

# CARESS Working Paper #95-04

## Justifying Rational Expectations

Stephen Morris<sup>a</sup>

Original Version: February 1993  
This Version: March 1995

### Abstract

In a static economy with symmetric information, the informational requirements for competitive equilibrium are very weak: markets clear and each agent is rational. With asymmetric information, the solution concept of competitive equilibrium has been generalized to rational expectations equilibrium. But now common knowledge of market clearing and rationality is required. This paper proves versions of these results in a formal model of knowledge.

## 1. Introduction

In a static economy with symmetric information, the informational requirements for competitive equilibrium are very weak. If each agent knows market prices and maximizes utility subject to his budget constraint, and markets clear, then we have a competitive equilibrium. It is not necessary to assume anything about what agents believe or know about other agents' behavior, market clearing or anything else. This observation, which has its roots in Adam Smith, is one of the most remarkable features of competitive equilibrium.

---

<sup>a</sup> Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia PA 19104-6297. E-Mail: smorris@econ.sas.upenn.edu. I am grateful for financial support to visit Cambridge from an ESRC research grant (R000232865) and an EEC contract (SPESCT910057).

With asymmetric information, the solution concept of competitive equilibrium has been generalized to the notion of rational expectations equilibrium (Radner (1979)). Agents infer information from market prices. It is widely understood that this solution concept imposes much more stringent requirements on the knowledge of agents. It must be common knowledge that agents understand the structure of the model, and thus it must be common knowledge that all agents are maximizing utility subject to budget and informational constraints, and that markets clear. The equilibrium formulation of the solution concepts and the elegance of the theoretical extension mask a radical change in assumptions.

The above claims are well understood, but they cannot be stated as theorems in the standard framework. The purpose of this paper is to introduce a model of agents' knowledge of the market process which allows these claims to be stated as theorems. Thus it is necessary to have a description of the problem which includes agents' knowledge about, for example, market clearing and other agents' behavior. Thus each state must be understood as specifying not only the traditional data of the economy, i.e. exogenous events, but also endogenous events such as agents' actions and prices. This is analogous to and inspired by work looking behind equilibrium solution concepts in game theory (e.g. Aumann (1987), Brandenburger and Dekel (1987) and Aumann and Brandenburger (1993)).

There are two reasons for carrying out this exercise. Firstly, it is paradoxical that some of the most important conceptual claims about equilibrium solution concepts in general equilibrium theory have not been stated as theorems. It is of interest in its own right. Secondly, it is possible to refine the above statements in ways that are of some interest. It will be shown that, for a given state space, even assuming the common prior assumption and common knowledge of market clearing and rationality does not imply that prices and agents' choices constitute a rational expectations equilibrium with respect to that state space. Common knowledge of market clearing and rationality does not rule out the possibility of sunspots. This implies in turn that the relevant definition of rational expectations equilibrium cannot require that market prices reveal no information that is not known by at least one agent.

Many papers have addressed these issues in the language of equilibrium theory. Dutta and Morris (1995), following McAllister (1990), have emphasized that in order to justify standard rational expectations equilibria, it is necessary to assume degenerate beliefs about prices (in addition to the common knowledge assumption). More generally, work on sunspots and equilibrium price uncertainty

has been similarly motivated by the realization that existing solution concepts assume even more than common knowledge of the structure of the model. This paper provides a check that informal statements about knowledge and common knowledge in such approaches are correct.

The paper is organized as follows. Section 2 gives a definition of competitive equilibrium (in a certain economy) and rational expectations equilibrium (in an economy with asymmetric information). Some issues concerning sunspots and the amount of information revealed by prices are discussed. In section 3, I discuss what it means to think of the state space as containing a description of endogenous variables and agents' knowledge of endogenous variables. Two lemmas corresponding to the informal claims in the first two paragraphs of the paper are proved (they are trivial once the framework is set up). Examples show that the lemmas are tight, so that, for example, anything less than common knowledge of market clearing fails to deliver even rational expectations equilibria with sunspots.

## 2. Equilibrium Concepts

Consider an  $L$  good economy, with agents  $1; \dots; H$ . Suppose agent  $h$  has endowment  $e^h \in \mathbb{R}_+^L$  and utility function  $u^h : \mathbb{R}_+^L \rightarrow \mathbb{R}$ .

**Definition 2.1.** A competitive equilibrium [CE] consists of a price vector  $p \in \mathbb{R}_{++}^L$  and demand  $x^h \in \mathbb{R}_+^L$ , for each agent, such that the following conditions are satisfied:-

- [1] Feasibility:  $\sum_{h=1}^H p x^h = \sum_{h=1}^H p e^h$ , for all  $h$ :
- [2] Rationality:  $u^h(y) \geq u^h(x^h) \implies p y \geq p e^h$ , for all  $h; y \in \mathbb{R}_+^L$
- [3] Market Clearing:  $\sum_{h=1}^H x^h = \sum_{h=1}^H e^h$

This notion can be generalized to an economy with uncertainty and asymmetric information as follows. Let  $S$  be some finite state space and let  $\mu$  be some strictly positive prior on the state space. Each agent  $h$  has a utility function,  $u^h : \mathbb{R}_+^L \times S \rightarrow \mathbb{R}$ , endowment  $e^h : S \rightarrow \mathbb{R}_+^L$ , and an information partition  $I^h$ . Write  $I^h(s)$  for the (unique) element of  $I^h$  containing  $s$ . Assume agents know their own endowment, so that  $s^0 \in I^h(s) \implies e^h(s^0) = e^h(s)$ .

Definition 2.2. A rational expectations equilibrium [REE] consists of a price function,  $p : S \rightarrow \mathbb{R}^L_{++}$ , and, for each agent  $h$ , a state contingent commodity bundle,  $x^h : S \rightarrow \mathbb{R}^L_+$ , such that

[1] Feasibility:  $p(s)x^h(s) \cdot p(s)e^h(s)$  for all  $h; s$ :

[2] Rationality:  $\int_{s^0 \in I^h(p; s)} p(s^0) u^h(y; s^0) > \int_{s^0 \in I^h(p; s)} p(s^0) u^h(x^h(s); s^0)$   
 $\Rightarrow p(s)y > p(s)e^h(s)$ , for all  $h; s; y \in \mathbb{R}^L_+$ .

[3] Market Clearing:  $\sum_{h=1}^n x^h(s) = \sum_{h=1}^n e^h(s)$ , for all  $s$ .

[4] Demand Measurability:  $s^0 \in I^h(p; s) \Rightarrow x^h(s^0) = x^h(s)$ ; for all  $h$

where  $I^h(p; s) = \{s^0 \in S : p(s^0) = p(s)\}$ . Another restriction sometimes required in the definition of REE is the following:-

Definition 2.3. Price Join Measurability:  $s^0 \in I^h(p; s) \Rightarrow p(s^0) = p(s)$

This requires that prices can differ across states only if some agent has different knowledge at those states. An argument for imposing it is the following: suppose we think there is some deterministic process by which prices are chosen as a function of actions of agents. Then prices cannot depend on anything which is not known to some agent.

On the other hand, suppose an "auctioneer" randomizes to determine prices. Then states may include a description of the auctioneer's randomizing device, and prices need not satisfy price join measurability. It will be useful for our lemmas to have an explicit description of how uncertainty about equilibrium prices might require the state space to be expanded. Consider the following example.

Example 2.4. Suppose there are two agents, 1 and 2, and two goods, 1 and 2.  $S = \{a; b\}$ ;  $\frac{1}{2}(a) = \frac{1}{2}(b) = \frac{1}{2}$ ;  $I^1 = \{a; b\}$  and  $I^2 = \{a; b\}$ ;  $e^1(s) = (1; 0)$  and  $e^2(s) = (0; 1)$  for  $s = a; b$ .

$$u^1(x_1; x_2; s) = \frac{1}{2} \ln(x_1) + \frac{1}{2} \ln(x_2), \text{ for } s = a; b$$

$$u^1(x_1; x_2; s) = \begin{cases} \frac{2}{3} \ln(x_1) + \frac{1}{3} \ln(x_2), & \text{if } s = a \\ \frac{1}{3} \ln(x_1) + \frac{2}{3} \ln(x_2), & \text{if } s = b \end{cases}$$

This economy has two rational expectations equilibria (satisfying the definition above). Let good 1 be a numeraire good. There is a non-revealing equilibrium with  $p_2(a) = p_2(b) = 1$ ,  $x^1(a) = x^1(b) = (\frac{1}{2}; \frac{1}{2})$  and  $x^2(a) = x^2(b) = (\frac{1}{2}; \frac{1}{2})$ . But there is also a fully revealing equilibrium with  $p_2(a) = \frac{3}{4}$ ,  $p_2(b) = \frac{3}{2}$ ,  $x^1(a) = (\frac{1}{2}; \frac{2}{3})$ ,  $x^1(b) = (\frac{1}{2}; \frac{1}{3})$  and  $x^2(a) = (\frac{1}{2}; \frac{1}{3})$ ,  $x^2(b) = (\frac{1}{2}; \frac{2}{3})$ .

But it is at least consistent with common knowledge of this data and the model for there to be uncertainty about what the price would have been, contingent on another state occurring. Suppose that in state b, the relative price of good 2 is always  $\frac{5}{4}$ , but in state a, an auctioneer randomizes and sets the price equal to  $\frac{5}{4}$  with probability  $\frac{1}{4}$  and  $\frac{3}{4}$  with probability  $\frac{3}{4}$ . In this case, if agent 2 observes a price of  $\frac{3}{4}$ , he knows the state is a, but if he observes a price of  $\frac{5}{4}$ , he assigns (by Bayes updating) probability  $\frac{1}{5}$  to state a and probability  $\frac{4}{5}$  to state b. In this case his expected utility is

$$EU^2(x_1; x_2) = \frac{1}{5}u^2(x_1; x_2; a) + \frac{4}{5}u^2(x_1; x_2; b) = \frac{2}{5} \ln(x_1) + \frac{3}{5} \ln(x_2)$$

But now markets clear if agents 1 and 2 consume  $(\frac{1}{2}; \frac{2}{3})$  and  $(\frac{1}{2}; \frac{1}{3})$  respectively whenever the price is  $\frac{3}{4}$ ; and  $(\frac{1}{2}; \frac{2}{5})$  and  $(\frac{1}{2}; \frac{3}{5})$  respectively whenever the price is  $\frac{5}{4}$ .

Dutta and Morris (1995) allow for this outcome explicitly as follows. Write  $\Phi(T)$  for the set of simple (finite) probability distributions on any finite or infinite set  $T$ . A price randomization is a function  $\pm : S \rightarrow \Phi(<_{++}^L)$ . Write  $-_{\pm}$  for the set of feasible state price pairs under  $\pm$ , i.e.  $-_{\pm} = \{f(s; p) : \pm(pj_s) > 0\}$ ; and  $-_{\pm}^h$  for the set of state price pairs,  $(s; p)$ , where  $h$  believes  $p$  possible in state  $s$ , i.e.  $-_{\pm}^h = \{f(s; p) : \pm(pj_s^h) > 0 \text{ for some } s^h \in I^h(s)\}$ .

**Definition 2.5.** A common belief equilibrium consists of a price function,  $\pm : S \rightarrow \Phi(<_{++}^L)$ , and, for each agent  $h$ , a state contingent commodity bundle,  $x^h : -_{\pm}^h \rightarrow <_{++}^L$ , such that

[1] Feasibility:  $\sum_{h=1}^H p x^h(s; p) \cdot p e^h(s)$  for all  $h; (s; p) \in S$ :

[2] Rationality:  $\sum_{s^0 \in S^0} p \frac{p^{\frac{1}{4}}(s^0) \pm (p_j s^0)}{\frac{1}{4}(s^0) \pm (p_j s^0)} u^h(y; s^0) > \sum_{s^0 \in S^0} p \frac{p^{\frac{1}{4}}(s^0) \pm (p_j s^0)}{\frac{1}{4}(s^0) \pm (p_j s^0)} u^h(x^h; s^0)$   
 $) \quad p y > p e^h(s), \text{ for all } h, (s; p) \in S$

[3] Market Clearing:  $\sum_{h=1}^H x^h(s; p) = \sum_{h=1}^H e^h(s), \text{ for all } (s; p) \in S$

But notice that, because price joint measurability was not imposed, such common belief equilibria can always be thought of as REE on a larger state space, including the payoff irrelevant uncertainty reflected in prices. The common belief equilibrium in the example above can now be written formally as:  $\pm(\frac{5}{4}; b) = 1$ ,  $\pm(\frac{5}{4}; a) = \frac{1}{4}$  and  $\pm(\frac{3}{4}; a) = \frac{3}{4}$ ;  $x^1(a; \frac{3}{4}) = (\frac{1}{2}; \frac{2}{3})$ ,  $x^1(a; \frac{5}{4}) = x^1(b; \frac{5}{4}) = (\frac{1}{2}; \frac{2}{5})$  and  $x^2(a; \frac{3}{4}) = (\frac{1}{2}; \frac{1}{3})$ ,  $x^2(\frac{5}{4}; a) = x^2(\frac{5}{4}; b) = (\frac{1}{2}; \frac{2}{5})$ .

Notice that since agent 1 did not care whether state a or b occurred, the example is non-generic. Dutta and Morris (1995) show that a continuum of non-trivial common belief equilibria exist in a robust class of economies. This paper, however, focuses on the narrower question of the foundations for the equilibrium solution concepts discussed in this section, which I now address.

### 3. Epistemic Foundations for General Equilibrium solution concepts

#### 3.1. Framework

I now want to consider a state space which describes not only exogenous variables (the data of the economy), but also endogenous variables (prices and agents' consumption choices). It is necessary to do this in order to formalize agents' knowledge of things like market clearing and rationality. The description given here is formally very similar to the standard framework described in the previous section, but has a quite different interpretation. In particular, when I write, for example, the price  $p$  as a function of a state, this is part of the description of that state, not any kind of equilibrium statement. Equilibrium conditions, in the standard formulation, will be represented in this formulation by the set of states where they are true.

Write  $U$  for a set of possible utility functions,  $u : \mathbb{R}_+^L \rightarrow \mathbb{R}$ . Write  $\Omega$  for the finite state space, with common prior  $\mu$ . Each state is a complete description of the relevant aspects of the world. Thus, for each state  $\omega$ , there is a price  $p(\omega)$  and, for each agent  $h$ , a utility function  $u^h(\omega)$ , endowment  $e^h(\omega)$ , consumption bundle  $x^h(\omega)$  and set of possible states  $I^h(\omega)$ . All this information can be summarized by functions representing price,  $p : \Omega \rightarrow \mathbb{R}_+^L$ , and for each agent,  $h = 1, \dots, H$ , a utility selection  $u^h : \Omega \rightarrow U$ , an endowment vector  $e^h : \Omega \rightarrow \mathbb{R}_+^L$ , and a commodity bundle  $x^h : \Omega \rightarrow \mathbb{R}_+^L$ . Each agent  $h$  has a partition representing his information on the state space,  $I^h$ .

With some abuse of notation, write  $u^h(x; \omega)$  for  $f(x)$  where  $f = u^h(\cdot)$ . Finally, it is useful to introduce notation for an expected utility function:

$$v^h(\omega) = \sum_{\omega' \in I^h(\omega)} \mu[\omega' | \omega] u^h(\omega')$$

Thus  $v^h(\omega)$  is the  $h$ 's expected utility function from commodity bundles to utility if the true state is  $\omega$ . Note that  $v^h$  is  $I^h$  measurable by construction. The following notation will also be useful

$$v^h(x; \omega) = \sum_{\omega' \in I^h(\omega)} \mu[\omega' | \omega] u^h(x; \omega')$$

Now equilibrium restrictions from the standard framework are translated into sets in this framework. Thus write  $C$  for the set of states where markets clear:-

$$C = \left\{ \omega : \sum_{h=1}^H x^h(\omega) = \sum_{h=1}^H e^h(\omega) \right\}$$

Write  $F^h$  for the event "agent  $h$  makes a feasible choice":-

$$F^h = \left\{ \omega : p(\omega) x^h(\omega) \cdot p(\omega) e^h(\omega) \leq 0 \right\}$$

Write  $R^h$  for the event "agent  $h$  makes a rational choice":-

$$R^h = \left\{ \omega : v^h(y; \omega) > v^h(x^h(\omega); \omega) \implies p(\omega) y \cdot p(\omega) e^h(\omega) > 0 \right\}$$

Write  $E^\mu$  for the event "prices and commodity bundles constitute a competitive equilibrium of the actual economy":-

$$E^{\pi} = \{ \omega : p(\omega); x^h(\omega)_{h=1:H} \text{ is a competitive equilibrium of the economy } e^h(\omega); u^h(\omega)_{h=1:H} \}$$

Thus  $E^{\pi}$  represents the set of states where prices and demands would be an equilibrium if all agents knew the true state.

Write  $E^{\pi^e}$  for the event "prices and commodity bundles constitute a competitive equilibrium of the economy with expected utility functions":-

$$E^{\pi^e} = \{ \omega : p(\omega); x^h(\omega)_{h=1:H} \text{ is a competitive equilibrium of the economy } e^h(\omega); v^h(\omega)_{h=1:H} \}$$

Thus  $E^{\pi^e}$  represents the set of states where prices and demands would be an equilibrium of the economy if it was common knowledge that each agent's utility function was his expected utility function.

These are the crucial events characterizing fundamentals. But we need a description of agents' knowledge of fundamentals as well. Define knowledge operator  $K^h$  as follows:-

$$K^h(A) = \{ \omega : I^h(\omega) \cap A \neq \emptyset \}$$

Thus  $K^h$  defines knowledge of events: for example, we would say that  $h$  knows that markets clear if  $\omega \in K^h(C)$ . We will also be concerned with when agents know the realization of a function. Suppose  $f : \Omega \rightarrow S$  for some set  $S$ . Agent  $h$  knows  $f$  at state  $\omega$  if he knows the realization of  $f$ . Thus

$$K^h(f) = \{ \omega : \exists ! \omega^0 \in I^h(\omega) \text{ such that } f(\omega^0) = f(\omega) \}$$

Endowments, prices, demand, and utility functions are all functions of the state. Clearly, we will only be interested in states where agents know their own endowment, demand and current market prices.

### 3.2. Results

Now we have:-



Lemma 3.1. Suppose each agent knows prices, and his own endowment and demand. Suppose markets clear and each agent receives a feasible and rational commodity bundle. Then prices and commodity bundles constitute a competitive equilibrium in expected utilities. Formally:

$$C \setminus \bigcap_{h=1}^H \{F^h \setminus R^h \setminus \cdot^h(p) \setminus \cdot^h(e^h) \setminus \cdot^h(x^h)\} \supseteq E^{ex}$$

Proof.  $C \setminus \bigcap_{h=1}^H \{F^h \setminus R^h\} \supseteq E^{ex}$  immediately by definitions.

The assumptions that each agent knows prices and his own endowment and demand thus plays no formal role in the analysis. Notice that this result allowed for asymmetric information, because agents may not know their own utility function. What is the relation between actual CE and CE in expected utilities? If an agent knows his own utility function, then  $u^h(!) = v^h(!)$ ; thus conditional on all agents knowing their own utility functions,  $E^a$  and  $E^{ex}$  coincide. Thus:

Lemma 3.2. If all agents know their own utility function, then all CE in expected utilities are actual CE i.e.

$$E^{ex} \setminus \bigcap_{h=1}^H \cdot^h(u^h) \supseteq E^a$$

Proof.  $! \in \cdot^h(u^h) \Rightarrow u^h(!) = v^h(!)$  for all  $! \in I^h \Rightarrow u^h(!) = v^h(!)$ .

Corollary 3.3. Suppose each agent knows prices, his own endowment, demand and utility function. Suppose markets clear and each agent receives a feasible and rational commodity bundle. Then prices and commodity bundles constitute an actual competitive equilibrium. Formally:

$$C \setminus \bigcap_{h=1}^H \{F^h \setminus R^h \setminus \cdot^h(p) \setminus \cdot^h(e^h) \setminus \cdot^h(x^h) \setminus \cdot^h(u^h)\} \supseteq E^a$$

Proof. Immediate from lemma 3.1 and lemma 3.2.

The critical assumption in corollary 3.3 was that each agent knows his own utility function. In this case there is nothing worth learning from prices. At those states where the conditions of the corollary hold, there is no relevant asymmetric information. When we drop this assumption, stronger common knowledge

restrictions are required to restore rational expectations equilibrium. Common knowledge is formally defined as follows.

Refer to any function  $f : \Omega \rightarrow H^g$  as an indexing function of degree  $n$ . Then  $f(1); f(2); \dots; f(n)$  is some sequence of names of agents. Write  $F_n$  for the collection of such indexing functions of degree  $n$ . Then the set of states where event  $A$  is common knowledge -  $K^c A$  - is defined to be the set of states where  $f(1)$  knows that  $f(2)$  knows that...  $f(n)$  knows that  $A$ , for every indexing function of every length. Thus:-

$$K^c A = \bigcap_{f \in F_n; n \geq 1} K^{f(1)} K^{f(2)} \dots K^{f(n)} A$$

The only property of common knowledge that is required here is the following well-known fixed point characterization of common knowledge (see Aumann (1976)).

Lemma 3.4.  $K^h K^c A = K^c A$ , for all  $h$ , for every event  $A$ .

The following lemma will be useful. Say that function  $f : \Omega \rightarrow S$  is  $I^h$  measurable on event  $A$  if  $\{\omega \in \Omega \mid I^h(\omega) \cap A \neq \emptyset\} = f^{-1}(A)$  for all  $A \subseteq S$ .

Lemma 3.5.  $A \subseteq K^c \cdot^h(f) \Rightarrow f$  is  $I^h$  measurable on event  $A$ .

Proof.  $A \subseteq K^c \cdot^h(f) \Rightarrow A \subseteq \cdot^h(f) \Rightarrow f(I^h(\omega)) = f(\omega)$  for all  $\omega \in I^h(\omega) \cap A$  and  $\omega \in A \Rightarrow f$  is  $I^h$  measurable on event  $A$ .

Now we have the following lemma.

Lemma 3.6. Consider the set of states where it is common knowledge that [1] each agent knows the current price and his own endowment and demand and [2] markets clear and each agent receives a feasible and rational commodity bundle. Then prices and commodity bundles restricted to that set constitute a rational expectations equilibrium of the economy restricted to that set. Formally, consider the event

$$A = K^c \left\{ C \cap \bigcap_{h=1}^H \{F^h \cap R^h \cap \cdot^h(p) \cap \cdot^h(e^h) \cap \cdot^h(x^h)\} \right\}$$

Then  $p$  and  $\{x^h\}_{h=1; \dots; H}$  restricted to the event  $A$  constitute an REE according to the definition of the previous section.

Proof.  $A \frac{1}{2} K^c \bigwedge_{h=1}^H F^h$  implies part 1 of definition 2.1 holds for all states in A.

$A \frac{1}{2} K^c \bigwedge_{h=1}^H R^h$  implies part 2 of definition 2.1 holds for all states in A.

$A \frac{1}{2} K^c(C)$  implies part 3 of definition 2.1 holds for all states in A. By lemma 3.5 and  $A \frac{1}{2} K^c \cdot^h(e^h)$ , we have  $e^h$  measurable with respect to  $I^h$  for each h on event A; this was assumed to be a property of the data in the definition of REE. By lemma 3.5 and  $A \frac{1}{2} K^c \cdot^h(x^h) \setminus \cdot^h(e^h)$ , we have p and  $x^h$  measurable with respect to  $I^h$  for each h on event A; thus property 4 of definition 2.1 is satisfied.

### 3.3. Examples

I will give three examples illustrating the framework and results; they also show that the results are tight. Consider first example 2.4 from section 2. We can represent it in the language of this section. Write  $f^\otimes$  for the function

$$f^\otimes(x_1; x_2) = \otimes \ln(x_1) + (1 - \otimes) \ln(x_2)$$

Example 3.7.  $\otimes = f^1; !_1; !_2; !_3g; \frac{1}{4}(!_1) = \frac{3}{8}, \frac{1}{4}(!_2) = \frac{1}{8}$  and  $\frac{1}{4}(!_3) = \frac{1}{2}; p(!_1) = (1; \frac{3}{4}), p(!_2) = (1; \frac{5}{4})$  and  $p(!_3) = (1; \frac{5}{4}); e^1(!) = (1; 0)$  and  $e^2(!) = (0; 1)$  for all  $!$ ;  $u^1(!) = f_{\frac{1}{2}}$  for all  $!$ ,  $u^2(!_1) = u^2(!_2) = f_{\frac{2}{3}}$ , and  $u^2(!_3) = f_{\frac{1}{3}}$ ;  $x^1(!_1) = (\frac{1}{2}; \frac{2}{3}), x^1(!_2) = x^1(!_3) = (\frac{1}{2}; \frac{2}{5}), x^2(!_1) = (\frac{1}{2}; \frac{1}{3}), x^2(!_2) = x^2(!_3) = (\frac{1}{2}; \frac{3}{5}); I^1 = (f^1; !_1g; f^1; !_2g; f^1; !_3g)$  and  $I^2 = (f^1; !_1g; f^1; !_2; !_3g)$ .

We can summarize the example as follows:

	$!_1$	$!_2$	$!_3$
$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
$u^1$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$
$v^1$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$
$u^2$	$f_{\frac{2}{3}}$	$f_{\frac{2}{3}}$	$f_{\frac{1}{3}}$
$v^2$	$f_{\frac{2}{3}}$	$f_{\frac{2}{5}}$	$f_{\frac{2}{5}}$
$e^1$	(1; 0)	(1; 0)	(1; 0)
$e^2$	(0; 1)	(0; 1)	(0; 1)
p	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{5}{4}$
$x^1$	$(\frac{1}{2}; \frac{2}{3})$	$(\frac{1}{2}; \frac{2}{5})$	$(\frac{1}{2}; \frac{2}{5})$
$x^2$	$(\frac{1}{2}; \frac{1}{3})$	$(\frac{1}{2}; \frac{3}{5})$	$(\frac{1}{2}; \frac{3}{5})$

Consider lemma 3.6 first. Since each agent always knows his own endowment, demand and the current price, and markets always clear and agents always make feasible and rational choices, then the event that this is true is certainly common knowledge. So prices and commodity bundles constitute a REE of this economy. But notice that states  $\omega_1$  and  $\omega_2$  are identical with respect to fundamentals. If we fixed the economy as we originally motivated it in the previous section, this REE would require a violation of price joint measurability.

The requirements of lemma 3.1 are satisfied at all states, so CE in expected utilities are chosen at all states (i.e.  $E^{\pi} = -$ ).

Now consider corollary 3.3. At states  $\omega_2$  and  $\omega_3$ , agent 2 does not know his own utility function. So there is no requirement that prices and commodity bundles at states  $\omega_2$  and  $\omega_3$  are competitive equilibria of the economies at those respective states (as indeed they are not). At state  $\omega_1$ , the conditions of lemma are satisfied, and we have a competitive equilibrium (thus  $E^{\pi} = f!_1g$ ).

There was little value added in the above example, because we did not use our freedom to model situations where the assumptions of general equilibrium are not common knowledge. Consider the following example.

Example 3.8.  $\omega = (f!_1; !_2; :::; !_{2N}g; \frac{1}{2N}(!_i) = \frac{1}{2N}$  for all  $i$ ;  $p(!) = (1; 1)$ , for all  $!$ ;  $e^1(!) = (1; 0)$  and  $e^2(!) = (0; 1)$  for all  $!$ ;  $u^1(!_i) = f_{\frac{1}{3}}$  for all  $i$  odd,  $u^1(!_i) = f_{\frac{2}{3}}$  for all  $i \in 2N$  even and  $u^1(!_{2N}) = f_{\frac{1}{2}}$ ;  $u^2(!_i) = f_{\frac{1}{3}}$  for all  $i$  odd and  $u^2(!_i) = f_{\frac{2}{3}}$  for all  $i$  even;  $x^1(!_1) = (\frac{1}{3}; \frac{2}{3})$ ,  $x^1(!) = (\frac{1}{2}; \frac{1}{2})$  for all  $! \in !_1$ ;  $x^2(!) = (\frac{1}{2}; \frac{1}{2})$  for all  $!$ ;  $I^1 = (f!_1g; f!_2; !_3g; :::; f!_{2N_i-2}; !_{2N_i-1}g; f!_{2N}g)$  and  $I^2 = (f!_1; !_2g; f!_3; !_4g; :::; f!_{2N_i-1}; !_{2N}g)$

We can summarize the example as follows:

	$\omega_1$	$\omega_i$ i even, $i \in 2N$	$\omega_i$ i odd, $i \in 1$	$\omega_{2N}$
$\frac{1}{4}$	$\frac{1}{2N}$	$\frac{1}{2N}$	$\frac{1}{2N}$	$\frac{1}{2N}$
$u^1$	$f_{\frac{1}{3}}$	$f_{\frac{2}{3}}$	$f_{\frac{1}{3}}$	$f_{\frac{1}{2}}$
$v^1$	$f_{\frac{1}{3}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$
$u^2$	$f_{\frac{1}{3}}$	$f_{\frac{2}{3}}$	$f_{\frac{1}{3}}$	$f_{\frac{2}{3}}$
$v^2$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$	$f_{\frac{1}{2}}$
$e^1$	(1; 0)	(1; 0)	(1; 0)	(1; 0)
$e^2$	(0; 1)	(0; 1)	(0; 1)	(0; 1)
$p_2$	1	1	1	1
$x^1$	$(\frac{1}{3}, \frac{2}{3})$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$
$x^2$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$

Let us check the conditions of lemma 3.6 carefully. Both agents always choose feasible and rational commodity bundles, so  $F^1 = F^2 = R^1 = R^2 = -$ , and thus these events are always common knowledge. Similarly, both agents always know prices and their endowment and own commodity bundle. However, markets do not clear in state  $\omega_1$ . Thus the event that markets clear  $C = \{\omega_i; i \in 2N\}$ . Let us consider what is known about market clearing at other states:  $K^1 C = \{\omega_i; i \in 2N\}$ ,  $K^2 K^1 C = K^2 C = \{\omega_i; i \in 2N\}$ ,  $K^1 K^2 K^1 C = K^1 K^2 C = \{\omega_i; i \in 2N\}$  etc. Thus  $K^C C = \emptyset$ ; there is nowhere common knowledge of market clearing, so we have no prediction of rational expectation equilibria being played.

Let us elucidate this result by considering a particular state,  $\omega_{2N}$ . At state  $\omega_{2N}$ , both agents know prices, their own endowments and commodity bundles. They are both maximizing their utility function subject to their budget constraints. All this is common knowledge. In addition, markets are clearing, both agents know that markets are clearing, and each agent knows that the other knows that the market is clearing, and so on  $N$  times. But lack of common knowledge of market clearing means that no standard equilibrium interpretation can be given to the outcome.

Finally, notice that the conditions of lemma 3.1 are satisfied at all states in  $C$ , thus  $E^{\text{ra}} = C = \{\omega_i; i \in 2N\}$ . But agents never know their utility function, so corollary 3.3 has no content and  $E^{\text{ra}} = \emptyset$ .

Example 3.8 can easily be altered so that there is common knowledge of market clearing but not common knowledge of rationality.

Example 3.9. Identical to example 3.8, except that  $x^1(\omega_1) = \frac{1}{2}; \frac{1}{2}$ .

Now  $C = \emptyset$ , so market clearing is always common knowledge,  $F^1 = F^2 = R^2 = \emptyset$ , so these events are always common knowledge also. But now  $R^1 = \{\omega_2; \dots; \omega_{2N}\}$ . So again, we get no rational expectation equilibrium, no competitive equilibrium and expected competitive equilibria at states  $\{\omega_2; \dots; \omega_{2N}\}$  only.

#### 4. Discussion

The classical theory of markets shows that equilibrium can be decentralized: each participant in the market need only maximize his own utility subject to prices and market clearing will ensure a Pareto-efficient equilibrium. This important insight is not robust to its assumptions: if there is any asymmetric information whatsoever, then the natural equilibrium analogue of competitive equilibrium - that is, rational expectations equilibrium - requires that market participants understand the structure of the economy and equilibrium, and thus these are common knowledge. This paper gave a formal version of these claims.

The results did not have much mathematical content: they were proved by manipulation of the definitions. The results are useful to the extent that they clarify what lies behind equilibrium solution concepts. In this, they parallel the work of Aumann and others in game theory which was cited earlier. Indeed, an alternative approach to this paper would have started with the known game theory results and applied them to a game which had competitive equilibria and/or rational expectations equilibria as their Nash equilibria. For example, the "market clearing" assumption in this paper would correspond to the rationality of a price player (minimizing excess demands) in the artificial game of Arrow and Debreu (1954).

One difficulty with this approach is that there are a number of problems giving strategic foundations to general equilibrium. It was a deliberate modelling choice in this paper to stay as close as possible to the spirit of general equilibrium (for example, having "price taking" built into the definition of rationality). This is based on the belief that issues concerning strategic foundations are essentially orthogonal to the issues considered here. It should be noted, however, that there

are some particular problems with modelling agents' learning from contemporaneous prices in a game theoretic setting. Agents must be assumed to be able to condition their equilibrium actions on others' simultaneous actions. Krishna (1986) and Minelli and Polemarchakis (1993) explore ways of doing this in game theoretic contexts.

## References

- [1] Arrow, K. and G. Debreu (1954). "Existence of an Equilibrium for a Competitive Economy," *Econometrica*, 22, 14-22.
- [2] Aumann, R. (1976). "Agreeing to Disagree," *The Annals of Statistics*, 4, 1236-1239.
- [3] Aumann, R. (1987). "Correlated Equilibrium as an Expression of Bayesian Rationality," *Econometrica*, 55, 1-18.
- [4] Aumann, R. and A. Brandenburger (1993). "Epistemic Conditions for Nash Equilibrium."
- [5] Brandenburger, A. and E. Dekel (1987). "Rationalizability and Correlated Equilibria," *Econometrica*, 55, 1391-1402.
- [6] Dutta, J. and S. Morris (1995). "The Revelation of Information and Self-Fulfilling Beliefs," CARESS Working Paper #95-03, University of Pennsylvania.
- [7] Krishna, V. (1986). "Strategic Informational Equilibrium."
- [8] McAllister, P. (1990). "Rational Behavior and Rational Expectations," *J. Econ. Theory*, 52, 332-363.
- [9] Minelli, E. and H. Polemarchakis (1993). "Knowledge at Equilibrium," CORE Discussion Paper #9354.
- [10] Radner, R. (1979). "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," *Econometrica*, 47, 655-678.