Adverse Selection and Liquidity Distortion in Decentralized Markets*

Briana Chang†

First Draft: Nov 1, 2010
This version: Nov 8, 2011

Abstract

This paper develops a theory of market illiquidity driven by adverse selection in decentralized markets, in which traders care about both the trading price and how fast they can find a counterparty. The model captures two key notions of illiquidity, market thinness and price undervaluation, and demonstrates how each arises endogenously. When illiquidity manifests itself as market thinness, sellers face long delays in finding a buyer. In certain cases, illiquidity also generates a price discount. In particular, sellers who are relatively distressed financially choose to transact quickly, but accept a price below the fundamental value. The model rationalizes limited market participation, it accounts for fire sales, and it explains how trading volume dries up when dispersion in quality increases. The paper also provides conditions under which each type of liquidity distortion occurs and therefore separately identifies the effects of adverse selection on trading price as well as trading volume.

Key words*: Liquidity; Search frictions; Adverse selection; Fire Sales; Over-the-Counter.* ***JEL****: D82, G1*

*I am indebted to Andrea Eisfeldt, Kiminori Matsuyama, Dale Mortensen and Alessandro Pavan for their continuous support and encouragement. The paper has also benefited from the useful discussion with Matthias Doepke, Simone Galperti, Daniel Garrett, Veronica Guerrieri, Philipp Kircher, Arvind Krishnamurthy, Robert Shimer, Martin Szydlowski, and Randall Wright. I also thank the seminar participants at the 2011 Scociety of Economic Dynamic Annual Meeting, the 2011 North American Summer Meeting of the Econometric Society, and the 2011 Chicago Federal Workshop on Money, Banking, Payments, and Finance.

†Northwestern University, Dept. of Economics. Email: bri.c@northwestern.edu. For the latest version, please see: www.brianachang.com.

1 Introduction

Market illiquidity inhibits the efficient allocation of resources. An illiquid market makes it difficult for firms to raise capital, for households to sell homes, and for intermediaries to effectively manage their liabilities. It therefore has been seen as a catalyst for the recent financial crisis. While the notions of market illiquidity, limited market participation, and fire sales have been widely used, most analyses just assume their existence without considering their origins. As a result, little is yet known about why each occurs in the first place. In particular, the standard notion that there is always a price at which anything with positive value can be sold is not enough to explain the limited market participation of buyers and the infrequent trading observed recently in the market.[1] It also remains unclear whether sellers will choose to unload their assets if they have to do so at an undervalued price. In other words, when will fire sales actually arise?

To answer these questions in an equilibrium framework, this paper develops a dynamic model which takes into account both buyers' entry decisions and sellers' trading strategies. I consider a decentralized trading environment with search frictions and adverse selection, where sellers have private information about the quality of their assets (defined as the NPV of the cash flows), and also possibly about their trading motives (i.e., different needs for cash or financing costs). This paper shows that two possible market liquidity distortions could arise in such a framework: *market thinness* and *price undervaluation*. Market thinness, defined as the buyer-seller ratio in the market, is a measure of market participation. It determines how fast sellers can unload their assets. A downward-distorted market thickness then implies that the seller will have difficulty in finding a buyer. Such an equilibrium outcome rationalizes the limited market participation and implies a low trading volume. It also explains the claims often heard in the popular media that financial sectors are clogged with illiquid assets. The model further demonstrates in what situations sellers need to endure undervalued prices. Since sellers are allowed to hold on to their assets while awaiting a better price in this dynamic environment, whether a seller accepts a price below the fundamental value depends on the market outcome. Hence, the phenomenon of a fire sale, if it arises, is also endogenous.

The setup employs a dynamic competitive search framework with adverse selection. Uninformed principals (buyers) post prices to attract informed agents (sellers), and sellers direct their search toward their preferred submarket. Such a framework highlights the fact that traders care about both the trading price and how fast they can find a counter-

[1] The notion of a buyers' strike in connection to the recent crisis has been discussed in Tirole (2010) and Diamond and Rajan (2009).

party in decentralized markets.[2] The setup is therefore relevant for any asset traded in decentralized markets and current owners tend to have private information about asset quality, such as, current residents of the house, the banks who design the mortgage-backed securities, or the firms who own corporate assets, etc.[3] The market outcomes, i.e., market thickness and trading prices, are then determined by buyers' and sellers' equilibrium trading strategies. Although the presence of market distortions is expected, it is not clear ex ante whether adverse selection will lead to price undervaluation, a thin market, or both. One contribution of this framework is the ability to provide conditions under which each distinct distortion arises. In contrast to the standard lemon problem, which typically gives predictions on the trading price, the model separately identifies the effects of adverse selection on trading price, trading volume, and market segmentation.

The first result shows that if sellers have superior information regarding the asset quality (i.e., there is information asymmetry in the common value), the equilibrium market thickness will be distorted downward as compared to an environment with complete information. In this case, there exists a unique equilibrium which is fully separating: asset prices reflect their fundamental values while sellers with high-quality assets suffer a longer trading delay as a result of lower buyer participation in those submarkets. The intuition is that holding different quality assets results in different waiting preferences. Given that owning a high-quality asset generates a higher flow payoff, a high-type seller is more willing to wait longer in exchange for getting a better price. In equilibrium, the distortion on the market thickness works as a screening mechanism and an agent's type is revealed by the submarket he chooses. The pooling equilibrium with a price distortion, which is the typical outcome of the standard lemon model, cannot be sustained.[4] Furthermore, I establish a strong link between the equilibrium market thinness and the underlying uncertainty stemming from adverse selection. I show that the underlying dispersion, or more precisely, the possible range of underlying asset qualities, plays an important role in determining the equilibrium market thinness: the broader the range of underlying asset qualities, the thinner the market. This result then implies that an increase in the underlying range of asset qualities, or equivalently, an increase in the severity of adverse

[2]Such a trading structure has been developed in the literature on the over-the-counter market based on (Duffie et al. (2005)), and the work on monetary theory based on (Kiyotaki and Wright (1993)).

[3]In particular, the difficulty in assessing the fundamental value of asset-backed securities, which therefore leads to the adverse selection problem, has been one of the prevailing explanations for the recent liquidity crisis. For example, Gorton (2008) provides an analysis of the source of the adverse selection problem in asset-backed securities and mortagage-backed securities.

[4]This paper uses the equilibrium refinement developed in the competitive search literature; see, for example, Guerrieri et al. (2010).

selection, leads to a drop in the trading volume. Meanwhile, since low-quality assets are traded more frequently, one will also observe the drop in the aggregate price.

I further consider the environment in which sellers have different needs for trade, capturing the idea that some sellers need to unload their asset more quickly if they are relatively distressed financially. More important, such motivation for selling are also unknown to the market. In other words, I extend the model to accommodate private information about sellers' private valuation of the assets, as well as the quality of the assets (the common value). In such an environment, the seller who wants to unload his asset more quickly can be the type who has a low-quality asset, or the one who simply has a relatively urgent need for cash. As a result, the types who are willing to wait longer are not necessarily the ones with more valuable assets. Buyers are therefore not willing to pay more to the types who are willing to wait longer, which undermines the full screening mechanism. Hence, I identify conditions under which semi-pooling equilibria exist, which feature a combination of both price and thickness distortions. In each such equilibrium, a thick market with a pooling price coexists with thin submarkets with higher prices. Interestingly, in such an equilibrium I construct, facing the trade-off between a trading delay and a discounted price, it is optimal for a seller who is more financially distressed to unwind his asset *quickly* and accept a loss because of the *undervaluation price*. On the other hand, a seller who is less financially distressed rather waits longer for a higher price. Thus, financial distress leads to fire sales.

The model therefore provides a microfoundation for fire sales. In the well-known explanation for fire sales developed in Shleifer and Vishny (1992) and Shleifer and Vishny (2010), fire sales are forced sales of assets in which high-valuation bidders are *sidelined* and the price is therefore below value in its best use. In those frameworks, it is assumed that natural buyers of the assets also experience financial distress so that they can not enter the market. In contrast, in this framework, all potential buyers are unconstrained, and the entry decisions of potential buyers are endogenous. For an outside observer, the market features the following behavior: few buyers are willing to offer a high price and it is indeed optimal for distressed sellers to sell their assets quickly even though they must accept price discounts. Hence, such an equilibrium outcome rationalizes key features of fire sales without relying on an exogenous assumption about market participation.

The rest of the paper is organized as follows. Section 2 introduces the basic model and establishes my approach to characterizing equilibria. Section 3 extends the basic model to allow for general payoff functions, resale, and nonmonotonicity in the matching value. In Section 4, I consider a setup in which sellers' motives for selling are unobserved by the market. That is, I further allow for asymmetric information in sellers' private values of holding the asset. Section 5 discusses the model's implications and related applications. For example, I show why some financial securities are more liquid than others that pay similar cash flows. Furthermore, applying the method developed to explain firms' capital reallocation, my framework provides a microfoundation for the reallocation pattern documented in Eisfeldt and Rampini (2006) and allows for a richer analysis of how this market friction responds to varied economic shocks. Finally, Section 6 discusses efficiency and provides a comparative statics analysis on searching costs and the elasticity of the matching function.

Related Literature

Theoretically, my work is closest to Guerrieri et al. (2010), who apply the notion of a competitive search equilibrium to a static environment with adverse selection and uninformed principals who are allowed to post (exchange) contracts. As discussed in Guerrieri et al. (2010), this equilibrium concept is similar to the refined equilibrium concept developed in Gale (1992) and Gale (1996). My paper analyzes price posting in a dynamic trading environment and contributes a different approach to characterizing the equilibrium. Furthermore, I investigate the environment where sellers do not only have private information about the quality of their assets but also about their trading motives. The paper shows that the degree to which the market can screen agents depends on the expected asset quality inferred from sellers' waiting preferences. If a sellers' private value of the asset is also unobserved by the market and the types who are willing to wait longer are not necessarily the ones with better assets, semi-pooling equilibria arise. Such an equilibrium rationalizes key features of a fire sale. This result contrasts with Inderst and Muller (2002), Guerrieri et al. (2010), and Guerrieri and Shimer (2011), where the private information is on the *common* value only and therefore the types who are willing to wait longer are necessarily worth more to buyers. In that case, a full separating equilibrium is the unique outcome. This is the first paper to show that asymmetric information on both *private* and *common* values might lead to semi-pooling equilibria in a competitive search framework. Different equilibrium outcomes imply different market distortions. The paper therefore contributes to the literature by identifying the condition under which each distinct distortion arises.

The new characterization method developed in my framework is essential for characterizing the equilibrium. In particular, the constructing algorithm in Guerrieri et al. (2010) is designed for the case when a fully separating equilibrium is obtained, while my approach can be applied also to characterizing semi-pooling equilibria. I establish that the equilibrium can be solved directly as the problem of an imaginary market designer. The designer specifies the price and market tightness for each submarket, in order to match individually optimizing buyers and sellers. This approach not only simplifies the equilibrium characterization to solving a differential equation but further facilitates extending the analysis to a more general environment.

The existence of a semi-pooling equilibrium is novel compared to the previous literature on competitive (direct) search (e.g., Moen (1997), Burdett et al. (2001), Mortensen and Wright (2002) and Eeckhout and Kircher (2010)). Without asymmetric information on the common value, it is known that a fully separating equilibrium is always obtained regardless of whether the information on the private value is complete or asymmetric. This is because directed search separates agents into different submarkets according to their different waiting preferences (Mortensen and Wright (2002) and Eeckhout and Kircher (2010)). With adverse selection, this paper shows that whether the information on the private value is complete or asymmetric is crucial for the equilibrium outcome.

Building on Guerrieri et al. (2010), a contemporaneous work by Guerrieri and Shimer (2011) also emphasizes the idea that liquidity works as a screening mechanism, where they obtain a fully separating equilibrium. They construct a model without search, in which rationing is allowed instead. As shown in the discussion, an economy with rationing can be understood as a limit case of the matching technology developed in my model. My framework is designed to handle a more general trading environment in the decentralized market, which allows for a general payoff function, two-sided heterogeneity[5], and, more importantly, heterogeneity in the sellers' private values of the assets. The concept of illiquidity in Guerrieri and Shimer (2011) shares the same intuition; however, the notion of fire sale is manifested differently here. In Guerrieri and Shimer (2011), the drop in the price is essentially driven by a decrease in the resale value due to the thin market. Fire sales in my model, on the other hand, occur when a relatively distressed seller wants to sell his asset more quickly; he therefore enters a thick market that includes lower quality assets, and accepts a price below the fundamental value. This feature arises in a semi-pooling equilibrium; moreover, the notion of price discount reconnects to the previous literature on liquidity driven by adverse selection, as in Eisfeldt (2004).

This paper is related to two lines of literature which focus separately on search frictions and adverse selection in asset markets. The literature focusing on the effect of search frictions in asset markets includes the Over-the-counter literature put forth by Duffie et al. (2005) and Duffie et al. (2007),[6] and the monetary search literature[7] (for example,

[5] The extension with heterogeneous buyers is in the attached supplementary materials.

[6] Building on their model, the literature studies the effects of liquidity in search models of asset pricing. See, for example, Weill (2008), Lagos and Rocheteau (2009).

[7] Williamson and Wright (2008) provides a detailed survey of this line of literature.

Kiyotaki and Wright (1993), Trejos and Wright (1995)). This paper is also related to the literature on lemon markets, building on the seminal works of Akerlof (1970). Most of this literature assumes that trades take place at one price so a pooling equilibrium and therefore a price discount is obtained, (e.g., Eisfeldt (2004)). Some dynamic models with adverse selection (e.g., Janssen and Roy (2002) and Daley and Green (2009)), also emphasize the fact that different types of sellers have different waiting preferences and therefore delay can be used to sort between different types of sellers. For example, Janssen and Roy (2002) takes a Walrasian approach and show that every equilibrium involves a sequence of increasing prices and qualities traded over time.

The above notions of distortions, price discount and trading delay, are both captured in this framework. However, they are manifested differently. In particular, the trading delay is generated by a low market participation in this paper, which provides a notion of market thinness. More importantly, traders are allowed to sort themselves into different submarkets in this framework.[8] The question of interest here is how different market segmentation may arise, which in turn implies different types of liquidity distortion. As shown in the baseline model, a fully separating equilibrium is the unique outcome; hence, adverse selection leads to a trading delay but not to an undervalued price. In that case, the resulting trading delay is conceptually similar to Janssen and Roy (2002), even though the setup is different. In contrast, in a semi-pooling equilibrium, the behavior of the pooling submarket shares similar features with the standard model of adverse selection, where the assets are valued at the pooling price and therefore a high-type seller suffers a price discount. Being able to capture these two notions of liquidity distortions as an outcome of decentralized matching markets is what distinguishes my model from the rest of the literature. There are other differences regarding the model predictions, which I will discuss later in the paper. For example, in most works on adverse selection, there exists at most a single price in the market at each point in time. In my framework, different prices coexist at each point in time; hence, the model gives a distinct prediction on price dispersion.

[8] Note that the directed search framework is crucial to capture such market segmentation, in contrast to the framework with random search. The predictions and focus in this model are therefore different from the works which consider adverse selection in the random-matching framework (for example, Williamson and Wright (1994), and recent works by Chiu and Koeppl (2011), Lester and Camargo (2011)).

2 Baseline Model

Players There is a continuum of sellers and each owns a single asset; the assets vary in quality indexed by $s \in S$ which is the sellers' private information. Assume that $S = [s_L, s_H] \subset R_+$ and $G^0(s)$ denotes the measure of sellers with asset quality weakly below s at $t = 0$. The other side of the market consists of a large continuum of homogenous buyers; that is, the measure of buyers is strictly larger than the measure of sellers. The measure of buyers who decide to enter the market is endogenously determined by the free-entry condition. This emphasizes the idea that there is a large number of potential buyers out there; therefore, limited market participation, if it arises, is endogenous.

Payoffs While holding the asset s, the seller enjoys a *flow* payoff s but must pay a holding cost c. A buyer, on the other hand, does not need to pay the holding cost and therefore simply enjoys the flow payoff of the purchased asset. In order to buy the asset, the buyer must search for a seller, incurring a flow search cost, $k > 0$, for the duration of his search.

One can think of the holding cost as a simple way to model a seller's need to "cash" the asset. As explained in Duffie et al. (2007), we could imagine this holding cost to be a shadow price for ownership due to, for example, (a) low liquidity, that is, a need for cash; (b) high financing cost; (c) adverse correlation of asset returns with endowments; or (d) a relatively low personal value from using the asset, as in the case of certain durable consumption goods, e.g., homes. For now, one should think of the holding cost c as an easy way to generate the gain from trades. As shown in the general model, the main result holds for a general payoff.

Setup All agents are risk-neutral, infinitely lived and discount at the interest rate, r. Time is continuous. The setup employs a dynamic competitive search framework. Buyers (uninformed principals) post a trading price p and sellers direct their search toward their preferred market. All traders have rational expectations about the equilibrium market tightness (i.e., the buyer-seller ratio) associated with each market; the market tightness in each market p is denoted by $\theta(p)$ and will be endogenously determined in equilibrium. As is standard, in each market, matching is bilateral and traders are subject to a random matching function. A seller who enters the market $(p, \theta(p))$ matches a buyer with the Poisson rate $m(\theta(p))$. Assuming $m(\cdot)$ is a strictly increasing function of θ captures the idea that relatively more buyers make it easier to sell. On the other side, a buyer in market $(p, \theta(p))$ meets a seller at the rate $q(\theta(p))$, where $q(\cdot)$ is a strictly decreasing function of θ. In other words, a higher buyer-seller ratio makes it harder for a buyer to meet a seller. Trading in pairs further requires that $m(\theta) = \theta \cdot q(\theta)$. For the baseline model, I further assume that traders leave the market once the trade takes place. For simplicity, I assume that the matching function takes the Cobb-Douglas form so that $m(\theta) = \theta^\rho$ where $1 > \rho > 0$ throughout this paper. The results, however, are robust to a different form of search technology with standard assumptions.[9]

2.1 Benchmark: Complete Information

I first establish the complete information enviorment as the benchmark, which is the canonical competitive search model put forth by Moen (1997). In such an environment, buyers simply post a trading price and sellers direct their search toward their preferred market. Moreover, following the interpretation of Mortensen and Wright (2002), one can imagine the competitive search equilibrium as if there were a market maker who can costlessly set up a collection Θ of submarkets. Each market is characterized by a pair $(\theta(p), p)$, which is known ex ante to all participants. Given the posted price and the market tightness in each market, each trader then chooses to search within his preferred submarket.

Sellers' and buyers' expected utilities when entering the market (θ, p) can be expressed respectively as follows:

$$rU^s(s, \theta, p) = s - c + m(\theta)(p - U^s(\theta, p, s))$$

$$rU^b(s, \theta, p) = -k + \frac{m(\theta)}{\theta}(\frac{s}{r} - p - U^b(\theta, p, s))$$

Because of the free entry condition, buyers' entry and exit decisions are instantaneously adjusted so that each buyer is indifferent between entering the market or not. Therefore, the market tightness is stationary in each market and so are the traders' utilities. Assuming perfect competition among market makers, the market maker's problem then reduces to maximizing traders' utilities. With perfect information, one can solve the equilibrium independently for each asset s. The market maker's optimization problem for each asset s is:

$$V^{FB}(s) = \max_{p,\theta} U^s(s, \theta, p) = \max_{p,\theta} \frac{s - c + pm(\theta)}{r + m(\theta)}$$

$$st \quad : \quad U^b(s) = \frac{m(\theta)(\frac{s}{r} - p) - \theta k}{r\theta + m(\theta)} = 0$$

[9] That is, $m(\cdot)$ is twice continuously differentiable and strictly concave.

One can easily show that θ_{FB} solves the following FOC[10]:

$$\frac{c}{k} = \frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)} \tag{1}$$

Notice that θ_{FB} is an increasing function of the cost ratio,$\frac{c}{k}$. Namely, it is relatively easier for sellers to meet buyers, and it takes longer for the buyer to find the seller when the holding cost is higher. Also, the first best solution is independent of the asset quality. This is true because the gain from trade is simply the holding cost, which is independent of the asset quality. The price of each asset is then: $p^{FB}(s) = \frac{s}{r} - \frac{k\theta_{FB}}{m(\theta_{FB})}$, the expected value of the asset minus the expected searching cost paid by buyers. Note that the first-best allocations can not be implemented in the environment with adverse selection. This is because that facing the same market tightness θ^{FB}, the low-type seller always wants to pretend to be a higher type so that they can get a higher payment.

2.2 Equilibrium with Adverse Selection

We now turn to an environment with adverse selection, in which sellers have private information about the asset quality. As in the complete information environment, buyers/sellers choose the price p they would like to offer/accept, and all traders have rational beliefs about the ratio of buyers to sellers $\theta(p)$ in each market p. The key difference is that buyers now form rational beliefs about the distribution of sellers' unobserved types in each market p, which determines the expected asset quality they receive in each market. Note that, given the expected asset quality in each market, the free-entry condition determines the measure of active buyers in each market independently of the distributions of sellers in other markets. As a result, the equilibrium market tightness function $\theta(\cdot)$ does not depend on the distributions of sellers in other markets. This property is important as it simplifies our analysis by focusing on stationary equilibria where the set of offered prices P^* and the market tightness function $\theta(\cdot)$ are time invariant even though the aggregate distribution evolves over time.

I now elaborate on how to construct stationary equilibria in this framework and focus on such equilibria throughout the paper. Consider the set of time-invariant offer prices and the market tightness function $\theta(\cdot)$. Each market is then characterized by the pair $(p, \theta(p))$. Clearly, sellers' strategies are stationary when facing the time invariant $(p, \theta(p))$. Sellers' trading decisions then determine the expected asset quality in each submarket.

[10]The maximization problem can be rewritten as $\max_{\theta \in R^+} V(s, \theta)$, after substituting the free entry condition. One can show that, given a concave matching function, $V(s, \theta)$ is a concave function in θ and therefore $\theta^{FB}(s)$ is a global maximizer of $V(s, \theta)$.

It thus pins down the buyers' expected value of buying an asset in each market p : $\bar{J}(p) = \int J(\tilde{s})\mu(\tilde{s}|p)d\tilde{s}$, where $\mu(s|p)$ denotes the probability of a type-s seller conditional on a match in the market p and $J(s)$ denotes the buyers' value of buying the asset from type-s seller. Given that the sellers' searching decisions are stationary and the matching is random in each submarket, notice that the composition of sellers' types is therefore stationary as well as buyers' expected matching value $\bar{J}(p)$. Furthermore, the free entry condition guarantees that, at each point in time, the measure of active buyers generates the correct ratio $\theta(p)$ in each submarket such that $p = \bar{J}(p) - \frac{k\theta(p)}{m(\theta(p))}$ for all $p \in P^*$ and therefore the market tightness function $\theta(p)$ is then stationary. Finally, one still needs to show that the set of offered prices P^* is time invariant. As it will become clear later, the set of offer prices depends on the sellers' equilibrium utilities and the *range* of underlying asset quality: both are time invariant in the constructed environment. Hence, the above discussion suggests a possible stationary equilibrium $(p, \theta(p), \mu(\cdot|p))$, where traders' *strategies* are stationary, and both the set of offered prices P^* and the market tightness function $\theta(\cdot)$ are time invariant.

Note that in a setting of competitive search models with heterogeneous agents, it is well-known that the type distribution does not play a role, as the standard result in the literature is a full separation (for example, Moen (1997)).[11] In an enviorment with adverse selection, there are two main differences. First of all, the possibility of (semi-) pooling is allowed. In this case, the distribution of sellers' types in other submarkets does not play a role, but the distribution within each market does matter and is governed by $\mu(\cdot|p)$. Second, as will become clear, the equilibrium market tightness of each market depends on the *range* of the underlying distribution, which is the key consequence of adverse selection.

Nevertheless, as shown above, it is enough to characterize the set of active markets P^*, the equilibrium market tightness function $\theta(\cdot)$, and the composition $\mu(s|p)$ in each market. This property makes our dynamic environment tractable as it eliminates the role of the aggregate distribution. The following section then characterizes the traders' decisions in such stationary equilibria, neglecting the role of aggregate distribution. The aggregate distribution, however, does evolve over time and affect aggregate statics (such as the aggregate price, the price dispersion, etc). Nevertheless, one can easily back out the aggregate dynamics afterward.

I define the set of feasible prices as $P = [0, J(s_H)]$ since no trade takes place at prices above $J(s_H)$ and below zero (under the assumption that $s_L - c > 0$). A stationary equilibrium consists of a set of offer prices P^*, a market tightness function $\theta(\cdot)$ and traders'

[11] Shi (2009) further establishes the block recursive property in an environment when on-the-job search is allowed.

trading decisions. The buyers' and sellers' expected payoffs in markets $(p, \theta(p), \mu(\cdot|p))$ can be expressed as:

$$
\begin{aligned}
rU^b(p, \mu(\cdot|p), \theta(p)) &= -k + \frac{m(\theta(p))}{\theta(p)}\left(\int \frac{\tilde{s}}{r}\mu(\tilde{s}|p)d\tilde{s} - p - U^b\right) \\
rU^s(p, \theta(p), s) &= s - c + m(\theta(p))(p - U^s(p, \theta(p), s)).
\end{aligned}
$$

In the market $(p, \theta(p), \mu(\cdot|p))$, a buyer pays a flow searching cost k until he meets a seller with a Poisson rate $\frac{m(\theta(p))}{\theta(p)}$. He will then pay price p for the asset, expecting the asset quality to be $\int \frac{\tilde{s}}{r}\mu(\tilde{s}|p)d\tilde{s}$. Facing the *active* markets, which can be characterized as $(p, \theta(p))_{p\in P^*}$, sellers direct their search toward their preferred markets and can always choose the option of no trade, denoted by $\varnothing$[12]. The equilibrium expected utilities of seller s then must satisfy:

$$
V^*(s) = \max_{p\in P^*\cup\varnothing} U^s(p, \theta(p), s)
$$

We now need to specify the belief off the equilibrium path. Our equilibrium concept adopts Guerrieri et al. (2010), which ressembles the refined Walrasian general-equilibrium approach developed in Gale (1992).[13] The spirit follows the market utility property used in the competitive search equilibrium literature.[14] That is, when a buyer contemplates a deviation and offers a price p which has not been posted, $p \notin P^*$, he has to take sellers' equilibrium utilities $V^*(s)$ as given and forms a belief about the market tightness and the types he will attract. First of all, a buyer expects a positive market tightness only if there is a type of seller who is willing to trade with him. Moreover, he expects to attract the type s who is most likely to come until it is no longer profitable for them to do so. Formally, define:

$$
\begin{aligned}
\theta(p, s) &\equiv \inf\{\tilde{\theta} > 0 : U^s(p, \tilde{\theta}, s) \geq V^*(s)\} \\
\theta(p) &\equiv \inf_{s\in S} \theta(p, s)
\end{aligned}
\tag{2}
$$

[12] To make the choice of no trade consistent with the rest of our notation, let $\varnothing_p = \bar{P} > J(s_H)$ denote a nonexistent price which is higher than the feasible pirce and the trading probability at $\varnothing_p$ is zero, $\theta(\varnothing_p) = 0$. Hence, a seller achives his outside option $\frac{s-c}{r}$ if $\varnothing = \arg\max V(p, \theta(p), s)$.

[13] See Guerrieri et al. (2010) for the detailed discussion regarding its relationship with different refinement developed in the previous literature.

[14] Burdett, Shi and Wright (2001) prove that a competitive search equilibrium is the limit of a two stage game with finite numbers of homogeneous buyers and sellers, which can be understood as a microfoundation for the market utility property.

By convention, $\theta(p,s) = \infty$ when $U^s(p,\tilde{\theta},s) \geq V^*(s)$ has no solution, specifically $\theta(p,s) = \infty$ for any $p < V^*(s)$. Intuitively, we can think of $\theta(p)$ as the *lowest* market tightness for which the buyer can find a seller. Now let $T(p)$ denote the set of types which are most likely to choose p:

$$T(p) = \arg\inf_{s\in S}\{\theta(p,s)\}$$

Therefore, given $\theta(p)$, p is optimal for every type $s \in T(p)$ but not optimal for $s \notin T(p)$. Hence, the buyer's assessment about $\mu(s|p)$ for any posted price p needs to satisfy the following restriction:

$$\text{For any price } p \notin P^* \text{ and type } s,\ \mu(s|p) = 0 \text{ if } s \notin T(p) \tag{3}$$

In the case when $T(p)$ is unique, a buyer then expects this deviation will only attract seller $T(p)$ and therefore $\mu(s|p) = 1$ if $s = T(p)$ and $\mu(s|p) = 0$ for $\forall\ s \notin T(p)$. To simplify the notation, let μ_p denote the sellers' distribution $\mu(\cdot|p)$ conditional on the market p.

Definition 1 *A stationary equilibrium consists of a set of offer prices P^*; a market tightness function in each market p, $\theta(\cdot) : P \to [0,\infty]$; and the conditional distribution of sellers in each submarket $\mu : S \times P^* \to [0,1]$, such that the following conditions hold*:

$E1$ *(optimality for sellers): let*

$$V^*(s) = \max\{\frac{s-c}{r}, \max_{p'\in P^*} U^s(p',\theta(p'),s)\}$$

and for any $p \in P^*$ and $s \in S$, $\mu(s|p) > 0$ implies $p \in \arg\max_{p'\in P^*\cup\varnothing} U^s(p',\theta(p'),s)$

$E2$ *(optimality for buyers): $E2(a)$ Free-entry, for any $p \in P^*$*

$$0 = U^b(p,\theta(p),\mu_p);$$

$E2(b)$ optimality of price posting: there does not exist any $p' \notin P^*$ such that $U^b(p',\theta(p'),\mu_{p'}) > 0$, where $\theta(p')$ and $\mu(s|p')$ satisfies (2) and (3)

As explained earlier, the aggregate distribution of sellers does not play a role. The law of motion of the stock of sellers in each market is given by the transaction outflow. On the other hand, buyers' participation must generate the *correct* buyer-seller ratio $\theta(p)$ at each point time in all the submarkets according to $E2$. To characterize the equilibrium, one does not need to track the aggregate distribution; therefore, the role of the traders' distribution is eliminated in the above definition. Nevertheless, one can back out the traders' distribution over time after solving the equilibrium above as shown in discussion below.

2.3 Characterization

I now show that the equilibrium outcome can be characterized as the solution to a mechanism design problem which takes into account both sellers' and buyers' optimality conditions. Intuitively, one can think of a market designer who promises a price and a market tightness for each market so that sellers truthfully report their type, that is, condition $E1$ has to hold. Moreover, a *feasible* mechanism must satisfy the market-clearing condition, i.e, $E2(a)$. In other words, the market tightness must equal the ratio of the measure of buyers who are willing to pay p to the measure of type-s sellers who are willing to accept p. Meanwhile, given that buyers can post the price freely in the decentralized markets, any price schedule recommended by the market designer has to be optimal for buyers. Otherwise, buyers will deviate by posting a price other than the ones recommended by the mechanism designer. This point is captured by condition $E2(b)$.

Overview of the solution: Our approach therefore follows two steps: First, we characterize the set of feasible mechanisms $\alpha \in A$ that satisfy $E1$ and the the free-entry condition $E2(a)$ (Lemma 1). Second, we use $E2(b)$ to identify a necessary conditions for the solution to step 1 to be decentralized in equilibrium. This second step enables us to show that the unique equilibrium allocation rule that is consistent with $E2(b)$ is the least-cost separating outcome.

To characterize the set of mechanisms that satisfy the sellers' IC constraints, I set up the problem as a mechanism-design problem (of an imaginary market designer). By the revelation principle, one can focus direct revelation mechanisms without loss of generality. A direct mechanism is a pair (θ, p) where $\theta : S \to R_+$ is the market tightness function and $p : S \to R_+$ is the price function. The mechanism is interpreted as follows. A seller who reports his type $\hat{s} \in S$ will then enter the market with the pair $(\theta(\hat{s}), p(\hat{s}))$. Denoting by $V(\hat{s}, s) = V(p(\hat{s}), \theta(\hat{s}), s)$ the payoff that type s obtains when he reports $\hat{s}$, we have that $V(\hat{s}, s)$ can be expressed as:

$$rV(\hat{s}, s) = s - c + m(\theta(\hat{s}))(p(\hat{s})) - V(\hat{s}, s)).$$

The sellers' IC condition then requires that $s \in \arg\max_{\hat{s}} \frac{s-c+p(\hat{s})m(\theta(\hat{s}))}{r+m(\theta(\hat{s}))}$. A seller's utility can be rearranged as:

$$V^*(s) = \max\{\frac{s-c}{r}, \max_{\hat{s}} \frac{s-c+p(\hat{s})m(\theta(\hat{s}))}{r+m(\theta(\hat{s}))}\}.$$

Notice that a seller can always choose not to participate and get his autarky utility $\frac{s-c}{r}$. For convenience, we can think of not entering the market as choosing a market where the matching rate is zero. Since the mechanism has to satisfy the sellers' IR constraint, we set

$\theta(s) = 0$ whenever the IR constraint is binding. The following lemma then characterizes any mechanism $\alpha = \{p(\cdot), \theta(\cdot)\}$ which satisfies $E1$:

Lemma 1 *The pair of functions $\{\theta(\cdot), p(\cdot)\}$ satisfies the sellers' optimality condition (E1) if and only if following conditions are satisfied:*

$$\frac{1}{r + m(\theta(s))} \textit{ is non-decreasing ;} \tag{M}$$

$$V^*(s) = \frac{u(s) + p(s) \cdot m(\theta(s))}{r + m(\theta(s))} = V^*(s_l) + \int_{s_l}^{s} V_s(p(\tilde{s}), \theta(\tilde{s}), \tilde{s}) d\tilde{s} \; ; \tag{ICFOC}$$

$$V^*(s) \geq \frac{u(s)}{r}. \tag{IR}$$

where $u(s) = s - c$ in the baseline model.

Proof. The proof follows from standard arguments in the mechanism-design literature (Milgrom and Segal (2002)). See Appendix. ■

Define $B(p) \equiv \{s \in S | \; p(s) = p\}$. Buyers' expected asset quality in market p is given by the conditional expectation $E[s|s \in B(p)]$. Given any feasible mechanism, the the free-entry condition for buyers implies that the following condition must hold:

$$p = \frac{E[s|s \in B(p)]}{r} - \frac{k\theta(p)}{m(\theta(p))}. \tag{4}$$

Definition 2 *Let A be the set of feasible mechanisms, such that for $\forall\alpha = (p^\alpha, \theta^\alpha) \in A$, the pair of functions $\{\theta^\alpha(\cdot), p^\alpha(\cdot)\}$ satisfies (M), $(ICFOC)$, (IR) and the free entry condition* (4).

Hence, Lemma 1 and the free-entry condition define the set of *feasible* mechanisms A. Now let $V^*(s; \alpha)$ denote the expected payoff to a type-s seller under mechanism α. Each mechanism $\alpha \in A$ is then composed of a price schedule $p^\alpha(\cdot)$ and market tightness $\theta^\alpha(\cdot)$. This set includes all possible pooling equilibria as well as separating ones. Nevertheless, not all of these mechanism can be sustained as a decentralized outcome. A decentralized equilibrium has to satisfy the buyers' optimality condition. Hence, $\alpha \equiv (p^\alpha, \theta^\alpha)$ is an equilibrium only if there is no profitable deviation for buyers to open a new market p',where the off-equilibrium belief is specified by (2) and (3), as discussed earlier. When a buyer considers opening a new market $p' \notin range$ of P^α, he expects to attract only the type who is most likely to come, $T(p')$, as defined by (3). To facilitate the analysis, we first prove the following lemma which identifies the type who is mostly likely to come to this new market.

Lemma 2 *Given any mechanism $\alpha \in A$, for any price $p' \notin$range of p^α, the unique type who will come to this market p' is given by*

$$\begin{aligned} T(p') &= s^+ \cup s^- \\ \textit{where } s^- &= \inf\{s \in S | p' < p^\alpha(s)\} \textit{ and } s^+ = \sup\{s \in S | p' > p^a(s)\}. \end{aligned}$$

Proof. Notice that $p^\alpha(\cdot)$ is nondecreasing for $\forall \alpha \in A$ given (M). Therefore, $T(p')$ is uniquely defined.[15] That is, the type who is most likely to come is unique. For any $p' \notin$ range of p^α, by definition, $\theta(p', s) \equiv \inf\{\tilde{\theta} > 0 : U^s(p, \tilde{\theta}, s) \geq V^*(s; \alpha)\}$. Therefore, for any $p' > V(s; \alpha)$,which is the relevant case,[16] the market tightness $\theta(p', s)$ solves: $G(p', \tilde{\theta}, s) \equiv U^s(p', \tilde{\theta}, s) - V^*(s; \alpha) = 0$,

$$\begin{aligned} \frac{d\theta(p', s)}{ds} &= -(\frac{dG/ds}{dG/d\theta}) \propto \frac{1}{r + m(\theta^\alpha(s))} - \frac{1}{r + m(\theta(p', s))} \\ &= \begin{Bmatrix} < 0 \text{ if } p' > p^\alpha(s), (\because \theta(p', s) < \theta^\alpha(s)) \\ > 0 \text{ if } p' < p^\alpha(s), (\because \theta(p', s) > \theta^\alpha(s)) \end{Bmatrix}. \end{aligned}$$

Recall that, when posting a new price p', a buyer should expect the lowest market tightness, $\theta(p') = \inf_s\{\theta(p', s)\}$, and the type most likely to come, $T(p') = \arg\inf\{\theta(p', s)\}$. The above result then implies that $T(p') = s^+ \cup s^-$. ■

With this condition, we can then show that there is no pooling in such environment and in fact, this is true for a more general payoff function, as long as the buyers' flow payoff, denoted by $h(s)$, is increasing in s.

Claim 1 *(Full Separation) When the buyers' value $h(s)$ is strictly increasing, there exists no submarket where seller types are pooled.*

Proof. See Appendix. ■

Intuitively, a buyer can post a new price p' which is only slightly higher that the original pooling price. In that case, he only pays a little bit more but gets the best type in the original pooling for sure (as implied from lemma 2), which therefore generates a

[15] The only exception is when some types of sellers are out of the market. In this case, there then exists a marginal type s^* such that $\theta(s) = 0$ for $\forall s > s^*$. For any $s > s^*$ and $p' > \frac{u(s)}{r}$, type-s will come to the market even when $\theta(p', s) \to 0$. Hence, $T(p')$ is then a set of these types of sellers. Nevertheless, such an exception is not relevant for the equilibrium result, as it will become clear later that a buyer will deviate even when he expects the *worst* type within this set.

[16] In the case where $p' < V^*(s; \alpha)$, $\theta(p', s) = \infty$ as $U^s(p', \theta, s) \geq V(s; \alpha)$ has no solution. In words, if the deviating price is lower than a type's equilibrium utility, obviously, this type will not come to this market.

profitable deviation. This result allows us to focus on a fully separating equilibrium. In each market, (θ, p, s),the price schedule then has to satisfy:

$$p(s) = \frac{s}{r} - \frac{k\theta(s)}{m(\theta(s))} \tag{5}$$

Substituting this payment schedule into $(ICFOC)$:

$$V^*(s) = \frac{s - c + (\frac{s}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))})m(\theta^*(s))}{r + m(\theta^*(s))} = V^*(s_l) + \int_{s_l}^{s} V_s(p^*(\tilde{s}), \theta^*(\tilde{s}), \tilde{s})d\tilde{s} \ ;$$

Take the derivative with respect to s on both side, one can then get the following differential equation for $\theta^*(s)$:

$$[c - k(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)})]\frac{d\theta}{ds} = -\frac{\theta}{\rho r}(r + m(\theta)) \tag{6}$$

Therefore, in order to satisfy the incentive compatibility constraints and free-entry condition, the market tightness function $\theta^*(\cdot)$ has to satisfy the above differential equation, subject to the monotonic condition (M). Left hand side of (6) is monotonically decreasing in θ and reaches zero at θ^{FB}. Therefore, for any initial condition $\theta_0(s_L) > \theta_{FB}$, the solution will be increasing in s and violate the monotonicity of the condition. (6) is a separable nonlinear first-order differential equation with a family solution form: $s = C + \int \frac{1}{f(\theta)} d\theta$, where $f(\theta) = \frac{-\frac{\theta}{\rho r}(r+m(\theta))}{[c+\frac{k}{\rho}((\rho-1)\theta - \frac{r\theta}{m(\theta)})]}$.One can understand the qualitative properties of the solutions through a simple phase diagram: for any $\theta \in (0, \theta^{FB})$, $f(\theta) < 0$ and $f'(\theta) > 0$; furthermore, $\lim_{\theta \to 0} f(\theta) = 0$. Hence, with any initial condition $\theta_0 < \theta_{FB}$, the solution $\theta(s; \theta_0)$ will be strictly decreasing. With the following claim, we further pin down the initial condition and therefore achieve the unique candidate of the equilibrium market tightness.[17]

Claim 2 *In any fully separating equilibrium, the lowest type achieves his first-best utility and*

$$\theta(s_L) = \theta^{FB}(s_L) \tag{7}$$

Proof. See Appendix for detail. ■

The intuition is clear: a downward distorted market tightness is to preventing a lower-type from mimicking a higher-type. Therefore, it should be clear that there is no reason to distort θ for the lowest type.

[17] One can see that standard condition of the uniqueness does not hold at $\theta_0(s_L) = \theta^{FB}$. In fact, there will be two solutions. However, the other solution increases with s and therefore violates the monotonic condition.

The solution of (6) with the initial condition θ^{FB} , denoted by $\theta(s;\theta^{FB})$, is illustrated in Fig 1 below:

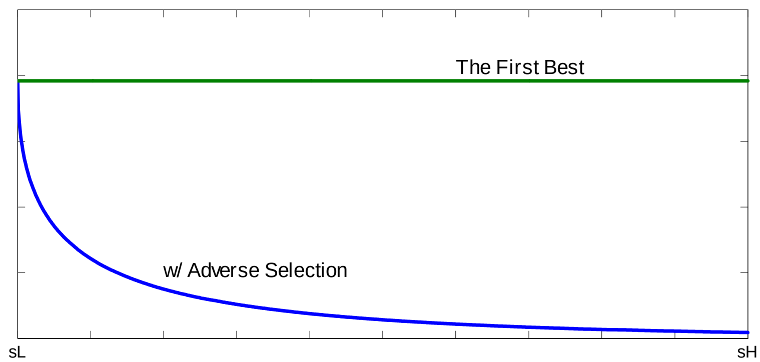

Fig 1: Equilibrium market tightness $\theta^*(s)$

Given $\theta^*(s) = \theta(s;\theta^{FB})$, the equilibrium price is also determined by (5). The mechanism can be summarized as follows. Because of asymmetric information, sellers face a lower meeting rate, $\theta^*(s) < \theta^{FB}$, for all s, but will get a higher payment $p^*(s) = \frac{s}{r} - \frac{k\theta^*}{m(\theta^*)} > p^{FB}(s)$. In each market, there are fewer buyers, who meet a seller with a relatively high meeting rate but pay a higher price. Sellers' equilibrium utilities are lower than in the first best benchmark due to the liquidity distortion: $V^*(s) < V^{FB}(s)$ for $\forall s > s_L$.

One can easily verify that the IR constraint holds for all sellers since there is no cost for entering the market and the trading price is higher than the outside option: $p^*(s) > p^{FB}(s) > \frac{s-c}{r}$. Furthermore, buyers do not find it profitable to open markets other than those that are already open. The argument is the following. First, note that the price function is continuous. Denote by (p_L, p_H) the lower bound and the upper-bound, respectively, of the support of the function $p(s)$ constructed as above. From Lemma 2, if buyers post a price $p' > p^H$, they will attract only the highest type. The corresponding (p', θ') has to provide the highest type the same utility; however, such a pair (p', θ') involves more distortion. As a result, a buyer's utility is lower due to the additional distortion. Such a deviation is therefore not profitable. Similarly, if posting $p^{'} < p^L$, a buyer will attract the lowest type with a pair of (p', θ'). Conditional on giving the lowest type his first best utility, any pair of (p', θ') other than (p^{FB}, θ^{FB}) implies a negative utility of a buyer. The above argument thus confirms that no profitable deviation exists for buyers. Hence, we have the following proposition:

Proposition 1 *The unique solution to the mechanism design problem subject to condition E1 and E2 is given by the market tightness function $\theta^* : S \to R_+$, and the price function*

$p^* : S \to R_+$, where $\theta^*(s)$ is the unique solution to (6) with the initial condition $\theta(s_L) = \theta^{FB}(s_L)$, and where $p^*(s)$ is given by (5).

Corollary 1 *The unique decentralized equilibrium outcome is characterized by:*

1) a set of offered prices (active submarkets) $P^* = \{p \in R_+ | p = p^*(s) \text{ for } s \in S\}$, where the price function $p^*(\cdot)$ is as given in Proposition 1;

2) the market tightness for each submarket $\Theta^* : P^* \to R_+ : \Theta^*(p) \equiv \theta(p^{*-1}(p))$, where $p^{*-1} : P^* \to S$ denotes the inverse of p^*;[18]

3) the share of type s in each submarket: $\mu(s|p) = I\{p^*(s) = p\}$;

3 Generalization

3.1 General Payoff

The goal of this section is to allow for a more general traders' payoff function. As before, there is a mass of heterogenous sellers. Each seller has one asset and the asset quality is indexed by $s \in S$ that is sellers' private information. The flow payoff of the asset s to the seller is now given by $u(s)$, where u is a continuously differentiable function, $u : S \to R_+$. The indices s are ordered so that $u(s)$ is increasing in s, i.e., $u'(s) > 0$. On the other side of the market, there is a large mass of buyers. The flow payoff of an asset s is given by $h(s)$ and h is a strictly positive function. I make the following assumptions about the traders' preferences and will discuss how these assumptions can be relaxed in Section 3.3.

Assumption 1: $h(s)$ is (1a) a continuously differentiable function and (1b) strictly increasing in s, $h_s(s) > 0$.

Assumption 2: $g(s) = h(s) - u(s) > 0$ for $\forall s \in S$.

Monotonicity in the buyers' value (1b) is an important assumption for our basic result. In particular, it is crucial for Claim 1 which establishes that there is no pooling submarket, and there is a unique fully separating equilibrium. As in the baseline model, one can think of s as representing the quality of the asset. Higher quality gives both sellers and buyers a higher payoff. In Section 3.3, I consider an environment where this assumption does not hold and show how the equilibrium outcome may behave differently. The second assumption simply guarantees that there are gains from trade for all s. The baseline model is nested as $h(s) = s$ and $u(s) = s - c$, where $c > 0$.

Given these two assumptions, it is straightforward to see that all our previous results hold. The only difference is that the equilibrium market tightness $\theta^*(s)$ now needs to

[18] Note that the price function p is strictly increasing and therefore invertible.

solve a more general differential equation, given by

$$[(h(s) - u(s) - k(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)})]\frac{d\theta}{ds} = -(r + m(\theta)) \cdot \frac{\theta}{\rho}\frac{h_s(s)}{r} \tag{8}$$

and the corresponding price schedule now satisfies

$$p(s) = \frac{h(s)}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))} \tag{9}$$

Claim 2 also remains intact and therefore the initial condition is given by $\theta^*(s_L) = \theta^{FB}(s_L)$, where the first best market tightness $\theta^{FB}(s)$ now solves:

$$\frac{h(s) - u(s)}{k} = \frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)} \tag{10}$$

Claim 3 *The first best solution* $\{\theta^{FB}(s), p^{FB}(s)\}$ *is not implementable when* $h_s > 0$*;*

Claim 4 *The equilibrium market tightness* $\theta^*(s)$ *is downward distorted compared to the first best, that is,*

$$\theta^*(s) < \theta^{FB}(s) \text{ for } \forall s > s_L$$

Proof. See Appendix. ■

From (8), one can see $\frac{d\theta^*(s)}{ds} < 0$ given that $\theta^*(s) < \theta^{FB}(s)$ and $h_s(s) > 0$. Furthermore, one can see that the slope of $h(s)$ determines the slope of $\theta^*(s)$: the faster buyers' value increases with the asset in quality s, the larger the distortion. As in the baseline model, the equilibrium can be summarized by Corollary 1. The transaction outflow for each type-s asset is then determined by the matching rate $\theta^*(s)$, with the law of motion $\frac{dg^t(s)}{dt} = -m(\theta(s))g^t(s)$, where $g^t(s)$ is the size of sellers with asset-s at time t. On the other hand, the measure of active buyers in each market is endogenously determined by the ratio $\theta^*(s)$, that is, $\mu_B^t(s) = \theta^*(s) \cdot g^t(s)$. Hence, as discussed earlier, one can back out the aggregate distribution of traders from the stationary value of the market tightness $\theta^*(s)$. Furthermore, because the only equilibrium is fully separating, the distribution of sellers' types does not have any impact on the equilibrium price and market tightness. This in turn means that, as long as the support of the distribution remains fixed, the equilibrium price and the market tightness $(p(s), \theta(s))$ for each market remains the same even if in each period there is an inflow of sellers.

3.2 Resale

The basic model assumes that once a buyer buys the asset, he keeps it forever. If a buyer is financially constrained in the future, he may have motives to sell the asset in exchange for cash and will then re-enter the market as a seller. Clearly, taking this into account, the buyer's expected profit will then depend on the resale value. This section extends the model to allow for resale to capture the possibility that preferences for asset ownership may change over time and to examine the impact of such a preference shock on the equilibrium price and market tightness. The flow value of owning the asset drops from $h(s)$ to $u(s)$ when the owner is hit by a preference shock which arrives at the Poisson arrival rate δ. In our basic model, this simply means that the owner now needs to pay the holding cost and hence he naturally becomes a seller in the market. In other words, such a shock can also be interpreted as a liquidity shock, which reflects the fact that owners become financially constrained. In the case of installed capital market, one can interpret this shock as a firm specific negative technology shock that forces the firm to disinvest. Given the market is designed in an incentive-compatible way, the owner of the asset s then enters the market $(p(s), \theta(s))$ as a seller. The contingent value of ownership can then be rewritten as:

$$rJ(s) = h(s) + \delta(V^*(s) - J(s))$$

where $V^*(s)$ is the equilibrium expected value of a type-s seller. All methods developed in the baseline model remain valid. The only key difference is that the value of holding the asset is now a function of the equilibrium resale value $V^*(s)$ and hence must be determined in equilibrium. Nevertheless, one can see that Assumption 1 still holds given that $h(s) + \delta V^*(s)$ strictly increases with s.[19] As discussed above, this means that the equilibrium is unique and fully separating. Our previous approach can be applied directly with the modified free entry condition:

$$p(s) = J(s) - \frac{k\theta}{m(\theta)} = \frac{h(s) + \delta V^*(s)}{r + \delta} - \frac{k\theta(s)}{m(\theta(s))} \tag{11}$$

The only difference is that we now have a different differential equation and, of course, a different first best solution, i.e, a different initial condition. The differential equation can be derived by substituting the above price schedule into $(ICFOC)$ and differentiating with respect to s on both sides, which yields:

[19] This is true because $V^*(s)$ is necessarily increasing in s : for any $s' > s$, $V^*(s') \geq U^s(s', p^*(s), \theta^*(s)) > U^s(s, p(s), \theta(s)) = V^*(s)$.

$$[(h(s)-u(s)-k(\frac{r+\delta+m(\theta)-\theta m'(\theta)}{m'(\theta)})]\frac{d\theta}{ds}=-(\frac{r+\delta+m(\theta)}{r+\delta})\cdot\frac{\theta}{\rho}(h_s(s)+\frac{\delta u_s(s)}{r+m(\theta)}) \tag{12}$$

One can easily check that the basic version (equation (8)) simply corresponds to the case where $\delta=0$ in (12) . Furthermore, (12) can be derived from (8) by setting the *effective* discount rate to $\tilde{r}=r+\delta$ and the equilibrium buyers' values $\tilde{h}(s)=h(s)+\delta V^*(s)$, where $\tilde{h}_s(s)$ then corresponds to $h_s(s)+\frac{\delta u_s(s)}{r+m(\theta^*(s))}$ as shown in the RHS of (12) . The initial condition is then given by the first best solution $\theta_\delta^{FB}(s)$ for such an environment, which is defined as follows:

$$\begin{aligned} V_\delta^{FB}(s) &= \max_\theta \frac{r+\delta}{r}(\frac{u(s)}{r+\delta}+\frac{m(\theta)(\frac{h(s)-u(s)}{r+\delta})-k\theta}{r+\delta+m(\theta)}) \\ \theta_\delta^{FB}(s) &= \arg\max V_\delta^{FB}(s) \end{aligned}$$

Notice that the first best solution $\theta_\delta^{FB}(s)$ continues to solve the analog of condition (10) but with the *effective* discount rate $\tilde{r}=r+\delta$. Interestingly, note that $\theta_\delta^{FB}(s)$ is decreasing in δ. The intuition is that fewer buyers want to enter the market when they are likely to sell the asset again soon. This is because the trading surplus is decreasing in δ. As a result, the equilibrium market tightness is lower due to less entries. Given the solution $(p_\delta^*(s),\theta_\delta^*(s))$, one can solve for the steady state in such an environment. In particular, for any type of asset, the steady state ratio of the holders to the sellers is pinned down by $\frac{m(\theta^*(s))}{\delta}$. That is, due to the downward distortion of $\theta^*(s)$, the better the asset, the larger the portion of the asset held in the "wrong" hands.

Remark: As before, adverse selection causes market tightness to be distorted downward compared to the first best outcome. In addition, the possibility of resale introduces an additional distortion by placing downward pressure on asset prices. In particular, according to (11), price decreases due to a lower equilibrium resale value.

3.3 Nonmonotonicity in the Matching Value

The previous analysis shows that there are no equilibria where partial pooling occurs in some of the submarkets. It is important to note that this result relies on the assumption that the buyers' valuations are increasing in the sellers' types $h_s(\cdot)>0$. The intuition is that if a buyer strictly prefers a higher type s, he can post a price which is slightly higher than the original pooling price and will attract the highest type for sure. Such a deviation is profitable; hence, pooling cannot exist. If this assumption is violated, a pooling equilibrium can then be sustained. Furthermore, as shown in Lemma 1, any IC allocation $\theta^*(s)$ has to satisfy the monotonicity condition (M). As one can observe from (8) , the assumption $h_s(\cdot) > 0$ is crucial to guarantee that the solution $\theta^*(s)$ is decreasing so that condition (M) is satisfied. If this condition fails, the allocation is no longer incentive-compatible. That is, a fully separating equilibrium can not be sustained.[20] The reason is that the screening mechanism is a combination of a downward-distorted liquidity and an upward-rising price scheme. To generate such a schedule, the market designer must make sure that buyers are willing to pay the price, given the expected value. However, buyers' willingness decreases if the types who are willing to wait longer have assets that are worth less.

To elaborate more on why a fully separating equilibrium can not be sustained, consider that buyers' value function $h(\cdot)$ is strictly increasing in s up to $\hat{s}$, i.e., $h_s(\cdot) > 0$ for $\forall s \in [s_L, \hat{s}]$ and let $s^+ = \hat{s} + \varepsilon$ such that $h(s^+) < h(\hat{s})$. As discussed before, in order to separate all types below $\hat{s}$, $\theta(\hat{s})$ has to be distorted downward and the pair $\{p(\hat{s}), \theta(\hat{s})\}$ has to satisfy buyers' free entry condition: $p(\hat{s}) = \frac{h(\hat{s})}{r} - \frac{k\theta(\hat{s})}{m(\theta(\hat{s}))}$. The utility of type-$\hat{s}$ seller $V(\hat{s})$ in market $\{p(\hat{s}), \theta(\hat{s})\}$ is represented by the blue line in the figure below and buyers' free entry condition is represented by the dash blue line.

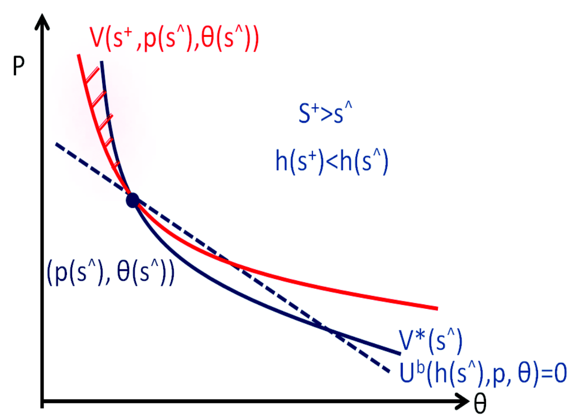

Fig Traders' indifference Curves in $p - \theta$ space

In order to prevent s^+ from going to the market $(p(\hat{s}), \theta(\hat{s}))$, the IC condition requires $V(s^+, p(s^+), \theta(s^+)) \geq V(s^+, p(\hat{s}), \theta(\hat{s}))$ and $\theta(s^+) \leq \theta(\hat{s})$. However, as illustrated in the above figure, for any point $\{p(s^+), \theta(s^+)\}$ satisfying the above conditions, which is in the area of shaded region, buyers' payoff is negative given that $h(s^+) < h(\hat{s})$. Hence, the full

[20]Notice that, different from the standard mechanism problems, the set of feasible mechanisms A is solved subject to the free-entry condition. A fully separating allocation therefore has to solve (8) . If the solution $\theta^*(s)$ to (8) does not satisfy (M), a fully separating scheme is no longer in the set of feasible mechanisms A.

separation can no longer be sustained when the types who are willing to wait longer are not necessarily those with better assets.[21]

This situation could arise when, for example, the holding cost is an arbitrary function of s. More interestingly, as will be shown in section 4, this situation is relevant when sellers' motives for selling are also unobserved by the market. The goal of this section is to establish how the previous analysis can be adjusted to accommodate for more general and abstract settings in which the monotonicity condition, by assumption, is violated exogenously. In Section 4, I apply the results from this section to an environment where such non-monotonicity arises endogenously from the fact that sellers' motives for selling are their own private information.

Since both Lemma 1 and 2 remain intact regardless of the assumption about $h(\cdot)$, one can apply both lemmas to construct semi-pooling equilibria. In particular, any mechanism $\alpha = (p^\alpha, \theta^\alpha)$ that promises the same price and market tightness to a subset of sellers $S' = [s_1, s_2] \subset S$ (i.e., semi-pooling) has to satisfy Lemma 1 and the free entry condition, which describes the set of feasible mechanisms A. Hence, a semi-pooling equilibrium can be found by picking a mechanism $\alpha \in A$ such that buyers, expecting to attract the type which are most likely to come (as implied by Lemma 2), will not deviate by posting any price $p' \notin$ range of p^α. In the following section, I use this logic to establish two approaches to constructing the semi-pooling equilibrium. Notice that, depending on the distribution, the equilibrium is generally not unique. The condition for its existence is identified below for each construction.

3.3.1 Pooling Types

Recall that any feasible mechanism is the one where the allocation satisfies Lemma 1 and where the buyers' free-entry condition holds. For any pooling on the sellers' side, there must be a corresponding expected value of the buyers over such a set of sellers. With this observation, the pooling equilibrium can then be found by *reconstructing* the buyers' valuations. In particular, if we can flatten the buyer's valuations over s (by bunching certain types together on the sellers' sides) in a way such that the buyers' valuation function is (weakly) monotonic in s, as shown in Fig 2a, one can then apply the previous analysis. However, although flattening $h(\cdot)$ directly is convenient to work with, but it

[21] Note that the above argument applies to the case when $h(\cdot)$ has a local maximum so that $\theta(\hat{s})$ is distorted downward. If, on the other hand, the function $h(\cdot)$ has only one local minimum $\hat{s}$ and monotonically increase in s after $\hat{s}$, it is possible to construct a full separating equilibrium with a combination of upward distortion (up to $\hat{s}$) and a downward distorted $\theta(s)$ afterward. Such a special case, however, is excluded in the analysis since it is not relevant for the environment I consider in Section 4.

does not necessarily guarantee a nonincreasing function $\theta^*(s)$. Therefore, the sufficient condition for which $\theta^*(s)$ is non-increasing is also identified below.

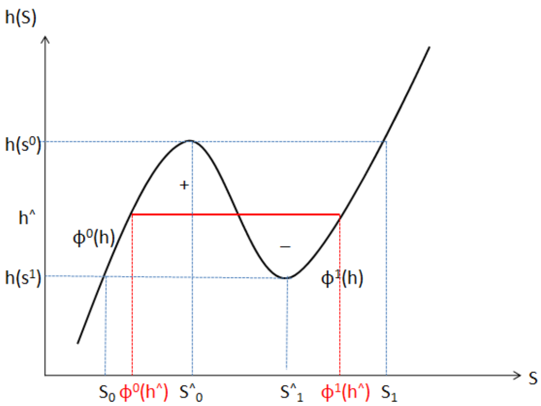

Fig 2a: Reconstructing $h(\tilde{s})$ by bunching types

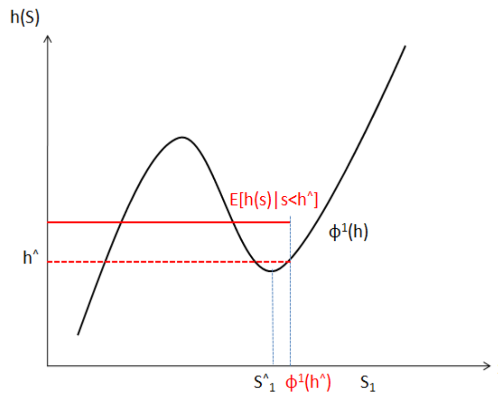

Fig 2b: Equilibrium with upward distorted market tightness

Formally, consider a continuously differentiable function $h(s)$ and assume that $h(s_L) < h(s_H)$ and the function has a finite number of interior peaks on $[s_L, s_H]$. Consider first a function with a single interior peak $\hat{s}_0$ and a single interior trough $\hat{s}_1$. As shown in the Fig 2a, the inverse image of the interval $[h(\hat{s}_1), h(\hat{s}_0),]$ is composed of two intervals, $[s_0, \hat{s}_0]$ and $[\hat{s}_1, s_1]$, over which $h(\cdot)$ is increasing, and one interval, $[\hat{s}_0, \hat{s}_1]$ over which $h(\cdot)$ is decreasing. Let $\phi_0(h)$ and $\phi_1(h)$ denote the inverse functions of h over the intervals $[s_0, \hat{s}_0]$ and $[\hat{s}_1, s_1]$. Let $\hat{h} \in [h(\hat{s}_1), h(\hat{s}_0)]$ solve:

$$H(h) \equiv h - \int_{\phi_0(h)}^{\phi_1(h)} h(\tilde{s}) \frac{dG(\tilde{s})}{G(\phi_1(h)) - G(\phi_0(h))} = 0$$

Given $\hat{h}$[22], a non-decreasing function $\tilde{h}(\cdot)$ can then be reconstructed as follows: let $\tilde{h}(s) = \hat{h}$ for $s \in [\phi_0(\hat{h}), \phi_1(\hat{h})]$ and $\tilde{h}(s) = h(s)$ otherwise. Now suppose that there are two interior peaks. If one could independently design different bunching levels where $\hat{h}_1 \leq \hat{h}_2$, a non-decreasing function $\tilde{h}(\cdot)$ can be constructed in a similar way. If treating the two bunching regions separately yields $\hat{h}_1 > \hat{h}_2$, we must then merge the two into a single bunching level. as long as a non-decreasing function $\tilde{h}(\cdot)$ can be obtained, the solution can be found as follows:

Step 1: For the first interval, $[s_L, \phi_0(\hat{h})]$, let $\theta^*(s)$ solve (8) with the initial condition $\theta^*(s_L) = \theta^{FB}(s_L)$. That is, given that $\tilde{h}(s)$ is strictly increasing over such an interval by construction, the market tightness function can be obtained as before.

[22] $\hat{h}$ might not be unique. It's existence is guaranteed by $h(s_1) > h(s_L)$.

Step 2: At the pooling interval, set $\theta^*(s) = \theta^*(\phi_0(\hat{h}))$ for $\forall s \in [\phi_0(\hat{h}), \phi_1(\hat{h})]$. This means that the allocation is the same among this set of sellers. The free entry condition is satisfied automatically since the expected value of such set of sellers equals $\hat{h}$ by construction and $\theta^*(s)$ equal $\theta^*(\phi_0(\hat{h}))$.

Step 3: In the last region, where $\tilde{h}(s)$ is strictly increasing again, let $\theta^*(s)$ solve (8) and set $\theta^*(\phi_1(\hat{h})) = \theta^*(\phi_0(\hat{h}))$. To ensure that the such solution is non-increasing, the initial condition for $\theta(\phi_1(\hat{h}))$ has to satisfy the following condition:

$$\theta^*(s_0) < \theta^{FB}(\phi_1(\hat{h}))$$

This condition can be understood from (8) , for the solution to be non-increasing, the initial condition must guarantee LHS of (8) is non-negative. In general, this condition then depends on the underlying $h(s)$ and underlying parameters. When this condition holds, our characterization method can be extended to an environment with such non-monotonicity. Note that although the function $\tilde{h}(\cdot)$ is not differentiable at the kinks, i.e., at the boundary points $\phi_0(\hat{h})$ and $\phi_1(\hat{h})$, the left and the right limit exists; hence, the differential equation (8) still applies. In other words, the previous method applies to any monotonically increasing $h(\cdot)$, even when $h(\cdot)$ has kinks. As a result, if the function $\tilde{h}(s)$ can be reconstructed as above, the equilibrium can then be characterized by the constructed $\theta^*(s)$ and the corresponding price, $p^*(s) = \frac{\tilde{h}(s)}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))}$. Such a construction then characterizes a semi-pooling equilibrium.

3.3.2 Equilibrium with upward distorted market tightness

We now focus on a particular type of semi-pooling equilibrium, which has distinct features from the previous analysis. In particular, upward distortion in the market tightness arises in such an equilibrium, which implies sellers can *quickly* unwind their assets (faster than the first best benchmark). Consider the function $h(s)$,which is strictly increasing in s after some point $\hat{s}_1 \in [s_L, s_H]$ and let $\phi_1(h)$ denote the inverse function of h mapping to $s \geq \hat{s}_1$. Define $q(\hat{h}) \equiv \int_{s_L}^{\phi_1(\hat{h})} h(\tilde{s}) \frac{dG(\tilde{s})}{G(\phi_1(\hat{h}))}$ and h^* solves $q(h) = h$. If $q(h) = h$ admits a solution h^*, then any $\hat{h} \in (h(\hat{s}_1), h^*) \Longrightarrow q(\hat{h}) > \hat{h}$, since $h(s)$ is strictly increasing after $\hat{s}_1$ (as shown in Fig 2b). A semi-pooling equilibrium, denoted by $\hat{h}$, can then be constructed as follows:

1) Pool all types below the marginal types which is given by $s^* = \phi_1(\hat{h})$;

2) Set the marginal type s^* indifferent between trading in the pooling market (p_q, θ_q) and his own market with the *first best outcome* $(p^{FB}(s^*), \theta^{FB}(s^*))$;

3) Separate markets for each type above the marginal type with downward distorted market tightness, which solves (8) as before.

Define the pair of functions $(\theta_q(s,\hat{h}), p_q(s,\hat{h}))$ which solves the following equations for any $q(\hat{h}) > h(s)$:

$$\begin{aligned}\theta_q(s,\hat{h}) &= \max_{\theta}\{\theta | V(s,p,\ \theta) = V^{FB}(s) \text{ and } p = \frac{q(\hat{h})}{r} - \frac{k\theta}{m(\theta)}\} \\ p_q(s,\hat{h}) &= \frac{q(\hat{h})}{r} - \frac{k\theta_p}{m(\theta_p)}\end{aligned}$$

That is, the pair $(\theta_q(s,\hat{h}),\ p_q(s,\hat{h}))$ gives a type-s seller his first-best utility. Note that, by construction, $\theta_q(s,\hat{h}) > \theta^{FB}(s)$ and $p_q(s,\hat{h}) < p^{FB}(s)$. A construction for an equilibrium with upward distorted market tightness is formally described in the following proposition:

Proposition 2 *If there exists $\hat{h}$ satisfying*

$$\begin{aligned}q(\hat{h}) &> h(s^*) &\text{(H1)}\\ V^*(s_L, \theta_q(s^*,\hat{h}), p_q(s^*,\hat{h})) &\geq V^{FB}(s_L) &\text{(H2)}\end{aligned}$$

where the marginal type is given by $s^* = \phi_1(\hat{h})$, then a semi-pooling equilibrium $\hat{h}$ with upward distorted market tightness exists. Such an equilibrium is characterized by the following equilibrium market tightness function and the price function:

$$\begin{aligned}\theta^*(s) &= \left\{\begin{array}{cc}\theta_q(s^*,\hat{h}) & \forall s \in [s_L, \phi_1(\hat{h})] \\ \theta(s;\theta_0(s^*)) & \forall s \geq \phi_1(\hat{h}) = s^*\end{array}\right\} \\ p^*(s) &= \left\{\begin{array}{cc}p_q(s^*,\hat{h}) & \forall s \in [s_L, \phi_1(\hat{h})] \\ \frac{h(s)}{r} - \frac{k\theta(s)}{m(\theta(s))} & \forall s \geq \phi_1(\hat{h}) = s^*\end{array}\right\}\end{aligned}$$

where $\theta(s;\theta_0(s^*))$ denotes the solution of (8) with the initial condition $\theta_0(s^*) = \theta^{FB}(s^*)$.

Being able to separate the marginal type s^* from the pooling market is crucial to construct such an equilibrium. In particular, the market tightness θ_q in the pooling market is upward-distorted, $\theta_q > \theta^{FB}(s^*)$ so that the monotonic condition (M) is satisfied. Intuitively, since the incentive constraint always binds with the type with the poor quality asset, the distortion is expected to occur for the types with better quality assets. Notice that in this case, by construction, according to $(H1)$, the average asset quality in such pooling market $q(\hat{h}) \equiv E[h(s)|s_L < s < \phi_1(\hat{h})]$ is higher than the value of the marginal type $h(s^*)$; therefore, the market tightness for the pooling market (better quality assets on average) is then upward-distorted[23] so that the marginal type (with relatively worse quality assets $h(s^*)$) will not mimic a higher type. That is, the marginal type is indifferent between entering the pooling market, in which he sells faster but with a lower price, and entering his own market $(p^{FB}(s^*), \theta^{FB}(s^*))$. Evidently, due to the single crossing property, types below the marginal type are strictly better off going to the pooling market. It is also straightforward to show that such a scheme is incentive compatible for all types above the marginal type, given that $h_s(s) > 0$ for $\forall s > s^*$ and therefore our previous result follows immediately.

The above argument shows that such a scheme is feasible, that is $\alpha \in A$. What is left to show is that buyers do not find it profitable to post any price $p' \notin range\ P^\alpha$. In particular, notice that there is a jump up in the equilibrium price at s^* from p_q to $p^{FB}(s^*)$. However, applying Lemma 2, a buyer will not benefit from raising the price $(p' = p_q + \varepsilon)$ to attract $\phi_1(\hat{h}) = s^*$, nor by lowering the price $(p' = p_q - \varepsilon)$ to attract s_L as he obviously can not do better given $V^*(s^*) = V^{FB}(s^*)$ and $(H2) : V(s_L, \theta_q, p_q) \geq V^{FB}(s_L)$.[24] Furthermore, for the same reason as before, any price $p' = p(s_H) + \varepsilon$ is not profitable as it attracts s_H while resulting in more distortion. As a result, the scheme in the above proposition satisfies both $E1$ and $E2$; therefore, it can be decentralized as a competitive equilibrium outcome.

4 Obscure Motives for Selling

In our baseline model, market liquidity essentially acts as a screening mechanism. Since holding an asset of different quality results in different liquidity preferences, an agent's type is revealed by his choice of which market to trade in. The crucial assumption for this result is that agents' liquidity positions (i.e., the holding costs in the baseline model) are observed. I now analyze the environment in which sellers' exact liquidity positions are not known by the market. For example, as pointed out by Tirole (2010), this situation is relevant when there are difficulties involved in discovering banks' liquidity positions. Any incentive-compatible mechanism must then accommodate this effect. Otherwise, sellers would benefit from appearing fragile in order to get a better price. In order to understand

[23] See Appendix for details.

[24] In the pooling market, the lowest type gets subsidies from the high type and therefore can achieve higher utility than his first best. Furthermore, $V(s_L, p^{FB}(s^*), \theta^{FB}(s^*)) > V^{FB}(s_L)$ is sufficient to guarantee $V(s_L, p^q, \theta^q) > V(s_L, p^{FB}(s^*), \theta^{FB}(s^*))$ as implied by the single crossing property and $\theta^q > \theta^{FB}(s^*)$.

how market liquidity might be affected not only by adverse selection but also by sellers' motives perceived by the market, this section considers an extension where the sellers' holding cost is not known to the market. In other words, there are two dimensions in sellers' types: the asset quality (the common value component) and the liquidity position (the private value component). The goal of this section is to understand how market thickness and equilibrium prices are affected by the combination of these two components. I first show how this setup can be nested into our general model and then discuss how the equilibrium might behavior differently because of the unobserved trading motives.

The setup is similar to our basic model but with the extension that a seller's type now has two components: $z^i = (s^i, c^i) \in Z \equiv S \times C$. As before, the support of s^i is the real interval $S \equiv [s_L, s_H] \subset R_+$, but the support of c^i is some arbitrary set C which can assume discrete or continuous values. A seller's payoff for holding an asset is then governed by both the cash flow s and his liquidity position c. Define type x as $x = s - c \in X \equiv \{x \in R|\ x = s - c,\ s \in S \text{ and } c \in C\}$, representing the seller's value for holding an asset. Intuitively, the mechanism discriminates only on the basis of the sellers' payoffs for owning an asset. Two agents with the same type x must obtain the same utility, irrespective of any other unobservable characteristics that might differentiate the two agents in terms of their attractiveness to buyers. Therefore, this setup can be reconducted to our general model with the following two reinterpretations. First, x is now the relevant seller's type.[25] The utility of seller x, when reporting his type to be $\hat{x}$, thus entering the market $(\theta(\hat{x}), p(\hat{x}))$, is then given by

$$rV(\hat{x}, x) = \frac{x + m(\theta(\hat{x})) \cdot p(\hat{x})}{r + m(\theta(\hat{x}))}.$$

Second, since buyers care only about the asset quality (i.e., the common value component of the seller's type), a buyer's expected value for buying the asset from type x is given by $h(x) = E[s|s - c = x]$,where $h : X \to R_+$. With the above interpretation, we can the apply our previous analysis. Obviously, what matters is the property of the function $h(\cdot)$. If $h(\cdot)$ is monotonically increasing, the sellers' private information about their liquidity positions essentially generates some noise, but the equilibrium continues to exhibit full separation with respect to type x. For a simple illustration, suppose c^i is distributed uniformly over the interval $[c_L, c_H]$ and s^i is distributed uniformly over $[s_L, s_H]$; one can then show that $h(\cdot)$ is increasing over the interval $[s_L - c_H, s_H - c_L]$. Hence, $\theta^*(x)$ can be

[25] The IC condition requires that $V(x, s) = V(x, s')$ and imposes that the allocation $\theta(x, s)$ varies with s only over a countable set of x. In this particular setup, any mechanism in which the allocation is conditional on s besides x necessarily reduces traders' utilities as it leads to more distortions on the sellers' sides.

obtained from the unique solution to the differential equation (8) with initial condition $\theta^{FB}(x_L)$, and the corresponding price is given by $p(x) = \frac{h(x)}{r} - \frac{k\theta^*(x)}{m(\theta(x))}$. The interesting case is when the function $h(\cdot)$ is not monotonically increasing, which is what we consider below.

4.1 An Example of Non-monotonicity

In this section, we consider a simple example in which the type who is willing to wait longer (a higher x) does not necessarily have a better asset (a violation of assumption A1). Consider the simple case in which there are two possible liquidity positions for sellers $C \equiv \{c_H, c_L\}$, where $c_H > c_L > 0$, and let λ denote the probability that the seller who owns the asset s has liquidity position c_H (a higher holding cost). For simplicity, assume that c and s are independent distributedand s is uniformly distributed over the interval $[s_L, s_H]$. The value of $h(\cdot)$ can then be understood as in the left figure below. Observe that the buyers' value function $h(\cdot)$ is not strictly increasing in x. In particular, the expected value of the asset drops in the overlapping region, since the asset could be either a low-quality one owned by a seller with a low holding cost or a high-quality one owned by a seller with a high holding cost. Observe that this drop in the expected value happens whenever c is discrete, regardless of the underlying distribution of s.Notice that the set X is the domain of both functions $h(\cdot)$ and $\tilde{G}(\cdot)$. I start with the case in which $s_H - s_L > c_H - c_L$ so that both $h(\cdot)$ and $\tilde{G}(\cdot)$ have full support. As it will become clear, a fire sale equilibrium always exists when $s_H - s_L < c_H - c_L$. Therefore similar results obtain when this opposite assumption is made.

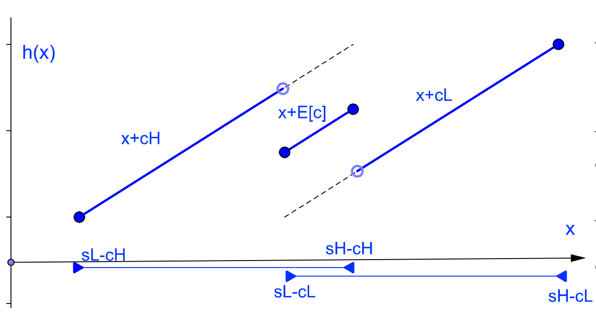

Fig 3a: Buyers' value: $h : X \rightarrow R_+$

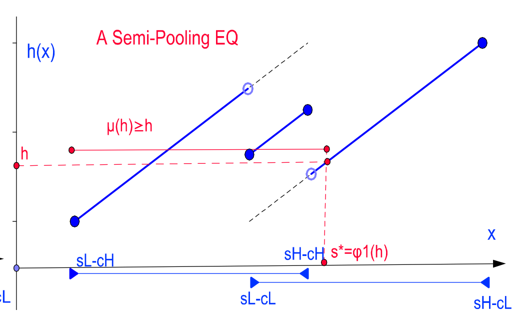

Fig 3b: Constructing a Fire Sale EQ

4.2 Endogenous Fire Sale

According to the previous discussion, multiple semi-pooling equilibria can exist since $h(\cdot)$ does not satisfy assumption A1. In particular, we are interested in the equilibrium in which upward-distorted market tightness occurs, as established in section 3.3.2. Such an equilibrium exhibits the following distinct features: certain types of sellers sell their assets more *quickly at a* price below the fundamental value. I refer to this situation as a *fire sale* of the assets.

In the above example, since $h(\cdot)$ is strictly increasing in x after $x_1 \equiv s_H - c_H$, then, in light of Proposition 2, the semi-pooling equilibrium $\hat{h}$ can be found when there exists an h^* which solves $q(h) = h$. I now provide an example that satisfies this condition.[26] Such pooling scheme $\hat{h} \in (h(x_1), h^*)$ is illustrated in Figure 3b. The corresponding characterization for the equilibrium prices and the market tightness are demonstrated in Figure 4a and 4b.

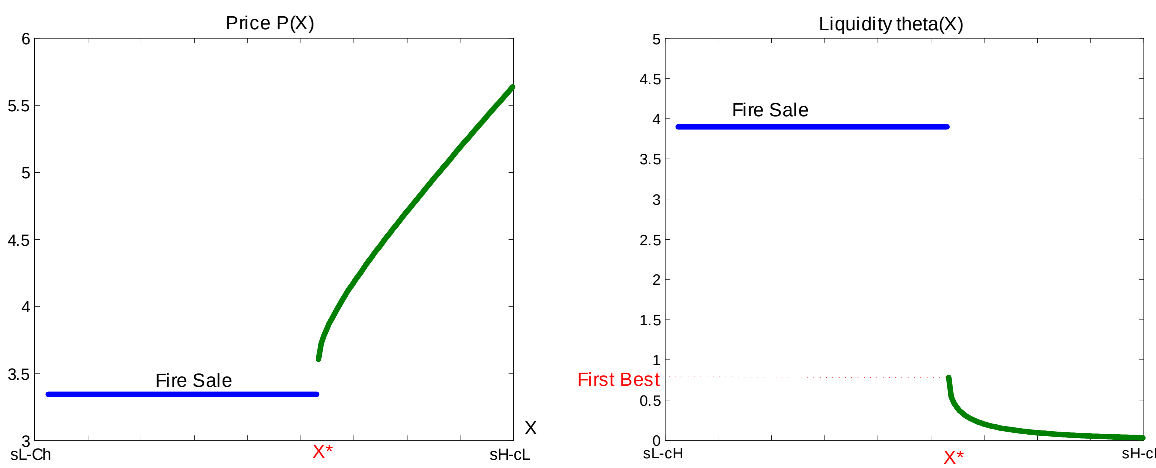

Fig 4: Equilibrim Price $p(x)$ and market tightness $\theta(x)$

The above figures shows the equilibrium price and market tightness ($p : X \to R_+$ and $\theta : X \to R_+$, respectively) in each submarket, with respect to the relevant type x. The *flat* schedule then represents the *pooling* submarket. As stated in proposition 2, the pair (θ_q, p_q) corresponding to in the pooling market has to solve simultaneously (a) the free-entry condition and (b) $V(x^*, \theta_q, p_q) = V^{FB}(x^*)$, subject to $\theta_q \geq \theta^{FB}(x^*)$. In this equilibrium, sellers entering the pooling market can unwind their assets quickly at the

[26] In this simple case with only two values for c, one can show that given $\lambda \equiv \Pr(c = c_H)$, there exists $\bar{\triangle}(\lambda) > 0$ such that a fire sale equilibrium always exists when $c_H - c_L > \bar{\triangle}(\lambda)$. For any $\hat{h} \in (h(x_1), h^*]$, such an equilibrium exists. On the other hand, as shown in Section 3.3.1, other types of semi-pooling equilibria may arise if $h(\cdot)$ can be reconstructed in a way that is weakly increasing.

pooling price $p_q = \frac{q(\hat{h})}{r} - \frac{k\theta^p}{m(\theta^p)}$, which is buyers' expected value from purchasing an asset in this pooling market, $\frac{q(\hat{h})}{r}$, minus the buyers' expected searching cost.

In the canonical model of fire sales, the undervalued price arises because the natural buyers of the assets experience financial distress at the same time as the sellers (see Shleifer and Vishny (1992) and the recent survey by Shleifer and Vishny (2010)). Hence, sellers who have an urgent need for cash will prefer to sell their asset quickly to the low-valuation buyers. The equilibrium outcome in our framework, instead, generates such patterns without relying on an exogenous assumption about market participation. In fact, all potential buyers are unconstrained in our framework and the number of buyers willing to offer a certain price is endogenously determined in equilibrium. For an outside observer, the market features the following behavior: (a) *few* buyers enter the markets for relatively high prices; (b) sellers with a high holding cost choose to sell their assets *fast* at an *undervalued price*, by pooling with worse quality assets.

The model can therefore explain why sellers who become relatively financially constrained may suffer a large drop in their selling price. Note that such an outcome exists only in an environment with adverse selection. Recall that in the first-best benchmark, the trading price is always equate to $p^{FB}(s) = \frac{s}{r} - \frac{k\theta^{FB}(s;c)}{m(\theta^{FB}(s;c))}$, where $\theta^{FB}(s;c)$ denotes the first-best market tightness, given the holding cost c. For any continuous in the holding cost c. In contract, with asymmetric information, the price may be discontinues in the holding cost. To be more specific, consider a seller (s, c_L) such that $s - c_L > x^*$. This is a seller with a relatively good-quality asset who trades at a price $p^*(x) = \frac{x+c_L}{r} - \frac{k\theta^*(x)}{m(\theta(x))}$. Now suppose this seller becomes financially constrained, i.e., his holding cost increases to $c_H > c_L$. This seller will then prefer to sell fast in a pooling market at the price $p_q = \frac{q(\hat{h})}{r} - \frac{k\theta^p}{m(\theta^p)}$. Hence, the price drops and the resulting price difference is larger than the change in the holding cost. In other words, an increase in the holding cost leads to a fire sale.

Remark 1) There are two ways to understand why our result is different from Guerrieri et al. (2010), where the least-cost separating equilibrium is the unique outcome. There are two main assumptions in Guerrieri et al. (2010): monotonicity and sorting. If sellers are ranked by their value for holding the asset x as in the above analysis, sellers can be sorted into different market; however, in such case the buyers' monotonicity condition is not satisfied, given that the buyers' utility is not monotone in x. In particular, in the asset market example given by Guerrieri et al. (2010), the private information is interpreted as the asset quality (the common value components) and the monotonicity condition is assumed to be satisfied. Hence, as in the baseline model, the types who are willing to wait longer are necessarily the ones with better assets. In that case, the monotonicity condition necessarily holds and therefore the equilibrium exhibits full separation. However, as demonstrated in this section, this condition does not necessarily hold when there is asymmetric information on the private value components.

On the other hand, if sellers are ranked by the quality of their asset s, then the monotonicity condition holds but the sorting condition does not in our trading environment (with limited contract space). That is, the only way to screen agents is through their waiting preference, and there is no way of separating agents with better asset quality but a high holding cost from those with a low-quality asset but a low holding cost, given that the net value of holding the asset, and hence the seller's waiting preference is the same. Note that, however, even though all types can be separated, the monotonicity condition still plays an important role. For example, consider $s^i \in \{s_L, s_H\}$ and $c^i \in \{c_L, c_H\}$ and thus there are four values of x. In this case, different sellers can therefore be sorted into different market. However, a semi-pooling equilibrium is obtained if and only if the monotonicity condition is not satisfied.

Remark 2) Notice that, in an environment where traders' liquidity positions are unobserved and where resale is allowed, the screening mechanism discussed in the previous section breaks down even if the original sellers are all homogeneous. The reason is that buyers can renter the market as sellers with a low holding cost. In this case, a full pooling equilibrium might arise, which will then share similar features with the standard lemon problem.

5 Implications/ Application

5.1 The Dispersion of Asset Quality

As shown in the previous analysis, in any separating equilibrium, the market tightness for each s depends only on the support of the distribution $G(s)$ but not on the specific shape of the distribution. The effect of a variation in the support of $G(\cdot)$ on the equilibrium market tightness for each s is summarized in the following proposition.

Proposition 3 *In any separating equilibrium, given any submarket with asset quality s, the resulting equilibrium market tightness $\theta^*(s)$ increases with the quality of the worst asset in the whole market s_L. Formally, let $\theta^*(s; s_L)$ be the solution of (6) satisfying the initial condition $\theta^*(s_L) = \theta^{FB}(s_L)$. For any $s'_L < s_L$, we have that*

$$\theta(s; s'_L) < \theta(s; s_L) \text{ for } \forall s$$

Proof. Let $\theta^*(s_L; s'_L)$ be the equilibrium liquidity of market s_L when the worst asset quality is s'_L. Given that $\theta^*(s_L; s'_L) < \theta_{FB}(s_L) = \theta^*(s_L; s_L)$, the result follows from the Comparison Theorem. ■

It is important to note that this effect only exists in an environment with adverse selection: With complete information, market liquidity is independent of the support $G(\cdot)$. In contrast, with adverse selection, the market thickness for each asset s is pinned down by the quality of the lowest asset in the market s_L as specified by the differential equation (8) . This condition guarantees that (a) sellers have incentives to sort themselves into different markets and (b) buyers are indifferent when it comes to choosing which market to join.

This result has implications for the effect of transparency on market tightness. To see this, suppose that there are two different asset markets $i \in \{a, b\}$ for each asset. The buyers' payoff for each market is given by

$$d_i = y_i + \sigma_i s$$

where $(y_i\ , \sigma_i)$ are publicly observed and s is the sellers' private information, where $s \in [s_L, s_H]$ with some distribution $G(s)$. Since $(y_i\ , \sigma_i)$ are publicly observed, we can imagine there are two separate markets for each asset, and buyers can choose which one to go. Each market is now characterized by (p^i, θ^i), where $i \in \{a, b\}$. We can then use similar methods as in the baseline model to solve for $p^i(s; \sigma^i)$ and $\theta^i(s; \sigma^i)$. Clearly, a higher σ_i has a similar effect as a higher dispersion. It can be easily shown that the more transparent an asset is, the more liquid it is. That is, for any $\sigma_b < \sigma_a$, $\theta^*(s; \sigma_b) > \theta^*(s; \sigma_a)$ $\forall s$.

This result further sheds light on the patterns in cross market liquidity. One important question which has been asked in the literature is why assets paying similar cash flow can have significant differences in their liquidity. We can answer this question using the comparative static exercise above. That is, consider two different asset $(y_i, \sigma_i), i \in \{a, b\}$, such that $E[d_a] = E[d_b]$. Loosely speaking, higher uncertainty about underlying asset quality has a determinantal effect on the overall liquidity of the market, even when the expected value remains unchanged. This implies that assets paying identical cash flows on average can differ significantly in their liquidity, transaction cost, and price, which are all determined endogenously in equilibrium and can be understood as follows:

$$p^i(s) = \underbrace{\frac{d_i}{r}}_{\substack{present\\ value}} - \underbrace{\frac{k\theta^i(s;\sigma^i)}{m(\theta^i(s;\sigma^i))}}_{\substack{transaction\\ \text{cost}}} + \underbrace{\frac{\delta}{r}\left(\frac{c + k\theta^i(s;\sigma^i)}{r + \delta + m(\theta^i;\sigma^i)}\right)}_{\substack{resale\\ value}}$$

Two assets paying the same cash flow $E[d^i]$ may thus have a different liquidity $\theta^i(s;\sigma^i)$. The model therefore predicts different trading patterns (i.e. trading volume, the aggregate prices, and price dispersion) in these two markets.

5.2 Policy Implication: Buyback

This section focuses on the effect of a buyback policy, consisting in cleaning up the market from toxic assets thus improving its liquidity. The idea of cleaning up a toxic asset in the market is not new. In particular, Tirole (2011) shows that government intervention needs to take into account traders' participation constraints and that the government always ends up overpaying for the worst assets. The intuition is the same in this framework. In particular, traders' alternative to joining the government program is trading in an over-the-counter market. Hence, though we do know that cleaning up toxic assets would improve market liquidity, what is important is to understand what *price* has to be paid in order to clean the market. Suppose that the government offers a price p_g to whomever shows up in the discount windows. Anticipating that traders' utilities would increase in the future after government intervention, the original price and market tightness is then no longer incentive-compatible for sellers. This is because a seller can now choose to hold onto the asset and claim to be a higher type in the future. Therefore, to solve for the equilibrium, a mechanism designer needs to take into account the fact that sellers participate in the scheme only if they get at least as much as what they can obtain in the decentralized market.

In equilibrium, the set of types who join the government's intervention program, Ω_g, and the set of types who stay in the decentralized market, Ω_d, are disjoint and satisfy $\Omega_g \cup \Omega_d \equiv S$. Sellers' payoffs can be expressed as:

$$V(s) = \max\{\frac{s-c}{r}, \max_{p'} U^s(p', \theta(p'), s), p_g\}$$

The condition for the marginal type s^* who participates in the government's scheme is then given by

$$V(s^*) = p_g$$

That is, the marginal type has to be indifferent between trading in the OTC market and obtaining the transfer from the government right away. Let (p^*, θ^*) denote the price and the market tightness that the marginal type will face if he goes to the market. Obviously,

$$\begin{aligned} U_g(s) &= p_g > \max_{p'} U(p', \theta(p'); \Omega_d) \text{ for } \forall s < s^* \\ U_g(s) &= p_g < \max_{p'} U(p', \theta(p'); \Omega_d) \text{ for } \forall s > s^* \end{aligned}$$

Also, from the previous discussion, we know how to solve the equilibrium outcome $p(\cdot), \theta(\cdot)$, given Ω_d. The key task is to pin down the marginal type, which is characterized as follows:

Proposition 4 *A competitive search equilibrium in the presence of a government buyback policy with price p_g, is characterized by a threshold s^* which solves*

$$V(s^*) = V^{FB}(s^*) = p_g$$

and a pair of functions $(p^*(s;\Omega_d), \theta^*(s;\Omega_d))$ for $s > s^*$ that jointly satisfy (5), (6), (7), such that all sellers with asset quality $s < s^*$ participate in the government's program; while all sellers with asset quality $s > s^*$ trade in the decentralized market with market tightness $\theta^*(s;\Omega_d)$ at a price $p^*(s;\Omega_d)$.

The red line in the figure below represents traders' utilities under the buyback policy p_g. Given any price offered by the government, the marginal types s has to solve $V^{FB}(s^*) = p_g$, represented by the intersection of p_g and the blue line, which in turn is represented by $V^{FB}(s)$. The black line represents sellers' utilities without intervention.

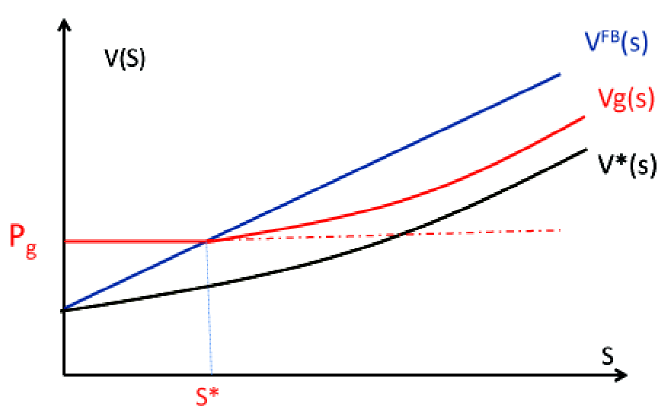

Fig 5: Sellers' utilities with Buyback Policy P_g

5.3 Reallocation and Macroeconomic Performance

As factor reallocation is crucial to aggregate performance, there is a strong link between how an economy performs and how well factors markets function. This section shows how the model delivers implications for the reallocation of firms' corporate assets. As documented in the empirical literature, changes in the ownership of firms' corporate assets—for example, product lines, plants, machines, and other business units—affect productivity. More precisely, capital typically flows from less to more productive firms, and this results in an increase in productivity. Furthermore, as suggested by the empirical findings, market thinness generates frictions that are a large impediment to the efficient reallocation of capital, even within a well-defined asset class, in which capital is moderately specialized. This model then provides a possible explanation as to why the market for used capitals is relatively thin. Severe trading delays would result in resource mismatch and have a negative impact on aggregate performance. This paper then elucidates the underlying source of market frictions and its link to economic fluctuation

To illustrate our result, suppose that there are two possible technologies $j \in \{H, L\}$. More productive firms which will play the role of buyers, produce according to the production function $h(s) = a_H s$, while less productive firms, in the role of sellers, produce according to the production function $u(s) = a_L s$, with $a_H > a_L$. Firms who are initially more productive receive a negative shock at the rate δ and become less productive. Let $b_t(s)$ represent the measure of capital s owned by highly productive firms and $g_t(s)$ denote the measure of capital s in the hands of less productive firms. Aggregate production can then be defined as follows:

$$\bar{A}_t = \int \{a_L s g_t(s) + a_H s b_t(s)\} ds \tag{13}$$

Both aggregate production and the cross-sectional distribution of active firms are endogenously determined in equilibrium. Interestingly and probably counter-intuitively, at the macro level, Eisfeldt and Rampini (2006) documented that capital reallocation is procyclical while the cross-sectional dispersion of productivity is countercyclical. Based on this finding, they then suggested that the reallocation friction is countercyclical and indicates that it is important to have a good foundation for such countercyclicality. Contrary to most macro models assuming exogenous adjustment costs, one advantage of our framework is allows for a richer analysis of how this market friction responds to various economic shocks and a better understanding of its aggregate implications.

First of all, consider the effect of a shock to the underlying dispersion of capital quality. From the previous analysis, it is clear that an increase in dispersion would increase market frictions. Hence, the resulting resource mismatch would generate a drop in TFP and further increase the cross-sectional dispersion of productivity. It therefore provides an explanation for the coexistence of the countercyclical dispersion and procyclical reallocation, as documented in the empirical literature. It is definitely an interesting extension to endogenize the underlying dispersion and worth further exploration.

In line with the growing literature on uncertainty (e.g., Bloom (2009)), the model can also be used to examine the effects of a shock affecting the probability that firms' productivity drops. Intuitively, a higher δ, that is a higher level of instability of business condition would decrease investors' willingness to enter. Hence, the market liquidity also decreases in δ, resulting in worse aggregate performance and a higher level of dispersion across firms. The story here, however, is different from Bloom (2009). Bloom (2009) shows that in the presence of capital adjustment costs, higher uncertainty (measured as a shock to the second moment) expands firms' inactive regions because it increases the real-option value of waiting. This in turn slows down the reallocations from low to high productivity firms. By endogenizing the adjustment cost, one reason for the difference from my result is that Bloom (2009) assumes exogenous adjustment costs. In my framework, firms who receive a negative shock will want to exit but have a hard time finding an investor who is willing to buy their capital. This idea also explains why few firms exit in bad time, as documented in Lee and Mukoyama (2008).[27]

6 Discussion

6.1 Discussion on Efficiency

Does the decentralized equilibrium outcome in our baseline model also solve the centralized planner's problem? The answer can be seen from our solution method. As explained above, among the set of feasible mechanisms A defined by Lemma 1, the decentralized outcome is the one satisfying the buyers' optimality constraint, $E2$. That is, given that buyers have the freedom to post new prices in a decentralized market, condition $E2$ is an

[27] It is important to note, however, that an increase in the downward uncertainty, also results in a higher demand for reallocation. Hence, the net effect on capital reallocation is ambiguous. As emphasized by Bachmann and Bayer (2009), a large countercyclical second moment shock would be incompatible with procyclical investment dispersion. However, the shock to the downward uncertainty considered here is different from the second moment shock. It is important to understand that it has two opposing effects and that its net effect will depend on parameters values. As a full calibration is beyond the scope of this paper, the main purpose of this exercise is to understand how uncertainty affects market frictions, as it can generate significant fluctuations in an environment with adverse selection.

additional constraint that is absent in the planner's problem. This implies that a social planner can always do (weakly) better than the market. In fact, in our baseline model, a fully pooling equilibrium always yields first best welfare as long as it is sustainable. The reason is that the first best level of market tightness is independent of asset quality. A pooling equilibrium, simply subsidizing some sellers at the expense of others, therefore does not induce any distortion as long as the participation constraint of the highest types is satisfied.

The above discussion then leads to the following question: Is the fully separating equilibrium outcome Pareto efficient? Or, is it Pareto dominated by the outcome of a (partial) equilibrium? The answer depends on the asset distribution, as reflected in the baseline model. First of all, recall that the outcome of separating equilibrium depends only on the range of the distribution of asset value. On the other hand, traders' utilities in any kind of pooling equilibrium will obviously depend on the shape of the distribution. In particular, from a high-type seller's viewpoint, the lower the mean, the more distortion on the price. Hence, given the range of the asset distribution, whether a high-type seller is better off in the separating equilibrium or in a pooling equilibrium depends on the shape of the distribution. This point then explains why the competitive search equilibrium is Pareto inefficient for some parameter values, as shown in Guerrieri et al. (2010). The important lesson is that the equilibrium outcome of a fully separating equilibrium is not necessarily constrained Pareto efficient. The main reason is that (partial) pooling cannot be sustained even when it is desirable. The resulting distortions in market tightness can be rather costly.

6.2 Asymmetric information on Trading Motives

As shown in Section 4, a (semi) pooling equilibrium can be sustained when the private value of holding an asset is also sellers' private information. According to the above discussion, asymmetric information on the sellers' trading motives can therefore increase welfare when the pooling outcome is desirable. For example, in the model with fire sales, only the types with *low* holding cost and relatively good asset qualities are separated while the rest are pooled. Sellers' utilities in such an equilibrium are shown in Figure 6 below and corresponding to the red line.

Now compare with the environment where sellers' holding cost is observable. The outcome can then be solved as in the baseline model with different holding cost. The utility of type-s seller with holding cost c is then denoted by $V^*(s;c)$, which is represented by the black line. First of all, all sellers with *low* holding cost are better off without the disclosure of the holding cost: the ones with relatively high-quality assets ($s - c_L > x^*$) are better off as the underlying range effectively decreases. (Notice that equilibrium outcome for this subset can be solved as if the worst asset is $c_L + x^*$ instead of s_L). The ones with relatively low-quality asset ($s - c_L < x^*$) are also better off compared to benchmark as they effectively receive subsidies from sellers with better assets. For the same reason, the types with worse assets and a high holding cost are also better off. Hence, the only one who might suffer when the holding cost is unobserved is the type with good assets but a *high* holding cost.

Nevertheless, as discussed earlier, depending on the asset distribution, the high-type seller in such a pool can be better off than the separated equilibrium. If that is the case, counterintuitively, all types of sellers achieve *higher* equilibrium utilities when the holding cost is *unobserved*. Whether the high-type seller is better off or not depends again on whether a price discount (pooling) or distorted market tightness is more costly.

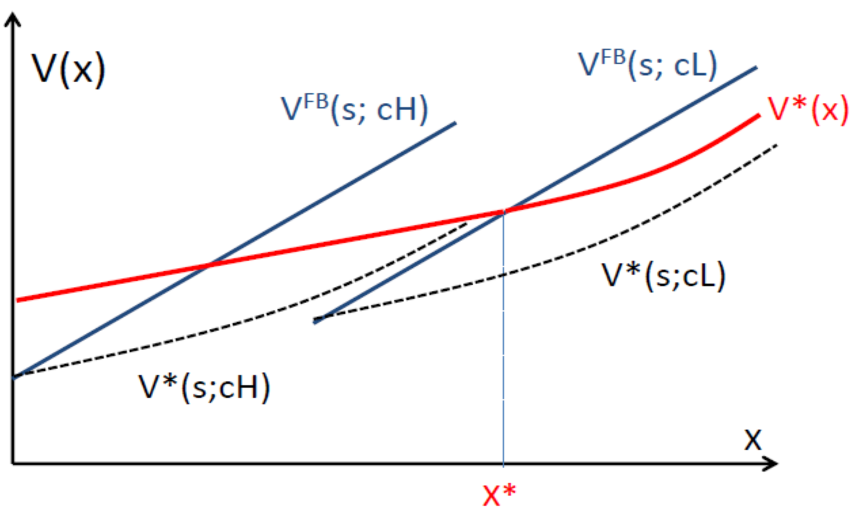

Fig 6: Sellers' Utilities

6.3 On the Searching Cost and the Elasticity of Matching function

In this section, we are interested how the degree of the distortion on market liquidity corresponds to the change in the searching cost k and the parameter ρ, which controls the concavity of the matching function $m(\theta) = \theta^\rho$ and $0 < \rho < 1$. Since, from sellers' view point, what matters is really the the arrival rate of a buyer, given by $m(\cdot) \equiv m(\theta(s))$, both κ and ρ do not play direct roles when solving sellers' IC constraint, as shown in

$(ICFOC)$. However, these two parameters control buyers' marginal rate of substitution of m and p: $\frac{dp(s)}{dm}|_{Ub=0} = -\frac{k(1-\rho)}{m}$. Due to the nature of the matching, the equilibrium function $p(\cdot)$ and $m(\cdot)$ are solved subject to buyers' free-entry condition and, therefore, through this channel, κ and ρ have impacts on the resulting distortion of m.

The level of the first best liquidity is a function of k and ρ, according to equation (1). However, the real interest is to see how the degree of distortion relative to the first best changes. To this end, I define $d(s, \rho, k) \equiv 1 - \frac{m^*(s,\rho,k)}{m^{FB}(\rho,k)}$, which represents the percentage decrease of the market liquidity as a result of adverse selection. Note that, in our basic model, $m^{FB}(\rho, k)$ is independent of type.

As shown in the proposition below, the degree of the liquidity distortion decreases with k and increases with ρ. The formal proof is in the appendix while the intuition is the following: A higher search cost k (or a lower ρ) means a buyer is more willing to pay for a higher price in exchange for a lower m. Hence, facing the schedule $\{p(\cdot), m(\cdot)\}$, if buyers with a low k are indifferent among all submarkets, buyers with a higher k will be strictly better off in the market with lower m. Hence, the decrease in the distortion has to be lower in the economy with a higher k.

Proposition 5 *The degree of the liquidity distortion* $d(s, \rho, k) \equiv 1 - \frac{m^*(s,\rho,k)}{m^{FB}(\rho,k)}$ *decreases with the searching cost* k *and increases with* ρ.

Same intuition holds for the parameter ρ. Notice that in the model of rationing, as in Guerrieri and Shimer (2011), buyers match with probability one as they are on the short side of the market. That is, effectively, buyers do not care about the market tightness as there is no congestion effect on the buyers' side (the short side of the of the market). One can therefore understand the result in Guerrieri and Shimer (2011) as the limit economy in our model when $\rho \to 1$. As implied by the above proposition, the degree of the resulting distortion is expected to be larger in Guerrieri and Shimer (2011).

7 Conclusion

Using a competitive search framework, this paper analyzes how asymmetric information on the common value, as well as the private value, leads to limited market participation or an undervalued price. It provides an explicit and exact meaning for different notions of liquidity– trading price and market thickness– and therefore captures the idea that sellers care about the selling price as well as how fast they can unload the asset in the decentralized market. Contrary to the standard lemon model, in which trade is usually assumed to take place at one price, in my framework trade is allowed to take place at different prices, and the possible set of prices offered and their corresponding market thickness are jointly determined. In equilibrium, market thickness works as a screening device since owning assets of different quality generates different waiting preferences. How much the market can screen the agents depends on the matching between sellers' waiting preferences and buyers' willingness to pay. If the types willing to wait longer are necessarily the ones with better assets (i.e. only the information on the common value is asymmetric), then a unique full-separating equilibrium is obtained. In that case, the market thickness of the submarkets with better-quality assets, is downward-distorted so that sellers with low-quality assets will not mimic ones with high-quality assets. Market illiquidity arises endogenously as an equilibrium outcome and manifests as market thinness. It therefore predicts a sharp decrease in trading volume and a drop in the aggregate trading price when information quality worsens.

On the other hand, if a sellers' private value of the asset is also unobserved by the market and, depending on the underlying distribution, the types who are willing to wait longer are not necessarily the ones with better assets, then the full screening cannot be sustained. Instead, a set of semi-pooling equilibria arise. In particular, we are interested in the equilibrium which depicts the phenomenon of a fire sale: as an equilibrium outcome, buyers who offer a relatively high price are *sidelined*; sellers who are in relatively urgent need for cash unload their assets more *quickly* at an *undervalued price* since they end up pooling with the low-quality assets. In this pooling submarket, sellers with high-quality assets will then suffer a price discount in line with the standard lemon model. Compared to the previous literature, the main contribution of this work is to show how different market distortions on price and market thickness arise in different informational settings. It therefore separately identifies the effects of adverse selection on trading price, trading volume, and market segmentation, and further sheds light on the limited market participation of buyers and the infrequent trading observed in the recent asset markets.

8 Appendix

8.1 Supplemental material on Heterogeneous Buyers

8.1.1 Theoretical extension with heterogeneous buyers

The setup is now chosen to allow for heterogenous buyers in the market so that it can be easily applied to trading environments with two-sided heterogeneity. Many decentralized markets have this feature. Understanding the trading pattern is crucial since it determines the allocation and therefore welfare. For example, in the factor market, the resource allocation determines aggregate productivity. Different companies might have different technology to utilize the assets (machine or capital). Productivity of the assets is determined by assets allocation, which is mainly governed by both the pattern of trade and the equilibrium liquidity. With this generalization, the model further sheds light on the sorting pattern. We show that, supermodularity in the matching value is enough to guarantee positive sorting, which is a distinct feature compared to the environment without adverse selection.

Consider that there are two types of buyers, $b^i \in \{b^h, b^l\}$ and buyers' type are observable. For simplicity, we assume that the measure of each type is larger than the one of sellers and the outside option of buyer b^i is given by $\phi(b^i)$[28]. The flow payoff of an asset owned by buyer b^i and bought from seller s is given by $h(b^j, s)$, where h shares the assumption as our basic model. The indices s and b^i that are ordered such that they increase the utility of sellers: $h(b^h, s) > h(b^l, s)$.For example, $h(b^i, s)$ represents the payoff produced by the firm with technology b^i and asset quality s.The simple functional form widely used for a macroeconomic model with heterogenous firms is usually given by $h(b^j, s) = b^j s$, which can be seen as the productivity. Furthermore, we assume that there is complementarity in the matching function.

Assumption 3: $h_s(b^h, s) - h_s(b^l, s) > 0$

Obviously, if $\phi(b^h) < \phi(b^l)$, sellers then always obtain higher value if trading with the higher type buyer and one can easily show that, facing resulting (p, θ), lower type buyer will not enter the market. In that case, the environment can be trivially solved just like as with homogenous buyers. The following analysis focus on the relevant environment in which both type of buyers are *active* in the market when there is no adverse selection. One can establish the benchmark similar as before. It is well known that the equilibrium outcome can be thought of as a competitive market maker who promises traders the price, the market tightness, as well as the trading pattern. The equilibrium will then consist of a price function $p^{FB}(s)$, a market tightness function $\theta^{FB}(s)$,trading pattern $j^{FB}(s)$ and the corresponding sellers' utility function $V^{FB}(s)$, which solve following the optimization problem:

$$V^{FB}(s) = \max_{j,p,\theta}\{\frac{u(s) + m(\theta)p}{r + m(\theta)} : U_b(p, \mu, \theta, b^j) = \phi(b^j)\}$$

[28]This assumption is made to simplify the analysis. One can interpret this as a partial equilibrium where we take the level of buyers' utilites as given.

The algebra detail is left in the appendix. In words, given buyer type b^i, one can solve the optimization problem as before. Similarly, the first best market tightness should be a function of the ratio of the gain from trade over the searching cost, which are type dependent and is denoted as $R(j,s) \equiv \frac{g(j,s)}{k+r\phi(b^j)}$. Let the function $V^{FB}(s,b^j)$ represent seller's utilities if traded with type b^j under perfect information. The first best utilities $V^{FB}(s)$ can be seen as a upper envelope of $V^{FB}(s,b^h)$ and $V^{FB}(s,b^l)$. That is, $V^{FB}(s) = \max_j\{V^{FB}(j,s)\}$. As it will become clear later, the interesting case is when there exists a marginal type $s^{FB} \in S$ who is indifferent to trading with high type and low type buyers and for a seller with assets $s < s^{FB}$, he will only trade with a lower type buyer and vice versa for sellers with assets $s > s^{FB}$[29]. The following section therefore focuses on such an environment and discusses how other cases can be solved accordingly.

As sellers do not care about buyers' types, sellers' expected value in each market is the same as before. On the other hand, type-j buyers' expected payoff can be expressed as follows:

$$rU^b(p,\mu,\theta(p),b^j) = -k + \frac{m(\theta(p))}{\theta(p)}(\int \frac{h(b^j,\tilde{s})}{r}\mu(\tilde{s}|p)d\tilde{s} - p - U_b)$$

Definition 3 *An equilibrium consists of a set of offered price P^*, a market tightness function in each market p, $\theta(\cdot) : P \to [0,\infty]$, the conditional distribution of sellers in each submarket $\mu : S \times P^* \to [0,1]$ and the trading pattern $j^* : P \to \{h,l\}$, such that the following conditions hold:*

$E1$ (optimality for sellers): let

$$V^*(s) = \max\{\frac{s-c}{r}, \max_{p' \in P^*} U^s(p',\theta(p'),s)\}$$

and for any $p \in P^*$ and $s \in S$, $\mu(s|p) > 0$ implies $p \in \arg\max_{p' \in P^* \cup \varnothing} U^s(p',\theta(p'),s)$

$E2$ (optimality for buyers and free-entry): for any $p \in P^*$ and $j \in \{h,l\}$

$$U_b(p,\mu_p,\theta(p),b^j) \le \phi^j$$

with equality if $p \in P^*$ and $j = j^*(p)$; and there does not exist any $p' \in P$ such that $U_b(p',\theta(p'),\mu_{p'},b^j) > \phi^j$, where $\theta(p')$ and $\mu(s|p')$ satisfies (2) and (3)

[29]It is clear that $R(h,s^{FB}) < R(l,s^{FB})$, given $\phi^h > \phi^l$. Therefore, $\theta(h,s^{FB}) < \theta(l,s^{FB})$ and $p(h,s^{FB}) < p(l,s^{FB})$. Namely, there will be two separating markets for the asset s^{FB}. These two markets are different from the trading price and the liquidity, between which the seller s^{FB} is indiscriminate. High type buyers will pay more for the good with shorter waiting time in one market and, vice versa for the low type buyers in the other market.

Clearly, IC constraints for sellers are the same as before, that is, Proposition 1 still holds. The only difference is that we need to make sure the buyers' optimality condition will hold for both types. In particular, facing the price and market tightness recommended by the market maker, a buyer will benefit neither from going to the markets which belong to the other buyers, nor from opening a market which has not been open. The mechanism can be interpreted as follows: given $(p(s), \theta(s))$,a seller reports his type $\hat{s}$ optimally; meanwhile, $j^*(s)$ denotes the sorting pattern recommended by the market maker, who recommends buyers $j^*(s)$ post the price $p(s)$, that is, entering the market $(p(s), \theta(s))$. The sets of types who trade with the lower type buyer, $\Omega_L = \{s : j^*(s) = l\}$, and of those who trade with the high type, $\Omega_H = \{s : j^*(s) = h\}$, are disjointed and satisfy $\Omega_L \cup \Omega_H \equiv S$. Then, define s^* as the marginal type $j^*(s^*) = \{l, h\}$. Obviously, some lessons learned from the basic model are still applied: there is no submarket involving pooling under assumption 1b) and, hence, we can focus on the full separation on the sellers' sides. From buyers' view points, each market can therefore be characterized as a pair of (p, θ, s). Given (p, θ, s),buyers will choose to go to the preferred markets and expect to trade with seller s.

Moreover, once we identify the set of sellers who trade with buyers j, $\Omega_{\ j}$, the market tightness can be solved as in the case in which there is only one type of buyer j. Given Ω_j, the solution of $\theta(s; j)$ needs to the following differential equation, which is similar to (8) but taking into account that buyers' heterogeneity

$$[(h(b^j, s) - u(s) - r\phi(b^j)) + \frac{k + r\phi(b^j)}{\rho}((\rho - 1)\theta - \frac{r\theta}{m(\theta)})]\frac{d\theta}{ds} = -(r + m(\theta)) \cdot \frac{\theta}{\rho}\frac{h_s(b^j, s)}{r} \quad (14)$$

As before, the corresponding price schedule $p(j, s)$ is then pinned down with the free entry condition:

$$p(s, j) = \frac{h(a^j, s)}{r} - \frac{(k + r\phi^j)\theta(s; j)}{m(\theta(s; j))} - \phi_j \quad (15)$$

Notice that solutions can be easily characterized once we have the initial condition for $\theta(s; j)$. Therefore, the key remaining task is essentially finding out the set Ω_j, that is, the marginal type s^* and identifying the initial condition $\{\theta_L^0, \theta_H^0\}$, which gives $\theta(s_L; j) = \theta_L^0$ and $\theta(s^*; j) = \theta_H^0$. For notation convenience, let $p^j(s), \theta^j(s)$ denote the price and the market tightness in the market with buyer type j. In equilibrium, it must be the case that the buyer j will not enter the market where $j^*(s) \neq j$. Hence, following constraints must be satisfied:

$$\begin{aligned} U_b(p^h, \theta^h, s, b^l) &< \phi^l \text{ for } j^*(s) = h \\ U_b(p^l, \theta^l, s, b^h) &< \phi^h \text{ for } j^*(s) = l \end{aligned}$$

To facilitate the analysis, define $\tilde{\theta}(s)$ to solve the following:

$$\begin{aligned} \phi^l &= U_b(p^h, \theta, s, b^l) \\ &= U_b(p^h, \theta, s, b^h) - \frac{q(\theta)}{r+q(\theta)}\left(\frac{h(b^h,s) - h\left(b^l,s\right)}{r}\right) \\ &= \phi^h - \frac{q(\theta)}{r+q(\theta)}\left(\frac{h(b^h,s) - h\left(b^l,s\right)}{r}\right) \end{aligned} \tag{16}$$

where $q(\theta) = \frac{m(\theta)}{\theta}$. Given that $h(b^h,s) - h\left(b^l,s\right)$ increases with s, $\tilde{\theta}(s)$ increases with s. This function then plays an important role in determining buyers' incentive constraint. Entering the high-type buyers' markets, the difference in utilities gain is characterized by the second term, $\frac{q(\theta)}{r+q(\theta)}\left(\frac{h(b^h,s)-h\left(b^l,s\right)}{r}\right)$, which captures low types' disadvantage. The impact of this disadvantage is higher when the expected waiting time for buyers is shorter, that is, for the higher $q(\theta)$ and hence the lower θ. As a result, for any $\theta < \tilde{\theta}(s)$, the low type will not mimic high type to enter the market. Similarly, when a high-type buyer contemplates entering a low-type market, he will only enter when $\theta < \tilde{\theta}(s)$ so that his advantage is high enough to compensate[30]. Hence, we can conclude the following claim:

Claim 5 *In equilibrium, the market (p, θ, s) attracts high-type buyers but not low-type buyers if $\theta < \tilde{\theta}(s)$; similarly, the market (p, θ, s) attracts low-type buyers but not high-type buyers if $\theta > \tilde{\theta}(s)$.*

Denote the function $\theta_j^{FB}(s), V_j^{FB}(s)$ as the market tightness and sellers' utility, respectively, when trading with buyer j with *complete information.* We next prove that the equilibrium can be characterized by following proposition.

Proposition 6 *The unique solution to the mechanism is a market tightness function $\theta : S \to R_+$, a price schedule $P : S \to R_+$, a marginal type s^*, a pair of initial condition $\{\theta_L^0\ , \theta_H^0\}$, where:*

$$\begin{aligned} \theta^*(s) &= \begin{Bmatrix} \theta(s,l;\theta_L^0), \ \textit{for } s \le s^* \\ \theta(s,h;\theta_H^0), \textit{for } s \ge s^* \end{Bmatrix}; p^*(s) = \begin{Bmatrix} p(s,l), \ \textit{for } s \ \le s^* \\ p(s,h), \textit{for } s \ge s^* \end{Bmatrix} \\ V^*(s) &= V_L^{FB}(s_L) + \int_{s_L}^{s} \frac{u'(\tilde{s})}{r+m(\theta^*(\tilde{s}))} d\tilde{s} \end{aligned}$$

where $\theta(s,j;\theta_j^0)$ is the solution to (8) *with the initial condition: $\theta(s_L,l) = \theta_L^0, \theta(s^*,h) = \theta_H^0$, and corresponding $p(j,s)$ is defined in* (15).

[30]One can show that the utility of a high-type buyer entering a low-type market is:$U_b(p^l,\theta,s,a^h) = \phi^l + \frac{q(\theta)}{r+q(\theta)}\frac{h(a^h,s)-h(a^l,s)}{r}$,which is bigger that ϕ^h iff $\theta < \tilde{\theta}(s)$.

a) The initial condition θ_l^0*:*

$$\theta_l^0 = \theta_l^{FB}(s_L)$$

b) The marginal types:

$$s^* = \begin{Bmatrix} s^A, \text{ if } \tilde{\theta}(s^A) \geq \ \theta_H^{FB}(s) \\ s^B, \text{ if } \tilde{\theta}(s^A) < \theta_H^{FB} \ (s) \end{Bmatrix}$$

where (s^A, s^B) is the unique[31] solution to the following equation:

$$\begin{aligned} s^A &: V(l,s) = V_L^{FB}(s_L) + \int_{s_L}^{s} \frac{u'(\tilde{s})}{r + m(\theta^*(\tilde{s}, l; \theta_L^0))} d\tilde{s} = V_H^{FB}(s) \\ s^B &: \tilde{\theta}(s) = \theta(s, l; \theta_L^0) \end{aligned}$$

c) The initial conditions θ_H^0:

$$\theta_H^0 = \begin{Bmatrix} \theta_H^{FB}(s^*), & \text{if } s^* = s^A \\ \tilde{\theta}(s^*), & \text{if } s^* = s^B \end{Bmatrix}$$

See omitted proof in the next section. As explained earlier, once we can separate the buyers from different markets, we can apply the method for homogenous buyers separately. Therefore, the equilibrium solution is expected to be a combination of two. However, it has to be combined in a particular way so that traders' optimality conditions hold. In appendix, we prove that the constructed solution above is the unique solution.

8.1.2 On Sorting Behavior

Shi (2001) and Eeckhout and Kircher (2010) have shown that the complementarity in production is not enough to guarantee positive assortative matching (PAM) in an environment with complete information. The intuition is that, given that the social surplus increases with types, it could be optimal to match high-type seller with a low-type buyer by promising him a tight market, that is, a higher utilization. The above intuition still holds in our framework with complete information. However, with adverse selection, we prove that the supermodularity of the matching value necessarily induces PAM in the equilibrium.

Proposition 7 *In the competitive search equilibrium with adverse selection, the equilibrium trading pattern* $j^*(s)$ *satisfies PAM, that is, for* $s' > s$, $j^*(s) = h \Longrightarrow j^*(s') = h$ *under the assumption* $h_s(a^h, s) - h_s(a^l, s) > 0$,

[31] Observe that (s^A, s^B) is unique (and all smaller than s^{FB}). Notice that, $V(l,s), V_H^{FB}(s), \tilde{\theta}(s), \theta_H^{FB}(s)$ are all well defined and monotonically increases in s and $\theta(s, l; \theta_L^0)$ is strictly decreasing in s.Given that $\tilde{\theta}(s_L) \leq \theta_L^{FB}(s_L)$ under the assumption $V_L^{FB}(s_L) > V_H^{FB}(s_{L.}) \Longrightarrow s^B$ always exists and is unique.

Proof. Suppose Not. There exists $s' > s$ such that $j^*(s) = h$ and $j^*(s) = l$. According to Claim 1, the equilibrium market tightness must satisfy: $\theta^*(s) \leq \tilde{\theta}(s)$ and $\theta^*(s') \geq \tilde{\theta}(s')$. Moreover, from the monotonic condition, (M), $\theta^*(s) \geq \theta^*(s')$. The above relation then implies $\tilde{\theta}(s) \geq \tilde{\theta}(s')$. This is a contradiction to the fact that $\tilde{\theta}(s)$ is strictly increasing with s under the assumption $h_s(a^h, s) - h_s(a^l, s) > 0$. (Recall $\tilde{\theta}(s)$ solves $\phi^l = \phi^h - \frac{q(\theta)}{r+q(\theta)}(\frac{h(a^h,s)-h\left(a^l,s\right)}{r}))$ ■

To understand this result, recall that the reason as to why a higher type can be better off when trading a low-type buyer is that he can be compensated by a higher utilization. That is, given that a lower type buyer is more willing to wait, it could be optimal for a high type seller to choose to trade with a lower type buyer, enjoying a lower gain from trade but a tighter market compared to trading with a high type buyer. Hence, contingent on negative assortative matching (NAM), a high type seller must be compensated with a higher market tightness compared to low type sellers. This situation, however, can not be sustained in an environment with adverse selection, as it violates the monotonic condition. Namely, it is not incentive compatible for the sellers. Notice that in an environment with complete information, a high type seller prefer to trader faster as his gain from trade is higher. Nevertheless, with adverse selection, when all sellers are facing the same market price schedule and market tightness, a high type seller becomes the one who is more patient in the sense that he will prefer the combination of a higher price and a lower market tightness as contrary to a low type seller. This implies that it would be optimal to match a high type seller with a buyer who is more willing to offer a higher price and less willing to wait. Obviously, a high type buyer is more willing to do this. Hence, a lower type buyer no longer has his advantage to trade with a high type seller as in the case with complete information.

Our solution developed earlier starts with the environment with PAM and $V^{FB}(l, s_L) > V^{FB}(h, s_L)$. However, according to the above Proposition, one should expect that those conditions can be relaxed. First of all, suppose $V^{FB}(h, s_L) > V^{FB}(l, s_L)$, so it is clear that $j^*(s_L) = h$ from Lemma 3 and, clearly, from the above Proposition, $j^*(s) = h$ for $\forall s \in S$. Hence, we can simply solve the model as if there are only high-type buyers in the market, regardless of positive or negative assortative matching under complete information. Suppose that we are now in the environment with NAM, that is, for $s' > s, V^{FB}(l, s) - V^{FB}(h, s) > 0 \Longrightarrow V^{FB}(l, s') - V^{FB}(h, s') > 0$ and $V(l, s_L) > V(h, s_L)$, implying that only low type buyers are active in the case with complete information. Although we do not provide the formal solution for this case, our conjecture tells us that the solution should take similar pattern as the developed method. And, depending on the range of S, it could be the case that $j^*(s) = h$ for some $s' > s$. The above argument shows that adverse selection essentially makes a higher type buyer more likely to stay the market compared to the case with complete information. Notice that this phenomenon can be understood for our main results as well, given that the marginal type decreases in the case of adverse selection, that is, $s^* < s^{FB}$. Hence, more sellers end up trading with high-type buyers with adverse selection.

This result might seem at first counter-intutive but, in fact, it is simply the flip side of market illiquidity. Adverse selection creates a downward distortion of market liquidity, that is, a low ratio of buyers over sellers in the market. This distortion makes it hard for a seller to find a buyer; on the other hand, it also makes it easier for a buyer to find a seller, shortening a buyer's wait time and expected search cost. Given that high-type buyers like to secure trade with high probability and they are willing to pay for this, the environment effectively makes a high type buyer more competitive, compared to low-type buyers.

8.2 Omitted Proof

(A) Proof of Lemma 1:

Proof. Let $V(\theta, s)$ denote $V(p(\theta), \theta, s)$.Observe that $V(\theta, s)$ satisfies following properties: that 1) $V_2(\theta, s)$ exists; 2) has an integrable bound:$\sup_{s\in S} |V_2(\theta, s)| \leq \frac{M}{r}$ for all s, where $M = u'(s_L)$ for; 3) $V(\theta, \cdot)$ is absolutely continuos (as a function of s) for all θ;4) $\theta^*(s)$ is nonempty. Following the mechanism literature, (see Milgrom and Segal (2002)), let

$$V^*(s) = \max_{\hat{s}} V(\theta(\hat{s}), s) = \max \frac{u(s) + p(\theta(\hat{s}))m(\theta(\hat{s}))}{r + m(\theta(\hat{s}))}$$

then any selection $\theta(s)$ from $\theta^*(s) \in \arg\max_{\theta'} V(\theta', s)$,

$$V^*(s) = V^*(s_l) + \int_{s_l}^{s} V_s(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s}$$

Namely, $(ICFOC)$ is the necessary condition for any IC contract. To prove the sufficiency, define function: $x = q(\theta) = \frac{1}{r+m(\theta)}$ and $q^{-1}(x) = \theta$. Also, since $\theta > 0$, $0 < x \leqslant \frac{1}{r}$. One can then easily see $V(x, s)$ satisfies the strict single crossing difference property under the assumption $u'(s) > 0$. For any $x' > x$ and $s' > s$:

$$V(x', s') - V(x', s) + V(x, s) - V(x, s') = x'(u(s') - u(s)) - x(u(s') - u(s)) > 0$$

Therefore, $V(x', s') - V(x, s') > V(x', s) - V(x, s)$. Given that $V(x, s)$ satisfies SSCD condition, then any non-decreasing $x(s)$ combined with $(ICFOC)$ are also sufficient conditions for the achievable outcome. Hence, $x(s) = \frac{1}{r+m(\theta^*(s))}$ has to solve subject to the non-decreasing constraint. Namely, the market tightness function $\theta^*(\cdot)$ has to be non-increasing. ■

B) Proof of Claim 1: No pooling

Proof. Suppose Not: There exists a subset of sellers $s \in S' = [s_1, s_2] \subset S$ are in the same market (p_p, θ_p).From the free entry condition,

$$p_p = \frac{E[s|s \in S']}{r} - \frac{k\theta_p}{m(\theta_p)}$$

and denote $V^{\alpha}(s_2) = V(p_p, \theta_p, s_2)$ as the utilities of the highest type seller in the market. Note that Given $\frac{E[s|s\in S']}{r} < \frac{s_2}{r}$, free entry condition implies $V^{\alpha}(s_2) < V^{FB}(s_2)$. First of all, consider the case that there is a separated market (p_2, θ_2) for s_2 and s_2 is indifferent between going to the this market and the pooling market. That is, define the pair (p_2, θ_2) solves:

$$\begin{Bmatrix} p_2 = \frac{s_2}{r} - \frac{k(\theta_2)}{m(\theta_2)} \\ V(p_2, \theta_2, s_2) = V^{\alpha}(s_2) \end{Bmatrix}$$

Given that $\frac{E[s|s\in S']}{r} < \frac{s_2}{r}$, there exists $p' = p_p + \varepsilon$ such that $p' \notin$range P^{α} and $p_p < p' < p_2$,where θ' solve $V(p', \theta', s_2) = V^{\alpha}(s_2)$ and $\theta_2 < \theta' < \theta^{FB}$.Namely, if a buyer deviates to posting p' , only the highest type in the original semi-pooling market will come. Since buyers attract s_2 and only need to provide $V^{\alpha}(s_2) < V^{FB}(s_2), U_b(p', \theta', s_2) > U_b(p_2, \theta_2, s_2) > 0$. Contradiction. This result can be easily generalized to the environment when the other market also involves pooling, i.e, $p_2' = \frac{E[s|s_2<s<s_h']}{r} - \frac{k(\theta_2')}{m(\theta_2')}$. Note that, given $\theta_2' > \theta_p$ and $\frac{E[s|s_2<s<s_h']}{r} > \frac{s_2}{r} > \frac{E[s|s\in S']}{r} \Longrightarrow p_2' > p_2 > p_p$.Hence, for the same reason, there exists a profitable deviation $p' = p_p + \varepsilon$ for buyers. Lastly, suppose that there are no other markets open for all $s > s_2$. Evidently, posting $p' = p_P + \varepsilon$ is also profitable, given that all the possible types who will come to this market are all weakly better than s_2,i,e, $T(p') = [s_2, s']$, where s' solves $p' = \frac{u(s').}{r}$. ■

C) Proof of Claim 2: the lowest type always receives his first best utility in a separated equilibrium

Proof. Suppose not, pick any initial condition $\theta_0' \in (0, \theta^{FB}(s_L))$ and denote its corresponding market tightness as $\theta'(s; \theta_0')$ and price schedule $p'(s)$. One can easily show that there exists $\tilde{p} = p'(s_L) - \varepsilon$ and, from Lemma 1, $T(\tilde{p}) = s_L$. That is, a buyer can open a new market with lower price and expect the lowest type to come. Due to the violation of the tangent condition at $(p'(s_L), \theta'(s_L))$ when $\theta'(s_L) \neq \theta^{FB}(s_L)$ and $V'(s_L) < V^{FB}(s_L)$, buyers' utility can be improved, $U(\tilde{p}, \theta(\tilde{p}), s_L) > \phi^L$. Contradiction. ■

D) Proof of Claim 3 and 4: From the FOC of the first best solution, one can solve $\frac{d\theta^{FB}(s)}{ds} = \frac{h'(s)-u'(s)}{k(\frac{1-\rho}{\rho})(1+\theta^{-\rho})} \equiv f_2(\theta, s)$. Observe from the differential equation, $\frac{d\theta^*(s)}{ds} =$

$f(\theta,s) \to -\propto$ at $f(\theta^{FB}(s),s)$ given $h_s > 0$. Hence, we know that $\theta^*(s) \leq \theta^{FB}(s)$ for some $s_1 > s_L$. Suppose now $\theta^*(s) > \theta^{FB}(s)$ for some s, which implies that these two function must cross at some point $(\hat{s}, \theta^{FB}(\hat{s}))$ and the slope of the crossing point must be the case that $f_2(\theta^{FB}(\hat{s}),\hat{s}) < f(\theta^{FB}(\hat{s}),\hat{s}) = -\propto$. Contradiction.

E) Proof of Proposition 5 on the degree of distortion:

Proof. Let $\triangle(s;\rho,k) = \frac{m^*(s,\rho,k)}{m^{FB}(s;\rho,k)}$ and rewrite (6) respect to m :

$$\frac{dm(s)}{ds} = \frac{-h_s(s) \cdot m(r+m)/r}{[g(s) - \frac{k}{\rho}(m^{1/\rho}(1-\rho) + rm^{(1-\rho)/\rho})} = F(m;\rho,k)$$

$$\begin{aligned} \frac{d\triangle(s;\rho,k)}{ds} &= \frac{1}{m^{FB}(\rho,k)}\frac{dm^*(s,\rho,k)}{ds} \\ &= F(\triangle \cdot m^{FB}(\rho,k),\rho,k) \end{aligned}$$

Given (a) $\frac{dF(\triangle \cdot m^{FB}(\rho,k),\rho,k)}{dk} = \frac{\partial F}{\partial k} + \frac{\partial F}{\partial m^{FB}}\frac{dm^{FB}}{dk} > 0$ and (b) the initial condition $\triangle(s_L;\rho,k') = \triangle(s_L;\rho,k) = 1$, by the comparison theorm, $\triangle(s;\rho,k') > \triangle(s;\rho,k)$ for $\forall s > s_L$. Hence, the distortion $1 - \frac{m^*(s,\rho,k)}{m^{FB}(\rho,k)}$ decreases with k. Similarly, $\frac{dF(\triangle \cdot m^{FB}(\rho,k),\rho,k)}{d\rho} = \frac{\partial F}{\partial \rho} + \frac{\partial F}{\partial m^{FB}}\frac{dm^{FB}}{d\rho} < 0$ and therefore $1 - \frac{m^*(s,\rho,k)}{m^{FB}(\rho,k)}$ increases with ρ

■

F) Note on the non-monotonicity and upward distortion market tightness:

I now introduce following notations. Let $\tilde{V}(s,\theta,\tilde{h}) = \frac{u(s)+m(\theta)(\frac{\tilde{h}}{r})-k\theta}{r+m(\theta)}$. Given any $r\tilde{h} > u(s)$, one can show that $\tilde{V}$ is strictly increasing in θ at the interval $[0,\theta^{FB}(s,\tilde{h})]$ and decreasing in $[\theta^{FB}(s,\tilde{h}),\infty]$, where $\theta^{FB}(s,\tilde{h})$ solves $\frac{r\tilde{h}-u(s)}{k} = \frac{r+m(\theta)-\theta m'(\theta)}{m'(\theta)}$. Define $\theta^d(s,\tilde{h},\tilde{V})$ and $\theta^U(s,\tilde{h},\tilde{V})$ the inverse functions of $\tilde{V}(s,\tilde{h},\cdot)$ over the intervals $[0,\theta^{FB}(s,\tilde{h})]$ and $[\theta^{FB}(s,\tilde{h}),\infty]$. Furthermore, one can see that $\theta^d(s,\tilde{h},\tilde{V}) < \theta^U(s,\tilde{h},\tilde{V})$.

Suppose that there is downward distortion $\theta^*(s) = \theta^d(s,h(s),V) < \theta^{FB}(s)$ for the type s while there exists $s^+ > s$ such that $h(s^+) < h(s)$. Consider a separated market for s and s^+.For s^+ be indifferent between these two markets, the only possible different values for market tightness are given by $\theta^d(s,h(s^+),V)$ or $\theta^U(s,h(s^+),V)$. However, $\theta^d(s,\tilde{h},V)$ is decreasing in $\tilde{h}$ and therefore $\theta^*(s) < \theta^d(s^+,h(s^+),V)$ and $\theta^*(s) < \theta^U(s,\tilde{h},\tilde{V})$.Both of them are contractions to the monotonic condition in Lemma 1. On the other hand, if $\theta^*(s) = \theta^U(s,h(s),V)$, one can set $\theta^d(s,h(s^+),V)$ without violating the monotonic condition. This then explains, in Proposition 2, it is necessary to set $\theta_p = \max_\theta\{\theta | V(s^*,p_{\mu,}\theta) = V^{FB}(s^*)$ and $p_\mu = \frac{\tilde{h}}{r} - \frac{k\theta_p}{m(\theta_p)}\}$. That is, $\theta_p = \theta^U(s^*,\tilde{h},V^{FB}(s^*))$.

References

Akerlof, G. (1970). The market for lemons. *Quarterly journal of Economics 84*(3), 488–500.

Bachmann, R. and C. Bayer (2009). *The cross-section of firms over the business cycle: new facts and a DSGE exploration*. Dt. Bundesbank, Press and Public Relations Div.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*(3), 623–685.

Burdett, K., S. Shi, and R. Wright (2001). Pricing and matching with frictions. *Journal of Political Economy 109*(5), 1060–1085.

Chiu, J. and T. Koeppl (2011). Trading dynamics with adverse selection and search: Market freeze, intervention and recovery. *Working Papers*.

Daley, B. and B. Green (2009). Waiting for news in the dynamic market for lemons. Technical report, mimeo.

Diamond, D. and R. Rajan (2009). Fear of fire sales and the credit freeze.

Duffie, D., N. Garleanu, and L. Pedersen (2005). Over-the-counter markets. *Econometrica 73*(6), 1815–1847.

Duffie, D., N. Gârleanu, and L. Pedersen (2007). Valuation in over-the-counter markets. *Review of Financial Studies 20*(6), 1865.

Eeckhout, J. and P. Kircher (2010). Sorting and decentralized price competition. *Econometrica 78*(2), 539–574.

Eisfeldt, A. (2004). Endogenous liquidity in asset markets. *The Journal of Finance 59*(1), 1–30.

Eisfeldt, A. and A. Rampini (2006). Capital reallocation and liquidity. *Journal of Monetary Economics 53*(3), 369–399.

Gale, D. (1992). A Walrasian theory of markets with adverse selection. *The Review of Economic Studies*, 229–255.

Gale, D. (1996). Equilibria and Pareto optima of markets with adverse selection. *Economic Theory 7*(2), 207–235.

Gorton, G. (2008). The panic of 2007.

Guerrieri, V. and R. Shimer (2011). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse selection in competitive search equilibrium. *Econometrica 78*(6), 1823–1862.

Inderst, R. and H. Muller (2002). Competitive search markets for durable goods. *Economic theory 19*(3), 599–622.

Janssen, M. and S. Roy (2002). Dynamic trading in a durable good market with asymmetric information*. *International Economic Review 43*(1), 257–282.

Kiyotaki, N. and R. Wright (1993). A search-theoretic approach to monetary economics. *The American Economic Review 83*(1), 63–77.

Lagos, R. and G. Rocheteau (2009). Liquidity in asset markets with search frictions. *Econometrica 77*(2), 403–426.

Lee, Y. and T. Mukoyama (2008). Entry, exit and plant-level dynamics over the business cycle.

Lester, B. and B. Camargo (2011). Trading dynamics in decentralized markets with adverse selection.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Moen, E. (1997). Competitive search equilibrium. *Journal of Political Economy 105*(2), 385–411.

Mortensen, D. and R. Wright (2002). Competitive Pricing and Efficiency in Search Equilibrium*. *International Economic Review 43*(1), 1–20.

Shi, S. (2001). Frictional Assignment. I. Efficiency* 1. *Journal of Economic Theory 98*(2), 232–260.

Shi, S. (2009). Directed search for equilibrium wage–tenure contracts. *Econometrica 77*(2), 561–584.

Shleifer, A. and R. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–1366.

Shleifer, A. and R. Vishny (2010). Fire sales in finance and macroeconomics. Technical report, National Bureau of Economic Research.

Tirole, J. (2010). Illiquidity and all its friends. *Fondazione Eni Enrico Mattei Working Papers*, 467.

Tirole, J. (2011). Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning.

Trejos, A. and R. Wright (1995). Search, bargaining, money, and prices. *The Journal of Political Economy 103*(1), 118–141.

Weill, P. (2008). Liquidity premia in dynamic bargaining markets. *Journal of Economic Theory 140*(1), 66–96.

Williamson, S. and R. Wright (1994). Barter and monetary exchange under private information. *The American Economic Review 84*(1), 104–123.

Williamson, S. and R. Wright (2008). New monetarist economics. *Manuscript dated September 17*, 2008.