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WORD-OF-MOUTH COMMUNICATION AND COMMUNITY ENFORCEMENT

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Abstract

We study community enforcement in a private information, random matching setting, where buyers privately "network" for information and sellers have a short term incentive to supply low quality. We also show that high quality can be sold in a sequential equilibrium with population M even when each buyer periodically interacts with only $N^*(M)$ players where $0 < \lim_{M\to\infty} N^{*2}/M < \infty$. We show that when networking is costly and M is large, low quality is supplied with positive probability in any Nash equilibrium. For this case, we characterize conditions for a sequential equilibrium in which both high and low quality are supplied.

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1. INTRODUCTION

It has long been recognized that community enforcement can make sellers behave cooperatively even when they meet particular buyers only infrequently and have a short term incentive to cheat, e.g., to supply low quality or to shirk in a labor contract. For instance, Klein and Leffler (1981) study the problem of credibly committing to offer high quality in a model where a continuum of buyers are randomly matched with several sellers and each seller has a short term incentive to supply low quality at a lower cost. In their model community enforcement by the buyers, through a coordinated boycott after observing low quality, provides incentives for the seller to produce high quality. The results of Klein and Leffler (1981), along with most of the existing literature on community enforcement, depend upon the assumption that past quality choices of the seller are public information.

When the number of sellers and buyers is large and particular sellers and buyers meet only infrequently or only once, the assumption of public information seems rather demanding. Recently this observation has led to a number of articles looking at community enforcement with less stringent informational assumptions. These papers include Milgrom et. al., (1990), Okuno-Fujiwara and Postlewaite (1995), Kandori (1992) and Ellison (1994). However, partially due to the difficulties of dealing with private information, all of the above mentioned papers have made extreme informational assumptions: either complete anonymity of players together with the assumption that players observe only the actions chosen in their own games, or alternatively, locally complete information, which allows a player to perfectly observe the status of his current opponent, based on the opponents past behavior. See also Greif (1993), Harrington (1995), Greif (1994) and Greif et. al., (1994).

Kandori (1992) and Ellison (1994) study a repeated prisoners' dilemma in a large but finite-population random-matching setting, where players are unable to recognize their opponents. They show that even then there exist sequential equilibria where all players play cooperatively in every period. Cooperation is supported by community enforcement based on contagious strategies: all players who are cheated immediately start cheating their opponents, understanding that the whole society is in a process of switching into non-cooperative actions. In the equilibrium, players behave cooperatively to avoid initiating a general switch to non-cooperative actions. An important factor underlying Kandori's and Ellison's results is that defection is a dominant strategy of the stage game. Whether contagious strategies would work in a repeated random matching game that does not share this property, such as the buyer seller game we are about to study, is still unknown. In addition, at least under their informational assumptions, the cooperative equilibria based on contagious strategies are unstable in the sense that a single "insane" player who does not cooperate can destroy the good equilibrium for all agents (Ellison, 1994).

Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992) consider games with local information processing: 1) Each player carries a label observable to her opponent, 2) when two players are matched they observe each other's label before choosing their actions, 3) a player and his partner's actions and labels today determine their labels tomorrow. The information processing is "local" in the sense that the actions chosen by a pair of players are based only on their labels, not on the entire distribution of labels across the population, and the updating of each player's label depends only on the previous labels and the outcome of the stage game. When the population is large and players are randomly matched, the observability of the current trading partner's label and the updating of the labels require the existence of some efficient information transmission and processing mechanism. This could be a medieval law judge (Milgrom et. al., 1990) or institutions like credit bureaus which track the transactions of every agent.

In many real life situations, however, social norms and informal information transmission mechanisms can replace formal institutions and still facilitate cooperation. In this paper, we present a model of community enforcement that is based on word-of-mouth communication. The information transmission is highly imperfect in the sense that information about each defection spreads only to part of the player population and defectors can not always be immediately punished. Despite this, since players can be identified, *private* reputations evolve. This allows equilibria where only defectors are punished; making our equilibria more stable with respect to "insane" players, who do not cooperate, than those based on contagious strategies. In fact, when information is privately costly, only some of the sellers can cooperate in any equilibrium. Nonetheless, word-of-mouth communication is shown to be surprisingly efficient in facilitating cooperation.

We assume there are M sellers and M buyers, where M is large but finite number, and that in each time period $t = 0, 1, 2, ...$, the sellers and buyers are randomly matched to play a stage game that contains an opportunity for a mutually beneficial trade. The sellers have a short-term incentive to supply low quality and will supply high quality only if the gains from maintaining good reputation outweigh the short-term loss. For simplicity, it is assumed that buyers would not knowingly purchase low quality at any price. We assume that buyers have networks of communication that, roughly speaking, work in the following manner: In each period each buyer observes N trades in addition to his own and N buyers, called spectators, observe his trade and send him signals regarding his current trading partner. We can think of these N spectators as friends of the buyer or just people who happen to pass by. We assume that the identities of the spectators can change from period to period.

Throughout the paper we consider two kinds of strategy profiles, which differ in the informativeness of the signals. The strategy profile with the less informative (actually totally uninformative) signals is equivalent to a model where signalling is not allowed. For these two strategy profiles we provide sufficient conditions on N and the discount factor for a sequential equilibrium where good quality is supplied by all sellers in every period. Assuming the existence of a public randomization device and high enough discount factor, these conditions can be stated as $N \geq N^*$ where N^* is a constant determined by the population size, discount factor and the payoff matrix. As one of our main results, we show that with informative signalling N^* is a diminishing fraction of the population size.

We then study a model where "networking" (i.e., setting up N connections) is costly. In this case, when M is large, we show that there must be a positive probability of sellers producing low quality goods in any equilibrium in order to give buyers an incentive to network. When the costs of networking are below a threshold value, we find a sequential equilibrium in which sellers initially randomize between high and low quality and continue to produce high quality if and only if they produced high quality in the first period. In this equilibrium the probability of buying low quality goods increases in M and the cost of networking. When the cost of networking reaches the threshold value, trade collapses because the probability of low quality goods that would provide agents with sufficient incentive to network is so large that each buyer no longer wishes to experiment with an unknown seller.

In the existing literature on quality provision, it is assumed that agents instantaneously learn about a seller's defection, see e.g. Klein and Leffler (1981) and Allen (1984). In this paper we show that word-of-mouth communication can spread information rather quickly and make such assumptions reasonable approximations in some settings. When the population is large and information privately costly, our results suggest, however, that both high and low quality would be produced in equilibrium. To further understand the role of institutions in transmitting information, as in Milgrom et. al. (1990), we feel it is useful to understand the workings of informal channels of information transmission. We hope that our formalization of word-of-mouth communication has interest on its own.

The rest of the paper is organized as follows: Section 2 gives the formal description of the model. Section 3 discusses players' strategies and presents sufficient conditions for sequential equilibria with informative and uninformative signalling, where high quality is produced by every seller in every period. Section 4 shows that with informative signalling as M goes to infinity, trade can be sustained with buyers networking with a diminishing proportion of other buyers. Section 5 studies costly networking and section 6 concludes the paper.

2. THE MODEL

There are two finite sets of players $M_k = \{1, 2, ..., M\}$, $k = S$, B. Denote by M_S the set of sellers and by M_B the set of buyers. We envisage the sellers as being positioned at fixed locations around a circle, where the locations are numbered clockwise from 1 to M. We refer to seller i as the seller at location i. It is assumed the buyers can identify the sellers by the number of their location, but the sellers can not recognize the identities of the buyers.[1]

Let Θ be the set of all permutations of M_B. In each period $t = 0, 1, 2, ...$, a permutation $\theta_t \in \Theta$ is chosen with uniform probability, independent of previous realizations. Buyer $\theta_t(i) \in M_B$ is placed at location i to play with seller $i \in M_S$ the following simultaneous move "trade" game:

$$\begin{array}{llcc} & & \multicolumn{2}{c}{Buyer\ \theta_t(i)} \\ & & B & NB \\ Seller\ i & H & 1,1 & 0,0 \\ & L & 1+g,-\ell & 0,0 \end{array},$$

where both g and ℓ are taken to be strictly positive numbers. The first (second) number in each entry indicates the seller's (buyer's) payoff. Seller's actions H and L refer to providing "*high-quality*" and "*low-quality*" while buyer's action B refers to "*buy*" and NB to "*not buy*". With g and ℓ strictly positive, L is the dominant strategy for the seller and (L, NB) is the only Nash equilibrium of the trade game. The sellers and buyers have a common discount factor $\delta \in (0, 1)$ and their overall payoffs are the discounted sum of payoffs from the trade games.

[1] The more general assumption that sellers can also recognize the identities of buyers does not change our results. The two strategy profiles that we consider would be equilibria of such a game under conditions slightly different from ours.

In each period $t = 0, 1, 2, ...$, there is preplay communication among neighboring buyers before the trade games. To be precise, the stage game proceeds as follows:

1. After θ_t is realized, buyer j observes θ_t, recognizes the identity of his opponent, $\theta_t^{-1}(j)$, as well as the identities of his "neighboring" sellers, $\theta_t^{-1}(j) + 1, \theta_t^{-1}(j) + 2, ..., \theta_t^{-1}(j) + N$, where $N \leq M/2 - 1$.[2,3,4] Let us denote by $S_j(\theta_t) = \{\theta_t^{-1}(j) + k\}_{1 \leq k \leq N}$ the subset of neighboring sellers, whom buyer j observes at period t, and by $N_j(\theta_t) = \{\theta_t(\theta_t^{-1}(j) + k)\}_{1 \leq k \leq N}$ their period t matches. We call $N_j(\theta_t)$ buyer j's neighboring buyers at period t. Also, let us denote by $N_j^s(\theta_t) = \{\theta_t(\theta_t^{-1}(j) - k)\}_{1 \leq k \leq N}$ the subset of buyers, who observe the interaction between j and $\theta_t^{-1}(j)$ at period t. We call $N_j^s(\theta_t)$ the spectators to buyer j's game at period t. Notice that the identities of the spectators, neighboring sellers and buyers depend on θ_t.

2. Buyer j sends a payoff-irrelevant signal to each of his neighboring buyers $n \in N_j(\theta_t)$ and receives a message from each spectator in $N_j^s(\theta_t)$. Let us introduce the following notation.

   - $C = \{\gamma, \beta\}$: the set of possible signals. We can interpret a signal γ or β as meaning respectively "*Good*" or "*Bad*".
   - $m_t^j(\ell) \in \{\gamma, \beta\}$: the signal from buyer j to buyer $\ell \in N_j(\theta_t)$.
   - $m_t^j \in \{\gamma, \beta\}^N$: the N-tuple of the signals from j to each of his neighboring buyers in $N_j(\theta_t)$.
   - $m_t(j) \in \{\gamma, \beta\}^N$: the N-tuple of the signals from j's spectators, $N_j^s(\theta_t)$, to buyer j.

3. Seller $\theta_t^{-1}(j)$ and buyer j play the 2×2 simultaneous move trade game described above. Denote the outcome (or the realized action profile) of that game by $(a_t^S(\theta_t^{-1}(j)), a_t^B(j))$, where $a_t^S(\theta_t^{-1}(j)) \in A_S = \{H, L\}$, $a_t^B(j) \in A_B = \{B, NB\}$.

[2] The assumption that buyers observe θ_t is made to ease the notation. An alternative assumption that does not change our results would be that buyer j is able to recognize only the identities of his opponent $\theta_t^{-1}(j)$ as well as his neighboring sellers in period t.

We also make the assumption that $N \leq M/2 - 1$. The extension of our analysis to $M/2 \leq N \leq M - 1$ is trivial. The case $N = M - 1$ would then correspond to the game with perfect observability, while the case $N = 0$ to the game where each buyer observes only the outcome of his trade game and the identity of his opponent.

[3] Sellers are female and buyers are male.

[4] These locations are of Mod M.

4. In addition to his own outcome, buyer j observes the realized action profiles of the period t trade games played by the sellers $i \in S_j(\theta_t)$ and buyers $n \in N_j(\theta_t)$. Denote this observation by $o_t(j) = \Big((a_t^S(i), a_t^B(\theta_t(i)))\Big)_{i \in S_j(\theta_t) \cup \{\theta_t^{-1}(j)\}} \in (A_S \times A_B)^{N+1}$.

The information that buyer j receives in period t can now be written as $(\theta_t, m_t(j), o_t(j))$. We denote with $H^t(j)$ the set of all possible histories for buyer j up to but not including period t. By convention, let $H^0(j) = \emptyset$. An element $h_t(j) \in H^t(j)$ includes all past realizations of θ_s, all past messages to player j, $m_s(j)$, all past messages from player j, m_s^j, and all past observations of player j, $o_s(j)$, where $0 \leq s < t$. Hence $h_t(j)$ is:

$$h_t(j) = (\theta_\tau, m_\tau(j), m_\tau^j, o_\tau(j))_{\tau=0}^{t-1}.$$

A pure strategy for buyer j is then a sequence $\{\widehat{m}_t^j, \widehat{b}_t^j\}_{t=0}^{\infty}$, where

$$\widehat{m}_t^j : \Theta \times H^t(j) \to \{\gamma, \beta\}^N$$
$$\widehat{b}_t^j : \Theta \times H^t(j) \times \{\gamma, \beta\}^N \to \{B, NB\}.$$

$\widehat{m}_t^j(\theta_t, h^t(j))$ specifies the N-tuple of signals that buyer j with private history $h^t(j)$ sends to his neighboring buyers $n \in N_j(\theta_t)$ in period t. $\widehat{b}_t^j(\theta_t, h^t(j), m_t(j))$ specifies the choice of action for buyer j in the period t trade game against seller $\theta_t^{-1}(j)$, when j has private history $h^t(j)$ and he receives signals $m_t(j)$ in the period t communication stage. Correspondingly, let $\{\widehat{\mu}_t^j, \widehat{\beta}_t^j\}_{t=0}^{\infty}$ denote a behavioral strategy for buyer j, where

$$\widehat{\mu}_t^j : \Theta \times H^t(j) \to \Delta\{\gamma, \beta\}^N$$
$$\widehat{\beta}_t^j : \Theta \times H^t(j) \times \{\gamma, \beta\}^N \to \Delta\{B, NB\}.$$

For seller i we define pure and behavioral strategies as sequences of maps $\{\widehat{s}_t^i\}_{t=0}^{\infty}$ and $\{\widehat{\sigma}_t^i\}_{t=0}^{\infty}$, where

$$\widehat{s}_t^i : (\{H, L\} \times \{B, NB\})^t \to \{H, L\}.$$
$$\widehat{\sigma}_t^i : (\{H, L\} \times \{B, NB\})^t \to \Delta\{H, L\}.$$

Note that the assumption that a seller does not recognize the identity of a buyer is implicit in this notation.

Because of the private histories that players have, the equilibrium concept that we apply is sequential equilibrium. Sequential equilibrium requires that after any history player's equilibrium strategy maximize his (her) expected payoff, taking as given all other player's strategies and his beliefs about the signals and actions taken by all other players in all previous periods. Furthermore, his beliefs should be "consistent" with the equilibrium strategy profile and private history, in the sense of Kreps and Wilson (1982).[5] A trivial sequential equilibrium is one where sellers play L and buyers NB after any history: the repetition of the *only* Nash equilibrium of the trade game. We are interested, however, in sequential equilibria that support the efficient outcome where (H, B) is played by all players in every period.

For the most part we confine our analysis on a particular class of strategy profiles, which we call "unforgiving". These strategy profiles require sellers to sell high quality in period zero (in section 5 with some probability), and sell high quality thereafter if and only if 1) they have always done so, and 2) buyers have always purchased their goods. Under the unforgiving strategy profile buyers play B, except to punish a seller by playing NB when they are informed of her defection. The strategy profiles are unforgiving in the sense that informed buyers punish a defector whenever they meet her.

There are two reasons for focusing on these strategy profiles. First, they are simple: In fact, because of the private information that players have, it is difficult to imagine other strategies that could support the efficient outcome as a sequential equilibrium in this game. For instance, it is not obvious whether contagious strategy profiles, where a seller's defection affects how buyers treat other sellers, would be equilibria in this game. Checking the incentives of a buyer to follow such a strategy on off-the-equilibrium paths is very complicated, because his incentives depend on his belief about the previous plays, which in turn depend on his private history.[6] Strategies with less severe, finite punishments are also difficult to implement because buyers typically do not know the time of the first defection and therefore cannot synchronize the last period of a punishment phase. Unforgiving strategies avoid these problems and the buyers incentives are easily shown to be satisfied. The second reason for focusing on these strategy profiles is that, in the class of non-contagious strategy profiles (i.e., where one sellers action does not affect how the other sellers are treated), these strategy profiles provide the maximum punishment for the seller. This is important because the conditions for the efficient outcome that we derive then characterize the minimum

[5] In Kreps and Wilson (1982), the definition of sequential equilibrium requires the specification of beliefs system as well as a strategy profile. Because the beliefs system which is consistent with our strategy profiles is simple, we refer only to the strategy profile when describing a sequential equilibrium.

[6] See Kandori (1992) for a discussion on the difficulties of private information.

N that is necessary for the efficient outcome in any sequential equilibrium based on non-contagious strategy profiles.

3. EXOGENOUS CONNECTIONS AND TRADE

In this section we provide sufficient conditions in terms of N for a sequential equilibrium where (H, B) is played by all players in every period. In our model information about sellers' behavior may spread through two possible sources, the effectiveness of which depends on the number of spectators, N. First, by observing the outcomes of $N + 1$ trade games in each period, a buyer receives information about $N + 1$ sellers: he observes their current actions and may infer knowledge of their past defections from the actions of their opponents. Second, the information can be transmitted through direct communication among neighboring buyers. The effectiveness of direct communication depends, however, on the information content of the signals.

We now introduce two unforgiving strategy profiles that differ with respect to the informativeness of the buyer's signals. For obvious reasons we refer to the first as the *Uninformative Strategy Profile* and to the second as the *Informative Strategy Profile*. In all periods $t = 0, 1, 2....$, after θ_t is realized:

The strategy for seller i is same under both strategy profiles and is:

(I) In the first period play H. After that, if the outcome in seller i's past games was always (H, B), play H.

(II) Play L otherwise.

The *Uninformative Strategy* for buyer j is:

(III) Signal randomly γ or β with equal probabilities to all $n \in N_j(\theta_t)$, irrespective of the private history $h_t(j)$.

(IV) If j has ever observed $\theta_t^{-1}(j)$ play L or someone (including himself) play NB against her, play NB regardless of the messages $m_t(j)$.

(V) Play B otherwise.

The *Informative Strategy* for buyer j is:

(III)' If j has ever observed $\theta_t^{-1}(n)$, where $n \in N_j(\theta_t)$, play L or someone (including himself) play NB against her, signal β. Signal γ otherwise.

(IV)' If j previously observed $\theta_t^{-1}(j)$ play L or someone play NB against her, or if he received a message β from any of his current spectators, $h \in N_j^s(\theta_t)$, play NB.

(V)' Play B otherwise.

Under both strategies, in the beginning of each period t each buyer j categorizes sellers into two different status groups based on his private history $H^t(j)$. If he has observed a seller play L or someone (including himself) play NB against her by period $t-1$, he gives her the "*Bad*" status β. Otherwise, he gives her the "*Good*" status γ. During the communication stage in period t (phase 2 of the stage game), buyer j is given the opportunity to signal to his neighboring buyers $n \in N_j(\theta_t)$ the statuses that he has assigned to their opponents $\theta_t^{-1}(n)$, and to revise the status that he assigns to his current opponent by taking into account the signals that he receives from the period t spectators to his game $h \in N_j^s(\theta_t)$.

As can be seen from the conditions (III), (IV) and (V), under the *Uninformative Strategy Profile* buyers merely "babble", disregard their neighbors' signals and base their choices of action against their period t opponents on the statuses that they assigned to them after the period $t-1$. So the communication stage is totally uninformative. Under the *Informative Strategy Profile*, on the other hand, signalling reveals all the relevant information (about receiver's opponent) of the senders, given the seller's strategy. Under this profile each buyer j sends a signal γ or β to each of his neighboring buyers $n \in N_j(\theta_t)$, depending on the statuses that he assigned to their opponents $\theta_t^{-1}(n)$ after period $t-1$. He also fully respects the messages that his spectators $h \in N_j^s(\theta_t)$ send to him before the period t trade game, and revises the status that he has assigned to his current opponent $\theta_t^{-1}(j)$ accordingly, basing his period t trade game choice of action on the revised status of $\theta_t^{-1}(j)$. Both strategy profiles are unforgiving since once a buyer assigns a particular seller a status β, he never upgrades her status to γ.

Before proceeding it is convenient to introduce some additional notation. Take any two time periods t' and t, where $t' < t$. Under the *Uninformative Strategy Profile* denote by b the probability that $\theta_t(i)$ was among the $N+1$ buyers who observed seller i at period t'. That is, let b denote the probability

$$\Pr\left\{i \in S_{\theta_t(i)}(\theta_{t'}) \bigcup \{\theta_{t'}^{-1}(\theta_t(i))\}\right\}.$$

Correspondingly, under the *Informative Strategy Profile* denote by b the probability that either $\theta_t(i)$ or some of the N $\theta_t(i)$'s time t spectators, $h \in N_{\theta_t(i)}^s(\theta_t)$, were among the $N+1$ buyers who observed i at period t'. In this case, b is the probability

$$\Pr\left\{ i \in \bigcup_{j\in\{\theta_t(i)\}\cup N^s_{\theta_t(i)}(\theta_t)} [S_j(\theta_{t'}) \bigcup \{\theta_{t'}^{-1}(j)\}] \right\}.$$

It is straightforward to check that

$$b = \begin{cases} \dfrac{N+1}{M} & \text{with the } \textit{Uninformative Strategy Profile, and} \\[2ex] 1 - \dfrac{\binom{M-N-1}{N+1}}{\binom{M}{N+1}} & \text{with the } \textit{Informative Strategy Profile} \end{cases}$$

For both strategies, note that since θ_t is i.i.d., b is time independent and does not depend on t and t'. Note also that for both strategies b increases in the number of spectators, N. In the proofs of Propositions 1 and 2 we need the unconditional probability of $\theta_t(i)$ assigning the seller i a status γ after the period t communication stage, given that seller i has defected in every period $t' \in \{ t_D, \ldots\ t-1\}$. With our notation, this probability can be written as $(1-b)^{t-t_D}$.

We are now ready to state our first two propositions. These propositions provide the conditions under which the *Uninformative* and the *Informative Strategy Profiles* are sequential equilibria of the random matching game. Notice that under both strategy profiles (H, B) is played in every period at each location along the equilibrium path.

Proposition 1: Define two constants δ^* and b^* as follows:

$$\delta^* = \left(1 + \frac{(\frac{M+g}{M})^2}{4(1+g)g(\frac{M-1}{M})} \right)^{-1},$$

$$b^* = \frac{g(1-\delta)}{\delta}.$$

i) If $\frac{g}{1+g} \leq \delta \leq \delta^*$, the *Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game if $b \geq b^*$.

ii) If $\delta \geq \max[\frac{g}{1+g}, \delta^*]$, there exists constants b_L and b_H, where $b^* \leq b_L \leq b_H < 1$, such that the *Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game if either $b^* \leq b \leq b_L$ or $b \geq b_H$. The constants b_L and b_H are given by

$$b_L = \frac{\frac{M+g}{M} - \sqrt{(\frac{M+g}{M})^2 - 4(1+g)\frac{g(1-\delta)}{\delta}(\frac{M-1}{M})}}{2(1+g)}$$

and

$$b_H = \frac{\frac{M+g}{M} + \sqrt{(\frac{M+g}{M})^2 - 4(1+g)\frac{g(1-\delta)}{\delta}(\frac{M-1}{M})}}{2(1+g)}.$$

The condition $\delta \geq g/(1+g)$ is necessary because with δ less than this, the efficient outcome could not be sustained by any equilibrium even with $M = 1$.[7]

Buyers' incentives to follow the unforgiving strategy profiles are easily satisfied: A buyer should expect his current opponent with status β to play L, regardless of his beliefs about the outcomes of her previous games, in which case NB is his best choice of action. If, on the other hand, he assigns her a status γ, playing B is optimal both on and off the equilibrium path given the consistent belief that she has never defected and will play H. Also the incentives for signalling are trivially satisfied.

Sellers' incentives to follow the uninformative strategy profile are characterized by two conditions: one preventing her from playing L on the equilibrium path and one that guarantees that sellers who have defected keep defecting irrespective of their private history. To give a seller an incentive to play H along the equilibrium path, the short-term gain from cheating, g, must be outweighed by the long-term loss resulting from the gradual loss of reputation among buyers. Given the buyers' strategies, this occurs if b (N) is sufficiently large that information about a seller's defection spreads quickly enough among the buyers. This results in the condition that $b \geq b^*$. On off-the-equilibrium paths, the strategy profile requires sellers to

[7] When $M = 1$, our random matching game is equivalent to a two-player standard repeated game with observable actions. In this case, the *Uninformative Strategy Profile*, which is identical with the *Informative Strategy Profile,* provides the maximum punishment for defection. This profile supports the efficient outcome as a Nash and a subgame perfect equilibrium if and only if $\delta \geq g/(1+g)$.

keep defecting rather than play H in an attempt to slow down the deterioration of her reputation. This off-the-equilibrium path constraint is satisfied when δ is small, $\delta \leq \delta^*$, since the short term gain g from selling low quality will then outweigh the future reward from trying to maintain a good reputation. When $\delta > \delta^*$, the condition is satisfied if either b is very large or b is small enough. If b is very large, $b \geq b_H$, it does not pay to slow down the deterioration of one's reputation, since with several informed buyers already playing NB against the seller, all buyers are soon likely to learn about seller's bad status anyway. On the contrary, if b is small enough, $b \leq b_L$, playing L is better than playing H simply because the information about her defections is not spreading very quickly. It can be shown that the off-the-equilibrium path constraint is always satisfied when $b = b^*$, implying that $b_L \geq b^*$.

This strategy profile is not a sequential equilibrium when $b \in (b_L, b_H)$. In proposition 3, however, by using a public randomization device we construct a sequential equilibrium which supports the efficient outcome for any b greater than b^*.

Proof. When (II) holds a seller has incentive to follow (I) if and only if the following inequality holds:

$$\frac{1}{1-\delta} \geq \frac{1+g}{1-(1-b)\delta}. \tag{3.1}$$

The left-hand side is the payoff from playing H in every period whereas the right-hand side is the payoff from playing L in every period. Since b must be less than one, the inequality can hold only when $\delta \geq g/(1+g)$. In that case equation (3.1) can be written as:

$$b \geq b^*. \tag{3.2}$$

By the principle of dynamic programming to verify that (II) is optimal, it is enough to check that a one time switch to H is not profitable after any history in which the seller has obtained a bad status, i.e., she has played L or some buyer has played NB against her. We can show that the seller's incentives to follow (II) increase in the number of buyers who are aware of her bad status. Since consistency requires this number to be at least $N+1$ (in states that (II) is concerned with), it will be sufficient for us to show that a seller has incentive to follow (II) when exactly $N+1$ players assign her a bad status.

Define $P_s(K)$ as the probability that $K+s$ buyers know about i's bad status after period t if i plays H in period t and K players knew about i's bad status after the period $t-1$. Let $\alpha = K/M$. It is straightforward to show that:

$$P_0(K) = (1-\alpha) + \frac{\alpha \binom{K-1}{N}}{\binom{M-1}{N}}$$

and for $\alpha < 1$,

$$P_s(K) = \frac{\alpha \binom{K-1}{N-s} \binom{M-K}{s}}{\binom{M-1}{N}}, \forall\ 1 \leq s \leq \min[N, M-K].$$

Denote with $u_s(K) = 1 - (K+s)/M$ the associated conditional probability that seller i's period $t+1$ match $\theta_{t+1}(i)$ does not know about i's bad status (after the period $t+1$ communication stage), given that $K+s$ buyers know about i's bad status after the period t.

Then, assuming that K buyers are aware of a seller's bad reputation before the current period, a seller has incentive to follow (II) if and only if:

$$\frac{(1+g)(1-\alpha)}{1-(1-b)\delta} \geq (1-\alpha) + \frac{\delta(1+g)}{1-(1-b)\delta} \sum_{s=0}^{\min[N,M-K]} P_s(K) u_s(K). \tag{3.3}$$

The left hand side is the payoff from playing L in each of the remaining periods, whereas the right hand side is the payoff for playing H in one period and then L thereafter. Realizing that s follows a hypergeometric distribution for $s = 1, 2, ...N$, it can be shown that

$$\sum_{s=0}^{\min[N,M-K]} P_s(K) u_s(K) = (1-\alpha)^2 + \alpha(1-\alpha)(1-b)\left(\frac{M}{M-1}\right).$$

It is now easy to see that equation (3.3) is relaxed as α is increased. The intuition for this result is quite simple: A seller who is matched with a buyer that is not aware of his bad status may by playing H keep his reputation among at most $N+1$ players and benefit from this reputation later. When α is large many of her current spectators are likely to know about his defection already, which reduces the benefit to playing H. Since $\alpha \geq b$ by consistency, it is sufficient to check that equation (3.3) holds when $\alpha = b$. Setting $\alpha = b$ and rearranging, equation (3.3) can be written:

$$b^2 - \left(\frac{M+g}{(1+g)M}\right) b + \frac{(1-\delta)g}{(1+g)\delta}\frac{M-1}{M} \geq 0.$$

This quadratic inequality holds if either $\delta \leq \delta^*$ or if $\delta \geq \delta^*$ and either $0 \leq b \leq b_L$ or $b_H \leq b \leq 1$. Combined with equation (3.2) this result implies the equilibrium conditions stated in proposition 1.

(III) is trivially satisfied given (IV) and (V). It is also easy to see that (IV) and (V) are satisfied given (I),(II) and (III). If buyer j observed his current match, $\theta_t^{-1}(j)$, play L in the past or some buyer play NB against her, he should believe she will play L by (II); so playing NB is his best response. If he has never observed $\theta_t^{-1}(j)$ play L, nor some buyer play NB against her, then given (I), (II) and (III), he should believe she will play according to (I) regardless of the messages he has received. This being the case, B is his optimal choice of action. This establishes (IV) and (V) and completes the proof. ■

As one would expect, the equilibrium conditions are very similar for the *Informative Strategy Profile.* In this case, however, it is not possible to state the off-the-equilibrium path conditions in terms of b as was true for the *Uninformative Strategy Profile.*

Proposition 2: For $\delta \geq g/(1+g)$, the *Informative Strategy Profile is* a sequential equilibrium of the above random matching game if

$$1 - \frac{\binom{M-N-1}{N+1}}{\binom{M}{N+1}} \geq \frac{g(1-\delta)}{\delta} \tag{3.4}$$

and

$$\frac{g}{\delta(1+g)} \geq \left(\frac{1+2g}{1+g}\right)\frac{\binom{M-N-1}{N+1}}{\binom{M}{N+1}} - I_{\{3(N+1)\leq M\}}\frac{\binom{M-2N-2}{N+1}}{\binom{M}{N+1}}, \qquad (3.5)$$

where I_A is a function that takes value 1 if A is true and 0 otherwise.

Equation (3.4) concerns on-the-equilibrium path behavior and can be written as $b \geq b^*$. Equation (3.5) is the constraint for off-the-equilibrium path behavior. As before, it is always satisfied when δ is small enough and when δ is large it is satisfied when b is either very large or b is small enough. Although the intuition for both conditions is exactly the same as under the *Uninformative Strategy Profile*, the second condition is not exactly the same in terms of b under the two strategy profiles. This constraint concerns a seller's possible deviation to H when playing on off-the-equilibrium paths, and is different for the two strategy profiles because the dissemination of a seller's bad reputation when playing H is different in terms of b under the two strategy profiles.

Proof. As was shown in the proof of proposition 1, assuming that (II) holds, a seller has incentive to follow (I) if and only if $b \geq b^*$. This is stated in equation (3.4).

Showing that she has incentive to follow (II) when equation (3.5) holds proceeds much as the proof of proposition 1. Denote with α the probability that seller i's period t opponent $\theta_t(i)$ is aware of i's bad status after the period t communication stage, if K buyers are aware of sellers i's bad status after the period $t-1$. Correspondingly, denote with ξ the probability that $\theta_t(i)$ is aware of i's bad status after the period t communication stage, if $K+N+1$ buyers are aware of sellers i's bad status after period $t-1$. Then

$$\alpha = 1 - I_{\{(N+1)\leq M-K\}}\frac{\binom{M-K}{N+1}}{\binom{M}{N+1}}$$

and

$$\xi = 1 - I_{\{2(N+1)\leq M-K\}} \frac{\binom{M-K-N-1}{N+1}}{\binom{M}{N+1}}.$$

Let P_s and u_s represent the same probabilities as in the proof of proposition 1.

Seller i has an incentive to follow (II) if and only if:

$$(1+g)\left((1-\alpha) + \frac{\delta((1-\alpha)(1-\xi) - (1-\alpha)^2 + \sum_{s=0}^{\min[N,M-K]} P_s u_s)}{1-(1-b)\delta}\right) \geq$$

$$(1-\alpha) + (1+g)\left(\frac{\delta \sum_{s=0}^{\min[N,M-K]} P_s u_s}{1-(1-b)\delta}\right) \tag{3.6}$$

The left hand side is the payoff from playing L in each of the remaining periods, whereas the right hand side is the payoff from playing H in one period and then L thereafter. Seller i's action makes a difference only when neither i's opponent $\theta_t(i)$ nor any of the spectators to i's game have assigned the bad status to i. This happens with probability $(1-\alpha)$. In that case, playing L results in a larger payoff by g, but $N+1$ new buyers learn about i's bad status, reducing the probability that i receives $(1+g)$ in the next period from $(1-\alpha)$ to $(1-\xi)$. If $\theta_t(i)$ or some of the spectators to i's game know about i's bad status, which happens with probability α, i receives nothing in that period and her reputation deteriorates similarly irrespective of the action that he takes.

This inequality can be written as:

$$\frac{g(1-(1-b)\delta)}{\delta(1+g)} \geq \xi - \alpha. \tag{3.7}$$

It is now straightforward to check that equation (3.6) is relaxed as K increases. Since $K \geq N$ by consistency, it is sufficient to check that equation (3.6) holds when $K = N$ or $\alpha = b$. Setting $\alpha = b$ and rearranging gives equation (3.5).

If a buyer j has ever observed a seller $i \in S_j(\theta_t)$ play L or her opponent play NB against her, j is indifferent between signalling γ or β to $\theta_t(i)$, since given (II) he expects i to play L in all his future games, including those with j himself, irrespective of the signal that he sends. So we may assume that he sends a truthful signal β in this case. If j has never observed i's game or if j has observed i but the outcome in i's games were always (H, B) he strictly prefers to send the message γ instead of β. Sending the message β would result in $\theta_t(i)$ playing NB against seller i, giving her a bad status and making her play L in the future. This would reduce buyer j's future payoffs from games where he is matched with seller i. This establishes condition (III)'. Given (I), (II) and (III)' conditions (IV)' and (V)' are trivial. ■

In order to create sequential equilibria that support trade for all $b \geq b^*$, let us now extend our basic model to include a public randomization device. The idea of using a public randomization device to adjust the severity of punishments is borrowed from Ellison (1994). In particular, we assume that before players choose their actions in period t, they observe a public random variable f_t which is drawn independently from a uniform distribution on $[0, 1]$. Let $f \in [0, 1]$ and consider adjusting the *Uninformative* and *Informative Strategy Profiles* as follows: In period t, sellers play according to the original strategies as long as $f_t \leq f$, but return immediately to the equilibrium path of the original strategies if $f_t > f$; buyers play according to the original strategies, except whenever $f_t > f$, at which point they forget all past actions of sellers and assign each seller status γ. Assuming that such a public randomization device is available, we can state the following proposition.

Proposition 3: *For $\delta \geq g/(1+g)$ there exists a function $f(\delta)$ such that the adjusted Informative and Uninformative Strategy Profiles with $f = f(\delta)$ are a sequential equilibrium of the random matching game if $b \geq b^*$, where $b^* \equiv g(1-\delta)/\delta$.*

The idea in the proof is that whenever $b \geq b^*$, we can by an appropriate choice of f adjust the severity of punishment for a seller so that she becomes indifferent between playing H (following (I)) and deviating on the equilibrium path. Because at off-the-equilibrium paths a seller has less incentive to protect her reputation than on-the-equilibrium path, as her reputation is deteriorating anyway, this indifference can be shown to imply that the off-the-equilibrium path condition always holds.

Proof. For any f_t, a seller has incentive to follow (I) if and only if

$$\frac{1}{1-\delta} \geq \frac{1+g}{1-(1-b)f\delta} + \sum_{t=1}^{\infty} f^{t-1}(1-f)\frac{\delta^t}{1-\delta}$$

or

$$\frac{1}{1-\delta f} \geq \frac{1+g}{1-(1-b)f\delta} \tag{3.8}$$

The left-hand side of the first inequality is the seller's payoff from following (I), while the right-hand side is her payoff from deviating and following (II), as long as $f_\tau \leq f$, and following (I) thereafter. For all $\delta \geq g/(1+g)$ and $b \geq b^*$, there exists $f(\delta) \in [0,1]$ such that equation (3.8) holds as an equality when $f = f(\delta)$. From now on, let us assume that $f = f(\delta)$.

A seller who is playing on the equilibrium path is now indifferent between playing H in every period and deviating. She is also indifferent between playing H in the current period and then deviating and deviating right away. By playing H in the current period she can keep her good reputation among $N+1$ buyers, until they, in some way, learn about her defections in the future. Now consider a seller who is following (II). If her opponent and all the N spectators to her game happen to assign her a good status, she also can keep her good reputation among $N+1$ buyers by playing H. This reputation, however, is worth less to her than if she were on the equilibrium path because, with some buyers already assigning her a bad status, these $N+1$ buyers are more likely to learn about her bad status before playing against her in the future. Since the short term gain from deviating is same in both cases, we conclude that playing L is optimal off the equilibrium path.

More formally, consider the sellers incentives to follow (II) in period t when $f_t \leq f(\delta)$. By the principle of dynamic programming, it is sufficient to show that a single-period deviation to H is unprofitable. Let $\alpha, P_s(K)$, and $u_s(K)$ denote the same probabilities as in the proof of proposition 1. A seller has an incentive to follow (II) if and only if:

$$\frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} + \sum_{t=1}^{\infty} f(\delta)^{t-1}(1-f(\delta))\frac{\delta^t}{1-\delta} \geq$$

$$(1-\alpha) + \frac{\delta f(\delta)(1+g)(\sum_{s=0}^{\min[N,M-K]} P_s u_s)}{1-(1-b)f(\delta)\delta} + \sum_{t=1}^{\infty} f(\delta)^{t-1}(1-f(\delta))\frac{\delta^t}{1-\delta} \tag{3.9}$$

or

$$\frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} \geq (1-\alpha) + \frac{\delta f(\delta)(1+g)(\sum_{s=0}^{\min[N,M-K]} P_s u_s)}{1-(1-b)f(\delta)\delta}. \tag{3.10}$$

We can show that this inequality holds as follows:

$$\frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} = \frac{(1-\alpha)}{1-\delta f(\delta)} =$$

$$(1-\alpha) + \frac{(1+g)\delta f(\delta)(1-\alpha)}{1-(1-b)f(\delta)\delta} \geq$$

$$(1-\alpha) + \frac{(1+g)\delta f(\delta)(\sum_{s=0}^{\min[N,M-K]} P_s u_s)}{1-(1-b)f(\delta)\delta}.$$

The first two equalities come from the fact that (3.8) holds as an equality with $f = f(\delta)$. The inequality follows since

$$\sum_{s=0}^{\min[N,M-K]} P_s u_s \leq \sum_{s=0}^{\min[N,M-K]} P_s u_0 = u_0 = 1-\alpha.$$

When $f_t \leq f(\delta)$ a buyer's problem is similar to that in propositions 1 and 2 so he is better off following the original strategies. On the other hand, given that a past defector plays H after $f_t > f(\delta)$ it is optimal for the buyer to assign her a status γ and treat her like the seller who never defected. ■

4. LARGE POPULATION RESULTS

In this section we study how fast N must grow in relation to M in order to sustain (H, B) as the outcome of the trade game for our strategies. If we denote $N^*(M)$ as the smallest integer N that satisfies the constraint $b \geq g(1-\delta)/\delta$ (i.e., $b \geq b^*$), then given propositions 1,2 and 3, the question can be reformulated as how fast $N^*(M)$ grows in relation to M.

If seller i has defected in the previous period, the $N+1$ buyers who observed the defection are the only ones who are informed of the defection before the current period's trade game starts. Then b, the probability that seller i meets a buyer who is informed of her previous defection, is simply $(N+1)/M$ under the *Uninformative Strategy Profile.* With this strategy profile it is clear that N^* and M must grow at the same rate in the limit.

This is not true, however, with the *Informative Strategy Profile.* In this strategy profile i's current opponent $\theta_t(i)$ assigns her a bad status if he observed this defection or he received an informative signal based on this defection during the current period's communication stage. With N spectators to $\theta_t(i)'$s game each of whom observes $N+1$ games, $\theta_t(i)$ obtains information from N^2+2N+1 possibly overlapping games and assigns i a bad status if any of these games was played at location i. This suggests that N^* may grow more slowly than M. Below we show that in order to sustain trade, N^* must grow only at a rate $\sqrt{M}$. To prove this result formally we need the following lemma.

Lemma 1: Under the *Informative Strategy Profile* $\lim_{M\to\infty} \dfrac{N^*(M)}{M} = 0.$

Proof. By the definition of $N^*(M)$ we have the following inequalities

$$\frac{\binom{M-N^*(M)-1}{N^*(M)+1}}{\binom{M}{N^*(M)+1}} \leq (1-b^*) < \frac{\binom{M-N^*(M)}{N^*(M)}}{\binom{M}{N^*(M)}} \tag{4.1}$$

First note $\limsup_{M\to\infty} N^*(M)/M < 1/2$. For the subsequences of $N^*(M)/M$ whose limits are $1/2$, the numerator of the right-hand side of the strict inequality in (4.1) approaches 1 and the denominator goes to the infinity, leading to a contradiction since b^* is assumed to be strictly less than one.

Using Stirling's formula

$$\sqrt{2\pi n}n^n e^{-n} \leq n! \leq \sqrt{2\pi n}n^n e^{-n} e^{\frac{1}{12n}}$$

the second inequality in (4.1) implies that

$$(1-b^*) <$$

$$\frac{(M-N^*)^{2(M-N^*)+1}}{M^{M+\frac{1}{2}}(M-2N^*)^{M-2N^*+\frac{1}{2}}}e^{\frac{1}{6(M-N^*)}} \tag{4.2}$$

Given that $\limsup_{M\to\infty} N^*/M < 1/2$, it is easy to see that the exponential term that appears at the right-hand side of (4.2) goes to 1 as M approaches infinity. The nonexponential term then has to be bounded away from zero for large M. In what follows we show this implies $\lim_{M\to\infty} N^*/M = 0$.

Let $q = N^*/M$. The nonexponential terms in the right hand side of (4.2) can now be written as:

$$\left(\frac{(1-q)^{2-2q}}{(1-2q)^{1-2q}}\right)^M \frac{(1-q)}{(1-2q)^{1/2}}.$$

Since $0 < q < 1/2$ and $\limsup_{M\to\infty} q < 1/2$ the second term of the above expression is bounded. Furthermore, we can show the first term is strictly decreasing with respect to q for $0 \le q \le 1/2$ and for that range of q it is one if and only if $q = 0$. Thus if $\lim_{M\to\infty} q \ne 0$, the first term is very close to zero for large M, which is a contradiction. Hence it must be that $\lim_{M\to\infty} N^*/M = 0$. ■

Proposition 4: Under the *Informative Strategy Profile* $0 < \lim\limits_{M\to\infty} \dfrac{N^*(M)^2}{M} < \infty$.

Proof. If we rewrite the inequalities (4.1) using Stirling's formula we get

$$\frac{(M-N^*-1)^{2(M-N^*)}}{M^M(M-2N^*-2)^{M-2N^*}}\left(\frac{(M-2N^*-2)^3}{M(M-N^*-1)^2}\right)^{\frac{1}{2}} e^{-\frac{1}{12(M-2N^*-2)}-\frac{1}{12M}}$$

$$\le (1-b^*) <$$

$$\frac{(M-N^*)^{2(M-N^*)}}{M^M(M-2N^*)^{M-2N^*}}\left(\frac{(M-N^*)^2}{M(M-2N^*)}\right)^{\frac{1}{2}} e^{\frac{1}{6(M-N^*)}} \tag{4.3}$$

Since the last two terms of both sides of the inequalities (4.3) approach one as $M \to \infty$ by lemma 1 and both of the first terms exhibit the same behavior in the limit, we must have

$$\frac{(M-N^*)^{2(M-N^*)}}{M^M(M-2N^*)^{M-2N^*}} \to (1-b^*) \text{ } as \text{ } M \to \infty. \tag{4.4}$$

Next we show that $0 < \lim\limits_{M\to\infty} N^{*2}/M < \infty$ in order for (4.4) to hold.

First of all, note that we can write

$$\frac{(M-N^*)^{2(M-N^*)}}{M^M(M-2N^*)^{M-2N^*}} =$$

$$\left[\left(1-\left(\frac{N^*}{M-N^*}\right)^2\right)^{-\left(\frac{M-N^*}{N^*}\right)^2}\right]^{\frac{N^{*2}}{M-N^*}} \left[\left(1-\frac{2N^*}{M}\right)^{\frac{-M}{2N^*}}\right]^{\frac{-2N^{*2}}{M}} =$$

$$(e+\alpha_M)^{\frac{N^{*2}}{M-N^*}} (e+\beta_M)^{\frac{-2N^{*2}}{M}}$$

where

$$\alpha_M = \left(1-\left(\frac{N^*}{M-N^*}\right)^2\right)^{-\left(\frac{M-N^*}{N^*}\right)^2} - e,$$

$$\beta_M = \left(1-\frac{2N^*}{M}\right)^{\frac{-M}{2N^*}} - e.$$

Since $\lim_{M\to\infty} N^*/M = 0$, both sequences $\{\alpha_M\}, \{\beta_M\}$ converge to zero from above. Now define new sequences $\{a_M\}, \{b_M\}$ such that $e^{a_M} = e + \alpha_M$, $e^{b_M} = e + \beta_M$. We can then write

$$\frac{(M-N^*)^{2(M-N^*)}}{M^M(M-2N^*)^{M-2N^*}} = e^{a_M \frac{N^{*2}}{M-N^*} - b_M \frac{2N^{*2}}{M}}. \tag{4.5}$$

Given (4.4) and (4.5), all that remains to show is that $\{N^{*2}/M\}$ must converge to some positive number in order for $\{a_M N^{*2}/(M-N^*) - b_M 2N^{*2}/M\}$ to converge to $\ln(1-b^*)$. If $\lim_{M\to\infty} N^{*2}/M = \infty$, $a_M N^{*2}/(M-N^*) - b_M 2N^{*2}/M$ approaches minus infinity as M goes to infinity. This can be easily shown using the fact that $\{a_M\}$ and $\{b_M\}$ converge to one from above. If $\lim_{M\to\infty} N^{*2}/M = 0$, we obtain another contradiction since $a_M N^{*2}/(M-N^*) - b_M 2N^{*2}/M \to 0$ as $M \to \infty$. Since $\{N^{*2}/M\}$ is bounded, it is straightforward to show $\lim_{M\to\infty} N^{*2}/M = -\ln(1-b^*)$. ■

5. ENDOGENOUS CONNECTIONS

In this section we extend our model by assuming that networking is costly. Let us say that strategies are non-contagious when only the sellers who have produced low-quality are punished. When either inviting spectators, $N_j^s(\theta_t)$, observing neighboring buyers, $N_j(\theta_t)$, or both are costly to buyer j, the following result holds:

Proposition 5: In any Nash equilibrium of the random matching game with costly networking that uses non-contagious strategies, low quality is produced with positive probability when $M > \delta/[g(1-\delta)]$.[8]

Proof. The proposition is proved by contradiction. Assume that there is a Nash equilibrium with non-contagious strategies in which every seller produces high quality with probability one in every period. Buyers then do not have any incentive to network and the optimal N_j must be zero for all buyers j. If, however, N_j is zero for all buyers and $M > \delta/[g(1-\delta)]$ all sellers have incentive to unilaterally deviate and produce low-quality. Contradiction. ■

There clearly exists the equilibrium where $N_j = 0$ for all buyers and every seller produces low quality. More interestingly, we show that if the costs of networking are small enough, there exist sequential equilibria with strictly positive probability of trade.

Let us concentrate on the case where observing neighboring buyers $n \in N_j(\theta_t)$ is costly to buyer j and where buyers are unable to affect the number of spectators to their game. This assumption corresponds to the idea that buyers network to gather information about their trading environment. An alternative - that leads to similar results - would be that buyers invited other buyers to their games in an attempt to obtain information regarding their current opponents. Clearly the

[8]A similar proposition could be stated allowing for contagious strategies for $M > \overline{M}(\delta, g, \ell)$, where $\overline{M} < \infty$.

second alternative would make sense only under the *Informative Strategy Profile*. More specifically, let us extend our basic model by assuming that in period zero, before θ_0 is realized, all buyers j can invest in N_j connections that allow them to observe the games in N_j consecutive locations to their own in every future period. To obtain N_j connections j must pay $N_j c$, where $c > 0$, in period zero.

With these assumptions, whenever the costs of networking are less than some threshold value $\overline{c}(M)$, we can find sequential equilibria with slightly modified *Informative and Uninformative Strategy Profiles* such that the sellers initially randomize between high and low quality and produce that level of quality in the future. A positive probability of low quality goods is necessary to provide buyers with an incentive to network. This probability tends to increase with M. When the costs of networking exceed the threshold value $\overline{c}(M)$, trade collapses because the probability of low quality goods that would provide buyers sufficient incentives to network is so high that buyers are unwilling to buy from unknown sellers. For simplicity, let us confine our analysis to the *Uninformative Strategy Profile*.

Consider the following *modified Uninformative Strategies*:

For the seller i, in all periods $t = 0, 1, 2....$, after θ_t is realized:

(I) In the first period play H with probability $1-p$ and L with probability p. After that, if the outcome in all the trade games where seller i played was (H, B), play H.

(II) Play L otherwise.

For the buyer j:

(III) In period 0, before θ_0 is realized, invest in $N^* - 1$ connections with probability r and in N^* connections with probability $1 - r$,

and in all periods $t = 0, 1, 2....$, after θ_t is realized:

(IV) Signal randomly γ or β with equal probabilities to all $n \in N_j(\theta_t)$, irrespective of the private history $h_t(j)$.

(V) If j has ever observed $\theta_t^{-1}(j)$ play L or someone (including himself) play NB against her, play NB regardless of the messages $m_t(j)$.

(VI) Play B otherwise.

Before proceeding, we need some additional notation. If buyer j invested in N_j connections, the probability that he observed $\theta_t^{-1}(j)$ at period t', where $t' < t$, is $(N_j + 1)/M$. Let us denote this probability by $b^e(N_j)$.[9] Then the probability that buyer j has never observed $\theta_t^{-1}(j)$ until period t is simply $(1 - b^e(N_j))^t$.

Proposition 6: The *modified Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game with

$$p = cM\left(\frac{(1-\delta(1-b^e(N^*-1)))\,(1-\delta(1-b^e(N^*)))}{\ell\delta}\right),$$

where

$$c \leq \overline{c} \equiv \frac{1}{M}\left(\frac{\ell\delta\,(M(1-\delta)+\delta)}{(1-\delta(1-b^e(N^*-1))\,(1-\delta(1-b^e(N^*)))\,((\ell+1)M(1-\delta)+\delta)}\right).$$

Proof. To have a sequential equilibrium in this extended game for our strategies, the following three equations must hold:

$$\frac{1}{1-\delta} = \frac{r(1+g)}{1-(1-b^e(N^*-1))\delta} + \frac{(1-r)(1+g)}{1-(1-b^e(N^*))\delta} \tag{5.1}$$

$$N^*, N^*-1 \in \arg\max_{N_j} \frac{1-p}{1-\delta} - \frac{p\ell}{1-\delta(1-b^e(N_j))} - N_j c, \tag{5.2}$$

$$(1-p) + \frac{\delta(1-p)}{M(1-\delta)} \geq p\ell. \tag{5.3}$$

A seller is willing to randomize in the initial period between providing high quality goods forever and providing low quality goods forever if and only if equation (5.1) holds. The left-hand side of (5.1) is the payoff from providing high quality goods forever and the right-hand side is the expected payoff from providing low quality goods forever. To see this, note that the probability that $\theta_t(i)$ has never observed i is just $(1 - b^e(N_{\theta_t(i)}))^t$. Given this indifference in the initial period, the seller who once provided low quality good can be shown to keep on providing low quality goods; the proof is exactly the same as that of proposition 3. So (I) and (II) are established.

[9] More precisely, $b^e(N_j) = \Pr[\theta_t^{-1}(j) \in S_j(\theta_{t'}) \cup \{\theta_{t'}^{-1}(j)\}]$

Equation (5.2) requires that buyers are willing to randomize between N^* and $N^* - 1$ connections. We can show that the right hand side of equation (5.2) has a single peak if N_j is treated as a positive real number. Hence if the expected payoff to buyer j is the same with N^* and $N^* - 1$, then N^* and $N^* - 1$ both solve j's maximization problem. Equation (5.2) is therefore satisfied if

$$\frac{p\ell}{(1-\delta(1-b^e(N^*)))} = \frac{p\ell}{(1-\delta(1-b^e(N^*-1)))} + c,$$

or

$$p = cM\left(\frac{(1-\delta(1-b^e(N^*-1)))\,(1-\delta(1-b^e(N^*)))}{\ell\delta}\right) \tag{5.4}$$

Given that other buyer's actions do not depend on the messages, (IV) is obvious. If a buyer has observed his current opponent play L or someone play NB against her, he should play NB against her given (II). And if a buyer has observed his current opponent and the outcomes have always been (H, B), he should believe she will play H and he should play B. If the buyer has never observed his current opponent, he should believe that she will play H with probability $(1-p)$. For a buyer to play B against her rather than give her a bad status by playing NB, equation (5.3) has to be satisfied. This equation can be rewritten as

$$p \leq \frac{M(1-\delta)+\delta}{(\ell+1)M(1-\delta)+\delta}. \tag{5.5}$$

Therefore the *modified Uninformative Strategy Profile* with p defined by equation (5.4) is a sequential equilibrium if p satisfies equation (5.5). This happens when $c \leq \overline{c}$. ■

Several interesting results now follow: First, for $c > 0$ equation (5.4) requires that p is strictly positive as was shown in proposition 5. L has to be played with positive probability to provide buyers with the incentive to network. Secondly, this probability is increasing in M. Under the *Uninformative Strategy Profile* approximately proportionally and with *Informative Strategy Profile* (it can be shown) less than proportionally. More striking result is the knife edge property of our equilibria: If even one more buyer invested in one more connection there would be no low quality at all (increasing the utility of all buyers and sellers discontinuously). But networking to reduce production of low quality is a public good and, as usual, everyone wants to free ride in its production. Because of this, the economy is stuck in an inefficient equilibrium.

Another interesting observation is that when equation (5.5) fails, trade collapses even when it might be beneficial for buyers to keep trading. This occurs because a buyer, who considers whether to trade with an unknown seller or to give her a bad status by playing NB, does not take into account the future trading opportunities of other buyers with her. With the *Informative Strategy Profile* there would still be another externality because of the informative signalling to neighbors. When choosing the number of locations to observe the buyers would not take into account the learning by their neighbors, but would only be interested in their own learning.

6. CONCLUSION

In many real life situations particular sellers and buyers trade with each other infrequently, or only once. In such instances, when one or both parties have short-term incentives to cheat, community enforcement may be needed to facilitate cooperation and trade. This paper studied community enforcement in the absence of institutions to transmit information.

We studied a large population, random matching game between buyers and sellers, where the sellers have a short-term incentive to cheat and supply low quality. We studied informal networks of communication as the mechanism that spreads information about sellers' behavior and facilitates trade. We looked at both informative and uninformative signalling and for the latter we showed that high quality can be sold in a sequential equilibrium with population M where each buyer networks with only $N^*(M)$ players with $\lim_{M\to\infty} N^*/M = 0$.

We studied the case of costly networking and showed that in this case, when M is large, low-quality goods must be supplied with positive probability in any equilibrium to provide buyers with an incentive to network. When the costs of networking were below a threshold value, we found a sequential equilibrium in which sellers initially randomize between high and low quality with probabilities $(1-p)$ and p respectively and then continue to produce high quality if and only if they did so in the first period. In this equilibrium p is strictly positive and increasing in both M and the costs of networking.

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