The value of switching costs

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Abstract

We extend the standard dynamic model with switching costs by assuming that different consumers have different switching costs. We show that this changes considerably the type of strategies used by the firms, with some unexpected consequences. In particular, we show that there are cases where an increase in the switching costs of all consumes can lead to a decrease in the profits of the incumbent.

This is a very preliminary and incomplete version. It certainly contains a number of mistakes and many typos. Comments are very welcome.

1 Introduction

On February 6, 2007, in the very same statement in which he called for an end to DRM (Digital Rights Management) for music distributed in electronic form, Steve Jobs discussed [1] the incumbency benefits that the iPod enjoyed thanks to iTunes' proprietary format.

He noticed that "[s]ome have argued that once a consumer purchases a body of music from one of the proprietary music stores, they are forever locked into only using music players from that one company. Or, if they buy a specific player, they are locked into buying music only from that company's music store." His response was that on average there are "22 songs purchased from the iTunes store for each iPod ever sold", and that this implied that "under 3% of the music on the average iPod is purchased from the iTunes store and protected with a DRM." His conclusion was that there was no lock-in as it is "hard to believe that just 3% of the music on the average iPod is enough to lock users".

This statement was heavily discussed by many commentators. In particular, John Lech Johansen[2] made the following interesting points.

"Many iPod owners have never bought anything from the iTunes Store. Some have bought hundreds of songs. Some have bought thousands. At the 2004 Macworld Expo, Steve revealed that one customer had bought $29,500 worth of music." Therefore, the lock-in is non negligible as "it's the customers who would be the most valuable to an Apple competitor that get locked in. The kind of customers who would spend $300 on a set-top box."

Johansen's point is that the consumers that matter, those who buy lots of online music, have high switching costs, and therefore that an entrant in the market will face large obstacles attracting them. In the simplest economic model with switching costs, Johansen is wrong: heterogeneity of switching costs does not matter. Indeed, assume that proportion α of the population has strictly positive switching costs σ, while the others have no switching costs. There is an incumbent, who was a monopolist in the past, and which therefore supplied all of the customers. Then arrives potential entrants, who, like the monopolist, have zero cost of production. If the consumers have high enough willingness to pay, the incumbent will charge σ and its profits will be $\alpha\sigma$, the total value of switching costs in the economy. Steve Jobs is not

[1] See Jobs (2007).

[2] See Johansen (2007).

underestimating the value of incumbency by assuming that all consumers have the same switching costs. [3]

The result changes, if we take into account the fact that the value of the clientele that the entrants are trying to acquire depends on these same switching costs. We will show [4] that Johansen is right. Decreasing α and increasing σ while keeping $\alpha\sigma$ constant increases the value of incumbency: the high switching costs consumers are more valuable, and the presence of more low switching costs consumers makes it more difficult for an entrant to attract the high value consumers.

There exists a significant body of theory which explores the consequences of consumer switching costs.[5] It shows that firms will compete aggressively to attract consumers with high switching costs, in the hope of extracting rents in the future. However, to the best of our knowledge, the fact that the distribution of switching costs plays an important role has not been highlighted. [6] We hope the present paper will contribute to close this gap.

Of course, it is mostly policy concerns [7]

that drive the interest on switching costs: the higher the switching costs, the less competitive the outcome and the higher the profits of the incumbent. This suggests that vigorous enforcement of antitrust laws is specially important in these cases, but has also other policy consequences. Actions by incumbent firms which increase consumer switching costs are often interpreted as anticompetitive, and as a way of increasing profits. We will show that increasing switching costs can actually *lower* the incumbents profits. This suggests that such actions should be subject to a rule of reason and not be considered *per se* anticompetitive.

We conduct our analysis by constructing a series of models that share the

[3] If we let $F(\sigma)$ be the distribution of switching costs and fix the average switching costs, $\int \sigma dF(\sigma)$, it is easy to verify that the incumbent's profits are maximized for the distributions of types assumed in this paragraph. By assuming that all customers have switching costs, Jobs chose one of the scenarios that maximized the value of incumbency for Apple.

[4] See footnotes **??** and 17.

[5] See Annex A of of Fair Trading (2003)for an excellent literature review.

[6] Taylor (2003) studies a dynamic model with heterogenous switching costs, but the switching cost of each consumer changes from period to period. This yields very different predictions, which we will discuss below. We believe that our assumptions are more realistic.

[7] See of Fair Trading (2003) for an indication of policy concerns, especially the case studies in Annexe C.

following features: a) each consumer has a switching cost which is invariant over time and is forward looking in the sense that they know make correct predictions about what prices they will face in the future; b) at the start of the "game" there is a single incumbent firm; and c) there is free entry by competing firms. Following much of the literature, we assume that only short term contracts are used and that a consumer's switching cost does not depend on the firm from which it is purchasing (this seems to be a fair idealization of many industries).

In section 3, we introduce our analysis by considering the case where all consumers have the same switching costs σ. In a one period model, the incumbent would charge σ, and, assuming that the mass of consumers is equal to 1, its profit would also be equal to σ. Then, in a multi-period model, the equilibrium profit of the incumbent is also equal to σ. Thus, one should be careful when using the formula "profit = marginal cost + switching cost" to interpret "switching cost" as the switching cost per period, σ/δ.

We begin our analysis of the heterogeneity of switching costs in section 4, where we study the model which we sketched above when describing the Jobs-Johansen debate: some consumers have a switching cost equal to $\sigma > 0$, while others have no switching cost. We identify the (stationary) equilibrium of the infinite horizon model. The intertemporal profit of the incumbent is greater than the one period profit, although smaller than the value of an infinite stream of one period profits. The presence of low switching cost customers hinder entrants who find it more costly to attract the high switching cost customers. As a consequence, we also find that the presence of low switching cost customers increase the price paid by the high switching cost customers.

In order to conduct more complete comparative statics, in section 5, we generalize the model of section 4 by assuming that the low switching cost consumers can have a strictly positive switching costs. For technical reasons, we turn to a two period model. For a large class of parameters decreasing the switching costs of *all* consumers *increases* the profits of the incumbent: the decrease in the low switching cost makes entrants less aggressive as they are, in the mixed strategy equilibrium, more likely to be the only ones attracted by this offer. The resulting increase in the profits of the incumbent is greater than its loss from the decrease in the high level switching costs. In section 6, we study a two-period model with a continuous distribution of switching costs and show the same economic results as in the two type model. We conclude and offer direction for future research in section 7.

2 Literature

The literature has made a distinction between switching cost models proper and subscription models: in switching cost models, a firm must charge the same price to both current and new consumers, while in subscription models it can offer different prices to consumers depending on the history of its purchases of its products. Switching cost models were introduced in the economics literature by Klemperer (1987*b*) (see also his excellent survey in Klemperer (1995)). Chen (1997) initiated the investigation of subscription models, to which Taylor (2003) made a notable contribution. With free entry, in each period an incumbent firm behave towards the past customers of other firms exactly like one of the entrants. It can never generate positive profits off of these consumers. There is no difference between switching cost models and subscription models.

Most of the switching cost literature focusses on two-period duopsony models in which firms can either charge a high price and extract rents from its current customers or charge a low price in order to attract customers from its rival. Klemperer (1987*a*) shows that higher switching costs may make entry more likely, as an incumbent will price less aggressively to attract to new consumers and hence be willing to lose some customers to entrants. With our free entry assumption, entrants will always be present and an incuumbent will never price to attract new customers, since free entry causes this to be a zero profit activity.

Farrell & Shapiro (1988), Beggs & Klemperer (1992), Padilla (1995), and Anderson, Kuman and Rajiv (2004) examine infinite horizon switching cost. All these authors considers models with two firms and homogenous switching costs;[8] they focus their analysis on the evolution of market shares and on the effect of switching cost on prices. By contrast, we focus our analysis on the consequences of the heterogeneity of switching costs in the presence of free entry. Dube, Hitsch, and Rossi (2006) present an infinite horizon model where a single consumer has random utility and firms have differentiated products. While their focus in on empirical part of the paper, they demonstrate that prices may fall when switching costs are present.

Taylor (2003) analyzes a finite horizon subscription model where consumers have different switching costs. However, the switching cost of a given

[8] In Beggs and Klemperer, consumers are horizontally differentiated, but once they join a firm, then they never buy from another firm.

consumer is random in each period and independent from period to period. If there are at least three firms, the expected value of a new consumer is zero due to Bertrand competition. Our assumption that switching costs are constant over time implies that it is harder for an entrant to attract the more valuable consumers, since an entrant will always attract the low switch cost consumers first. This generates our comparative statics results on the incumbent's profits that an incumbent's profits may actually fall with an increase in switching costs.

3 When consumers have the same switching cost: You cannot get rich on switching costs alone

There is a continuum of consumers with mass normalized to 1, and a good which can be supplied by a number of firms, as we will describe below. Consumers have a totally inelastic demand for one unit of the good, and therefore always buy one unit from some firm or the other. For the time being, all consumers have the same switching cost σ.

We assume that in previous periods, the consumers have bought from an incumbent, firm I. Let us consider first a one period model with a denumerable number of entrants who can enter the market at zero cost. The game that we consider is a two stage games:

Stage 1: The incumbents and the entrants set prices.

Stage2: The consumers choose from which firm to buy.

Assuming, as we will throughout this paper that all firms have zero marginal cost, it is standard to prove that there is essentially one equilibrium of this game, where the incumbent charges a price of σ, the entrants a price of 0, and all consumers buy from the incumbent.

It is from this model that much of our standard intuition about switching costs is derived. However, we have not found in the literature a clear statement of what happens when this game is repeated, with new entrants in every period; almost all of the literature focuses on the case of duopsony. As we will show, the discounted profit of the incumbent is only equal to σ, the same as in the one period model.

It is easy to see this if the game is repeated twice. Formally, we expand the game above by assuming that every entrant that has sold to a positive measure of consumers in the first period is a second period incumbent, *and* that there are new entrants, at least two, in the second period. Because we have a dynamic model, we need to introduce a discount factor; let us call it δ, and assume that it is the same for all firms and consumers.

It is clear that all second period incumbents will charge σ, and make profits equal to σ times the mass of customersthey had in the first period. Therefore, Bertrand competition between first-period entrants will push the price that they charge down to $-\delta\sigma$. Consumers know that all firms will charge the same price in the second period. Hence, firm I will be able to keep its customers only if it charges a price less than or equal to $-\delta\sigma + \sigma$. It is straightforward to show that this is the price that it will charge, and that it will "keep" all its customers. Hence its discounted profit is

$$(-\delta\sigma + \sigma) + \delta\sigma = \sigma.$$

An easy proof by induction shows that the same result holds for a model with any finite number of periods. [9]

We now show that the same result holds true in the infinite horizon version of this model. In each period, we assume that are a finite number of active entrants who offer the good. We look for subgame perfect equilibria, that satisfy set of conditions which which we describe informally here and are formally defined in the appendix. The first conditions are in place to eliminate bad coordination equilibrium. The second condidions are a set of stationarity conditions. Finally, we assume that players do not use weakly dominated strategies.

We want to prevent the following type of situation: an entrant firm makes a much better offer than the incumbent, taking into account the fact that the consumers have to pay the switching cost σ. However, all consumers think that the others will refuse the offer. Therefore, every consumer feels that if he accepts the offer, he will be the only one, which since we assume that entrants that firms who do not have a positive measure of consumers at the end of the period are not active in the future, implies that the consumer will have to pay the switching cost once again in the following period. Therefore, it is an equilibrium for all consumers not to accept the offer.

[9] Technically, the analysis is a special case of Taylor (2003). We are simply focusing attention on an important economic consequence of his analysis.

To eliminate this ”bad equilibrium” we assume that “consumers have mass”. Informally, we will allow small groups of consumers to coordinate on a strategy, such that if a positive measure of consumers would be strictly better off purchasing from the entrant than the incumbent if they all moved to the entrant (in the sense that their discounted disutility is lower after this deviation from the equilibrium), then they will leave the incumbent.

We will make three assumptions which can be thought of as stationarity assumptions. First, we will use a “coordination” requirement on the consumers. We look for equilibria where consumers purchase as much as possible from the same firm. A consequence of this assumption is that on the equilibrium path, there will be a single incumbent. Our second requirement is a stationarity requirement on the pricing of the incumbents, whether, there is one, as would be the case along the equilibrium path, or several, as could be the case out of the equilibrium path. We assume that incumbents all set the same price, whatever the history. Third, we assume that consumers switch as little as possible. That is, if the incumbent price plus the switching cost does not exceed an entrant's price, then a consumer will not switch.

Finally, as in one period Bertrand models with different costs for the different firms, [10] we assume firms play undominated strategies.

Call Π the present discounted profit of a incumber firm from which all the consumers bought in the previous period. This profit is independent of the firm's name and of the date. Entrants are willing to charge $-\delta\Pi$ to attract all the buyers. As in the two period model, consumers know that they welfare in subsequent periods will be the same whichever firm they purchase from, and the incumbent will set a price equal to $-\delta\Pi$ plus σ in order to keep its customers.[11] Hence, the equilibrium profit of the incumbent satisfies

$$\Pi = -\delta\Pi + \sigma + \delta\Pi = \sigma.$$

Therefore, as in the two period model, the incumbent can only collect the

[10]If a firm has marginal cost equal to 1 and the other marginal cost equal to 0, the equilibrium should be that both firms charge 1 and that consumers charge choose to buy from the low cost firm. However, without elimination of dominated strategies, there are also equilibria where both firms charge $p \in (0, 1)$ and consumers buy from the low cost firm. The strategy of charging p is dominated for the high cost firm, as it could make a negative profit if consumers chose to purchase its product.

[11]Technically, the incumbent will charge $-\delta\Pi + \sigma$, and the continuation equilibrium if one or several entrants charged $-\delta\Pi$ would be for all the consumers to buy from the incumbent.

switching cost once: he only gets one bite at the apple.[12] To implement this profit, in every period the entrants charge $-\delta\sigma$ while the incumbent charges $\sigma(1-\delta)$, which yields a present discounted profit equal to

$$\frac{\sigma(1-\delta)}{(1-\delta)} = \sigma.$$

We summarize these results in the following proposition.

Proposition 1. *When all consumers have the same switching costs σ, for any discount factor δ, the profit of the incumbent is σ in the one period model, in the finite horizon model (whatever the horizon) and in the stationary equilibrium of the infinite horizon model.*

4 Heterogeneity of switching costs hurts consumers

We now turn to the main theme of the article: the consequences of the fact that different consumers have different switching costs. In this section, we begin this analysis by considering an infinite horizon model with two types of consumers. High switching cost consumers, who are a fraction α of the population, have a switching cost equal to $\sigma > 0$, while low switching cost consumers, who are a fraction $(1-\alpha)$ of the population, have a switching cost equal to 0. (For ease of exposition, we will sometimes drop the "switching" and refer to high (resp. low) cost consumers).

We will analyze this model with two types of price setting games. One is where firms simultaneously set prices in each period (Bertrand) and the other is where the incumbent first sets its price and then the entrants set their prices (Stackelberg). It turns out that the analysis in the infinite horizon game is simpler (no mixed strategies!) with Stackelberg competition, and we therefore first focus on this case. We demonstrate that both forms of competition generate the same profits for the incumbent.

[12] In a companion paper, we prove that there exist other equilibria of this game, even if we impose that the equilibrium outcome is stationary. We identify equilibria where the profit of the incumbent is as low as 0 and as high as $\sigma/(1-\delta)$ (that is the incumbent charges σ in every period.

4.1 Analysis and results

As a benchmark, in the one period model, competition drives the prices of entrants to 0, while the incumbent charges a price of σ, and obtains a profit of $\alpha\sigma$.

As in the model where consumers have the same switching costs, we restrict attention to equilibria that satisfy the consumer coordination and the stationarity conditions.[13]

The following proposition summarizes the consequences for the incumbent of the fact that consumers do not all have the same switching costs.

Proposition 2. *In the infinite horizon model, where α of the consumers have switching costs equal to $\sigma > 0$, while the others have zero switching costs, under either Stackelberg or Bertrand competition*

1. *The expected profit of the incumbent,*
$$\Pi = \frac{\alpha\sigma}{1-\delta+\alpha\delta}, \tag{1}$$
is increasing in α and σ and decreasing in δ.

2. *This profit is greater than the profit $\alpha\sigma$ that it obtains in the one period model, but smaller than value, $\alpha\sigma/(1-\delta)$ of an infinite stream of one period profits.*

3. *Π is smaller than σ, but we have, for all α*
$$\lim_{\delta\to 1} \Pi = \sigma.$$

Parts 1 and 2 of the proposition shows that, contrary to what happens when all consumers have the same switching costs, the intertemporal profit is not equal to the one period profit, but is *greater*; however the per period profit is smaller in the infinite horizon model than in the one period model. Finally, part 3 shows that when economic agents become very patient, the

[13] Since there are multiple consumer types, we also make a stationariy assumption on the entrant's pricing distribution, when the low cost consumers are at a different firm than the high swich cost consumers. In particular, the distribution of the lowest entrant price In or out of equilibrium, the distribution of p^E, the minimum of the prices charged by entrants and the firms which sold only to lsc customers in the previous period, is independent of history.

profit of the incumbent are independent of the proportion of high switching cost consumers, whereas in the one period model profits are proportional to the proportion of high switching cost consumers. As we will explain below, low switching cost customers makes attracting profitable, high switch cost customers more difficult since the unprofitable buyers accept the offers made by entrants to attract the profitable customers.

The next proposition summarizes the consequences of the form of competition for the welfare of consumers.

Proposition 3. *In the infinite horizon model, where α of the consumers have switching costs equal to $\sigma > 0$, while the others have zero switching costs*

1. *With Stackelberg competition, the utility of buyers with high switching cost in is an increasing function of α.*
2. *Consumers surplus and welfare is lower under Bertrand competition than under Stackelberg competition.*

Part 2 is a consequence of the fact that Bertrand competition introduces some inefficiencies, whose costs are entirely borne by the consumers, as the profit of the incumbent is independent of the mode of competition and the (expected) profits of entrants are always zero.

As the proposition shows, results are essentially similar with Stackelberg and Bertrand competition. We first study the simpler Stackelberg setup. We modify the game so that in each period the incumbents set and announce their prices first before the entrants all simultaneously set and announce their prices.

As in section 3, let Π be the profit of the incumbent if it has all the consumers at the start of the period. An entrant will be willing to underbid the incumbent by (slightly more than) σ as long as the incumbent's price is greater than $-\delta\Pi + \sigma$. Hence, the incumbent will charge $-\delta\Pi + \sigma$ and sell to the α high cost customers at this price.[14]

We therefore have

$$\begin{aligned} \Pi &= \alpha \times (-\delta\Pi + \sigma) + \delta\Pi \\ &\Longrightarrow \Pi = \frac{\alpha\sigma}{1 - \delta + \alpha\delta}. \end{aligned} \tag{2}$$

[14] Technically, in equilibrium the incumbent charges $-\delta\Pi + \sigma$ and the entrants charge 0. In any continuation equilibrium after one or several entrants charge $-\delta\Pi$, the high switching costs consumers buy from the incumbent.

This implies that the price charged by the incumbent is equal to

$$\sigma \frac{1-\delta}{1-(1-\alpha)\delta}.$$

The price paid by the consumers with a high switching cost increases when the proportion $(1-\alpha)$ of low cost consumers increases. It would be a mistake to assume that because the average switching cost is decreasing, the price charged by the incumbent is decreasing.

It is worthwhile pointing out that if the number of low switching cost consumers is increasing, while the number of high switching costs consumers remain constant, the profits of the incumbent increase. Indeed, assume a mass $\eta > 0$ of low switching cost consumers is added. The proportion of high level consumers becomes $\alpha' = \alpha/(1+\eta)$ and the profit of the incumbent becomes

$$(1+\eta) \times \frac{\alpha'\sigma}{1-\delta+\alpha'\delta} = \frac{\alpha\sigma}{1-\delta+\frac{\alpha}{1+\eta}\delta},$$

which increases with η. Low switching costs consumers deter entrants, as they accept the offers intended to attract high switching costs customers, while leading to no increase in profits in subsequent periods.

4.2 Analysis of Bertrand competition

In the Stackelberg equilibrium the incumbent charges $-\delta\Pi + \sigma = \frac{\sigma(1-\delta)}{1-\delta+\alpha\delta}$, in order to avoid being underbid by the entrants by more than σ. In the Bertrand game, it is not an equilibrium for the incumbent to charge $-\delta\Pi+\sigma$ and for at least one entrant charges $-\delta\Pi$: the entrant would attract only the low switching cost consumers, which yield no profit in future periods, at a negative price. Indeed, there is no pure strategy equilibrium of the game, but we will still be able to show that the profits of the incumbent are equal to the profits in Stackelberg competition.

We will do this by proving that $-\delta\Pi + \sigma$ belongs to the support of the distribution of prices announced by the incumbent, and that when it chooses this price, it keeps the high switching costs customers with probability 1. This will imply that equation (2) holds. (More precisely, we will show that $-\delta\Pi+\sigma$ is the lower bound on the support of prices charged by the incumbent, and that when it chooses a prices arbitrarily close to this lower bound, it keeps the high switching cost customers with probability close to 1.)

As with the previous two models, we restrict attention to equilibria where consumer can coordinate when they all gain from such coordination and the stationarity assumptions on players' strategies. We note that since only mixed strategy equilibria exists, this means that firms who attract consumers with high switch costs will all use the same equilibrium pricing distribution. Furthermore, the distribution of the lowest priced entrant price does not depend on the incumbent name. We sketch the proof here and provide the formal proof in the appendix.

It is clear that consumers with zero switching cost will always purchase from one of the lowest price sellers, and also, because of the Markov hypothesis that high level consumers will switch from the incumbent whenever $p_I > p_E + \sigma$ and that they not switch when $p_I < p_E + \sigma$.

Let $\underline{b}_E$ be the lower bound of the support of the strategies of entrants. We now show that $\underline{b}_E$ is equal to $-\delta\Pi$. First, there can be no equilibrium with $\underline{b}_E < -\delta\Pi$, basically because firms who would choose such a low price would make a profit of $\underline{b}_E - \delta\Pi < 0$, as they would sell the good to all the consumers at a price equal to $\underline{b}_E$, and become incumbent in the next period. [15] Second, if we had $\underline{b}_E > -\delta\Pi + \sigma$, the incumbent would never charge less than $\underline{b}_E + \sigma$. Then, by charging slightly less than $\underline{b}_E$, an inactive entrant would be sure to attract all the consumers and would make strictly positive profits.

This implies that $\underline{b}_E + \sigma = -\delta\Pi + \sigma$ is the lower bound of the prices charged by the incumbent. To show the result, it is sufficient to show that if the incumbent charges (close to) this price, its high switching cost customers "stay with him" with probability (close to) 1. This will be the case if the distribution of the prices charged by the entrants does not have a mass point at $\underline{b}_E$. We are therefore left with the task of showing that there is not such a mass point. If there was one, there would exist $\eta > 0$ such that the incumbent announces a price in $(\underline{b}_E + \sigma, \underline{b}_E + \sigma + \eta]$ with probability 0: for

[15] The formal proof must distinguish between the case where the distribution of prices of the entrants has a mass point at t, and the case where it does not. If it does not, by choosing p close enough to t, the entrant would be the low bidder with probability close to 1, and make a strictly negative profit when it is, which proves the result. If the distribution does have a mass point, it is easy to see that the expected value of the sum of the profits of the firms who announce t is negative (one has to be a bit careful, as it could be that if both firms announce t, one attracts all the low switching cost consumers and the other all the high switching cost consumers - the second one could have positive profits). Hence, at least one of the firm makes strictly negative expected profits when announcing t, which proves the contradiction.

any price in this interval, he would be better off announcing slightly less than $\underline{b}_E + \sigma$ and keeping its high switching cost customers with probability 1. In this case, each entrant would be better off announcing a price in the interval $(\underline{b}_E, \underline{b}_E + \eta)$ rather than $\underline{b}_E$, as it would then have a strictly positive probability of obtaining a strictly positive profit. Therefore, $\underline{b}_E + \sigma$ is indeed the lower bound of the support of the prices of the incumbent, and that it keeps its high switching cost customers with probability close to 1 when announcing a price close to this lower bound. This establishes that equation (2) holds.

The preceding reasoning establishes that any equilibrium which satisfies our assumptions would yield profits for the incumbent equal to those of the Stackelberg equilibrium. We still have to show that such an equilibrium does exist. To do this, we begin by computing the distribution of prices announced by the incumbent. Let p_E be the lowest price announced by an entrant. It must the case that the expected profit of an entrant who knows that he has announced a lower price that the other entrant is equal [16] to 0. Therefore, we have

$$\begin{aligned}(1 - F_I(p_E + \sigma))[p_E + \delta\Pi] + F_I(p_E + \sigma) \times (\alpha p_E) = 0 \\ \Longrightarrow F_I(p_I) = \frac{p_I - \sigma + \delta\Pi}{(1-\alpha)(p_I - \sigma) + \delta\Pi}.\end{aligned}$$

F_I is equal to 0 when $p_I = -\delta\Pi + \sigma$ and to 1 when $p_I = \sigma$. Similarly, the distribution of p_E is determined by the fact that the profits of the incumbent are equal to Π, for all prices:

$$\begin{aligned}0 \times F_E(p_I - \sigma) + (\alpha p_I + \delta\Pi)(1 - F_E(p_I - \sigma)) = \Pi \\ \Longrightarrow F_E(p_E) = 1 - \frac{\Pi}{\alpha(p_E + \sigma) + \delta\Pi}.\end{aligned}$$

F_E is equal to 0 when $p_E = -\delta\Pi$. On the other hand,

$$F_E(0) = \frac{\alpha\sigma - (1-\delta)\Pi}{\alpha\sigma + \delta\Pi} < 1;$$

there is a mass point in the distribution of p_E at 0. It is then easy to compute the distribution of prices of each active entrant by the formula It is then easy

[16] The high price active entrant is making a profit of 0. If the low price active entrant made a positive expected profit for some prices, the expected profits of the active entrants would be strictly positive, which is impossible.

to check that the behavior of the different agents is a best response to the strategies of the others.

In this section, we have assumed that only one set of consumers had positive switching costs. This enabled us to easily characterize the equilibrium set, since the zero switching cost consumers always purchased from the lowest priced firm in every period. Thus, a firm that ever attracted these consumers would always ignore them in their subsequent pricing strategies: they can never make a profit from them. If this were not the case, then it is much more difficult to solve for the equilibrium in the Bertrand version of the infinite horizon model. This is due to the following facts. First, there is no pure strategy equilibrium for a large set of parameters. Second, there will be subgames where no high switching cost buyers move, subgames where a fraction of the high switching cost buyers will move, and subgames where all the high switching cost buyers move. This makes it much more difficult to characterize the equilibrium. Thus, we turn to a two period model where there are at least two types of buyers who have positive switching costs to show that lowering switching cost may actually lead to higher incumbent profits. More importantly, the distribution of the buyer's switching costs are important for the firms' profits.

5 Lower switching costs can lead to higher profits

5.1 Results and intuition

In this section, there are still two levels of switching costs, but they are both strictly positive and the game lasts two periods. The main point of this section is to show that, for a large range of parameters, the incumbent's profits can *fall* as the consumer's switching costs *rise*. In particular, we will examine the case when the low buyer switching costs, σ_L, are small relative to the high buyer switching costs, σ_H. We use the two period model due to the difficulties discussed above in the infinite horizon model. We think that the main economic points hold if the time horizon is extended. First, we will present the main proposition that state the unique incumbent equilibrium payoffs for the parameter set of interest. Next, we provide an explanation for the equilibrium and then derive comparative statics results that demonstrate how incumbent payoffs change with the parameters. Finally, we prove the main proposition of the section and briefly discuss the equilibria and comparative statics for the other parameter values.

Proposition 4. *If $\sigma_L/\sigma_H < \alpha\delta/(1+\delta)$, then the equilibrium profit of the incumbent is*

$$\pi^I = \sigma_H \left[\frac{\alpha\sigma_H - \sigma_L}{(\sigma_H - \sigma_L)} (1 + \delta - \alpha\delta) \right]. \tag{3}$$

Thus, we find a unique equilibrium payoff that gives the incumbent more than the single period switching cost from the highest buyer types, H buyers, $\alpha\sigma_H$, since

$$\left[\frac{\alpha\sigma_H - \sigma_L}{(\sigma_H - \sigma_L)} (1 + \delta - \alpha\delta) \right] > \alpha$$

whenever $\sigma_L/\sigma_H < \alpha\delta/(1+\delta)$. As in the infinite horizon model, where some buyers have 0 switching cost, the presence of these low switching cost L buyers, enables the incumbent to generate these higher profits than the single switching cost for these parameter values. [17]

There are only mixed strategy equilibria for this set of parameters. We provide a sketch here of the proof for the derivation of the incumbent's equilibrium payoff. First, the lowest price that an entrant would ever charge in period 1 is $-\alpha\delta\sigma_H$; this is because the largest period 2 payoff for an entrant is by it attracting all the H buyers in period 1 and then charging them σ_H in period 2. This gives the entrant a profit of $\alpha\sigma_H$. Since the L buyers will always go to the lowest priced entrant in period 1 if they switch, it will cost them σ_L no matter who they are with in period 2, the lowest priced entrant will attract these consumers also. Thus, $-\delta\alpha\sigma_H$ is the most that a period 1 entrant would give away to attract buyers in period 1. Second, H buyers know that if they *all* go to the lowest priced entrant, then they will pay σ_H tomorrow. Thus, if an incumbent charges any price less than $\sigma_H - \alpha\delta\sigma_H = \sigma_H(1-\alpha\delta)$, then he can keep a fraction of the high switching cost buyers with probability 1. This fraction is $\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}$. So, an incumbent who charges $\sigma_H(1-\alpha\delta)$ will keep $\alpha \left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]$ H buyers in period 1 and be able to charge them σ_H in period 2; he will lose all the low switching cost buyers at such a price. This generates a payoff of π^I. The incumbent must charge a price of $\sigma_H(1-\alpha\delta)$ or less, since if he did not an entrant could make

[17] If $\sigma_L = 0$, equation (3 becomes $\pi^I = \alpha\sigma_H(1+\delta-\alpha\delta)$, which for constant $\alpha\sigma_H$ is decreasing in α and therefore increasing in δ, and justifies the statement of footnote 4.

positive profits by attracting all the buyers. Since this price must be in the support of the incumbent's pricing distribution, this is his equilibrium profit.

We now derive some interesting comparative statics results for the incumbent's profits. First, we allow the L buyers' costs to increase.

$$\operatorname{sign} \frac{\partial \pi^I}{\partial \sigma_L} = \operatorname{sign} \left[-(\sigma_H - \sigma_L) + \alpha\sigma_H - \sigma_L \right] < 0.$$

Thus, in equilibrium, the incumbent's profits actually fall as the L buyers' switching costs rise. The reason is that as these cost rise, the entrants, for a given ratio of low and high switching cost buyers are more willing to charge σ_L instead σ_H. This means that more H buyers can switch and face the lower period 2 price. Another way to view this is that the entrants are more aggressive in trying to attract buyers in period 1, since if they do not acquire enough H buyers to charge σ_H in period 2, their payoff will be larger due to the increase in σ_L. This lowers the incumbent's payoffs.

On the other hand, if the switching costs grows only for the H buyers, then the incumbent's profit rises. This can be seen by examining how the incumbent's profit changes with σ_H.

$$\operatorname{sign} \frac{\partial \pi^I}{\partial \sigma_H} = \operatorname{sign} \left[\alpha\sigma_H^2 - 2\alpha\sigma_H\sigma_L + \sigma_L^2 \right] > 0,$$

since

$$\alpha\sigma_H^2 - 2\alpha\sigma_H\sigma_L + \sigma_L^2 > (\alpha\sigma_H - \sigma_L)^2.$$

There are two effects on the incumbent's profit by raising σ_H. First, is the pricing effect: higher σ_H means higher first and second period prices. Second, a higher σ_H increases the fraction of high switching cost buyers that the incumbent can keep by charging $\sigma_H(1-\alpha\delta)$, since the entrant will charge σ_H in period 2 for a lower fraction of H buyers. Similarly, if α increases, then the incumbent's profit increases

$$\operatorname{sign} \frac{\partial \pi^I}{\partial \alpha} = \operatorname{sign} \left[\sigma_H + \delta\sigma_H - 2\alpha\delta\sigma_H + \sigma_L \right] > 0.$$

Finally, suppose that all switching costs increase by ε, then

$$\operatorname{sign} \frac{\partial \pi^I}{\partial \varepsilon} = \operatorname{sign} \left[\sigma_H(2\alpha - 1) - \sigma_L - \varepsilon(2 - \alpha) \right]$$

A sufficient condition for this to be negative is $\alpha \leq 1/2$. Thus, as long as, on average the low switching cost rise more than the high switching costs, the incumbent's profit is lower due to the increase in switching costs.

Now, we turn to the proof of the proposition of this section.

5.2 Proof of Proposition 4

To find the equilibrium profit, we first solve the period 2 subgame. We know that in period 2, new entrants will charge a price of 0, due to the free entry hypothesis. A firm that has consumers at the beginning of period 2, will charge either σ_L and keep all consumers or charge σ_H and keep only his high switch consumers. Clearly, if consumers switch firms, they will go to a period 2 entrant, since it charges a lowest price. If ϕ is the proportion of high switch cost consumers that a firm has at the beginning of period 2, then its pricing behavior is: if $\phi < \sigma_L/\sigma_H$, the firm will charge σ_L; if $\phi > \sigma_L/\sigma_H$, it will charge σ_H; and if $\phi = \sigma_L/\sigma_H$, it will charge either σ_L or σ_H. This is because charging more that σ_H cannot be optimal, as an entrant could attract the consumers and make a positive profit. Charging less that σ_L cannot be optimal, as increasing the price will keep the consumers and increase profits. Charging any price in (σ_L, σ_H) is not optimal, as all L buyers are lost and increasing the price will keep the H buyers and increase profits. In equilibrium, it must be the case that the firm keeps all consumers if it charges σ_L and only the H consumers if it charges σ_H.

It is useful to define the following cut-off value:

$$\gamma = \frac{(1-\alpha)\sigma_L}{\alpha(\sigma_H - \sigma_L)} \tag{4}$$

If all L buyers and a fraction γ of H buyers go to the lowest priced period 1 entrant, then the entrant is indifferent between charging σ_L in period 2 and retaining all buyers and obtaining profits $\sigma_L\left[(1-\alpha)+\alpha\gamma\right]$ and charging σ_H and just keep the H buyers to obtain profits $\gamma\sigma_H$.

Next, we examine period 1. First, we derive the L buyer behavior. No matter which firm an L buyer is with in period 2, it will always cost the buyer σ_L in that period; either they will pay that price with their current firm or they will switch and pay 0 with an entrant. Thus, if the L buyer switches firms in period 1, then it will want to minimize first period cost which implies that if it switches it will go to a lowest priced entrant. Furthermore, if p_E is the lowest priced entrant, and p_I is the incumbent's period 1 price, than all

L buyers will switch to the lowest priced entrant in period 1 if $p_I - p_E > \sigma_L$, while if $p_I - p_E < \sigma_L$ all L buyers will stay with the incumbent.

Second, we determine the H buyers' behavior in period 1. We note two facts. First, if not all the L buyers leave the incumbent in period 1, then none of the H buyers will leave, since if an L buyer is not willing to pay the switching costs in period 1, then neither would an H buyer. Also, given the pricing behavior of firms in period 2, it will never be the case that entrant i has a fraction of H buyers greater than γ and that entrant j has a fraction less than γ of H buyers. Entrant i will charge σ_H in period 2, while j will charge σ_L. Since entrant j obtains some L buyers, it must be one of the lowest priced period 1 entrants. An H buyer could improve its payoff by switching to entrant j instead of firm i.

In period 1, the H buyer behavior is thus: If $p_I - p_E > \sigma_H$, then all H buyers go a lowest priced entrant, since all L buyers will leave the incumbent and the buyer will pay at least as much in period 2 with the incumbent as with as entrant. If $p_I - p_E < \sigma_H(1-\delta) + \delta\sigma_L$, then all H buyers stay with the incumbent, since the lowest price that an entrant will charge in period 2 is σ_L and thus the minimum total cost for an H buyer who switches, $\sigma_H + p_E + \delta\sigma_L$, is greater than his cost of staying with the incumbent $p_I + \delta\sigma_H$. Finally, if $\sigma_H(1-\delta) + \delta\sigma_L < p_I - p_E < \sigma_H$, γ of the H buyers go to a lowest priced entrant if the prices are in this region, since the entrants will still charge only σ_L in period 2. This will be a lower cost than staying with the incumbent and paying σ_H in period 2. On the other hand, if any more H buyers switch to an entrant then its price will be σ_H in period 2, and this will yield a lower utility for a switching buyer than staying with the incumbent.

Now, we determine the sellers' period 1 pricing behavior. First, we characterize the pricing interval of period 1 entrants. Next, we characterize properties of the incumbent's first period pricing distribution. This will lead to bounds on the incumbent's profits. Finally, we complete the proof with derivation of the incumbent's equilibrium profits.

Lemma 1. *In period* 1*, the prices of an entrant who obtains consumers will be in the interval* $[-\delta\alpha\sigma_H, -\delta\sigma_L]$*.*

Proof. First, we establish the upper bound of entrant's price support. If an entrant ever attracts consumers, it will charge at least σ_L in period 2. In period 1, no firm charging a price greater than $-\delta\sigma_L$ has a positive measure of customers with positive probability, since if they did, then another firm could undercut the price and make a positive profit for the entire game which contradicts the free entry assumption.

Next, we establish the lower bound. Let $-\delta\alpha\sigma_H - \epsilon$, be the lowest entrant price in some state of nature. All firms that charge such a price will, in aggregate, attract all the the low customers and any H customers that switch. Given that the proportion of H customers that are spread across the lowest priced firms cannot exceed α, the firms will on average make losses. Thus, at least one firm will have an incentive to deviate. □

Now, we characterized the incumbent's first period pricing distribution. This is found by examining what happens to the incumbent's H buyers for different prices that he charges. First, we show that if the price is above the one period "flow value" of switching costs, $\sigma_H(1-\delta)$, then the incumbent loses at least γ of the H buyers. On the other hand, if the price is less than $\sigma_H(1-\alpha\delta)$, then the incumbent loses at most γ of its H buyers. This demonstrates that there is no price in the interval $(\sigma_H(1-\delta), \sigma_H(1-\delta+\alpha\delta))$.

Lemma 2. *If the incumbent charges any price strictly greater than $\sigma_H(1-\delta)$ in period 1, then it will lose at least a proportion γ of the H buyers.*

Proof. By Lemma 1, the highest entrant price is $-\delta\sigma_L$. If an incumbent charges a price greater than $\sigma_H(1-\delta)$, then an H's payment if he stays with the incumbent is more than σ_H, while if less than γ of the H buyers go to an entrant, his costs are at most σ_H, which includes his switching cost in period plus the maximum payment in period 1, $-\delta\sigma_L$, plus his payment of σ_L next period. Thus, he improves his welfare by moving to an entrant. □

Lemma 3. *If the incumbent charges any price less than $\sigma_H(1-\alpha\delta)$ in period 1, then it will lose at most γ of the H buyers.*

Proof. Let p_I be the price charged by the incumbent. A H buyer who stays with the incumbent has a total cost (over two periods) of

$$p_I + \delta\sigma_H < \sigma_H(1-\alpha\delta) + \delta\sigma_H = \sigma_H(1+\delta-\alpha\delta). \tag{5}$$

If more than proportion γ of H buyers go to the entrants, the entrant will charge σ_H in the second period, and the total cost of a H buyer who has moved is

$$p_E + \sigma_H + \delta\sigma_H \geq -\delta\alpha\sigma_H + \sigma_H + \delta\sigma_H = \sigma_H(1+\delta-\alpha\delta) \tag{6}$$

¿From these two equations, we see that a H buyer who moved would be worse off than if he did not move, which establishes the contradiction. □

The next corollaries follow from the above two lemmas, and demonstrate that the incumbent never charges a price in the interval $(\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$ and can guarantee itself a profit of $(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$.

Corollary 1. *If the incumbent charges any price in $(\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$, he will loose exactly γ of the H buyers. Thus, the incumbent will never charge a price in this interval since there is always a higher price he can charge which will give him the same demand and higher prices in the first period.*

Corollary 2. *If the incumbent charges $\sigma_H(1-\alpha\delta)$, he looses exactly γ of the H buyers. By charging slightly less than $\sigma_H(1-\alpha\delta)$, the incumbent can guarantee itself profits as close as it wants to $(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$.*

We will now demonstrate that the incumbent's price distribution must contain prices of $\sigma_H(1-\alpha\delta)$ or less. This is the highest incumbent price that guarantees the incumbent keeps at least $1-\gamma$ of the H buyers. That is, even if the lowest entry price is $-\alpha\delta\sigma_H$, not all the H buyers will leave the incumbent. If the entrants only attract γ of the H buyers, then it will only charge σ_L. If the incumbent's price was always greater than $\sigma_H(1-\alpha\delta)+\varepsilon$, then an entrant who chose a price of $-\alpha\delta\sigma_H+\varepsilon/2$ would attract all the buyers in period 1, charge σ_H in period 2 and always make a positive profit.

Proposition 5. *The incumbent charges $\sigma_H(1-\alpha\delta)$ or less with positive probability.*

Proof. Suppose the incumbent *always* charged prices greater than $\sigma_H(1-\alpha\delta)$. An entrant can find an $\epsilon>0$, and charge a price of $-\alpha\delta\sigma_H+\epsilon$ such that all buyers will go to the entrant if it is the lowest priced firm. This is because the incumbent loses all L buyers, since $\sigma_H(1-\alpha\delta)>\sigma_L(1-\delta)$. Furthermore, all the H buyers prefer to go an entrant than the incumbent charging a price of P, since

$$\alpha\delta\sigma_H-\epsilon-\sigma_H-\delta\sigma_H > -P-\delta\sigma_H$$

for small enough $\epsilon>0$.

Let the probability that an entrant is the lowest priced entrant be q_L. This gives the entrant a profit of $q_L\left[-\delta\alpha\sigma_H+\epsilon+\delta\alpha\sigma_H\right]=q_L\epsilon>0$. Thus, the incumbent must charge a price with positive probability of $\sigma_H(1-\alpha\delta)$ or lower in any equilibrium for an entrant to make 0 expected economic profit. $\square$

We note, up to now, we have not used our assumption on the relative switching cost of $\frac{\sigma_L}{\sigma_H} < \frac{\delta\alpha}{1+\delta}$. Now, we will invoke the assumption. To simplify notation, in the sequel we define

$$x = \frac{\sigma_L}{\sigma_H}$$

Lemma 4. *If*

$$x < \frac{\delta\alpha}{1+\delta},$$

then the incumbent never charges $\sigma_H(1-\delta)$ or less.

Proof. Giving the possibility of deviating to a higher first period price and still keeping a proportion $1-\gamma$ of the H buyers, the incumbent will only charge $\sigma_H(1-\delta)$ or less only if can keep more than $1-\gamma$ of these buyers, and hence be able to charge σ_H in the second period. Let η be the minimum number of H buyers that it looses when it charges $\sigma_H(1-\delta)$ or less. Its profits are bounded above by

$$\alpha(1-\eta)[\sigma_H(1-\delta)+\delta\sigma_H]$$

(because the proportion of H buyers that he has in the second period is greater than γ, it will charge σ_H.[18]

The incumbent will never charge $\sigma_H(1-\delta)$ or less if this profit is less than the profit it can guarantee if it charges$\sigma_H(1-\alpha\delta)$. For it to charge such a price, we must have

$$\alpha(1-\eta)[\sigma_H(1-\delta)+\delta\sigma_H] > \alpha(1-\gamma)[\sigma_H(1-\alpha\delta)+\delta\sigma_H]$$

which is equivalent to

$$1-\eta > \frac{\alpha - x}{\alpha(1-x)}\left[1+\delta-\alpha\delta\right] \tag{7}$$

Noting that the left hand side of (7) is falling in η, if we take the $\eta = 0$ and take the largest possible $x = \frac{\alpha\delta}{1+\delta}$ we obtain that for (7) to hold

$$1 > \frac{\alpha}{\alpha(1+\delta-\alpha\delta)}(1+\delta-\alpha\delta) = 1$$

which proves the lemma. □

[18]We note that the incumbent would never charge a price low enough to keep L buyers for these parameters. Such a price could not exceed $(1-\delta)\sigma_L$.

Thus, we have proven the Proposition, since the incumbent will always charge a price in period 1 of $\sigma_H(1-\alpha\delta)$, and never a lower price.

Now, we briefly discuss the equilibrium for the rest of the parameter set. If σ_L is large relative to σ_H, in particular if $x \geq \alpha$, then the unique equilibrium payoff for the incumbent is σ_L. In the unique pure strategy equilibrium, the incumbent charges $\sigma_L(1-\delta)$ in period 1 and no buyers switch. Clearly, if the switching cost are increased, this increases the incumbent's payoff.

For $x \in (\frac{\delta\alpha}{1+\delta}, \alpha)$, there are two regions, which we examine in the Appendix. In both regions, there is the same mixed strategy equilibrium where the incumbent seller's payoff is the same as when $x < \frac{\delta\alpha}{1+\delta}$, namely $\sigma_H \left[\frac{\alpha\sigma_H - \sigma_L}{(\sigma_H - \sigma_L)}(1+\delta-\alpha\delta)\right]$. If $x \in (x_C, \alpha)$, where x_C implicitly solves

$$\sigma_H\sigma_L \left[1+\delta+\alpha\delta-\alpha\right] \geq \delta\left[\alpha\sigma_H^2 + \sigma_L^2\right]$$

then there is also a pure strategy equilibrium where the incumbent charges $\sigma_H(1-\delta)$ in period 1, keeps all the H buyers and has an equilibrium payoff of $\alpha\sigma_H$. Note that $x_C > \frac{\delta\alpha}{1+\delta}$. So, we still have the mixed strategy equilibrium in this intermediate region and thus the same comparative statics results as were derived above along with a pure strategy equilibrium where a change in the buyer switching costs weakly improves the incumbent's profits.

6 Two period Model with Continuous switching cost

Forthcoming

References

Beggs, A. & Klemperer, P. (1992), 'Multiperiod competition and switching costs', *Econometrica* **60**, 651–666.

Chen, Y. (1997), 'Paying constumers to switch', *Journal of Economics and Management Strategy* **6**, 877–897.

Farrell, J. & Shapiro, C. (1988), 'Dynamic competition with switching costs', *The RAND Journal of Economics* **19**, 123–137.

Jobs, S. (2007), 'Thoughts on music', Statement of 6 February 2006. visited on ??
URL: *http://www.apple.com/hotnews/thoughtonmusic*

Johansen, J. L. (2007), 'Steve's misleading statistics'. visited on ??
URL: *http://nanocrew.net/2007/02/06/steves-misleading-statistics/*

Klemperer, P. D. (1987*a*), 'Entry deterrence in markets with switching costs', *Economic Journal* **97**, 99–117.

Klemperer, P. D. (1987*b*), 'Markets with consumer switching costs', *Quarterly Journal of Economics* **102**, 375–394.

Klemperer, P. D. (1995), 'Competition when consumers have switching costs', *Review of Economic Studies* **62**, 515–539.

of Fair Trading, O. (2003), 'Switching costs'. Report of the UK government Office of Fair Trading.

Padilla, A. (1995), 'Revisiting dynamic duopoly with consumer switching costs', *Journal of Economic Theory* **67**, 520–530.

Taylor, C. R. (2003), 'Supplier surfing: Competition and consumer behavior in subscription markets', *The RAND Journal of Economics* **34**(2), 223–246.

7 Conclusion

Forthcoming

8 Appendix

8.1 Infinite horizon

The aim of this appendix is just internal: to be a bit pedantic about the definition of equilibrium. At this point, it has just the Stackelberg equilibrium, for simplicity.

As part of the rules of the game, a firm which has not sold in a previous period has to drop out (except, of course, entrants in their first period).

We need first to define the equilibrium. Histories are defined as usual as the past prices and purchases of all consumers. Each agent maximizes its profits, and subgame perfection is well defined. We will identify equilibria, hence we do not need to prove existence.

We look for equilibria, in which the moves of a firm depend only on the masses of consumers of each type that it has, in a stationary way (except that the entrants can have their move depend on the prices charged by the incumbents, and on the distribution of types among these incumbents — we are in Stackelberg!).

We assume that if for any $\eta > 0$, there exists $\varepsilon \in (0, \eta]$ such that a mass of consumers would all be better off if they deviated from a putative equilibrium, they do deviate. This gives “mass” to the consumers.

Now we look for stationary equilibrium in which:

- In equilibrium, and also for any way consumers are distributed across firms due to deviations, all firms who have had a positive mass of consumers σ_H consumers charge the same price.

  We will demonstrate that in equilibrium no positive mass of σ_H consumer leaves the incumbent and that the price that the incumbent charges in each period is

  $$p^S = \frac{(1-\delta)\sigma}{1-\delta+\alpha\delta}. \tag{8}$$

  The difficulty that we face is the following: assume that there are several incumbents (because of deviations from the equilibrium path in the past); in equilibrium they should all charge p^S. Assume that one of the incumbents raise its price. The entrants will fear that if they keep their prices at $p^S - \sigma$ they will attract all the lsc (= low switching cost) consumers and only a proportion of the hsc consumers. Hence, they will raise their prices so as to attract no hsc consumer! Therefore, for all incumbents to charge p^S is not an equilibrium if only the hsc consumers at the deviating incumbent leave for the entrant. The restriction to equilibria where that all the hsc customers stay with one firm is not sufficient to eliminate this problem, as we have to prove that this is an equilibrium - we have to show that if they all move to one firm, it is individually rational for all of them to do so (*i.e.*, there is no incentives to deviate from this coordinated move).

To rephrase the problem, we have to handle what happens after the incumbent has charged p^S, one entrant has charged $p^S - \sigma$ and some, but not all, hsc consumers have moved. I propose that we look for equilibria that satisfy this generalization of the "all consumers stay together" property:

Assumption 1. *If, given the strategies of the firms, there exists a continuation equilibrium such that a) all hsc consumers purchase from the same firm and b) some hsc consumers are made better off while none are made worse off, then the hsc consumers choose this continuation equilibrium. (This clearly would have to be formalized better).*

Now, we want to show that any equilibrium has the incumbent charging p^S in every period, and hsc consumers never switching.

In equilibrium we cannot have a price such that

$$p - \sigma + \alpha\delta\frac{p}{1-\delta} > 0 \Longleftrightarrow p > p^S.$$

If we did, it would not be an equilibrium, since an entrant could charge less than $p - \sigma$ attract all buyers and make a positive (discounted), since all the hsc consumers will purchase from that entrant, as the price they would pay would be the same ever after, and they gain more than σ this period. (In this case, they would all have strict incentives to move).

We cannot have

$$p - \sigma + \alpha\delta\frac{p}{1-\delta} < 0 \Longleftrightarrow p < p^S.$$

If this inequality holds, no entrant attracts any of the hsc consumers (which they can only do by charging at most $p-\sigma$). Assume they did, the sum of the profits of the lowest price entrants would equal to $p-\sigma+\alpha\delta p/(1-\delta)<0$ — at least one of them would be loosing money, hence this cannot be an equilibrium.

Clearly, it cannot be the case that an incumbent would lose a positive meaure of consumers if it charges p^S, since it could slightly lower its price and keep all the hsc consumers.

Hence we are left with the equilibrium in the main text. We just have to prove that it is indeed an equilibrium, and this is the case, since no incumbent can raise its price and keep consumers and clearly a lower price would be unprofitable. Also, no entrant can gain by charging a price different than 0.

I think that we can use the same logic for the Bertrand game; actually it may even be less restrictive. In particular, we do not need the consumer mass assumption. This is because in equilibrium, an incumbent will lose some or even all of the hsc consumers (if he does not charge the price p^S) with positive probability. Thus, if an incumbent raises its price then the mixed strategy equilibrium takes care of this issue automatically. Since, in equilibrium, the only time that entrants will charge the same price with positive probability is a price of 0, we do not have to assume that all buyers who switch will switch to the same firm since lsc always go to the lowest priced firm and the hsc consumers will either follow the lsc consumers or stay with an incumbent except when all entrants charge a price of 0. Since $p^S>0$, all lsc consumers always leave an incumbent. Clearly, it would not pay for hsc to switch to a higher priced firm if no lsc consumers are going to the entrant.

8.2 Two period model

We will derive the entire set of equilibria for the two period, two type model. We divide the parameter set up in terms of x.

If $x \geq \alpha$, then the unique pure strategy equilibrium incumbent payoff is σ_L. Period 1 prices are $\sigma_L(1-\delta)$ and $-\delta\sigma_L$, and no consumers switch.

There is a unique *pure* strategy incumbent equilibrium payoff of $\alpha\sigma_H$ for values of x between α and for a value of $x=x_C$, which we will implicitly define below, strictly greater than $\frac{\delta\alpha}{1+\delta}$. In the equilibrium the incumbent charges $\sigma_H(1-\delta)$, the entrants charge $-\sigma_L$ and only the L buyers switch. For this to be an equilibrium, the incumbent must not be willing to raise his price to $\sigma_H-\delta\sigma_L$ and only keep $1-\gamma$ of the H buyers. It is straightforward to show that this is the most profitable possible deviation.

For this to be the case,

$$\alpha\sigma_H \geq [\sigma_H - \delta\sigma_L + \delta\sigma_H]\,\alpha\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]$$

which holds if

$$\sigma_H\sigma_L\left[1+\delta+\alpha\delta-\alpha\right] \geq \delta\left[\alpha\sigma_H^2 + \sigma_L^2\right] \tag{9}$$

Notice that at $x = \alpha$, (9) holds strictly. Thus, there is a set of x, in which this is an equilibrium. Clearly, as x falls, (9) becomes harder to meet. Call the x where (9) is an equality x_C. So, for $x \in (x_C, \alpha)$, we have an equilibrium.

If $x \leq \frac{\delta\alpha}{1+\delta}$, then the unique equilibrium payoff is $\pi^I = \sigma_H(1+\delta-\alpha\delta)\left[\frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L}\right]$, from the proof of Proposition 4. What we want to demonstrate is that for $\frac{\delta\alpha}{1+\delta} < x < x_C$, this is also an equilibrium payoff. For this to be an equilibrium, the incumbent must be indifferent for every price along its equilibrium pricing distribution. If $G(p_E)$ is an entrant's pricing distribution, then it must satisfy

$$(1 - G(p_E))\left[p_E + \sigma_H + \delta\sigma_H\right]\alpha\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right] = \pi^I$$

where $p_I = p_E + \sigma_H$. Solving for $G(p_E)$, we obtain

$$G(p_E) = \frac{p_E + \alpha\delta\sigma_H}{p_E + \sigma_H + \delta\sigma_H}$$

note that there is a mass point at $-\delta\sigma_L$ of $\frac{\sigma_H(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L}$.

Suppose $\frac{\delta\alpha}{1+\delta} < x \leq \alpha$ and assume the same equilibrium strategies as were used when $x < \frac{\delta\alpha}{1+\delta}$. If the incumbent deviates from the equilibrium with a price of $p_I \in [\sigma_L(1-\delta), \sigma_H(1-\delta)]$, then its profits are

$$(p_I + \delta\sigma_H)\alpha\left[1 - G(p_I - \sigma_H) + G(p_I - \sigma_H)\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]\right] \tag{10}$$

Plugging in for $G(p_I - \sigma_H)$ and simplifying we obtain that maximizing

$$(p_I + \delta\sigma_H)\alpha(\sigma_H - \sigma_L) - (p_I - \sigma_H + \alpha\delta\sigma_H)\sigma_L(1-\delta) \tag{11}$$

is equivalent to maximizing (10). The first order condition with respect to p_I is

$$\alpha\sigma_H - \sigma_L(1 + \alpha - \delta).$$

This is positive when $x < \frac{\alpha}{1+\alpha-\delta}$. This is the leading case since we are looking in the set of parameters when $x \in \left(\frac{\delta\alpha}{1+\delta}, \alpha\right)$ and $\frac{\alpha}{1+\alpha-\delta} > \alpha$ whenever $\delta > \alpha$. The most profitable deviation for the incumbent in this case is a price of $\sigma_H(1-\delta)$ where he hopes to keep all the H consumers. This will only occur if $p_E = -\delta\sigma_L$, the highest entrant price and price which has a mass point. For this not to be a profitable deviation

$$\pi^I \geq \alpha\sigma_H \left[1 - G(-\delta\sigma_L) + G(-\delta\sigma_L)\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]\right]$$

must hold. The r.h.s. is composed of the chance that he keeps all the H buyers $1-G(-\delta\sigma_L)$ plus if he only keeps $1-\gamma$ of them $G(-\delta\sigma_L)\left[\frac{\alpha\sigma_H-\sigma_L}{\alpha(\sigma_H-\sigma_L)}\right]$. Simplifying and substituting we get

$$[1+\delta-\alpha\delta]\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right] \geq \left[1 - \frac{\sigma_H(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L}\left(1 - \left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]\right)\right]$$

$$[1+\delta-\alpha\delta]\left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right] \geq \left[1 - \frac{\sigma_H(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L}\left[\frac{\sigma_L(1-\alpha)}{\alpha(\sigma_H - \sigma_L)}\right]\right]$$

$$[1+\delta-\alpha\delta]\left[\alpha\sigma_H - \sigma_L\right] \geq \left[\alpha(\sigma_H - \sigma_L) - \frac{\sigma_H(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L}\left[\sigma_L(1-\alpha)\right]\right]$$

$$\frac{\sigma_H(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L}\left[\sigma_L(1-\alpha)\right] \geq \alpha(\sigma_H - \sigma_L) - [1+\delta-\alpha\delta]\left[\alpha\sigma_H - \sigma_L\right]$$

$$\frac{\sigma_H\sigma_L(1+\delta-\alpha\delta)}{\sigma_H + \delta\sigma_H - \delta\sigma_L} \geq \sigma_L(1+\delta) - \alpha\delta\sigma_H$$

$$\alpha\sigma_H^2 + \sigma_L^2 \geq \sigma_H\sigma_L(1+\alpha)$$

Dividing by σ_H^2, we get

$$\alpha + x^2 \geq x(1+\alpha)$$

Note that the inequality is an equality at $x = \alpha$ and it strictly holds for smaller x. Thus, we have proved the result for when $\frac{\delta\alpha}{1+\delta} < x < \frac{\alpha}{1+\alpha-\delta}$. Thus, if $\frac{\alpha}{1+\alpha-\delta} > \alpha$ we have proven the result.

Suppose that $\frac{\alpha}{1+\alpha-\delta} < \alpha$, so that (11) is decreasing in p_I. That means that the best deviation price is a price that does not exceed $\sigma_L(1-\delta)$. At these prices, the incumbent may retain some L buyers if the entrants' prices are not too low.