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Intra Firm Bargaining and Shapley Values

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by

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Abstract

We study two wage bargaining games between a firm and multiple workers. We revisit the bargaining game proposed by Stole and Zwiebel (1996a). We show that, in the unique Subgame Perfect Equilibrium, the gains from trade captured by workers who bargain earlier with the firm are larger than those captured by workers who bargain later, as well as larger than those captured by the firm. The resulting equilibrium payoffs are different from those reported in Stole and Zwiebel (1996a) as they are not the Shapley values. We propose a novel bargaining game, the Rolodex game, which follows a simple and realistic protocol. In the unique no-delay Subgame Perfect Equilibrium of this game, the payoffs to the firm and to the workers are their Shapley values.

JEL Codes: D21, J30.

Keywords: Intra firm bargaining; Shapley value.

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1 Introduction

We revisit the problem of wage bargaining between a firm and multiple employees. The standard axiomatic approach to this type of multilateral bargaining problem is provided by Shapley (1953), who shows that there exists a unique solution satisfying certain desirable properties. The classic game-theoretic approach to the bargaining problem between a firm and multiple employees is provided by Stole and Zwiebel (1996a), who obtain three results. First, they propose a notion of stability and characterize the stable bargaining outcome (Theorem 1). Second, they advance an extensive-form game which they claim to implement the stable bargaining outcome in the unique Subgame Perfect Equilibrium (Theorem 2). Finally, they show that workers' wages and the firm's profit in the stable outcome coincide with the Shapley values (Theorem 4). Stole and Zwiebel (1996b) analyze the implications of their bargaining outcome for a wide variety of substantive issues related to the technology choice and organizational design of the firm. Since then, the Stole and Zwiebel bargaining outcome has been applied widely in search-theoretic models of the labor market.[1]

In this paper, we analyze two perfect-information bargaining games between a firm and multiple employees. First, we re-examine the bargaining game proposed by Stole and Zwiebel. We solve for the unique Subgame Perfect Equilibrium of the game. We characterize the equilibrium payoffs accruing to firm and workers and show that they are different from those reported in Stole and Zwiebel (1996a) and, hence, they are different from the Shapley values and from the stable bargaining outcome. Second, we propose a novel bargaining game between a firm and multiple workers that follows a simple and realistic protocol and delivers the Shapley values in the unique no-delay Subgame Perfect Equilibrium. We refer to this game as the "Rolodex game," after the rotating file device used to store business contact information.

In the first part of the paper, we characterize the solution to the Stole and Zwiebel game (henceforth, the SZ game). The game includes a firm and n workers, who are placed in some arbitrary queue. The game proceeds as a finite sequence of pairwise bargaining sessions between the firm and one of the workers. Each bargaining session follows the same protocol as in Binmore, Rubinstein and Wolinsky (1986, henceforth the BRW game), i.e. the worker and the firm alternate in making proposals about the worker's wage and, after every rejection, there is some probability of a breakdown. Each bargaining session ends either with an agreement over some wage or with a breakdown. In case of agreement, the firm enters a bargaining session with the next worker in the queue. In case of breakdown, the worker exits the game and the whole bargaining process starts over with one less worker. When the firm reaches an agreement with all the workers who are left in the game, the agreed-upon

[1] See, e.g., Cahuc, Marque and Wasmer (2008), Ebell and Haefke (2009), Itskhoki and Redding (2010), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), Helpman and Itskhoki (2015).

wages are paid out and production takes place.

We prove that the unique Subgame Perfect Equilibrium (SPE) of the SZ game is such that the firm reaches an agreement with every worker without delay. The gains from trade (and, hence, the wages) captured by a worker are decreasing with the worker's position in the queue, i.e. the first worker captures more gains from trade than the second worker, who captures more gains from trade than the third, etc... The gains from trade captured by the firm are equal to those captured by the last worker. The relevant notion of gains from trade is given by the output produced by the firm and n workers net of the sum of the payoffs that each of the n workers would obtain if he were excluded from the game and the payoff that the firm would obtain if it were to bargain with $n-1$ rather than n workers. We show that the equilibrium payoffs to the workers imply intra-firm wage inequality. We show that the equilibrium payoff to the firm implies that the firm has an incentive to hire more workers than it would if it were to take the wage as given.

The structure of the equilibrium payoffs of the SZ game is easy to explain. In the bargaining session between the firm and the last worker, the wage transfers utility from the firm to the worker at a rate of 1 to 1. That is, if the last worker gets paid an extra dollar, the worker's payoff increases by a dollar and the firm's payoff falls by a dollar. When the wage transfers utility at the rate of 1 to 1, the outcome of the BRW bargaining session is a wage that equates the gains from trade accruing to the worker and the firm. In the bargaining session between the firm and the i-th worker in the queue, with $i < n$, the wage transfers utility from the firm to the worker at a rate of less-than-1 to 1. Indeed, if the i-th worker gets paid an extra dollar, the firm's profit falls by less than a dollar because the firm will end up paying a lower wage to the subsequent workers. When the wage transfers utility at a rate of less-than-1 to 1, the outcome of the BRW bargaining session is a wage such that the gains from trade accruing to the worker exceed those accruing to the firm. And the further ahead in the queue is the worker, the lower is the firm's marginal cost of increasing the worker's wage, the higher is the rate at which the wage transfers utility from the firm to the worker and, ultimately, the higher are the worker's gains from trade relative to the firm's.

The equilibrium payoffs of the SZ game are not the Shapley values. The Shapley values are such that the firm and every worker capture the same share of the gains from trade, where the gains from trade are defined as the output produced by the firm with n workers net of the sum of the workers' payoffs if they were excluded from production and the firm's Shapley value with $n-1$ rather than n workers. Therefore, while the equilibrium payoffs of the SZ game have the same structure as the Shapley values–as they can be represented as shares of the same notion of gains from trade–they are different from the Shapley values–as the shares of the gains from trade accruing to the various players are different. In particular, the shares accruing to workers at the head of the queue are higher than in the Shapley values, while the shares accruing to workers at the end of the queue and to the firm are lower.

In the second part of the paper, we study the Rolodex game, a novel extensive-form game between a firm and multiple workers. The Rolodex game follows the same protocol as the SZ game, with one modification. In the Rolodex game, when a worker rejects a counteroffer from the firm, he moves to the end of the queue and the firm enters a bargaining session with the worker who, among those without agreement, is now at the top of the queue. In contrast, in the SZ game, a bargaining session between a firm and a worker continues until it reaches either an agreement or a breakdown. All other aspects of the Rolodex game are the same as in the SZ game. In particular, in the Rolodex game as in the SZ game, a breakdown in negotiations between the firm and a worker causes the worker to exit the game and the firm to restart the entire bargaining process with one less worker. We refer to this game as the Rolodex game because the firm cycles through the workers without agreement, rather than bargaining with one of them until it reaches an agreement or a breakdown.

We prove that there exists a unique no-delay SPE of the Rolodex game. In this equilibrium, the firm and every worker–irrespective of his position in the initial queue–capture an equal share of the gains from trade. The notion of gains from trade is the output produced by the firm with n workers net of the sum of the workers' payoffs if they were to be excluded from the game and the payoff of the firm if it were to bargain with $n-1$ rather than n workers. Since the notion of gains from trade and the share of the gains from trade accruing to firm and workers are the same in the Rolodex game as in the Shapley values, the equilibrium payoffs of the Rolodex game coincide with the Shapley values. The equilibrium payoffs to the workers in the Rolodex game imply no intra-firm wage inequality. The equilibrium payoff to the firm in the Rolodex game implies that the firm has an incentive to hire more workers than it would if it were to take the wage as given, but fewer than if wages were determined according to the SZ game.

It is easy to understand why the equilibrium payoffs of the Rolodex game coincide with the Shapley values. In the bargaining session between the firm and the last worker in the queue, the wage transfers utility from the firm to the worker at the rate of 1 to 1. Given this rate of transformation, the outcome of the bargaining session is a wage that equates the gains from trade captured by the firm and the last worker. In the bargaining session between the firm and the i-th worker in the queue, with $i<n$, the wage transfers utility from the firm to the worker at a rate of less-than-1 to 1. However, the i-th worker cannot take advantage of this higher rate of transformation. This is because, if the i-th worker rejects a counteroffer from the firm, he becomes the last worker in the queue. Thus, the firm can successfully offer to the i-th worker the same wage that is earned by the last worker and, in turn, the i-th worker can only successfully demand the same wage that is earned by the last worker. Overall, the firm and every worker capture an equal share of the gains from trade, as per the Shapley values. Moreover, the notion of gains from trade is the same as for the Shapley values because, if there is a breakdown in negotiations between the firm and a

worker, the worker exits the game and the firm restarts the whole bargaining process with $n-1$ rather than n workers.

The paper is a contribution to the literature on wage bargaining between a firm and multiple employees. The first part of the paper is devoted to revisiting the SZ game. The analysis of this game is of natural interest given its sensible protocol and its widespread application in the labor-search literature. We find that the unique SPE of the game is such that workers are paid different wages depending on the order in which they bargain with the firm. We relate this finding to the empirical literature on intra-firm wage inequality and the return to seniority. We find that, given this wage setting game, the firm has an incentive to hire more workers than it would in a competitive labor market, as well as more workers than it would if wages were given by the Shapley values. These findings are novel because the equilibrium payoffs to the firm and to the workers are not those reported in Stole and Zwiebel (1996a). In particular, the equilibrium payoffs of the SZ game are different from the Shapley values, while the equilibrium payoffs reported in Theorem 2 of Stole and Zwiebel (1996a) are the Shapley values. The mistake in the proof of Theorem 2 of Stole and Zwiebel (1996a) is to argue that the outcome of the bargaining session between the firm and a worker is always such that the gains from trade accruing to the two parties are equalized. However, this is only true for the bargaining session between the firm and the last worker in the queue. As Theorems 1 and 4 of Stole and Zwiebel (1996a) are correct though, they should still be credited for providing a stability-based foundation to the Shapley values.

The second part of the paper is devoted to the analysis of the Rolodex game. As the SZ game, the Rolodex game is a generalization of BRW to an environment in which the firm bargains with multiple workers. As the SZ game, the Rolodex game features a protocol that is plausible in the context of wage negotiations between a firm and its employees, in the sense that the firm is involved in every negotiation and workers are involved only in negotiations regarding their own wage. In contrast to the SZ game, the players' payoffs in the unique no-delay SPE of the Rolodex game are the Shapley values. The Rolodex game contributes to the theoretical literature on bargaining by identifying a novel protocol that yields the Shapley values in the context of a wage bargain in a multi-worker firm. In follow-up work (Brügemann et al. 2017), we show that the Rolodex game also yields the Myerson-Shapley values in more general contexts. The similarity between the Rolodex and the SZ game reveals that Stole and Zwiebel (1996a) had the right insight about some of the aspects of the type of extensive-form game that would yield the Myerson-Shapley values. The Rolodex game also contributes to the applied literature on search-theoretic models of the labor market. In fact, the Rolodex game is a game with a simple, perfect-information and plausible protocol which may be referred to by the labor-search literature as a justification for using the Shapley values as the outcome of the wage bargain between a firm and multiple employees.[2]

[2] There are other perfect-information games that deliver the Shapley values. Two important games are

The SZ game does not yield the Shapley values because workers who bargain earlier with the firm are in a superior strategic position relative to workers who bargain later, as they understand that the marginal cost to the firm from giving them a higher wage is lower. The protocol of the Rolodex game makes it impossible for workers who bargain earlier to exploit their strategic position because, upon rejecting an offer from the firm, they are automatically moved to the end of the queue. Another way to prevent workers at the head of the queue from exploiting their strategic position is to assume that the outcome of their bargain is not observed by other workers. This idea is formalized by De Fontenay and Gans (2014). They study a game between agents in a network. Agents bargain bilaterally with each of the agents with whom they are linked following the same protocol as in BRW. The game is one of imperfect information as the history of a bargaining session is privately observed by the two parties involved in it. Under some assumptions about off-equilibrium beliefs (i.e. passive beliefs), the unique Perfect Bayesian Equilibrium of the de Fontenay and Gans game is such that the agents' payoffs are equal to their Myerson-Shapley values. A special case of the game is a private-information version of the SZ game. Therefore, under imperfect information and passive beliefs, the equilibrium payoffs of the SZ game coincide with the Shapley values.

2 The Stole and Zwiebel Game

In this section, we study the bargaining game between a firm and multiple workers proposed by Stole and Zwiebel (1996a). In Section 2.1, we describe the extensive form of the SZ game. In Section 2.2, we characterize the unique SPE of the SZ game. We find that, in equilibrium, the gains from trade captured by a worker are decreasing in the order in which he bargains with the firm, and that the last worker in the order captures the same gains from trade as the firm does. In Section 2.3, we show that the equilibrium payoffs to the workers and to the firm are different from the Shapley values. We also discuss substantive implications of the solution to the SZ game, such as intra-firm wage inequality and over-hiring.

2.1 Environment and Preliminaries

We begin by describing the extensive form of the SZ game. The players in the game are a firm and $n \geq 1$ workers. If the firm employs $j \geq 0$ of the n workers and pays them wages w_1, w_2, ... w_j, it attains a payoff of $F(j) - w_1 - w_2 - ...w_j$, where $F(j)$ denotes

those developed by Gul (1986) and Hart and Mas-Colell (1996). However, the protocol of these games does not provide a plausible description of wage negotiations in a multi-worker firm. In the context of a wage negotiation between a firm and its workers, the protocol of the game by Gul features situations in which a worker buys labor from a coworker and then sells it to the firm, as well as situations in which a worker buys the assets of the firm and then hires additional workers. Similarly, in the context of a wage bargain in a multi-worker firm, the protocol of the game by Hart and Mas-Colell features situations in which a worker proposes a wage not only for himself, but also for his coworkers.

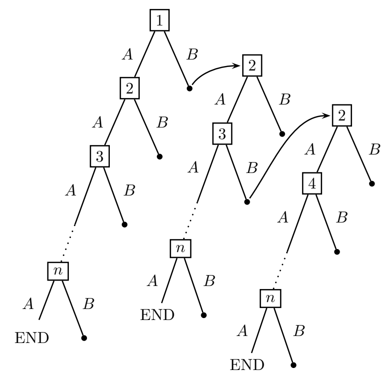

Figure 1: Sequence of bargaining sessions in SZ game

the value of the output produced by the firm and j employees. We assume that $F(j)$ is strictly positive, strictly increasing and strictly concave in j, i.e. $F(j) < F(j+1)$ and $F(j+1) - F(j) > F(j+2) - F(j+1)$ for $j = 0, 1, 2...$ Workers are ex-ante identical. If a worker is hired at the wage w, he attains a payoff of w. If the worker is not hired, he attains a payoff of $\underline{w} \geq 0$, where $\underline{w}$ might represent the value that the worker can obtain from some other firm or the value of unemployment.

The game consists of a finite sequence of bilateral bargaining sessions between the firm and one of the workers. Workers are placed in some arbitrary order from 1 to n. The game starts with a bargaining session between the firm and the first worker in the order. A bargaining session involves the determination of the worker's wage and ends either with an agreement over a particular wage or with a breakdown. If the session ends with an agreement, the firm enters a bargaining session with the worker who, among those still in the game, is next in the order. If the session ends with a breakdown, the worker permanently exits the game and the entire bargain process starts over. That is, all prior agreements between firm and workers are erased and the firm enters a bargaining session with the worker who, among those still in the game, is first in the order. The game ends when the firm has reached an agreement with all the workers still in the game. When the game ends, production takes place and the firm pays the agreed-upon wages. Figure 1 illustrates the sequence of the bargaining sessions in the SZ game, where the number in the box denotes the position in the order of the worker with whom the firm is bargaining at that time.

Each bargaining session between the firm and a worker follows the alternating-offer protocol of BRW. The session starts with the worker making a wage offer to the firm. If the firm accepts the offer, the session ends and the firm starts bargaining with the next worker. If the firm rejects the offer, the negotiation breaks down with probability q and continues with probability $1-q$, with $q \in (0, 1)$. If the negotiation continues, the firm makes a counteroffer to the worker. If the worker accepts the counteroffer, the bargaining session comes to an end. If the worker rejects the counteroffer, the negotiation breaks down with probability q and continues with probability $1-q$. As long as the session continues, the worker and the firm take turns in making offers while facing a probability q of breakdown after every rejection.

The SZ game is a natural generalization of the BRW bargaining game to an environment in which the firm is connected to multiple workers. The firm negotiates sequentially with each individual worker over that worker's wage. The negotiation between the firm and a worker follows the same protocol as in BRW. The game ends when the firm reaches an agreement with all the workers who, at that point in time, are still connected to the firm. The game starts over whenever a negotiation breaks down and the firm and the worker lose contact. The assumption is meant to capture the idea that–when the connection between the firm and a worker breaks down–the physical environment of the game changes and, for this reason, all prior agreements are annulled. The SZ game seems like a plausible description of the bargaining process between a firm and its workforce.

As in Stole and Zwiebel (1996a), we shall focus on SPE in the limit for $q \to 0$. Before embarking on the characterization of the equilibrium of the SZ game, it is useful to remind our readers of the solution of the BRW bargaining game between a firm and a worker in which the worker's marginal benefit of a higher wage is 1, the firm's marginal cost of a higher wage is $1-\beta$, and, thus, the wage transfers utility from the firm to the worker at the rate of $1-\beta$ to 1.

Lemma 1: *Consider the BRW alternating-offer game between a firm and a worker. If the firm and the worker reach an agreement at the wage w, the payoff to the worker is w and the payoff to the firm is $y-w-t(w)$, where $t(w)$ is a linear function of w with derivative $-\beta$, $\beta \in [0,1)$. If the firm and the worker do not reach an agreement, the payoff to the worker is $\underline{w}$ and the payoff to the firm is z. (i) If $y-z-\underline{w}-t(\underline{w})<0$, any SPE is such that the firm and the worker do not reach an agreement. (ii) If $y-z-\underline{w}-t(\underline{w}) \geq 0$, the unique SPE[3] is such that the firm and the worker immediately reach an agreement at the wage*

$$w = \underline{w} + \frac{1}{(2-q)(1-\beta)} \left[y - z - \underline{w} - t(\underline{w}) \right]. \tag{1}$$

Proof: The result follows immediately from Proposition 4.2 in Muthoo (1999). ■.

In the BRW game described in Lemma 1, the gains from trade are $y-z-\underline{w}-t(\underline{w})$. The lemma states that, if the gains from trade are negative, any SPE is such that the firm and the worker do not reach an agreement. If the gains from trade are positive, the unique SPE is such that trade takes place without delay at the wage (1). In this case, the gains from trade accruing to the worker, $w-\underline{w}$, and the gains from trade accruing to the firm,

[3] To be precise, there are multiple SPE of the BRW game when the gains from trade are zero. All of the SPEs are payoff equivalent, but some of them involve agreement and some do not. For the remainder of the section, we restrict attention to the SPE in which agreement takes place instantaneously when the gains from trade are zero.

$y - z - w - t(w)$, are respectively given by

$$\begin{aligned} w - \underline{w} &= \frac{1}{(2-q)(1-\beta)}\left[y - z - \underline{w} - t(\underline{w})\right], \\ y - w - t(w) - z &= \frac{1-q}{2-q}\left[y - z - \underline{w} - t(\underline{w})\right]. \end{aligned} \tag{2}$$

Two features of the equilibrium of the BRW game are worth pointing out. First, in the limit as the breakdown probability q goes to zero, the equilibrium wage and payoffs converge to the wage and payoffs under the axiomatic Nash bargaining solution. In fact, for $q \to 0$, the equilibrium wage converges to

$$w = \underline{w} + \frac{1}{2(1-\beta)}\left[y - z - \underline{w} - t(\underline{w})\right]. \tag{3}$$

This is the wage that maximizes the Nash product $(w - \underline{w}) \cdot (y - z - w - t(w))$. As the equilibrium wage converges to the wage that maximizes the Nash product, it follows that the equilibrium payoffs to firm and to worker converge to the payoffs under axiomatic Nash bargaining. This is a well-known result. We wished to restate it because it implies that our characterization of the SZ game would be the same if we replaced the BRW alternating-offer bargaining game with axiomatic Nash bargaining.

Second, in the limit for $q \to 0$, the ratio of the equilibrium gains from trade accruing to the worker to those accruing to the firm is given by

$$\frac{w - \underline{w}}{y - w - t(w) - z} = \frac{1}{1-\beta}. \tag{4}$$

That is, the ratio of the worker's gains from trade to the firm's gains from trade is equal to the ratio $1/(1-\beta)$ of the worker's marginal benefit of a higher wage to the firm's marginal cost of a higher wage. For $\beta = 0$, *utility is perfectly transferrable*, in the sense that the marginal cost to the firm of a higher wage is 1, the marginal benefit to the worker of a higher wage is 1 and, thus, the wage transfers utility at a rate of 1 to 1. In this case, (4) implies that the equilibrium of the BRW game is such that the gains from trade accruing to the worker and those accruing to the firm are equated. When $\beta > 0$, *utility is not perfectly transferrable*, in the sense that the marginal cost to the firm of a higher wage is $1 - \beta < 1$, the marginal benefit to the worker of a higher wage is 1 and, thus, the wage transfers utility at a rate of $1 - \beta$ to 1. In this case, (4) implies that the equilibrium of the BRW game is such that the gains from trade accruing to the worker are $1/(1-\beta) > 1$ times those accruing to the firm. The higher is β, the lower is the firm's marginal cost of a higher wage, the higher is the rate at which the wage transfers utility from the firm to the worker, and the higher is the ratio of the gains from trade accruing to the worker relative to those accruing to the firm. Again this is a well-known and well-understood result. However, we wished to restate it to point out that the outcome of the BRW game is such that the firm's and worker's gains from trade are equal only when utility is perfectly transferrable.

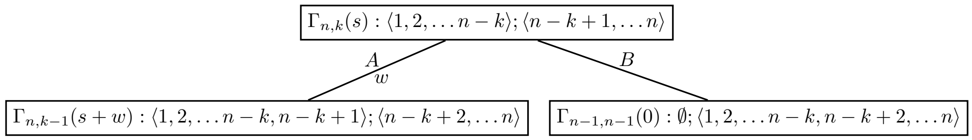

Figure 2: Structure of generic subgame in SZ game

2.2 Equilibrium of the SZ Game

We begin the analysis of the SZ game by introducing some notation. We denote as $\Gamma_{n,k}(s)$ the subgame in which there are n workers in the game, $n-k$ of these workers have already reached an agreement with the firm for wages summing up to s, k workers have yet to reach an agreement with the firm, and the firm is about to enter a bargaining session with the first of those k workers. We denote as $w^i_{n,k}(s)$ the wage of the i-th of k workers without agreement in an SPE of the subgame $\Gamma_{n,k}(s)$. The SZ game between the firm and n workers is the subgame $\Gamma_{n,n}(0)$. We find it useful to adopt some shorthand notation for the equilibrium outcomes of $\Gamma_{n,n}(0)$. In particular, we denote as $\tilde{\pi}_n$ the payoff to the firm and with $\tilde{w}^i_n$ the payoff to the i-th worker in an SPE of $\Gamma_{n,n}(0)$. Clearly, $\tilde{\pi}_0 = F(0)$.

Figure 2 illustrates the structure of a generic subgame $\Gamma_{n,k}(s)$, in which workers 1, 2, … $n-k$ have reached an agreement with the firm for wages summing up to s and $n-k+1$, $n-k+2$, ..., n is the order of the workers who have yet to reach an agreement with the firm. The subgame $\Gamma_{n,k}(s)$ starts with a bargaining session between the firm and worker $n-k+1$. If the session ends with an agreement at the wage w, the game enters the subgame $\Gamma_{n,k-1}(s+w)$, in which workers 1, 2, …, $n-k$, $n-k+1$ have reached an agreement with the firm for wages summing up to $s+w$ and $n-k+2$, $n-k+3$, ..., n is the order of workers who have yet to reach an agreement with the firm. If the session ends with a breakdown, the worker exits and the game continues with the subgame $\Gamma_{n-1,n-1}(0)$, in which workers 1, 2, …, $n-k$, $n-k+2$,..., n have yet to reach an agreement with the firm.

The intuition behind the equilibrium properties of the SZ game can be gained by studying the game between the firm and two workers. We solve for the SPE of the SZ game $\Gamma_{2,2}(0)$ by backward induction. First, we solve for the SPE of the subgame $\Gamma_{1,1}(0)$ which is reached if the bargaining session between the firm and one of its two workers ends with a breakdown. Second, we solve for the SPE of the subgame $\Gamma_{2,1}(w_1)$ which is reached if the bargaining session between the firm and the first of its two workers ends with an agreement at the wage w_1. Third, we solve for the SPE of the game $\Gamma_{2,2}(0)$.

Consider the subgame $\Gamma_{1,1}(0)$. It consists of an alternating-offer bargaining session between the firm and the one worker left in the game. If the session ends with the firm and the worker agreeing to the wage w_1, the payoff to the firm is $F(1) - w_1$ and the payoff to the worker is w_1. If the session ends with a breakdown, the payoff to the firm is $\tilde{\pi}_0$ and the payoff to the worker is $\underline{w}$. The protocol and payoff structure of $\Gamma_{1,1}(0)$ is the same as in the BRW game characterized in Lemma 1 for $y = F(1)$, $z = \tilde{\pi}_0$ and $t(w_1) = 0$, which is obviously a linear function of w_1 with derivative $-\beta_0 = 0$. Thus, assuming the gains from trade $F(1) - \tilde{\pi}_0 - \underline{w}$ are positive, the unique SPE is such that the firm and the worker reach an agreement without delay at the wage

$$\tilde{w}_1^1 = \underline{w} + \frac{1}{2-q}\left[F(1) - \tilde{\pi}_0 - \underline{w}\right]. \tag{5}$$

In turn, this implies that the unique SPE is such that the payoff to the firm is

$$\tilde{\pi}_1 = \tilde{\pi}_0 + \frac{1-q}{2-q}\left[F(1) - \tilde{\pi}_0 - \underline{w}\right]. \tag{6}$$

Note that, in the bargaining session between the firm and the only worker left in the game, utility is perfectly transferrable, as the marginal cost to the firm of a higher wage is $1-\beta_0 = 1$ and the marginal benefit to the worker of a higher wage is 1 and, hence, the wage transfers utility from the firm to the worker at a rate of 1 to 1.

Next, consider the subgame $\Gamma_{2,1}(w_1)$. It starts with an alternating-offer bargaining session between the firm and the second worker. If the session ends with the firm and the second worker agreeing to the wage w_2, the game comes to an end. In this case, the payoff to the firm is $F(2) - w_1 - w_2$ and the payoff to the second worker is w_2. If the session ends with a breakdown, the second worker leaves and attains a payoff of $\underline{w}$, while the firm enters the subgame $\Gamma_{1,1}(0)$ in which it renegotiates its prior agreement with the first worker. In the unique SPE of $\Gamma_{1,1}(0)$, the payoff to the firm is $\tilde{\pi}_1$. The set of SPE of $\Gamma_{2,1}(w_1)$ coincides with the set of SPE of a reduced-form version of $\Gamma_{2,1}(w_1)$ where the subgames following the end of the bargaining session between the firm and the second worker are replaced with their associated SPE payoffs (see, e.g., Proposition 9.B.3 in Mas-Colell, Whinston and Green 1995). The reduced-form version of $\Gamma_{2,1}(w_1)$ has the same protocol and payoff structure as the BRW game described in Lemma 1 for $y = F(2) - w_1$, $z = \tilde{\pi}_1$ and $t(w_2) = 0$, which is obviously a linear function of w_2 with derivative $-\beta_0 = 0$. Thus, if the gains from trade $F(2) - w_1 - \tilde{\pi}_1 - \underline{w}$ are negative, any SPE is such that the firm and the worker do not reach an agreement. If the gains from trade are positive, the unique SPE is such that the firm and the worker reach an agreement without delay at the wage

$$w_{2,1}^1(w_1) = \underline{w} + \frac{1}{2-q}\left[F(2) - w_1 - \tilde{\pi}_1 - \underline{w}\right]. \tag{7}$$

Note that, also in the bargaining session between the firm and the second worker, utility is perfectly transferrable, as the marginal cost to the firm of a higher wage is $1 - \beta_0 = 1$ and the marginal benefit to the worker of a higher wage is 1.

Finally, consider the game $\Gamma_{2,2}(0)$. The game starts with an alternating-offer bargaining session between the firm and the first worker. There are three qualitatively different outcomes of the session. First, the session may end with an agreement at the wage $w_1 \leq F(2) - \tilde{\pi}_1 - \underline{w}$.

In this case, the game continues with the subgame $\Gamma_{2,1}(w_1)$. For $w_1 \leq F(2) - \tilde{\pi}_1 - \underline{w}$, the unique SPE of $\Gamma_{2,1}(w_1)$ is such that the firm and the second worker reach an immediate agreement at the wage $w_{2,1}^1(w_1)$ and, hence, the payoff to the firm is $F(2) - w_1 - w_{2,1}^1(w_1)$ and the payoff to the first worker is w_1. Second, the bargaining session between the firm and the first worker may end with an agreement at the wage $w_1 > F(2) - \tilde{\pi}_1 - \underline{w}$. Also in this case, the game continues with the subgame $\Gamma_{2,1}(w_1)$. However, for $w_1 > F(2) - \tilde{\pi}_1 - \underline{w}$, any SPE of the subgame $\Gamma_{2,1}(w_1)$ is such that the bargaining session between the firm and the second worker ends with a breakdown. Hence, the second worker leaves and the firm renegotiates with the first worker. The unique SPE of the renegotiation is such that the firm attains a payoff of $\tilde{\pi}_1$ and the first worker attains a payoff of $\tilde{w}_1^1$. Third, the bargaining session between the firm and the first worker may end with a breakdown. In this case, the first worker leaves and attains a payoff of $\underline{w}$, while the firm enters the subgame $\Gamma_{1,1}(0)$. In the unique SPE of $\Gamma_{1,1}(0)$, the payoff to the firm is $\tilde{\pi}_1$.

The set of SPE of $\Gamma_{2,2}(0)$ coincides with the set of SPE of a reduced-form version of $\Gamma_{2,2}(0)$ in which the subgames following the bargaining session between the firm and the first worker are replaced with their associated SPE payoffs. The protocol of the reduced-form version of $\Gamma_{2,2}(0)$ is the same as the protocol of the game described in Lemma 1. The payoff structure of the reduced-form version of $\Gamma_{2,2}(0)$ is not the same as in Lemma 1, because the agreement payoffs for the firm and the first worker depend on whether w_1 leads to a breakdown with the second worker or not. However, assume that, whenever indifferent, the firm rejects any wage offer from the first worker that would lead to a breakdown in negotiations with the second worker. Similarly, assume that, whenever indifferent, the firm chooses not to make any counteroffer to the first worker that would lead to a breakdown in negotiations with the second worker. Under these tie-breaking assumptions, we show in Appendix C that the outcome of the reduced-form version of $\Gamma_{2,2}(0)$ is the same as if the agreement payoffs were $F(2) - w_1 - w_{2,1}^1(w_1)$ and w_1 for all w_1. We can then apply Lemma 1 for $y = F(2)$, $z = \tilde{\pi}_1$ and $t(w_1) = w_{2,1}^1(w_1)$, where $t(w_1)$ is a linear function of w_1 with derivative $-\beta_1$ for $\beta_1 = 1/(2-q)$. Thus, if $F(2) - \tilde{\pi}_1 - \underline{w} - w_{2,1}^1(\underline{w}) \geq 0$ or equivalently $F(2) - \tilde{\pi}_1 - 2\underline{w} \geq 0$, the firm and the first worker reach an immediate agreement at the wage

$$\tilde{w}_2^1 = \underline{w} + \frac{1}{1-q}\left[F(2) - \tilde{\pi}_1 - \underline{w} - w_{2,1}^1(\underline{w})\right]. \tag{8}$$

In turn, this implies that the payoff to the firm is

$$\tilde{\pi}_2 = \tilde{\pi}_1 + \frac{1-q}{2-q}\left[F(2) - \tilde{\pi}_1 - \underline{w} - w_{2,1}^1(\underline{w})\right]. \tag{9}$$

Note that, in the bargaining session between the firm and the first of its two workers, utility is not perfectly transferrable. The marginal benefit to the first worker from receiving a higher wage is 1. The marginal cost to the firm from paying the first worker a higher wage is $1 - \beta_1 = 1 - 1/(2-q)$ because, if the firm pays the first worker an extra dollar, the gains from trade between the firm and the second worker fall by 1 dollar and, for this reason, the firm ends up paying the second worker $1/(2-q)$ dollars less. Therefore, in the bargaining session between the firm and the first of its two workers, the wage transfers utility at the rate of $1-1/(2-q)$ to 1, where $1-1/(2-q)<1$. As we will see, this observation has a crucial bearing on the equilibrium payoffs and is the reason why the SZ-game payoffs are not the Shapley values.

We are now in the position to summarize the outcome of the SZ game $\Gamma_{2,2}(0)$ between the firm and two workers. Assuming $F(2)-\tilde{\pi}_1-2\underline{w}\geq 0$, there is a unique SPE of $\Gamma_{2,2}(0)$ in which the firm reaches an immediate agreement with the first worker for the wage $\tilde{w}_2^1$ in (8). Since $\tilde{w}_2^1\leq F(2)-\tilde{\pi}_1-\underline{w}$, the firm also reaches an immediate agreement with the second worker for the wage $\tilde{w}_2^2=w_{2,1}^1(\tilde{w}_2^1)$. Substituting out $w_{2,1}^1(\underline{w})$ in (8), we find that the wage (and payoff) of the first worker is

$$\tilde{w}_2^1=\underline{w}+\frac{1}{2-q}\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]. \tag{10}$$

Evaluating $w_{2,1}^1(w_1)$ at $\tilde{w}_2^1$, we find that the wage (and payoff) of the second worker is

$$\tilde{w}_2^2=\underline{w}+\frac{1-q}{(2-q)^2}\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]. \tag{11}$$

Substituting out $w_{2,1}^1(\underline{w})$ in (9), we find that the payoff of the firm is

$$\tilde{\pi}_2=\tilde{\pi}_1+\frac{(1-q)^2}{(2-q)^2}\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]. \tag{12}$$

For $q\to 0$, the equilibrium payoffs to the workers and to the firm are given by

$$\begin{aligned}\tilde{w}_2^1&=\underline{w}+\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]/\,2,\\ \tilde{w}_2^2&=\underline{w}+\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]/\,4,\\ \tilde{\pi}_2&=\tilde{\pi}_1+\left[F(2)-\tilde{\pi}_1-2\underline{w}\right]/\,4,\end{aligned} \tag{13}$$

where $\tilde{\pi}_1$ is given by (6). The expressions in (13) are easy to interpret. The term $F(2)-\tilde{\pi}_1-2\underline{w}$ is a measure of the overall gains from trade between the firm and the two workers. The expressions in (13) then state that the first worker in the order captures 1/2 of these gains from trade, while the second worker in the order and the firm each capture 1/4 of the gains from trade. There is a simple logic behind this division of the gains from trade. In the bargaining session between the firm and the second worker, utility is perfectly transferrable. Hence, Lemma 1 implies that the outcome of the session is such that the gains from trade accruing to the second worker and the firm are equalized. In the bargaining session between the firm and the first worker, utility is not perfectly transferrable. In particular, the wage transfers utility from the firm to the first worker at the rate of 1/2 to 1. Hence, Lemma 1 implies that the outcome of the session is such that the gains from trade accruing to the first worker are twice as large as those accruing to the firm. From these observations, it follows that the first worker captures 1/2, while the second worker and the firm each capture 1/4 of the gains from trade. Intuitively, the first worker captures a larger fraction of the gains from trade than the second worker because he can take advantage of the fact that the firm's marginal cost of paying him a higher wage is lower. The first worker captures a larger fraction of the gains from trade than the firm because the gains from trade accruing to the firm and the second worker are equal.

The following theorem generalizes the characterization of the SPE of the SZ game between the firm and two workers in the limit for $q \to 0$ to the case of n workers.

Theorem 1. (Stole and Zwiebel game). *Consider the SZ game $\Gamma_{n,n}(0)$ between the firm and n workers. Assume that the overall gains from trade are positive, i.e. $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$. In the limit for $q \to 0$, the unique SPE of the game is such that the firm reaches an agreement with all of the workers without delay. The payoff $\tilde{\pi}_n$ to the firm is given by the difference equation*

$$\tilde{\pi}_j = \tilde{\pi}_{j-1} + [F(j) - \tilde{\pi}_{j-1} - j\underline{w}]/\,2^j, \text{ for } j = 1, 2, ...n, \tag{14}$$

with initial condition $\tilde{\pi}_0 = F(0)$. The payoff to the i-th of n workers is given by

$$\tilde{w}_n^i = \underline{w} + [F(n) - \tilde{\pi}_{n-1} - n\underline{w}]/\,2^i. \tag{15}$$

Proof: In Appendix A. ■

Theorem 1 states that the SPE payoffs of the SZ game $\Gamma_{n,n}(0)$ are such that the first worker in the order captures 1/2 of the overall gains from trade $F(n) - \tilde{\pi}_{n-1} - n\underline{w}$. The second worker in the order captures 1/4 of the gains from trade. More generally, the i-th worker in the order captures $1/2^i$ of the gains from trade. The firm captures $1/2^n$ of the gains from trade. The logic behind these payoffs is simple. In the bargaining session between the firm and the i-th worker, the wage transfers utility at the rate of $1/2^{n-i}$ to 1. In fact, the i-th worker's marginal benefit from receiving a higher wage is 1. The firm's marginal cost of paying the i-th worker a higher wage is only $1/2^{n-i}$ because, if the firm pays the i-th worker 1 extra dollar, it will lower the sum of wages paid to the following workers by $1 - 1/2^{n-i}$ dollars less. Hence, the outcome of the bargaining session between the firm and the i-th worker is such that the gains from trade accruing to the worker are 2^{n-i} as large as those accruing to the firm. These observations imply that the i-th worker captures $1/2^i$ of the gains from trade and the firm captures $1/2^n$ of the gains from trade.

2.3 Properties of the Solution of the SZ Game

Having characterized the SPE of the SZ game, we now turn to discuss some of its properties. The first property we wish to discuss is the relationship between its solution and the axiomatic bargaining solution of Shapley: The SPE payoffs of the SZ game are not the Shapley values. When the firm has n workers, the Shapley value π_n^* of the firm is given by the difference equation[4]

$$\pi_j^* = \pi_{j-1}^* + \left[F(j) - \pi_{j-1}^* - j\underline{w}\right] / \,(1+j), \text{ for } j = 1, 2, ...n, \tag{16}$$

with initial condition $\pi_0^* = F(0)$. The Shapley value w_n^* of each of the n workers is given by

$$w_n^* = \underline{w} + \left[F(n) - \pi_{n-1}^* - n\underline{w}\right] / \,(1+n). \tag{17}$$

In words, the Shapley values are such that the firm and every worker each capture the same share $1/(1+n)$ of the gains from trade, where the gains from trade are defined as $F(n) - \pi_{n-1}^* - n\underline{w}$. The SZ-game payoffs differ from the Shapley values in two dimensions. First, the SZ-game payoff to the firm is different from the Shapley value of the firm for any $n \geq 2$. To see this, notice that the difference equations defining the firm's equilibrium payoff and the difference equation defining the firm's Shapley value have the same structure but different coefficients in front of the term $F(j) - \pi_{j-1} - j\underline{w}$. Namely, $1/2^j$ for the equilibrium payoff and $1/(1+j)$ for the Shapley value. Since the coefficients are different for all $j \geq 2$, the firm's equilibrium payoff is different from the firm's Shapley value for all $n \geq 2$. Second, the equilibrium payoff to a worker depends on the worker's position in the bargaining order, while the Shapley value is the same for every worker. This implies that the workers' SZ-game payoffs are different from the workers' Shapley values for any particular ordering of $n \geq 2$ workers. Moreover, since the SZ-game payoff to the firm is different from its Shapley value while the sum of all the players' SZ-game payoffs is the same as the sum of the Shapley values, the workers' SZ-game payoffs are different from the workers' Shapley values also in expectation over any distribution of orderings for any $n \geq 2$.

The second property of the SZ game that we wish to discuss is substantive: Workers are paid different wages even though they are identical and they are employed by the same firm. There is empirical evidence (see, e.g., Postel-Vinay and Robin 2002) documenting intra-firm wage inequality, i.e. inequality among workers who appear to be similar and who are employed by the same firm. The SZ game provides a possible explanation for this type of inequality based on the idea that workers who bargain earlier with the firm are in a better strategic position than workers who bargain later and earn higher wages. A compelling empirical test of this explanation would require information on the order in which workers bargain with the firm. Such data is not easily available. However, Buhai et al. (2014) have data on the seniority of workers within a firm (where seniority is defined as the tenure of a worker relative to the tenure of his coworkers), which is perhaps the most natural order in which a firm might approach its employees when negotiating wages. If, indeed, seniority is related to the bargaining order, the finding in Buhai et al. (2014) that seniority has a positive effect on wages even after controlling for tenure is consistent with the SZ game explanation for intra-firm wage inequality.

[4] Theorem 4 in Stole and Zwiebel (1996a) shows that the Shapley values can be written as (16) and (17).

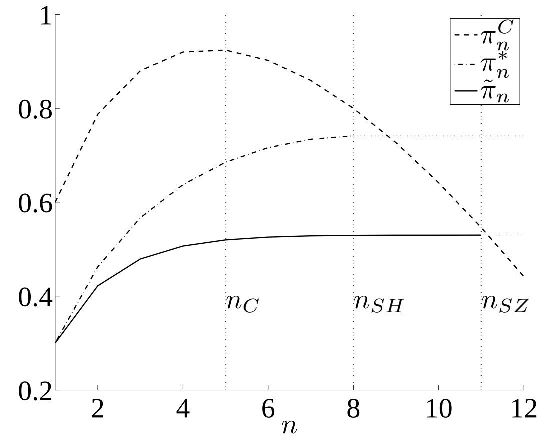

Figure 3: Profits and hiring for $F(n) = n^{\frac{2}{3}}$ and $\underline{w} = 0.4$

The third property of the SZ game that we wish to discuss is also substantive: If wages are determined by the SZ game, the firm has an incentive to hire more workers than it would in a competitive labor market where it takes the wage $\underline{w}$ as given.[5] First, consider the case in which the firm hires workers in a competitive market where the wage $\underline{w}$ is given. If the firm hires n workers, it attains a payoff of

$$\pi_n^C = F(n) - n\underline{w}. \tag{18}$$

Since $F(n)$ is strictly concave in n, so is the firm's payoff π_n^C. Therefore, there exists a unique number n_C of workers that maximizes the payoff of the firm. Moreover, the payoff to the firm π_n^C is strictly increasing in n for all $n \leq n_C$ and strictly decreasing in n for all $n \geq n_C$. The properties of π_n^C and n_C are illustrated in Figure 3.

Now, consider the case in which wages are determined by the SZ game. If the firm hires n workers, it attains a payoff $\tilde{\pi}_n$ which satisfies the difference equation (14). The solution to this difference equation is a weighted average of the competitive payoffs π_i^C for $i = 0, 1, \ldots n$. Namely,

$$\tilde{\pi}_n = \textstyle\sum_{i=0}^{n} 2^{-i} \left[\prod_{j=i+1}^{n} \left(1 - 2^{-j}\right)\right] \pi_i^C. \tag{19}$$

The payoff $\tilde{\pi}_n$ is strictly smaller than the competitive payoff π_n^C for all $n \leq n_C$. This property follows from the fact that $\tilde{\pi}_n$ is a weighted average of π_i^C for $i = 0, 1, \ldots n$ and π_i^C is strictly increasing in i for all $i \leq n_C$. The increase $\tilde{\pi}_n - \tilde{\pi}_{n-1}$ in the payoff from hiring n rather than $n-1$ workers is strictly positive for all $n \leq n_C$. This property follows from the observations that $\tilde{\pi}_n - \tilde{\pi}_{n-1}$ has the same sign as $\pi_n^C - \tilde{\pi}_{n-1}$ and $\pi_n^C - \tilde{\pi}_{n-1} > \pi_{n-1}^C - \tilde{\pi}_{n-1} > 0$ for all $n \leq n_C$. From this property of $\tilde{\pi}_n - \tilde{\pi}_{n-1}$, it follows that the number n_{SZ} of workers that maximizes the firm's payoff $\tilde{\pi}_n$ when wages are set as by the SZ game is greater than n_C. From the fact that $\tilde{\pi}_n - \tilde{\pi}_{n-1}$ has the same sign as $\pi_n^C - \tilde{\pi}_{n-1}$, it follows that n_{SZ} is such

[5] Following Stole and Zwiebel (1996a), we assume that hiring is costless.

that the firm's payoff $\tilde{\pi}_n$ is equal to the competitive payoff π_n^C up to integer rounding[6], i.e. $\tilde{\pi}_{n_{SZ}} \doteq \pi_{n_{SZ}}^C$. The properties of $\tilde{\pi}_n$ and n_{SZ} are illustrated in Figure 3.

It is also useful to consider the case in which wages are given by the workers' Shapley values. If the firm hires n workers, it attains a payoff of π_n^* which satisfies the difference equation (16). The solution to this difference equation is

$$\pi_n^* = \sum_{i=0}^{n}(1+n)^{-1}\pi_i^C. \tag{20}$$

The payoff π_n^* is strictly smaller than the competitive payoff π_n^C for all $n \leq n_C$, as π_n^* is a weighted average of π_i^C for $i = 0, 1, \dots n$. The increase $\pi_n^* - \pi_{n-1}^*$ in the payoff from hiring n rather than $n-1$ workers is strictly positive for all $n \leq n_C$, as $\pi_n^* - \pi_{n-1}^*$ has the same sign as $\pi_n^C - \pi_{n-1}^*$ and $\pi_n^C - \pi_{n-1}^* > \pi_{n-1}^C - \pi_{n-1}^* > 0$ for all $n \leq n_C$. Therefore, the number of workers n_{SH} that maximizes the firm's payoff π_n^* when wages are set according to the Shapley values is greater than n_C. Moreover, n_{SH} is such that the firm's payoff π_n^* is equal to the competitive payoff π_n^C up to integer rounding, i.e. $\tilde{\pi}_{n_{SH}} \doteq \pi_{n_{SH}}^C$. In order to compare n_{SH} and n_{SZ}, notice that, while both π_n^* and $\tilde{\pi}_n$ are weighted averages of π_i^C for $i = 0, 1, \dots n$, π_n^* places more weight on high values of i and less weight on low values of i. Using this observation, we can show that $\pi_n^* \geq \tilde{\pi}_n$ for all $n \leq n_{SH}$ and, hence, $\pi_n^C - \tilde{\pi}_{n-1} > \pi_n^C - \pi_{n-1}^* > 0$ for all $n \leq n_{SH}$. Since $\tilde{\pi}_n - \tilde{\pi}_{n-1}$ has the same sign as $\pi_n^C - \tilde{\pi}_{n-1}$, it then follows that n_{SZ} is greater than n_{SH}. The properties of π_n^* and n_{SH} are illustrated in Figure 3.

There is a simple intuition behind the finding that $n_C \leq n_{SH} \leq n_{SZ}$. For all wage setting mechanisms, the firm's benefit from hiring an n-th worker is positive as long as the gains from trade $F(n) - n\underline{w} - \pi_{n-1}$ are positive. For different wage setting mechanisms, the firm captures a different share of the gains from trade, which implies that the firm's payoff π_{n-1} from having $n-1$ workers is different. If wages are set competitively, the firm captures all the gains from trade, which implies $\pi_{n-1}^C = F(n-1) - (n-1)\underline{w}$. If wages are given by the Shapley values, the firm captures only a fraction of the gains from trade, which implies $\pi_{n-1}^* < \pi_{n-1}^C$. If wages are given by the SZ game, the firm captures an even smaller fraction of the gains from trade, which implies $\tilde{\pi}_{n-1} < \pi_{n-1}^*$. Since $\tilde{\pi}_{n-1} < \pi_{n-1}^* < \pi_{n-1}^C$ and hiring continues as long as $F(n) - n\underline{w} - \pi_{n-1} > 0$, it follows that the firm hires more workers if wages are set by the SZ game than if they are set by the Shapley values and, in turn, more workers if wages are set by the Shapley values than if they are set competitively.

Up to this point, we have stressed the differences between the SZ-game payoffs and the Shapley values. Notwithstanding these differences, it is important to recognize that the SZ-

[6] The payoff-maximizing employment level n_{SZ} is such that $\tilde{\pi}_n = \pi_n^C$ up to integer rounding. From (14), it then follows that n_{SZ} is such that the gains from trade $F(n) - \tilde{\pi}_{n-1} - n\underline{w}$ are equal to zero up to integer rounding and, hence, all workers earn approximately a wage equal to their outside option $\underline{w}$. Thus, at the payoff-maximizing level of employment n_{SZ} the outcome of the SZ game does not feature intra-firm wage dispersion. However, this prediction is an artifact of the assumption of costless hiring. If, as is the case in labor-search models, hiring is costly, the firm would hire fewer than n_{SZ} workers, the gains from trade would be strictly positive, and the outcome of the SZ game would feature intra-firm wage inequality.

game payoffs and the Shapley values share a common structure in which the payoff π_n to the firm is given by a difference equation of the form

$$\pi_j = \pi_{j-1} + \lambda_j[F(j) - \pi_j - j\underline{w}], \text{ for } j = 1, 2, ...n, \tag{21}$$

and the payoff w_n^i to the i-th worker is given by an expression of the type

$$w_n^i = \underline{w} + \mu_n^i[F(n) - \pi_{n-1} - n\underline{w}], \tag{22}$$

with $\lambda_n + \sum_{i=1}^n \mu_n^i = 1$. The key feature of this common structure is that firm and workers share a notion of gains from trade given by $F(n) - \pi_{n-1} - n\underline{w}$, i.e. the output $F(n)$ that the firm and n workers produce together net of the sum $n\underline{w}$ of the payoffs that each worker can attain if he were excluded from production and the payoff π_{n-1} that the firm could attain if it were to bargain with $n-1$ rather than n workers. The SZ-game payoffs share this structure with the Shapley values because of the assumption that–when the bargaining session between the firm and one of the n workers breaks down–the worker exits the game and the firm starts the game over with $n-1$ workers. The SZ-game payoffs are different from the Shapley values because the division of the gains from trade among players is different. In light of their common structure, one could say that the SZ game delivers a generalized version of the Shapley values.[7]

The above observations are important as they provide a recipe to construct an extensive-form bargaining game that yields the Shapley values. Such a game should maintain the assumption that a breakdown in a bargaining session between firm and worker causes the worker to exit and the firm to restart the whole bargaining process, but it should modify the protocol of the bargaining session so that the firm and the worker always capture an equal share of the gains from trade. The Rolodex game in the next section follows this recipe.

3 The Rolodex Game

In this section, we propose a novel bargaining game between a firm and n workers. We refer to it as the Rolodex game. In Section 3.1, we describe the extensive form of the Rolodex game and relate it to the extensive form of the SZ game. In Section 3.2, we characterize the unique no-delay SPE of the Rolodex game. We find that, in equilibrium, a worker captures the same fraction of the gains from trade as the firm, irrespective of the order in which he bargains with the firm. Moreover, the relevant notion of gains from trade is given by the

[7]In Section 3, Stole and Zwiebel (1996a) consider a generalized version of their payoff equations, derived under the assumption that the outcome of the bargaining session between the firm and a worker is such that the two parties divide the gains from trade according to some arbitrary share, which is identical for every worker but allowed to depend on the total number of workers n. The firm's generalized payoff is the same as in (21). The worker's generalized payoff is not the same as in (22), because (22) allows for the possibility–a possibility that materializes in the equilibrium of the SZ game–that workers capture a different share of the gains from trade depending on their position in the bargaining queue.

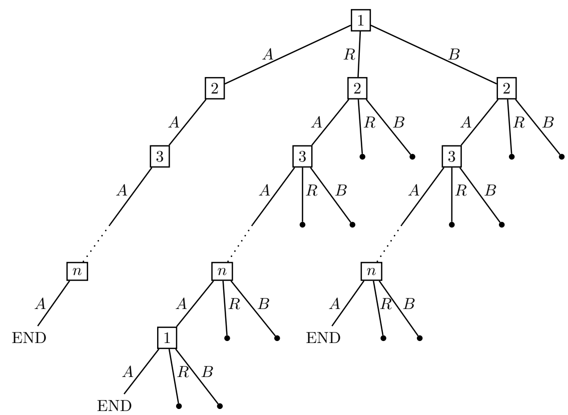

Figure 4: Sequence of bargaining sessions in Rolodex game

difference between the output produced by the firm and n workers net of the sum of the payoffs that the workers can attain if they are excluded from the game and the payoff that the firm can attain by bargaining with $n-1$ rather than n workers. We show that these properties of equilibrium imply that the payoffs to the workers and to the firm are equal to their Shapley values.

3.1 Environment

We start by describing the extensive form of the Rolodex game. The game consists of a sequence of bilateral bargaining sessions between the firm and one of the workers. Workers are placed in some arbitrary queue from 1 to n. The game starts with a bargaining session between the firm and the first worker in the initial queue. The bargaining session involves the determination of the worker's wage and ends with an agreement, a rotation of the rolodex, or a breakdown. If the session ends with agreement, the firm enters a bargaining session with the worker who, among those without an agreement, is next in the queue. If the session ends with a rotation of the rolodex, the worker moves to the end of the queue of workers without agreement. Then the firm enters a bargaining session with the worker who, among those who have yet to reach an agreement, is next in the updated queue. If the session ends with a breakdown, the worker exits the game and the entire bargain process starts over.[8] The game ends when the firm has reached an agreement with all the workers still in the game. Figure 4 illustrates the sequence of the bargaining sessions in the Rolodex game, where the number in the box denotes the position in the original queue of the worker with whom the firm is bargaining at that time.

Each bargaining session between the firm and a worker involves, at most, one offer and one counteroffer. The session starts with the worker making a wage offer to the firm. If the firm accepts the offer, the session ends with an agreement and the firm starts bargaining

[8] The order in which the remaining workers are placed when the bargaining process starts over is irrelevant.

with the worker who, among those without an agreement, is next in the queue. If the firm rejects the offer, the session ends with a breakdown with probability q and continues with probability $1-q$. If the session continues, the firm makes a wage counteroffer to the worker. If the worker accepts the counteroffer, the session ends with an agreement and the firm starts bargaining with the worker without agreement who is next in the queue. If the worker rejects the counteroffer, the session ends with a breakdown with probability q and ends with a rotation of the rolodex with probability $1-q$.

There is only one difference between the protocols of the Rolodex and SZ games. In the SZ game, a bargaining session between a firm and a worker continues until the firm and the worker reach an agreement or the worker leaves the game. That is, a worker can keep rejecting the counteroffers of the firm without losing his place in the queue. In the Rolodex game, a worker moves to the end of the queue when he rejects a firm's counteroffer. Under the SZ protocol, a worker at the head of the queue can take advantage of the fact that–if the firm pays him an extra dollar–it will pay the workers who follow him in the queue a lower wage. In contrast, under the Rolodex protocol, any worker is in the same strategic position as the last worker in the queue. In fact, if any worker rejects the firm's counteroffer, he becomes the last worker and, thus, the firm can successfully offer him the same wage that is earned by the last worker and the worker can only successfully demand the same wage that is earned by the last worker. In equilibrium, any worker captures the same share of the gains from trade as the last worker in the queue. Since the last worker in the queue captures the same share of the gains from trade as the firm, all players capture an equal share of the gains from trade, as per the Shapley values. Moreover, since the Rolodex protocol maintains the assumption that a breakdown in a bargaining session causes the worker to exit and the firm to restart the whole bargaining process, the notion of the gains from trade is the same as in the Shapley values. Taken together, these two features imply that the Rolodex game yields the Shapley values.

3.2 Solution of the Rolodex Game

We are interested in the no-delay Subgame Perfect Equilibria of the Rolodex game, i.e. equilibria with the property that the firm and the workers reach an agreement without delay in any subgame in which the gains from trade are positive.[9] We shall use the same notation for the Rolodex game as for the SZ game. That is, we denote as $\Gamma_{n,k}(s)$ the subgame in which there are n workers in the game, $n-k$ of these workers have already reached an agreement with the firm for wages summing up to s, k workers have yet to reach an agreement with the

[9] For some configurations of the parameters, we were able to construct Subgame Perfect Equilibria of the Rolodex game with delay. For this paper, however, we decided to restrict the analysis to no-delay SPE in order to facilitate the comparison between the Rolodex game, the SZ game and, more generally, the literature, since most perfect-information bargaining games only admit equilibria without delay. We leave the analysis of delay SPE of the Rolodex game for further research.

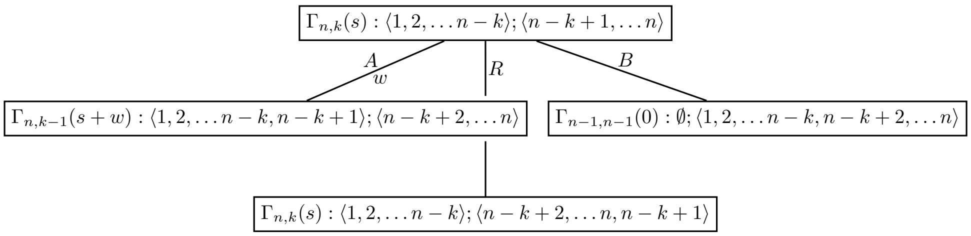

Figure 5: Structure of generic subgame in Rolodex game

firm, and the firm is about to enter a bargaining session with the first of those k workers. We denote as $w^i_{n,k}(s)$ the wage of the i-th of the k workers without agreement in a no-delay SPE of the subgame $\Gamma_{n,k}(s)$. The Rolodex game between the firm and n workers is then $\Gamma_{n,n}(0)$. We use the shorthand $\tilde{\pi}_n$ to denote the payoff to the firm and $\tilde{w}^i_n$ to denote the payoff to the i-th worker in a no-delay SPE of $\Gamma_{n,n}(0)$.

Figure 5 illustrates the structure of a generic subgame $\Gamma_{n,k}(s)$, in which workers 1, 2, ..., $n-k$ have reached an agreement with the firm for wages summing up to s and $n-k+1$, $n-k+2$, ... , n is the queue of workers who have yet to reach an agreement with the firm. The subgame $\Gamma_{n,k}(s)$ starts with a bargaining session between the firm and worker $n-k+1$. If the session ends with agreement at the wage w, the game enters the subgame $\Gamma_{n,k-1}(s+w)$, in which workers 1, 2, ..., $n-k+1$ have reached an agreement with the firm for wages summing up to $s+w$ and $n-k+2$, $n-k+3$, ... , n is the queue of workers who have yet to reach an agreement with the firm. If the session ends with a rotation of the rolodex, the game enters the subgame $\Gamma_{n,k}(s)$, in which workers $1, 2, ...n-k$ have reached an agreement with the firm for wages summing up to s and $n-k+2$, $n-k+3$,... , n, $n-k+1$ is the queue of workers who have yet to reach an agreement with the firm. If the session ends with a breakdown, the worker exits and the game continues with the subgame $\Gamma_{n-1,n-1}(0)$, in which workers 1, 2, ..., $n-k$, $n-k+2$, ... , n have yet to reach an agreement with the firm.

The intuition behind the equilibrium properties of the Rolodex game can be gained by studying the game between the firm and two workers. We solve for the no-delay SPE of the Rolodex game $\Gamma_{2,2}(0)$ by backward induction. First, we solve for the no-delay SPE of the subgame $\Gamma_{1,1}(0)$ which is reached if the bargaining session between the firm and one of its two workers ends with a breakdown. Second, we solve for the no-delay SPE of the subgame $\Gamma_{2,1}(w_1)$ which is reached if the bargaining session between the firm and one of its two workers ends with an agreement at the wage w_1. Third, we solve for the no-delay SPE of the game $\Gamma_{2,2}(0)$.

Consider the subgame $\Gamma_{1,1}(0)$. It begins with the worker making a wage offer to the firm. If the offer is rejected, the firm makes a counteroffer to the worker. If the counteroffer is rejected, a rotation of the rolodex takes place and the worker is moved from the top to the bottom of the queue. However, as there are no other workers in the game, the rotation leaves the worker at the top of the queue. Thus, the worker and the firm continue in alternating offers and counteroffers until either they reach an agreement or until the negotiation breaks down. If the firm and the worker agree to the wage w_1, the subgame ends. In this case, the payoff to the firm is $F(1)$ and the payoff to the worker is w_1. If the negotiation breaks down, the subgame also ends. In this case, however, the payoff to the firm is $\tilde{\pi}_0$ and the payoff to the worker is $\underline{w}$. Overall, the protocol and the payoff structure of the subgame $\Gamma_{1,1}(0)$ are the same as in BRW and the SPE of the subgame is given by Lemma 1 for $y = F(1)$, $z = \tilde{\pi}_0$ and $t(w_1) = 0$. Therefore, assuming the gains from trade $F(1) - \tilde{\pi}_0 - \underline{w}$ are positive, the unique SPE of the subgame $\Gamma_{1,1}(0)$ is such that the firm and the worker immediately agree to the wage

$$\tilde{w}_1^1 = \underline{w} + \frac{1}{2-q}[F(1) - \tilde{\pi}_0 - \underline{w}]. \tag{23}$$

In turn, this implies that the payoff to the firm is

$$\tilde{\pi}_1 = \tilde{\pi}_0 + \frac{1-q}{2-q}[F(1) - \tilde{\pi}_0 - \underline{w}]. \tag{24}$$

Now, consider the subgame $\Gamma_{2,1}(w_1)$. It starts with a bargaining session between the firm and the worker with whom the firm has yet to reach an agreement. As there are no other workers without an agreement, the rotation of the rolodex does not affect the worker's position in the queue. Hence, the bargaining session is such that the worker and the firm alternate in making offers and counteroffers until either they reach an agreement or until the negotiation breaks down. If the firm and the worker agree to the wage w_2, the game ends, the payoff to the firm is $F(2) - w_1 - w_2$ and the payoff to the worker is w_2. If the negotiation breaks down, the worker exits and the payoff to the worker is $\underline{w}$. The firm enters the subgame $\Gamma_{1,1}(0)$, in which it renegotiates its prior agreement with the other worker. In the unique SPE of $\Gamma_{1,1}(0)$, the payoff to the firm is $\tilde{\pi}_1$. The reduced-form version of the subgame $\Gamma_{2,1}(w_1)$ in which the continuation subgames are replaced with the associated SPE payoffs has the same protocol and the same payoff structure as in BRW for $y = F(2) - w_1$, $z = \tilde{\pi}_1$ and $t(w_2) = 0$. Therefore, if the gains from trade $F(2) - w_1 - \tilde{\pi}_1 - \underline{w}$ are negative, any SPE of $\Gamma_{2,1}(w_1)$ is such that the firm and the worker do not reach an agreement. If the gains from trade are positive, the unique SPE of $\Gamma_{2,1}(w_1)$ is such that the firm and the worker immediately reach an agreement at the wage

$$w_{2,1}^1(w_1) = \underline{w} + \frac{1}{2-q}[F(2) - w_1 - \tilde{\pi}_1 - \underline{w}]. \tag{25}$$

Next, consider the game $\Gamma_{2,2}(0)$. Assume that the gains from trade $F(2) - \tilde{\pi}_1 - 2\underline{w}$ are positive. To characterize the unique no-delay equilibrium, we use the standard method of proof for Rubinstein-style models developed by Shaked and Sutton (1984), which proceeds by characterizing the suprema and infima of the sets of equilibrium payoffs (see also Section

3.2.2 in Muthoo (1999)). Let m_W and M_W denote the infimum and the supremum among all no-delay SPE of the offer made by a worker to the firm, given that the other worker has yet to reach an agreement with the firm. Similarly, let m_F and M_F denote the infimum and the supremum among all no-delay SPE of the counteroffer made by the firm to a worker, given that the firm has yet to reach an agreement with the other worker. A worker can always attain the payoff $\underline{w}$ by making offers greater than $\underline{w}$ and rejecting all offers smaller than $\underline{w}$. Since we are looking for SPEs in which all equilibrium offers and counteroffers are immediately accepted, $M_W \geq m_W \geq \underline{w}$ and $M_F \geq m_F \geq \underline{w}$. If the firm agrees with the worker to a wage $w_1 > \overline{w}$, with $\overline{w} \equiv F(2) - \tilde{\pi}_1 - \underline{w}$, the firm enters the subgame $\Gamma_{2,1}(w_1)$ where it does not reach an agreement with the other worker. Since we are looking for no-delay SPE, $m_W \leq M_W \leq \overline{w}$ and $m_F \leq M_F \leq \overline{w}$. Further, note that, if the firm and the worker reach an agreement at any wage $w_1 \leq \overline{w}$, the firm enters the subgame $\Gamma_{2,1}(w_1)$ where it reaches an immediate agreement with the other worker at a wage of $w^1_{2,1}(w_1)$.

Consider any no-delay SPE of $\Gamma_{2,2}(0)$. First, we characterize the worker's equilibrium payoff from accepting/rejecting a counteroffer $\hat{w}_c \in [\underline{w}, \overline{w}]$ from the firm, given that the other worker is without agreement.[10] If the worker accepts the counteroffer, he attains a payoff of $\hat{w}_c$. In fact, if the worker accepts, the firm reaches an agreement with the other worker at the wage $w^1_{2,1}(\hat{w}_c)$, the game comes to an end, and the worker is paid the wage $\hat{w}_c$. If the worker rejects the counteroffer, he attains a payoff between[11] $q\underline{w} + (1-q)w^1_{2,1}(M_W)$ and $q\underline{w} + (1-q)w^1_{2,1}(m_W)$. In fact, if the worker rejects, he exits the game with probability q. In this case, the worker attains a payoff of $\underline{w}$. With probability $1-q$, the worker moves to the end of the queue. In this case, the firm and the worker agree to the wage $w^1_{2,1}(w_o)$ after the firm and the other worker reach an agreement to some wage $w_o \in [m_W, M_W]$. Since $w^1_{2,1}(w_o)$ is strictly decreasing in w_o, the payoff to the worker from rejecting $\hat{w}_c$ is between $q\underline{w} + (1-q)w^1_{2,1}(M_W)$ and $q\underline{w} + (1-q)w^1_{2,1}(m_W)$.

Second, we characterize the firm's equilibrium counteroffer $w_c \in [\underline{w}, \overline{w}]$. The equilibrium counteroffer w_c is accepted by the worker. The worker's decision can be optimal only if the payoff from accepting w_c is greater than the payoff from rejecting w_c. Since the infimum of the firm's counteroffer w_c is m_F, the worker's acceptance payoff is w_c, and the rejection payoff is greater than $q\underline{w} + (1-q)w^1_{2,1}(M_W)$, we have

$$m_F \geq q\underline{w} + (1-q)w^1_{2,1}(M_W). \tag{26}$$

Similarly, note that, in equilibrium, the worker finds it optimal to accept any counteroffer $\hat{w}_c \in [\underline{w}, \overline{w}]$ greater than $q\underline{w} + (1-q)w^1_{2,1}(m_W)$. Thus, if the firm makes such a counteroffer, its payoff is $F(2) - \hat{w}_c - w^1_{2,1}(\hat{w}_c)$. Since $F(2) - \hat{w}_c - w^1_{2,1}(\hat{w}_c)$ is strictly decreasing in $\hat{w}_c$

[10] In what follows, we use $\hat{w}_o$ and $\hat{w}_c$ to denote generic offers and counteroffers, and w_o and w_c to denote offers and counteroffers in an arbitrary no-delay SPE. Later, we use w^*_o and w^*_c to denote offers and counteroffers in the unique no-delay SPE.

[11] The interval is non-empty as $w^1_{2,1}(w_1)$ is strictly decreasing in w_1.

and $w_c \in [\underline{w}, \overline{w}]$, the firm's equilibrium counteroffer w_c can be optimal only if it is no greater than $q\underline{w} + (1-q)w^1_{2,1}(m_W)$. As the supremum of the firm's counteroffer w_c is M_F, we have

$$M_F \leq q\underline{w} + (1-q)w^1_{2,1}(m_W). \tag{27}$$

Third, we characterize the firm's equilibrium payoff from accepting/rejecting an offer $\hat{w}_o \in [\underline{w}, \overline{w}]$ from a worker, given that the other worker is without agreement. If the firm accepts the offer, it attains a payoff of $F(2) - \hat{w}_o - w^1_{2,1}(\hat{w}_o)$. If the firm rejects the offer, its payoff is between[12] $q\tilde{\pi}_1 + (1-q)[F(2) - M_F - w^1_{2,1}(M_F)]$ and $q\tilde{\pi}_1 + (1-q)[F(2) - m_F - w^1_{2,1}(m_F)]$. In fact, if the firm rejects the offer, the worker exits the game with probability q. In this case, the firm attains a payoff of $\tilde{\pi}_1$. With probability $1-q$, the firm makes a counteroffer. In this case, the firm and the worker agree to some wage $w_c \in [m_F, M_F]$ and then the firm and the other worker agree to the wage $w^1_{2,1}(w_c)$. Since $F(2) - w_c - w^1_{2,1}(w_c)$ is strictly decreasing in w_c, the firm's payoff from rejecting $\hat{w}_o$ is between $q\tilde{\pi}_1 + (1-q)[F(2) - M_F - w^1_{2,1}(M_F)]$ and $q\tilde{\pi}_1 + (1-q)[F(2) - m_F - w^1_{2,1}(m_F)]$.

Fourth, we characterize the worker's equilibrium offer $w_o \in [\underline{w}, \overline{w}]$. The equilibrium offer w_o is accepted by the firm. The firm's decision can be optimal only if the payoff from accepting w_o is greater than the payoff from rejecting w_o. Since the supremum of the worker's offer is M_W, the firm's acceptance payoff is $F(2) - w_o - w^1_{2,1}(w_o)$, and the firm's rejection payoff is greater than $q\tilde{\pi}_1 + (1-q)[F(2) - M_F - w^1_{2,1}(M_F)]$, we have

$$F(2) - M_W - w^1_{2,1}(M_W) \geq q\tilde{\pi}_1 + (1-q)[F(2) - M_F - w^1_{2,1}(M_F)]. \tag{28}$$

Similarly, note that, in equilibrium, the firm accepts any offer $\hat{w}_o \in [\underline{w}, \overline{w}]$ such that the acceptance payoff is greater than the upper bound on the rejection payoff $q\tilde{\pi}_1 + (1-q)[F(2) - m_F - w^1_{2,1}(m_F)]$. Hence, if the worker makes such an offer, his payoff is $\hat{w}_o$. Since the firm's acceptance payoff is strictly decreasing in $\hat{w}_o$ and $w_o \in [\underline{w}, \overline{w}]$, the worker's equilibrium offer $w_o \in [\underline{w}, \overline{w}]$ can be optimal only if the firm's acceptance payoff is no greater than the upper bound on the firm's rejection payoff. As the infimum of the worker's offer w_o is m_W, we have

$$F(2) - m_W - w^1_{2,1}(m_W) \leq q\tilde{\pi}_1 + (1-q)[F(2) - m_F - w^1_{2,1}(m_F)]. \tag{29}$$

The inequalities (26)-(29) provide bounds on the equilibrium offers m_W and M_W and counteroffers m_F and M_F. Combining these inequalities, it is easy to show that $m_F = M_F = w^*_c$, where

$$w^*_c = \underline{w} + \frac{(1-q)^2}{1 + (1-q) + (1-q)^2}[F(2) - \tilde{\pi}_1 - 2\underline{w}]. \tag{30}$$

Similarly, it is easy to show that $m_W = M_W = w^*_o$ where

$$w^*_o = \underline{w} + \frac{1}{1 + (1-q) + (1-q)^2}[F(2) - \tilde{\pi}_1 - 2\underline{w}]. \tag{31}$$

[12] The interval is non-empty as $w_1 + w^1_{2,1}(w_1)$ is strictly increasing in w_1.

We are now in the position to construct a candidate no-delay SPE of $\Gamma_{2,2}(0)$. In the candidate SPE, a worker makes the offer w_o^* to the firm whenever the other worker is without agreement. The firm accepts any offer $\hat{w}_o$ such that $\hat{w}_o \leq w_o^*$ and rejects any other offer. The firm makes the counteroffer w_c^* to a worker whenever the other worker is without agreement. The worker accepts any counteroffer $\hat{w}_c$ such that $\hat{w}_c \geq w_c^*$ and $\hat{w}_c \leq \overline{w}$ and rejects any counteroffer $\hat{w}_c < w_c^*$.[13] The strategies and payoffs associated with the subgames following an agreement or a breakdown with the first of two workers without agreement are those implied by the no-delay SPE of $\Gamma_{2,1}(w_1)$ and $\Gamma_{1,1}(0)$. It is easy to verify that this candidate no-delay SPE is indeed an equilibrium (see Appendix B). Moreover, it is obvious that the no-delay SPE constructed above is unique with respect to outcomes and payoffs, as $m_W = M_W = w_o^*$ implies that, in any no-delay SPE, the worker makes the offer w_o^* and the firm accepts it.

We can now compute the equilibrium outcomes and payoffs in the unique no-delay SPE of $\Gamma_{2,2}(0)$. Assuming $F(2) - \tilde{\pi}_1 - 2\underline{w} \geq 0$, the firm and the first worker reach an immediate agreement at the wage $\tilde{w}_2^1 = w_o^*$, while the firm and the second worker reach an immediate agreement at the wage $\tilde{w}_2^2 = w_{2,1}^1(w_o^*)$. Therefore, the wage (and payoff) of the first worker is given by

$$\tilde{w}_2^1 = \underline{w} + \frac{1}{1 + (1-q) + (1-q)^2}[F(2) - \tilde{\pi}_1 - 2\underline{w}]. \tag{32}$$

The wage (and payoff) of the second worker is

$$\tilde{w}_2^2 = \underline{w} + \frac{1-q}{1 + (1-q) + (1-q)^2}[F(2) - \tilde{\pi}_1 - 2\underline{w}]. \tag{33}$$

The payoff of the firm is

$$\tilde{\pi}_2 = \tilde{\pi}_1 + \frac{(1-q)^2}{1 + (1-q) + (1-q)^2}[F(2) - \tilde{\pi}_1 - 2\underline{w}]. \tag{34}$$

For $q \to 0$, the payoffs to the workers and the firm are

$$\begin{aligned} \tilde{w}_2^1 &= \underline{w} + [F(2) - \tilde{\pi}_1 - 2\underline{w}]/\,3, \\ \tilde{w}_2^2 &= \underline{w} + [F(2) - \tilde{\pi}_1 - 2\underline{w}]/\,3, \\ \tilde{\pi}_2 &= \tilde{\pi}_1 + [F(2) - \tilde{\pi}_1 - 2\underline{w}]/\,3, \end{aligned} \tag{35}$$

where $\tilde{\pi}_1$ is given by (24). It is easy to understand why the SPE payoffs of the Rolodex game are given by (35). A breakdown in the bargaining session between the firm and either worker causes the worker to exit the game and the firm to restart the entire negotiation process with only one worker. For this reason, the overall gains from trade are given by $F(2) - \tilde{\pi}_1 - 2\underline{w}$. The bargaining session between the firm and the second worker is as in the BRW game with perfectly transferrable utility. For this reason, the gains from trade accruing to the firm are the same as the gains from trade accruing to the second worker. The bargaining session

[13] We do not need to specify the worker's response to a counteroffer $\hat{w}_c > \overline{w}$ because the firm never finds it optimal to make such a counteroffer independently of the response of the worker.

between the firm and the first worker is such that, if the worker rejects the counteroffer of the firm, he moves to the end of the queue. For this reason, the first worker is in the same strategic position and captures the same gains from trade as the second worker. Taken together, these observations imply that the firm and every worker capture the same fraction (1/3) of the gains from trade $F(2) - \tilde{\pi}_1 - 2\underline{w}$. These are the Shapley values.

The following theorem generalizes the characterization of the no-delay SPE of the Rolodex game between the firm and two workers to the case of n workers.

Theorem 2: (Rolodex game). *Consider the Rolodex game $\Gamma_{n,n}(0)$ between the firm and n workers. Assume that the gains from trade are positive, i.e. $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$. In the limit for $q \to 0$, the unique no-delay SPE of the game is such that the payoff to the firm is given by the difference equation*

$$\tilde{\pi}_j = \tilde{\pi}_{j-1} + [F(j) - \tilde{\pi}_{j-1} - j\underline{w}] / (1 + j), \text{ for } j = 1, 2, ...n, \tag{36}$$

with initial condition $\tilde{\pi}_0 = F(0)$. The payoff to the i-th of n workers is given by

$$\tilde{w}_n^i = \underline{w} + [F(n) - \tilde{\pi}_{n-1} - n\underline{w}] / (1 + n). \tag{37}$$

Proof: In Appendix B. ■

Three comments about Theorem 2 are in order. First, note that the SPE payoffs of the Rolodex game between the firm and n workers are equal to the Shapley values. The firm's Rolodex payoff is given by the solution to (36) which is the same difference equation that characterizes the Shapley value of the firm. Thus, $\tilde{\pi}_j = \pi_j^*$ for $j = 0, 1, \ldots n$. Each worker's Rolodex payoff is given by (37) which is the Shapley value of the worker. Thus, $\tilde{w}_n^i = w_n^*$ for $i = 1, 2, ...n$. The Rolodex payoffs are equal to the Shapley values because, in equilibrium, the firm and every worker capture an equal share of the gains from trade–exactly as in the Shapley values–and the notion of gains from trade is given by $F(n) - \tilde{\pi}_{n-1} - n\underline{w}$–exactly as in the Shapley values. The reason why the Rolodex payoffs are such that every player captures an equal share $1/(n + 1)$ of the gains from trade and the reason why the notion of the gains is given by $F(n) - \tilde{\pi}_{n-1} - n\underline{w}$ are exactly the same as in the case of two workers.

Second, if wages are determined by the Rolodex game, the firm has an incentive to hire more workers than it would in a competitive labor market where it takes the wage $\underline{w}$ as given. However, the firm has an incentive to hire fewer workers if wages are determined by the Rolodex game than if wages were determined by the SZ game. These findings follow immediately from the analysis in Section 2.3 of the firm's hiring decision when wages are given by the Shapley values, together with the observation that the equilibrium payoffs of the Rolodex game are the Shapley values. For the same reason, many of the results about the firm's organizational design and technology choice in Stole and Zwiebel (1996a and 1996b) and in follow-up papers–results that are derived under the premise that the equilibrium payoffs of the SZ game are the Shapley values–can be applied directly to an environment in which wages are determined by the Rolodex game.

Third, Theorem 2 shows that the Rolodex game yields the Shapley values in the context of the wage bargaining between a firm and its workers. The context is restrictive along two dimensions: (i) the firm is essential in production; (ii) the network in which negotiations occur has a star graph, i.e. one central player bargains with a number of other players who do not interact directly with each other. In a follow-up paper (Brügemann et al. 2017) we analyze the Rolodex game in a more general context. First, we remove the assumption that one player is essential in production and instead allow the worth of coalitions to be described by a general characteristic function. We show that the equilibrium payoffs are the Myerson-Shapley values for the corresponding star graph. Second, we adapt the extensive form to accommodate a general graph of connections between players: instead of a central player rotating through the other players, the game rotates through bilateral connections without agreement. We show that, for this graph, the equilibrium payoffs are the Myerson-Shapley values.[14]

4 Conclusions

In this paper, we analyzed two perfect-information wage bargaining games between a firm and its workers. We first revisited the bargaining game of Stole and Zwiebel (1996a). We showed that, in the unique SPE of this game, the payoffs to the workers and the firm are different from those reported in Stole and Zwiebel (1996a) and, hence, different from the Shapley values. We then proposed a novel bargaining game, which we dubbed the Rolodex game. We showed that, in the unique no-delay SPE of this game, the payoffs to the workers and the firm are the Shapley values. Thus, the Rolodex game results offers a game-theoretic foundation for those who wish to use the Shapley values as the outcome of wage negotiations between a firm and its workers.

[14] The generalized Rolodex game yields the same equilibrium payoffs as those obtained by de Fontenay and Gans (2014) in their imperfect information game with passive beliefs.

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Appendix

A Proof of Theorem 1

The following proposition contains the characterization of the unique SPE of the subgame $\Gamma_{n,n}(0)$ in which the firm has yet to reach an agreement with all of the n workers remaining in the game.

Proposition A.1: *Consider the subgame $\Gamma_{n,n}(0)$. (i) If $F(n) - \tilde{\pi}_{n-1} - n\underline{w} < 0$, any SPE is such that the firm does not reach an agreement with all of the n workers. The payoff to the firm is given by $\tilde{\pi}_n = \tilde{\pi}_{n-1}$. (ii) If $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$, the unique SPE is such that the firm immediately reaches an agreement with all of the n workers. The payoff to the firm is given by*

$$\tilde{\pi}_n = \tilde{\pi}_{n-1} + \left(\frac{1-q}{2-q}\right)^n \left[F(n) - \tilde{\pi}_{n-1} - n\underline{w}\right]. \tag{A1}$$

The payoff to the i-th worker is given by

$$\tilde{w}_n^i = \underline{w} + \frac{1}{2-q}\left[F(n) - \left(\sum\nolimits_{j=1}^{i-1} \tilde{w}_n^j\right) - \tilde{\pi}_{n-1} - (n+1-i)\underline{w}\right]. \tag{A2}$$

For $n = 1$, Proposition A.1 holds as the payoffs in (A1) and (A2) boil down to the equilibrium payoffs of the BRW game. For $n = 2$, Proposition A.1 holds as the payoffs in (A1) and (A2) are those derived in Section 2.2. In what follows we are going to prove that Proposition A.1 holds for a generic n by induction. That is, we are going to prove that if the proposition holds for the subgame $\Gamma_{n,n}(0)$, it also holds for the subgame $\Gamma_{n+1,n+1}(0)$ where the firm has yet to reach an agreement with all of the $n+1$ workers left in the game.

Central to the characterization of the equilibrium of $\Gamma_{n+1,n+1}(0)$ is the following proposition.

Proposition A.2: *Consider the subgame $\Gamma_{n+1,k}(s)$ in which the firm has $n+1$ workers, it has yet to reach an agreement with $k \leq n+1$ workers, and it has agreed to wages summing up to s with the first $n+1-k$ workers. (i) If $F(n+1) - s - \tilde{\pi}_n - k\underline{w} < 0$, any SPE is such that the firm does not reach an agreement with all the k remaining workers; (ii) If $F(n+1) - s - \tilde{\pi}_n - k\underline{w} \geq 0$, the unique SPE is such that the firm reaches an immediate agreement with each of the k remaining workers. The sum of the wages paid to the k remaining workers is*

$$t_{n+1,k}(s) = k\underline{w} + \left[1 - \left(\frac{1-q}{2-q}\right)^k\right]\left[F(n+1) - s - \tilde{\pi}_n - k\underline{w}\right]. \tag{A3}$$

For $k = 1$, Proposition A.2 holds as the payoffs in (A3) are the same as those in the BRW game. We prove that Proposition A.2 holds for any $k \leq n+1$ by induction. That is, we prove that, if Proposition A.2 holds for some arbitrary $k \leq n$, then it also holds for $k+1$. To this aim, we consider the subgame $\Gamma_{n+1,k+1}(s)$, in which the firm has $n+1$ employees, it has yet to reach an agreement with $k+1$ of them and it has agreed to wages summing up to s with the first $n-k$ workers. As usual, we characterize the solution to this subgame by backward induction.

First, consider the subgame $\Gamma_{n,n}(0)$ in which, after a breakdown in negotiations between the firm and the first of the $k+1$ workers without agreement, bargaining starts over between the firm and the n workers left in the game. Since we have conjectured that Proposition A.1 holds when the firm has n workers, the SPE payoff of the firm in this subgame is $\tilde{\pi}_n$.

Second, consider the subgame $\Gamma_{n+1,k}(s+w_1)$ in which, after the firm has reached an agreement at some wage w_1 with the first worker without an agreement, the firm starts bargaining with the other k workers without an agreement. Since we conjectured that Proposition A.2 holds when the firm has $n+1$ workers and has yet to reach an agreement with k of them, there is a unique SPE to this subgame. In particular, if $w_1 > \overline{w} \equiv F(n+1) - s - \tilde{\pi}_n - k\underline{w}$, the SPE is such that the firm does not reach an agreement with all of the k remaining workers. In this case, the firm's payoff is $\tilde{\pi}_n$. If $w_1 \leq \overline{w}$, the SPE is such that the firm immediately reaches an agreement with all of the k remaining workers. In this case, the firm's payoff is $F(n+1) - s - w_1 - t_{n+1,k}(s+w_1)$.

Third, we characterize the solution to the subgame $\Gamma_{n+1,k+1}(s)$. Consider the bargaining session between the firm and the first of the $k+1$ workers without an agreement. If the firm and the worker do not reach an agreement, the worker exits the game and the firm enters the subgame $\Gamma_{n,n}(0)$. In this case, the payoff to the firm is $\tilde{\pi}_n$ and the payoff to the worker is $\underline{w}$. If the firm and the worker agree to a wage $w_1 > \overline{w}$, the firm enters the subgame $\Gamma_{n+1,k}(s+w_1)$ with negative gains from trade. In this case, the payoff to the firm is $\tilde{\pi}_n$ and the payoff to the worker is the wage earned by the $(n-k+1)$-th worker in the game with n workers. Finally, if the firm and the worker agree to a wage $w_1 \leq \overline{w}$, the firm enters the subgame $\Gamma_{n+1,k}(s+w_1)$ with positive gains from trade. In this case, the firm reaches an agreement with all the other workers, the payoff to the firm is $F(n+1) - s - w_1 - t_{n+1,k}(s+w_1)$ and the payoff to the worker is w_1. Notice that $t_{n+1,k}(s+w_1)$ is linear and of the form

$$t_{n+1,k}(s+w_1) = \alpha_k - \beta_k w_1, \tag{A4}$$

where β_k given by

$$\beta_k = 1 - \left(\frac{1-q}{2-q}\right)^k. \tag{A5}$$

Consider the reduced-form version of the subgame $\Gamma_{n+1,k+1}(s)$ in which the subgames following the bargaining session between the firm and the first of the $k+1$ workers without agreement are replaced with the associated SPE payoffs. The reduced form of $\Gamma_{n+1,k+1}(s)$ has the same protocol as the BRW game, but not the same payoff structure. However, assume that, whenever indifferent, the firm rejects any wage offer from the worker that would lead to a breakdown in negotiations with one of the subsequent workers. Similarly, assume that, whenever indifferent, the firm chooses not to make any counteroffer to the worker that would lead to a breakdown with subsequent workers. Under these tie-breaking assumptions, we prove in Appendix C that the outcome of the reduced form of $\Gamma_{n+1,k+1}(s)$ is the same as the outcome of the BRW game with $y = F(n+1) - s$, $z = \tilde{\pi}_n$ and $t(w_1) = t_{n+1,k}(s + w_1)$, where $t_{n+1,k}(s + w_1)$ is a linear function of w_1 with derivative $-\beta_k$. It then follows from Lemma 1 that, if $F(n+1) - s - \tilde{\pi}_n - \underline{w} - t_{n+1,k}(s + \underline{w}) < 0$ or equivalently $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} < 0$, any SPE is such that the firm and the worker do not reach an agreement. In contrast, if $F(n+1) - s - \tilde{\pi}_n - \underline{w} - t_{n+1,k}(s + \underline{w}) \geq 0$ or equivalently $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \geq 0$, the unique SPE is such that firm and the first worker immediately reach an agreement over the wage

$$w^1_{n+1,k+1}(s) = \underline{w} + \frac{1}{(2-q)(1-\beta_k)} \left[F(n+1) - s - \tilde{\pi}_n - \underline{w} - t_{n+1,k}(s + \underline{w}) \right]. \tag{A6}$$

We can now summarize the characterization of the subgame $\Gamma_{n+1,k+1}(s)$. If $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} < 0$, any SPE is such that the firm and the first worker do not reach an agreement. If $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \geq 0$, any SPE is such that the firm and the first worker immediately reach an agreement over the wage $w^1_{n+1,k+1}(s)$ in (A6). Substituting $t_{n+1,k}(s + \underline{w})$ with (A3) and β_k with (A5) into (A6), we can write $w^1_{n+1,k+1}(s)$ as

$$w^1_{n+1,k+1}(s) = \underline{w} + \frac{1}{2-q} \left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \right]. \tag{A7}$$

Since $w^1_{n+1,k+1}(s) \leq \overline{w}$, the firm then reaches an immediate agreement with the following k workers for wages totaling up to

$$\begin{aligned} & t_{n+1,k}(s + w^1_{n+1,k+1}(s)) \\ = \; & k\underline{w} + \frac{1-q}{2-q} \left[1 - \left(\frac{1-q}{2-q} \right)^k \right] \left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \right]. \end{aligned} \tag{A8}$$

The sum $t_{n+1,k+1}(s)$ of the wage paid by the firm to the first worker, $w^1_{n+1,k+1}(s)$, and the wages paid to the following k workers, $t_{n+1,k}(s + w^1_{n+1,k+1}(s))$, is given by

$$t_{n+1,k+1}(s) = (k+1)\underline{w} + \left[1 - \left(\frac{1-q}{2-q} \right)^{k+1} \right] \left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \right]. \tag{A9}$$

These results establish that, if Proposition A.2 holds for some $k \leq n$, it also holds for $k+1$. Since the proposition holds for $k = 1$, this implies that it holds for any generic $k \leq n+1$. This completes the proof of Proposition A.2.

Letting $k = n+1$ and $s = 0$ in Proposition A.2, we can characterize the payoffs of the subgame $\Gamma_{n+1,n+1}(0)$. In particular, if $F(n+1) - \tilde{\pi}_n - (n+1)\underline{w} < 0$, any SPE is such that the firm does not reach an agreement with all of its $n+1$ workers. In this case, the payoff to the firm is given by $\tilde{\pi}_{n+1} = \tilde{\pi}_n$. If $F(n+1) - \tilde{\pi}_n - (n+1)\underline{w} \geq 0$, the unique SPE is such that the firm immediately reaches an agreement with all of its $n+1$ workers. In this case, the payoff to the firm is given by

$$\tilde{\pi}_{n+1} = \tilde{\pi}_n + \left(\frac{1-q}{2-q}\right)^{n+1} \left[F(n+1) - \tilde{\pi}_n - (n+1)\underline{w}\right]. \tag{A10}$$

The payoff to the i-th worker is given by

$$\tilde{w}_{n+1}^i = \underline{w} + \frac{1}{2-q}\left[F(n+1) - \left(\sum\nolimits_{j=1}^{i-1} \tilde{w}_{n+1}^j\right) - \tilde{\pi}_n - (n+2-i)\underline{w}\right]. \tag{A11}$$

The above results show that, if Proposition A.1 holds for some n, it also holds for $n+1$. Since the proposition holds for $n = 1$, this means that it holds for any generic $n = 2, 3...$ This completes the proof of Proposition A.1.

We are now in the position to prove Theorem 1, which we restate for the reader's convenience.

Theorem 1: (Stole and Zwiebel game). *Consider the SZ game* $\Gamma_{n,n}(0)$ *between the firm and* n workers. *Assume that the overall gains from trade are positive, i.e.* $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$. *In the limit for* $q \to 0$*, the unique SPE of the game is such that the payoff* $\tilde{\pi}_n$ *to the firm is given by the difference equation*

$$\tilde{\pi}_j = \tilde{\pi}_{j-1} + [F(j) - \tilde{\pi}_{j-1} - j\underline{w}]/\, 2^j, \text{ for } j = 1, 2, ...n, \tag{A12}$$

with initial condition $\pi_0 = F(0)$. *The payoff to the* i*-th of* n *workers is given by*

$$\tilde{w}_n^i = \underline{w} + [F(n) - \tilde{\pi}_{n-1} - n\underline{w}]/\, 2^i. \tag{A13}$$

Proof: It is straightforward to show that if $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$ then $F(j) - \tilde{\pi}_{j-1} - j\underline{w} \geq 0$ for $j = 1, 2, ...n-1$. From this observation and Proposition A.1, it follows that $\tilde{\pi}_j$ is given by (A1) for $j = 1, 2, ...n$ and $\tilde{w}_n^i$ is given by (A2) for $i = 1, 2, ...n$. Taking the limit of (A1) and (A2) for $q \to 0$, we obtain (A12) and (A13). ■

B Proof of Theorem 2

The following proposition contains the characterization of the unique no-delay SPE of the subgame $\Gamma_{n,n}(0)$ in which the firm has yet to reach an agreement with all of the n workers remaining in the game.

Proposition B.1: *Consider the subgame* $\Gamma_{n,n}(0)$. *(i) If* $F(n) - \tilde{\pi}_{n-1} - n\underline{w} < 0$, *any SPE is such that the firm does not reach an agreement with all of the* n *workers. The payoff to* *the firm is given by $\tilde{\pi}_n = \tilde{\pi}_{n-1}$. (ii) If $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$, the unique no-delay SPE is such that the firm immediately reaches an agreement with all of the n workers. The payoff to the firm is given by*

$$\tilde{\pi}_n = \tilde{\pi}_{n-1} + \frac{(1-q)^n}{\sum_{j=0}^{n}(1-q)^j}\left[F(n) - \tilde{\pi}_{n-1} - n\underline{w}\right]. \tag{B1}$$

The payoff to the i-th worker is given by

$$\tilde{w}_n^i = \underline{w} + \frac{1}{\sum_{j=0}^{n+1-i}(1-q)^j}\left[F(n) - \sum_{j=1}^{i-1}\tilde{w}_n^j - \tilde{\pi}_{n-1} - (n+1-i)\underline{w}\right]. \tag{B2}$$

For $n = 1$, Proposition B.1 holds as the payoffs in (B1) and (B2) boil down to the equilibrium payoffs of the BRW game. For $n = 2$, Proposition B.1 holds as the payoffs in (B1) and (B2) coincide with those derived in Section 3.2. In the next pages, we are going to prove that Proposition B.1 holds for a generic n by induction. That is, we are going to prove that if the proposition holds for the subgame $\Gamma_{n,n}(0)$, it also holds for the subgame $\Gamma_{n+1,n+1}(0)$ in which the firm has yet to reach an agreement with all of the $n+1$ workers left in the game.

Central to the characterization of the subgame $\Gamma_{n+1,n+1}(0)$ is the following proposition.

Proposition B.2: *Consider the subgame $\Gamma_{n+1,k}(s)$ in which the firm has $n+1$ workers, it has yet to reach an agreement with $k \leq n+1$ workers, and it has agreed to wages summing up to s with the other $n+1-k$ workers. (i) If $F(n+1) - s - \tilde{\pi}_n - k\underline{w} < 0$, any SPE is such that the firm does not reach an agreement with all of the k remaining workers; (ii) If $F(n+1) - s - \tilde{\pi}_n - k\underline{w} \geq 0$, the unique no-delay SPE is such that the firm reaches an immediate agreement with each of the k remaining workers. The sum of the wages paid to the k remaining workers is*

$$t_{n+1,k}(s) = k\underline{w} + \frac{\sum_{j=0}^{k-1}(1-q)^j}{\sum_{j=0}^{k}(1-q)^j}\left[F(n+1) - s - \tilde{\pi}_n - k\underline{w}\right]. \tag{B3}$$

The wage paid to the first of the k remaining workers is

$$w_{n+1,k}^1(s) = \underline{w} + \frac{1}{\sum_{j=0}^{k}(1-q)^j}\left[F(n+1) - s - \tilde{\pi}_n - k\underline{w}\right]. \tag{B4}$$

The wage paid to the last of the k remaining workers is

$$w_{n+1,k}^k(s) = \underline{w} + \frac{(1-q)^{k-1}}{\sum_{j=0}^{k}(1-q)^j}\left[F(n+1) - s - \tilde{\pi}_n - k\underline{w}\right]. \tag{B5}$$

For $k = 1$, Proposition B.2 holds as the payoffs in (B3)-(B5) are the same as in the BRW game. We prove that Proposition B.2 holds for any $k \leq n+1$ by induction. That is, we prove that, if Proposition B.2 holds for some arbitrary $k \leq n$, then it also holds for $k+1$.

To this aim, we consider the subgame $\Gamma_{n+1,k+1}(s)$, in which the firm has $n+1$ employees, it has yet to reach an agreement with $k+1$ of them and it has agreed to wages summing up to s with the other $n-k$. As usual, we characterize the solution to this subgame by backward induction.

First, consider the subgame $\Gamma_{n,n}(0)$ in which, after a breakdown in negotiations between the firm and the first of the $k+1$ workers without agreement, bargaining starts over between the firm and the n workers left in the game. Since we have conjectured that Proposition B.1 holds when the firm has n workers, the no-delay SPE payoff of the firm in this subgame is uniquely determined and given by $\tilde{\pi}_n$.

Second, consider the subgame $\Gamma_{n+1,k}(s+w_1)$ in which, after the firm has reached an agreement at some wage w_1 with the first worker without an agreement, the firm starts bargaining with the other k workers without an agreement. Since we conjectured that Proposition B.2 holds when the firm has $n+1$ workers and has yet to reach an agreement with k of them, there is a unique no-delay SPE to this subgame. In particular, if $w_1 > \overline{w} \equiv F(n+1) - s - \tilde{\pi}_n - k\underline{w}$, any SPE is such that the firm does not reach an agreement with all of the k remaining workers. In this case, the firm's payoff is $\tilde{\pi}_n$. If $w_1 \leq \overline{w}$, the unique no-delay SPE is such that the firm immediately reaches an agreement with all of the k remaining workers. In this case, the firm's payoff is $F(n+1) - s - w_1 - t_{n+1,k}(s+w_1)$.

Third, we characterize the outcome of the subgame $\Gamma_{n+1,k+1}(s)$. Assume that the gains from trade $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}$ are positive. Let m_W and M_W denote the infimum and the supremum among all no-delay SPE of the offer made by a worker to the firm, given that there are $k+1$ workers without agreement. Similarly, let m_F and M_F denote the infimum and the supremum among all no-delay SPE of the counteroffer made by the firm to a worker, given that there are $k+1$ workers without agreement. A worker can always attain the payoff $\underline{w}$ by making offers greater than $\underline{w}$ and rejecting all offers smaller than $\underline{w}$. Since we are looking for SPEs in which all equilibrium offers and counteroffers are immediately accepted, $M_W \geq m_W \geq \underline{w}$ and $M_F \geq m_F \geq \underline{w}$. If the firm agrees with the worker to a wage $w_1 > \overline{w}$, the firm enters the subgame $\Gamma_{n+1,k}(s+w_1)$ where it does not reach an agreement with all the other worker. Since we are looking for no-delay SPE, $m_W \leq M_W \leq \overline{w}$ and $m_F \leq M_F \leq \overline{w}$. Further, note that, if the firm and the worker reach an agreement at any wage $w_1 \leq \overline{w}$, the firm enters the subgame $\Gamma_{n+1,k}(s+w_1)$ where it reaches an immediate agreement with all the other workers.

Consider any no-delay SPE of $\Gamma_{n+1,k+1}(s)$. First, we characterize the worker's equilibrium payoffs from accepting/rejecting a counteroffer $\hat{w}_c \in [\underline{w}, \overline{w}]$ from the firm, given that there are $k+1$ workers without agreement. If the worker accepts the counteroffer, he attains a payoff of $\hat{w}_c$. In fact, if the worker accepts, the firm reaches an agreement with all the other workers, the game comes to an end and the worker is paid the wage $\hat{w}_c$. If the worker rejects the counteroffer, he attains a payoff between $q\underline{w} + (1-q)w^k_{n+1,k}(s+M_W)$

and $q\underline{w} + (1-q)w^k_{n+1,k}(s+m_W)$. In fact, if the worker rejects, he exits the game with probability q. In this case, the worker attains a payoff of $\underline{w}$. With probability $1-q$, the worker moves to the end of the queue. In this case, the firm and the worker agree to the wage $w^k_{n+1,k}(s+w_o)$ after the firm and the worker who is first in the updated queue reach an agreement at some wage $w_o \in [m_W, M_W]$. Since $w^k_{n+1,k}(s+w_o)$ is strictly decreasing in w_o, the payoff to the worker from rejecting $\hat{w}_c$ is between $q\underline{w} + (1-q)w^k_{n+1,k}(s+M_W)$ and $q\underline{w} + (1-q)w^k_{n+1,k}(s+m_W)$.

Second, we characterize the firm's equilibrium counteroffer $w_c \in [\underline{w}, \overline{w}]$. The equilibrium counteroffer w_c is accepted by the worker. The worker's decision can be optimal only if the payoff from accepting w_c is greater than the payoff from rejecting w_c. Since the infimum of the firm's counteroffer w_c is m_F, the worker's acceptance payoff is w_c, and the rejection payoff is greater than $q\underline{w} + (1-q)w^k_{n+1,k}(s+M_W)$, we have

$$m_F \geq q\underline{w} + (1-q)w^k_{n+1,k}(s+M_W). \tag{B6}$$

Similarly, note that, in equilibrium, the worker finds it optimal to accept any counteroffer $\hat{w}_c \in [\underline{w}, \overline{w}]$ greater than $q\underline{w} + (1-q)w^k_{n+1,k}(s+m_W)$. Thus, if the firm makes such a counteroffer, its payoff is $F(n+1)-s-\hat{w}_c-t_{n+1,k}(s+\hat{w}_c)$. Since $F(n+1)-s-\hat{w}_c-t_{n+1,k}(s+\hat{w}_c)$ is strictly decreasing in $\hat{w}_c$, the firm's equilibrium counteroffer w_c can be optimal only if it is no greater than $q\underline{w} + (1-q)w^k_{n+1,k}(s+m_W)$. As the supremum of the firm's counteroffer w_c is M_F, we have

$$M_F \leq q\underline{w} + (1-q)w^k_{n+1,k}(s+m_W). \tag{B7}$$

Third, we characterize the firm's equilibrium payoff from acepting/rejecting an offer $\hat{w}_o \in [\underline{w}, \overline{w}]$ from a worker, given that there are $k+1$ workers without agreement. If the firm accepts the offer, it attains a payoff of $F(n+1) - s - \hat{w}_o - t_{n+1,k}(s+\hat{w}_o)$. If the firm rejects the offer, its payoff is between $q\tilde{\pi}_n + (1-q)[F(n+1) - s - M_F - t_{n+1,k}(s+M_F)]$ and $q\tilde{\pi}_n + (1-q)[F(n+1) - s - m_F - t_{n+1,k}(s+m_F)]$. In fact, if the firm rejects the offer, the worker exits the game with probability q. In this case, the firm attains a payoff of $\tilde{\pi}_n$. With probability $1-q$, the firm makes a counteroffer. In this case, the firm and the worker agree to some wage $w_c \in [m_F, M_F]$ and then the firm and the other workers agree to wages summing up to $t_{n+1,k}(s+w_c)$. Since $F(n+1) - s - w_c - t_{n+1,k}(s+w_c)$ is strictly decreasing in w_c, the firm's payoff from rejecting $\hat{w}_o$ is between $q\tilde{\pi}_n + (1-q)[F(n+1) - s - M_F - t_{n+1,k}(s+M_F)]$ and $q\tilde{\pi}_n + (1-q)[F(n+1) - s - m_F - t_{n+1,k}(s+m_F)]$.

Fourth, we characterize the worker's equilibrium offer $w_o \in [\underline{w}, \overline{w}]$. The equilibrium offer w_o is accepted by the firm. The firm's decision can be optimal only if the payoff from accepting w_o is greater than the payoff from rejecting w_o. Since the supremum of the worker's offer is M_W, the firm's acceptance payoff is $F(n+1) - s - w_o - t_{n+1,k}(s+w_o)$, and the firm's rejection payoff is greater than $q\tilde{\pi}_n + (1-q)[F(n+1) - s - M_F - t_{n+1,k}(s+M_F)]$, we have

$$F(n+1) - s - M_W - t_{n+1,k}(s+M_W) \geq q\tilde{\pi}_n + (1-q)[F(n+1) - s - M_F - t_{n+1,k}(s+M_F)]. \tag{B8}$$

Similarly, note that, in equilibrium, the firm accepts any offer $\hat{w}_o \in [\underline{w}, \overline{w}]$ such that the acceptance payoff is greater than the upper bound $q\tilde{\pi}_n + (1-q)[F(n+1) - s - m_F - t_{n+1,k}(s+m_F)]$ on the rejection payoff. Hence, if the worker makes such an offer, he attains a payoff of $\hat{w}_o$. Since the firm's acceptance payoff is strictly decreasing in $\hat{w}_o$, the worker's equilibrium offer w_o can be optimal only if the firm's acceptance payoff is no greater than the upper bound on the firm's rejection payoff. As the infimum of the worker's offer w_o is m_W, we have

$$F(n+1) - s - m_W - t_{n+1,k}(s+m_W) \geq q\tilde{\pi}_n + (1-q)[F(n+1) - s - m_F - t_{n+1,k}(s+m_F)]. \quad \text{(B9)}$$

Subtracting (B7) from (B6) and simplifying using (B5) gives $m_F - M_F \geq \gamma(m_W - M_W)$ with $\gamma = (1-q)^k / \sum_{j=0}^{k}(1-q)^j$. Subtracting (B9) from (B8) and simplifying using (B3) yields $m_W - M_W \geq (1-q)(m_F - M_F)$. Combining these inequalities gives $m_F - M_F \geq (1-q)\gamma(m_F - M_F)$, and $q\gamma < 1$ implies $m_F = M_F = w_c^*$ and $m_W = M_W = w_o^*$. Thus (B6)–(B9) hold as equalities, and it is straightforward to solve for w_c^* and w_o^*:

$$w_c^* = \underline{w} + \frac{(1-q)^{k+1}}{\sum_{j=0}^{k+1}(1-q)^j}[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}], \quad \text{(B10)}$$

$$w_o^* = \underline{w} + \frac{1}{\sum_{j=0}^{k+1}(1-q)^j}[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}]. \quad \text{(B11)}$$

The above observations imply that, in any no-delay SPE, a worker makes the offer w_o^* to the firm whenever there are $k+1$ workers without agreement, and the firm makes the counteroffer w_c^* to a worker whenever there are $k+1$ workers without agreement.

We are now in the position to construct a candidate no-delay SPE of $\Gamma_{n+1,k+1}(s)$. In the candidate SPE, a worker makes the offer w_o^* to the firm whenever there are $k+1$ workers without agreement. The firm accepts any offer $\hat{w}_o$ such that $\hat{w}_o \leq w_o^*$ and rejects any other offer. The firm makes the counteroffer w_c^* to a worker whenever there are $k+1$ workers without agreement. The worker accepts any counteroffer $\hat{w}_c$ such that $\hat{w}_c \geq w_c^*$ and $\hat{w}_c \leq \overline{w}$ and rejects any offer $\hat{w}_c < w_c^*$. We do not need to specify the worker's response to a counteroffer $\hat{w}_c > \overline{w}$ because, as we shall see, the firm never finds it optimal to make such a counteroffer. Since any no-delay SPE is such that the worker makes the offer w_o^*, the firm makes the counteroffer w_c^* and these proposals are accepted, the candidate no-delay SPE described above is unique with respect to payoffs and outcomes.

First, we verify that the acceptance/rejection strategy of the worker is optimal. If the firm makes a counteroffer $\hat{w}_c \leq \overline{w}$, the worker's acceptance payoff is $\hat{w}_c$ and the rejection payoff is $q\underline{w} + (1-q)w_{n+1,k}^k(s + w_o^*)$. By construction, the acceptance payoff equals the rejection payoff for $\hat{w}_c = w_c^*$ and, hence, the worker finds it optimal to accept w_c^*. The acceptance payoff is strictly smaller than the rejection payoff for $\hat{w}_c < w_c^*$ and, hence, the worker finds it optimal to reject $\hat{w}_c$. The acceptance payoff is strictly greater than the rejection payoff for $\hat{w}_c > w_c^*$ and, hence, the worker finds it optimal to accept $\hat{w}_c$.

Second, we verify that the firm's counteroffer w_c^* is optimal. If the firm makes the counteroffer w_c^*, the worker accepts and the firm attains a payoff of $F(n+1) - s - w_c^* - t_{n+1,k}(s + w_c^*)$. If the firm makes a counteroffer $\hat{w}_c < w_c^*$, the worker rejects and the firm's payoff is $q\tilde{\pi}_n + (1-q)[F(n+1) - s - w_o^* - t_{n+1,k}(s + w_o^*)]$, which is smaller than $F(n+1) - s - w_c^* - t_{n+1,k}(s + w_c^*)$ since $w_o^* > w_c^*$. If the firm makes a counteroffer $\hat{w}_c > w_c^*$ with $\hat{w}_c \leq \overline{w}$, the worker accepts and the firm's payoff is $F(n+1) - s - \hat{w}_c - t_{n+1,k}(\hat{w}_c)$, which is smaller than $F(n+1) - s - w_c^* - t_{n+1,k}(s + w_c^*)$. If the firm makes a counteroffer $\hat{w}_c > \overline{w}$ and the worker accepts, the firm attains a payoff of $\tilde{\pi}_n$. If the firm makes a counteroffer $\hat{w}_c > \overline{w}$ and the worker rejects, the firm attains a payoff of $q\tilde{\pi}_n + (1-q)[F(n+1) - s - w_o^* - t_{n+1,k}(s + w_o^*)]$. In either case, the firm's payoff is smaller than $F(n+1) - s - w_c^* - t_{n+1,k}(s + w_c^*)$. From the above observations, it follows that the firm finds it optimal to make the counteroffer w_c^*.

Third, we verify that the acceptance/rejection strategy of the firm is optimal. If the worker makes an offer $\hat{w}_o \leq \overline{w}$, the firm's acceptance payoff is $F(n+1) - s - \hat{w}_o - t_{n+1,k}(s + \hat{w}_o)$ and the rejection payoff is $q\tilde{\pi}_n + (1-q)[F(n+1) - s - w_c^* - t_{n+1,k}(s + w_c^*)]$. By construction, the acceptance payoff equals the rejection payoff for $\hat{w}_o = w_o^*$ and, hence, the firm finds it optimal to accept w_o^*. The acceptance payoff is strictly greater than the rejection payoff for $\hat{w}_o < w_o^*$ and, hence, the firm finds it optimal to accept $\hat{w}_o < w_o^*$. The acceptance payoff is strictly smaller than the rejection payoff for $\hat{w}_o > w_o^*$ and, hence, the firm finds it optimal to reject $\hat{w}_o > w_o^*$. If the worker makes an offer $\hat{w}_o > \overline{w}$, the firm's acceptance payoff is $\tilde{\pi}_n$, which is strictly smaller than the rejection payoff. Hence, the firm rejects such an offer.

Finally, we verify that the worker's offer w_o^* is optimal. If the worker makes the offer w_o^*, he attains a payoff of w_o^*. If the worker makes an offer $\hat{w}_o < w_o^*$, the firm accepts and the worker attains a payoff of $\hat{w}_o < w_o^*$. If the worker makes an offer $\hat{w}_o > w_o^*$, the firm rejects and the worker attains a payoff of $q\underline{w} + (1-q)w_c^*$, which is smaller than w_o^* since $w_c^* < w_o^*$. From the above observations, we conclude that the worker finds it optimal to make the offer w_o^*.

We are now in the position to summarize our characterization of the SPE of the Rolodex game $\Gamma_{n+1,k+1}(s)$. For $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} < 0$, it is straightforward to show that any SPE is such that the firm does not reach an agreement with all the $k+1$ workers without agreement. For $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} \geq 0$, the unique no-delay SPE is such that the firm reaches an immediate agreement with all of the $k+1$ workers who did not have an agreement at the beginning of the subgame. The firm and the first of these $k+1$ workers reach an agreement at the wage $w_{n+1,k+1}^1(s) = w_o^*$, i.e.

$$w_{n+1,k+1}^1(s) = \underline{w} + \frac{1}{\sum_{j=0}^{k+1}(1-q)^j}[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}]. \tag{B12}$$

The firm and the last of the $k+1$ workers reach an agreement at the wage $w_{n+1,k+1}^{k+1}(s) =$

$w^k_{n+1,k}(s+w^*_o)$, i.e.

$$w^{k+1}_{n+1,k+1}(s) = \underline{w} + \frac{(1-q)^k}{\sum_{j=0}^{k+1}(1-q)^j}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \qquad \text{(B13)}$$

The sum of the wages paid by the firm to the $k+1$ workers without agreement is $t_{n+1,k+1}(s) = w^*_o + t_{n+1,k}(s+w^*_o)$, i.e.

$$t_{n+1,k+1}(s) = (k+1)\underline{w} + \frac{\sum_{j=0}^{k}(1-q)^j}{\sum_{j=0}^{k+1}(1-q)^j}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \qquad \text{(B14)}$$

These observations show that, if Proposition B.2 holds for some $k < n+1$, it also holds for $k+1$. Since the proposition holds for $k = 1$, this means that it holds for any generic k. This concludes the proof of Proposition B.2.

Letting $k = n+1$ and $s = 0$ in Proposition B.2, we can characterize the payoffs of the subgame $\Gamma_{n+1,n+1}(0)$. In particular, if $F(n+1) - \tilde{\pi}_n - (n+1)\underline{w} \geq 0$, the unique no-delay SPE is such that the firm immediately reaches an agreement with all the $n+1$ workers. The payoff to the firm is given by

$$\tilde{\pi}_{n+1} = \tilde{\pi}_n + \frac{(1-q)^{n+1}}{\sum_{j=0}^{n+1}(1-q)^j}\left[F(n+1) - \tilde{\pi}_n - (n+1)\underline{w}\right]. \qquad \text{(B15)}$$

The payoff to the i-th worker is given by

$$\tilde{w}^i_{n+1} = \underline{w} + \frac{1}{\sum_{j=0}^{n+2-i}(1-q)^j}\left[F(n+1) - \textstyle\sum_{j=1}^{i-1}\tilde{w}^j_{n+1} - \tilde{\pi}_n - (n+2-i)\underline{w}\right]. \qquad \text{(B16)}$$

The above results show that, if Proposition B.1 holds for some n, it also holds for $n+1$. Since the proposition holds for $n = 1$, this means that it holds for any generic $n = 2, 3...$ This completes the proof of Proposition B.1.

We are now in the position to prove Theorem 2, which we restate for the reader's convenience.

Theorem 2: (Rolodex game). *Consider the Rolodex game $\Gamma_{n,n}(0)$ between the firm and n workers. Assume that the gains from trade are positive, i.e. $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$. In the limit for $q \to 0$, the unique no-delay SPE of the game is such that the payoff to the firm is given by the difference equation*

$$\tilde{\pi}_j = \tilde{\pi}_{j-1} + \left[F(j) - \tilde{\pi}_{j-1} - j\underline{w}\right]/\left(1+j\right), \text{ for } j = 1, 2, ...n, \qquad \text{(B17)}$$

with initial condition $\tilde{\pi}_0 = F(0)$. The payoff to the i-th of n workers is given by

$$\tilde{w}^i_n = \underline{w} + \left[F(n) - \tilde{\pi}_{n-1} - n\underline{w}\right]/\left(1+n\right). \qquad \text{(B18)}$$

Proof: It is straightforward to show that if $F(n) - \tilde{\pi}_{n-1} - n\underline{w} \geq 0$ then $F(j) - \tilde{\pi}_{j-1} - j\underline{w} \geq 0$ for $j = 1, 2, ...n-1$. From this observation and Proposition B.1, it follows that $\tilde{\pi}_j$ is given by (B1) for $j = 1, 2, ...n$ and $\tilde{w}^i_n$ is given by (B2) for $i = 1, 2, ...n$. Taking the limit of (B1) and (B2) for $q \to 0$, we obtain (B17) and (B18). ■

C Outcome of Bargaining Session in SZ Game

Consider the subgame $\Gamma_{n+1,k+1}(s)$ in which there are $n+1$ workers left in the game, $n-k$ of them have reached an agreement with the firm for wages summing up to s, $k+1$ workers have yet to reach an agreement with the firm, and the firm is about to start a bargaining session with the first of those $k+1$ workers.

We want to characterize the outcome of the bargaining session between the firm and the first of the $k+1$ workers without agreement. As discussed in the main text and in Appendix A, if the bargaining session ends with the firm and the worker agreeing to a wage $w \leq \overline{w}$, with $\overline{w} \equiv F(n+1) - s - \tilde{\pi}_n - k\underline{w}$, the firm reaches an agreement with all the k remaining workers for wages summing up to $t_{n+1,k}(s+w)$. Hence, in this case, the payoff to the firm is $F(n+1) - s - w - t_{n+1,k}(s+w)$ and the payoff to the worker is w. If the bargaining session ends with the firm and the worker agreeing to a wage $w > \overline{w}$, the firm does not reach an agreement with the following worker. In this case, the bargaining process starts over with n workers; the payoff to the firm is $\tilde{\pi}_n$ and the payoff to the worker is $\tilde{w}_n^{n+1-k}$. Finally, if the bargaining session ends with a breakdown, the payoff to the firm is $\tilde{\pi}_n$ and the payoff to the worker is $\underline{w}$.

We focus on SPE subject to a tie-breaking rule. In particular, whenever the firm is indifferent between accepting and rejecting an offer $w_o > \overline{w}$ that leads to a breakdown with one of the other workers without agreement, the firm rejects w_o. Similarly, whenever the firm is indifferent, we assume that it does not make a counteroffer $w_c > \overline{w}$ that leads to a breakdown with one of the other workers without agreement. In order to carry out the analysis, it is useful to introduce some additional notation. Consider the subgame that starts with the worker making an offer to the firm, and denote as m_W and M_W the infimum and the supremum payoffs to the worker among all SPEs that satisfy the tie-breaking rule. Consider the subgame that starts with the firm making a counteroffer to the worker, and denote as m_F and M_F the infimum and the supremum payoffs to the firm among all SPEs that satisfy the tie-breaking rule. Finally, it is useful to define the function $\phi(x)$ as the solution with respect to w of the equation

$$F(n+1) - s - w - t_{n+1,k}(s+w) = x.$$

That is,

$$\phi(x) = \underline{w} + [F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}] - \left(\frac{2-q}{1-q}\right)^k [x - \tilde{\pi}_n]. \tag{C1}$$

In what follows, we show that there is a unique SPE satisfying the tie-breaking rule and that such SPE has the same solution as the BRW game. The proof involves three intermediate claims, which closely follow the steps in Chapter 3 of Muthoo (1999). We carry out the analysis under the assumption that there are strictly positive gains from trade, i.e. $F(n+1) - s - \tilde{\pi}_n - \underline{w} - t_{n+1,k}(s+\underline{w}) > 0$ or, equivalently, $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} > 0$.

Claim C.1: *The payoff bounds m_W, M_W, m_F and M_F are such that*

$$\begin{aligned} m_W &\geq \underline{w}, \quad M_W \leq \overline{w}, \\ m_F &\geq \tilde{\pi}_n, \quad M_F \leq F(n+1) - s - \underline{w} - t_{n+1,k}(s + \underline{w}). \end{aligned} \tag{C2}$$

Proof: First, notice that the worker can guarantee himself a payoff of at least $\underline{w}$ by making offers equal to $\underline{w}$ and rejecting all counteroffers smaller than $\underline{w}$. Hence, $m_W \geq \underline{w}$. Similarly, the firm can guarantee itself a payoff of $\tilde{\pi}_n$ by making counteroffers equal to $\overline{w}$ and rejecting all offers greater than $\overline{w}$. Hence, $m_F \geq \tilde{\pi}_n$. Next, suppose $M_W > \overline{w}$. Then, there exists an SPE in which either the worker makes an offer $w_o > \overline{w}$ that is accepted by the firm, or the firm makes a counteroffer $w_c > \overline{w}$ that is accepted by the worker. In the first case, if the firm accepts the offer it attains a payoff of $\tilde{\pi}_n$, and if it rejects the offer it attains a payoff greater than $q\tilde{\pi}_n + (1-q)m_F \geq \tilde{\pi}_n$. Under the assumption that, in case of indifference, the firm rejects an offer that causes a breakdown with a subsequent worker, the firm rejects $w_o > \overline{w}$. If the firm makes a counteroffer $w_c > \overline{w}$ which is accepted by the worker, it attains a payoff $\tilde{\pi}_n$. If the firm makes instead a counteroffer $w_c' < \overline{w}$, it attains a payoff non-smaller than $\tilde{\pi}_n$. Under the assumption that, in case of indifference, the firm does not make a counteroffer that causes a breakdown with a subsequent worker, the firm never makes a counteroffer $w_c > \overline{w}$. Hence, $M_W \leq \overline{w}$. Finally, suppose $M_F > F(n+1) - s - \underline{w} - t_{n+1,k}(s + \underline{w})$. Then, there exists an SPE in which either the firm makes a counteroffer offer $w_c < \underline{w}$ that is accepted by the worker, or the worker makes an offer $w_o < \underline{w}$ that is accepted by the firm. If the firm makes a counteroffer $w_c < \underline{w}$, the worker's payoff from accepting is $w_c < \underline{w}$ and the payoff from rejecting is $q\underline{w} + (1-q)m_W \geq \underline{w}$. Hence, the worker will never accept a counteroffer w_c. If the worker makes an offer $w_o < \underline{w}$ that is accepted, the worker's payoff is $w_o < \underline{w} \leq m_W$. Hence, the worker will never make an offer $w_o < \underline{w}$. Therefore, $M_F \leq F(n+1) - s - \underline{w} - t_{n+1,k}(s + \underline{w})$.
■

Claim C.2: *The payoff bounds m_W, M_W, m_F and M_F are such that*

$$\begin{aligned} & m_W \geq \phi(q\tilde{\pi}_n + (1-q)M_F), \\ & m_F \geq F(n+1) - s - q\underline{w} - (1-q)M_W - t_{n+1,k}(s + q\underline{w} + (1-q)M_W). \end{aligned} \tag{C3}$$

Proof: Consider a subgame starting with the worker making an offer. The firm accepts any offer $w \leq \overline{w}$ such that

$$F(n+1) - s - w - t_{n+1,k}(s + w) \geq q\tilde{\pi}_n + (1-q)M_F. \tag{C4}$$

Since $q\tilde{\pi}_n + (1-q)M_F \geq \tilde{\pi}_n$ and $F(n+1) - s - w - t_{n+1,k}(s + w)$ is strictly decreasing in w and equals $\tilde{\pi}_n$ for $w = \overline{w}$, it follows that the offer w_o that equates the left and the right-hand sides of (C4) is smaller than $\overline{w}$. Hence, if the worker makes the offer w_o, his payoff is

$$u_W = w_o \equiv \phi(q\tilde{\pi}_n + (1-q)M_F) \leq m_W. \tag{C5}$$

Next, consider a subgame starting with the firm making a counteroffer. The worker accepts any counteroffer $w \leq \overline{w}$ such that

$$w \geq q\underline{w} + (1-q)M_W. \tag{C6}$$

Since $q\underline{w}+(1-q)M_W < \overline{w}$, the counteroffer w_c that equates the left and the right-hand sides of (C6) is smaller than $\overline{w}$. Hence, the payoff to the firm when making the counteroffer w_c is

$$u_F = F(n+1) - s - q\underline{w} - (1-q)M_W - t_{n+1,k}(s + q\underline{w} + (1-q)M_W). \tag{C7}$$

Since $m_F \geq u_F$, we have established the second part of the claim. ■

Claim C.3: *The payoff bounds m_W, M_W, m_F and M_F are such that*

$$\begin{aligned} &M_F \leq F(n+1) - s - q\underline{w} - (1-q)m_W - t_{n+1,k}(s + q\underline{w} + (1-q)m_W), \\ &M_W \leq \phi(q\tilde{\pi}_n + (1-q)m_F). \end{aligned} \tag{C8}$$

Proof: Consider a subgame starting with the firm making a counteroffer. In any SPE, the worker rejects a counteroffer $w_c < \underline{w}_c$, where

$$\underline{w}_c = q\underline{w} + (1-q)m_W \leq \overline{w}. \tag{C9}$$

If the SPE involves the worker accepting the counteroffer, the payoff to the firm cannot be greater than

$$u_F^a = F(n+1) - s - \underline{w}_c - t_{n+1,k}(s + \underline{w}_c). \tag{C10}$$

If the SPE involves the worker rejecting the counteroffer and the continuation payoffs are u_W and v_F, the payoff to the firm is

$$\begin{aligned} u_F^r &= q\tilde{\pi}_n + (1-q)v_F \\ &\leq q\tilde{\pi}_n + (1-q)[F(n+1) - s - u_W - t_{n+1,k}(s+u_W)] \\ &\leq F(n+1) - s - u_W - t_{n+1,k}(s+u_W) \\ &\leq F(n+1) - s - m_W - t_{n+1,k}(s+m_W) \\ &\leq F(n+1) - s - \underline{w}_c - t_{n+1,k}(s+\underline{w}_c), \end{aligned} \tag{C11}$$

where the second line follows from the fact that $v_F \leq F(n+1) - s - u_W - t_{n+1,k}(s+u_W)$, the third line follows from the fact that $u_W \leq M_W \leq \overline{w}$ and hence $\tilde{\pi}_n \leq F(n+1) - s - u_W - t_{n+1,k}(s+u_W)$, the fourth line follows from the fact that $m_W \leq u_W$, and the last line from the fact that $\underline{w}_c \leq m_W$. Overall, the payoff to the firm when making a counteroffer cannot be greater than

$$u_F \leq M_F \leq \max\{u_F^a, u_F^r\} = F(n+1) - s - \underline{w}_c - t_{n+1,k}(s + \underline{w}_c). \tag{C12}$$

Next, consider a subgame starting with the worker making an offer. In any SPE, the firm rejects every offer $w_o > \overline{w}_o$, where

$$F(n+1) - s - \overline{w}_o - t_{n+1,k}(s + \overline{w}_o) = q\tilde{\pi}_n + (1-q)m_F. \tag{C13}$$

If the SPE involves the firm accepting the counteroffer, the payoff to the worker cannot be greater than

$$u_W^a = \overline{w}_o = \phi(q\tilde{\pi}_n + (1-q)m_F). \tag{C14}$$

If the SPE involves the firm rejecting the offer and the continuation payoffs are u_F and v_W, the payoff to the worker is

$$\begin{aligned} u_W^r \quad &= q\underline{w} + (1-q)v_W \\ &\leq q\underline{w} + (1-q)\phi(u_F) \\ &\leq \phi(u_F) \\ &\leq \phi(m_F) \\ &\leq \phi(q\tilde{\pi}_n + (1-q)m_F), \end{aligned} \tag{C15}$$

where the second line follows from the fact that $v_W \leq \phi(u_F)$, the third line follows from the fact that $\underline{w} \leq \phi(u_F)$, the fourth line follows from the fact that $m_F \leq u_F$, and the last line from the fact that $\tilde{\pi}_n \leq m_F$. Overall, the payoff to the worker when making an offer cannot be greater than

$$u_W \leq M_W \leq \max\{u_W^a, u_W^r\} = \overline{w}_o. \tag{C16}$$

This completes the proof of the claim. ■

From Claims C.2 and C.3 and the definitions of $t_{n+1,k}$ and ϕ, it follows that

$$\begin{aligned} m_W &\geq \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right], \\ M_W &\leq \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right], \\ m_F &\geq \tilde{\pi}_n + \left(\frac{1-q}{2-q}\right)^{k+1}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right], \\ M_F &\leq \tilde{\pi}_n + \left(\frac{1-q}{2-q}\right)^{k+1}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \end{aligned} \tag{C17}$$

Since $m_W \leq M_W$ and $m_F \leq M_F$, the above inequalities imply

$$\begin{aligned} m_W = M_W &= \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right], \\ m_F = M_F &= \tilde{\pi}_n + \left(\frac{1-q}{2-q}\right)^{k+1}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \end{aligned} \tag{C18}$$

That is, all SPE's starting with the worker making an offer give the same payoff for the worker, and all SPE's starting with the firm making a counteroffer give the same payoff to the firm.

We can now derive the equilibrium outcome of the bargaining session. The session starts with the worker making an offer w_o to the firm. The firm finds it optimal to accepts the offer if $w_o \leq \overline{w}$ and

$$w_o \leq \phi(q\tilde{\pi}_n + (1-q)M_F). \tag{C19}$$

The worker chooses the offer taking as given the firm's acceptance strategy above. Denote w_o^* as $\phi(q\tilde{\pi}_n + (1-q)M_F)$. If the worker makes the offer w_o^*, the firm accepts it as w_o^* satisfies (C19) and it is strictly smaller than $\overline{w}$. Hence, the worker's payoff is

$$\begin{aligned} w_o^* &= \phi(q\tilde{\pi}_n + (1-q)M_F) \\ &= \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \end{aligned} \tag{C20}$$

If the worker makes an offer $w_o < w_o^*$, the firm accepts it as w_o satisfies (C19) and it is smaller than $\overline{w}$. Hence, the worker's payoff is

$$\begin{aligned} w_o &< \phi(q\tilde{\pi}_n + (1-q)M_F) \\ &= \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \end{aligned} \tag{C21}$$

If the worker makes an offer $w_o > w_o^*$, the firm rejects it as w_o either violates (C19) or it is greater than $\overline{w}$. Hence, the worker's payoff is no greater than

$$\begin{aligned} & q\underline{w} + (1-q)\phi(M_F) \\ = \quad & \underline{w} + \frac{(1-q)^2}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right] \\ < \quad & \underline{w} + \frac{1}{2-q}\left[F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w}\right]. \end{aligned} \tag{C22}$$

From the above expressions, it follows that the worker finds it optimal to make the offer w_o^*, which is immediately accepted by the firm. Clearly, this is true for any SPE that satisfies our tie-breaking rules. It is then straightforward to verify that an SPE that satisfies our tie-breaking rules exists.

We can now summarize our findings. When the gains from trade are strictly positive, i.e. $F(n+1) - s - \tilde{\pi}_n - (k+1)\underline{w} > 0$, the unique SPE (subject to the tie-breaking rules) is such that the bargaining session ends immediately with an agreement at the wage w_o^*. When the gains from trade are strictly negative, it is straightforward to verify that the unique SPE is such that the bargaining session ends with a breakdown. When the gains from trade are zero, there are several payoff equivalent SPEs. As standard in the literature, we assume that the firm and the worker reach an immediate agreement at the wage w_o^*. These are the same outcomes as for the BRW game in Lemma 1 for $y = F(n+1) - s$, $z = \tilde{\pi}_n$, and $t(w) = t_{n+1,k}(s+w)$.