

Asymmetric Information, Firm Size and Promotion Policy ¹

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Abstract

This paper analyzes an economy in which firms cannot observe the ability of their employees upon hiring them. We augment the Waldman (1984a) framework by endogenising firm size and show that in an environment with free entry, in the only Strong Nash Equilibrium, each firm employs workers with different amounts of ability. These discrepancies lead to variance in firm size and a positive correlation between firm size and wages. We also show that the each firm's assignment policy is efficient; however, firms that employ workers with greater ability are above optimal size.

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1 Introduction

Why are similar firms not of the same size? Can we characterize an economy with no search friction and firms with identical (decreasing returns-to-scale) technology, in which we observe a non-degenerate size distribution of firms? Can we also obtain a positive correlation between firm size and wages in such economy?

In this paper we show that asymmetric information regarding employees' ability results in variety in firm size. We analyze an economy with no search friction and free entry of firms with the same technology. We assume that firms cannot observe their employees' ability upon hiring them and obtain two types of equilibria. In the first equilibrium, employees are equally distributed across firms; in the second, which is the only Strong Nash Equilibrium (an equilibrium that is stable against the deviation of any given coalition of players), individuals with similar ability join the same firm. The above result is in line with De Melo (2008), who finds a positive correlation between a worker's wage fixed effect and the average fixed effects of his co-workers. We also show that firms employing better employees are larger in equilibrium; hence, we obtain a positive correlation between firm size and wages.

A simple numerical example will illustrate this point. Consider a two-period economy, in which employees observe their own ability at the beginning of the first period. At the beginning of the second period, each firm observes her employees' abilities and decides which (if any) to promote in the second period (Note that a firm does not observe the ability of the employees employed by another firm). Also assume that employees' alternative wages equal their expected ability and that employees' abilities are drawn from a uniform distribution within the support $[0, 1]$.

The economy is composed of three firms. The first one promotes individuals with abilities $[0.9, 1]$, another firm promotes individuals with abilities $[0.8, 0.9]$ and a third promotes individuals with abilities $[0.7, 0.8]$. Each firm also employs non promoted individuals as raw labor.

Outside firms (who cannot observe the ability of each employee) use the fact that an individual was promoted in a given firm to form expectations regarding his ability. We obtain that the alternative wage of an individual promoted in the first firm is 0.95 (which

is the expected ability of the employees promoted in that firm), in the second firm 0.85 and in the third 0.75. Also assume that the wage of a non promoted individual is 0.5.

Consider an individual with ability 0.88; he cannot be promoted in the first firm (the first firm will not promote him at the second period) but his wage is higher in the second firm than in the third. Hence, in equilibrium, he joins the second firm.

Furthermore, under the assumption that the cost associated with hiring an employee is not a function of the employee's ability, we obtain that the cost "per ability" is lower in a firm that employs individuals with the higher ability. For example, if any individual needs an office and the cost per office is 0.1, then the cost per employee in the first firm is 1.05 (his wage plus the cost associated with hiring him) while the cost per ability is $1.05/0.95$, while the cost per ability in the second firm is $0.95/0.85$. Hence, a firm that employs individuals with higher ability also faces lower "ability cost" hence employs more "ability units". As a result a firm which employs better individuals is larger on equilibrium. Note that if promoted and non promoted individuals are complementary production function a firm that employs more ability units also employs more "raw labor". We therefore obtain a positive correlation between firm size and wages, Brown and Medoff (1989).

To illustrate, look at Bank of America and Commerce Bank. Previous explanations claim that Bank of America is larger than Commerce Bank due to different production functions or some friction. We offer a different explanation: We show that, in equilibrium, each bank enjoys a different "reputation" that allows it to employ individuals who differ in their abilities. As a result, the bank that employs better employees is also larger.

We also analyze promotion policies in the current paper. Previous research has mainly focus on cases of a firm employing one individual. In a seminal paper, Waldman (1984a) shows that a firm that employs only one manager has an inefficient promotion policy. Our paper contributes to the above literature by showing how the above result is affected by the number of the firm's employees.

We show that, each firm pays her marginal employee (i.e., the employee with the least ability) a wage that is higher than his product. The firm uses the promotion of such an employee to decrease the average ability and wages of her promoted workers. For example, consider the second firm from the above example, she promotes individuals with abilities $[0.8, 0.9]$ and pays a wage of 0.85 upon promotion. If she promotes individuals with abilities

[0.85,0.9] she would pay a wage of 0.875 to her promoted workers. Hence, by promoting additional individuals, the firm reducing the expected ability and the alternative wage of whole her promoted workers. Note that, the total wage paid to promoted workers is a function of their expected ability. Hence, the marginal cost of promoting another worker is a function of his expected ability and the promotion policy is efficient.

In the standard Nash equilibrium to the one-period game (Waldman 84a), the firm takes the outside wage as fixed for both promoted and non-promoted workers in choosing a promotion policy. In that case the firm will not take into account how the promotion policy affects wages and promotions will not be efficient. In the presented paper, the firm takes into account how the policy affects the wage of a promoted worker. Note that this result stems from the assumed ability of other firms to observe the proportion of promoted individuals. Under this assumption, each firm can use the promotion of workers with low ability to decrease the wage paid to her other promoted workers.

In this paper we combine two theoretical approaches to explain the size distribution of firms and the positive correlation between wage levels and firm size. We adopt the Waldman (1984a) structure, where promotion decisions are analyzed while taking into account the information revealed by a worker's level in the the hierarchy, together with the literature on hierarchies in firms. The main new result is the following: If, at the beginning of their working lives, employees observe their own abilities while firms do not, the only equilibrium that is a Strong Nash Equilibrium is the one in which employees with similar abilities join the same firm.

The paper is structured as follows: In Section 2 we provide a review of the related literature. Section 3 develops a benchmark in which neither the firm nor the employee observe the employee's ability. We use this setup to analyze the promotion policy. In Section 4 we assume that only employees observe their own ability and obtain the paper's main results of the paper. Section 5 concludes and suggests further research directions.

2 Related Literature

Viner (1932), while assuming that each firm has the same U-shaped long-run average cost function, concludes that in equilibrium, each firm produces at the minimum point of this

curve. However, a large body of empirical evidence shows that the size distribution of firms is neither degenerate nor constant (Evans (1987), Petrunia (2008))¹.

Lucas (1978) as well as Waldman (1984b) assume that individuals differ in their ability. They conclude that the larger the firm, the more worthwhile it is for her to employ a more capable manager. Both papers analyze an economy composed of firms employing one manager and an endogenous number of laborers. In the present paper we endogenous the number of managers.

An additional explanation concerns the recruitment of workers as well as the dampening of the quitting rate of current workers. According to Burdett and Mortensen (1998), larger firms may have more difficulty in recruiting and retaining of workers, thus leading to the need to pay higher wages. Postal-Vinay and Robin (2002) construct and estimate a similar search model in which firms differ in productivity while employees differ in ability.

Jovanovic (1982) considers a perfectly competitive industry where firms have different but time-invariant efficiency levels. Firms only gradually learn their types by observing their noisy cost realizations. Firms that learn that they are efficient grow and survive, while firms that obtain negative information decline and eventually exit the market.

Financial frictions that are motivated by limited enforceability provide another explanation for the observed variety in firm's size (Albuquerque and Hopenhayn (2004), Cooley and Quadrini (2001), Cooley, Marimon and Quadrini (2004)).

Our paper also relates to a large body of empirical evidence showing that larger firms pay higher wages than do smaller ones (Abowd et al. (1999), Bayard and Troske (1999), Brown and Medoff (1989)). Our model also explains the existence of sectoral wage differences (Abowd et al., (1999), Gibbons and Katz (1992)).

Several theoretical justifications for the observed wage gaps have appeared in the literature. One class of explanations considers the moral hazard problem and the related issue of supervision costs (Becker and Stigler (1974), Bulow and Summers (1986)). Another justification is offered by the adverse selection problem, created by information asymmetries (Weiss and Landau (1984)). Technological factors may also explain the wage gaps observed in that larger firms are likely to be more innovative and adopt more advanced

¹Caves (1998) provides an excellent survey of the empirical literature on turnover and mobility of firms.

technologies (Reilly (1995) and Idson and Oi (1999)).

The presented model also relates to the literature on hierarchies in firms. This literature analyzes the relationships between employees at different levels of a hierarchy. In a seminal paper Keren and Levhary (1979) show that the span of control (a variable determined by each firm when choosing its hierarchical structure) is independent of firm size. Calvo and Wellisz (1978) as well as Qian (1994) derive an optimal hierarchical structure for a firm. Their analysis indicates that wages should increase when moving up the hierarchy. This motivates greater effort on the part of senior managers, mitigating the loss-of-control problem as already noted by Williamson (1967). The relations between the span of control and the quality of workers employed in different hierarchy level was investigated by Rosen (1982). He shows that workers' ability should increase when moving up the hierarchy.

In the current paper we assume that all firms have the same production function; however, also we assume that firms cannot observe the ability of their employees upon hiring them. We obtain that in the only strong Nash Equilibrium possible, each firm employs individuals with different abilities and that a firm employing better individuals is larger in equilibrium. We also discuss the promotion policy of firms in that economy. We show that as long as outside firms, who cannot observe the ability of individuals, can observe the proportion of promoted individuals each firm use the promotion of workers with low ability to decrease the wage paid to her other promoted workers and the promotion policy is efficient.

3 A benchmark

As a benchmark, we analyze an economy in which neither the firm nor the employee observe the employees' ability at the beginning of the first period. The analyzed economy consists of two sectors. In the first sector, each firm employs one employee while in the second sector, one finds identical firms exhibiting decreasing returns to scale. The price of the good produced in the first sector is normalized and equal to 1.

Since individuals cannot observe their own ability at the beginning of the first period, individuals join different firms at random and each firm employs individuals with the

same distribution of ability. We thus obtain that all firms in the same sector are of the same size. Hence, We use the benchmark to show the basic ingratitude of the model and analyze promotion policy in each sector.

In the next section, we analyze a similar economy in which individuals do not observe their own ability in the beginning of the first period and obtain the same result regarding the promotion policy. However, we obtain variety in firm size.

We also make the following assumptions:

1. Individuals live for two periods; in each period, the labor supply is perfectly inelastic and fixed at one unit for each individual.

2. The ability of each individual, A_i , is drawn from a distribution $G(A)$ within the support $[0, 1]$.

3. Each individual observes his own ability at the end of the first period (in section (4) we relax this assumption and assume that each employee observes his own ability at the beginning of the first period).

4. Each firm observes the ability of her employees at the end of the first period. Outsides firms cannot observe the ability of individuals employed in other firms².

Note that as a result of the above assumption the alternative wage of each employee cannot be a function of his own ability.

5. An individual (employed in either sector) can be assigned to either of two jobs, which we denote as job 1 and job 2. In the first period, all employees are employed in job 1. In order to be a candidate for job 2, it is necessary to have accumulated experience in job 1. We also assume firm-specific human capital so that an old individual's output is higher if he has not switched firms during his lifetime.

6. There is free entry of firms into both sectors.

7. The job assignment-wage rate pair offered to an old individual by his first period employer is public information.

8. The proportion of promoted individuals is observable.

9. Firms cannot commit to future wages (i.e., the firm cannot commit to paying a wage that is higher than each employee's alternative wage). As a result, the wage of each

²Empirical evidence supporting asymmetric employer learning can be found at Gibbons and Katz (1990), Schoenberg (2007), Pinkston (Forthcoming) and DeVaro and Waldman (2007).

employee equals his alternative wage.

Before the second period begins, the firm must decide upon its assignment policy, i.e., who will be assigned to job 2. When making this decision, the firm also considers the wage that it will have to pay each worker, a wage derived from the wage offers of competing firms. We are looking at a Perfect Bayesian Equilibria, hence, competing firms (which cannot observe the ability of each worker) use the assignment policy and the fact that the worker has been assigned to job 2, to calculate the worker's expected ability.

As we show below, the profit generated by an employee in job 2 rises with his level of ability. This leads to an optimal assignment policy characterized by a threshold level, A^{L1} (A^{L2}), the ability level from and above which a worker is assigned to job 2 in Sector 1 (Sector 2). The wage that a worker assigned to job 2 will be paid is derived from the highest wage that he can receive from a competing firm.

We continue by analyzing each sector in a different subsection.

3.1 Sector 1

Recall that each firm in sector 1 employs only one employee and that A^{L1} denotes the ability level from and above which a worker is assigned to job 2 and that the ability of each individual, A_i , is drawn from a distribution $G(A)$ within the support $[0, 1]$. Hence, $G(A^{L1})$ actually represents the proportion of employees assigned to job 1 in the second period, whereas $1 - G(A^{L1})$ represents the probability of being assigned to job 2.

Each individual employed in Sector 1 produces:

k if he is assigned to job 1 and this is the first period of employment by his current employer;

A_i if he is assigned to job 2 and this is the first period of employment by his current employer;

αA_i ($\alpha > 1$) if he is assigned to job 2 and this is the second period of employment by his current employer.

βk ($\beta \geq 1$) if he is assigned to job 1 and this is the second period of employment by his current employer.

Waldman (1984a), who analyzes an economy similar to the one above, shows the

following:

Proposition 1 *The wage of individuals assigned to job 1 in the second period is k while the wage of an individual assigned to job 2 is $\int_{A^{L1}}^1 Ag(A) dA$, representing his expected ability.*

where A^{L1} denotes the threshold needed for job assignment in Sector 1.

The intuition behind the above results is the following: The alternative wage of an individual employed in job 1 in the second period equals his product in an outside firm, k .

Recall that firms do not observe the ability of workers who are not employed by them. As a result from this assumption, the alternative wage of an individual assigned to job 2 equals his expected product in an alternative firm, which equal his expected ability, $\int_{A^{L1}}^1 Ag(A)$. Note that employees assigned to the same job enjoy the same wage. The intuition behind this result is the following: Only the assignment of an individual to a job reveals information. Hence, there are no incentives to pay higher wages to some employees rather than to other.

The profits from an employee employed in job 1 equal his product, (βk) , minus his wage, (k) , or $(\beta - 1)k$. The profits from an employee employed in job 2 are given by his product, αA_i , minus his wage, $A_i - \int_{A^{L1}}^1 Ag(A)$. The product of each employee is higher than his wage, hence all individuals remain with their first-period employer in the second period.

One can see that the profits from an individual assigned to job 2 are an increasing function of his ability, while the profits from an individual assigned to job 1 are independent of his ability. Hence, we obtain that if the firm assigns an individual with ability A^{L1} to job 2, then all individuals with ability A_i such that $A_i > A^{L1}$, are also assigned to job 2.

A^{L1} is calculated to make the firm employing an individual with such ability indifferent as to whether to promote him or not. Hence:

$$(\beta - 1)k = \alpha A^{L1} - \int_{A^{L1}}^1 Ag(A) dA \quad (1)$$

Note that the LHS of the above equation represents the profits from an individual assigned to job 1, while the RHS represents the profits from an individual assigned to job 2.

The condition for maximization of the firm's product is assignment of each employee to the job in which he produces the highest product, hence, $A^{L1} = k$. Using the above equations, we can conclude that:

Proposition 2 *The assignment policy is not efficient.*

Proof. Using equation (1). ■

The intuition behind the above proposition is the following: All individuals who are assigned to job 2 enjoy the same wage (their wage depends only on the average ability of an individual assigned to job 2). Hence, the profits from the individual whose product is the same in each assignment (i.e., the individual with ability A_i , such that $\alpha A_i = k$) are higher if he is assigned to job 1 (that is, an individual with such ability produces the same product in both assignments although his wage is lower if he is assigned to job 1).

Due to the free entry assumption, The wage paid to first-period employees, w_1 , equals the employee's product in the first period plus expected profits, based on his product, in the second period.

$$w_1 = k + G(A^{L1}) (\beta - 1) k + (1 - G(A^{L1})) (\alpha - 1) \int_{A^{L1}}^1 Ag(A) dA. \quad (2)$$

where k denotes the product in the first period, the second expression represents the expected profits from an individual who is assigned to job 1 in the second period and the third represents the expected profits from an employee assigned to job 2 in the second period.

After a discussion of wages and job assignments in sector 1, we now turn to an examination of the same variable for individual employed in Sector 2.

3.2 Sector 2

Recall that firms in Sector 2 have a decreasing returns-to-scale production function and that each individual can be assigned to either of two jobs, which we denote as job 1 and

job 2. Output in job 1 is independent of ability, while output in job 2 is a function of ability. Specifically, the production function, which is identical for each firm, is given by

$$F(q, m) - FC \tag{3}$$

where q represents the number of employees assigned to job 1, counted in efficiency units, m is the sum of the abilities of individuals assigned to job 2 and FC denotes a fixed cost. We assume that F is continuously differentiable and that $F_1, F_2, F_{12} > 0$ ³. find the condition

We also assume that there is a cost to employing individuals in job 2. The total cost of opening job 2 vacancies is given by $c(v)$, where v denotes the number employees in job 2. We assume that $c'(v) \geq 0$ and do not impose any restrictions on the sign of the second derivative. We also discuss the case in which $c(v) = 0$ for all v .

We denote the price of the good produced in Sector 2 by p and calculate it below.

The firm's employees in job 1 are divided into two groups. One group contains second-period laborers, consisting of workers who are employed for a second period, of size $G(A^{2L})L$, where A^{2L} denotes the lower ability assigned to job 2 in Sector 2 and L denotes the size of the firm's cohort. The other groups contains first-period workers, consisting of "newcomers", employed by the firm for the first time, and is of size L . In terms of efficiency units, each first-period worker equals 1, whereas each second-period worker in the other group equals β , $\beta \geq 1$ efficiency units. Hence:

$$q = (1 + \beta G(A^{2L}))L$$

To obtain an expression representing the sum of the abilities of the firm's managers (denoted by m). Note that,

$$m = \alpha L \int_{A^{2L}}^1 g(A) AdA + M$$

The parameter $\alpha > 1$ reflects the relative advantage of a promoted individual and M represents the expected ability of new individuals who are hired into job 2.

We assume that at the end of the first period, the host firm observes the employees' ability level and subsequently offers the worker a job assignment-wage rate pair for the

second period. Potential employers do not observe the ability of each worker. However, they do observe each worker's job assignment-wage rate and make an offer. Potential employers are willing to bid up to the worker's expected marginal product (making use of A^{2L} to calculate it).

The firm's strategy for the second period consists of a threshold level for assignment to different jobs and the wage level of each employee (as a function of his assignment). In equilibrium, each firm's strategy maximizes its profits, taking into account the alternative wage of her employees, which is a function of the chosen assignment policy. The workers' strategy is to choose the firm that offers the highest wage. We assume that in the case of a tie, workers will remain with their host firm.

To find the threshold level a worker needs to be assigned to job 2, A^{2L} , we first calculate the wage of an employee assigned to job 2 as a function of the threshold level, A^{2L} . The wage of an individual assigned to job 2 results from the competition between his current and potential employers.

We obtain that each firm's wage offer (which equals the external wage the manager can achieve) is given by

$$w_A \int_{A^{2L}}^1 g(A) A dA \quad (4)$$

This wage is the product of two components: the expected ability of an employee assigned to job 2, and w_A , the alternative wage per ability.

The alternative wage per ability of each worker equals his alternative product in Sector 1. This observation results from the assumptions that there is a free entry of firms into Sector 1 and that each firm employs only one individual. Using this observation we obtain that $w_A = 1$ (recall that the price of Sector 1's product is normalized to 1 and that the product of a worker with ability A_i who is employed in job 2 in sector 1 equals A_i).

To simplify matters, we assume that in equilibrium, the expected ability of an employee assigned to job 2 in Sector 2 is higher than that required for assignment to job 2 in Sector 1 (i.e. $\int_{A^{2L}}^1 g(A) A dA > k$). This assumption is satisfied if differences among sectors are not too pronounced⁴.

⁴In the absence of this assumption, the alternative of managers employed in Sector 2 is k .

The wage of an individual who is assigned to job 1 in the second period is also given by his alternative wage, which equals k . We denote by w_1 the wage of a first-period employee, which is determined by equilibrium considerations as will become apparent later.

In order to obtain an internal solution we assume that:

$$pF_1 \left(L\beta, \alpha L \int_0^1 g(A) AdA \right) - k > pF_2 \left(\beta L, L\alpha \int_0^1 g(A) AdA \right) - w_m \quad \text{for all } L \quad (5)$$

where $w_m = \int_{A^{2L}}^1 g(A) AdA$, the wage paid, in equilibrium, to an employee assigned to job 2.

With this condition assigning of all second-period employees to job 2 (choosing $A^{2L} = 0$) cannot be optimal because in this case, the firm makes higher profits from employees who are assigned to job 1 than from employees assigned to job 2. We also assume that $F_2(L(1+\beta), 0) = \infty$, hence in equilibrium $A^{2L} < 1$.

We show later that the firm-specific human capital acquired during the first period results in a positive profit from each second-period employee. However, the result of competition over first-period employees is a zero profit from each employee. A firm therefore continues to employ all her second-period employees.

We now turn to calculating the values of the variables that maximize the firm's product. I.e. L and A^{2L} , the number of employees and the threshold ability needed for promotion are chosen by a firm whose objective is to maximize her net product (i.e., her production minus her employees' alternative product) instead of a competitive firm that maximizes profits.

The net total product of each firm is given by

$$pF \left((\beta + G(A^{2L})) L, \alpha L \int_{A^{2L}}^1 G(A) AdA \right) - Lw_1 - (1 + G(A^{2L})) Lk \quad (6) \\ - \int_{A^{2L}}^1 Ag(A) dA - kL(1 - G(A^{2L})) - c(L(1 - G(A^{2L}))).$$

where w_1 denotes the expected product of a first-period employee in sector 1 and c denotes the cost of opening a vacancy in job 2.⁵

⁵Note that in equilibrium, each firm employs the same number of employees in both periods. To make the exposition simpler we use L to denote firm size in both periods.

We now analyze the competitive environment.

To find the optimal ability threshold level, A^{2L} , we consider firm profit as a function of A^{2L} (an optimum exists since profits are continuous in A^{2L} , which is also bounded between zero and one).

$$\pi = pF(q, m) - G(A^{2L})Lk - L(1 - G(A^{2L}))w_A \int_{A^{2L}}^1 g(A) dA - Lw_{11} - c(L(1 - G(A^{2L}))) \quad (7)$$

The first expression represents the firm's product, the second the total wage paid to second-period employees assigned to assignment 1, the third the total wage paid to second-period employees assigned to job 2, the fourth the total wage paid to first-period employees and the fifth the total cost of opening job 2 vacancies.

Using the FOC of equation (7) with respect to A^{2L} , we can show that the profits from an individual assigned to job 2 are an increasing function of his ability, whereas profits from an employee assigned to job 1 are not a function of ability. Hence, all employees with ability A_i , $A_i > A^{2L}$ are assigned to job 2 in the second period while all employees with ability A_i , $A_i < A^{2L}$ are assigned to job 1 in the second period.

One can also show that all firms are identical because they all face the same production function and employee's ability distribution. In the next section, in which employees observe their own ability while firms do not, this observation does not hold.

In equilibrium, the wage paid to individuals employed in Sector 2 in the first period, w_{12} , makes them indifferent among the sectors. We calculate this wage by reducing the expected second-period wage in Sector 2 from W , the expected wage paid in Sector 1. We obtain that w_{12} , the wage paid to individuals employed in Sector 1 in the first period, is given by:

$$w_{12} = W - G(A^{2L})k - (1 - G(A^{2L})) \int_{A^{2L}}^1 g(A) dA$$

Next we analyze the efficiency of the assignment policy

We start by showing that a firm strictly prefers to assign an employee with ability A^{2L} (the employee with the lowest ability who is assigned, in equilibrium, to job 2) to job 1.

If the firm is indifferent to assigning an individual with ability A^{2L} between job 1 and 2, the following equation should hold:

$$A^{2L}F_1 - \int_{A^{2L}}^1 g(A) AdA - c'(L(1 - G(A^{2L}))) = F_2 - w_k \quad (8)$$

The LHS represents the profits from a worker assigned to job 2, which equals his marginal product minus his wage minus the cost of opening a vacancy. The RHS represents the profits from the same individual if he is assigned to job 1.

Using the FOC of equation (7) with respect to A^{2L} and equation (8), one can show that the firm makes negative profits from individuals with ability A^{2L} .

However, we show the following:

Proposition 3 *The policy assignment is efficient.*

Proof. Using the F.O.C of equations (6) and (7). ■

Note that we obtain this result even though a worker's wage is not a function of his own ability but of the average ability of workers assigned to job 2 in his host firm. Hence, the "regular" equilibrium condition, in which the value of the marginal product equals the wage, does not hold.

Note that the above proposition contradicts the result we obtain in Subsection (3.1). The intuition behind this contradiction is straightforward. An increase in the number of employees in job 2 decreases their average ability. As a result, each firm decreases the wage paid to all her promoted employees.

The total wage paid by a firm to workers assigned to job 2 is given by

$$L(1 - G(A^{2L})) \int_{A^{2L}}^1 Ag(A) dA. \quad (9)$$

Recall that L denotes the number of firm employees; $1 - G(A^{2L})$ is the proportion of workers who are assigned to job 2 and $\int_{A^{2L}}^1 Ag(A) dA$ is the average ability of a worker assigned to job 2 (which equals his wage).

We obtain that the total wage paid to workers assigned to job 2 is a function of the total ability of those individuals. Hence, the marginal cost of assigning additional workers to job 2 is given by differencing equation (9) (the total wage paid by a firm to

workers assigned to job 2) with respect to A^{2L} . Note that this derivative is given by $L(1 - G(A^{2L}))w_a Ag(A^{2L})$. This derivative equals the increase in the total ability of workers assigned to job 2, which in turn equals the ability of the last worker assigned (the workers with the least ability assigned to job 2 in each firm).

In other words, the total wage paid to workers assigned to job 2 in Sector 2 is a function of the sum of their ability. Hence, the marginal cost of assigning another worker to job 2 equals his own ability. In Sector 1 (analyzed in Subsection (3.1)), each firm employs only one employee; hence, it cannot use the assignment of such an employee to decrease the wage paid to other employees.

The intuition behind the previous observation is opposite to the intuition governing monopsonist behavior. The firm uses the assignment of the employee with ability A^{2L} to decrease the total expected ability of all her all employees who are assigned to job 2. As a result of this behavior, the firm decreases the wage paid to each employee assigned to job 2.

To complete the discussion of the equilibrium, we need to determine p , the price of the good produced in Sector 2. We calculate p by calculating the profits of a firm in Sector 2 and equating them to 0. We obtain that

$$pF\left((1 + \beta G(A^{2L}))L, \alpha L \int_{A^{2L}}^1 G(A) AdA\right) - LW - FC = 0$$

We solve the benchmark under the assumption that all the economy's agents (firms and employees) do not observe employee ability at the first period. We turn now to the main discussion of the paper, while relaxing this assumption.

4 The Model

Here we relax one of the assumptions made in the previous section. We assume that workers observe their own ability at the beginning of the first period while firms do not. If we relax this assumption we obtain that the economy has two types of equilibria: the first one is identical to that obtained in the previous section; in the second type of equilibrium,

which is the only strong Nash Equilibrium⁶, workers are not equally distributed across firms. Workers with similar ability join the same firm and each firm employs workers with the same ability.

We show that, in equilibrium, firms that employ better employees are larger (they employ employees with a larger amounts of abilities in job 2 and more individuals in job 1). This result is in line with a large body of empirical literature showing a positive correlation between firm size and wages and wage differences across sectors.

In this section we use the same technology as in the previous one, that is, the description of the production function of a firm in Sector 1 is given in the beginning of Subsection (3.1), while the production function of firm in Sector 2 is given by equation (3). We denote by A_j^L (A_j^h) the lower (upper) ability that is assigned to job 2 in firm j .

Recall that, we are looking at a Perfect Bayesian Equilibria. Outside firms (who cannot observe the ability of each employee) use the fact that an individual was promoted in a given firm to form expectetions regarding his ability. The expected ability (and the alternative wage wage) of each employee is a function only of his job and employer, the only observed variables.

The second's period alternative wage of a worker employed in job 2 equals his expected ability, given by $\int_{A_j^L}^{A_j^h} Ag(A) dA$ (recall that A_j^L (A_j^h) denote the lower (upper) ability that is assigned to job 2 in firm j). The alternative wage of a worker employed in job 1 equals k .

We assume that a firm and a worker cannot sign a long term contract and the wage of each employee equals his alternative wage.

Hence, we can conclude:

Proposition 4 *There are two types of equilibria in the economy. In the first equilibrium $A_h^j = A_h^i$ and $A_l^j = A_l^i$. In the second $A_l^j = A_h^i$.*

Proof. If Workers overlap such that $A_l^j > A_h^i$, workers with A_k , such that $A_l^j > A_k > A_h^i$, can increase their wage by moving from firm i to firm j . If there are two firms, such

⁶A strong Nash equilibrium (or a coalition-proof equilibrium) is stable against the deviation of any given coalition of players.

that $A_h^j = A_h^i$ and $A_l^j = A_l^i$, a worker who moves from one firm to another does not change the ability distribution of the employees in the new firm nor his own wage. ■

In the first type of equilibrium, there are identical firms, i.e., firms that employ workers with the same distribution of ability. In the second type of equilibrium, there is a clear ability threshold level that separates workers employed in different firms (i.e., $A_l^j = A_h^i$).

As an example consider three firms. The first and the second ones promote individuals with abilities $[0.8, 1]$, and a third one promotes individuals with abilities $[0.7, 0.9]$.

We obtain that the wage of a promoted individual employed in the first and the second firms is 0.9 (which is the average ability of the employees employed in that firm), in the second firm 0.85 and in the third 0.8. An individual with ability 0.85 who is employed in the third firm can join the third one and enjoy a higher wage.

Proposition 5 *Only the second equilibrium is a Strong Nash equilibrium. That is, no two firms employ workers with the same ability.*

I.e., No two firms employ individuals with the same ability.

Proof. Consider two firms that employ workers with the same distribution of ability, such that $A_j^H = A_i^H$ and $A_j^L = A_i^L$. If all workers with ability above A^k , $A^H > A^k > A^L$ move to firm j , their wage is increased. ■

The difference between the above proofs is the following: If firms overlap, then different firms employ workers with the same ability but with different ability bounds (i.e., $A_j^H \neq A_i^H$ or $A_j^L \neq A_i^L$). Furthermore, some workers can increase their own wage by moving to another firm in contradiction to the assumption of equilibrium. However, if the bounds of the distribution of employees' ability are equal, a worker does not increase his own wage by moving to another firm. Only a mass of workers moving from one firm to another can increase their wage.

Note that the above propositions hold for firms in both sectors. No two firms in Sector 2 employ workers with the same ability. However, because every firm in Sector 1 employs only one employee, the above corollary holds for a mass of firms in Sector 1. We obtain that the ability of employees employed in each mass of firms lies within a subset of $[0, 1]$.

There can be one mass of firms in Sector 1 employing workers with abilities $A_k \in [A_1, A_2]$; or more than one mass, meaning that firms in one mass employ workers with

ability $A_k \in [A_1, A_2]$ while firms in the other mass employ workers with ability $A_k, A_k \in [A_3, A_4]$ where $A_3 > A_2$. The higher and lower bounds of the abilities of workers assigned to job 2 in Sector 1, as well as the number of masses, result from profit maximization and are analyzed below.

Under the assumed production function, all firms in Sector 2 employ workers in job 1. The second-period wage of such employee is k . In order for those workers to be indifferent among various firms, their wage in the first period must be equal as well. We denote the lifetime wage of an employee with ability below A^{2L} (an employee assigned to job 1 in the second period) as W_1 , which is calculated below. Hence, w_1 , the first-period wage of all workers employed in sector 2, is given by $W_1 - k$. A firm that offers a higher wage will hire all workers who cannot be assigned to job 2 in equilibrium.

We denote the firm employing the worker with Ability 1 (the highest ability) as s_1 and the lowest ability that is assigned to job 2 in that firm as A_1^L . The firm that assigns A_1^L to job 2 is denoted as s_2 , and so forth (recall that the lower ability assigned to job 2 in one firm equals the higher ability assigned to job 2 in another firm). We denote as s_0 the firm employing workers with the lowest ability.

We refer to a firm employing workers with ability higher than in another firm as a better firm. A firm that assigns workers with a lower ability to job 2 is referred to as a worse firm.

In equilibrium the highest ability worker that joins each firm, A_h^j , is given and equal to the lowest ability worker who can be assigned to job 2 in firm $j - 1$. Workers with higher ability than A_h^j can join a better firm (and be assigned to job 2 in that firm). Workers with lower ability can be assigned to job 2 in a worse firm (while they cannot be assigned to job 2 in a better one). We obtain that all the employees in each firm, have the same ability.

The profits of each firm (for a given A^h , the highest ability that joins that firm) are given by

$$\pi = F(q, m) - W_{A^h} - Kw_k - c(L(G(A^h) - G(A^+))) \quad (10)$$

where W_{A^h} denotes the total wage paid by a firm that employs workers with ability A^h , K denotes the number of second-period employees employed in job 1.

The worker with the highest ability who joins each firm is given in equilibrium. Given his ability, each firm has two choice variables: the lowest ability worker assigned to job 1 in firm j , (A_j^L) and the size of the firm (K).

Note that, in the economy analyzed in Section (3), the firm has two choice variables, A^L and L . By choosing A^L , the firm chooses both the sum of abilities assigned to job 2 and the number of employees assigned to job 1 for a given firm size, L . In the economy analyzed in the current section, the firm has to decide which workers to assign to the different jobs (i.e., choosing A^l does not determines the size of K). In this economy, the choice variables of the firm are K and A^L .

We can now turn to the size distribution of firms. Using the FOC of equation (10) with respect to A^L and some algebra, one can show that

$$F_1 - w_A = \frac{c' (L (G (A^h) - G (A^l)))}{A^l} \quad (11)$$

recall that c denotes the total cost of opening job 2 vacancies.

Before discussing the main results of the paper, we analyze a case in which

$$c' = 0 \quad \text{for all firms}$$

Hence, there is no cost to opening a vacancy. This case provides the intuition for the next proposition.

Proposition 6 *Firm s_j is the same size as firm s_{j+1} . Hence, both firms employ the same number of employees in job 1 and the same amount of abilities in job 2.*

Proof. Using equation (11). ■

Intuitively, in the absence of costs of opening a vacancy, we solve the problem faced by a firm employing two production factors (rearranging equation (11) yields $F_1 = w_A$). However, even though all firms are of the same size (i.e., they employ the same number of m , sum of abilities of workers assigned to job 2, and K , the number of employees assigned to job 1), they differ in the number of employees employed by them (a firm that employs better employees employs a smaller number of workers to obtain the same m).

However, under the assumption that $c' > 0$:

Proposition 7 *Firm s_j is larger than firm s_{j+1} (the better firm is larger than the worse one), i.e., it employs more employees in job 1 and employees with larger amounts of abilities in job 2.*

Proof. Without loss of generality, we denote the better firm as h and the worse one as j (recall that the better firm has a higher A^h than the worse one). We analyze three cases separately using equation (11).

$c'_j = c'_h$, an increase in A^l decreases F_1 , hence increases m .⁷

$c'_h < c'_j$, an increase in A^l decreases F_1 , hence increases m .

$c'_h > c'_j$, a firm with a higher ability threshold for assigning workers to job 2 assigns more employees to job 2 (due to the higher marginal cost of opening a vacancy).

Using the FOC of equation (10) with respect to K and the assumption that $F_{12} > 0$, one can show that a firm with a higher m (the sum of the abilities of workers assigned to job 2) also employs more employees in job 1. ■

Proposition 8 *The distribution of workers across firms is inefficient, i.e., the economy's production can be increased by reassigning workers across firms.*

Proof. Without loss of generality, assume that firm h is better than firm j . If we equate the production factors among those firms (i.e., K , m and the number of employees assigned to job 2), the sum of their product is increased due to the concavity of the production function. We obtain a higher increased in their product due to the assumption that $c' > 0$. ■

After the discussion of the promotion policy and the size distribution of firms, we turn to the discussion of the firms' profit and the entrance of a new firm.

Firms that are better than others use their "reputation" to hire better employees. Note that in equilibrium, each worker maximizes his own wage by joining a specific firm. A better firm than the one he joins will not promote him while his wage in a worse firm is lower upon promotion.

⁷We use the same proof for two separate cases. The first is $c''(v) = 0$ for all v . The second is that two firms with the same number of employees assigned to job 2. The second case can also be proven using a method similar to the one used in the case $c'_j > c'_h$.

Recall that a firm cannot commit toward a future wages and that, by offering a first-period wage higher than the equilibrium first-period wage, she hires all the workers who will not be promoted in equilibrium.

Consider the decision faced by a worker upon the entrance of a new firm. The first period wage of each worker as well as the second period wage of a non promoted worker are constant in equilibrium across firms (the entrance of a new firm does not change it). An individual who can be assigned to job 2 in an old firm j , has no incentive to join a new firm. The new firm cannot commit toward his future wages and he will enjoy the same wage upon promotion in the old and the new firm only if all the workers who were previously employed by firm j join the new firm.

The only firm that offers him the same wage as his previous employer is the worst firm (the one employing workers with the lowest ability in job 2). Because no other workers join the new firm, we obtain that the expected ability of a worker assigned to job 2 in a new firm equals the expected ability of a worker employed in the worst firm.

The worst firm makes zero profits while other firms make a positive profit. The intuition is straightforward: Only workers who can be assigned to job 2 in the worst firm join a new firm. Hence, due to the free entry assumption, if firm s_0 makes positive profits, a new firm will open and be able to hire workers who were previously employed by the current firm s_0 .

Proposition 9 *The profits of firm s_j (a better firm) are higher than those of firm s_{j+1} (s_j is better than s_{j+1}).*

Proof. Firm j (the better one) can decrease both m and K in order to reach the size of $j + 1$. However, due to the lower number of workers employed in job 2, her profits are higher than $j + 1$'s profit (due to the cost of opening a vacancy). Since the firm chooses to be larger, her profits also increase. ■

Intuitively, using proposition (7), one can show that a better firm chooses to hire more employees for both jobs. Since the number of employees assigned to each job is a choice variable, we obtain that the profits of firm h are higher than the profits of the firm j . This observation results from the lower cost per unit of ability, i.e., the cost of opening a vacancy is equal among firms regardless of the difference in employee ability.

We obtain an economy in which firms make positive profits. This observation results from the assumption that firms cannot commit to future wages. If we relax this assumption, a new firm can commit at the beginning of the first period to wages and an assignment policy that are identical to the best firm (which also make the highest profits). However, we assume that a firm cannot commit to such contract; hence, workers with high ability will not join such a firm.

We find p , the price of the product produced in Sector 2, by using the zero profit condition with respect to s_0 , the firm employing workers with the lowest ability:

$$\pi = pF(q, m) - W_{A_0^h} - Kw_k - c(L(G(A_0^h) - G(A_0^l))) - FC = 0.$$

where A_0^h denotes the employee with the greatest ability joining the worst firm and A_0^l the employee with the lowest ability that is promoted in that firm.

Note that the previous equation has only one variable, p , the price of the product produced in Sector 2.

We turn to the profit maximization of firm s_0 (i.e., the firm employing workers with the lowest ability).

This firm's profit maximization differs from the other firms' profit maximization in the following way: Choosing A^l in that firm determines the sum of abilities of employees assigned to job 2 and the number of employees assigned to job 1.

We now turn to firms and employees in Sector 1.

As stated in proposition (4), each mass of Sector 1 firms employs workers within a continuous subset of $[0, 1]$ (worker abilities). The highest ability in each subset is given in equilibrium. That is, workers who can be assigned to job 2 in one subset do not join worse firms since they can join better ones and receive a higher wage in the second period.

We analyze one subset (all the different subsets can be analyzed in the same way). Denote by A^H the highest ability of a worker who joins that subset in equilibrium. We obtain that the worker with the lowest ability that is assigned to job 2 in each equilibrium, A^L , is given by the following implicit equation:

$$\alpha \int_{A^L}^{A^H} Ag(A) dA - A^L = (\beta - 1)k$$

where the LHS represents the profits from an employee with the lowest ability assigned to job 2, while the RHS represents the profits from an employee assigned to job 1.

We denote firms that employ workers who are not promoted in equilibrium as worst firms.

The lifetime wage of a worker employed in sector 1 who is promoted in equilibrium equals his second-best lifetime wage. This observation results from the following argument: Assume, by way of contradiction, that the above observation does not hold. If firms in a certain subset pay their employees a higher wage than their second-best alternative, they can increase their profits, in contradiction to being in equilibrium. If wages are lower than their second-best alternative, no employee will join such firm. Hence, we obtain that the first-period wage of a worker employed in Sector 1 who is assigned to job 2 in equilibrium equals his second-best life time wage minus his second-period wage.

The intuition behind the above observation is the following: If a firm hires only one worker who will enjoy a higher wage later in life, she can decrease his wage at the start of his working life. However, if the firm employs workers who enjoy both high and low wages in the second period, workers with a higher future wages can imitate workers with lower future wages in the first period and enjoy higher wages in both periods.

To conclude, the first-period wage of a worker employed in Sector 1 (in which each firm employs only one worker) is negatively correlated with his wage in the second period. However, the first period wage of a worker employed in Sector 2 (in which each firm employs a large number of workers in both jobs) is not a function of his wage in the second period.

Note that the second-best alternative of a worker who can be assigned to job 2 only in the worst firm equals the lifetime wage of a worker who is not promoted in equilibrium.

Hence, we can conclude that:

Proposition 10 *The worst firms employ workers who can and cannot be assigned to job 2 in equilibrium.*

Assume, by a way of contradiction, that the above observation does not hold. Hence, there are firms in Sector 1 that employ only workers who are not assigned to job 2 in equilibrium; we denote those firms as *firmk*. Consider a worker who can be assigned to job 2 in equilibrium and joins *firmk*. His alternative wage upon promotion is given by

his product at the worst firm (if he can be assigned to job 2 in any other firm, he would join her and enjoy a higher lifetime wage).

We calculate the first-period wage of a worker employed in the worst firms, w_1 , using the zero profit condition and obtain the following:

$$w_1 = k + pr(\beta - 1)k + (1 - pr)(\alpha - 1) \int_{A^L}^{A^H} Ag(A) dA$$

where pr denotes the probability of being promoted in such firms. This probability is a function of the production function (it is calculated using the marginal product of workers in equilibrium) and the ability distribution function.

The first-period wage of an employee in Sector 2 also equals w_1 . Employees who cannot be assigned to job 2 are indifferent among both sectors in equilibrium and enjoy the same wage in each period.

Recall that all first period workers in sector 2 enjoy the same wage in the first period, hence the correlation between the wage in the first and the second period wage is 0. However, we obtain a negative correlation between the wage paid in the first and the second period in Sector 1.

5 Conclusions

A large body of empirical literature documents the size distribution of firms. Many theoretical models have tried to explain this distribution under a variety of assumptions, including technological shocks that differ across firms as well as friction in the labor and the financial markets.

Another line of research shows a positive correlation between the number of workers employed in a firm and the quality of her managers. However, those studies assume that only one manager is needed for production to proceed, with no need for another.

In the current paper we analyzed an economy consisting of two jobs. In job 1, the product of each worker employed is not a function of his ability, whereas in job 2, the product of each worker is a function of his ability. We show that in the only strong Nash equilibrium, different firms enjoy a "reputation" that allow them to hire workers with different amount of ability.

We also show that under the assumption that firms bear costs when opening a vacancy, firms that employ workers with higher ability (and pay them higher wages) are larger in equilibrium. This result is in line with a large body of research showing a positive correlation between firm size and wages.

The main empirical prediction of the present paper is that a positive correlation is obtained between the wage fixed effect of the of employees which are employed in the same firm (De Melo (2008)).

Another empirical prediction of the proposed model – is the following: Employees in larger firms enjoy larger wage increases upon promotion. Since all employees in Sector 2 enjoy the same wage in the first period and employees employed in larger firms receive a higher wage in the second period, we obtain that employees in larger firms receive higher wage increases upon promotion.

The main direction for further research is analysis of cases in which employees cannot perfectly observe their ability. We believe that such cases result in the overlap of employee abilities among the different firms.

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