Joint Dynamics of Costs and Prices in Markets with Search Frictions

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March 2007

Abstract

This paper studies price dynamics in a product markets characterized by: (a) *search frictions*—in the sense that it takes time for a buyer to find a seller that produces a version of the good he likes; (b) *anonymity*—in the sense that sellers cannot price discriminate between first-time buyers and returning costumers; (c) *asymmetric information*—in the sense that sellers are subject to idiosyncratic shocks to their marginal cost of production and privately observe the shocks' realizations. I find that the joint dynamics of costs and prices may be very different than in a standard Walrasian market. When shocks are i.i.d., the price remains constant in the face of fluctuations in a seller's marginal cost. When shocks are moderately persistent, the price adjusts slowly and imperfectly in response to changes in a seller's cost. Finally, when shocks are sufficiently persistent, the price adjusts instantaneously and efficiently as soon as a seller's production cost varies.

JEL Codes: L11, D83

Keywords: Search Frictions, Asymmetric Information, Rigid Prices, Sticky Prices

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1 Introduction

Consider a product market populated by buyers—each demanding one unit of the good at regular intervals of time—and sellers—each producing a different variety of the good. In this market, buyers do not know whether they like the variety produced by a particular seller when they invest some time researching it. Therefore, at any point in time, some of the buyers are matched—they have found a seller they like—and some of them are searching—they have yet to find a seller. Also, in this market, buyers are anonymous and sellers cannot price discriminate between first-time and returning customers. Finally, each seller is subject to idiosyncratic shocks to its cost of production and privately observes it.

In this frictional market, the price demanded by a seller plays three conceptually different roles. First, it determines the distribution of gains from trade between the seller and returning customers. Secondly, it directs the search strategy of unmatched buyers across different sellers and markets. And finally, the price contains information about the seller's cost of production and, in turn, it affects the buyers' expectations about future prices.

In this paper, I develop a simple model of this type of product market and use it to understand the relationship between the dynamics of fundamentals and the dynamics of prices. The analysis is carried out under the maintained assumption that—perhaps because of reputation considerations—sellers can commit in advance to a price schedule which specifies, for each date and public history, a menu of terms of trade from which to choose.

As a preliminary step, I study the optimal price schedule and the equilibrium price dynamics when the realization of productivity shocks is public information. I find that the optimal schedule is time-inconsistent, i.e. after any history and date, the actual price is strictly lower than the one that would maximize the seller's profits from that date on. Intuitively, the optimal schedule is time-inconsistent because part of the benefit of demanding a lower price at some date t is attained before date t—namely, the increase in the number of unmatched buyers searching the seller at dates $0, 1, 2...t - 1$—while all of the cost of demanding a lower price is borne at date t. Moreover, I find that the price is increasing in the contemporaneous realization of the seller's production cost. Because it is time-inconsistent and dependent on the cost of production, this first-best schedule is, in general, infeasible when the seller has private information about his productivity shocks. In particular, the seller would typically want to pretend the cost of production is high in order to increase the price and extract more of the gains from trade from the repeated costumers.

The qualitative properties of the optimal incentive-compatible price schedule turn out to depend on the persistence and magnitude of productivity shocks. When shocks are small and i.i.d. overtime, the incentive-compatible schedule doesn't give any discretion to the seller—i.e. at every date, the menu of choices contains just one item—and the equilibrium prices are rigid—i.e. while the seller's cost of production changes over time, the terms of trade remain constant. When shocks are somewhat persistent, the incentive-compatibly schedule gives the seller limited and history-dependent discretion—i.e. the menu of options has a narrow range and depends on past choices—and the equilibrium prices are sticky—i.e. the adjustment of prices to productivity shocks is spread out over an extended period of time. Finally, when shocks are sufficiently persistent, the incentive-compatible schedule gives the seller maximum discretion—in the sense that the menu contains the same prices available in the first-best schedule—and prices are fully flexible—i.e. as soon as the production cost changes, the price adjusts to its first-best level.

There is a common intuition behind these findings. If a seller with a low cost of production deviates from the equilibrium and chooses the price that buyers expect to observe when the cost is high, we can identify two channels through which this deviation affects profits. First, profits change because the seller charges a higher price in that period. Because of the time-inconsistent nature of the prices problem, this effect increases profits. Secondly, profits change because buyers believe the seller has a high cost and expects higher prices in the future. This second effect reduces profits. When shocks are i.i.d., the second effect is not active and the only way to satisfy the incentive-compatibility constraint is by eliminating discretion. When shocks are somewhat persistent, the second effect is active and it is possible to give some discretion to the seller. When shocks are sufficiently persistent, the effect of current price choices on buyer's expectations is so strong that the first-best price schedule is incentive compatible.

Related Literature. First, this paper contributes to the literature on pricing in markets with search frictions. Diamond (1971) analyzes the case where—because of lack of communication between buyers and sellers before the search decisions are made—prices have exclusively a distributive role. He finds that all sellers set their price equal to the buyers' valuation of the good and extract all the gains from trade. Montgomery (1991), Moen (1997) and Burdett, Shi and Wright (2001) analyze the case where—because buyers perfectly observe prices before deciding where to search and sellers can perfectly identify costumers—prices play only the role of allocating the search effort of unmatched buyers across sellers. They find that sellers compete for prospective buyers and equilibrium prices lie between the competitive and the monopoly levels. In the context of the labor market, Burdett and Mortensen (1998) study the case where—because employers cannot price discriminate between employees with different tenure—the wage plays both the roles of distributing gains from trade and allocating workers across firms. Just like in my paper, they find that wages are higher when the expected duration of employment relationships is longer and when search frictions are smaller. Unlike in my paper, they focus on the no-discounting case and find that the wage schedule is time-consistent. Coles (2001) generalizes the Burdett-Mortensen model to allow for discounting and finds out that the optimal wage schedule is indeed time-inconsistent. Nevertheless, Coles does not analyze the effect of either public or private productivity shocks on prices.

Secondly, my paper is related to the literature on pricing in markets where costumers have a cost of switching from one provider to another. In fact, also in these markets, the price plays the dual role of distributing rents between the seller and the "locked-in" buyers and directing the flows of "uncommitted" buyers. And, also in these markets, the seller faces, in every period, a trade-off between increasing his future costumer base (by charging a low price) and exploiting his existing base (by charging a high price). Unlike in my paper, the literature on switching costs analyzes this trade-off under the assumption that sellers cannot commit to future prices (see Klemperer (1987, 1995) or Beggs and Klemperer (1992)). Moreover, this literature has not analyzed the dynamics of prices when sellers have private information about their production costs.

Structure of the Paper. In Section 2, I describe the physical environment. In Section 3, I formulate the seller's problem when productivity shocks are perfectly observable and characterize the first-best price schedule. In Section 4, I begin by formulating the pricing problem when productivity shocks are privately observed by the seller. Then, I identify a condition on the persistence of productivity shocks which guarantees that the first-best schedule is incentive compatible under asymmetric information. Finally, I characterize the qualitative properties of the second-best price schedule when the incentive compatibility constraints are binding. Section 5 briefly concludes. All proofs are relegated in the Appendix.

2 The Environment

The market for an indivisible and perishable consumption good is populated by a finite number of sellers and a continuum of buyers with large measure. In period t each seller i can produce the good at the constant marginal cost $c_{i,t}$. This cost is an idiosyncratic random variable that can take either the relatively low value c_ℓ or the relatively high value c_h, $0 < c_\ell < c_h$. The probability of each realization depends on the seller's past productivity—namely, $\Pr(c_{i,t+1} = c_{i,t}) = \rho \geq \frac{1}{2}$. The seller maximizes the expected sum of profits discounted at rate $\beta \in (0,1)$. In period t, each buyer j can participate to the market by paying an opportunity cost of $z > 0$ utils. If the buyer visits the market and purchases one unit of the good at the price $p_{i,t}$, he receives utility $u - p_{i,t}$. If the buyer visits the market and doesn't purchase the good, he receives zero utility. The buyer maximizes the expected sum of utilities discounted at rate β.

Buyers and sellers come together through a search and matching process. If buyer j searches seller i, the two parties match successfully with probability $\lambda_{i,t}$ and fail to match with probability $1 - \lambda_{i,t}$. In the first case, the buyer has the option to purchase the good from the seller in the current period and, as long as the match survives, in future periods. In the second case, the buyer cannot trade in the current period and has to search again for a seller in the next period. A match is dissolved either when the buyer is displaced (an event that occurs with probability $\sigma \in (0,1)$ in each period) or when he chooses to stop trading with the seller.[1] Because of congestion effects, we assume that the probability $\lambda_{i,t}$ is a decreasing function of the measure $q_{i,t}$ of buyers searching seller i in period t. Because of network effects, we assume that the probability $\lambda_{i,t}$ is an increasing function of the measure $n_{i,t}$ of buyers who visited seller i in period $t-1$. For the sake of analytical tractability, we assume that $\lambda_{i,t}$ only depends on the ratio between $q_{i,t}$ and $n_{i,t}$. In particular, the function λ says $\mathbb{R}_t$ into $[0,1]$ and is such that $\lambda' (q/n) < 0, \lambda(0) = 1$ and $\lambda(\infty) = 0$.

In period t, events unfold in the following order. First, each seller realizes its productivity shock and publishes its terms of trade. Than, each buyer observes the entire distribution of terms of trade and decides whether to visit the market. Furthermore, matched buyers decide whether to remain with their current provider or to search elsewhere. Unmatched buyers only choose which seller to search. Finally, new matches are formed and production takes place. Throughout the paper, we assume that sellers cannot price discriminate among buyers because buyers are anonymous in this market.

3 Pricing with Observable Productivity Shocks

The purpose of this paper is to formulate and solve the pricing problem of a seller that enters the market in period $t = 0$ with the cost of production c_0 and a base of costumers of measure $n(c_0) > 0$. I assume that—perhaps because of reputational concerns—the seller can pre-commit to a sequence of state-contingent prices $\mathbf{p} = \{p(h^t)\}_{t=0}^{\infty}$, where h^t is the seller's public history up to date t. In this section, I also assume that productivity shocks are public and therefore $h^t = c^t = \{c_0, c_1, ...c_t\}$.

[1] This no recall assumption is costumarily adopted in search theory to simplify the dynamics of the buyers' problem (cf Burdett and Mortensen (1998), Burdett and Coles (2003), Fishman and Rob (1995)).

3.1 Seller's Problem

Denote with $U(c^t)$ the expected lifetime utility for a buyer who is matched with the seller in period t, after the history c^t has been realized. In period t, the buyer trades with the seller and receives the periodical utility $u - p(c^t)$. With probability $1-\sigma$, in period $t+1$ the buyer has the option of remaining matched to the seller and receiving the continuation utility $U(c^{t+1})$ or searching some other seller/market and receiving the continuation utility Z. With probability σ, in period $t+1$ the buyer is exogenously displaced and he receives the continuation utility Z. Therefore, $U(c^t)$ is equal to

$$U(c^t) = u - p(c^t) + \beta \sum_{c^{t+1}} \Pr(c^{t+1}|c^t) \left[(1-\sigma) \max\left\{U(c^{t+1}), Z\right\} + \sigma Z\right]. \tag{1}$$

Notice that the value Z of the outside option is greater than $(1-\beta)^{-1}z$ because the buyer is free to opt out of the market. Also, notice that Z is smaller than $(1-\beta)^{-1}z$ because, at this value, the entry of new buyers in the market is infinitely elastic. Therefore, Z is equal to the present value of the flow cost of entry z.

Next, consider a buyer who decides to search the seller in period t, after the history c^t has been realized. With probability $\lambda(q(c^t)/n(c^t))$, the buyer matches successfully with the seller and receives the expected lifetime utility $U(c^t)$. With probability $1 - \lambda(q(c^t)/n(c^t))$, the buyer does not match with the seller and receives the lifetime utility βZ. In expectation, the value of searching the seller in period t is smaller than Z—because buyers are free to enter the market and search any particular seller they like—and is greater than Z whenever $q(c^t) > 0$—because those $q(c^t)$ buyers are free to search elsewhere. Therefore, in equilibrium, the measure of buyers $q(c^t)$ searching the seller is such that

$$\lambda\left(q\left(c^t\right)/n\left(c^t\right)\right)\left[U\left(c^t\right) - \beta Z\right] + \beta Z \leq Z \tag{2}$$

and $q(c^t) \geq 0$ with complementary slackness condition. It is convenient to denote with $\theta(U(c^t))$ the ratio of buyers searching the seller $q(c^t)$ to old costumers $n(c^t)$ that solves the equilibrium condition (2).

If the value $U(c^t)$ of being matched to the seller is smaller than the outside option Z, the entirety of the $n(c^t)$ old costumers leaves and no new costumers arrive. If $U(c^t)$ is greater than Z, a fraction $1-\sigma$ of the seller's $n(c^t)$ old costumers returns and $n(c^t) \cdot \theta(U(c^t)) \cdot \lambda\left(\theta(U(c^t))\right)$ new costumers arrive. Overall, the law of motion for the seller's costumer base can be written as

$$n(\{c^t, c_{t+1}\}) = n\left(c^t\right) \cdot \left[1 - \sigma + \eta\left(U\left(c^t\right)\right)\right], \tag{3}$$

where $\eta : \mathbb{R} \to \mathbb{R}$ is a function that takes the value $\sigma - 1$ if $U(c^t) < Z$ and $\theta(U(c^t)) \cdot \lambda\left(\theta(U(c^t))\right)$ otherwise. From the properties of λ, it follows that η is continuous and monotonically increasing in $U(c^t)$. In addition, for the sake of analytical tractability, I assume that η is concave and $\eta(\infty) \le \beta^{-1} - (1-\sigma)$[2].

In period $t = 0$, the seller commits to the price schedule $\mathbf{p}$ that maximizes the expected discounted sum of profits taking as given the law of motion for the costumer base, i.e.

$$\max_{\mathbf{p}} \sum_{t=0}^{\infty} \beta^t \left[\sum_{c^t} \Pr(c^t|c_0) n(c_t)[1-\sigma+\eta(U(c^t))][p(c^t)-c_t] \right], \text{ s.t.} \tag{SP1}$$

$$(1),\ (3) \text{ and } c_0,\ n(c_0) \text{ given.}$$

The sequence problem (SP1) has two remarkable properties. First, after any history c^t, the optimal schedule $\mathbf{p}$ maximizes the seller's profits subject to providing the buyers' at least the lifetime utility $U(c^t)$. Secondly, after any history c^t, the price schedule that maximizes the seller's profits subject to providing the buyers with $U(c^t)$ is independent from the costumer base $n(c^t)$ and the maximized profits are proportional to $n(c^t)$. Using these two properties, in the Appendix, I prove that the sequence problem (SP1) has an equivalent recursive-form representation. In the recursive problem, the state variables are the seller's cost of production c_i and the buyers' promised value U. The choice variables are the value V actually delivered to the buyers, $V \ge U$, the current price p and next period's promised values U'_j, $j = \{\ell, h\}$. The objective function is the sum of current profits $(1-\sigma+\eta(V)) \cdot (p - c_i)$ and future profits $(1-\sigma+\eta(V)) \cdot \beta \cdot E[\Pi_j(U'_j)|c_i]$.

Lemma 1: (Recursive Formulation) *Denote with $\Pi_i(U)$ the value function associated to the sequence problem (SP1) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be greater or equal than U. Then $\Pi_i(U)$ solves the Bellman equation*

$$\Pi_i(U) = \max_{p,V,U'_j \ge Z} (1-\sigma+\eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i) \Pi_j(U'_j) \right], \text{ s.t.} \tag{BE1}$$

$$U \le V = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i) \left[(1-\sigma) U'_j + \sigma Z \right].$$

Let $\{V_i(U), p_i(U), U'_{i|j}(U)\}$ be the policy functions associated to the solution to the Bellman equation above. Then, for all histories $c^t = \{c^{t-2}, c_i, c_j\}$, the optimal price schedule is such that $p(c^t)$ is equal to $p_j(U(c^t))$, where $U(c^t) = U'_{j|i}(U(c^{t-1}))$ and $U(c_0) = Z$.

[2] The assumption on the concavity of η is needed to guarantee that the seller's maximization problem is strictly quasi-concave. The assumption on $\eta(\infty)$ guarantees that the seller's value function is finite.

3.2 First-Best Price Schedule

After substituting out the price p, the recursive problem (BE1) can be broken down in two stages, i.e.

$$\begin{aligned} \Pi_i(U) &= \max_V (1-\sigma+\eta(V)) \cdot \pi_i(V), \\ \pi_i(V) &= u - c_i - V + \beta\sigma Z + \max_{U'_j \geq Z} \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U'_j) + (1-\sigma)U'_j\right]. \end{aligned} \tag{4}$$

In the first stage, the seller decides how much lifetime utility V its costumers should be offered subject to the *promise-keeping* constraint $V \geq U$. In the second stage, the seller decides how the lifetime utility V should be allocated over time and across states.

How much lifetime utility should the seller offer to its costumers? If V is smaller than the outside option Z, the seller does not have any costumers and its profits are equal to zero. If V is greater than Z, the seller has $1-\sigma+\eta(V)$ costumers and obtains the profit $\pi_i(V)$ from each one of them. Over this region, the seller's total profits $(1-\sigma+\eta(V))\cdot\, \pi_i(V)$ are first positive and increasing and then decreasing in the lifetime utility V. They are maximized at $\underline{U}_i$, where the benefit of attracting $\eta'(\underline{U}_i)$ additional new costumers is equal to the cost of lowering the current price by 1 dollar, i.e.

$$\eta'(\underline{U}_i) \cdot \pi_i(\underline{U}_i) = 1 - \sigma + \eta(\underline{U}_i) \tag{5}$$

The seller's offer is subject to the promise-keeping constraint $V \geq U$. If U is lower than $\underline{U}_i$, the constraint is moot and the seller offers the profit-maximizing value $\underline{U}_i$. If U is greater than $\underline{U}_i$, the constraint binds and the seller offers its costumers the value it had promised them.

How should the seller allocate the buyers' lifetime utility V over time and across states? The seller can backload any feasible allocation by reducing the utility $u-p$ offered to its costumers in the current period by $\Pr(c_j|c_i)\beta(1-\sigma)$ dollars and increasing their continuation value U'_j by 1 dollar. Then, in the current period, the seller collects $\Pr(c_j|c_i)\beta(1-\sigma)$ extra dollars per unit of output sold. And, in the next period, it attracts $\eta'(U'_j)$ additional costumers and lowers the price by 1 dollar. If the seller frontloads a feasible allocation, the effects on current and future profits have the same magnitude and the opposite sign. The optimal allocation $(u-p, U'_\ell, U'_h)$ is such that the seller's profits cannot be increased by tilting the timing of benefits neither back nor forth, i.e.

$$-(1-\sigma) = \eta'(U'_j) \cdot \pi_j(U'_j) - (1-\sigma+\eta(U'_j)), \text{ for } j = \ell, h, \tag{6}$$

$$p(V) = u - V + \beta\sigma Z + \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U'_j + \sigma Z\right]. \tag{7}$$

Notice that, because an increase in U'_j by 1 util allows the seller to not only attract $\eta'(U'_j)$ additional costumers in the next period but also raise its current price, the optimal continuation value U'_j is greater than $\underline{U}_j$.

Using the solution to the first and second stage problems, I can recover the structure of the first-best price schedule $\boldsymbol{p}$ and its qualitative properties. In period $t = 0$, the seller enters the market with no prior obligations, $U(c^0) = Z$, and the production cost $c_0 = c_i$. The seller offers its costumers the profit-maximizing lifetime utility $\underline{U}_i$ by setting the current period's price to $p(\underline{U}_i)$ and committing to the continuation values (U'_ℓ, U'_h). In period $t \geq 1$, after the history c^t has been realized, the seller has the production cost $c_t = c_j$ and an obligation to deliver its costumers at least U'_j. The seller offers them the promised lifetime utility U'_j by setting the current period's price to $p(U'_j)$ and committing to the continuation values (U'_ℓ, U'_h). Because U'_i is greater than $\underline{U}_i$, prices are decreasing over time. Also, because $\underline{U}_\ell$ is greater than $\underline{U}_h$ and U'_ℓ is greater than U'_h, prices are increasing in the contemporaneous realization of the cost of production.

Proposition 1: (Pricing with observable costs). *The first-best price schedule $\boldsymbol{p} = \{p\left(c^t\right)\}_{t=0}^{\infty}$ has the following properties:*

1. *If $t = 0$ and $c_t = c_i$, $p(c^t) = p(\underline{U}_i)$. If $t \geq 1$ and $c_t = c_i$, $p(c^t) = p(U'_i)$;*
2. *For all c^t and c^0 such that $c_t = c_0$, $p(c^t) < p(c^0)$;*
3. *For all c^t and $\tilde{c}^t$ such that $c_t = c_\ell$ and $\tilde{c}_t = c_h$, $p(c^t) \leq p(\tilde{c}^t)$.*

The first-best price schedule characterized in Proposition 1 is *time-inconsistent*. At date $t = 0$, the seller finds optimal to charge its costumers the high price $p(\underline{U}_i)$ and promise them the low price $p(U'_i)$ for the subsequent period. When date $t = 1$ arrives, the seller has already obtained part of the benefit of promising $p(U'_i)$—i.e. the increase in the inflow of new costumers at $t = 0$—but has still to bear its entire cost. Then, the seller would like to renege the original schedule and, once again, charge its costumers the high price $p(\underline{U}_i)$ and promise them the low price $p(U'_i)$ in the future.

4 Pricing with Unobservable Costs

Consider a seller that enters the market in period $t = 0$ with the cost of production c_0 and a costumer base of measure $n(c_0) > 0$. Assume that the seller can commit to a sequence of state-contingent prices $\boldsymbol{p} = \{p(h^t)\}_{t=0}^{\infty}$, where h^t is the seller's public history up to date

t. Assume that, in every period $t \geq 1$, the seller privately observes the realization of its cost of production c_t and makes a public announcement $\widehat{c}_t \in \{c_\ell, c_h\}$ about it. Hence, h^t is $\widehat{c}^t = \{c_0, \widehat{c}_1, ...\widehat{c}_t\}$. In this section, I formulate and solve the pricing problem of the seller subject to the restriction that, after any history $\widehat{c}_t$, the costumer's beliefs about the cost of production c_t are degenerate.

4.1 Seller's Problem

Without loss in generality, I can assume that the buyers interpret the seller's reports as truthful, i.e. $\Pr(c_t = \hat{c}_t | \widehat{c}^{t-1}) = 1$. Denote with $U(\widehat{c}^t)$ the expected lifetime utility for a buyer who is matched with the seller in period t, after the history of announcements $\widehat{c}^t = \{\widehat{c}^{t-1}, c_j\}$ has been reported. In period t, the buyer trades with the seller and receives the periodical utility $u - p(\widehat{c}^t)$. In period $t+1$,the buyer expects that the seller will report the production cost c_j and offer him the continuation utility $U(\{\widehat{c}^t, c_j\})$ with probability $\rho \geq 1/2$. The buyer expects that the seller will report the production cost c_{-j} and offer him the continuation utility $U(\{\widehat{c}^t, c_{-j}\})$ with probability $1-\rho$. Given those beliefs, $U(\widehat{c}^t)$ is equal to

$$U\left(\widehat{c}^t\right) = u - p\left(\widehat{c}^t\right) + \beta\left[\textstyle\sum_{\widehat{c}^{t+1}} \Pr(\widehat{c}^{t+1}|\widehat{c}^t)[(1-\sigma)\max\{U(\widehat{c}^{t+1}), Z\} + \sigma Z]\right]. \tag{8}$$

Along the equilibrium path, the seller's reporting strategy must be consistent with the buyers' inference of the production cost c_t from the announcement $\hat{c}_t$. Therefore, for all $\widehat{c}^{t-1} = c^{t-1}$ and $c_t = c_i$, the price schedule $\boldsymbol{p}$ must induce the seller to report its type correctly, i.e.

$$\begin{aligned}&\sum_{\tau=t}^{\infty} \beta^{\tau-t}\left\{\textstyle\sum_{c^\tau} \Pr(c^\tau|c^t) n\left(c^\tau\right)\left[1-\sigma+\eta(U\left(c^\tau\right))\right]\left[p\left(c^\tau\right)-c_\tau\right]\right\} \geq \\ &\sum_{\tau=t}^{\infty} \beta^{\tau-t}\left\{\textstyle\sum_{c^\tau} \Pr\left(c^\tau|c^t\right) n\left(\widehat{c}\left(c^\tau\right)\right)\left[1-\sigma+\eta(U\left(\widehat{c}\left(c^\tau\right)\right))\right]\left[p\left(\widehat{c}\left(c^\tau\right)\right)-c_\tau\right]\right\},\end{aligned} \tag{IC}$$

where $\widehat{c}(c^\tau)$ is the public history $\{c^{t-1}, c_{-i}, c_{t+1}, ...c_\tau\}$. In writing the *incentive compatibility* constraint (IC), I have assumed that—independently from its period$-t$ announcement—the seller will find optimal to report its type correctly in any subsequent period $\tau \geq t$. This is the right assumption to make because the seller's expected profits from reporting its true type and from lying depend on the public history $\widehat{c}^{\tau-1}$ and on the cost of production c_τ but not on the previous realizations of productivity shocks $c^{\tau-1}$. Therefore, the same incentive compatibility constraint (IC) which guarantees that the seller will truthfully report c_τ after the history $\widehat{c}^{\tau-1}$ has been realized and reported, also guarantees that the seller will truthfully report c_τ after the history $\widehat{c}^{\tau-1}$ has been reported and some different history has been realized.

In general, the optimal incentive-compatible price schedule $\boldsymbol{p}^*$ need not be renegotiation proof, i.e. there may exist some histories after which the seller and its costumers would agree to modifying $\boldsymbol{p}^*$. In order to rule out this possibility, I restrict attention to price schedules $\boldsymbol{p}$ such that, for any reported history $\hat{c}^t = c^t$ and for any feasible $\hat{\boldsymbol{p}}$[3], if $U(c^t|\hat{\boldsymbol{p}}) > U(c^t|\boldsymbol{p})$ then

$$\begin{aligned} &\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{\textstyle\sum_{c^\tau} \Pr\left(c^\tau|c^t\right) n(c^\tau|\boldsymbol{p})[1-\sigma+\eta\left(U\left(c^\tau|\boldsymbol{p}\right)\right)][p(c^\tau)-c_\tau]\right\} > \\ &\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{\textstyle\sum_{c^\tau} \Pr\left(c^\tau|c^t\right) n(c^\tau|\hat{\boldsymbol{p}})[1-\sigma+\eta\left(U\left(c^\tau|\hat{\boldsymbol{p}}\right)\right)][\hat{p}(c^\tau)-c_\tau]\right\}. \end{aligned} \tag{RP}$$

Notice that—because the value of a price schedule depends on the reported history $\widehat{c}^{t-1}$ and on the production cost c_t, but does not depend on the realized history c^{t-1}—the *renegotiation proofness* constraint (RP) guarantees that $\boldsymbol{p}$ is ex-post efficient even if the seller has lied in some previous period $\tau \leq t-1$.

In period $t = 0$, the seller commits to the price schedule $\boldsymbol{p}$ that maximizes the expected discounted profits subject to the incentive compatibility and renegotiation proofness constraints, i.e.

$$\begin{aligned} &\max_{\boldsymbol{p}} \sum_{\tau=t}^{\infty} \beta^t \left\{\textstyle\sum_{c^t} \Pr(c^t|c_0) n(c_t)[1-\sigma+\eta(U(c^t))][p(c^t)-c_t]\right\}, \text{ s.t.} \\ &\text{(IC), (RP) and } c_0,\ n(c_0) \text{ given.} \end{aligned} \tag{SP2}$$

The sequence problem (SP2) has two remarkable features. First, after any realized history c^t and reported history $\widehat{c}^t$, the optimal price schedule $\boldsymbol{p}$ satisfies (IC) and (RP) at all subsequent dates $\tau \geq t+1$. And, among all the feasible schedules, $\boldsymbol{p}$ is the one that maximizes the profits of a seller with production cost $\hat{c}_t$ subject to providing the buyers with a lifetime utility non-smaller than $U(\widehat{c}^t)$. Secondly, after any realized history c^t and reported history $\widehat{c}^t$, the feasible schedule that maximizes the seller's profits subject to providing the buyers with $U(\widehat{c}^t)$ is independent from the costumer base $n(\hat{c}^t)$ and the maximized profits are proportional to $n(\hat{c}^t)$. Using these two properties, in the Appendix, I prove that the sequence problem (SP2) has an equivalent recursive-form representation.

Lemma 2: (Recursive Formulation) *Denote with $\Pi_i(U)$ the value function associated to the sequence problem (SP2) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be greater or*

[3] The schedule $\hat{\boldsymbol{p}}$ is feasible if it satisfies the incentive-compatibility constraint (IC) and the renegotiation-proofness condition (RP) in all periods $\tau \geq t+1$.

equal than U. Then $\Pi_i(U)$ solves the Bellman equation

$$
\begin{aligned}
&\Pi_i(U) = \max_{p,V,U'_j \geq Z} (1-\sigma+\eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\Pi_j(U'_j)\right], \text{ s.t.} \\
&U \leq V = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i) \left[(1-\sigma)U'_j + \sigma Z\right], \\
&\Pi_j(U'_j) \geq \tilde{\Pi}_{-j}(U'_{-j}) \text{ for } j = \ell, h, \\
&\tilde{\Pi}_i(U) = (1-\sigma+\eta(V_i(U))) \left[p_i(U) - c_{-i} + \beta \textstyle\sum_j \Pr(c_j|c_{-i})\Pi_j(U'_{j|i}(U))\right].
\end{aligned}
\tag{BE2}
$$

Let $\{V_i(U), p_i(U), U'_{i|j}(U)\}$ be the policy functions associated to the solution to the Bellman equation above. Then, for all histories $c^t = \{c^{t-2}, c_i, c_j\}$, the optimal price schedule is such that $p(c^t)$ is equal to $p_j(U(c^t))$, where $U(c^t) = U'_{j|i}(U(c^{t-1}))$ and $U(c_0) = Z$.

4.2 Very Persistent Shocks: Fully Flexible Prices

When it satisfies the incentive compatibility and renegotiation proofness constraints, the first-best schedule is the solution to the pricing problem under asymmetric information. Because the first-best schedule is ex-post efficient, the renegotiation proofness constraint (RP) is certainly satisfied. But because the schedule is time-inconsistent, the incentive compatibility constraint (IC) need not hold. In this subsection, I identify a necessary and sufficient condition on the persistence of productivity shocks which guarantees that the first-best schedule will be incentive compatible. For the sake of simplicity, I carry out the analysis under the assumption that η is linear over the range of values promised by the seller.

Imagine that the seller realizes the high cost of production c_h after having announced the history $\hat{c}^{t-1}$. If it chooses to report the low cost c_ℓ instead of c_h, the seller lowers its price by $p(U'_h) - p(U'_\ell)$ dollars and attracts $\eta'(U'_\ell - U'_h)$ additional costumers in period t. Because the first-best price schedule is history independent, the report $\hat{c}_t$ does not affect the dynamics of prices and costumers in subsequent periods. Therefore, the seller reports its actual cost of production c_h if and only if

$$
\begin{aligned}
&\Pi_h(U'_h) - \tilde{\Pi}_\ell(U'_\ell) = (1-\sigma+\eta(U'_\ell))\left[1-\beta(1-\sigma)(2\rho-1)\right] - \eta'\pi_h(U'_h) = \\
&(1-\sigma)\left[1-\beta\left(1-\sigma+\eta(U'_\ell)\right)(2\rho-1)\right] + \eta'\left[1-\beta\left(1-\sigma\right)(2\rho-1)\right](U'_\ell - U'_h) \geq 0,
\end{aligned}
\tag{9}
$$

where the second line is obtained after substituting in the first order condition (6). Analytically, it is immediate to verify that the incentive compatibility constraint (9) is always satisfied. Intuitively, the seller has no incentive to report c_ℓ instead of c_h because this would imply lowering a price that, from its perspective in period t, is already too low.

Next, imagine that the seller realizes the low cost of production c_ℓ after having announced the history $\hat{c}^{t-1}$. If it chooses to report the high cost c_h instead of c_ℓ, the seller increases its price by $p(U'_h) - p(U'_\ell)$ dollars and attracts $\eta'(U'_\ell - U'_h)$ fewer costumers in period t. The report $\hat{c}_t$ does not affect the dynamics of prices and costumers in subsequent periods. Therefore, the seller reports its actual cost of production c_ℓ if and only if

$$\begin{aligned}&\Pi_\ell(U'_\ell) - \tilde{\Pi}_h(U'_h) = \eta'\pi_\ell(U'_\ell) - (1-\sigma+\eta(U'_h))\left[1-\beta(1-\sigma)(2\rho-1)\right] = \\ &\eta'\left[1-\beta(1-\sigma)(2\rho-1)\right](U'_\ell - U'_h) - (1-\sigma)\left[1-\beta(1-\sigma+\eta(U'_h))(2\rho-1)\right] \geq 0.\end{aligned} \tag{10}$$

The incentive compatibility constraint (10) may be satisfied or violated depending on parameter values. In particular, there exists a critical level of persistence ρ^* of the productivity shocks such that the constraint (10) is satisfied if ρ is greater than ρ^* and is violated if ρ is below ρ^*. Intuitively, because the schedule is time inconsistent, the low cost seller would like to raise the current price. But by reporting c_h instead of c_ℓ, the seller not only increases the current price, it also makes costumers irrationally pessimistic about the future terms of trade. And this side effect becomes stronger the more persistent productivity shocks are.

PROPOSITION 2: (Fully Flexible Prices) *There exists a* $\rho^* \in \left[\frac{1}{2}, 1\right]$ *such that, for all* $\rho > (<)\rho^*$, *the first-best price schedule is feasible and optimal* (not feasible) *when the seller has private information about its productivity shocks.*

4.3 IID Shocks: Rigid Prices

When the persistence of productivity shocks is lower than the critical level ρ^*, the first-best schedule violates the incentive compatibility constraint (IC). In order to characterize the second-best schedule, it is convenient to break down the recursive problem (BE2) in two stages

$$\begin{aligned}&\Pi_i(U) = \max_{V \geq U}(1-\sigma+\eta(V))\cdot\pi_i(V), \\ &\pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U'_j \geq Z} \quad \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.} \\ &\qquad\qquad \Pi_j(U'_j) \geq \tilde{\Pi}_j(U'_j) \text{ for } j \in \{\ell, h\}.\end{aligned} \tag{11}$$

In the first-stage problem, the choice variable is the costumers' lifetime utility V. The objective function is the expected discounted profit for a seller with the current cost of production c_i. The function is first increasing and then decreasing in V and attains its unique maximum at $\underline{U}_i$, where $\underline{U}_i$ is the solution to the equation (5). The choice of V is limited by the promised-keeping constraint $U \leq V$. Therefore, if $U \leq \underline{U}_i$, the solution to the first-stage problem is to provide costumers with the profit-maximizing value $\underline{U}_i$. If $U > \underline{U}_i$, the solution is to provide costumers with the promised value U.

In the second-stage problem, the choice variables are the costumers' continuation values U'_ℓ and U'_h. The objective function is the profit per costumer for a seller that provides them with the lifetime utility V. The function is quasi-concave in (U'_ℓ, U'_h) and attains its unique maximum at (U'^*_ℓ, U'^*_h), where U'^*_j is the solution to the equation (6). The choice of (U'_ℓ, U'_h) is limited by the incentive-compatibility constraint $\Pi_j(U'_j) \geq \tilde{\Pi}_{-j}(U'_{-j})$. Because V enters the objective function separately from the choice variables, the solution to the second-stage problem is independent from the lifetime utility V and can be denoted with $(U'_{\ell|i}, U'_{h|i})$. Because the objective function is increasing in U'_j and the constraint is independent from U'_j for all $U'_j \leq \underline{U}_j$, the solution to the second-stage problem $(U'_{\ell|i}, U'_{h|i})$ is greater than $(\underline{U}_\ell, \underline{U}_h)$.

Using the qualitative properties of the solution to the first and second stage problems, I can express the incentive compatibility constraint as

$$\begin{gathered}(1-\sigma+\eta(U'_j))\cdot\pi_j(U'_j) \geq (1-\sigma+\eta(U'_{-j}))\cdot\tilde{\pi}_{-j}(U'_{-j}), \\ \tilde{\pi}_i(U) \equiv u - c_{-i} - U + \beta\sum_j \left\{\Pr(c_j|c_i)\left[(1-\sigma)U'_{j|i} + \sigma Z\right] + \Pr(c_j|c_{-i})\Pi_j(U'_{j|i})\right\}.\end{gathered} \tag{12}$$

Notice that, if the seller realizes the production cost c_{-i} but announces c_i, its expected profits per costumer $\tilde{\pi}_i(U)$ are *generally* different from $\pi_{-i}(U)$. First, when the seller misreports its type, the costumers' expectations about the value of the match are not correct. In particular, while the costumers expect to receive the continuation value $U'_{i|i}$ with probability ρ and $U'_{-i|i}$ with probability $1-\rho$, the seller offers them $U'_{i|i}$ a fraction $1-\rho$ of the time and $U'_{-i|i}$ a fraction ρ of the time. Secondly, when the seller misreports its type, the continuation values $U'_{\ell|i}$ and $U'_{h|i}$ prescribed by the second-best schedule are not its preferred way to allocate the costumers' lifetime utility over time and across states.

Only when shocks are i.i.d., $\tilde{\pi}_i(U)$ is equal to $\pi_{-i}(U)$. In this case, a seller that realizes the cost of production c_j correctly reports its type if it attains higher profits by offering to its costumers the lifetime utility U'_j rather than U'_{-j}, i.e.

$$(1-\sigma+\eta(U'_j))\cdot\pi_j(U'_j) \geq (1-\sigma+\eta(U'_{-j}))\cdot\pi_j(U'_{-j}). \tag{13}$$

Since U'_h and U'_ℓ are both greater than $\underline{U}_h$, a seller that realizes the cost of production c_h announces its true type if and only if U'_h is smaller than U'_ℓ. Since U'_ℓ is greater than $\underline{U}_\ell$ but

U_h' need not be greater than $\underline{U}_\ell$, a seller that realizes the cost of production c_ℓ announces its true type if either U_h' is greater than U_ℓ' or sufficiently smaller than the profit-maximizing value $\underline{U}_\ell$. The set of continuation values (U_ℓ', U_h') that induces both seller's types to report their actual costs is illustrated in Figure 1.

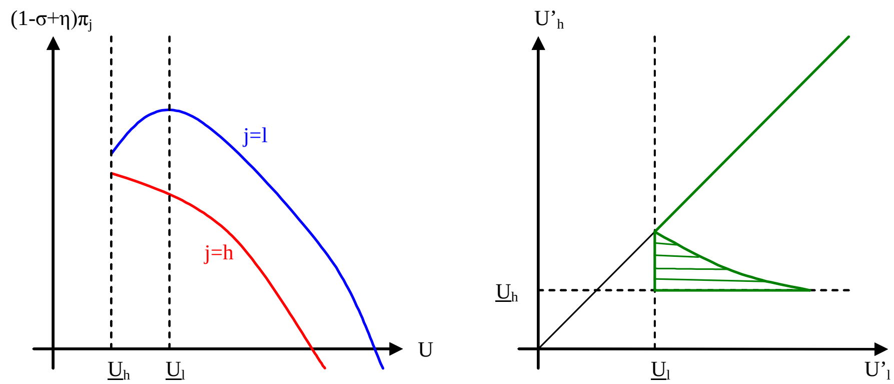

Figure 1: Incentive compatible continuation values (IID Shocks)

Given the characterization of the incentive compatibility constraint (13), I can conclude that there exist two candidate solutions to the second-stage problem. The first solution prescribes that the continuation value offered to the costumers should be independent from the seller's announcement about its cost of production, i.e. $U_h' = U_\ell' = U'$, and such that the allocation of costumers' utility over time is on average efficient

$$-(1-\sigma) = \sum_j \tfrac{1}{2} \left[\eta'(U') \cdot \pi_j (U') - 1 - \sigma + \eta(U')\right]. \tag{14}$$

The second solution prescribes that the continuation value should be lower when the cost of production announced by the seller is higher, i.e. $U_h' < U_\ell'$, and that U_h' should be sufficiently far below $\underline{U}_\ell$ to induce the low-cost seller to truthfully report its type. When productivity shocks are small, the state-independent solution is optimal because it closely approximates the first-best $(U_\ell'^*, U_h'^*)$ while the state-contingent solution approximates the no commitment outcome $(\underline{U}_\ell, \underline{U}_h)$. This leads to the following proposition.

Proposition 3 (Rigid Prices) *Let $c_\ell(\Delta) = c - \Delta$ and $c_h(\Delta) = c + \Delta$ for some $c \in (0, u-z)$. If $\rho = 1/2$, there exists a $\Delta^* > 0$ such that, for all $\Delta \in (0, \Delta^*)$, the second-best price schedule has the following properties:*

1. *If $c_0 = c_i$, the price $p(c_0)$ is $p(\underline{U}_i)$;*

2. *For $t \geq 1$ and all $\hat{c}^t$, the price $p(\hat{c}^t)$ is $p(U')$;*

3. *The function $p(U)$ is given by $u - U + \beta[(1-\sigma)U' + \sigma Z]$.*

4.4 Moderately Persistent Shocks: Sticky Prices

In this subsection, I characterize the optimal price schedule under asymmetric information when production costs are positively correlated over time, but not to the point where the first-best schedule becomes feasible. In order to develop the analysis, I find convenient to first solve a version of the second-stage problem (11) that abstracts from the incentive compatibility constraint for the high-cost seller and to later verify that the constraint is satisfied.

Let the persistence ρ of production costs be anywhere in the interval $(0, \rho^*)$. The relaxed version of the second-stage problem in (11) is

$$\begin{aligned} \pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U'_j \geq \underline{U}_j} \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.} \\ (1 - \sigma + \eta(U'_\ell)) \cdot \pi_\ell(U'_\ell) \geq (1 - \sigma + \eta(U'_h)) \cdot \widetilde{\pi}_h(U'_h). \end{aligned} \tag{15}$$

Consider the incentive compatibility constraint for the low-cost seller. On the one hand, the seller's profits from truthfully reporting its type are monotonically decreasing with the continuation value U'_ℓ promised to its costumers. On the other hand, the seller's profits from misreporting its type are first increasing and then decreasing in the continuation value U'_h expected by the customers and they are maximized at $\tilde{\underline{U}}_h \in [Z, \underline{U}_\ell]$. Moreover, when $U'_\ell = U'_h$, the seller makes higher profits by correctly reporting its type rather than lying. Therefore, the incentive compatibility constraint is satisfied either when U'_h is not much smaller than the alternative continuation value U'_ℓ or when U'_h is sufficiently far below the profit-maximizing value $\tilde{\underline{U}}_h$. The set of incentive-compatible continuation values is illustrated in Figure 2.

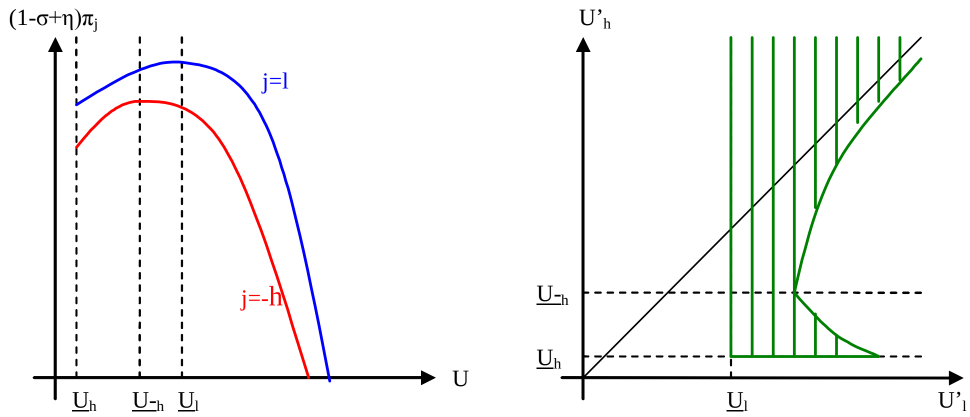

Figure 2: Incentive compatible continuation values (Persistent Shocks)

When $\rho < \rho^*$, the first-best solution $(U_\ell'^*, U_h'^*)$ to the second-stage problem does not satisfy the low-cost seller's incentive compatibility constraint. The second-best solution distorts the continuation values (U_ℓ', U_h') away from $(U_\ell'^*, U_h'^*)$ in order to make the low-cost seller indifferent between correctly reporting its type and lying. More specifically, when productivity shocks are small, the second-best solution distorts the continuation value U_ℓ' downward and U_h' upward. And, if in the previous period the seller has reported the low cost of production, then U_ℓ' is distorted less and U_h' is distorted more than if the seller had reported c_h. Overall, the second-best continuation values $(U_{\ell|i}', U_{h|i}')$ are such that $U_h'^* \leq U_{h|i}' < U_{\ell|i}' \leq U_{\ell|i}'^*$, $U_{\ell|\ell}' > U_{\ell|h}'$ and $U_{h|h}' < U_{h|\ell}'$.

Now, I am in the position to recover the structure of the second-best price schedule $\boldsymbol{p} = \{p(\hat{c}^t)\}_{t=0}^{\infty}$. In period $t = 0$, the seller enters the market with no prior obligations, $U(c_0) = Z$, and the production cost c_i. The seller offers its costumers the profit-maximizing lifetime utility $\underline{U}_i$ by setting the current period's price to $p_i(\underline{U}_i)$ and committing to the continuation values $(U_{\ell|i}', U_{h|i}')$, where

$$p_i(U) = u - U + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)\, U_{j|i}' + \sigma Z\right]. \tag{16}$$

In period $t \geq 1$ and after the public history $\hat{c}^{t-1} = \{\hat{c}^{t-2}, c_i\}$ has been realized, the seller reports its actual production cost c_j and offers its costumers the promised lifetime utility $U_{j|i}'$ by setting the current period's price to $p_j(U_{j|i}')$ and committing to the continuation values $(U_{\ell|j}', U_{h|j}')$.

From the properties of the continuation values $U'_{j|i}$, I can characterize the joint dynamics of costs and prices. First, "steady-state" prices are increasing in the production cost. That is, if the seller realizes the production cost c_ℓ for a sufficiently long period of time, it charges the price $p_\ell(U'_{\ell|\ell})$ which is strictly lower than the price $p_h(U'_{h|h})$ it would have charged if it had realized c_h instead. Secondly, prices are "sticky." That is, when the seller first realizes the high production cost, it charges a price $p_h(U'_{h|\ell})$ which is strictly lower than the steady-state level $p_h(U'_{h|h})$. Conversely, when the seller first realizes the low production cost, it charges a price $p_\ell(U'_{\ell|h})$ which is strictly greater than $p_\ell(U'_{\ell|\ell})$.

Proposition 4: (Sticky Prices) *Let $c_\ell = c - \Delta$ and $c_h = c + \Delta$ for some $c \in (0, u - z)$. There is a $\Delta^* > 0$ such that, for all $\Delta \in (0, \Delta^*)$, the second-best price schedule has the following properties:*

1. *If $c_0 = c_i$, $p(c_0)$ is $p_i(\underline{U}_i)$. If $t \geq 1$ and $\hat{c}^t = \{\hat{c}^{t-2}, c_i, c_j\}$, $p(\hat{c}^t)$ is $p_j(\underline{U}'_{j|i})$;*
2. *If $\hat{c}_1^t = \{\hat{c}_1^{t-2}, c_h, c_h\}$ and $\hat{c}_2^t = \{\hat{c}_1^{t-2}, c_\ell, c_\ell\}$, $p(\hat{c}_1^t)$ is strictly greater than $p(\hat{c}_2^t)$;*
3. *If $\hat{c}_1^t = \{\hat{c}_1^{t-2}, c_\ell, c_h\}$ and $\hat{c}_2^t = \{\hat{c}_1^{t-2}, c_h, c_h\}$, $p(\hat{c}_1^t)$ is strictly smaller than $p(\hat{c}_2^t)$. If $\hat{c}_1^t = \{\hat{c}_1^{t-2}, c_h, c_\ell\}$ and $\hat{c}_2^t = \{\hat{c}_1^{t-2}, c_\ell, c_\ell\}$, $p(\hat{c}_1^t)$ is strictly greater than $p(\hat{c}_2^t)$.*

5 Conclusions

In this paper, I have studied price dynamics in product markets characterized by: (a) *search frictions*—in the sense that it takes time for a buyer to find a seller that produces a version of the good he likes; (b) *anonymity*—in the sense that sellers cannot price discriminate between first-time buyers and returning costumers; (c) *asymmetric information*—in the sense that sellers are subject to idiosyncratic shocks to their marginal cost of production and privately observe the shocks' realizations.

In these frictional markets, prices play three conceptually distinct roles: (a) *allocative*—prices direct the search effort of unmatched buyers across different sellers; (b) *distributive*—prices divide the gains from trade between sellers and their repeated costumers; (c) *informative*—prices may signal the seller's cost of production and, indirectly, about the future terms of trade.

The main finding of the paper is that, in these frictional markets, the joint dynamics of costs and prices may be qualitatively very different than in a standard Walrasian market. More specifically, when shocks are i.i.d., the price remains constant in the face of fluctuations in a seller's marginal cost. When shocks are moderately persistent, the price adjusts slowly and imperfectly in response to changes in a seller's cost. Finally, when shocks are sufficiently persistent, the price adjusts instantaneously and efficiently as soon as a seller's production cost varies.

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A Appendix

A.1 Proof of Lemma 1

Claim 1: After any history c^t, the optimal price schedule $\mathbf{p}$ is such that $U(c^t|\mathbf{p}) \geq Z$.
Proof: On the way to a contradiction, let c_1^t be the earlier history at which $U(c_1^t|\mathbf{p}) \geq Z$. Then, if the history c_1^t is realized (an event which occurs with positive probability), the seller loses all its current customers and can't attract any any new customers in the future. Its expected discounted profits are equal to zero. Now, consider an alternative schedule $\widehat{\mathbf{p}}$ such that $\widehat{p}(c^\tau) = p(c^\tau)$ if c^τ is not a subsequent of c_1^t and $\widehat{p}(c^\tau) = u - z > 0$ otherwise. For all $\tau < t$, the seller's periodical profits are the same with $\widehat{\mathbf{p}}$ and $\mathbf{p}$ because prices and customers are the same. Similarly, for all $c^t \neq c_1^t$, the seller's continuation profits are the same with $\widehat{\mathbf{p}}$ and $\mathbf{p}$. Finally for $c^t = c_1^t$, the continuation profits are strictly positive. Overall, in period $t = 0$, the seller strictly prefers to commit to the schedule $\widehat{\mathbf{p}}$ that $\mathbf{p}$, which contradicts the optimality of the latter. $\|$

Denote with $\Pi_i(U)$ the value function associated to the sequence problem (SP1) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be *greater or equal* than U. Denote with $\Pi_i^+(U)$ the value function associated to (SP1) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be *equal* to U.

Claim 2: The value functions $\Pi_i(U)$ and $\Pi_i^+(U)$ are such that

$$\begin{aligned} &\Pi_i(U) = \max_{V,p,U_j' \geq Z} (1 - \sigma + \eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\Pi_j^+(U_j')\right], \text{ s.t.} \\ &U \leq V \equiv u - p + \beta \textstyle\sum_j \Pr(c_j|c_i) \left[(1-\sigma)U_j' + \sigma Z\right]. \end{aligned} \tag{A1}$$

Proof: Making use of Claim 1, I can write the value function $\Pi_i(U)$ as

$$\Pi_i(U) = \max_{\substack{p(c_0),\mathbf{p}_1,\\ U(c_0),U(c^1)}} n(c^1)\left[p(c^0) - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\sum_{c^\tau} \Pr(c^t|\{c_0,c_j\}) \frac{n(c^{t+1})}{n(c^1)}[p(c^t) - c_t]\right]\right]$$

$$U \le U(c^0) \equiv u - p(c^0) + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U(\{c_0,c_j\}) + \sigma Z\right],$$

$$Z \le U(c^t) \equiv u - p(c^t) + \beta \textstyle\sum_{c^{t+1}} \Pr(c^{t+1}|c^t)\left[(1-\sigma)U(c^{t+1}) + \sigma Z\right],$$

$$n(c^1) = (1-\sigma+\eta(U(c^0))),\ \frac{n(c^{t+1})}{n(c^1)} = \frac{n(c^t)}{n(c^1)}(1-\sigma+\eta(U(c^t))).$$

The maximization problem above can be broken down in two stages. In the first stage, the seller chooses $p(c^0), U(c^0)$ and $U(\{c_0,c_j\})$ subject to the first and third constraints. In the second stage, the seller chooses $\mathbf{p}_1 = \{p(c^t)\}_{t=1}^{\infty}$ in order to maximize its continuation profits subject to delivering *exactly* $U(\{c_0,c_j\})$ to the customers and given an initial cost of production c_j and a customer base with measure $n(c^1)/n(c^1) = 1$. Therefore, the value function associated to the second-stage problem is $\Pi_j^+(U(\{c_0,c_j\}))$. $\|$

Claim 3: The function $\Pi_i(U)$ satisfies the Bellman equation (BE1).

Proof: First, notice that the solution to the maximization problem in (A1) is a continuation value U_j^+ that belongs to the set $U_j^{PF} = \{U : \Pi_j^+(U) = \Pi_j(U)\}$, i.e. the set of continuation values U such that the seller could not increase its profits by delivering more than U. Therefore, I can restrict attention to continuation values in U_j^{PF} and replace $\Pi_j^+(U)$ with $\Pi_j(U)$ in (A1). Secondly, notice that, if the continuation function in (A1) is $\Pi_j(U)$ and the choice of continuation values is not restricted to U_j^{PF}, the solution to the maximization problem is $U_j' \in U_j^{PF}$. Therefore, I can relax the choice set and obtain (BE1). $\|$

Claim 4: If $P_i(U)$ is a solution to the functional equation $(BE1)$, then $P_i(U)$ is equal to $\Pi_i(U)$.

Proof: Denote with T the mapping associated to $(BE1)$. It is immediate to verify that T satisfies the Blackwell's sufficient conditions for a contraction mapping. Therefore, T has a unique fixed point. $\|$

To conclude the proof of Lemma 1, notice that the value function associated to the sequence problem (SP1) when $c_0 = c_i$ and $n(c_0) > 0$ is given by $n(c_0) \cdot \Pi_i(0)$.

A.2 Proof of Proposition 1

Consider the first-stage problem in (4). For $V < Z$, the objective function $(1 - \sigma + \eta(V)) \cdot \pi_i(V)$ is equal to zero because $\eta(V) = \sigma - 1$. For $V = Z$, the function is strictly positive because $\eta(V) = 0$ and, as proved in Lemma 1, $\pi_i(Z) > 0$. For $V \geq Z$, the function is quasi-concave because it is concave wherever increasing and may be convex only when strictly decreasing. The function attains its maximum for $V = \underline{U}_i$, where $\underline{U}_i$ satisfies

$$[\eta'(\underline{U}_i) \cdot \pi_i(\underline{U}_i) - (1 - \sigma + \eta(\underline{U}_i))] \cdot (\underline{U}_i - Z) = 0. \tag{A2}$$

From the properties of the objective function, it follows that the solution $V_i(U)$ to the first-stage problem is $\underline{U}_i$ whenever $U \leq \underline{U}$ and U otherwise. In turn, the value function $\Pi_i(U)$ associated to the first-stage problem is constant at $(1 - \sigma + \eta(\underline{U}_i)) \cdot \pi(\underline{U}_i)$ whenever $U \leq \underline{U}_i$ and is strictly decreasing and quasi-concave otherwise.

Next, consider the second-stage problem in (4). As proved in Lemma 1, the choice set can be restricted to those continuation values that are greater than the profit-maximizing values $(\underline{U}_\ell, \underline{U}_h)$. Over this domain, the objective function is quasi-concave and attains is maximum at (U'_ℓ, U'_h), where U'_j satisfies

$$\eta'(U'_j) \cdot \pi_j(U'_j) - (1 - \sigma + \eta(U'_j)) = -(1 - \sigma). \tag{A3}$$

Because $\eta' \pi_j - (1 - \sigma + \eta)$ is non-negative for all $U \leq \underline{U}_j$, the optimal continuation value U'_j is strictly greater than the profit-maximizing value $\underline{U}_j$.

Finally, I want to compare the solution to the first and second stage problem under high and low cost of production. Denote with T the contraction mapping associated to the Bellman equation (BE1). Since $(TP)_h < (TP)_\ell$ whenever $P_h \leq P_\ell$, the unique fixed point Π of the contraction mapping T associated to the Bellman equation (BE1) is such that the profit function is strictly decreasing in the cost of production. In turn, this implies that $\pi_h(U) < \pi_\ell(U)$ and, through the first order conditions (A2) and (A3), that $\underline{U}_h \leq \underline{U}_\ell$ and $U'_h < U'_\ell$.

A.3 Proof of Lemma 2

Claim 1: After any history $\widehat{c}^t$, the optimal price schedule $\boldsymbol{p}$ is such that $U(\widehat{c}^t|\boldsymbol{p}) \geq Z$.

Proof: Suppose that, at $\widehat{c}_1^t$, the optimal schedule $\boldsymbol{p}$ is such that the buyers' lifetime utility $U(\widehat{c}^t|\boldsymbol{p})$ is strictly smaller than Z and, consequently, the seller's expected profits are zero.

Consider the alternative schedule $\widehat{\mathbf{p}}$ which prescribes the constant price $\widehat{p}(\widehat{c}^{\tau}) = u - z$ for all histories $\widehat{c}^{\tau}$ that are subsequents of $\widehat{c}_1^t$. At $\widehat{c}_1^t$, the alternative schedule $\widehat{\boldsymbol{p}}$ is such that the buyers' lifetime utility $U\left(\widehat{c}^t|\widehat{\boldsymbol{p}}\right)$ is equal to Z and the seller's expected profits are strictly positive. The schedule $\widehat{\boldsymbol{p}}$ satisfies the incentive-compatibility constraint (IC). If it is also renegotiation-proof, then $\widehat{\boldsymbol{p}}$ is a feasible Pareto improvement over $\boldsymbol{p}$ after the history $\widehat{c}_1^t$ is realized. Therefore, $\boldsymbol{p}$ violates the constraint (RP) and is not an optimum. If $\widehat{\boldsymbol{p}}$ is not renegotiation-proof, then there exists a feasible schedule $\tilde{\boldsymbol{p}}$ which is a Pareto improvement over $\widehat{\boldsymbol{p}}$ and, a fortiori, over $\boldsymbol{p}$. Again, $\boldsymbol{p}$ violates the constraint (RP) and is not an optimum. $\|$

Denote with $\Pi_i\left(U\right)$ the value function associated to (SP2) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c^0)$ is constrained to be *greater or equal* than U. Denote with $\Pi_i^+(U)$ the value function associated to (SP2) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c^0)$ is constrained to be *equal* to U. Finally, let U^{PF} be the set of promised values U such that the seller could not increase its profits by delivering more than U, i.e. $U_i^{PF} = \{U : \Pi_i^+(U) = \Pi_i(U)\}$.

Claim 2: The value function $\Pi_i^+(U)$ satisfies the Bellman equation

$$\Pi_i^+(U) = \max_{p_i, U_j' \in U_j^{PF}} (1 - \sigma + \eta(U)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\Pi_j^+(U_j')\right], \text{ s.t.}$$

$$U = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U_j' + \sigma Z\right], \tag{A4}$$

$$\Pi_j^+(U_j') \geq \tilde{\Pi}_{-j}^+(U_{-j}'),$$

where

$$\tilde{\Pi}_i^+(U) = (1 - \sigma + \eta\left(U\right)) \left[p_i^+\left(U\right) - c_{-i} + \beta \textstyle\sum_j \Pr(c_j|c_{-i})\Pi_j^+(U_{j|i}'^+\left(U\right))\right].$$

Moreover, the value functions $\Pi_i(U)$ and $\Pi_i^+(U)$ satisfy the functional equation

$$\Pi_i(U) = \max_{V, p, U_j' \in U_j^{PF}} (1 - \sigma + \eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\Pi_j^+(U_j')\right], \text{ s.t.}$$

$$U \leq V = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U_j' + \sigma Z\right], \tag{A5}$$

$$\Pi_j^+(U_j') \geq \tilde{\Pi}_{-j}^+(U_{-j}').$$

Proof: The proof of this claim follows directly from the analysis of the seller's problem in Section 4.1. $\|$

Claim 3: The function $\Pi_i(U)$ satisfies the Bellman equation (BE2).
Proof: The continuation value U_j' that solves the maximization problem in (A5) belongs to the set U_j^{PF}. For all $U \in U_j^{PF}$, the profit function $\Pi_j^+(U)$ is equal to $\Pi_j(U)$ and the function $\tilde{\Pi}_j^+(U)$ is equal to $\tilde{\Pi}_j(U)$, where $\tilde{\Pi}_j(U)$ is defined in (BE2). Therefore, I can replace the continuation profit $\Pi_j^+(U)$ with $\Pi_j(U)$ in the objective function of (A5) and substitute the incentive compatibility constraint $\Pi_j^+(U_j') \geq \tilde{\Pi}_{-j}^+(U_{-j}')$ with $\Pi_j(U_j') \geq \tilde{\Pi}_{-j}(U_{-j}')$. Moreover, if the constraint $U_j' \in U^{PF}$ is removed from the modified problem, the optimal continuation value U_j' belongs to U_j^{PF}. Therefore, I can also substitute the constraint $U_j' \in U_j^{PF}$ with the constraint $U_j \geq Z$. $\|$

To conclude the proof of Lemma 2, notice that the value function associated to the sequence problem (SP2) when $c_0 = c_i$ and $n(c_0) > 0$ is given by $n(c_0) \cdot \Pi_i(0)$.

A.4 Proof of Proposition 2

For $\hat{c}^{t-1} = c^{t-1}$ and $c_t = c_h$, the first-best schedule satisfies the incentive compatibility constraint (IC) if and only if

$$(1-\sigma)\left[1-\beta\left(1-\sigma+\eta(U_\ell')\right)(2\rho-1)\right]+\eta'\left[1-\beta\left(1-\sigma\right)(2\rho-1)\right](U_\ell'-U_h') \geq 0.$$

The first term on the LHS is positive because $\beta\left(1-\sigma+\eta\left(U\right)\right)$ is strictly smaller than 1. The second term on the LHS is positive because U_ℓ' is strictly greater than U_h'. Therefore, the incentive compatibility constraint is satisfied.

For $\hat{c}^{t-1} = c^{t-1}$ and $c_t = c_h$, the first-best schedule satisfies the incentive compatibility constraint (IC) if and only if

$$\eta'\left[1-\beta\left(1-\sigma\right)(2\rho-1)\right](U_\ell'-U_h')-(1-\sigma)\left[1-\beta\left(1-\sigma+\eta(U_h')\right)(2\rho-1)\right] \geq 0.$$

The derivative of the LHS with respect to the persistence ρ of productivity shocks is given by

$$\beta\left(1-\sigma\right)\left[2(1-\sigma+\eta\left(U_h'\right))+(2\rho-1)\,\eta'\right]+\left[1-\beta\left(1-\sigma\right)(2\rho-1)\right]\eta'\frac{d(U_\ell'-U_h')}{d\rho}.$$

If $dU_\ell'/d\rho > 0$ and $dU_h'/d\rho < 0$, the derivative is strictly positive and, hence, there exists a critical level of persistence $\rho^* \in [1/2, 1]$ such that the incentive compatibility constraint is satisfied when and only when $\rho \geq \rho^*$.

In order to identify the sign of $dU_i'/d\rho$, it is convenient to let $P_i(U_\ell, U_h; \rho)$ denote the profits of a seller that has realized the production cost c_i and has committed to providing its costumers with the lifetime utility U_ℓ whenever $c_t = c_\ell$ and with U_h whenever $c_t = c_h$, i.e.

$$P_i(U_\ell, U_h; \rho) = (1 - \sigma + \eta(U_i)) [p_i - c_i + \beta\rho P_i + \beta(1-\rho)P_{-i}]$$

$$p_i(U_\ell, U_h; \rho) = u - U_i + \beta\sigma Z + \beta(1-\sigma) [\rho U_i + (1-\rho)U_{-i}] .$$

For a generic couple (U_ℓ, U_h), P_i is smaller than the value function Π_i. For (U_ℓ, U_h) equal to the optimal continuation values (U'_ℓ, U'_h), P_i is equal to Π_i. The derivative of P_i with respect to the persistence of productivity shocks is given by

$$\frac{\partial P_i}{\partial \rho} = \Delta^{-1}\beta [(1-\sigma)(U_i - U_{-i}) + (P_i - P_{-i})] ,$$

where Δ is a positive constant. When evaluated at (U'_ℓ, U'_h), $\partial P_\ell/\partial\rho$ is strictly positive and $\partial P_h/\partial\rho$ is strictly negative because $U'_\ell > U'_h$ and $P_\ell = \Pi_\ell > \Pi_h = P_h$.

The value function $\pi_i(V; \rho)$ associated to the second-stage problem in (4) is equal to

$$\pi_i(V; \rho) = u - c_i - V + \beta\sigma Z + \beta \max_{U_\ell, U_h} \textstyle\sum_j \Pr(c_j|c_i) [P_j(U_\ell, U_h; \rho) + (1-\sigma)U_j]$$

and its derivative with respect to the persistence ρ of productivity shocks is

$$\frac{\partial \pi_i}{\partial \rho} = \beta \left[(1-\sigma)(U'_i - U'_{-i}) + (P_i - P_{-i}) + \rho\frac{\partial P_i}{\partial \rho} + (1-\rho)\frac{\partial P_{-i}}{\partial \rho}\right] =$$

$$= \frac{\beta}{1 - \sigma + \eta(U'_i)} \frac{\partial P_i}{\partial \rho}.$$

From the second line, it follows that $\partial \pi_\ell/\partial\rho$ is strictly positive and $\partial \pi_h/\partial\rho$ is strictly negative. In turn, from the first order condition (6) for the continuation value, it follows that $dU'_\ell/d\rho$ is strictly positive and $dU'_h/d\rho$ is strictly negative.

A.5 Preliminaries to Propositions 3 and 4

Consider the first-stage problem in (11). For $V < Z$, the objective function $(1 - \sigma + \eta(V)) \cdot \pi_i(V)$ is equal to zero because $\eta(V) = \sigma - 1$. For $V = Z$, the function is strictly positive because $1 + \sigma = \eta(V) = 1 - \sigma > 0$ and $\pi_i(Z) > 0$ as proved in Lemma 2. For $V \geq Z$, the function is quasi-concave. The function attains its maximum for $V = \underline{U}_i$, where $\underline{U}_i$ is the solution to equation (A2). From the properties of the objective function, it follows that the solution $V_i(U)$ to the first-stage problem is $\underline{U}_i$ whenever $U \leq \underline{U}_i$ and U otherwise. In turn, the value function $\Pi_i(U)$ associated to the first-stage problem is constant at $(1 - \sigma + \eta(\underline{U}_i)) \cdot \pi(\underline{U}_i)$ whenever $U \leq \underline{U}_i$ and is strictly decreasing and quasi-concave otherwise.

Next, consider the second-stage problem in (11). As proved in Lemma 2, the choice set can be restricted to the continuation values (U'_ℓ, U'_h) that are greater than the profit-maximizing values $(\underline{U}_\ell, \underline{U}_h)$. Over this domain, the objective function is jointly quasi concave in (U'_ℓ, U'_h). Also, for any given U'_h, the objective function is maximized at U'^*_ℓ, where U'^*_ℓ is the solution to equation (A3) for $j = \ell$ and is strictly greater than $\underline{U}_\ell$. For any given U'_ℓ, the objective function is maximized at U'^*_h, where U'^*_h is the solution to equation (A3) for $j = h$ and is strictly greater than $\underline{U}_h$. The choice of the continuation values is limited by the incentive compatibility constraint $\Pi_j(U'_j) \geq \widetilde{\Pi}_{-j}(U'_{-j})$. Since V enters separately from (U'_ℓ, U'_h) in the objective function and does not enter the constraints, the solution $U'_{j|i}(V)$ to the second-stage problem is independent from V and can be denoted with $U'_{j|i}$.

A.6 Proof of Proposition 3

When $\rho = 1/2$, the second-stage problem in (12) can be reformulated as

$$\pi_i(V) = u - c_i - V + \beta\sigma Z + \frac{\beta}{2} \max_{U'_j \geq \underline{U}_j} \textstyle\sum_j \left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.}$$

$$(1 - \sigma + \eta(U'_j)) \cdot \pi_j(U'_j) \geq (1 - \sigma + \eta(U'_{-j})) \cdot \pi_j(U'_{-j}). \tag{A6}$$

Since $U'_h \geq \underline{U}_h$, $U'_\ell \geq \underline{U}_\ell \geq \underline{U}_h$ and the function $(1 - \sigma + \eta) \cdot \pi_h$ is strictly decreasing for all $U \geq \underline{U}_h$, the high-cost seller's incentive compatibility constraint (A6) is equivalent to $U'_\ell \geq U'_h$. Since the function $(1 - \sigma + \eta) \cdot \pi_\ell$ is strictly increasing for $U \in [\underline{U}_h, \underline{U}_\ell]$ and strictly decreasing for $U \geq \underline{U}_\ell$, the low-cost seller's incentive compatibility constraint (A6) is satisfied either if $U'_h \geq U'_\ell$ or if U'_h is sufficiently lower than $\underline{U}_\ell$. Overall, a couple of continuation values (U'_ℓ, U'_h) is feasible if either $U'_\ell = U'_h \geq \underline{U}_\ell$ or $U'_\ell \neq U'_h$ and $U'_h \leq \underline{U}_\ell$, $U'_\ell \geq \underline{U}_\ell$.

Let $c_\ell(\Delta) = c - \Delta$ and $c_h(\Delta) = c + \Delta$ for some $c \in (0, u - z)$ and $\Delta \geq 0$. If the solution to the second-stage problem is such that $U'_\ell = U'_h$, the seller's profits per costumer are bounded below by

$$\pi_i^{\mathcal{P}}(V; \Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2} \textstyle\sum_j \left[\Pi_j\left(U'^*_\ell(\Delta); \Delta\right) + (1-\sigma)\, U'^*_\ell(\Delta)\right].$$

If the solution to the second-stage problem is such that $U'_\ell \neq U'_h$, the seller's profits per costumer are bounded above by

$$\pi_i^{\mathcal{S}}(V; \Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2} \textstyle\sum_j \left[\Pi_j(U_j^{\mathcal{S}}(\Delta); \Delta) + (1-\sigma)\, U_j^{\mathcal{S}}(\Delta)\right],$$

where $U_h^{\mathcal{S}}(\Delta) = \min\{U'^*_h(\Delta), \underline{U}_\ell(\Delta)\}$ and $U_\ell^{\mathcal{S}}(\Delta) = U'^*_\ell(\Delta)$. Independently from the nature of the solution to the second-stage problem, the seller's profits per costumer are bounded above by

$$\pi_i^{\mathcal{P}}(V; \Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2} \textstyle\sum_j \left[\Pi_j(U'^*_j(\Delta); \Delta) + (1-\sigma)\, U'^*_j(\Delta)\right].$$

For $\Delta = 0$, $\pi_i^{\mathcal{P}}(V;\Delta)$ is equal to $\pi_i^*(U;\Delta)$ because $\Pi_\ell(U;\Delta) = \Pi_h(U;\Delta)$ and $U_\ell'^*(\Delta) = U_h'^*(\Delta)$. For $\Delta = 0$, $\pi_i^{\mathcal{S}}(U;\Delta)$ is strictly smaller than to $\pi_i^*(U;\Delta)$ because $U_h^{\mathcal{S}}(\Delta) = \underline{U}_h(\Delta)$ and $\underline{U}_h(\Delta) < U_h'^*(\Delta)$. By continuity, I conclude that the solution to the second-stage problem is such that $U_\ell' = U_h'$ for all $\Delta \in (0, \Delta^*)$.

A.7 Proof of Proposition 4

In order to characterize the second-best price schedule when productivity shocks are persistent, I start by conjecturing that the solution to the problem (SP2) is such that: (i) the high-cost seller's incentive compatibility constraint is moot, i.e. $\Pi_h(U_{h|i}') > \tilde{\Pi}_\ell(U_{\ell|i}')$ for $i = \ell, h$; (ii) the low-cost seller prefers to report its true type rather than lying whenever $U_\ell' = U_h' = U$, i.e. $\pi_\ell(U) \geq \widetilde{\pi}_h(U)$.

If the high-cost seller's incentive compatibility constraint is moot, the second-stage problem in (12) can be reformulated as

$$\pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U_j' \geq \underline{U}_j} \sum_j \Pr(c_j|c_i)\left[\Pi_j(U_j') + (1-\sigma)U_j'\right], \text{ s.t.}$$

$$(1 - \sigma + \eta(U_\ell')) \cdot \pi_\ell(U_\ell') \geq (1 - \sigma + \eta(U_h')) \cdot \widetilde{\pi}_h(U_h'). \tag{A7}$$

For all $U_\ell' \geq \underline{U}_\ell$, the LHS of (A7) is strictly decreasing. For all $U_h' \geq \underline{U}_h$, the RHS of (A7) is quasi-concave because it is concave whenever increasing and convex only when strictly decreasing. The RHS attains its maximum for $U_h' = \widetilde{\underline{U}}_h \leq \underline{U}_\ell$, where $\widetilde{\underline{U}}_h$ satisfies

$$\left[\eta'(\widetilde{\underline{U}}_h) \cdot \widetilde{\pi}_h(\widetilde{\underline{U}}_h) - (1 - \sigma + \eta(\widetilde{\underline{U}}_h))\right] \cdot (\widetilde{\underline{U}}_h - Z) = 0.$$

It is useful to partition the feasible set of the second-stage problem into the subsets $\mathcal{P}$ and $\mathcal{S}$. Specifically, $\mathcal{P}$ is the set of continuation values (U_ℓ', U_h') that are feasible and such that $U_h' \geq \widetilde{\underline{U}}_h$, while $\mathcal{S}$ is the set of continuation values that are feasible and such that $U_h' \leq \widetilde{\underline{U}}_h$. The set $\mathcal{P}$ contains the state-independent continuation values $U_\ell' = U_h' \geq \underline{U}_\ell$ because $\pi_\ell(U') \geq \widetilde{\pi}_h(U')$. The set $\mathcal{S}$ does not contain any continuation values (U_ℓ', U_h') such that U_h' is greater than $\underline{U}_\ell$ because $\widetilde{\underline{U}}_h \leq \underline{U}_\ell$.

Let $c_\ell(\Delta) = c - \Delta$ and $c_h(\Delta) = c + \Delta$ for some $c \in (0, u - z)$ and $\Delta \geq 0$. If the solution to the second-stage problem belongs to the subset $\mathcal{P}$, the seller's profits per costumer are bounded below by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \sum_j \Pr(c_j|c_i)\left[\Pi_j(U_\ell'^*(\Delta);\Delta) + (1-\sigma)U_\ell'^*(\Delta)\right].$$

If the solution to the second-stage problem belongs to the subset $\mathcal{S}$, the seller's profits per costumer are bounded above by

$$\pi_i^{\mathcal{S}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \sum_j \Pr(c_j|c_i)\left[\Pi_j(U_j^{\mathcal{S}}(\Delta);\Delta) + (1-\sigma)U_j^{\mathcal{S}}(\Delta)\right],$$

where $U_h^{\mathcal{S}}(\Delta) = \min\{U_h'^*(\Delta), \underline{U}_\ell(\Delta)\}$ and $U_\ell^{\mathcal{S}}(\Delta) = U_\ell'^*(\Delta)$. Independently from the nature of the solution to the second-stage problem, the seller's profits per costumer are bounded above by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U_j'^*(\Delta);\Delta) + (1-\sigma)\,U_j'^*(\Delta)\right].$$

For $\Delta = 0$, $\pi_i^{\mathcal{P}}(V;\Delta)$ is equal to $\pi_i^*(U;\Delta)$ because $\Pi_\ell(U;\Delta) = \Pi_h(U;\Delta)$ and $U_\ell'^*(\Delta) = U_h'^*(\Delta)$. For $\Delta = 0$, $\pi_i^{\mathcal{S}}(U;\Delta)$ is strictly smaller than to $\pi_i^*(U;\Delta)$ because $U_h^{\mathcal{S}}(\Delta) = \underline{U}_h(\Delta)$ and $\underline{U}_h(\Delta) < U_h'^*(\Delta)$. By continuity, I conclude that the solution to the second-stage problem belongs to $\mathcal{P}$ for all $\Delta \in (0, \Delta_1)$.

When Δ is sufficiently small, i.e. $\Delta \in (0, \Delta_2)$, the unconstrained maximum of the second-stage problem $(U_\ell'^*, U_h'^*)$ is not feasible because it violates the low-cost seller's incentive compatibility constraint (cf. condition (10)). When this is the case, the constraint (A7) holds with equality because the objective function of the second-stage problem is quasi concave. Therefore, for all $\Delta \in (0, \Delta^*)$, where $\Delta^* = \min\{\Delta_1, \Delta_2\}$, the solution to the second-stage problem belongs to the subset $\mathcal{P}$ and satisfies the constraint (A7) with equality.

For all $\Delta \in (0, \Delta^*)$, the solution $(U'_{\ell|i}, U'_{h|i})$ to the second-stage problem has the following properties:

1. The continuation value $U'_{\ell|i}$ is smaller than $U_\ell'^*$. Proof: If $U'_{\ell|i}$ is strictly greater than $U_\ell'^*$, then $(U_\ell'^*, U'_{h|i})$ is feasible because $\Pi_\ell(U_\ell'^*) > \Pi_\ell(U'_{\ell|i})$. Also, $(U_\ell'^*, U'_{h|i})$ is preferable because the objective function is quasi concave in U'_ℓ and is maximized at $U_\ell'^*$.

2. The continuation value $U'_{h|i}$ is greater than $U_h'^*$. Proof: If $U'_{h|i}$ is strictly smaller than $U_h'^*$, then $(U'_{\ell|i}, U_h'^*)$ is feasible because $\tilde{\Pi}_h(U_h'^*) < \tilde{\Pi}_h(U'_{h|i})$. Also, $(U'_{\ell|i}, U_h'^*)$ is preferable because the objective function is quasi concave in U'_h and is maximized at $U_h'^*$.

3. The continuation value $U'_{h|i}$ is strictly smaller than $U'_{\ell|i}$. Proof: If $U'_{h|i}$ is greater than $U'_{\ell|i}$, then $\Pi_\ell(U'_{\ell|i}) \geq \tilde{\Pi}_h(U'_{\ell|i}) > \tilde{\Pi}_h(U'_{h|i})$. This is not possible because (A7) holds with equality for all $\Delta \in (0, \Delta^*)$.

4. If $U'_{h|j} > U'_{h|i}$, then $U'_{\ell|j}$ is strictly greater than $U'_{\ell|i}$. Proof: Since the LHS and RHS of (A7) are strictly decreasing in U'_ℓ and U'_h and the constraint (A7) holds with equality, if $U'_{h|j} > U'_{h|i}$ then $U'_{\ell|j} > U'_{\ell|i}$.

5. The continuation values are such that $U'_{\ell|\ell} \geq U'_{\ell|h}$ and $U'_{h|\ell} \geq U'_{h|h}$. Proof: Since the objective function puts more weight on $\Pi_\ell(U'_\ell) + (1-\sigma)\,U'_\ell$ and less weight on $\Pi_h(U'_h) + (1-\sigma)\,U'_h$ when $c_i = c_\ell$ than when $c_i = c_h$, $\Pi_\ell(U'_{\ell|\ell}) + (1-\sigma)\,U'_{\ell|\ell}$ is greater than $\Pi_\ell(U'_{\ell|h}) + (1-\sigma)\,U'_{\ell|h}$. In light of property (1), this implies that $U'_{\ell|\ell}$ is greater than $U'_{\ell|h}$. In light of property (2), this implies that $U'_{h|\ell}$ is greater than $U'_{h|h}$.

6. The continuation values are such that $E\left[(1-\sigma)U'_{j|i}+\Pi_i(U'_{j|i})|c_k\right]$ is greater for $k=i$ than $-i$. Proof: This result follows immediately from property (5).

These six properties of the optimal continuation values lead immediately to Proposition 4.

In the last step of the analysis, I have to verify my initial conjectures. In order to verify that the high-cost seller's incentive compatibility constraint is moot, it is convenient to rewrite $\Pi_h(U'_{h|i}) \geq \tilde{\Pi}_\ell(U'_{\ell|i})$ as

$$\eta'(U'_{h|i}-U'_{\ell|i})\cdot\pi_h(U'_{h|i}) \geq$$

$$(1-\sigma+\eta(U'_{\ell|i}))\left[\begin{array}{l} U'_{h|i}-U'_{\ell|i}+\beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|\ell}-U'_{h|\ell})+ \\ \\ \beta\left\{E\left[(1-\sigma)U'_{i|\ell}+\Pi_i(U'_{i|\ell})|c_h\right]-E\left[(1-\sigma)U'_{i|h}+\Pi_i(U'_{i|h})|c_h\right]\right\}. \end{array}\right] \tag{A8}$$

First, notice that $U'_{h|i}\geq U'^{*}_{h}$ implies that $\eta'\pi_h(U'_{h|i})$ is smaller than $\eta(U'_{h|i})$ and that the LHS of (A8) is bounded below by

$$(U'_{h|i}-U'_{\ell|i})\cdot\eta(U'_{h|i}). \tag{A9}$$

Secondly, notice that $E\left[(1-\sigma)U'_{i|h}+\Pi_i(U'_{i|h})|c_h\right]$ greater than $E\left[(1-\sigma)U'_{i|\ell}+\Pi_i(U'_{i|\ell})|c_h\right]$ and $U'_{\ell|h}-U'_{h|h}$ greater than $U'_{\ell|\ell}-U'_{h|\ell}$ (a fact that can be derived from the low-cost seller's incentive compatibility constraint) imply that the RHS of (A8) is bounded above by both

$$\begin{aligned} &(1-\sigma+\eta(U'_{\ell|i}))\left[U'_{h|i}-U'_{\ell|i}+\beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|\ell}-U'_{h|\ell})\right], \\ &(1-\sigma+\eta(U'_{\ell|i}))\left[U'_{h|i}-U'_{\ell|i}+\beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|h}-U'_{h|h})\right]. \end{aligned} \tag{A10}$$

Overall, the high-cost seller's incentive constraint (A8) is satisfied if (A9) is greater than (A10) or, equivalently, if

$$(1-\sigma)\left[1-\beta(1-\sigma+\eta(U'_{h|i}))(2\rho-1)\right]+\eta'\left[1-\beta(1-\sigma)(2\rho-1)\right](U'_{\ell|i}-U'_{h|i})\geq 0. \tag{A11}$$

Because $\beta(1-\sigma+\eta(U'_{h|i}))$ is smaller than 1 and $U'_{\ell|i}$ is greater than $U'_{h|i}$, the sufficient condition (A11) is satisfied.

Finally, I have to verify the conjecture that the low-cost seller prefers to report its true type rather than lying whenever $U'_\ell=U'_h=U$, i.e. $\pi_\ell(U)\geq\tilde{\pi}_h(U)$ or

$$\begin{aligned} &E\left[\Pi_j(U'_{j|\ell})+(1-\sigma)\,U'_{j|\ell}|c_\ell\right]\geq \\ &E\left[\Pi_j(U'_{j|h})+(1-\sigma)\,U'_{j|h}|c_\ell\right]+(2\rho-1)\,(1-\sigma)\,(U'_{h|h}-U'_{\ell|h}). \end{aligned} \tag{A12}$$

Since $E\left[\Pi_j(U'_{j|\ell}) + (1-\sigma)\, U'_{j|\ell}|c_\ell\right]$ is greater than $E\left[\Pi_j(U'_{j|h}) + (1-\sigma)\, U'_{j|h}|c_\ell\right]$ and $U'_{h|h}$ is smaller than $U'_{\ell|h}$, condition (A12) is satisfied.