Aggregate Implications of a Credit Crunch

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Abstract

We take an off-the-shelf model with financial frictions and heterogeneity, and study the mapping from a credit crunch, modeled as a shock to collateral constraints, to simple aggregate wedges. We study three variants of this model that only differ in the form of underlying heterogeneity. We find that in all three model variants a credit crunch shows up as a different wedge: efficiency, investment, and labor wedges. Furthermore, all three model variants have an undistorted Euler equation for the aggregate of firm owners. These results highlight the limitations of using representative agent models to identify sources of business cycle fluctuations.

Keywords: financial frictions, business cycles, heterogeneity, aggregation

What are the sources of aggregate fluctuations? To answer this question, macroeconomists often rely on aggregate data and the representative agent framework, thereby abstracting from underlying heterogeneity in the economy. One common approach is to use aggregate productivity shocks, preference shocks, or more generally wedges on the optimality conditions of the representative agent to account for aggregate fluctuations. An obvious advantage of this approach is its simplicity, and it has, for example, been used to infer the relative importance of financial frictions as a driver of business cycles.[1] To evaluate the usefulness of this exercise, we take an off-the-shelf model with financial frictions and heterogeneity, and study the mapping from a credit crunch, modeled as a shock to collateral constraints, to simple aggregate efficiency, investment and labor wedges. We study three variants of this model that only differ in the form of underlying heterogeneity.

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[1]Examples include Chari, Kehoe and McGrattan (2007), Smets and Wouters (2007), Ohanian (2010), and Justiniano, Primiceri and Tambalotti (2010, 2011). We discuss these and other examples in more depth in the "Related Literature" section at the end of this introduction.

Our first result is that in all three model variants a credit crunch shows up as a different wedge. A credit crunch shows up as an efficiency wedge if there is heterogeneity in the productivity of final goods producers. In contrast, it shows up as an investment wedge if we replace heterogeneity in the productivity of final goods producers with heterogeneous investment costs. Finally, a credit crunch shows up as a labor wedge in an economy with heterogeneous recruitment costs. Our second result is that all three model variants have an undistorted Euler equation for the aggregate of firm owners. We show that this is due to a general equilibrium effect and argue that investment wedges from financial frictions are largely an artifact of partial equilibrium reasoning. Taken together, our two results imply that it is impossible to identify a credit crunch from standard aggregate data like output, labor and investment.

Our model features entrepreneurs that have access to three constant returns to scale technologies: a technology to produce final goods, another technology to transform final goods into capital, and a third technology for transforming recruitment effort today into workers in the following period. The three model variants we study only differ in the technology in which entrepreneurs are heterogeneous. In all three model variants, entrepreneurs face collateral constraints that limit their ability to acquire capital or recruit workers.

In addition to entrepreneurs, the economy is populated by a continuum of homogeneous workers. We consider two alternative assumptions regarding workers' access to asset markets: the case of financial autarky and the case where they are allowed to save in a risk-free bond. The first assumption allows for a sharper theoretical characterization of the model's transition dynamics. We also consider an extension where workers face shocks to their efficiency units of labor.

We first study the model variant with heterogeneous final goods productivity, and no heterogeneity in investment and recruitment costs. Aggregate TFP evolves endogenously as a function of the collateral constraint and the distribution of entrepreneurial wealth. Under the assumption of logarithmic preferences, a credit crunch is exactly isomorphic to a TFP shock. In addition, while individual investment decisions are distorted, aggregate investment can be characterized in terms of the Euler equation of a representative entrepreneur that is undistorted. This result is due to a general equilibrium effect: in response to a credit crunch, the interest rate adjusts in such a way that bonds remain in zero net supply; this implies that the aggregate return to wealth equals the aggregate return to capital, and the credit crunch is entirely absorbed by a decrease in TFP. While these results are exact only for the case of logarithmic utility, we show by means of numerical simulations that they hold approximately for the case of general Constant Relative Risk Aversion preferences under standard parameter values.

Once we aggregate entrepreneurs, the economy consist of two types of agents, a representative entrepreneur and a representative worker. If workers are in financial autarky, an investment wedge is needed to characterize aggregate data in terms of a representative agent. However, we show that this investment wedge is negative: a credit crunch looks like an episode in which investment is subsidized, not taxed. Furthermore, we show by means of simulations that the investment is negligible under the alternative assumption that workers face idiosyncratic labor income risk and save in a risk-free bond.

Having studied our first model variant with heterogeneous final goods productivity, we consider two variants with heterogeneity along two other dimensions. In the second model variant entrepreneurs face heterogeneous investment costs – meaning they differ in their technologies to transform final goods into investment goods – but are homogeneous in their final goods production and recruitment technologies. In the third model economy entrepreneurs face heterogeneous recruitment costs – meaning they differ in their technologies to transform recruitment effort today into workers in the following period.

In these model variants, a credit crunch shows up as an investment wedge and a labor wedge respectively. While a credit crunch maps into different wedges in all three model variants, the logic is always the same: a credit crunch worsens the allocation of resources across heterogeneous entrepreneurs and this misallocation decreases the average efficiency of the technology in which entrepreneurs are heterogeneous. In the case of heterogeneous investment technologies, for instance, a credit crunch leads to a worse aggregate investment technology. This shows up as an investment wedge even though the credit crunch has no direct effect on aggregate investment, if the productivity of the aggregate investment technology is not accounted for. A similar intuition applies to the model with heterogeneous recruitment technologies.

Related Literature Our paper is most closely related to the literature that uses wedges in representative agent models to summarize aggregate data (Mulligan, 2002; Chari, Kehoe and McGrattan, 2007).[2] Chari, Kehoe and McGrattan find that the investment wedge did not fluctuate much over the business cycle in postwar aggregate data. They show that in popular theories such as Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1998), financial frictions manifest themselves primarily as investment wedges and conclude that such theories are therefore not promising for the study of business cycles. This finding has been challenged by Christiano and Davis (2006), Justiniano, Primiceri and Tambalotti (2010, 2011), mainly on the grounds that changes in the empirical implementation of Chari

[2]The idea of using such wedges to draw inferences about the sources of aggregate fluctuations goes back at least to Parkin (1988) who studies the labor wedge.

et al.'s procedure overturn the result that the investment wedge did not fluctuate much.[3]

Our paper instead questions the usefulness of wedges on a more basic level. Wedges have been used for at least two purposes. First, they have been used as a "diagnostic" for identifying the primitive shocks driving business cycles (Cole and Ohanian, 2002; Ohanian, 2010). This approach is invalidated by our finding that the same shock – a credit crunch – shows up as a different wedge depending on the form of underlying heterogeneity. Second, wedges have been used as a "guide" to build better models: *given knowledge* of a specific primitive shock, say a credit crunch, the observed wedges are used to narrow down the class of mechanisms through which this shock leads to economic fluctuations. This more nuanced approach is for example advocated by Chari, Kehoe and McGrattan (2007). In this sense a wedge is "just another moment" that a model can be calibrated to. We agree with this characterization. However, it is then unclear why wedges would have any superiority over other moments.[4] Further, micro rather than aggregate data may be better suited to narrow down the mechanisms through which a given shock operates.[5]

A growing recent literature argues that financial frictions can cause aggregate productivity losses (Khan and Thomas, 2010; Gilchrist et al., 2010) or manifest themselves in a labor wedge (Jermann and Quadrini, 2009; Arellano, Bai and Kehoe, 2011).[6] We view our paper as complementary to these, but novel along two dimensions. First, we stress that one main reason why financial frictions may show up in different aggregate variables is their interaction with different forms of underlying heterogeneity. It should be clear that this is a generic feature of all models with financial frictions, a point we emphasize by working with a relatively standard and off-the-shelf model in which we have mainly enriched the underlying heterogeneity. Second, we argue that the intuition that financial frictions should manifest

[3]Christiano and Davis (2006) show that this result is, for example, not robust to the introduction of investment adjustment costs or to an alternative formulation of the investment wedge in terms of a tax on the gross return on capital rather than a tax on the price of investment goods. Justiniano, Primiceri and Tambalotti (2010, 2011) view the data through the lens of a "New Keynesian" model instead of an RBC model, and argue that most business cycle fluctuations are driven by shocks to the marginal efficiency of investment, the equivalent of an investment wedge. They then point out that these investment shocks might proxy for financial frictions.

[4]For instance, why is it more appealing to match the labor wedge rather than, say, aggregate hours worked and/or the unemployment rate?

[5]In our framework, for instance, observed wedges in combination with knowledge of a credit crunch could, in principle, be used to assess the relative importance of our three forms of underlying heterogeneity. However, the statement "if only there were a credit crunch so that we could find out where the heterogeneity is" seems backwards at best. Examining micro data is the much more obvious strategy for identifying sources of heterogeneity.

[6]That financial frictions cause aggregate productivity losses is a popular theme in the growth and development literature. Among others, see Banerjee and Duflo (2005), Jeong and Townsend (2007), Buera and Shin (2010), Buera, Kaboski and Shin (2010), Moll (2010). Buera, Kaboski and Shin (2010) and Moll (2010) also argue that aggregate capital accumulation – as measured by the steady state capital-to-output ratio – is unaffected in their models with heterogeneous final goods producers.

themselves as investment wedges is an artifact of partial equilibrium reasoning. This follows from our result that our three model variants have an undistorted Euler equation for the aggregate of firm owners.[7]

None of our criticisms are special to wedges. They apply one-for-one to other papers that try to learn about the sources of business cycle fluctuations using a representative agent framework and aggregate data alone, say most of the "New Keynesian" literature as exemplified by Smets and Wouters (2007) and Galí, Smets and Wouters (2011).[8] In raising these concerns, our paper has much in common with the work by Chang and Kim (2007) and Chang, Kim and Schorfheide (2010) who examine heterogeneous-agent economies with incomplete capital markets and indivisible labor. They show that a macroeconomist examining aggregate time-series generated by their model with neither distortions nor labor-supply shocks, would conclude that their economy features a time-varying labor wedge or preference shock, and that therefore abstracting from cross-sectional heterogeneity can potentially mislead policy predictions. See Geweke (1985) and Blinder (1987) for earlier critiques of representative agent models when heterogeneity is important.

Following Bernanke and Gertler (1989), a large theoretical literature studies the role of credit market imperfections in business cycle fluctuations. Most papers are similar to ours in that they study heterogeneous entrepreneurs subject to borrowing constraints. In light of our finding that the exact form of heterogeneity matters, we note that most of them assume that entrepreneurs are heterogeneous in their investment technologies (Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1998; Kiyotaki and Moore, 1997, 2005, 2008; Christiano, Motto and Rostagno, 2009; Gertler and Kiyotaki, 2010; Kurlat, 2010).[9] Models with entrepreneurs that are heterogeneous in their final goods productivity are rarer. Exceptions are the papers by Kiyotaki (1998), Kocherlakota (2009), Bassetto, Cagetti and De Nardi (2010), Brunnermeier and Sannikov (2011), Gilchrist et al. (2010) and Khan and

[7] Chari, Kehoe and McGrattan (2007) themselves feature an example of an economy with financial frictions that show up as both an investment wedge and an efficiency wedge (see their Proposition 1), and in a knife-edge case, *only* as an efficiency wedge. We view our results as substantial generalizations of theirs because our results hold in an off-the-shelf model of financial frictions and we clarify that the absence of an investment wedge should be considered a generic feature of general equilibrium models with collateral constraints rather than a knife-edge case.

[8] Smets and Wouters (2007) use aggregate time series and a representative agent model with various structural shocks, including a risk premium shock and an investment-specific technology shock, to understand the sources of business cycle fluctuations. Similarly, Drautzburg and Uhlig (2011) argue that a "financial friction wedge" is the key to understanding the recession of 2007 to 2009.

[9] Kiyotaki and Moore (1997, 2005, 2008) and Gertler and Kiyotaki (2010) make the assumption that each period "investment opportunities" arrive randomly to some exogenous fraction of entrepreneurs. Only entrepreneurs with an "investment opportunity" can acquire new investment goods; others cannot. In our framework, this corresponds to an extreme, binary, form of heterogeneous investment costs: either investment costs are zero, corresponding to the arrival of an investment opportunity, or infinite.

Thomas (2010).[10]

An important distinctive feature of our model is an undistorted Euler equation for the aggregate of firm owners. In most of the literature, this result does not hold because it is assumed that borrowers and lenders differ in their rates of time preference so as to guarantee that entrepreneurs are constrained in equilibrium. Instead, we explicitly model the stochastic evolution of the productivity of entrepreneurs, and their decision to be either active and demand capital, or inactive and supply their savings to other entrepreneurs. Our analysis shows that these alternative modeling assumptions have very different aggregate implications.[11]

One of the main contributions of this paper is to derive analytic expressions for the various wedges despite the rich underlying heterogeneity. To deliver such tractability, we build on work by Angeletos (2007) and Kiyotaki and Moore (2008). Their insight is that heterogeneous agent economies remain tractable if individual production functions feature constant returns to scale because then individual policy rules are linear in individual wealth.[12]

Our paper is organized according to the different dimensions of heterogeneity we consider: heterogeneous productivity (Section 1), heterogeneous investment costs (Section 2), and heterogeneous recruitment costs (Section 3). In Section 4, we discuss how the use of more disaggregated data might allow for identification of a credit crunch. Section 5 is a conclusion.

1 Benchmark Model: Heterogeneous Productivity

1.1 Preferences and Technology

Time is discrete. There is a continuum of entrepreneurs that are indexed by $i \in [0,1]$. Entrepreneurs are heterogeneous in their productivity, z_{it}, their capital holdings, k_{it} and their debt, d_{it}. Each period, entrepreneurs draw a new productivity from a distribution $\psi(z)$. Importantly, this productivity shock is not only iid across entrepreneurs but also iid

[10] Our paper and the majority of the literature focus on credit constraints on the production side of the economy, more precisely those faced by entrepreneurs. In contrast, Guerrieri and Lorenzoni (2011) and Midrigan and Philippon (2011) focus on borrowing constraints at the household level and Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) on those faced by financial intermediaries.

[11] In addition to assuming that individuals differ in their discount factors, some of the papers in the literature (e.g. Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1998) assume that entrepreneurs are identical ex-ante and only heterogeneous ex-post and that there is a real cost of default. This assumption implies that entrepreneurs face a wedge between their ex-ante cost of funds and the risk-free rate.

[12] In contrast to the present paper, Angeletos focuses on the role of "uninsured idiosyncratic investment risk" and does not feature collateral constraints (except for the so-called "natural" borrowing constraint). Kiyotaki and Moore analyze a similar setup with borrowing constraints but their focus is on understanding the implications of monetary factors for aggregate fluctuations.

over time.[13] We assume a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic. Entrepreneurs have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{1}$$

Each entrepreneur owns a private firm which uses k_{it} units of capital and l_{it} units of labor to produce

$$y_{it} = f(z_{it}, k_{it}, l_{it}) = (z_{it} k_{it})^{\alpha} l_{it}^{1-\alpha} \tag{2}$$

units of output, where $\alpha \in (0, 1)$. Entrepreneurs also have access to the following linear technology to transform final goods into investment goods

$$k_{it+1} = x_{it} + (1-\delta) k_{it} \tag{3}$$

where x_{it} is investment and δ is the depreciation rate.

There is a unit mass of workers. Workers have preferences over consumption and hours worked

$$\sum_{t=0}^{\infty} \beta^t [u(C_t^W) - v(L_t)] \tag{4}$$

where u is as in (1) and v is increasing and convex. For most of our results, we restrict the analysis to the case where workers do not have access to assets, and therefore, are hand-to-mouth consumers. We later present numerical results for the case where workers have the same preferences as (4), can accumulate risk-free bonds, and face idiosyncratic labor endowment shocks.

1.2 Budgets

Entrepreneurs hire workers in a competitive labor market at a wage w_t. They also trade in risk-free bonds. Denote by d_{it} the stock of bonds issued by an entrepreneur, that is his debt. When $d_{it} < 0$ the entrepreneur is a net lender. The budget constraint is

$$c_{it} + x_{it} = y_{it} - w_t l_{it} - (1 + r_t) d_{it} + d_{it+1}. \tag{5}$$

Entrepreneurs face borrowing constraints

$$d_{it+1} \leq \theta_t k_{it+1}, \quad \theta_t \in [0, 1]. \tag{6}$$

This formulation of capital market imperfections is analytically convenient. It says that at most a fraction θ_t of next period's capital stock can be externally financed. Or alternatively,

[13] In appendix C we analyze the case where productivity is persistent. The conclusions for the case of logarithmic utility function are unaffected by relaxing the assumption that shocks are iid over time.

the down payment on debt used to finance capital has to be at least a fraction $1-\theta_t$ of the capital stock. Different underlying frictions can give rise to such borrowing constraints, for example limited commitment. Finally, note that by varying θ_t, we can trace out all degrees of efficiency of capital markets; $\theta_t = 1$ corresponds to a perfect capital market, and $\theta_t = 0$ to the case where it is completely shut down. The implications of variations in θ_t over the business cycle for aggregate GDP and capital are the main theme of this paper.

Timing: In order for there to be an interesting role for credit markets, an entrepreneur's productivity next period, z_{t+1}, is revealed at the end of period t, before the entrepreneur issues his debt d_{t+1}. That is, entrepreneurs can borrow to finance investment corresponding to their new productivity. Besides introducing a more interesting role for credit markets, a second purpose of this assumption is to eliminate "uninsured idiosyncratic investment risk". This is the focus of Angeletos (2007) and is well understood.

The budget constraint of entrepreneurs can be simplified slightly. The capital income of an entpreneur is

$$\Pi(z_{it}, k_{it}, w_t) = \max_{l_{it}} \, (z_{it}k_{it})^{\alpha} l_{it}^{1-\alpha} - w_t l_{it} \tag{7}$$

Maximizing out over labor, we obtain the following simple and *linear* expression for profits:

$$\Pi(z_{it}, k_{it}, w_t) = z_{it}\pi_t k_{it}, \quad \pi_t = \alpha \left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)/\alpha}. \tag{8}$$

This implies that the budget constraint of an entrepreneur reduces to

$$c_{it} + k_{it+1} = z_{it}\pi_t k_{it} + (1-\delta)k_{it} - (1+r_t)d_{it} + d_{it+1}. \tag{9}$$

1.3 Equilibrium

An *equilibrium* in this economy is defined in the usual way. That is, an equilibrium are sequences of prices $\{r_t, w_t\}_{t=0}^{\infty}$, and corresponding quantities such that (i) entrepreneurs maximize (1) subject to (6) and (9), taking as given $\{r_t, w_t\}_{t=0}^{\infty}$, and (ii) markets clear at all points in time:

$$\int d_{it} di = 0, \tag{10}$$

$$\int l_{it} di = L. \tag{11}$$

Summing up entrepreneurs' and workers' budget constraints and using these market clearing conditions, we also obtain the aggregate resource constraints of the economy which we find useful to state here.

$$C_t + X_t = Y_t, \quad K_{t+1} = X_t + (1-\delta)K_t \tag{12}$$

$$C_t = C_t^E + C_t^W \tag{13}$$

Here, K_t, Y_t and X_t are the aggregate capital stock, output and investment. C_t is aggregate consumption which is the sum of total consumption by entrepreneurs, C_t^E, and workers, C_t^W.

1.4 Aggregate Wedges

The main goal of this paper is to study the mapping from a credit crunch to aggregate wedges. We follow the literature, in particular Chari, Kehoe and McGrattan (2007), and define these wedges as follows.

Definition 1 *Consider aggregate data* $\{K_t, L_t, Y_t, C_t\}_{t=0}^{\infty}$ *generated by our model economy. The* efficiency wedge *is defined as* $A_t = Y_t K_t^{-\alpha} L_t^{-(1-\alpha)}$*. The* labor wedge*,* τ_{Lt}*, is defined by*

$$\frac{v'(L_t)}{u'(C_t)} = (1-\tau_{Lt})(1-\alpha)\frac{Y_t}{L_t} \tag{14}$$

Finally, the investment wedge*,* τ_{Xt}*, is defined by*

$$u'(C_t)(1+\tau_{Xt}) = \beta u'(C_{t+1})\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)(1+\tau_{Xt+1})\right], \quad \textit{all } t. \tag{15}$$

These wedges have the natural interpretation of productivity, and labor and investment taxes in a representative agent economy with resource constraint (12), Cobb-Douglas aggregate production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ and preferences of the representative consumer given by $\sum_{t=0}^{\infty}\beta^t[u(C_t) - v(L_t)]$. Equation (14) has the interpretation of the labor supply and labor demand conditions with the labor wedge corresponding to a labor income tax. Equation (15) has the interpretation of the Euler equation of the representative consumer and the investment wedge, τ_{Xt}, then resembles a tax rate on investment.[14]

In our economy, by assumption only entrepreneurs invest; workers only supply labor. In answering the question whether aggregate investment is distorted, it will therefore sometimes be useful to examine what we term the *entrepreneurial investment wedge*. This object is analogous to the investment wedge just defined, but uses only aggregate data on quantities pertaining to entrepreneurs. The definition of a *worker labor wedge* will be similarly useful below.

[14] More precisely, consider the following competitive equilibrium in this economy. The representative consumer maximizes his utility function subject to the budget constraint

$$C_t + (1+\tau_{Xt})X_t = (1-\tau_{Lt})w_t L + R_t K_t + T_t$$

and the capital accumulation law $K_{t+1} = X_t + (1-\delta)K_t$, where R_t is the rental rate and T_t are lump-sum transfers. Equation (15) is the corresponding Euler equation. Further, a representative firm maximizes profits given by $A_t K_t^{\alpha} L^{1-\alpha} - w_t L - R_t K_t$ so $R_t = \alpha Y_t/K_t$ and $w_t = (1-\alpha)Y_t/L_t$. Chari, Kehoe and McGrattan (2007) term this the "benchmark prototype economy".

Definition 2 *Consider aggregate data* $\{K_t, Y_t, C^E_t\}_{t=0}^{\infty}$ *generated by the model economy. The* entrepreneurial investment wedge, τ^E_{Xt}, *is defined by the equation*

$$u'(C^E_t)(1+\tau^E_{Xt}) = \beta u'(C^E_{t+1})\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)(1+\tau^E_{Xt+1})\right], \quad \textit{all } t. \tag{16}$$

The worker labor wedge, τ^W_{Lt}, *is defined by*

$$\frac{v'(L_t)}{u'(C^W_t)} = (1-\tau^W_{Lt})(1-\alpha)\frac{Y_t}{L_t}.$$

As we will show below, it turns out that the investment wedge, τ_{Xt}, and labor wedge, τ_{Lt}, do not necessarily equal the *entrepreneurial* investment wedge, τ^E_{Xt}, and *worker* labor wedge, τ^W_{Lt}.[15]

1.5 Log Utility

We find it instructive to first present our model and main result for the special case of log utility, $\sigma = 1$.

1.5.1 Individual Behavior

The problem of an entrepreneur can be written recursively as:

$$\begin{aligned} &V_t(k, d, z_{-1}, z) = \max_{c,d',k'} \ \log c + \beta\mathbb{E}[V_{t+1}(k', d', z, z')] \quad \text{s.t} \\ &c + k' - d' = z_{-1}\pi_t k + (1-\delta)k - (1+r_t)d, \quad d' \le \theta_t k', \quad k' \ge 0. \end{aligned} \tag{17}$$

Here we denote by z_{-1} the productivity of an entrepreneur in the current period, by z his productivity in the next period, and by z' his productivity two periods ahead. The expectation is taken over z' only, because – as we discussed above – we assume that an entrepreneur knows z at the time he chooses capital and debt holdings. This problem can be simplified. To this end define an entrepreneur's "cash-on-hand", m_{it}, and "net worth", a_{it}, as

$$m_{it} \equiv z_{it}\pi_t k_{it} + (1-\delta)k_{it} - (1+r_t)d_{it}, \quad a_{it} \equiv k_{it} - d_{it} \tag{18}$$

Lemma 1 *Using the definitions in* (18)*, the following dynamic program is equivalent to* (17)*:*

$$v_t(m, z) = \max_{a'} \ \log(m - a') + \beta\mathbb{E}v_{t+1}(\tilde{m}_{t+1}(a', z), z')$$
$$\tilde{m}_{t+1}(a', z) = \max_{k',d'} \ z\pi_{t+1}k' + (1-\delta)k' - (1+r_{t+1})d', \quad \textit{s.t.}$$

[15] It is easy to see that $\tau_{Xt} \neq \tau^E_{Xt}$ if the marginal rate of substitution of the "representative worker", $u'(C^W_t)/[\beta u'(C^W_{t+1})]$, is different from that of the "representative entrepreneur", $u'(C^E_t)/[\beta u'(C^E_{t+1})]$. This is what will happen below.

$$k' - d' = a', \quad k' \leq \lambda_t a', \quad \lambda_t \equiv \frac{1}{1-\theta_t} \in [1, \infty)$$

The interpretation of this result is that the problem of an entrepreneur can be solved as a two-stage budgeting problem. In the first stage, the entrepreneur chooses how much net worth, a', to carry over to the next period. In the second stage, conditional on a', he then solves an optimal portfolio allocation problem where he decides how to split his net worth between capital, k' and bonds, $-d'$. The borrowing constraint (6) immediately implies that the amount of capital he holds can be at most a multiple $\lambda_t \equiv (1-\theta_t)^{-1}$ of this net worth. λ_t is therefore the maximum attainable leverage. From now on, a credit crunch will interchangeably mean a drop in θ_t or λ_t.

Lemma 2 *Capital and debt holdings are* linear *in net worth, and there is a productivity* cutoff *for being active* $\underline{z}_{t+1}$.

$$k_{it+1} = \begin{cases} \lambda_t a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ 0, & z_{it+1} < \underline{z}_{t+1} \end{cases}, \quad d_{it+1} = \begin{cases} (\lambda_t - 1) a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ -a_{it+1}, & z_{it+1} < \underline{z}_{t+1}. \end{cases} \tag{19}$$

The productivity cutoff is defined by $\underline{z}_{t+1}\pi_{t+1} = r_{t+1} + \delta$*.*

Both the linearity and cutoff properties follow directly from the fact that individual technologies (2) display constant returns to scale in capital and labor. We have already shown that maximizing out over labor in (7), profits are linear in capital, (8). It follows that the optimal capital choice is at a corner: it is zero for entrepreneurs with low productivity, and the maximal amount allowed by the collateral constraints, $\lambda_t a'$, for those with high productivity. The productivity of the marginal entrepreneur is $\underline{z}_{t+1}$. For him, the return on one unit of capital $z\pi_{t+1}$ equals the user cost of capital, $r_{t+1} + \delta$. The linearity of capital and debt delivers much of the tractability of our model.

Lemma 3 *Entrepreneurs save a constant fraction of cash-on-hand:*

$$a_{it+1} = \beta m_{it+1}, \tag{20}$$

or using the definitions of cash-on-hand and net worth in (18)

$$k_{it+1} - d_{it+1} = \beta[z_{it}\pi_t k_{it} + (1-\delta)k_{it} - (1+r_t)d_{it}]. \tag{21}$$

1.5.2 Aggregation

Aggregating (21) over all entrepreneurs, we obtain our first main result:

Proposition 1 *Aggregate quantities satisfy*

$$Y_t = Z_t K_t^\alpha L^{1-\alpha} \tag{22}$$

$$K_{t+1} = \beta \left[\alpha Y_t + (1-\delta) K_t\right] \tag{23}$$

where

$$Z_t = \left(\frac{\int_{\underline{z}_t}^\infty z\psi(z)dz}{1-\Psi(\underline{z}_t)} \right)^\alpha = \mathbb{E}[z|z \geq \underline{z}_t]^\alpha \tag{24}$$

is measured TFP. The cutoff is defined by

$$\lambda_{t-1}(1-\Psi(\underline{z}_t)) = 1. \tag{25}$$

Corollary 1 *Aggregate entrepreneurial consumption is given by* $C_t^E = (1-\beta)[\alpha Y_t + (1-\delta)K_t]$ *and satisfies an Euler equation for the "representative entrepreneur":*

$$\frac{C_{t+1}^E}{C_t^E} = \beta \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \tag{26}$$

Aggregate consumption of workers is given by $C_t^W = (1-\alpha)Y_t$.

1.5.3 A Credit Crunch

In this section, we conduct the following thought experiment: consider an economy that is in steady state at time, $t = 0$, with a given degree of financial friction, λ_0 (equivalently, $\theta_0 = 1 - 1/\lambda_0$). At time $t = 1$, there is a credit crunch: λ_t falls and then recovers over time according to

$$\lambda_{t+1} = (1-\rho)\lambda_0 + \rho\lambda_t, \quad \rho \in (0,1) \tag{27}$$

until it reaches the pre-crunch level of λ_0. We ask: what are the "impulse responses" of aggregate output, consumption and capital accumulation to this credit crunch?

Proposition 2 *In our benchmark economy and under the assumption of log-utility, a credit crunch*

(i) is isomorphic to a drop in total factor productivity as can be seen from (24) *and* (25).

(ii) does not not distort the Euler equation of a "representative entrepreneur" which is given by (26), *and hence the* entrepreneurial *investment wedge defined in* (16) *is zero,* $\tau_{Xt}^E = 0$ *for all* t.

(iii) results in an investment wedge, τ_{Xt}, *defined recursively by*

$$\frac{C_{t+1}}{C_t}\tau_{Xt} - \beta(1-\delta)\tau_{Xt+1} = \frac{C_t^W}{C_t}\left[\frac{C_{t+1}^E}{C_t^E} - \frac{C_{t+1}^W}{C_t^W}\right], \ t \geq 1 \quad \tau_{X0} = 0. \tag{28}$$

(iv) results in a worker *labor wedge* $\tau^W_{Lt} = 0$*, and a labor wedge given by* $\tau_{Lt} = -C^E_t / C^W_t$*.*

A credit crunch distorts the investment decisions of individual entrepreneurs. One may have expected that therefore also the investment decision of a "representative entrepreneur" is distorted. Part (ii) of the proposition states that this is not the case: a credit crunch lowers aggregate investment only to the extent that it lowers TFP and therefore the aggregate marginal product of capital; the wedge in the Euler equation of a representative entrepreneur is identically zero. This result is not straightforward. Much of the next subsection – which also covers the more general case of CRRA utility – will be concerned with discussing the intuition behind it. Part (iii) of the Proposition states that while aggregate investment is not distorted, there is nevertheless a non-zero investment wedge as in Definition 1. This is because, while the Euler equation of the "representative entrepreneur" is not distorted, the "representative worker" is borrowing constrained and has consumption $C^W_t = (1-\alpha)Y_t$. Aggregate consumption is the sum of the consumption of workers and entrepreneurs. The aggregate investment wedge is found by matching up two equations: the growth rate of aggregate consumption and the equation defining the aggregate investment (15). It can easily be seen that a non-zero investment wedge is needed to match up these two equations. Its size depends on relative consumption growth of entrepreneurs and workers. We will argue momentarily that this investment wedge is actually "upside down", in the sense of looking like a *subsidy* to investment as opposed to a tax. Furthermore, this investment wedge is really an artifact of one of the modeling assumptions we make to obtain closed forms, namely that workers cannot save. We show that under the alternative assumption that workers can save in a risk-less asset and face idiosyncratic labor income risk, the investment wedge becomes negligible. Finally, part (iv) shows that there is also a labor wedge. This is the case even though workers are on their labor supply curve (the worker labor wedge is zero), and – as was the case for the investment wedge – results from our assumption that entrepreneurs and workers are two distinct classes of agents.

Figures 1 and 2 graphically illustrate Proposition 2. Figure 1 displays the time-paths for the degree of financial frictions λ_t and the implied TFP path.[16] Since the two are isomorphic, we choose the initial drop in λ_t so as to cause a ten percent decline in productivity. Figure 2 shows the effect of a credit crunch on aggregate TFP (panel a), the entrepreneurial investment wedge (panel b), the investment wedge (panel c), and the labor wedge (panel d). Panel (a) simply restates the productivity drop from Figure 1. Panel (b) shows the *entrepreneurial* investment wedge, τ^E_{Xt}, which is zero throughout the transition as discussed in the Proposition. Panel (c) shows the investment wedge, τ_{Xt}. It is positive at first, and

[16] We use the following parametrization of the model: $\beta = 0.95$, $\delta = 0.06$, $\alpha = 0.33$, $\lambda_0 = 3$, and assume that the distribution of productivity of entrepreneurs is Pareto, $\eta z^{-\eta-1}$, with tail parameter $\eta = 2.1739$.

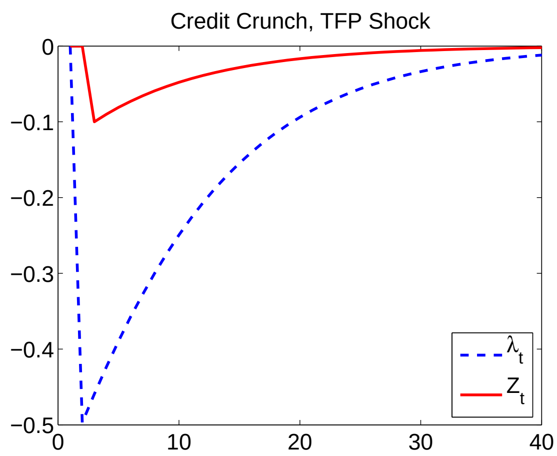

Fig. 1: Response to a Credit Crunch

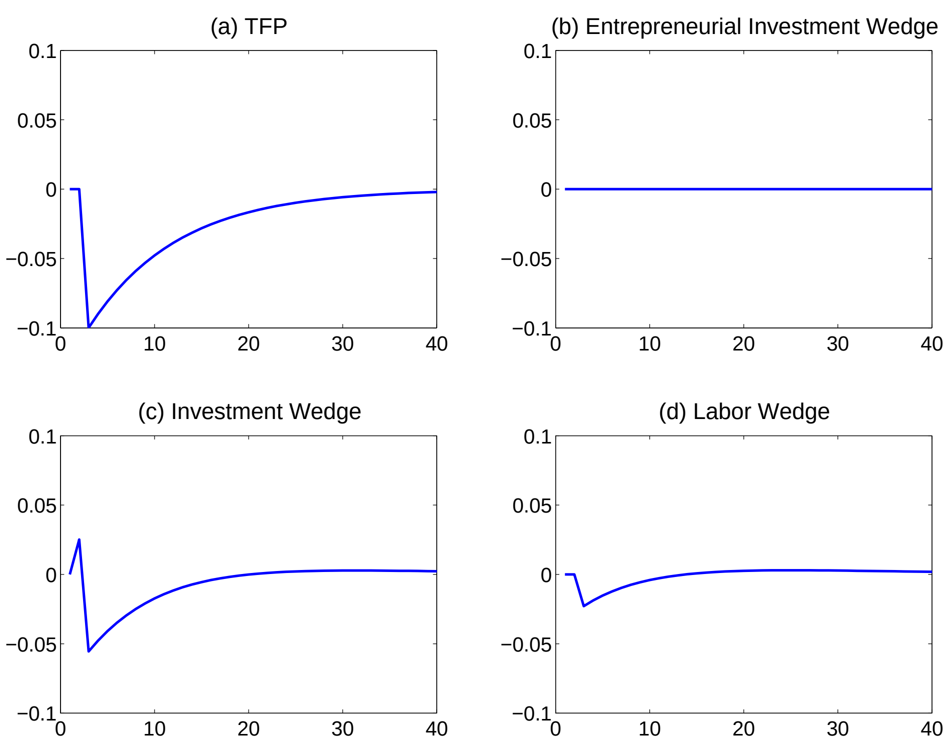

Fig. 2: Response to a Credit Crunch

negative throughout most of the transition; in steady state, it is zero because consumption growth for both workers and entrepreneurs is zero (see equation (28)). Importantly, and contrary to what the reader may have expected, the investment wedge is *negative*, meaning it looks like a subsidy. Finally, panel (d) shows the labor wedge defined in (14) which also looks like a subsidy.[17] That both the investment and the labor wedge do not equal zero is mainly due to our modeling assumptions, an issue we discuss now.

In order to obtain closed form solutions, we have separated individuals into "entrepreneurs" and "workers" and have assumed that the latter cannot save. Since workers are by assumption not "on their Euler equation", it is this assumption that delivers a zero *entrepreneurial* investment wedge, but a non-zero investment wedge. The left panel of Figure 3 presents the investment wedge under two alternative assumptions on the savings behavior of workers: they save in a risk-free bond; and they save in a risk-free bond and additionally face some labor income risk as in Aiyagari (1994). In both cases we assume that they need to hold non-negative wealth, i.e. they cannot borrow. Details are in Appendix B. When workers save in a risk-free bond but face no labor income risk (green,

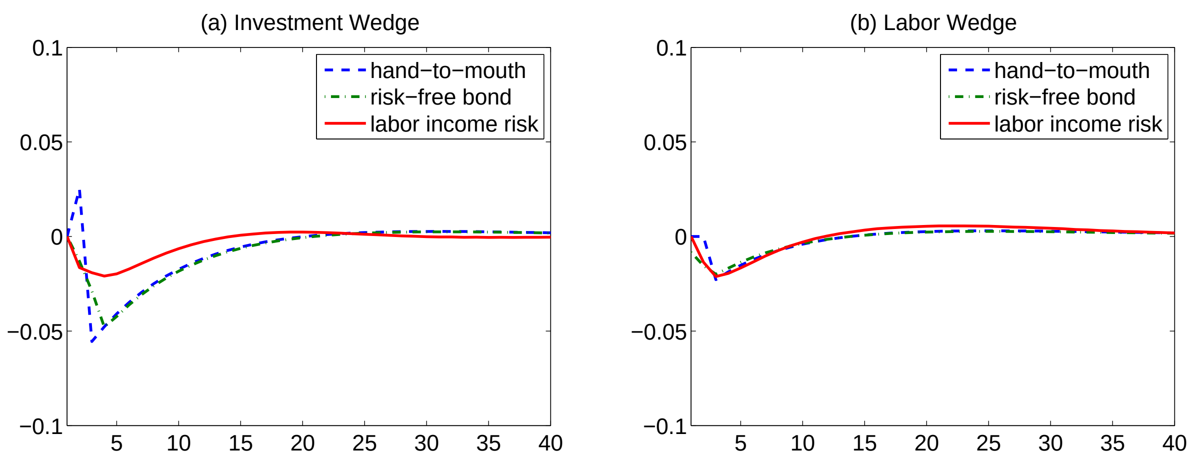

Fig. 3: Alternative Assumptions about Workers' Savings: Investment and Labor Wedges

dash-dotted line), the investment wedge is negative throughout the entire transition. That the investment wedge is not zero comes from the fact that while workers can save, they are still borrowing constrained. This is because the interest rate in our economy is less than the rate of time preference and therefore, in the absence of risk, workers hold zero wealth in the initial steady state. A negative TFP shock triggered by a credit crunch decreases the wage and only worsens this borrowing constraint. This implies that their consumption growth rate is higher than that of entrepreneurs and hence from (28) that the investment wedge is negative. In contrast, with labor income risk (red, solid line), workers in the initial

[17] In contrast, the *worker* labor wedge, which we choose not to display here is identically zero throughout the transition.

steady state hold positive wealth due to precautionary motifs. This means that only a small fraction of them end up borrowing-constrained when their wage falls after a credit crunch. Most workers are therefore on their unconstrained Euler equations and the investment wedge becomes negligible.

Panel (b) of Figure 3 presents the labor wedge under two alternative assumptions on savings behavior. As discussed in Proposition 2, the labor wedge is a function of the consumption of entrepreneurs relative to that of workers. In the two extensions where workers accumulate assets, the difference in the growth rate of the consumption of workers and entrepreneurs is smaller, and therefore, the movements in the labor wedge is smoother.[18]

1.6 General CRRA Utility and Intuition for Undistorted Aggregate Euler Equation

This section presents the case where individuals' preferences are given by the general CRRA utility function (1). It also presents an alternative and more intuitive derivation of the result in Proposition 2 that a credit crunch does not distort the Euler equation of a representative entrepreneur, $\tau^E_{Xt} = 0$. We show that the result follows from a general equilibrium effect that comes from bonds being in zero net supply. The analysis of the saving problem of individual entrepreneurs with CRRA utility is similar to the log case analyzed in the preceding section.[19] We therefore relegate the details to Appendix C.

1.6.1 Individual Euler Equations

The Euler equation of an individual entrepreneur (with respect to net worth, a_{it+1}) is[20]

$$\frac{u'(c_{it})}{\beta\mathbb{E}[u'(c_{it+1})]} = R^a_{it+1} \tag{29}$$

where

$$R^a_{it+1} \equiv 1 + r_{t+1} + \lambda_t \max\{R^k_{it+1} - 1 - r_{t+1}, 0\} = \frac{R^k_{it+1}k_{it+1} - (1 + r_{t+1})d_{it+1}}{a_{it+1}} \tag{30}$$

[18]Ultimately, the labor wedge in our benchmark model stems from the fact that entrepreneurs do not supply labor. We conjecture that a relatively straightforward extension of our model where entrepreneurs supply labor will feature a negligible labor wedge.

[19]For $\sigma \neq 1$, the saving policy function cannot be solved in closed form anymore. While the saving policy function can still be shown to be linear in cash-on-hand, the saving rate now depends on future productivity, z_{it+1} (which is known at time t): $a_{it+1} = s_{t+1}(z_{it+1})a_{it}$. With log-utility $s_{t+1}(z_{it+1}) = \beta$ is constant because the income and substitution effects of a higher productivity draw exactly offset each other.

[20]The Euler equation (29) is $u'(c_{it}) = \beta\mathbb{E}[u'(c_{it+1})R^a_{it+1}]$. The return to wealth R^a_{it+1} can be taken out of the expectation because of our assumption that next period's productivity z_{it+1} and therefore R^a_{it+1} is known at the time a_{it+1} is chosen. Further, the second equality in (30) uses the complementary slackness condition $(R^k_{it+1} - 1 - r_{t+1})(\lambda_t a_{i+1} - k_{it+1}) = 0$.

is the return to wealth and

$$R^k_{it+1} \equiv \alpha \frac{y_{it+1}}{k_{it+1}} + 1 - \delta \tag{31}$$

is the return to capital. Note that for credit constrained entrepreneurs, the return to capital is greater than the interest rate, $R^k_{it+1} > 1 + r_{t+1}$. Therefore also their return to savings is higher than the interest rate, $R^a_{it+1} > 1 + r_{t+1}$, which is to say that individual Euler equations are distorted.[21] In contrast and as we have shown in Proposition 2, aggregate investment is *undistorted* under certain conditions. The goal of this section is to show how *distorted individual* Euler equations can be aggregated to obtain an *undistorted aggregate* Euler equation of the form (26). This alternative derivation of (26) has the advantage that directly working with individual Euler equations is more intuitive and also underlines that the logic behind our result is, in fact, quite general.

1.6.2 Euler Equation of Representative Entrepreneur

We aggregate (29) by taking a wealth weighted average to obtain:

$$\int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di = \int R^a_{it+1} \frac{a_{it+1}}{K_{t+1}} di \tag{32}$$

It is useful to separately analyze the left-hand-side and right-hand-side of this equation. We denote these by

$$\text{LHS} \equiv \int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di \qquad \text{and} \tag{33}$$

$$\text{RHS} \equiv \int R^a_{it+1} \frac{a_{it+1}}{K_{t+1}} di. \tag{34}$$

Right-Hand Side. By manipulating the right-hand side, (34), we obtain the following Lemma whose proof is simple and therefore stated in the main text.

Lemma 4 (RHS) *A wealth weighted average of the return to wealth accumulation across entrepreneurs equals the aggregate marginal product of capital:*

$$RHS = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta.$$

Proof From (30) we have

$$\int R^a_{it+1} a_{it+1} di = \int R^k_{it+1} k_{it+1} di - (1 + r_{t+1}) \int d_{it+1} di = \int R^k_{it+1} k_{it+1} di,$$

[21] However, note that the distortion at the individual level takes the form of a *subsidy* rather than a tax, that is investment wedges at the individual level are *negative*. This is because for a constrained entrepreneur, each dollar saved has an additional shadow value because it relaxes his borrowing constraint.

where the second equality uses that bonds are in zero net supply, (10). Using the definition of R^k_{it+1}, (31), we get

$$\text{RHS} = \int R^a_{it+1} \frac{a_{it+1}}{K_{t+1}} di = \int R^k_{it+1} \frac{k_{it+1}}{K_{t+1}} di = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta. \square$$

Lemma 4 will be the main building block of the result that the Euler equation of a representative entrepreneur is not distorted (Proposition 3). The proof of the Lemma has two main steps: the first step is to show that the aggregate return to *wealth* equals the aggregate return to *capital*. Entrepreneurs can allocate their wealth between two assets, capital and bonds. But in the aggregate, bonds are in zero net supply. Therefore the aggregate return to wealth must equal the aggregate return to capital. This result is remarkably general. It does *not* in any way depend on the form of utility or production functions. For example, the latter could display decreasing returns to scale. We spend some more time discussing this result in the next paragraph. The second step in the proof is to show that a capital weighted average of the returns to capital, (31), equals the aggregate marginal product of capital:

$$\int R^k_{it+1} \frac{k_{it+1}}{K_{t+1}} di = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta.$$

The assumption of Cobb-Douglas production functions is crucial for this step because it implies that the marginal product of capital is proportional to the average product. Given the Cobb-Douglas assumption, this second step is relatively mechanical and we will not discuss it further.

The key to understanding Lemma 4 is a general equilibrium effect that comes from bonds being in zero net supply. To gain some intuition, consider an economy that starts in equilibrium with $(\lambda_t, r_{t+1}) = (\lambda, r)$. At time t, a credit crunch hits and leverage decreases to $\lambda^* < \lambda$. We index variables by (λ, r) and trace out the economy's response. We suppress time subscripts for notational simplicity. When r is fixed in partial equilibrium, an immediate effect of the credit crunch is that credit is restricted and hence aggregate capital demand drops below aggregate capital supply

$$K(\lambda^*, r) = \int k_i(\lambda^*, r) di < \int a_i di \equiv A \tag{35}$$

Following similar steps as in Lemma 4, the wealth weighted average of individual returns to wealth can be shown to be

$$\begin{aligned} \text{RHS}(\lambda^*, r) &= \left[\alpha \frac{Y(\lambda^*, r)}{K(\lambda^*, r)} + 1 - \delta\right] \frac{K(\lambda^*, r)}{A} + (1 + r)\left[1 - \frac{K(\lambda^*, r)}{A}\right] \\ &< \alpha \frac{Y(\lambda^*, r)}{K(\lambda^*, r)} + 1 - \delta \end{aligned} \tag{36}$$

In partial equilibrium, a credit crunch causes the aggregate return to wealth to *fall below* the aggregate return to capital. This is because the credit crunch results in a positive share of the aggregate portfolio being allocated towards bonds which earn a lower return than capital. The implication is that a credit crunch looks like the introduction of a tax on the returns to capital, with the second line of (36) corresponding to the pre-tax return and the first line to the after-tax return. Put another way: in partial equilibrium, the entrepreneurial investment wedge is positive. In general equilibrium, however, things look quite different. An immediate implication of (35) is that the interest rate must fall until bonds are in zero net supply, or equivalently $K(\lambda^*, r^*) = A$. This immediately implies that

$$\text{RHS}(\lambda^*, r^*) = \alpha \frac{Y(\lambda^*, r^*)}{K(\lambda^*, r^*)} + 1 - \delta$$

Bonds being in zero net supply means that the share of the aggregate portfolio invested in bonds equals zero as before the credit crunch. Therefore the aggregate return to wealth again equals the aggregate return to capital, and the effect of the credit crunch is entirely absorbed by a decrease in TFP.

This general equilibrium effect obviously hinges on our economy being closed. In an open economy a credit crunch would lead to an increase in the entrepreneurial investment wedge. We find it worthwhile to note that the sign of the *level* of the investment wedge is generally ambiguous. In particular it will often be *negative*, meaning it looks like a subsidy to investment.[22] Another crucial assumption is that the borrowing constraint takes the form (6). Consider instead a more general borrowing constraint $k_{it+1} \leq b_{it+1}(a_{it+1}, z_{it+1}, r_{t+1}, w_{t+1}, ...)$. One can show that Lemma 4 holds if and only if the elasticity of the borrowing limit, b_{it+1}, with respect to wealth, a_{it+1}, is one. Apart from that, the borrowing constraint can be a general function of, say, individual productivities, prices and so on.

Left-Hand Side. By manipulating the left-hand side (33), we obtain the following Lemma.

[22] In an open economy, and similar to (36), the Euler equation of a representative entrepreneur is

$$\frac{C^E_{t+1}}{\beta C^E_t} = \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right) \frac{K_{t+1}}{A_{t+1}} + (1 + r)\left(1 - \frac{K_{t+1}}{A_{t+1}}\right)$$

and therefore the entrepreneurial investment wedge as defined in (16) is negative whenever the economy's aggregate capital stock, K_{t+1}, is greater than its aggregate wealth A_{t+1}. Depending on the degree of heterogeneity, a negative investment wedge may, in fact, be the only possibility. To see this consider the degenerate case with homogenous entrepreneurs who all face the same collateral constraints $K_{t+1} \leq \lambda_t A_{t+1}, \lambda_t \geq 1$. Since everyone is alike, the constraint can only bind if the economy as a whole is borrowing, $K_{t+1} > A_{t+1}$. The investment wedge must therefore be negative in this degenerate case. The intuition is straightforward: for a constrained entrepreneur, each dollar saved has an additional shadow value because it relaxes his borrowing constraint.

Lemma 5 (LHS)

$$LHS = \frac{C^E_{t+1}}{C^E_t}\frac{1}{\bar{s}_{t+1}}\frac{1-\bar{s}_{t+1}}{1-\bar{s}_{t+2}} \quad \text{where} \quad \bar{s}_{t+1} = \int_0^\infty s_{t+1}(z)\psi(z)dz \tag{37}$$

and $s_{t+1}(z)$ is the saving rate of type z.

For the special case of log-utility, $\sigma = 1$, all entrepreneurs save the same fraction of their cash-on-hand regardless of their type, $s_t(z) = \beta$. Hence (37) specializes to

$$\text{LHS} = \frac{C^E_{t+1}}{\beta C^E_t} \tag{38}$$

Combining Left-Hand Side and Right-Hand Side. In the case of log-utility, (38) and Lemma 4 together immediately imply the undistorted aggregate Euler equation in (26).[23] In the more general case of CRRA utility, we can still combine Lemmas 4 and 5 to obtain

Proposition 3 *In our benchmark economy with general CRRA utility, a credit crunch*

(i) *results in an* entrepreneurial *investment wedge, τ^E_{Xt} defined by*

$$\frac{1}{\beta}\left(\frac{C^E_{t+1}}{C^E_t}\right)^\sigma (1+\tau^E_{Xt}) - (1-\delta)\tau^E_{Xt+1} = \frac{C^E_{t+1}}{C^E_t}\frac{1}{\bar{s}_{t+1}}\frac{1-\bar{s}_{t+1}}{1-\bar{s}_{t+2}} \tag{39}$$

where the initial (steady state) value is $\tau^E_{X0} = (\beta/\bar{s}-1)/(1-\beta(1-\delta))$.

(ii) *results in an investment wedge, τ_{Xt}, defined by*

$$\left[\frac{C^E_{t+1}}{C^E_t} + \frac{C^W_t}{C_t}\left(\frac{C^W_{t+1}}{C^W_t} - \frac{C^E_{t+1}}{C^E_t}\right)\right]^\sigma (1+\tau_{Xt}) - \left(\frac{C^E_{t+1}}{C^E_t}\right)^\sigma (1+\tau^E_{Xt}) = \beta(1-\delta)(\tau_{Xt+1} - \tau^E_{Xt+1}), \tag{40}$$

where the initial (steady state) investment wedge is $\tau_{X0} = \tau^E_{X0}$.

Consistent with Proposition 2, the entrepreneurial investment wedge in (i) collapses to $\tau^E_{Xt} = 0$ for the case of log-utility $\sigma = 1$. This is because in that case $\bar{s}_t = \beta$. For $\sigma \neq 1$ the entrepreneurial investment wedge can be either positive or negative. We illustrate this in Figure 4 which shows the effect of a credit crunch for three different values of the inverse of the intertemporal elasticity of substitution, σ. A value of $\sigma = 1$ corresponds to log-utility and therefore the transition dynamics for that case are identical to Figure 2. The

[23]Similarly, the law of motion of the aggregate capital stock in the economy with CRRA utility is

$$K_{t+1} = \bar{s}_{t+1}[\alpha Y_t + (1-\delta)K_t], \quad \bar{s}_{t+1} = \int_0^\infty s_{t+1}(z)\psi(z)dz$$

For the special case $\sigma = 1$, and hence $s_t(z) = \beta$, we obtain (23).

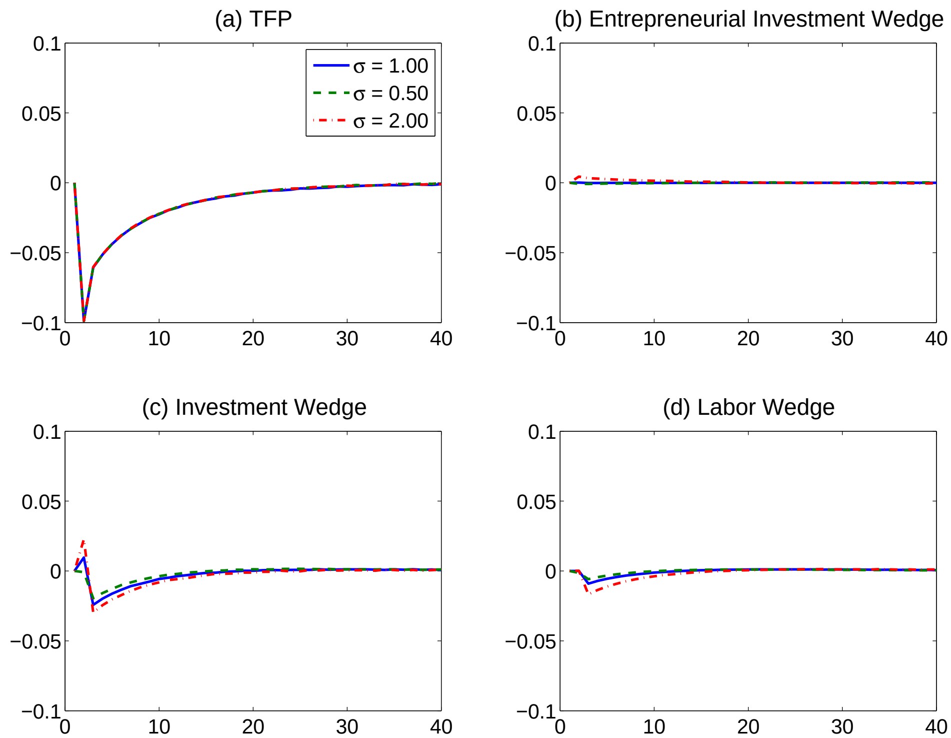

Fig. 4: Response to a Credit Crunch: General CRRA Utility

entrepreneurial investment wedge (panel b) is positive for the case where $\sigma < 1$ and negative for the case $\sigma > 1$. This is intuitive: if entrepreneurs are relatively unwilling to substitute intertemporally (σ is high), they overaccumulate assets. In aggregate data, this looks like a subsidy to savings. The wedges further depend on σ in a continuous fashion: for values of σ that are "close" to one such as the ones chosen in the Figure, the wedges are "similar" to the log-case. Finally, the non-zero entrepreneurial investment wedge for the case $\sigma \neq 1$ is best thought of as arising from individual marginal utilities not being equalized under incomplete markets, rather than from the presence of borrowing constraints. The parameter governing borrowing constraints, λ_t, only enters the aggregate Euler equation (32) through the right-hand side (34). But this equals the aggregate marginal product of capital regardless of σ (Lemma 4). In contrast, the left-hand-side (33) encodes individual marginal utilities and hence aggregation effects due to incomplete insurance and so on.

2 Heterogeneous Investment Costs

We have argued in the previous two sections that in an economy with heterogeneity in productivity, a credit crunch shows up in TFP; in contrast, the investment wedge is either zero or small. The purpose of the next two sections is to argue that this is by no means necessarily the case. If heterogeneity takes a different form, a credit crunch can show up as either an investment or a labor wedge. In this section, we consider the case of heterogeneous investment costs and show that a credit crunch manifests itself as an investment wedge while aggregate TFP is unaffected by construction.[24]

The economy is essentially the same as in section 1 but differs in one important aspect: we replace heterogeneity in the productivity of final goods producers with heterogeneity in investment costs. To obtain one unit of investment goods, different entrepreneurs have to give up different amounts of consumption goods. The role of credit markets is then to reallocate funds towards those entrepreneurs with low investment costs.

Besides allowing us to make the point that different forms of heterogeneity have different aggregate implications, the case of heterogeneous adjustment is also useful to relate to much of the existing literature on financial frictions and business cycles. In particular, a number of papers make the assumption that each period "investment opportunities" arrive randomly to some exogenous fraction of entrepreneurs. Only entrepreneurs with an "investment opportunity" can acquire new investment goods; others cannot.[25] In our framework, this corresponds to an extreme form of heterogeneous investment costs: either investment costs are zero, corresponding to the arrival of an investment opportunity, or infinite.

2.1 Preferences, Technology and Budgets

There is a representative final goods producer with technology $Y_t = AK_t^{\alpha}L^{1-\alpha}$. Hence there is no heterogeneity in final goods production.[26] Since TFP is exogenous, an immediate implication is that a credit crunch cannot result in an efficiency wedge by assumption. Final goods producers rent capital from entrepreneurs at a rental rate R_t. In equilibrium,

[24] As pointed out by Kurlat (2010) who analyzes a similar model with heterogeneity in the efficiency of investment, a credit crunch in models like his and ours may manifest itself as an efficiency wedge in addition to an investment wedge if capital formation is measured inaccurately. If capital formation measures fail to take into account decreases in the efficiency of investment due to a worse allocation of resources, a credit crunch in one period would show up as decreased aggregate TFP in future periods. Related, a decline in current TFP would arise if GDP were measured using the relative prices of consumption and investment for some base year, as is commonly done in practice. We here instead operate under the assumption that capital is measured correctly and that GDP is measured in units of consumption at current prices.

[25] The following papers all feature such heterogeneous "investment opportunities": Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998), Kiyotaki and Moore (2005), Kiyotaki and Moore (2008), Gertler and Kiyotaki (2010), and Kurlat (2010). Exceptions with heterogeneous productivity are Kiyotaki (1998), Brunnermeier and Sannikov (2011), and Khan and Thomas (2010).

[26] An alternative assumption that also implies that final goods production can be summarized by an aggregate production function is that there is heterogeneity in productivity but final goods producers do not face any credit (or other) constraints. The fact that homogeneity of final goods producers is equivalent to perfect credit markets for final goods producers underlines again that the important feature of a model is how credit constraints *interact* with heterogeneity.

$R_t = \alpha Y_t / K_t$.

There is still a continuum of entrepreneurs indexed by $i \in [0, 1]$. These entrepreneurs have the same preferences as before, (1), but to make our point in the simplest way, we restrict the analysis to the case of log-utility $\sigma = 1$. They own and accumulate capital, and rent it to the representative firm. Entrepreneurs differ in their investment costs which we denote by ω_{it}. To increase the capital stock by x_{it} units of capital, an entrepreneur has to give up $\omega_{it} x_{it}$ units of the final good where $\omega_{it} \geq 1$. Each period, entrepreneurs draw a new investment cost from a distribution $\psi(\omega)$. The budget constraint of an entrepreneur is therefore

$$c_{it} + \omega_{it} x_{it} = R_t k_{it} - (1 + r_t) d_{it} + d_{it+1}$$

The law of motion for capital and the borrowing constraint are unchanged and given by (3) and (6). As before, entrepreneurs simply maximize their utility subject to these constraints. We also continue to assume that workers don't save and simply consume their labor income.

2.2 Aggregation and Credit Crunch

To answer the question whether there will be an investment wedge in this economy, we can aggregate individual Euler equations in a similar fashion to section 1.6.

Proposition 4 *In the economy with heterogeneous adjustment costs, the Euler equation of the "representative entrepreneur" takes the form*

$$\frac{C_{t+1}^E}{\beta C_t^E} \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di = \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta) \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di \tag{41}$$

Therefore, a credit crunch results in an entrepreneurial investment wedge, τ_{Xt}^E, defined recursively by

$$\frac{C_{t+1}^E}{\beta C_t^E} \tau_{Xt}^E - (1 - \delta) \tau_{Xt+1}^E = \frac{C_{t+1}^E}{\beta C_t^E} \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di - (1 - \delta) \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di \tag{42}$$

In contrast to the case with heterogeneous productivity, heterogeneous investment costs imply that the Euler equation of a representative entrepreneur (41) appears distorted. With imperfect credit markets, some entrepreneurs with investment costs, $\omega_{it} > 1$ will be active and hold positive capital stocks, $k_{it+1} > 0$ and therefore

$$\int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di > 1, \quad \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di > 1.$$

Comparing this aggregate Euler equation to the equation defining the entrepreneurial investment wedge, (15), it is obvious that $\tau_{Xt}^E \neq 0$. The second part of the proposition makes this intuition precise. It is in fact tempting to set the entrepreneurial investment wedge equal to $1 + \tau_{Xt}^E = \int \omega_{it}(k_{it}/K_t)di$. However, this would be incorrect because the weights on ω_{it} are given by k_{it+1}/K_{t+1} rather than k_{it}/K_t. Hence the more complicated definition of τ_{Xt}^E in (42) is needed.

Summarizing, in a model with heterogeneous investment costs the results from the model with heterogeneous productivities are reversed: a credit crunch results in an entrepreneurial investment wedge and – by construction – in no efficiency wedge. This is illustrated in Figure 5 (but see the discussion in footnote 24 on capital measurement issues and their implications for wedges).[27]

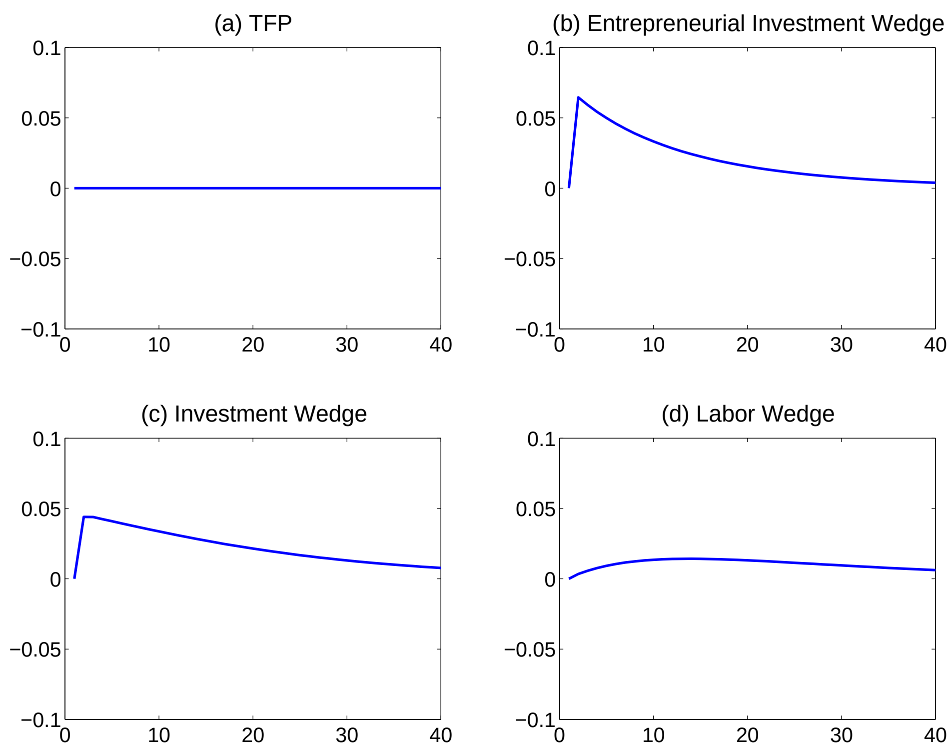

Fig. 5: Response to a Credit Crunch: Heterogenous Investment Costs

3 Heterogenous Recruitment Costs

We have shown that two different assumptions on the dimension along which individual entrepreneurs are heterogeneous can lead to a credit crunch resulting in either an efficiency or an investment wedge. In this section, we show that with heterogeneity in yet another

[27]We assume that the investment cost is uniformly distribution over $[1, 1.1]$. We consider the same shock to the collateral constraint as in the benchmark model.

dimension, namely labor recruitment costs, a credit crunch can also show up as a labor wedge.

Our starting point is the observation that with some form of labor search frictions, labor looks very much like capital. In particular, search models typically have the feature that, in order to increase their labor force, firms have to post vacancies one period in advance, exactly in the same way they invest to increase their stock of physical capital.[28] This implies that financial frictions have the potential to affect employment and hence the labor wedge.[29]

We show in this section that an extension of our previous model that features labor search frictions, in combination with heterogeneity across entrepreneurs in the cost of recruiting, can indeed deliver a labor wedge. The result follows exactly the same logic as our previous results on the investment and efficiency wedges. A credit crunch affects the allocation of labor across entrepreneurs with different recruitment costs in such a way that the aggregate cost of recruiting increases which delivers a drop in employment and hence an increase in the labor wedge. If instead, our model were to feature heterogeneity in productivity, a credit crunch would show up as a TFP wedge (see Appendix E where we work out such a model).

Heterogeneous recruitment costs are not merely a theoretical construct that we use to make our point. For instance, Davis, Faberman and Haltiwanger (2010) examine US data and find substantial heterogeneity in the cross-section of the "vacancy yield" of firms (the number of realized hires per reported job opening).

3.1 Preferences, Technology and Budgets

There is again a continuum of entrepreneurs indexed by $i \in [0,1]$. They have the preferences in (1). Each entrepreneur employs l_{it} workers and produces $y_{it} = Al_{it}$ units of output. Note that, in contrast to the previous sections, there is no capital for simplicity. With search frictions, labor becomes a state variable so dropping capital from the model allows us to work with only one state variable and retain closed form solutions. Furthermore productivity, A, is homogenous across firms. Therefore there is no efficiency wedge by assumption. An entrepreneur's employment evolves according to

$$l_{it+1} = x_{it} + (1-\delta)l_{it}, \tag{43}$$

where x_{it} is the number of new hires and δ is the exogenous rate of job separations. In order to hire a worker, an entrepreneur has has to post a costly vacancy. We assume that in order to attract x_{it} workers, an entrepreneur has to post $\omega_{it}x_{it}$ vacancies. We refer to $1/\omega_{it}$ as

[28] For a formulation where this is very apparent see Shimer (2010).

[29] For other frameworks in which financial frictions result in a labor wedge, see Jermann and Quadrini (2009), Arellano, Bai and Kehoe (2011).

the "vacancy yield". ω_{it} is drawn from $\psi(\omega)$, and is assumed to be iid across entrepreneurs and over time. Posting one vacancy costs one unit of the consumption good and hence the budget constraint of an entrepreneur is

$$c_{it} + \omega_{it} x_{it} - d_{it+1} = A l_{it} - w_t l_{it} - (1 + r_t) d_{it} \tag{44}$$

Note that we assume that all entrepreneurs pay a common wage, w_t. Given that search frictions introduce the possibility of different wage determination mechanisms and that these search frictions are heterogeneous across firms, this is not necessarily the case. However, we show below that such a common wage is consistent with individual rationality. We therefore proceed using the assumption of a common wage.

We change our borrowing constraint slightly. We assume that an entrepreneur can issue debt worth at most a fraction θ_t of output in the next period:[30]

$$d_{it+1} \leq \theta_t A l_{it+1}. \tag{45}$$

The reason for working with this slightly different constraint is that our previous constraint (6) has capital on the right-hand side. The result that a credit crunch shows up as a labor wedge if recruitment costs are heterogeneous would remain unchanged, if we reintroduced capital into the model and worked with the constraint (6). However, we could no longer obtain closed form solutions in this case. That being said, entrepreneurs maximize their utility, (1), subject to (43), (44) and (45).

Workers have preferences (4) which we specialize to

$$\sum_{t=0}^{\infty} \beta^t [u(C_t^W) - v(L_t)], \quad u(C) = \log C, \quad v(L) = \frac{\gamma \varepsilon}{1 + \varepsilon} L^{\frac{1+\varepsilon}{\varepsilon}} \tag{46}$$

where $\gamma > 0$ measures the disutility of working, and $\varepsilon > 0$ is the Frisch (constant marginal utility of wealth) elasticity of labor supply. We continue to assume that workers cannot save and simply consume their labor income, $C_t^W = w_t L_t$. With the preferences in (46), the marginal rate of substitution between leisure and consumption is given by $v'(L)/u'(C) = \gamma L^{1/\varepsilon} C$. Using that in our economy without capital, $\alpha = 0$ and $Y_t = A L_t$, the labor wedge – as defined in (14) – reduces to

$$\tau_{Lt} = 1 - \gamma L_t^{1/\varepsilon} C_t / A. \tag{47}$$

[30]This can again be motivated with a limited commitment problem: entrepreneurs can default on their loans. In this case, a creditor can obtain a fraction θ_t of output y_{it+1}. Knowing this, the creditor restricts his loan to be less than $\theta_t y_{it+1}$.

3.2 Wages

In models with search frictions, wages are typically determined through Nash-bargaining between employers and employees. We work out the Nash bargaining solution in Appendix D and show that the fact that entrepreneurs are heterogeneous in their recruitment costs, ω_{it}, results in entrepreneur-specific wages being paid. This makes the Nash solution somewhat complicated to work with, in particular given that our stated goal is to derive simple characterizations of aggregate variables. We therefore pursue a different approach in the main text, exploiting the well-known fact that search models typically feature a *set* of wages that workers are willing to accept and that employers are willing to pay (Hall, 2005). Any such wage satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement. This is useful because there is, in particular, a *common* wage that is in this bargaining set.

Lemma 6 *A sufficient condition for a common wage, w_t, to be in the bargaining set is*

$$\gamma L_t^{1/\varepsilon} C_t^W \leq w_t \leq A.$$

This Lemma simply states that any wage greater than the marginal rate of substitution, $\gamma L_t^{1/\varepsilon} C_t^W$, but smaller than the marginal product of labor, A, is in the bargaining set.[31] We then simply impose an ad-hoc wage rule, namely that the wage always lies exactly halfway between the bounds in Lemma 6:

$$w_t = \frac{\gamma L_t^{1/\varepsilon} C_t^W + A}{2}$$

Since workers are hand-to-mouth workers, $C_t^W = w_t L_t$, we immediately get that the common wage is $w_t = A/(2 - \gamma L_t^{(1+\varepsilon)/\varepsilon})$.

3.3 Individual Behavior

We obtain the following characterization of an entrepreneur's optimal choice of recruiters and hence workers next period.

Lemma 7 *The optimal labor choice of an entrepreneur satisfies*

$$\omega_{it} l_{it+1} - d_{it+1} = \beta \left[A l_{it} \left(1 + (1-\delta)\omega_{it}\right) - w_t l_{it} - (1+r) d_{it} \right] \tag{48}$$

Note that this expression is of the same form as the optimal savings policy function in the case with debt-constrained capital accumulation, (21). The term in brackets on the

[31] The same condition is made use of in Blanchard and Gali (2010).

right-hand-side of (48) is an entrepreneur's "cash-on-hand". The assumption of log-utility then implies that he then "saves" a constant fraction β of this "cash-on-hand". Here, one of the entrepreneur's assets is his stock of workers, valued by their opportunity cost in terms of final goods, $\omega_{it} l_{it+1}$.

3.4 Aggregation and Credit Crunch

We want to show that in the present model with heterogeneous recruitment costs, a credit crunch results in a labor wedge. To do so, we aggregate (48) over all entrepreneurs and obtain the following characterization of the evolution of employment and hence the labor wedge.

Proposition 5 *Aggregate employment evolves according to*

$$L_{t+1} = \beta \Omega_t^{-1} \left[A + (1-\delta) \int \omega_{it} \frac{l_{it}}{L_t} di - w_t \right] L_t, \qquad w_t = \frac{A}{2 - \gamma L_t^{(1+\varepsilon)/\varepsilon}}$$

where $\Omega_t \equiv \omega_{it} l_{it+1} / L_{t+1} di$ *is the "aggregate recruitment cost". A credit crunch increases* Ω_t *and hence decreases employment,* L_{t+1}*, resulting in an increase of the labor wedge,* τ_{Lt+1}*, defined in* (47)*.*

Figure 6 graphically illustrates the response to a credit crunch in the economy with heterogeneous recruitment costs.

4 Other Implications of a Credit Crunch

Up to this point we have focused on the implications of a credit crunch for standard aggregate variables, seen through the lens of a representative agent model. We have shown that the same fundamental shock has very different aggregate implications, depending on the nature of the underlying heterogeneity. These results raise the natural question: Does the use of more disaggregated data help to disentangle the source of aggregate fluctuations?

We now discuss how a credit crunch materializes in terms of various relatively more disaggregate variables: measures of external finance, the differential between the aggregate marginal product of capital and the interest rate, and the distribution of productivity of active entrepreneurs.

4.1 External Finance Measures

A variable that naturally contains information about the extent to which credit conditions have contracted is the use of external funds to finance investment or recruitment costs.

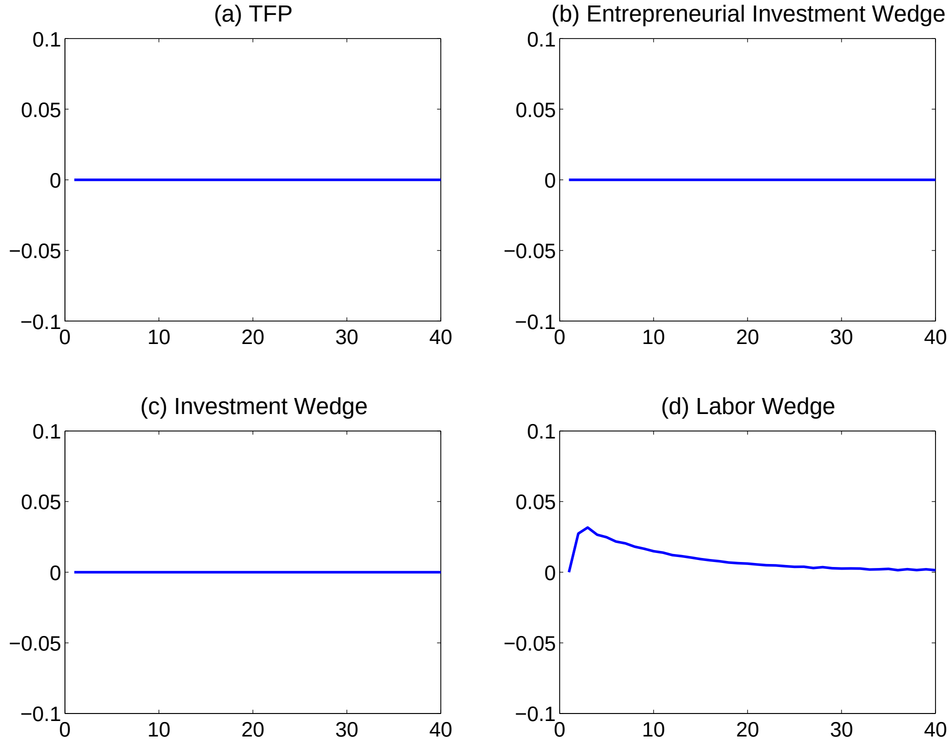

Fig. 6: Response to a Credit Crunch: Heterogenous Recruitment Costs

For instance, the ratio of (gross) aggregate debts D_t relative to the aggregate capital stock directly identifies the collateral constraint parameter θ_{t-1} in our first two models:[32] $D_t/K_t = \theta_{t-1}$, which uses the fact that entrepreneurs are either inactive so employ zero capital and lend, or are active in which case they use capital and exhaust their borrowing limit (6). If there are no capital markets, $\theta_t = 0$, there is no external finance: $D_t/K_t = 0$. If capital markets are perfect, $\theta_t = 1$, the entire capital stock of the economy is financed externally: $D_t/K_t = 1$. A related measure, which can be calculated more easily as it does not require information on the aggregate capital stock, is the ratio of (gross) aggregate debt relative to GDP. In our first two models, this ratio equals the product of the collateral constraint parameter and the capital to output ratio: $D_t/Y_t = \theta_{t-1}K_t/Y_t$.

In panel (a) of Figure 7 we show how this measure behaves in response to a credit crunch for the three models we consider. In all three cases we see that a credit crunch is associated with a decline in the ratio of external finance to GDP. In the models with heterogeneous investment or recruitment cost the ratio of external finance to GDP trivially contracts, at least on impact, as the capital to output ratio is constant. In the model with heterogeneous productivities the overall effect is ambiguous as it depends on the value of the elasticity of

[32] In the model with heterogeneous recruitment costs, the collateral constraint parameter equals the ratio of (gross) aggregate debt to GDP, $D_t/Y_t = \theta_{t-1}$.

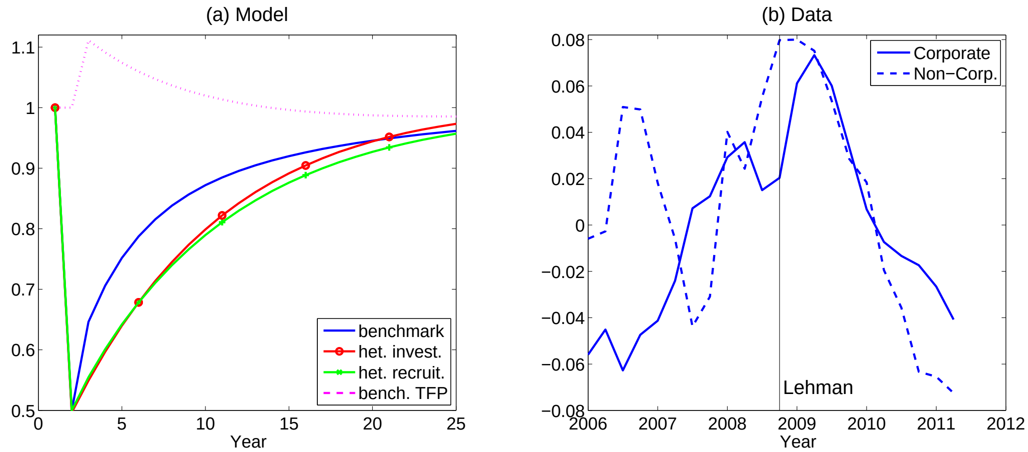

Fig. 7: Response of External-Finance-to-GDP to a Credit Crunch

TFP, Z_t, with respect to θ_{t-1}, but we can show that the ratio of external finance to GDP unambiguously declines in a credit crunch provided θ_{t-1} is small.

Panel (a) of Figure 7 also shows the behavior of external finance to GDP ratio in the benchmark model in response to a negative, pure TFP shock (of the same magnitude as the decline in TFP cause by the credit crunch). In contrast to a credit crunch, a negative TFP shock results in an increase in the external finance to GDP ratio. This is because θ_{t-1} is constant, and therefore, the behavior of the ratio of external finance to GDP is the mirror image of the capital to output ratio, which increases in response to a negative TFP shock.

In panel (b) of Figure 7 we present related measures for the US economy during the credit contraction of 2008, which for the business sector followed the bankruptcy of Lehman Brothers. In particular, we plot the ratio of credit market liabilities to the value-added for the non-farm, non-financial non-corporate (dashed line) and corporate (solid line) sectors. We present deviations of the series from a Hodrick-Prescott trend.[33] Since the collapse of Lehman the credit to GDP of the non-corporate sector declined by more than ten percent. For the corporate sector there was a slightly smaller decline, which started with a lag of two quarters.[34] To put these numbers into perspective, note that in the US National Income and Product Accounts, about 25-30% of business GDP is generated by the non-corporate sector. The behavior of external finance to GDP in the data is therefore broadly consistent with its behavior in our model following a credit crunch. Moreover, the response of this statistic to a credit crunch is consistent across model variants, and at the same time different from the one to a pure TFP shock.

[33] We use a value for the smoothing parameters of 1600, commonly used for quarterly data.

[34] Shourideh and Zetlin-Jones (2011) document a fact that may be related to this asymmetric behavior of external finance for the corporate and non-corporate sectors: for publicly traded firms (in Compustat), almost all investment is financed internally while most investment by privately held firms (from the Amadeus database) is financed through borrowing. This fact is related because all publicly traded firms are corporations and most privately held firms are not.

4.2 Return Premium

The differential between the aggregate marginal product of capital and the interest rate (return premium) is another variable that could in principle provide useful information to identify a credit crunch.

In panel (a) of Figure 8 we show the behavior of the return premium in a credit crunch in the benchmark model (solid line) and the model with heterogeneous investment cost (dashed line). In the benchmark model, a credit crunch results in a sharp decline of the interest rate

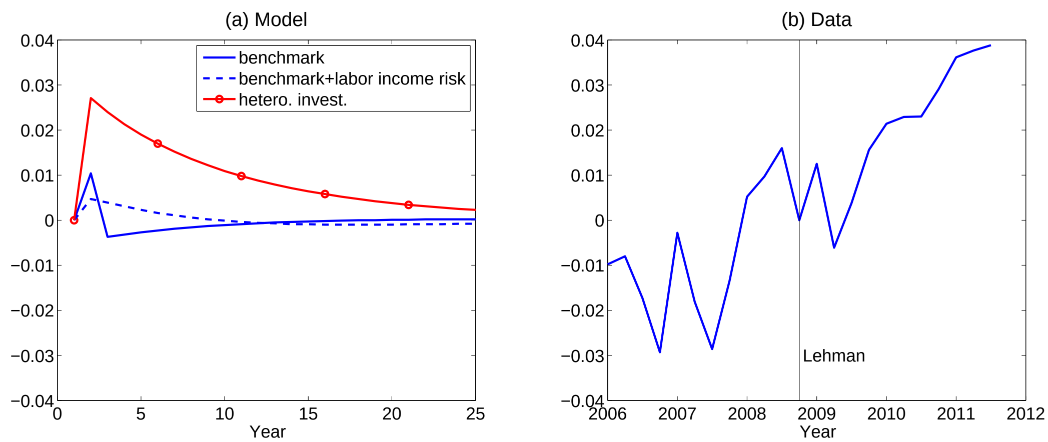

Fig. 8: Response of Return Premium to a Credit Crunch

that is greater than the fall in the future marginal product of capital. This leads to an increase in the return premium (solid line). The initial effect is eventually reversed, and the return premium turns negative. A smoother and monotonic version of this response is obtained in the version of the benchmark model where workers face labor income risk (dashed line, see Appendix B for details of the model). In the model with heterogeneous investment cost the return premium mimics the behavior of the investment wedge, which translates into an increase, and gradual decline, in the return premium. In the benchmark model with Pareto distributed shocks, the behavior of the return premium in response to a TFP shock is identical to the behavior of the return premium in response to a credit crunch. Thus, the return premium is not necessarily a very useful statistic to separate a pure TFP shock from a shock to collateral constraints.

In panel (b) of Figure 8 we show the evolution of the return premium for the US economy during the credit contraction of 2008.[35] Consistent with the broad implications of the model shown in the left panel, the differential between the aggregate marginal product of capital and the interest rate widened in the period that followed the fourth quarter of 2008.

[35]To measure the aggregate marginal product of capital we use $\alpha = 0.33$, real GDP data and capital stock constructed using the permanent inventory method, real investment data, and $\delta = 0.06$. We initialized the capital stock by $K_{1946.75} = I_{1947}/(0.06 + 0.032)$. For the real interest rate we use the 3-month Treasury bill secondary market annual rate minus the quarterly inflation rate of the GDP deflator.

4.3 Productivity and Firm Size Distribution and Reallocation

Given that we emphasize the importance of heterogeneity, it is natural to attempt to identify a credit crunch from the evolution over the business cycle of certain *distributions* of variables at the micro level. Consider our first model where entrepreneurs are heterogeneous in their productivity. As shown in Proposition 1, a credit crunch results in a decrease of the productivity threshold for being active. That is, there is entry of unproductive firms which causes a drop in TFP. This is consistent with evidence in Kehrig (2011) who documents that the dispersion of productivity in U.S. durable manufacturing is greater in recessions than in booms, which primarily reflects a relatively higher share of unproductive firms. This is in contrast to the so-called "cleansing effect" of recessions (Caballero and Hammour, 1994). That a productivity threshold for being active decreases is also a feature of our two other model variants. However, we do not know of any evidence that have documented this for the case of investment or recruitment costs.

Our model also predicts that a credit crunch results in a decrease of the share of employment of the, say, top ten percent most productive firms. As less productive entrepreneurs become active and use labor and capital, the share of factors employed by the most productive entrepreneurs declines. This implication is consistent with the evidence in Moscarini and Postel-Vinay (2010), provided that we interpret large firms in the data as more productive.

Finally, in all our three model variants, a credit crunch has real effects because it worsens the allocation of resources across heterogeneous firms. Measures of the reallocation of resources are therefore obvious statistics to examine as part of any attempt to identify a credit crunch. Eisfeldt and Rampini (2006) find that the amount of capital reallocation between firms (sales of property, plant and equipment, and acquisitions) decreases in recessions. For the Great Depression, Ziebarth (2011) documents that increases in the amount of resource misallocation can explain a substantial fraction of the TFP decline in two particular industries (50% for manufactured ice and 10 to 15% for cement). In a similar spirit, Sandleris and Wright (2011) argue that resource misallocation accounts for roughly half of the ten percent decline in manufacturing TFP during the Argentine financial crisis.

5 Conclusion

The main message of this paper is that while trying to learn about the sources of business cycles using a representative agent framework and aggregate data alone may seem appealing, this approach is invalidated by the presence of heterogeneity. This follows from our result that the mapping from a credit crunch in a heterogeneous agent economy to the aggregate variables in a representative agent economy depends crucially on the form of underlying heterogeneity; depending on where an economy features heterogeneity, a credit crunch can show up in very different aggregate variables. To make this argument concrete, we have examined the implications of a credit crunch for simple aggregate wedges. We have shown that a credit crunch shows up as an efficiency wedge if there is heterogeneity in the productivity of final goods producers. In contrast, it shows up as an investment wedge if investment costs are heterogeneous; or as a labor wedge if recruitment costs are heterogeneous.

In addition, we have argued that going beyond data on standard aggregates such as output, labor, and investment and instead examining more disaggregated data may allow for the identification of a credit crunch. An obvious candidate is the use of information on the amount of externally financed capital relative to GDP, as a statistic that tells an unambiguous story across models of financial friction.

Appendices

A Proofs

Proof of Lemma 1 The Lemma follows directly from using the definitions of cash-on-hand, m_t and net worth, a_t in the dynamic programming problem (17).

Proof of Lemma 2 The Lemma follows from the linearity of the portfolio allocation problem, i.e. the maximization problem defining the function $\tilde{m}_{t+1}(a', z)$ in Lemma 1.

Proof of Lemma 3 Consider the Bellman equation in (1) which can be written as

$$V_t(m,z) = \max_{a'} \; \log(m-a') + \beta \mathbb{E} V_{t+1}(m_{t+1}(a',z),z')$$

$$m_{t+1}(a',z) = \tilde{m}_{t+1}(z)a', \quad \tilde{m}_{t+1}(z) = \max\{z\pi_{t+1} - r_{t+1} - \delta, 0\}\lambda_t + 1 + r_{t+1}$$

The proof proceeds with a guess and verify strategy. Guess that the value function takes the form $V_t(m,z) = v_t(z) + B\log m$, and substitute into the Bellman equation. In particular, note that $\mathbb{E}V_t(m',z') = \mathbb{E}v_t(z') + B\log m'$. The first order equation is

$$\frac{1}{m-a'} = \beta \frac{B}{\tilde{m}_{t+1}(z)a'}\tilde{m}_{t+1}(z) \quad \Rightarrow \quad a' = \frac{\beta B}{1+\beta B}m$$

The Bellman equation becomes

$$v_t(z) + B\log m = \log\left[\frac{1}{1+\beta B}m\right] + \beta\left[\mathbb{E}v_{t+1}(z') + B\log\frac{\beta B}{1+\beta B}m\right]$$

Collecting the terms involving $\log m$, we see that $B = 1/(1-\beta)$ and $a' = \beta m$ as claimed.$\Box$

Proof of Proposition 1 Consider first the bond market clearing condition. Using (19) and (20), we have that individual debt is $d_{it+1} = (\lambda_t - 1)\beta m_{it}$ if $z_{it+1} \geq \underline{z}_{t+1}$ and $d_{it+1} = -\beta m_{it}$ otherwise. Using that z_{it+1} is independent of m_{it}, (10) becomes

$$(\lambda_t - 1)\int_{\underline{z}_{t+1}}^{\infty} \psi(z)dz - \int_0^{\underline{z}_{t+1}} \psi(z)dz = 0 \quad \text{or} \quad \lambda_t(1 - \Psi(\underline{z}_{t+1})) = 1. \tag{49}$$

Labor demand is

$$l_{it} = \left(\frac{\pi_t}{\alpha}\right)^{1/(1-\alpha)} k_{it}z_{it} \tag{50}$$

It follows that output is $y_{it} = (\pi_t/\alpha)z_{it}k_{it}$. Aggregate output is then

$$Y_t = \int y_{it}di = \frac{\pi_t}{\alpha}\int z_{it}k_{it}di.$$

Since $k_{it} = \lambda_{t-1}a_{it} = \lambda_{t-1}\beta m_{it-1}$ if $z_{it} \geq \underline{z}_t$ and zero otherwise, we have

$$\int z_{it}k_{it}di = \lambda_{t-1}X_t\beta M_{t-1} = \lambda_{t-1}X_tK_t, \quad X_t \equiv \int_{\underline{z}_t}^{\infty} z\psi(z)dz \tag{51}$$

Hence $Y_t = (\pi_t/\alpha)\lambda_{t-1}X_tK_t$. Next, consider the labor market clearing condition. Integrating (50) over all i,

$$L = \left(\frac{\pi_t}{\alpha}\right)^{1/(1-\alpha)} \lambda_{t-1}X_tK_t. \tag{52}$$

Rearranging $\pi_t = \alpha(\lambda_{t-1}X_t)^{\alpha-1}K_t^{\alpha-1}L^{1-\alpha}$ and using it the expression for output $Y_t = (\lambda X_t)^\alpha K_t^\alpha L^{1-\alpha}$. Eliminating λ_{t-1} using (49), we obtain (22). The law of motion for aggregate capital is derived by integrating (21) over all entrepreneurs:

$$K_{t+1} = \beta\left[\pi_t\int z_{it}k_{it}di + (1-\delta)K_t\right] \tag{53}$$

Using (51) and (52),

$$K_{t+1} = \beta\left[\alpha Z_tK_t^\alpha L^{1-\alpha} + (1-\delta)K_t\right], \quad Z_t = (\lambda_tX_t)^\alpha,$$

which is equation (23) in Proposition 1.□

Proof of Proposition 2

Part (i): That $\tau^E_{Xt} = 0$ follows directly from inspection of (16) and (26).

Part (ii): Aggregate consumption is $C_t = C_t^W + C_t^E$. Hence

$$\frac{C_{t+1}}{C_t} = \frac{C^E_{t+1}}{C^E_t}\frac{C^E_t}{C_t} + \frac{C^W_{t+1}}{C^W_t}\frac{C^W_t}{C_t} = \frac{C^E_{t+1}}{C^E_t} + \frac{C^W_t}{C_t}\left(\frac{C^W_{t+1}}{C^W_t} - \frac{C^E_{t+1}}{C^E_t}\right) \tag{54}$$

Using (26),

$$\frac{C_{t+1}}{C_t} = \beta\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] + \frac{C^W_t}{C_t}\left(\frac{C^W_{t+1}}{C^W_t} - \frac{C^E_{t+1}}{C^E_t}\right)$$

Subtracting (15) from both sides and rearranging, we obtain (28).□

Proof of Lemma 5 (LHS) We show in Appendix C that the saving policy function takes the form $a_{it+1} = s_{t+1}(z_{it+1})m_{it}$ or $k_{it+1} - d_{it+1} = s_{t+1}(z_{it+1})m_{it}$. Aggregating over all types:

$$K_{t+1} = \bar{s}_{t+1}M_t, \quad C_t^E = (1 - \bar{s}_{t+1})M_t, \quad \bar{s}_{t+1} \equiv \int_0^\infty s_{t+1}(z)\psi(z)dz$$

Since $R^a_{it+1} = m_{it+1}/a_{it+1}$, the individual Euler equations (29) can be written as

$$u'(c_{it}) = \beta\mathbb{E}[u'(c_{it+1})]\frac{m_{it+1}}{a_{it+1}}$$

Therefore

$$\int \frac{u'(c_{it})}{\beta\mathbb{E}[u'(c_{it+1})]}\frac{a_{it+1}}{K_{t+1}}di = \frac{M_{t+1}}{K_{t+1}} = \frac{C^E_{t+1}}{C^E_t}\frac{1}{\bar{s}_{t+1}}\frac{1-\bar{s}_{t+1}}{1-\bar{s}_{t+2}}$$

where the last equality uses that $C_t^E = (1 - \bar{s}_{t+1})M_t$ and $K_{t+1} = \bar{s}_{t+1}M_t$.□

Proof of Proposition 3

Part (i) Combining Lemmas 4 and 5,

$$\frac{C^E_{t+1}}{C^E_t}\frac{1}{\bar{s}_{t+1}}\frac{1-\bar{s}_{t+1}}{1-\bar{s}_{t+2}} = \alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta.$$

Combining with the definition of the entrepreneurial investment wedge (15) we obtain (39).

Part (ii) Subtract (16) from (15) and use that preferences are CRRA

$$\left(\frac{C_{t+1}}{C_t}\right)^\sigma (1 + \tau_{Xt}) - \left(\frac{C^E_{t+1}}{C^E_t}\right)^\sigma (1 + \tau^E_{Xt}) = \beta(1-\delta)(\tau_{Xt+1} - \tau^E_{Xt+1}) \tag{55}$$

Substituting (54) into (55), we obtain (40).□

Proof of Proposition 4 Denote the Lagrange multiplier on (6) by μ_{it} and that on the constraint $k_{it+1} \geq 0$ by ψ_{it}. The two Euler equations with respect to capital and debt are

$$\frac{1}{c_{it}}\omega_{it} = \beta\mathbb{E}\left[\frac{1}{c_{it+1}}\right][R_{t+1} + (1-\delta)\omega_{it+1}] + \mu_{it}\theta_t + \psi_{it} \tag{56}$$

$$\frac{1}{c_{it}} = \beta\mathbb{E}\left[\frac{1}{c_{it+1}}\right](1 + r_{t+1}) + \mu_{it} \tag{57}$$

Multiply (56) by k_{it+1} and (57) by $-d_{it+1}$ and add them

$$\frac{1}{c_{it}}[\omega_{it}k_{it+1} - d_{it+1}] = \beta\mathbb{E}\left[\frac{1}{c_{it+1}}\right][R_{t+1}k_{it+1} + (1-\delta)\omega_{it+1}k_{it+1} - (1+r_{t+1})d_{it+1}] + \mu_{it}[\theta k_{it+1} - d_{it+1}] + \psi_{it}k_{it+1}$$

The complementary slackness condition corresponding to (6) is $\mu_{it}[\theta k_{it+1} - d_{it+1}] = 0$ and $\psi_{it}k_{it+1} = 0$. It can then be verified that this Euler equation is satisfied by $k_{it+1}\omega_{it} - d_{it+1} = \beta m_{it}$ and $c_{it} = (1-\beta)m_{it}$ where $m_{it} \equiv R_t k_{it} + (1-\delta)\omega_{it}k_{it} - (1+r_t)d_{it}$. Therefore

$$C_t = (1-\beta)\left[R_t K_t + (1-\delta)\int \omega_{it}k_{it}di\right] \tag{58}$$

$$K_{t+1} = \beta \left[\int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di \right]^{-1} \left[R_t K_t + (1-\delta) \int \omega_{it} k_{it} di \right] \tag{59}$$

Combining (58) and (59) and using that $R_{t+1} = \alpha Y_{t+1}/K_{t+1}$, yields (41).

Proof of Lemma 6 The steps described here follow Shimer (2010). First, consider entrepreneurs who solve

$$V_t(l, d, \omega) = \max_{c,x,d'} \log c + \beta \mathbb{E} V_{t+1}(l', d', \omega') \quad \text{s.t.}$$
$$c + \omega x - d' = Al - w_t l - (1 + r_t)d, \quad l' = (1-\delta)l + x, \quad x \geq 0, \quad d' \leq \phi A l'$$

The envelope condition gives us the marginal value to an entrepreneur of having an extra worker paid w_t

$$V_{lt}(l_{it}, d_{it}, \omega_{it}) = \frac{A + (1-\delta)\omega_{it} - w_t}{c_{it}}. \tag{60}$$

Next, consider workers. From their point of view, employment evolves exogenously as $L_{t+1} = (1-\delta)L_t + f_t(1 - L_t)$. Here f_t is the probability of finding a job which is defined by the requirement that the number of workers finding jobs, $f_t(1 - L_t)$, is equal to the number of workers recruited by firms $\int x_{it} di$ and hence $f_t = \int x_{it} di/(1 - L_t)$. The value of a worker is

$$W_t(L_t) = u\left(w_t L_t\right) - v(L_t) + \beta W_{t+1}[(1-\delta)L_t + f_t(1 - L_t)]$$

The marginal value for workers at the equilibrium level of employment of having one worker employed at a wage w_t in period t rather than unemployed is

$$W_t'(L_t) = \frac{w_t}{C_t^W} - \gamma L_t^{1/\varepsilon} + \beta(1 - \delta - f_t) W_{t+1}'(L_{t+1}). \tag{61}$$

Entrepreneurs are willing to pay all wages for which $V_{lt}(l_{it}, d_{it}, \omega_{it}) \geq 0$ in (67). Workers are willing to accept all wages for which $W_t'(L_t) \geq 0$ in (61). It is easy to see that a wage satisfying the condition in Lemma 6 satisfies both requirements.$\square$

Proof of Lemma 7 Defining "cash-on-hand" $m_{it} \equiv Al_{it} + (1-\delta)\omega_{it} l_{it} - w_t l_{it} - (1 + r_t)d_{it}$, the budget constraint of an entrepreneur becomes $c_{it} - d_{it+1} + \omega_{it} l_{it+1} = m_{it}$. The problem of an entrepreneur can then be stated in recursive form as

$$V(m, \omega) = \max_{l',d'} \log\left(m - \omega l' + d'\right) + \beta \mathbb{E} V(m', \omega') \quad \text{s.t.}$$

$$m' = Al' + (1-\delta)\omega' l' - wl' - (1 + r)d', \quad d' \leq \phi A l'$$

Following similar steps as in the proof of Lemma 3, entrepreneurs save a constant fraction β of their cash-on-hand, m_{it}, and hence their optimal labor choice satisfies (48).$\square$

Proof of Proposition 5 The proposition follows directly from aggregating (48) across all entrepreneurs.

B Alternative Modeling of Workers

We consider an extension where workers are allow to save in a risk-free asset and they face shocks to their efficiency units of labor h. The recursive problem of a worker is summarized by the Bellman equation:

$$\begin{aligned} V_t^W(a,h) &= \max_{c,l,a'} u(c) - v(l) + \beta \mathbb{E} V_{t+1}^W(a',h') \\ &\quad \text{s.t.} \\ &\quad c + a' = w_t h l + (1+r_t)a \end{aligned}$$

In the simulations presented in Figure 3 we consider a simple two state process for the efficiency units of labor, $h \in \{0,1\}$, with transition probabilities [.2 .8; .05 .95]. In addition, we assume that workers with zero efficiency units of labor receive a transfer equals to $0.4w_t$. We interpret this model as roughly capturing an unemployment shock in a world with unemployment insurance that offers a 40% replacement ratio.

C Analysis of Economy with CRRA Preferences and Persistent Shocks

In this appendix, we analyze the case with CRRA preferences. For sake of generality and to show that the assumption of iid shocks in the main text is not crucial for our main results, we also allow for persistence in the stochastic process of entrepreneurial productivity. In particular, we assume that in each period entrepreneurs retain their productivity with probability γ. With the complementary probability $1-\gamma$ entrepreneurs draw a new productivity from the distribution $\psi(z)$.

C.1 Characterization of Individual's Saving Problem

The value function of an entrepreneur with cash-in-hand m and ability z solve

$$V_t(m,z) = \max_{a'} \frac{(m-a')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V_{t+1}(m_{t+1}(a',z),z')|z\right]$$

where $m_{t+1}(a',z) = \tilde{m}_{t+1}(z)a', \quad \tilde{m}_{t+1}(z) = \max\{z\pi_{t+1} - r_{t+1} - \delta, 0\}\lambda_t + 1 + r_{t+1}$.
The proof proceeds with a guess and verify strategy. Guess that the value function takes

the form $V_t(m,z) = v_t(z)\frac{m^{1-\sigma}}{1-\sigma}$, and substitute into the Bellman equation.

$$v_t(z)\frac{m^{1-\sigma}}{1-\sigma} = \max_{a'} \frac{(m-a')^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}\left[v_{t+1}(z')|z\right]\frac{[\tilde{m}_{t+1}(z)a']^{1-\sigma}}{1-\sigma}$$

It will be useful to define the auxiliary variable

$$\nu_{t+1}(z) = \beta\mathbb{E}[v_{t+1}(z')|z]\tilde{m}_{t+1}(z)^{1-\sigma} \tag{62}$$

so that the Bellman equation is

$$v_t(z)\frac{m^{1-\sigma}}{1-\sigma} = \max_{a'} \frac{(m-a')^{1-\sigma}}{1-\sigma} + \nu_{t+1}(z)\frac{(a')^{1-\sigma}}{1-\sigma} \tag{63}$$

The first order condition is

$$(m-a')^{-\sigma} = \nu_{t+1}(z)(a')^{-\sigma}$$

or

$$a' = s_{t+1}(z)m, \quad s_{t+1}(z) \equiv \frac{1}{1+\nu_{t+1}(z)^{-1/\sigma}}$$

Consumption is

$$c = \frac{\nu_{t+1}(z)^{-1/\sigma}}{1+\nu_{t+1}(z)^{-1/\sigma}}m$$

Substituting into the Bellman equation (63) and canceling the terms involving $m^{1-\sigma}/(1-\sigma)$,

$$v_t(z) = \left(\frac{\nu_{t+1}(z)^{-1/\sigma}}{1+\nu_{t+1}(z)^{-1/\sigma}}\right)^{1-\sigma} + \nu_{t+1}(z)\left(\frac{1}{1+\nu_{t+1}(z)^{-1/\sigma}}\right)^{1-\sigma}$$

which after some manipulation becomes

$$v_t(z) = \left(1+\nu_{t+1}(z)^{1/\sigma}\right)^{\sigma}$$

or using the definition of $\nu_{t+1}(z)$ in (62),

$$v_t(z) = \left(1+\left\{\beta\mathbb{E}\left[v_{t+1}(z')|z\right]\tilde{m}_{t+1}(z)^{1-\sigma}\right\}^{1/\sigma}\right)^{\sigma}$$

This is a functional equation in $v_t(z)$ that can be solved numerically.

C.2 Evolution of the Wealth Density, Aggregate Capital and Productivity

The evolution of the wealth density $\xi_t(z)$ is described by the following functional equation

$$\xi_{t+1}(z) = \frac{K_t}{K_{t+1}}\left[\gamma s_{t+1}(z)\tilde{m}_t(z)\xi_t(z) + (1-\gamma)\psi(z)s_{t+1}(z)\int\tilde{m}_t(z_{\text{-}1})\xi(z_{\text{-}1})dz_{\text{-}1}\right] \tag{64}$$

Using Lemma 4 and integrating over all z we obtain a law of motion for aggregate capital

$$K_{t+1} = \gamma K_t \int s_{t+1}(z)\tilde{m}_t(z)\xi_t(z)dz + (1-\gamma)\bar{s}_{t+1}\left[\alpha Y_t + (1-\delta)K_t\right]. \tag{65}$$

There are two cases for which the model allows for a simple aggregation, given the evolution of aggregate productivity Z_t. First, if we assume that entrepreneurs' productivity is iid over time, equation C.2 specializes to

$$K_{t+1} = \bar{s}_{t+1}\left[\alpha Y_t + (1-\delta)K_t\right].$$

The second correspond to the case of log preferences. Using that $s_{t+1}(z) = \bar{s}_{t+1} = \beta$ and applying Lemma 4 to the first term in the right hand side of equation C.2 we obtain a simple equation describing the evolution of aggregate capital:

$$K_{t+1} = \beta\left[\alpha Y_t + (1-\delta)K_t\right].$$

While we can aggregate the model given the evolution of aggregate productivity, in the more general model the evolution of aggregate productivity is itself a function of the wealth density. Defining

$$\Xi(z) \equiv \int_0^z \xi(x)dx,$$

aggregate productivity is a capital weighted average of entrepreneurs' productivity

$$Z_t = \left(\frac{\int_{\underline{z}_t}^{\infty} z\xi(z)dz}{1-\Xi(\underline{z}_t)}\right)^{\alpha}.$$

Finally, the cutoff is defined by

$$\lambda_{t-1}(1-\Xi(\underline{z}_t)) = 1.$$

D Generalized Nash Bargaining: Entrepreneur-Specific Wage

Instead of a common wage, we could have worked with entrepreneur-specific wages that are determined by Nash bargaining. We here derive these wages for completeness. The steps described here follow Shimer (2010). We modify his derivations to allow for heterogeneity on the side of employers. Let $V_l(l_{it}, d_{it}, \omega_{it}, t)$ denote the marginal utility for entrepreneur i with employment l_{it}, debt d_{it}, and recruitment cost, ω_{it} of employing a worker at wage w_{it}. Let $W_i(\{l_{it}\}, t)$ denote the marginal utility for workers at the equilibrium level of employment of having one worker employed at a wage w_{it} in period t rather than unemployed.[36].

[36] As shown below, this value depends on the entire distribution of employment, $\{l_{it}\}$

Consider first the value of an entrepreneur which is given by

$$
\begin{aligned}
V(l,d,\omega) = \max_{c,x,d'} \log c + \beta \mathbb{E} V(l',d',\omega') \quad \text{s.t.} \\
c + \omega x - d' = Al - w_t l - (1+r_t)d, \\
l' = (1-\delta)l + x, \quad x \geq 0, \quad d' \leq \phi A l'
\end{aligned}
$$

The first order condition for recruiting, x_{it}, is

$$
\omega_{it}\frac{1}{c_{it}} = \beta \mathbb{E} V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1) \tag{66}
$$

and the envelope condition

$$
V_l(l_{it}, d_{it}, \omega_{it}, t) = \frac{A_t + (1-\delta)\omega_{it} - w_{it}}{c_{it}} \tag{67}
$$

This is the marginal value to an entrepreneur of having an extra worker paid w_{it}

Next, consider workers. Workers take as given the distribution of employment and its evolution of employment. In particular, they take as given the (exogenous) job separation rate δ and the (endogenous) probability of finding a job at firm i, f_{it}. This job finding rate is defined by the requirement that the number of workers finding jobs, $f_{it}(1-L_t)$, is equal to the number of workers recruited by firms x_{it} and hence $f_{it} = x_{it}/(1-L_t)$. From the point of view of workers employment then evolves as $l_{it+1} = (1-\delta)l_{it} + f_{it}(1-L_t)$. The value of a worker can then be written in recursive form as

$$
W(\{l_{it}\},t) = \log\left(\int w_{it} l_{it} di\right) - \gamma \int l_{it} di + \beta \mathbb{E} W(\{l_{it+1}\}, t+1)
$$

The envelope condition is

$$
W_i(\{l_{it}\},t) = \frac{w_{it}}{C_t^W} - \gamma + \beta(1-\delta)\mathbb{E}W_i(\{l_{it+1}\},t+1) - \beta \int f_{jt} \mathbb{E} W_j(\{l_{it+1}\},t+1) dj \tag{68}
$$

Following the same analysis as in Shimer (2010), it can easily be shown that if wages are determined by generalized Nash bargaining, the entrepreneur-specific wage w_{it} satisfies

$$
(1-\phi) W_i(\{l_{it}\},t) C_t^W = \phi V_l(l_{it}, d_{it}, \omega_{it}, t) c_{it} \tag{69}
$$

where $\phi \in [0,1]$ represents the worker's bargaining power. Multiply (68) by $(1-\phi)C_t^W$ to obtain

$$
\begin{aligned}
(1-\phi) W_i(\{l_{it}\},t) C_t^W = (1-\phi)(w_{it} - \gamma C_t^W) \\
+ \frac{C_t^W}{C_{t+1}^W}\left[\beta(1-\delta)C_{t+1}^W(1-\phi)\mathbb{E}W_i(\{l_{it+1}\},t+1) - \beta \int f_{jt} C_{t+1}^W (1-\phi) \mathbb{E} W_j(\{l_{it+1}\},t+1) dj\right]
\end{aligned}
$$

Substitute in from (69)

$$\phi V_l(l_{it}, d_{it}, \omega_{it}, t)c_{it} = (1-\phi)(w_{it} - \gamma C_t^W)$$

$$+\frac{C_t^W}{C_{t+1}^W}\left[\beta(1-\delta)\phi \mathbb{E}V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1)c_{it+1} - \beta\int f_{jt}\phi \mathbb{E}V_l(l_{jt+1}, d_{jt+1}, \omega_{jt+1}, t+1)c_{jt+1}dj\right]$$

From (66)

$$\mathbb{E}V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1)c_{it+1} = \frac{c_{it+1}}{c_{it}}\omega_{it}$$

to eliminate $V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1)$ and (67) to eliminate $V_l(l_{it}, d_{it}, \omega_{it}, t)$,

$$\phi\left[A + (1-\delta)\omega_{it} - w_{it}\right] = (1-\phi)(w_{it} - \gamma C_t^W) + \frac{C_t^W}{C_{t+1}^W}\phi\left[(1-\delta)\frac{c_{it+1}}{c_{it}}\omega_{it} - \int \frac{c_{jt+1}}{c_{jt}} f_{jt}\omega_{jt}dj\right]$$

Rearranging

$$\phi\left[A + (1-\delta)\omega_{it}\left(1 - \frac{C_t^W}{C_{t+1}^W}\frac{c_{it+1}}{c_{it}}\right) + \frac{C_t^W}{C_{t+1}^W}\int \frac{c_{jt+1}}{c_{jt}} f_{jt}\omega_{jt}dj - w_{it}\right] = (1-\phi)(w_{it} - \gamma C_t^W)$$

And hence

$$w_{it} = \phi\left(A + (1-\delta)\omega_{it}\left(1 - \frac{C_t^W}{C_{t+1}^W}\frac{c_{it+1}}{c_{it}}\right) + \frac{C_t^W}{C_{t+1}^W}\int \frac{c_{jt+1}}{c_{jt}} f_{jt}\omega_{jt}dj\right) + (1-\phi)\gamma C_t^W$$

This is the Nash-bargaining solution. Note that wages are entrepreneur-specific because of heterogeneity in recruitment costs, and also because financing constraints imply that consumption growth rates differ across entrepreneurs.[37]

E Model with Homogenous Recruitment Costs and Heterogenous Productivity

Consider the same model as in section 3 but where entrepreneurs are heterogeneous in their productivity, $y_{it} = z_{it}l_{it}$, z_{it} is drawn from $\psi(z)$ iid over time and across entrepreneurs. Everything remains unchanged except the budget constraint of an entrepreneur which now is

$$c_{it} + x_{it} - d_{it+1} = z_{it}l_{it} - w_t l_{it} - (1+r_t)d_{it}$$

The equilibrium has the feature that there is a productivity cutoff for being active $\underline{z}_t$. Only entrepreneurs who are above this cutoff are active. Hence the equivalent of the sufficient

[37] Without heterogeneity and with perfect financial markets (implying $c_{it+1}/c_{it} = C_{t+1}^W/C_t^W$), the wage would simply be $w_t = \phi(A + f\omega) + (1-\phi)\gamma C_t^W$.

condition in Lemma 6 for a common wage, w_t, to be in the bargaining set is $\gamma L_t^{1/\varepsilon} C_t^W \leq w_t \leq \underline{z}_t$. We again impose that the wage lies halfway between these bounds:

$$w_t = \frac{\gamma L_t^{1/\varepsilon} C_t^W + \underline{z}_t}{2} \quad \Rightarrow w_t = \frac{\underline{z}_t}{2 - \gamma L_t^{(1+\varepsilon)/\varepsilon}}$$

where the second equality follows because $C_t^W = w_t L_t$. Defining cash-on-hand $m_{it} = z_{it} l_{it} + (1-\delta) l_{it} - w_t l_{it} - (1+r) d_{it}$ and net worth $a_{it+1} = l_{it+1} - d_{it+1}$, the Bellman equation of an entrepreneur is

$$V(m,z) = \max_{a',l',d'} \log(m - a') + \beta \mathbb{E} V(\tilde{m}(a',z), z')$$

$$\tilde{m}(a',z) = \max_{l',k'} \; zl' + (1-\delta)l - w'l' - (1+r')d', \quad l' - d' = a', \quad l' \leq \lambda(z) a', \quad \lambda(z) = \frac{1}{1-\theta z}$$

Optimal labor choice therefore satisfies

$$l_{it+1} = \begin{cases} \lambda(z_{it+1}) a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ 0, & z_{it+1} < \underline{z}_{t+1} \end{cases} \tag{70}$$

where $\underline{z}_{t+1} = w_{t+1} - 1 + \delta$. We can again show that the assumption of log-utility implies that agents save a constant fraction of cash-on-hand, $a_{it+1} = \beta m_{it}$ or

$$l_{it+1} - d_{it+1} = \beta \left[z_{it} l_{it} + (1-\delta) l_{it} - w_t l_{it} - (1+r) d_{it} \right] \tag{71}$$

Next we can find an expression for the productivity cutoff, $\underline{z}$. From (70), we have

$$L_t = \int l_{it} di = \int_{\underline{z}_t}^{\infty} \lambda(z) \psi(z) dz \beta M_{t-1} = \int_{\underline{z}_t}^{\infty} \lambda(z) \psi(z) dz L_t$$

Hence the cutoff, $\underline{z}_t$, is pinned down from $\int_{\underline{z}_t}^{\infty} \lambda(z) \psi(z) dz = 1$. Aggregating over all entrepreneurs and using (70) gives

$$L_{t+1} = \beta \left[Z_t + 1 - \delta - w_t \right] L_t, \quad w_t = \frac{\underline{z}_t}{2 - \gamma L_t^{(1+\varepsilon)/\varepsilon}} \quad \text{where} \quad Z_t = \int_{\underline{z}_t}^{\infty} z \lambda(z) \psi(z) dz$$

is TFP. Note that employment, and hence the labor wedge, only move because of movements in TFP.

F Behavior of External Finance Relative to GDP

This section derives the effect of a credit crunch in period 1 on the external finance to GDP ratio in period 2 (the first period where a credit crunch have an effect on the economy), and contrasts it with the effect of a pure TFP shock in period 2.

Proposition 6

$$\frac{\partial \log\left(\frac{D_2}{Y_2}\right)}{\partial \log Z_1} = -1 < \frac{1}{\lambda_1 - 1}\left(\frac{1}{\lambda_1} - (1 - \frac{1}{\lambda_1})\alpha\left(1 - \frac{\underline{z}_2}{\frac{\int_{\underline{z}_2}^{\infty} z\psi(z)dz}{1-\Psi(\underline{z}_2)}}\right)\right) = \frac{\partial \log\left(\frac{D_2}{Y_2}\right)}{\partial \log \lambda_1}.$$

Proof

$$\begin{aligned}\frac{D_2}{Y_2} &= \frac{\int_{z_{i,2}\geq \underline{z}_t} d_{i,2}di}{Y_2} \\ &= \frac{(\lambda_1 - 1)\left(1 - \Psi(\underline{z}_2)\right) K_2}{Z_2 K_2^{\alpha} L^{1-\alpha}}\end{aligned}$$

$$\frac{\partial\left(\frac{D_2}{Y_2}\right)}{\partial \lambda_1} = \left(\frac{K_2}{L}\right)^{1-\alpha}\frac{1}{Z_2}\left[1 - \Psi(\underline{z}_2) - (\lambda_1 - 1)\psi(\underline{z}_2)\frac{\partial \underline{z}_2}{\partial \lambda_1} - \frac{(\lambda_1 - 1)\left(1 - \Psi(\underline{z}_2)\right)}{Z_2}\frac{\partial Z_2}{\partial \lambda_1}\right] \quad (72)$$

Differentiating (24) with respect to λ_1

$$\frac{\partial Z_2}{\partial \lambda_1} = \alpha\left(\frac{\int_{\underline{z}_2}^{\infty} z\psi(z)dz}{1 - \Psi(\underline{z}_2)}\right)^{\alpha-1}\frac{\psi(\underline{z}_2)}{1 - \Psi(\underline{z}_2)}\left[\frac{\int_{\underline{z}_2}^{\infty} z\psi(z)dz}{1 - \Psi(\underline{z}_2)} - \underline{z}_2\right]\frac{\partial \underline{z}_2}{\partial \lambda_1}$$

Applying the Implicit Function Theorem to (25)

$$\frac{\partial \underline{z}_2}{\partial \lambda_1} = \frac{1 - \Psi(\underline{z}_2)}{\lambda_1 \psi(\underline{z}_2)}$$

Substituting the expressions for $\partial Z_2/\partial\lambda_1$ and $\partial \underline{z}_2/\partial\lambda_1$ into (72)

$$\frac{\partial\left(\frac{D_2}{Y_2}\right)}{\partial \lambda_1}\frac{\lambda_1}{\frac{D_2}{Y_2}} = \frac{1}{\lambda_1 - 1}\left(\frac{1}{\lambda_1} - (1 - \frac{1}{\lambda_1})\alpha\left(1 - \frac{\underline{z}_2}{\frac{\int_{\underline{z}_2}^{\infty} z\psi(z)dz}{1-\Psi(\underline{z}_2)}}\right)\right).$$

It is straightforward to see that the elasticity of external finance to GDP with respect to an exogenous change in TFP equals

$$\frac{\partial\left(\frac{D_2}{Y_2}\right)}{\partial Z_2}\frac{Z_2}{\frac{D_2}{Y_2}} = -1.$$

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