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PIER Working Paper 18-002

Matching to Produce Information

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http://ssrn.com/abstract_id=3113594

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January 30, 2018

Abstract

We study endogenous team formation inside research organizations through the lens of a one-sided matching model with non-cooperative after-match information production. Using our characterization of the equilibria of the production game, we show that equilibrium sorting of workers into teams may be inefficient. *Asymmetric effort inefficiency* occurs when a productive team is disrupted by a worker who chooses to join a less productive team because there is an equilibrium played inside that team in which she exerts relatively less effort. *Stratification inefficiency* occurs when a productive team forms, but generates a significant negative externality on the productivity of other teams.

Keywords: Matching, Teams, Correlation, Information Acquisition.

JEL Clasifications: C78, L23, D83.

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In many research-oriented organizations, workers produce information in teams. Hierarchical theories of the firm assume management assigns workers to teams (Coase (1937), Williamson (1996)). But recent management literature suggests a trend towards less-structured organizational forms (Bloom, Sadun and Van Reenen (2010), Foss (2003), Grandori (2001)). At their extreme, these forms mirror the internal organization of a University: workers choose their productive partners and partnerships are self-enforced.

The history of the Danish hearing aid manufacturer Oticon provides evidence that unstructured organizations are vulnerable to incentive problems. In 1987, Oticon lost almost half of its equity when its competitors began selling cosmetically superior devices. In response, Oticon replaced vertical, hierarchical production with horizontal, project-based production. At first, these changes were profitable. Eliminating hierarchies and allowing workers to lead their own teams enabled the firm to take advantage of the private information of its workers.[1] However, new problems arose. First, management could not keep track of the number of hours workers dedicated to each project (Foss, 2003). Second, some teams were better than others "in terms of how well the team members worked together and what the outcome of team effort was" (Larsen, 2002). Third, competition meant that "anybody [at a project] could leave at will, if noticing a superior opportunity in the internal job market" (Foss, 2003). Due to these problems, Oticon's organizational form "has been superceded by something far more structured" (Foss, 2000).

We posit a model of endogenous team formation to study the incentives of workers inside organizations that decentralize information production. Our model provides a plausible explanation for Oticon's decision to revert back to a hierarchical structure after experimenting with decentralization. Specifically, if workers no longer had private information after decentralization, then continuing to allow workers to sort themselves into teams may have been inefficient. We characterize two inefficiencies that resemble those observed in Oticon. The first, which we call *asymmetric effort inefficiency*, occurs when a productive team is disrupted by a worker who chooses to join a less productive team because there is an equilibrium played inside that team in which she exerts relatively less effort. As a result, in an asymmetric effort-inefficient partition, there is too much

[1] Dessein (2002) shows that pushing decision-rights down a hierarchy can improve profitability if it allows the firm to exploit "local knowledge". Consistent with this finding, Oticon's CEO commented that decentralization "improved markedly [Oticon's] ability to invent new ideas, concepts, and make use of what [Oticon] actually [had]" (Kao, 1996). In particular, the firm was able to revive old projects that later turned out to be profitable.

asymmetry of effort *inside* teams. The second, which we call *stratification inefficiency*, occurs when a productive team forms, but generates a significant negative externality on the productivity of other teams. As a result, in a stratification-inefficient partition, there is too much inequality in productivity *across* teams.

In our model, there are a finite number of workers who want to predict a Gaussian state. Each may acquire Gaussian signals about it at a cost. Signals have the same precision, but may be correlated in any way. First, workers form teams. Second, workers simultaneously produce signals. Finally, after observing the signals of their teammates, each worker guesses the state.

Throughout the paper, we focus on the case in which teams have at most two members.[2] With this restriction, the two-stage game is a one-sided matching problem with an (after-match) non-cooperative *Production Subgame*. In the Production Subgame, in order to account for pre-play communication, we study pure-strategy *Pareto-Efficient Nash Equilibria (PEN)*, which are Nash equilibria Pareto-undominated by any other Nash equilibrium. In the two-stage game, we introduce and study *Coalitional Subgame Perfect Equilibria (CSPE)*, each of which is a collection of PEN, one for every feasible team, and a partition of workers into teams such that there is no deviating team yielding each worker in that team a higher payoff given the prescribed off-path PEN.[3]

To study equilibrium sorting patterns, we first characterize the equilibria of the Production Subgame. Our characterization consists of cutoff values on pairwise correlations of worker signals that order PEN in terms of their symmetry. Under a mild assumption on the cost of producing a signal, there is a cutoff above which there is a unique asymmetric PEN and another cutoff below which there is a unique symmetric PEN. Higher (lower) correlations imply signals are substitutable (complementary), and strategic substitutability (complementarity) drives equilibria to be asymmetric (symmetric). When signals are not too revealing, we obtain an even stronger characterization: there is an intermediate cutoff above which all PEN are asymmetric and below which there is at least one symmetric and one asymmetric PEN.[4]

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[2]Our definitions extend immediately to the case in which team size is unrestricted, though it complicates our characterizations. See Section 5 for a detailed discussion.

[3]Our notion of stability contrasts with the *Core*. A partition of workers into teams is in the Core if there is no deviating team in which workers can *choose a PEN* and each be made better off. In our environment, the Core may be empty due to cycles of re-negotiation. In Section 2.2, we provide an example reminiscent of the Roommate Problem of Gale and Shapley (1962) illustrating these cycles.

[4]The intuition for the latter result, described in detail in Section 3.2, is more subtle due to multiplicity of equilibria in the Production Subgame and a non-monotonicity in the marginal value of information.

We consider three efficiency criteria to assess CSPE outcomes: Pareto Efficiency, Welfare Efficiency, and Information Efficiency. An outcome is *Pareto-Efficient* if no worker can be made strictly better off without making another worse off, an outcome is *Welfare-Efficient* if it maximizes the sum of worker payoffs, and an outcome is *Information-Efficient* if the sum of the precisions of information created by each team is greater than in any other outcome with the same number of teams.

We first show that a CSPE may be Pareto-Inefficient, though a Pareto-Efficient CSPE always exists. Off-path PEN that enforce stability may also prevent mutually beneficial re-negotiation. We then characterize asymmetric effort and stratification inefficiencies, which reduce welfare and information production. Asymmetric effort inefficiency happens when a worker playing a symmetric equilibrium has a profitable outside option to join a less productive team playing an asymmetric equilibrium.[5] Stratification inefficiency happens if breaking up a productive team is beneficial; the loss in productivity of separating workers in productive teams may be outweighed by the gain in productivity obtained by re-matching workers in unproductive teams.[6] Finally, we show that both inefficiencies occur in an open set of correlation matrices and argue that our qualitative results are robust to relaxations of our assumptions.

Related Literature

Team Formation. A general lesson of the team formation literature (e.g. Chade and Eeckhout (2017), Page (2008), Prat (2002)) is that forming heterogeneous teams is optimal when workers are complementary. We show that such teams may not form endogenously. Teams may be comprised of similar individuals even when heterogeneous teams are available.

Chade and Eeckhout (2017), the most closely related paper to ours, analyze welfare optimal sorting patterns. In their environment, individuals have quadratic payoffs and form teams to share information. Within teams, each individual obtains exactly one signal. Signals have different precisions, but have a constant pairwise correlation. Their main result is that negative assortative

[5] If an asymmetric equilibrium is selected so that the deviating worker must exert more effort than her partner, then the efficient partition can be supported in a CSPE. However, symmetry of the Production Subgame means that there is also an asymmetric equilibrium supporting the (inefficient) deviation.

[6] We require the existence of non-trivial asymmetric equilibria in order to generate asymmetric effort inefficiency. In contrast, no such restriction is required to generate stratification inefficiency. Stratification inefficiency is driven by rich cross-complementarities across workers and does not depend on the details of the Production Subgame.

matching is optimal.

In our environment, we cannot order individuals by their productivity; the productivities of the individuals in a team are determined by their pairwise correlations. More crucially, we allow individuals to choose the number of signals they produce while Chade and Eeckhout (2017) abstract from strategic considerations. In Section 5.1, we fix the signal structure of Chade and Eeckhout (2017) and show that negative assortative matching is no longer optimal when individuals can choose the number of draws they take; if two individuals obtain signals with different enough precision, the individual obtaining less precise signals may not take *any* draws.

Matching. Since teams have at most two individuals, the assignment of workers to teams is a one-to-one one-sided matching problem. A worker's payoff from a match depends on strategically-taken actions after the match has been formed. Gale and Shapley (1962) show that the existence of a stable matching is not guaranteed in one-sided matching problems. Introducing an ex-post matching stage allows us to prove existence by fixing equilibria played off-path.

The only paper we are aware of that has considered a matching problem with ex-post concerns is Kaya and Vereshchagina (2014). In their environment, individuals match and decide how much effort to exert in the production of output. If effort is not observable, a manager can provide incentives by destroying output or hiring a budget-breaker. Kaya and Vereshchagina (2014) ask which organization is optimal ex-ante; studying symmetric equilibria, they derive conditions under which hiring a budget-breaker is advantageous. Instead, we focus on the incentives to form teams after fixing an organizational structure.

Strategic Experimentation. As in Bolton and Harris (1999) and Cripps, Keller and Rady (2005), individuals have incentives to free-ride off of the information produced by others. Information is a public good and hence, they do not take into account the positive externality of producing information. In our model, we focus on a different effect: individuals can choose to form a team in which their partners provide larger quantities of the public good.

1 Model

A finite number of workers, indexed by the set $\mathscr{N} := \{1, ..., N\}$, are uncertain about a state θ. Each has a Gaussian prior with mean μ_θ and variance σ_θ^2, and can obtain costly Gaussian signals about

θ from a worker-specific technology. Technologies are unbiased and have the same precision, but produce correlated signals. Then, the joint distribution of signals is:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} \sim N\left(\begin{pmatrix} \theta \\ \theta \\ \vdots \\ \theta \end{pmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \cdots & \cdots & 1 \end{bmatrix}\right).^{7}$$

We define a two-stage game. In the first stage, workers form teams. Teams have at most two members; the set of feasible teams is $\mathscr{S} := \mathscr{N} \times \mathscr{N}$, where $(i, j) \in \mathscr{S}$ denotes a team with two members if $i \neq j$ and one member if $i = j$.[8] Workers may only be a member of one team; the assignment of workers to teams is a partition $\Pi : \mathscr{N} \to \mathscr{S}$, where $\Pi(i) = (i, j)$ implies $\Pi(j) = (i, j)$. Adding a teammate has a positive, fixed cost; the cost of forming team $S = (i, j)$ is $K * \mathbb{I}^S$, where $K > 0$ is a constant and $\mathbb{I}^S$ is one if $i \neq j$ and zero otherwise.

In the second stage, each worker simultaneously chooses a positive integer of conditionally independent signals.[9] Fix a team $S = (i, j)$. Suppose Worker i chooses a positive integer m_i and Worker j chooses a positive integer m_j. If $i = j$, so that the team has one member, and $m_i = m_j$ is greater than 0, nature produces m_i conditionally independent signals from $N(\theta, 1)$. If $i \neq j$, so that the team has two members, and either m_i or m_j is greater than zero, then nature produces signals according to the following algorithm:

1. Set $n = 1$.

2. If $m_i \geq n$ and $m_j \geq n$, draw two conditionally independent signals from the distribution,

[7] We justify the assumption that signals are drawn from a normal distribution by appealing to the Central Limit Theorem. If a signal represents the outcome of an experiment and each experiment involves a large number of repetitions, the normal distribution is a good approximation of the distribution of the signal. Even without a large sample justification, we show the qualitative analysis of our model is largely unchanged when modeling draws as outcomes of a binomial experiment. See Online Appendix F.2. We fix variances and vary the correlation matrix because introducing heterogeneous variances complicates without illuminating. However, to facilitate direct comparison between our environment and that of Chade and Eeckhout (2017), we consider the case in which correlations are fixed across workers and variances are heterogeneous in Section 5.1.

[8] Online Appendix E extends the analysis to any team size. We summarize the main findings in Section 5.2.

[9] Our qualitative results do not change when allowing for serial correlation–see footnote 15. Discreteness in the number of draws is not an important assumption–see Online Appendix F.3. We discuss the assumption of simultaneous draws in Section 5.3.

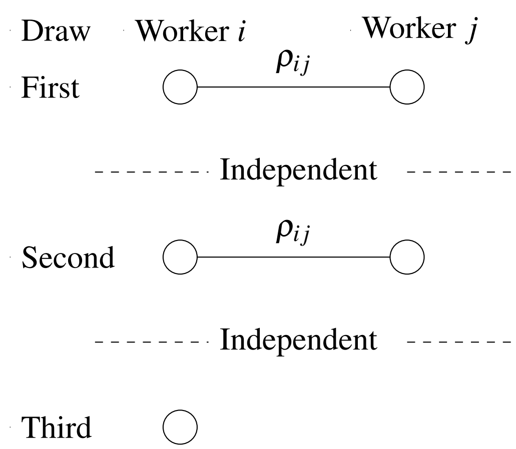


Figure 1: Draw Procedure when $m_i = 3$ and $m_j = 2$.

$$N\left(\begin{pmatrix}\theta \\ \theta\end{pmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho_{ij} \\ \rho_{ji} & 1 \end{bmatrix}\right).$$

3. If either $m_i \geq n$ or $m_j \geq n$, but not both, draw one signal from $N(\theta, 1)$.

4. If $m_i \leq n$ and $m_j \leq n$, exit the procedure. Else, replace n with $n+1$ and return to step 2.

The procedure is depicted graphically in Figure 1 for a team (i, j) in which $m_i = 3$ and $m_j = 2$.

Finally, after observing the signal realizations of every team member, each worker takes an action $a^* \in \mathbb{R}$ to maximize the expected value of a quadratic payoff function.[10] Letting x_i denote the realized vector of length m_i and the concatenation of signals observed in team S by x^S,

$$a^* \in \arg\max_{a \in \mathbb{R}} -E_\theta \left[(a - \theta)^2 \mid x^S\right].$$

We present the timeline of the model in Figure 2.

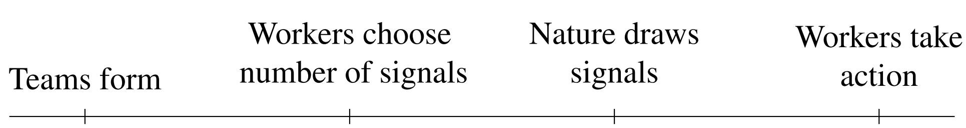


Figure 2: Timeline.

[10]Since all workers have the same payoff function and set of information, this is equivalent to workers jointly taking an action a.

1.1 Pareto-Efficient Nash Equilibrium

We call the second-stage of the two-stage game the *Production Subgame*. In a team $S = (i, j)$, given a strategy profile (m_i, m_j), the payoff of Worker i conditional on membership in team S is,

$$v^S(m_i, m_j) = -E_{x^S}\left[\max_{a\in\mathbb{R}} E_\theta\left[(a-\theta)^2 \mid x^S\right]\right] - c(m_i), \tag{1}$$

where the cost function $c : \mathbb{Z}_+ \to \mathbb{R}$ is increasing, marginal costs are increasing, and $c(0) = 0$. Then, the action space $\mathbb{Z}_+$ and payoff function v^S for each worker define a normal form game. To account for pre-play communication, in each couple $S = (i, j)$, we require that the strategy profile $m^*(S) := (m_i^*(S), m_j^*(S))$ is a pure strategy **Pareto-Efficient Nash Equilibrium (PEN)**.[11]

1.2 Core and Coalitional Subgame Perfection

We define two solution concepts for the two-stage game: *Coalitional Subgame Perfect Equilibrium (CSPE)* and the *Core*.[12] A partition Π is part of a CSPE if, fixing a PEN in *every* feasible team, there is no profitable deviating team $S' \in \mathscr{S}$. For notational convenience, denote $S_\Pi(i)$ as the team in partition Π to which Worker i belongs.

Definition 1. *Let Π be a partition of $\mathscr{N}$ and $M^* = \{m^*(S)\}_{S\in\mathscr{S}}$ be a collection of PEN, one for every feasible team. The tuple (Π, M^*) is a* ***Coalitional Subgame Perfect Equilibrium (CSPE)*** *if there does not exist a team $S' \in \mathscr{S}$ such that for all $i \in S'$,*

$$v_i(m^*(S')) - K * \mathbb{I}^{S'} > v_i(m^*(S_\Pi(i))) - K * \mathbb{I}^{S_\Pi(i)}.$$

A partition in the Core must be robust to another round of negotiation. After teams are formed, we require that workers cannot form a deviating team $S' \in \mathscr{S}$ and *choose* a PEN that makes each worker better off.[13]

[11] A PEN is a Nash Equilibrium that is Pareto undominated by any other Nash Equilibrium. One reason we focus on pure strategies is that there is no mixed strategy Nash Equilibrium that Pareto dominates all pure strategy Nash Equilibria. Further, the results are not heavily dependent on the restriction to pure strategies.

In the general analysis in Online Appendix E, we use **Coalition-Proof Nash Equilibrium (CPNE)** as our solution concept for the Production Subgame. When teams have at most two members, the set of CPNE is equivalent to the set of Pareto-Efficient Nash Equilibria (Bernheim, Peleg and Whinston (1987)).

[12] Our definitions may be written using a matching function. We write them in terms of partitions so that they extend easily to the case in which there are no restrictions on team size. We discuss this extension in Section 5.2.

[13] Our definition may be interpreted as the Core of a coalition game in which the valuation of a coalition is determined by the equilibrium correspondence of the Production Subgame.

Definition 2. *Let* Π *be a partition of* $\mathscr{N}$ *and* $\hat{M} = \{\hat{m}(S)\}_{S\in\Pi}$ *be a collection of PEN, one for each team in* Π*. The tuple* $(\Pi, \hat{M})$ *is in the* ***Core*** *if there does not exist a team* $S' \in \mathscr{S}$ *and a PEN* $m'(S')$ *such that for all* $i \in S'$*,*

$$v_i(m'(S')) - K * \mathbb{I}^{S'} > v_i(\hat{m}(S_\Pi(i))) - K * \mathbb{I}^{S_\Pi(i)}.$$

Notice, both definitions coincide when there is a unique PEN within every feasible team and every Core partition is a CSPE partition.

2 Existence

We now check that an equilibrium of the two-stage game exists. We first characterize the optimal action taken by each worker to prove there is a pure-strategy PEN within each team. Next, we show that the Core may be empty for reasons reminiscent of the Roommate Problem of Gale and Shapley (1962). Finally, we show that a CSPE *always* exists and at least one of them is Pareto Efficient. In the process, we illuminate the difference between the two solution concepts.

2.1 Pure Strategy PEN

Because each worker's payoff function is quadratic, her optimal action given any signal realization is the posterior mean and her expected payoff when signals are costless is the negative posterior variance. In Lemma 1, we provide a closed-form solution for the posterior variance in a two-worker team and re-state the preceding observations.[14]

Lemma 1. *The optimal action for a worker in team S is,*

$$a = E(\theta \mid x^S).$$

If signals are costless, the expected payoff of a worker is the negative posterior variance. The posterior variance in a two-worker team acquiring (m,n) *signals with pairwise correlation* $\rho \in (-1,1]$ *is,*

$$f(\rho,m,n) := \left(\left(\min\{n,m\}\frac{2}{1+\rho} + |m-n|\right)\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1}.^{15}$$

[14]The details of this proof, and all others that are not in the main text, can be found in Appendix A.

[15]Fix $n \leq m$ and suppose that the drawing procedure follows an AR(1) process with correlation $\hat{\rho}$. Then the

The next Lemma states that we may bound the action space without loss of generality. The reason is that diminishing marginal returns to information production implies that, eventually, the marginal value of a draw must be less than the marginal cost *regardless* of the behavior of one's partner.

Lemma 2. *There is a positive integer $\bar{M}$ such that for each positive integer $m \geq \bar{M}$, m is a not best response by Worker i to any strategy by Worker j.*

Since we can bound the action space, we may redefine the game as a finite potential game to show that there exists a pure strategy Nash equilibrium.

Proposition 1. *There exists a pure strategy Nash equilibrium of the Production Subgame.*

Proof. Given that no worker optimally chooses a number of draws larger than $\bar{M}$, we can redefine, without loss of generality, the Production Subgame as the normal form game $(\{0,1,\ldots,\bar{M}\}^2, \{v_i, v_j\})$. Define the function,

$$\Phi(m,n,\rho) = -f(m,n,\rho) - c(m) - c(n).$$

It is a potential function since

$$v_1(m,n,\rho) - v_1(m',n,\rho) = -f(m,n,\rho) - c(m) + f(m',n,\rho) + c(m') = \Phi(m,n,\rho) - \Phi(m',n,\rho)$$

$$v_2(m,n,\rho) - v_2(m,n',\rho) = -f(m,n,\rho) - c(n) + f(m,n',\rho) + c(n') = \Phi(m,n,\rho) - \Phi(m,n',\rho).$$

Hence, the redefined game is a finite potential game and is guaranteed to have a pure strategy Nash equilibrium by Corollary 2.2 of Monderer and Shapley (1996). □

To prove a pure strategy PEN exists, we show that no mixed strategy Nash equilibrium dominates all pure strategy Nash equilibria.

two-worker team variance after (m,n) signals is

$$Var(\theta \mid (m,n)) = \begin{cases} \left(\frac{m-(m-2)\hat{\rho}}{1+\hat{\rho}}\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1} & \text{if } n = 0 \\ \left(\left[\frac{n-(n-2)\hat{\rho}}{1+\hat{\rho}}\frac{2}{1+\rho_{ij}} + \frac{(m-n)(1-\hat{\rho})}{1+\hat{\rho}}\right]\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1} & \text{if } n > 0. \end{cases}$$

A derivation is available upon request. The extra information generated by a joint or individual signal is modified by the correlation $\hat{\rho}$. As the functional form with respect to ρ_{ij} is similar, our subsequent analysis would be similar if we assumed there was serial correlation.

Corollary 1. *There exists a pure strategy PEN of the Production Subgame.*

2.2 Empty Core Example

Suppose there are three workers and their technologies are correlated according to the network depicted in Figure 3. The number inside each circle is the identity of the worker and the numbers next to the edges connecting the circles are pairwise correlations.

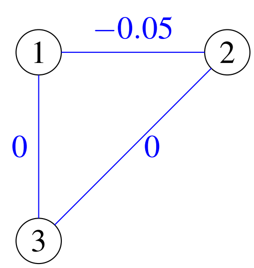


Figure 3: Correlation Matrix for an Empty Core.

Table 1: Parameters for Section 2.2 and example in section 4.1.

Parameter	Interpretation	Value
σ^2	Signal Variance	2
σ_θ^2	Prior Variance	1
$c(m)$	Cost of m Draws	$0.002m^2$
K	Cost of Teammate	0.01

Given the parameters in Table 1, the equilibria and payoffs of the Production Subgame for each feasible team are as follows. In team $(1,2)$, there is a unique, symmetric equilibrium $(4,4)$. Workers obtain a payoff of -0.22. In teams $(1,3)$ and $(2,3)$, there is a unique equilibrium, up to identity, $(5,4)$. The worker taking five draws obtains a payoff of -0.23 and the worker taking four draws obtains a payoff of -0.21. Finally, if any worker remains alone, she takes seven draws and obtains a payoff of -0.32.

Define a **leader** as a teammate taking weakly more draws than her partner and a **follower** as a teammate taking strictly fewer draws than her partner. By exhibiting a preference cycle, we show the Core is empty. Suppose the team $(1,2)$ is formed. Then, Worker 3 and Worker 1 can form a mutually beneficial deviating team in which Worker 3 is the leader. Suppose the team $(1,3)$ is formed. Then, Worker 2 can make an offer to its leader and form a mutually beneficial deviating team in which Worker 2 is the leader. Suppose the team $(3,2)$ is formed. Then, Worker 1 can form a mutually beneficial deviating team with its leader in which Worker 1 is the leader. Finally, if all teams are singletons, Worker 1 and Worker 2 can form a team and be made better off. Hence, there is no partition in the Core.[16,17]

2.3 Existence CSPE

In the empty Core example, non-existence is driven by incentives to re-negotiate off-path equilibria. In a CSPE, fixed off-path PEN eliminate incentives to deviate. Consider the previous example and the partition $\Pi = \{(1,2),(3)\}$. If the equilibrium in teams $(1,3)$ and $(2,3)$ is fixed to be $(5,4)$, Worker 1 and Worker 2 cannot join another team in which they exert less effort than their partner. Allowing such enforcement is not too permissive; the suggested partition is the only one compatible with a CSPE. Further, it is Pareto-Efficient.

Theorem 1. *A CSPE exists. At least one must be Pareto-Efficient.*

The proof of existence is constructive. We first order teams sequentially in decreasing order of the payoff attainable to a leader. We then choose PEN off-path so that each teammate is the leader in any other feasible team. Then, there are no profitable deviations.[18] To prove that there is a Pareto-Efficient CSPE, we fix a CSPE outcome and show that if there is a Pareto-improving outcome, then it has to be a CSPE as well. As there are only a finite number of equilibria, we may iterate until there are no Pareto improvements.[19]

[16] The same logic would hold if all pairwise correlations were 0. We add the correlation of -0.05 so that there is a unique CSPE, which we discuss below.

[17] We provide sufficient conditions for the Core to be nonempty in Online Appendix A. The sufficient conditions eliminate all possible cycles. They are not necessary because the Core may be non-empty even when cycles exist.

[18] We extensively exploit the symmetry of the game. In particular, in any couple any worker can be selected to be the team leader. Any other environment where the after-match game is symmetric will have at least one CSPE.

[19] We provide an example in Section 4.1 in which there is a CSPE that is not Pareto-Efficient.

3 Production Subgame Analysis

Recall that the posterior variance in a two-worker team is,

$$f(\rho, m_i, m_j) = \left(\left(\min\{m_i, m_j\}\frac{2}{1+\rho} + \left|m_i - m_j\right|\right)\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1},$$

where m_i and m_j are the number of draws taken by Worker i and Worker j, and $\rho \in (-1, 1]$ is their pairwise correlation.[20] Then, the marginal value of draw m_i by Worker i given that Worker j takes m_j draws is,

$$MV(m_i; m_j, \rho) \equiv [-f(\rho, m_i, m_j)] - [-f(\rho, m_i - 1, m_j)] = f(\rho, m_i - 1, m_j) - f(\rho, m_i, m_j).$$

We analyze $MV(m_i; m_j, \rho)$ in Lemma 3 through Lemma 6.

3.1 The Marginal Value of Information

Lemma 3 states that the marginal value of a draw by a leader is increasing in ρ.[21] The reason is that the information left to learn, fixing m_i and m_j, is increasing in ρ.

Lemma 3 (Leader Comparative Statics in ρ)**.** *For $m_i > m_j$, $MV(m_i; m_j, \rho)$ is increasing in ρ.*

The same property does not hold for a follower.[22] While the amount of information left to learn, fixing the number of draws by each teammate, increases in ρ, the value of matching a leader's draw *decreases* in ρ because the information produced is more redundant. Hence, the marginal benefit of a draw by a follower is non-monotonic. We prove it is strictly concave in the pairwise correlation ρ and has a unique maximizer.

Lemma 4 (Follower Comparative Statics in ρ)**.** *For $m_i \leq m_j$ where $m_j \geq 1$, $MV(m_i; m_j, \rho)$ is strictly concave in ρ with unique maximizer,*

$$\tilde{\rho}(m_i, m_j, \gamma) = \frac{\left(m_j + \gamma - \sqrt{m_i(m_i+1)}\right)^2}{-(m_j+\gamma)^2 + m_i(m_i+1)}.$$

[20] If ρ is -1, and each worker takes at least one draw, the team learns the state perfectly since the signals are located symmetrically around it. Hence, the posterior variance is zero.

[21] Recall, a *leader* is a teammate taking weakly more draws than her partner.

[22] Recall, a *follower* is a teammate taking strictly fewer draws than her partner.

We now make stepwise comparisons between the marginal value of a draw by a leader and the marginal value of a draw by a follower. Workers initially take $m-1$ draws. The leader's marginal value is the payoff of taking an m-th draw. The follower's marginal value is the payoff of taking an m-th draw, given that the leader already took an m-th draw. Lemma 5 states that for any number $m \geq 1$ and signal-to-prior variance ratio $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$, there is a unique correlation, $\hat{\rho}(n,\gamma)$, below which the marginal value of the leader is less than the marginal value of the follower, and above which the opposite holds.

Lemma 5 (Leader-Follower MV Comparison 1)**.** *Fix $m_i \geq 1$ and γ. Then,*

$$\underbrace{MV(m_i; m_i - 1, \rho)}_{\text{Marginal Value Leader}} < \underbrace{MV(m_i; m_i, \rho)}_{\text{Marginal Value Follower}}$$

if and only if,

$$\rho < \hat{\rho}(m_i,\gamma) = \frac{-(\gamma - 1 + 2m_i) + \sqrt{(\gamma - 1 + 2m_i)^2 - 4\gamma}}{2\gamma} < 0.$$

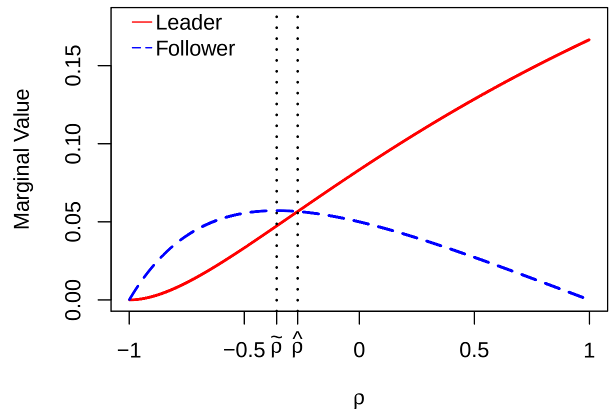
Lemma 6 states that if γ is sufficiently large the pairwise correlation at which the marginal value of a follower is maximized, $\tilde{\rho}(m_i, m_j, \gamma)$, must be less than $\hat{\rho}(m_i,\gamma)$. We use this property in the next section to order equilibria in terms of their symmetry.

Lemma 6 (Leader-Follower MV Comparison 2)**.** *Fix $m_i \geq 2$. Then, there exists $\gamma^*(m_i) \in [\frac{1}{2}, 1)$ such that $\gamma \geq \gamma^*(m_i)$ if and only if*

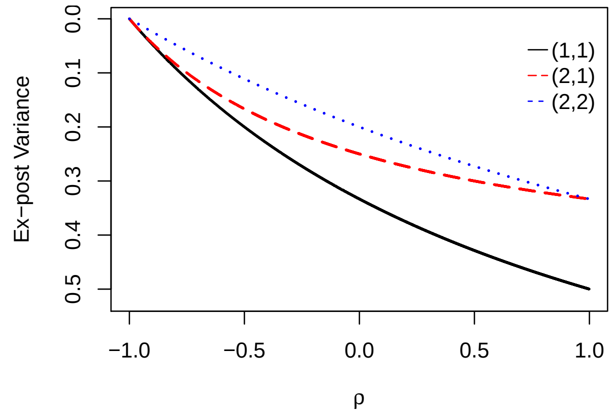
$$\tilde{\rho}(m_i, m_i - 1, \gamma) \leq \hat{\rho}(m_i,\gamma).^{23}$$

Figure 4 illustrates Lemma 3 through Lemma 6 when $\gamma = 1$ and $m_i = 2$. By Lemma 3, the marginal value of the leader is increasing in ρ because the information left to learn increases. Lemma 4 implies the follower's marginal value has the hump-shape depicted in 4a. Figure 4b illustrates how diminishing marginal returns to information production generates it; when ρ is higher the follower has more to learn, but produces more redundant information. By Lemma 5, the marginal value of the leader is less than the marginal value of the follower below a negative cutoff $\hat{\rho}$. Finally, since $\gamma = 1$, the condition in Lemma 6 is satisfied. Hence, the correlation that maximizes the follower's marginal value, $\tilde{\rho}$, is less than $\hat{\rho}$.

[23] For $m_i = 1$, $\tilde{\rho}(m_i, m_i - 1, \gamma) = -1$, so the inequality is satisfied for any γ.



(a) Marginal value of second draw for leader and follower.



(b) Ex-post variance for different strategy profiles.

Figure 4: Ex-post variances and marginal values when $\sigma = \sigma_\theta = 1$.

3.2 PEN Characterization

We now characterize the equilibrium correspondence. Assumption 1 bounds the cost of the first draw so that the following properties hold in equilibrium:

1. In any team, at least one worker takes at least one draw.

2. In a two-worker team with pairwise correlation -1, both workers take one draw.

Assumption 1. $c(1) < \frac{\min\{\sigma_\theta^2, \sigma^2\}}{1+\gamma}$.

By Lemma 3, the marginal value of a leader is increasing monotonically in ρ and by Lemma 5, the marginal value of a follower is decreasing monotonically after $\tilde{\rho}$. Hence, a high enough pairwise correlation generates a unique asymmetric equilibrium and a low enough pairwise correlation generates a unique symmetric equilibrium.

Proposition 2. *Suppose Assumption 1 is satisfied. Then, there exist cutoff values ρ^* and ρ^{**} such that,*

$$-1 < \rho^* \leq \rho^{**} < 1$$

and the following properties hold:

1. *For $\rho \leq \rho^*$, there is a unique PEN. It is symmetric.*

2. *For $\rho > \rho^{**}$, there is a unique PEN up to the identity of each worker. In it, one worker takes a strictly positive number of draws and the other takes none.*

The Proposition does not characterize equilibria for correlations between ρ^* and ρ^{**}. Without additional restrictions, there are two complications. First, there may be multiple PEN. Second, there may be a correlation at which the unique equilibrium is asymmetric and a higher correlation at which there is a symmetric equilibrium. Each is illustrated in the equilibrium correspondence presented in Figure 5. To see why, note that for correlations around $\rho = 0$, there are two asymmetric equilibria. Further, when $\rho = -0.29$ the unique PEN is asymmetric, but there is a unique PEN for a slightly higher correlation and it is symmetric.

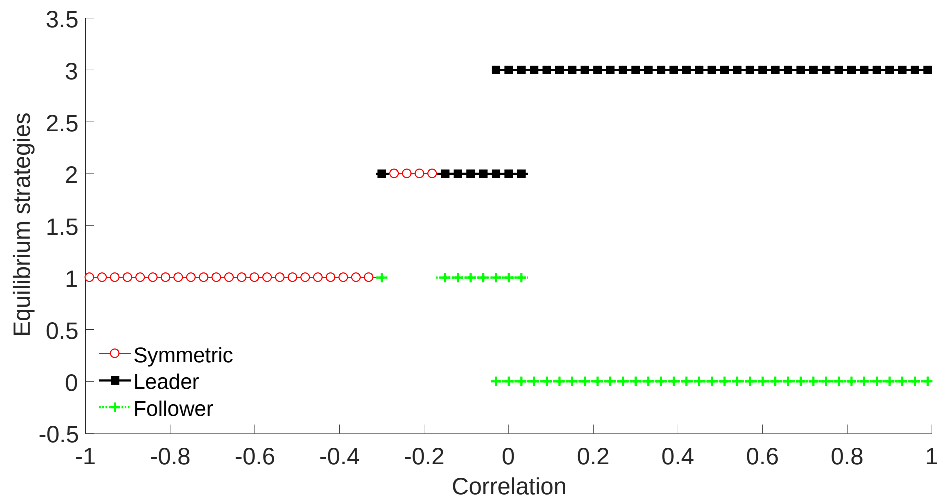


Figure 5: Equilibrium correspondence when $c(m) = 0.019m$, $\sigma^2 = \frac{1}{4}$ and $\sigma_\theta^2 = 1$.

The second phenomenon deserves further attention. The key to the example is that $\hat{\rho} < \tilde{\rho}$, in contrast to the situation depicted in Figure 4. For $\rho = -0.29 \in (\hat{\rho}, \tilde{\rho})$ and $n = 1$, the marginal value of a draw for a leader is greater than the marginal value of a draw for a follower because $\rho > \hat{\rho}$. Here, we fix the marginal cost of a second draw so that the asymmetric equilibrium $(2,1)$ is played. If ρ increases, however, the marginal value of the follower *increases* and may exceed the chosen marginal cost. The former phenomena occurs because the increase in the value of information left to learn for the follower offsets the declining value of matching the leader's draw. If the follower takes a second draw, however, the leader has no incentive to take a third draw because the information left to learn decreases sufficiently. Hence, a symmetric equilibrium $(2,2)$ is played.

The previous phenomena could not have happened if $\tilde{\rho} < \hat{\rho}$. In this case, when an asymmetric equilibria is played, the marginal value of the leader is greater than the marginal value of the follower at the leader's last draw. And increasing ρ past $\hat{\rho}$ must *decrease* the marginal value of the follower because $\rho > \hat{\rho} > \tilde{\rho}$. Hence, the follower will not be induced to match the leader's draw.

In general, as we have seen in Lemma 6, the relationship between $\hat{\rho}$ and $\tilde{\rho}$ depends on the relationship between σ and σ_θ. There, we proved that if $\gamma = \frac{\sigma^2}{\sigma_\theta^2} \geq 1$ and $m_i \geq 2$, then $\tilde{\rho}(m_i, m_i - 1, \gamma) \leq \hat{\rho}(m_i, \gamma)$. If $\gamma \geq 1$, we then obtain a stronger characterization: there is an intermediate cutoff below which there is at least one symmetric and one asymmetric PEN and above which all PEN are asymmetric.[24]

Proposition 3. *Suppose Assumption 1 is satisfied and $\gamma = \frac{\sigma^2}{\sigma_\theta^2} \geq 1$. There exist cutoff values ρ^*, ρ^{**}, and ρ^{***} such that,*

$$-1 < \rho^* \leq \rho^{***} \leq \rho^{**} < 1$$

and the following properties hold:

1. *For $\rho \leq \rho^*$, there is a unique PEN. It is symmetric.*

2. *For $\rho \in (\rho^*, \rho^{***}]$, there is at least one symmetric and one asymmetric PEN.*

3. *For $\rho > \rho^{***}$, all PEN are asymmetric.*

4. *For $\rho > \rho^{**}$, there is a unique PEN up to the identity of each worker. In it, one worker takes a strictly positive number of draws and the other takes none.*

4 Inefficiency

Using our characterization of the Production Subgame, we study the efficiency of CSPE outcomes. An **outcome** is a partition, Π^*, and a collection of PEN within that partition, $(m^*(S))_{S \in \Pi^*}$. We measure the efficiency of outcomes using three criterion. First, an outcome is **Pareto-Efficient** if no worker can be made strictly better off without making another worse off. Second, an outcome is

[24] We provide further characterizations of PEN in Online Appendix C. There, we show that if $\gamma \leq \frac{1}{2}$, the equilibrium correspondence of the leader (follower) is increasing (decreasing) in ρ in the weak set order sense of Topkis (2011). Again, the key idea is that there is an unambiguous relationship between $\tilde{\rho}$ and $\hat{\rho}$ when $\gamma \leq 1/2$; Lemma 6 implies that $\tilde{\rho} > \hat{\rho}$ if $\gamma \leq 1/2$ regardless of the strategy profile.

Welfare-Efficient if it maximizes the sum of worker payoffs. Third, an outcome is **Information-Efficient** if the sum of the precision of information created by each team is greater than in any other outcome with the same number of teams.

Formally, an outcome is Information-Efficient if,

$$\{\Pi^*, (m^*(S))_{S\in\Pi^*}\} \in \underset{\Pi\in\mathscr{P}^{|\Pi^*|},(m(S))_{S\in\Pi}}{\arg\max} \sum_{S\in\Pi} \frac{1'_{m(S)}\Sigma^{-1}_{m(S)}1_{m(S)}}{\sigma^2},$$

where $\mathscr{P}^{|\Pi^*|}$ is the set of partitions containing the same number of teams as Π^*, $1_{m(S)}$ is a vector of ones with length given by the equilibrium number of draws taken in team S, and $\Sigma_{m(S)}$ is the correlation structure of these draws. We justify our information criterion as follows. For a planner sharing the common prior of the workers, selecting an Information Efficient outcome is equivalent to selecting an outcome that minimizes a quadratic loss function. To see why, note that Lemma 1 implies each team reports their posterior mean. If the planner knows the equilibrium strategy profile, but not the identity of each worker, it can construct a bias-corrected report from each team S:

$$\tilde{x}_S := \frac{1'_{m(S)}\Sigma^{-1}_{m(S)}x_{m(S)}}{1'_{m(S)}\Sigma^{-1}_{m(S)}1_{m(S)}} \sim N\left(\theta, \frac{\sigma^2}{1'_{m(S)}\Sigma^{-1}_{m(S)}1_{m(S)}}\right).$$

Assuming independence of reported estimates across teams, the planner's ex-ante loss is

$$-\left(\sigma_\theta^{-2} + \sum_{S\in\Pi} \frac{1'_{m(S)}\Sigma^{-1}_{m(S)}1_{m(S)}}{\sigma^2}\right)^{-1}.$$

Then, maximizing $\sum_{S\in\Pi} \frac{1'_{m(S)}\Sigma^{-1}_{m(S)}1_{m(S)}}{\sigma^2}$ is equivalent to minimizing the planner's ex-ante loss.

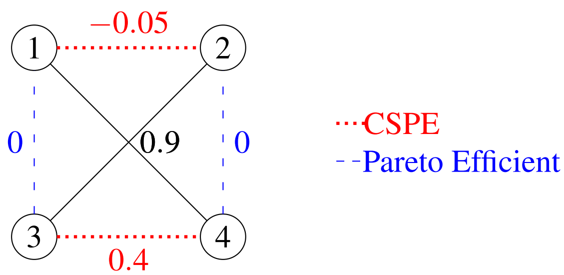
4.1 Pareto Inefficiency[25]

While a Pareto-Efficient CSPE always exists, not every CSPE is Pareto- Efficient. Fix the parameters in Table 1 and suppose technologies are correlated according to the network in Figure 6. Then, there is a Pareto-Inefficient CSPE with partition $\Pi^* = \{(1,2),(3,4)\}$ and on-path PEN $(m^*(S))_{S\in\Pi^*} = \{(4,4),(4,5)\}$.[26]

Why is the outcome Pareto-Inefficient? Worker 1 and Worker 2 may only be matched in a

[25]We thank Yeon-Koo Che for drawing our attention to this issue.

[26]We report equilibrium strategies and payoffs below the network. There is a unique PEN in each size-two team.



Correlation	PEN	Payoff
-0.05	$(4,4)$	$(-0.22,-0.22)$
0	$(4,5)$	$(-0.21,-0.23)$
0.4	$(4,5)$	$(-0.23,-0.29)$
0.9	$(0,7)$	$(-0.22,-0.32)$

Figure 6: Correlation Matrix and PEN for Pareto Inefficiency Example.

CSPE if Worker 1 is forced to be the leader in an off-path team with Worker 3. Otherwise, Worker 1 and Worker 3 can form a profitable deviating team. Should they do so, however, not only would both be made better off, so would Worker 2 and Worker 4 if Worker 2 is the follower in such a pair. We see that Pareto inefficiency arises because off-path PEN that enforce stability may prevent mutually beneficial re-negotiation.

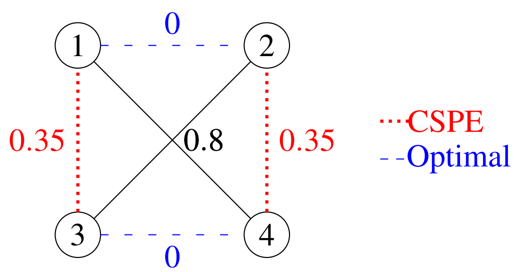
4.2 Asymmetric Effort Inefficiency

We next describe asymmetric effort inefficiency. We first consider a numerical example with parameters presented in Table 2 and signal structure described by the network in Figure 7. The PEN in all size-two teams are unique up to identity and each is presented below the Figure.

Table 2: Parameters for Examples in Section 4.2 and Section 4.3.

Parameter	Interpretation	Value
σ^2	Signal Variance	1
σ_θ^2	Prior Variance	1
$c(m)$	Cost of m Draws	$0.01m^2$
K	Cost of Teammate	0.01

In the example, there is a Welfare and Information-Inefficient CSPE in which $(1,3)$ and $(2,4)$ are matched. In contrast, the efficient matching is $(1,2)$ and $(3,4)$.[27] Even though $(1,2)$ is the most productive feasible team, as measured by the equilibrium posterior precision $1_{m(S)}\Sigma_{m(S)}^{-1}1'_{m(S)}$, asymmetric equilibria within teams $(1,3)$ and $(2,4)$ disrupt it. Why? Worker 1 can take fewer draws in a team with Worker 3 than in a team with Worker 2. Despite learning less, Worker 1 obtains a higher payoff because she exerts less effort. Analogously, Worker 4 can take fewer draws in a team with Worker 2 than in a team with Worker 3. Then, if the cost of forming a pair K is small, Worker 2 and Worker 3 are left with no better option than to accept a team in which each works more than her partner.



Correlation	PEN	Payoff	$1_{m(S)}\Sigma_{m(S)}^{-1}1'_{m(S)}$
0	$(2,2)$	$(-0.24,-0.24)$	4
0.35	$(1,3)$	$(-0.23,-0.31)$	3.48
0.8	$(0,3)$	$(-0.25,-0.34)$	3

Figure 7: Correlation Matrix and PEN for Asymmetric Effort Inefficiency Example.

We call the CSPE with partition $\{(1,3),(2,4)\}$ an **Asymmetric Effort-Inefficient CSPE**, defined as a CSPE in which an efficient CSPE is disrupted by a worker who chooses to join an inefficient team because there is an equilibrium played inside that team in which she exerts relatively less effort. We show that such inefficiency can happen as long as non-trivial asymmetric equilibria exist in the Production Subgame. Recall, if Assumption 1 holds, by Proposition 2 there exists a cutoff, ρ^*, below which there is a unique symmetric equilibrium, and a cutoff, ρ^{**}, above which there is a unique asymmetric equilibrium in which one teammate takes zero draws. If $\rho^* < \rho^{**}$, for ρ greater than but close to ρ^*, there is an asymmetric equilibrium in which each worker takes

[27]It turns out that the efficient matching can be supported as a CSPE.

a positive number of draws. Then, there is an open set of correlation matrices for which there is a Welfare-Inefficient CSPE.

To see why, consider a four-person economy $\{i, j, i', j'\}$ in which teams (i, j) and (i', j') are efficient. Shirking may disrupt an efficient outcome if the following two phenomena occur: first, (i, j) is not formed because i obtains a higher payoff as a follower in (i, i') than in a symmetric equilibrium in (i, j); second, (i', j') is not formed because j' obtains a higher payoff as a follower in (j, j') than in a symmetric equilibrium in (i', j'); third, i' and j' have no better option than to be a leader in a team with i and j, respectively. The first property is satisfied if ρ_{ij} is close to, but less than, ρ^* and $\rho_{ii'}$ is close to, but greater than, ρ^*. The second property is satisfied if $\rho_{i'j'}$ is close to, but less than, ρ^* and $\rho_{jj'}$ is close to, but greater than, ρ^*. Finally, if cross correlations $\rho_{ij'}$ and $\rho_{i'j}$ are above ρ^{**}, and the cost of forming a pair K is not prohibitively large, i' and j' have no superior outside options than to form a team together.[28] We prove that there is an open set of correlation matrices satisfying the previous properties and obtain the following Theorem.

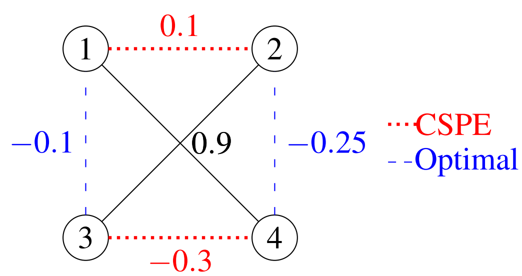
Theorem 2 (Asymmetric Effort Inefficiency)**.** *Suppose Assumption 1 holds, K is small, and $\rho^* < \rho^{**}$. Then, there is an open set of correlation matrices for which there is an Asymmetric Effort-Inefficient CSPE which does not maximize welfare.*

Notice, the Theorem does not state that the outcome is Information-Inefficient. Although in the numerical example above welfare maximizing teams maximize information production, this is not true in general. The reason is that there are two ways a welfare-optimal team playing a symmetric equilibrium can be broken. First, the deviating teammate may take fewer draws in another asymmetric equilibrium. In this case, the asymmetric equilibrium generates less information and Welfare Efficiency coincides with Information Efficiency. Second, a worker may take the *same* number of draws in a welfare-inefficient team as in the welfare-efficient team. The asymmetric-effort temptation arises because the worker's partner in the welfare-inefficient team may take more draws than her partner in the welfare-efficient team. In this case, better information may be produced in the welfare-inefficient partition. We provide such an example in Online Appendix D.

[28]There is another kind of inefficiency that also occurs in an open set. It may be the case that (i) (i, j) is not formed because i obtains a higher payoff as a follower in (i, i') than in a symmetric equilibrium in (i, j) and (ii) (i', j') is not formed because i' obtains a higher payoff as a leader in (i, i') than as a leader in (i', j'). A proof is available upon request.

4.3 Stratification Inefficiency[29]

We now illustrate stratification inefficiency. Suppose technologies are correlated according to the network in Figure 8. Then, the PEN in each size-two team is unique: a symmetric equilibrium $(2,2)$ is played in each team except when the pairwise correlation is 0.9. The partition in which the most productive pair $(3,4)$ match and the least productive pair $(1,2)$ match is the unique CSPE and Core partition. Nonetheless, individual incentives are misaligned with planner objectives to exploit the entire correlation matrix and the outcome is Welfare and Information-Inefficient.



Correlation	Equilibrium	Payoff
-0.3	$(2,2)$	$(-0.19,-0.19)$
-0.25	$(2,2)$	$(-0.20,-0.20)$
-0.1	$(2,2)$	$(-0.22,-0.22)$
0.1	$(2,2)$	$(-0.26,-0.26)$
0.9	$(0,3)$	$(-0.34,-0.25)$

Figure 8: Correlation Matrix and PEN for Stratification Inefficiency Example.

To see why, note that Worker 3 and Worker 4 obtain the highest feasible payoff and always prefer to form a team together. Given that Worker 3 and Worker 4 form a team, when K is small enough, Worker 1 and Worker 2 have no better option than to form a team. But together, Worker 1 and Worker 2 are relatively unproductive; a joint draw is positively correlated. Then, the loss of information from disrupting the productive team comprised of Worker 3 and Worker 4 is out-weighed by the gain from re-matching the unproductive team comprised of Worker 1 and Worker

[29]In a simulation study, available upon request, we provide a lower bound on the measure of inefficient correlation matrices. The economy has four workers, $c(m) = 0.01m^2$, and $\sigma = \sigma_\theta = 1$. We restrict matrices to those that have unique CSPE partitions; hence, inefficiencies are driven by the stratification problem discussed in detail in here. We find that 21.06% and 18.28% of the correlation matrices with a unique CSPE partition do not maximize welfare and information production, respectively.

2.

We call the CSPE with partition $\{(3,4),(1,2)\}$ a **Stratification-Inefficient CSPE**, defined as a CSPE in which a productive team matches, but whose outcome can be improved upon by re-assigning its members to less productive teams. The inefficiency identified in the example holds for an open set of covariance matrices. To see why, consider a four-person economy $\{i, j, i', j'\}$. Select $\rho_{i'j'}$ larger than ρ_{ij} so that (i, j) is more productive than (i', j') and such that in both teams the same PEN exists and in it both workers take a strictly positive number of draws. If ρ_{ij} and the cost of forming a pair K are small enough, (i, j) and (i', j') are part of a CSPE. Select $\rho_{ij'}$ and $\rho_{i'j}$ slightly larger than ρ_{ij}, but less than $\rho_{i'j'}$. Then, a planner benefits from breaking up the productive team (i, j) and matching i with j' (and j with i'). Notice, the inefficiency increases in $\rho_{i'j'}$ as the benefit from re-matching the unproductive team increases. Further, notice that we do not require that the PEN in the CSPE and Efficient outcomes are symmetric, as in the numerical example. We prove that there is an open set of correlation matrices satisfying the previous properties and obtain the following Theorem.

Theorem 3 (Stratification Inefficiency)**.** *Suppose Assumption 1 holds and K is small. Then, there is an open set of correlation matrices for which there is a Stratification-Inefficient CSPE that does not maximize welfare or information production.*

5 Extensions

5.1 Heterogeneous Variances

We fix the precision of each individual's signals to be constant across them and consider heterogeneous correlations. Chade and Eeckhout (2017) fix pairwise correlations and consider heterogeneous precisions. The key difference in our environment, however, is that there is an after-match game in which individuals choose the number of signals they produce. In contrast, Chade and Eeckhout (2017) assume that each individual exogenously receives one signal. This distinction is important even if we consider the signal structure in Chade and Eeckhout (2017). In particular, their main result that negative assortative matching is Welfare-Efficient need not hold.

Suppose signals have different precisions, but are conditionally independent across individuals.

Consider a team (i, j) in which i produces signals with a variance of 1 and j produces signals with a variance greater than 1. In Figure 9, we show how the equilibria of the Production Subgame change as j's variance increases, holding fixed a common cost function. From the Figure, we see that there is a symmetric PEN only if j's variance is close to 1. As j's variance increases, her incentive to take a draw decreases and at some point j has no incentive to produce *any* signals.

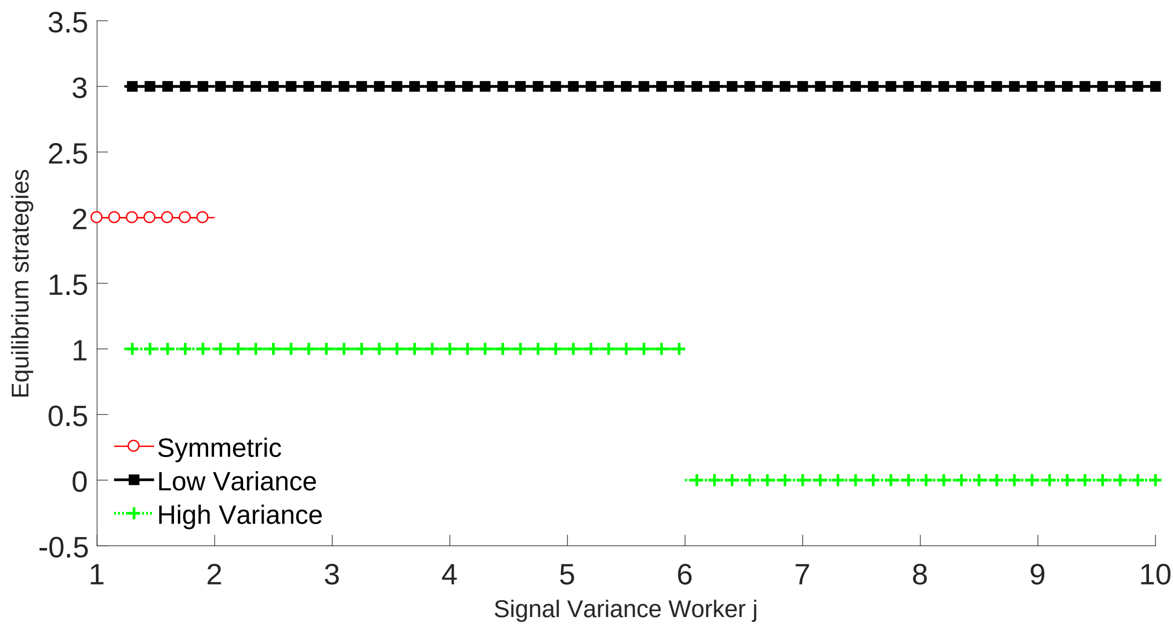


Figure 9: Equilibrium correspondence when $c(m) = 0.001m^2$, $\sigma_\theta = 1$, $\rho_{ij} = 0$, and the variance of Worker i's signals is 1.

This observation has stark implications for team formation. Suppose there are four individuals with variances 0.25, 0.5, 1 and 1.25. The negative assortative matching pairs the individual with variance 0.25 with the individual with variance 1.25. However, in this team, the unique equilibrium of the Production Subgame is $(2,0)$.[30] It is then welfare-dominated by the matching $\{(0.25,1),(0.5,1.25)\}$, the most negative assortative matching in which all individuals take a positive number of draws.[31]

[30]Notice that if there is an increasing membership cost, the worker with variance 0.25 does not want to be part of this team either.

[31]Another interesting observation is that a smaller variance is not always beneficial. If an individual reduces her variance, she may have to acquire more information or may induce her teammate to acquire less information. Hence, pre-match investments may not be profitable.

5.2 Unrestricted Team Size

We briefly summarize how our analysis changes when we allow for larger teams.[32] Existence of equilibrium of the two-stage game is complicated. The existence of a pure strategy Nash equilibrium inside each team is guaranteed because the best response of any worker is bounded and the modified game is a finite Potential Game. Nonetheless, a CSPE need not exist. The intuition for the failure of Theorem 1 is that the game inside each team is no longer symmetric. Hence, off the path of play, it may not be possible to force a worker to take more draws than her teammates.

When a CSPE does exist, we observe that there are many correlation matrices for which the grand coalition does *not* form. There are two reasons for this. First, in equilibrium, a new team member may decide to take zero draws. Second, even if the teammate produces a signal, cross correlations may be such that the information added is negligible. The interaction of membership fees and strategic considerations then discourages teams from adding new members.

We also note that, as a general matter, it is neither easier nor more difficult to enforce stability when allowing for larger teams. In Online Appendix E.3, we present an example in which the Core is empty when teams have at most two members, but is empty when allowing for larger teams. We present another example, however, in which the Core is empty when teams are unrestricted in size and non-empty when teams are restricted to have at most two members.

5.3 Summary of Robustness Checks[33]

In Online Appendix F.1, we study a finite extensive game with sequential decisions. The main conclusion is that for many, but not all, correlations there is a Subgame Perfect Equilibrium of the sequential game that coincides with the most symmetric equilibrium of the simultaneous game. The cases in which there may not be a Subgame Perfect Equilibrium that coincides with any equilibrium of the simultaneous game occur when pairwise correlations lie in a region $(\bar{\rho}, \bar{\bar{\rho}})$ for fixed $\bar{\bar{\rho}} \geq \bar{\rho} > 0$. However, it need not be the case that all equilibria of the extensive game are more symmetric than all equilibria of the simultaneous game. We provide an example in which the extensive game has an equilibrium that is more asymmetric than the most symmetric equilibrium of the simultaneous game, and does not coincide with any equilibrium of the simultaneous game.

[32] See Online Appendix E for a more detailed discussion.
[33] See Online Appendix F for a more detailed discussion.

We have assumed throughout the paper that the prior and signals are Gaussian. We could have instead assumed that the state and the signal are binary. In this case, the ex-post variance does not admit a simple closed-form solution. In Online Appendix F.2, we show that when the number of draws is small, the marginal value of a draw satisfies the same qualitative properties as the ones described in section 3. Hence, all subsequent results hold.

Finally, in Online Appendix F.3, we consider a variation of the after-match game in which the action space is continuous. We show that the equilibrium is unique and symmetric for negative correlations and asymmetric, but not necessarily unique, for positive correlations. This is consistent with Proposition 3.

6 Applications

We have introduced a two-stage game to study how workers form teams to produce information. In the second stage, workers acquire signals simultaneously and non-cooperatively. In the first stage, given common knowledge of the signal structure, workers cooperatively form teams. Our main contribution is to characterize the inefficiencies that arise when team formation is decentralized. Asymmetric effort inefficiency occurs when a worker may exert different effort levels in different teams; even when a more productive team is available, a worker may prefer to form a less productive team in which she can exert less effort than her partner. Stratification inefficiency occurs when the best teams form, but create a negative externality on other teams.

We have studied a case in which information production is decentralized. However, in many organizations managers intervene in information production. We believe the inefficiencies we identify can be used to rationalize this observed practice. Beyond Oticon, we provide two such examples.

Lazear (1998) argues that multinational firms benefit from the complementarities of its workers across countries. A firm capable of expanding across countries can take advantage of complementarities in knowledge by facilitating information exchanges. Our model illustrates that such interactions may be profitable, but may not occur endogenously. Hence, we suggest an informational rationale for the multinational corporation.

Krehbiel (1992) portrays the committee system in the House of Representatives as a specializationof-labor arrangement with committee composition to be determined by informational objectives. In practice, senior party leaders allocate legislators to committees and both major parties are well-represented in each committee. Our model suggests an efficiency rationale for this observed practice; endogenous sorting into committees may be inefficient. Asymmetric effort inefficiency may be particularly relevant as the workload of congresspeople has increased greatly over time (Schickler and Wawro (2013)).

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A Proofs

Proof Lemma 1

For any function $g: X \to \mathbb{R}$, where X is the set of possible realizations of signals,

$$-\mathbb{E}_{x,\theta}\left[(g(x)-\theta)^2\right] \leq -\mathbb{E}_x\left[(\mathbb{E}(\theta \mid x)-\theta)^2\right] = -\mathbb{E}_x\left[\mathbb{E}_\theta\left[(\mathbb{E}(\theta \mid x)-\theta)^2 \mid x\right]\right] = -Var(\theta \mid x).$$

The inequality follows because $\mathbb{E}\left[(b-\theta)^2|x\right]$ is minimized by setting $b=\mathbb{E}[\theta|x]$. The first equality follows from the Law of Iterated Expectations. The second equality follows from the definition of conditional variance.

To solve for the closed form of the conditional variance we calculate the posterior distribution. Denoting $p(x|\theta)$ as the likelihood function and $p(\theta)$ as the prior density function,

$$\begin{aligned}
p(x|\theta)p(\theta) &\propto \exp\left(-\frac{1}{2}\left[(\theta-\mu_\theta)^2\sigma_\theta^{-2}+(\theta\cdot 1_N-x)'\sigma^{-2}\Sigma^{-1}(\theta\cdot 1_N-x)\right]\right)\\
&\propto \exp\left(-\frac{1}{2}\left[\theta^2(\sigma_\theta^{-2}+\sigma^{-2}1_N'\Sigma^{-1}1_N)-\theta(2\mu_\theta\sigma_\theta^{-2}+\sigma^{-2}(x'\Sigma^{-1}1_N+1_N'\Sigma^{-1}x)\right]\right)\\
&\propto \exp\left(-\frac{1}{2}\left[\theta-A\right]'B\left[\theta-A\right]\right),
\end{aligned}$$

where $B=(\sigma_\theta^{-2}+\sigma^{-2}1_N'\Sigma^{-1}1_N)$, $A=B^{-1}(\mu_\theta\sigma_\theta^{-2}+\sigma^{-2}1_N'\Sigma^{-1}x)$, and the proportionality operator eliminates positive constants. Since the derived expression is the kernel of a normal distribution, $Var(\theta \mid x)=B^{-1}$.

We construct B^{-1} when $N=2$, Worker 1 takes $m \geq n$ draws, and Worker 2 takes n draws. The prior covariance matrix, Σ^{-1}, is block diagonal with n blocks of the form,

$$\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and $m-n$ scalar blocks each equal to 1. The inverse of a block diagonal matrix is equal to the block diagonal matrix formed by inverting each block. Since,

$$\Sigma_0^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix},$$

we have,

$$1_2'\Sigma_0^{-1}1_2 = \frac{2}{1+\rho}.$$

Then, $1_N'\Sigma^{-1}1_N$ is the sum of n of these blocks and $m-n$ times 1. Hence,

$$B^{-1} = \left(\sigma^{-2}1_N'\Sigma^{-1}1_N + \sigma_\theta^{-2}\right)^{-1} = \left(\sigma^{-2}\left(n\frac{2}{1+\rho} + (m-n)\right) + \sigma_\theta^{-2}\right)^{-1}.$$

The general expression when m need not be greater than n is:

$$Var(\theta \mid x) = \left(\left(\min\{n,m\}\frac{2}{1+\rho} + |m-n|\right)\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1}.$$

Proof Lemma 2

We show that if $n_i \neq n_j$,

$$MV(n_i;n_j,\rho) - MV(n_i-1;n_j,\rho) < 0,$$

and if $n_i \neq n_j + 1$,

$$MV(n_i;n_j+1,\rho) - MV(n_i;n_j,\rho) < 0.$$

For $n_i \leq n_j$,

$$f(\rho,n_i-1,n_j) - f(\rho,n_i,n_j) = \frac{\left(\frac{1-\rho}{1+\rho}\right)\sigma^{-2}}{\left(\left(n_i\frac{1-\rho}{1+\rho}+n_j+1-\frac{2}{1+\rho}\right)\sigma^{-2}+\sigma_\theta^{-2}\right)\left(\left(n_i\frac{1-\rho}{1+\rho}+n_j\right)\sigma^{-2}+\sigma_\theta^{-2}\right)}$$

is strictly decreasing in n_j and in n_i because $\frac{1-\rho}{1+\rho} > 0$. For $n_i \geq n_j + 1$,

$$f(\rho,n_i-1,n_j) - f(\rho,n_i,n_j) = \frac{\sigma^{-2}}{\left(\left(n_j\frac{1-\rho}{1+\rho}+n_i-1\right)\sigma^{-2}+\sigma_\theta^{-2}\right)\left(\left(n_j\frac{1-\rho}{1+\rho}+n_i\right)\sigma^{-2}+\sigma_\theta^{-2}\right)}$$

is strictly decreasing in n_i and in n_j, again because $\frac{1-\rho}{1+\rho} > 0$.

The preceding observations imply that the best response by Worker i is decreasing in n_j, since the marginal value is strictly decreasing. We only need to prove that the best response by Worker i to 0 draws by Worker j is finite. It suffices to show that there is an $n \in \mathbb{Z}_+$ such that $f(\rho,n-1,0) - f(\rho,n,0)$ is smaller than $c(1)$.

$$f(\rho,n-1,0) - f(\rho,n,0) \quad = \frac{1}{(n-1)\sigma^{-2}+\sigma_\theta^{-2}} - \frac{1}{n\sigma^{-2}+\sigma_\theta^{-2}} < \frac{\sigma^2}{n(n-1)},$$

implies that for $n > \frac{\sigma^2}{c(1)} + 1$, we obtain the required inequality. Define $\bar{M}$ as the smallest such n.

Proof Corollary 1

Suppose there are multiple pure strategy equilibria and all of them are asymmetric. Let (m_i^*, m_j^*)

be the preferred equilibrium of Worker i. Take a mixed strategy equilibrium (σ_i, σ_j) and suppose that it is a Pareto improvement over (m_i^*, m_j^*). Then, there has to be a strategy (k_i, k_j) for which $\sigma_i(k_i) > 0$, $\sigma_j(k_j) > 0$, $u_i(k_i, k_j) \geq u_i(m_i^*, m_j^*)$, and $u_j(k_i, k_j) > u_j(m_i^*, m_j^*)$.

Since the game is symmetric, $k_i \neq m_i^*$ and $k_j \neq m_j^*$. Hence, there are four cases to consider:

1. $k_i > m_i^*$ **and** $k_j < m_j^*$: If $k_j < m_j^*$ the inequality $u_i(k_i, k_j) \geq u_i(m_i^*, m_j^*)$ cannot hold.

2. $k_i > m_i^*$ **and** $k_j > m_j^*$: The proof of Lemma 2 implies that the marginal value of a draw by Worker j decreases in the number of draws by Worker i. If $k_i < k_j$, it has to be that $u_j(k_i, k_j) < u_j(m_i^*, m_j^*)$, contradicting $u_j(k_i, k_j) > u_j(m_i^*, m_j^*)$. A similar argument holds if $k_i \geq k_j$.

3. $k_i < m_i^*$ **and** $k_j > m_j^*$: If $k_j > m_j^*$ the inequality $u_j(k_i, k_j) > u_j(m_i^*, m_j^*)$ cannot hold.

4. $k_i < m_i^*$ **and** $k_j < m_j^*$: Since the marginal value of a draw by Worker i decreases in the number of draws by Worker j, if $k_i < k_j$ we get $u_i(k_i, k_j) < u_i(m_i^*, m_j^*)$, a contradiction to $u_i(k_i, k_j) \geq u_i(m_i^*, m_j^*)$. A similar argument holds if $k_i \geq k_j$.

As all cases lead to contradiction, we conclude that the mixed strategy equilibrium (σ_i, σ_j) is not a Pareto improvement over (m_i^*, m_j^*).

Proof Theorem 1

First, note that only $\binom{N}{2}$ couples can be formed. For each couple, each equilibrium played inside the couple yields a (finite) payoff for each teammate. Order every such equilibrium and couple by the payoff obtained by the leader. Choose the couple, say (i, j), and equilibrium such that the leader, say i, obtains the highest payoff compared to any leader in any couple playing any equilibrium. If there is more than one such couple, equilibrium, leader combination, choose one arbitrarily.

Fix both workers to be leaders in any other team. If i and j know that in any other couple they will be a leader neither will want to deviate; Worker i knows that in any other couple the leader gets at most what she is getting now. As both i and j acquire the same information and i is taking more draws, we know that the payoff of j is larger than the payoff of i. Hence, j also cannot do any better.

Set i and j aside and repeat the process with the workers that are left. The only difference is that the workers picked in the second round will be the leaders in any couple they can form not including i or j. By induction, we find a partition and strategy profile comprising a CSPE.

As there exists at least one CSPE and since the number of possible partitions and equilibria for each team are finite, there is a finite, strictly positive number of CSPE. Choose a CSPE $(\Pi, \{m^*(S)\}_S)$. If it is Pareto-Efficient we are done. Suppose it is not. Then, there is another feasible partition $\hat{\Pi} = \{T_1, \dots, T_m\}$ and on-path equilibria, $\hat{m}(T)_{T\in\hat{\Pi}}$, such that, for each worker,

$$v_i(\hat{m}(T_{\hat{\Pi}(i)})) - K * \mathbb{I}^{T_{\hat{\Pi}(i)}} \geq v_i(m^*(S_\Pi(i))) - K * \mathbb{I}^{S_\Pi(i)}$$

and the inequality is strict for at least one of them. Consider the profile $(\hat{\Pi}, \{\tilde{m}(S)\}_S)$ where $\tilde{m}(S) = m^*(S)$ if $S \notin \hat{\Pi}$ and $\tilde{m}(S) = \hat{m}(S)$ if $S \in \hat{\Pi}$. This profile is a CSPE; on-path, each worker obtains a higher payoff than in the original CSPE and each worker has the same possible deviations. As there is a finite number of CSPE and in every step we are weakly increasing the payoff of all worker and strictly increasing it for at least one, we can repeat the process until a Pareto-Efficient CSPE is found.

Proof Lemma 3

For $n_i > n_j$,

$$\frac{\partial f(\rho, n_i - 1, n_j) - f(\rho, n_i, n_j)}{\partial \rho} \propto \left(\left(n_j \frac{1-\rho}{1+\rho} + n_i\right)\sigma^{-2} + \sigma_\theta^{-2}\right) + \left(\left(n_j \frac{1-\rho}{1+\rho} + n_i - 1\right)\sigma^{-2} + \sigma_\theta^{-2}\right) > 0.$$

Proof Lemma 4

If Worker 1 takes t draws, the marginal benefit of taking draw $n+1$ for Worker 2 is,

$$f(\rho, t, n) - f(\rho, t, n+1) = \frac{\left(\frac{1-\rho}{1+\rho}\right)\sigma^{-2}}{\left(\left(n\frac{2}{1+\rho} + t - n\right)\sigma^{-2} + \sigma_\theta^{-2}\right)\left(\left((n+1)\frac{2}{1+\rho} + t - n - 1\right)\sigma^{-2} + \sigma_\theta^{-2}\right)}.$$

Differentiating with respect to ρ,

$$\frac{\partial f(\rho, t, n) - f(\rho, t, n+1)}{\partial \rho} = \frac{2\sigma^2\left(-(t+\gamma)^2(1+\rho)^2 + n(n-1)(1-\rho)^2\right)}{\left(2n + (t - n + \gamma)(1+\rho)\right)^2\left(2(n+1) + (t - n - 1 + \gamma)(1+\rho)\right)^2}.$$

Differentiating again with respect to ρ,

$$\frac{\partial^2 f(\rho,t,n)-f(\rho,t,n+1)}{\partial\rho^2} \propto 4n(n+1)\left(-(t+\gamma)^2(1+\rho)-n(n-1)(1-\rho)\right)$$

$$+n(n+1)\left[2n(t-n-1+\gamma)+2(n+1)(t-n+\gamma)+(t-n-1+\gamma)(t-n+\gamma)\right](2\rho-2)<0.$$

Hence, the marginal value $f(\rho,t,n)-f(\rho,t,n+1)$ is strictly concave. The unique maximizer $\tilde{\rho}(t,n,\gamma)$ must satisfy,

$$(t+\gamma)^2(1+\tilde{\rho}(t,n,\gamma))^2 = n(n+1)(1-\tilde{\rho}(t,n,\gamma))^2,$$

a quadratic equation in ρ with roots,

$$\rho^+ = \frac{\left(t+\gamma+\sqrt{n(n+1)}\right)^2}{-(t+\gamma)^2+n(n+1)}$$

$$\rho^- = \frac{\left(t+\gamma-\sqrt{n(n+1)}\right)^2}{-(t+\gamma)^2+n(n+1)}.$$

Both roots are negative because the denominator is negative. However, ρ^- is less than -1 and therefore is infeasible. Since $n+1\leq t$, ρ^+ is greater than -1. Set $\tilde{\rho}(t,n,\gamma)=\rho^+$.

Proof Lemma 5

$$f(\rho,t-1,t)-f(\rho,t,t)\geq f(\rho,t-1,t-1)-f(\rho,t-1,t)$$

$$\Leftrightarrow \frac{1}{\left((t-1)\frac{2}{1+\rho}+1\right)\sigma^{-2}+\sigma_\theta^{-2}}-\frac{1}{t\frac{2}{1+\rho}\sigma^{-2}+\sigma_\theta^{-2}}\geq\frac{1}{(t-1)\frac{2}{1+\rho}\sigma^{-2}+\sigma_\theta^{-2}}-\frac{1}{\left((t-1)\frac{2}{1+\rho}+1\right)\sigma^{-2}+\sigma_\theta^{-2}}$$

$$\Leftrightarrow \frac{1-\rho}{1+\rho}\left((t-1)\frac{2}{1+\rho}\sigma^{-2}+\sigma_\theta^{-2}\right)\geq\left(t\frac{2}{1+\rho}\sigma^{-2}+\sigma_\theta^{-2}\right)$$

$$\Leftrightarrow 0\geq\gamma\rho^2+(\gamma-1+2t)\rho+1.$$

The last inequality involves a quadratic concave function in ρ. The roots are:

$$\rho^+(t)=\frac{-(\gamma-1+2t)+\sqrt{(\gamma-1+2t)^2-4\gamma}}{2\gamma}$$

$$\rho^-(t)=\frac{-(\gamma-1+2t)-\sqrt{(\gamma-1+2t)^2-4\gamma}}{2\gamma}.$$

We first show that $\rho^-(t)$ is infeasible by showing that $\rho^-(t) < -1$ for $t > 1$. Note,

$$\rho^-(t) - \rho^-(t+1) = \frac{2 + \sqrt{(\gamma - 1 + 2(t+1))^2 - 4\gamma} - \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}}{2\gamma},$$

is positive since $(\gamma - 1 + 2t)^2 - 4\gamma$ is increasing in $t \geq 1$ and, at $t = 1$, $(\gamma - 1 + 2t)^2 - 4\gamma = (\gamma - 1)^2 > 0$. If $\gamma < 1$, then $\rho^-(1) < -1$ and hence $\rho^-(t) < -1$ for all $t \geq 1$. If $\gamma \geq 1$, then $\rho^-(1) = -1$, but $\rho^-(t) < -1$ for all $t > 1$.

Second, we show that $\rho^+(t) \in [-1, 0)$ so that $\hat{\rho}(t, \gamma) = \rho^+(t)$ as stated in the Lemma. $\rho^+(t) < 0$ since $\sqrt{(\gamma - 1 + 2t)^2} > \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}$. Further,

$$\rho^+(t+1) - \rho^+(t) = \frac{-2 + \sqrt{(\gamma - 1 + 2(t+1))^2 - 4\gamma} - \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}}{2\gamma}$$

is positive since,

$$-2 + \sqrt{(\gamma - 1 + 2(t+1))^2 - 4\gamma} > \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}$$

$$\Leftrightarrow (\gamma + 1 + 2t)^2 > (\gamma - 1 + 2(t+1))^2 - 4\gamma$$

$$\Leftrightarrow 4\gamma > 0.$$

Then, $\rho^+(t)$ is increasing in t and,

$$\rho^+(1) = \frac{-(\gamma + 1) + \sqrt{(\gamma - 1)^2}}{2\gamma} = \begin{cases} \frac{-2}{2\gamma} > -1 & \text{if } \gamma \geq 1 \\ \frac{-2\gamma}{2\gamma} = -1 & \text{if } \gamma < 1. \end{cases}$$

Proof Lemma 6

Let $g(t, \gamma) = \tilde{\rho}(t, t-1, \gamma) - \hat{\rho}(t, \gamma)$. For any $t \in [2, \infty)$, we show that $g(t, \gamma)$ has a unique zero in the interval $[\frac{1}{2}, 1)$. First, note that,

$$g(t, \tfrac{1}{2}) = \frac{-8t^2 - 1 + 8(t + \frac{1}{2})\sqrt{t(t-1)}}{8t + 1} - (\tfrac{1}{2} - 2t + \sqrt{4t^2 - 2t - \tfrac{7}{4}}) > 0,$$

if and only if

$$8t^2 - 2t - \frac{3}{2} + 4(2t+1)\sqrt{t(t-1)} > (8t+1)\sqrt{4t^2 - 2t - \frac{7}{4}}$$
$$\Leftrightarrow 64t^3 - 48t^2 - 15t - 1 > 0,$$

which holds for $t \geq 2$.

Second, when $t \geq 2$, $(t+1)(t-1+2\sqrt{t(t-1)}) < (3t+1)\sqrt{(t+1)(t-1)}$ and so,

$$g(t,1) = \frac{-(t+1)(t-1)-2(t+1)\sqrt{t(t-1)}+(3t+1)\sqrt{(t+1)(t-1)}}{-(3t+1)} < 0 \cdot$$

By the intermediate value theorem, there exists a $\gamma^* \in [\frac{1}{2}, 1)$ such that $g(t,\gamma^*) = 0$.

To prove that $g(\cdot,\gamma)$ is strictly decreasing in γ we show that $\frac{\partial \hat{\rho}(t,\gamma)}{\partial \gamma} > 0$ and $\frac{\partial \tilde{\rho}(t,t-1,\gamma)}{\partial \gamma} < 0$. To show $\frac{\partial \hat{\rho}(t,\gamma)}{\partial \gamma} > 0$, note,

$$\begin{aligned}
\frac{\partial \hat{\rho}(t,\gamma)}{\partial \gamma} &= \frac{\Big(-1+\big((\gamma-1+2t)^2-4\gamma\big)^{-0.5}(\gamma-3+2t)\Big)2\gamma-2\Big(-(\gamma-1+2t)+\sqrt{(\gamma-1+2t)^2-4\gamma}\Big)}{4\gamma^2} \\
&\propto 3\gamma-2t\gamma-1+4t-4t^2+(2t-1)\sqrt{(\gamma-1+2t)^2-4\gamma}.
\end{aligned}$$

Then, $\frac{\partial \hat{\rho}(t,\gamma)}{\partial \gamma} > 0$ if and only if,

$$\begin{aligned}
&(2t-1)\sqrt{(\gamma-1+2t)^2-4\gamma} > \gamma(2t-3)+(2t-1)^2 \\
\Leftrightarrow\quad & \gamma^2(2t-1)^2 > \gamma^2(2t-3)^2.
\end{aligned}$$

As the last inequality holds, $\frac{\partial \hat{\rho}(t,\gamma)}{\partial \gamma} > 0$. Finally, since $\gamma > 0$ and $t-1 < \sqrt{t(t-1)}$,

$$\frac{\partial \tilde{\rho}(t,t-1,\gamma)}{\partial \gamma} = \frac{4(t+\gamma)\Big(t(t-1)-(t+\gamma)\sqrt{t(t-1)}\Big)-2\sqrt{(t(t-1))^3}-4(t+\gamma)^2\sqrt{t(t-1)}}{(-(t+\gamma)^2+t(t-1))^2} < 0.$$

Proof Proposition 2

A sufficient condition for a symmetric equilibrium is:

$$f(\rho,n-1,n)-f(\rho,n,n) \geq f(\rho,n-1,n-1)-f(\rho,n-1,n).$$

From Lemma 5, the condition is satisfied for correlations smaller than $\hat{\rho}(t,\gamma)$. Lemma 5's proof shows $\hat{\rho}(t,\gamma)$ is increasing in t and in γ. $\hat{\rho}(1,\gamma) = -1$ for any γ. However, by Assumption 1, for $\rho = -1$ and correlations close to it, both workers take at least one draw. Hence, there is a threshold $\rho^* > -1$ below which the unique equilibrium is symmetric.

As ρ approaches 1, the marginal benefit of completing the first draw by the follower, $f(\rho,0,1) - f(\rho,1,1)$, approaches zero. By continuity and monotonicity of the marginal value in ρ, there exists a unique $\rho^{**} < 1$ such that $f(\rho^{**},0,1) - f(\rho^{**},1,1) = c(1)$. By the proof of Lemma 2, the marginal value is decreasing with respect to the number of draws by the leader so for any correlation higher than ρ^{**} the follower does not take any draw.

Proof Proposition 3

Suppose that for correlation ρ there is an asymmetric equilibrium (m_1, m_2) in which $m_1 > m_2$ and there is no symmetric equilibrium. Then, $\rho > \hat{\rho}(m_1, \gamma)$. We need to argue that for correlation $\rho' > \rho$ there is no symmetric equilibrium. We can use the *Sequential Research Response Algorithm*:

1. Fix a team $S = (i, j)$.

2. Set $(m_1^0, m_2^0) = (0, 0)$.

3. In iteration t, if $f(\rho, t-1, m^{t-1}) - f(\rho, t, m^{t-1}) > c(t) - c(t-1)$, set $m_1^t = m_1^{t-1} + 1$ so that Worker 1 takes one more draw and move to Step 4. If not, set $m_1^t = m_1^{t-1}$ so that Worker 1 does not take another draw and move to Step 5.

4. In iteration t, Worker 2 chooses $m_2^t = \arg\max_{m \leq t} f(\rho, t, m) - c(m)$. Go back to Step 3 and replace t with $t+1$.

5. (Complement Effect) Workers consider choosing $(m_1^{t+1}, m_2^{t+1}) = (m_1^t + 1, m_2^t + 1)$. They choose it only when it is a Nash equilibrium and makes each worker weakly better off. Iterate until the condition is not satisfied. Then, move to Step 6.

6. (Substitution Effect) Consider $(m_1^t + 1, m_2^t - n)$ where $m_2^t - n$ is a best-response by Player 2 given $m_1^t + 1$ and subject to the constraint that $0 \leq n \leq m_2^t$. If $(m_1^t + 1, m_2^t - n)$ is a Nash equilibrium and makes each worker weakly better off set $(m_1^{t+1}, m_2^{t+1}) = (m_1^t + 1, m_2^t - n)$. Iterate until the condition is not satisfied and stop the algorithm.

In Online Appendix B, we prove the following Proposition.

Proposition. *The Sequential Research Response Algorithm finds a unique PEN of the Production Subgame up to the identity of the workers. It finds the most symmetric equilibrium, i.e. one which minimizes* $|m_i - m_j|$.

Notice that by Lemma 4 and Lemma 6 at any iteration s, we can conclude that $m_2^s(\rho') \leq m_2^s(\rho)$ since the marginal value of a draw by the follower is always smaller under ρ'. To complete the argument, Lemma 3 implies that when the process reaches iteration s, if Worker 1 decides to take draw s when the correlation is ρ she would take it when the correlation is ρ' as well. This implies that the procedure will continue for more iterations for higher correlations and the most symmetric equilibrium will be more asymmetric under ρ' than under ρ. If there is a symmetric equilibrium, we can repeat the argument by starting the *Sequential Research Response Algorithm* from the asymmetric equilibrium (m_1, m_2). Then, for $\rho > \rho'$ there has to be an equilibrium more asymmetric than (m_1, m_2).

Proof Theorem 2

Consider a four-worker economy $\{i, j, i', j'\}$. Define three sets of correlations using our characterization in Proposition 2. First, define the set of correlations for which the only PEN is symmetric:

$$P^1 = [-1, \rho^*).$$

Second, define the set of correlations for which there may be multiple equilibria:

$$P^2 = [\rho^*, \rho^{**}].$$

Third, define the set of correlations for which the only PEN is symmetric, but in which each worker can obtain a higher payoff in an asymmetric equilibrium:

$$P^3 = \left\{ \rho \in P^1 : \bar{v}_i(\rho) < \sup_{\tilde{\rho} \in (P^1)^c} (\bar{v}_i(\tilde{\rho})) \right\}.$$

Note, $\rho^* < \rho^{**}$ not only ensures that P^2 is non-empty, but also ensures that P^3 is non-empty and open.

We select correlations so that (i, j) and (i', j') are efficient, but (i, i') and (j, j') match in a CSPE because of asymmetric-effort opportunities. Select $\rho_{ii'}$ and $\rho_{jj'}$ in $[\rho^*, \rho^{**}]$ close to ρ^* so that the payoff of the follower in the most asymmetric equilibrium is larger than the payoff obtained in an open set contained in P^3. Let $\delta_{ii'}$ and $\delta_{jj'}$ be the difference in payoffs between the follower and leader in the most asymmetric equilibria in teams (i, i') and (j, j'). Pick $\rho_{ij}, \rho_{i'j'} \in P^3$ so that the payoff of each worker in (i, j) and (i', j') is less than $\frac{min\{\delta_{ii'}, \delta_{jj'}\}}{2}$ from the maximum payoff a follower can attain in team (i, i') or (j, j'). Finally, pick all other correlations including these four workers in $(\rho^{**}, 1]$ so that these teams are never formed.

Since the sum of payoffs of workers i and j in (i,j) is larger than the sum of the payoffs of workers i and i' in (i,i'), and analogously for (i',j') and (j,j'), matching (i,j) and (i',j') is efficient. However, there is a CSPE matching (i,i') and (j,j'). Suppose i and j are the followers in these pairs, respectively. Then, each receives the highest obtainable payoff and has no incentive to deviate. Further, i' and j' do not have a better outside option; they do not want to form a deviating team and as long as K is small enough being alone gives each a lower payoff.

Proof Theorem 3

Consider a four-worker economy $\{i,j,i',j'\}$. We select six correlations so that forming teams (i,i') and (j,j') is efficient, but for which there is a CSPE matching (i,j) and (i',j'). Pick $\rho_{ij} < \rho_{i'j'} < \rho^{**}$ such that (m,n) where $m \geq n > 0$ is a PEN in (i,j) and (i',j'). Next, take $\rho_{ii'}, \rho_{jj'} \in [\rho_{ij}, \rho_{i'j'}]$ close enough to ρ_{ij} such that (m,n) is a PEN in (i,i') and (j,j').[34] Finally, choose $\rho_{ij'}$ and $\rho_{i'j}$ greater than ρ^{**} so that (i,j') and (i',j) never formed in a CSPE or efficient outcome.

Fix (m,n) in (i,j), (i',j'), (i,i'), and (j,j'). Then, if K small enough, $\{(i,j),(i',j')\}$ is a CSPE partition; i and j obtain the highest possible payoff in a team together and, if K small enough, i' and j' prefer to be in team together than to be alone. Next, note that each worker's payoff is decreasing in the pairwise correlation ρ,

$$\frac{\partial v_i(m,n,\rho)}{\rho} = \frac{-2m\sigma^{-2}}{\left((2m+(m-n)(1+\rho))\,\sigma^{-2}+(1+\rho)\sigma_\theta^{-2}\right)} < 0.$$

Hence, by picking $\rho_{ii'}, \rho_{jj'}$ close enough to ρ_{ij} the sum of payoffs from matching (i,i') and (j,j') is greater than the sum of payoffs from matching (i,j) and (i',j'). As the same equilibrium is played in each of these teams, the latter result implies that the amount of information created in the Welfare- Efficient matching is greater than in the CSPE matching.

[34]To understand why, consider the intuition from Figure 4. As long as ρ_{ij} is not the largest correlation in an interval in which (m,n) is played, these correlations are guaranteed to exist by the continuity of the marginal value function. If ρ_{ij} is the largest such correlation, select a new ρ_{ij} to the left of the original one. Notice that it is not true in general that for all correlations between ρ_{ij} and $\rho_{i'j'}$ the same equilibrium exists.

Online Appendix to "Matching to Produce Information"

Ashwin Kambhampati, Carlos Segura-Rodriguez and Peng Shao *

A Sufficient Conditions for a Non-Empty Core

Define three sets of correlations using our characterization in Proposition 2. First, define the set of correlations for which the only PEN is symmetric:

$$P^1 = [-1, \rho^*].$$

Second, define a set P^2 that is the union of (i) the elements in P^1 for which a worker can obtain a higher payoff in an asymmetric equilibrium and (ii) the set of correlations for which there may be multiple equilibria:

$$P^2 = \left\{ \rho \in P^1 : \bar{v}_i(\rho) < \sup_{\tilde{\rho} \in (P^1)^c} (\bar{v}_i(\tilde{\rho})) \right\} \cup [\rho^*, \rho^{**}],$$

where $\bar{v}_i(\rho)$ is the maximum equilibrium payoff of an worker inside a team with correlation ρ. Third, define the set of correlations for which there is a unique asymmetric equilibrium in which one worker takes zero draws:

$$P^3 = (\rho^{**}, 1].$$

Theorem 4 *Suppose Assumption 1 and one of the following two conditions holds:*

1. *All correlations are in $P^1 \cup P^3$.*

2. *If Worker j has more than one pairwise correlation in P^2, then for all i such that $\rho_{ij} \in P^2$, ρ_{ij} is the only pairwise correlation in P^2 for Worker i.*

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Then, the Core is non-empty.

The logic of the first condition is that if all PEN are symmetric or include complete free riding, we can form teams with pairwise correlations in P^1 sequentially in decreasing order of payoffs. Since those with correlations in P^1 will be matched with their preferred feasible choice and nobody wants to be in a team with a partner that takes zero draws, there are no profitable deviations. The second condition in Theorem 4 ensures there are no cycles of profitable deviations to non-trivial asymmetric equilibria, i.e. to teams with correlations outside of P^3. Hence, the example in Figure 3 is ruled out.

B Sequential Research Response Algorithm

While we have proved that there exists a Pareto-Efficient Nash Equilibrium (PEN) of the Production Subgame, we have not specified a procedure to find one. We accomplish this task with the *Sequential Research Response Algorithm*. It identifies a unique PEN up to the worker identity. The algorithm proceeds as follows:

1. Fix a team $S = (i, j)$.

2. Set $(m_1^0, m_2^0) = (0, 0)$.

3. In iteration t, if $f(\rho, t-1, m^{t-1}) - f(\rho, t, m^{t-1}) > c(t) - c(t-1)$, set $m_1^t = m_1^{t-1} + 1$ so that Worker 1 takes one more draw and move to Step 4. If not, set $m_1^t = m_1^{t-1}$ so that Worker 1 does not take another draw and move to Step 5.

4. In iteration t, Worker 2 chooses $m_2^t = \arg\max_{m \leq t} f(\rho, t, m) - c(m)$. Go back to Step 3 and replace t with $t+1$.

5. (Complement Effect) Workers consider choosing $(m_1^{t+1}, m_2^{t+1}) = (m_1^t + 1, m_2^t + 1)$. They choose it only when it is a Nash equilibrium and makes each worker weakly better off. Iterate until the condition is not satisfied. Then, move to Step 6.

6. (Substitution Effect) Consider $(m_1^t + 1, m_2^t - n)$ where $m_2^t - n$ is a best-response by Player 2 given $m_1^t + 1$ and subject to the constraint that $0 \leq n \leq m_2^t$. If $(m_1^t + 1, m_2^t - n)$ is a Nash

equilibrium and makes each worker weakly better off set $(m_1^{t+1}, m_2^{t+1}) = (m_1^t + 1, m_2^t - n)$. Iterate until the condition is not satisfied and stop the algorithm.

The algorithm finds a Nash Equilibrium without Step 5 and Step 6. Step 5 and Step 6 allow us to find a PEN by considering possible Pareto improvements. Step 5 is important only when the strategy profile found in the first four steps is symmetric and when draws are strategic complements. Step 6 can affect the final outcome only if the correlation between teammates is positive.

Lemma 7 *Step 5 in the Sequential Research Response Algorithm can affect the outcome selected only if at the end of Step 3 $m_1^t = m_2^t \equiv m$ and $MV(m+1; m+1, \rho) > MV(m+1; m, \rho)$. Step 6 can affect the outcome only if $\rho > 0$.*

Using the preceding Lemma, we show that our algorithm finds a PEN.

Proposition 4 *The Sequential Research Response Algorithm finds a unique PEN of the Production Subgame up to the identity of the workers. It finds the most symmetric equilibrium, i.e. one which minimizes $|m_i - m_j|$.*

C Additional PEN Characterization

Define the equilibrium correspondence of a leader (Worker 1) and follower (Worker 2):

$$M_1(\rho) = \{m_1 : (m_1, m_2) \text{ is a PEN and } m_1 \geq m_2\}$$

$$M_2(\rho) = \{m_2 : (m_1, m_2) \text{ is a PEN and } m_1 > m_2\}.$$

When $\gamma \leq 1/2$, we obtain a sharp characterization of the equilibrium correspondence. The key is that, by Lemma 6, $\tilde{\rho} > \hat{\rho}$. Following Topkis (2011), we say that the equilibrium correspondence for worker i, $M_i(\rho)$, is **increasing in the weak set order sense** when the following two properties are satisfied:

1. If $\rho' > \rho$ and $m_i \in M_i(\rho)$, then there is $m_i' \in M_i(\rho')$ such that $m_i' \geq m_i$.

2. If $\rho' > \rho$ and $m_i' \in M_i(\rho')$, then there is $m_i \in M_i(\rho)$ such that $m_i \leq m_i'$.

Proposition 5 *Suppose Assumption 1 is satisfied and $\gamma \leq \frac{1}{2}$. Then:*

- *$M_1(\rho)$ is increasing with respect to ρ in the weak set order sense.*
- *$M_2(\rho)$ is decreasing with respect to ρ in the weak set order sense on the interval(s) for which it is defined.*

Figure 10 presents an example that shows that we cannot strengthen the monotonicity condition we use in the corollary. In the Figure, for $\rho = 0$ there are three equilibria: $(3,2)$, $(4,1)$ and $(5,0)$. However, for smaller correlations there are no equilibria in which the leader takes 3 or 4 draws.

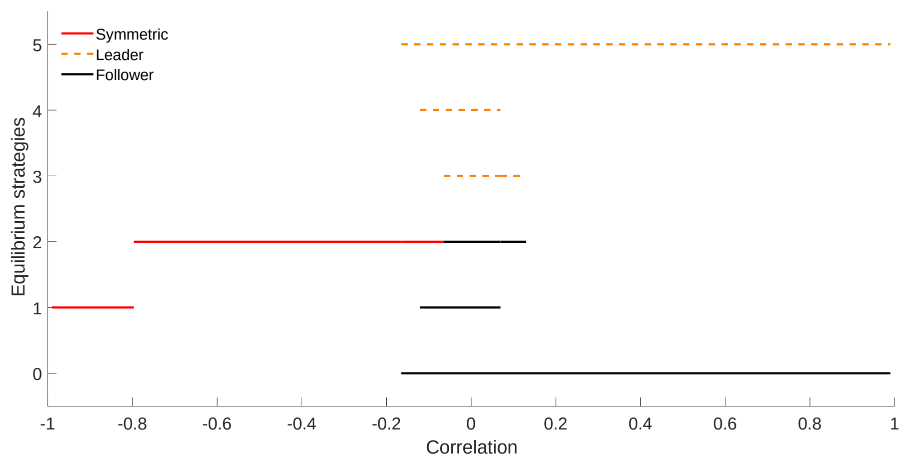

Figure 10: Equilibrium strategies when $c = 0.01m$, $\sigma = \frac{1}{2}$, and $\sigma_\theta = 1$.

D Asymmetric Effort and Information Efficiency

In the main text, we presented an example in which asymmetric effort leads to an Information-Inefficient outcome. In this section, we present an example in which it leads to an Information-Efficient outcome. We fix parameters in Table 10. Because $c(1) = 0.002 < \frac{1}{3} = \frac{\min\{\sigma_\theta^2, \sigma^2\}}{1+\gamma}$ and $\gamma = \frac{\sigma^2}{\sigma_\theta^2} = 2$, Proposition 3 holds. The cutoff values are: $\rho^* = \rho^{***} = -0.006$ and $\rho^{**} = 0.849$.

Table 10: Parameters for Example 2.2 and Example 4.1.

Parameter	Interpretation	Value
σ^2	Signal Variance	2
σ_θ^2	Prior Variance	1
$c(m)$	Cost of m Draws	$0.002m^2$
K	Cost of Teammate	0.01

Suppose signals are correlated according to the network in Figure 11. Then, all PEN are unique up to identity. The equilibrium actions and information in all possible size-two teams are presented in Table 11. In the partition $\{(1,2),(3,4)\}$, the unique PEN in each team, $(4,4)$, is symmetric. However, there is a CSPE with outcome partition $\{(1,3),(2,4)\}$ and PEN $(4,5)$ in each team.

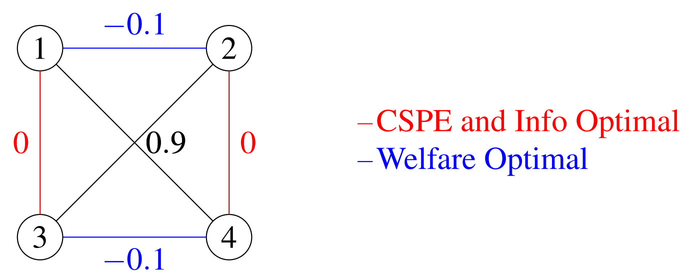

Figure 11: Correlation Matrix.

Table 11: Equilibrium actions and payoff in all size-two groups.

Correlation	Equilibrium	Payoff	$1_S'\Sigma_S^{-1}1_S$
-0.1	$(4,4)$	$(-0.216,-0.216)$	4.44
0.0	$(4,5)$	$(-0.214,-0.232)$	4.5
0.9	$(0,7)$	$(-0.222,-0.32)$	3.5

In this outcome, in contrast to the example in the main text, asymmetric effort increases information production. The reason is that Worker 1 (Worker 4) takes the same number of draws in a team with Worker 2 (Worker 3) as she does in a team with Worker 3 (Worker 2). However, Worker 1 (Worker 4) exerts *relatively* less effort in a team with Worker 3 (Worker 2) than in a team with Worker 2 (Worker 3). Notice, this means that seemingly worse teams may be superior in terms of information production given after-match strategic considerations; $(1,2)$ and $(3,4)$ each have a pairwise correlation of -0.1 and play a unique and symmetric PEN, while the teams $(1,3)$ and $(2,4)$ have a higher correlation and play a unique and asymmetric PEN. Nonetheless, $(1,2)$ and $(3,4)$ produce superior information as measured by $1_S'\Sigma_S^{-1}1_S$ since Worker 3 (Worker 2) is induced to take more draws than Worker 2 (Worker 3) when matched with Worker 1 (Worker 4).

E Extension to Any Team Size

In this section, we allow workers to form teams of any size. First, we show that a Nash equilibrium within each team always exists. Second, we show a CSPE need not exist. Third, we show that increasing the maximum allowable team size can work in favor or against the existence of the Core.

E.1 Existence Nash Equilibrium

Lemma 1 still applies, so the optimal decision given a realization of signals is the conditional expected value of θ and the expected payoff is the negative expected posterior variance. We simplify the expected posterior variance in the following Lemma.

Lemma 8 *Suppose there are P Workers in a team and they choose, without loss of generality, $m_1 \geq m_2 \geq \ldots \geq m_P$ draws. Let Φ^p be the correlation matrix of Workers $1,\ldots,p$ and $(a^p_{i,j})^P_{i=1}$ denote the entries of the inverse of each of these matrices. The integrated variance is then,*

$$f(m_1,m_2,\ldots,m_P) = \left(\left((m_1-m_2)+(m_2-m_3)\sum_{i=1,j=1}^{2,2} a^2_{i,j}+(m_3-m_4)\sum_{i=1,j=1}^{3,3} a^3_{i,j}+\ldots \right.\right.$$
$$\left.\left. +(m_{P-1}-m_P)\sum_{i=1,j=1}^{P-1,P-1} a^{P-1}_{i,j}+m_P\sum_{i=1,j=1}^{P,P} a^P_{i,j}\right)\sigma^{-2}+\sigma_\theta^{-2}\right)^{-1}.$$

As there is still a finite bound on the best response of any worker, we can bound the action space and show that the modified finite game is a Potential game. It follows that there is a pure-strategy Nash Equilibrium. With many workers the natural generalization of a Pareto-Efficient Nash Equilibrium (PEN) is a Coalition-Proof Nash Equilibrium (CPNE). In some cases, the algorithm presented in section B can be modified to find a CPNE. Details are available upon request.

E.2 Existence CSPE

We next show that a CSPE need not exist. Fix $\sigma = 1$, $\sigma_\theta = 1$, $c(m) = 0.01m^2$, and $K = 0.01$. Suppose signals are correlated according to the covariance matrix,

$$\Sigma = \begin{bmatrix} 1 & -0.20 & 0.15 & 0.15 & 0.15 & 0.40 \\ -0.20 & 1 & 0.15 & 0.40 & 0.40 & 0.40 \\ 0.15 & 0.15 & 1 & 0.15 & 0.40 & -0.20 \\ 0.15 & 0.40 & 0.15 & 1 & -0.20 & 0.15 \\ 0.15 & 0.40 & 0.40 & -0.20 & 1 & 0.40 \\ 0.40 & 0.40 & -0.20 & 0.15 & 0.40 & 1 \end{bmatrix}.$$

In this example, there are 20 possible teams with 3 workers. We categorize teams into seven classes based on their correlation structure. Table 12 presents all CPNE and payoffs in each class.

Table 12: Equilibrium strategies and payoff in all size-three teams.

Class	Equilibrium	Payoff
$(1,2,3)$	$(2,2,1)$	$(-0.196,-0.196,-0.156)$
$(1,2,4)$	$(2,2,0)$	$(-0.207,-0.207,-0.167)$
$(1,2,6)$	$(2,2,0)$	$(-0.207,-0.207,-0.167)$
$(1,4,6)$	$(2,2,1)$	$(-0.248,-0.248,-0.248)$
$(1,5,6)$	$(2,2,0)$	$(-0.263,-0.263,-0.233)$
$(1,3,4)$	Any permutation of $(2,2,1)$	$(-0.238,-0.238,-0.208)$
$(2,5,6)$	Any permutation of $(3,1,1)$	$(-0.304,-0.224,-0.224)$

Suppose we start with the partition $\{(1,2,3),(4,5,6)\}$. $(4,5,6)$ is in the class $(1,2,4)$ and Workers 4 and 5 would prefer to deviate and form a team with Worker 1, since $(4,5,1)$ is in the class $(1,2,3)$. Suppose we start with the partition $\{(4,5,1),(3,6,2)\}$. $(3,6,2)$ is in the class $(1,2,4)$ and Worker 3 and 6 would prefer to form a team with Worker 4, since team $(3,6,4)$ is in the class $(1,2,3)$. Finally, suppose we start with the partition $\{(3,6,4),(1,2,5)\}$. $(1,2,5)$ is in the class $(1,2,4)$ and Workers 1 and 2 would prefer to form a team with Worker 3, since team $(1,2,3)$ is in the class $(1,2,3)$. As the last deviating partition is the one we started with, none of these partitions are part of a CSPE.

We next observe that no other team in another class of three workers can be part of a CSPE since workers can always deviate to a team in the class $(1,2,3)$ and obtain a higher payoff. Further, the only two-worker teams in which workers obtain a higher payoff than in a three-worker team are those in which the pairwise correlation is -0.2. However, the only equilibrium in these teams is $(2,2)$ and workers obtain $(-0.21,-0.21)$. Hence, each worker would also like to deviate to a three-worker team in the class $(1,2,3)$. To complete the argument, we note that any single worker can be made better off by creating a team in the class $(1,2,3)$ with two other workers.

E.3 Larger Teams Can Either Enforce or Prevent Stability

Consider the example in Section 2.2. There we showed that if teams have at most two members, then the Core is empty. However, if size-three teams are allowed workers can form the grand coalition and play an equilibrium $(4,4,3)$ yielding payoffs $(-0.182,-0.182,-0.168)$. Since we assume that the cost of adding a worker is small ($K = 0.01$), there is no profitable deviating team. Hence, the grand coalition of three workers is in the Core.

The previous example shows that larger teams can enforce stability. Unfortunately, they may prevent stability as well. Suppose $\sigma = \sigma_\theta = 1$ and $c(m) = 0.04m^2$. Suppose the signals are correlated according to the network in Figure 12. If teams have at most two members, the partition $\{(1,4),(2,3)\}$ is in the Core. In both teams, the PEN $(1,1)$ is played. Workers in $(1,4)$ obtain a payoff of -0.415 and workers in $(2,3)$ obtain a payoff of -0.434.

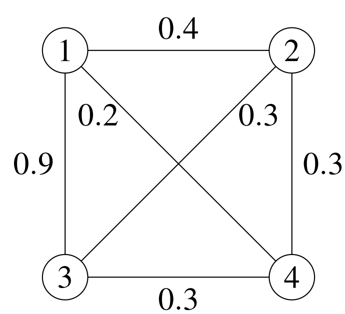

Figure 12: Correlation matrix for which allowing large teams prevents stability.

If we allow for larger teams, however, the team $(2,3,4)$ is a profitable deviation for Worker 2, 3, and 4. In this team, the unique PEN is $(1,1,1)$ and each worker obtains a payoff of -0.387. If this team is formed, however, Worker 1 is alone and obtains a payoff of -0.493. Then, Worker 1 can then form a profitable deviating team with Worker 2 and play a PEN $(2,1)$; Worker 1 obtains a payoff of -0.452 and Worker 2 obtains a payoff of -0.332 and hence it is profitable for both. If Worker 1 and Worker 2 form a team, however, Worker 3 and 4 will also form a team and obtain -0.434 each. But if such a team were to form, Worker 1 and 4 could form a profitable deviating team. As we end with the partition we began with, there is a cycle and the Core is empty.[1]

F Robustness Checks

F.1 Sequential versus Simultaneous Decision

In this section, we present a finite sequential version of the game played within each team. We assume that the total number of periods $T \geq 2\bar{M}$, where $\bar{M}$ is the upper bound on best responses described in Lemma 2. In each period, each worker chooses whether or not to take a draw $a_t^i \in \{0,1\}$. Draws across periods are conditionally independent and draws in the same period are correlated according to the pairwise correlation of teammates, ρ. In period t, all workers observe all actions a_{t-1} and signals x_{t-1} in periods $1,...t-1$; the public history at period t is given by $h^{t-1} = (a_r, x_r)_{r=1}^{t-1}$ where $a_r = (a_r^1, a_r^2)$.

Let H^{t-1} denote the set of feasible histories up to period t. Then, a strategy for Worker i is a function $s_i : \bigcup_{t=1}^{T} H^{t-1} \to \{0,1\}$. The expected payoff of Worker i given the history $(a_r, x_r)_{r=1}^{T}$ is:

$$v_i(((a_r)_{r=1}^{T})) = -\frac{1}{\left(\frac{2}{1+\rho}\sum_{r=1}^{T} a_r^1 a_r^2 + \sum_{r=1}^{T}(a_r^1 + a_r^2 - 2a_s^1 a_r^2)\right)\sigma^{-2} + \sigma_\theta^{-2}} - c\left(\sum_{r=1}^{T} a_r^i\right),$$

where we drop the signal realizations as an input in the payoff function because they do not affect payoffs. We refer to the equilibrium outcome number of draws as (n_1, n_2), where $n_i = \sum_{r=1}^{T} a_r^i$.

We consider Subgame Perfect Equilibria that are not Pareto Dominated by any other Subgame Perfect Equilibrium– call such an equilibrium a Pareto-Efficient Subgame Perfect Equilibrium (PESP). The next proposition states that, if there is a PEN in the simultaneous game such that

[1] The grand coalition is not in the Core either. In all PEN, some worker takes zero draws.

strategies differ by at most 1, there is an identical PESP outcome of the sequential game.

Proposition 6 *Let (m_1, m_2) be the most symmetric PEN in the simultaneous game. If $|m_1 - m_2| < 2$, there is a PESP of the sequential game with outcome (n_1, n_2), where $n_1 = m_1$ and $n_2 = m_2$.*

The following example shows why we cannot extend the proposition to all correlations. Suppose $\sigma = \sigma_\theta = 1$ and $c(m) = 0.05m$. If $\rho = 0.15$, the only equilibrium in the simultaneous game is $(3,0)$. However, in the sequential game this cannot be a Subgame Perfect Equilibrium. Suppose Worker 1 deviates and decides to take only one draw in each of the last two periods. Then, the best response of Worker 2 is to take a draw in period $T-1$ or period T. This gives to Worker 1 a payoff of -0.367 instead of -0.4.[2] This observation illustrates that, for intermediate correlations, inefficiency due to asymmetric equilibria may be smaller in the extensive game than in the simultaneous game.

Although our intuition suggests that all equilibria of the simultaneous game are more asymmetric than all equilibria of the sequential game, this may not be true. In the following example, there is an asymmetric equilibrium of the sequential game that is more asymmetric than the most symmetric equilibrium of the simultaneous game. Further, it is not an equilibrium of the simultaneous game. Reconsider the example in Figure 10. In the simultaneous game, for correlation $\rho = 0.1$, the profile $(3,2)$ is the most symmetric equilibrium and $(4,1)$ is not an equilibrium. However, in the sequential game, the on-path sequence $(a_r)_{r=1}^{T}$, with $a_T^2 = 1$, $a_r^1 = 1$ for $r = T-4, T-3, T-2, T-1$ and $a_r^i = 0$ in any other period, is consistent with a PESP. Notice, all draws are taken in different periods and $(4,1)$ is the outcome number of draws. A deviation by Worker 1 at period $T-4$ is not necessarily followed by an increase in the number of draws by Worker 2, since an extra draw by her implies acquiring correlated information. It can be shown that a Nash equilibrium of the Subgame following such a deviation is $(3,1)$. As $(4,1)$ is preferred by Worker 1 to $(3,1)$, Worker 1 does not have incentive to deviate at $T-4$. A similar argument follows for deviations in other periods.[3]

[2] In the unique Subgame Perfect Equilibrium, up to identity, Worker 1 takes 2 draws and Worker 2 takes 1 draw, with no draws taken in the same period.

[3] It can also be shown that $(5,0)$ is the outcome number of draws of a PESP of the sequential game.

		Worker i	
		H	L
Worker j	H	$p^2+\rho_{ij}p(1-p)$	$p(1-p)(1-\rho_{ij})$
	L	$p(1-p)(1-\rho_{ij})$	$(1-p)^2+\rho_{ij}p(1-p)$

Figure 13: Joint distribution when state is High (H).

F.2 Other Stochastic Processes

The normal information environment, which we characterize in the main section of the paper, is only one of many we could consider. In this section, we analyze another environment that has been extensively studied in the learning literature. Instead of having a continuum of possible states, we assume that there are two possible states, High (H) and Low (L).

For simplicity we assume that workers share a common prior over the state θ in which $Pr(\theta = H) = \frac{1}{2}$. Each worker can purchase a signal who's realization is either L or H and equals the true state with probability $p > \frac{1}{2}$. Figure 13 presents the probability of the four possible realizations of two correlated signals, one by Worker i and one by Worker j, when the state is H. If the state is L, probabilities are given by the matrix in Figure 13 in which elements on the main diagonal switched.

Notice that in this environment the feasible set of correlations is bounded below. In particular, it has to be that $\rho_{ij} \geq -\frac{1-p}{p}$. In the normal case, if workers have correlation -1 they learn the state perfectly after one draw. In the two-state environment, when a couple compares draws and has the most feasible negative correlation they need not learn the state; the state is revealed if HH (or LL) is observed, but not given any other realization. Further, for any correlation, there is a positive probability that HL or LH is observed.

When workers share an arbitrary number of signals, there is no simple expression to calculate the expected posterior variance. It can still be found analytically, but its calculation becomes considerably more complicated when a larger number of signals is considered. Table 13 gives the integrated variance for a number of cases. Detailed calculations are available upon request.

These values are enough to find the equilibrium of the game when each worker's best response is bounded by three. We define $\tilde{\rho}(t,p)$ and $\hat{\rho}(t,p)$ as in the main text. Figure 14 displays the value

Table 13: Expected Posterior Variance in the two-state model for some strategies.

# signals i	# signals j	Expected Posterior Variance
0	0	$\frac{1}{4}$
1	0	$p(1-p)$
1	1	$p(1-p)\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}+\frac{1}{2}(1-\rho_{ij})\right)$
2	0	$p(1-p)\left(\frac{p(1-p)}{p^2+(1-p)^2}+\frac{1}{2}\right)$
2	1	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^3+(1-p)^3+\rho_{ij}p(1-p)}+2(1-\rho_{ij})+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{(1+\rho_{ij})p(1-p)}\right)$
2	2	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))^2(1-p+\rho_{ij}p)^2}{(p^2+\rho_{ij}p(1-p))^2+((1-p)^2+\rho_{ij}p(1-p))^2}+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{2p(1-p)}\right.$ $\left.+(1-\rho_{ij})^2+\frac{4(1-\rho_{ij})(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}\right)$
3	0	$p^2(1-p)^2\left(\frac{p(1-p)}{p^3+(1-p)^3}+3\right)$
3	1	$p^2(1-p)^2\left(\frac{p(p-1)(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2(p^2+\rho_{ij}p(1-p))+(1-p)^2((1-p)^2+\rho_{ij}p(1-p))}+(1-\rho_{ij})+\frac{2p(1-p)(1-\rho_{ij})}{p^2+(1-p)^2}\right.$ $\left.+\frac{2(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p(1-p+\rho_{ij}p)+(1-p)(p+\rho_{ij}(1-p))}\right)$

of these functions when $t=2$. It shows it is still true that we have $\tilde{\rho}(2,p)>\hat{\rho}(2,p)$ if and only if the precision of the signal is high enough. We suspect a similar result is true for larger t. We can then find similar examples of inefficiency and an empty Core.

Finally, the proof of existence of a CSPE depends only on the symmetry of the equilibrium correspondence; that is, given a cross correlation ρ, if (m,m') is an equilibrium then the profile (m',m) is an equilibrium, as well. As this property holds in the two-state environment, the existence of a CSPE is still guaranteed.

F.3 Is discreteness of the action space important?

We modify our environment so that Worker i decides the precision of her signal, a non-negative number r, and pays a cost dr.[4] Then, if Worker i is alone she chooses r to maximize,

$$\frac{-1}{r+\sigma_\theta^{-2}}-dr.$$

[4]Notice that this equivalent to the worker obtaining signals with variance 1 and choosing a number of draws, r.

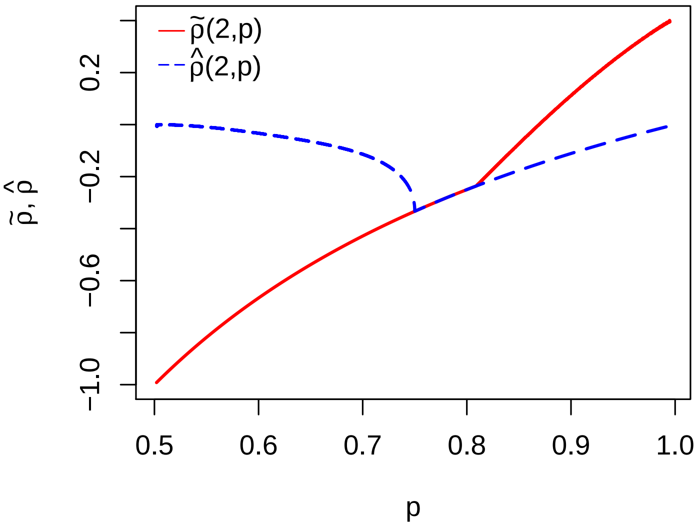

Figure 14: Values $\tilde{\rho}(2,p)$ and $\hat{\rho}(2,p)$ for different signal precisions p.

The first-order condition implies that the optimum is $r = \frac{1}{\sqrt{d}} - \sigma_\theta^{-2}$.[5] This result is consistent with our previous ones: the cheaper the information is and/or the larger the uncertainty the worker faces, the more precision she buys.

If Worker i and j are in a team together, payoffs depend on a factor $\gamma_{ij} \in [1, \infty)$ that specifies the team's productivity. The payoff of Worker i in this team is given by

$$v_i(r_i, r_j) = \frac{-1}{\underline{r}_{ij}(\gamma_{ij} - 1) + \bar{r}_{ij} + \sigma_\theta^{-2}} - dr_i,$$

where $\underline{r}_{ij} = \min\{r_i, r_j\}$ and $\bar{r}_i j = \max\{r_i, r_j\}$. Notice, if the cost function is linear, we can rewrite $\gamma_{ij} = \frac{2}{1+\rho_{ij}}$ and obtain the payoff function in our original environment.

The equilibrium correspondence is similar to the one described in the main text and characterized in the following proposition.

Proposition 7

- *If $\gamma_{ij} < 2$, the unique Nash equilibrium, up to the identity of the workers, is $(0, \frac{1}{\sqrt{d}} - \sigma_\theta^{-2})$.*
- *If $\gamma_{ij} = 2$, any strategy profile such that $r_i + r_j = \frac{1}{\sqrt{d}} - \sigma_\theta^{-2}$ is a PEN.*

[5] Assume parameter values so that the optimal r is a strictly positive number. Assume also that the cost function is linear.

- *If $\gamma_{ij} \geq 2$, the only PEN is*

$$r_i = r_j = \frac{\sqrt{\frac{\gamma_{ij}-1}{d}} - \sigma_\theta^{-2}}{\gamma_{ij}}.$$

The proposition implies that for negative correlations the only equilibrium is symmetric, for conditionally independent signals there is multiplicity, and for positive correlations the only equilibrium is fully asymmetric. We obtain similar results for convex cost functions. In these cases, if $\gamma_{ij} < 2$ there is an asymmetric PEN and for $\gamma_{ij} > 2$ there is a symmetric PEN.

G Proofs

Proof of Theorem 4

Recall that forming a team of size 2 costs $K > 0$. Hence, no team will form if its members have a pairwise correlation in P^3. If they did, the leader of the team would have a profitable deviation; she could obtain the same information alone without paying K.

The set P^1 satisfies the top coalition property (see Banerjee, Konishi and Sönmez (2001)) when teams have at most two members. Hence, if all members have pairwise correlations in P^1 there is a Core partition. As no couple with a pairwise correlation in P^3 will form a team, the result follows if all correlations are in $P^1 \cup P^3$.

Now, suppose the second condition holds. Form the team with pairwise correlation in $P^1 \setminus P^2$ that gives each member the highest payoff. By definition, no teammate wants to form a deviating team in which members have a pairwise correlation outside $P^1 \setminus P^2$. We may inductively construct teams with pairwise correlations in $P^1 \setminus P^2$ in this manner.

After we have finished creating all teams with pairwise correlations in $P^1 \setminus P^2$, set aside these workers. Consider the resulting correlation matrix in which some workers may have multiple pairwise correlations in P^2. Pick any such worker and allow her to choose a teammate from the subset of her possible partners that prefers being the leader in a team with a pairwise correlation in P^2 over being alone. Select a PEN in which the worker with multiple pairwise correlations in P^2 is the follower. As she chooses this teammate, she does not have an incentive to deviate and the worker she picks does not have an incentive to deviate either since all her other pairwise correlations are in P^3. Iterate until all workers with multiple pairwise correlations $\rho_{ij} \in P^2$ have been paired. There may be an even number of remaining workers with one correlation in P^2. Form these couples if they prefer them over being alone.

Proof Lemma 7

First, suppose $m_1^t > m_2^t$ at the end of Step 3. Then, the marginal value of $m_2 + 1$ given m_1 is less than its marginal cost. From Lemma 2, the marginal value of draw $m_2 + 1$ is even smaller when Worker 1 is taking $m_1 + 1$ draws. Hence, Worker 2 will never agree to a joint increase. Second, Step 5 can effect the outcome only if $MV(m+1; m+1, \rho) > MV(m+1; m, \rho)$ since, if not, Worker 1 should have increased her number of draws in Step 3.

Step 6 can affect the final outcome only if $f(m_1+1, m_2-1, \rho) + c(m_1+1) - c(m_1) < f(m_1, m_2, \rho)$ and $f(m_1, m_2, \rho) + c(m_2) - c(m_2-1) > f(m_1+1, m_2-1, \rho)$. These inequalities are satisfied only if

$$2f(m_1+1, m_2-1, \rho) < 2f(m_1, m_2, \rho),$$

since c satisfies increasing marginal costs. The above expression holds only if $\rho > 0$.

Proof Proposition 4

We first show that the algorithm finds a Nash equilibrium by arguing that no worker has an incentive to deviate. Partition the algorithm into three sections. In the first section, possibly with zero iterations, both workers strictly increase the number of draws they take. In the second section, Worker 1 strictly increases the number of draws she takes while Worker 2 weakly decreases them. In the third section, corresponding to Steps 5 and 6, both workers decide simultaneously whether or not to deviate.

In the first section, Worker 1 does not want to weakly decrease after both workers increase. By symmetry, Worker 2's incentive to increase her number of draws by one is the same as Worker 1's incentive to decrease her number of draws by one. Denote t' as the period in which the first section ends.

In the second section, Worker 1 never wants to decrease her number of draws when Worker 2 decreases her number of draws in the previous period. The proof of Lemma 2 implies that the benefit of a draw decreases in the number of draws of the other worker. Therefore, the marginal benefit of the last draw by Worker 1 is larger than what it was before. Denote t'' as the period in which the second section ends.

In the third section, no worker wants to deviate as deviations must, by definition, be mutually beneficial. Finally, note that during the whole procedure Worker 1 is always increasing her number of draws. By Lemma 7, we know that we cannot alternate between steps 5 and 6. Then, Lemma 2 implies that the procedure cannot go beyond iteration $\bar{M}$. Therefore, times t' and t'' are well defined and the algorithm finds a Nash equilibrium.

To see the outcome selected is a PEN, note that the only profitable deviations left to consider after t'' are those in which Worker 1 decreases and Worker 2 increases. But Worker 1 and Worker 2 will only want to deviate jointly if they can coordinate and produce more information, or if redundancy of information implies a profitable deviation to an asymmetric equilibrium. Both deviations are accounted for in Step 5 and Step 6. Uniqueness and symmetry follow from construction.

Proof of Proposition 5

For a given ρ, we consider three possible cases:

1. (Case 1) All PEN (m_1, m_2) are such that $|m_1 - m_2| \geq 2$.
2. (Case 2) There is a PEN (m_1, m_2) such that $|m_1 - m_2| \geq 2$ and a PEN (m'_1, m'_2) such that $|m'_1 - m'_2| \leq 2$
3. (Case 3) There is no PEN (m_1, m_2) such that $|m_1 - m_2| \geq 2$.

In each case, we show that the following two properties hold:

1. (Property 1) If $\rho' > \rho$ and $m_1 \in M_1(\rho)$, then there is a $m'_1 \in M_1(\rho')$ such that $m'_1 \geq m_1$. If $\rho' > \rho$ and $m_2 \in M_2(\rho)$, then there is a $m'_2 \in M_2(\rho')$ such that $m'_2 \leq m_2$.
2. (Property 2) If $\rho' > \rho$ and $m'_1 \in M_1(\rho')$, then there is a $m_1 \in M_1(\rho)$ such that $m_1 \geq m'_1$. If $\rho' > \rho$ and $m'_2 \in M_2(\rho')$, then there is a $m_2 \in M_2(\rho)$ such that $m_2 \leq m'_2$.

The following claim states that Property 1 holds in Case 1 and 2. Let $m_i(\rho)$ denote a PEN strategy for Worker i given a correlation ρ.

Claim 5 *Suppose that $(m_1(\rho), m_2(\rho))$ is a PEN with $|m_1(\rho) - m_2(\rho)| \geq 2$. Then for any $\rho' > \rho$ there is a PEN $(m_1(\rho'), m_2(\rho'))$ such that $m_1(\rho') \geq m_1(\rho)$ and $m_2(\rho') \leq m_2(\rho)$.*

Proof Let $(m_1(\rho), m_2(\rho))$ be a PEN such that $|m_1(\rho) - m_2(\rho)| \geq 2$. Without loss of generality, assume that $m_1(\rho) > m_2(\rho)$. Now, start the *Sequential Research Response Algorithm* by initializing $(m_1^0, m_2^0) = (m_1(\rho), m_2(\rho))$ and fixing $\rho' > \rho$. We only need to prove that Worker 2 does not want to deviate up, since Lemma 3 implies Worker 1 does not want to deviate down.

The fact that the equilibrium is asymmetric for correlation ρ implies that $\rho > \hat{\rho}(m_1(\rho), \gamma)$. We only need to prove that $\tilde{\rho}(m_1(\rho), m_1(\rho) - k, \gamma) < \hat{\rho}(m_1(\rho), \gamma)$; if this inequality holds, the marginal benefit of increasing from $m_1(\rho) - k$ to $m_1(\rho) - k + 1$ is larger at ρ than at ρ'.

Let $t = m_1(\rho)$. From the proof of Lemma 5 we know that $\frac{\hat{\rho}(t,\gamma)}{\partial \gamma} > 0$ and from the proof of Lemma 2 we know that $\lim_{\gamma \to 0} \hat{\rho}(2, \gamma) = \frac{-1}{3}$. Finally,

$$\frac{\partial \hat{\rho}(t,\gamma)}{\partial t} = \frac{-2+4(\gamma-1+2t)\left(\sqrt{(\gamma-1+2t)^2-4\gamma}\right)^{-1}}{2\gamma} \propto -1 + \frac{2(\gamma-1+2t)}{\sqrt{(\gamma-1+2t)^2-4\gamma}} > -1+2 > 0.$$

Then, $\hat{\rho}(t, \gamma) \geq \frac{-1}{3}$ for any $t \geq 2$ and γ.

The quadratic equation $1 - 5a + 4a^2$ is negative for $a < 1$. Plugging in $a = \frac{(t-1)(t-2)}{(t+\gamma)^2}$ gives,

$$(t+\gamma)^4 + 4((t-1)(t-2))^2 > 5(t-1)(t-2)(t+\gamma)^2 \Leftrightarrow \tilde{\rho}(t, t-2, \gamma) < -\tfrac{1}{3}.$$

Since $\tilde{\rho}(t, t-k, \gamma)$ is decreasing in k we conclude that for any $t \geq k$, $k \geq 2$ and γ, $\tilde{\rho}(t, t-k, \gamma) < \hat{\rho}(t, \gamma)$. ■

The following claim implies that Property 1 is satisfied for case 3.

Claim 6 *Suppose, for some ρ, there is a PEN (m_1, m_2) in which $|m_1 - m_2| = k$. Then, there cannot be another PEN (m_1', m_2') in which $|m_1' - m_2'| = k+1$. Hence, if for ρ there is a unique PEN (m_1, m_2) with $absm_1 - m_2 < 2$, for $\rho'' < \rho$ there must be a unique PEN (m_1'', m_2'') such that $|m_1'' - m_2''| < 2$ and $m_1'' \leq m_1$.*

Proof Without loss of generality, assume that $m_1 > m_2$. Towards contradiction, consider another PEN (m_1', m_2') in which $|m_1' - m_2'| = k+1$. Then, either $m_1' = m_1 + 1$ or $m_2' = m_2 - 1$, but not both.

Suppose $m_1' = m_1 + 1$. If both (m_1, m_2) and (m_1', m_2') are Nash equilibria, it means that Worker 1 is indifferent between both of them. But Worker 2 strictly prefers (m_1', m_2'), so (m_1, m_2) cannot be a PEN.

If instead $m_2' = m_2 - 1$ and both profiles are Nash equilibria, Worker 2 is indifferent but Worker 1 strictly prefers (m_1, m_2), so (m_1', m_2') cannot be a PEN.

At ρ, if there is a unique equilibrium in which $|m_1(\rho)-m_2(\rho)|<2$, then for any lower correlation there is a unique equilibrium in which $|m_1(\rho)-m_2(\rho)|<2$. This follows from a contradiction argument using Claim 5.

To conclude, notice that we can find the unique equilibrium by using the *Sequential Research Response Algorithm*. Since by Lemma 3 the marginal value of a draw for the leader is strictly increasing in ρ, the algorithm stops earlier for a lower correlation and hence, $m_1'' \leq m_1$. ■

The following claim implies Property 2 holds in Case 1 since, for a given ρ, (m_1,m_2) and (m_1',m_2') such that $m_1 > m_1'$ and $m_2 > m_2'$ cannot both be PEN.

Claim 7 *Suppose that for ρ the most symmetric PEN (m_1,m_2) is such that $|m_1-m_2|\geq 2$. Then for $\rho'>\rho$ the most symmetric equilibrium, (m_1',m_2'), is such that $m_1'\geq m_1$ and $m_2'\leq m_2$.*

Proof Suppose for ρ, (m_1,m_2) with $m_1-m_2\geq 2$ is the most symmetric PEN. Then, it can be found using the *Sequential Research Response Algorithm*. We only need to prove that for $\rho'>\rho$ the algorithm takes at least as many iterations.

We will show first that $\rho>0$. Take $\bar{\rho}\leq 0$. Suppose that when applying the algorithm with correlation $\bar{\rho}$, the follower stops increasing the number of draws at iteration s. First, $m_2^s = s-1$, since

$$f(s,s-2,\bar{\rho})-f(s,s-1,\bar{\rho})\geq f(s-1,s-1,\bar{\rho})-f(s,s-1,\bar{\rho}) \Leftrightarrow \tfrac{2}{1+\bar{\rho}}\geq 2.$$

where the inequality holds since $\bar{\rho}\leq 0$. This implies that if Worker 1 wants to increase to s draws, then Worker 2 wants to keep at least $s-1$ draws. Second, Worker 1 is going to stop at iteration $s+1$, since his marginal value of taking draw $s+1$ is smaller than that of Worker 2 of increasing to s draws:

$$f(s,s-1,\bar{\rho})-f(s,s,\bar{\rho})\geq f(s,s-1,\bar{\rho})-f(s+1,s-1,\bar{\rho}) \Leftrightarrow \tfrac{2}{1+\bar{\rho}}\geq 2.$$

Since by Lemma 7 the last step of the algorithm can be only applied for positive correlations, we conclude that $\rho>0$.

We know that $\tilde{\rho}(t,n,\gamma)\leq 0$ for all t, $n\leq t-1$ and γ. This means that for any step in the algorithm the marginal value of the follower is larger when facing correlation ρ than when facing correlation ρ'. Then, for each iteration s, $m_2^s(\rho)\geq m_2^s(\rho')$. But this implies, by the proof of Lemma

2, that the marginal value of a draw for the leader at each iteration is larger when facing correlation ρ'. Hence, $m_1^s(\rho) \leq m_1^s(\rho')$.

If Worker 1 does not want to continue increasing her number of draws unilaterally at the same iteration when facing correlations ρ and ρ', Lemma 7 implies that they can only increase the asymmetricity of the outcome when the correlation is ρ'. ■

Finally, the following claim implies that Property 2 is satisfied in Cases 2 and 3.

Claim 8 *Suppose for ρ the most symmetric equilibrium is such that $|m_1 - m_2| = 1$. If for $\rho' > \rho$ the most symmetric equilibrium is such that $|m_1' - m_2'| = 1$ then $m_1 \leq m_1'$ and $m_2 \geq m_2'$.*

Proof Suppose $m_1' < m_1$, so $m_2' < m_2$. Lemma 3 and the proof of Lemma 2 imply that $MV(m_1'; m_2', \rho') > MV(m_1 + 1; m_2, \rho)$, so both profiles cannot be equilibria, a contradiction. Therefore, $m_1' \geq m_1$.

If $m_1' = m_1$, by assumption it has to be that $m_2' = m_2$ and we are done. Suppose $m_1' > m_1$, so $m_2' > m_2$. If these are equilibria it has to be that $MV(m_1 + 1; m_2', \rho') > MV(m_1 + 1; m_2, \rho)$ contradicting Lemma 3 and the proof of Lemma 2. Therefore, $m_2' \leq m_2$. ■

Proof Proposition 6

After every history h^{t-1} each worker knows the posterior variance of θ, which we denote $\sigma^t(h^{t-1})$. We define three automaton states: W_N, W_{D_1}, W_{D_2}. W_N is the state at which no worker deviates, W_{D_1} is the state at which Worker 1 is the last deviator, and W_{D_2} is the state at which Worker 2 is the last deviator. Consider the strategy profile

$$s_i(h^{t-1}) = \begin{cases} 1 \text{ if } m_i(\sigma^t(h^{t-1})) \geq T - t \\ 0 \text{ otherwise} \end{cases},$$

where $m_i(\sigma^t(h^{t-1}))$ is the most symmetric equilibrium given $\sigma^t(h^{t-1})$ and $m_1(\sigma^t(h^{t-1})) \geq m_2(\sigma^t(h^{t-1}))$. Off the path of play choose any Nash equilibrium of the Subgame. Make the last worker that has deviated from the prescribed strategy profile take the largest number of draws implied by this Nash equilibrium.

To see why no worker has an incentive to deviate, notice if Worker 1 does not take a draw when she is prescribed to do so, then she can never take as many draws as she was initially prescribed. But as $|m_1 - m_2| < 2$, Worker 2 cannot compensate for Worker 1's deviation. As Worker 1 prefers to take m_1 instead of $m_1 - 1$ draws in the simultaneous game, she has no incentive to deviate. A similar argument applies for Worker 2.

Proof Proposition 7

Suppose $r_i > r_j$. Then, the marginal value of r_i for Worker i is,

$$\frac{1}{\left(r_j(\gamma_{ij}-1)+r_i+\sigma_\theta^{-2}\right)^2}$$

and the marginal of r_j for Worker j is,

$$\frac{\gamma_{ij}-1}{\left(r_j(\gamma_{ij}-1)+r_i+\sigma_\theta^{-2}\right)^2}.$$

If $\gamma_{ij} < 2$, there is a corner solution in which $r_j = 0$. Given r_j, i's best-response is $r_i = \frac{1}{\sqrt{d}} - \sigma_\theta^{-2}$.

If $\gamma_{ij} = 2$, the marginal value of a draw is the same for both workers. Optimally, each chooses r so that the marginal value equals the marginal cost. Since any investment division between the workers does not affect the marginal output, any profile (r_i, r_j) such that $r_i + r_j = \frac{1}{\sqrt{d}} - \sigma_\theta^{-2}$ is an equilibrium.

If $\gamma_{ij} > 2$, it cannot be the case that $r_i > r_j$ since the marginal benefit for Worker j is strictly larger and both workers face the same marginal cost. Hence, all equilibria are symmetric. For (r, r) to be an equilibrium, it must be the case that:

$$\left.\frac{1}{\left(r_j(\gamma_{ij}-1)+r_i+\sigma_\theta^{-2}\right)^2}\right|_{r_i=r_j} \leq d,$$

and,

$$\left.\frac{\gamma_{ij}-1}{\left(r_j(\gamma_{ij}-1)+r_i+\sigma_\theta^{-2}\right)^2}\right|_{r_i=r_j} \geq d.$$

The only PEN is the profile in which $r = r_i = r_j$ is maximized and satisfies the previous constraints. Hence, the second inequality binds. Re-arranging yields the equation stated in the Proposition.

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