# A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy

By

Dale F. Gray, Robert C. Merton and Zvi Bodie

September 25, 2006

Dale F. Gray

Sr. Risk Expert, International Monetary Fund

President of MF Risk, Inc.

Robert C. Merton

John and Natty McArthur University Professor
Harvard Business School

Zvi Bodie

Professor of Finance
School of Management
Boston University

Expanded version of paper presented at C.V. Starr Confernce on Finance and Macroeconomics

October 2, 2002, New York University

(Draft being submitted for publication. Please do not quote without permission from the authors.)

## Abstract

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of national economic and financial systems. This paper proposes a new comprehensive approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use the CCA approach to model and measure sectoral and national risk exposures, and analyze policies to offset their potentially harmful effects. This new framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability. It also provides a new framework for sovereign capital structure analysis. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improve price and volatility discovery, and expand international risk intermediation activities.

# A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy

By

Dale F. Gray, Robert C. Merton, Zvi Bodie

## Introduction

Vulnerability of a national economy to volatility in the global markets for credit, currencies, commodities, and other assets has become a central concern of policymakers, credit analysts, and investors everywhere. This paper describes a new framework for analyzing a country's exposure to macroeconomic risks based on the theory and practice of contingent claims analysis.[1] In this framework, the sectors of a national economy are viewed as interconnected portfolios of assets, liabilities, and guarantees, that can be analyzed like puts and calls. We measure the sensitivities of the market values of these portfolios to "shocks" in underlying market risk factors, and we illustrate how to use contingent claims analysis to quantify sovereign credit risk. This framework makes it transparent how risks are transferred across sectors, and how they can accumulate in the balance sheet of the public sector and ultimately lead to a default by the government. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. The framework also facilitates the quantification of these risk relationships and highlights their non-linear character.

## Contingent Claims Analysis

A contingent claim is any financial asset whose future payoff depends on the value of another asset. The prototypical contingent claim is an option - the right to buy or sell the underlying asset at a specified exercise price by a certain expiration date. A call is an option to buy; a put is an option to sell. Contingent claims analysis is a generalization of the option pricing theory pioneered by Black-Scholes (1973) and Merton (1973). Since 1973, option pricing methodology has been applied to a

[1] See Merton (1974, 1977, 1992, 1998). When applied to debt and equity securities, contingent claims analysis is commonly called the "Merton Model."

wide variety of contingent claims. In this paper we focus on its application to the analysis of credit risk and guarantees against the risk of default.

The contingent claims approach is based on three principles: (i) the values of liabilities are derived from assets; (ii) liabilities have different priority (i.e. senior and junior claims); and, (iii) assets follow a stochastic process. The liabilities consist of senior claims (such as senior debt), subordinated claims (such as subordinated debt) and the junior claims (equity or the most junior claim). As total assets decline, the value of risky debt declines, and credit spreads on risky debt rise. (See Annex 1 for more details.)

Balance sheet risk is the key to understanding credit risk and crisis probabilities. Default happens when assets cannot service debt payments. Uncertain changes in future asset value, relative to promised payments on debt, is the driver of default risk. Shocks to flows, prices, or liquidity frequently end up being converted into credit risk in a crisis. But macro models do not handle credit risk well. Financial fragility is intimately related to probability of default. Default is hard to handle in traditional macro models in part due to assumptions which usually exclude the possibility of default. In addition, flow-of-funds and accounting balance sheets cannot provide measures of risk exposures which are forward-looking estimates of losses.

## Contingent Claim Balance Sheets for Sectors

We view an economy as a set of interrelated balance sheets with three types of aggregate sectors -- corporate, financial, and public sector.[2] The same general principles of contingent claims that apply to analysis of a single firm can also be applied to an aggregation of firms. The liabilities of a firm, a portfolio of firms in a sector, or the public sector (combined government and monetary authorities) can be valued as contingent claims on the assets of the respective firm, sector or public sector. The corporate sector refers to an aggregation of all non-financial firms. A more accurate model of a sector would be CCA models for each individual firm or financial institution and then group them into an aggregrate portfolio. Treating the sector as one large firm is simpler and capture certain risk characteristics of the sector for the purposes of this analysis, but data permitting, a portfolio of CCA models of individual firms or institutions provides a richer model.

Governments and central banks typically provide explicit or implicit financial support to large financial institutions in the case of serious deposit runs, illiquidity or insolvency. The financial guarantee from the government is a contingent asset, which is modeled as a put option. Interlinked CCA balance sheets for the corporate sector, the financial sector and the public sector are shown in Figure 1.[3]

[2] Gray, Merton, Bodie (2002); Draghi, Merton, Giavazzi, (2002); Gray (2002).

[3] The household sector balance sheet can be added, with household income and assets comprising assets. Household non-discretionary expenditures are the senior liability, debt as a subordinated obligation and discretionary expenditures of households being the junior claim.

## Figure 1 – Interlinked CCA Balance Sheets for the Economy

| Assets | Liabilities |
|---|---|
| **CORPORATE SECTOR** | |
| **Corporate Assets** | **Debt**<br>(default-free value minus put option)<br><br>**Equity**<br>(call option on corporate assets) |
| **FINANCIAL SECTOR** | |
| **Loans and other Assets**<br>(including loans to corporate sector and public sector)<br><br>**Financial Guarantees**<br>(modeled as put option) | **Debt / Deposits / Liabilities**<br>(default-free value minus put option)<br><br>**Equity**<br>(call option on financial sector total assets) |
| **PUBLIC SECTOR (Government and Monetary Authorities)** | |
| **TOTAL ASSETS**<br><br>**Foreign Currency**<br>(including contingent foreign reserves)<br><br>**Net Fiscal Asset and Other Public Assets**<br><br>**Value of Other Public Sector Assets** | **Financial Guarantees** (modeled as put options related to too-important-to-fail financial and other entities)<br><br>**Foreign-currency Debt**<br>(default-free value of debt minus put option)<br><br>**Base Money and Local-currency Debt**<br>Held Outside of the Government and Monetary Authorities<br>(call options on public sector assets) |

The public sector balance sheet is an analytical economic balance sheet of the combined government and the monetary authorities.[4] The goal is to construct the liability side of the balance sheet so that the liabilities can be valued and linked to the value of total assets. The combined balance sheet for the public sector is shown in Figure 2. The numeraire can be in local or foreign currency units. The CCA balance sheets for large developed economies with "hard" currencies are measured in units of local currency. The CCA balance sheets of emerging market countries with "soft" currencies are usually measured in a "hard" currency (e.g. US dollar) because it simplifies the analysis and we are interested in valuation and credit risk associated with claims denominated in hard currencies, such as foreign-currency denominated debt.

## Figure 2 – Stylized Balance Sheet for the Public Sector

| **Assets** | **Liabilities** |
| --- | --- |
| | **Guarantees** |
| **Foreign Reserves** | **Foreign-currency Debt** |
| **Net Fiscal Asset** | **Local-currency Debt** (held outside of monetary authorities and government) |
| **Other Public Assets** | **Base Money** |

[4] This analytical combined balance sheet includes the monetary authority activities related to foreign currency reserves and "net domestic credit" to government but excludes the direct activities of the

Assets include: foreign currency reserves and contingent foreign currency reserves;[5] net fiscal asset (present value of taxes and revenues, including seignorage, less present value of government expenditures); and other public assets (e.g. equity in public enterprises, value of the public sector's monopoly on the issue of money, and other financial and non-financial assets). Liabilities include: local-currency debt; foreign-currency debt; financial guarantees; and base money. See Appendix 2 for details.

It is useful to look into the relationships between items in four categories: fiscal activities, monetary and foreign currency reserve activities, risky debt, and financial guarantees.

*Fiscal Assets and Liabilities* - In this framework, the items related to fiscal assets and liabilities are taxes, revenues and expenditures. Expenditures can be divided into non-discretionary expenditures which are senior claims, and discretionary expenditures which are junior claims. Non-discretionary expenditures are core expenditures (e.g., defense, education, core infrastructure, welfare, etc.) that will not be given up before giving up on paying the debt. Operationally, discretionary expenditures are ones that are subordinated to the explicit liability claims against the government. Discretionary expenditures may become especially significant in situations of high fiscal revenue such as windfalls from oil or natural resources.

Under stress situations, the government maintains the non-discretionary expenditures and cuts the discretionary expenditures. Under these assumptions, we can subtract the present value of non-discretionary expenditures from the present value of tax capability to obtain the *net fiscal asset*,[6] given that non-discretionary expenditures are clearly senior claims and have the same maturity patterns as taxes and fiscal revenues. The *net fiscal asset* is thus similar to the present value of the primary fiscal surplus over time (the present value of fiscal surplus minus interest payments). This step also simplifies the process of constructing the CCA balance sheet because it is much easier to

monetary authority with the banking sector, such as credit and liquidity support activities that do not go through the government balance sheet or affect foreign exchange reserves.

[5] The total foreign reserves of the public sector include actual reserves plus *contingent* reserves from international financial institutions, such as the IMF, other governments or contingent credit lines.

[6] The value of assets of an operating firm can be considered as the present value of stochastic future cash flow from income minus net new investment expenditures to create that income. For the public sector, the net fiscal asset is the present value of stochastic future fiscal flows from taxes and revenues minus non-discretionary expenditures equivalent to the present value of the primary fiscal surplus.

obtain market values for the other non-expenditure related liabilities, as will be discussed in more detail later.

*Monetary and Foreign Reserve Assets and Liabilities* - Base money is a liability of the monetary authorities and thus a liability of the public sector.[7] Base money consists of currency in circulation, bank reserves (required reserves, excess reserves, vault cash). The counterpart of base money liabilities are the assets of the monetary authorities net foreign assets and net domestic assets (including credit to government less government deposits, claims on banks and other items). Changes in base money correspond to changes in net foreign assets and net domestic assets.

*Sovereign Local Currency Debt Liabilities* - Local-currency debt of the government is a claim on sovereign assets. On the combined balance sheet of the public sector, the local-currency debt is the portion of debt held outside of the monetary authorities and the government.

*Sovereign Foreign Currency Debt Liabilities* – Foreign currency denominated debt is risky debt includes foreign-currency debt.

*Financial Guarantees* – As described earlier, implicit or explicit guarantees to "too-important-to-fail" banks and other financial institutions or pension obligations are liability items on the public sector's balance sheet which are modeled as put options.

Base money does not pay a "dividend"; it provides a convenience yield of money for transactions. The quantity of base money can be increased with subsequent consequences for inflation. Sovereign local currency debt is a claim on the public sector balance sheet, paying a "dividend" equal to the promised interest payment. A risk associated with holding sovereign local currency debt is that the government may dilute (or inflate away) part of the value or the debt, or may forcibly restructure some of the debt. The "dilution/inflation risk premium" is an extra premium demanded by the holders of local currency debt.[8]

---

[7] Base money is also known as high-powered money or reserve money. As is the common practice, it is the main liability of the monetary authorities (IMF, 2000, Buiter, 1993, Blejer and Schumacher, 2000). Base money is "multiplied" by the banking system; the multipliers relate base money to M1, M2, etc. When a country joins a currency union (i.e. merges with another sovereign or dollarizes) base money is exchagned for foreign currency reserves.

[8] See Gray, Lim and Malone for more details.

## Measuring Implied Asset Value and Volatilities Using Market Prices

From the observed prices and volatilities of market-traded securities, one can estimate the implied values and volatilities of the underlying assets. [9], [10] These implied asset values and asset volatilities can be used to calibrate the pricing and risk model of major sectors in the economy.

Domestic equity markets provide pricing and volatility information for the calculation of corporate, bank and non-bank financial assets, and asset volatilities. The Merton model is widely used to estimate implied assets and asset volatility for firms and financial institutions with traded equity. The method uses solves two equations for two unknowns, asset value and asset volatility (details in Annex 1, Merton (1974), KMV (1999), and Crouhy et. al. 2001). [11]

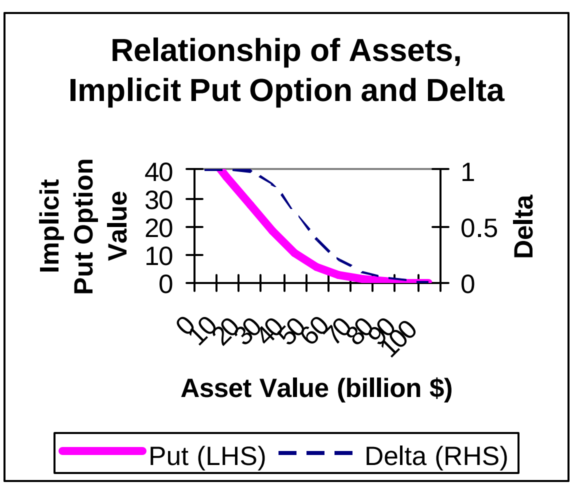
For the sovereign balance sheet, the prices in the international markets (including foreign currency market, debt market, and credit derivatives market), together with information from domestic market prices, provide the market information for the value and volatility of liabilities on the public sector balance sheet.[12] This information can be used to calculate implied asset values, volatilities, and higher moments of implied asset distributions for the sovereign (details are in Annex 2). Applications to a wide range of countries is described in Gray (2001 and 2002), Gapen et. al. (2004 and 2005), and IMF (2006). The key sectors of and economy can be calibrated and linked into an economy-wide CCA balance sheets framework. Subsequently, we can do "forward" simulations to estimate the impact of "shocks" and policy changes on the economic and financial system.[13]

[9] An implied value refers to an estimate derived from other observed data. Techniques for using implied values are widely practiced in options pricing and financial engineering applications.
[10] See Bodie and Merton (1995).
[11] The CCA approach is used to calibrate balance sheets for listed corporates and banks. For unlisted corporates and banks, information from balance sheets is used along with proxies and comparables from CCA type models of similar firms in the same sector. In the household sector, data permitting, the portfolio of assets is constructed (pension, mutual funds, deposits, PV of labor income and other estimated assets) and the CCA model is used to get implied household net worth and its volatility.
[12] Gray, 2000, 2001.
[13] As in the MfRisk model

## Measuring Risk Exposures

So far, we have discussed how to calculate the value of debt, guarantees, and equity using the CCA approach. We now turn to how to measure the risk exposures. The values of the contingent claims on the CCA balance sheets contain embedded implicit options which can be used to obtain certain risk measures. These include risk exposures in risky debt, probabilities of default, spreads on debt, the sensitivity of the implicit option to the underlying asset (the delta), sensitivity to other parameters, distance to distress, value-at-risk and other measures. The implicit put option changes in a non-linear way as the underlying asset changes. The slope of the put option vs. asset is the sensitivity of the put option value to the underlying asset value which is the delta. The delta measures the change in the value of an option per unit change in the value of the underlying asset. For example, the government's exposure to its guarantee to the banking sector changes as banking assets change. Figure 3 illustrates the value of the implicit put option and the delta for the implicit put option plotted against the banking system asset. This is simply (the absolute value of) the slope of the tangent to the function defining the value of the option at any point. The implicit put option can be the risk exposure that the holders of debt of a sector have, e.g. holders of sovereign debt. The implicit put option can also be a measure of the government's exposure, acquired through implicit or explicit guarantees, if the government provides such guarantees. The implicit put option increases in a non-linear way as the market value of the sector's assets decline.

**Figure 3:**



## Linkages in a Simple Three-Sector Framework

To show how we can apply the CCA framework, we focus on a simplified model with three aggregate sectors -- corporate, financial, and public sectors. In this model, the corporate sector's liabilities include bank loans which are the banking sector's assets. The system's financial stability depends on the government's financial guarantee to the banks.

The debt of the corporate sector can be described as default-free debt combined with a short of a put option on corporate assets. The economic balance sheet of the banking sector has assets consisting of corporate loans (default-free debt minus the value of a put option). The banking sector also includes guarantees from the government as an asset, which is a liability on the government's economic balance sheet (Figure 4).

### Figure 4 Balance Sheets for Simple Three-Sector Framework

**Corporate Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Corporate Assets | Debt (=Default-free value of debt minus implicit put option) |
| | Equity (Implicit call option) |

**Banking Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Loans (Debt of Corporate Sector) | Debt |
| Other Assets | Deposits |
| Financial Guarantee (Implicit Put Option) | Equity (Implicit call option) |

**Public Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Foreign Reserves | Financial Guarantee (Implicit put option) |
| Net Fiscal Asset and Other Assets | Foreign Debt (Default-free value of debt minus implicit put option) |
| Value of Monopoly on Issue of Money | Base Money and Local-currency Debt (Implicit call options) |

These three economic balance sheets demonstrate the interdependence among sectors; with one sector "long" a certain implicit option and another sector "short" the same implicit option. The economic balance[14] sheets for each of the three sectors with illustrative numbers are shown in Annex 3.

The sector CCA balance sheets can be integrated together as shown in Figure 5. Each sector contains assets adjusted for guarantees and when the junior claims/equity and risky debt are subtracted the net is zero (columns). When shocks affect the corporate sector it feeds into the financial sector and then could transmit risk to the government. The sum of all positions can be calculated and broken down into the portfolio of the claims of foreigners and other domestic residents. The foreigner's portfolio of claims is the value of what could be viewed as the present value of the risk adjusted current account. The framework provides a relative valuation tool for market and credit risk within sectors (e.g. sovereign foreign debt/CDS , local debt, foreign exchange instruments, and interest rates) and across sectors (e.g. sovereign debt, exchange rates, stock market index and banking sector equity or deposits).

## Figure 5 Economy-Wide CCA Balance Sheet Matrix

**CCA Balance Sheet (billion US$)**

| | Sovereign | Banking Sector | Non-Bank Financial | Corporate | Horizontal Sum** |
|---|---|---|---|---|---|
| *Asset without Guarantee* | $V_{Sovereign}$ | $V_{Banks}$ | $V_{NBank Financial}$ | $V_{Corporate}$ | S V |
| *Implicit Guarantee* | minus G | +G | +G | +G | 0 |
| Asset (including Guarantee) | $A_{Sovereign}$ | $A_{Banks}$ | $A_{NBank Financial}$ | $A_{Corporate}$ | S A |
| Jr Claim or Equity | Jr Claim | $E_{Banks}$ | $E_{NBank Financial}$ | $E_{Corporates}$ | S (J+E) |
| *Default-free Debt Value* | $B_S$ | $B_B$ | $B_{NBF}$ | $B_C$ | S B |
| *Expected Loss* | EL | EL | EL | EL | S EL |
| Risky Debt (Default-free - EL) | $B_S$-EL | $B_B$-EL | $B_{NBF}$-EL | $B_C$-EL | S Risky Debt |
| Assets minus Liabilities* | 0 | 0 | 0 | 0 | 0 |
| **Risk Indicators** | | | | | |
| Distance to Distress | $D2D_S$ | Ave $D2D_B$ | Ave $D2D_{NB}$ | Ave $D2D_C$ | |
| Default Probability | $DP_S$ | Ave $DP_B$ | Ave $DP_{NB}$ | Ave $DP_C$ | |
| Spread (BPS) | $Spr_S$ | ... | ... | ... | |

**Equals Asset + Guarantee - Jr Claim - (Default-free Value of Debt minus EL)*

*** Can be broken down into risk adjusted positions of residents and foreigners*

[14] The economic balance sheet is the "mark-to-market" balance sheet of the sector's assets and liabilities, including the economic values of other relevant contingent assets and contingent liabilities. This is in contrast to a traditional GAAP accounting balance sheet. For example, the government financial guarantee to the banking system is not a GAAP entry.

## Integrated Value and Risk Transmission between Sectors

The framework described above is versatile and can be used to understand many types of crises and risk shifting that cannot as easily be analyzed with other techniques. The risk-transmission patterns can be dampened or may be magnified depending on the capital structure and linkages. The framework can help identify situations where volatility gets magnified and negative feedback loops that can trigger severe crises. The patterns of value and default correlation across different asset classes, sectors and sovereign debt values depend on these structures and links, unique to a particular economy. Below are some examples of risk transmission between sectors. Actual risk transfer in an economy is likely to involve several risk-transmission channels.

### Risk Transmission from the Corporate Sector to the Banking Sector and to the Government

The corporate sector's financial distress – possibly caused by stock market declines which reduce the value of corporate assets, recession, commodity price drops, or excessive unhedged foreign debt accompanied by currency devaluation – can be transmitted to the financial sector.

*Risk Transfer*

**Corporate Sector** → **Banking Sector** → **Government**

We can use the three-sector framework to show how the risk can be transmitted from the corporate sector to the banking sector and to the public sector through implicit and explicit guarantees. The example of a negative shock to the corporate sector is a drop in assets as a result of recession, equity sell-offs, the combination of currency devaluation and unhedged foreign debt, or other negative shocks (shown in Annex 3, Figure A3-2). The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. As the corporate assets decline, the government guarantee to the banking sector increases in a non-linear way.

**Risk Transmission from the Banking Sector to the Government**

The banking sector's financial distress, such as systemic banking crises due to deposit runs, a decline in asset value or mismanagement, can be transmitted to the government through guarantees.

*Risk Transfer*

**Banking Sector** → **Government**

The example of the impact of a deposit run on the balance sheets of the three-sector model in Annex 3, Figure A3-3 shows that the banking sector's default barrier rises, when the banking sector faces a large deposit run resulting in a large increase in the implicit guarantee.

**Risk Transmission from the Government to the Banks and Feedback**

The public sector's financial distress or default can transmit risk to the financial system. When the banking sector is holding a significant proportion of government securities, and there is a negative shock to the government financial position, it can have a detrimental impact on the banks. The government's implicit guarantee is also likely to increase. This, in turn, makes the government financial position worse, creating a compounding effect, which may result in the government's failure to honor its guarantee obligations and cause a collapse of the banking system.

*Risk Transfer*

**Banking/Financial System** ⇄ **Government**

Figure A3-4 in Annex 3 shows the impact of a decline in government assets resulting in lower value of sovereign debt relative to the base case. In Figure A3-5, we assume the same decline in government assets but that the banking sector's assets consist of *half* government securities and *half* loans to the corporate sector (as compared to 100% corporate loans in the base case). The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above. In some situations, this vicious cycle can spiral out of control, eventually resulting in the inability of the government to provide sufficient guarantees to banks and leading to a systemic financial crisis.

**Risk Transmission from the Pension System to the Government**

The financial distress related to pension plans can result in the transmission of risk to the government.

*Risk Transfer*

**Pension System** ⟶ **Government**

Figure A3-6 shows an example of this type or risk transmission. We assume that the pension system contains one-half of corporate sector equity (in a defined benefit plan which has an implicit government guarantee). A decline in corporate assets would cause the corporate equity value to drop. This, in turn, increases the government guarantee to the pension system and the implicit guarantee to banks.

**Risk Transmission from the Public Sector to Holders of Public Sector Debt**

Fiscal, banking and other problems can cause distress for the government which can transmit risk to holders of government debt.

*Risk Transfer*

**Public Sector** ⟶ **Debt Holders (sovereign foreign currency denominated debt or sovereign local currency denominated debt)**

Holders of foreign-currency debt have a claim on the value of the debt minus the potential credit loss, which is dependent on the level of assets of the public sector (in foreign currency terms) compared to the foreign-currency default barrier. Thus, we can use the CCA approach to analyze the value of public sector foreign-currency debt by comparing how the volatility of the public sector assets (measured in US dollar terms) changes relative to the foreign-currency default barrier. A large component of the spread on sovereign foreign-currency debt is the credit spread to compensate for the risk of default over the horizon. The credit spread on sovereign foreign-currency debt is a function of: (i) the ratio of sovereign asset, A, to the default barrier, $DB_F$ (associated with default free debt value of foreign debt; (ii) the volatility of sovereign assets, $\sigma_A$; and, (iii) horizon and risk-free interest rate.[15] As the term ($A/DB_F$) declines and/or $\sigma_A$ increases, the spread increases in a *non-linear* way and eventually turns sharply higher. The total public sector asset includes foreign currency reserves, the net fiscal asset, and the value of seigniorage in US dollar terms. Thus a decline in foreign currency reserves, lower fiscal revenues, and/or a rise the foreign debt default barrier will raise spreads.[16]

The value of (risky) local currency debt is influenced by the risk that the government may dilute (or inflate away) part of the value or the debt, or may forcibly restructure some of the debt. The "dilution/inflation risk premium" is an extra premium demanded by the holders of local currency debt.

The volatility of the public sector asset is heavily influenced by exchange rate and fiscal volatilities. In the crisis periods, the fiscal volatility and exchange rate volatility can combine to produce a higher volatility of the sovereign asset . This means that the risk premium on local currency debt is very likely to be higher and lead to an increase sovereign spreads on foreign currency debt. A stylized distress scenario for an emerging market is a decline in the sovereign asset, rolling over local currency debt which becomes more difficult as the holders of the local currency debt demand a higher premium, likely monetization of the deficit leading to higher inflation and depreciation of the exchange rate. This lower foreign currency value of sovereign assets and higher volatility increases spreads on foreign currency debt as default probability can increase. A sovereign can, in principle always issue more money but foreign currency cannot be printed. This is somewhat analogous to a firm that can dilute stock holders, e.g. stock splits, and issue shares but cannot print hard cash needed to service debt.

[15] Spread = $-1/T \ln[\, N(d_2) + (A/(DB_F\, e^{-rt}))\, N(-d_1)]$, see Annex 1.

[16] See IMF GFSR April 2006, Box 3.6 for sovereign CCA and impact of changes in debt structure

## Interrelationship of Macro Financial Contingent Claim Balance Sheets, Risk Exposures and Traditional Macroeconomic Flows

The combined accounts – income/flow, mark-to-market balance sheets, and risk exposure measures -- comprise the three important sets of interrelated accounts in the economy which are somewhat similar to those in large modern financial institutions. Risk managers would find it difficult to analyze the risk exposure of their financial institution by relying solely on the income and cash flow statements, and not taking into account (mark-to-market) balance sheets or information on their institution's derivative or option positions. The country risk analysis that relies only on macroeconomic flow-based approach is deficient in a similar way, given that the traditional analysis does not take into account the volatility of assets. Note that when the volatility of assets in the CCA balance sheet equations is set to zero the values of the implicit put and call options go to zero. Something very similar to the traditional macroeconomic flow of funds is the result since the change in assets is equal to changes in cash and book value of debt. Flow of Funds can be seen as a special deterministic case of the CCA balance sheet equations when volatility is set to zero and annual changes are calculated. The risk transmission between sectors is lost.

## Controlling and Transferring Risk

The application of CCA to analyze risk exposures in the sectors of an economy offers a rich framework for comparing alternative ways to control and transfer risk. There are several benefits. First, it gives the interrelated values and risk exposure measures across sectors. Understanding of these values and risk exposures can help identify particularly vulnerable situations and potential chain reactions of default. This allows formulation of various alternative ways to control and transfer risk. Second, the framework dovetails with risk-management strategies involving *explicit derivatives and swaps* used by the private and public sectors to control, hedge or transfer risk.[17]

Four broad categories of strategies are: a direct change in the financial structure (the structure of assets and liabilities within the existing institutional structure); managing

[17] One example, in Blejer and Schumacher (2000), includes central bank forward contracts.

guarantees; risk transfer (diversification, hedging and insurance); and, in the longer run, an institutional change to tailor the institutional structure to fulfill financial functions more efficiently within the specific geopolitical environment.

**Direct Change in Financial Structure**

Increases in assets and declines in default barriers can reduce the vulnerability to distress, reduce spreads on debt, and reduce the value and the deltas of put options (whether they are embedded in risky debt or financial guarantees from the government). CCA, by its nature, shows how the changes in value of assets relate to changes in values of liabilities. Thus, it provides a natural framework for analysis of mismatches, such a currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability.

**Management of Guarantees**

CCA provides the key to measuring the value and understanding the risk of guarantees. The three basic methods that a guarantor of liabilities has to manage the risks of guarantees are:

*Monitoring* – The method requires frequent marking-to-market of assets and liabilities of the insured party and collateral that can be seized when the insured party's assets fall below a predetermined target.

*Asset Restrictions* – This method of controlling costs and managing the cost of the guarantee requires the insured party to (at least partially) hedge its guaranteed liabilities with restrictions on assets in a manner that limits the volatility of net worth.

*Risk-based premiums* – Under this method, the guarantor charges a fee that is commensurate with the riskiness of the guarantee.[18]

Guarantees on the debt of financial institutions, whether explicit or implicit, should be openly recognized in the government's balance sheet. This framework provides a way to measure the value of the guarantee and the risk exposures associated with the guarantee.

---

[18] See Merton and Bodie (1992, 1993) and Bodie and Merton (1993).

**Risk Transfer**

There are three ways to transfer risk, *diversification, hedging and insurance*. Much of the risk described here results from concentration risk and diversification to parties who have a comparative advantage in bearing various risks. If the balance sheets of corporations and financial institutions are weak when the economy is weak—as it is generally the case—then it is precisely when tax revenue is low, and the cost of debt service is high because sovereign risk has increased. In this case, the value of the guarantees will be particularly high. This observation offers a powerful argument for diversification of the government exposure to local shocks (see Box 1).

The financial markets, especially in emerging markets, are often "incomplete", meaning that they provide only limited possibilities to shift risk across various entities and groups. In such situations, diversification through international capital mobility is the obvious alternative. However, the transfer across borders of the ownership of real and financial assets is a rather inflexible way to achieve diversification (as it is costly to reverse); often it also runs against political constraints.

**Box 1**

## Examples of Diversification, Hedging and Risk Mitigation

- *Asset Diversification in Banking Sector* – Asset diversification would suggest that a bank which invests part of its assets in domestic government bonds enhances its exposure to local macro shocks; the value of government bonds will be low precisely when the value of the loan book is low. Therefore, in such economies, banks should hedge the exposure of their loan book by investing in non-domestic assets—such as bonds.
- *Equity Swaps as a Method of Diversifying Internationally* – An equity swap would enable a small country to diversify internationally without violating possible restrictions on investing capital abroad. Suppose that small-country pension funds who already own the domestic equity were to enter into swaps with a global pension intermediary (GPI). In the swap, the total return per dollar on the small country's stock market is exchanged annually for the total return per dollar on a market-value weighted-average of the world stock markets. The swap effectively transfers the risk of the small-country stock market to foreign investors and provides the domestic investors with the risk-return pattern of a well-diversified world portfolio. Since there are no initial payments between parties, there are no initial capital flows in or out of the country. Subsequent payments, which may be either inflows or outflows, involve only the *difference* between the *returns* on the two stock market indices, and no "principal" amount flow.
- *Contingent Reserves or Contingent Sovereign Capital* – Corporations sometimes contract for contingent equity or debt purchases triggered under pre-agreed conditions. Similarly, governments could make arrangements with external public or private sector entities for pre-agreed purchase of government local-currency debt under specific circumstances such as a sudden stop in capital flows or certain revenue losses, commodity price drops or natural disasters. The value of such contingent capital can be compared to the costs of increasing paid-in capital reserves via debt issues. This macrofinance framework could be used to do value-at-risk for the sovereign balance sheet to help determine the appropriate level of foreign currency reserves and contingent reserves or contingent sovereign capital.
- *Sovereign Bonds with Special Features* – GDP-linked bonds or bonds with specific roll-over clauses can help manage risk. Indexed bonds. Commodity linked bonds linked to major exports such as oil or copper. Catastrophy bonds (CAT) and similar instruments.
- *Diversification and Hedging Related to Management of Foreign Reserves* – A sovereign holds foreign currency reserves, in part, to as a cushion against potential losses of the monetary authorities or government. The framework described here can be used to assess the costs of increasing reserves via issue of foreign debt, local currency debt, money or contingent capital contracts against the benefits of having a cushion to mitigate losses (e.g. mitigate the risks posed by the implicit put options associated with government foreign and local currency debt and contingent obligations of the monetary authority). If there are excess reseves, the framework here could be used to assess the investments or strategies that provide the likely optimal hedging, diversification or risk mitigation tailored for the specific risk characteristics of the country's sovereign balance sheet.
- *Others* – Other types of swaps could include assets, equity, or debt of the corporate sector, the financial sector, and the public sector. Weather derivatives. Credit derivatives. Postitions taken by various public organizations to lay-off risk in adverse circumstances and/or or to mitigate tax revenve and spending risks.

The macro finance analytical framework could be useful for the design of new risk intermediation and risk transfer products, whereby various risks in one economy could be packaged and sold internationally to improve the efficiency of risk sharing and enhance returns.

## Institutional Change

In the longer-term, institutional changes to satisfy certain financial functions more efficiently can reduce risk. In the case of the banking sector, this is particularly important, given the vulnerabilities and costs of crisis in the banking system. Fiscal costs of banking crisis show no sign of declining and can range from costs of 3% to 80% of GDP, not to mention the inefficiencies caused before, during and after the crises. The potential for very costly government guarantees to the banking system, which can arise quickly and can have large associated risk exposures and costs, support the arguments *that it may be best to safely shrink the banking system*.[19] Structural reform, over time, could aim to reduce the size of the banking system and increase the role of institutions that can fulfill the key functions of banks (payments functions and pooling and investment of resources) but do so in an efficient and less risky manner.

The combination of a smaller banking system, improved management of guarantees, equity swaps between the pension system and international counterparties, and direct change in the financial structure would dramatically reduce risk exposures and systemic vulnerability.

[19] Posen, A. 2001, "*A Strategy to Prevent Financial Crises: Safely Shrink the Banking Sector*"

## Conclusions

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of countries' economic and financial systems. This paper proposes a new approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use CCA to model and measure sectoral and national risk exposures, and we analyze policies to offset their potentially harmful effects. The framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. It provides a new framework adapting the CCA model to the sovereign balance sheet which can help forecast credit spreads and a framework for relative valuation of credit and market risks for the sovereign and across economic sectors. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improved price and volatility discovery, and expanding international risk intermediation activities.

## Annex 1 - Framework for Contingent Claims Analysis, Risk Measures, and Spreads Using Black-Scholes-Merton Formula

### Black-Scholes-Merton Equations for Pricing Contingent Claims

CCA defines these fundamental relationships between the value of assets and the value of claims. The total market value of assets, A(t), of an entity financed with debt and junior claims (most junior claim or equity) is equal to the market value of junior claims and market value of risky debt, J(t) + D(t). Assets are stochastic and thus assets, at time t, in the future may decline below the point where debt payments on scheduled dates cannot be made. The junior claim (equity in the case of firms) can be viewed as an implicit call option.

Assets = Junior Claim + Risky Debt
= Junior Claim + Default-Free Debt – Debt Guarantee

The value of the junior claim, the debt guarantee embedded in the value of risky debt, and the financial guarantee can all be formulated in terms of *implicit options* (Merton, 1974). The value of "risky" debt is the default-free value of the debt minus the debt guarantee.

Risky Debt = Default-Free Debt – Debt Guarantee
= Default-Free Debt – Implicit Put Option

Financial Guarantee = Implicit Put Option

The implicit put option is $P = Be^{-rt}(N(-d_2)) - A_0(N(-d_1))$. The value of the risky debt, D, is thus the default-free value minus the expected loss: $D = Be^{-rt} - P = Be^{-rt} - (Be^{-rt}N(-d_2) - A_0 N(-d_1))$. The value of the junior claim, J, is equal to the value of call option, $J = A_0 \mathrm{N}(d_1) - Be^{-r_f T}\mathrm{N}(d_2)$

where $d_2 = \dfrac{\ln\left(\dfrac{A_0}{B}\right) + \left(r_f - \dfrac{s_A^2}{2}\right)t}{s_A\sqrt{t}}$, and $d_1 = d_2 + s_A\sqrt{t}$, r is the risk free rate and t is the horizon period. N(d) = the probability that a random draw from a standard normal distribution will be less than d.

$N(-d_2)$ is the risk-neutral default probability and $N(-d_2^*)$ is the "real world or physical" default probability. $N(-d_2^*) = N(-d_2^* - l\sqrt{t})$ where $l$ is the market price of risk. The credit spread, s, is the premium required to compensate for the expected loss. The get the formula for the spread note that the yield-to-maturity for the risky debt D is $y_t$, then

$\exp(-y_t t) = \frac{D}{B} = \frac{Be^{-rt} - P}{B}$ This can be rewritten to get the spread, s.

$$s = y_t - r = -\frac{1}{t}\ln\left(1 - \frac{P}{Be^{-rt}}\right) = -\frac{1}{t}\ln\left(N(d_2) + \left(\frac{A}{Be^{-rt}}\right)N(-d_1)\right)$$

The delta of the put option is $N(d_1) - 1$

**Example:** Assuming that:

Asset value A=$100,
Asset return volatility of $\sigma$ = 0.40 (40%),
Default-free value of debt = default barrier = DB = $ 75
(derived from short-term debt, $30, plus one-half of long-term debt, $90)
Risk-free rate = 0.05 (5%)
Time horizon = 1, one year

The value of the junior claim/equity is $32.2 and the value of risky debt is $67.8 (equal to the present value of the default barrier minus put option = 75*0.95 – 3.55 = 67.8). The "delta" for the call option is N($d_1$), is 0.89 in the above example. The "delta" for the put option (implicit guarantee) is N($d_1$) – 1, or -0.11 in the example. Using the spread formula above, the one-year spread for the example is calculated as: 0.0510, or 510 basis points over the risk-free rate. The *probability of default*, using this model, is N(–$d_2$), or 0.20 (20%) in the example above.

**Calculating Implied Assets and Implied Asset Volatility for Firms, Banks, Non-bank Financials with traded equity**

In the Merton model for firms, banks and non-bank financials with traded equity the following two equations are used to solve for the two unknowns A, asset value, and $\sigma_A$, asset volatility.

$$J = A_0 \mathrm{N}(d_1) - Be^{-r_f T}\mathrm{N}(d_2)$$

$$A\sigma_A = J\sigma_J \frac{\partial J}{\partial A} = J\sigma_J N(d_1)$$ [20]

[20] For a recently published book explaining these concepts applied to credit risk, see Crouhy, Galai, and Mark, 2000.

## Annex 2 - Public Sector CCA Balance Sheet and its Calibration Using the Contingent Claims Approach

This Annex describes a framework how the segregated contingent claim balance sheet of the monetary authorities and the government can be combined together and how the implied sovereign assets and asset volatility can be calculated and risk indicators estimated.[21] Useful insights can be obtained when one views relationship between the assets and liabilities of the public sector[22] in a similar way as separate balance sheets of the government and monetary authorities where there are cross-holdings and financial guarantees between these two public sector "partners." Under this structure, the assets of the monetary authority include foreign reserves, credit to the government and other claims. The liabilities of the monetary authority partner are base money and financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency denominated debt. The assets of the government partner include the net fiscal asset, other assets, while the liabilities include credit to the monetary authority (and could include local-currency debt held by the monetary authority) , local-currency debt held outside of the government and monetary authority, financial guarantees and foreign currency denominated debt.

Figure A2-1 shows the structure of this segregated balance sheet structure. This simplified framework is not meant to be a comprehensive catalogue of all the guarantees, the nature of which varies by country and by the detailed structure of the relationship between monetary authorities and the government. There may also be implicit financial support from the monetary authorities to the government via purchase of government local-currency debt under certain circumstances, but this is not shown here. The action of the monetary authority "partner" of buying additional government local-currency debt entails issue of additional base money. There are also "options," that the government and the monetary authorities have to "default" on the obligations to convert local currency into foreign currency. Similarly the government could "forcibly" restructure local-currency debt or to dictate "mandatory" purchases of government bonds by certain public or private institutions or the option to inflate to cover potential shortfalls. Also, in some countries, banks may have deposits with the monetary authorities that receive a higher priority claim on foreign currency reserves than the holders of local currency, which could be junior to claims on foreign currency for payment of external foreign-currency debt.

[21] See Gapen, Gray, Lim, Xiao, 2005.

[22] See Buiter, W. 2000.

## Figure A2-1 – Segregated Balance Sheet for the Public Sector

| Assets | Liabilities |
|---|---|
| **MONETARY AUTHORITY "PARTNER"** | |
| **Foreign Reserves** | ***Obligation to supply FX to Government to pay FX Debt*** |
| ***Credit to Government*** | **Base Money** |
| **Credit to other Sectors** | |
| **GOVERNMENT "PARTNER"** | |
| **Net Fiscal Asset** | **Guarantees (to too-important-to-fail entities)** |
| **Other Public Sector Assets** | **Foreign-currency Debt** |
| ***Obligation from Monetary Authority to supply FX to Government to pay FX Debt*** | **Local-currency Debt Held Outside of the Government & Monetary Authorities** |
| | ***Credit from Monetary Authorities*** |

Note that the cross-holdings of government debt and guarantees from monetary authorities to Government are in italics. Liquidity operations of the monetary authorities are not included.

The priority of the claims on the assets certainty can vary from country to country. In many cases, though, we can think of the guarantees to banks or other "too big to fail entities" as senior claims. Also most governments find it easier to inflate or dilute local currency debt in a distress situation before defaulting of foreign currency debt. Thus a case can be made that foreign currency debt is senior to local currency debt. The government may certainly take the view that credit from the monetary authorities is the most junior obligation and many governments may or may not honor that claim. The credit from the monetary authorities is an asset on the side of the monetary authority partner and a liability of the government partner. Similarly, the financial guarantees to the government partner are an asset on its balance sheet and a liability of the monetary authority partner. When the balance sheets are combined these two items drop out. The segregated balance sheet above reduces to the combined balance sheet in Figure 1. Contingent claims approach can be applied to the segregated or the combined balance sheets, the choice of which depends on the purposes of the analysis.

**Calculating Implied Sovereign Assets and Implied Sovereign Asset Volatility Using CCA for the Public Sector Balance Sheet**

This section describes how the CCA framework can be used to estimate implied assets and asset volatility, since the market value of sovereign assets cannot be observed directly. The public sector balance sheet in Figure A2-2 has liabilities structured in a way that we can observe the market value of the junior claims and the distress barrier of foreign currency debt so as to be able to adapt the Merton model to the sovereign. One adjustment needed is to subtract the "senior" guarantee to too-big-to-fail entities from both sides the balance sheet as shown in Figure A2-2. On the simplified balance sheet, the local-currency debt of the government, held outside of the monetary authorities, and base money are local currency liabilities which can be modeled as a call option on the public sector assets with the default barrier derived from the foreign-currency debt .

## Figure A2-2 Example Public Sector Contingent Claims Balance Sheet with Liabilities Modeled with Options (all items in $ terms)

| Assets | Liabilities |
|---|---|
| **$Foreign Reserves**<br><br>**$Net Fiscal Asset** (Stochastic Present Value of Taxes minus Expenditures)<br><br>**$Other Public Assets**<br><br>**minus $Guarantees (Modeled as a Put Option associated with Banks/Entities receiving guarantee)** | **$Foreign-currency Debt (Default Free Value of Debt minus Put Option )**<br><br>**$ Base Money plus LC Debt Held Outside of the Government & Monetary Authorities (Call Option)** |

The units in which the balance sheet is measured can be nominal local currency units, in real terms in local currency units, or in foreign currency. Any numeraire can be used. Since one of the goals of the analysis here is to analyze the value of the CCA balance sheet in the international context (including effects of changes in FX reserves and the credit risk embedded in foreign debt), the units are in US $.

Local currency debt is a claim on sovereign assets whose value, in principle, can be diluted or inflated. Money is a claim; its issue can be used to increase sovereign assets, via increase in reserves or credit to government. It can in most cases, be exchanged for foreign currency and in the event of dollarization reserves are used to buy out the monetary base. In most cases it is a long-term claim and in that sense junior to debt obligations. Local currency debt and money have some similarities to "shares" and the value of money and local currency debt times the exchange rate can be seen a sort of "market cap" of the sovereign.

This model combines money and local currency debt together to get Local Currency Liabilities (LCL) in a simple two claim CCA framework in order to calibrate the sovereign balance sheet by calculating implied sovereign assets and asset volatility. Value of local currency liabilities in foreign currency terms, $LCL_{\$}$, is a call option of sovereign assets in foreign currency terms, $V_{\$Sov}$, with strike price tied to the distress barrier for foreign currency denominated debt $B_f$ derived from the promised payments on foreign currency debt and interest payments up to time t..

$$LCL_{\$} = V_{\$Sov} \mathrm{N}(d_1) - B_f e^{-r_f T} \mathrm{N}(d_2)$$

The formula for the value of local currency liabilities in foreign currency terms is:

$$LCL_{\$} = M + B_{d\$,t=0} = \frac{(M_{LC} e^{r_d T} + B_d) e^{-r_f T}}{X_F}$$

The volatility of the local currency liabilities is:

$$s_{\$LCL} = f\left(M, B_d, r_d, s_M, s_d, X_F, r_{M,Dd\$}\right)$$

The definition of the variables is shown below.

$M_{LC}$ base money in local currency terms; $r_d$ domestic interest rate; $r_f$ foreign interest rate; Domestic currency denominated debt is $B_d$ (derived from the promised payments on local currency debt and interest payments up to time t); $X_F$ forward exchange rate; $s_{X_F}$ volatility of forward exchange rate; $s_{D_d}$ volatility of domestic debt in local currency terms; $r_{D_d, X_F}$ correlation of forward exchange rate and vol of domestic debt in local currency terms; $r_{M, D_{d\$}}$ correlation of money (in foreign currency terms) and local currency debt (in fc terms); $s_{MLC}$ Volatility of money (in lc terms); $s_M$ volatility of money (in fc terms); and, $s_{D_{d\$}}$ volatility of local currency debt (in fc terms).

The two key equations relating assets and local currency liabilities are:

$$LCL_{\$} = V_{\$\mathrm{Sov}}\mathrm{N}(d_1) - D_f e^{-r_f T}\mathrm{N}(d_2)$$

$$V_{\$Sov}\sigma_{\$Sov} = LCL_{\$}\sigma_{\$LCL}\frac{\partial L_{\$LCL}}{\partial A_{\$Sov}} = LCL_{\$}\sigma_{\$LCL}N(d_1)$$

These two equations can be used to calculate the two unknowns, sovereign asset value and sovereign asset volatility. The sovereign default probabilities, spreads and other risk indicators can be calculated.[23] Stochastic interest rates (both domestic and foreign) can be incorporated in the model with frameworks such as Shimko et. al. (1993) which integrate a Vasicek term-structure model into the Merton model.

**Breaking Down Sovereign Assets into Key Components**

The sovereign asset value can be broken down into its key components, Reserves (R), net fiscal asset or present value of the primary fiscal surplus (PVPS), Implicit Guarantees (G), and Other remainder items.

$$V_{Sov\$,0} = R + PVPS_{\$} - G_{\$} + Other_{\$}$$

The value of the foreign currency reserves can be observed and the guarantee can be estimated from the banking and corporate sector CCA model. Subtracting these from the Implied sovereign asset we can calculate the residual which includes the primary fiscal surplus. If we estimate the expected present value of the primary fiscal surplus (an obvious approximation) the remainder "Other" can be estimated. "Other" may be due to various factors, including contingent financial support from other governments or multilaterals.

$$V_{Sov\$,0} - R + G_{\$} - E[PVPS_{\$}] = Other_{\$}$$

We can use this valuation formula to evaluate the effects of changes in reserves, the primary fiscal balance, and the implicit guarantee on the sovereign asset value. This can be used with changes in the composition of short-term and long-term debt and with money and the exchange rate for sensitivity and stress tests to evaluate changes in soverign credit spreads and other values and risk indicators.

[23] Xu,D and Ghezzi, P. 2002 develop a stochastic debt sustainability model and show how it is related to the CCA model described in Gray, Merton, Bodie 2002 and this paper.

## Annex 3 – Examples of Risk Transmission Channels between Economic Balance Sheets of Sectors

This Annex describes the CCA equations and inter-linkages among sectors for a simple three sector framework and numerical examples for a number of different scenarios which demonstrate value and risk transmission and calculation of risk exposures. Figures in Annex 3:

1. Base case sectoral balance sheet.

(Changes in parentheses in all figures are all relative to base case)

2. Negative shock to corporate sector assets and the subsequent impact on banking assets and increase in government implicit guarantee.

3. Deposit run and subsequent rise in government implicit guarantee.

4. Negative shock to government assets resulting in lower value of sovereign debt.

5. Negative shock to government assets and thus a decline in the value of government securities held by banks leading to an increase in implicit financial guarantee (and feedback loop).

6. Negative shock to corporate sector assets (as in 2. above) in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has an implicit government guarantee). The negative shock to corporate sector assets results in lower pension system's asset value with higher implicit guarantee for pension system, in conjunction with higher implicit guarantee to banks.

## Illustrative Equations for Inter-linked Sectors

*MV stands for market value, A is assets, E is equity, J is junior claim, D is debt, FS is financial guarantee from government, C means call option, P means put option. (Subscripts C, B, and G refer to the corporate, banking and financial, and public sector, respectively.)*

***Corporate Sector***

Market value balance sheet equation for the corporate sector(s) using contingent claims.

$MV(A_C) = MV(E_C) + MV(D_C)$ *[Market Value Balance Sheet Equation]*

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[ Equation for MV(D$_C$)]*

$MV(A_C) = C[MV(A_C), DB_C, \sigma_C, T, r] + DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$

***Bank and Financial Institutions Sector***

Using the market value balance sheet equation, we substitute variables representing the contingent claims:

$MV(A_B) + FS_G = MV(E_B) + MV(D_B)$ *[Market Value Balance Sheet Equation]*

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support, i.e. Guarantee]*

$MV(A_B) + FS_G = C[MV(A_B), DB_B, \sigma_B, T, r] + MV(D_B)$

Banking sector assets contain loans to the corporate sector

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[Loans Made to the Corporate Sector]*

***Public sector (Government and Monetary Authority) Sector***

Using the market value balance sheet equation, we substitute in variables representing the contingent claims:

$MV(A_G) = MV(J_G) + MV(D_G) + FS_G$ *[Market Value Balance Sheet Equation]*

$$MV(A_G) = C[MV(A_G), DB_G, \sigma_G, T, r] + DB_G - P[MV(A_G), DB_G, \sigma_G, T, r] + P[MV(A_B), DB_B, \sigma_B, T, r]$$

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support to Banks/Financial Institutions]*

$MV(D_G) = DB_G - P[MV(A_G), DB_G, \sigma_G, T, r]$ *[Equation for MV(D$_G$)]*

## Annex 3 (cont.) Figure A3-1

Example sectoral economic balance sheets base case:

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity (call option) | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 81.3 |
| Financial Guarantee | 7.4 | Equity | 13.3 |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 7.4 |
| Foreign reserves | 40 | Foreign Debt | 82.15 |
| | | Local-currency Debt & Other Liabilities | 50.45 |
| Total | 140 | | 140 |

(Units are in Billions of $)

The *delta* of the guarantee is -0.35 in this base case.

## Annex 3 (cont.) - Figure A3-2

Negative Shock to Corporate Sector Assets, decline of $40 billion (from $120 to $80 billion) as compared to base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 80<br>(- 40) | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option of 15.8) | 74.1<br>(- 13.1) |
| | | Corporate Equity | 5.9<br>(- 26.9) |
| Total | 80<br>(- 40) | | 80<br>(- 40) |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -15.8) | 74.1<br>(- 13.1) | Deposits | 81.3 |
| Financial Guarantee | 13.3<br>(+ 5.7) | Equity | 6.1<br>(-7.2) |
| Total | 87.4<br>(- 7.2) | | 87.4<br>(- 7.2) |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 13.3<br>(+ 5.7) |
| Foreign reserves | 40 | Foreign Debt | 80.4<br>(- 1.75) |
| | | Local Currency Debt & Other Liabilities | 46.2<br>(- 4.25) |
| Total | 140 | | 140 |

(Units are in Billions of $)

*Risk Transmission*: Lower corporate assets $\rightarrow$ lower corporate equity and debt $\rightarrow$ lower bank assets $\rightarrow$ higher guarantee from government required $\rightarrow$ higher cost of government guarantee lowers value of sovereign debt. The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. The *delta* of the guarantee is -0.56 in this case, as compared to -0.35 in the base case.

## Annex 3 (cont.) - Figure A3-3

Deposit Run - Default barrier rises for banks as $36 billion of long term savings and time deposits become short-term liabilities with deposit run. In this contingent claims framework for banks, the default barrier for banks includes deposits calculated as default-free value of short and long-term deposits which is approximated by demand deposits plus a fraction of time and saving deposits. In a deposit run, a portion of the long-term time and savings deposits shift to the short-term category, thus raising the overall default barrier and raising the size of the implicit guarantee of the government. As $36 billion of long term savings and time deposits become short-term liabilities, the result is a significant increase in implicit financial guarantees from $7.4 billion to $32.6 billion, an increase of $25.2 billion from the base case. The *delta* of the guarantee is -0.83 in this case, more than double the -0.35 delta value in the base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 117.3 (+ 36) |
| Financial Guarantee | 32.6 (+ 25.2) | Equity | 2.5 (- 10.8) |
| Total | 119.8 (+ 25.2) | | 119.8 (+ 25.2) |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 32.6 (+ 25.2) |
| Foreign reserves | 40 | Foreign Debt | 73.7 (- 8.4) |
| | | Local-currency Debt & Other Liabilities | 33.7 (- 16.8) |
| Total | 140 | | 140 |

(Units are in Billions of $)

## Annex 3 (cont.) Figure A3-4

Negative Shock to Public Sector Assets of $20 billion, result is lower "market value of liabilities" as compared to base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 81.3 |
| Financial Guarantee | 7.4 | Equity | 13.3 |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 90<br>(- 10) | Financial Guarantee to Banks | 7.4 |
| Foreign reserves | 30<br>(- 10) | Foreign Debt | 79.1<br>( - 3.1) |
| | | Local Currency Debt & Other Liabilities | 33.5<br>( - 16.9) |
| Total | 120<br>(- 20) | | 120<br>(- 20) |

(Units are in Billions of $)

*Risk Transmission*: Negative shock to government assets → lower value of foreign debt (senior debt in this example) and lower value of local-currency debt and other liabilities.

## Annex 3 (cont.) Figure A3-5

Negative Shock to Government Assets and Decline in Value of Government Securities Held by Banks - Banking sector assets consist of half government securities and half loans to corporate sector (as compared to 100% corporate loans in the base case). The market value of government securities (local-currency debt) declines due to decline in government assets of $20 billion relative to the base case. The decline in government assets of $20 billion increases the guarantee to banks by $6.2 billion to $13.6 billion. The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 (43.6 loans from banks, and 43.6 to non-banks) |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (43.6) and Govt. Securities (30) | 73.6 ( -16.4) | Deposits | 81.3 |
| Financial Guarantee | 13.58 (+ 6.18) | Equity | 6.1 ( -7.2) |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 90 (- 10) | Financial Guarantee to Banks | 13.58 (+ 6.18) |
| Foreign reserves | 30 (- 10) | Foreign Debt | 76.8 ( - 5.35) |
| | | Local-currency Debt & Other Liabilities | 29.9 ( - 20.55 ) |
| Total | 120 (- 20) | | 120 (- 20) |

(Units are in Billions of $)

## Annex 3 (cont.) - Figure A3 - 6

This scenario describes a negative shock to corporate sector assets (as in A5-2. above), in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has a government guarantee). The results are shown below in Figure A5-6. A decline in corporate assets by $40 billion (from $120 to $80 billion) would cause the corporate equity value to drop by $26.9 billion to $5.9 billion (as compared with the base case). This increases the government guarantee to the pension system by $9 billion and the implicit guarantee to banks by $5.7 billion. In total, the government guarantees to pension system and banking system would increase to $22.3 billion, significantly higher than $7.4 billion in the base case).

## Figure A3 - 6

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 80<br>(- 40) | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option of 15.8) | 74.1<br>(- 13.1) |
| | | Corporate Equity | 5.9<br>(- 26.9) |
| Total | 80<br>(- 40) | | 80<br>(- 40) |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -15.8) | 74.1<br>(- 13.1) | Deposits | 81.3 |
| Financial Guarantee | 13.3<br>(+ 5.7) | Equity | 6.1<br>(-7.2) |
| Total | 87.4<br>(- 7.2) | | 87.4<br>(- 7.2) |

**Pension System**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Equity (initially 16.4) | 3<br>(- 13.4) | Defined Benefit (Present Value) | 12 |
| Financial Guarantee | 9<br>(+ 9) | | |
| Total | 12 | | 12 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks & Financial Guarantee to Pension System | 13.3<br>+ 9<br>=22.3<br>(+ 14.7) |
| Foreign reserves | 40 | Foreign Debt | 78.9<br>(- 3.25) |
| | | Local-currency Debt & Other Liabilities | 39<br>(- 11.45) |
| Total | 140 | | 140 |

(Units are $ Billions.)

## References


Allen, M., C. Rosenberg, C. Keller, B. Sester, and N. Roubini, "A Balance Sheet Approach to Financial Crisis," IMF Working Paper 02/210, 2002.

Beckerman, P. and A. Solimano, Crisis and Dollarization in Ecuador, World Bank, 2002.

Black, F. and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (May-June): 637-54.

Black, Fischer, *Business Cycles and Equilibrium*, Basil Blackwell, 1987.

Blejer, M and A. Cheasty, Chapter 14 in How to Measure the Fiscal Deficit, edited by, IMF, 2000.

Blejer, Mario and Liliana Schumacher (2000), "Central banks use of derivatives and other contingent liabilities: Analytical issues and policy implications", Working Paper 00/66. Washington, IMF.

BIS Financial Stability Forum, *Report on Capital Flows*, April 2000, Chairman Mario Draghi.

BIS Working Paper No 92, *"Recent Initiatives to Improve Regulation and Supervision of Private Capital Flows,"* October 2000.

Bodie, Z. and R. C. Merton. "Pension Benefit Guarantees in the United States: A Functional Analysis." In The Future of Pensions in the United States, edited by R. Schmitt. Philadelphia: University of Pennsylvania Press, 1993.

Bodie, Z. and Merton R., 2000 *Finance*, Prentice Hall, Upper Saddle River, N.J.

Bodie, Zvi and Robert C. Merton (2002), "International pension swaps", *Journal of Pension Economics and Finance*, vol.1 (January).

**Bodie, Zvi, "On Asset-Liability Matching and Federal Deposit and Pension Insurance," *Federal Reserve Bank of St. Louis Review*, July/August 2006.**

Buiter, W., *"Measurment of the Public Sector Deficit and Its Implications for Policy Evaluation and Design,"* Chapter 14 in How to Measure the Fiscal Deficit, edited by Blejer, M. and A. Cheasty,, IMF, 2000.

Chan-Lau, Jorge, "Anticipating Credit Events Using Credit Default Swaps, with an Application to Sovereign Debt Crises," IMF Working Paper 03/106, 2003.

Crouhy, Michel, Dan Galai and Robert Mark (2001) *Risk Management*. New York: Mc Graw Hill.

Draghi, Mario, F. Giavazzi, and R. C. Merton, "Transparency, Risk Management and International Financial Fragility." Paper prepared for the Fourth Geneva Conference on

"Financial Markets: Shock Absorbers or Shock Creators?" Geneva, Switzerland, May 10, 2002. http://www.people.hbs.edu/larricale/Trans%20Risk%20Management.pdf

Gapen, Michael T., Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao, 2004, "The Contingent Claims Approach to Corporate Vulnerability Analysis: Estimating Default Risk and Economy-Wide Risk Transfer," IMF Working Paper 04/121 (Washington: International Monetary Fund).publications@imf.org.

Gapen, Michael T., Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao, 2005, "Measuring and Analyzing Sovereign Risk with Contingent Claims" IMF Working Paper 05/155. (Washington: International Monetary Fund). publications@imf.org

Gray, D., Robert C. Merton, Zvi Bodie, "A New Framework for Analyzing and Managing Macrofinancial Risks," Conference on Finance and the Macroeconomy, October 2002, NYU.

Gray, D. and Stone, M., *"Corporate Balance Sheets and Macroeconomic Policy,"* Finance and Development, September 1999.

Gray, Dale F. (2001) "Macro financial risk country report: Thailand, MfRISK and Macro Financial Risk Framework." (Macro Financial Risk Framework – Patent Pending).

Gray, D, "Macro Finance: The Bigger Picture," Risk Magazine, June 2002.

Gray, D., Presentations at Credit Risk Summit in NY and London in 2000; GARP Convention Presentation Feb. 2001; Presentations at several Risk Conferences, including June 2001 Boston, April 2002 Paris and June 2002 Boston.

Gray, D., "Raise the Flags: Applying Risk Analysis Tools to Economies," GARP Risk Review, Issue 11, March/April 03.

Gray, D., Lim, C. H., Malone, S., "A New Debt Sustainability Framework: Incorporating Balance Sheets and Uncertainty", Forthcoming IMF Working Paper 2006/7. IMF.

IMF, 2006, Global Financial Stability Report IGFSR) Box 3.6 page 120.

IMF, August 2006, Gray, D. and Jones, M., Indonesia: Selected Issues Paper, Chapter III. "Measuring Sovereign and Banking Risk in Indonesia: An Application of the Contingent Claims Approach" in IMF Country Report No. 06/318, publications@imf.org.

Jarrow, R., and Madan, D., *"Arbitrage, Martingales, and Private Monetary Value,"* Journal of Risk, Vol 3, No. 1, Fall 2000.

KMV Corporation, *"Modeling Default Risk,"* 1999 KMV Corp, Crosbie, Peter, KMV.

Merton, R.C. (1973), *"Theory of Rational Option Pricing,"* Bell Journal of Economics and Management Science, 4 (Spring): 141-83.(Chapter 8 in Continuous-Time Finance)

Merton, R.C., *"On the Pricing of Corporate Debt: The Risk Structure of Interest Rates."* Journal of Finance 29 (May 1974), pp. 449-70. ( Chapter12 in Continuous-Time Finance.).

Merton, R.C., *"An Analytic Derivation of the Cost of Loan Guarantees and Deposit Insurance: An Application of Modern Option Pricing Theory."* Journal of Banking and Finance 1 (June 1977), pp. 3-11 (Chapter 19 in Continuous-Time Finance)

Merton, R. C. (1992), Continuous-Time Finance. Oxford, U.K.: Basil Blackwell, (Rev. ed.)

Merton, R.C. (1995), *"A Functional Perspective of Financial Intermediation,"* Financial Management, Vol. 24, No. 2, (Summer): 23-41.

Merton, R.C. (1998), *"Applications of Option-Pricing Theory: Twenty-Five Years Later,"* Les Prix Nobel 1997, Stockholm: Nobel Foundation; reprinted in American Economic Review, (June): 323-349.

Merton, R.C., "Commentary: Finance Theory and the Shift to Integration", Risk, July 1999, P. 50

Merton, R.C., *"Future Possibilities in Finance Theory and Finance Practice."* In Mathematical Finance: Bachelier Congress 2000, edited by H.Geman, D. Madan, S. Pliska and T. Vorst. Berlin Springer-Verlag, 2002.

Merton, R.C. and Z. Bodie (1992), *"On the Management of Financial Guarantees,"* Financial Management, 21 (Winter): 87-109.

Merton, R.C. and Bodie, Z. *"Deposit Insurance Reform: A Functional Approach."* In Carnegie-Rochester Conference Series on Public Policy. Vol. 38, edited by A. Meltzer and C. Plosser. Amsterdam: Elsevier N.V., 1993.

Moody's Investors Service, Global Credit Research, *"Moody's Public Firm Risk Model: A Hybrid Approach to Modeling Short Term Default Risk."*

Neftci, S. N., An Introduction to the Mathematics of Financial Derivatives, Academic Press, USA, 2000.

Pettis, Michael, The Volatility Machine, Oxford, 2001.

Posen, A., "A Strategy to Prevent Future Crises: Safely Shrink the Banking Sector," Institute for International Economics, 2001. (http://www.iie.com/papers/posen0601-2.htm)

Schaechter, A., "Implementation of Monetary Policy and the Central Bank's Balance Sheet," IMF Working Paper 01/149, 2001.

Shimko, David, Tejima, N. van Deventer, D. "The Pricing of Risky Debt When Interest Rates are Stochastic", Journal of Fixed Income, September 1993.

Xu, David, and Piero Ghezzi, 2002, "From Fundamentals to Spreads: A Fair Spread Model for High Yield Emerging Market Sovereigns," Global Markets Research, Deutsche Bank.

Dale Gray is President of MF Risk, Inc. and Sr. Risk Expert for the IMF Monetary and Capital Markets Department. MFRisk board members include Robert C. Merton and Zvi Bodie. Contact information for Dale Gray is email Dgray@MFRisk.com or Dgray@imf.org. Contact information for Robert Merton is www.people.hbs.edu/rmerton/ and Zvi Bodie is zbodie@bu.edu.

The authors would like to thank Carlos Medeiros, Michael Gapen, Yingbin Xiao, Cheng Hoon Lim, Matt Jones, Paul Mills (all IMF), Stephen Kealhofer (Moody'-KMV), Simon Jiang (Moody's Investors Service), Sam Malone (Oxford) and Joe Zou (Taconic) for useful discussions and comments on the ideas presented in the paper.

WP/05/155

# IMF Working Paper

# Measuring and Analyzing Sovereign Risk with Contingent Claims

*Michael T. Gapen, Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao*

INTERNATIONAL MONETARY FUND

 WP/05/155

**IMF Working Paper**

International Capital Markets Department

**Measuring and Analyzing Sovereign Risk with Contingent Claims**

Prepared by Michael T. Gapen, Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao[1]

Authorized for distribution by Carlos Medeiros

August 2005

**Abstract**



This paper develops a comprehensive new framework to measure and analyze sovereign risk. Since traditional macroeconomic vulnerability indicators and accounting-based measures do not address risk in a comprehensive and forward-looking way, the contingent claims approach is used to construct a marked-to-market balance sheet for the sovereign, and derive a set of credit-risk indicators that serve as a barometer of sovereign risk. Applications to 12 emerging market economies show the risk indicators to be robust and highly correlated with market spreads. The framework can help policymakers design risk mitigation strategies and rank policy options using a calibrated structural model unique to each economy.

JEL Classification Numbers: E61, G13, G15, H63

Keywords: Sovereign risk, contingent claims, debt sustainability, risk management

Author(s) E-Mail Address: mgapen@imf.org; dgray@imf.org; clim@imf.org; yxiao2@imf.org

---

[1] The authors would like to thank Zvi Bodie, Robert Merton, Linda Tesar, and participants of the JP Morgan Chase seminars at the 2005 Annual Meetings of the Inter-American Development Bank in Okinawa and the Asian Development Bank in Istanbul, as well as of the Institute of International Finance Country Risk Workshop, for their comments and useful discussions. We are also grateful to Hung Tran, Eliot Kalter, Carlos Medeiros, and participants of the Research Department seminar in the IMF for their comments.

Contents Page

Tables


Appendix

# I. INTRODUCTION

As economies have become more reliant on private capital flows, they have also become more vulnerable to the volatility of capital flows, and to price and other shocks. A comprehensive framework is needed to analyze—and hopefully help prevent—large scale capital account crises and associated financial distress. A useful approach that has been gaining popularity since the Asian crisis is to assess the risk posed by potentially unstable positions in sectoral balance sheets, including in the corporate, financial, and public sectors. Shocks to interest rates, exchange rates, or market sentiment that bring about a deterioration in the value of a sector's assets compared to its liabilities lead to a reduction of its net worth. In the extreme case, net worth turns negative and the sector may become insolvent. In these cases, risks are transferred across balance sheets, triggering widespread distress. Risk transfer can be "bottom-up" from the corporate sector to the banking system and ultimately to the sovereign balance sheet, as was the case during the Asian crisis, or it can be "top-down," as was seen more recently in Latin America. Developing an effective approach to detect and assess balance sheet vulnerabilities before they become severe is essential to minimize risks and protect the stability of the overall economy. In this paper, the contingent claims approach (CCA) is used to measure and analyze risk on the public sector, or sovereign, balance sheet.[2] Estimating risk using such an approach has a long tradition in modern financial theory and has been widely applied in the analysis of corporate sector credit risk.[3] It is increasingly being used to estimate risk in the financial sector, but has yet to be broadly applied at the sovereign level.[4] This paper represents a first step in this direction.

Effective risk analysis must meet three objectives. First, it needs to identify existing balance sheet mismatches. Second, it must incorporate uncertainty inherent in balance sheet components since uncertain changes in future asset value relative to promised payments on debt obligations ultimately drive default risk. Third, effective risk analysis must translate uncertainty into quantifiable risk indicators that measure risk exposures to reveal whether balance sheet risks are building or subsiding. Such quantitative risk indicators should also incorporate forward-looking information.

The contingent claims approach meets all three objectives. It uses the basic structure of a balance sheet, adding market prices and uncertainty as key inputs, to derive simple risk indicators that are forward looking. In effect, this framework provides a marked-to-market balance sheet for the sovereign. In measuring sovereign risk, the contingent claims approach derives estimates for sovereign asset value and asset volatility—which are not directly

---

[2] The contingent claims approach was applied to estimate balance sheet risk in the aggregated corporate sector in Gapen, Gray, Lim, and Xiao (2004). The analysis also provided estimates of risk transfer across the corporate, financial, and public sectors.

[3] Black and Scholes (1973), Merton (1973), and Merton (1998).

[4] Merton (1977); Gray (2002); Gray, Merton, and Bodie (2003); Draghi and Merton (2003); Gray (2004); and Chan Lau (2004).

observable—from the value and volatility of sovereign liabilities that are observable. These values are then weighed against existing contractual liabilities to provide a market-based assessment of sovereign default risk. Unlike traditional macroeconomic vulnerability indicators and accounting-based measures, which cannot address risk or uncertainty in a comprehensive or forward-looking manner and rely mainly on static ratios, the contingent claims approach provides a richer, dynamic way to measure and analyze risk.

This paper develops a set of key *credit risk indicators* to measure sovereign balance sheet risk. These include: distance to distress, probability of default, credit spreads, and the market value of risky foreign currency denominated debt. These indicators are closely related since they are all derived from the core contingent claim relationships. Associated with these risk indicators are *sensitivity measures* that report how responsive the credit risk indicators are to changes in underlying model parameters, such as changes in the value of sovereign assets and volatility. Importantly, the sensitivity measures capture nonlinear changes in value which are often observed during crisis periods.

To illustrate the usefulness of the credit risk indicators as a collective barometer of sovereign risk, they are subjected to robustness tests using observed market data for a sample of emerging market countries. The tests suggest a high degree of correlation between the credit risk indicators and the observed market data on spreads. As market credit spreads were not used as inputs in deriving the risk indicators, the high correlation suggests that the risk indicators can be confidently used as reasonable measures of sovereign credit risk, thus lending support to the contingent claims structural model developed in this paper. The risk indicators can be examined in individual country cases to evaluate whether market expectations of sovereign vulnerabilities are increasing or decreasing over time or they can be examined across countries to rank relative riskiness.

As a further demonstration of the applicability of the contingent claims approach in evaluating sovereign risk, the paper uses the model calibrated to market data to evaluate how risk indicators change given specific scenarios. Through scenario analysis, policymakers can observe the extent to which negative economic shocks could worsen sovereign financial soundness through capital outflows, a depreciating exchange rate or slower economic growth. As an additional step, Monte Carlo simulations are used in conjunction with the contingent claims approach to yield probability distributions and confidence intervals for the set of sovereign credit risk indicators. Since simulations allow for the assessment of many potential market scenarios, it provides for a more comprehensive risk analysis that includes probability distributions and value-at-risk (VaR) measures. Policymakers can use these tools to help them design and implement risk mitigation strategies to reduce balance sheet risk and to rank competing policy choices.

Finally, the paper points to two promising areas in which the contingent claims approach can be usefully applied: reserve management and debt sustainability. On reserve management, the contingent claims approach can be used to derive an appropriate target for reserve adequacy, where an adequate level of reserves could be defined as the level of reserves that keep the credit risk indicators above a specified threshold. On debt sustainability, the contingent claims approach offers several advantages over the traditional debt sustainability analysis which has tended to focus on ratios of debt-to-GDP as the primary criterion for deciding whether public debt is on a sustainable path. In particular, the approach provides a structural framework that relates debt payments with the capacity to pay, and threshold levels for sovereign credit risk.

The paper is structured as follows. Section II defines sovereign risk and introduces the contingent claims approach of measuring the value and volatility of sovereign assets. Section III constructs the contingent claims balance sheet. Section IV shows how the credit risk indicators are developed. Section V applies several robustness checks to the credit risk indicators to assess their correlation with actual market data. Section VI presents a calibrated baseline balance sheet for a hypothetical sovereign. Scenario and simulation analysis is conducted on the baseline balance sheet to assess the impact on sovereign risk. The section discusses briefly how this approach can be used to evaluate potential policy choices. Section VII details the next steps in the application of this approach to evaluating reserve management and debt sustainability. Section VIII concludes. Further details on the use of option pricing techniques to derive the credit risk indicators are provided in the appendix.

## II. A Practical Approach to Sovereign Risk

Of the different types of sovereign risk, one of the most important is the risk of default. Sovereign default is often a culmination of accumulated distress, where the risk of sovereign default is effectively driven by the interplay of three main elements: the value of sovereign assets, asset volatility, and leverage. Sovereign asset value is defined as the combined *market value* of all sovereign assets. Asset value is an aggregate of different components which are dependent on the country's future economic prospects and policy decisions. Since future economic prospects are uncertain, asset volatility captures the inherent uncertainty, or variability, of future sovereign asset value. Leverage measures the size of the sovereign's contractual liabilities. Contractual liabilities are measured in *book value* terms since these are the amounts that the sovereign is obligated to pay.

The approach to sovereign risk outlined in this paper closely mirrors a similar process that has successfully been applied to estimate firm credit risk.[5] There are sufficient similarities between individual firm risk and sovereign risk to suggest a reasonable transfer of the contingent claims approach from corporate to sovereign risk analysis. These similarities are examined in more detail in Box 1. In the next section, the concept of sovereign distress is defined, and the interplay between the sovereign asset, its volatility and debt obligations in the determination of sovereign distress is discussed.

[5] The contingent claims approach has been widely applied by financial market participants to measure the default probability of corporations and banks based on the market prices of the equity and book values of debt. See KMV (1993); Crosbie and Bohn (2001); Crouhy, Galai, and Mark (2000); and Cossin and Pirotte (2001) for the contingent claims approach to individual firm credit risk.

## A. Defining Sovereign Distress: The Concept

Sovereign distress increases when the market value of sovereign assets declines relative to its contractual obligations on debt. Default ultimately occurs when the sovereign assets fall below the contractual liabilities. Contractual liabilities, therefore, constitute a *distress barrier, and* sovereign distress is measured by the relationship between sovereign assets relative to this distress barrier. Default risk increases when the value of sovereign assets declines towards the distress barrier or when asset volatility increases such that the value of sovereign assets becomes more uncertain and the probability of the value falling below the distress barrier becomes higher.[6]

Evidence from the universe of corporate defaults indicates that the market value of firm assets can sometimes trade below the book value of total liabilities for a significant period of time.[7] This is most often the case when the majority of liabilities are long-term, allowing the firm to continue servicing debt payments while undertaking steps to improve the financial health of the firm. A similar argument can be applied to sovereign credit risk, whereby the probability of distress is increased when most of the liabilities are short-term, or when rollover risk is highest. Therefore, the approach adopted in this paper follows the well-established procedure in estimating corporate default risk, namely that the value of sovereign assets that triggers an incidence of sovereign distress lies somewhere in-between the book value of total liabilities and short-term liabilities. This adjusted value of liabilities is defined as the distress barrier, and is commonly denoted as the sum of short-term debt, interest payments for one year, and half of long-term debt. [8]

The market value of sovereign assets in relation to the distress barrier is illustrated in Figure 1. The uncertainty in future sovereign asset value is represented by a probability distribution at the time horizon. At the end of the period, the value of sovereign assets may be above the distress barrier, indicating that debt service can be made, or below the distress barrier, leading to default. The probability that sovereign assets will fall below the distress barrier is simply the area of the distribution that lies below the distress barrier. The bottom panels in Figure 1 detail the effects of a decline in the value of sovereign assets and an increase in uncertainty over future sovereign asset value. In the first case, the lower expected

---

[6] Volatility of sovereign assets can differ across countries for many reasons, including, but not limited to, the level of international reserves on the government's balance sheet, the exchange rate, and variations in government revenue and expenditures. Countries with lower asset volatility are generally able to use larger amounts of leverage with relative comfort while countries with higher asset volatility would be better-off taking on less leverage.

[7] Crosbie and Bohn (2003). Moody's KMV maintains a database with over 250,000 company-years of data and 4,700 incidents of default or bankruptcy.

[8] This definition of the distress barrier is identical to that used by Moody's KMV in corporate sector default risk analysis (Crosbie and Bohn, 2003). Short-term is defined as one year or less by residual maturity.

sovereign asset value means more of the probability distribution lies below the unchanged distress barrier, which results in a higher probability of default. In the second case, the lower expected sovereign asset value is also accompanied by an increase in asset volatility. The increased volatility widens the probability distribution, leading to an even higher probability of default as more of the area under the probability distribution now lies below the distress barrier.

### B. Estimating the Value of Sovereign Assets

Given the conceptual definition of sovereign distress, how does one go about estimating it? The main challenge would be deriving an accurate estimate for the market value and volatility of sovereign assets. While the levels and amounts of contractual liabilities are relatively easy to determine from balance sheet information, the same is not true when measuring the value of sovereign assets or its volatility.[9] The market value of sovereign assets is not directly observable and must therefore be estimated. With this in mind, there are several ways to value an asset:

- Determining value from observed market prices of all or part of the asset. This can be from a market price quote, direct observation, bid-ask quote or other similar direct measures;
- Determining value by a comparable or adjusted comparable. A sophisticated version of obtaining a comparable value is the present value of a discounted expected cash flows—such as the primary surplus—with an appropriate discount rate;
- Determining value from an implied value where the balance sheet relationships between assets and liabilities allow the observed prices of liabilities to be used to obtain the implied value of the assets.

The three methods have different advantages and disadvantages. The first method is straightforward but difficult to apply because only a few components of sovereign assets have directly observable market prices. International reserves are both observable and have a market value, yet the remaining items lack observable market prices.[10] The second method using comparables is commonly used, but also has shortcomings. These are related to the difficultly of projecting future cash flows, deciding the appropriate discount rate, and determining all of the relevant components that underlie the cash flow projections for

---

[9] Foreign currency debt in global markets is predominantly fixed-rate, "bullet" maturity debt which results in easily defined contractual flows. Some global debt is amortizing, but these payments are usually well-specified. The main difficulties in estimating debt payments arise when the debt payments are linked to changes in interest rates, exchange rates, or inflation. These forms are more often found in domestic as opposed to global capital markets.

[10] Buiter (1993) discusses in detail the many items on the balance sheet of the public sector, including nonmarketable items, such as social overhead capital.

tangible and intangible items included in the asset value estimation. For example, determining the present value of the net fiscal asset requires estimates of future economic performance, the political commitment to a variety of programs including social security and other entitlement programs, and the use of an appropriate discount rate. Estimates for the value of other assets like the value of the public sector monopoly on money issuance run into similar problems. Furthermore, it is unclear how asset volatility should be best measured under the first two methods.

The third method, which is the approach adopted in this paper, circumvents the problems in the first two methods by estimating sovereign asset value and volatility indirectly with information on observable values of the liability side of the balance sheet. This approach relies on the relationship between assets and liabilities. Since liabilities are claims on current or future assets, this approach is often referred to as "contingent claims" analysis and yields an "implied" estimate for sovereign assets. The calculation of implied values is a very common technique in the finance world. The collective view of many market participants is incorporated in the observable market prices of liabilities and the change in the market price of these liabilities will determine its volatility. This contingent claim approach implicitly assumes that market participants' views on prices incorporate forward-looking information about the future economic prospects of the sovereign. This does not imply that the market is always right about its assessment of sovereign risk, but that it reflects the best available collective forecast of the expectations of market participants.

Implementing contingent claims analysis to derive the implied sovereign asset value and volatility requires several steps and assumptions. These are discussed in the next section.

## III. Contingent Claims Analysis of the Sovereign Balance Sheet

The contingent claims sovereign balance sheet is constructed from the basic accounting balance sheet of the government and monetary authorities. Figure 2 shows the balance sheets of the government and monetary authorities as two segregated yet linked balance sheets. Government liabilities include foreign currency debt, domestic currency debt, and obligations owed by the government to the monetary authorities and the guarantees to "too-important-to-fail entities." Government assets include a claim on a portion of the foreign currency reserves held by the monetary authority and other public sector assets such as the present value of the primary fiscal surplus. The balance sheet of the monetary authority in Figure 2 has assets consisting of international reserves (net foreign assets) and credit to government (net domestic assets). Liabilities of the monetary authority are base money and a claim of the government on a portion of foreign currency reserves.

In order to use the contingent claims balance sheet to estimate the asset and volatility of sovereign assets, three steps are needed:

- First, a sovereign balance sheet need to be constructed with the liability side containing only elements with observable quantities and market prices, and in a common currency.

- Second, assumptions on the seniority of sovereign liabilities need to be defined to use standard contingent claim relationships.
- Third, option pricing techniques are used to estimate the value and volatility of sovereign assets from the observable market value and volatility of sovereign liabilities.

## A. Consolidating the Sovereign Balance Sheet

The two segregated balance sheets of the government need to be consolidated so that every entry on the liability side can be traced to observable data and the entire balance sheet is denominated in a common currency. Balance sheets for the country case studies presented in this paper are measured in U.S. dollars for ease of comparison, but the analysis holds even if they are valued in domestic currency.[11] Through the consolidation process the government claim on foreign currency reserves and credit to government net out and guarantees to too-important-to-fail entities are subtracted from the sovereign asset.[12] Figure 3 shows the consolidated sovereign balance sheet denominated in a common foreign currency. All the entries on the liability side of the contingent claim sovereign balance sheet in Figure 3 are now directly observable from market prices.

## B. Seniority of Consolidated Balance Sheet Liabilities

Seniority of sovereign liabilities is not defined through legal status as in the corporate sector, but may be inferred from examining the behavior of government policymakers during periods of stress. In times of stress, governments often make strenuous efforts to remain current on their foreign currency debt, efforts that effectively make such debt senior to domestic currency liabilities.[13] The payment of foreign currency debt requires the acquisition

---

[11] Measuring the balance sheet in U.S. dollars results in variable sovereign assets versus a fixed distress barrier. Measuring the balance sheet in domestic currency will result in both variable sovereign assets and a variable distress barrier. In either configuration, the contingent claim formulas will produce the same results.

[12] The implicit guarantees to the financial sector, or other entities, could remain on the liability side of the consolidated public sector balance sheet and modeled as implicit put options. For more details see Merton (1977); Gray, Merton, and Bodie (2002 and 2003); Gapen, Gray, Lim, and Xiao (2004); and Van den End (2005). These papers link the sovereign to the contingent claim balance sheets of the banking or corporate sectors. The detailed analysis of the links to other sectors is beyond the scope of this paper.

[13] Support for viewing foreign currency debt as senior can be found in the literature on "original sin" in Eichengreen and others (2002). Support for modeling domestic currency liabilities as junior claims can be found in Sims (1999) who argues that local currency debt has many similarities to equity issued by firms. He models domestic currency debt as "equity" and in this setting, domestic currency debt becomes an important absorber of fiscal risk, just as equity is a cushion and risk absorber for firms. As long as there is some probability that the government will run a primary surplus in the future and/or will engage in the repurchase of domestic currency debt then such debt has value.

of foreign currency, which the government has a more limited capacity to produce. In contrast, the government has much more flexibility to issue, repurchase and restructure local currency debt. For this reason, governments sometimes introduce capital controls to prevent convertibility and preserve remaining international reserves to service sovereign external debt obligations. In other instances, governments have insisted on the mandatory rollover or restructuring of domestic currency debt during periods of distress without simultaneously engaging foreign currency creditors. In these circumstances, holders of domestic currency liabilities will see the value of their claim greatly reduced since sovereign distress are often accompanied by instances of exchange rate depreciation, which reduces the value of the domestic currency liabilities in terms of its value in foreign currency.

Three recent examples of sovereign debt restructuring illustrate this implicit seniority structure. Russia in 1998–99 introduced capital controls, forced a lengthening of maturities on domestic currency government debt, and declared a unilateral moratorium on private sector external debt obligations while still publicly stating their intention to honor sovereign external debt.[14] In March 1999, Ecuador froze all checking, savings, and time deposits to limit further exchange rate depreciation.[15] In August 1998, Ukraine imposed convertibility restrictions in the foreign exchange market and selectively restructured domestic debt held by banks.[16] (Other examples include government restructuring of debt held by domestic banks or pension funds, thereby reducing their present value, prior to the restructuring of foreign currency denominated external debt).

For these reasons, this paper models foreign currency debt as a senior claim and domestic currency liabilities as junior claims.[17] Default in this paper, therefore, means default on foreign currency debt and the distress barrier, *DB*, defines the level at which payments on foreign currency debt cannot be made. The distress barrier is assumed to equal to the book value of short-term external debt plus interest and one-half of long-term external debt.[18] Default is assumed to occur when the value of sovereign assets declines below the distress barrier. As the junior claim, the value of domestic currency liabilities is dependent on the level of sovereign assets above and beyond what is necessary to service senior foreign currency debt. Senior claims, however, are risky because asset value may not be sufficient to

---

[14] Ariyoshi and others (2000).

[15] Gule and others (2003); Allen (2002).

[16] Shadman-Valavi (1999); Allen (2002).

[17] It should be noted that this ordering can be flexible and the contingent claims framework can be adapted to any number of different seniority structures. In future work, the seniority assumption will be relaxed to take into account multiple layers of liabilities, described in more detail in the appendix.

[18] An alternative procedure would be to use all of short-term external debt and interest, plus long-term external debt discounted by the risk-free rate.

meet promised payments. The value of senior claims, therefore, can be seen as having two components, the *default-free value* (promised payment value) and the *expected loss* associated with default when the assets are insufficient to meet the promised payments. The value of junior claims is the residual value of sovereign assets after the promised payments to senior claims have been made. Thus, in financial terminology, the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, while the value of *risky* foreign currency debt can be modeled as default-free value of debt—equivalent to the distress barrier—minus the expected loss in the event of default.

The next section discusses how changes in the values of observed market variables—the value and volatility of sovereign liabilities—are used to infer changes in unobserved variables—the value of sovereign assets based on the contingent claims balance sheet and seniority structure above.

### C. Calculating Implied Sovereign Asset Value and Volatility

Since the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, standard option pricing techniques can be applied to derive implied estimates for sovereign asset value and volatility. The balance sheet states that the value of assets is the sum of the value of domestic and foreign currency liabilities, $V_A = V_{DCL} + V_{FCL}$, where $V_{FCL}$ represents the value of risky foreign currency liabilities and $V_{DCL}$ represents the value of domestic currency liabilities.[19] The option pricing formulas employed to estimate sovereign asset value and volatility rely on a few select variables: the value and volatility of domestic currency liabilities ($V_{DCL}$ and $Vol_{DCL}$, respectively), the distress barrier ($DB$), the risk-free interest rate ($r_f$), and time ($t$). As shown in more detail in the appendix, these variables can be combined into two equations,

$$V_{DCL} = Function\ (\ \boldsymbol{V_A},\ \boldsymbol{Vol_A},\ DB,\ t,\ r_f\ ), \qquad (1)$$

$$Vol_{DCL} = Function\ (\ \boldsymbol{V_A},\ \boldsymbol{Vol_A},\ DB,\ t,\ r_f\ ), \qquad (2)$$

which can be solved simultaneously to derive the two unknowns, which are the implied market value ($V_A$) and volatility ($Vol_A$) of sovereign assets. Thus the information embedded in the value and volatility of domestic currency liabilities (in units of foreign currency) and the distress barrier derived from the book value of foreign currency debt yield estimates of implied sovereign asset value and implied asset volatility over a given time horizon. The volatility of the domestic currency liabilities (domestic currency debt and base money) come from a variety of sources, including the volatility of the exchange rate and of the quantities issued. The volatility of the exchange rate process is relatively more important in a floating

---

[19] The value of senior foreign currency liabilities can also be obtained using the implicit put option in risky debt (Gray, Merton, Bodie 2003; Gapen et al., 2004; Gray 2004).

exchange rate environment while the quantities of domestic currency liabilities may vary substantially under a fixed or heavily managed exchange rate system.

## IV. Sovereign Credit Risk Indicators

Having derived the estimates of implied asset value and volatility, this section details how they can be used to develop useful indicators of sovereign risk. These risk indicators are the distance to distress, the risk-neutral probability of default, the value of senior foreign currency debt and the sovereign credit risk premium or sovereign risk-neutral credit spread.[20] While price and spread information may be easily observable from the market, the market information itself does not reveal the rationale underlying the risk premium nor does it reveal what is often the most valuable piece of information in risk analysis—how much risk exposures could change as the health of the sovereign improves or declines on the margin. The contingent claims approach links the credit risk premium to the balance sheet framework, allowing for an evaluation of the structural determinants of credit risk.

### *1. Distance to Distress and Risk-Neutral Probability of Default*

The implied value and volatility of sovereign assets can be combined with the distress barrier to produce an indicator of default risk, referred to here as the distance to distress. This measure computes the difference between the implied forward market value of sovereign assets and the distress barrier scaled by a one standard deviation move in sovereign assets. The distance to distress is defined conceptually as:

$$\frac{\textit{Implied market value of sovereign assets} - \textit{Distress barrier}}{\textit{Implied market value of sovereign assets} * \textit{Sovereign asset volatility}}.$$

The numerator above measures the distance between the expected one-year ahead market value of sovereign assets and the distress barrier. This amount is then scaled by a one-standard deviation move in sovereign assets. The distance to distress therefore yields the number of standard deviations sovereign asset value is from distress. Lower market value of sovereign assets, higher levels of foreign currency debt, and higher levels of sovereign asset volatility all serve to decrease the distance to distress.

In formula representation,

---

[20] Risk-neutral valuation is an important factor underlying the derivation of the Black-Scholes option pricing formula whereby the value of the option can be derived by forming a riskless hedge portfolio. Thus, option values do not depend on the investor's or decision maker's attitude toward risk, which is a major benefit of this approach. Alternative balance sheet approaches based on discounted cash flows are subject to serious error not only from errors in cash flow projections, but from errors in choosing the discount rate. See Hull (1993, pp. 221–222) and Chriss (1997, pp.190–193) for additional discussion of risk-neutral valuation.

$$\text{distance to distress} = d_2 = Function\ (V_A, DB, r_f, Vol_A, t), \tag{3}$$

where $d_2$ is from the Black-Scholes option pricing formula (see appendix). Distance to distress for a hypothetical sovereign is illustrated in Figure 1.

The option pricing relationships in equations (1) and (2) above also yield a measure of probability of default, commonly referred to as the risk-neutral default probability. The probability of default is simply the probability that future sovereign asset value will fall below the distress barrier. The option pricing formula used in this analysis assume that future sovereign asset value is distributed log-normally and the risk-neutral probability of default is therefore the shaded area that lies below the distress barrier as shown in Figure 1. The risk-neutral default probability (RNDP) is,

$$RNDP = N(-d_2), \text{ where } d_2 = Function(V_A, DB, r_f, Vol_A, t), \tag{4}$$

and $N(.)$ is the cumulative normal distribution at the distance to distress, $d_2$.

*2. Value of Foreign Currency Liabilities and Sovereign Credit Risk Premium*

The other two useful sovereign risk indicators that can be obtained using the contingent claims approach are the sovereign credit spread or credit risk premium, and the market value of foreign currency liabilities. The value of risky senior foreign currency liabilities ($V_{FCL}$) can be derived using the implied value and volatility of sovereign assets, equations ( 1 ) and (2), and the balance sheet identity noted above, that is, the value of assets is the sum of the value of domestic and foreign currency liabilities. Using these relationships together yields the value of risky foreign currency liabilities,[21]

$$V_{FCL} = function\left(\left(\frac{V_A}{DBe^{-r_f t}}\right), Vol_A, t, r_f\right). \tag{5}$$

The term, $DBe^{-r_f t}$, is the distress barrier discounted to the present by the risk-free rate over the time horizon $t$. Since the distress barrier is based on the book value of foreign currency debt, it is equivalent to the default-free value of foreign currency liabilities. If the ratio of sovereign assets to the default-free value of foreign currency liabilities rises or the volatility of sovereign assets declines, the value of risky foreign currency liabilities increases. Conversely, as the ratio of sovereign assets over the discounted distress barrier falls or asset volatility rises, the market value of risky debt will decline, possibly falling below its default-free value. In other words, if the sovereign becomes more wealthy and the stream of its income less uncertain, the market value of its foreign currency debt will become more

[21] See appendix.

valuable, and vice versa. Therefore, the value of foreign currency liabilities is a useful indicator of the expected gain/loss in asset value that the sovereign is likely to experience if the market prices of its debt increase/decrease.

Manipulating equation ( 5 ) results in an estimate of the risk neutral credit spread (RNS) of,

$$RNS = y - r_f = function\left(\left(\frac{V_A}{DBe^{-r_f t}}\right), Vol_A, t, r_f\right), \qquad (6)$$

where $y = -(1/t)\ln(V_{FCL}/DB)$. The left hand side of equation ( 6 ) represents the yield to maturity on risky foreign currency debt less the risk-free rate of interest and is therefore equivalent to a credit risk premium, or risk-neutral credit spread. In addition to the risk-free rate and time, the sovereign risk premium is a function of only two variables: the volatility of sovereign assets and the ratio of the value of sovereign assets to the distress barrier. Both increases in the ratio of sovereign assets to foreign currency liabilities and decreases in sovereign asset volatility reduce the sovereign risk premium. Conversely, as the ratio of sovereign assets to foreign currency liabilities decreases or sovereign asset volatility increases, the risk premium widens. The intuition is similar as before. The sovereign's credit risk declines if it has a cushion of assets to protect it from negative shocks and the cushion is relatively stable.

It is useful to note that no market information on foreign currency denominated debt, namely bond spreads or credit default swap spreads, have been used while computing the value of risky foreign currency liabilities and credit risk premium in the model. Only information on the book value of payments on existing foreign currency debt is used in construction of the distress barrier. This is combined with market information from domestic currency liabilities and the exchange rate to estimate the value of foreign currency liabilities and the credit risk premium. This is noteworthy since the model output can be then compared with readily available market information to evaluate the robustness of this approach.

*3. Sensitivity Measures*

Associated with the sovereign risk indicators are sensitivity measures, which reveal how responsive the set of risk indicators are to changes in model parameters, namely changes in the value of sovereign assets and asset volatility. This paper focuses on eight relevant sensitivity measures. The first four are the changes in distance to distress, risk-neutral default probability, risk-neutral credit spreads, and value of foreign currency debt from a 1 percent change in the value of sovereign assets. The second four are changes in the same risk indicators from a 1 percent change in sovereign asset volatility. These sensitivity measures are critical in risk analysis because they capture nonlinear changes in value, and equally important, they look beyond the current level of distance to distress, spreads or probability of default. In other words, they provide an indication of the potential *risk exposure* of the sovereign. The sensitivity measures are highest when sovereign asset value is in the neighborhood of the distress barrier, reflecting magnified default risk. In this instance, small changes in underlying asset value in either direction will have proportionately larger impacts on the balance sheet risk indicators. In sum, while the credit risk indicators yield current estimates of sovereign balance sheet risk, the sensitivity measures point to how sovereign risk could further change if the balance sheet improves or weakens on the margin.

## V. Robustness of Sovereign Credit Risk Indicators

The degree to which the contingent claims risk indicators closely parallel actual market data will indicate their usefulness as early warning indicators of sovereign risk. To this end, a historical time series of the risk indicators is compared with actual market data for a number of emerging market countries. The historical data for the sovereign risk indicators in equations (3) – (6) were obtained from the Macrofinancial Risk (M*f*Risk) model while the historical market data were obtained from credit default swap and external debt markets.[22] Robustness of the sovereign credit risk indicators is examined through their correlation and relationship with actual data and through regression analysis.

### A. Correlation with Market Data

If the model output is robust, distance to distress should be negatively correlated with actual sovereign credit spreads. As distance to distress increases, credit risk should decline and be reflected in lower credit default swap spreads. Figure 4 displays the relationship between the distance to distress indicator for twelve emerging market sovereign balance sheets versus that country's observed credit default swap (CDS) spread.[23] Table 1 reports the correlation of the risk indicators with the observed sovereign credit default swap spreads and EMBI spreads.[24] As can be seen there is a very high correlation for most countries between the two risk indicators—distance to distress and risk-neutral spread—and the observed CDS spreads and EMBI+ spreads from January 2003 to August 2004. The reported correlations confirm the expected negative relationship between distance to distress and both CDS and EMBI+ spreads. The correlations also display a high degree of significance as 29 of the 34 reported correlations between distance to distress and CDS spreads are significant at the 95 or 99 percent level. In many cases, correlation is highest with the 5-year CDS spread that likely reflects the greater liquidity in this market relative to the shorter maturity CDS market. A

---

[22] The M*f*Risk model was developed by Macro Financial Risk, Inc. and applied to 17 countries under a joint research effort between Moody's and Macro Financial Risk, Inc. Access to M*f*Risk is only available through subscription.

[23] In Figure 4 the distance to distress scale is inverted. While this visually depicts a positive correlation between distance to distress and credit spreads, this implies that increases in distance to distress should result in lower spreads on credit default swaps.

[24] The reported correlations in Table 1 were computed using Spearman's rank correlation instead of conventional correlation. Conventional correlation is inappropriate in this case since it implicitly assumes linear relationships among variables, an assumption which contradicts the nonlinear relationship between variables as found in this paper. Spearman's rank correlation is a less restrictive measure to gauge relationships among variables since it does not impose any linearity assumptions.

similar level of significance is found between distance to distress and country EMBI+ spreads with 8 of the 9 reported correlations significant at the 99 percent level.

As a second check on robustness, the risk-neutral sovereign credit spread for each country is compared with the EMBI+ spread and CDS spread.[25] Figure 5 displays the expected positive relationship between the risk-neutral sovereign credit spread and each EMBI+ country spread for nine emerging markets for the sample period from January 2003 to August 2004. The correlation between the risk-neutral sovereign credit spread and the respective EMBI+ spread during the same time period is also reported in Table 1. The correlations show the expected positive relationship between the risk-neutral credit spread and EMBI+ country and CDS spreads. The correlations between the risk neutral credit spread and the CDS spread display a high degree of significance at the 95 and 99 percent levels for 30 out of 34 reported correlations. The correlations between the risk-neutral credit spread and EMBI+ spread display significance at similar levels in 8 out of 9 cases.

## B. Regression Analysis

Two fixed effects panel regressions are used to estimate the relationship between risk-neutral spreads and EMBI+ country spreads and CDS spreads. The mapping from risk neutral spreads to actual market spreads is important because the risk neutral spreads tend to underestimate actual market credit spreads. The fixed effects model treats differences across countries in the sample as parametric shifts of the regression function (i.e., differences across countries are captured in differences in the constant term). This approach yields the following relationships:

- **Risk-neutral credit spreads and observed CDS spreads.** The relationship between risk-neutral credit spreads and observed CDS spreads is estimated by applying a fixed effects panel regression to a combined cross-country sample of 981 observations from April 2002 to August 2004.[26] Results, which are reported in Table 2, indicate that the coefficient and constants are highly significant at all confidence intervals and the R-squared from the panel regression is 88 percent.

- **Risk-neutral credit spreads and EMBI+ spreads.** The relationship between risk-neutral credit spreads and EMBI+ spreads is estimated by applying a similar fixed effects panel regression to the same cross-country sample of 981 observations from April 2002 to August 2004. Results, which are reported in Table 3, indicate that the coefficient and constants are highly significant at all confidence intervals and the R-squared from the panel regression is 96 percent.

---

[25] The EMBIG index has replaced the EMBI+ index as the preferred index for tracking emerging market credit spreads, but historical EMBIG index data was not available at the time of the writing of this paper.

[26] The countries in the sample include Brazil, Bulgaria, Colombia, Malaysia, Mexico, the Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

Given the goodness of fit of the above regressions, the individual country panel equations can be used to map sovereign risk-neutral credit spreads into: (i) actual CDS spreads, and (ii) actual EMBI+ spreads. For example, the estimated equation for Mexico used to map sovereign risk-neutral credit spread into the actual spread on credit default swaps is,

$$\ln(CDS_t) = 1.72 + 0.52 * \ln(RNS_t). \tag{7}$$

The similar estimated equation used to map risk-neutral credit spreads into actual EMBI+ spreads for Mexico is,

$$\ln(EMBI_t) = 4.78 + 0.15 * \ln(RNS_t). \tag{8}$$

As a numerical example, suppose that application of equation ( 6 ) results in risk-neutral spreads on foreign currency debt for Mexico of 200 basis points. Inserting this value into equations ( 7 ) and ( 8 ) results in a credit default swap spread of 88 basis points and an EMBI+ spread of 263 basis points.

The relationship between sovereign risk-neutral default probability and estimated actual default probability can also be determined. This procedure is necessary since risk-neutral default probabilities overstate the actual probability of default. To implement this comparison, some estimate of actual default probability is needed. In the application of the contingent claims approach to corporate credit risk, the standard adjustment mechanism is to map firm risk-neutral default probabilities against a database of actual corporate defaults.[27] However, a sufficiently large dataset of sovereign defaults is not available, meaning some other approach is necessary. A second best approach is to use estimates of actual default probability, or market implied default probabilities (MIDP), which can be obtained from credit default swap spreads assuming a specific loss given default and time horizon (a recovery rate of 30 percent was used in this analysis).[28] Using this approach, a fixed effects panel regression is applied to a cross-country sample of 935 observations from January 2003 to August 2004 in order to examine the relationship between risk-neutral default probabilities

---

[27] For example, Moody's KMV (MKMV) utilizes 30 years of historical data over 6,000 public and 70,000 private company default events to derive the firm specific probability of default, which is referred to as the Expected Default Frequency™ (EDF).

[28] Market implied default probabilities (MIDP) can be obtained from CDS spreads through the following equation:

$$MIDP = \frac{1 - \exp(-spread * t)}{1 - R},$$

where *spread* is the net 1-year credit default swap spread, $t$ is the time horizon (equal to 1 in this case), and $R$ = 30 percent is the recovery rate. If the 1-year CDS spread is 180 basis points, the implied default probability is 2.5 percent. See appendix.

and market implied default probabilities.[29] The fixed effects panel regression displays high explanatory power with an R-squared of 93 percent. Results are reported in Table 4.

However, there is a problem with the panel regression results. As can be seen through close examination of Table 4, the regression equations for Korea, Malaysia, Mexico, Poland, and South Africa result in market implied default probabilities that are higher than risk-neutral default probability, which is a contradiction. This problematic result is likely due to two factors: (i) the assumption of a constant loss given default for all countries regardless of credit risk; and (ii) lack of a sufficiently long time series for credit default swaps. In practice, loss given default may change as probability of default and credit default swap spreads change. More sophisticated methods of estimating MIDP from credit default swap data are therefore needed, including methods that allow the recovery rate to vary with probability of default. These advanced methods are beyond the scope of this paper and are left for future research.

An alternative approach is to pool the individual country data into one regression to increase the number of observations, while maintaining the assumption of a constant loss given default.[30] The estimated equation using data from the pooled countries in the sample is,

$$\begin{array}{ccccccc} \ln(MIDP_t) & = & -1.24 & + & 1.01 & * & \ln(RNDP_t) \\ & & (-15.54) & & (23.23) & & R^2 = 0.735, \end{array} \quad (9)$$

where the numbers in parenthesis represent the relevant t-statistic. While the explanatory power of this pooled regression falls slightly relative to the panel regression reported in Table 4 (R-squared declines from 0.93 to 0.74), the level of explanatory power remains high and the relationship is highly significant. Furthermore, equation ( 9 ) produces a market implied probability of default that is lower than the risk-neutral probability of default. For example, suppose that application of equation ( 4 ) results in a risk-neutral probability of default equal to 8 percent. Inserting this value into equation ( 9 ) results in a market implied default probability of 2.3 percent. In other words, actual probability of default is approximately one-third of risk-neutral probability of default.

The sensitivity of sovereign spreads in response to a change in the distance to distress is a nonlinear relationship. Figure 6 plots observed market spreads (one-year CDS spreads) versus model distance to distress for all twelve countries in the sample (889 data pairs from the period mid-2002 to mid-2004). The figure shows the relationship aggregated across countries. The solid line is a best fit regression, with an R-squared of 0.80. It shows the spread going up exponentially as the distance to distress declines. The sensitivity of changes in spreads, for a given change in distance to distress, is much lower for countries with a high

[29] The countries in the sample include Brazil, Colombia, Korea, Malaysia, Mexico, the Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

[30] Crouhy, Galai, and Mark (2000) also assume constant loss given default.

distance to distress. As the distance to distress declines from 1.5 to 1.4, the spread increases on average by 35 basis points. However, if the distance to distress drops from 0.5 to 0.4 the spread increases on average by 375 basis points.

The robustness checks in this section suggest that the distance to distress, risk-neutral credit spread, and risk-neutral probability of default are useful for evaluating sovereign vulnerabilities. The evidence indicates that the book value of foreign currency liabilities along with market information from domestic currency liabilities and the exchange rate contain important information about changes in the value of foreign currency liabilities and credit risk premium. The nonlinearities and inclusion of volatility in the option pricing relationship used in this analysis contributes to the high degree of explanatory power and correlation with actual data. Finally, the estimated relationships in equations ( 7 ) – ( 9 ) allow for straightforward transformation of model outputs into estimates of observable market data.

## VI. Scenario and Simulation Analysis: Hypothetical Sovereign

With robustness verified, the structural models calibrated using the contingent claims framework and unique to each economy can be used with scenario and simulation analysis to evaluate shocks and policies. The goal is to estimate the potential effects of changes in economic conditions and impact of government policies on sovereign credit risk and sensitivity indicators. To begin with, a baseline balance sheet for a hypothetical sovereign is calibrated and the resulting baseline risk indicators and sensitivity measures are reported (Table 6). Scenario analysis is then conducted using two capital flow examples and the resulting point estimates for the credit risk indicators and sensitivity measures are compared to the baseline set of indicators. Next, the scenario analysis is extended using Monte Carlo simulations, which provides one way of generating a large number of market outcomes and produces probability distributions for each risk indicator. Unlike the scenario analysis which is intended to investigate the effects of a specific market outcome, the Monte Carlo simulation draws randomly from sample interest rate and exchange rate distributions to compute probability distributions and confidence intervals for a set of market outcomes. This process allows for the “stress testing” of the risk levels of the sovereign risk indicators to derive what are commonly known as value-at-risk (VaR) measures.

### A. The Baseline

The starting point is the baseline balance sheet as displayed in Table 5:

- *Calibrated values.* The distress barrier is assumed to be US$100 billion, comprising short-term foreign currency debt plus interest of US$40 billion and one-half of long-term debt of US$60 billion. The value and volatility of domestic currency liabilities in dollar terms are US$82 billion and 0.76 (76 percent), respectively. Using equations ( 1 ) and ( 2 ), the implied value of sovereign assets is US$175 billion and the implied volatility of sovereign assets is 0.38 (38 percent). Foreign currency reserves are assumed to make up US$40 billion of implied sovereign assets.

- *Credit risk indicators*. From equation ( 3 ), the resulting distance to distress is 1.4 standard deviations. The distance to distress results in a risk-neutral probability of default of 8 percent in equation ( 4 ). Equations ( 5 ) and ( 6 ) yield the value of risky foreign currency debt as US$95 billion and risk-neutral credit spread of 115 basis points, respectively.[31] The market value of risky foreign currency debt implies a present value expected loss of US$1 billion. This value is derived from the difference between the discounted present value of the distress barrier (using a risk-free rate of 4 percent yields a present value distress barrier of US$96 billion) and the implied market value of foreign currency debt.

- *Sensitivity measures*. Sensitivity measures are calculated from a 1 percent change in sovereign asset value and volatility. For example, when the value of sovereign assets decreases by 1 percent, Table 5 shows that the distance to distress falls by 0.03 standard deviations (i.e., from 1.4 to 1.37 standard deviations), risk-neutral default probability increases by 0.41 percent, risk-neutral credit spreads increase by 7 basis points, and the expected loss on foreign currency debt increases by US$70 million. Sensitivity measures are also reported for a 1 percent change in sovereign asset volatility.

As previously discussed, the sensitivity measures capture the nonlinearity present within the contingent claims relationships. An example of the presence of nonlinearities is shown in Figure 6, which plots how risk-neutral spreads change in response to changes in sovereign asset value. As the value of sovereign assets approach the distress barrier, a 1 percent reduction in sovereign assets results in a 25 basis point increase in credit spreads compared with the baseline calibration, where the same 1 percent reduction in sovereign asset value only leads to a 7 basis point increase in credit spreads. The nonlinearity implies that the drop in sovereign assets has a proportionately larger impact on credit risk when sovereign assets are close to the distress barrier. The converse is true when the implied value of sovereign assets is well-above the distress barrier.

## B. Scenario Analysis

Two scenarios are examined and compared with the baseline. Scenario 1 represents the potential negative effects associated with capital outflows and Scenario 2 illustrates the positive effects from capital inflows. First, suppose that economic conditions deteriorate so that capital outflows occur. Capital outflows are normally associated with some combination of an exchange rate depreciation, a drop in domestic debt prices (possibly associated with a rise in domestic interest rates), and an increase in volatility of both debt prices and the exchange rate. The impact of capital outflows on the sovereign balance sheet risk indicators depends in part on the response of policymakers. The assumption in this example is that policymakers accommodate some, but not all, of the shock. This would include some loss of

---

[31] At this point, the risk-neutral default probability and spread would then be mapped into market implied probability of default and market spreads using equations (7) – (9), but this step is ignored here since this exercise is only for a hypothetical sovereign balance sheet.

international reserves, tighter interest rate policy, and increase in the net fiscal asset. Under this scenario, sovereign asset value is assumed to fall by US$25 billion to US$155 billion, with international reserves falling from US$40 to US$35 billion. Sovereign asset volatility increases from 38 percent to 43 percent. The left column in Table 6 (Scenario 1) displays the new contingent claim sovereign balance sheet, balance sheet risk indicators, and sensitivities after capital outflows.

In sum, capital outflows worsen the credit risk indicators and risk exposure of the sovereign has increased relative to the baseline. Distance to distress falls from 1.4 to 1.0 standard deviations and risk-neutral probability of default increases from 8 to 17 percent. Risk-neutral spreads on foreign currency debt rise to reflect the increased risk of nonrepayment as the expected loss has increased from US$1.5 billion to US$3 billion. In addition to a worsening of the credit risk indicators, the sensitivity measures have increased since implied sovereign asset value is fewer standard deviations from the distress barrier. For example, a 1 percent decline in sovereign asset value under the baseline scenario increased risk-neutral default probability by 0.41 (from 8 to 8.41 percent) and risk-neutral spreads by 7 basis points, while in this capital outflow scenario these values are now 0.63 and 16, respectively. The higher sensitivities reflect the higher degree of nonlinearity within the option pricing formula as sovereign assets move closer to the distress barrier. This is indicative of observed nonlinear value changes in actual credit events.

A similar procedure can be applied to illustrate the opposite effects of capital inflows. Sustained capital inflows typically result in some exchange rate appreciation, improvement in domestic debt prices, and lower financial market volatility. Capital inflows may also provide space for an increase in international reserves which may necessitate sterilization operations. Based on this scenario, the value of sovereign assets is assumed to rise US$195 billion while its volatility drops to 37 percent. Also, international reserves is assumed to rise by US$5 billion and the increase in the dollar value of domestic currency liabilities is a reflection of both sterilization and exchange rate appreciation. The right column of Table 6 (Scenario 2) displays the contingent claim sovereign balance sheet, credit risk indicators, and sensitivities after capital inflows. The increase in sovereign asset value and reduction in volatility yield the expected decrease in credit risk and sensitivity relative to the baseline. Distance to distress rises above two standard deviations and risk-neutral probability of default decreases by half to 4 percent. Risk-neutral spreads on foreign currency debt decline as the value of risky foreign currency debt approaches its default free value. Each of the sensitivity measure decreases relative to the baseline from the improved sovereign asset value and volatility with respect to the distress barrier.

### C. Monte Carlo Simulations

The scenario analysis above yields three related point estimates for the credit risk indicators, one from the baseline calibration and two from a negative and positive shock. While such scenario analysis may be useful in examining a specific event, it only reveals a very small view of the possible set of market disturbances. Scenario analysis to recreate a specific event is always subject to the criticism that market stress scenarios, in fact, rarely repeat themselves. Monte Carlo simulation methods can be used to systematically deal with multiple scenarios, yielding probability distributions for risk indicators and value-at-risk

(VaR) measures.[32] The Monte Carlo procedure implemented in this section takes random draws from hypothetical forward distributions for domestic interest rates and the exchange rate. Following the process outlined in Box 2, Monte Carlo simulations are conducted on the baseline sovereign balance sheet presented in the previous section.

Probability distributions for distance to distress, risk-neutral default probability, risk-neutral spreads, and the value of sovereign assets resulting from the simulation are reported in Figure 7. While the mean distance to distress remains 1.4 standard deviations, the same value as reported in Table 4, the distribution reveals a confidence interval for distance to distress based on the sample exchange rate and interest rate distributions. For example, from Figure 7, the lower 5 percent probability for distance to distress is 0.9 standard deviations, the upper 5 percent probability for risk neutral default probability is 18 percent, and the upper 5 percent probability for risk neutral spreads is 387 basis points. In other words, given the assumed exchange rate and interest rate distributions and correlation, distance to distress remains above 0.9 standard deviations, risk neutral default probability remains below 18 percent, and risk neutral spreads remain below 387 basis points 95 percent of the time. Finally, the 5 percent lower bound on sovereign assets is US$160 billion, making the implied sovereign asset VaR equal to US$15 billion.

VaR measures are often used to evaluate both market and credit risk in the financial sector.[33] In the financial sector, VaR typically defines a level of capital which, for a high degree of confidence, is an upper bound on the amount of gains or losses to a portfolio from market or credit risk. On the sovereign balance sheet, VaR by corollary, could be defined as the upper bound on the amount of gains or losses to implied sovereign asset value from market risk.[34] Just as a bank or asset manager is required to hold capital in reserve to protect against market

[32] While Monte Carlo simulations are able to handle many thousands of possible events, they produce a random set of outcomes based on the market characteristics assumed, which may or may not predict potential shocks. The simulation process will only produce as many extreme events as dictated by the distribution assumption of the market variables. To be comprehensive, simulation procedures should be combined with various scenario assumptions to produce a set of stress outcomes.

[33] See Jorian (2000). VaR models estimate the exposure of a portfolio, or the equivalent set of positions, to market risk. The measure captures the expected maximum loss and is usually expressed within a confidence interval.

[34] Two other sovereign VaR measures can be calculated. The first, sovereign capital-at-risk, is an extension of sovereign VaR for the central bank. The probability distribution of the residual value of "capital" or junior claim of the monetary authorities is calculated and a confidence level attached to the risk that the monetary authorities cannot meet its commitments. Blejer and Schumacher (1999) use a similar construction. The second, sovereign credit-at-risk, is the upper bound on gains or losses due to credit risk, which in this case is the value of the guarantee to the banking system. See Gapen, Gray, Lim, and Xiao (2004) for an example of how this could be modeled.

or credit loss, governments often identify a need to acquire sufficient levels of foreign currency reserves or insurance arrangements to protect against adverse market developments. The sovereign value-at-risk measure can be used as a tool to gauge whether the level of reserves is sufficient to protect against the risk of "sudden stops," or to maintain debt sustainability against adverse economic shocks.

## D. Evaluating Policy Design

Using the Monte Carlo baseline simulation as a starting point, potential policy choices can be evaluated. For example, changes in the level of reserves, alternative debt structures, or the use of risk mitigation instruments like insurance contracts can be tested. The new policy option modifies the sovereign balance sheet and simulations using draws from the same interest rate and exchange rate distributions will reveal new distributions of risk indicators which can be evaluated against the original baseline configuration.[35] The example of debt management with alternative debt structures is examined first followed by a strategy for reserve accumulation. Finally, a combination of debt and reserve management is considered.

- **Debt management.** Panel 1in Figure 8 illustrates an example of liability management, whereby US$10 billion in foreign currency debt is replaced with an equal amount of interest rate linked domestic currency debt to examine the impact of reduced exchange rate exposure. As a result, the distress barrier falls to US$90 billion while domestic currency liabilities increase by US$10 billion. The new Monte Carlo simulations on this adjusted balance sheet yield improvements in the risk indicators. The mean values and confidence intervals for distance to distress, risk-neutral default probability, and mean risk-neutral spreads all improve.

- **Reserve management**. Panel 2 of Figure 8 illustrates the example of reserve accumulation financed with an equal amount of domestic currency debt such that the level of sovereign assets and interest rate linked domestic currency liabilities both increase by US$10 billion. This scenario could be viewed as a proactive strategy to accumulate reserves or reflect capital inflows, and therefore, the increase in domestic currency debt is the result of sterilization. The operation yields improvements in the risk indicators—as seen in the mean values and lower 5 percent probability distributions—although the margin of improvement is less than that found in the example on debt management.

---

[35] Simulating the adjusted sovereign balance sheet under the same exchange rate and interest rate distributions and correlation is subject to the Lucas critique; namely that these distributions and correlations are derived from market expectations which are likely to change with the shift in policy. In this debt management example, the expected future exchange rate could be more appreciated with lower volatility given the lower levels of foreign currency debt. Consequently, the simulations conducted in this paper should only be viewed as illustrative of potential impacts from policy changes. See Best (2000) for additional discussion on the limits of stress testing.

- **Asset and liability management**. The reserve and debt management strategies above can also be implemented simultaneously, as shown in Panel 3 of Figure 8. The distress barrier declines to US$90 billion, the amount of domestic currency debt increases by US$20 billion, and the level of foreign currency reserves increases by US$10 billion. The effect of simultaneously enacting both strategies yields improvements in the risk indicators by an amount equal to more than the sum of the two strategies individually, reflecting model nonlinearity. Combining the two strategies is advantageous since the debt management operation reduces the distress barrier while the reserve accumulation strategy leaves the mean value of sovereign assets nearly unchanged relative to the baseline.

Deciding on the efficacy of any of the above strategies involves a systematic weighting of the trade-offs inherent in each case. There are clear elements in each of the three alternative strategies that are beneficial from a policy perspective (the reduction in exchange rate exposure and increase in reserves) which need to be balanced against the clear negatives from a balance sheet perspective (the increase in domestic currency interest bearing obligations). The contingent claims balance sheet risk indicators can therefore be useful in guiding policy design given its ability to compare different policy options using quantifiable risk indicators.

## VII. Next Steps

### A. A Robust Framework for Reserve Management

The application of contingent claims analysis and sovereign VaR to reserve management is a stark departure from accounting indicators commonly used for reserve management. One widely used indicator of reserve adequacy is the ratio of foreign currency reserves to total public and private short-term foreign currency debt. Both public and private sector debt is included since reserves of the public sector must facilitate transactions related to economy-wide financing requirements. However, the simple accounting ratio of reserves to total short-term foreign currency debt is deficient when it comes to risk analysis because it does not take uncertainty of balance sheet risks into account. Applying a broad-based rule for an appropriate ratio of reserve coverage uniformly across countries implicitly assumes all sovereign balance sheet risks are similar and neglects cross-country differences in sovereign balance sheet risk.[36]

In contrast, an appropriate target for reserve adequacy could be based on a level of reserves that minimizes instances of distress using the contingent claims risk indicators. For example, an adequate level of reserves could be defined as the level of reserves that keeps distance to distress above a desired standard deviation 95 percent of the time based on the likely

[36] IMF (2000) examines three ratios: reserves to imports, reserves to monetary aggregates, and reserves to public and private short-term foreign currency debt by residual maturity. The report concludes that reserves to short-term foreign currency debt is a superior measure and recommends a ratio of 1 be a lower bound for adequate reserve coverage.

exchange rate process. Adequate reserve coverage could also target a basket of credit risk indicators by setting reserve levels to maintain the combined set of indicators at target levels for a specific confidence interval. In sum, reserve management using this framework examines the impact of the level and volatility of reserves as a component of the wider sovereign asset value and volatility with a link to the balance sheet risk indicators. The application of contingent claims in analyzing sovereign credit risk can be adapted to include many different aspects of reserve management, including the currency composition of reserves, or various other risk mitigation techniques.[37]

## B. A Robust Framework for Debt Sustainability

In addition to providing a framework for reserve management, the use of contingent claims to analyze sovereign risk is well-suited for robust debt sustainability analysis. Traditional debt sustainability analysis has focused on ratios of current and forecasted debt-to-GDP as the primary criterion for deciding whether the public sector debt remains on a sustainable path, usually without explicitly incorporating uncertainty in a systematic, coherent framework. The following elements indicate why the approach in this paper could provide the basis for a more robust framework for debt sustainability analysis:

- The contingent claims sovereign balance sheet translates balance sheet risks into quantifiable risk indicators. In this framework, debt sustainability could be defined as the debt structure which keeps key credit risk indicators below (or above) certain threshold levels for a given confidence level. In contrast, the debt-to-GDP ratio identifies an element of sovereign risk, but is not part of a structural framework that measurably relates debt payments with the capacity to pay. For example, the contingent claims structural framework is able to assess the impact of changes in the level of reserves on sovereign risk whereas the debt-to-GDP ratio remains invariant to such changes.

- The quantitative sovereign credit risk indicators described in this paper incorporate uncertainty and volatility. Higher market uncertainty is often translated into higher interest rate and exchange rate volatility, widening the forward distributions on both variables and increasing the volatility of sovereign assets. Distance to distress will fall, probability of default will rise along with spreads on foreign currency debt, and the expected loss on risky foreign currency debt will increase. However, the debt-to-GDP ratio does not change with an increase in sovereign asset volatility and would therefore miss an important component of risk analysis

---

[37] Caballero and Panageas (2005) examine various instruments and risk mitigation strategies that policymakers could implement in addition to traditional reserve accumulation in a model of sudden stops in capital flows.

- The contingent claims sovereign balance sheet includes an assessment of maturity or rollover risk through construction of the distress barrier.[38] The debt-to-GDP ratio does not change if a decrease in short-term foreign currency debt is matched by an equal book value increase in long-term foreign currency debt. The use of the contingent claims sovereign balance sheet reflects this change by signaling a decrease in sovereign risk due to the more favorable debt profile.

- Finally, the contingent claims approach incorporates nonlinear value changes. The use of nonlinear modeling in a structural framework captures complex relationships and more accurately conveys the nonlinear nature of credit events. During periods of stress, small changes in interest rates, exchange rates, and/or volatility can result in large changes in sovereign risk on the margin. An accounting ratio like debt-to-GDP is not capable of this level of complexity, nor is it released with enough frequency to enable its use during periods of stress where vulnerabilities may build or subside rapidly.

Using contingent claims to model sovereign credit risk therefore offers several advantages over the traditional debt-to-GDP analysis. Additional research in this direction could prove useful and would require extension of the framework to a medium-term setting while incorporating the outlook for relevant economic and policy variables.

## VIII. Conclusions

This paper develops a comprehensive new framework to measure and analyze sovereign risk by applying the contingent claims approach to the balance sheet of the combined government and monetary authorities. A marked-to-market balance sheet is constructed that provides a structural framework that identifies balance sheet risks, incorporates uncertainty, and yields quantifiable risk indicators. The main outputs of this framework include sovereign credit risk indicators, sensitivity measures, and sovereign value-at-risk. These sovereign risk indicators incorporate both forward-looking market prices and nonlinear changes in values, and should consequently have greater predictive power in estimating sovereign credit risk than would traditional macroeconomic vulnerability indicators or accounting based ratios.

Application to 12 emerging market economies show the risk indicators to be robust and significant when compared to market observed credit spreads on foreign currency debt, even

[38] This is true whether one uses the simplified distress barrier in this paper (short-term foreign currency debt and interest plus one-half long-term foreign currency debt) or a more sophisticated approach (short-term foreign currency debt and interest plus the present discounted value of long-term foreign currency debt and interest). Both approaches would reflect an increase in sovereign risk if long-term foreign currency debt was traded for equal book value amounts of short-term foreign currency debt. The distress barrier under the second approach, however, would be more sensitive to near-term repayment humps that would carry a higher weight in the distress barrier than a similar payment profile further out the maturity scale.

though the spreads were not used as inputs. This lends support for the approach, as well as illustrates that the level and variation of forward exchange rates and other market variables contain valuable information for analyzing sovereign credit risk.

Using the contingent claims approach to analyze sovereign risk has several merits from a policy perspective. The ability of the contingent claims approach to provide a structural interpretation of the sovereign balance sheet, unique to each economy, is a valuable contribution in the area of policy design and risk management, translating policy choices and changing economic conditions directly into quantitative indicators of financial soundness. This paper describes how the framework can be used with scenario and simulation analysis to evaluate shocks and the impact of corrective policies. Policymakers can observe how negative economic shocks worsen sovereign financial soundness through capital outflows, depreciating the exchange rate, or slower economic growth. Policymakers can use this tool to design and implement risk-mitigation strategies to reduce sovereign balance sheet risk. Equally important, the set of tools available to policymakers have direct links to the assets and liabilities in the sovereign balance sheet and can influence market expectations. In response to changing economic conditions, active policy decisions to alter the primary fiscal surplus, level of interest rates, structure of debt, or reserve intervention policy can mitigate or offset shocks to the government balance sheet. The ability of the contingent claims balance sheet risk to measurably assess the potential policy mix is an important element of strategic planning, and offers policymakers the valuable opportunity to rank policy options.

The contingent claims framework can be adapted and extended in several important directions. It is well-suited for a more robust analysis of debt sustainability as compared with the widely used debt-to-GDP ratio which is a static, backward-looking indicator. The framework can also be used to estimate an appropriate target for reserve adequacy, where adequacy could be defined as the level of reserves that minimizes the probability of distress.

### Box 1. From Corporate to Sovereign Risk Analysis

**It is useful to point out the key similarities between the sovereign balance sheet and balance sheets of individual firms. On the asset side,**

- **Firms** have main assets consisting of cash and the present value of future earnings (stream of revenues minus expenditures). The firm has other assets including property, plant, equipment, and inventory. The firm may have also contingent assets from a parent company, or implicit guarantees to subsidiaries.

- The **sovereign** has, on its consolidated balance sheet, international reserves and present value of the net fiscal surplus (stream of revenues minus expenditures). Sovereign assets may increase due to contingent financing arrangements with multilateral or other sources, or may be reduced by the cost of implicit or explicit guarantees to financial institutions or other too-important-to fail entities. The sovereign has land and other assets, which are unlikely to be sold, so they do not enter into the expected government revenue stream and thus are not included in this definition of asset.

**On the liability side,** firm liabilities may include senior debt, subordinated debt, and equity. Market capitalization of the firm is equal to price of equity multiplied by the number of shares issued. Sovereign liabilities include foreign currency debt. The sovereign also has local currency debt and base money, which when multiplied by the exchange rate yields the foreign currency value of domestic currency liabilities.

**Default risk in the corporate and sovereign sectors are also similar from a valuation perspective:**

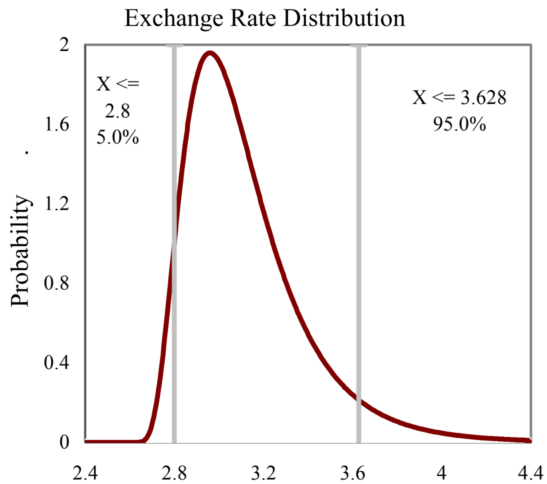
- **Corporate default.** Corporate sector defaults trigger a bankruptcy process, well-defined in some countries and less well-defined in others, whereby creditors are assigned their claim to firm assets based on the legally defined seniority of liabilities in the capital structure. Since debt is senior to equity in the firm liability structure, bondholders have senior legal claims to remaining firm assets in the event of default. A review of corporate sector defaults reveals that senior bondholders exercise their control in various ways post default. In some cases, bondholders choose to sell liquidate remaining assets to obtain cash payment. In other cases, bondholders choose to replace management while receiving new claims, oftentimes in the form of equity.

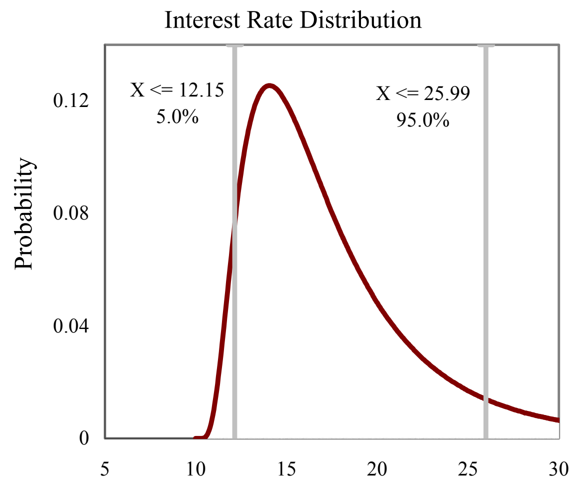
- **Sovereign default.** While sovereign defaults do not trigger a well-defined bankruptcy process that applies equally across countries, instances of sovereign default trigger a restructuring process whereby predefault liabilities are exchanged for postdefault liabilities. In this restructuring process, holders of sovereign liabilities do not receive similar legal claims to ownership of sovereign assets in the event of a sovereign default (e.g., bondholders cannot assume control of the policy apparatus, possess public sector entities, or liquidate assets). Instead, the holders of sovereign debt have a claim on restructured debt of lower value in the event of default.

From a valuation perspective, default risk in a sovereign setting is similar to default risk in the corporate sector. The present value of the expected loss in the sovereign setting is associated with receiving restructured debt of lower value after default while in the corporate setting it is associated with post-default cash payouts or new claims at lower value. Bondholders in both cases value their claims at their default-free value minus the present value of expected loss, which can be estimated using an implicit put option.

**Box 2. Implementing the Monte Carlo Simulation**

The Monte Carlo simulation procedure applied in this section takes random draws from hypothetical forward distributions for both domestic interest rates and exchange rates and calculates the effects of these variables on balance sheet values and risk indicators. The one-year forward exchange rate is assumed to be 3 units of the domestic currency to 1 U.S. dollar and the one-year forward interest rate is assumed to be 17 percent. Lognormal distributions for each were constructed based on recently observed market patterns in several emerging market economies, as shown in the figures below. The correlation between exchange rates and interest rates was set at 0.6, meaning that the Monte Carlo simulation conducts sample draws such that exchange rate depreciations are generally associated with higher interest rates and vice versa.




The Monte Carlo simulation procedure then selects random draws from these hypothetical distributions. The sample forward exchange rate is applied to the contingent claims sovereign balance sheet in the translation of domestic currency assets and liabilities into their respective U.S. dollar values. In contrast to exchange rate variations, simulating the effect of interest rate changes requires additional assumptions. Broad money is assumed to comprise half of domestic currency liabilities with the remainder in interest rate-linked domestic debt. The interest rate draw is applied to the existing domestic currency debt for a period of three years and then is assumed to return to 17 percent. If the realization of interest rates in the random draw is above the assumed 17 percent forward interest rate, the discounted marginal increase in interest costs are subtracted from the value of sovereign assets to reflect higher debt service costs. Alternatively, if the interest rate draw is below the assumed forward interest rate, then this discounted decrease in debt service costs is added to the value of sovereign assets.

The resulting sovereign balance sheet values from each random draw are then used to compute the new set of risk indicators. In contrast to the point estimates for the balance sheet risk indicators that result from scenario analysis, the process of conducting random samples from distributions of exchange rates and interest rates results in probability distributions for the relevant risk indicators.

Figure 1. Distribution of Sovereign Asset Value and the Distress Barrier

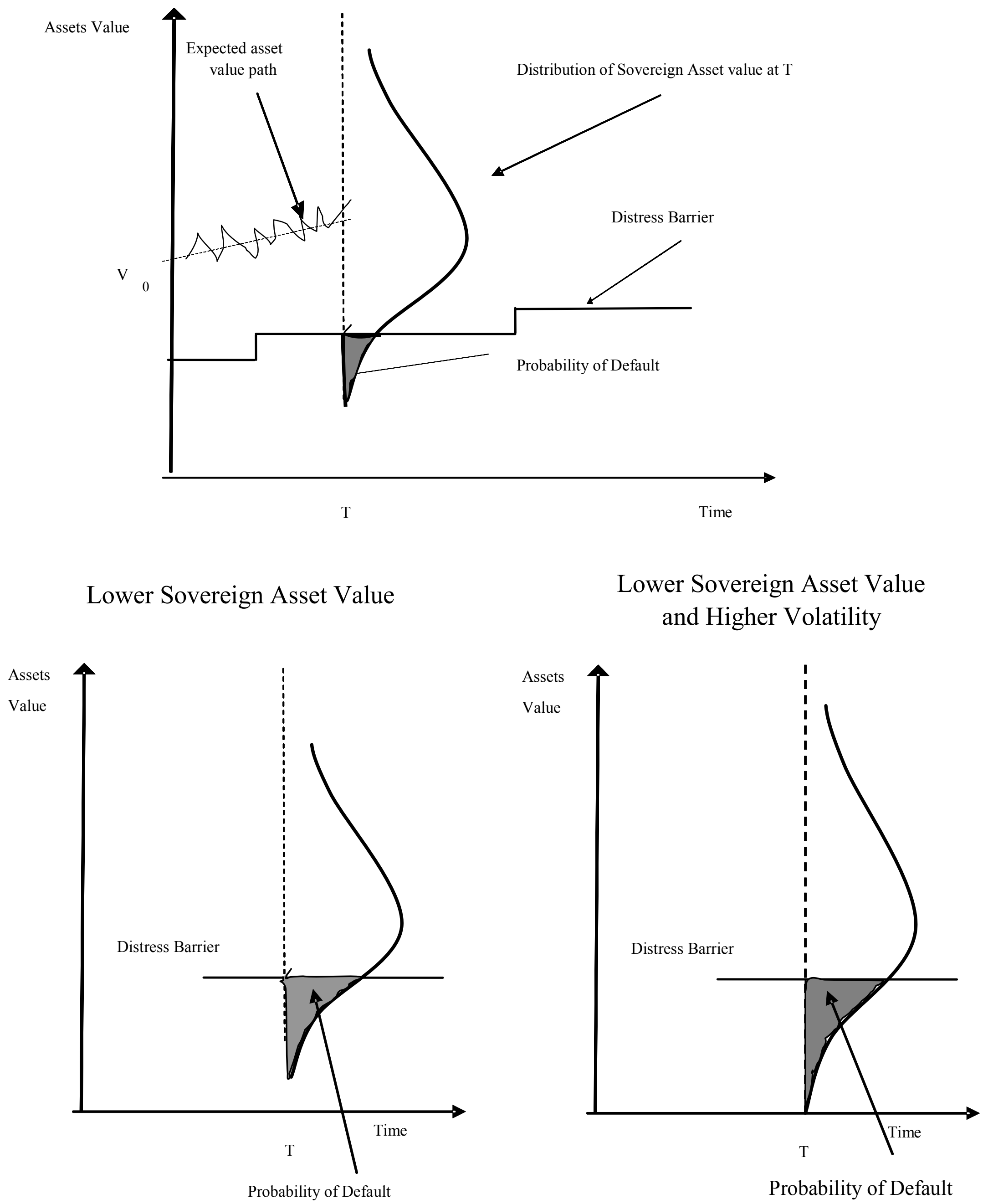


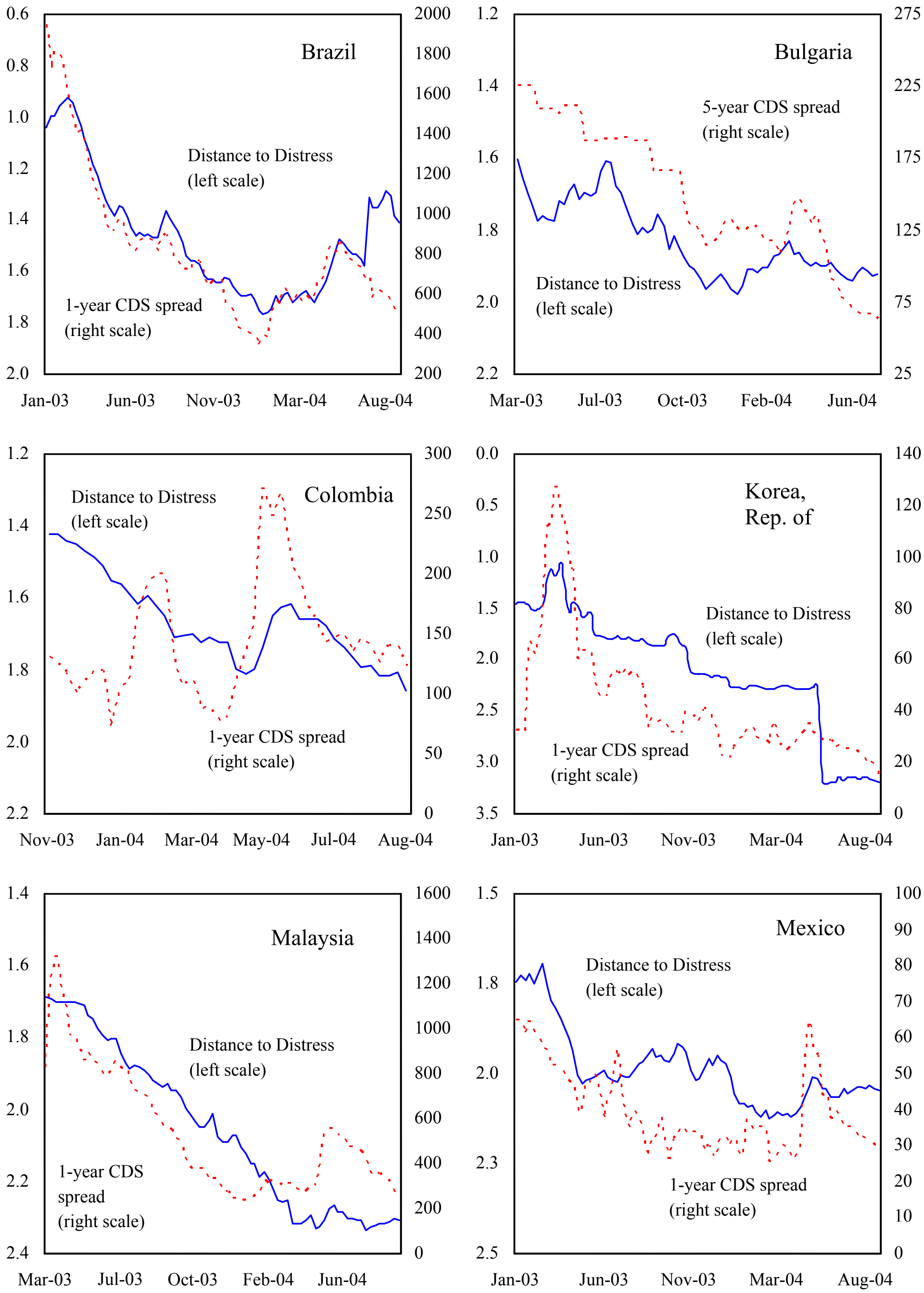
Figure 2. Segregated Balance Sheets of the Government and Monetary Authority

| **GOVT ASSETS** | **GOVT LIABILITIES** |
| --- | --- |
| *Claim on a Portion of International Reserves (of Monetary Authority)*<br><br>Other Public Sector Assets | *Credit to Government (from Monetary Authority)*<br><br>Domestic Currency Debt<br><br>Foreign Currency Debt<br><br>Implicit and Explicit "Too-important-to-fail" Guarantees |
| **MONETARY AUTHORITY ASSETS** | **MONETARY AUTHORITY LIABILITIES** |
| International Reserves<br><br>*Credit to Government* | Base Money<br><br>*Government Claim on Portion of International Reserves* |

Figure 3. The Consolidated Contingent Claims Public Sector Balance Sheet

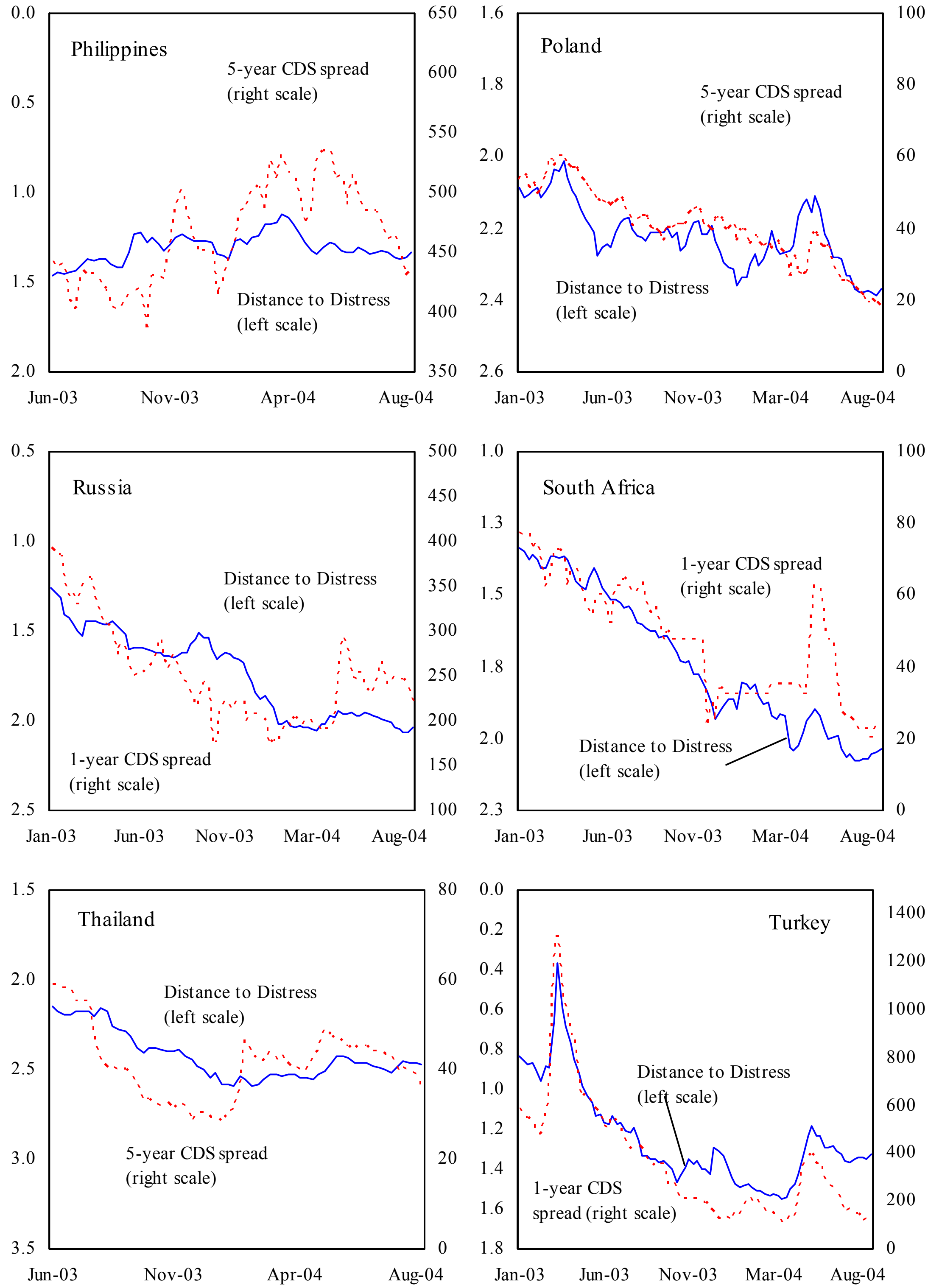
| **ASSETS** | **LIABILITIES** |
| --- | --- |
| **International Reserves**<br><br>**Domestic Currency Assets, in Foreign Currency Terms**<br>[ = Other Assets – Guarantees ] | **Value of Domestic Currency Liabilities, in Foreign Currency Terms**<br>[ = Domestic Currency Debt + Base Money ]<br><br>**Foreign Currency Debt** |

Figure 4. Distance to Distress and Credit Default Swaps



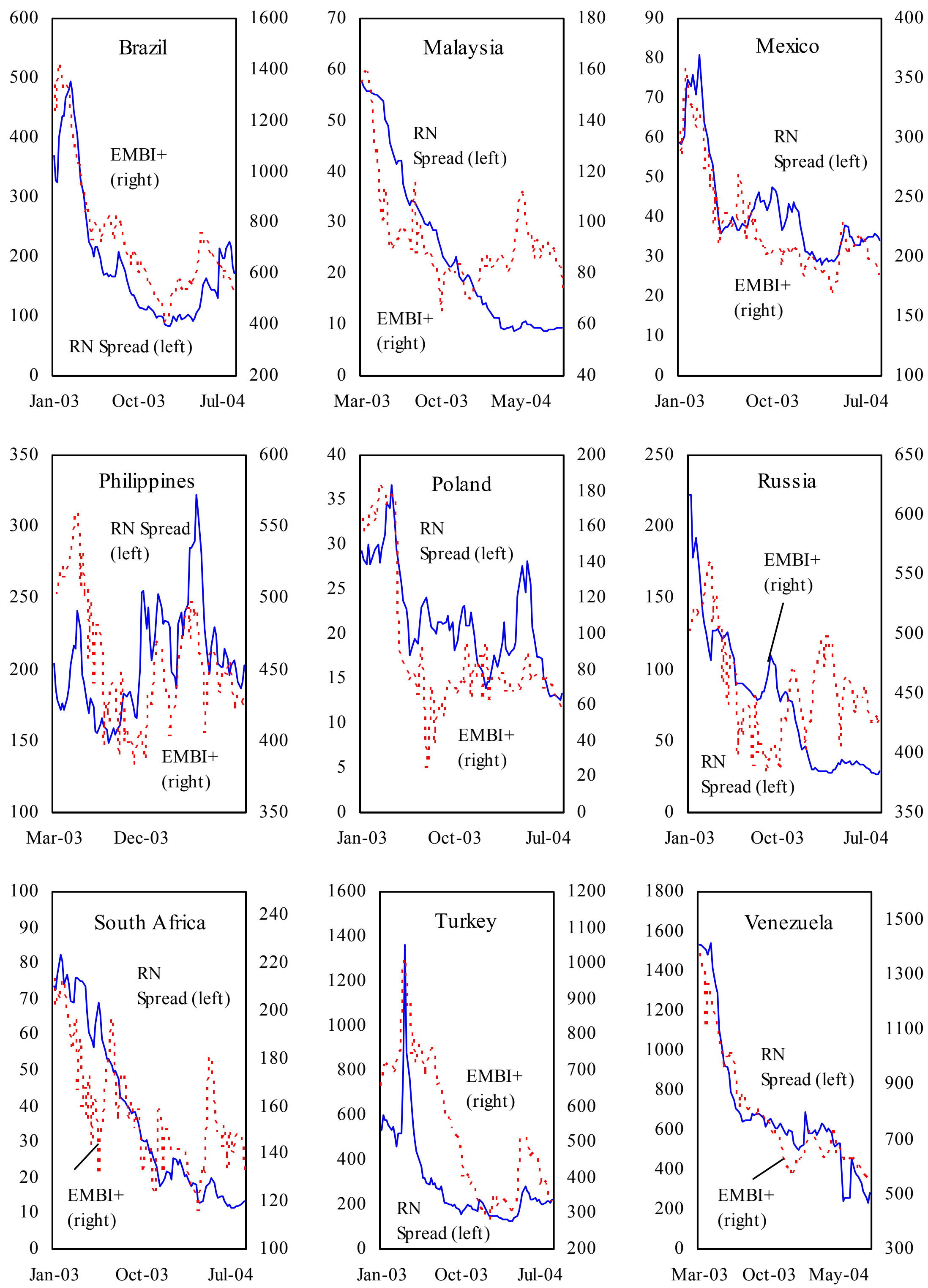

Source: Moody's MfRisk, Bloomberg.

Figure 4, contd. Distance to Distress and Credit Default Swaps




Source: Moody's MfRisk, Bloomberg.

Figure 5. Model Credit Spread and J.P. Morgan EMBI+ Country Index




Source: Moody's MfRisk, Bloomberg.

Figure 6. Market Observed Spreads and Model Distance to Distress

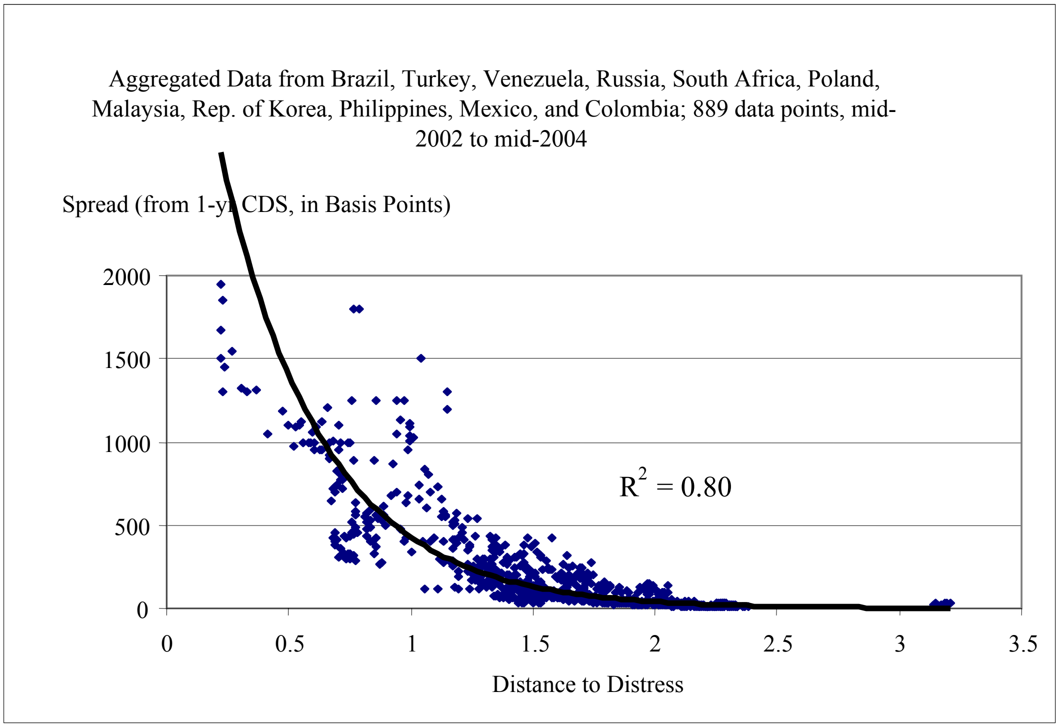



Figure 7. Monte Carlo Simulations: Hypothetical Sovereign

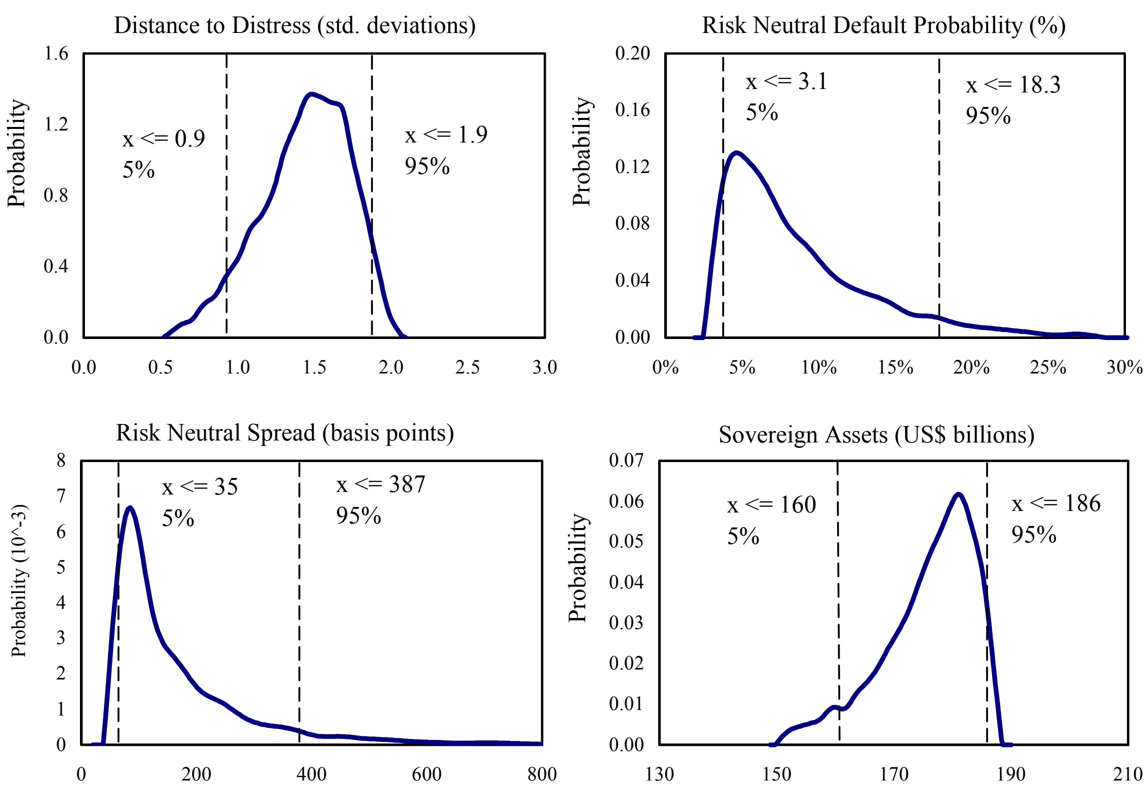
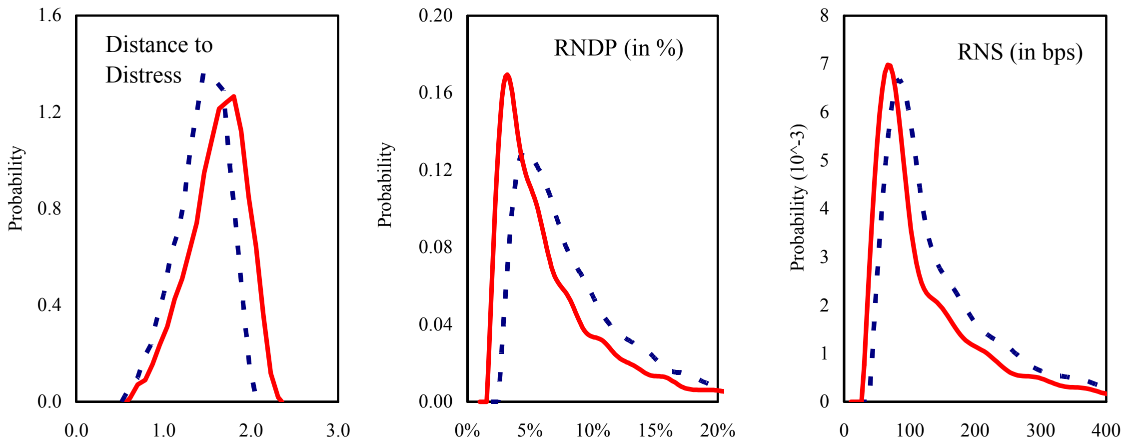
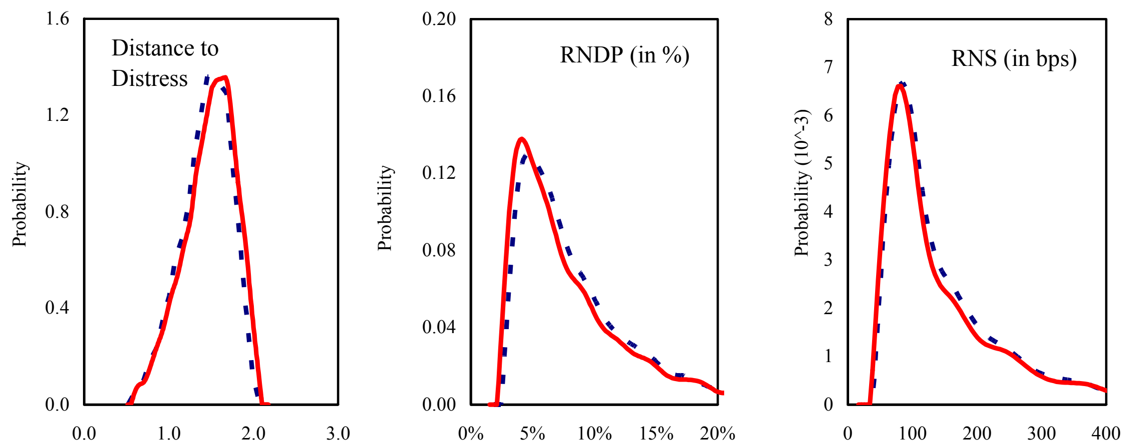


Figure 8. Evaluation of Policy Options

Panel 1. Debt Management Risk Indicators (solid) Versus Baseline Risk Indicators (dashed).




Panel 2. Reserve Management Risk Indicators (solid) Versus Baseline Risk Indicators (dashed).




Panel 3. Combined Debt and Reserve Management Risk Indicators (solid) vs. Baseline Risk Indicators (dashed).

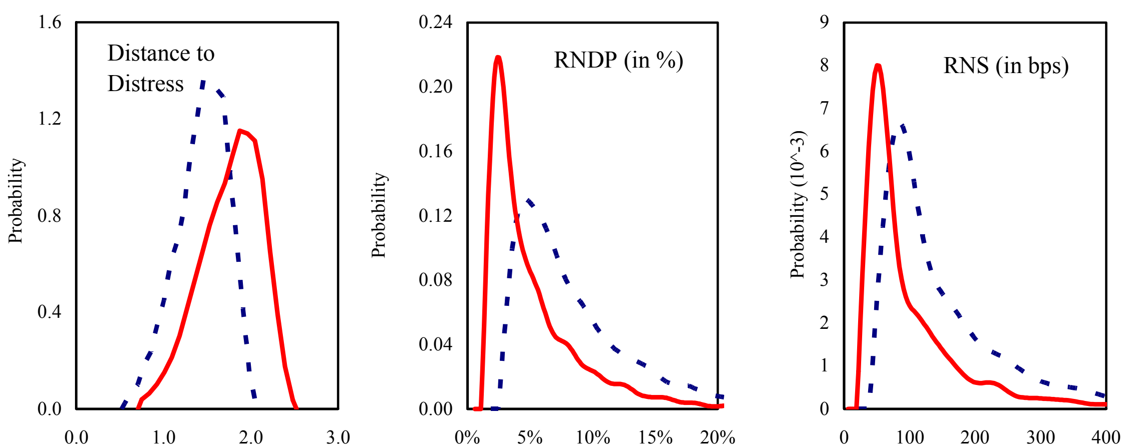


Table 1. Spearman Rank Correlation: Sovereign Risk Indicators and Actual Data

| Country | Distance to Distress and: 1-year CDS Spread | 3-year CDS Spread | 5-year CDS Spread | Country EMBI+ Spread | Risk Neutral Spread and: 1-year CDS Spread | 3-year CDS Spread | 5-year CDS Spread | Country EMBI+ Spread |
|---|---|---|---|---|---|---|---|---|
| Brazil | -0.68** | -0.79** | -0.80** | -0.81** | 0.70** | 0.82** | 0.82** | 0.83** |
| Bulgaria | n/a | -0.72** | -0.91** | n/a | n/a | 0.72** | 0.83** | n/a |
| Korea, Rep of | -0.83** | -0.85** | -0.88** | n/a | 0.84** | 0.85** | 0.89** | n/a |
| Malaysia | -0.72** | -0.73** | -0.14 | -0.36** | 0.72** | 0.73** | 0.15 | 0.39** |
| Mexico | -0.44** | -0.62** | -0.73** | -0.72** | 0.44** | 0.62** | 0.73** | 0.73** |
| Philippines | -0.33* | -0.43** | -0.53** | -0.20 | 0.33* | 0.43** | 0.54** | 0.17 |
| Poland | -0.16 | -0.68** | -0.69** | -0.44** | 0.06 | 0.67** | 0.69** | 0.45** |
| Russia | -0.29** | -0.54** | -0.66** | -0.47** | 0.30** | 0.54** | 0.67** | 0.47** |
| South Africa | -0.80** | -0.76** | -0.75** | -0.47** | 0.86** | 0.77** | 0.75** | 0.64** |
| Thailand | -0.29 | n/a | -0.28* | n/a | 0.41* | n/a | 0.27* | n/a |
| Turkey | -0.83** | -0.84** | -0.84** | -0.85** | 0.82** | 0.83** | 0.83** | 0.85** |
| Venezuela | -0.29* | -0.22 | -0.20 | -0.89** | 0.33* | 0.27 | 0.22 | 0.90** |

** Denotes significance at 1 percent level.

* Denotes significance at 5 percent level.

Table 2. Regression Output: Risk-Neutral Spreads and CDS Spreads

$$\ln\left(CDS_i\right) = i\alpha_i + \beta \ln\left(RNS_i\right) + \varepsilon_i$$

| | |
|---|---|
| R-squared | 0.88 |
| Adjusted R-squared | 0.88 |
| Log likelihood | -645.02 |
| F-statistic | 640.78 |
| Prob(F-statistic) | 0.00 |

| Indep. Variables / Country | Constant | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|---|
| ln(RNS) | | 0.52 | 0.02 | 24.53 | 0.00 |
| Brazil | 3.43 | | 0.12 | 28.09 | 0.00 |
| Colombia | 2.54 | | 0.11 | 22.34 | 0.00 |
| Korea, Rep. of | 2.59 | | 0.07 | 38.50 | 0.00 |
| Malaysia | 1.54 | | 0.08 | 18.57 | 0.00 |
| Mexico | 1.72 | | 0.09 | 18.42 | 0.00 |
| Philippines | 2.53 | | 0.12 | 20.50 | 0.00 |
| Poland | 1.20 | | 0.08 | 14.55 | 0.00 |
| Russia | 2.72 | | 0.10 | 26.94 | 0.00 |
| South Africa | 2.09 | | 0.09 | 23.63 | 0.00 |
| Turkey | 2.98 | | 0.13 | 23.04 | 0.00 |
| Venezuela | 2.94 | | 0.15 | 20.14 | 0.00 |

Table 3. Regression Output: Risk-Neutral Spreads and EMBI+ Spreads

$$\ln\left(\boldsymbol{EMBI}_i\right) = \boldsymbol{i}\alpha_i + \boldsymbol{\beta}\ln\left(\boldsymbol{RNS}_i\right) + \boldsymbol{\varepsilon}_i$$

| | |
|---|---|
| R-squared | 0.96 |
| Adjusted R-squared | 0.96 |
| Log likelihood | 340.48 |
| F-statistic | 1548.00 |
| Prob(F-statistic) | 0.00 |

| Indep. Variables / Country | Constant | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|---|
| ln(RNS) | | 0.15 | 0.02 | 6.30 | 0.00 |
| Brazil | 5.69 | | 0.11 | 50.86 | 0.00 |
| Colombia | 4.68 | | 0.08 | 60.98 | 0.00 |
| Korea, Rep. of | 5.41 | | 0.11 | 48.94 | 0.00 |
| Malaysia | 4.04 | | 0.07 | 60.30 | 0.00 |
| Mexico | 4.78 | | 0.08 | 58.74 | 0.00 |
| Philippines | 5.29 | | 0.12 | 42.88 | 0.00 |
| Poland | 3.79 | | 0.08 | 47.83 | 0.00 |
| Russia | 5.05 | | 0.09 | 54.28 | 0.00 |
| South Africa | 4.52 | | 0.07 | 61.41 | 0.00 |
| Turkey | 5.26 | | 0.12 | 43.85 | 0.00 |
| Venezuela | 5.61 | | 0.14 | 39.41 | 0.00 |

Table 4. Regression Output: Default Probability

$$\ln\left(\boldsymbol{MIDP}_i\right) = \boldsymbol{i}\alpha_i + \boldsymbol{\beta}\ln\left(\boldsymbol{RNDP}_i\right) + \boldsymbol{\varepsilon}_i$$

| | |
|---|---|
| R-squared | 0.93 |
| Adjusted R-squared | 0.93 |
| Log likelihood | -157.18 |
| F-statistic | 770.00 |
| Prob(F-statistic) | 0.00 |

| Indep. Variables / Country | Constant | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|---|
| ln(RNDP) | | 0.23 | 0.03 | 7.38 | 0.00 |
| Brazil | 0.61 | | 0.08 | 7.94 | 0.00 |
| Colombia | 0.24 | | 0.07 | 3.37 | 0.00 |
| Korea, Rep. of | -0.77 | | 0.04 | -21.63 | 0.00 |
| Malaysia | -1.44 | | 0.03 | -49.84 | 0.00 |
| Mexico | -0.90 | | 0.04 | -25.61 | 0.00 |
| Philippines | 0.50 | | 0.09 | 5.52 | 0.00 |
| Poland | -1.65 | | 0.04 | -38.67 | 0.00 |
| Russia | 0.16 | | 0.04 | 3.82 | 0.00 |
| South Africa | -0.79 | | 0.05 | -17.13 | 0.00 |
| Turkey | 0.64 | | 0.08 | 8.39 | 0.00 |
| Venezuela | 1.08 | | 0.08 | 12.76 | 0.00 |

Table 5. Example Contingent Claims Sovereign Balance Sheet Risk Indicators

| **Contingent Claim Sovereign Balance Sheet** | (US$ billion, unless indicated) |
|---|---|
| Value of sovereign assets (implied) | 175 |
| Foreign reserves (observed value) | 40 |
| Sovereign asset less reserves (implied) | 135 |
| Value of risky foreign currency debt | 94.5 |
| Distress barrier 1/ | 100 |
| PV of distress barrier 1/ | 96 |
| PV of expected losses (= implict put option) | 1.5 |
| Value of local currency liabilities 1/ | 80.5 |
| Volatility of asset (implied) | 38% |
| **Credit Risk Indicators** | |
| Distance to distress 2/ | 1.4 |
| Risk-neutral default probability (RNDP) | 8% |
| Risk-neutral spread (RNS) 3/ | 115 |
| **Sensitivity Measures** 4/ | |
| Change in distance to distress / 1% change in assets 2/ | -0.03 |
| Change in distance to distress / 1% change in asset vol. 2/ | -0.05 |
| Change in RNDP / 1% change in assets | 0.41 |
| Change in RNS / 1% change in assets 3/ | 7 |
| Change in RNS / 1% change in asset vol. 3/ | 16 |
| Change in PV expected loss / 1 % change in assets | 0.07 |
| Change in PV expected loss / 1 % change in asset vol. | 0.15 |

1/ Model inputs. Remainder are model outputs.
2/ In standard deviation of sovereign asset value.
3/ Spread in basis points.
4/ Based on a 1 percent change in sovereign asset value (e.g. from 175 to 176.75) and sovereign asset volatility (e.g. from 38 percent to 39 percent).

Table 6. Alternative Scenarios and Contingent Claim Sovereign Balance Sheet Risk Indicators

| | Scenario 1 | Baseline | Scenario 2 |
|---|---|---|---|
| **Contingent Claim Sovereign Balance Sheet** | | (US$ billion, unless indicated) | |
| Value of sovereign assets | 155 | 175 | 195 |
| Foreign reserves 1/ | 35 | 40 | 45 |
| Sovereign asset less reserves | 120 | 135 | 150 |
| Value of risky foreign currency debt | 93 | 94.5 | 96 |
| Distress barrier 1/ | 100 | 100 | 100 |
| PV of distress barrier 1/ | 96 | 96 | 96 |
| PV of expected losses (= implict put option) | 3 | 1.5 | 0.5 |
| Value of local currency liabilities 1/ | 62 | 80.5 | 99 |
| Volatility of asset | 43% | 38% | 37% |
| **Credit Risk Indicators** | | | |
| Distance to distress 2/ | 1.0 | 1.4 | 2.1 |
| Risk-neutral default probability (RNDP) | 17% | 8% | 4% |
| Risk-neutral spread (RNS) 3/ | 325 | 115 | 60 |
| **Sensitivity Measures** 4/ | | | |
| Change in distance to distress / 1% decrease in assets 2/ | -0.02 | -0.03 | -0.03 |
| Change in distance to distress / 1% increase in asset vol. 2/ | -0.03 | -0.05 | -0.06 |
| Change in RNDP / 1% decrease in assets | 0.63 | 0.41 | 0.23 |
| Change in RNS / 1% decrease in assets 3/ | 16 | 7 | 3 |
| Change in RNS / 1% increase in asset vol. 3/ | 28 | 16 | 9 |
| Change in PV expected loss / 1 % decrease in assets | 0.15 | 0.07 | 0.03 |
| Change in PV expected loss / 1 % increase in asset vol. | 0.26 | 0.15 | 0.08 |

1/ Model inputs for baseline.
2/ In standard deviation of sovereign asset value.
3/ Spread in basis points.
4/ Based on a 1 percent change in sovereign asset value (e.g. from 175 to 176.75) and sovereign asset volatility (e.g. from 38 percent to 39 percent).

## Appendix. Black-Scholes Option Pricing Formula in Contingent Claim Analysis

The Black-Scholes option pricing formula was originally applied to the valuation of options on traded equity and quickly spread to a variety of applications.[39] The original paper by Merton (1974) extended the option pricing formula from explicit option to implicit options in the context of corporate liabilities. He pointed out how equity can be modeled as a call option on firm assets and risky debt has an embedded put option. This work has been applied extensively (KMV 1993, Crosbie and Bohn 2001) and extended in many different directions to include multiple layers of debt and integrated with interest rate models (Cossin and Pirotte, 2001), to basket assets in multiple currencies.

### A. Implied Sovereign Asset Value and Volatility

In this paper, the Black-Scholes option pricing formula is used to relate the value and volatility of domestic currency liabilities to the value and volatility of sovereign assets. The value of domestic currency liabilities as a call option on sovereign assets is,

$$V_{DCL} = V_A N(d_1) - DBe^{-r_f t} N(d_2), \qquad (\text{A1})$$

where $V_A$ is the value of sovereign assets, $V_{DCL}$ is the value of domestic currency liabilities, $DB$ is the distress barrier or value of default-free debt, $r_f$ is the risk-free rate of interest, and $t$ is the time to maturity on a default-free bond in years. $N(d)$ is the cumulative probability distribution function for a standard normal variable (i.e., the probability that a random draw from a standard normal distribution will be below $d$) where,

$$d_1 = \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r_f + \frac{1}{2}\sigma_A^2\right)t}{\sigma_A \sqrt{t}}, \qquad (\text{A2})$$

$$d_2 = d_1 - \sigma_A \sqrt{t},$$

and $\sigma_A$ is the standard deviation of return on sovereign assets.

The Black-Scholes formula above contains two unknowns, sovereign assets and volatility of sovereign assets. The relationship between volatility of sovereign assets and volatility o domestic currency liabilities is given by,

[39] For readers interested in a more explicit derivation of the Black-Scholes option pricing formula, see Black and Scholes (1973) and Merton (1973, 1974). While the derivations in these studies use continuous-time mathematics, Hull (1993) and Baxter and Rennie (1996) detail how binomial models can be used to develop discrete-time representations.

$$V_{DCL} = \frac{\sigma_A}{\sigma_{DCL}} \mathrm{V_A} N(d_1), \qquad (A3)$$

where $\sigma_{DCL}$ is the standard deviation of domestic currency liabilities.[40] Here, $N(d_1)$ is the change in the price of domestic currency liabilities with respect to a change in sovereign assets, or $\partial V_{DCL} / \partial V_A$. This ratio is also referred to as the option *delta*. However, the main implication of the above relationship is that the standard deviation of domestic currency liabilities can be derived from historical data, including exchange rate data, and used to solve for sovereign asset volatility. Using standard iterative techniques, equations (A1) and (A3) can be solved simultaneously for the implied value of sovereign assets and sovereign asset volatility. Using this output, the precise measure of distance to distress is $d_2$ in equation (A2).

$$d_2 = \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r_f - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A \sqrt{t}} = \frac{\ln\left(V_A * \exp\left(\left(r_f - \frac{1}{2}\sigma_A^2\right)t\right)\right) - \ln(DB)}{\sigma_A \sqrt{t}}.$$

### B. Probability of Default and Sovereign Risk Premium

The probability of default is the likelihood that future sovereign asset value will fall below the distress barrier. Therefore, computing probability of default requires calculating the cumulative normal distribution function, *N*(.). This can be done using numerical methods or polynomial approximation. Tables that compute *N*(.) are also found in many financial and econometric texts. Using one of these methods will yield the probability of default as,

$$\text{Risk-Neutral Probability of Default} = N(-d_2). \qquad (A4)$$

The face value of senior foreign currency debt can be derived from equation (A1) and the balance sheet relationship, $V_A = V_{DCL} + V_{FCL}$, where $V_{FCL}$ represents the value of foreign currency liabilities.[41] Using these relationships together yields the value of foreign currency liabilities as,

$$V_{FCL} = V_A(1 - N(d_1)) + DBe^{-r_f t} N(d_2), \qquad (A5)$$

which is also equal to,

[40] See Hull (1993, p. 38).

[41] Merton (1974) derives similar measures for the pricing of corporate debt.

$$V_{FCL} = DBe^{-r_f t} - \left[ DBe^{-r_f t} N(-d_1) - V_A N(-d_2) \right], \qquad (A6)$$

when modeled as the default free value minus the implicit put option (present value of expected loss). The term, $DBe^{-rt}$, is the distress barrier discounted to the present by the risk-free rate.

Equation (A5) can also be expressed in terms of a credit risk premium,

$$y_t - r_f = \frac{-1}{t} \ln \left\{ \frac{V_A}{DBe^{-rt}} N\left(-d_1\right) + N\left(d_2\right) \right\}, \qquad (A7)$$

where $y_t = -\left(1/t\right) \ln\left(V_{FCL} / DB\right)$. The left hand side represents the yield to maturity on risky debt less the risk-free rate of interest and is therefore equivalent to a risk premium. In addition to the risk-free rate and time, examination of equation (A7) reveals that sovereign risk premium is a function of only two variables: the volatility of sovereign assets and the ratio of the value of sovereign assets to the present value of the promised payments on foreign currency liabilities, discounted by the risk free rate. Increases in the ratio of sovereign assets to foreign currency liabilities and decreases in sovereign asset volatility both decrease the sovereign risk premium.[42]

As described in the body of the paper there is a strong relationship of the sovereign risk neutral default probabilities with the market implied default probabilities (MIDP). The risk-neutral probability of default is $N(-d_2)$. Its relationship with the estimated default probability (EDP) is,

$$N(-d_2) = N\left(N^{-1}(MIDP) + \lambda\sqrt{T}\right), \text{ where } MIDP = \frac{1 - e^{-st}}{1 - R} \approx EDP, \qquad (A8)$$

where $\lambda$ is the market price of risk, $s$ is the observed spread, and $R$ is the assumed recovery rate. If we use the market implied default frequencies (MIDP) implied from observed sovereign CDS spreads as a proxy for the estimated default probability (EDP), then,

$$N^{-1}\left(N(-d_2)\right) - N^{-1}\left(\frac{1 - e^{-st}}{1 - R}\right) = \lambda\sqrt{T} = \frac{\mu_{Sov} - r}{\sigma_{sov}} \sqrt{T}, \qquad (A9)$$

[42] While the BSM model assumes constant volatility, the MfRisk model is more realistic including adjustments to volatility and deviations from strictly lognormal distributions of asset value and other values such as exchange rate.

where $\mu_{Sov}$ is the return on sovereign assets, $r$ is the risk-free rate, and $\sigma_{sov}$ is the volatility of sovereign assets.

## C. Extensions to Multiple Layer of Liabilities

Contingent claims analysis (CCA) can be extended to multiple layers of liabilities. Instead of one distress barrier there can be multiple distress barriers tied to the different layers of debt. With three layers of liabilities, the implicit options that make up the liabilities becomes, as shown in the table below: [43]

| | Distress Barrier | CCA Implicit Options |
|---|---|---|
| Most Junior (equity-like) | Senior Plus Sub-ordinated Debt Default Barrier ($DB_{Sr+Sub}$) | Call Option 1 (Assets, $DB_{Sr+Sub}$, r, t, Asset volatility) |
| Sub-ordinated Debt or Preferred Equity | Senior Debt Default Barrier ($DB_{Sr}$) | Call Option 2 (Assets, $DB_{Sr}$, r, t, Asset volatility) minus Call Option 1 (Assets, $DB_{Sr+Sub}$, r, t, Asset volatility) |
| Senior Debt | | Assets minus Call Option 2 = $DB_{Sr}$ minus Put Option (Assets, $DB_{Sr}$, r, t, Asset volatility) |
| Total | | Sum Equals Assets |

The Moodys-MfRisk model uses two layers of sovereign liabilities, local currency liabilities as junior claims and foreign currency denominated debt as the senior claim. This assumption appears to be a reasonable approximation from anecdotal evidence, from the observed robustness of the model, and from the behavior of spreads during periods of stress.[44]

CCA balance sheets with three or more layers, as described above, can be constructed, calibrated and tested to refine the model further. For a consolidated CCA sovereign balance

---

[43] See Cossin and Pirotte (2001) for a discussion on how the framework can handle multiple layers of liabilities or default sequences.

[44] Assuming that *all* of money and local currency debt are senior and that all of foreign currency debt is junior leads to inconsistencies. Crises resulting in depreciation of the exchange rate cause the “foreign currency junior claim” to grow large compared to domestic currency debt. This is inconsistent with the observation that CDS spreads on foreign currency debt increase with sharp depreciations. In situations of large exchange rate appreciation, usually considered beneficial from a credit risk perspective, the value of the “foreign currency debt junior claim” would be very small relative to domestic currency debt, indicating a large expected loss is associated with the domestic currency debt. Observed spreads on local and foreign currency debt during periods of stress is not consistent with assuming all local currency debt and money are senior to foreign currency debt.

sheet with three layers, the seniority structure could be: foreign currency debt as senior, part of local currency debt as subordinated and the rest of local currency liabilities as junior. Alternatively, the priority could be: part of local currency debt as senior, foreign currency debt is subordinated, followed by the remainder of local currency liabilities as the most junior claim. The three-layer model could be used to analyze the segregated *government* balance sheet. For example, the most senior claim on the government balance sheet could be foreign currency denominated debt, the subordinated claim could be local currency debt and the most junior claim could be government obligations to the monetary authorities.

**References**

Allen, Mark, and others, 2002, “Sovereign Debt Restructurings and the Domestic Economy Experience in Four Recent Cases” (Washington: International Monetary Fund). Available via the Internet:#http://www.imf.org/external/NP/pdr/sdrm/2002/022102.pdf.

Allen, Mark, Christoph Rosenberg, Christian Keller, B. Sester, and N. Roubini, 2002, “A Balance Sheet Approach to Financial Crisis,” IMF Working Paper 02/210 (Washington: International Monetary Fund).

Ariyoshi, Akira, Karl Habermeier, Bernard Laurens, Inci Ötker-Robe, Jorge Iván Canales-Kriljenko, and Andrei Kirilenko, 2000, *Capital Controls: Country Experiences with Their Use and Liberalization*, IMF Occasional Paper No. 190 (Washington: International Monetary Fund.

Baxter, Martin, and Andrew Rennie, 1996, *Financial Calculus: An Introduction to Derivative Pricing* (Cambridge: Cambridge University Press).

Best, Philip, 2000, “Stress Testing” in *The Professional’s Handbook of Financial Risk Management*, ed. by M. Lore and Lev Borodovsky (Oxford: Reed Educational and Professional Publishing).

Black, Fischer, and Myron S. Scholes, 1973, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, Vol. 81, pp. 637–654.

Blejer, M., and L. Schumacher, 1999, “Central Bank Vulnerability and the Credibility of its Commitments: A Value at Risk Approach,” *The Journal of Risk*, Vol. 2, No. 1, (November), pp. 37–55.

Bohn, J., 1998, “A Survey of Contingent-Claims Approaches to Risky Debt Valuation,” Working Paper, (Berkeley: University of California, Haas School of Business).

Buiter, Willem H., 1993, “Measurement of the Public Sector Deficit and Its Implications for Policy Evaluation and Design,” in *How to Measure the Fiscal Deficit*, ed. by Mario I. Blejer and Adrienne Cheasty (Washington: International Monetary Fund).

Caballero, Ricardo J., and Stavros Panageas, 2005, “Insurance and Reserves Management in a Model of Sudden Stops,” paper presented at the IMF Institute seminar “Emerging Markets Macroeconomics: An Insurance Perspective,” Washington, February 7–9.

Chan-Lau, J., Jorbert, A. and Janet Kong, 2004, “An Option-Based Approach to Bank Vulnerabilities in Emerging Markets,” IMF Working Paper 04/03 (Washington: International Monetary Fund).

Chriss, Neil A., 1997, *Black-Scholes and Beyond* (New York: McGraw-Hill).

Cossin D., and H. Pirotte, 2001, *Advanced Credit Risk Analysis,* (New York: John Wiley and Sons).

Crosbie, Peter J., and Jeffrey R. Bohn, 2001. "Modeling Default Risk," Moody's KMV. Available via the Internet: #http://www.moodyskmv.com.

Crouhy, Michel, Dan Galai, and Robert Mark, 2000, "A Comparative Analysis of Current Credit Risk Models," *Journal of Banking & Finance*, Vol. 24, pp. 59–117.

Draghi, Mario, Giavazzi, F., Robert Merton, "Transparency, Risk and International Financial Fragility," Fourth Geneva Conference on the World Economy: Financial Markets: Shock Absorbers or Shock Creators?, International Center for Monetary and Banking Studies, Geneva, Switzerland, May 10, 2002. #http://www.people.hbs.edu/larricale/Trans%20Risk%20Management.pdf.

Eichengreen, Barry, Ricardo Hausmann, and Ugo Panizza, 2002, "Original Sin: The Pain, the Mystery, and the Road to Redemption," paper presented at Inter-American Development Bank conference, "Currency and Maturity Matchmaking: Redeeming Debt from Original Sin," Washington, November 21–22, 2002.

Gapen, Michael T., Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao, 2004, "The Contingent Claims Approach to Corporate Vulnerability Analysis: Estimating Default Risk and Economy-Wide Risk Transfer," IMF Working Paper 04/121 (Washington: International Monetary Fund).

Gray, Dale F., 2004, "Modeling Sovereign Default Risk and Country Risk Using Moody's-M*f*Risk Framework with Specific Country Applications," M*f*Risk Working Paper No. 1–04.

Gray, Dale F., Robert C. Merton, and Zvi Bodie, 2003, "A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy," M*f*Risk Working Paper No. 1–03. Available via the Internet: #http://www.moodys-mfrisk.com and www.defaultrisk.com/pp_sov02.htm.

Gray, Dale F., Robert C. Merton, and Zvi Bodie, 2002, "A New Framework for Analyzing and Managing Macrofinancial Risks," Presentation at NYU CV Starr/RED Conference on Finance and the Macroeconomy, October 11–12 2002.

Gray, Dale F., 2005, "Macrofinance: New Contingent Claims Framework for Valuation and Risk Analysis of Economies and its Real-world Applications," paper presented at the IEOR Department Seminar Series, Columbia University, November 15, 2005. Available via the Internet: www.ieor.columbia.edu/fe_seminars_Gray.html.

Gregory, Jon, 2003, *Credit Derivatives: The Definitive Guide*, The Debt and Equity Linkage and the Valuation of Credit Derivatives by Keenan, S., Sobehart, J., and Benzschawel, pp. 67–90, Risk Books, London.

Gule, Anne-Marie, David Hoelscher, Alain Ize, Alfredo Leone, David Marston, and Marina Moretti, 2003, "Dealing with Banking Crises in Dollarized Economies," in *Managing Financial Crises: Recent Experiences and Lessons for Latin America*, ed. by Charles Collyns and G. Russell Kincaid, IMF Occasional Paper No. 217 (Washington: International Monetary Fund).

Hull, John C., 1993, *Options, Futures, and Other Derivative Securities* (Englewood Cliffs, New Jersey: Prentice Hall).

International Monetary Fund, 2000, "Debt- and Reserve-Related Indicators of External Vulnerability," (Washington).

Jorion, Philippe, 2000, *Value at Risk: The New Benchmark for Managing Financial Risk* (New York: McGraw-Hill).

KMV, 1993, "A Comment on Market vs. Accounting-Based Measures of Default Risk," pp. 2–4, McQuown, J. of KMV Corporation, online at www.KMV.com.

Lore, M., and Lev Borodovsky editors, 2000, *The Professional's Handbook of Financial Risk Management* (Oxford: Reed Educational and Professional Publishing).

Merton, Robert C., 1973, "Theory of Rational Option Pricing," *The Bell Journal of Economics and Management Science*, Vol. 4, pp. 141–183.

______, 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance*, Vol. 29, pp. 449-470.

______, 1998, "Applications of Option-Pricing Theory: Twenty-Five Years Later," *The American Economic Review*, Vol. 88, pp. 323–349.

______, 2002, "Future Possibilities in Finance Theory and Finance Practice," *Mathematical Finance: Bacheilier Congress, 2000,* Edited by Geman, Madan, Pliska and Vorst, Springer.

______, 2005, "Swapping your Country's Risks," Harvard Business Review, Feb. 2005.

Schaechter, A., "Implementation of Monetary Policy and the Central Bank's Balance Sheet," IMF Working Paper 01/149, 2001.

Shadman-Valavi, Mohammed, and others, 1999, *Ukraine: Recent Economic Developments,* IMF Staff Country Report No. 99/42 (Washington: International Monetary Fund).

Sims, Christopher A., 1999, "Domestic Currency Denominated Government Debt as Equity in the Primary Surplus," paper presented at the Latin American meetings of the Econometric Society, Cancun, Mexico. August. 19.

Van den End, W., M. Tabbae, 2005, "Measuring Financial Stability; Applying the MfRisk Model to the Netherlands" De Nederlandsche Bank, WP No. 30, March.

WP/04/121

# IMF Working Paper

# The Contingent Claims Approach to Corporate Vulnerability Analysis: Estimating Default Risk and Economy-Wide Risk Transfer

*Michael T. Gapen, Dale F. Gray,*
*Cheng Hoon Lim, and Yingbin Xiao*

INTERNATIONAL MONETARY FUND

 WP/04/121

**IMF Working Paper**

International Capital Markets Department

# The Contingent Claims Approach to Corporate Vulnerability Analysis: Estimating Default Risk and Economy-Wide Risk Transfer

Prepared by Michael T. Gapen, Dale F. Gray, Cheng Hoon Lim, and Yingbin Xiao[1]

Authorized for distribution by Carlos Medeiros

July 2004

**Abstract**



In this paper, we examine the ability of the contingent claims approach (CCA) to identify corporate sector and economy-wide vulnerabilities. We apply the Moody's M*f*Risk model, which uses aggregated CCA principles, to assess vulnerabilities retroactively in two historical country cases. The results indicate that the method may prove helpful in identifying corporate sector vulnerabilities and estimating the associated value of risk transfer across interrelated balance sheets of the corporate, financial, and public sectors.

JEL Classification Numbers: G13, G32, G34

Keywords: Contingent claims approach, Corporate sector vulnerability, Credit risk

Author's E-Mail Address: mgapen@imf.org; dale.gray@moodys.com; clim@imf.org; yxiao2@imf.org

---

[1] Michael T. Gapen and Yingbin Xiao are Economists in the Capital Markets Financing Division of the IMF's International Capital Markets Department. Dale F. Gray is President of the Macro Financial Risk Corporation and Consultant to the Macro Financial Risk Project, Moody's Investors Service, New York, New York. Cheng Hoon Lim is the Deputy Division Chief in the IMF's Capital Markets Financing Division of the International Capital Markets Department. This paper was presented at the conference on "Corporate Restructuring: International Best Practices," hosted by the World Bank, Washington, D.C., March 22, 2004, and will be published in the forthcoming conference volume under the same name.

Contents Page

Appendices

# I. INTRODUCTION

Many studies have documented the importance of monitoring weaknesses in the corporate sector and their impact on the wider financial system. The motivation for close monitoring derives from the fact that corporate failures are usually expensive in terms of the cost of employment, lost output, and banking sector distress in cases where the corporate sector is heavily dependent on bank financing. Moreover, to the extent that the banking sector enjoys full or partial financial guarantee from the government, the losses incurred by the banking sector could translate into a higher debt burden for the public sector as the government issues bonds to cover the cost of recapitalizing banks. In cases where the risk transmission among sectors is magnified because of the capital structure linkages between sectors, problems that appeared isolated in the corporate sector could have far-reaching consequences, triggering severe economy-wide financial crises—what we refer to as "macrofinancial" risk. The cost of corporate failures tends to be more acute in emerging markets than in mature markets because corporate financing is less diversified and is more vulnerable to sudden capital outflows and sharp changes in both world interest rates and the exchange rate. Moreover, there are fewer avenues available to hedge or absorb financial losses.[2] Developing an effective approach to detect corporate vulnerabilities before they become severe is essential in minimizing macrofinancial risks, thereby protecting the stability of the financial system and overall economy.

The contingent claims approach (CCA) was developed from modern finance theory and has been widely applied by financial market participants to measure the default probability of a firm based on the market prices of the firm's debt and equity.[3] In this paper we apply the contingent claims approach on an aggregated level to estimate corporate sector credit risk and evaluate the potential costs of macrofinancial risk transfers. In particular, we examine the ability of CCA to estimate probability of default within the corporate sector, assess and value the potential for risk transfer, and serve as an early-warning indicator.

Since market prices represent the collective views and forecasts of many investors, the contingent claims methodology is forward looking—unlike analysis based only on a review of past financial statements—and helps increase the predictive power of the estimates of default risk. The ability to translate continuously adjusting financial market price information into current market value estimates of asset value is especially important, given the speed with which

---

[2] The Asian crisis during the 1990s is often cited as an example where widespread bankruptcies of highly leveraged companies contributed to sharp declines in output and significant losses in the banking sector. Eventually, the public sector was forced to recapitalize the banking sector at a significant fiscal cost.

[3] See Crouhy, Galai, and Mark (2000), Merton (1998). Perhaps the most widespread application of CCA has come from Moody's KMV (MKMV) whose products are used by more than 2,000 leading financial institutions and firms in over 80 countries. Utilizing 30 years of historical data over 6,000 public and 70,000 private company default events for a total of 70,000 public and 1 million private companies, healthy and distressed, around the world. MKMV uses firm asset value, future asset distribution, asset volatility and the level of the default barrier to derive the firm specific probability of default, which they refer to as the Expected Default Frequency™ (EDF).

economic conditions change relative to the time span between releases of consolidated accounting balance-sheet information. In contrast, accounting-based approaches to assessing corporate credit risk rely on historical balance sheet information which arrives with a significant lag, usually 90 days after the end of the quarter or annual period.[4] Furthermore, CCA takes into account the volatility of assets when estimating default risk. The volatility of assets is crucial in this process, since firms may have similar levels of equity and debt, but very different probabilities of default if underlying asset volatility differs.

We use CCA to estimate risk indicators at the aggregated industry level for the nonfinancial corporate sector, including distance to distress and probability of default. This is the first and central purpose of the paper. However, focusing solely on the corporate sector ignores important possibilities for risk transfer across the consolidated balance sheets of the corporate, financial, and public sectors. Therefore, the second purpose of this paper is to extend the contingent claim methodology to a multisector framework, in which linkages between the corporate, financial, and public sectors can be examined. A multisector analysis allows for a more thorough understanding of the potential feedback effects between sectors and implications, if any, for the health of the financial and nonfinancial corporate sectors.

Through two historical case studies, we apply the CCA using the Moody's Macro Financial Risk (M*f*Risk) model to assess the potential value of such models to act as early-warning indicators of vulnerability.[5] We apply the framework to retroactively assess the likelihood of corporate failure at the industry level and compare the results with actual events. We then extend the framework to a multisector analysis to estimate the level of risk transmission at the macro level between the corporate, banking, and public sectors. We conclude with a discussion of the results, including potential measures to mitigate risk through enhanced surveillance and policy design.

## II. Contingent Claims Analysis

Initial theoretical work on contingent claims focused both on the pricing of options and the application of option theory to the analysis of the corporate capital structure.[6] Since the total

---

[4] The accounting based approach maps a reduced set of financial accounting variables to a risk scale to discriminate between repayment and non-repayment. A prominent accounting based approach was developed by Altman (1968) and used a linear combination of five accounting and market variables to produce a credit score—the so-called "Z-score." A subsequent seven factor "Zeta model" was later introduced by Altman et al. (1977).

[5] The M*f*Risk model was developed under a joint research effort between Moody's and Macro Financial Risk, Inc. Access to M*f*Risk is only available through subscription.

[6] Black-Scholes (1973) and Merton (1973, 1974) use no-arbitrage principles to derive the theoretical valuation formula for options commonly known as the "Black-Scholes Option Pricing Model." The authors also discuss the application of option pricing to corporate liabilities. In subsequent work, Geske (1979) shows how options on equity are really compound options on firm value and expands the Black-Scholes formula to cover such cases.

(continued…)

value of the firm is equal to the sum of the value of the securities in the capital structure, the securities in the capital structure can be viewed as contingent claims on the underlying value of the firm.[7] CCA can be used to analyze how the value of the contingent claim changes as the value of the firm changes through time. Therefore, contingent claims analysis should be viewed as a generalization of option pricing theory with the aim of specifying a framework within which all contingent claims can be valued.

Contingent claims analysis is based on three simple principles: (i) the value of liabilities flows from assets, (ii) liabilities have different seniority (and thus have different risks related to their seniority), and (iii) there is a random element to the way asset value evolves over time. Debt is a senior claim on the asset value and equity has a junior or residual claim on the asset value. Debt is risky because asset value may not be sufficient to meet the promised debt payments. The value of risky debt, therefore, can be seen as having two components, the *default-free value* of the debt (promised payment value) and the *expected loss* associated with default when the assets are insufficient to meet the promised payments on the debt. The value of the junior claim (equity in the case of firms) is derived from the residual value after the promised debt payments have been made.

If the value of assets has a random component (e.g., price changes, shocks and other factors affect asset value), higher asset volatility means there is a greater probability that assets will fall below the level necessary to meet the senior debt payments over the horizon period. Consequently, higher volatility means higher expected loss and a lower value of risky debt, other things equal. Financial techniques, namely option pricing relationships, have been developed to measure the expected losses as a function of the asset value, asset volatility, the default free value of debt, and the time horizon. Similarly, the value of equity and junior claims can be measured as a function of the same variables. The expected loss in risky debt is an implicit put option. Equity and junior claims are implicit call options.[8]

The essence of CCA is that changes in observed variables—the value of securities in the capital structure—are used to infer changes in unobserved variables—the value of the firm. The application of CCA to the capital structure derives from the seniority of liabilities in the capital

---

[7] The general definition of a contingent claim is any asset whose future payoff is contingent on the outcome of an uncertain event. In this context, it is the right to receive the residual value of the asset or the obligation to receive (residual) value of the asset.

[8] Government guarantees to the banking and financial sector increase with the volatility of assets in that sector. The government guarantees are also implicit put options which are a function of the banking and financial sector asset value, the associated asset volatility, the associated deposits and debt obligations, and the time horizon.

structure and the balance sheet identity that the total market value of debt plus equity must equal the current market value of the firm.[9]

## A. The CCA Methodology

In this section we briefly illustrate the contingent claims methodology as applied to a simplified corporate balance sheet consisting of senior debt and junior equity.[10] At any point in time, the total market value of assets, $A$, of a firm financed with debt, $D$, and equity, $E$, is equal to the market value of equity plus market value of risky debt. Fundamental analysis dictates that firm asset value is derived from the stochastic discounted present value of income minus expenditures with the potential for asset value to decline below the point where scheduled debt payments can be made. If assets fall to a level where debt cannot be serviced, then default is the result. This level is often referred to as distress barrier, $DB$, and is equal to or close to the default-free value of debt.[11]

Equity holders have a junior contingent claim on the residual value of assets in the future. In this manner, the value of equity can be viewed as an option where holders of equity receive the maximum of either assets minus the distress barrier, or nothing in the case of default. The value of equity, therefore, is,

$$E = max\ [\ A - DB, 0\ ]. \qquad (1)$$

The standard option pricing formulas can then be used to relate changes in the price of firm assets to changes in equity.[12] Given the relationship between firm equity and firm assets, changes in the value and volatility of traded equity can be used via option pricing relationships to infer changes in the market value and volatility of firm assets.

The case of risky debt, however, is slightly more complex. Holders of debt are obligated to absorb losses in the event of default and the guarantee of repayment by the lender can be modeled as an implicit put option since debt holders receive assets of the defaulted firm (or equivalently, the assets of the firm get "put" to the debt holders). Thus, holders of risky debt receive either the default-free value or, in the event of default, the senior claim on assets. Since the value of default-free debt is equal to the distress barrier and the implicit put option on the assets of the firm yields $max\ [\ DB - A, 0\ ]$, the market value of risky debt can be modeled as,

---

[9] Securities on the liability side of the corporate balance sheet could also include senior collateralized debt, convertible securities, and preferred equity.

[10] See Appendix I for additional detail on contingent claims analysis and the application of the Black-Scholes option pricing formula.

[11] Analysis by MKMV based primarily on U.S. companies shows that from an empirical point of view the distress barrier is best approximated by short-term debt and one-half of long-term debt plus interest.
[12] See Appendix I for details.

$$D = min\ [\ A, DB\ ] = DB - max\ [\ DB - A, 0\ ]. \qquad (2)$$

Inserting these option pricing relationships into the economic balance sheet identity results in a market value of firm assets at time *t* of,

$$A = D + E, \qquad (3)$$

$$A = DB - max\ [\ DB - A, 0\ ] + max\ [\ A - DB, 0\ ]. \qquad (4)$$

The option pricing formula is used in a two step process. First, the observed market value of equity and the distress barrier are used with the call option formula to derive the value of firm assets. The value of firm assets and the distress barrier are then used with the put option formula to derive the implied market value of risky debt. Thus, the CCA uses call and put option pricing formulas to develop a market value balance sheet based on observed financial market variables and financial statement information.

### B. Distance to Distress and Probability of Default

Two useful credit risk indicators that arise from the implementation of CCA are the distance to distress and probability of default.[13] The option pricing formulas applied in CCA to estimate credit risk rely on only a few select variables: the value and volatility of equity, the distress barrier, the risk-free interest rate, and time. These variables can be combined into a measure of default risk, called the distance to distress, which computes the difference between the implied market value of firm assets and the distress barrier scaled by a one standard deviation move in firm assets. In the application of CCA to actual firm capital structures, most practitioners compute the distress barrier as the sum of the book value of total short-term debt and one-half of long-term debt plus interest on long-term debt. This computation is used since historical instances of firm defaults have shown that it is possible for the value of firm assets to trade below the book value of total debt for significant periods of time without a default if most of the debt is long-term. Short-term debt, however, is more binding since the firm faces rollover risk in a shorter period of time. Thus, an adjustment is made to reduce the weight of long-term debt in the distress barrier.

The distance to distress combines the difference between assets and distress barrier with the volatility of assets into one measure,

---

[13] Although not explicitly discussed in this paper, a third useful credit risk indicator that can be obtained from CCA is the credit spread, or the additional risk premium required by bondholders to compensate for expected loss. See Appendix I for additional details.

$$\frac{\text{(Market value of assets – Distress barrier)}}{\text{(Market value of assets)*(Asset volatility)}}, \quad (5)$$

which yields the number of standard deviations of asset value from distress. The distance to distress for a hypothetical firm is illustrated in Figure 1. The numerator above measures the distance between the expected one-year ahead market value of firm assets and the distress barrier. This amount is then scaled by a one-standard deviation move in firm assets. Lower market value of assets, higher levels of leverage, and higher levels of asset volatility all serve to decrease the distance to distress.

The final step to estimate the probability of default consists of a mapping between the distance to distress measure from the equation above and actual probabilities of default based on historical data.[14] Using historical information of a large sample of firms and given a distance to distress, Moody's KMV is able to estimate the proportion of these firms that actually defaulted in a one-year ahead time horizon.

### C. Moody's MfRisk Model: Contingent Claims Analysis in a Multisector Framework

While the CCA approach as applied to individual corporate balance sheets has become a useful and widely applied tool in risk analysis, the focus on the corporate sector alone is too narrow to fully assess vulnerabilities. The corporate, financial, and public sectors are linked and changes in value in one sector can transmit risk from one sector to another. Changes in the value of the assets of one sector lead to changes in value of the liabilities of that sector which in turn affect the value of assets and liabilities in other sectors. For example, a decline in the assets of the corporate sector leads to a decline in the value of risky corporate debt as the expected losses increase. The lower value of debt held by banks lowers bank assets and causes the value of bank liabilities to fall and increases the likelihood of banking sector financial difficulties. Since governments frequently provide explicit or implicit financial guarantees to banks and large financial institutions out of concerns of systemic risk, the guarantee to the financial sector is a contingent liability on the sovereign balance sheet and a contingent asset on the financial sector balance sheet. As recent history has shown, these guarantees can become very large—from 20 to 50 percent of GDP in recent emerging market crises. These guarantees can be modeled as implicit put options.[15]

---

[14] This step is necessary since the option pricing formula used in the derivation of the distance to distress generally overstates actual probability of default. The Black-Scholes option pricing formula results in risk-neutral probabilities of default since the formula is derived from no-arbitrage conditions. Moody's KMV has demonstrated using historical instances of default that the actual probability distribution has fatter tails than the normal distribution applies. See Crosbie and Bohn (2003) and Jarrow and Turnbull (1997) for additional information.

[15] See Merton (1977), for example.

The existence of these linkages via risky debt and guarantees allows for risk transfer across balance sheets, meaning corporate sector vulnerabilities may come from "inside" the business environment or from "outside" if the value of the guarantee were to change suddenly. Therefore, a complete analysis of corporate sector vulnerabilities and potential for risk transfer requires a set of interrelated balance sheets across the corporate, financial, and public sectors.[16]
The ability to recognize and price both the expected losses in corporate debt and implicit guarantees is essential for conducting a full assessment of corporate sector vulnerabilities.

The Moody's M*f*Risk model is a practical application of CCA to the major sectors of a national economy in order to measure and analyze macrofinancial risks.[17] The model constructs market value balance sheets of assets, debt liabilities, and junior claims for the aggregated corporate, financial, and public sectors to measure risk exposures and analyze risk transmissions between sectors. Hence, the M*f*Risk model builds upon previous CCA credit risk methods by analyzing the probability of private and public debt defaults, currency crises, convertibility risk, cross-balance sheet vulnerability, and the values of the government guarantees provided to the financial and/or corporate sectors.

## III. ASSESSING CORPORATE SECTOR VULNERABILITIES

To assess the ability of contingent claims methods to predict corporate sector vulnerabilities, we apply the Moody's M*f*Risk model to calculate retroactively the distance to distress and estimated actual default probabilities of the nonfinancial corporate sector for two case studies: Thailand and Brazil.[18] CCA can be applied at the sector level in one of two ways. The sector can be built up from individual firm CCA models (e.g., Moody's KMV) or the principles of CCA can be applied to the aggregate balance sheet for each of the main industry sectors of the nonfinancial corporate sector. Under the latter, CCA treats each industry sector as if it were one large firm. A disadvantage of aggregating across industry sector is that it may be possible for individual firm weakness to be masked in the aggregation process since the ability of any one firm to impact the industry market value balance sheet is dependent on firm size relative to the industry as a whole. However, the aggregation process should be indicative of possible systemic vulnerabilities since a critical mass of firms is needed to influence the overall industry sector balance sheet. Aggregation maintains the fundamental premise underlying the CCA approach: (i) that the value of sector liabilities flows from sector assets, (ii) the liabilities have different seniority, and (iii) there is a random element to the way sector asset value evolves over time.

---

[16] Gray, Merton, and Bodie (2003); Gray (2002).

[17] See Appendix II for additional information on the M*f*Risk multisector model and consolidated balance sheets.

[18] Unless otherwise indicated, all reported values from applying CCA to the country cases were obtained from the M*f*Risk model.

The next sections describe the application of the Moody's M*f*Risk model to the corporate sectors in Brazil and Thailand during 2002 and 1997, respectively. Historical industry balance sheet information was combined with actual market price information to compute distance to distress and probability of default. The resulting vulnerability indicators across industry sectors are then compared with the results of actual corporate sector defaults to test the ability of the aggregated contingent claims approach to serve as an early-warning indicator of corporate sector vulnerabilities.

### A. The Brazilian Corporate Sector

Demand for capital in Brazil during the 1990s was spurred by large-scale privatizations accomplished through the use of leverage and, in some cases, existing leverage was sold with the asset itself. Consequently, corporate sector leverage increased during this time in the form of dollar-denominated, short- and medium-term securities (Figure 2).[19] The sectors that were most leveraged were transportation, metals and mining, food and beverage, pulp and paper, and vehicle and parts (Table 1). Official data indicated that the overall corporate sector had a sizable negative foreign exchange (FX) position on its balance sheet before hedging through derivative operations.[20]

Some industry sectors are naturally hedged since they have export proceeds. However, the export base of Brazil is fairly small—at the time, merchandise exports of goods and services amounted to about US$70 billion (15 percent of GDP)—in relation to total external debt.[21] While FX debt ratios were high in the metals, mining, and pulp and paper sectors, these sectors have the ability to generate FX revenues and, consequently were better positioned to weather FX shocks. However, currency mismatches were likely present in sectors that operate primarily in the domestic economy, such as electric utilities, telecommunications, and retail trade. FX hedging was accomplished by the use of FX derivatives intermediated through the financial system and facilitated by the provision of FX-linked domestic public sector debt.[22]

---

[19] Data for the corporate sector represents quoted companies on the São Paulo stock exchange (BOVESPA) from Economatica, a private data provider. The companies in the sample hold about seventy percent of the overall nonfinancial private sector external debt and includes some public sector companies, but Petrobras has been excluded.

[20] Official data indicates the corporate sector had US$63 billion in assets at end-2001 against a debt level of about US$90 billion. Corporate sector assets included US$51 billion registered as direct investment abroad and were likely less liquid.

[21] Total external debt to total exports of goods and nonfactor services was about 300 percent.

[22] FX hedging via derivatives is wide-spread and Brazilian firms hedged foreign exchange risk, mainly by the use of FX derivatives. The public sector provided the corporate sector with interest rate and currency protection through the highly indexed structure of public debt. Banks acted as intermediaries by buying the dollar-linked bonds or FX swaps issued by the public sector and then selling the FX hedge to the

(continued…)

## B. CCA and Financial Market Uncertainty in Brazil in 2002

Applying the CCA framework retroactively to the aggregated industry sectors of publicly listed companies on the Brazil stock exchange suggests that this approach would have provided an accurate view of the pending financial difficulties within the Brazilian corporate sector. Using the historical balance sheet capital structures in conjunction with historical traded equity prices as inputs, the option pricing framework relates these to the value of industry assets. Prior to the financial market volatility, most industries of the corporate sector were two or more standard deviations away from the sector distress barrier, implying a relatively low probability of default over a one-year ahead time horizon (Figure 3).[23] During the second half of 2002, fears over policy continuity following the upcoming presidential elections combined with weakening sentiment in external capital markets caused a deterioration in financial market conditions in Brazil. Access to international capital markets by both the public and private sector came to a halt beginning in July of 2002 and pressure on the currency intensified. The *real* depreciated from R$2.3 per U.S. dollar in April 2002 to nearly R$4.0 in October before ending the year at R$3.5.

Equity valuations began to decline in April of 2002 and did so on a near-continuous basis before bottoming out in October. As equity valuations declined significantly, the distance to distress decreased for many firms and resulting probabilities of default were at their highest at end-September 2002 (Figure 4) when financial uncertainties peaked. Despite the high volatility of asset prices and capital outflows, equity markets remained selective. In particular, while the ensuing market uncertainty affected all industries, equity price declines were more concentrated in industries that had operations primarily in the domestic economy as opposed to export led sectors. Industries that operated in the domestic economy; such as retail trade, textiles, home appliances (electronics), utilities, and food and beverage; were more likely to have dollar liabilities on their balance sheets versus local currency revenue streams. At the height of the crisis, the distance to distress for these industries ranged from (0.3) standard deviations for home appliances to 0.4 standard deviations for chemicals, indicating high levels of balance sheet distress (when distance to distress turns negative, the CCA suggests that some firms in the industry sector are in default). In contrast, the main export industries; mining, oil and gas, steel, pulp and paper, and petroleum chemicals; were less affected since currency mismatches between assets and liabilities were less prevalent. Distance to distress for these sectors averaged 1.1 standard deviations, well below their levels in March, but well above the distance to distress of their nonexport based counterparts. Therefore, markets discriminated against nonexport sectors more heavily when weighing the impact of the depreciation and economic slowdown.

---

corporate sector. Firms also hedged themselves on the BM&F, the local derivative market, which is very active.

[23] Private sector credit risk analysts and Moody's M*f*Risk associates view a one-year ahead distance to distress measure below 0.5 as an indication of severe stress.

The electric utility industry is an example of an industry that operates primarily in the local market, with revenues in local currency and some liabilities in foreign currency. The electric utility sector raised large amounts of funds during 1998–99 in dollar loans largely related to privatization efforts. The subsequent devaluations in the currency in 1999 and 2002 along with the rationing of power in 2001 resulted in balance sheet weaknesses. As shown in Figure 5, the distance to distress in the utility sector narrowed steadily and reached its lowest level at end-September. Prior to the financial market volatility, the utility sector had a distance to distress of slightly above 2 standard deviations. The subsequent mapping of this distance to distress into probability of default indicated that the aggregated industry had a one-year ahead probability of default equal to 5 percent (Figure 6). In September, when equity price declines were largest and implied asset volatility had reached its peak (Figure 7), the distance to distress for the sector had fallen to a standard deviation of 0.2 at its lowest recorded level and averaged a standard deviation of 0.6 for the entire month. This average standard deviation was equivalent to a one-year ahead probability of default of around 30 percent for the sector as a whole.

The highest profile case of distress within the electric utility sector was Eletropaulo, Latin America’s largest power distributor and a subsidiary of AES. On August 26, 2002, Eletropaulo was placed in selective default by Standard & Poor’s after missing a debt payment and approached holders of R$700 million (US$223 million) in debt with a plan to extend payments by two years. Other instances of weakness in the utility sector were partially resolved by an infusion of government lending or capital infusion by the parent company. In addition to the utility sector, there were other selected high profile cases of corporate defaults that confirm the level of distress indicated in the M*f*Risk model output. For example, BCP Telecommunications, the wireless unit of BellSouth and Safra Group, defaulted on a US$375 million loan in late March. Globopar, Latin America’s largest media company, defaulted on US$1.5 billion worth of debt in October 2002. This decision also pushed one of its subsidiaries, Globo Cabo S.A., into default on nearly US$100 million of debt in early November. Varig, the largest airline carrier in Latin America, agreed with creditors in September to reschedule debt payments after returning some of its leased aircraft and renegotiating more favorable terms on remaining leases.

Outside of some systemic weakness in utilities and media-telecommunications, the Brazilian corporate sector weathered the financial volatility in 2002 relatively well due to, in part, the rapid improvement in the economic environment after the election-related uncertainties passed. There were instances of capital infusion from parent entities or public sector sources, but this process was more the exception rather than the rule and widespread corporate defaults were avoided. In contrast to the Brazil case study, the Asian crisis in 1997–1998 is often cited as an example of bankruptcies of highly leveraged companies that eventually posed systemic vulnerabilities to the financial and public sector. The next section uses the M*f*Risk model to retroactively assess the aggregated CCA on the Thai corporate sector during the Asian crisis.

### C. The Thai Corporate Sector

Strong economic growth, along with a fixed nominal exchange rate and capital account liberalization, contributed to a surge in capital inflows in Thailand and most of South East Asia from 1992–1996. Private sector investment in manufacturing and real estate was complemented by public sector investment, especially in infrastructure relating to transportation, telecommunications, and utilities. Private sector capital inflows into the five most affected economies from the Asian financial crisis increased from US$30 billion in 1992 to US$73 billion in 1996.[24] Unlike in Latin America, the bulk of the capital inflows into Asia came in the form of bank lending, which accounted for US$120 billion out of the total US$234 billion (51 percent) of private capital flows.

Much of the bank lending went to the nonfinancial private sector in the form of short-term U.S. dollar denominated loans. About 65 percent of all external borrowing was conducted by the nonfinancial corporate sector from 1992–1996 (Table 2) and the majority of this debt was contracted at short maturities. Nearly 75 percent of all short-term debt had original maturities of less than six months, with 1–3 months being the most popular time horizon (Table 3). Successive years of a heavy reliance on debt relative to equity financing resulted in a debt-to-equity ratio of nearly 160 percent in 1996, the highest in Asia at that time.[25] Even with increasing quantities of short-term U.S. dollar denominated debt on corporate sector balance sheets, the widespread perception that the peg would remain stable contributed to low hedging positions. While data on overall corporate sector hedging practices are unavailable, studies are available that provide insight into hedging behavior. For example, one study examined 29 large nonfinancial firms and found that 85 percent of the total foreign debt positions were unhedged.[26]

The real estate and asset price bubble came under pressure beginning in 1996 as growth began to slow significantly in the second half of the year. Successive years of capital inflows and an appreciating U.S. dollar after 1994 caused an appreciation in the real exchange rate.[27] A higher real exchange rate combined with decreasing external demand led to a sharp reduction in export growth from 25 percent in 1995 to a 2.0 percent decline in 1996. Manufacturing output, especially in the areas of durable goods and associated inputs, fell sharply. Declines were also registered in beverages and textiles. Overall, measures of capacity utilization in the manufacturing sector declined from 80 percent in 1995 to 76 percent in 1996.

---

[24] IMF (1998). The five economies are Indonesia, Korea, Malaysia, the Philippines, and Thailand. Capital flows are measured as net foreign direct investment plus net portfolio investment plus net other investment.

[25] Pomerleano (2001) shows debt/equity ratios increased from 70 percent in 1992 to 155 percent by end-1995.

[26] SBC Warburg Dillion Read (1997). The level reported is the weighted average of unhedged short-term and long-term debt and should only be viewed as an indicative measure of hedge since firm-by-firm statistics in the sample varied widely. The firms in the sample accounted for US$16 billion in U.S. dollar denominated debt, of which 15 percent was classified as short-term.

[27] Radelet and Sachs (1998) and IMF (2002).

## D. CCA and the Crisis in Thailand in 1997

While the generalized slowdown should have signaled an increase in the probability of an economic adjustment from built-up imbalances, traditional signals of vulnerability did not register much forewarning.[28] However, aggregated CCA provided some indication of weakness prior to the severe financial market volatility that ensued. Equity prices began to decline in early 1996, nearly 18 months before the floating of the Baht, decreasing the distance to distress for many firms. Since much of the accumulated debt during previous years was short-term and dollar denominated, the distress barrier became more binding since firms were vulnerable to both rollover and exchange rate risk.[29] By the end of 1996, the distance to distress had fallen in three of the four main nonfinancial industry sectors. Manufacturing, trade and services, and real estate and construction, all registered lower distance to distress relative to their 1992 positions while agriculture registered a slight increase. Industry level assets relative to distress barriers in 1992 and 1996 are displayed in Figure 8. The distance to distress was lowest in the real estate and construction sectors, reflecting the increasing likelihood of a boom-bust cycle.

Serious FX market pressures began in early 1997 with the release of poor fiscal and export data. Concerns over increased monetization, a deteriorating current account, and nonperforming assets in the financial sector caused foreign investors to begin unwinding carry-trade positions.[30] The default by Samprasong Land in February and rising short-term interest rates in developed economies further fueled capital flight and FX pressures. Extensive intervention by the Bank of Thailand in support of the currency and the imposition of capital controls in May failed to stem capital flight and speculative positions by market participants. The baht was subsequently floated on July 2, 1997.

By the end of July, the baht had depreciated nearly 24 percent and equity prices 20 percent relative to their end-1996 levels. The depreciation substantially raised distress barriers for firms with dollar-denominated liabilities and equity prices continued to reflect a worsening economic outlook and capital outflows. By the end of July, distance to distress indicators showed a disturbing picture for the real estate and construction sectors, and substantial strains on the manufacturing, trade, and service sectors. As shown in Figure 9, standard deviations from the distress barrier were significantly negative for real estate and construction. Manufacturing, trade, and services were all within one-half of one standard deviation from distress.

---

[28] See Radelet and Sachs (1998) for an evaluation of traditional early warning analysis on the Asian crisis countries.

[29] Recall that the distress barrier is the sum of the book value of short-term debt plus one-half of long-term debt and interest. Higher ratios of short-term debt to total debt imply a more stringent barrier.

[30] A carry trade is a short-term position put in place to take advantage of persistent interest rate differentials and a fixed-exchange rate. See IMF (1998, p. 44) for additional details.

Following the initial turbulence, Thailand agreed on a US$17 billion stabilization program with the IMF.[31] However, the currency continued to depreciate and equity prices continued to fall. By the end of October the baht had fallen 58 percent from 25 to 40 B/US$ and the SET Index stood at 447, or 46 percent below its end-1996 value. At these levels, CCA implies a very serious crisis with widespread corporate defaults as distance to distress becomes negative for all corporate sectors except agriculture (Figure 10). The CCA approach effectively predicted widespread corporate defaults after the floating of the baht. According to reports by financial institutions to the Bank of Thailand, around US$5 billion of corporate debt, or 3 percent of GDP, was engaged in restructuring in June of 1998. This amount rose to around US$19 billion, or 16 percent of GDP, in June 1999. Of this total, an estimated two-thirds was held by domestic commercial banks with the balance evenly split between foreign banks and finance companies.

## IV. Multisector Contingent Claims Analysis

While the examples of the Brazilian and Thailand corporate sectors highlights the usefulness of CCA as a tool in risk analysis, the focus on the nonfinancial corporate sector alone may be too narrow to fully assess vulnerabilities. As previously indicated, third party guarantees like implicit government guarantees of the financial system are present or implicit across the balance sheets of the public sector, financial sector, and nonfinancial corporate sector. Therefore, a complete analysis of corporate sector vulnerabilities and potential for macrofinancial risk transfer requires a set of interrelated balance sheets across the corporate, financial, and public sectors (the composition of the consolidated balance sheets is detailed in Appendix II with special emphasis on application of CCA to the public sector). The following sections discuss two examples of application of CCA to a multisector framework. First, the Brazil example illustrates risk transfer from the public sector to the corporate sector balance sheet. Second, the Thailand example illustrates risk transfer from the corporate sector to the balance sheets of the financial and public sectors.

### A. Multisector CCA—Brazil

In the context of Brazil, the multisector CCA framework can be used analyze risk transmission during 2002 between the public sector and the corporate sector since the public sector previously played an active role in providing currency hedges to the corporate sector. The financial market instability during 2002 caused some market participants to call into question the sustainability of the public sector debt and, in turn, the value of the currency hedge provided to the corporate sector. Public sector distress was therefore transmitted to the asset side of the balance sheet of the corporate sector.[32]

---

[31] International Monetary Fund (1998).

[32] To the extent that the corporate sector holds government debt directly, changes in the value of these assets would appear on the balance sheet.

At end-2001, total external debt of the financial and nonfinancial corporate sectors amounted to US$117 billion and external debt service costs for the year were estimated at US$8 billion in interest and US$30 billion of principal. The private sector hedged this foreign currency exposure through government issued dollar-linked debt and FX swaps, intermediated through the financial system.[33] Since the hedge is denominated and settled in domestic currency, the holders of the hedge must still convert *reais* into foreign currency to complete the transaction.

CCA can be used to estimate the convertibility risk faced by holders of domestic currency since they have only a residual claim on the foreign currency assets of the public sector.[34] This claim can be viewed as a call option on government foreign currency assets. The payoff is equal to the maximum of residual FX reserves above one-year ahead external debt service or zero if, (i) FX reserves fall below one-year ahead external debt service costs, or (ii) capital controls are put in place to restrict convertibility.[35] Restrictions on the ability to convert local currency to foreign currency would prohibit the ability of the corporate sector to service external debt.

During the financial market volatility in 2002, access to international capital markets by the public and private sectors dried up and rollover rates on external debt fell. Demand for dollars increased and public sector international reserves fell. According to CCA, the result was an increased probability of convertibility risk as a subset of public sector assets (international reserves) approached an implicit distress barrier (one-year ahead foreign currency liabilities of the public sector). Local markets were not seen as providing an effective hedge partly because all local contracts were settled in the domestic currency. As shown in Figure 11, CCA estimated that the probability of restrictions on currency convertibility rose from 5 percent in March to 30 percent in September.[36] As the perception of convertibility risk increased, some firms either stopped rolling over their FX hedge position or even sold existing hedge in order purchase dollars in the spot market.[37] The decline in the demand for forward hedge coincided with

---

[33] Risk transmission from the corporate sector to the banking sector is relatively mild. Private sector credit as a percent of GDP is only 25 percent and the banking sector intermediates the FX exposure to the private sector. Therefore, net FX exposure remains on the public sector balance sheet as opposed to the banking sector.

[34] This assumes that sovereign external debt payments have first claim on the foreign currency assets of the public sector, thus only residual foreign currency reserves in excess of scheduled external debt payments are available for conversion from domestic to foreign currency holdings.

[35] External debt service costs in 2002 for the nonfinancial public sector were estimated at US$7.4 billion of interest and US$6.5 billion of principal. At end-2001, gross international reserves were US$36 billion.

[36] The convertibility risk measure in Figure 11 is displayed as a monthly average.

[37] The notional value of registered outstanding hedge positions registered on the CETIP, a securities clearing registry, declined from US$50 billion as of end-May 2002 to US$37 billion at end-October. Amounts on CETIP include financial companies and trading within own accounts and serves more as an indication of hedging practices as opposed to actual amounts.

prepayments and capital outflows of nearly US$7 billion during the same time period.[38] The use of CCA to estimate convertibility risk appears to explain some of the rationale behind the capital outflows and corporate sector behavior during the second half of 2002.

The Brazilian government took many steps to address the market uncertainty, focusing on a set of core policies that maintained discipline and restored market confidence. The government increased the primary surplus while tax and pension reform were given priority in the reform agenda. Consistent policy implementation and improving fundamentals led to a rapid normalization of financial markets. For example, nine months after external access was halted and six months following the peak of financial market turbulence, the sovereign reaccessed external capital markets in April 2003. Widespread access by the corporate sector to external markets returned shortly thereafter. As uncertainties subsided, both convertibility risk and capital outflows declined rapidly.

### B. Multisector CCA—Thailand

In the context of Thailand and the Asian crisis, the multisector CCA framework can be used to analyze risk transmission from the corporate sector to the financial sector to the public sector. The size of the risk transfer across the balance sheets is captured in two steps. First, equity of the financial sector is modeled as a call option on total financial sector assets, yielding estimates of changes in the market value of financial sector assets over time. Second, the financial guarantee from the public sector is modeled as a put option based on the derived market value of financial sector assets. The value of the put option also requires an assumption over recovery rates, which are normally less than full. For this analysis, a recovery rate of 80 percent was assumed for the banking sector and 60 percent for the non-bank financing companies. Based on this structure, declines in the value of financial sector equity from the depreciation of the baht and increases in loan delinquency rates cause the market value of assets to decline, increasing probability of default. As the probability of default rises, the value of the government guarantee adjusted for recovery rates increases.

As discussed above and displayed in Figures 9 and 10, distance to distress indicators for the industry sectors were low or negative by end-July 1997, especially in real estate and construction. By the end of October, corporate sector distance to distress measures were negative for all sectors except for agriculture, implying a high probability of widespread corporate defaults. Corporate sector weakness was transferred to the balance sheet of the financial sector, reflected in lower equity prices, increased asset volatility, and decreasing distance to distress. As shown in Figure 12, distance to distress for the financing companies was already significantly negative in July 1997. In August 1997, around the same time Thailand agreed on the stabilization program with the IMF, the Bank of Thailand suspended operations in 42 financial companies. By October, the distance to distress for both commercial banks and financial companies were

[38] Capital outflows are registered flows through so-called CC5 accounts and were obtained from the Central Bank of Brazil.

negative. The problems in the balance sheets of the financial companies became so severe that the Bank of Thailand closed 56 finance companies in December 1997.

The decreasing market value of financial sector assets relative to the distress barrier increased the value of the financial sector guarantee. Figure 13 plots the evolution of the one-year ahead estimate of the value of the financial sector guarantee throughout 1997. The value is listed as a percent of 1996 nominal GDP for scale purposes. As shown in the figure, the value of the guarantee was relatively small during early 1997, but had already increased to nearly 10 percent of GDP prior to the float, representing the building vulnerabilities on the balance sheets of the financial companies in particular. After the floating of the baht in July and subsequent turbulence, CCA estimated the value of the financial sector guarantee at between 30 and 40 percent of GDP by October 1997.

Subsequent restructurings in the financial sector indicate that the one-year ahead estimated value of the financial sector guarantee was relatively accurate. In October 1998, reports by private sector specialists estimated that 45 percent of loans were nonperforming and that loan losses amounted to 27 percent of GDP or 350 percent of financial sector capital.[39] The slow pace of the restructuring process meant that the actual number of restructured loans was not known until several years later. Fortunately, the Bank of Thailand has published extensive data on the debt restructuring carried out by financial institutions after the 1997 financial market crisis.[40] The database includes number of cases and amounts restructured in total and by type of loan. Based on this data, financial sector debt equivalent to 23 percent of GDP had completed the restructuring process by end-1999. By end-2000, this value had reached 40 percent of GDP. Both numbers roughly correspond to model estimates.

## V. Conclusions

In this paper, we have examined the application of the contingent claims approach to identifying corporate sector and macrofinancial vulnerabilities. In doing so, we applied the Moody's M*f*Risk model, which uses aggregated CCA principles, to assess vulnerabilities retroactively in the corporate sector as well as in a multisector setting using two historical cases. The results presented here indicate that the method holds the promise of identifying corporate sector vulnerabilities and estimating the associated value of macrofinancial risk transfer across interrelated balance sheets of the corporate, financial, and public sectors.

The main output of the CCA is an estimated probability of default that is a function of the capital structure of the balance sheet, the volatility of asset returns, and the current asset value. Since the information contained in the firm's balance sheet and equity price can be translated into a probability of default, the CCA has underpinned the development of credit-risk and risk-

[39] Armstrong and Spencer (1998).

[40] See the website of the Bank of Thailand *http://www.bot.or.th/* and Dasri (2000) for additional information.

management techniques. In particular, the CCA is used by (i) major credit rating agencies to monitor and assign credit ratings, (ii) financial institutions to inform interest rate pricing on loans and set adequate levels of regulatory capital, and (iii) investment banks and insurance companies to assess value-at-risk. For example, when applied across a portfolio of firms, the probability of default multiplied by weighting within the portfolio creates a value-at-risk (VaR) indicator that is then used in conjunction with other VaR indicators to adjust capital adequacy or allow the firm to offset risk exposure by entering into offsetting financial transactions.

Although the CCA has most often been applied to individual firm balance sheets, this paper has used the Moody's M*f*Risk model to demonstrate how the CCA also applies to aggregated sector balance sheets or across balance sheets in a multisector framework. The evolution of the CCA into a multisector framework incorporates the potential for risk transfer across balance sheets, providing an additional avenue for vulnerability analysis. The conclusions presented here show that such analysis is warranted given (i) a full or partial financial guarantee from the government to the financial sector, (ii) government provision of other guarantees, such as currency hedges to the corporate sector, (iii) the presence of a high degree of correlation across firm balance sheets (the *chaebol* structure in Korea is an example of the potential for interconnectivity across firm balance sheets), and (iv) an environment of rapid capital flows where the effect of a currency crisis can quickly be transmitted between the balance sheets of the sovereign, financial, and corporate sectors.

## A. Advantages of CCA

The advantages of the contingent claims methodology are numerous, and only some of these are discussed here. The main advantage of the CCA is that it uses observable balance sheet and financial market data along with volatility to construct a measure of default risk. The ability to translate continuously adjusting financial market price information into current market value estimates of asset value is especially important given the speed with which economic conditions change relative to the time span between releases of consolidated-accounting-balance-sheet information. Furthermore, balance sheet information arrives with a significant lag, usually 90 days after the quarter or annual ending period. The CCA combines the capital structure of the balance sheet with current price information from financial markets to construct a market value estimate of the current balance sheet along with forward looking indicators of vulnerability. In addition, the CCA distinguishes itself from other vulnerability analysis by recognizing the important role that volatility has in determining default probabilities. Increases in volatility increase the option value and benefits equity holders at the expense of bondholders. By capturing volatility, the CCA accounts for the fact that firms with same capital structures may have different distance to distress and default probabilities.

Although not explicitly detailed in this paper, the contingent claims methodology incorporates nonlinearities which yield significant improvements over traditional linear relationships in vulnerability analysis. In option-pricing theory, the value of the option is dependent on changes in the underlying asset. The rate of change of the price of the option relative to changes in the underlying asset is referred to as the *delta.* The equivalent measure in bond pricing is *duration*—how much the bond price changes in response to a change in interest rates. However, the duration measure is only accurate for small changes in the interest rate. In fact, duration changes as the interest rate changes, and this measure is referred to as *convexity*. The same is true of option prices. The option delta is only accurate over small changes in asset price. The equivalent measure to convexity in option pricing is called the option *gamma*, or the rate at which the option delta changes as the price of the underlying asset changes. Therefore, the nonlinearity of the Black-Scholes methodology allows for a more accurate description of changes in vulnerabilities from large changes in asset prices just as both duration and convexity are needed to compute accurate changes in bond prices from large movements in interest rates.[41] Linear relationships that ignore higher orders could dramatically underestimate potential vulnerability and, consequently, fail to be adequate indicators for surveillance purposes.

### B. Hurdles to Overcome

There are several hurdles to overcome when implementing the CCA methodology, but most can be mitigated. First, the CCA uses on-balance-sheet information to construct the capital structure of the balance sheet, and off-balance-sheet items, such as hedging or derivative positions may not be included. This fact, however, is not a disadvantage of the CCA alone—it is a shortcoming of all vulnerability indicators that rely on balance-sheet information. Rather, the CCA relies heavily on financial markets to synthesize the current economic situation and its potential impact on the balance sheet and reflects this in current equity or junior claim prices. Thus, with respect to off-balance-sheet positions, the CCA is implicitly assuming that financial markets are the best source for information regarding the current value of a firm. This is not an unrealistic assumption since industry analysts, equity specialists, and other local-market participants are likely to be in the best position to assess current trends and developments. However, the CCA does not assume that market forecasts are always correct. Financial markets can be taken by surprises and the degree to which this happens limits the ability of the CCA to accurately forecast vulnerabilities.

The CCA option-pricing relationships derive a risk-neutral default probability based on cumulative normal distributions that must be mapped into actual expected default probability. The use of risk-neutral default probabilities generally overstates the actual probability of default, requiring a mapping to estimated actual probability of default based on historical data. Fortunately, Moody’s KMV has been able to use historical observations of public and private company defaults to map risk-neutral default probabilities into actual expected default probabilities. The resulting distribution has slightly fatter tails than a cumulative normal distribution. The mapping, however, is mainly representative of U.S. firms, although it has been successfully used in a wide variety of applications. Some emerging markets have poor-quality information and many unlisted firms. Moody's has tried to address this problem with a blended approach using an option model and financial data (Riskcalc). Comparative studies have shown that the CCA-related models are much more accurate than Z-score accounting-ratio models or

[41] Advances in option-pricing methodology also include the option *vega* which can allow for changing volatility of the underlying asset. This feature is important since volatility of assets often increases significantly during periods of financial stress. Traditional linear relationships are, therefore, significantly inferior to their nonlinear counterparts.

accounting ratios in predicting default (around 70 percent accuracy ratios for the CCA-type models and about 50 to 56 percent for the Z-score and accounting-ratio models).[42]

Finally, application of the CCA to the financial sector and public sector balance sheets is still a relatively new phenomenon. Credit risk within financial institutions is often difficult to disentangle given the inherent problems in analyzing and pricing illiquid loan portfolios and understanding true liability positions. However, market analysts understand these issues well, and preliminary results from this analysis and Moody’s KMV suggest that the CCA does well in assessing vulnerabilities within the financial sector. With respect to applying the CCA to the public sector balance sheet, occurrences of sovereign defaults are few in number, making testing and calibrating of the model on sovereign balance sheets difficult. Moody's-MfRisk has overcome this problem by correlating the public sector risk indicators with credit spreads instead of a pool of defaults, allowing for calculation of sovereign spreads and default probabilities for a variety of different scenarios. Therefore, although application of the CCA to the public sector is new, early results indicate that this application holds promise.

### C. Implications for Macroeconomic Risk Management

At a minimum, application of the CCA in a multisector setting allows for the ability to value the potential for risk transfers across balance sheets and, in particular, the ability to value the public sector guarantee to the financial sector, as is shown for the case of Thailand. However, just as the CCA is used by the private sector to analyze risk and implement risk-management strategies, the CCA applied in a multisector framework provides policymakers with the same ability at the public sector level. The multisector setting in the Moody’s M*f*Risk model provides an interconnected framework within which policymakers can analyze potential policy mixes and evaluate which may be more adept at countering vulnerabilities.

Policymakers can follow several strategies in order to transfer or mitigate risk, including (i) a direct change in the balance sheet through policy action (e.g., increasing the primary surplus increases the assets of the public sector relative to the distress barrier), (ii) managing implicit or explicit guarantees (e.g., providing or withdrawing credit guarantees to the banking sector or a currency hedge to the corporate sector), (iii) risk transfer strategies (e.g., signing contracts with foreign banks or insurance companies), (iv) institutional changes in markets (e.g., capital requirements, Basel Core Principles, payment systems, and mark-to-market requirements), and (v) mitigating risk by diversification, hedging or insurance. Any of these strategies, either individually or in a combination can be analyzed in a multisector framework to manage risk in a dynamic economy. Although this is not the subject of this paper, we suggest it as a useful avenue for further study.

---

[42] Sobehart and Stein (2000).

Table 1.
Brazil: Non-financial Companies, Leverage and Debt Structure, December 2001

| Sector | Debt/Equity | Liabilities/Equity | Current Liab/Liabilities | ST Debt/Total Debt | FX Debt/Total Debt | ST FX Debt/ST Debt | LT FX Debt/LT Debt | |
|---|---|---|---|---|---|---|---|---|
| Basic & Fab Metal | 1.50 | 2.44 | 0.39 | 0.43 | 0.76 | 0.76 | 0.75 | |
| Chemical | 0.75 | 1.74 | 0.48 | 0.32 | 0.62 | 0.81 | 0.52 | |
| Construction | 0.64 | 1.84 | 0.42 | 0.66 | 0.15 | 0.00 | 0.44 | |
| Electric Electron | 0.75 | 2.27 | 0.59 | 0.72 | 0.63 | 0.70 | 0.47 | |
| Electric Power | 0.58 | 1.15 | 0.33 | 0.23 | 0.70 | 0.48 | 0.76 | |
| Food & Beverage | 1.34 | 2.39 | 0.50 | 0.48 | 0.61 | 0.61 | 0.61 | |
| Industrial Machin | 0.61 | 1.35 | 0.70 | 0.68 | 0.61 | 0.75 | 0.34 | |
| Mining | 0.75 | 1.18 | 0.38 | 0.32 | 0.93 | 0.87 | 0.96 | |
| Oil & Gas | 0.34 | 1.51 | 0.60 | 0.52 | 0.63 | 0.89 | 0.35 | |
| Pulp & paper | 1.09 | 1.47 | 0.43 | 0.42 | 0.78 | 1.76 | 0.07 | |
| Telecommunication | 0.55 | 1.10 | 0.45 | 0.25 | 0.61 | 1.23 | 0.40 | |
| Textile | 0.66 | 1.35 | 0.56 | 0.58 | 0.44 | 0.55 | 0.30 | |
| Trade | 1.00 | 2.59 | 0.69 | 0.49 | 0.72 | 0.75 | 0.69 | |
| Transportat Serv | -41.22 | -82.84 | 0.37 | 0.21 | 0.29 | 0.13 | 0.33 | 1/ |
| Vehicle & parts | 1.05 | 2.51 | 0.70 | 0.66 | 0.64 | 0.49 | 0.91 | |

Sources: Economatica; and IMF staff estimates.

1/ This value was the result of a large loss incurred on a small negative shareholders equity.

## Table 2. Thailand: Gross External Borrowing by Nonbank Sector 1992–1996[1]

| | 1992 | | 1993 | | 1994 | | 1995 | | 1996 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Short-Term | Total | Short-Term | Total | Short-Term | Total | Short-Term | Total | Short-Term | Total |
| Total | 14,870 | 17,730 | 16,076 | 18,816 | 14,757 | 18,061 | 16,606 | 22,064 | 18,228 | 25,943 |
| Non-bank financial institutions | 4,632 | 4,807 | 6,578 | 6,900 | 6,169 | 6,569 | 8,528 | 9,685 | 7,338 | 8,745 |
| Non-financial sector | 10,238 | 12,923 | 9,498 | 11,916 | 8,588 | 11,492 | 8,078 | 12,379 | 10,890 | 17,198 |
| Trade | 3,832 | 3,982 | 4,099 | 4,264 | 3,251 | 3,394 | 3,147 | 3,460 | 3,806 | 4,148 |
| Construction | 151 | 199 | 156 | 194 | 91 | 185 | 110 | 145 | 200 | 791 |
| Industry | 5,195 | 7,035 | 4,478 | 6,167 | 2,917 | 4,414 | 3,331 | 6,174 | 4,178 | 6,216 |
| Food | 348 | 506 | 173 | 262 | 180 | 286 | 87 | 352 | 84 | 414 |
| Textiles | 257 | 362 | 302 | 584 | 237 | 309 | 145 | 282 | 125 | 252 |
| Metals | 388 | 531 | 447 | 592 | 296 | 472 | 316 | 523 | 235 | 492 |
| Electrical appliances | 1,425 | 1,692 | 1,139 | 1,319 | 619 | 744 | 1,101 | 1,402 | 1,172 | 1,548 |
| Machinery and transport | 2,025 | 2,209 | 1,392 | 1,488 | 1,054 | 1,099 | 937 | 1,079 | 1,990 | 2,015 |
| Chemicals | 257 | 357 | 313 | 604 | 221 | 374 | 237 | 922 | 264 | 906 |
| Petroleum products | 145 | 595 | 515 | 634 | 123 | 528 | 219 | 562 | 107 | 292 |
| Others | 350 | 783 | 197 | 684 | 187 | 602 | 289 | 1,052 | 201 | 297 |
| Services 2/ | 111 | 162 | 152 | 215 | 83 | 190 | 157 | 504 | 294 | 623 |
| Others | 949 | 1,545 | 613 | 1,076 | 2,246 | 3,309 | 1,333 | 2,096 | 2,412 | 5,420 |

Source: Reprinted from IMF (2002). Data provided by the Thai authorities.
1/ Includes borrowing from affiliates; excludes commercial banks and BIBFs.
2/ Excludes real estate.

## Table 3. Thailand: Borrowing Terms of Private External Loans, 1992–1996[1]

| | 1992 | | 1993 | | 1994 | | 1995 | | 1996 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Amount of Borrowing | | Amount of Borrowing | | Amount of Borrowing | | Amount of Borrowing | | Amount of Borrowing | |
| | Millions US$ | Percent of Total | Millions US$ | Percent of Total | Millions US$ | Percent of Total | Millions US$ | Percent of Total | Millions US$ | Percent of Total |
| Total | 17,730 | 100.0 | 18,816 | 100.0 | 18,061 | 100.0 | 22,064 | 100.0 | 25,943 | 100.0 |
| Short-term | 14,870 | 83.9 | 16,076 | 85.4 | 14,757 | 81.7 | 16,606 | 75.3 | 18,228 | 70.3 |
| < 1 month | 375 | 2.1 | 721 | 3.8 | 2,698 | 14.9 | 4,486 | 20.3 | 4,546 | 17.5 |
| 1-3 months | 10,922 | 61.6 | 11,051 | 58.7 | 9,200 | 50.9 | 8,495 | 38.5 | 9,604 | 37.0 |
| 4-6 months | 2,605 | 14.7 | 3,481 | 18.5 | 1,727 | 9.6 | 1,980 | 9.0 | 2,522 | 9.7 |
| 7-11 months | 59 | 0.3 | 25 | 0.1 | 54 | 0.3 | 61 | 0.3 | 216 | 0.8 |
| 12 months | 909 | 5.1 | 798 | 4.2 | 1,078 | 6.0 | 1,584 | 7.2 | 1,340 | 5.2 |
| Long-term | 2,860 | 16.1 | 2,740 | 14.6 | 3,304 | 18.3 | 5,458 | 24.7 | 7,715 | 29.7 |
| No age | 237 | 1.3 | 263 | 1.4 | 276 | 1.5 | 621 | 2.8 | 892 | 3.4 |
| More than 1-3 years | 689 | 3.9 | 809 | 4.3 | 1,224 | 6.8 | 1,893 | 8.6 | 2,994 | 11.5 |
| More than 3-5 years | 748 | 4.2 | 619 | 3.3 | 818 | 4.5 | 1,461 | 6.6 | 1,994 | 7.7 |
| More than 5-10 years | 1,097 | 6.2 | 725 | 3.9 | 863 | 4.8 | 1,002 | 4.5 | 1,243 | 4.8 |
| More than 10 years | 89 | 0.5 | 324 | 1.7 | 123 | 0.7 | 481 | 2.2 | 592 | 2.3 |

Source: Reprinted from IMF(2002). Data provided by the Thai authorities.
1/ Includes borrowings from affiliates; excludes commercial banks and BIBFs.

Figure 1.  Distance to Distress

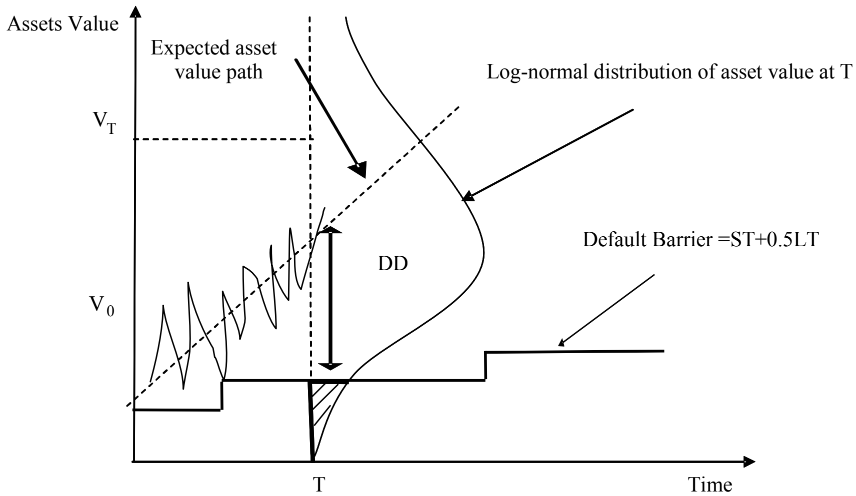



Figure 2. Brazil: Corporate Sector Leverage Indicators

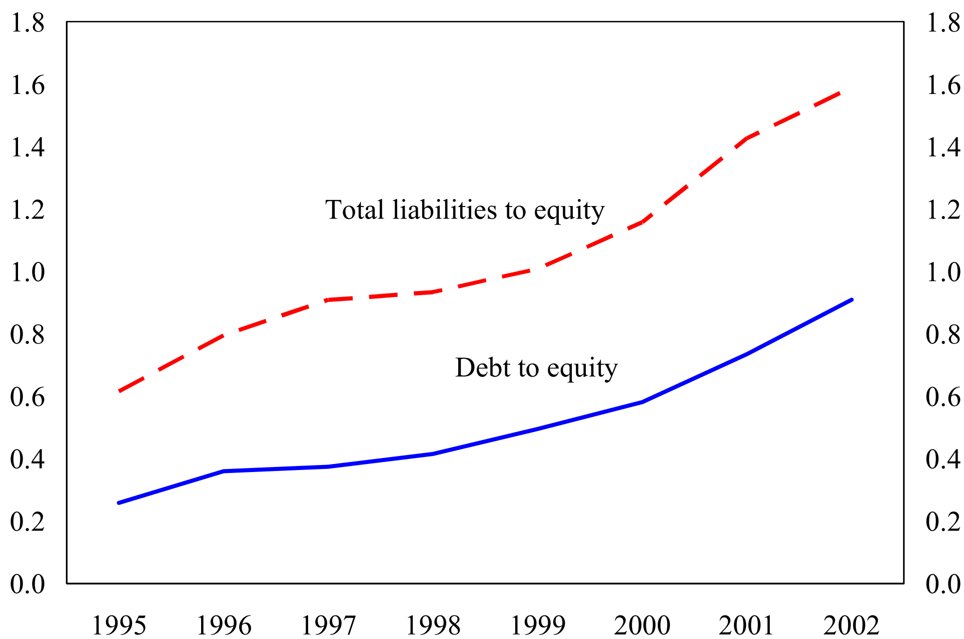


Figure 3. Brazil: Distance to Distress by Sector in March 2002

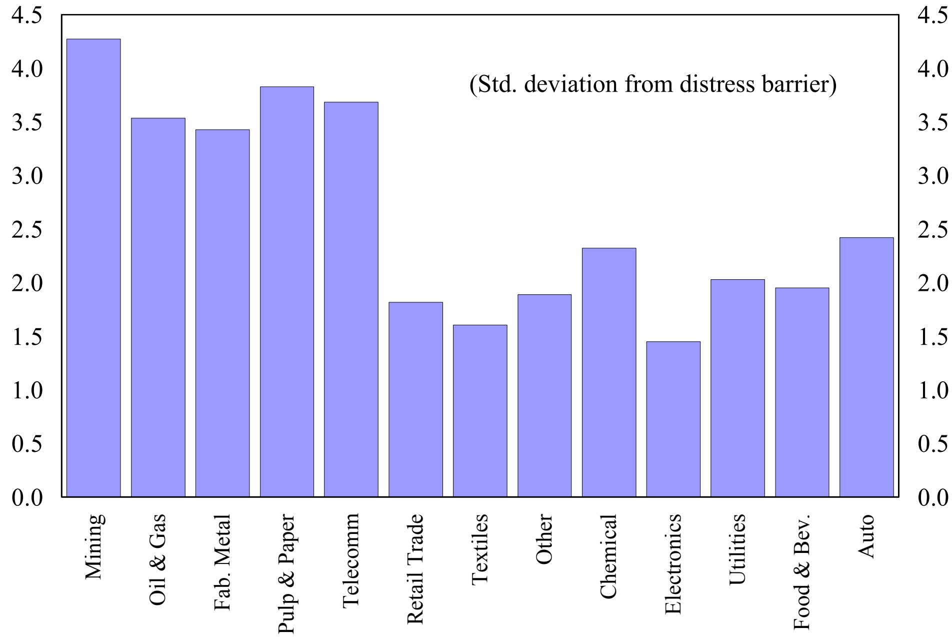



Figure 4. Brazil: Distance to Distress by Sector in September 2002

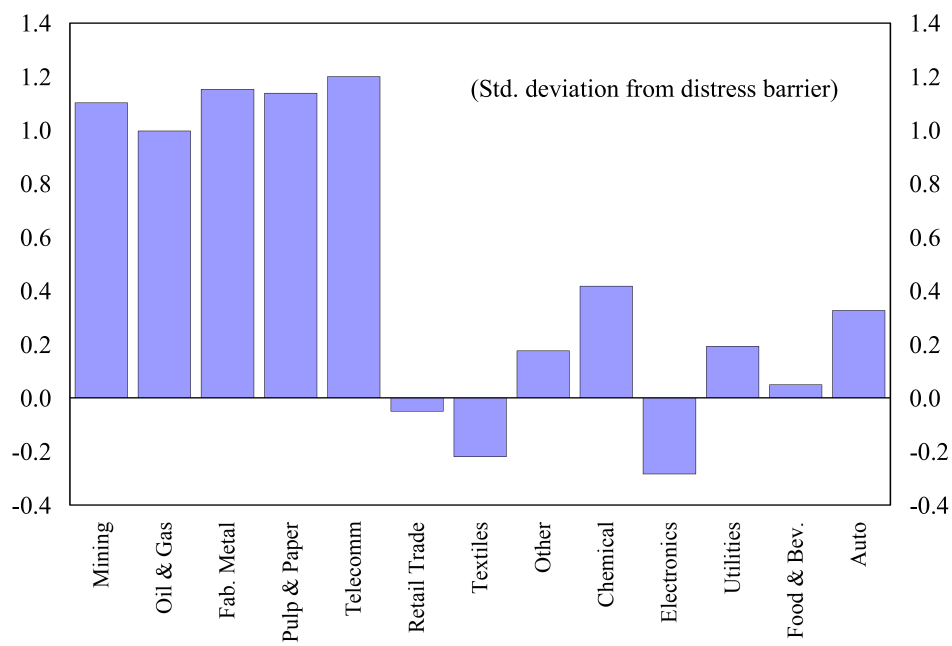


Figure 5: Brazil Utility Sector: Assets Relative to Distress Barrier

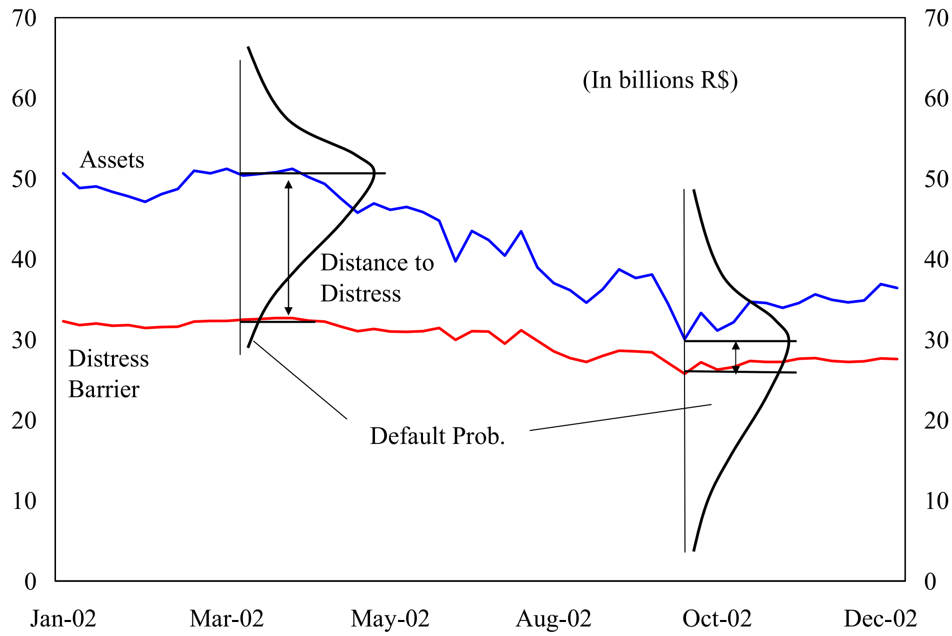



Figure 6. Estimated Actual Default Probability Versus Distance to Distress
Utility Sector

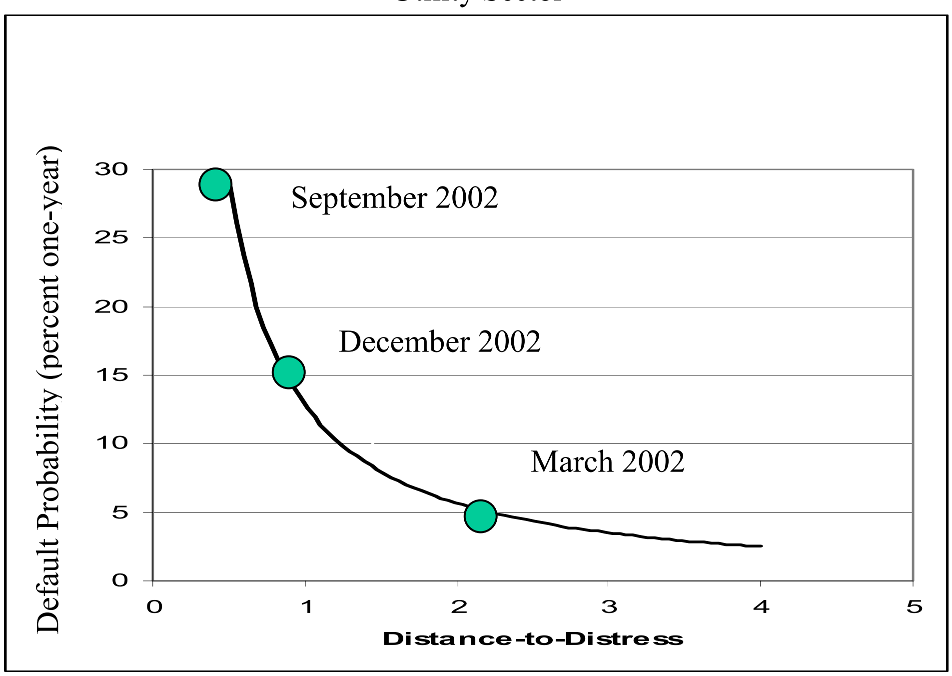


Figure 7. Brazil Utility Sector: Implied Asset Volatility

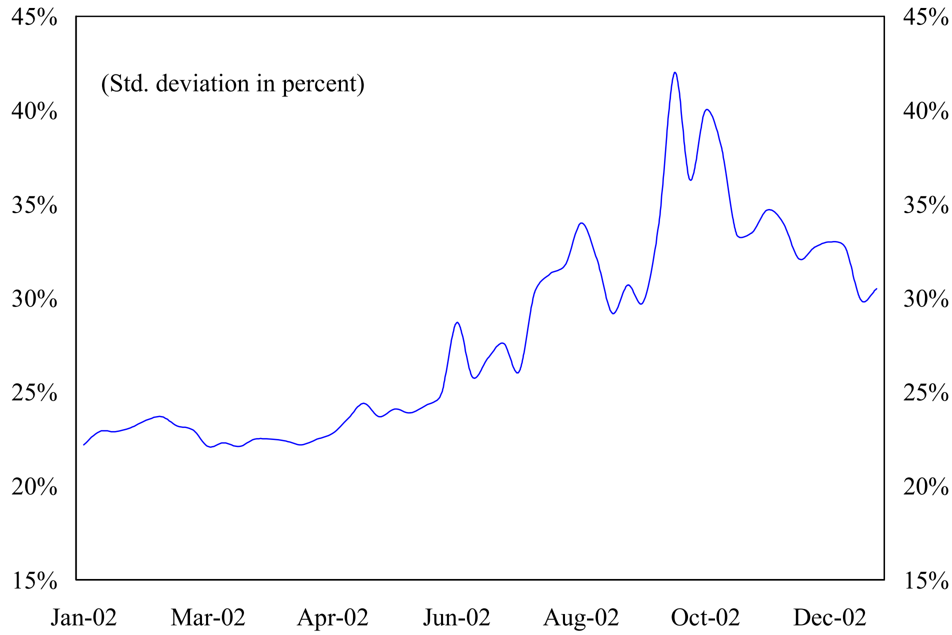



Figure 8. Thailand: Assets Minus Distress Barrier: 1992 vs. 1996

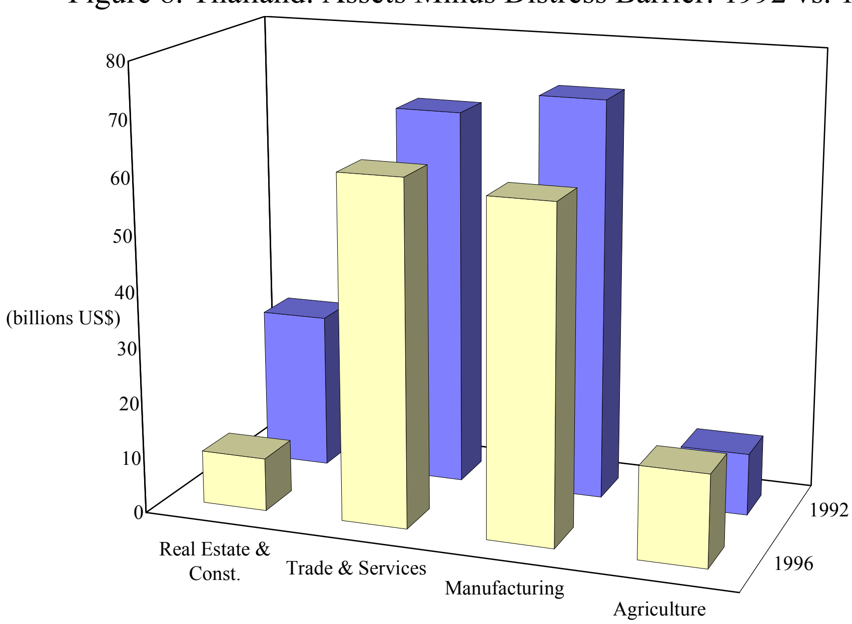


Figure 9. Thailand: Distance to Distress by Sector: July 1997


Figure 10. Thailand: Distance to Distress by Sector: October 1997

Figure 11. Brazil: Convertibility Risk and Capital Outflows through CC5 Accounts

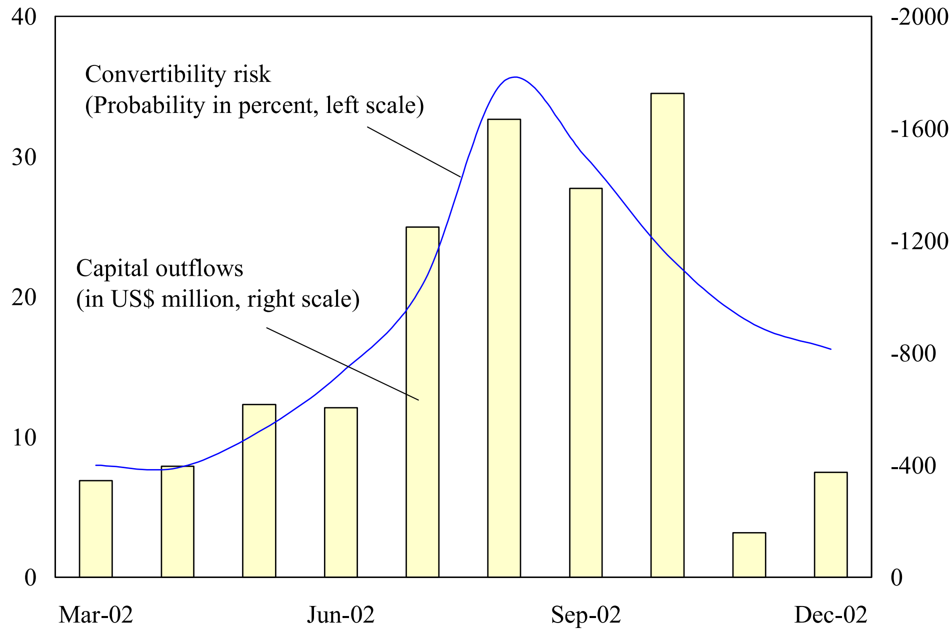



Figure 12. Thailand: Financial Sector Distance to Distress 1997

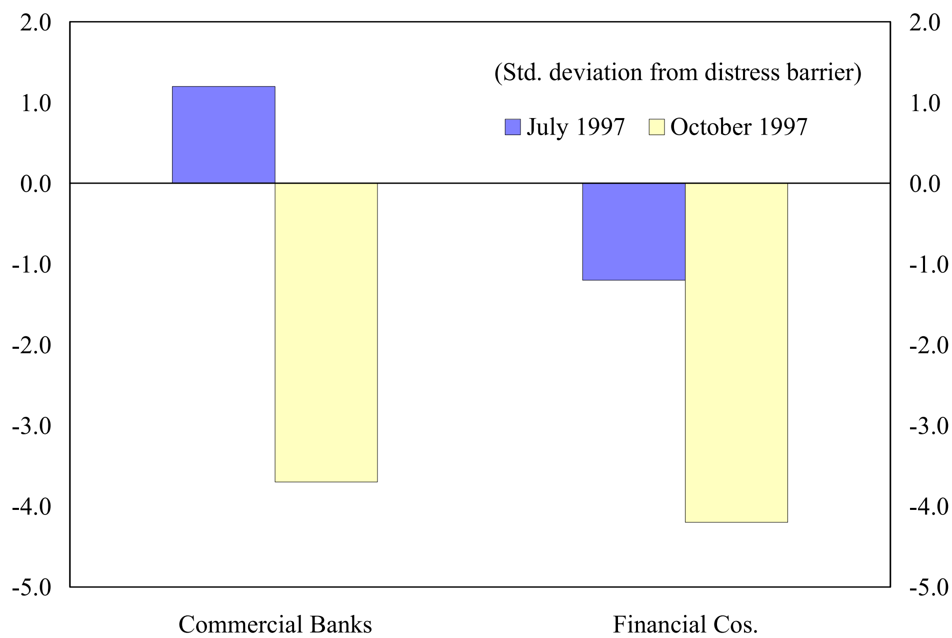
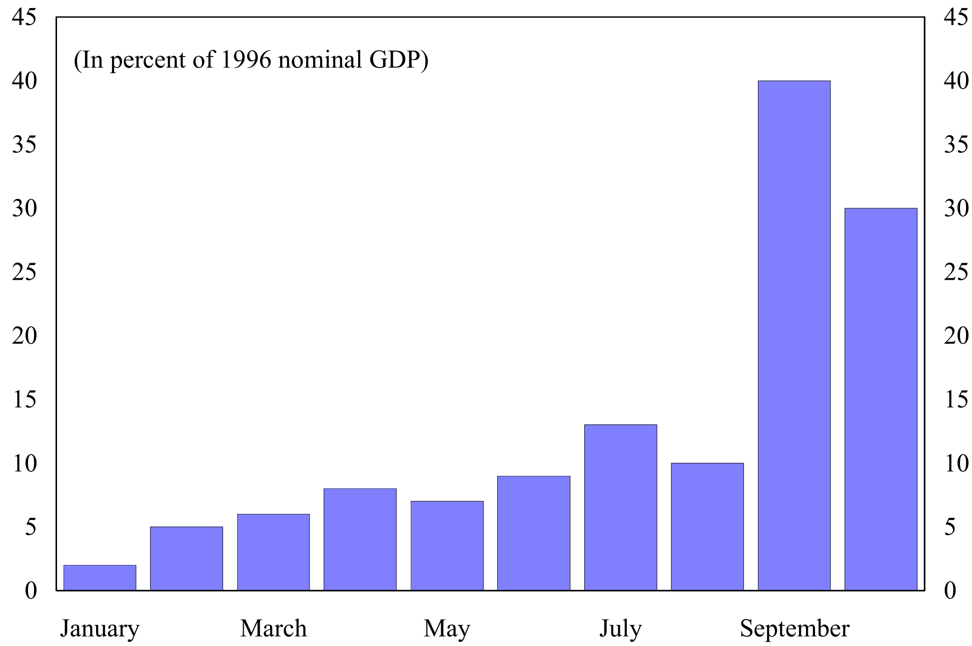


Figure 13. Thailand: Value of the Financial Sector Guarantee 1997



**BLACK-SCHOLES OPTION PRICING FORMULA IN CONTINGENT CLAIMS ANALYSIS**

The contingent claims approach (CCA) defines fundamental relationships between the value of assets and the value of claims. CCA can be applied to simplified balance sheets with liabilities composed of senior debt and junior equity, or used with more complicated balance sheets that include senior collateralized debt and other classes of junior claims, such as convertible securities and preferred equity. The purpose of CCA is to analyze how the value of these claims on firm assets change as the value of the firm changes through time. Here, we consider the simplified balance sheet of senior debt and junior equity and discuss the application of the Black-Scholes option pricing formula.

## A. Assets and Liabilities as Implicit Options

The total market value of assets, $A$, of a firm financed with debt, $D$, and equity, $E$, is equal to the market value of equity plus market value of risky debt. Asset value is derived from the stochastic discounted present value of income minus expenditures with the potential for asset value to decline below the point where scheduled debt payments can be made. If assets fall to or below this level, then default is the result. This level is often referred to as distress barrier, $DB$ and represents the default-free value of debt.

The holders of equity are holders of a junior claim and have a contingent claim on the residual value of assets in the future. In this manner, the value of equity can be viewed as an option where holders of equity receive the maximum of either assets minus debt, or nothing in the case of default. The value of equity, therefore, is,

$$E = max\,[\,A - DB, 0]. \qquad (\text{A1})$$

The case of risky debt, however, is slightly more complex. Holders of debt are obligated to absorb losses if there is a default and the guarantee of repayment by the lender can be modeled as an implicit put option (i.e., in the event of default, the bondholders have a right to sell the remaining assets of the firm). Thus, holders of risky debt receive either the default-free value or, in the event of default, the senior claim on assets. Beginning with the relationship between risky and default-free debt, this can be modeled as,

Value of default-free debt = Value of risky debt + Value of the guarantee, ( A2 )

Value of risky debt = value of default-free debt – implicit put option ( A3 )

Since the value of default-free debt is the distress barrier and the implicit put option on the assets of the firm yields $max\,[\,DB - A, 0\,]$, the market value of risky debt can be modeled as,

$$D = min\,[\,A, DB\,] = DB - max\,[\,DB - A, 0\,]. \qquad (\text{A4})$$

Inserting these option pricing relationships into the economic balance sheet identity results in a value of firm assets at time *t* of,

$$A = D + E, \qquad (\text{A5})$$

$$A = DB - max\,[\,DB - A, 0\,] + max\,[\,A - DB, 0\,]. \qquad (\text{A6})$$

## B. Black-Scholes Formula

Given that assets and liabilities in the firm balance sheet can be related using implicit options, the standard option pricing formula can be used to price these relationships. The main insight behind the methodology of the Black-Scholes formula is that the value of the option can be derived by forming a riskless hedge portfolio.[43] A riskless portfolio is created consisting of a position in a derivative security and a position in a stock. The risk-free nature of the portfolio is derived from the fact that both the derivative and stock price are affected by the same underlying source of uncertainty. Over any short period of time, the two must be perfectly correlated. If the appropriate positions are established, the gain (loss) from the stock position always offsets the loss (gain) from the derivative security, so that the overall value of the position at the end of the period is known with certainty. Therefore, the return of the hedge portfolio is equal to the risk-free rate of interest. By forming a riskless hedge portfolio, the derivation relies primarily on no-arbitrage principles as opposed to equilibrium relationships. The return must be equal to the risk-free rate over the holding period.

Using the Black-Scholes formula, the value of equity as a call option on firm assets is,

$$E = AN(d_1) - DBe^{-rT} N(d_2), \qquad (\text{A7})$$

where *A* is the value of the assets, *E* is the value of equity, *DB* is the distress barrier or value of default-free debt, *r* is the risk-free rate of interest, *T* is the time to maturity on the default-free bond in years. *N(d)* is the cumulative probability distribution function for a standard normal variable (i.e., the probability that a random draw from a standard normal distribution will be below *d*) where,

---

[43] For readers interested in a more explicit derivation of the Black-Scholes option pricing formula, see Black and Scholes (1973), and Merton (1973, 1974). While the derivations in these studies use continuous-time mathematics, Hull (1993) and Baxter and Rennie (1996) detail how binomial models can be used to develop discrete-time representations.

$$d_1 = \frac{\ln\left(\frac{A}{DB}\right) + \left(r + \frac{1}{2}\sigma_A{}^2\right)T}{\sigma_A\sqrt{T}}, \qquad (A8)$$
$$d_2 = d_1 - \sigma_A\sqrt{T},$$

and $\sigma_A$ is the standard deviation of return on firm assets. Following a similar process, the value of the implicit put option from the guarantee on debt is,

$$P = DBe^{-rT}N(-d_2) - AN(-d_1), \qquad (A9)$$

where $P$ is the put option and remaining variables are defined as above. The implied market value of risky firm debt is the value of the implicit put option plus the distress barrier. The value of the put option can also be viewed as the expected loss if default occurs. In this way, the credit spread is the risk premium required by the bondholders to compensate for the expected loss. That is,

$$Spread = -\frac{1}{T}\ln\left[N(d_2) + \frac{A}{DBe^{-rT}}N(-d_1)\right]. \qquad (A10)$$

Each of the Black-Scholes formulas above contain two unknowns, firm assets and volatility of firm assets. The relationship between volatility of firm assets and volatility of equity is given by,

$$E = \frac{\sigma_A}{\sigma_E}AN(d_1), \qquad (A11)$$

where $\sigma_E$ is the standard deviation of equity.[44] Here, $N(d_1)$ is the change in the price of equity with respect to a change in the underlying assets of the firm, or $\partial E/\partial A$. This ratio is also referred to as the option *delta*. However, the main implication of the above relationship is that the standard deviation of equity can be derived from historical data and used to solve for asset volatility. If historical data is not viewed as a good predictor of future asset volatility and if local derivative markets were well-developed, then an alternative method would be to use individual firm equity option data to derive the implied volatility of equity. In addition, option data on an overall equity market index and firm or sectoral betas could be combined to derive implied equity volatility at the firm or sectoral levels.

[44] See Hull (1993, pp. 38).

## C. Example of Distance to Distress and Probability of Default

This section provides a brief numerical example of estimating distance to distress and probability of default. Assume that using the Black-Scholes option pricing formula and the volatility of firm assets, a firm can be characterized by the following,

Asset Value = 1000,
Annualized Asset Volatility = $\sigma_A$ = 0.36,
Distress barrier = 600,
Risk-free Rate = 0.05,

Distance to distress expressed in relation to the standard deviation of firm assets over the time horizon $T$ is measured through evaluation of $d_2$,

$$\text{Distance to distress} = \frac{\ln\left(\frac{1000}{600}\right)+\left(0.05-\frac{0.13}{2}\right)}{0.36}=1.4\,. \qquad (\text{A12})$$

Thus, the distance to distress over a one-year horizon for a firm with these characteristics is 1.4 standard deviations of firm asset value from the distress barrier. Converting this measure to a probability of default requires calculating the cumulative normal distribution function, $N(.)$. This can be done using numerical methods or polynomial approximation. Tables that compute $N(.)$ are also found in many financial and econometric texts. Using one of these methods will yield the probability of default as,

$$\text{Probability of Default} = N(-1.4) = 0.08 \text{ or } 8 \text{ percent.} \qquad (\text{A13})$$

As discussed in the text, however, the probability of default as calculated above relies on risk-neutral pricing and generally overstates the true probability of default. Moody's KMV uses actual historical data on corporate defaults to map the distance to distress in (5) to an Estimated Default Frequency™ (EDF). Nevertheless, the example provided here is instructive of the procedure as a whole.

## D. Properties of Black-Scholes

When firm assets become very large, both $d_1$ and $d_2$ also become very large, meaning $N(d_1)$ and $N(d_2)$ are close to 1. When this happens, equity behaves more like a forward contract with delivery price equal to the distress barrier. The price of equity becomes,

$$E = A - DBe^{-rT}. \qquad (\text{A14})$$

As firm assets become very large, the price of the implicit put option from the guarantee on debt approaches zero since $N(-d_1)$ and $N(-d_2)$ both approach zero. This is merely a recognition of the fact that risky debt approaches default-free debt as assets of the firm continue to grow. The likelihood that default will occur, necessitating bondholders to exercise the put option on firm assets, becomes very low. Conversely, as firm assets decline, both $d_1$ and $d_2$ also decline, meaning $N(d_1)$ and $N(d_2)$ approach zero and equity becomes increasingly worthless. At the same time, however, the implicit put option from the guarantee on debt becomes more valuable since $N(-d_1)$ and $N(-d_2)$ both approach 1.

As volatility of firm assets approaches zero, both $d_1$ and $d_2$ become very large and $N(d_1)$ and $N(d_2)$ approach 1. As volatility declines to zero, the firm looks more like a riskless asset and the value of equity looks more like a forward contract, as was the case under very large firm assets. As firm asset volatility rises, the implicit put option on debt becomes more valuable since the probability that firm assets fall below the distress barrier increases. Since firm assets are fewer standard deviations away from the distress barrier, the distance to distress decreases, making equity less valuable and the implicit put option more valuable.

## MULTISECTOR CCA MODEL

The same general principles of CCA as applied to firm or industry level balance sheets can also be applied to the public and financial sectors to construct interrelated consolidated balance sheets. The multisector CCA balance sheets are displayed in Figure A1. The liabilities of a sector are contingent claims on the assets of the sector. Corporate sector liabilities are risky debt and equity which are contingent claims on corporate sector assets. Financial sector liabilities—debt, deposits, and equity—are contingent claims on banking sector assets including loans, reserves and other assets. The major public sector liabilities are guarantees to "too-big-to-fail" entities, foreign debt, local currency debt and money. These represent contingent claims on public sector assets.

Risk can be transmitted from one sector to another via value changes in the value of risky debt, guarantees, and junior claims which are linked across sectors. For example, debt obligations exist between corporate sectors and banks, between the sovereign and banks. Assets of banks include risky debt to the corporate sector and sovereign debt. Changes in the assets of one sector lead to changes in value of the liabilities of that sector which in turn affect the value of assets and liabilities in other sectors. Foreign and domestic investors hold debt and equity claims in these various sectors. Changes in prices—exchange rate, interest rate, and other prices—affect the value of assets and liabilities, and thus have an impact on risk transmission between sectors. While only three sectors are shown in Figure A1 for simplicity of exposition, a typical Moody's-MfRisk macrofinancial model has about 15 corporate industry sectors, 5 to 10 bank and financial sectors and one sovereign sector made up of a combined market value balance sheet of the government and monetary authorities.

Within the financial sector, CCA can be applied to subsectors including banks, pension funds, and insurance companies. Like the nonfinancial corporate sector, equity of the financial sector is a contingent claim on total assets and is modeled as a call option with strike price equal to the default-free value of debt. This relationship is used to derive the implied market value of financial sector assets which is then used to derive the value of the financial guarantee from the public sector. The value of this financial guarantee can be modeled as a put option, giving the financial sector the "right to sell" firm assets to the government sector if financial sector assets fall below the distress barrier.[45] As the health of the financial sector declines, the market value of assets approaches the distress barrier resulting in a lower distance to distress and higher value on the financial guarantee.

Application of CCA to the public sector is more complex since the public sector combines the functions of the government and monetary authorities. The goal is to construct the liability side of the balance sheet so that the liabilities can be valued and linked to the value of total assets. Securities issued by the public sector and the financial guarantees provided by it give the holders contingent claims on the assets of the government and monetary authorities.

[45] The value of the put option should be adjusted to reflect partial recapitalization instead of full recapitalization by assuming an estimated recover rate below unity.

To apply CCA to combined balance sheet of the government and monetary authorities, it is useful to begin by defining eligible assets and liabilities.[46] The main entries on the asset side of the public sector balance sheet include: (i) foreign currency reserves,[47] (ii) the present value of future taxes and revenues, (iii) public assets, such as equity in public enterprises, land, and mineral assets, and (iv) value of the public sector's monopoly on the issue of money. The main entries on the liability side of the balance sheet include[48] (i) the present value of government expenditures, including social insurance and entitlement programs, (ii) domestic currency debt, (iii) foreign currency or external debt, (iv) relevant financial guarantees, and (v) base money.

Establishing the interrelationship and priority within assets and liabilities is essential in applying CCA to the public sector balance sheet. While the priority of debt service obligations varies among countries and over time within any one country, this analysis assumes the most senior liabilities of the public sector are foreign currency debt plus financial guarantees.[49] Foreign currency debt is viewed as senior since experience indicates that governments take exceptional steps to meet such payments, subjugating other elements of the policy agenda. Domestic currency debt and the guarantees of currency convertibility are then junior claims in the liability structure.[50,51] Based on this ordering, sustained declines in the value of public sector assets would lead initially to restrictions on currency convertibility and a restructure of domestic currency debt in order to maintain payments on foreign-currency debt. However, if public sector assets continue to decline value, a default on foreign currency debt becomes more likely.

In the M*f*Risk model, public sector liabilities are contingent claims modeled with implicit options. The outstanding amounts of domestic currency debt of the government and base money are modeled as a call option on total public sector assets with the distress barrier

[46] See Buiter (2000) for the accounting balance sheet of the public sector.

[47] The total foreign reserves of the public sector in the M*f*Risk model include actual reserves plus contingent reserves from international financial institutions like the IMF or contingent credit lines.

[48] Liabilities only refer to those held outside of the combined government and monetary authorities.

[49] Changing the ordering of priority does not eliminate the use of CCA, but instead simply reorders the option pricing relationships.

[50] Base money is issued by the monetary authority with the associated obligation to exchange base money for foreign currency reserves in the absence of capital controls. When domestic currency is exchanged for foreign currency, foreign currency reserves of the banking system and monetary authority are reduced.

[51] In some cases, market participants have traded protection against the imposition of capital controls which restrict convertibility of domestic currency. For example, Tavakoli (2001) examined convertibility protection prices and found that they can trade at 50 to 100 percent higher than credit default protection, indicating that participants feel convertibility risk is greater than default risk.

composed of senior foreign currency debt and financial guarantees.[52] As discussed above, the financial guarantee is modeled as a put option using assets and the distress barrier of the financial sector. The implied market value of external debt is modeled as the default-free value minus an expected loss which is modeled as an implicit put option (debt holders receive the minimum of the default-free value or, in the event of default, lower value after debt restructuring).

The balance sheet of the public sector can be constructed using pricing information from the international financial market. Using this data, the M*f*Risk model can be used to derive the implied asset value of the public sector. For example, if measured in U.S. dollars, the M*f*Risk model uses the value and volatility of the junior claims of the sovereign (i.e., money and local currency debt) *in dollar terms*. When based in dollars, the volatility of the exchange rate becomes an important element. The book value of short-term and long-term sovereign foreign debt in dollar terms gives the public sector foreign debt distress barrier.[53] Using the sovereign contingent claim balance sheet relationships, this allows for the calculation of the implied sovereign asset value and the implied volatility of sovereign assets. For example, for Brazil in early 2003 the value of the junior claims of the sovereign in dollar terms was US$104 billion and its volatility in dollar terms was 0.98. The book value of foreign and dollar linked debt give a default barrier of US$100 billion. Using this information, and two finance equations, the implied sovereign asset value was US$204 billion and implied sovereign asset volatility is estimated to be 0.6.

Therefore, changes in the distress barrier for the public sector come from two sources: (i) changes in the liability structure of external debt from, for example, changes in maturity structure or currency movements; or, (ii) changes in the value of the financial guarantee due to changes in the health of the financial system. The implied market value of external debt is modeled as the default-free value minus a put option since debt holders receive the minimum of the default-free value or, in the event of default, the senior claim on public sector assets.

---

[52] The balance sheet of the public sector is the only balance sheet that contains accounts denominated in different currencies. In order to compare claims in a common currency, the M*f*Risk model takes the value of (base money + debt in local currency + interest costs)*(FXrate) to represent the "equity" call option on public sector assets. The distress barrier is composed of foreign currency denominated external debt. This is similar to the corporate sector analysis where equity is valued as (# of shares)*(price) = market capitalization, which is then a call option on (assets—distress barrier).

[53] In the case of Brazil, both domestic dollar-linked sovereign debt and external dollar-denominated debt are used to construct the distress barrier.

Figure A1. Multisector CCA Balance Sheets

**Nonfinancial Corporate Sector**

| ASSETS | LIABILITIES |
| --- | --- |
| **Corporate Sector Assets** | **Debt**<br>(default-free value - put option)<br><br>**Equity**<br>(call option on corporate assets) |

**Financial Sector**

| ASSETS | LIABILITIES |
| --- | --- |
| **Loans and other Assets**<br>(including loans to corporate sector and public sector)<br><br>**Financial Guarantees**<br>(modeled as put option) | **Debt / Deposits / Liabilities**<br>(default-free value minus put option)<br><br>**Equity**<br>(call option on financial sector total assets) |

**Combined Government and Monetary Authorities**

| ASSETS | LIABILITIES |
| --- | --- |
| **Foreign Currency**<br><br>**Net Fiscal Asset**<br><br>**Value of Monopoly on Issue of Money**<br><br>**Other Public Assets** | **Foreign-currency Debt**<br>(default-free value - put option)<br><br>**Financial Guarantees**<br>(put options related to financial sector)<br><br>**Base Money and Local-currency Debt**<br>(call options on public sector assets) |

## References

Altman, Edward I., 1968, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*: pp. 589–609.

Altman, Edward I., Robert G. Haldeman and P Narayanan, 1977, " A New Model to Identify Bankruptcy Risk of Corporations," *Journal of Banking and Finance*: pp. 29–54.

Armstrong, Angus and Michael Spencer, 1998. "Will the Asian Phoenix Rise Again?" Emerging Markets Research—Global Emerging Markets. Deutsche Bank Research, October.

Banco Central do Brasil, n.d., Economia e Finançias [online]. Available on the World Wide Web at *http://www.bcb.gov.br/?ECONOMIA*.

Bank of Thailand, n.d., [online]. Available on the World Wide Web at *http://www.bot.or.th/*.

Baxter, Martin, and Andrew Rennie, 1996, *Financial Calculus: An Introduction to Derivative Pricing* (Cambridge: Cambridge University Press).

Black, Fischer, and Myron S. Scholes, 1993, "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81: pp. 637–654.

Bodie, Z., and R. Merton, 2000 *Finance.* Upper Saddle River, New Jersey: Prentice Hall.

Bohn, Jeffrey R., and Peter J. Crosbie, 2002, "Modeling Default Risk." KMV, LLC.

Buiter, W., 2000, "Measurement of the Public Sector Deficit and Its Implications for Policy Evaluation and Design," *How to Measure the Fiscal Deficit*, ed. by M. Blejer, and A. Cheasty, Washington: International Monetary Fund.

Crosbie, Peter J., and Jeffrey R. Bohn, 2003, "Modeling Default Risk." Moody's KMV [online]. Available on the World Wide Web at *http://www.moodyskmv.com*.

Crouhy, Michel, Dan Galai, and Robert Mark, 2000, "A Comparative Analysis of Current Credit Risk Models," *Journal of Banking and Finance*, Vol. 24: pp. 59–117.

Dasri, Tumnong, 2000, "Out-of-Court Corporate Debt Restructuring in Thailand," Presented at "International Conference on Systemic Resolution of Corporate and Bank Distress in Crisis-Affected East Asian Countries." Hosted by The World Bank, Asian Development Bank Institute, and Ministry of Finance, Japan. January 11–12, 2000, Tokyo, Japan.

Geske, R., 1979, "The Valuation of Compound Options," *Journal of Financial Economics*, Vol. 7: pp. 63–81.

Gray, Dale F., 2001, "Macro Financial Risk Country Report," M*f*Risk.

———, 2002, "Macro Finance: The Bigger Picture." *Risk Magazine*, June.

———, Robert C. Merton, and Zvi Bodie, 2003, "A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy," M*f*Risk Working Paper No. 1–03. Available on the World Wide Web at *http://www.moodys-mfrisk.com*.

Haksar, Vikram, and Piyabha Kongsamut, 2003, "Dynamics of Corporate Performance in Thailand," IMF Working Paper 03/214 (Washington: International Monetary Fund).

Holden, Craig W., n.d., "Investment Theory and Practical Modeling Skills," Unpublished.

Hull, John C., 1993, *Options, Futures, and Other Derivative Securities* (Englewood Cliffs, New Jersey: Prentice Hall).

International Monetary Fund, 1998, *International Capital Markets*: *Developments, Prospects, and Key Policy Issues.* World Economic and Financial Surveys (Washington).

———, 2001, "Thailand: Selected Issues," IMF Country Report No. 01/147 (Washington: International Monetary Fund).

———, 2001, "Thailand: Statistical Appendix," IMF Country Report No. 01/155 (Washington: International Monetary Fund).

———, 2002, "Thailand: Selected Issues and Statistical Appendix," IMF Country Report No. 02/195 (Washington: International Monetary Fund).

———, 2003, "Thailand: Statistical Appendix," IMF Country Report No. 03/338 (Washington: International Monetary Fund).

———, 2004, "Thailand: Selected Issues," IMF Country Report No. 04/1 (Washington: International Monetary Fund).

Jarrow, Robert A., and Stuart M. Turnbull, 1997, *Derivative Securities.* South-Western College Publishing.

Jones, E. Philip, Scott P. Mason, and Eric Rosenfeld, 1984, "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation," *Journal of Finance, Vol.* 39: pp. 611–625.

Merton, Robert C., 1973,. "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, Vol. 4: 141–83.

———, 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance,* Vol. 29: 449–70.

———, 1977, "An Analytic Derivation of the Cost of Loan Guarantees and Deposit Insurance: An Application of Modern Option Pricing Theory," *Journal of Banking and Finance,* Vol. 1: 3–11.

———, 1998, "Applications of Option-Pricing Theory: Twenty-Five Years Later," *The American Economic Review,* Vol. 88: 323–349.

Pomerleano, Michael, 1998, "The East Asia Crisis and Corporate Finances: The Untold Microeconomic Story," *Emerging Markets Quarterly* (Winter).

Radelet, Steven, and Jeffrey Sachs, 1998, "The Onset of the East Asian Financial Crisis," NBER Working Paper 6680 (Cambridge, Massachusetts: National Bureau of Economic Research).

SBC Warburg Dillion Read, 1997, "Valuation Issues: Reality Check 2," November.

Sobehart, Jorge R., and Roger M. Stein, 2000, "Moody's Public Firm Risk Model: A Hybrid Approach to Modeling Short-Term Default Risk," Moody's Rating Methodology, March.

Tavakoli, Janet M., 2001, *Credit Derivatives and Synthetic Structures: A Guide to Instruments and Applications, 2nd edition,* (New York: John Wiley and Sons, Inc).

Vajragupta, Yos, and Pakorn Vichyanond, 1998, "Thailand's Financial Evolution and the 1997 Crisis," Thailand Development Research Institute, October.