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Property Taxation, Zoning, and Efficiency: A Dynamic Analysis*

Abstract

This paper revisits the argument that a system of local governments financing public service provision via property taxes will produce an efficient allocation of both housing and services if communities can implement zoning ordinances. The novel feature of the analysis is a dynamic model in which housing stocks and public policies are endogenously determined. While in principle zoning ordinances can result in efficiency, the analysis suggests that communities are unlikely to choose the right policies. Indeed, long run welfare may be lower with zoning than without.

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1 Introduction

In the U.S., public services like education, police, and libraries, are provided by local governments whose primary source of revenue has traditionally been the property tax.[1] Given this, the efficiency properties of a system of local governments financing public service provision via property taxes have long been of central interest. In a classic paper, Hamilton (1975) argued that such a system could produce an efficient allocation of both housing and services if local governments could implement zoning ordinances specifying minimum housing qualities. This was a striking conclusion since economic intuition suggests property taxes will distort both housing choices and public service levels. Hamilton's insight was that, through zoning, local governments could establish a minimum property tax for living in their communities. Assuming households choose among communities in Tiebout fashion, the property tax then becomes a fee for public services and efficiency results. This optimistic vision of this system of local public finance has been very influential, giving rise to the so-called *Benefit View* of the local property tax.[2]

Hamilton's analysis did not specify a precise model of how communities set their zoning ordinances and public service levels. Rather, his basic argument simply assumed that households faced a set of communities offering a full range of policies. As noted by White (1975), this begs the question of what options would be available to households in equilibrium. While subsequent literature has attempted to clarify the issue, the task is difficult because the problem has a natural dynamic structure. Zoning ordinances are chosen by existing residents and impact only new construction. It is therefore through its effect on new construction that zoning determines the composition of communities and housing prices. However, the literature employs static models in which distinctions between existing and future residents and old and new construction are hard to capture.

This paper presents a novel dynamic model of a system of local governments financing public services by property taxes in which policies are chosen by existing residents and housing stocks evolve over time. It then uses this model to analyze the policies communities choose and the

[1] For a history of the use of the property tax in the U.S. see Wallis (2001).

[2] The *Benefit View* argues that the property tax is a non-distortionary and non-redistributive user charge for public services. It is distinct from the *New View* that sees the property tax as a distortionary tax on the local use of capital. For a formal exposition of the New View see Zodrow and Mieszkowski (1986). For discussion and further debate see Fischel (1992, 2001a, 2001c), Mieszkowski and Zodrow (1989), Nechyba (2001) and Zodrow (2001a, 2001b).

efficiency of the resulting allocations of housing and services. While in principle zoning ordinances can result in efficiency, the analysis suggests that communities are unlikely to choose the right policies. Indeed, long run welfare may be lower with zoning than without. These findings challenge the Benefit View of the property tax.

The model considers a geographic area consisting of two communities. There is a constant population of households who need to reside in the area, but there is turnover so that in each period new households arrive and old ones leave. The only way to live in the area is to buy a house in one of the communities. Houses come in two types, small and large, and households differ with respect to their housing preferences. Houses are durable, but a fixed fraction of the housing stock is destroyed in each period. New houses can be constructed making the housing stock endogenous. Each community provides a public service to its residents which is financed by a proportional tax on the value of property. The level of this service is chosen in each period by the residents of the community.

Without zoning, in steady state, each community has an identical fraction of large houses and the same tax rate and level of public services. The fraction of large houses is below the first best level as housing decisions are distorted downwards by the property tax. Services may be over or under-provided relative to first best levels depending on whether the majority of residents own large or small houses. The tax price of services faced by owners of large houses exceeds the true price, while that faced by small house owners is below it. There can be multiple equilibrium steady states, with the belief that property taxes will be high being self-fulfilling.

Following Hamilton's logic, if a zoning ordinance requiring that all new houses be large is exogenously imposed in one community, the resulting steady state will be efficient. Households with stronger housing preferences live in the zoned community in large houses and the remainder live in small houses in the unzoned community. The price of all houses is uniform within each community and hence the tax price of public services equals the true price. Accordingly, residents choose efficient service levels.

If zoning ordinances are chosen in each period by residents, this efficient outcome is unlikely to result. Residents' preferences over zoning depend on how it impacts the price of their homes and the surplus they expect to get from public services. The redistributive incentives that housing prices and public service surplus provide do not guide residents to make efficient decisions. Indeed, there exists no equilibrium with endogenous zoning which has a steady state that is both efficient and satisfies a local stability property. This reflects the fact that at an efficient state with one zoned community with large houses and another unzoned community with small houses, residents in the unzoned community can increase the price of their homes with no adverse effect on the tax price they pay for services by introducing zoning.

Not only is the efficient outcome unlikely to result, but long run welfare may actually be lower with zoning than without. Under some conditions there exist equilibria which involve both communities *always* imposing zoning. In these over-zoning equilibria, all new construction is in large homes and, in steady state, the entire stock of houses is large. The distortion in the housing market is therefore of the opposite form to that arising without zoning. While in equilibrium the property tax is a benefit tax and service levels are efficient, the distortions in housing offset these gains.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature and Section 3 introduces the model. Section 4 discusses equilibrium without zoning and Section 5 considers exogenous zoning. Section 6 identifies the problems that emerge with endogenous zoning and Section 7 argues these are robust to allowing more communities. Section 8 concludes with a discussion of the implications of the results for state and local public finance.

2 Related literature

The paper relates to three distinct literatures. The first is the state and local public finance literature on *Tiebout models*. In a seminal paper, Tiebout (1956) suggested that the mechanism of households "voting with their feet" by choosing between communities on the basis of their public service-tax packages could improve allocative efficiency. His idea was that households would sort into communities with others who had similar demands for services and this sorting would create gains in public service surplus. Since then, a large theoretical literature has developed exploring this basic idea.[3]

Tiebout's analysis assumed that local governments financed service provision by head taxes. Since head taxes are rarely part of the public finance landscape, the literature quickly developed Tiebout models incorporating property tax finance. There are two varieties, distinguished by their assumptions about housing supply. Both assume that households have preferences defined over

[3] For an excellent review of this literature see Ross and Yinger (1999).

housing, private consumption, and public services, that communities finance service provision by a proportional tax on housing, and that service levels are chosen collectively by residents. The first variety assumes that housing is supplied by absentee landlords according to exogenously given supply schedules (for example, Epple, Filimon and Romer (1984)). In these models, households are best interpreted as renters. The second variety assumes that the housing stock is fixed and owned by residents (for example, Hamilton (1976) and Nechyba (1997)). Property taxation complicates Tiebout sorting because the tax price of services is below the true price for households who consume relatively less housing. This makes it attractive for households to live in communities where they consume relatively less housing. In the first variety, this force makes it difficult to find stable allocations of households across communities. In the second, it results in *capitalization*: small houses in communities with a larger fraction of large houses cost more.[4]

The model presented here builds on these Tiebout models with property taxation. It follows the second variety in assuming existing houses are owned by residents. However, new houses can be built by competitive construction firms. In the spirit of the first variety, the location of new construction is influenced by the existing mix of homes in the communities as this determines the tax price of services. The main advance incorporated in the model is its dynamic structure. The importance of introducing dynamics into Tiebout models has long been recognized, but there appear to have been few successful attempts to do so.[5] While used here to study zoning, the model could be employed to analyze other policy decisions with dynamic consequences, such as public infrastructure investments.

The second related literature is that on zoning.[6] The traditional justification for zoning is to deal with externalities. Such *externality zoning* can, for example, prevent over-crowding of communities. However, it has long been recognized that zoning can be employed to alter the

[4] The difficulties in finding stable allocations in the first variety of model can be overcome with appropriate assumptions. Epple, Filimon and Romer (1993) prove the existence of equilibrium in a model in which households have identical preferences and different income levels. Equilibrium involves communities stratified by income levels. Lower income households do not wish to live in higher income communities even though the tax price of services is lower, because the overall spending on services is higher than they would like. As shown by Nechyba (1997), in the second variety of model equilibrium exists under general conditions.

[5] One exception is Epple, Romano and Sieg (2009) who extend the Epple et al (1984) model to incorporate overlapping generations. Their key concern is to understand how households switch communities over the life cycle as their children grow up and services like education become less salient. Their model assumes that households rent housing in each period and that the supply of housing is time invariant, so they are not concerned with the political behavior of owner occupiers and the regulation of new construction.

[6] For an excellent introduction to zoning and other land-use regulations see Fischel (1999).

allocation of the costs of public services between existing residents and newcomers or outsiders, a practice known as *fiscal zoning*.[7] Hamilton's work highlights a normatively attractive aspect of such zoning by showing how it could be used to overcome the problem of newcomers paying less than their fair share of services by buying cheaper houses. But, as emphasized by White (1975), zoning might also be used to force newcomers to pay more than their fair share, a practice she refers to as *fiscal-squeeze zoning*. In addition, zoning could be used by existing residents to increase the value of their homes by restricting supply, which White calls *scarcity zoning*. These abuses of zoning might give rise to *over-zoning* (Davis (1963)), whereby the supply of housing is inefficiently restricted. This paper's dynamic model captures the distributional conflict between existing residents and newcomers, and permits a unified treatment of the use of zoning to manipulate both housing prices and the surplus obtained from public services.

There are two prior studies of zoning in Tiebout models with property taxes.[8] Fernandez and Rogerson (1997) study the impact of zoning in a two-community model of the variable housing supply variety. They model zoning as a minimum housing level and assume that only one community imposes it. They first study the impact of an exogenous zoning requirement and then analyze an endogenously determined level. They assume residents first choose a community to live in, then choose property taxes and zoning, and finally choose a level of housing. They analyze how the incorporation of zoning impacts allocations and welfare, uncovering a number of subtle effects.

A limitation of the Fernandez and Rogerson analysis, is that communities are fixed by the time zoning decisions are made so that their impact on community composition is not captured. To address this, Calabrese, Epple and Romano (2007) incorporate zoning in a different way. Households first choose an initial community of residence, which is committed by a purchase of land and then residents collectively choose zoning and property taxes for their communities.[9] After these policy choices, households revisit their choice of community and purchase housing

[7] See, for example, Margolis (1956). Zoning may also be motivated by the desire to change the type of household entering the community. Residents may believe requiring new houses to have large lots will attract a better class of resident. This may reduce crime and yield better peer groups in schools. This is referred to as *exclusionary zoning* and is analyzed in Oates (1977) and Calabrese, Epple and Romano (2006).

[8] See also Pogodzinski and Sass (1994) who present a theoretical framework to underpin their empirical investigation of the impact of zoning on housing values.

[9] While Fernandez and Rogerson assume that residents choose taxes and zoning sequentially, Calabrese et al assume a simultaneous choice. To get around the potential difficulties created by a two dimensional policy space, they employ the citizen-candidate approach and assume that residents elect a citizen to choose policy rather than directly vote over policies.

and consume public services in their new community. This quasi-dynamic structure means that residents anticipate the impact of their decisions on land prices and community composition. Calabrese et al show that, without zoning, all communities are identical. With zoning, there is stratification, with higher income communities having stricter minimum housing requirements. Zoning leads to aggregate welfare gains because it reduces distortions in both housing and public service provision. However, in contrast to Hamilton's analysis, it does not result in first best outcomes.

The main advantage of this paper's model over these works lies in its dynamic structure. This allows the impact of zoning on the value of the existing stock of housing to be captured. While in Calabrese et al's quasi-dynamic set-up households anticipate how zoning impacts the value of their land, all housing is produced after zoning ordinances have been decided. The dynamic structure also allows the key "grandfathering" characteristic of zoning whereby existing property is exempt from regulation to be captured. By contrast, in the models of Fernandez and Rogerson and Calabrese et al, households are bound by the constraints that they impose.

Zoning is a particular type of land-use regulation. The importance of such regulations in understanding housing markets in the U.S. has been demonstrated in a number of recent studies (see, for example, Glaeser and Gyourko (2003), Glaeser, Gyourko and Saks (2005), and Green, Malpezzi, and Mayo (2005)). This has led to increasing interest in the determinants of these regulations (see, for example, Glaeser and Ward (2009)). Ortalo-Magne and Prat (2010) develop an innovative theoretical model which sheds light on the simultaneous determination of households' location decisions, their housing choices, and collective decisions on housing restrictions. They study an infinite horizon overlapping generation model of an economy with a rural and urban area. Households must live where they work and households in the urban area can rent or own housing. The urban wage is uncertain which makes rental rates and house prices uncertain. The number of urban houses is initially fixed but supply can be expanded. However, new construction requires a permit and permits are controlled by urban residents. Residents are more likely to oppose new construction if they own rather than rent, so political choices depend on the endogenous extent of home ownership. For a broad range of parameter choices, the supply of urban houses is inefficiently restricted. In its focus on dynamic inefficiencies in housing supply generated by political choices, this paper is complementary to Ortalo-Magne and Prat's work.

The third related literature on dynamic models of political economy. The last two decades have seen considerable progress in this area, with the literature moving from two period models with simple underlying policy spaces to infinite-horizon models with rich fundamentals. Important examples can be found in the macroeconomic literature which has explored the political determination of capital and labor taxes in the neoclassical growth model (see, for example, Krusell, Quadrini and Rios-Rull (1997) and Krusell and Rios-Rull (1999)). In these analyses, citizens' accumulation of assets is impacted by current and future taxes which in turn depend upon the distribution of assets. Similarly, in this paper's analysis, citizens' housing decisions are impacted by current and future policies which in turn depend upon housing stocks. Accordingly, the notion of political equilibrium used here follows the approach taken in these works.

3 The model

Consider a geographic area consisting of two communities, indexed by $i \in \{1, 2\}$. The time horizon is infinite, with periods indexed by $t \in \{0, ..., \infty\}$. A constant population of households of size 1 need to reside in the area, but there is turnover, so that in each period new households arrive and old ones leave. The probability that a household residing in the area will need to remain there in the subsequent period is μ. Thus, in each period, a fraction $1 - \mu$ of households leave the area and are replaced by an equal number of new ones.[10]

The only way to live in the area is to own a house in one of the communities.[11] Houses come in two types, large and small. Houses are durable, but a fixed fraction d of the stock in each community is destroyed at the end of each period.[12] This fraction is assumed to be less than $1 - \mu$, so that households face a higher probability of having to leave the area than of having their houses destroyed. New houses can be built in each period and the cost of building a house of type $H \in \{L, S\}$ is C_H where $C_L > C_S$. Each community has enough land to accomodate a population of size 1 and land has no alternative use.[13] The stock of old houses of type H in

[10] We have in mind that households leave for reasons to do with employment opportunities or changes in family circumstance.

[11] The model does not "micro-found" why households cannot rent houses. The usual assumption is that moral hazard issues in the maintenance of the house make owning the more efficient arrangement. Obviously, if households were renters they would have different incentives with respect to property values. On these issues see Ortalo-Magne and Prat (2010).

[12] This assumption follows Glaeser and Gyourko (2005) and is necessary to get turnover of the housing stock. Given the constant population, if all houses were infinitely durable there would be no dynamics. By housing being "destroyed", we have in mind both literal destruction by floods, hurricanes, fires, or termites, and also houses being torn down because of decay due to the passage of time.

[13] This implies that the supply of housing in each community is perfectly elastic over the relevant range which

community i at the beginning of a period is denoted O_{Hi} and new construction is denoted N_{Hi}. New construction is completed at the beginning of each period and new and old houses are perfect substitutes. Thus, post construction, there are $O_{Hi} + N_{Hi}$ type H houses in community i.

A public service is provided in each community. The service level in community i is denoted g_i. The cost of the service is cg_i per household.[14]

Each household receives an exogenous income of y per period.[15] When living in the area, households have preferences defined over housing, public services, and private consumption. They differ in their preferences for large houses which are measured by the parameter θ. A household of type θ with private consumption x and services g obtains a period payoff of $\theta + x + B(g)$ if it lives in a large house and $x + B(g)$ if it lives in a small house. The service benefit function $B(g)$ is non-decreasing and concave.[16] When not living in the area, a household's payoff just depends on its private consumption. Households discount future payoffs at rate δ and can borrow and save at rate $1/\delta - 1$. The fraction of households with type less than or equal to θ is $F(\theta)$.

There are competitive housing markets in both communities which open at the beginning of each period. Demand comes from new households moving into the area and remaining residents who need new houses or who want to move. Supply comes from owners leaving the area, residents who want to move, and new construction. New construction is supplied by competitive construction firms. The price of houses of type H in community i is denoted P_{Hi}.

Service provision in each community i is financed by a proportional tax τ_i on the value of property $\sum_H P_{Hi}(O_{Hi} + N_{Hi})$. Each community must balance its budget in each period implying that

$$\tau_i \sum_H P_{Hi}(O_{Hi} + N_{Hi}) = cg_i \sum_H (O_{Hi} + N_{Hi}) \qquad i \in \{1, 2\}. \tag{1}$$

The level of service provision in any period is chosen collectively by the residents of the community

is in the spirit of Hamilton's assumptions. The model can be extended to allow land not used for housing to have some constant productivity in agricultural use. This complicates notation without fundamentally changing the insights from the analysis.

[14] This specification is consistent with Hamilton (1975) who assumed that the average cost of providing services equals the marginal cost. This assumption eliminates concern about optimal community size. In static models, introducing either public good features in service provision or congestion externalities leads to additional sources of inefficiency as discussed by, among others, Buchanan and Goetz (1972). An interesting topic for future research is to see how these problems play out in dynamic models.

[15] There are no income effects so income heterogeneity can be introduced without changing the results.

[16] This preference specification implies that households have identical preferences for public services and, in this sense, the model differs from standard Tiebout models. It is possible to add heterogeneous service preferences to the model and it would be interesting to study how this impacts equilibrium.

that period. The service level preferred by a majority of residents is implemented.

The timing of the model is as follows. Each period begins with a stock of old houses $\mathbf{O} = (O_{L1}, O_{S1}, O_{L2}, O_{S2})$ of aggregate size $1 - d$. Existing residents learn whether they will be remaining in the area and new households arrive. Housing markets open, housing prices $\mathbf{P} = (P_{L1}, P_{S1}, P_{L2}, P_{S2})$ are determined, and new construction $\mathbf{N} = (N_{L1}, N_{S1}, N_{L2}, N_{S2})$ takes place. The total amount of new construction must equal d. The housing market activity determines the post-construction housing stocks $\mathbf{O}+\mathbf{N}$. Residents then choose the public service levels g_1 and g_2 which determine the property tax rates τ_1 and τ_2. Finally, at the end of the period, a fraction d of the housing stock in each community is destroyed. All houses, new and old, are equally likely to be destroyed, implying that next period's stock of old houses is given by $\mathbf{O}' = (1-d)(\mathbf{O}+\mathbf{N})$.[17]

4 Equilibrium without zoning

We begin by analyzing what would happen without zoning. We first clarify what is meant by an equilibrium and then discuss some properties of equilibrium. Next we study steady states and discuss the existence of equilibrium and convergence to these steady states. Finally, we discuss the efficiency of these steady states.

4.1 Definition of equilibrium

The model has a recursive structure. The state can be summarized by the stock of old houses $\mathbf{O}$.[18]

Given this stock, the housing market determines prices and new construction and we recognize this dependence by writing $\mathbf{P}(\mathbf{O})$ and $\mathbf{N}(\mathbf{O})$. The prices $\mathbf{P}(\mathbf{O})$ and post-construction housing stock $\mathbf{O}+\mathbf{N}(\mathbf{O})$ then determine the tax bases of the two communities and these in turn determine public service levels $(g_1(\mathbf{O}), g_2(\mathbf{O}))$ and tax rates $(\tau_1(\mathbf{O}), \tau_2(\mathbf{O}))$. Households understand what prices, new construction, services, and taxes will be given any initial state $\mathbf{O}$. They also understand that next period's stock of old houses will be given by $\mathbf{O}' = (1-d)(\mathbf{O}+\mathbf{N}(\mathbf{O}))$. They treat all these aggregate relationships as exogenous and beyond their control.

[17] While it would be more realistic to assume that older houses were more likely to be destroyed, introducing this feature would require keeping track of the age of each house and allowing for age-dependent housing prices. This would make the analysis intractable.

[18] Under our assumptions of no income effects and costless mobility, the allocation of homes among households does not impact market outcomes or policy determination.

Decisions of households At the beginning of any period, households fall into two groups: those who resided in the area in the previous period and those who did not, but must in the current period. The first group is differentiated by the homes they own. There are five possible home ownership states represented by $o \in \{L1, S1, L2, S2, \varnothing\}$; $o = Hi$ means that the household owns a type H house in community i and $o = \varnothing$ means that it does not own a house (which would be the case if its house was destroyed). The second group of households will not own homes, so that $o = \varnothing$ for all these households.

Households in the first group who need to leave the area will sell their houses and obtain a continuation payoff of

$$P_o(\mathbf{O}) + \frac{y}{1-\delta}, \tag{2}$$

where $P_\varnothing(\mathbf{O}) = 0$. The remaining households in the first group and all those in the second must decide in which community to live and in what type of house. Formally, they must make a home ownership decision $a \in \{L1, S1, L2, S2\}$. Since selling a house and moving is costless and houses of the same type are perfect substitutes, there is no loss of generality in assuming that all households owning houses at the beginning of any period sell them. This makes each household's home ownership decision independent of its home ownership state o. It also means that the only future consequences of the current period choice of housing is through the selling price in the subsequent period.

To make this more precise, let $V_\theta(\mathbf{O})$ denote the expected payoff of a household of type θ at the beginning of a period in which it has to live in the area, does not own a house, and the aggregate state is $\mathbf{O}$. Then, the expected payoff of a household of type θ at the beginning of a period in which it has to live in the area and is in home ownership state o is

$$P_o(\mathbf{O}) + V_\theta(\mathbf{O}). \tag{3}$$

The value function $V_\theta(\mathbf{O})$ satisfies the functional equation

$$V_\theta(\mathbf{O}) = \max_{a \in \{L1,S1,L2,S2\}} \left\{ \begin{array}{c} y + \theta I_{H(a)} + B(g_{i(a)}(\mathbf{O})) - \tau_{i(a)}(\mathbf{O}) P_a(\mathbf{O}) - P_a(\mathbf{O}) \\ +\delta[(1-d)P_a(\mathbf{O}') + \mu V_\theta(\mathbf{O}') + (1-\mu)\frac{y}{1-\delta}] \end{array} \right\}, \tag{4}$$

where I_H is an indicator function equal to 1 if H equals L, $H(a)$ is the house type associated with home ownership choice a, and $i(a)$ is the community associated with a. Let $\alpha_\theta(\mathbf{O})$ be the set of optimal home ownership choices. This will contain more than one element if, for example, households are indifferent between communities. The household's home ownership choice will determine probabilistically its home ownership state in the next period. For example, if $a = L1$, then $o' = L1$ with probability $1 - d$ and $o' = \varnothing$ with probability d.

Housing market equilibrium Construction firms are competitive and the production costs of new homes are constant. Accordingly, the supplies of large and small homes are perfectly elastic at the prices C_L and C_S, respectively. This means that if $P_{Hi}(\mathbf{O}) = C_H$, firms will willingly supply any number of new homes of type H in community i but if $P_{Hi}(\mathbf{O}) < C_H$ none will be supplied.

Let $\xi_{Hi}(\theta, \mathbf{O})$ be the fraction of type θ households selecting houses of type H in community i and let $\boldsymbol{\xi}(\theta, \mathbf{O})$ denote the vector $(\xi_{L1}(\cdot), \xi_{S1}(\cdot), \xi_{L2}(\cdot), \xi_{S2}(\cdot))$. If a positive fraction of type θ households are selecting houses of type H in community i it must be the case that Hi is in the set of optimal choices for these households; i.e., $Hi \in \alpha_\theta(\mathbf{O})$. In equilibrium, it must be the case that the total fraction of households selecting houses of type H in community i is equal to the supply of such houses; that is,

$$\int_\theta \xi_{Hi}(\theta, \mathbf{O}) dF(\theta) = O_{Hi} + N_{Hi}(\mathbf{O}). \tag{5}$$

In addition, it must be the case that all households of type θ are selecting some type of housing, so that for all types θ we have that

$$\sum_H \sum_i \xi_{Hi}(\theta, \mathbf{O}) = 1. \tag{6}$$

Choice of public service levels and tax rates All households get the same benefit from public services. However, residents living in different houses face different tax prices for services, which may give rise to different preferred service levels. Using (1), the preferred service level for residents of type H houses in community i is

$$g^*(\rho_{Hi}(\mathbf{O})) = \arg\max_g \{B(g) - \rho_{Hi}(\mathbf{O})g\}, \tag{7}$$

where $\rho_{Hi}(\mathbf{O})$ is the tax price of services that residents of type H houses in community i face when the state is $\mathbf{O}$.[19] This tax price is given by

$$\rho_{Hi}(\mathbf{O}) = \frac{cP_{Hi}(\mathbf{O})}{P_{Li}(\mathbf{O})\lambda_i(\mathbf{O}) + P_{Si}(\mathbf{O})(1 - \lambda_i(\mathbf{O}))}, \tag{8}$$

[19] The simplicity of these optimal public service levels reflects the assumption that policies are chosen *after* the market for housing has cleared. At that point, the housing stock and its value are predetermined. When it is

where $\lambda_i(\mathbf{O})$ is the fraction of post-construction houses that are large in community i.[20] The tax price is determined by the *relative* price of type H houses in community i and the fraction of large houses. If large houses are more expensive than small houses, the tax price is lower for those owning small houses and is decreasing in the fraction of large houses. The majority preferred level of public services in community i is

$$g_i(\mathbf{O}) = \begin{cases} g^*(\rho_{Li}(\mathbf{O})) & \text{if } \lambda_i(\mathbf{O}) \geq 1/2 \\ g^*(\rho_{Si}(\mathbf{O})) & \text{if } \lambda_i(\mathbf{O}) < 1/2 \end{cases}. \tag{9}$$

Using (1), the associated tax rate is

$$\tau_i(\mathbf{O}) = \frac{cg_i(\mathbf{O})}{P_{Li}(\mathbf{O})\lambda_i(\mathbf{O}) + P_{Si}(\mathbf{O})(1 - \lambda_i(\mathbf{O}))}. \tag{10}$$

Equilibrium An *equilibrium without zoning* consists of a price rule $\mathbf{P}(\mathbf{O})$, a new construction rule $\mathbf{N}(\mathbf{O})$, public service rules $(g_1(\mathbf{O}), g_2(\mathbf{O}))$, tax rules $(\tau_1(\mathbf{O}), \tau_2(\mathbf{O}))$, and, for each household type, a value function $V_\theta(\mathbf{O})$, a housing demand correspondence $\alpha_\theta(\mathbf{O})$ and housing selection functions $\boldsymbol{\xi}(\theta, \mathbf{O})$, such that three conditions are satisfied. First, *household optimization*: for each household type θ the value function $V_\theta(\mathbf{O})$ satisfies (4) and, for all $\mathbf{O}$, every element of $\alpha_\theta(\mathbf{O})$ solves the maximization problem described in (4). Second, *housing market equilibrium*: the housing selection functions and new construction rules satisfy (5) and (6), and, in addition, $\xi_{Hi}(\theta, \mathbf{O})$ is positive only if Hi is an element of $\alpha_\theta(\mathbf{O})$ and $N_{Hi}(\mathbf{O})$ is positive only if $P_{Hi}(\mathbf{O}) = C_H$. Third, *majority rule*: the public service and tax rules satisfy (9) and (10).

chosen, the property tax is therefore like a non-distortionary tax on capital and equilibrium responses are irrelevant for the calculus of citizen decision-making. While taxes and public services do impact the housing market, it is the expectation of these taxes and services that are relevant and the taxes chosen today do not influence expectations concerning tomorrow's taxes. This contrasts with Tiebout models with property taxation of the variable housing supply variety which assume taxes are chosen *before* housing choices are made. In this spirit, an alternative modelling assumption would be that in each period contemporaneous property taxes are fixed and households vote on next period's taxes. Households would then anticipate how next period's taxes would impact next period's housing market equilibrium. While this assumption is perhaps less natural than the assumption made here (and certainly more complicated), it would be interesting to work out its implications.

[20] That is, $\lambda_i(\mathbf{O}) = \frac{O_{Li}+N_{Li}(\mathbf{O})}{O_{Li}+N_{Li}(\mathbf{O})+O_{Si}+N_{Si}(\mathbf{O})}$.

4.2 Some properties of equilibrium

Inspecting the household's problem (4) and using the definitions in (8) and (10), it is clear that a household choosing a type H house will prefer to live in the community that maximizes[21]

$$[B(g_i) - \rho_{Hi} g_i] - P_{Hi} + \delta(1-d)P'_{Hi}. \tag{11}$$

The term in square brackets is the public service surplus associated with community i, which is the difference between service benefits and the tax cost. The second term is the current price of a type H house in community i and the final term is the discounted expected value of the house next period. Notice that (11) is independent of θ, so that all households choosing type H houses have the same preferences over communities. Thus, in equilibrium, if type H houses are available in both communities, all those choosing them must be indifferent between communities. It follows from (11) that for $H \in \{L, S\}$

$$[B(g_1) - \rho_{H1} g_1] - P_{H1} + \delta(1-d)P'_{H1} = [B(g_2) - \rho_{H2} g_2] - P_{H2} + \delta(1-d)P'_{H2}. \tag{12}$$

This arbitrage equation implies that differences in public service surplus across communities must be capitalized into differences in housing prices as argued by Hamilton (1976).

From the household's problem (4), we also see that a household will prefer a large house in community i to a small house if its preference θ exceeds

$$P_{Li} - P_{Si} - \delta(1-d)(P'_{Li} - P'_{Si}) + (\rho_{Li} - \rho_{Si})g_i. \tag{13}$$

This expression represents the higher cost of a large house and includes both price and tax differences. Note that (12) implies that (13) is equalized across communities. Thus, letting

$$\theta_c = P_{Li} - P_{Si} - \delta(1-d)(P'_{Li} - P'_{Si}) + (\rho_{Li} - \rho_{Si})g_i, \tag{14}$$

it follows from (12) and (13) that all households with preference larger than θ_c will prefer a large house and all those with preference less than θ_c a small house. In equilibrium, therefore, we must have that

$$F(\theta_c) = \sum_{i=1}^{2}(O_{Si} + N_{Si}), \tag{15}$$

[21] The term P'_{Hi} is short-hand for $P_{Hi}(\mathbf{O}')$. Similarly, P_{Hi} is short-hand for $P_{Hi}(\mathbf{O})$, g_i is short-hand for $g_i(\mathbf{O})$, etc.

and that

$$1 - F(\theta_c) = \sum_{i=1}^{2} (O_{Li} + N_{Li}). \tag{16}$$

Exactly how types with preference larger than θ_c are allocated across the two communities does not matter, provided that (5) and (6) are satisfied. Similarly, for types with preference smaller than θ_c.

4.3 Steady states

Given an equilibrium, a stock of old houses $\mathbf{O}^*$ is a *steady state* if new construction at $\mathbf{O}^*$ is such as to maintain the stock constant. The following proposition tells us what steady states look like.

Proposition 1 *Let O^* be a steady state of an equilibrium without zoning. Then, the fraction of large houses in each community is the same; that is, $\lambda_1(O^*) = \lambda_2(O^*) = \lambda^*$. If this fraction exceeds $1/2$, the public service level in each community is $g_L^* \equiv g^*(\frac{cC_L}{C_L\lambda^* + C_S(1-\lambda^*)})$ and households live in large houses if their preference exceeds*

$$(1 - \delta(1-d))(C_L - C_S) + \frac{c(C_L - C_S)}{C_L\lambda^* + C_S(1-\lambda^*)} g_L^*. \tag{17}$$

If the fraction is less than $1/2$, the service level is $g_S^* \equiv g^*(\frac{cC_S}{C_L\lambda^* + C_S(1-\lambda^*)})$ and households live in large houses if their preference exceeds

$$(1 - \delta(1-d))(C_L - C_S) + \frac{c(C_L - C_S)}{C_L\lambda^* + C_S(1-\lambda^*)} g_S^*. \tag{18}$$

Proof: If $\mathbf{O}^*$ is an equilibrium steady state, then, $\mathbf{N}(\mathbf{O}^*) = d\mathbf{O}^*/(1-d)$. Thus, since $O_{Hi}^* > 0$ for all Hi, it must be the case that there is new construction of both types of houses in both communities. Accordingly, housing prices must equal construction costs so that $\mathbf{P}(\mathbf{O}^*) = (C_L, C_S, C_L, C_S)$. It must also be the case that the fraction of large houses in each community is the same; that is, $\lambda_1(\mathbf{O}^*) = \lambda_2(\mathbf{O}^*) = \lambda^*$. For if one community had a greater fraction of large houses, the public service surplus enjoyed by large house owners in that community would be higher than in the other which would violate (12). Since both house prices and the fraction of large houses are the same across the two communities, it follows from (9) and (10) that service levels and taxes are also the same. If a majority of households own large houses ($\lambda^* \geq 1/2$), then, from (8) and (9), the public service level will be g_L^* and, from (13), households live in large houses only if their preference exceeds the expression in (17). If a majority of households own small houses ($\lambda^* < 1/2$), the public service level is g_S^* and households live in large houses only if their preference exceeds the expression in (18). ■

Note that the steady state stock of houses in the two communities is not tied down by this proposition. It tells us only that the fraction of large houses in each community must be the same.[22] The communities can be of different sizes in long run equilibrium.[23]

4.4 Existence of equilibrium and convergence to steady states

Proposition 1 assumes an equilibrium exists and tells us what equilibrium steady states must look like. It tells us nothing about the existence of equilibrium or equilibrium steady states. Nor does it tell us whether in equilibrium the housing stock must converge to a steady state. We now briefly discuss these issues.

Discussing convergence requires some additional terminology. For any initial state $\mathbf{O}$, define the sequence of old housing stocks $\langle \mathbf{O}_t(\mathbf{O}) \rangle_{t=0}^{\infty}$ inductively as follows: $\mathbf{O}_0(\mathbf{O}) = \mathbf{O}$ and $\mathbf{O}_{t+1}(\mathbf{O}) = (1-d)[\mathbf{O}_t(\mathbf{O}) + \mathbf{N}(\mathbf{O}_t(\mathbf{O}))]$. Intuitively, if we start in period 0 with old housing stock $\mathbf{O}$, in period t the stock will be $\mathbf{O}_t(\mathbf{O})$. Then, the sequence of housing stocks $\langle \mathbf{O}_t(\mathbf{O}) \rangle_{t=0}^{\infty}$ *converges* to the steady state $\mathbf{O}^*$ if $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}) = \mathbf{O}^*$.

It is straightforward to find equilibria in which the housing stock converges to a steady state. The first task is to find a steady state. If there exists $\lambda^* \geq 1/2$ satisfying the equation

$$\lambda^* = 1 - F((1 - \delta(1-d) + \frac{cg_L^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)), \tag{19}$$

then there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_L^*. Similarly, if there exists $\lambda^* < 1/2$ satisfying the equation

$$\lambda^* = 1 - F((1 - \delta(1-d) + \frac{cg_S^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)), \tag{20}$$

there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_S^*. Under mild conditions, there must exist either a $\lambda^* \geq 1/2$ satisfying (19) or a $\lambda^* < 1/2$ satisfying (20). Indeed, both could be true. For if small home owners are choosing

[22] In his static model with two housing types and a proportional property tax, Hamilton (1976) conjectured that in long run equilibrium it must be the case that the proportionate mix of housing in each community is the same. Like us, he assumed that households differed in their demand for housing. With income effects or different preferences for public services, it should be possible to get stratification as in Epple et al (1984).

[23] The aggregate stock of large old houses $O_{L1}^* + O_{L2}^*$ must equal $(1-d)\lambda^*$ and the aggregate stock of small old houses $O_{S1}^* + O_{S2}^*$ must equal $(1-d)(1-\lambda^*)$.

services, property taxes will typically be higher than if large home owners are choosing. All else equal, higher property taxes lead less households to choose large houses. It is perfectly possible, therefore, to have one steady state in which large home owners are a majority and choose low taxes, and another in which small home owners are a majority and choose high taxes.

Having found a steady state, the next step is to construct an equilibrium in which the housing stock converges to this steady state. To illustrate, suppose there exists $\lambda^* < 1/2$ satisfying equation (20) and consider constructing an equilibrium in which the fraction of large houses in each community converges to λ^*. Consider first starting from an initial state with a symmetric allocation of old houses and assume the intial fraction of large houses is less than λ^*. Then, initially, all new construction will be in the form of large houses and will be balanced across the communities. Once the fraction λ^* can be reached with new construction, construction of small homes will begin and the steady state will be reached. During the adjustment to steady state the price of small homes will be less than C_S, but it will be increasing over time as the relative fraction of small homes decreases.

Now suppose we start with one community (say, community 1) with a larger fraction of large homes than the other, but with both still less than the steady state. In this case, all new construction of large homes will occur in community 1. This is because of the more favorable tax base. Eventually, community 2 will become so small that one period's new construction of large homes will be sufficient to equate the fraction of large homes in the two communities. After that, new construction of both types of homes in the two communities can start. Community 2 can remain a small size or increase in size relative to community 1. This is immaterial. During the adjustment to steady state, the price of both large and small homes in community 2 will be lower than in community 1. The superior tax base will therefore be capitalized into housing prices.

Finally, suppose that we start with one community (say, community 1) not only with a larger fraction of large homes than the other, but with a larger fraction than the steady state. This is the most complicated case. Here, there are a number of possible paths to the steady state, depending on the aggregate stock of small homes. If there is an aggregate shortage of small homes, construction of small homes occurs in community 1, reducing the fraction of large houses in that community. In the meantime, community 2 shrinks. Eventually, community 2 becomes so small that one period's new construction of large homes will be sufficient to equate the fraction of large homes in the two communities. If there is an aggregate glut of small homes, new construction of large homes will occur in community 1. For a while, the fraction of large homes in community 1 will increase futher away from the steady state as the stock of large homes is built up. Eventually, however, the stock of large homes will be sufficiently large that new construction of small homes will begin and the fraction of large homes in community 2 will return towards the steady state level. Again, new construction will only occur in community 1 once it has got sufficiently small that that one period's new construction of large homes will be sufficient to equate the fraction of large homes in the two communities.

4.5 Efficiency

The steady states described in Proposition 1 are not efficient.[24] From an efficiency perspective, households should own a large house if their preference θ exceeds

$$\theta^e \equiv (1 - \delta(1-d))(C_L - C_S). \tag{21}$$

From (17) and (18) of Proposition 1, the steady state fraction of large houses will be too low.[25] Property taxation means that owners of large houses face a higher tax price of services and this encourages households to purchase cheaper homes. Moreover, the efficient level of public services is

$$g^e \equiv g^*(c) = \arg\max \{B(g) - cg\}. \tag{22}$$

From Proposition 1, when $g^*(\cdot)$ is increasing, public services will be under-provided if large home owners are in the majority and over-provided if small home owners are in the majority. This reflects the fact that property taxation drives the tax price of services below the true cost for small home owners and above it for large home owners.

Less obviously, when there are multiple equilibrium steady states, they may be Pareto ranked. Suppose there exists two equilibrium steady states, λ_a^* and λ_b^*, one in which large home owners form a majority and the other in which small home owners are in the majority. All households are better off when large home owners are a majority if small home owners are better off. This requires that the public service surplus enjoyed by small home owners is higher when large home owners form the majority. This is possible because, even though small home owners are not obtaining

[24] By an efficient steady state we mean one which maximizes aggregate surplus. This is the standard notion of efficiency in the literature.

[25] This is consistent with the arguments in Hamilton (1976).

their ideal service level when large home owners are a majority, they benefit from a larger tax base.[26]

5 Equilibrium with exogenous zoning

Suppose that from period 0 onwards, one community enforces a zoning requirement that requires all newly constructed houses be large.[27] What would happen? This section addresses this question.

Introducing zoning in this way does not substantially complicate the definition of equilibrium. We only have to recognize that the zoning requirement impacts the housing market equilibrium by limiting the supply of small homes in the zoned community. In particular, the supply of small houses in any period is perfectly inelastic and just equals the old stock. This means that the price of small houses in the zoned community can exceed the cost C_S. An *equilibrium with exogenous zoning* is defined to be an equilibrium that recognizes this constraint.

It is straightforward to characterize steady states with exogenous zoning.

Proposition 2 *In a steady state of an equilibrium with exogenous zoning, all houses in the zoned community are large and all houses in the unzoned community are small. The public service level in each community is the efficient level g^e defined in (22) and households live in the zoned community only if their preference exceeds the efficient cut-off θ^e defined in (21).*

Proof: Suppose that community 1 is the zoned community. If $\mathbf{O}^*$ is a steady state, then, under zoning, it must be the case that $O^*_{S1} = 0$ and hence $\lambda_1(\mathbf{O}^*) = 1$. It must also be the case that $O^*_{L2} = 0$ and hence that $\lambda_2(\mathbf{O}^*) = 0$. To see why, suppose, to the contrary, that $O^*_{L2} > 0$. Then it must be the case that the steady state price of large houses in both communities is C_L. Since the price of small houses in community 2 is C_S, the tax price of public services is lower for large house owners in community 1. But this means public service surplus enjoyed by large house owners in community 1 is higher than in community 2 which would violate (12). Since $P_{L1}(\mathbf{O}^*) = C_L$ and

[26] Interestingly, when there do exist multiple Pareto ranked equilibria, citizens could be better off with a property tax limit which guaranteed that the inefficient high tax steady state could not be reached.

[27] In reality, zoning requirements must be backed by some type of externality justification or they may be challenged in court. Accordingly, zoning requirements are specified in terms of quantity constraints like minimum lot size rather than simply minimum construction costs. There has been debate in the literature about how precisely such quantity constraints allow communities to regulate new construction. Fischel (1992) argues persuasively for the view that communities are able to regulate very precisely.

$\lambda_1(\mathbf{O}^*) = 1$ and $P_{S2}(\mathbf{O}^*) = C_S$ and $\lambda_2(\mathbf{O}^*) = 0$, it follows from (9) and (10) that

$$(g_1(\mathbf{O}^*), \tau_1(\mathbf{O}^*)) = (g^*(c), \frac{cg^*(c)}{C_L})$$

and that

$$(g_2(\mathbf{O}^*), \tau_2(\mathbf{O}^*)) = (g^*(c), \frac{cg^*(c)}{C_S}).$$

From (4), it follows that a household of type θ will prefer living in a large house in community 1 to a small house in community 2 if

$$\theta + B(g^*(c)) - \frac{cg^*(c)}{C_L}C_L - C_L + \delta(1-d)C_L \geq B(g^*(c)) - \frac{cg^*(c)}{C_S}C_S - C_S + \delta(1-d)C_S$$

or, equivalently, if their preference θ exceeds θ^e as defined in (22). ■

The key point to note about Proposition 2 is that the allocation of housing and public services in the steady state is efficient. This is simply Hamilton's logic at work in a dynamic context. In contrast to the case without zoning, the steady state stocks of housing are uniquely defined: there are $(1-d)(1-F(\theta^e))$ old large houses in the zoned community and $(1-d)F(\theta^e)$ old small houses in the unzoned community. As in the case without zoning, it is straightforward to construct an equilibrium in which the housing stock converges to the steady state.[28]

6 Equilibrium with endogenous zoning

The previous section showed that if one community had a zoning ordinance requiring that all newly constructed houses be large, then there is a unique steady state and it is efficient. Moreover, there exist equilibria in which housing stocks converge to this efficient steady state. However, this begs the question of whether communities would actually choose to implement zoning in this way. Specifically, if in each period the communities could choose whether to implement zoning, would the efficient outcome arise?

To analyze this question, suppose that at the end of each period, after choosing taxes, the residents of each community vote whether to impose a zoning ordinance which requires that all

[28] If at the time zoning is imposed there are initially both types of houses in the two communities, then convergence to the steady state will be asymptotic. In the approach to the steady state, both large and small houses in the zoned district will be worth more than those in the unzoned district. Moreover, tax rates will be lower in the zoned district. These two properties mean there should be a negative relationship between tax rates and house prices. On the other hand, there will be a positive relationship between zoning and housing values.

construction in the subsequent period be large houses.[29] The vote takes place before housing stock is destroyed, so that at the time of voting, all residents own homes. Let $Z_i \in \{0, 1\}$ denote community i's zoning regulation, with $Z_i = 1$ meaning the ordinance is imposed.

With endogenous zoning, each period now begins with a stock of old houses $\mathbf{O}$ and the zoning regulations $\mathbf{Z} = (Z_1, Z_2)$ chosen by residents in the prior period. The housing market determines prices and new construction under these regulations. As before, after the housing market has cleared, residents choose service levels. Following this, residents choose next period's zoning regulations $\mathbf{Z}'$. Finally, at the end of the period, a fraction of the housing stock is destroyed.

6.1 Definition of equilibrium

We begin by clarifying what is meant by equilibrium in the extended model. Prices and new construction will now depend on both the initial housing stock and zoning regulations and so we write $\mathbf{P}(\mathbf{O}, \mathbf{Z})$ and $\mathbf{N}(\mathbf{O}, \mathbf{Z})$. Similarly, we write the service levels as $(g_1(\mathbf{O}, \mathbf{Z}), g_2(\mathbf{O}, \mathbf{Z}))$ and tax rates as $(\tau_1(\mathbf{O}, \mathbf{Z}), \tau_2(\mathbf{O}, \mathbf{Z}))$. Next period's stock of old houses is given by $\mathbf{O}' = (1 - d)(\mathbf{O} + \mathbf{N}(\mathbf{O}, \mathbf{Z}))$ and next period's zoning regulations by $\mathbf{Z}'(\mathbf{O}, \mathbf{Z})$.

Households' decision-making with respect to home ownership is not fundamentally altered by the presence of zoning. We just let the value functions and optimal policy correspondences depend upon $\mathbf{Z}$ by writing $V_\theta(\mathbf{O}, \mathbf{Z})$ and $\alpha_\theta(\mathbf{O}, \mathbf{Z})$.[30] Zoning impacts the housing market equilibrium by limiting the supply of small homes as discussed in the previous section. Let $\xi_{Hi}(\theta, \mathbf{O}, \mathbf{Z})$ be the fraction of type θ households selecting houses of type H in community i. Zoning only affects the choice of public service levels and tax rates through its impact on prices and new construction.

The additional work in defining equilibrium comes in modelling the zoning decision. Suppose

[29] In some New England towns changes in zoning rules must be approved by the majority of citizens at town meetings (Gyourko, Saiz and Summers (2008)). However, more generally, zoning decisions are made by a combination of local politicians and administrative boards (typically designated the Planning and Zoning Commission and the Board of Zoning Adjustment) rather than determined by direct democracy. These boards are composed of local citizens appointed by local politicians. Our assumption is that this decision-making process will produce decisions that will reflect the will of the majority of residents. The mechanism we have in mind is that the electoral process will select local politicians whose preferences are congruent with those of the majority of voters and they will in turn appoint like-minded board members. Readers who object to this assumption can take comfort in the fact that there will often be no significant disagreement among residents. See Fischel (2001b) for more discussion of local government decision making and a defense of the median voter assumption.

[30] Modifying (4), the value function $V_\theta(\mathbf{O}, \mathbf{Z})$ satisfies the functional equation

$$V_\theta(\mathbf{O}, \mathbf{Z}) = \max_{a \in \{L1, S1, L2, S2\}} \left\{ \begin{array}{c} y + \theta I_{H(a)} + B(g_{i(a)}(\mathbf{O}, \mathbf{Z})) - \tau_{i(a)}(\mathbf{O}, \mathbf{Z}) P_a(\mathbf{O}, \mathbf{Z}) - P_a(\mathbf{O}, \mathbf{Z}) \\ +\delta[(1-d) P_a(\mathbf{O}', \mathbf{Z}') + \mu V_\theta(\mathbf{O}', \mathbf{Z}') + (1-\mu)\frac{y}{1-\delta}] \end{array} \right\}$$

where $\mathbf{O}' = (1 - d)(\mathbf{O} + \mathbf{N}(\mathbf{O}, \mathbf{Z}))$ and $\mathbf{Z}' = \mathbf{Z}'(\mathbf{O}, \mathbf{Z})$.

the state is $(\mathbf{O},\mathbf{Z})$. At the time the zoning decision is made, the housing stock will be $\mathbf{O}+\mathbf{N}(\mathbf{O},\mathbf{Z})$ and the fraction of type θ households with type H houses in community i will be $\xi_{Hi}(\theta,\mathbf{O},\mathbf{Z})$. Using (2) and (3) and taking expectations, a type θ household with a type H house in community i will support imposing zoning if

$$\begin{array}{c}(1-d)P_{Hi}(\mathbf{O}',(1,Z'_{-i}))+\mu V_\theta(\mathbf{O}',(1,Z'_{-i}))+(1-\mu)\frac{y}{1-\delta} \\ \geq (1-d)P_{Hi}(\mathbf{O}',(0,Z'_{-i}))+\mu V_\theta(\mathbf{O}',(0,Z'_{-i}))+(1-\mu)\frac{y}{1-\delta}\end{array}, \tag{23}$$

where $\mathbf{O}'$ denotes next period's stock of old houses and Z'_{-i} denotes the zoning decision of the other community.[31] The two sides of the inequality are next period's expected utility with and without zoning. Taking expectations is necessary because at the time of voting the household is uncertain whether its house will be destroyed and whether it will need to leave the area. Let $I_\theta(Hi;\mathbf{O}',Z'_{-i})=1$ if the household favors zoning and 0 otherwise. If $\mathbf{Z}'(\mathbf{O},\mathbf{Z})$ are equilibrium zoning decisions, then, for each community, $Z'_i(\mathbf{O},\mathbf{Z})=1$ if a majority of the community's residents favor zoning given that the other community is choosing $Z'_{-i}(\mathbf{O},\mathbf{Z})$. More formally, we require that

$$Z'_i(\mathbf{O},\mathbf{Z})=1 \Leftrightarrow \sum_H \int_\theta \xi_{Hi}(\theta,\mathbf{O},\mathbf{Z})[I_\theta(Hi;\mathbf{O}',Z'_{-i}(\mathbf{O},\mathbf{Z}))-\frac{1}{2}]dF(\theta)>0. \tag{24}$$

Following the definition of equilibrium without zoning, an *equilibrium with endogenous zoning* consists of a price rule $\mathbf{P}(\mathbf{O},\mathbf{Z})$, a new construction rule $\mathbf{N}(\mathbf{O},\mathbf{Z})$, public service rules $(g_1(\mathbf{O},\mathbf{Z}),g_2(\mathbf{O},\mathbf{Z}))$, tax rules $(\tau_1(\mathbf{O},\mathbf{Z}),\tau_2(\mathbf{O},\mathbf{Z}))$, zoning rules $\mathbf{Z}'(\mathbf{O},\mathbf{Z})$, and, for each type of household, value functions $V_\theta(\mathbf{O},\mathbf{Z})$, housing demand correspondences $\alpha_\theta(\mathbf{O},\mathbf{Z})$, and housing selection functions $\boldsymbol{\xi}(\theta,\mathbf{O},\mathbf{Z})$, satisfying household optimization, housing market equilibrium, and majority rule. The notion of housing market equilibrium defined earlier is amended to include the requirement that $N_{Si}(\mathbf{O},\mathbf{Z})$ is positive only if $P_{Si}(\mathbf{O},\mathbf{Z})=C_S$ *and* $Z_i=0$, and the notion of majority rule is extended to include the requirement that the zoning rule satisfies (24).

6.2 Understanding household preferences over zoning

To get a feel for household preferences over zoning, consider when a type θ household with a type H house in community i will favor imposing zoning. Assume first that it will remain optimal for the household to live in a type H house in the next period regardless of the community's regulation

[31] The prices P_{Hi} and value functions V_θ assume that whatever this period's zoning decision, in all subsequent periods zoning policies will be determined by the equilibrium rule $\mathbf{Z}'(\cdot)$.

decision. In equilibrium, the household will be indifferent between communities so there is no loss of generality in assuming that the household continues to locate in community i. Thus, using (4), (8), and (10), we have that

$$V_\theta(\mathbf{O}', (1, Z'_{-i})) - V_\theta(\mathbf{O}', (0, Z'_{-i})) = B(g_i^1) - \rho_{Hi}^1 g_i^1 - P_{Hi}^1 - \left(B(g_i^0) - \rho_{Hi}^0 g_i^0 - P_{Hi}^0\right) + \delta\Delta', \quad (25)$$

where the 1 and 0 superscripts denote variables with and without zoning and Δ' denotes the difference in continuation payoffs with and without zoning. Substituting (25) into (23), we can rewrite condition (23) as

$$(1 - \mu - d)\left[P_{Hi}^1 - P_{Hi}^0\right] + \mu[B(g_i^1) - \rho_{Hi}^1 g_i^1 - (B(g_i^0) - \rho_{Hi}^0 g_i^0)] + \mu\delta\Delta' \geq 0. \quad (26)$$

The first term of (26) is the welfare impact resulting from changing next period's price of type Hi houses. Given that $1 - \mu > d$, price increases are valued by households. Intuitively, when households are more likely to leave the area than to have to replace their houses, a higher price for the house type they own is beneficial. Since zoning restricts the supply of small homes, it is natural to expect P_{Si}^1 to be at least as large as P_{Si}^0. The impact on the price of large homes is less clear as there are both supply and demand effects.

The second term of (26) is the change in next period's public service surplus resulting from imposing zoning. This change could be positive or negative. Imposing zoning may increase the fraction of large houses which will increase surplus by reducing tax prices. However, it will also impact relative housing prices. If, for example, it raises the relative price of small houses, the tax price paid by small home owners could increase. Imposing zoning may also have political consequences that impact service levels. If large home owners become a majority, service levels may be reduced, lowering surplus for small home owners.

The final term of (26), which is the difference in continuation values, can be decomposed in exactly the same way as (23) if it is the case that the household will remain in a type H house in the period after next. There will be an impact arising from changing housing prices, an impact resulting from changing public service surplus, and a future impact. Repeated application of this logic reveals that a household who will remain in a type H house as long as it remains in the area will favor zoning if

$$\sum_{t=0}^{\infty} (\mu\delta)^t \left\{(1 - \mu - d)\left[P_{Hit}^1 - P_{Hit}^0\right] + \mu[B(g_{it}^1) - \rho_{Hit}^1 g_{it}^1 - (B(g_{it}^0) - \rho_{Hit}^0 g_{it}^0)]\right\} \geq 0. \quad (27)$$

Here, P^1_{Hit} denotes the price of type Hi houses t periods after the decision to impose zoning becomes effective, P^0_{Hit} denotes the price of type Hi houses t periods after the decision not to impose zoning becomes effective, etc. While this expression is useful in clarifying how zoning impacts household welfare, evaluating its sign is complex because the consequences of today's zoning decision for future prices are unclear in general. In particular, today's zoning decision could alter the entire path of future new construction.

The preceding analysis assumes it will remain optimal for the household to live in the same type of house regardless of the community's zoning decision. However, if zoning expands the supply of large homes, it must cause some households to live in large houses who would otherwise have lived in small houses. In this case, condition (23) can be written as:

$$(1-d)\left[P^1_{Hi} - P^0_{Hi}\right] + \mu[\theta - (P^1_{Li} - P^0_{Si})] + \mu[B(g^1_i) - \rho^1_{Li} g^1_i - (B(g^0_i) - \rho^0_{Si} g^0_i)] + \mu\delta\Delta' \geq 0. \quad (28)$$

The first term is the change in the value of the household's current house, the second term is the benefit from owning a large home less the incremental cost,[32] and the third term is the change in public service surplus. The latter will likely be negative because the household faces a higher tax price of services with a large house.

6.3 Endogenous zoning and efficiency

We now investigate whether efficient results will obtain when communities choose their zoning regulations in a decentralized manner period by period. To pose the question clearly, we first clarify terminology. Given an equilibrium with endogenous zoning, a stock of old housing and zoning rules $(\mathbf{O}^*, \mathbf{Z}^*)$ is a *steady state* if new construction at $(\mathbf{O}^*, \mathbf{Z}^*)$ is such as to maintain the stock at $\mathbf{O}^*$ and if residents vote to maintain the zoning rules $\mathbf{Z}^*$. Drawing on the discussion in Section 5, the steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ is *efficient* if *either* $\mathbf{Z}^*$ equals $(1, 0)$ and $\mathbf{O}^*$ equals $(1-d)(1-F(\theta^e), 0, 0, F(\theta^e))$, *or* $\mathbf{Z}^*$ equals $(0, 1)$ and $\mathbf{O}^*$ equals $(1-d)(0, F(\theta^e), 1-F(\theta^e), 0)$. Generalizing the terminology of Section 3, for any initial state $(\mathbf{O}, \mathbf{Z})$ consider the sequence of old housing stocks and zoning rules $\langle \mathbf{O}_t(\mathbf{O}, \mathbf{Z}), \mathbf{Z}_t(\mathbf{O}, \mathbf{Z})\rangle_{t=0}^{\infty}$ defined inductively as follows: $(\mathbf{O}_0(\mathbf{O}, \mathbf{Z}), \mathbf{Z}_0(\mathbf{O}, \mathbf{Z})) = (\mathbf{O}, \mathbf{Z})$ and $(\mathbf{O}_{t+1}(\mathbf{O}, \mathbf{Z}), \mathbf{Z}_{t+1}(\mathbf{O}, \mathbf{Z})) = ((1-d)(\mathbf{O}_t(\cdot) + \mathbf{N}(\mathbf{O}_t(\cdot), \mathbf{Z}_t(\cdot))), \mathbf{Z}'(\mathbf{O}_t(\cdot), \mathbf{Z}_t(\cdot)))$. Starting in period 0 with old housing stock $\mathbf{O}$ and zoning rules $\mathbf{Z}$, in period t the stocks will be $\mathbf{O}_t(\mathbf{O}, \mathbf{Z})$ and the zoning rules will be $\mathbf{Z}_t(\mathbf{O}, \mathbf{Z})$. The sequence of housing stocks and zoning rules $\langle \mathbf{O}_t(\mathbf{O}, \mathbf{Z}), \mathbf{Z}_t(\mathbf{O}, \mathbf{Z})\rangle_{t=0}^{\infty}$

[32] Note that this difference in purchase prices overstates the true increase in housing cost because it neglects the benefits of owning a more valuable asset next period, which show up in Δ'.

converges to the steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ if (i) there exists t^* such that for all t bigger than t^*, $\mathbf{Z}_t(\cdot)$ equals $\mathbf{Z}^*$, and (ii) $\lim_{t\to\infty} \mathbf{O}_t(\cdot) = \mathbf{O}^*$.

Given this terminology, the question of interest is do there exist equilibria with endogenous zoning in which the housing stock and zoning rules converge to an efficient steady state for any initial condition? We will demonstrate that there exists no equilibrium with endogenous zoning which has a steady state that is efficient and satisfies a stability property we call "strong local stability". The steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ is *strongly locally stable* if there exists $\varepsilon > 0$ such that for any initial state $(\mathbf{O}, \mathbf{Z})$ with the property that $\|\mathbf{O} - \mathbf{O}^*\| < \varepsilon$ we have that $\mathbf{Z}_t(\mathbf{O}, \mathbf{Z}) = \mathbf{Z}^*$ for $t = 1, ..., \infty$ and $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}, \mathbf{Z}) = \mathbf{O}^*$. Intuitively, what this means is that if we start with an initial stock of old houses close to the steady state level and perturb the zoning rules from their steady state levels, then in the next period citizens will return the zoning rules to their steady state levels. Moreover, thereafter, the zoning rules will remain at $\mathbf{Z}^*$ and the housing stock will gradually return to its steady state level. The concept is slightly stronger than local stability which would require only that *eventually* zoning rules return to their steady state values. Since by the definition of a steady state, $\mathbf{Z}^*$ are the equilibrium zoning rules when the post-construction housing stock is $\mathbf{O}^* + \mathbf{N}(\mathbf{O}^*, \mathbf{Z}^*)$, if d is small, the requirement is just that zoning choices are not sensitive to small changes in post-construction housing stocks in the neighborhood of $\mathbf{O}^* + \mathbf{N}(\mathbf{O}^*, \mathbf{Z}^*)$.[33]

Proposition 3 *There exists no equilibrium with endogenous zoning which has a steady state that is both efficient and strongly locally stable.*

Proof: See Appendix.

To understand the result recall that at an efficient steady state, one community consists of large homes and the other of small. Moreover, the large home community imposes zoning and the small home community does not. Suppose the small home community were to deviate and impose zoning. What would be the response? Given the large home community is already imposing zoning, the short run effect would be to restrict the supply of small houses and raise their price. There would be no detrimental effect on public service surplus for home owners in the small home community because their community already contains no large homes. The tax price for small home owners can therefore only go down. Thus, the short run impact for residents of the small home community is positive. The long run impact is more complex because it depends on how the

[33] Given that new construction must sum to d, we know that $\|\mathbf{O} + \mathbf{N}(\mathbf{O}, \mathbf{Z}) - (\mathbf{O}^* + \mathbf{N}(\mathbf{O}^*, \mathbf{Z}^*))\|$ must be less than $\varepsilon + d$.

equilibrium responds to the deviation. In principle, the deviation could set into motion a series of changes that end up causing harm to the residents of the small home community.[34] This is impossible to know without knowing the full path of equilibrium play. Nonetheless, if the efficient steady state is strongly locally stable, it can be shown that there will be no long run impact of the deviation on the small home community. In the period following the one in which the deviation becomes effective, construction of small homes resumes and prices return to C_S. All large home construction takes place in the zoned community, so that the small home community remains homogeneous. In this case, therefore, the small home community must benefit from a deviation.

This Proposition does not rule out the possibility that there exists an equilibrium with endogenous zoning which has a steady state that is efficient but not strongly locally stable. Nonetheless, the logic underlying the Proposition suggests that it will be difficult to find equilibria with endogenous zoning which have efficient steady states. Moreover, even if such an equilibrium exists, it seems unlikely that the housing stock and zoning rules will converge to the efficient steady state.

6.4 Welfare-reducing zoning

The previous sub-section suggests that endogenous zoning decisions are unlikely to produce an efficient outcome. However, it still could be the case that zoning allows efficiency improvements relative to the case without zoning. This sub-section presents a pair of examples showing how zoning can reduce long run welfare.

6.4.1 No preferences for large houses

Consider first the case in which all households have housing preference parameter 0 and so are indifferent between large and small houses.

Proposition 4 *Suppose that all households are indifferent between large and small houses. Then, there exists an equilibrium with endogenous zoning in which both communities always impose zoning.*

Proof: See Appendix.

[34] For example, suppose that community 2 deviating leads community 1 to relax its zoning requirement. This may lead low θ households to build in community 1 to take advantage of the better fiscal externality. This in turn may reduce the price of small houses in community 2. This is an unlikely scenario, to be sure, because it is not clear why community 1 would wish to remove zoning. But it is conceivable that there could be long run effects on the residents of community 1 that might justify the decision.

The key to the tractability of this example is that large and small houses are perfect substitutes and hence must have the same price. Given this, public service surplus in each community is independent of the housing stock. In an equilibrium in which both communities always impose zoning, on the equilibrium path all houses are priced at C_L and services in each community are set at the efficient level. If one community deviates by removing zoning, all new construction takes place in that community in the form of small houses. The price of houses drops to C_S but there is no change in public service surplus. Following the deviation, all prices revert back to C_L. Accordingly, the deviation is undesirable.

Without zoning, all houses would be small in steady state. In the equilibrium of Proposition 4, the stock of old houses converges to the steady state $\mathbf{O}^* = ((1-d)/2, 0, (1-d)/2, 0)$ so that, in the limit, all houses are large. This is highly inefficient given that no household gets any additional benefit from living in a large house.

6.4.2 Two preference types and a threshold public service

Now suppose that there are two types of households: *low types* with preference 0 who are indifferent between large and small houses, and *high types* with preference $\overline{\theta}$ who prefer large houses. High types make up a fraction ζ of the population. Further suppose that the public service is a *threshold service* that yields benefits up to some level $\overline{g}$ and none beyond that. Formally, $B(g) = \beta \min\{g, \overline{g}\}$ where β is large relative to c. With this benefit function, each community will provide the service level $\overline{g}$ independent of the mix of housing and housing prices. Nonetheless, households still have an incentive to care about the composition of their communities because this will impact their tax prices.

Proposition 5 *Suppose that there are two types of households and that the public service is a threshold service. Further suppose that*

$$\overline{\theta} > (1 + \frac{cg}{C_S})(C_L - C_S). \tag{29}$$

Then, if $d < 1/2$ and

$$1 - \mu - d > \mu \frac{c\overline{g}}{C_L}, \tag{30}$$

there exists an equilibrium with endogenous zoning in which both communities always impose zoning.

Proof: See Appendix.

To understand this result, note that since both communities always impose zoning, on the equilibrium path the stock of old large houses gradually increases as small houses are replaced by large. When the stock of old large houses has grown sufficiently large so that it exceeds $\zeta - d$, the price of *all* houses in that period and beyond is C_L. This is because, after new construction, there will be sufficient large houses for high types and the marginal buyer is a low type. Since low types are indifferent between large and small houses, their prices must be the same. In periods before the stock of old large houses has reached $\zeta - d$ there will be insufficient large houses for high types. To induce high types to own small houses their prices must be below C_L. This in turn means that all new construction of large houses will be concentrated in the community with the largest fraction of large houses to benefit from the lower tax price of services. The price of large houses in the other community will therefore be less than C_L.

If one community deviates from equilibrium behavior by relaxing its zoning requirement, the impact will depend on what the stock of old large houses is in the period the deviation becomes effective. If the stock exceeds ζ so that there are already sufficient large houses for high types, all new construction will take place in the deviating community in the form of small houses. The price of all houses will drop to C_S but there will be no change in public service surplus. Following the period in which the deviation becomes effective, all prices will revert back to C_L. Accordingly, the deviation will be undesirable.

If the stock of old large houses is less than $\zeta - d$, the deviation will create no change in behavior under condition (29). All new construction will be in large houses just as it would be with zoning. If this is the case, there is no incentive to deviate in this range.

The tricky case is when the stock of old large houses lies between $\zeta - d$ and ζ. Under condition (29), new construction of large houses will be such as to meet the demands of high types and the remaining new construction will be in small houses in the deviating community. Since with zoning the price of all houses would be C_L, the impact of the deviation is to reduce the price of small houses in the deviating community to C_S. The deviation may also reduce the price of large houses in the deviating community below C_L if new construction of large houses takes place in the other community. In addition to this impact on the value of property, there is also an impact on public service surplus. By reducing the relative price of small homes, the deviation reduces the tax price for small home owners in the deviating community. Following the period in which the deviation becomes effective, all prices will revert back to C_L. This is because the inital stock of old large houses will be $(1-d)\zeta$ which exceeds $\zeta - d$.

In order for prospective owners of small homes not to want to deviate, the one period gain in public service surplus must be less than the cost of a lower home value. That this is true for low types who own small homes is guaranteed by (30). To see why, note from (26) that such an individual will favor keeping zoning if

$$(1-\mu-d)\left[C_L - C_S\right] + \mu(\rho^0_{Si} - c)\overline{g} \geq 0 \tag{31}$$

where ρ^0_{Si} is the tax price for small home owners if zoning is removed. Next observe that $\rho^0_{Si} \geq cC_S/C_L$ and hence

$$(1-\mu-d)\left[C_L - C_S\right] + \mu(\rho^0_{Si} - c)\overline{g} \geq \left[C_L - C_S\right]\left[1-\mu-d-\mu\frac{c\overline{g}}{C_L}\right]. \tag{32}$$

Condition (30) guarantees that the right hand side of (32) is positive. High types will not be prospective owners of small homes and therefore will not benefit from removing zoning.[35]

The above argument relies on condition (29). Without this assumption, the implications of a community deviating when the stock of old large houses is less than ζ are more complicated. Consider again the case in which the stock lies between $\zeta - d$ and ζ. Under the behavior just described, in the period in which the deviation becomes effective, the price of small houses in the deviating community will be C_S. In the following period, the price of these houses will jump to C_L, implying a large speculative gain from purchasing a small house. High types may be willing to forego the benefits of large home ownership for one period to benefit from this gain. If this is the case, all households will demand small homes and no new large homes will be constructed. This will imply a reduction in the price of large homes in both communities. Moreover, in the next period the stock of old large houses may fall below $\zeta - d$ and hence prices will not all equal C_L.

More generally, the possibility of this speculative gain may change the implications of a community deviating when the stock of old large houses is less than $\zeta - d$. There is no guarantee that without zoning all new construction will be in the form of large houses. High types may prefer small houses because they know they will appreciate in price. If small houses are constructed when zoning is removed then this will delay the time at which the stock of old large houses has

[35] The only group who might benefit are low types who own large homes. But under the assumption that $d < 1/2$, this group must be a minority of residents.

grown sufficiently large so that it exceeds $\zeta - d$ and all house prices equal C_L. While this point will eventually be reached, the deviation will change the dynamic adjustment and prices of both small and large houses will be different along the entire path.

The bottom line is that with a smaller willingness to pay of high types, a community deviating may change housing prices in more than just the period in which the deviation becomes effective. This makes tracing out the implications of a deviation more involved. If removing zoning reduces all housing prices, the basic trade off is unchanged. The costs of lower house values must be weighed against any benefit from a more favorable tax price caused by a relative price change. A condition similar to (30) would guarantee that cost exceeds benefit in each period for a majority of residents. However, the direction of future price changes is not clear and, in principle, it is possible that removing zoning could increase house values in the deviating community in some future period.[36]

As in the previous example, in this equilibrium the stock of old houses converges to the steady state $\mathbf{O}^* = ((1-d)/2, 0, (1-d)/2, 0)$ so that, in the limit, all houses will be large. With no zoning, in steady state a fraction ζ of houses will be large. Zoning therefore reduces welfare.

6.4.3 Discussion

In both these examples, in the long run, the property tax is a benefit tax in the sense that each household's tax bill equals the value of services it consumes. However, the important lesson is that in the presence of zoning this does not imply there are no distortions in the housing market. Indeed, housing decisions may be severely distorted. The distortion arises directly from communities' zoning decisions which inefficiently restrict households' housing options.

7 More communities

In the environment of this paper, two communities are sufficient to achieve efficiency with exogenous zoning.[37] Thus, it seems reasonable to study endogenous zoning with only two communities.

[36] In particular, it is not obvious that removing zoning will reduce future housing prices in the deviating community. When the stock of old houses is less than $\zeta - d$, on the equilibrium path new construction of large homes will take place in the community with the largest fraction of large homes. If the other community deviates by removing zoning, it will attract new construction of small homes at the expense of new construction of large homes in the zoned community. This will simultaneously reduce the future fraction of large houses in both communities. These reductions will have opposing effects on house prices in the deviating community.

[37] More communities will be necessary with a larger number of types of housing and/or heterogeneity in public service tastes.

However, this does raise the question of whether additional communities might be helpful in mitigating the problems that arise. In particular, the difficulties arise from residents using zoning to manipulate the prices of their houses. In principle, more communities might reduce communities' market power and dampen these incentives. This section briefly considers this point.

Introducing an additional community creates no real changes in the model without zoning or with exogenous zoning: Propositions 1 and 2 apply appropriately generalized. For the purposes of Proposition 2, efficiency would result if one or two communities had zoning. In the former case, there would be two small house communities and, in the latter, two large house communities. The way in which homes are allocated across the homogeneous communities in the long run is immaterial.

With endogenous zoning, an additional community may permit the existence of equilibria which have an efficient steady state. Consider, for example, an efficient steady state in which community 1 is zoning, communities 2 and 3 are not zoning, and all small homes are located in community 2. Suppose that at this efficient steady state, community 2 deviates by imposing zoning. This will have no impact on housing prices in community 2, since new construction of small houses will simply switch to community 3.

Nonetheless, adding more communities does not change the fundamental message of the analysis. For while community 2 cannot influence the price of its homes by introducing zoning at the efficient steady state, it can for housing stocks arbitrarily close to the efficient steady state. To illustrate consider for ε small and positive the stock $\mathbf{O} = (1-d)(1-F(\theta^e), 0, \varepsilon, F(\theta^e) - \varepsilon, 0, 0)$. This differs from the efficient steady state in that community 2 has a small number of large houses. If the efficient steady state is strongly locally stable, the equilibrium path starting from this stock will have community 1 implementing zoning and communities 2 and 3 not. New construction of small homes will occur only in community 2 because of the beneficial fiscal externality created by the large homes. If community 2 deviates by imposing zoning all new construction of small homes will switch to community 3. However, the price of small homes in community 2 must be higher than that in community 3 and so the deviation boosts prices in community 2. Moreover, as a result of the deviation, the value of large homes in community 2 will also be higher because the fraction of large homes will be greater.

The short run impact of this deviation must be positive for those owning small homes in community 2 for ε sufficiently small. To see this, let the prices of housing in community 2 in the period in which the deviation takes effect be (P^1_{L2}, P^1_{S2}) with the deviation and (P^0_{L2}, C_S) without. Let λ^1_2 and λ^0_2 denote the fractions of large houses in community 2 with and without the deviation and let g^1_2 and g^0_2 be the service levels. As we have argued, it will be the case that $P^1_{L2} > P^0_{L2}$, $P^1_{S2} > C_S$, and $\lambda^1_2 > \lambda^0_2$. From (26), a resident living in community 2 with a small house will gain from introducing zoning in the period the deviation becomes effective if

$$(1-\mu-d)\left[P^1_{S2} - C_S\right] + \mu[B(g^1_2) - \frac{cP^1_{S2}}{\lambda^1_2 P^1_{L2} + (1-\lambda^1_2)P^1_{S2}} g^1_2 - \left(B(g^0_2) - \frac{cC_S}{\lambda^0_2 P^0_{L2} + (1-\lambda^0_2)C_S} g^0_2\right)] > 0. \tag{33}$$

It is straightforward to show that the difference in public service surplus is greater than

$$-\mu c g^0_2 \left(\frac{\left[P^1_{S2} - C_S\right]\lambda^1_2 C_L}{C^2_S}\right). \tag{34}$$

Thus, since $\lambda^1_2 \leq \varepsilon/F(\theta^e)$, inequality (33) must hold for sufficiently small ε given that $1-\mu > d$.

The long run impact is also positive. Assuming the efficient steady state is strongly locally stable, new construction of small homes will resume in community 2 in the period following the one in which the deviation becomes effective. The price of small homes will therefore return to the construction cost C_S. However, both the price and fraction of large homes in community 2 will be marginally higher than on the equilibrium path. This increases the public service surplus of small home owners.

It follows that while community 2 has no incentive to introduce zoning at the efficient steady state, it does at stocks arbitrarily close to the efficient steady state. This means that Proposition 3 can be generalized to the case of three communities and the message of Section 6.3 that the housing stock and zoning rules are unlikely to converge to an efficient steady state remains.[38] In addition, Propositions 4 and 5 generalize so that the message of Section 6.4 concerning welfare-reducing zoning also remains. Thus, the difficulties with endogenous zoning are not mitigated by introducing more communities.

8 Conclusion

This paper has presented a novel dynamic model that can be used to analyze theoretical issues in state and local public finance. The model has been employed to revisit a classic question in public finance concerning the ability of a system of local governments financing public services

[38] The generalization of Proposition 3 can be found in the Appendix.

with property taxes to produce efficient allocations of both housing and services. The analysis confirms Hamilton's insight that, with the right zoning ordinances, such a system can yield efficient results. However, it also suggests that local governments will be unlikely to choose the right zoning ordinances. Indeed, allowing local governments to implement zoning may actually reduce long run welfare.

The results undermine the *Benefit View* of the property tax in the sense that they contradict the argument that combining decentralized property taxation and zoning will yield first best results. However, they do not contradict the narrower position that, with zoning, property taxes will be benefit taxes. It is just that this narrower claim is besides the point. Property taxes being benefit taxes does not imply that housing decisions are undistorted, since such decisions can be directly distorted by the supply restrictions embodied in zoning.

It should nonetheless be stressed that the results of this paper do not imply that property taxation is a bad way of financing local government.[39] In particular, if we take it as an institutional constraint that communities are able to implement zoning ordinances, then property taxation may be an excellent system of finance. To illustrate, suppose that to keep housing prices high, communities would implement zoning whether they financed public services with head taxes or property taxes. Then, property taxes will be equivalent to head taxes. The point is that housing is distorted by zoning under both methods of finance and property taxes create no additional distortions relative to head taxes. Sales or income taxation, on the other hand, will distort consumption and labor supply decisions and will not obviously change housing distortions by influencing zoning decisions.

[39] For a nice overview of the debate about whether local governments should use property or income taxes see Oates and Schwab (2004). For alternative political economy perspectives see Glaeser (1996) and Hoxby (1999).

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9 Appendix

9.1 Proof of Proposition 3

Suppose, to the contrary, that there exists an equilibrium with endogenous zoning with a steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ which is both efficient and strongly locally stable. Assume that community 1 is the community who zones so that $\mathbf{O}^* = (1-d)\,(1-F(\theta^e), 0, 0, F(\theta^e))$ and $\mathbf{Z}^* = (1, 0)$. Since $(\mathbf{O}^*, \mathbf{Z}^*)$ is a steady state, it must be that $\mathbf{N}(\mathbf{O}^*, \mathbf{Z}^*) = d\,(1-F(\theta^e), 0, 0, F(\theta^e))$ and $\mathbf{Z}'(\mathbf{O}^*, \mathbf{Z}^*) = (1, 0)$. However, we claim that $Z_2'(\mathbf{O}^*, \mathbf{Z}^*) \neq 0$, which will establish a contradiction.

We will show that *all* residents of community 2 would be better off imposing zoning if the state were $(\mathbf{O}^*, \mathbf{Z}^*)$. Recall that community 2 consists of all households of type $\theta \leq \theta^e$ and that at the time of voting they all own small houses. The continuation payoff for a resident of type θ if zoning is not implemented is

$$(1-d)C_S + \mu V_\theta(\mathbf{O}^*, \mathbf{Z}^*) + (1-\mu)\frac{y}{1-\delta}.$$

Since $(\mathbf{O}^*, \mathbf{Z}^*)$ is a steady state, we have that for all $\theta \leq \theta^e$

$$V_\theta(\mathbf{O}^*, \mathbf{Z}^*) = y + B(g^e) - cg^e - C_S + \delta[(1-d)C_S + \mu V_\theta(\mathbf{O}^*, \mathbf{Z}^*) + (1-\mu)\frac{y}{1-\delta}],$$

which implies that

$$V_\theta(\mathbf{O}^*, \mathbf{Z}^*) \equiv V^* = \frac{y + B(g^e) - cg^e - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

Note that this continuation payoff is independent of θ.

The continuation payoff if zoning is implemented for a household of type $\theta \leq \theta^e$ is

$$(1-d)P_{S2}(\mathbf{O}^*, (1,1)) + \mu V_\theta(\mathbf{O}^*, (1,1)) + (1-\mu)\frac{y}{1-\delta}.$$

To evaluate this, we need to know what happens following community 2's deviation to impose zoning. Consider the sequence $\langle \mathbf{O}_t, \mathbf{Z}_t \rangle_{t=0}^{\infty}$ defined inductively as follows: $(\mathbf{O}_0, \mathbf{Z}_0) = (\mathbf{O}^*, (1,1))$ and $(\mathbf{O}_{t+1}, \mathbf{Z}_{t+1}) = ((1-d)(\mathbf{O}_t + \mathbf{N}(\mathbf{O}_t, \mathbf{Z}_t)), \mathbf{Z}'(\mathbf{O}_t, \mathbf{Z}_t))$. In addition, for all $t = 0, ..., \infty$, let $\mathbf{P}_t = \mathbf{P}(\mathbf{O}_t, \mathbf{Z}_t)$ and $\mathbf{N}_t = \mathbf{N}(\mathbf{O}_t, \mathbf{Z}_t)$. Then, $(\mathbf{O}_0, \mathbf{Z}_0, \mathbf{N}_0, \mathbf{P}_0)$ describes variables in the period in which the deviation becomes effective, $(\mathbf{O}_1, \mathbf{Z}_1, \mathbf{N}_1, \mathbf{P}_1)$ describes variables in the first period following the deviation, etc. Since $(\mathbf{O}^*, \mathbf{Z}^*)$ is strongly locally stable and $\|\mathbf{O}^* - \mathbf{O}^*\| = 0$, we know that for all $t \geq 1$, $\mathbf{Z}_t = \mathbf{Z}^*$ and that $\lim_{t\to\infty} \mathbf{O}_t = \mathbf{O}^*$.

Claim 1: For sufficiently large t, $P_{L1t} = C_L$ and $P_{S2t} = C_S$.

Proof of Claim 1: To prove this it is enough to show that for sufficiently large t, $N_{L1t} > 0$ and $N_{S2t} > 0$. But this follows from the fact that $\lim_{t\to\infty} \mathbf{O}_t = \mathbf{O}^*$ and that $\mathbf{O}^* = (1 - d)(1 - F(\theta^e), 0, 0, F(\theta^e))$. ■

Claim 2: For all $t = 0, ..., \infty$, $\lambda_1(\mathbf{O}_t, \mathbf{Z}_t) = 1$ and $\lambda_2(\mathbf{O}_t, \mathbf{Z}_t) = 0$.

Proof of Claim 2: We know that $\lambda_1(\mathbf{O}_0, \mathbf{Z}_0) = 1$ because $(O_{L10}, O_{S10}) = ((1-d)(1-F(\theta^e)), 0)$ and, since zoning is in place in community 1, we have that $N_{S10} = 0$. Moreover, by strong local stability, zoning is in place in all future periods and thus it must be that $\lambda_1(\mathbf{O}_t, \mathbf{Z}_t) = 1$ for all t.

We know that $(O_{L20}, O_{S20}) = (0, (1-d)F(\theta^e))$. We also claim that for all $t \geq 0$, $N_{L2t} = 0$. To see this, suppose, to the contrary, that $N_{L2t} > 0$. Then, in order for households to want to buy these houses it must be that

$$[B(g_{2t}) - \tau_{2t}C_L] - C_L + \delta(1-d)P_{L2t+1} \geq [B(g^e) - cg^e] - P_{L1t} + \delta(1-d)P_{L1t+1}.$$

But because community 2 has small houses and community 1 does not, it must be that

$$B(g_{2t}) - \tau_{2t}C_L < B(g^e) - cg^e.$$

Thus, for the above inequality to hold, we must have that $P_{L2t+1} > P_{L1t+1}$. But we know that community arbitrage implies that

$$[B(g_{2t+1}) - \tau_{2t+1}P_{L2t+1}] - P_{L2t+1} + \delta(1-d)P_{L2t+2} = [B(g^e) - cg^e] - P_{L1t+1} + \delta(1-d)P_{L1t+2}.$$

But again because community 2 has small houses, it must be that

$$[B(g_{2t+1}) - \tau_{2t+1}P_{L2t+1}] < [B(g^e) - cg^e].$$

Thus, we require $P_{L2t+2} > P_{L1t+2}$. Continuing this line of argument, we conclude that $P_{L2t} > P_{L1t}$ for all $t = 1, ..., \infty$. But we know by the previous claim that for sufficiently large t, it must be that $P_{L1t} = C_L$. It follows that for all $t \geq 0$, $\lambda_t(\mathbf{O}_0, \mathbf{Z}_0) = 0$. ■

Claim 3: $P_{S20} > C_S$ and for all $t \geq 1$, $P_{S2t} = C_S$.

Proof of Claim 3: By Claim 1 we know that for sufficiently large t it must be that $(P_{L1t}, P_{S2t}) = (C_L, C_S)$. Let $\widehat{t}$ be the largest period in which $(P_{L1t}, P_{S2t}) \neq (C_L, C_S)$. Suppose first that $\widehat{t} = 0$. Then all we need to show is that $P_{S20} > C_S$. We know that $\mathbf{O}_0 = (1-d)(1 - F(\theta^e), 0, 0, F(\theta^e))$, that $\mathbf{N}_0 = (d, 0, 0, 0)$ and that $P_{L10} = C_L$. We also know that $P_{L11} = C_L$ and that $P_{S21} = C_S$. Suppose to the contrary that $P_{S20} \leq C_S$. Then, it must be that all types with preferences less than θ^e strictly prefer small houses in community 2 implying that demand is at least equal to $F(\theta^e)$. Supply, however, is equal to $(1-d)F(\theta^e)$.

Now suppose that $\widehat{t} \geq 1$. Since $\mathbf{Z}_{\widehat{t}} = (1,0)$, there are two possibilities in period $\widehat{t}$: (i) $P_{L1\widehat{t}} = C_L$, $P_{S2\widehat{t}} < C_S$, and $N_{L1\widehat{t}} = d$, and (ii) $P_{L1\widehat{t}} < C_L$, $P_{S2\widehat{t}} = C_S$, and $N_{S2\widehat{t}} = d$. We now show that possibility (i) cannot arise. Suppose, to the contrary, that it does arise. Then, given that $(P_{L1\widehat{t}+1}, P_{S2\widehat{t}+1}) = (C_L, C_S)$, we know that it must be that $O_{L1\widehat{t}} + d \leq 1 - F(\theta^e)$ and that $O_{S2\widehat{t}} \geq F(\theta^e)$. This is because all households with types θ less than θ^e will strictly prefer to purchase a small house at these prices. Thus, we must have that $O_{S2\widehat{t}} \geq F(\theta^e)$ in order for the housing market to clear. Now consider period $\widehat{t}-1$. Suppose that $O_{S2\widehat{t}-1} < F(\theta^e)$. Then, since $O_{S2\widehat{t}} \geq F(\theta^e)$, there must be new construction of small houses in community 2 in period $\widehat{t}-1$. In that case, $P_{S2\widehat{t}-1} = C_S$, but since the price of small houses falls in period $\widehat{t}$, no households with types greater than θ^e will want to purchase a small house in community 2. Accordingly, we have that $O_{S2\widehat{t}-1} + N_{S2\widehat{t}-1} \leq F(\theta^e)$. But then we have that

$$O_{S2\widehat{t}} = (1-d)[O_{S2\widehat{t}-1} + N_{S2\widehat{t}-1}] < F(\theta^e),$$

which is a contradiction. Thus, $O_{S2\widehat{t}-1} \geq F(\theta^e)$. This in turn implies that $P_{L1\widehat{t}-1} = C_L$, $P_{S2\widehat{t}-1} < C_S$, and that $N_{L1\widehat{t}-1} = d$. Again, there can be no new construction of small houses, because all households of type greater than θ^e will want large houses. Continuing this line of argument, we conclude that for all $t = 1, ..., \widehat{t}$, we must have that $O_{S2t} \geq F(\theta^e)$. But since $O_{S20} = (1-d)F(\theta^e)$ and $N_{S20} = 0$, we have that

$$O_{S21} = (1-d)[O_{S20} + N_{S20}] = (1-d)^2 F(\theta^e) < F(\theta^e)$$

which is a contradiction. We conclude therefore that it cannot be that $P_{L1\widehat{t}} = C_L$, $P_{S2\widehat{t}} < C_S$, and $N_{L1\widehat{t}} = d$.

We have therefore established that in period $\widehat{t}$, $P_{L1\widehat{t}} < C_L$, $P_{S2\widehat{t}} = C_S$, and $N_{S2\widehat{t}} = d$. If $\widehat{t} \geq 2$, consider period $\widehat{t}-1$. Again, there are two possibilities: (i) $P_{L1\widehat{t}-1} = C_L$, $P_{S2\widehat{t}-1} < C_S$, and $N_{L1\widehat{t}-1} = d$, and (ii) $P_{L1\widehat{t}-1} < C_L$, $P_{S2\widehat{t}-1} = C_S$, and $N_{S2\widehat{t}-1} = d$. Using similar logic, we can again show that possibility (i) cannot arise.

Continuing on in this way, we conclude that for all $t = 1, ..., \widehat{t}$, we have that $P_{L1\widehat{t}} < C_L$, $P_{S2\widehat{t}} = C_S$, and $N_{S2\widehat{t}} = d$. Now consider period 0, the period the deviation becomes effective. We know that $\mathbf{O}_0 = (1-d)(1-F(\theta^e), 0, 0, F(\theta^e))$, that $\mathbf{N}_0 = (d, 0, 0, 0)$ and that $P_{L10} = C_L$.

We also know that $P_{L11} \leq C_L$ and that $P_{S21} = C_S$. We now argue that $P_{S20} > C_S$. Suppose to the contrary that $P_{S20} \leq C_S$. Then, it must be the case that all types with preferences less than θ^e strictly prefer small houses in community 2 implying that demand is at least equal to $F(\theta^e)$. Supply, however, is equal to $(1-d)F(\theta^e)$. ■

We can now complete the proof of the Proposition. Consider the payoff of a household of type 0 under the deviation. As the household with the lowest preference for large houses, this household can expect to remain in small houses in community 2 for as long as it remains in the area. Thus, given that for all $t \geq 1$, $P_{S2t} = C_S$ and $\lambda_2(\mathbf{O}_t, \mathbf{Z}_t) = 0$, we have that

$$V_0(\mathbf{O}_0, (1,1)) = y + B(g^e) - cg^e - P_{S20} + \delta[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}].$$

This household will favor imposing zoning if

$$(1-d)P_{S20} + \mu V_0(\mathbf{O}_0, (1,1)) + (1-\mu)\frac{y}{1-\delta} - \left[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}\right] > 0.$$

This difference equals

$$(1-\mu-d)\left[P_{S20} - C_S\right],$$

which is positive given that $1-\mu > d$. It follows that households of type 0 are in favor of imposing zoning.

Now consider households of type $\theta \in (0, \theta^e]$. As noted, the continuation payoff for these residents if zoning is not implemented is exactly the same as for a type 0 household. On the other hand, since a type θ household can always make the same choices as a type 0 household, it must be the case that $V_\theta(\mathbf{O}_0, (1,1)) \geq V_0(\mathbf{O}_0, (1,1))$. It therefore follows that

$$\begin{aligned} & (1-d)P_{S20} + \mu V_\theta(\mathbf{O}_0, (1,1)) + (1-\mu)\frac{y}{1-\delta} - \left[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}\right] \\ \geq \quad & (1-\mu-d)\left[P_{S20} - C_S\right] > 0. \end{aligned}$$

Thus, households of type $\theta \in (0, \theta^e]$ also favor imposing zoning. ■

9.2 Proof of Proposition 4

We first describe the components of our proposed equilibrium and then argue that it is indeed an equilibrium. The new construction and price rules are given by

$$\mathbf{N}(\mathbf{O},\mathbf{Z}) = \begin{cases} (0,d/2,0,d/2) & \text{if } \mathbf{Z}=(0,0) \\ (0,0,0,d) & \text{if } \mathbf{Z}=(1,0) \\ (0,d,0,0) & \text{if } \mathbf{Z}=(0,1) \\ (d/2,0,d/2,0) & \text{if } \mathbf{Z}=(1,1) \end{cases},$$

and

$$\mathbf{P}(\mathbf{O},\mathbf{Z}) = \begin{cases} (C_S,C_S,C_S,C_S) & \text{if } \mathbf{Z}\neq(1,1) \\ (C_L,C_L,C_L,C_L) & \text{if } \mathbf{Z}=(1,1) \end{cases}.$$

For each community i, the public service rule is $g_i(\mathbf{O},\mathbf{Z}) = g^e$ and the tax rule is

$$\tau_i(\mathbf{O},\mathbf{Z}) = \begin{cases} \frac{cg^e}{C_S} & \text{if } \mathbf{Z}\neq(1,1) \\ \frac{cg^e}{C_L} & \text{if } \mathbf{Z}=(1,1) \end{cases}.$$

The zoning rules are $\mathbf{Z}'(\mathbf{O},\mathbf{Z}) = (1,1)$.

The household value function is

$$V_0(\mathbf{O},\mathbf{Z}) = \begin{cases} y + B(g^e) - cg^e - C_S + \delta[(1-d)C_L + \mu V_0 + (1-\mu)\frac{y}{1-\delta}] & \text{if } \mathbf{Z}\neq(1,1) \\ y + B(g^e) - cg^e - C_L + \delta[(1-d)C_L + \mu V_0 + (1-\mu)\frac{y}{1-\delta}] & \text{if } \mathbf{Z}=(1,1) \end{cases}$$

where

$$V_0 = \frac{y + B(g^e) - cg^e - C_L + \delta[(1-d)C_L + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

The housing selection functions are $\boldsymbol{\xi}(0,\mathbf{O},\mathbf{Z}) = \mathbf{O} + \mathbf{N}(\mathbf{O},\mathbf{Z})$.

To establish that our proposed equilibrium is indeed an equilibrium we need to verify the three requirements of equilibrium: household optimization, housing market equilibrium, and majority rule. Household optimization is satisfied because all houses cost the same and, because $\theta = 0$, households are indifferent between the available choices. Housing market equilibrium is satisfied by construction. All new houses will be small unless both communities are zoning. Given that all housing prices are the same, it does not matter where the new construction takes place. The public service rules satisfy majority rule because house prices are all the same and hence all households face a tax price of c. They therefore demand a level of services $g^*(c) = g^e$. Given (10), this implies that the tax rules are as described.

It remains to check that the zoning decisions are optimal. We show only that the residents of community 1 will support imposing zoning. The argument for community 2 is similar. Note from (23) that a household with a type H house in community 1 will support imposing zoning when community 2 is imposing zoning if

$$(1-d)\left[P_{H1}(\mathbf{O}',(1,1)) - P_{H1}(\mathbf{O}',(0,1))\right] > \mu\left[V_0(\mathbf{O}',(0,1)) - V_0(\mathbf{O}',(1,1))\right],$$

where $\mathbf{O}' = (1-d)\left[\mathbf{O} + \mathbf{N}(\mathbf{O},\mathbf{Z})\right]$. But we have that

$$P_{H1}(\mathbf{O}',(1,1)) - P_{H1}(\mathbf{O}',(0,1)) = C_L - C_S$$

and that

$$V_0(\mathbf{O}',(0,1)) - V_0(\mathbf{O}',(1,1)) = C_L - C_S.$$

Thus, since $1 - \mu > d$, all households in community 1 will support imposing zoning as required. ■

9.3 Proof of Proposition 5

The task is to construct an equilibrium with endogenous zoning in which both communities always impose zoning. We first describe the components of the equilibrium and then show the zoning decisions are optimal.

9.3.1 The components of the equilibrium

Policy rules The public service rules in our proposed equilibrium are $(g_1(\mathbf{O},\mathbf{Z}), g_2(\mathbf{O},\mathbf{Z})) = (\overline{g},\overline{g})$ and the tax rules are given by (10). The zoning rules are $\mathbf{Z}'(\mathbf{O},\mathbf{Z}) = (1,1)$.

New construction and prices For any initial stock of old houses $\mathbf{O}$ there are four possible pairs of zoning regulations.

Case 1: $\mathbf{Z} = (1,1)$

In this case, we distinguish two sub-cases. The first is when $O_{L1} + O_{L2} + d \geq \zeta$ so that post-construction there are more large houses than there are high types. In this sub-case, we assume that $\mathbf{N}(\mathbf{O},(1,1)) = (d/2, 0, d/2, 0)$ and that $\mathbf{P}(\mathbf{O},(1,1)) = (C_L, C_L, C_L, C_L)$. Because there are more large houses than there are high types, the marginal buyer is a low type and is indifferent between small and large houses. This indifference reflects not only preferences but also the fact that, under this price rule, on the equilibrium path the price of all houses will equal C_L in the next period. As a consequence, it is consistent with housing market equilibrium to have the price of all houses equal to C_L in the current period. Given these uniform prices, households are indifferent between communities and hence new construction can be allocated evenly across communities.

The second sub-case is when $O_{L1} + O_{L2} + d < \zeta$ so that post-construction there are less large houses than there are high types in which case the marginal buyer is a high type. In this sub-case, the price of small houses must be lower than that of large houses to induce high types to buy. Given this price differential, new construction of large homes will take place in the community with the largest fraction of large homes. We assume that new construction is concentrated in community 1 unless community 2 has a larger fraction of large houses; that is,

$$\mathbf{N}(\mathbf{O},(1,1)) = \begin{cases} (d,0,0,0) & \text{if } \frac{O_{L1}}{O_{L1}+O_{S1}} \geq \frac{O_{L2}}{O_{L2}+O_{S2}} \\ (0,0,d,0) & \text{if } \frac{O_{L1}}{O_{L1}+O_{S1}} < \frac{O_{L2}}{O_{L2}+O_{S2}} \end{cases}.$$

Constructing the prices for this sub-case is more involved. Let $t^*(\mathbf{O})$ be the number of periods it takes for the post-construction stock of large houses to exceed ζ assuming that all new construction is of large houses. Formally, $t^*(\mathbf{O})$ is the smallest t such that

$$(1-d)^t(O_{L1} + O_{L2}) + \sum_{\tau=0}^{t}(1-d)^\tau d \geq \zeta.$$

If both communities impose zoning in all future periods, in $t^*(\mathbf{O})$ periods time the post-construction stock of large houses will exceed the fraction of high types. Now define the sequence of prices $(P_{L1t}(\mathbf{O}), P_{S1t}(\mathbf{O}), P_{L2t}(\mathbf{O}), P_{S2t}(\mathbf{O}))_{t=0}^{t^*(\mathbf{O})}$ as follows.

If $O_{L1}/(O_{L1} + O_{S1})$ is at least as large as $O_{L2}/(O_{L2} + O_{S2})$, let $P_{L1t}(\mathbf{O}) = C_L$ for all t and let $(P_{S1t}(\mathbf{O}), P_{L2t}(\mathbf{O}), P_{S2t}(\mathbf{O}))_{t=0}^{t^*(\mathbf{O})}$ be the solution to the system of difference equations

$$\overline{\theta} + \delta(1-d)(C_L - P_{S1t+1}) = (1+\tau_{1t})[C_L - P_{S1t}]$$

$$\overline{\theta} + \delta(1-d)(P_{L2t+1} - P_{S2t+1}) = (1+\tau_{2t})[P_{L2t} - P_{S2t}]$$

$$(1+\tau_{1t})C_L - \delta(1-d)C_L = (1+\tau_{2t})P_{L2t} - \delta(1-d)P_{L2t+1}$$

with end point condition $(P_{S1t^*(\mathbf{O})}, P_{L2t^*(\mathbf{O})}, P_{S2t^*(\mathbf{O})}) = (C_L, C_L, C_L)$, where the tax rates in the above system are given by

$$(\tau_{1t}, \tau_{2t}) = (\frac{c\overline{g}}{C_L\lambda_{1t} + P_{S1t}(1-\lambda_{1t})}, \frac{c\overline{g}}{P_{L2t}\lambda_{2t} + P_{S2t}(1-\lambda_{2t})})$$

with

$$(\lambda_{1t}, \lambda_{2t}) = (\frac{(1-d)^t O_{L1} + \sum_{\tau=0}^{t}(1-d)^{\tau} d}{(1-d)^t O_{S1} + (1-d)^t O_{L1} + \sum_{\tau=0}^{t}(1-d)^{\tau} d}, \frac{O_{L2}}{O_{L2}+O_{S2}}).$$

Intuitively, these prices represent the future path of prices up to and including the first period in which prices equal C_L assuming that both communities impose zoning. The first equation says that high types are indifferent between large and small houses in community 1. The second equation says the same thing for community 2 and the third equation says that high types are indifferent between buying large houses in the two communities. These equations imply that low types prefer small houses and also that they are indifferent between communities.

If $O_{L1}/(O_{L1}+O_{S1})$ is less than $O_{L2}/(O_{L2}+O_{S2})$, let $P_{L2t}(\mathbf{O}) = C_L$ for all t and let $(P_{L1t}(\mathbf{O}), P_{S1t}(\mathbf{O}), P_{S2t}(\mathbf{O}))_{t=0}^{t^*(\mathbf{O})}$ be the solution to the system of difference equations

$$\overline{\theta} + \delta(1-d)(P_{L1t+1} - P_{S1t+1}) = (1+\tau_{1t})[P_{L1t} - P_{S1t}]$$

$$\overline{\theta} + \delta(1-d)(C_L - P_{S2t+1}) = (1+\tau_{2t})[P_{L2t} - P_{S2t}]$$

$$(1+\tau_{1t})P_{L1t} - \delta(1-d)P_{L1t+1} = (1+\tau_{2t})C_L - \delta(1-d)C_L$$

with end point condition $(P_{L1t^*(\mathbf{O})}, P_{S1t^*(\mathbf{O})}, P_{S2t^*(\mathbf{O})}) = (C_L, C_L, C_L)$, where the tax rates in the above system are given by

$$(\tau_{1t}, \tau_{2t}) = (\frac{c\overline{g}}{C_L\lambda_{1t} + P_{S1t}(1-\lambda_{1t})}, \frac{c\overline{g}}{P_{L2t}\lambda_{2t} + P_{S2t}(1-\lambda_{2t})}),$$

with

$$(\lambda_{1t}, \lambda_{2t}) = (\frac{O_{L1}}{O_{L1}+O_{S1}}, \frac{(1-d)^t O_{L2} + \sum_{\tau=0}^{t}(1-d)^{\tau} d}{(1-d)^t O_{S2} + (1-d)^t O_{L2} + \sum_{\tau=0}^{t}(1-d)^{\tau} d}).$$

Given these price sequences, we set

$$\mathbf{P}(\mathbf{O}, (1,1)) = (P_{L10}(\mathbf{O}), P_{S10}(\mathbf{O}), P_{L20}(\mathbf{O}), P_{S20}(\mathbf{O})).$$

This reflects the fact that both communities will continue to impose zoning on the equilibrium path.

Case 2: $\mathbf{Z} = (1,0)$

There are three sub-cases to consider. The first is when $O_{L2} + O_{L1} \geq \zeta$ so that the stock of old large houses is larger than the number of high types. In this case, all new construction takes place in community 2 and it is all in small houses; i.e., $\mathbf{N}(\mathbf{O},(1,0)) = (0,0,0,d)$. Moreover, prices are all equal to C_S; i.e., $\mathbf{P}(\mathbf{O},(1,0)) = (C_S, C_S, C_S, C_S)$. This reflects the fact that the marginal buyer is a low type who is indifferent between large and small houses. This indifference reflects not only preferences but also the fact that, because $(1-d)(O_{L2}+O_{L1})+d \geq \zeta$, on the equilibrium path the price of all houses will equal C_L in the next period.

The second sub-case is when $O_{L2}+O_{L1}+d \leq \zeta$ so that even if all new construction is in the form of large houses, the stock of houses post-construction can not exceed the number of high types. In this case, if condition (29) is satisfied, things are exactly the same as with both communities zoning; that is, all new construction is in the form of large houses and is located in the community with the largest fraction of large homes. This means that $\mathbf{N}(\mathbf{O},(1,0)) = \mathbf{N}(\mathbf{O},(1,1))$ and that $\mathbf{P}(\mathbf{O},(1,0)) = \mathbf{P}(\mathbf{O},(1,1))$. Intuitively, condition (29) guarantees that high types strictly prefer large to small houses and hence new construction will respond to their demands.

The third sub-case is when $O_{L2}+O_{L1} < \zeta < O_{L2}+O_{L1}+d$. In this case, we get sufficient new construction of large houses to meet the demands of high types and the remaining new construction in the form of small houses. Specifically, we assume

$$\mathbf{N}(\mathbf{O},(1,0)) = \begin{cases} (\zeta - O_{L2} - O_{L1}, 0, 0, O_{L2}+O_{L1}+d-\zeta) & \text{if } \frac{O_{L1}}{O_{L1}+O_{S1}} \geq \frac{O_{L2}}{O_{L2}+O_{S2}+O_{L2}+O_{L1}+d-\zeta} \\ (0, 0, \zeta - O_{L2} - O_{L1}, O_{L2}+O_{L1}+d-\zeta) & \text{if } \frac{O_{L1}}{O_{L1}+O_{S1}} < \frac{O_{L2}}{O_{L2}+O_{S2}+O_{L2}+O_{L1}+d-\zeta} \end{cases}$$

Thus, new construction of small houses takes place in community 2 and new construction of large houses takes place in which ever community has the largest fraction of large houses after the new small houses are accounted for.

Turning to prices, if $\frac{O_{L1}}{O_{L1}+O_{S1}} \geq \frac{O_{L2}}{O_{L2}+O_{S2}+O_{L2}+O_{L1}+d-\zeta}$ then $\mathbf{P}(\mathbf{O},(1,0)) = (C_L, P_{S1}, P_{L2}, C_S)$ where the prices (P_{S1}, P_{L2}) are such as to make small homeowners indifferent between buying in community 1 or community 2 and large homeowners indifferent between buying in community 1 or community 2. Given that prices of all houses in the next period on the equilibrium path will be C_L, this requires that

$$(1+\tau_1)P_{S1} = (1+\tau_2)C_S \;\; \& \;\; (1+\tau_1)C_L = (1+\tau_2)P_{L2},$$

where the tax rates in these expressions are given by

$$(\tau_1, \tau_2) = (\frac{c\overline{g}}{C_L\lambda_1 + P_{S1}(1-\lambda_1)}, \frac{c\overline{g}}{P_{L2}\lambda_2 + C_S(1-\lambda_2)})$$

with

$$(\lambda_1, \lambda_2) = (\frac{\zeta - O_{L2}}{\zeta - O_{L2} + O_{S1}}, \frac{O_{L2}}{O_{L2} + O_{S2} + O_{L2} + O_{L1} + d - \zeta}).$$

If $\frac{O_{L1}}{O_{L1}+O_{S1}} < \frac{O_{L2}}{O_{L2}+O_{S2}+O_{L2}+O_{L1}+d-\zeta}$ then $\mathbf{P}(\mathbf{O}, (1,0)) = (P_{L1}, P_{S1}, C_L, C_S)$ where the prices (P_{L1}, P_{S1}) are such as to make small homeowners indifferent between buying in community 1 or community 2 and large homeowners indifferent between buying in community 1 or community 2. This requires that

$$(1 + \tau_1)P_{S1} = (1 + \tau_2)C_S \;\;\&\;\; (1 + \tau_1)P_{L1} = (1 + \tau_2)C_L,$$

where the tax rates in these expressions are given by

$$(\tau_1, \tau_2) = (\frac{c\overline{g}}{P_{L1}\lambda_1 + P_{S1}(1 - \lambda_1)}, \frac{c\overline{g}}{C_L\lambda_2 + C_S(1 - \lambda_2)})$$

with

$$(\lambda_1, \lambda_2) = (\frac{O_{L1}}{O_{L1} + O_{S1}}, \frac{\zeta - O_{L1}}{O_{S2} + O_{L1} + d}).$$

Case 3: $\mathbf{Z} = (0, 1)$

This is just symmetric to the case in which $\mathbf{Z} = (1, 0)$.

Case 4: $\mathbf{Z} = (0, 0)$

This case is similar to that in which only one community is zoning, except that new construction can take place in both communities. The first sub-case is when $O_{L2} + O_{L1} \geq \zeta$ so that the stock of old large houses is larger than the number of high types. Then all new construction is in small houses and is divided uniformly across communities; i.e., $\mathbf{N}(\mathbf{O}, (0,0)) = (0, d/2, 0, d/2)$. Moreover, prices are all equal to C_S; i.e., $\mathbf{P}(\mathbf{O}, (0,0)) = (C_S, C_S, C_S, C_S)$. Again, this reflects the fact that the marginal buyer is a low type who is indifferent between large and small houses and also that on the equilibrium path all prices will equal C_L in the next period.

The second sub-case is when $O_{L2} + O_{L1} + d \leq \zeta$ so that even if all new construction is in the form of large houses, the stock of houses post-construction can not exceed the number of high types. In this case, if condition (29) is satisfied, things are exactly the same as with both communities zoning, so $\mathbf{N}(\mathbf{O}, (0,0)) = \mathbf{N}(\mathbf{O}, (1,1))$ and $\mathbf{P}(\mathbf{O}, (0,0)) = \mathbf{P}(\mathbf{O}, (1,1))$.

The third sub-case is when $O_{L2} + O_{L1} < \zeta < O_{L2} + O_{L1} + d$. In this case, we get sufficient new construction of large houses to meet the demands of high types and the remaining new construction in the form of small houses. Where this new construction takes place depends upon the initial allocation of old houses. Specifically, we assume that if it would result in community 1 having at least as large a fraction of large homes, then all new construction occurs in community 1.

Thus, if $\frac{\zeta - O_{L2}}{O_{L1}+O_{S1}+d} \geq \frac{O_{L2}}{O_{L2}+O_{S2}}$, $\mathbf{N}(\mathbf{O},(0,0)) = (\zeta - O_{L2} - O_{L1}, O_{L2}+O_{L1}+d-\zeta, 0, 0)$. Prices are given by $\mathbf{P}(\mathbf{O},(0,0)) = (C_L, C_S, P_{L2}, P_{S2})$ where (P_{L2}, P_{S2}) are such that small homeowners are indifferent between buying in community 1 or community 2 and large homeowners are indifferent between buying in community 1 or community 2; i.e.,

$$(1+\tau_1)C_S = (1+\tau_2)P_{S2} \ \ \& \ \ (1+\tau_1)C_L = (1+\tau_2)P_{L2}.$$

The tax rates in these expressions are given by

$$(\tau_1, \tau_2) = (\frac{c\overline{g}}{C_L\lambda_1 + C_S(1-\lambda_1)}, \frac{c\overline{g}}{P_{L2}\lambda_2 + P_{S2}(1-\lambda_2)})$$

with

$$(\lambda_1, \lambda_2) = (\frac{\zeta - O_{L2}}{O_{L1}+O_{S1}+d}, \frac{O_{L2}}{O_{L2}+O_{S2}}).$$

If $\frac{\zeta - O_{L2}}{O_{L1}+O_{S1}+d} < \frac{O_{L2}}{O_{L2}+O_{S2}}$ then if all new construction is allocated in community 1 this would result in a lower fraction of large houses in community 1 which would create incentives to build in community 2. There are two possibilities. If $\frac{\zeta - O_{L1}}{O_{L2}+O_{S2}+d} \geq \frac{O_{L1}}{O_{L1}+O_{S1}}$, then all new construction takes place in community 2. Thus, $\mathbf{N}(\mathbf{O},(0,0)) = (0, 0, \zeta - O_{L2} - O_{L1}, O_{L2} + O_{L1} + d - \zeta)$ and $\mathbf{P}(\mathbf{O},(0,0)) = (P_{L1}, P_{S1}, C_L, C_S)$ where (P_{L1}, P_{S1}) are such that small homeowners are indifferent between buying in community 1 or community 2 and large homeowners are indifferent between buying in community 1 or community 2; i.e.,

$$(1+\tau_1)P_{S1} = (1+\tau_2)C_S \ \ \& \ \ (1+\tau_1)P_{L1} = (1+\tau_2)C_L.$$

The tax rates in these expressions are given by

$$(\tau_1, \tau_2) = (\frac{c\overline{g}}{P_{L1}\lambda_1 + P_{S1}(1-\lambda_1)}, \frac{c\overline{g}}{C_L\lambda_2 + C_S(1-\lambda_2)})$$

with

$$(\lambda_1, \lambda_2) = (\frac{O_{L1}}{O_{L1}+O_{S1}}, \frac{\zeta - O_{L1}}{O_{S2}+O_{L2}+d}).$$

If $\frac{\zeta - O_{L1}}{O_{L2}+O_{S2}+d} < \frac{O_{L1}}{O_{L1}+O_{S1}}$ then putting all new construction in either community will not be consistent with equilibrium. In this case, we assume that new construction is allocated across the communities in such a way to equalize the fraction of large homes in each community. Thus,

$$\mathbf{N}(\mathbf{O},(0,0)) = (N_{L1}, N_{S1}, N_{L2}, N_{S1})$$

where

$$\frac{O_{L1}+N_{L1}}{O_{L1}+N_{L1}+O_{S1}+N_{S1}} = \frac{O_{L2}+N_{L2}}{O_{L2}+N_{L2}+O_{S2}+N_{S2}}$$

$$N_{L1} + N_{S1} + N_{L2} + N_{S1} = d$$

$$O_{L1} + N_{L1} + O_{L2} + N_{L2} = \zeta$$

Prices are $\mathbf{P}(\mathbf{O},(0,0)) = (C_L, C_S, C_L, C_S)$.

Household value functions We deal first with the case in which $\mathbf{Z} = (1,1)$. If the initial stock of old houses $\mathbf{O}$ is such that $O_{L2} + O_{L1} + d \geq \zeta$, then since on the equilibrium path zoning will be in place in all future periods, in that period and in all future periods, prices will equal C_L. Thus, we have that

$$V_0(\mathbf{O},(1,1)) = V_0 \equiv \frac{y + B(\overline{g}) - c\overline{g} - C_L(1-\delta(1-d)) + \delta(1-\mu)\frac{y}{1-\delta}}{1-\delta\mu},$$

and

$$V_{\overline{\theta}}(\mathbf{O},(1,1)) = V_{\overline{\theta}} \equiv \frac{y + \overline{\theta} + B(\overline{g}) - c\overline{g} - C_L(1-\delta(1-d)) + \delta(1-\mu)\frac{y}{1-\delta}}{1-\delta\mu}.$$

If the initial stock $\mathbf{O}$ is such that $O_{L2} + O_{L1} + d < \zeta$, then it will take $t^*(\mathbf{O})$ periods before the stock of large houses is such that all prices are C_L. In the current period prices are $(P_{L10}(\mathbf{O}), P_{S10}(\mathbf{O}), P_{L20}(\mathbf{O}), P_{S20}(\mathbf{O}))$, in the following period prices are $(P_{L11}(\mathbf{O}), P_{S11}(\mathbf{O}), P_{L21}(\mathbf{O}), P_{S21}(\mathbf{O}))$, etc. Thus, we have that

$$\begin{aligned} V_0(\mathbf{O},(1,1)) &= \sum_{t=0}^{t^*(\mathbf{O})-1} (\delta\mu)^t \left[y + B(\overline{g}) - \rho_{Sit}(\mathbf{O})\overline{g} - P_{Sit}(\mathbf{O}) + \delta(1-d)P_{Sit+1}(\mathbf{O})\right] + (\delta\mu)^{t^*(\mathbf{O})} V_0 \\ &+ \sum_{t=0}^{t^*(\mathbf{O})-1} (\delta\mu)^t \delta(1-\mu)\frac{y}{1-\delta} \end{aligned}$$

and

$$\begin{aligned} V_{\overline{\theta}}(\mathbf{O},(1,1)) &= \sum_{t=0}^{t^*(\mathbf{O})-1} (\delta\mu)^t \left[y + \overline{\theta} + B(\overline{g}) - \rho_{Lit}(\mathbf{O})\overline{g} - P_{Lit}(\mathbf{O}) + \delta(1-d)P_{Lit+1}(\mathbf{O})\right] + (\delta\mu)^{t^*(\mathbf{O})} V_{\overline{\theta}} \\ &+ \sum_{t=0}^{t^*(\mathbf{O})-1} (\delta\mu)^t \delta(1-\mu)\frac{y}{1-\delta}. \end{aligned}$$

For zoning decisions not on the equilibrium path $\mathbf{Z} \neq (1,1)$ we have that:

$$\begin{aligned} V_0(\mathbf{O},\mathbf{Z}) &= y + B(\overline{g}) - \rho_{Si}(\mathbf{O},\mathbf{Z})\overline{g} - P_{Si}(\mathbf{O},\mathbf{Z}) + \delta(1-d)P_{Si0}((1-d)(\mathbf{O}+\mathbf{N}(\mathbf{O},\mathbf{Z}))) \\ &+\delta[\mu V_0((1-d)(\mathbf{O}+\mathbf{N}(\mathbf{O},\mathbf{Z})),(1,1)) + (1-\mu)\frac{y}{1-\delta}] \end{aligned}$$

and

$$V_{\overline{\theta}}(\mathbf{O},\mathbf{Z}) = y+\overline{\theta}+B(\overline{g})-\rho_{Li}(\mathbf{O},\mathbf{Z})\overline{g}-P_{Li}(\mathbf{O},\mathbf{Z})+\delta(1-d)P_{Li0}((1-d)(\mathbf{O}+\mathbf{N}(\mathbf{O},\mathbf{Z})))$$
$$+\delta[\mu V_{\overline{\theta}}((1-d)(\mathbf{O}+\mathbf{N}(\mathbf{O},\mathbf{Z})),(1,1))+(1-\mu)\frac{y}{1-\delta}].$$

Thus, we have a temporary departure from the equilibrium path and then return to it with a possibly different initial stock of old houses.

Household demand and housing selection functions If $O_{L1}+O_{L2}+N_{L1}(\mathbf{O},\mathbf{Z})+N_{L2}(\mathbf{O},\mathbf{Z})<\zeta$, then we have that $\alpha_0(\mathbf{O},\mathbf{Z})=\{S1,S2\}$ and $\alpha_{\overline{\theta}}(\mathbf{O},\mathbf{Z})=\{L1,S1,L2,S2\}$. If $O_{L1}+O_{L2}+N_{L1}(\mathbf{O},\mathbf{Z})+N_{L2}(\mathbf{O},\mathbf{Z})\geq\zeta$, then we have that $\alpha_0(\mathbf{O},\mathbf{Z})=\{L1,S1,L2,S2\}$ and $\alpha_{\overline{\theta}}(\mathbf{O},\mathbf{Z})=\{L1,L2\}$.

If $O_{L1}+O_{L2}+N_{L1}(\mathbf{O},\mathbf{Z})+N_{L2}(\mathbf{O},\mathbf{Z})\geq\zeta$, the housing selection functions for high types are

$$\begin{array}{l}(\xi_{L1}(\overline{\theta},\mathbf{O},\mathbf{Z}),\xi_{S1}(\overline{\theta},\mathbf{O},\mathbf{Z}))=(\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})},0)\\(\xi_{L2}(\overline{\theta},\mathbf{O},\mathbf{Z}),\xi_{S2}(\overline{\theta},\mathbf{O},\mathbf{Z}))=(\frac{O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})},0)\end{array},\tag{35}$$

and for low types are

$$\begin{array}{l}(\xi_{L1}(0,\mathbf{O},\mathbf{Z}),\xi_{S1}(0,\mathbf{O},\mathbf{Z}))=(\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})}{1-\zeta}\right]\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}\right],\left[\frac{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})}{1-\zeta}\right])\\(\xi_{L2}(0,\mathbf{O},\mathbf{Z}),\xi_{S2}(0,\mathbf{O},\mathbf{Z}))=(\left[\frac{O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}{1-\zeta}\right]\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}\right],\left[\frac{O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})}{1-\zeta}\right])\end{array}.\tag{36}$$

If $O_{L1}+O_{L2}+N_{L1}(\mathbf{O},\mathbf{Z})+N_{L2}(\mathbf{O},\mathbf{Z})<\zeta$, the housing selection functions for high types are

$$\begin{array}{l}(\xi_{L1}(\overline{\theta},\mathbf{O},\mathbf{Z}),\xi_{S1}(\overline{\theta},\mathbf{O},\mathbf{Z}))=(\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})}{\zeta}\right],\left[\frac{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})}{\zeta}\right]\left[\frac{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})-(1-\zeta)}{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})}\right])\\(\xi_{L2}(\overline{\theta},\mathbf{O},\mathbf{Z}),\xi_{S2}(\overline{\theta},\mathbf{O},\mathbf{Z}))=(\left[\frac{O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}{\zeta}\right],\left[\frac{O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})}{\zeta}\right]\left[\frac{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})-(1-\zeta)}{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})}\right])\end{array},\tag{37}$$

and for low types are

$$\begin{array}{l}(\xi_{L1}(0,\mathbf{O},\mathbf{Z}),\xi_{S1}(0,\mathbf{O},\mathbf{Z}))=(0,\frac{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})}{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})})\\(\xi_{L2}(0,\mathbf{O},\mathbf{Z}),\xi_{S2}(0,\mathbf{O},\mathbf{Z}))=(0,\frac{O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})}{O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})+O_{S2}+N_{S2}(\mathbf{O},\mathbf{Z})})\end{array}.\tag{38}$$

9.3.2 The zoning decisions are optimal

Having specified all the components of the equilibrium, it remains to show that $\mathbf{Z}'(\mathbf{O},\mathbf{Z})=(1,1)$ is consistent with equilibrium. We show that community 1 will support zoning if it expects community 2 to impose zoning. The proof of the converse is similar. Let $(\mathbf{O}, \mathbf{Z})$ be given and note from (23) that a type θ household with a type H house in community 1 will support imposing zoning when community 2 is imposing zoning if

$$(1-d)\left[P_{H1}(\mathbf{O}', (1,1)) - P_{H1}(\mathbf{O}', (0,1))\right] > \mu\left[V_\theta(\mathbf{O}', (0,1)) - V_\theta(\mathbf{O}', (1,1))\right], \tag{39}$$

where $\mathbf{O}' = (1-d)(\mathbf{O} + \mathbf{N}(\mathbf{O}, \mathbf{Z}))$.

Suppose first that $O'_{L1} + O'_{L2} \geq \zeta$. In this case, we know that $O_{L1} + N_{L1}(\mathbf{O}, \mathbf{Z}) + O_{L2} + N_{L2}(\mathbf{O}, \mathbf{Z})$ exceeds ζ and hence, at the time of voting, all high types own large homes while low types own small and large homes. We also know that both $P_{S1}(\mathbf{O}', (1,1))$ and $P_{L1}(\mathbf{O}', (1,1))$ equal C_L and that both $P_{S1}(\mathbf{O}', (0,1))$ and $P_{L1}(\mathbf{O}', (0,1))$ equal C_S. Thus,

$$P_{H1}(\mathbf{O}', (1,1)) - P_{H1}(\mathbf{O}', (0,1)) = C_L - C_S. \tag{40}$$

Further, we have that $V_0(\mathbf{O}', (1,1)) = V_0$ and that $V_{\overline{\theta}}(\mathbf{O}', (1,1)) = V_{\overline{\theta}}$. In addition, since it is the case that

$$(1-d)[O'_{L1} + O'_{L2} + N_{L1}(\mathbf{O}', (0,1)) + N_{L2}(\mathbf{O}', (0,1))] + d \geq \zeta,$$

we have that

$$V_0(\mathbf{O}', (0,1)) = y + B(\overline{g}) - c\overline{g} - C_S + \delta(1-d)C_L + \delta[\mu V_0 + (1-\mu)\frac{y}{1-\delta}]$$

and that

$$V_{\overline{\theta}}(\mathbf{O}', (0,1)) = y + \overline{\theta} + B(\overline{g}) - c\overline{g} - C_S + \delta(1-d)C_L + \delta[\mu V_{\overline{\theta}} + (1-\mu)\frac{y}{1-\delta}].$$

This means that for $\theta \in \{0, \overline{\theta}\}$

$$V_\theta(\mathbf{O}', (0,1)) - V_\theta(\mathbf{O}', (1,1)) = C_L - C_S. \tag{41}$$

Since $1 - \mu > d$, (40) and (41) imply that (39) holds for both types of households, whatever type of house they own. Intuitively, community 1 removing zoning in this range will simply result in a one period reduction in the price of all houses from C_L to C_S. Because prices remain uniform, there will be no change in any household's tax price of services.

Next suppose that $O'_{L1} + O'_{L2} + d \leq \zeta$. In this case, relaxing zoning has no effect on prices or new construction, so accordingly, all residents of community 1 favor keeping it in place.

Finally, suppose that $O'_{L1} + O'_{L2} < \zeta < O'_{L1} + O'_{L2} + d$. In this case, it could be that $O_{L1} + N_{L1}(\mathbf{O}, \mathbf{Z}) + O_{L2} + N_{L2}(\mathbf{O}, \mathbf{Z})$ exceeds ζ and hence, at the time of voting, all high types own large homes while low types own small and large homes. It could also be that $O_{L1} + N_{L1}(\mathbf{O}, \mathbf{Z}) + O_{L2} + N_{L2}(\mathbf{O}, \mathbf{Z})$ is less than ζ, so that all low types own small houses and high types own both large and small homes. We also know that both $P_{S1}(\mathbf{O'}, (1,1))$ and $P_{L1}(\mathbf{O'}, (1,1))$ equal C_L and that $P_{S1}(\mathbf{O'}, (0,1))$ equals C_S. In addition, we know that $P_{L1}(\mathbf{O'}, (0,1)) \leq C_L$. Thus,

$$P_{S1}(\mathbf{O'}, (1,1)) - P_{S1}(\mathbf{O'}, (0,1)) = C_L - C_S \tag{42}$$

and

$$P_{L1}(\mathbf{O'}, (1,1)) - P_{L1}(\mathbf{O'}, (0,1)) = C_L - P_{L1}(\mathbf{O'}, (0,1)) \geq 0. \tag{43}$$

Turning to the value functions, we again have that $V_0(\mathbf{O'}, (1,1)) = V_0$ and that $V_{\overline{\theta}}(\mathbf{O'}, (1,1)) = V_{\overline{\theta}}$. In addition, we have that

$$V_0(\mathbf{O'}, (0,1)) = y + B(\overline{g}) - \rho_{S1}(\mathbf{O'}, (0,1))\overline{g} - C_S + \delta(1-d)C_L + \delta[\mu V_0 + (1-\mu)\frac{y}{1-\delta}],$$

and that

$$V_{\overline{\theta}}(\mathbf{O'}, (0,1)) = y + \overline{\theta} + B(\overline{g}) - \rho_{L1}(\mathbf{O'}, (0,1))\overline{g} - P_{L1}(\mathbf{O'}, (0,1)) + \delta(1-d)C_L + \delta[\mu V_{\overline{\theta}} + (1-\mu)\frac{y}{1-\delta}].$$

Thus, we have that

$$V_0(\mathbf{O'}, (0,1)) - V_0(\mathbf{O'}, (1,1)) = (c - \rho_{S1}(\mathbf{O'}, (0,1)))\overline{g} + C_L - C_S, \tag{44}$$

and that

$$V_{\overline{\theta}}(\mathbf{O'}, (0,1)) - V_{\overline{\theta}}(\mathbf{O'}, (1,1)) = (c - \rho_{L1}(\mathbf{O'}, (0,1)))\overline{g} + C_L - P_{L1}(\mathbf{O'}, (0,1)). \tag{45}$$

To show that (39) holds for low types who own small homes, we must demonstrate that

$$(1 - \mu - d)(C_L - C_S) > \mu(c - \rho_{S1}(\mathbf{O'}, (0,1)))\overline{g}.$$

Intuitively, the impact of removing zoning is going to be a one period reduction in the price of small homes from C_L to C_S. The relative price of small homes must fall and hence the tax price of services for small home owners could rise. The cost of the housing price decrease must exceed any benefit from a lower tax price of services.

We know that

$$\rho_{S1}(\mathbf{O'},(0,1)) = \frac{cC_S}{P_{L1}(\mathbf{O'},(0,1))\lambda_1(\mathbf{O'},(0,1)) + C_S(1-\lambda_1(\mathbf{O'},(0,1)))}$$

and thus

$$c - \rho_{S1}(\mathbf{O'},(0,1)) = c\left[\frac{\lambda_1(\mathbf{O'},(0,1))[P_{L1}(\mathbf{O'},(0,1)) - C_S]}{P_{L1}(\mathbf{O'},(0,1))\lambda_1(\mathbf{O'},(0,1)) + C_S(1-\lambda_1(\mathbf{O'},(0,1)))}\right].$$

In order for low types with small homes to want to keep zoning we require that

$$(1-\mu-d)(C_L - C_S) > \mu c\overline{g}\left[\frac{\lambda_1(\cdot)[P_{L1}(\cdot) - C_S]}{P_{L1}(\cdot)\lambda_1(\cdot) + C_S(1-\lambda_1(\cdot))}\right].$$

Note that the right hand side is increasing in $P_{L1}(\cdot)$ and $\lambda_1(\cdot)$ and hence is less than

$$\mu c\overline{g}\left[\frac{C_L - C_S}{C_L}\right].$$

Thus, it suffices to show that

$$(1-\mu-d)(C_L - C_S) > \mu c\overline{g}\left[\frac{C_L - C_S}{C_L}\right].$$

This follows from (30).

To show that (39) holds for high types who own large houses, we need to show that

$$(1-\mu-d)(C_L - P_{L1}(\mathbf{O'},(0,1))) > \mu(c - \rho_{L1}(\mathbf{O'},(0,1)))\overline{g}.$$

But we know that

$$\rho_{L1}(\mathbf{O'},(0,1)) = \frac{cP_{L1}(\mathbf{O'},(0,1))}{P_{L1}(\mathbf{O'},(0,1))\lambda_1(\mathbf{O'},(0,1)) + C_S(1-\lambda_1(\mathbf{O'},(0,1)))} > c$$

and hence this follows from the assumption that $1-\mu > d$. Intuitively, for high types owning large homes, removing zoning both reduces the value of their homes and raises the tax price of services. To show that (39) holds for high types who own small houses, we need to show that

$$(1-d)(C_L - C_S) > \mu\left[(c - \rho_{L1}(\mathbf{O'},(0,1)))\overline{g} + C_L - P_{L1}(\mathbf{O'},(0,1))\right].$$

But this follows immediately since $C_S < P_{L1}(\mathbf{O'},(0,1))$. This formula reflects the fact that high types who own small homes will, with or without zoning, own large homes next period.

The only remaining group is low types who own large homes. This group may indeed prefer to relax zoning. We may assume that, with or without zoning, households in this group sell their large homes and buy small homes in the next period. Relaxing zoning may decrease the value of their homes somewhat but may reduce their tax price of services by a greater amount. However, we claim that this group must be a minority. This requires showing that

$$\xi_{L1}(0,\mathbf{O},\mathbf{Z})(1-\zeta) < \xi_{L1}(\overline{\theta},\mathbf{O},\mathbf{Z})\zeta + \xi_{S1}(0,\mathbf{O},\mathbf{Z})(1-\zeta).$$

If low types own large homes it must be the case that $O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})$ exceeds ζ. From the selection rules (35) and (36), we know that

$$\xi_{L1}(0,\mathbf{O},\mathbf{Z})(1-\zeta) = [O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})]\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}\right]$$

and that

$$\xi_{L1}(\overline{\theta},\mathbf{O},\mathbf{Z})\zeta+\xi_{S1}(0,\mathbf{O},\mathbf{Z})(1-\zeta) = (\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})})\zeta+O_{S1}+N_{S1}(\mathbf{O},\mathbf{Z})$$

Thus, it suffices to show that

$$(\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})})\zeta \geq [O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})]\left[\frac{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta}{O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})}\right]$$

which is equivalent to

$$\zeta \geq O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta.$$

But we know that

$$O'_{L1}+O'_{L2} = (1-d)\left[O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})\right] < \zeta$$

which implies that

$$O_{L1}+N_{L1}(\mathbf{O},\mathbf{Z})+O_{L2}+N_{L2}(\mathbf{O},\mathbf{Z})-\zeta < \zeta(\frac{d}{1-d}).$$

Thus, all we need is that $d < 1/2$ which holds by assumption. This completes the proof. ■

9.4 Extension of Proposition 3 to three communities

Note first there is a sense in which Proposition 3 is trivially true when we have three communities. Take an efficient steady state in which, say, communities 2 and 3 are unzoned and the small houses are allocated in community 2 so that $\mathbf{Z}^* = (1,0,0)$ and $\mathbf{O}^* = (1-d)(1-F(\theta^e),0,0,F(\theta^e),0,0)$. Consider for ε small and positive the stock $\mathbf{O} = (1-d)(1-F(\theta^e),0,0,F(\theta^e)-\varepsilon,\varepsilon,0)$. Thus, compared with $\mathbf{O}^*$, $\mathbf{O}$ features a small number of large houses in community 3. Define $\langle \mathbf{O}_t(\mathbf{O}^*,\mathbf{Z}^*),\mathbf{Z}_t(\mathbf{O}^*,\mathbf{Z}^*)\rangle_{t=0}^{\infty}$

in the usual way and assume that $\mathbf{Z}_t(\mathbf{O}^*, \mathbf{Z}^*) = \mathbf{Z}^*$ for all t. Then it cannot be the case that $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}^*, \mathbf{Z}^*) = \mathbf{O}^*$. This is because, by virtue of having a small fraction of large houses, community 3 is now a more attractive place to build new small houses than community 2. This means the fraction of small houses in community 3 will grow relative to that in community 2 and $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}^*, \mathbf{Z}^*) = (1-d)(1-F(\theta^e), 0, 0, 0, 0, F(\theta^e))$. It follows that $(\mathbf{O}^*, \mathbf{Z}^*)$ is not strongly locally stable in the sense defined earlier.

Nonetheless, from an efficiency perspective there is nothing troubling about this example. Whether in the long run all the small housing is located in community 2, community 3, or both, is immaterial. The difficulty is that the division of construction across the two unzoned communities is arbitrary and small differences between the two communities will force it in one or the other direction. To reflect this, we modify our definition of strong local stability. For any given zoning rules $\mathbf{Z}$, let $\Phi(\mathbf{Z})$ is the set of housing stocks $\mathbf{O}$ that would, given equilibrium play, be steady states if the zoning rules were fixed at $\mathbf{Z}$; i.e., $\Phi(\mathbf{Z}) = \{\mathbf{O}' \,|\, \mathbf{O}(\mathbf{O}', \mathbf{Z}) = \mathbf{O}'\}$. For example,

$$\Phi((1,0,0)) = \{\mathbf{O}' \,|\, \mathbf{O}' = (1-d)(1-F(\theta^e), 0, 0, F(\theta^e) - x, 0, x) \text{ for } x \in [0, F(\theta^e)]\}.$$

Then, we say that the steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ is strongly locally stable if there exists $\varepsilon > 0$ such that for any initial state $(\mathbf{O}, \mathbf{Z})$ with the property that $\|\mathbf{O} - \mathbf{O}^*\| < \varepsilon$ we have that $\mathbf{Z}_t(\mathbf{O}, \mathbf{Z}) = \mathbf{Z}^*$ for $t = 1, ..., \infty$ and $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}, \mathbf{Z}) \in \Phi(\mathbf{Z}^*)$. This definition reduces to our earlier one in the case in which there is a unique steady state associated with a particular set of zoning rules.

We now demonstrate that Proposition 3 holds with this new more general definition of strong local stability. Suppose, to the contrary, that there exists a political equilibrium with endogenous zoning with an equilibrium steady state $(\mathbf{O}^*, \mathbf{Z}^*)$ which was both efficient and strongly locally stable. The logic from Proposition 3 applies if two communities are zoning, since the unzoned community is a monopoly supplier of new small homes. So we can assume that only one community is zoning. Assume that community 1 is the community who zones so that $\mathbf{O}^* = (1-d)(1-F(\theta^e), 0, 0, F(\theta^e) - x, 0, x)$ for some $x \in [0, F(\theta^e)]$ and $\mathbf{Z}^* = (1,0,0)$. If $(\mathbf{O}^*, \mathbf{Z}^*)$ is strongly locally stable it must be the case that there exists $\varepsilon > 0$ such that for any initial state $(\mathbf{O}, \mathbf{Z})$ with the property that $\|\mathbf{O} - \mathbf{O}^*\| < \varepsilon$ we have that $\mathbf{Z}_t(\mathbf{O}, \mathbf{Z}) = \mathbf{Z}^*$ for $t = 1, ..., \infty$ and $\lim_{t\to\infty} \mathbf{O}_t(\mathbf{O}, \mathbf{Z}) \in \Phi(\mathbf{Z}^*)$.

Now consider for ε small and positive the stock

$$\mathbf{O} = (1-d)(1-F(\theta^e), 0, \varepsilon, F(\theta^e) - x - \varepsilon, 0, x).$$

We will show that for sufficiently small ε, $Z_2'(\mathbf{O},\mathbf{Z}^*) \neq 0$ which will contradict the assumption $(\mathbf{O}^*,\mathbf{Z}^*)$ is strongly locally stable.

Suppose first that the residents of community 2 follow the postulated equilibrium and do not impose zoning. As the efficient steady state is strongly locally stable, the future play of the equilibrium will have community 1 implementing zoning and communities 2 and 3 not. All new construction of small homes will occur in community 2 and the price of new small homes will be less than C_S in community 3. All new construction of large homes will occur in community 1 and the price of large homes in community 2 will be less than C_L. Let λ_{2t}^0 be the fraction of large homes in community 2 after $t = 0,....,\infty$ periods and let P_{L2t}^0 be the price of such houses.

Now suppose that the residents of community 2 were to deviate from the postulated equilibrium behavior by imposing zoning. By strong local stability, in all subsequent periods, households anticipate that zoning rules will return to steady state levels. In the period the deviation becomes effective, new construction of small homes will occur in community 3. However, the price of small homes in community 2 must be higher than those in community 3 because of the beneficial fiscal externality created by the presence of large homes. Let P_{S20}^1 be the price of small homes in community 2 in the period the deviation becomes effective. The value of large homes in community 2 will also be higher as will the fraction of large homes. Let P_{L20}^1 be the price of large homes in community 2 and λ_{20}^1 the fraction. Following the period of deviation, there will be a lower fraction of small homes in community 2 which will increase the price of large homes relative to the equilibrium. Let λ_{2t}^1 denote the fraction after t periods and let P_{L2t}^1 denote the price. The price of small homes in community 2 will return to C_S and the price of small homes in community 3 will be less than the construction cost.

Now consider the incentives to deviate for low types who own small homes in community 2. The payoff on the equilibrium path for a type θ who owns a small house in community 2 and will continue to live in a small house is

$$(1-d)C_S + \mu V_0^0 + (1-\mu)\frac{y}{1-\delta}.$$

The continuation value V_0^0 is the first element of the sequence $\left\langle V_t^0 \right\rangle_{t=0}^{\infty}$ defined inductively by

$$\begin{aligned} V_t^0 \quad = \quad & B(g^*(\frac{cC_S}{\lambda_{2t}^0 P_{L2t}^0 + (1-\lambda_{2t}^0)C_S})) - \frac{cC_S}{\lambda_{2t}^0 P_{L2t}^0 + (1-\lambda_{2t}^0)C_S} g^*(\frac{cC_S}{\lambda_{2t}^0 P_{L2t}^0 + (1-\lambda_{2t}^0)C_S}) - C_S \\ & + \delta[(1-d)C_S + \mu V_{t+1}^0 + (1-\mu)\frac{y}{1-\delta}] \end{aligned}$$

with end point condition

$$V_\infty^0 = \frac{B(g^*(c)) - cg^*(c) - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

The payoff under the deviation for a type θ who owns a small house in community 2 and will continue to live in a small house is

$$(1-d)P_{S20}^1 + \mu V_0^1 + (1-\mu)\frac{y}{1-\delta},$$

where

$$\begin{aligned} V_0^1 &= B(g^*(\frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1})) - \frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1} g^*(\frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1}) - P_{S20}^1 \\ &\quad + \delta[(1-d)C_S + \mu V_1^1 + (1-\mu)\frac{y}{1-\delta}]. \end{aligned}$$

The continuation value V_1^1 is the first element of the sequence $\langle V_t^1 \rangle_{t=1}^{\infty}$ defined inductively by

$$\begin{aligned} V_t^1 &= B(g^*(\frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S})) - \frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S} g^*(\frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S}) - C_S \\ &\quad + \delta[(1-d)C_S + \mu V_{t+1}^1 + (1-\mu)\frac{y}{1-\delta}] \end{aligned}$$

with end point condition

$$V_\infty^1 = \frac{B(g^*(c)) - cg^*(c) - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

The gain from deviating is

$$\Delta = (1-d)[P_{S20}^1 - C_S] + \mu\left[V_0^1 - V_0^0\right]$$

But we have that

$$V_0^1 - V_0^0 = S_0^1 - P_{S20}^1 + \delta\mu V_1^1 - \left[S_0^0 - C_S + \delta\mu V_1^0\right]$$

where S_t^0 and S_t^1 denote public service surplus on the equilibrium path and with the deviation. Thus, we have that

$$\Delta = (1-\mu-d)[P_{S20}^1 - C_S] + \mu\left[S_0^1 - S_0^0\right] + \delta\mu^2\left[V_1^1 - V_1^0\right]. \tag{46}$$

We now claim that

$$\mu\left[S_0^1 - S_0^0\right] \geq -\mu c g^*(\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S})\left(\frac{\left[P_{S20}^1 - C_S\right]\lambda_{20}^1 C_L}{C_S^2}\right). \tag{47}$$

To prove this, note that

$$g^*(\frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}) = \arg\max_g \left\{ B(g) - \frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}} g \right\}$$

and so

$$S^1_0 \geq B(g^*(\frac{cC_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S})) - \frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}} g^*(\frac{cC_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S}).$$

It follows that

$$\mu\left[S^1_0 - S^0_0\right] \geq \mu c g^*(\frac{cC_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S}) \left[\frac{cC_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S} - \frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right]$$

Moreover, since $P^1_{L20} \geq P^0_{L20}$, $P^1_{L20} \geq C_S$ and $\lambda^1_{20} > \lambda^0_{20}$, we have that

$$\begin{aligned} & \mu c g^*(\cdot)\left[\frac{C_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S} - \frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \\ \geq \quad & \mu c g^*(\cdot)\left[\frac{C_S}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})C_S} - \frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \end{aligned}$$

In addition,

$$\mu c g^*(\cdot)\left[\frac{C_S}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})C_S} - \frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \geq -\mu c g^*(\cdot)\left[\frac{\left(P^1_{S20}-C_S\right)\lambda^1_{20}P^1_{L20}}{C^2_S}\right].$$

Combining (46) and (47), we have

$$\Delta \geq (1-\mu-d)[P^1_{S20}-C_S] - \mu c g^*(\cdot)\left[\frac{\left(P^1_{S20}-C_S\right)\lambda^1_{20}P^1_{L20}}{C^2_S}\right] + \delta\mu^2\left[V^1_1 - V^0_1\right]. \qquad (48)$$

We also claim that $V^1_1 \geq V^0_1$. For this, it is enough to show that for all $t = 1, ..., \infty$, $S^1_t \geq S^0_t$. We have that

$$S^1_t \geq B(g^*(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S})) - \frac{cC_S}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S} g^*(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S}).$$

Thus,

$$S^1_t - S^0_t \geq \left[\frac{1}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S} - \frac{1}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S}\right] cC_S g^*(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S})$$

But we know that $\lambda^1_{2t} \geq \lambda^0_{2t}$, $P^1_{L2t} \geq P^0_{L2t}$ and $P^0_{L2t} \geq C_S$. Thus, we have that

$$\frac{1}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S} \geq \frac{1}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S},$$

which implies the result.

It now follows from (48) that

$$\Delta \geq [P^1_{S20}-C_S]\left((1-\mu-d) - \mu c g^*(\cdot)\left[\frac{\lambda^1_{20}P^1_{L20}}{C^2_S}\right]\right)$$

Since $\lambda^1_{20} \leq \varepsilon/\left(F(\theta^e)-x\right)$, this must be positive for sufficiently small ε given that $1-\mu > d$.

■