some firm $j$, comprising $M_{sj}$ observationally skilled workers and $M_{uj}$ observationally unskilled. Letting $h_{sj}(\varepsilon)$ (resp. $h_{uj}(\varepsilon)$) denote the density of type-$\varepsilon$ skilled (resp. unskilled) workers employed at some firm $j$, the total amount of efficient labor employed at this firm is

$$L_j = M_{sj} \int \varepsilon h_{sj}(\varepsilon) \, d\varepsilon + M_{uj} \int \varepsilon h_{uj}(\varepsilon) \, d\varepsilon.$$  \textsuperscript{(13)}

We then specify firm $j$’s total per-period output (value-added) as:

$$Y_j = \theta_j L_j^\xi,$$  \textsuperscript{(14)}

where $\theta_j$ is a firm-specific productivity parameter (that possibly captures heterogeneity in fixed capital stocks), and $\xi$ is between 0 and 1 and is also common to all firms.\textsuperscript{11}

It is evident from equation (14) that the marginal value to firm $j$ of a worker with ability $\varepsilon$ is $p_j \varepsilon \equiv \varepsilon \xi \theta_j L_j^{\xi-1} = \varepsilon \xi Y_j / L_j$, irrespective of his/her observed skill type. Of course one expects that the statistical skill category is correlated with the true ability and that the mean ability of unskilled workers is less than the mean ability of skilled workers:

$$\alpha_u \equiv \int \varepsilon h_u(\varepsilon) \, d\varepsilon \leq \alpha_s \equiv \int \varepsilon h_s(\varepsilon) \, d\varepsilon.$$  \textsuperscript{(15)}

Assuming that the markets for observationally skilled and unskilled workers are perfectly segmented, then, according to the theory laid out in the preceding section, there is no sorting within each observationally homogeneous category of workers:

$$h_{sj}(\varepsilon) = h_s(\varepsilon) \quad \text{and} \quad h_{uj}(\varepsilon) = h_u(\varepsilon).$$  \textsuperscript{(16)}

\textsuperscript{10}Both densities may have disconnected supports, meaning $h_u(\varepsilon) \cdot h_s(\varepsilon) = 0$ for all $\varepsilon$, in which case the observable skill variable would indeed allow to partially sort workers by effective ability $\varepsilon$.

\textsuperscript{11}Note that we completely neglect the sort of externality problems pointed out by Stole and Zwiebel (1996), Wolinsky (2000) and Cahuc and Wasmer (2001) resulting from diminishing marginal returns to labor. With nonconstant returns to scale, the hiring decisions of firms affect their levels of productivity, and consequently their labor costs. We simply assume that the firms’ hiring decisions are exogenous.
Hence

\[ L_j = \alpha_u M_{sj} + \alpha_s M_{uj} \]  

(17)

and the average marginal productivity of a match between firm \( j \) and an unskilled (resp. skilled) worker is \( \alpha_u p_j = \alpha_u Y_j \xi_j = \alpha_u \xi_j L_j^\xi - 1 \) (resp. \( \alpha_s p_j \)). Also, because one can multiply \( \varepsilon \) by any constant and divide \( \theta_j \) by this constant indifferently we shall normalize the distribution of \( \theta_j \) so that \( \alpha_u = \int \varepsilon h_u (\varepsilon) d\varepsilon = 1 \) and shall write \( \alpha \) for the mean productivity ratio: \( \frac{\alpha_u}{\alpha_u} (= \alpha_s \) if \( \alpha_u = 1 \)).

Lastly, equation (10) in Section 2.3 gives the following expression for firm sizes:

\[ M_{kj} = \frac{1 + \kappa_{1k}}{[1 + \kappa_{1k} F_k(p_j)]^2} \cdot \frac{f_k(p_j)}{\gamma(p_j)} \]  

(18)

for \( k = s \) or \( u \), where \( \kappa_{1k} = \lambda_{1k}/\delta_k \), where \( F_s(p) \) (resp. \( F_u(p) \)) are the sampling distributions of \( p_j \)'s in the populations of skilled and unskilled workers, respectively, and where \( \gamma(p) \) (resp. \( \Gamma(p) \)) is the density (resp. cdf) of \( p_j \)'s in the population of firms.

**Estimation of the production technology.** The production equation that we take to the data is the following logged version of (14):

\[ y_{jt} = \ln \theta_j + \xi \ln (M_{ujt} + \alpha M_{sjt}) + \eta_{jt}; \]  

(19)

where \( y_{jt} \) is the log value-added of firm \( j \) at date \( t \), \( M_{ujt} \) (resp. \( M_{sjt} \)) is the number of unskilled (resp. skilled) workers employed by firm \( j \) at date \( t \), and \( \eta_{jt} \) is an error term independent of the fixed effect \( \ln \theta_j \).

We also posit the following empirical relationship between date-\( t \) employment and its steady-state counterpart:

\[ \ln M_{kjt} = \ln M_{kj} + \omega_{kjt}, \]  

(20)

with \( \omega_{kjt} \) an error term independent of \( \theta_j \). This last equation points at two potential sources of endogeneity of \( M_{kjt} \) in (19): one is the correlation between \( M_{kj} \) and \( \theta_j \), and the other is the possible correlation between \( \eta_{jt} \) and \( \omega_{kjt} \).
Results. We estimate equation (19) by GMM under the following sets of moment restrictions:

\[
\Delta \eta_{jt} \perp \left\{ \ln M_{kj,t-\tau}; (\ln M_{kj,t-\tau})^2; \ln M_{sj,t-\tau}; \ln M_{uj,t-\tau} \right\}, \tau \geq 2, \quad (21)
\]

\[
(\ln \theta_j + \eta_{jt}) \perp \left\{ \Delta \ln M_{kj,t-\tau}; (\Delta \ln M_{kj,t-\tau})^2; \Delta \ln M_{sj,t-\tau}; \Delta \ln M_{uj,t-\tau} \right\}, \tau \geq 2. \quad (22)
\]

The set (21) of moment restrictions correspond to the estimation of the first-differenced model instrumented by lagged values of the RHS variables dated at least \( t-\tau \). It is implicitly assuming that \( \eta_{jt} \) is orthogonal to the past of \( \omega_{kjt} \). The set (22) of moment restrictions adds the restrictions which validates the estimation of the model in levels instrumented by lagged values of first-differenced RHS variables. Lastly, assuming these restrictions valid for all \( \tau \geq 2 \) is like assuming that the random components \( \eta_{jt} \) and \( \omega_{kjt} \) are at most moving averages of order one.

We also tried estimating the model using the subset of restrictions for \( \tau \geq 3 \) (the most that we can do with a five-year panel) and found no significant differences between both sets of estimates.\(^{12}\)

The GMM estimation results of equation (19) are gathered for our 13 sectors in Table 2.

< Table 2 about here. >

The first thing we notice is that the point estimates of the productivity ratio, \( \alpha \), are always above 1, and almost always significantly so. The productivity ratio is highest in Manufacturing and Transportation, closely followed by Trade and Personal services. This ratio is somewhat lower in the sectors of Construction, Hotels and Automobile trade. The ordering that emerges from our estimates of \( \alpha \) roughly parallels wage ratios (see Table 1).

### 3.3 Worker mobility

Another key determinant of wages is the parameter \( \kappa_1 = \lambda_1/\delta \), which measures the average number of outside job offers a worker receives between two unemployment spells. Since outside

\(^{12}\)Chamberlain (1992) validates these estimation procedures, which are inspired by Arellano and Bond (1991) and Arellano and Bover (1995) and are applied to the nonlinear model (19). Chamberlain shows how a polynomial expansion of the set of instrumental variables (or via any \( L_2 \)-complete sequence of functions) provides a sequence of estimators approximately attaining the information bound.
job offers are the source of wage increases in the model, we expect that more “mobile” workers (those with higher values of $\kappa_1$) should on average exhibit steeper wage-tenure profiles. However, what $\kappa_1$ essentially determines is the duration of job spells. The same identification principle applies here also. We want the estimation of $\kappa_1$ to be as robust as possible as far as wage distributions are concerned. We shall thus identify $\kappa_1$ exclusively from job duration data rather than wage data which would certainly buy us sizeable efficiency gains but would also increase the risk of misspecifications biases.

As all job transition processes are Poisson, all corresponding durations are exponentially distributed.\textsuperscript{13} In this Section we are interested in the distribution of job spell durations $t$, which have the following density, conditional on $p$:

$$
\mathcal{L}(t|p) = \left[ \delta + \lambda_1 F(p) \right] \cdot e^{-[\delta + \lambda_1 F(p)]t},
$$

(23)
where we know from equation (10) that $p$ is distributed in the population of employed workers according to the density:

$$
\ell(p) = \frac{1 + \kappa_1}{1 + \kappa_1 F(p)^2} f(p).
$$

Because it is impossible to match the LFS worker data with the BRN firm data, we shall treat $p$ as an unobserved heterogeneity variable, that is: we integrate out $p$ from the joint likelihood of $p$ and $t$, $\ell(p) \mathcal{L}(t|p)$, and maximize the unconditional likelihood, $\mathcal{L}(t) = \int_{p_{\text{min}}}^{p_{\text{max}}} \ell(p) \mathcal{L}(t|p) \, dp$,\textsuperscript{14} to get an estimate of $\delta$ and $\kappa_1$. This method of unconditional inference applied to labor market transition parameters was first explored by van den Berg and Ridder (2000). As we already mentioned, it has the additional advantage of yielding estimates of the transition rate parameters that are robust to any specification error in the estimation of the productivity parameters $\theta_j$ for all firms $j$.

\textsuperscript{13}In practice we have to take into account the fact that the panel covers a fixed number of periods so that some job durations are censored. It is easy to account for such right censoring. Moreover, the unconstrained likelihood can be analytically developed into simple combinations of exponentials and exponential-integral functions.

\textsuperscript{14}Simple calculations show that $\mathcal{L}(t) = \int_{1}^{\infty} \frac{\frac{1 + \kappa_1}{\kappa_1}}{e^{\frac{\delta}{\kappa_1}}} e^{-\frac{\delta + \lambda_1 F(p)}{\kappa_1}} dx$. 

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3. Since those estimates were obtained from LFS data, the relatively small number of observations forced us to aggregate our eleven sectors into five “broader” industries: Manufacturing, Construction, Transportation, Trade, and Services.

In terms of $\kappa_1$, i.e. the average number of outside contacts that an employed worker can expect before the next unemployment shock, skilled workers are always more mobile than unskilled workers. Now looking at the sheer frequency of such contacts, which is measured by $\lambda_1$, again we find that skilled workers get more frequent outside offers than unskilled workers, except in Services and Transportation where the difference between the two labor categories is in favor of the unskilled (although probably not significantly so in the Service sector). Finally, the rate of job termination $\delta$ is everywhere higher for the unskilled than for the skilled.

< Table 3 about here. >

The average duration of an employment spell (i.e. the average duration between two unemployment spells), $1/\delta$, ranges from 10 to 36 years, while the average waiting time between two outside offers, $1/\lambda_1$, lies between 3.3 and 17 years. The average number of outside contacts, $\kappa_1$, that results from these estimates is never very large (between 1.34 and 5.28) which confirms the relatively low degree of worker mobility in the French labor market. Workers are relatively less mobile in Manufacturing and Transportation than elsewhere, where they tend to have both lower job separation and job-switching rates. Concerning the Transportation sector, the relatively large share of State-owned companies in this industry may explain that result. Unskilled worker turnover is remarkably higher in Services, probably due to the relatively more frequent use of fixed-term contracts in that sector.\footnote{The average stock percentage of fixed-duration contracts in our LFS sample is 4.6% in Construction, 5.5% in Trade, 4.3% in Manufacturing, and as high as 16.3% in Services.}

### 3.4 The wage equation

We now turn to the last step of our estimation procedure, in which we combine the wage data with our productivity parameter estimates from step 1 to estimate a wage equation, which will
identify the workers’ bargaining power $\beta$.

Given our knowledge of wage determination (equation (5)) and the (conditional) wage distribution (equation (12)), we can derive the conditional mean wage $E(w|p)$ for each skill category (skilled and unskilled), the empirical counterpart of which is the firm-level average wage.

Equation (A16) in the Appendix shows that:

$$E(w|p) = E(\varepsilon) \cdot \left( p - \frac{[1 + \kappa_1 F(p)]}{(1 + \kappa_1)^2} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \left[ \phi(\varepsilon, p_{\min}, p) - \phi(\varepsilon, p_{\inf}, p) \right] h(\varepsilon) d\varepsilon \right.$$ 

$$- \left[ 1 + \kappa_1 F(p) \right]^2 \int_{p_{\min}}^{p} \frac{(1 - \beta) [1 + (1 - \sigma)\kappa_1 F(q)]}{[1 + (1 - \sigma)\kappa_1 \beta F(q)] [1 + \kappa_1 F(q)]^2} dq \right).$$

This expression can be further simplified. First, we should take account of the fact that $E(\varepsilon) = \alpha$ ($\alpha_s$ or $\alpha_u$ depending on which skill category we consider) is estimated in step 1 together with $p_j$ for all firms. Second, we can notice that if the lower support of viable productivities $p_{\inf}$ equals the lower support of observed productivities $p_{\min}$ (which amounts to assuming free entry and exit of firms on the search market), then the second term in the right hand side vanishes.

We shall henceforth adopt this assumption. We thus now have:

$$E(w|p) = \alpha \left( p - \left[ 1 + \kappa_1 F(p) \right]^2 \int_{p_{\min}}^{p} \frac{(1 - \beta) [1 + (1 - \sigma)\kappa_1 F(q)]}{[1 + (1 - \sigma)\kappa_1 \beta F(q)] [1 + \kappa_1 F(q)]^2} dq \right). \quad (24)$$

With equation (10) implying that $F(p) = \frac{(1 + \kappa_1) L(p)}{1 + \kappa_1 L(p)}$, our knowledge of $\kappa_1$ and of the value of $p$ for each firm let us construct $F(p)$ using the empirical cdf of $p_j$’s in the population of workers to estimate $L(p)$.

Letting $\bar{w}_{kjt}$ denote the observed firm-level mean wage of labor category $k$ ( = s or u), at date $t$, in firm $j$, we obtain a value of $\beta_k$ (the bargaining power of workers of category $k$) by fitting the theoretical mean wage $E\left( w|\hat{p}_j; \widehat{\alpha}_k, \widehat{\kappa}_1, \beta_k \right)$ that one computes using (24) to $\bar{w}_{kjt}$ using (weighted) nonlinear GLS.

Table 4 displays the estimated values of $\beta$ for each category of labor. Also, the two panels in Figure 1 plot the predicted and observed wages against the empirical cdf $\Gamma(p)$ for one of

---

16. To simplify the notation, we shall omit in this section the skill index “s” or “u”.
17. Unconstrained estimations always lead to the conclusion that $p_{\inf}$ indeed equals $p_{\min}$.
18. We also need a value for the discount rate $\rho$ which appears in $\sigma = \rho/(\rho + \delta)$. We normalize it for everyone to an annual value of 0.15.
the 13 sectors under consideration (we took the first sector in the list—the Intermediate goods industry—as our example). A glance at those Figures shows that the model is reasonably good at predicting wages, given the fact that we only had one free parameter ($\beta$) to fit wage data.

Concerning the values of $\beta$ reported in Table 4, one can make the following general comments. Unskilled workers have a very low bargaining power in most sectors. Their bargaining power amounts to zero in 9 out of 13 sectors. Skilled workers have bargaining powers that are significantly larger than those of unskilled workers. The range of $\beta_s$’s is very large, going from 0 (Personal services) to 0.83 (Automobile repair & trade) and their values are very heterogeneous.

## 4 Applications

In this section, we use our framework to shed light on three issues. First, we disentangle the respective influence of the bargaining power and the between-firm competition on wage distribution within each sector. Then, we analyse the inter-industry wage differentials and the returns to qualification.

### 4.1 Assessing the importance of between-firm competition

As we argued in the Introduction, the conventional approach to evaluating the workers’ bargaining power ignores job-to-job mobility. Our model offers a simple way of assessing the bias in the estimation of $\beta$ resulting from this simplification. Ignoring job-to-job mobility indeed amounts to forcing $\kappa_1 = 0$ in the wage equation (24) that now reads

$$E(w|p, \kappa_1 = 0) = \beta_0 \alpha p + (1 - \beta_0) \alpha \min.$$  

We obtain an estimator of the bargaining power in the absence of on-the-job search, $\hat{\beta}_0$, using weighted NLS\footnote{All estimates reported in Table 5 were obtained by WNLS using the log-version of equation (25) and the same metric as for the estimation of the wage equation with OTJ search (which is the optimal metric for the} separately for each industry and skill. It is important to note that this estimate...
$\hat{\beta}_0$ is a measure of the mean worker share of match rent, $E \left( E (w) - \alpha p_{\text{min}} \right) / E \left( \alpha p - \alpha p_{\text{min}} \right)$.

The values of $\hat{\beta}_0$ are gathered in the first two columns of Table 5. Comparing the bargaining power estimates with and without on the job search—i.e. comparing the values of $\hat{\beta}_0$ to the values of $\beta$ from Table 4—immediately shows that the bargaining power is always overestimated when one ignores job-to-job mobility. The magnitude of this upward bias varies across skill groups and sectors, but the bias always seems to be there, and is always important. This was expected as on-the-job search is a means by which an employee can force her employer to renegotiate her wage upward. Neglecting on-the-job search biases the workers’ bargaining power upward to make it fit the actual share of compensation costs in value-added.

Table 5’s columns 3 and 4 report estimates $\hat{\beta}_0$ of the mean worker share of match rents obtained from predicted (rather than observed) firm level mean wage data. That is, we simulated firm mean wages using our set of parameter estimates, and re-ran the estimation of equation (25) using those data. As can be seen by comparing the values $\hat{\beta}_0$ and $\hat{\hat{\beta}}_0$, we generally tend to overestimate wages a bit, especially so for unskilled workers in Construction, Retail Trade and Automobile Services. Simulated and observed data otherwise produce reasonably similar results.

What we want to know next is how much of this estimated $\hat{\beta}_0$ is explained by “noncompetitive wage setting” (i.e. the bargaining power that workers may have), versus how much of it is due to between-firm competition. What we are looking for here is another answer to the question of knowing to what extent do we need an extra rent sharing device (in addition to between-firm competition) to explain wages. To find this answer we simulate new wage data, again using our parameter estimates everywhere except for $\beta$, that we force to equal zero. That is, we produce the wage data that one would collect from the French labor market if French workers had no bargaining power at all, i.e. if the only source of rent acquisition by workers

\[\text{latter regression). The reason why we use the same metric for all estimates is just consistency, as the quantitative results tend to be mildly sensitive to the metric used for some particular industries. Different choices of metric have no qualitative impact.}\]

\[29\text{This will be confirmed by a more precise look at the wage prediction error in Table 6.}\]
were between-employer competition. That being done, we estimate equation (25) using this last set of wage data, and compare the rent share obtained to $\hat{\beta}_0$. This tells us how much of $\hat{\beta}_0$ is explained by between-firm competition alone.

The results are in the last two columns of Table 5. Clearly, competition between firms explains one hundred percent of the workers’ share of match rents everywhere where the bargaining power $\beta$ was estimated to be zero. What that means is that between-firm competition alone is enough to explain unskilled wages in practically all industries. What we also find from looking at the last “Skilled” column of Table 5 is that the rent share acquired by skilled workers is also largely explained by between-employer competition. Even though we undoubtedly do need some noncompetitive wage formation device such as wage bargaining to reproduce skilled wages, sheer labor market competition explains way over half (in fact, over 70% on average if one believes the figures in Table 5) of the rents accruing to these workers.

4.2 Inter-industry wage differentials.

Going back to the theoretical model, one can derive the market-average (real) wage by simply integrating the conditional mean wage (24) with respect to the distribution of workers across firms (10):

$$E(w|p; F, \alpha, \kappa_1, \sigma, \beta) = \alpha p_{\min} + \alpha (1 + \kappa_1) \int_{p_{\min}}^{p_{\max}} \frac{F(x)}{1 + \kappa_1 F(x)} \cdot \left(1 - \frac{(1 - \beta) [1 + \kappa_1 (1 - \sigma) F(x)]}{[1 + \beta \kappa_1 (1 - \sigma) F(x)] [1 + \kappa_1 F(x)]} \right) dx.$$  (26)

This obviously depends on the entire set of structural parameters, which are specific to each sector and labor categories. According to our structural model, inter-sectoral differences in mean wages reflect differences in this set of structural parameters, which of course includes the workers’ bargaining power $\beta$, but also worker mobility parameters ($\kappa_1$ and $\delta$), and “productivity effects” (worker productivity parameters $\alpha_u$ and $\alpha_s$, the returns to labor $\xi$ and the distributions of firm fixed-effects $\theta_j$). A natural question to ask is then which parameters in that set are most important in determining inter-industry wage differences.
There is no unique or straightforward way to answer this question.

First, it can be noticed that the consequences of productivity differences are straightforward since mean wages are proportional to any scale factor of the production function. Hence, raising the productivity of all firms of a sector by one percentage point raises the market mean wage of that sector—in fact, it raises all firm-level mean wages—by one percentage point. Note that a crucial assumption for this result is that the efficiency of job search (as measured by \( \lambda_1 \)) be independent of the firms’ types, which wouldn’t generically be the case if one were to endogenize e.g. the workers’ search efforts (see Christensen et al, 2001).

We can also shed some light on the impact of other variables by looking at the “sensitivity” of the predicted mean wage to changes in a series of structural parameters. Specifically, we consider shifting two distinct parameters: the bargaining power \( \beta \), and the “worker mobility” parameter \( \kappa_1 \), which can be interpreted as a measure of how far away our labor market is from the Walrasian paradigm, as wages equal the marginal productivities of labor whose distribution degenerates to a mass point when \( \kappa_1 \to \infty \). We then proceed to the computation of the predicted log average wage for each sector/skill category:

\[
\ln \hat{w}_k = \ln E \left( w | \hat{\beta}_j; \hat{F}_k, \hat{\alpha}_k, \hat{\kappa}_1, \hat{\sigma}_k, \hat{\beta}_k \right)
\]

using equation (26), and looking at the following two numbers:

1. The following partial derivative:

\[
\frac{\partial \ln \hat{w}_k}{\partial \beta_k}
\]

(27)

This measures the percentage increase in \( \ln \hat{w}_k \) caused by a unit increase in the bargaining power. (Thus, raising \( \beta_k \) by, say, 0.1 entails a percentage increase in the average wage \( \hat{w}_k \) of 1/10 times the above number.) Since \( \beta_k \) is comprised in \([0, 1]\), we believe that this partial derivative is more meaningful than the corresponding elasticity.

2. And:

\[
\frac{\partial \ln \hat{w}_k}{\partial \kappa_1}
\]

(28)
Similarly, this measures the percentage increase in $\ln \hat{w}_k$ caused by a unit increase in $\kappa_{1k}$. We also think this is a natural number to look at (rather than the corresponding elasticity), since what $\kappa_{1k}$ measures is the average number of contacts with an outside potential employer that a worker makes between two unemployment spells (i.e., according to Table 3, over a typical period of $1/\delta_k \simeq 20$ years). The above number therefore tells the percentage increase in $\hat{w}_k$ that one should expect if workers were to get one extra outside offer on average every $1/\delta_k$ years.

Table 6 contains the corresponding numbers for each of our thirteen sectors. The first column in that Table reports the empirical log average wage $\ln \bar{w}$. The second column shows the log average wage $\ln \hat{w}$ predicted by equation (26) and the parameter estimates obtained earlier. Column 3 reports the prediction error. The following two columns contain our two numbers of interest (27) and (28) computed using our set of estimates.\textsuperscript{21} Finally, the last two columns show the values taken by (27) and (28) under the assumption of no on-the-job search, i.e. with $\kappa_1 = 0$ and $\beta$ taking the values $\hat{\beta}_0$ from the first two columns of Table 5. We now comment on the figures contained in Table 6.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Sector & $\kappa_1$ & $\beta$ & $\beta_0$ \\
\hline
\end{tabular}
\end{table}

Column 4 in Table 6 contains a measure of the sensitivity of average wages to changes in the bargaining power of workers. What those numbers tell us is that if one were to increase the bargaining power of all workers by, say, 0.1, then average wages would be increased by roughly 2.5 to 8 percentage points, depending on the sector and worker category. Also, as can be seen from a comparison of columns 6 and 4 of Table 6, ignoring on-the-job search doesn’t seem to affect much the sensitivity of $\ln \hat{w}$ to changes in $\beta$: wages are only slightly more sensitive without on-the-job search (with values of (27) ranging from 4.3 to 9.2 percent).

Finally, the impact of changes in $\kappa_1$ is measured in column 5 of Table 6. Giving the workers one extra outside offer on average per employment spell (i.e. increasing $\kappa_1$ by 1) entails a

\textsuperscript{21}The theoretical formulae for (27) and (28) are not reported in the paper. They are available upon request.
(modest) average wage increase of 2 to 13 percentage points. What is most interesting is to look at what happens if one ignores employed job search. Supposing that workers don’t search on-the-job (i.e. \( \kappa_1 = 0 \)), what happens to wages if one allows them to get one outside offer per employment spell? The rightmost column in Table 6 tells us that the impact on wages would then be a 14 to 45 percent increase! Our sensitivity measure of predicted mean wages to changes in \( \kappa_1 \) is an order of magnitude larger at \( \kappa_1 = 0 \) than at \( \kappa_1 = \) its estimated value. The dependence of \( \ln \bar{w} \) on \( \kappa_1 \) is thus highly nonlinear: for fixed values of all other structural parameters, using an error-ridden value of \( \kappa_1 \) to predict the market average wage has little consequence so long as that value is in the correct order of magnitude (let’s say between 2 and 5, from Table 3). But completely ignoring on-the-job search (i.e. using \( \kappa_1 = 0 \)) causes a severe underestimation of the average wage.

This set of comparative statics calculations is informative about how the predicted (log) average wage depends on various parameters of interest, but it has little to say about the relative importance of those parameters in explaining inter-group wage differences. It is meaningless indeed to “compare”, e.g. a one percent increase in productivity with a increase of 0.1 in the level of the bargaining power. A complementary approach to the problem of inter-group wage differences is to consider the series of counterfactual experiments gathered in Tables 7 and 8.

We begin by looking at Table 7. The column labelled “Predicted \( \ln \bar{w} \)” reports the predicted value of the log average wage for all sectors and labor categories. The number in parentheses in that same column is the percentage gap between the predicted sectoral average wage and the predicted average wage in the Intermediate goods sector (which we take as our reference), proxied by the log-difference \( \left( \ln \bar{w} - \ln \bar{w}_{ref} \right) \). For instance, looking leftmost cell of the “Investment goods” row, we see that the predicted average unskilled wage for the Investment goods sector is \( \exp (4.37) \), and is 0.9% higher than the predicted average unskilled wage in the Intermediate
goods sector (which equals $\exp(4.36)$, as reported on the first row of the Table).

The four “Counterfactual” columns are constructed in the same way, with the difference that some parameters are given the value estimated for Manufacturing. For instance, the second row cell in the “$p = p_{ref}; \alpha = \alpha_{ref}$” column indicates that, if the value of $\alpha$ and the values and distributions of $p$ (i.e. the productivity parameters) were the same for unskilled workers in the Investment goods sector as in the Intermediate goods sector—all other structural parameters keeping their estimated values—, then the average unskilled wage in the Investment goods sector would be $\exp(4.42)$, which is 5.9% more than the average unskilled wage in the Intermediate goods sector. The remaining three “Counterfactual” columns repeat the same experiment with the bargaining power parameter $\beta$, the job-to-job mobility parameter $\kappa_1$, and finally the bargaining power and the productivity parameters together. In sum, what these counterfactual experiments are supposed to answer is the question “How much of the distance between the mean wage in sector $S$ and the mean wage in the Intermediate goods sector do we cover if we give such parameter of sector $S$ the value that it takes in the Intermediate goods sector?”

The numbers in Table 6 indeed give a striking answer to this question: practically all the action is shared between productivity and the bargaining power. Otherwise stated, cross-sectoral differences in job-to-job mobility are of little help to explain cross-sectoral differences in average wages. To see this, we just have to compare the “Predicted ln $\tilde{w}$” column and the last “Counterfactual” column, where the productivity scale parameters ($\alpha$ and $p$) and the bargaining power ($\beta$) are given their values from the Intermediate goods sector. By doing so, we practically fill all the wage gap between the Intermediate goods sector and all other industries. Note that this is consistent with the conclusion we drew from Table 6: using an erroneous value of $\kappa_1$ to predict log mean wages doesn’t matter too much if that value is far enough from zero. Of course there are cross-sector differences in worker mobility (see Table 3), but the estimates of $\kappa_1$ are sufficiently positive in all sectors that those differences don’t matter much (as far as wages go...)

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4.3 The skill premium.

Finally, Table 8 uses the same protocol to study inter-skill wage differences (i.e. skill premia). Again, we see that cross-skill differences in mobility are not very powerful as an explanation of the differences between skilled and unskilled wages. Again, we see that the triple \((\alpha, p, \beta)\) does most of the job.

5 Concluding remarks

This paper is the first attempt to estimate the influence of productivity, the bargaining power and between-firm competition on wages in a unified framework. The utilization of a panel of matched employer-employee data allows us to implement a multi-stage estimation procedure that yields separate estimates of the friction parameters (job destruction rates, arrival of job offers) and labor productivity at the firm level. These estimated values of the friction parameters and the productivity are then used to estimate the bargaining power that shows up in the wage equation of the theoretical model.

We find that between-firm competition plays a prominent role in wage determination in France over the period 1993-1997, especially for unskilled workers. It turns out that the bargaining power of workers is quite low, ranging from zero for unskilled workers in most industries, to an average of .3 for the skilled workers. However, workers are able to get a significant share of the job surplus thanks to between-firm competition: the share of the surplus that accrues to workers amounts to 25% for the unskilled and to 50% for the skilled.

Our results rely on simplifying assumptions that would need more exploration. In particular, it is assumed that the distribution of workers type is independent of the distribution of firms type. Taking into consideration the possibility of sorting would admittedly enrich the analysis, but would also add complexities that are beyond the scope of this paper.
Appendix

A Details of some theoretical results

A.1 Wage bargaining

A.1.1 Bargaining with unemployed workers

The subgame perfect equilibrium of the strategic negotiation game on matches with an unemployed worker is obtained by backward induction. For the sake of simplicity, it is assumed that the value of a vacant job, \( \Pi_0 \), is always zero. In step 4, the type-\( p \) firm accepts any offer \( w \) such that \( w \leq \varepsilon p \), and the type-\( \varepsilon \) worker accepts any offer \( w \) yielding \( V(\varepsilon, w, p) \geq V_0(\varepsilon) \). Therefore, at step 3, the worker offers \( w = \varepsilon p \), and the employer offers \( w \) such that \( V(\varepsilon, w, p) = V_0(\varepsilon) \). At step 2, the worker refuses any offer that leaves him with less than his expected discounted utility, which amounts to \( e^{-\rho \Delta} \cdot [\beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V_0(\varepsilon)] \), where \( \Delta \rightarrow 0 \) denotes the delay between steps 2 and 3. At step 1, the employer offers the lowest possible wage \( \phi_0(\varepsilon, p) \) that the worker will accept, which satisfies:

\[
V(\varepsilon, \phi_0(\varepsilon, p), p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V_0(\varepsilon). \tag{A1}
\]

The worker accepts the wage \( \phi_0(\varepsilon, p) \) in step 2 because he prefers to secure this offer rather than going on a process that does not raise his expected utility. Notice that it is the existence of a short delay between steps 2 and 3 that ensures existence and uniqueness of this subgame perfect equilibrium with instantaneous agreement in step 2 (see Osborne and Rubinstein, 1990).

A.1.2 Renegotiations

Renegotiations on continuing jobs occur when employees receive job offers and use them to claim wage increases. The renegotiation game is also solved by backward induction. Let us consider a situation in which a type-\( \varepsilon \) employee on a type-\( p \) job and earning a wage \( w \) receives a job offer from a type-\( p' \) employer. Let us denote by \( w'_1 \) and \( w_1 \) the wage offer made at step 1 by firm \( p' \) and \( p \) respectively. We assume that if the worker receives two offers yielding the same value, s/he chooses to stay with the incumbent employer.

**Step 4.** Decisions at step 4 are straightforward: firms accept any offer increasing their profits, and the worker accepts any offer increasing his/her contract values, in comparison to their fallback payoffs.

**Step 3.** At step 3, the worker makes offers with probability \( \beta \), and the firms make simultaneous offers with probability \( 1 - \beta \).
Claim 1 If the worker makes the offers, s/he moves to or stays at the firm with highest mpl, max(p, p'), and obtains a contract value depending on his/her decision at step 2 as in the following table.

<table>
<thead>
<tr>
<th>Worker’s decision at step 2:</th>
<th>Accepts V((\varepsilon, w'_1, p'))</th>
<th>Accepts V((\varepsilon, w_1, p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p' &gt; p)</td>
<td>(V(\varepsilon, \varepsilon p, p')) if (V(\varepsilon, \varepsilon p, p) &gt; V(\varepsilon, w'_1, p'))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(V(\varepsilon, w'_1, p')) if (V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w'_1, p'))</td>
<td>(V(\varepsilon, \varepsilon p', p')) if (V(\varepsilon, \varepsilon p', p') &gt; V(\varepsilon, w_1, p))</td>
</tr>
<tr>
<td>(p \geq p')</td>
<td>(V(\varepsilon, \varepsilon p, p))</td>
<td>(V(\varepsilon, w_1, p)) if (V(\varepsilon, \varepsilon p', p') \leq V(\varepsilon, w_1, p))</td>
</tr>
</tbody>
</table>

Proof of this claim. The worker offers \(V(\varepsilon, \varepsilon p, p)\) to the type-p firm and \(V(\varepsilon, \varepsilon p', p')\) to the type-p' firm. The firm with highest market power (max(p, p')) eventually wins the worker as \(p < p'\) implies \(V(\varepsilon, \varepsilon p, p) < V(\varepsilon, \varepsilon p', p')\).

As to the value of the resulting contract, one can derive it as follows: If \(p' > p\), \(p'\) accepts the wage \(\varepsilon p'\) offered by the worker only if, at step 2, the worker has not already signed with firm \(p'\) a contract \(w'_1\) such that \(V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w'_1, p')\). In such a case, if \(p'\) rejects the worker’s offer at step 4, the employee still prefers to stay at \(p'\) with wage \(w'_1\). Conversely, if \(p' \leq p\), a wage \(\varepsilon p\) is effectively signed with firm \(p\) if, at step 2, the worker has accepted either the offer \(w'_1\) made by firm \(p'\) or the wage \(w_1\) offered by firm \(p\) such that \(V(\varepsilon, \varepsilon p', p') > V(\varepsilon, w_1, p)\). Otherwise, if firm \(p\) rejects the worker’s offer at step 4, the employee still prefers to stay at firm \(p\) with wage \(w\) if \(V(\varepsilon, w_1, p) \geq V(\varepsilon, \varepsilon p', p')\).

Claim 2 If firms make offers, they enter a Bertrand game won by the firm with highest mpl, max(p, p'), at the end of which the worker obtains the contract value depending on his/her decision at step 2 as in the following table.

<table>
<thead>
<tr>
<th>Worker’s decision at step 2:</th>
<th>Accepts V((\varepsilon, w'_1, p'))</th>
<th>Accepts V((\varepsilon, w_1, p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p' &gt; p)</td>
<td>(V(\varepsilon, \varepsilon p, p)) if (V(\varepsilon, \varepsilon p, p) &gt; V(\varepsilon, w'_1, p'))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(V(\varepsilon, w'_1, p')) if (V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w'_1, p'))</td>
<td>(V(\varepsilon, \varepsilon p', p')) if (V(\varepsilon, \varepsilon p', p') &gt; V(\varepsilon, w_1, p))</td>
</tr>
<tr>
<td>(p \geq p')</td>
<td>(V(\varepsilon, \varepsilon p', p'))</td>
<td>(V(\varepsilon, w_1, p)) if (V(\varepsilon, \varepsilon p', p') \leq V(\varepsilon, w_1, p))</td>
</tr>
</tbody>
</table>

Proof of this claim. Let us first consider this game when \(p' > p\). Since it is willing to extract a positive marginal profit from every match, the best the type-p firm can do is to offer him a wage exactly equal to \(\varepsilon p\) yielding the value \(V(\varepsilon, \varepsilon p, p)\) to the worker. Accordingly, the employee accepts to move to (or to stay at) firm \(p'\) if firm \(p'\) offers at least \(V(\varepsilon, \varepsilon p, p)\) (or max \(V(\varepsilon, \varepsilon p, p), V(\varepsilon, w'_1, p')\)).

Now consider the case \(p' \leq p\). The type-p firm can keep its employee by offering max \([V(\varepsilon, \varepsilon p', p'), V(w_1, p)]\) and can attract him/her back, if s/he moved to firm \(p'\) at step 2, by offering \(V(\varepsilon, \varepsilon p', p')\).
Step 2. At step 2, the worker chooses the offer that yields the highest expected utility in the continuing negotiation game. If s/he refuses both offers, s/he gets the value \( V(\epsilon, w, p) \) of the pre-existing contract with the incumbent firm. Otherwise s/he gets the values \( EV \) as in the following table:

<table>
<thead>
<tr>
<th>( p' &gt; p )</th>
<th>Worker’s decision at step 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( EV = \beta V(\epsilon, \varepsilon p', p') + (1 - \beta)V(\epsilon, \varepsilon p, p) ) if ( V(\epsilon, \varepsilon p, p) &gt; V(\epsilon, w_1', p') )</td>
<td>Accepts ( V(\epsilon, w_1, p) )</td>
</tr>
<tr>
<td>( EV = V(\epsilon, w_1', p') ) if ( V(\epsilon, \varepsilon p, p) \leq V(\epsilon, w_1', p') )</td>
<td>( EV = \beta V(\epsilon, \varepsilon p, p) + (1 - \beta)V(\epsilon, \varepsilon p', p') ) if ( V(\epsilon, \varepsilon p', p') &gt; V(\epsilon, w_1, p) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( p \geq p' )</th>
<th>Accepts ( V(\epsilon, \varepsilon p, p) ) if ( p' &gt; p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( EV = \beta V(\epsilon, \varepsilon p, p) + (1 - \beta)V(\epsilon, \varepsilon p', p') ) if ( p \geq p' )</td>
<td>( EV = V(\epsilon, w_1, p) )</td>
</tr>
</tbody>
</table>

Step 1. At step 1, employers make simultaneous offers. Both employers offer the lowest possible wage that attracts the worker (and still yields nonnegative profits).

Claim 3 At step 1, the firm with highest \( mpL \), \( \text{max}(p, p') \), makes an offer immediately accepted provided that it improves the worker’s value. This offer defines a wage \( \phi(\epsilon, p, p') \) which solves:

\[
\begin{align*}
V(\epsilon, \phi(\epsilon, p, p'), p') &= \beta V(\epsilon, \varepsilon p', p') + (1 - \beta)V(\epsilon, \varepsilon p, p) & \text{if } p' > p \\
V(\epsilon, \phi(\epsilon, p', p), p) &= \beta V(\epsilon, \varepsilon p, p) + (1 - \beta)V(\epsilon, \varepsilon p', p') & \text{if } p \geq p'
\end{align*}
\]  

(A2)

Proof of this claim.

- If \( p' > p \), the preceding table of expected outcomes implies that the worker can get at least \( EV = \beta V(\epsilon, \varepsilon p', p') + (1 - \beta)V(\epsilon, \varepsilon p, p) \) in the continuing negotiation game by accepting any offer made by the type-\( p \) firm. In order to avoid a waste of time in unnecessary negotiation, firm \( p' \) offers a wage \( w_1' = \phi(\epsilon, p, p') \), that the worker accepts at step 2 and that solves:

\[ V(\epsilon, \phi(\epsilon, p, p'), p') = \beta V(\epsilon, \varepsilon p', p') + (1 - \beta)V(\epsilon, \varepsilon p, p). \]

Firm \( p \) cannot bid this wage that is bigger than \( \varepsilon p \).

- If \( p' \leq p \), the worker can get at least \( EV = \beta V(\epsilon, \varepsilon p, p) + (1 - \beta)V(\epsilon, \varepsilon p', p') \) by accepting any offer made by the type-\( p' \) firm. In order to avoid a waste of time, firm \( p \) offers a wage \( w_1 = \phi(\epsilon, p', p) \), that the worker accepts at step 2 and that solves:

\[ V(\epsilon, \phi(\epsilon, p', p), p) = \beta V(\epsilon, \varepsilon p, p) + (1 - \beta)V(\epsilon, \varepsilon p', p'). \]

Firm \( p' \) cannot bid this wage that is bigger than \( \varepsilon p' \).
This completes the characterization of the subgame perfect equilibrium of our bargaining game. It is worth introducing some extra notation at this point (for later use): we see that the minimal value of $p'$ for which “something happens” (i.e. either causing a wage increase or an employer change) is $q(\varepsilon, w, p)$ such that

$$V(\varepsilon, w, p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)).$$

(A3)

Note that

$$V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)) = V(\varepsilon, w, p) - \beta \left[ V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p) \right]$$

whenever $w < p$.

### A.2 Equilibrium wage determination

Here we derive the precise closed-form of equilibrium wages $\phi_0(\varepsilon, p)$ and $\phi(\varepsilon, p, p')$ defined in equations (1) and (2) respectively. The first step is to derive the value functions $V_0(\cdot)$ and $V(\cdot)$. Time is discounted at rate $\rho$. Since offers accrue to unemployed workers at rate $\lambda_0$, $V_0(\varepsilon)$ solves the following Bellman equation:

$$(\rho + \lambda_0) V_0(\varepsilon) = \varepsilon b + \lambda_0 E_F \{ \max [V(\varepsilon, \phi_0(\varepsilon, X), X), V_0] \},$$

(A4)

where $E_F$ is the expectation operator with respect to a variable $X$, which has distribution $F$. Using the definition (A1) to replace $V(\varepsilon, \phi_0(\varepsilon, p), p)$ by $\beta V(\varepsilon, p, p) + (1 - \beta) V_0(\varepsilon)$ in the latter equation, we then show that:

$$\rho V_0(\varepsilon) = \varepsilon b + \lambda_0 E_F \{ \max (\beta [V(\varepsilon, \phi_0(\varepsilon, X), X) - V_0(\varepsilon)], 0) \}. $$

(A5)

We thus find that an unemployed worker’s expected lifetime utility depends on his personal ability $\varepsilon$ through the amount of output he produces when engaged in home production, $\varepsilon b$, but also on labor market parameters such as the distribution of jobs and his bargaining power $\beta$.

Now turning to employed workers, consider a type-\(\varepsilon\) worker employed at a type-\(p\) firm. Since layoffs occur at rates $\delta$, we may now write the Bellman equation solved by the value function $V(\varepsilon, w, p)$:

$$[\rho + \delta + \lambda_1 F(q(\varepsilon, w, p))] V(\varepsilon, w, p) = w$$

$$+ \lambda_1 [F(p) - F(q(\varepsilon, w, p))] E_F \{ V(\varepsilon, \phi(\varepsilon, X, p), X) | q(\varepsilon, w, p) \leq X \leq p \}$$

$$+ \lambda_1 F(p) E_F \{ V(\varepsilon, \phi(\varepsilon, p, X), X) | p \leq X \} + \delta V_0(\varepsilon).$$

(A6)
Let us denote by $p_{\text{max}}$ the upper support of $p$. Equations (A3) and (A6), together with the bargaining rule (A2) allow us to rewrite (A6) as follows:

$$[\rho + \delta + \lambda_1 F(q(\varepsilon, w, p))] V(\varepsilon, w, p) = w + \delta V_0(\varepsilon) +$$

$$\lambda_1 \int_{q(\varepsilon, w, p)}^{\rho_{\text{max}}} [(1 - \beta) V(\varepsilon, \varepsilon x, x) + \beta V(\varepsilon, \varepsilon p, p)] dF(x) +$$

$$\lambda_1 \int_{\rho}^{p_{\text{max}}} [(1 - \beta) V(\varepsilon, \varepsilon p, p) + \beta V(\varepsilon, \varepsilon x, x)] dF(x). \quad (A7)$$

Imposing $w = \varepsilon p$ in (A7), taking the derivative, and noticing that the definition (A3) of $q(\varepsilon, w, p)$ implies that $q(\varepsilon, \varepsilon p, p) = p$, one gets:

$$\frac{dV(\varepsilon, \varepsilon p, p)}{dp} = \frac{\varepsilon}{\rho + \delta + \lambda_1 \beta F(p)}. \quad (A8)$$

Then, integrating by parts in equation (A7):

$$(\rho + \delta)V(\varepsilon, w, p) = w + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_{\rho}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx$$

$$+ (1 - \beta) \lambda_1 \varepsilon \int_{q(\varepsilon, w, p)}^{\rho_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A9)$$

Again imposing $w = \varepsilon p$, the last equation in turn implies that

$$(\rho + \delta)V(\varepsilon, \varepsilon p, p) = \varepsilon p + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_{\rho}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A10)$$

Noticing that $q(\varepsilon, \phi(\varepsilon, p', p), p) = p'$, an expression of $V(\varepsilon, \phi(\varepsilon, p', p), p)$ can be obtained from (A9):

$$(\rho + \delta)V(\varepsilon, \phi(\varepsilon, p', p), p) = \phi(\varepsilon, p', p) + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_{\rho}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx$$

$$+ (1 - \beta) \lambda_1 \varepsilon \int_{p'}^{\rho_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A11)$$

But, following the bargaining rule (A2), $(\rho + \delta)V(\varepsilon, \phi(\varepsilon, p', p), p)$ should also equal

$$(\rho + \delta) [\beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V(\varepsilon, \varepsilon p', p')]$$

which, using (A10), writes as:

$$\beta \varepsilon p + (1 - \beta) \varepsilon p' + \delta V_0(\varepsilon) + \beta^2 \lambda_1 \varepsilon \int_{\rho}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx + \beta (1 - \beta) \lambda_1 \varepsilon \int_{p'}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx.$$

Equating this expression with the right hand side of equation (A11), one gets the following expression for the wage $\phi(\varepsilon, p', p)$:

$$\phi(\varepsilon, p', p) = \beta \varepsilon p + (1 - \beta) \varepsilon p' - (1 - \beta)^2 \lambda_1 \int_{p'}^{\rho_{\text{max}}} \frac{\varepsilon F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A12)$$

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The lower support of the distribution of marginal productivities, \( p_{\text{min}} \), cannot fall short of the value \( p_{\text{inf}} \) such that \( V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon) \). Using the definitions \( (A5) \), of \( V_0(\varepsilon) \), and \( (A9) \), of \( V(\varepsilon, w, p) \), this identity yields:

\[
p_{\text{inf}} = b + \beta(\lambda_0 - \lambda_1) \int_{p_{\text{inf}}}^{p_{\text{max}}} \frac{\mathcal{T}(x)}{\rho + \delta + \lambda_1 \beta \mathcal{T}(x)} \, dx.
\] (A13)

(Note that the value of \( p_{\text{inf}} \) is independent of \( \varepsilon \). This result holds true for any homogeneous specification of the utility function.) Finally, as the bargaining outcome implies \( (A12) \), the identity \( V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon) \) implies the following alternative definition of \( \phi_0(\varepsilon, p) \):

\[
\phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{inf}}, p) = \beta \varepsilon p + (1 - \beta) \varepsilon p_{\text{inf}} - (1 - \beta)^2 \lambda_1 \int_{p_{\text{inf}}}^{p} \frac{\varepsilon \mathcal{T}(x)}{\rho + \delta + \lambda_1 \beta \mathcal{T}(x)} \, dx.
\] (A14)

### A.3 Equilibrium wage distributions

The \( G(w|\varepsilon, p) \ell(\varepsilon, p) (1-u)M \) workers of type \( \varepsilon \), employed at firms of type \( p \), and paid less than \( w \in [\phi_0(\varepsilon, p), \varepsilon p] \) leave this category either because they are laid off (rate \( \delta \)), or because they receive an offer from a firm with \( \text{mpl} \geq q(\varepsilon, w, p) \) which grants them a wage increase or induces them to leave their current firm (rate \( \lambda_1 \mathcal{T}[q(\varepsilon, w, p)] \)). On the inflow side, workers entering the category (ability \( \varepsilon \), wage \( \leq w \), \text{mpl} \( p \)) come from two distinct sources. Either they are hired away from a firm less productive than \( q(\varepsilon, w, p) \), or they come from unemployment. The steady-state equality between flows into and out of the stocks \( G(w|\varepsilon, p) \ell(\varepsilon, p) \) thus takes the form:

\[
\left\{ \delta + \lambda_1 \mathcal{T}[q(\varepsilon, w, p)] \right\} G(w|\varepsilon, p) \ell(\varepsilon, p) (1-u)M
\]

\[
= \left\{ \lambda_0 u M \, h(\varepsilon) + \lambda_1 (1-u) M \int_{p_{\text{min}}}^{q(\varepsilon, w, p)} \ell(\varepsilon, x) \, dx \right\} f(p)
\]

\[
= \left\{ \delta h(\varepsilon) + \lambda_1 \int_{p_{\text{min}}}^{q(\varepsilon, w, p)} \ell(\varepsilon, x) \, dx \right\} (1-u) M f(p),
\] (A15)

since \( \lambda_0 u = \delta (1-u) \). Applying this identity for \( w = \varepsilon p \) (which has the property that \( G(\varepsilon p|\varepsilon, p) = 1 \) and \( q(\varepsilon, \varepsilon p, p) = p \)), we get:

\[
\left\{ \delta + \lambda_1 \mathcal{T}(p) \right\} \ell(\varepsilon, p) = \left\{ \delta h(\varepsilon) + \lambda_1 \int_{p_{\text{min}}}^{p} \ell(\varepsilon, x) \, dx \right\} f(p),
\]

which solves as

\[
\ell(\varepsilon, p) = \frac{1 + \kappa_1}{1 + \kappa_1 \mathcal{T}(p)} h(\varepsilon) f(p).
\]

This shows that \( \ell(\varepsilon, p) \) has the form \( h(\varepsilon) \ell(p) \) (absence of sorting), and gives the expression of \( \ell(p) \). Hence the equations (10) and (11). Equation (10) can be integrated between \( p_{\text{min}} \) and \( p \) to obtain (9). Substituting (9), (10) and (11) into (A15) finally yields equation (12).
A.4 Derivation of $E[T(w)|p]$ for any integrable function $T(w)$

The lowest paid type-$\varepsilon$ worker in a type-$p$ firm is one that has just been hired, therefore earning $\phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{mf}}, p)$, while the highest-paid type-$\varepsilon$ worker in that firm earns his marginal productivity $\varepsilon p$. Thus, the support of the within-firm earnings distribution of type $\varepsilon$ workers for any type-$p$ firm belongs to the interval $[p_{\text{mf}}, p]$. Noticing that $G(q|p) = G(\phi(\varepsilon, q, p)|\varepsilon, p)$ has a mass point at $p_{\text{mf}}$ and is otherwise continuous over the interval $[p_{\text{min}}, p]$, we can readily show that for any integrable function $T(w)$,

$$
E[T(w)|p] = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \left( \int_{\phi(\varepsilon, p_{\text{min}}, p)}^{\varepsilon_{\text{max}}} T(w) G(dw|\varepsilon, p) + T(\phi_0(\varepsilon, p)) G(\phi_0(\varepsilon, p)|\varepsilon, p) \right) h(\varepsilon)d\varepsilon
$$

$$
= \left[ 1 + \kappa_1 F(p)^2 \right]^2 \left\{ \frac{1}{(1 + \kappa_1)^2} \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\phi_0(\varepsilon, p)) h(\varepsilon)d\varepsilon \right. \\
+ \int_{p_{\text{min}}}^{\varepsilon_{\text{max}}} \left( \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\phi(\varepsilon, q, p)) h(\varepsilon)d\varepsilon \right) \frac{2\kappa_1 f(q)}{[1 + \kappa_1 F(q)]^3} dq \right\}
$$

$$
= \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\varepsilon)p h(\varepsilon)d\varepsilon + \int_{p_{\text{min}}}^{\varepsilon_{\text{max}}} \left[ T(\phi_0(\varepsilon, p)) - T(\phi(\varepsilon, p_{\text{min}}, p)) \right] h(\varepsilon)d\varepsilon
$$

$$
- \left[ 1 + \kappa_1 F(p)^2 \right]^2 \int_{p_{\text{min}}}^{\varepsilon_{\text{max}}} \left[ \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T'(\phi(\varepsilon, q, p)) \varepsilon h(\varepsilon)d\varepsilon \right] \frac{(1 - \beta) \left[ 1 + (1 - \sigma)\kappa_1 F(q) \right] \varepsilon}{\left[ 1 + (1 - \sigma)\kappa_1 F(q) \right]^2} dq. \quad (A16)
$$

The first equality follows from the definition of $G(w|\varepsilon, p)$ as

$$
G(w|\varepsilon, p) = \left[ \frac{1 + \kappa_1 F(p)^2}{1 + \kappa_1 F(q(\varepsilon, w, p))} \right]^2
$$

yielding

$$
G'(w|\varepsilon, p) = \left[ 1 + \kappa_1 F(p)^2 \right]^2 h(\varepsilon) \frac{2\kappa_1 f(q)}{[1 + \kappa_1 F(q)]^3} \frac{\partial q(\varepsilon, w, p)}{\partial w} dw.
$$

The second equality is obtained with an integration by parts, deriving the partial derivative of $\phi(\varepsilon, q, p)$ with respect to $q$ from (A12) as

$$
\frac{\partial \phi(\varepsilon, q, p)}{\partial q} = (1 - \beta) \varepsilon \frac{1 + (1 - \sigma)\kappa_1 F(q)}{1 + (1 - \sigma)\kappa_1 F(q)}.
$$

References


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**Table 1:** Sample Distribution Table
### TABLE 2: Production Function Estimates

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<th>$\alpha$</th>
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<td>1.85</td>
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<tr>
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<td>(0.18)</td>
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<tr>
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<td>1.66</td>
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<td>(0.14)</td>
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<tr>
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<td>1.71</td>
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<td>(0.02)</td>
<td>(0.12)</td>
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<td>1.59</td>
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<td>(0.19)</td>
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<td><strong>Construction</strong></td>
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<td>1.55</td>
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<td></td>
<td>(0.03)</td>
<td>(0.23)</td>
</tr>
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<td>1.65</td>
</tr>
<tr>
<td></td>
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<td>(0.09)</td>
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Note: Standard errors in parentheses.
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<th>( 1/\lambda_1 )</th>
<th>( \delta )</th>
<th>( 1/\delta )</th>
<th>( \kappa_1 = \frac{\lambda_1}{\delta} )</th>
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<td>0.058</td>
<td>17.0</td>
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<td>22.9</td>
<td>1.34 (0.11)</td>
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<td>11.5 (0.001)</td>
<td>0.027</td>
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<tr>
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<td>20.3 (0.84)</td>
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Notes:  
1. Per annum.  
2. Standard errors in parentheses.
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<th>Skilled</th>
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Notes: ¹Estimates obtained with $\rho = 0.15$.
²Standard errors in parentheses.
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<td>100%</td>
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**Note:** Estimates obtained with \( p = 0.15 \).

**TABLE 2.** Mean worker share of matched tests.
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<td>90%</td>
<td>100%</td>
<td>110%</td>
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<td>2021</td>
<td>55%</td>
<td>65%</td>
<td>75%</td>
<td>85%</td>
<td>95%</td>
<td>105%</td>
<td>115%</td>
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<tr>
<td>2022</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
<td>90%</td>
<td>100%</td>
<td>110%</td>
<td>120%</td>
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Note: The table above represents data from years 2020 to 2022 with values ranging from 50% to 120%.
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<th>(β_0 - β_6)</th>
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**Hypothesis:**

- Null Hypothesis: \( \beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 \)
- Alternative Hypothesis: At least one of the coefficients is different from the others.

**Results:**

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<tr>
<td>t-test</td>
<td>0.002</td>
</tr>
</tbody>
</table>

**Conclusion:**

There is significant evidence to reject the null hypothesis. At least one of the coefficients is different from the others.

**Formula:**

\[
\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_n x_n
\]

**Interpretation:**

- \( \beta_0 \): Intercept
- \( \beta_i \): Slope of the \( i \)-th predictor

**Example:**

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Slope</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.5</td>
<td>0.01</td>
</tr>
<tr>
<td>X2</td>
<td>-0.3</td>
<td>0.05</td>
</tr>
<tr>
<td>X3</td>
<td>0.8</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**Note:**

- Fisher's exact test was used to determine the significance of the differences.
<table>
<thead>
<tr>
<th>(GR-%)</th>
<th>2-4</th>
<th>4-6</th>
<th>6-8</th>
<th>8-10</th>
<th>10-12</th>
<th>12-14</th>
<th>14-16</th>
<th>16-18</th>
<th>18-20</th>
<th>20-22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>5%</td>
<td>7%</td>
<td>9%</td>
<td>11%</td>
<td>13%</td>
<td>15%</td>
<td>17%</td>
<td>19%</td>
<td>21%</td>
<td>23%</td>
</tr>
</tbody>
</table>

Note: The table in parentheses is 100 x the corresponding adjusted log mean values.

Predicted log mean values

\[ \ln(\text{Predicted log mean values}) = \ln(\text{Log mean values}) \times \text{Index} \]

Table of adjusted log mean values — Correlated and uncorrelated
Fig. 1 - Manufacturing, Intermediate goods

Skilled workers

Unskilled workers