Earnings and Wealth Inequality and Income Taxation:

Quantifying the Trade-Offs of Switching to a Proportional Income Tax in the U.S.

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ABSTRACT

In this paper we develop a quantitative theory of earnings and wealth inequality that accounts for the U.S. earnings and wealth distributions almost exactly, and we use this theory to measure the steady-state trade-offs that arise when switching from the current progressive income tax system, to a tax system in which income is proportionally taxed. Our theory is based on households that face an uninsurable idiosyncratic process on wages, that go through the life cycle stages of retirement and death, and that have altruistic feelings towards their progenie. Moreover, unlike some of the recent research on wealth inequality, the households in our model economies are equally patient. The main steady-state trade-offs implied by our policy experiment are the following: on the one hand we find that output, wealth and, to a very small extent, the labor input are higher in the model economy with proportional income taxes (4.4, 11.4 and 0.9% higher, respectively). On the other hand we find that in this model economy the distributions of wealth and consumption are significantly more unequal (their Gini indices increase by 10.4 and 13.0%, respectively). Finally, we find that the inequality of earnings and the earnings and wealth mobility of households remain almost unchanged.

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1 Introduction

This paper quantifies the consequences for the U.S. economy of switching from the current progressive income tax system to a proportional income tax system. This switch reduces the distortions that result from the high income tax rates that are paid by households in the top income brackets, but it also reduces the redistributive properties of the current tax system. Consequently, this policy switch implies a trade-off between efficiency and equality. The main purpose of this paper is to quantify this trade-off.

If we want our findings to be meaningful, we need a theory of income and wealth inequality capable of accounting quantitatively for the extremely high concentration of wealth found in U.S. data. In this paper we provide such a theory. Our theory is based on households who have identical preferences, face uninsurable idiosyncratic shocks, go through the life-cycle stages of retirement and death, and have altruistic feelings towards their progenie. These households also pay progressive income and estate taxes, and they receive government transfers upon retirement. Consequently, they save to substitute for insurance against the household-specific risk, to finance their consumption when they retire, and to endow their estates.

Unlike other papers that use a stochastic process on earnings to generate an endogenous distribution of wealth across otherwise identical households, our theory generates a joint distribution of earnings and wealth that matches the one observed in the U.S. almost exactly. Furthermore, it succeeds in doing so while it also replicates the mildly progressive U.S. effective income tax system.[1] Unlike Hubbard, Skinner, and Zeldes (1995), who have already pointed out the importance of government policies in shaping the asset holdings of poor households, we use a general equilibrium model capable of accounting simultaneously for the asset holdings of both the rich and the poor households. Finally, unlike Krusell and Smith (1998), who rely on shocks to household preferences to produce cross-sectional differences in patience, in our model economy the time discount factors of every household are identical and time-invariant.[2]

Our ability to match the earnings and wealth inequality facts observed in the U.S. and the main U.S. macroeconomic aggregates arises from the explicit consideration of the following features: some aspects of the life-cycle, altruism, and a social security system that increases the income of many households upon retirement.[3] The explicit modeling of the life-cycle

[1] See Aiyagari (1994), Krusell and Smith (1998), Krusell and Smith (1997), Castañeda, Díaz-Giménez, and Ríos-Rull (1998), Quadrini (1997), Huggett (1996), to name a few of these papers. Quadrini (1997) had been the most successful attempt to date in replicating the observed wealth dispersion in the U.S. See Quadrini and Ríos-Rull (1997) for a review of the previous successes and failures of models that attempt to account for wealth inequality.

[2] Sarte (1995) explores a model where all income inequality comes from differences in discount rates interacting with a progressive tax system.

[3] Gokhale, Kotlikoff, Sefton, and Weale (1998) construct an overlapping generations economy with unin-

stages of retirement and death gives the households an extra motive to save. Altruism provides a reason for households to accumulate significantly larger amounts of wealth than those that are needed to maintain a high standard of living during their life-times. The explicit modeling of transfers makes it possible for many households to view retirement as a positive earnings shock. This feature implies that many low-earnings households run down their assets before retirement, and it allows for the existence of many households with very small, or even zero, asset holdings.[4,5]

Another important feature of our model economy is that we consider the labor decision explicitly. We do so because we think that this margin is important when measuring the distortionary effects of taxation. Furthermore, in our model economies the stochastic differences in earnings arise from the different realizations of a stationary Markovian stochastic process on wages, and from the labor decision.[6]

We calibrate our model economy to match the main U.S. macroeconomic aggregates, the main features of the U.S. public sector, and the main inequality properties of the U.S. economy as measured by the Lorenz curves of labor earnings and wealth. Once this is done, we switch from the current progressive tax system to a proportional tax system that generates the same steady state level of government revenues. We compute the steady states of these two economies and we find the following: *i.)* aggregate output and aggregate consumption increase by 4.4% and 4.1%, respectively; *ii.)* the aggregate capital stock and the aggregate labor input increase by 11.4% and 0.9%, respectively; *iii.)* earnings inequality as measured by the Gini index of the earnings distribution remains virtually unchanged; *iv.)* wealth inequality as measured by the Gini index of the wealth distribution increases by 10.4%;[7] *v.)* consumption inequality as measured by the Gini index of the consumption distribution increases by 13.0%; and *vi.)* the mobility of households between the different earnings and wealth groups remains virtually unchanged.

The main reason that justifies the effects of this policy switch on earnings, wealth and consumption inequality is that proportional income taxes reduce the marginal capital and

tentional bequests that arise from incomplete annuitization and mortality risk. In their paper social security also plays an important role in generating wealth inequality, but they consider only those households whose head's age is close to retirement.

[4]Note that this effect is exactly the opposite from the one predicted by the standard life-cycle model of savings for retirement. Furthermore, in the life-cycle model the only households with very low assets are either the very young or the very old.

[5]Also note that in our model economy the only recipients of public transfers are the retired households, and that these transfers correspond quantitatively to the sum of the U.S. Social Security and Medicare transfers.

[6]By wages we mean earning opportunities per unit of time. We do not mean a specific contractual arrangement. It should be noted that the households in our model economies do not engage in activities that change the characteristics of this stochastic process. Specifically, we abstract from human capital accumulation either for the parents, or for their progenie. We acknowledge that these issues might be important to understand the distribution of earnings and wealth, but we leave them for future research.

[7]This increase is specially significant given the high starting value of this index (0.78).

labor income tax rates paid by the highest earnings and wealthiest households. This increases the willingness of the rich both to work and to accumulate assets. It turns out that the increase in asset holdings is so large that the wealth effect of the policy switch —which reduces the desire of the rich to work— offsets its substitution effect —which induces them to want to work harder. Consequently, earnings inequality does not change, but wealth inequality and, more importantly, consumption inequality increase substantially.

Other researchers have measured the impact of similar tax changes using different general equilibrium models. For example, Altig, Auerbach, Kotlikoff, Smetters, and Walliser (1997) use an overlapping generations model with multiple earnings classes.[8] In their model, a switch to a proportional income tax increases steady state consumption by 6.9%, the steady state capital stock by 7.6% and the steady state labor input by 4.8%. Their aggregate findings differ from ours mainly in the large response of the labor input to the tax change. Of course, the differences between the distributional implications of their model and ours are also very large. This is because in their model economy all changes in asset holdings are due to the life-cycle motive, and this feature reduces their model's ability to generate wealth inequality.

The rest of the paper is organized as follows: in Section 2 we describe the model economies, in Section 3 we discuss our calibration targets and choices, in Section 4 we report our calibration results and we assess our model economy as a theory of inequality, in Section 5 we describe the policy experiment and we report the trade-offs of switching from the current tax system to a proportional income tax system. Finally, in Section 6 we offer some concluding comments.

2 The model economies

The model economies analyzed in this paper are modified versions of the stochastic neoclassical growth model with uninsured idiosyncratic risk and no aggregate uncertainty.[9] The key features of our model economies are the following: *i.)* they include a large number of heterogeneous households; *ii.)* the households face an uninsured, household-specific shock to their employment opportunities; *iii.)* they also face a positive probability of dying, and when they die they are replaced by a newly-born member of their dynasties; and *iv.)* the households have altruistic feelings towards their progenie.

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[8]This model relies on forced inheritance functions to generate wealth inequality à la Fullerton and Rogers (1993).

[9]Huggett (1993) and Aiyagari (1994) analyze model worlds that are similar to ours. We extend their model economies in two ways: *i.)* we include additional dimensions of household heterogeneity, namely, age; and *ii.)* we endogenize the choice between labor and leisure.

2.1 The private sector

2.1.1 Population dynamics and information

We assume that at each point in time our model economy is inhabited by a measure one continuum of households. Each period some households are born, and some households die. We assume that the measure of the newly-born is the same as the measure of the deceased. Consequently, the total measure of households remains constant.

We assume that the households in our model economy age stochastically. In the very first period of their lives, the newly-born households are already of working-age. Each period working-age households face a constant probability of moving into retirement. Once they retire, the households face a constant probability of dying. Two reasons lead us to model the demographics in this way: it is a very parsimonious modeling of the life-cycle, and it makes it possible for our calibrated model economy to display two features that we deem important, namely, short model periods, and a relative age distribution of working-age households and retirees that matches the one observed in U.S. data.[10]

We also assume that each period, each household faces an idiosyncratic random disturbance that determines its age and its individual employment opportunities. We use the compact notation s_t to denote the household's age and its employment opportunity jointly. We assume that these disturbances are independent and identically distributed across households, and that they follow a finite state Markov chain with conditional transition probabilities given by

$$\pi(s' \mid s) = Pr\{s_{t+1} = s' \mid s_t = s\} \tag{1}$$

where $s, s' \in S = \{1, 2, \ldots, n_s\}$.[11] Finally, we assume that newly-born households draw a shock from the stationary distribution of employment opportunities of working-age households. We denote this stationary distribution by $\Gamma^*(s)$.

2.1.2 Employment opportunities

The household-specific employment and age process, s, takes values in $S \equiv \{\epsilon, \varrho\}$, where ϵ is a J-dimensional vector and ϱ is a scalar. When a household draws shock ϵ_j, we say that it is of working-age, and we assume that it is endowed with ℓ units of time and $\epsilon_j > 0$ efficiency-labor units. When a household draws shock ϱ we say that it is retired, and we assume that is is endowed with ℓ units of time and zero efficiency-labor units. We use the notation ϵ_s to denote the endowment of efficiency-labor units received by a household that draws shock s.

[10]The relevance of these features is discussed below.

[11]Note that $\pi(. \mid s)$ is the probability of being alive for one more period, which is smaller than one.

2.1.3 Preferences

We assume that households derive utility only from their consumption and leisure when they are alive, and that they order their random streams of these goods according to

$$\sum_{t=0}^{\infty}\beta^t\left\{\sum_{s_{t+1}}\pi(s_{t+1}|s_t)u(c_t,\ell-l_t,s_t)+\eta\beta\left(1-\pi(.|s_t)\right)\sum_{s_{t+1}\in\epsilon}V(s_{t+1},k_{t+1})\Gamma^*(s_{t+1})\right\} \quad (2)$$

where u is a continuous and strictly concave utility function; $0<\beta<1$ is the time-discount factor; $c_t\geq 0$ is the households' consumption, ℓ is the households' endowment of productive time, and l_t is labor. Hence, $\ell-l_t$ is time allocated by the household to non-market activities, which we call leisure. The second term in equation (2) describes the utility derived by the households from their progenie, and parameter $\eta>0$ measures the households' altruism. To avoid a cumbersome representation of the utility derived by the households from the welfare of their progenie, we use the compact notation $V(s_{t+1},k_{t+1})$ to denote the utility of a newly-born household who draws idiosyncratic shock s_{t+1}, and who inherits a stock of wealth k_{t+1}.

2.1.4 Production possibilities

We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale aggregate production function, $Y_t = f\left(K_t,L_t\right)$. We also assume that the capital stock depreciates geometrically at a constant rate δ.

2.2 The government sector

We assume that the government in our model economies taxes households' income and estates and that it uses the proceeds of taxation to make real transfers to retired households and to finance government consumption. We assume that income taxes are described by function $\tau(y)$, where y denotes income, that estate taxes are described by function $\tau_E(k)$, where k denotes wealth, and that public transfers are described by function $\omega(s)$. Therefore, in our model economies a government policy rule is a specification of $\{\tau(y),\ \tau_E(k),\ \omega(s)\}$ and the process on government consumption, G. Since we also assume that the government must balance its budget every period, these policies must satisfy the following restriction:

$$G_t+\Omega_t=T_t \quad (3)$$

where Ω_t and T_t denote aggregate transfers and aggregate tax revenues, respectively.

Note that social security in our model economy takes the form of transfers to retired households, and that these transfers do not depend on past contributions made by the households.[12]

2.3 Market arrangements

We assume that there are no insurance markets for the household-specific shock, s.[13] To buffer their streams of consumption against these shocks, the households can accumulate assets in the form of real capital. These capital asset holdings are restricted to belonging to a compact set $\mathcal{K}$. The lower bound of this set can be interpreted as a form of liquidity constraints, or, alternatively, as a solvency requirement. The existence of an upper bound is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference.[14] This condition is satisfied in all the model economies that we study.

We also assume that firms rent factors of production from households in competitive spot markets. Consequently, factor prices are given by the corresponding marginal productivities.[15]

2.3.1 Initial endowment and liquidation of assets

We assume that newly-born households are endowed with an initial stock of assets which they inherit from their parents. When their time comes to die, retired households do so at the end of the period after the current labor, consumption, investment and savings have taken place. At the beginning of the following period, the deceased households' estates are liquidated. A fraction $1-\tau_E(k)$ of their assets, if any, is inherited by the deceased households' progenie. The remaining assets are transformed into the current period composite good and are taxed away by the government.

2.4 Equilibrium

In this paper we consider only recursive, *i.e.* stationary Markov, equilibria. This equilibrium concept might exclude other types of equilibria such as those that consider arrangements

[12] We make this assumption for technical reasons. Discriminating between the households according to their past contributions to a social security system requires the inclusion of a second asset-type state variable in the household decision problem, and this increases the computational costs significantly. These costs are already very high because the iterative nature of our calibration procedure makes it necessary to solve the model economies many times over. As we will see when we report our calibration results below, this assumption turns out to be a reasonable abstraction.

[13] This is the key feature of this class of model worlds. When insurance markets are allowed to operate, our model economies collapse to a standard representative household model, as long as the right initial conditions hold.

[14] See Huggett (1993) and Aiyagari (1994) for details.

[15] In this class of model economies firms do not play any intertemporal role for two main reasons: first, they do not make profits and, second, they cannot be used by the households who own them to substitute for insurance by choosing non-profit-maximizing strategies.

that implement history-dependent allocations like those described, for instance, in Atkeson and Lucas (1992) and Atkeson and Lucas (1995). The reason for this assumption is that in this paper we are interested in the aggregate consequences of a specific set of market arrangements, but we do not attempt to account for the reasons that justify the existence of those markets. Furthermore, in this paper we consider only steady states.[16]

Each period, the economy-wide state is the measure of households which is denoted by x, and is defined over $\mathcal{B}$, an appropriate family of subsets of $\{\mathcal{K} \times S\}$.[17]

2.4.1 The households' decision problem

The households' state variable is the pair (s, k) which includes the realization of the household-specific process, s, and the beginning-of-period capital stock, k. The dynamic program solved by a household in state (s, k) is the following:

$$v(s,k) = \max_{c \geq 0, k' \in \mathcal{K}, 0 \leq l \leq \ell} u(c, \ell - l, s) + \beta \left\{ \sum_{s'} v(s', k') \pi(s'|s) + [1 - \pi(.|s)] \eta \sum_{s' \in \epsilon} V \left[s', k' \left(1 - \tau_E \left(k' \right) \right) \right] \Gamma^*(s') \right\} \quad (4)$$

s.t.:

$$c + k' = y - \tau(y) + k(1 - \delta) \quad (5)$$

$$y = kr + l(s,k) w \epsilon_s + \omega(s) \quad (6)$$

where v denotes the households' value function, and r and w denote the factor prices. Since the households' decision problem is a finite-state, discounted dynamic program, it can be shown that an optimal stationary Markov solution to this problem exists always.

2.4.2 Definition of equilibrium

A steady state equilibrium for this economy is a household policy, $\{c(s,k),\ k'(s,k),\ l(s,\ k)\}$, a pair of household value functions $v(s,k)$, and $V(s,k)$, a government policy, $\{\tau(y),\ \tau_E(k),\ \omega(s),\ G\}$, a stationary measure of households, x^*, a vector of time invariant prices, (r, w), and a vector of aggregates, (K, L, T, Ω) such that:

[16] In a recent paper Cole and Kocherlakota (1997) have studied economies with unobservable private storage, and have concluded that in that class of economies the best achievable allocation is the equilibrium allocation that obtains in an economy where households have access to credit that is not state-contingent. We interpret their findings to imply that the market structure that we use in this paper could arise endogenously from certain lack of enforcement and unobservability features of the underlying environment.

[17] Note that we do not need to keep track of household names since the decisions of households in the same individual state are always the same.

i.) Factor inputs, tax revenues and total transfers are obtained aggregating over households.

$$K = \int_{S,\mathcal{K}} k \, dx^*, \qquad L = \int_{S,\mathcal{K}} l(s,k) \, \epsilon_s \, dx^* \tag{7}$$

$$T = \int_{S,\mathcal{K}} \tau(y) \, dx^* + \int_{S,\mathcal{K}} \tau_E(k') \, k'(s,k) \, [1 - \pi(. \mid s)] dx^*, \qquad \Omega = \int_{S,\mathcal{K}} \omega(s) \; dx^* \tag{8}$$

where household income, $y(s,k)$, is defined in equation (6).

ii.) Given x^*, K, L, r and w, the household policy solves the households' decision problem described in (4), and factor prices are factor marginal productivities:

$$r = f_1(K,L) + (1-\delta) \text{ and } w = f_2(K,L) \,. \tag{9}$$

iii.) The utility of a newly-born household, $V(s,k)$, is the same as that of a working-age household, $v(s,k)$.

$$V(s,k) = v(s,k). \tag{10}$$

iv.) The goods market clears:

$$\int_{S,\mathcal{K}} [c(s,k) + k'(s,k)] \, dx^* + G \leq f(K,L) + (1-\delta)K \tag{11}$$

v.) The government budget constraint is satisfied:

$$G + \Omega = T \tag{12}$$

vi.) The measure of households is stationary

$$x^*(S_0, \mathcal{K}_0) = \int_{S_0,\mathcal{K}_0} \left\{ \int_{S,\mathcal{K}} \left[\xi_{k'=k'(s,k)} \; \pi(s'|s) + \xi_{k'=[1-\tau_E(k')]k'(s,k)} \; [1 - \pi(.|s)] \, \Gamma^*(s') \right] dx^* \right\} dk' ds' \tag{13}$$

for all $(S_0, \mathcal{K}_0) \in \mathcal{B}$, and where ξ denotes the indicator function. Appendix 1 describes the procedure that we use to compute this equilibrium.

3 Calibration

Our strategy is to calibrate our benchmark model economy to the key ratios of the U.S. national income and product accounts, to the current U.S. tax system, and to the current U.S. earnings and wealth distributions.

In this section, first we describe our calibration targets, next we discuss our choices for the functional forms that help us implement these targets, and finally we report our parameter choices. In Section 4 we report our calibration findings, and we discuss the reasons that allow us to replicate the U.S. earnings and wealth inequality almost exactly.[18]

3.1 Targets

We choose the benchmark model economy's functional forms and parameters so that its steady state statistics mimic the following calibration targets as closely as possible.

3.1.1 Model period

Time aggregation matters for the cross-sectional distribution of flow variables such as earnings. Short model periods imply high wealth to income ratios and are, therefore, computationally costly. Consequently, computational considerations lead us to prefer long model periods. The longest model period that is consistent with the data collection procedures used by the 1992 Survey of Consumer Finances is a year, and this is the period that we choose for our model economies.

3.1.2 Macroeconomic aggregates

We want our model economy's output shares to mimic the output shares of the U.S. economy. Therefore we choose as our calibration targets an investment to output ratio, I/Y, of 16%, a government expenditures to output ratio, G/Y, of 19%, and a transfers to output ratio, Tr/Y, of 9%. In the post World War II U.S. data the first two of these ratios do not seem to display any trend, but the transfers to output ratio has been growing steadily. Our chosen value for this ratio, 9%, is a little lower than the 10.6% observed in 1995, because we exclude some income-dependent transfers that are not related to elderly households. Note that our definition of transfers includes both Social Security payments and Medicare. In the U.S., the first of these two items is quite progressive and the second item is lump-sum. For reasons

[18] Note that throughout this paper our definition of earnings both for the model economy and for the U.S. data includes only before-tax labor income. It does not include either capital income or government transfers. The sources for the data and the definitions of all the distributional variables used in this paper can be found in Díaz-Gimenez, Quadrini, and Ríos-Rull (1997).

discussed in Footnote 12 above, we choose our model economies' transfers to be entirely lump-sum.[19]

For the capital income share of output we choose a value of 0.376.[20] Finally, we want to match certain characteristics of the cross-sectional distribution of consumption and hours. Specifically, we target the cross-sectional variance of consumption to be four times the cross-sectional variance of hours worked. We summarize these statistics in Table 1.

Table 1: The U.S. macroeconomic aggregates that we target

	Y	C	I	G	Tr	Capital Share	σ_c/σ_l
U.S.	1.00	0.65	0.16	0.19	0.09	0.376	4.0

3.1.3 The U.S. tax system

We want the model economy's tax system to mimic some key features of the U.S. tax system. However, in our model economy we abstract from property, consumption and excise taxes, and we do not make any distinction between capital income and labor income for tax purposes.

Income taxes: Gouveia and Strauss (1994) have characterized the 1989 U.S. effective household income tax function with the following function:

$$\tau(y) = a_0(y - (y^{-a_1} + a_2)^{-1/a_1}) \tag{14}$$

with parameter values a_0=0.258, a_1=0.768 and a_2=0.031. Figure 1 shows the average and marginal tax rates implied by this function.

Estate Taxes: We want the model economy's estate tax function to mimic the U.S. estate tax schedule that we report in Table 2.

3.1.4 The distribution of earnings

We want the Lorenz curve for the model economy's distribution of earnings to mimic some points of the Lorenz curve for the U.S. economy's distribution of earnings. We use the measurement of the Lorenz curve of earnings for the U.S. economy reported in Díaz-Gimenez et al. (1997), and we include its key points in the second column of Table 3.

[19]See Krusell and Ríos-Rull (1997) for a discussion of these issues.
[20]See Castañeda et al. (1998) for details.

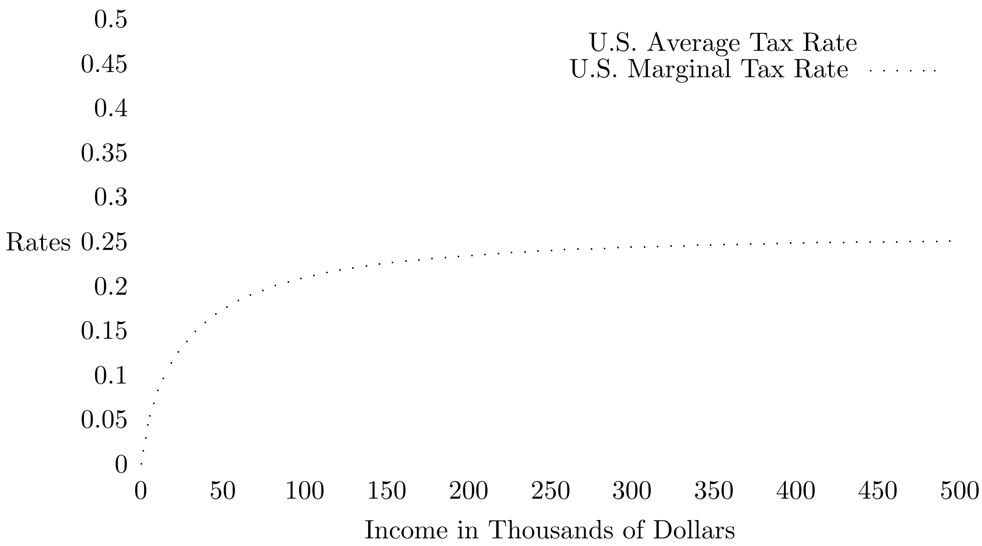

Figure 1: The 1989 U.S. average and marginal tax rate functions according to Gouveia and Strauss (1994).

Table 2: The U.S. estate tax

Capital brackets	Tax Rates
0–$600,000	0.000
More than $600,000	0.5

3.1.5 The distribution of wealth

We want the Lorenz curve for the model economy's distribution of wealth to mimic some points of the Lorenz curve for the U.S. economy's distribution of wealth. Again we use the measurement of the Lorenz curve of wealth for the U.S. economy reported in Díaz-Gimenez et al. (1997), and we include its key points in the third column of Table 3.

3.2 Functional forms

In this section we discuss our choices for the functional forms of the utility function, of the aggregate production function and of the income and estate tax functions.

3.2.1 Preferences

To characterize the household decision problem described in equation (4), we must choose a form for the utility function. We use a constant relative risk aversion utility function which

Table 3: The distributions of earnings and wealth in the U.S. economy

		Earnings	Wealth
Bottom	0–1	–0.40	–0.52
	1–5	0.00	–0.02
	5–10	0.00	0.01
Quintiles	0–20	–0.40	–0.39
	20–40	3.19	1.74
	40–60	12.49	5.72
	60–80	23.33	13.43
	80–100	61.39	79.49
Top	90–95	12.38	12.62
	95–99	16.37	23.95
	99–100	14.76	29.55
Gini Index		0.63	0.78

is separable in time and in consumption and leisure.

The utility function of private consumption for working-age households, *i.e.* for those who draw shock $s = \epsilon$, is the following:

$$u(c, l, \epsilon) = \frac{c^{1-\gamma_1}}{1-\gamma_1} + B\frac{(\ell - l)^{1-\gamma_2}}{1-\gamma_2} \tag{15}$$

This choice is relatively non-standard. We make this choice because we want our model economy to mimic the cross-sectional differences in earnings and wealth. Another reason that lead us not to choose Cobb-Douglas preferences is that the implied cross-sectional variability in hours worked would have been several orders of magnitude larger than the value observed in the U.S. For retired households, *i.e.* for those who draw shock $s = \varrho$, this utility function collapses to a constant relative risk aversion function in only consumption, since they always choose not to work and, consequently, the second term in expression (15) becomes a constant.

Finally, we assume that the households value the utility of their progenie in the same way as they value their own utility. Consequently, in expression (2) parameter $\eta = 1$.

3.2.2 Technology

In the U.S. after World War II, the real wage has increased at an approximately constant rate —at least until 1973— and factor income shares have displayed no trend. To account for these two properties, we choose the following Cobb-Douglas aggregate production function

$$f(K_t, L_t) = K_t^{\theta} L_t^{1-\theta}. \tag{16}$$

We also assume that the capital stock depreciates geometrically at a constant rate δ.

3.2.3 Tax functions

For our model economy's income tax function, we choose the income tax function obtained by Gouveia and Strauss (1994) for the U.S. economy, and reported in expression (14) above. As it stands, we cannot use this function in our model economy because the marginal tax rates are unit-dependent. To solve this problem we use the values reported by Gouveia and Strauss (1994) for a_0 and a_1 and we find the value of a_2 that equates the tax rates paid by a household that earns the mean household income both in the U.S. and in the model economies.

We want our model economy's estate tax function to mimic the estate tax schedule reported in Table 2 above. To do this, once again, we have to translate the \$600,000 limit into model units. In 1990 in the U.S. the mean household income was slightly above \$50,000. This implies that the limit on tax-free inheritances was about twelve times the mean household income. In our model economy, therefore, we set the tax-exempt stock of wealth, $\bar{k}$, to be twelve times the model economy's mean household income. The estate tax function that achieves these targets is the following:[21]

$$\tau_E(k) = \begin{cases} 1/2(k - \bar{k}) & \text{for } k > \bar{k} \\ 0 & \text{for } k < \bar{k} \end{cases} \tag{17}$$

Finally, to deal with the fact that the U.S. government also obtains tax revenues from sources other than income and estate taxes, we add constant a_3 to the marginal income tax rate function reported by Gouveia and Strauss (1994). Our choice for the model economy income tax function is therefore the following:[22]

$$\tau(y) = a_0(y - (y^{-a_1} + a_2)^{-1/a_1}) + a_3. \tag{18}$$

3.3 Parameters

The set of U.S. economy statistics that we attempt to match is very large, and so is the set of model economy parameters that we are free to choose to match those targets. The calibration of this model economy amounts to an exercise in solving a non-linear system with a large number of equations and of unknowns. Unfortunately, this is not as simple as it seems since non-linear systems are not guaranteed to have a solution.

Specifically, we solve a system of 36 equations in 36 unknowns. Our 36 equations include the equilibrium condition that sets the aggregate capital-labor ratio to be consistent with

[21] Estate taxes turn out to play an important role in our findings. An additional discussion of these taxes can be found in Section 4 below.

[22] Note that this is equivalent to assuming that the government in the model economy uses a proportional income tax to obtain all the non-income-tax revenues levied by the U.S. government.

relative factor prices and the condition that requires the government to balance its budget. The remaining 34 equations specify the steady-state values of the following 34 targets: the sizes of consumption, government expenditures, public transfers and the capital share of output and the relative standard deviation of consumption and hours, which give us 5 targets; the progressivity of the income tax function and the fact that we only consider income and estate taxes, which give us 4 targets; the marginal estate tax and the tax exempt estate tax bracket, which give us 2 targets; 9 independent points from the Lorenz curve of the distribution of earnings; 9 independent points from the Lorenz curve of the distribution of wealth;[23]; and the endowment of time, the relative coefficient of risk aversion, the average aggregate employment rate and the expected durations of both the working-life and retirement, which give us 5 additional targets.

Our 36 unknowns are the capital-labor ratio, the value of aggregate income that appears in equation (18) and the following 34 free model economy parameters: the 14 preference, technology and tax parameters reported in Table 4; the 4 independent relative productivities reported in the first row of Table 5; and the 16 independent non-zero transition probabilities reported in Table 6.[24]

3.3.1 Preferences, technology and tax function parameters

In Table 4 we report our choices for the preference, technology and tax parameters.[25]

3.3.2 The employment opportunities process

We report the normalized choices for the endowments of efficiency labor units in the first row of Table 5, and the choices for the transition probabilities of the household-specific process in Table 6. In the second row of Table 5 we report the stationary distribution of working-age households implied by this transition probability matrix.

4 A theory of inequality

In this section we report the calibration results, we discuss the reasons that allow us to replicate the U.S. earnings and wealth distributions almost exactly, and we assess our benchmark model economy as a theory of inequality.

[23]We target 9 instead of 10 points of the Lorenz curves because we lump together the bottom 1 and 5 percent of both distributions.

[24]Note that the zeros in the last row of Table 6 are part of our modeling of the life-cycle (specifically they arise because we assume that retired households never become working-age households). On the other hand, we impose the remaining six zeroes because in our initial calibration solution the equation solver chose negative values for the corresponding transition probabilities.

[25]Note that throughout the paper we follow the public finance conventions and we report the tax and government revenue ratios as a fraction of net output, which in our model economy coincides with national income.

Table 4: The calibrated preference and technology parameters of the benchmark model economy

Preferences		Technology		Government	
Parameter	*Value*	*Parameter*	*Value*	*Parameter*	*Value*
β	0.89	θ	0.376	a_0	0.258
γ_1	1.50	δ	0.092	a_1	0.768
γ_2	5.50			a_2	0.424
B	0.10			a_3	0.201
ℓ	1.70			τ_E	0.500
η	1.00			$\bar{k}$	$12\bar{y}$

Table 5: The relative productivities and the stationary sizes of the different employment opportunities types

	$s = \epsilon_1$	$s = \epsilon_2$	$s = \epsilon_3$	$s = \epsilon_4$	$s = \epsilon_5$
$\epsilon(s)$	1.0	2.92	7.30	35.05	1810.22
Γ^*	0.236	0.451	0.279	0.034	0.00022

4.1 Macroeconomic aggregates

In Table 7 we report the values of our aggregate targets for the U.S. and the benchmark model economies. We find that these values are almost identical in the two economies.

4.1.1 Taxation

As we have already discussed in Section 2.2 the tax function proposed by Gouveia and Strauss (1994) has to be normalized before it can be used in the model economy. In Figure 2 we compare the normalized model economy average income tax rates with those reported by Gouveia and Strauss (1994) for the U.S. economy. We find that the tax functions are very similar in the two economies.

4.2 The distribution of earnings

In Table 8 we report the key statistics of the distribution of earnings in the U.S. and in the benchmark model economies.

Table 6: The transition probabilities for the household-specific process

	To s' (in %)					
From s	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	r
ϵ_1	0.970	0.121E-06	0.819E-02	0.00	0.00	0.222E-01
ϵ_2	0.581E-02	0.962	0.100E-01	0.00	0.00	0.222E-01
ϵ_3	0.202E-02	0.134E-01	0.953	0.970E- 02	0.672E-07	0.222E-01
ϵ_4	0.518E-01	0.00	0.278E-01	0.897	0.103E-02	0.222E-01
ϵ_5	0.101	0.00	0.571E-01	0.335E- 05	0.820	0.222E-01
r	0.00	0.00	0.00	0.00	0.00	0.944

Table 7: The main aggregates of the U.S. and of the benchmark model economies

	Y	C	I	G	Tr
U.S.	1.00	0.65	0.16	0.19	0.09
Benchmark	1.00	0.64	0.17	0.19	0.09

We find that the distributions of earnings are very similar in both economies. The main differences between the model and the data are that the shares earned by the fifth quintile and the 80–90 percentiles are slightly higher in the model economy, and that the shares earned by the fourth quintile and the 90–95 percentiles are slightly higher in the data.

4.3 The distribution of wealth

In Table 9 we report the key statistics of the distribution of wealth in the U.S. and in the benchmark model economies. According to these statistics, the benchmark model economy accounts for the U.S. distribution of wealth almost exactly. The main differences between the model and the data are that the share of wealth owned by the fourth quintile is slightly higher in the model economy, and that the shares owned by the second and third quintiles are slightly higher in the data.

4.4 Mobility

People do not stay in the same earnings and wealth groups forever, and a convincing theory of inequality should also be able to account for at least some of the features of the observed earnings and wealth mobility. One way to measure this economic mobility is to compute the fractions of households of each quintile that move to the other quintiles. These measures of

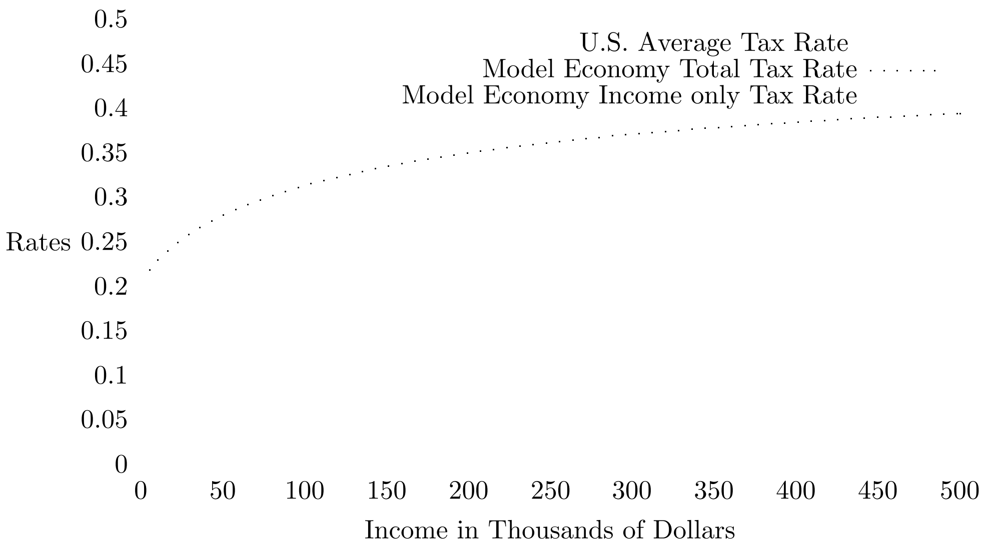

Figure 2: The U.S. and the benchmark model economies' income tax schedules (the latter with and without the addition of other government revenue)

mobility give us a total of 40 independent statistics. One way to summarize these statistics is to focus only on the diagonal elements of the resulting transition matrices. These diagonal elements measure the fraction of households of any given quintile that remain in the same quintile after a certain period of time. We call these fractions the persistence statistics. In Table 10 we report these persistence statistics for the earnings and wealth quintiles of both the U.S. and the benchmark model economies.[26]

We find that, even though the model economy mimics some qualitative aspects of the mobility of U.S. households, the earnings and wealth mobility statistics of the model economy differ significantly from those obtained from U.S. data. For example we find that every quintile of both earnings and wealth is relatively persistent, and that the first earnings quintile is more persistent than the fifth earnings quintile, both in the model and in the data. But we also find that the model economy's fifth earnings quintile is far too persistent, that every wealth quintile in the model economy is significantly more persistent than the corresponding quintile in the data, and that the relative persistence of the first and fifth wealth quintiles is reversed: in the model economy the first wealth quintile is relatively more persistent, and the opposite holds true in U.S. data.

Recall that, while the life-cycle features of our model economy are highly stylized, the PSID data used to measure earnings and wealth mobility in the U.S. economy shows a strong

[26]The U.S. persistence statistics are the same as those reported in Díaz-Gimenez et al. (1997). The source for their raw data was the Panel Study of Income Dynamics (PSID). The period considered was the five years between 1984 and 1989. To construct the quintiles they considered only the households that belonged to both the 1984 and the 1989 PSID samples.

Table 8: The distributions of earnings in the U.S. and in the benchmark model economies

		U.S.	Model
Bottom	0–1	–0.40	0.00
	1–5	0.00	0.00
	5–10	0.00	0.00
Quintiles	0–20	–0.40	0.00
	20–40	3.19	3.27
	40–60	12.49	14.35
	60–80	23.33	20.73
	80–100	61.39	61.65
Top	90–95	12.38	10.25
	95–99	16.37	16.51
	99–100	14.76	15.01
Gini Index		0.63	0.62

life-cycle component.[27] Consequently, we should not expect our benchmark model economy to match the mobility properties of the U.S. economy very closely. We conjecture that versions of our model economy that include a more detailed specification of the life-cycle will be able to mimic the U.S. mobility statistics better.

4.5 An assessment

Unlike previous models, we think that our benchmark model economy succeeds in replicating the U.S. distributions of earnings and wealth almost exactly. There are still some distributional features of the model economy that should be improved. The most important of these features are the relative shares of wealth owned by the third and the fourth quintiles, and the mobility properties.

As we have already mentioned, to improve the study of mobility issues, we think that we should develop models with a more sophisticated implementation of the age-profile of households. Those models should be able to simultaneously capture the earnings and wealth mobility that is due to the life-cycle and to other reasons, such as those represented by the shocks that are the gist of this paper. Finally, a more sophisticated specification of estate taxes could well be in order. Given their current parameterization, we assume that households cannot reduce their estate tax burden. In the U.S. we know that trusts funds and inter-vivos transfers reduce the amount that the government collects from estate taxes. Furthermore, our benchmark model economy generates a wealth to output ratio that is

[27]According to Auerbach and Kotlikoff (1987), Ríos-Rull (1996) and others, the age-earnings profile of the PSID sample displays a clear inverted-U shape.

Table 9: The distributions of wealth in the U.S. and in the benchmark model economies

		U.S.	Model
Bottom	0–1	–0.52	0.00
	1–5	–0.02	0.00
	5–10	0.01	0.00
Quintiles	0–20	–0.39	0.01
	20–40	1.74	0.36
	40–60	5.72	3.78
	60–80	13.43	16.60
	80–100	79.49	79.26
Top	90–95	12.62	11.67
	95–99	23.95	22.81
	99–100	29.55	29.54
Gini Index		0.78	0.79

slightly smaller than 2.0, while in the U.S economy this statistic is somewhere between 2.5 and 3.0. We conjecture that a more sophisticated modeling of the effective estate taxes will increase the wealth to output ratio without changing the other properties of the model economy.

5 The policy experiment

Once we have developed a convincing theory of earnings and wealth inequality, we use this theory to quantify the trade-offs of switching from the current progressive tax system, to a proportional income tax system. To this purpose, we compute the steady state of a model economy with the following properties: *i.)* its preferences, its technology and the properties of its employment opportunities process are identical to those in the benchmark model economy; *ii.)* its government expenditures, public transfers and total government revenues are identical to those in the benchmark model economy; *iii.)* its income taxes are different: in this model economy the government levies a proportional tax on income; and *iv.)* its estate tax function is also unchanged.

Once we have computed the stationary equilibrium of the model economy with progressive income taxes, computing the equilibrium for the model economy with proportional income taxes is relatively simple. It amounts to solving a system of four equations in four unknowns. The four equations are the equilibrium condition for the capital-labor ratio, the two conditions on the size of government, and the condition that determines the units for the estate tax. The four unknowns are the capital-labor ratio, the proportional tax rate,

Table 10: Mobility in the U.S. and in the benchmark model economies: fractions of households that remain in the same quintile after 5 years

	Earnings		Wealth	
	U.S.	Benchmark Model	U.S.	Benchmark Model
1st	0.86	0.88	0.67	0.96
2nd	0.41	0.68	0.47	0.93
3rd	0.47	0.64	0.45	0.82
4th	0.46	0.75	0.50	0.75
5th	0.66	0.81	0.71	0.88

aggregate wealth and aggregate output.

5.1 Aggregate trade-offs

In Table 11 we report the main macroeconomic aggregates of the benchmark and of the proportional income tax model economies. Specifically, we report the aggregates that we have used to calibrate the benchmark model economy, and we also report total capital, the labor input, total hours worked in the market, the wealth to output ratio, the fraction of government revenues over net output and the coefficients of variation of both consumption and hours worked.

We find that switching to a proportional income tax system increases the steady state level of output by 4.4%. This is due to a much higher steady-state level of capital —11.4%— and to a higher work effort that translates into a slight increase in the steady-state labor input —less than 1%. Obviously, the changes in output between the two economies do not occur immediately after the policy switch. They take place many periods after the policy change, once the transition to the new steady state has been completed.[28]

The increase in consumption after the switch to proportional income taxes is 4.1%, which is slightly smaller than that of output. The rest of the extra output goes to increase investment. This is needed because the steady-state stock of capital is significantly larger in the economy with proportional income taxes. Note that there is no change in the level of government consumption by construction, and that, therefore, the government's share of output is smaller in the economy with proportional income taxes. Also note that our assumptions about the behavior of the government imply that public expenditures and transfers do not

[28]This result assumes that the new steady state is stable, a conjecture that cannot be verified with the computational tools at our disposal.

Table 11: The steady state aggregates of the benchmark and the proportional income tax model economies

	Benchmark Model	Proportional Model	% Change
Y	1.000	1.044	4.4%
C	0.635	0.661	4.1%
I	0.166	0.185	11.4%
G	0.189	0.189	0.0%
Tr	0.087	0.087	0.0%
K	1.79	1.97	11.4%
Labor Input	1.00	1.009	0.9%
Total Hours	1.00	1.016	1.6%
K/Y	1.79	1.89	5.6%
Government Revenues/Net Output	32.80	31.63	– 3.7%
σ_c/c	2.627	3.535	34.5%
σ_l/l	0.639	0.641	0.3%

change.

The policy switch brings about a sizable increase in the stock of capital which accounts for most of the increase in steady-state output. This is the case because, as we have already mentioned, the increase in the labor input is very small —less than 1.0%. Consequently, the wealth to output ratio is higher in the proportional income tax model economy —about 6% higher. We also find that total hours worked in the market show a larger increase than the labor input. This can be the case only if the households who increase their work effort are mostly low-efficiency households.

Intuitively, it is surprising that a switch to proportional income taxes brings about a relative increase in the work effort of low-income households, but our results imply that the income effects of the switch, that induce earnings-poor households to work longer hours, seem to dominate. In the model economy, as in the data, there is a positive correlation between employment opportunities and wealth. The increase in total wealth —and the fact that it is more concentrated— results in a positive wealth effect that reduces the labor effort of the high-wage households. We find that this wealth effect turns out to be quite large. In fact, it is so large that it counteracts the substitution effect that results from the lower marginal income tax rates levied after the policy switch on the rich.

We find that the total reduction in government revenues as a fraction of net output implied by the policy switch is –3.7%. This reduction is smaller than the increase in output and it is justified by the facts that taxes are levied on Net National Product and not on

Gross National Product, and that the increase in the former is smaller than the increase in the latter due to the small change of the total labor input relative to that of aggregate capital.

Finally, we find that the policy switch brings about a very large increase in the cross-sectional coefficient of variation of household consumption —35%, and that the corresponding statistic for aggregate hours remains virtually unchanged. Once again, this is an immediate consequence of the relatively small response of the work effort to the policy switch.[29]

5.1.1 Taxation

In Figure 3 we illustrate the marginal income tax rates of the two model economies. We also illustrate the benchmark model economy's aggregate income tax revenue rate for comparison. Recall that in the benchmark model economy the mean household income is approximately \$50,000. Note that in the proportional income tax model economy total output is larger and, therefore, the average income tax rate is smaller than in the benchmark model economy. Also note that, in spite of this result, the average income tax rates levied on many low-income households increase after the policy switch.

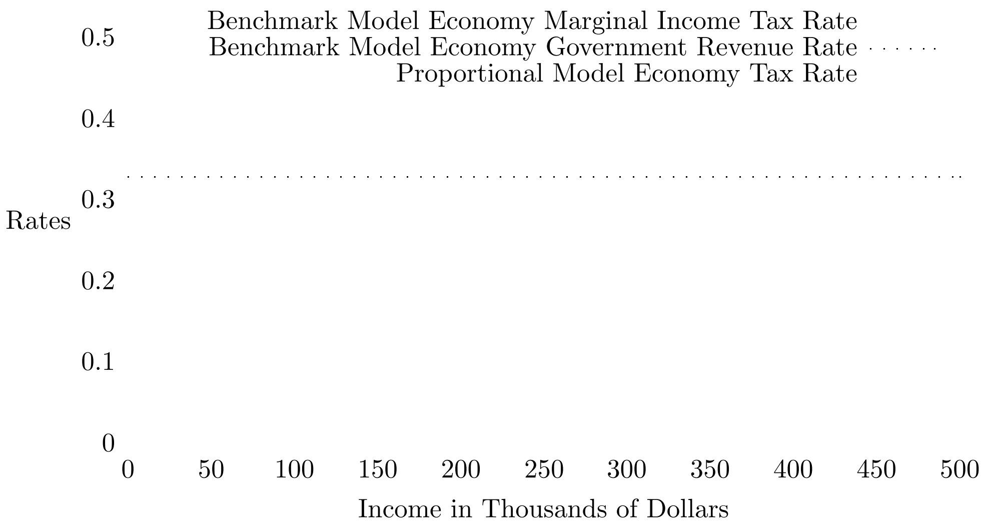

Figure 3: The marginal income tax rates in the benchmark and in the proportional income tax model economies.

[29]Recall that in the calibration of the benchmark model economy we targeted a cross-sectional variance of consumption that is four times that of hours. To attain this target, the curvature of leisure in the utility function is large relative to that of consumption. The small response of aggregate hours is partly a consequence of these choices.

5.2 Inequality trade-offs

In Table 12 we report the key statistics of the distribution of earnings in both model economies. We find that switching to a proportional income tax has virtually no effects on the

Table 12: The distributions of earnings in the benchmark and in the proportional income tax model economies.

		Benchmark Model	Proportional Model
Bottom	0–1	0.00	0.00
	1–5	0.00	0.00
	5–10	0.00	0.00
Quintiles	0–20	0.00	0.00
	20–40	3.27	3.31
	40–60	14.35	14.44
	60–80	20.73	20.87
	80–100	61.65	61.38
Top	90–95	10.25	10.24
	95–99	16.51	16.38
	99–100	15.01	14.80
Gini Index		0.615	0.613

distribution of earnings. This means that even though the aggregate work effort increases by slightly less than 1.0%, it does so in a way that does not affect earnings inequality. This finding is consistent with the very small increases in both the labor input and total hours that are brought about by the policy switch. This finding is confirmed by the fact that the increase in the cross-sectional standard deviation of hours is negligible —only 0.3%. Once again, the household responses to the policy switch are both very small and very similar along this margin. The low-wage households —who are also relatively wealth-poor— suffer a small increase in their income tax rates, and while the substitution effect of this increase makes them want to work less, its wealth effect makes them want to work more. In the case of the high-wage households —who are also wealthier— these two effects are reversed. Overall, there is a small increase in work effort that leaves earnings inequality almost unchanged.

In Table 13 we report the key statistics of the distribution of wealth in both model economies. We find that switching to a proportional income tax brings about a significant increase in wealth inequality. In the benchmark model economy the third quintile owns a non-negligible amount of wealth, and this is no longer the case in the model economy with proportional taxes. In fact, after the policy switch, the bottom 60% of the asset holders

Table 13: The distributions of wealth in the benchmark and in the proportional income tax model economies

		Benchmark Model	Proportional Model
	0–1	0.00	0.00
Bottom	1–5	0.00	0.00
	5–10	0.00	0.00
	0–20	0.01	0.00
	20–40	0.36	0.00
Quintiles	40–60	3.78	0.11
	60–80	16.60	10.44
	80–100	79.26	89.46
	90–95	11.67	11.86
Top	95–99	22.81	25.77
	99–100	29.54	39.09
Gini Index		0.791	0.874

own only about 1 per thousand of total wealth. We also find that proportional income taxes increase the share of wealth owned by only the previously wealthy. Specifically, the policy switch increases the share of wealth owned by only the top 10% of the wealth distribution. This does not mean that the remaining 90% of the population owns less assets. It means that their share of wealth is smaller. Nevertheless, it is also the case that most households do in fact experience a reduction in their steady-state asset holdings. This is because the equilibrium interest rate is smaller and because the policy switch increases the marginal income tax rate of some of the low-income households. The increase in the Gini index of the distribution of wealth is also very large —a whipping 10.5%. This increase seems even larger especially after we realize that an increase in the Gini index of 26% would have achieved maximum inequality, *i.e.*, that which obtains when one household owns all the wealth in the economy.[30]

Given this large increase in wealth inequality, and the fact that there is a positive correlation between employment opportunities and wealth, it is easy to understand why the policy switch has virtually no effects on earnings inequality. The income and wealth effects induced on the labor choice cancel each other out. High-earnings households face lower marginal income tax rates, but are also wealthier, and the overall effect of these two changes is to

[30]It should be understood that we are talking about the maximum Gini index that can be achieved with non-negative values of the variable of interest.

leave their willingness to work almost unchanged.

Our theory can be easily used to calculate many other distribution statistics. As an example, in Table 14 we report the key statistics of the distribution of consumption. As was

Table 14: The distribution of consumption in the benchmark and in the proportional income tax model economies

		Benchmark Model	Proportional Model	Proportional Model Levels
	0–1	0.23	0.22	0.23
Bottom	1–5	0.94	0.86	0.90
	5–10	1.32	1.08	1.13
	0–20	6.10	5.16	5.38
	20–40	9.41	7.60	7.93
Quintiles	40–60	13.66	12.95	13.51
	60–80	19.89	18.76	19.58
	80–100	50.94	55.54	57.96
	90–95	9.11	9.19	9.59
Top	95–99	13.45	14.82	15.46
	99–100	12.11	15.87	16.56
Gini Index		0.432	0.488	

the case with the distribution of wealth, proportional taxes bring about an increase in the share of consumption of only the top 10% of the households, and only a very small number of households experience an increase in their consumption levels. This can be seen by comparing the first and the last columns of Table 14. This last column shows the distribution of consumption of the proportional income tax model economy measured in the same units as those used for the benchmark model economy. We find that, except for a tiny increase in consumption experienced by the bottom one per cent of the distribution, only the households that belong to the top quintile —and a small proportion of those that belong to the fourth quintile— experience an actual increase in consumption. This finding has important implications. It means that a switch to proportional income taxes might actually reduce the steady-state levels of consumption of a large part of the population.

Still, we cannot, and we will not make any normative statement about the desirability of either tax system. The reasons for this are that we are comparing steady states, and that to evaluate the welfare properties of different policies we should also take into account the changes that take place along the transition paths from one steady state to the other. The computation of these transition paths is technically very demanding even in relatively simple models.[31]

5.3 Mobility trade-offs

In Table 15 we report the persistence statistics, *i.e.*, the fraction of households of each quintile that remain in the same quintile after five years. As discussed above, earnings and wealth mobility are not the main concerns of this paper. This notwithstanding, we find that the switch to a proportional income tax system has small effects on our measure of household mobility. The biggest change affects the earnings mobility of the households in the first earnings quintile —the earnings poorest. We find that the earnings mobility of this group decreases substantially after the policy switch. We believe, however, that this result is essentially meaningless. This is because in both economies the first households whose earnings are positive are close to the 20% mark, and their earnings are very small because their hours worked are tiny. Consequently, movements in and out of the first quintile are very sensitive to even very small changes in earnings. As further evidence of the fact that the change in the mobility properties is not significant, we find that the fraction of households that remain in the lowest 40% of the earnings distribution is 0.8366 for the benchmark model economy and 0.8358 for the model economy with proportional income taxes.

To summarize, we find that earnings and wealth mobilities of the model economy households are basically unaffected by the policy switch, and we insist in the fact that our theory was not primarily designed to account for household mobility and that, therefore, we consider our mobility findings to be less important than our aggregate and inequality findings.

Table 15: The fraction of households of each quintile that remain in the same quintile 5 years later in the benchmark and in the proportional income tax model economies.

	Earnings		Wealth	
	Benchmark Model	Proportional Model	Bench mark Model	Proportional Model
1st	0.88	0.70	0.96	0.98
2nd	0.66	0.64	0.93	0.93
3rd	0.64	0.64	0.82	0.83
4th	0.75	0.75	0.74	0.78
5th	0.81	0.81	0.88	0.88

[31]See, for example, Auerbach and Kotlikoff (1987) or Ríos-Rull (1994).

6 Concluding comments

In this paper we develop a quantitative theory of earnings and wealth inequality that accounts for the U.S. earnings and wealth distributions almost exactly, and we use this theory to measure the steady-state trade-offs that arise when switching from the current progressive income tax system, to a tax system in which income is proportionally taxed. On the one hand, we find that output, wealth and, to a very small extent, the aggregate work effort are higher in the model economy with proportional income taxes. On the other hand, we find that in this model economy the distributions of both wealth and consumption are significantly more unequal. We also find that this policy switch leaves earnings inequality virtually unchanged.

These findings lead us to conclude that uninsured shocks to the household earnings opportunities seem to be the essential ingredient to replicate the U.S. joint distribution of earnings and wealth. We also conclude that there are three other features that seem to be needed to model inequality successfully. These features are the following: *i.)* the prospects of retirement seem to be needed to induce the earnings-rich households to accumulate enough savings during their working-lives; *ii.)* the explicit consideration of a social security system that increases the income of many households upon retirement seems to be needed to account for the very low levels of assets held by the large number of the U.S. wealth-poor; and *iii.)* altruistic considerations towards the households' progenie seem to be needed to account for the large amount of assets accumulated by the wealthiest households. Without altruism, lifetime utility maximization would predict large dissavings upon retirement, and not enough wealth accumulation.

The shortcomings of our theory have suggested the following extensions of our research: *i.)* we should include a more detailed description of the life-cycle in order to improve our account for the earnings and wealth mobility of U.S. households; *ii.)* we should include a more detailed description of the U.S. effective estate taxes in order to increase the wealth to output ratios of our model economies; and, last but not least, *iii.)* we should move towards models that are more explicit about the ability of the households to affect their future earnings opportunities, especially through parental investments in the human capital of their children.

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Appendix

A Computation

This appendix describes the algorithm that we have used to compute the equilibrium allocations of the model economies. The outline of the algorithm is the following:

We use a standard non-linear equation solver (specifically a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package). We use this solver to search for a zero of a system of 36 equations in 36 unknowns. The equations include the steady-state equilibrium condition for the capital-labor ratio, the condition that requires the government to balance its budget, and the 34 equations that specify the steady-state values of our 34 targets. The unknowns include the guesses for the capital-labor ratio and for aggregate income, and the values of our 34 free model economy parameters.

For each specification of the model economy parameters, to compute the equilibrium we do the following

• *Step 1:* We compute the households' decision rules. We do this using a piecewise linear decision rule method. The decision rule grid is not equally spaced. Given the very large range of possible asset holdings needed to achieve the observed wealth concentration, we use a very large number of grid points. Specifically we use about 1,000 grid points per realization of the shock, which gives us a total of about 6,000 grid points. In every iteration and in every gridpoint, a non-linear equation has to be solved. Monotonicity of this equation and the assumed piecewise linearity of the decision rules simplify the computations. However, in our model economy the leisure decision is endogenous and this adds one extra level of complexity. For every evaluation of the Euler equation another non-linear equation has to be solved —the contemporaneous first order condition. Occasionally, this first order condition does not have a solution. Only when this happens, the feasibility constraint on the upper bound of leisure is binding. In this case we set the work effort to zero. See Ríos-Rull (1997) for details.

• *Step 2:* We compute the Markov process associated with the decision rules of each household-type that satisfies the necessary conditions for existence of a unique stationary distribution, x^* (see Aiyagari (1994) or Huggett (1995)). We approximate this distribution with a piecewise linearization of its associated distribution function. Again, the grid for this approximation has many points — about 12,000— and they are particularly close to each other near the origin. Again, see Ríos-Rull (1997) for details.

• *Step 3:* We compute the model economy's distributional and aggregate statistics. This step effectively involves the computation of integrals with respect to the stationary distribution, x^*. We evaluate these integrals directly using our approximation to the distribution function for every statistic except for those that measure mobility. To compute the mobility statistics we use a large sample of households drawn from x^*. Again see Ríos-Rull (1997) for details.