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Formulating Optimal Portfolios in South American Stock Markets*

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Abstract

This paper analyses the optimal investment strategy in the stock markest of a selected group of South American countries: Mexico, Brazil, Argentina, and Chile. The Markowitz efficiency frontiers are derived based on daily stock market index returns expressed in US dollars, for the period of January 1, 1988 through December 23, 1993. In addition to the Markowitz algorithm, the low partial moment algorithm is used. The benefits of internationa diversification are studied from the perspectives of an American investor who can invest both in the U.S. and in the South American stock markets. The paper assesses the risks and rewards of investing in these countries based on both foreign exchange as well as sovereign risks. It is shown that the optimal portfolio derived provides a risk-adjusted return that is better or, as good as, the return realizable from investing in stock markets with lesser degrees of risk. The optimal portfolio is calculated based on stock-market returns for the emerging South American countries mentioned, with the S&P 500 Index incorporated into the analysis. The portfolio's performance is then measured using various portfolio evaluation techniques.

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1. Introduction

International investment gradually increased during the late 1980s and the early 1990s, with the emergence of markets in newly industrialized countries in Latin America, like Mexico, Brazil, Argentina, and Chile. These infant stock markets have been profitable for investors who are willing to assume the additional risk. Yet, many market analysts have pointed out that such markets are somewhat of an abnormality, in that they tend to be characterized as markets that are thin, narrow and driven by poorly informed individuals rather than fundamentals. A prominent example that typifies this new type of market is Mexico. In December of 1994, after six years of hard-won stability, the Mexican market lost about 36 percent of its stock-index value in US dollars (and 50.6 percent in dollar terms in the year 1994) causing the Brazilian stock-index to plunge by 16.4 percent since the Mexican currency crisis started.

Despite this example and many others like it, it cannot be assumed that investing in emerging stock markets is, on the whole, more risky than investing in more developed countries, given the expected returns. What it does suggest, however, is that the international investor would do better by holding a diversified portfolio, rather than concentrate his investments in a single emerging market that is currently yielding high returns. The reasoning behind global diversification is that stock markets are not highly correlated and consequently do not move in lock step with one another. That is, a portfolio with a broad global mix yields a higher return and reduces overall risk, than, say, one concentrating in the US stock market alone, see Bailey and Stulz (1990). Global diversification has generally paid well: in a ten year period ending in October, 1994, the US portion of Morgan Stanley Capital International's world index was up by 296 percent, (with dividends reinvested but before subtracting withholding taxes), while Morgan Stanley's 20-stock-market Europe-Australia-Far-East (EAFE) Index was up fully 455 percent! Its world stock index-climbed 345 percent.

There are some that claim that, while international diversification could be a rational undertaking today, the same logic may not hold in the future[1]. Expanding links between national economies and increasing intra-regional trade, combined with the explosive growth in cross-border portfolio investment, could result in a world in which stock markets are synchronized, marching to the beat

[1] Wall Street Journal, "Going Global", December 9, 1994, page R8.

of the same drummer. Moreover, those at the helm of such funds are increasingly utilizing the same information or have access to identical sources of information. Given the proliferation of similar data bases and sources of information in the investment world, the argument goes, there will undoubtedly be a convergence in the decision-making process.

Furthermore, deregulation, including an easing of rules on foreign investment, has caused stock markets to become more accessible. Additionally, the trend for increased privatization of government-owned industries extends the list of blue-chip opportunities for investors interested in the markets of Mexico, Chile, Brazil and Argentina. These developments mean that the prices of foreign stocks are being influenced more and more by US investor sentiment. An example of this development is Telefonos de Mexico SA, Mexico's national telephone company and its largest stock in terms of market capitalization, the price of which is said to be set in New York, where the company's American Depository Receipts (ADRs) are listed, and not in Mexico City. While it is probable that some stock markets will become increasingly similar, it is not likely that different regions will be marching to the same tune at the same time. Perceptive investors will be able to both recognize and capitalize on interregional differences.

Another reason why an increase in correlation among stock markets is not likely to materialize is that countries are different in their industrial compositions and endowments. For example, while energy stocks represent merely 8 percent of the US stock market's total capitalization, they account for 43 percent of Netherlands' market and 41 percent of Norway's, but none in Germany's or Switzerland's markets. Consequently, when an investor invests in different stock markets he is effectively buying different sets of cash flows. Coffee and banana plantations are going to stay in South America; they are not going to sprout up in Germany or the United Kingdom.

The major world stock markets have been studied since the late 1960s (Grubel, 1968; Granger and Morgenstern, 1970; Levy and Sarnat, 1970; Grubel and Fadner, 1971; Agmon, 1972; Bertoneche, 1979; Hilliard, 1979) and more recently, (Schollhammer and Sands, 1985; Eun and Shim, 1989; Meric and Meric 1989; Von Furstenberg and Jeon, 1989, 1991; Hamao, Masulis and Ng, 1990; Koch and Koch, 1991; Birati and Shachmurove, 1992; Chan, Gup and Pan, 1992; Malliaris and Urrutia, 1992; Roll, 1992; and Friedman and Shachmurove, 1995). While some have studied the

Latin American economies, see Bhagwati (1993), Alonso (1994), and Gwyne (1994), this study is the first to investigate the national stock market indexes of the newly emerging countries of South America.

Determining which country takes the leading position in market size depends on how well each country's stock market and currency happens to be doing at the time, so it is not of great significance. Yet, the ordering presented is true for the period studied in this paper. Among the four markets analysed in this paper, the Mexican stock market is the largest, followed by the Brazilian, then the Chilean and finally the Argentinean, see Table 1.

The strategy of investing in South American markets assumes the absence of any regulations significant enough to affect the selection of the optimal portfolio. However, this is not always the case. For the countries studied in this paper, Mexican security markets have regulations which restrict the maximum percentage of foreign ownership. In Brazil, foreigners have to wait two years to re-sell stock in privatized companies or repatriate profits and dividends. Foreign investors are required to maintain their funds in Brazil for six years. In Argentina, the Foreign Investment Act was modified in 1989 to further equalize the treatment of foreign and domestic investors. Chilean markets are free of notable restrictions affecting foreign investors. In addition to different regulations for different investors, foreigners have differential tax treatments on both capital gains and dividend proceeds.

This paper examines the strategy of investing in South American markets, and the risk-return trade-off that is associated with such a strategy. An optimal portfolio is derived based on historic observations, and then evaluated utilizing performance measures. Section II discusses theoretical issues. Section III presents the empirical results derived. Section IV summarizes the performance of all the optimal portfolios throughout the time period of the study. Finally, section V provides a brief summary of the paper.

2. Theoretical Concepts

This section provides a brief survey of the theoretical concepts used in this paper. The theoretical concepts are optimization algorithms and portfolio evaluation techniques. Optimization algorithms are mathematical procedures that solve multiple variable problems simultaneously. The results are optimal given the information provided in the formulation of the problem. Funds are allocated into different investments in such a way that return is maximized and variability minimized. Evaluation techniques are used to assess the optimal portfolios of the S&P 500 and the South American markets by comparing them to other investment alternatives such as the S&P 500 alone, or a portfolio consisting of equally weighted initial allocations of the assets present in the optimal portfolio derived. The statistical measures used in this study are: geometric mean, variance, beta, and lower partial moment (LPM).[2]

2.1. Optimization Algorithms

Optimization algorithms consist of Strategic and Tactical Optimization, Markowitz Variance-Covariance Analysis, Lower Partial Moment Analysis, and Lower Partial Moment Algorithm.[3]

2.1.1. Strategic and Tactical Optimization of Portfolio Allocations

There are two decisions that need to be made during portfolio allocations: choosing between asset classes such as stocks, bonds, foreign currency, etc. (strategic optimization) and choosing between securities in any given asset class (tactical optimization). Most investors consistently optimize across asset classes, that is, they perform strategic optimization. Few, however, optimize within a given asset class, that is, they ignore tactical optimization. There is evidence to support the concept of tactical optimization. For example, an equity market index with optimized allocations will outperform indexes with equal or value weighted allocations, see Haugen (1990a,b).

2.1.2. Markowitz Variance-Covariance Analysis

[2] These measures are described in the Appendix which is available upon request.

[3] Other algorithms are also experimented with. These are Nawrocki (1991) Lower Partial Moment Heuristic Algorithm, Elton, Gruber, Padberg (1976) Single Index Model, and Elton, Gruber, Padberg (1976) Average Correlation Heuristic Algorithm. The results from the application of these algorithms are similar to those generated through the application of the algorithms mentioned above, and are available from the author upon request.

$$Vp = \sum_{i=1}^{k} \sum_{j=1}^{k} [X_i \times X_j \times Cov_{ij}] \tag{2.1}$$

where Vp is the portfolio variance, k is the number of assets in the portfolio, X is the share of asset i or j within the portfolio, and Cov_{ij} is the covariance between assets i and j, and is calculated by:

$$Cov_{ij} = \sigma_i \times \sigma_j \times r_{ij} \tag{2.2}$$

where σ_i is the standard deviation for asset i and r_{ij} is the correlation coefficient between assets i and j.

The expected return is determined by:

$$Ep = \sum_{i=1}^{k} [X_i \times E(R_i)] \tag{2.3}$$

where Ep is the return on the portfolio, and $E(R_i)$ is the expected return for security i.

Using the above formulas, quadratic programming is set up to maximize return and minimize variance:

$$Minz = Vp - \lambda Ep \tag{2.4}$$

$$s.t, \sum_{i=1}^{k} X_i = 1,$$

where λ is the slope of the objective function, and Ep is the expected return on the portfolio. λ can be varied from infinity to zero in order to solve for different points on the efficient frontier.[4] The result of these portfolios is that they map the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk, see Markowitz (1959).

[4] The algorithm used is the critical line algorithm. It starts with the highest return portfolio which, by definition, includes the asset with the highest return. Each asset is then evaluated using a critical value (pivot conditions) to determine which is the next asset to enter the portfolio. As assets enter into the portfolio, it becomes more diversified and will have lower risk as well as return. Each portfolio derived is called a corner portfolio. A corner portfolio is when an asset either enters or exits the portfolio. The result of these corner porfolios is that they map the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk, see Markowitz (1959).

2.1.3. Lower Partial Moment (LPM) Analysis

In Lower Partial Analysis (LPM), the variance is simply replaced with the semi-variance, which is a special case of lower partial moment with n=2.

$$LPM_{2,p} = \sum_{i=1}^{k} \sum_{j=1}^{k} [X_i \times X_j \times SD_i \times SD_j \times r_{ij}] \tag{2.5}$$

$$Minz = LPM_{2,p} - \lambda Ep \tag{2.6}$$

where $LPM_{2,p}$ is the semivariance of portfolio p, k is the number of assets, SD_i is the semideviation [square root of the semivariance] for asset i, and r_{ij} is the correlation between assets i and j, see Bawa (1975), Fishburn (1977), Nawrocki (1991).

2.1.4. Lower Partial Moment Algorithm

This algorithm computes the ratio of the Lower Partial Moment over the Co-Lower Partial Moment (LPM/CLPM) matrix given the degree of the LPM, which is determined by the degree of risk aversion.

$$LPM_{i,n} = (1/k) \times \sum_{t=1}^{k} [Max(0, (h - R_{it}))]^n \tag{2.7}$$

$$CLPM_{ij,n-1} = (1/k) \times \sum_{t=1}^{k} [Max(0, (h - R_{it}))]^{n-1} \times (h - R_{jt}) \tag{2.8}$$

where LPM_{in} is the n-degree LPM for security i, n is the degree of the LPM, k is the number of periods used, h is the target return, and R_{it} is the return for security i during period t. LPM analysis depends on the investor's risk attitude towards below-target returns. When $n < 1$, the investor is a risk-lover. When $n > 1$, the investor is risk-averse, and will favor lower LPM values, see Nawrocki (1991).

2.2. Portfolio Evaluation

Portfolio evaluation techniques consist of Terminal Wealth, Sharpe's Utility Measure, Sharpe, Treynor and Jensen Measures, Reward to Semivariance, and Stochastic Dominance.

2.2.1. Terminal Wealth

Terminal wealth is the k-th power of the geometric mean, or simply the product of the individual returns. It is the only important performance measure for long term evaluation. This is due to the fact that risk-return measures are not accurate because of the decreasing importance of liquidity risk as the investment horizon increases.

2.2.2. Sharpe's (1966) Utility Measure

$$Utility = Return - Variance/(RiskTolerance) \tag{2.9}$$

where the higher the investor's risk tolerance, the greater the proportion of his portfolio invested in risky assets, see Sharpe and Alexander (1990). Risk tolerance is defined as the amount of risk an investor is willing to assume. This amount, in turn, is determined by the nature of each particular investor. For investors who are risk-averse, lower amounts of risk will be tolerated compared to their risk-neutral and risk-loving counterparts. Risk-averse investors penalize the expected rate of return of a risky investment by a certain percentage to reflect the risk involved. Risk-neutral investors, on the other hand, look solely at the expected returns of investments, thus risk levels are not a factor for them. Finally, risk-loving investors adjust expected returns upwards when there is risk present, see Bodie, Kane and Marcus (1993).

2.2.3. Sharpe (1966), Treynor (1966) and Jensen (1968) Measures:

The Sharpe (1966), Treynor (1966), and Jensen (1968) measures are as follows:

$$Sharpe = (R_p - R_f)/\sigma_p \tag{2.10}$$

$$Treynor = (R_p - R_f)/ß_p \tag{2.11}$$

$$Jensen(a_p) = (R_p - R_f) - ß_p(R_m - R_f) - e_t \tag{2.12}$$

where R_f is the riskless rate of return, R_p is the return on the portfolio, σ_p is the standard deviation of the portfolio, and $ß_p$ is the portfolio$'sbeta$.

Both the Sharpe and the Treynor measures use reward to risk ratios. The Sharpe measure, uses standard deviation in its denominator, while the Treynor measure uses the beta value. The Jensen measure, which is based on the Capital Asset Pricing Model (CAPM), looks at investment performance by calculating the intercept a_p of the regression line: (R_p - R_f) = a_p + $ß_p$(R_m - R_f) + e_t. When the portfolio fares better than the market, a_p is greater than 0. When it under performs compared to the market, a_p is less than 0. If a_p is significantly different than zero and positive, the portfolio is considered successful. On the other hand, if a_p is less than zero, the portfolio is a failure. Consequently, the higher the value of ap, the greater the abnormal rate of return achieved by the portfolio in excess of the market, see Jensen (1968), Levy and Sarnat (1984).

2.2.4. Reward to Semivariance

$$Reward\ to\ Semivariance = (R_p - R_f)/SD_p \tag{2.13}$$

where SD_p is the semideviation of the portfolio. This ratio is preferred over alternative ones as studies have revealed that the Sharpe (1966), Treynor (1966), and Jensen (1968) measures are statistically biased. It has been shown that historically all three composite measures have exhibited systematically biased relationships with the risk measures. Various causes of the biases have been proposed. These causes are: the existence of unequal borrowing and lending rates, the failure to consider higher moments of return distributions, and the elusive "true" holding period, see Ang and Chua (1979). The shortcoming of this ratio is that it assumes a fixed utility function by setting n=2. To overcome this shortcoming, the more general reward-to-LPM ratio should be utilized as the degree (n) can be manipulated to match the investor's function, see Klemkosky (1973).

2.2.5. Stochastic Dominance

Stochastic dominance is a good evaluation technique for judging the performances of portfolios, due to the fact that it does not make any assumptions concerning the underlying probability distribution of security returns, and is based on a very general utility function. First Degree Stochastic

Dominance (FSD) places no restrictions on utility functions except that they be nondecreasing. Thus FSD acts as a preliminary screening that eliminates those options that no rational investor would choose. Second Degree Stochastic Dominance (SSD) applies only to risk-averse investors by assuming a concave utility function. All efficient sets included in SSD are also present in FSD, but not necessarily vice versa. Finally, Third Degree Stochastic Dominance (TSD) further assumes decreasing absolute risk aversion, and hence is only applicable to yet a smaller group of investors. Decreasing absolute risk aversion means the risk premium an investor is willing to pay to get rid of a given risk decreases as his wealth increases. This implies that he becomes less risk-averse at higher levels of wealth, see Saunders (1980), Levy and Sarnat (1984).

3. Empirical Results

The database constitutes observations from January 1, 1988 till December 23, 1993. In this section, to keep the exposition tractable, an optimal portfolio for the period ranging from July 9, 1993 till December 23, 1993 is used as the basis of the following discussion. Twelve additional periods are also analysed in the next section in order to extend the sample size in an attempt to minimize the possibilities of changing means and variances of the sample as pointed out by Fama (1965). The average Treasury bill interest rate, which is also the assumed risk free rate for that period, is 3.46 percent. The stock markets of the US, Argentina, Mexico, Brazil and Chile are analysed and their performance compared with the returns from the S&P 500 alone, which is the composite index of 500 US stocks, and is generally considered to accurately mirror the performance of the US stock market. All the South American stock market indices are expressed in terms of United States dollars. In this way, the analysis incorporates exchange rate risks, in order to concentrate on the benefits of international diversification, and yet take into account fluctuations of the domestic currencies versus the United States dollar. This period was chosen for discussion due to a number of reasons. One reason is that it constitutes the most recent data available at the time of writing. Another is that analysis conducted on it has resulted in the largest number of optimal portfolios for any period, and is thus more likely to reflect the optimal asset allocation. Moreover, it has one of the lowest T-bill rates recorded over the span of the study, which serves as an incentive for investors to seek higher yielding returns.

Table 2 shows that there are six possible portfolios all of which lie on the optimization frontier, based on their different characteristics. Each one is optimal due to its superiority as to its annual return, standard deviation, probability of loss, utility, R/SV ratio or periodic return. The algorithm utilized is the Markowitz critical line algorithm which computes various corner portfolios on the efficient frontier, see Markowitz (1959). Of the six optimal portfolios on the frontier, that with the highest Return/Semivariance ratio is designated as being most optimal. In this case, this is portfolio number 4. It has a R/SV ratio of 0.52 which is considerably higher than any of the five other portfolios' corresponding R/SV ratios. If one were to invest in such a portfolio, an annual return of 64.32 percent would be realized. Table 3 shows the composition of the optimal portfolio which consists of four components in the following proportions: Argentina 22.97 percent, Mexico 30.18 percent, Brazil 2.00 percent and Chile 44.85 percent. The S&P does not receive any allocation. The portfolio has a standard deviation of 0.68 percent. It has a shortfall probability, defined as the probability of realizing a return below the risk free rate, of 0.39.

Table 4 gives a brief report for individual assets. It shows that the annualized return for the S&P is 9.90 percent with a standard deviation of 0.46 percent; in Argentina the return is 71.70 percent with a standard deviation of 1.29 percent. In Mexico, the return is 115.56 percent and the deviation is 1.38 percent; in Brazil, the return is 43.37 percent annually with a deviation of 2.82 percent. Finally, in Chile, the return is 0.60 percent, with a standard deviation of 0.63 percent. Given the high return available in Mexico, it may seem surprising that Mexican market is not the most volatile of the markets. Consequently, one is led to conclude that there is a degree of risk inherent in investing in the Mexican market that is not reflected in the measured standard deviation. This risk lies in two forms: foreign exchange risk and sovereign risk. Foreign exchange risk is the risk that a foreign currency denominated return will have a decreased real value due to an adverse movement between two currencies. Sovereign risk is the risk of a foreign government intervening in the markets and undertaking actions that have a negative impact on one's investments, see Grabbe (1991). These risks are present in the Mexican market, as well as in the other South American markets to a lesser degree. Shortfall probabilities are 0.22 in Argentina, 0.31 in Mexico, 0.14 in

Brazil and 0.12 in Chile. The fact that Mexico has the highest shortfall risk, implying that Mexican market has the largest combination of foreign exchange risk, sovereign risk and market volatility, may serve to explain the magnitude of the risk premium demanded by investors in the Mexican market.

The portfolio beta is 0.32, which is significantly below the market beta of 1.0. The Sharpe measure is 0.28, the Treynor measure is 0.59 and the Jensen Alpha value is 0.002. These results are better understood when compared to the corresponding values for the market. The Sharpe measure for the S&P 500 is 0.06, the Treynor measure is 0.03 and the Jensen value is, by definition, 0. Thus, the portfolio offers significantly more reward per unit of risk, both variance and beta, than does the S&P 500 alone. Furthermore, the results are compared to those of a portfolio consisting of equally weighted initial allocations to all securities in the optimal portfolio. The optimal portfolio outperforms the equally weighted portfolio on all counts. The performance measures considered are periodic return, Sharpe measure, Treynor measure, Jensen Alpha, Beta, T-test, R-squared test, terminal wealth, utility of the portfolio and the R/SV ratio. Moreover, the portfolio realizes a higher return than predicted by the Capital Asset Pricing Model (CAPM), given its beta and the average market return. Since the Jensen measure is greater than zero, that means that the portfolio fares better than the market. These results can be seen in Table 5.

The conclusions mentioned above are summarized in Figures 1 through 10. Figure 1 describes the portfolio utility as a function of time for the optimal portfolio (EV4), the equally weighted portfolio, and the S&P 500 alone. Figure 2 deals with the Sharpe ratio, while Figures 3, 4 and 5 illustrate the Treynor measure, Jensen Alpha and R/SV ratio. Figures 6-10 show the projected returns on hypothetical investments of 100% capital allocations to just one country or the S&P 500 index. Figure 6 shows these results for the S&P, while figures 7-10 show how the investment performs in Brazil, Mexico, Argentina and Chile respectively.

Further insights may be gained by looking at additional figures that illustrate loss horizons, return horizons, target reports, wealth plots and efficient frontier representations, all of which are available from the author upon request. The loss horizons simulates the performance of the portfolio for different investment horizons. As the investment horizon increases, the probability of loss decreases accordingly. The return horizon provides the expected mean and variability of the portfolio over various time horizons. As the investment horizon gets larger, the perceived risk of the portfolio decreases. The target report provides the probability of the portfolio return being below a number of different returns, ranging from a maximum of the target return to a minimum equal to one half of the target return. The wealth plot provides a plot against the market index. There are also figures for all the portfolios that were reported to be on the optimization frontier by the optimization algorithm.

In addition to the simple variance-covariance analysis, the Lower Partial Moment algorithm is applied to the optimal portfolio. The Lower Partial Movement algorithm computes the LPM/CLPM (Lower Partial Moment/ Co. Lower Partial Moment) matrix, given the investor's degree for the LPM. Table 6 shows that applying this algorithm to the available data generates an optimal portfolio that yields an annual return of 65.94 percent, and has a R/SV ratio of 0.53. These are almost identical to the return and R/SV ratio generated by the critical line algorithm. Table 7 breaks the portfolio into its component securities; it consists of 15.87 percent Argentina, 36.08 percent Mexico, 1.08 percent Brazil and 46.97 percent Chile. It is interesting to note that, as previously seen, the allocation to the S&P is zero.

Furthermore, for a complete and thorough analysis, first, second and third degree dominance are utilized to evaluate the risk/return performance of the securities in the portfolios. The securities that are listed under third dominance provide the best risk/return performance. Under First Degree Dominance all the assets except Chile are included. The reason that Chile is not included is that it has a lower likelihood of the same level of return as the other assets, given a fixed level of risk. Under Second and Third Degree Dominance, only Mexico and the S&P 500 remain. The reason that Argentina and Brazil are not included under the Second Degree is that the cumulative probability of either Mexico or the S&P 500 each taken separately, minus the cumulative probabilities of either Argentina or Brazil, also taken separately, are always non-negative. Table 8 presents these results. It lists the assets for each degree of dominance, and displays their corresponding statistical variables. Table 9 shows the shows the exact dates and T-bill rates for the time periods that were used during this analysis. Table 10 is a summary of the essential characteristics by which the portfolios were evaluated. This is included so as to illustrate changes that are observed in these characteristics over time and how these parameters influence portfolio returns.

4. Performance Summary

While the discussion in Section III focused on analysing the performance of the optimal portfolio for a single period - July 9, 1993 till December 23, 1993, this section attempts to broaden the scope of analysis by examining the average performance of the thirteen optimal portfolios derived, the first twelve of which represent the twelve periods preceding the one discussed above.

Table 10 lists the performance characteristics for each period (s1-s13), as well as providing summary statistics such as the mean, median, minimum, maximum, and standard deviation across the thirteen periods. In addition, the optimal portfolio for each period is shown by its component allocations in the four markets, and the S&P 500. The mean allocations for all periods are given in Table 11. They are 6.11 percent Argentina, 14.12 percent Brazil, 30.91 percent Chile, 44.37 percent Mexico, and 4.48 percent S&P 500.

The mean annual return is 97.93 percent, with a maximum of 216.15 percent occurring in period 4, and a minimum of 12.36 percent in period 6. The standard deviation of the annual returns is 66.80 percent. The mean Sharpe Measure is 0.19, with a standard deviation of 0.11 percent. The maximum Sharpe Measure is realized in period 8, while the minimum occurs in period 6. Similarly, the mean Treynor Measure is 0.25, its standard deviation is 7.16 percent, the maximum occurs in period 2, and the minimum in period 4. The mean value for the Jensen Alpha is 0.002, with a standard deviation of 0.001 percent. The maximum was obtained in periods 1 and 4, while the minimum in period 6. The mean beta for the thirteen periods is 0.30, with a standard deviation of 0.22 percent.

The maximum beta measured is 0.66 in period 6, while the minimum, -0.02, was seen in period 4. The mean utility measure is 0.17, with a standard deviation of 0.13 percent. The maximum is 0.35 and the minimum is -0.04, occurring in periods 8 and 6 respectively. Finally, the mean R/SV is 0.34, with a standard deviation of 0.20 percent. The portfolio with the highest R/SV ratio was in period 8, while the lowest R/SV ratio was recorded for the optimal portfolio of period 6.

These results are better understood when compared to an appropriate benchmark, like the S&P 500. The average daily return for the time period covered in the paper, i.e. January 1, 1988 till December 23, 1993, is 0.037 percent, which translates to 9.69 percent annually, assuming that there are 250 business days in the year. This result is significantly lower than the return in the worst time period, that being time period 6, where it is 12.36 percent, with the daily return of 0.047. It is clear that on average all periods significantly outperform investments restricted to only the S&P 500. This supports the original premise that diversification holds higher rewards. The overall level of returns surpasses those from investing in any single market.

5. Conclusion

This paper analyses the daily stock market returns of four South American countries, and the strategy of investing in them for diversification purposes, for the period July 9, 1993 to December 23, 1993. An optimal portfolio is derived, and then evaluated using performance measures. The optimal portfolio, generated through the application of the Markowitz Critical Line algorithm, is one that allocates 22.97 percent in Argentina, 30.18 percent in Mexico, 2.00 percent in Brazil and 44.85 percent in Chile. It has an annualized return of 64.32 percent, a R/SV ratio of 0.52, a standard deviation of 0.68 percent and a shortfall probability of 0.39. The portfolio's beta is 0.32, significantly below the corresponding market beta of 1.0, which means that the portfolio is far less volatile than the market (as represented by the S&P 500). The Sharpe measure is 0.28, the Treynor measure is 0.59 and the Jensen Alpha is 0.002.

In addition to the Markowitz Critical Line algorithm, the Lower Partial Moment algorithm is applied to the optimal portfolio. The Lower Partial Moment algorithm yields similar results to the critical line algorithm utilized in terms of portfolio allocations.

While the portfolio chosen seems to be a good investment opportunity, one should be precise in evaluating its merits. Efficient markets do not inaccurately price securities. If an investment yields an annual return of 64.32 percent, for a risk level that appears to be below that of lower returning assets, then one of the premises is wrong. Either the return is not consistently as high as one initially believed, or one's perceived risk of the higher yielding asset is lower then it should actually be.

South American markets are a case in point. While the Reward-to-Risk ratios might be appealing, based on stock return volatilities, there are additional risk factors that need to be accounted for. Foreign investments are inherently risky. The risk lies in two forms: foreign exchange risk and sovereign risk. Foreign exchange risk is the risk that a foreign currency denominated return will have a decreased real value due to an adverse movement between two currencies. Sovereign risk is the risk of a foreign government intervening in the markets, and undertaking actions that have a negative impact on one's investments. Both these risks are present in the optimal portfolio derived above as it is based on allocation into foreign securities. Consequently, investors must demand a risk premium, in order to be compensated for the additional risk they are bearing.

Nonetheless, it is important to stress to the international investor the importance of holding a well diversified portfolio, rather than concentrating his investments in a single market. Stock markets are not highly correlated, and thus do not move lock step with one another. Consequently, by holding a portfolio consisting of allocations in several markets, the investor is able to diversify away the risk of an adverse movement in any given market having a substantial effect on his portfolio return.

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