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PIER Working Paper 15-031

“Distributional Incentives in an Equilibrium Model of
Domestic Sovereign Default

by

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<http://ssrn.com/abstract=2660247>

Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default*

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27 August 2015

Abstract

Europe's debt crisis resembles historical episodes of outright default on domestic public debt about which little research exists. This paper proposes a theory of domestic sovereign default based on distributional incentives affecting the welfare of risk-averse debt and non debtholders. A utilitarian government cannot sustain debt if default is costless. If default is costly, debt with default risk is sustainable, and debt falls as the concentration of debt ownership rises. A government favoring bond holders can also sustain debt, with debt rising as ownership becomes more concentrated. These results are robust to adding foreign investors, redistributive taxes, or a second asset.

*We acknowledge the generous support of the National Science Foundation under awards 1325122 and 1324740. Comments by Jesse Schreger, Fabrizio Perri, Vincenzo Quadrini and by participants at presentations at Columbia University, the 2014 Winter Meetings of the Econometric Society, and the NBER Conference on Sovereign Debt and Financial Crises are also gratefully acknowledged. We thank Jingting Fan for excellent research assistance. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

Keywords: public debt, sovereign default, European debt crisis

JEL Classifications: E6, E44, F34, H63

1. Introduction

The seminal study by Reinhart and Rogoff (2011) identified 68 episodes in which governments defaulted outright (i.e., by means other than inflation) on their domestic creditors in a cross-country database going back to 1750. These domestic defaults occurred via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspensions of payments. Reinhart and Rogoff also documented that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about two-thirds on average) and that domestic defaults were associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses, and full-blown economic crises. Despite these striking features, the authors also found that domestic sovereign default is a “forgotten history” that remains largely unexplored in economic research.

The ongoing European debt crisis also highlights the importance of studying domestic sovereign default. In particular, four features of this crisis make it more akin to a domestic default than to the typical external default that dominates the literature on public debt default. First, countries in the Eurozone are highly integrated, with the majority of their public debt denominated in their common currency and held by European residents. Hence, from a European standpoint,

default by one or more Eurozone governments means a suspension of payments to “domestic” agents, instead of external creditors. Second, domestic public-debt-GDP ratios are high in the Eurozone in general, and very large in the countries at the epicenter of the crisis (Greece, Ireland, Italy, Spain, and Portugal). Third, the Eurozone’s common currency and common central bank rule out the possibility of individual governments resorting to inflation as a means to lighten their debt burden without an outright default. Fourth, and perhaps most important from the standpoint of the theory proposed in this paper, European-wide institutions such as the European Central Bank (ECB) and the European Commission are weighting the interests of both creditors and debtors in assessing the pros and cons of sovereign defaults by individual countries, and creditors and debtors are aware of these institutions’ concern and of their key role in influencing expectations and default risk.¹ Hall and Sargent (2014) document a similar situation in the process by which the U.S. government handled the management of its debt in the aftermath of the Revolutionary War.

Table 1 shows that the Eurozone’s fiscal crisis has been characterized by rapid increases in public debt ratios and sovereign spreads that coincided with rising government expenditure ratios. The table also shows that debt ownership, as proxied

1. The analogy with a domestic default is imperfect, however, because the Eurozone is not a single country, and in particular, there is no fiscal entity with tax and debt-issuance powers over all the members.

by Gini coefficients of wealth distributions, is unevenly distributed in the seven countries listed, with mean and median Gini coefficients of around two-thirds.²

TABLE 1. Euro Area: Key Fiscal Statistics and Wealth Inequality

Moment (%)	Gov. Debt		Gov. Exp.		Spreads		Gini Wealth
	Avg.	2011	Avg.	“crisis peak”	Avg.	“crisis peak”	
France	34.87	62.72	23.40	24.90	0.08	1.04	0.73
Germany	33.34	52.16	18.80	20.00	-	-	0.67
Greece	84.25	133.09	18.40	23.60	0.37	21.00	0.65
Ireland	14.07	64.97	16.10	20.50	0.11	6.99	0.58
Italy	95.46	100.22	19.40	21.40	0.27	3.99	0.61
Portugal	35.21	75.83	20.00	22.10	0.20	9.05	0.67
Spain	39.97	45.60	17.60	21.40	0.13	4.35	0.57
Avg.	48.17	76.37	19.10	21.99	0.22	7.74	0.64
Median	35.21	64.97	18.80	21.40	0.17	5.67	0.65

Note: Author’s calculations are based on OECD Statistics, Eurostat, ECSB and Davies, Sandstrom, Shorrocks, and Wolff (2009). “Gov. Debt” refers to Total General Government Net Financial Liabilities (avg 1990-2007); “Gov. Exp.” corresponds to government purchases in National Accounts (avg 2000-2007); “Sov Spreads” correspond to the difference between interest rates of the given country and Germany for bonds of similar maturity (avg 2000-2007). For a given country i , they are computed as $(1 + r^i)/(1 + r^{Ger}) - 1$. “Crisis Peak” refers to the maximum value observed during 2008-2012 using data from Eurostat. “Gini Wealth” are Gini wealth coefficients for 2000 from Davies, Sandstrom, Shorrocks, and Wolff (2009) Appendix V.

Taken together, the history of domestic defaults and the risk of similar defaults in Europe pose two important questions: What accounts for the existence of domestic debt ratios exposed to default risk? And can the concentration of the ownership of government debt be a determinant of domestic debt exposed to default risk?

This paper aims to answer these questions by proposing a framework for explaining domestic sovereign defaults driven by distributional incentives. This framework is motivated by the key fact that a domestic default entails substantial

2. In Section A.1 of the Appendix, we present a more systematic analysis of the link between debt and inequality and show that government debt is increasing in inequality when inequality is low but decreasing for high levels of inequality.

redistribution across domestic agents, with all of these agents, including government debtholders, entering in the payoff function of the sovereign. This is in sharp contrast to what standard models of external sovereign default assume, particularly those based on the classic work of Eaton and Gersovitz (1981).

We propose a tractable two-period model with heterogeneous agents and noninsurable aggregate risk in which domestic default can be optimal for a government responding to distributional incentives. A fraction γ of agents are low-wealth (L) agents who do not hold government debt, and a fraction $1 - \gamma$ are high-wealth (H) agents who hold the debt. The government finances the gap between exogenous stochastic expenditures and endogenous taxes by issuing nonstate-contingent debt, retaining the option to default. In our benchmark case, the government is utilitarian, so the social welfare function assigns the weights γ and $1 - \gamma$ to the welfare of L and H agents, respectively.

If the government is utilitarian and default is costless, the model cannot support an equilibrium with debt. This is because for any given level of debt that could have been issued in the first period, the government always attains the second period's socially efficient levels of consumption allocations and redistribution by choosing to default, and if default in period two is certain the debt market collapses in the first period. An equilibrium with debt under a utilitarian government can exist if default entails an exogenous cost in terms of disposable income. When default is costly, repayment becomes optimal if the amount of period-two consumption dispersion that

the competitive equilibrium with repayment supports yields higher welfare than the default equilibrium net of default cost.

Alternatively, we show that an equilibrium with debt can be supported if the government's payoff function displays a "political" bias in favor of bondholders, even if default is costless. In this case, the government's weight on H -type agents is higher than the actual fraction of these agents in the distribution of bond holdings. In this extension, the debt is an *increasing* function of the concentration of debt ownership, instead of *decreasing* as in the utilitarian case. This is because incentives to default get weaker as the government's weight on L -type agents falls increasingly below γ . The model with political bias also yields the interesting result that agents who do not hold public debt may prefer a government that weighs bond holders more than a utilitarian government. This is because the government with political bias has weaker default incentives, and can thus sustain higher debt at lower default probabilities, which relaxes a liquidity constraint affecting agents who do not hold public debt by improving tax smoothing.

We also explore three other important extensions of the model to show that the main result of the benchmark model, namely, the existence of equilibria with domestic public debt exposed to default risk, is robust. We examine extensions opening the economy so that a portion of the debt is held by foreign investors, introducing taxation as another instrument for redistributive policy and adding a second asset as an alternative vehicle for saving.

This work is related to various strands of the extensive literature on public debt. First, studies on public debt as a self-insurance mechanism and a vehicle that alters consumption dispersion in heterogeneous agents models without default, such as Aiyagari and McGrattan (1998), Golosov and Sargent (2012), Azzimonti, de Francisco, and Quadrini (2014), Floden (2001) and Heathcote (2005).³

A second strand is the literature on external sovereign default in the line of the Eaton and Gersovitz (1981) model (e.g. Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2012), and Yue (2010)).⁴ Also in this literature, and closely related, Aguiar and Amador (2013) analyze the interaction between public debt, taxes, and default risk and Lorenzoni and Werning (2013) study the dynamics of debt and interest rates in a model in which default is driven by insolvency and debt issuance follows a fiscal rule.

3. A related literature initiated by Aiyagari, Marcet, Sargent, and Seppala (2002) studies optimal taxation and public debt dynamics with aggregate uncertainty and incomplete markets, but in a representative-agent environment. Pouzo and Presno (2014) extended this framework to incorporate default and renegotiation.

4. See Panizza, Sturzenegger, and Zettelmeyer (2009), Aguiar and Amador (2014), and Tomz and Wright (2012) for recent reviews of the sovereign debt literature. Some studies in this area have examined models that include tax and expenditure policies, as well as settings with foreign and domestic lenders, but always maintaining the representative agent assumption (e.g. Cuadra, Sanchez, and Sapriza (2010), Vasishtha (2010) and more recently Dias, Richmond, and Wright (2012) have examined the benefits of debt relief from the perspective of a *global* social planner with utilitarian preferences.

A third strand is the literature on political economy and sovereign default, which also focuses mostly on external default (e.g. Amador (2003), Dixit and Londregan (2000), D’Erasmus (2011), Guembel and Sussman (2009), Hatchondo, Martinez, and Sapriza (2009), and Tabellini (1991)). A few studies such as those of Alesina and Tabellini (1990) and Aghion and Bolton (1990) focus on political economy aspects of government debt in a closed economy, including default, and Aguiar, Amador, Farhi, and Gopinath (2013) examine optimal policy in a monetary union subject to self-fulfilling debt crises.

A fourth important strand of the literature focuses on the consequences of default on domestic agents, the role of secondary markets, discriminatory versus nondiscriminatory default, and the role of domestic debt in providing liquidity (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), Mengus (2014) and Di Casola and Sichlimiris (2014)).⁵ As in most of these studies, default in our setup is nondiscriminatory, because the government cannot

5. Motivated by the recent financial crisis and extending the theoretical work of Gennaioli, Martin, and Rossi (2014), a set of recent papers focuses on the interaction between sovereign debt and domestic financial institutions, such as Sosa-Padilla (2012), Bocola (2014), Boz, D’Erasmus, and Durdu (2014), and Perez (2015).

discriminate across any of its creditors when it defaults. Our analysis differs in that default is driven by distributional incentives.⁶

Finally, there is also a newer literature that is closer to our work in that it studies the trade-offs between distributional incentives to default on domestic debt and the use of debt in infinite-horizon models with heterogeneous agents (see, in particular, D’Erasmus and Mendoza (2014) and DAVIS, Golosov, and Shourideh (2014)). In D’Erasmus and Mendoza, debt is determined by a fiscal rule, while in this paper, we model public debt as an optimal choice and derive analytical expressions that characterize equilibrium prices and the solution of the government’s problem. Our work differs from DAVIS et al. in that they assume complete domestic asset markets, so their analysis abstracts from the role of public debt in providing social insurance, while the nonstate-contingent nature of public debt plays a central role in the distributional incentives we examine here and in the endogenous default costs studied in D’Erasmus and Mendoza (2014). In addition, DAVIS et al. focus on the solution to a Ramsey problem that supports equilibria in which default is not observed along the equilibrium path, while in our work default is an equilibrium outcome.⁷

6. Andreasen, Sandleris, and van der Gote (2011), Ferriere (2014) and Jeon and Kabukcuoglu (2014) study environments in which domestic income heterogeneity plays a central role in the determination of external defaults.

7. See also Golosov and Sargent (2012) who study debt dynamics without default risk in a similar environment.

2. Model Environment

Consider a two-period economy inhabited by a continuum of agents with aggregate unit measure. Agents differ in their initial wealth position, which is characterized by their holdings of government debt at the beginning of the first period. The government is represented by a social planner with a utilitarian payoff who issues one-period, nonstate-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents.

2.1. Household Preferences and Budget Constraints

All agents have the same preferences, which are given by:

$$u(c_0) + \beta E[u(c_1)], \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0, 1)$ is the discount factor and c_t for $t = 0, 1$ is individual consumption.

The utility function $u(\cdot)$ takes the standard CRRA form.

All agents receive a nonstochastic endowment y each period and pay lump-sum taxes τ_t , which are uniform across agents. Taxes and newly issued government debt are used to pay for government consumption g_t and repayment of outstanding government debt. The initial supply of outstanding government bonds at $t = 0$ is denoted by B_0 . Given B_0 , the initial wealth distribution is defined by a fraction γ of households that are the L -type individuals with initial bond holdings b_0^L , and a

fraction $(1 - \gamma)$ that are the H -types and hold $b_0^H > b_0^L$. These initial bond holdings satisfy market clearing: $\gamma b_0^L + (1 - \gamma)b_0^H = B_0$, which given $b_0^H > b_0^L$ implies that $b_0^H > B_0$ and $b_0^L < B_0$.

The budget constraints of the two types of households in the first period are given by:

$$c_0^i + q_0 b_1^i = y + b_0^i - \tau_0 \quad \text{for } i = L, H. \quad (1)$$

Agents collect the payout on their initial holdings of government debt (b_0^i), receive endowment income y , and pay lump-sum taxes τ_0 . This net-of-tax resources are used to pay for consumption and purchases of new government bonds b_1^i at price q_0 . Agents are not allowed to take short positions in government bonds, which is equivalent to imposing the no-borrowing condition often used in heterogeneous-agents models with incomplete markets: $b_1^i \geq 0$.

The budget constraints in the second period differ depending on whether or not the government defaults. If the government repays, the budget constraints take the standard form:

$$c_1^i = y + b_1^i - \tau_1 \quad \text{for } i = L, H. \quad (2)$$

If the government defaults, there is no repayment on the outstanding debt, and the agents' budget constraints are:

$$c_1^i = (1 - \varphi(g_1))y - \tau_1 \quad \text{for } i = L, H. \quad (3)$$

As is standard in the sovereign debt literature, we can allow for default to impose an exogenous cost that reduces income by a fraction φ . This cost is usually modeled as a function of the realization of a stochastic endowment income, but since income is constant in this setup, we model it as a function of the realization of government expenditures in the second period g_1 . In particular, the cost is a nonincreasing, stepwise function: $\varphi(g_1) \geq 0$, with $\varphi'(g_1) \leq 0$ for $g_1 \leq \bar{g}_1$, $\varphi'(g_1) = 0$ otherwise, and $\varphi''(g_1) = 0$. Hence, \bar{g}_1 is a threshold high value of g_1 above which the marginal cost of default is zero.⁸

2.2. Government

At the beginning of $t = 0$, the government has outstanding debt B_0 and can issue one-period, nonstate-contingent discount bonds $B_1 \in \mathcal{B} \equiv [0, \infty)$ at the price $q_0 \geq 0$. Each period, the government collects lumpsum revenues τ_t and pays for g_t . Since g_0 is known at the beginning of the first period, the relevant uncertainty with respect to government expenditures is for g_1 , which is characterized by a well-defined

8. This formulation is analogous to the stepwise default cost as a function of income proposed by Arellano (2008) and now widely used in the external default literature, and it also captures the idea of asymmetric costs of tax collection (see Barro (1979) and Calvo (1988)). Note, however, that for the model to support equilibria with debt under a utilitarian government all we need is $\varphi(g_1) > 0$. The additional structure is useful for the quantitative analysis and for making it easier to compare the model with the standard external default models. In external default models, the nonlinear cost makes default more costly in “good” states, which alters default incentives to make default more frequent in “bad” states, and it also contributes to support higher debt levels.

probability distribution function with mean μ_g . We do not restrict the sign of τ_t , so $\tau_t < 0$ represents lumpsum transfers.

At equilibrium, the price of debt issued in the first period must be such that the government bond market clears:

$$B_t = \gamma b_t^L + (1 - \gamma)b_t^H \quad \text{for } t = 0, 1. \quad (4)$$

This condition is satisfied by construction in period 0. In period one, however, the price moves endogenously to clear the market.

The government has the option to default at $t = 1$. The default decision is denoted by $d_1 \in \{0, 1\}$ where $d_1 = 0$ implies repayment. The government evaluates the values of repayment and default as a benevolent planner with a social welfare function. In the rest of this Section we focus on the case of a standard utilitarian social welfare function: $\gamma u(c_1^L) + (1 - \gamma)u(c_1^H)$. The government, however, cannot discriminate across the two types of agents when setting taxation, debt and default policies.

At $t = 0$, the government budget constraint is

$$\tau_0 = g_0 + B_0 - q_0 B_1. \quad (5)$$

The level of taxes in period one is determined after the default decision. If the government repays, taxes are set to satisfy the following government budget

constraint:

$$\tau_1^{d_1=0} = g_1 + B_1. \quad (6)$$

Notice that, since this is a two-period model, equilibrium requires that there are no outstanding assets at the end of period 1 (i.e., $b_2^i = B_2 = 0$ and $q_1 = 0$). If the government defaults, taxes are simply set to pay for government purchases:

$$\tau_1^{d_1=1} = g_1. \quad (7)$$

3. Equilibrium

The analysis of the model's equilibrium proceeds in three stages. First, we characterize the households' optimal savings problem and determine their payoff (or value) functions, taking as given the government debt, taxes, and default decision. Second, we study how optimal government taxes and the default decision are determined. Third, we examine the optimal choice of debt issuance that internalizes the outcomes of the first two stages.

3.1. Households' Problem

Given B_1 and γ , a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$v^i(B_1, \gamma) = \max_{b_1^i} \left\{ u(y + b_0^i - q_0(B_1, \gamma)b_1^i - \tau_0) + \right. \quad (8)$$

$$\beta E_{g_1} \left[(1 - d_1(B_1, g_1, \gamma))u(y + b_1^i - \tau_1^{d_1=0}) + \right.$$

$$\left. \left. d_1(B_1, g_1, \gamma)u(y(1 - \varphi(g_1)) - \tau_1^{d_1=1}) \right] \right\},$$

subject to $b_1^i \geq 0$. The term $E_{g_1}[\cdot]$ represents the expected payoff across the repayment and default states in period one.⁹

The first-order condition, evaluated at the equilibrium level of taxes, yields the following Euler equation:

$$u'(c_0^i) \geq \beta(1/q_0(B_1, \gamma))E_{g_1} \left[u'(y - g_1 + b_1^i - B_1)(1 - d_1(B_1, g_1, \gamma)) \right], = \text{ if } b_1^i > 0 \quad (9)$$

9. Notice in particular that the payoff in case of default does not depend on the level of individual debt holdings (b_1^i), reflecting the fact that the government cannot discriminate across households when it defaults.

In states in which, given (B_1, γ) , the value of g_1 is such that the government chooses to default ($d_1(B_1, g_1, \gamma) = 1$), the marginal benefit of an extra unit of debt is zero.¹⁰ Thus, conditional on B_1 , a larger default set (i.e., a larger set of values of g_1 such that the government defaults), implies that the expected marginal benefit of an extra unit of savings decreases. This implies that, everything else being equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes.¹¹

Given that income and taxes are homogeneous across agents, it has to be the case at equilibrium that $b_1^H > b_1^L$, which therefore implies that H types are never credit constrained. In contrast, whether L types are credit constrained or not depends on parameter values. This is less likely to happen the higher b_0^L , B_0 or B_1 . Whenever the L types are constrained, the H -types are the marginal investor and their Euler equation can be used to derive the equilibrium price. For the remainder of the paper we focus on equilibria in which L -types are constrained (i.e., $b_0^L = b_1^L = 0$), to capture the feature of heterogeneous agents models with incomplete markets that a fraction of agents is always credit-constrained endogenously, and public debt has the social benefit that it contributes to reduce the tightness of this constraint.

The equilibrium bond price is the value of $q_0(B_1, \gamma)$ for which, as long as consumption for all agents is nonnegative and the default probability of the

10. Utility in the case of default equals $u(y(1 - \varphi(g_1)) - g_1)$, and is independent of b_1^i .

11. Note also that from the agents' perspective, their bond decisions do not affect $d_1(B_1, g_1, \gamma)$.

government is less than one, the following market-clearing condition holds:

$$B_1 = \gamma b_1^L(B_1, \gamma) + (1 - \gamma) b_1^H(B_1, \gamma), \quad (10)$$

where B_1 in the left-hand-side of this expression represents the supply of public debt, and the right-hand-side is the aggregate government bond demand.

It is instructive to analyze the households' problem further assuming logarithmic utility ($u(c) = \log(c)$), because under these assumptions we can solve for $q_0(B_1, \gamma)$ in closed form and use the solution to establish some important properties of bond prices and default risk spreads. We show in Section A.4 of the Appendix that the equilibrium bond price is:

$$q_0(B_1, \gamma) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma} \right) B_0 \right) \Pi(B_1, \gamma)}{1 + \left(\frac{\gamma}{1-\gamma} \right) \beta B_1 \Pi(B_1, \gamma)}, \quad (11)$$

where $\Pi(B_1, \gamma) \equiv E_{g_1} \left[\frac{1 - d(B_1, g_1, \gamma)}{y - g_1 + (\gamma/(1-\gamma)) B_1} \right]$ is the expected marginal utility of H -type agents for the second period, which weights only non-default states because the marginal benefit of debt is zero in default states. Since, as also shown in the Appendix (Section A.4), $\partial \Pi(B_1, \gamma) / \partial B_1 < 0$, it follows that $\partial q_0(B_1, \gamma) / \partial B_1 < 0$ for $c_0^H > 0$. Moreover, since bond prices are decreasing in B_1 , it follows that the “revenue” the government generates by selling debt, $q_0(B_1, \gamma) B_1$, behaves much the same as the familiar debt Laffer curve of the Eaton-Gersovitz models derived by

Arellano (2008).¹² This Laffer curve will play a key role later in determining the government's optimal debt choice. In particular, the government internalizes that higher debt eventually produces decreasing revenues and that in the decreasing segment of the Laffer curve revenues fall faster as the debt increases and much faster as default risk rises sharply.

A similar expression can be obtained for the “risk-free” price (i.e., the bond price that arises in a model with full commitment) to show that the risk premium is nonnegative, and it is strictly positive if there is default in equilibrium. The premium is increasing in B_1 since the default set is increasing in B_1 . We also show in Section A.4 of the Appendix that the spread is a multiple of $1/\beta(y - g_0 + (\gamma/(1 - \gamma))B_0)$. As a result, the total date-0 resources available for consumption of the H types $(y - g_0 + (\gamma/(1 - \gamma))B_0)$ have a first-order *negative* effect on default risk spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. Changes in γ have a similar impact on this ratio also pushing spreads downward. However, as γ increases, default incentives also strengthen, as the welfare of debtholders is valued less.

12. It is straightforward to show that revenue $R(B_1) = q_0(B_1, \gamma)B_1$ follows a Laffer curve in the $[0, B_1^{max}]$ interval, where B_1^{max} is the upper bound of debt such that the government chooses default for any realization of g_1 and thus $q_0(B_1^{max}, \gamma) = 0$. Since $R(0) = 0$ with $R'(0) = q(0, \gamma) > 0$, and $R(B_1^{max}) = 0$ with $R'(B_1^{max}) = q'_0(B_1^{max}, \gamma)B_1^{max} < 0$, it follows by Rolle's theorem that $R(B_1)$ has at least one local maximum in $(0, B_1^{max})$.

Thus, in principle, the response of spreads to increases in the concentration of debt ownership is ambiguous.

3.2. Government's Problem

3.2.1. *Government Default Decision at $t = 1$.* At $t = 1$, the government chooses whether to default by solving this optimization problem:

$$\max_{d \in \{0,1\}} \{W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1)\}, \quad (12)$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1)$ denote the values of the social welfare function at the beginning of period one in the case of repayment and default, respectively. Using the government budget constraint to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government's utilitarian payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1) + (1 - \gamma)u(y - g_1 + b_1^H - B_1) \quad (13)$$

and

$$W_1^{d=1}(g_1) = u(y(1 - \varphi(g_1)) - g_1). \quad (14)$$

Notice that all households lose g_1 of their income to government absorption regardless of the default choice. Moreover, debt repayment reduces consumption and welfare of L types and rises them for H types (since $(b_1^L - B_1) \leq 0$ and $(b_1^H - B_1) \geq 0$), whereas default implies the same consumption and utility for both types of agents.

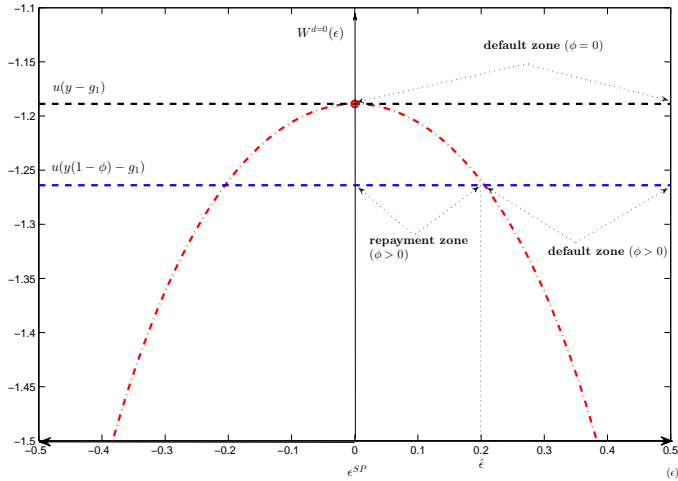
The distributional mechanism determining the default decision can be illustrated by means of a graphical tool. To this end, it is helpful to express the values of optimal debt holdings as $b_1^L = B_1 - \varepsilon$ and $b_1^H(\gamma) = B_1 + \gamma/(1 - \gamma)\varepsilon$, for some hypothetical decentralized allocation of debt holdings given by $\varepsilon \in [0, B_1]$. Consumption allocations under repayment would therefore be $c_1^L(\varepsilon) = y - g_1 - \varepsilon$ and $c_1^H(\gamma, \varepsilon) = y - g_1 + \gamma/(1 - \gamma)\varepsilon$, so ε also determines the decentralized consumption dispersion. The efficient dispersion of consumption that the social planner would choose is characterized by the value of ε^{SP} that maximizes social welfare under repayment, which satisfies this first-order condition:

$$u' \left(y - g_1 + \frac{\gamma}{1 - \gamma} \varepsilon^{SP} \right) = u' (y - g_1 - \varepsilon^{SP}). \quad (15)$$

Hence, the efficient allocations are characterized by zero consumption dispersion because equal marginal utilities imply $c^{L,SP} = c^{H,SP} = y - g_1$, which is attained with $\varepsilon^{SP} = 0$.

Consider now the government's default decision when default is costless ($\varphi(g_1) = 0$). This scenario is depicted in Figure 1, which plots the social welfare function under repayment as a function of ε as the bell-shaped curve, and the social welfare under default (which is independent of ε) as the black dashed line.

FIGURE 1. Default Decision and Consumption Dispersion



Clearly, the maximum welfare under repayment is attained when $\epsilon = 0$, which is also the efficient amount of consumption dispersion ϵ^{SP} .¹³ Given that the only policy instruments the government can use, other than the default decision, are nonstate contingent debt and lump-sum taxes, it is straightforward to conclude that default is always optimal. This is because default produces identical allocations in a decentralized equilibrium as the socially efficient ones, since default produces zero consumption dispersion with consumption levels $c^L = c^H = y - g_1$. This outcome is invariant to the values of B_1 , g_1 , and γ . This result also implies that the model without default costs cannot support equilibria with domestic debt subject to default risk because default is always optimal.

13. Recall also that we defined the relevant range of decentralized consumption dispersion for $\epsilon > 0$, so welfare under repayment is decreasing in ϵ over the relevant range.

The outcome is very different when default is costly. With $\varphi(g_1) > 0$, default still yields zero consumption dispersion but at lower levels of consumption and therefore utility, since consumption allocations in the default state become $c^L = c^H = (1 - \varphi(g_1))y - g_1$. This does not alter the result that the first-best social optimum is $\varepsilon^{SP} = 0$, but what changes is that default can no longer support the consumption allocations of the first best. Hence, there is now a threshold amount of consumption dispersion in the decentralized equilibrium, $\widehat{\varepsilon}(\gamma)$, which varies with γ and such that for $\varepsilon \geq \widehat{\varepsilon}(\gamma)$ default is again optimal, but for lower ε repayment is now optimal. This is because when ε is below the threshold, repayment produces a level of social welfare higher than the one that default yields. Figure 1 also illustrates this scenario.

3.2.2. Government Debt Decision at $t = 0$. We can now examine how the government chooses the optimal amount of debt to issue in the initial period. Before studying the government's optimization problem, it is important to emphasize that in this model, debt is a mechanism for altering consumption dispersion across agents, both within a period and across periods. In particular, since $b_0^L = b_1^L = 0$, consumption dispersion in each period and repayment state can be written as $c_0^H - c_0^L = 1/(1 - \gamma)[B_0 - q(B_1, \gamma)B_1]$, $c_1^{H,d=0} - c_1^{L,d=0} = (1/(1 - \gamma))B_1$, and $c_1^{H,d=1} - c_1^{L,d=1} = 0$. These expressions make it clear that, given B_0 , issuing at least some debt ($B_1 > 0$) reduces consumption dispersion at $t = 0$ compared with no debt ($B_1 = 0$) but increases it at $t = 1$ if the government repays (i.e., $d = 0$). Moreover, the debt Laffer curve that governs $q_0(B_1, \gamma)B_1$ limits the extent to which debt can reduce consumption dispersion at $t = 0$. Starting from $B_1 = 0$, consumption

dispersion in the initial period falls as B_1 increases, but there is a critical positive value of B_1 beyond which it increases with debt.

At $t = 0$, the government chooses its debt policy internalizing the aforementioned effects, including the dependence of bond prices on the debt issuance choice. The government chooses B_1 to maximize the “indirect” social welfare function:

$$W_0(\gamma) = \max_{B_1} \{ \gamma v^L(B_1, \gamma) + (1 - \gamma) v^H(B_1, \gamma) \}. \quad (16)$$

where v^L and v^H are the value functions obtained from solving the households’ problems defined in the Bellman equation (8) taking into account the government budget constraints and the equilibrium pricing function of bonds.

We can gain some intuition about the solution of this maximization problem by deriving its first-order condition and rearranging it as follows (assuming that the relevant functions are differentiable):

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta}{q(B_1, \gamma)\gamma} \{ \beta E_{g_1} [\Delta d \Delta W_1] + \gamma \mu^L \} \quad (17)$$

where

$$\eta \equiv q(B_1, \gamma) / (q'(B_1, \gamma)B_1) < 0,$$

$$\Delta d \equiv d(B_1 + \delta, g_1, \gamma) - d(B_1, g_1, \gamma) \geq 0, \text{ for } \delta > 0 \text{ small,}$$

$$\Delta W_1 \equiv W_1^{d=1}(g_1, \gamma) - W_1^{d=0}(B_1, g_1, \gamma) \geq 0,$$

$$\mu^L \equiv q(B_1, \gamma)u'(c_0^L) - \beta E_{g_1} [(1 - d^1)u'(c_1^L)] > 0.$$

In these expressions, η is the price elasticity of the demand for government bonds, $\Delta d \Delta W_1$ represents the marginal distributional benefit of a default, and μ^L is the shadow value of the borrowing constraint faced by L -type agents.

If both types of agents were unconstrained in their bonds' choice, so that in particular $\mu^L = 0$, and if there is no change in the risk of default (or assuming commitment to remove default risk entirely), so that $E_{g_1} [\Delta d \Delta W_1] = 0$, then the optimality condition simplifies to $u'(c_0^H) = u'(c_0^L)$. Hence, in this case, the social planner issues debt to equalize marginal utilities of consumption across agents at date 0, which requires simply setting B_1 to satisfy $q(B_1, \gamma)B_1 = B_0$.

If H -type agents are unconstrained and L -type are constrained (i.e., $\mu^L > 0$), which is the scenario we are focusing on, and still assuming no change in default risk or a government committed to repay, the optimality condition reduces to

$u'(c_0^H) = u'(c_0^L) + (\eta\mu^L)/q(B_1, \gamma)$. Since $\eta < 0$, this result implies $c_0^L < c_0^H$ because $u'(c_0^L) > u'(c_0^H)$. Thus, the government's debt choice sets B_1 as needed to maintain an optimal, positive level of consumption dispersion. Moreover, since optimal consumption dispersion is positive, we can also ascertain that $B_0 > q(B_1, \gamma)B_1$, which, using the government budget constraint, implies that the government runs a primary surplus at $t = 0$. The government borrows resources, but less than it would need to eliminate all consumption dispersion (which requires zero primary balance).

The intuition for the optimality of issuing debt can be presented in terms of tax smoothing and savings: Date-0 consumption dispersion without debt issuance would be $B_0/(1 - \gamma)$, but this is more dispersion than what the government finds optimal because by choosing $B_1 > 0$ the government provides tax smoothing (i.e., reduces date-0 taxes) for everyone, which in particular eases the L -type agents credit constraint and provides also a desired vehicle of savings for H types. Thus, positive debt increases consumption of L types (since $c_0^L = y - g_0 - B_0 + q(B_1, \gamma)B_1$) and reduces consumption of H types (since $c_0^H = y - g_0 + (\gamma/(1 - \gamma))(B_0 - q(B_1, \gamma)B_1)$). But issuing debt (assuming repayment) also increases consumption dispersion at $t = 1$, since debt is then paid with higher taxes on all agents, while H agents collect also the debt repayment. Thus, the debt is being chosen optimally to trade off the social costs and benefits of reducing (increasing) date-0 consumption and increasing (reducing) date-1 consumption for rich (poor) agents.

In the presence of default risk and if default risk changes near the optimal debt choice, the term $E_{g_1} [\Delta d \Delta W_1]$ enters in the government's optimality condition with a

positive sign, which means the optimal gap in the date-0 marginal utilities of the two agents widens even more. Hence, the government's optimal choice of consumption dispersion for $t = 0$ is greater than without default risk, and the expected dispersion for $t = 1$ is lower, because in some states of the world, the government will choose to default and consumption dispersion would then drop to zero. Moreover, the debt Laffer curve now plays a central role in the government's weakened incentives to borrow because as default risk rises the price of bonds drops to zero faster and the resources available to reduce date-0 consumption dispersion peak at lower debt levels. In short, default risk reduces the government's ability to use nonstate-contingent debt to reduce consumption dispersion.

3.3. Competitive Equilibrium with Optimal Debt and Default Policy

For a given value of γ , a competitive equilibrium with optimal debt and default policy is a pair of household value functions $v^i(B_1, \gamma)$ and decision rules $b^i(B_1, \gamma)$ for $i = L, H$, a government bond pricing function $q_0(B_1, \gamma)$ and a set of government policy functions $\tau_0(B_1, \gamma)$, $\tau_1^{d \in \{0,1\}}(B_1, g_1, \gamma)$, $d(B_1, g_1, \gamma)$, $B_1(\gamma)$ such that:

1. Given the pricing function and government policy functions, $v^i(B_1, \gamma)$ and $b_1^i(B_1, \gamma)$ solve the households' problem.
2. $q_0(B_1, \gamma)$ satisfies the market-clearing condition of the bond market (equation (10)).
3. The government default decision $d(B_1, g_1, \gamma)$ solves problem (12).

4. Taxes $\tau_0(B_1, \gamma)$ and $\tau_1^d(B_1, g_1, \gamma)$ are consistent with the government budget constraints.
5. The government debt policy $B_1(\gamma)$ solves problem (16).

4. Quantitative Analysis

In this section, we study the model's quantitative predictions based on a calibration using European data. The goal is to show whether a reasonable set of parameter values can produce an equilibrium with debt subject to default risk and to study how the properties of this equilibrium change with the model's key parameters. Since the two-period model is not well suited to account for the time-series dynamics of the data, we see the results more as an illustration of the potential relevance of the model's argument for explaining domestic default rather than as an evaluation of the model's general ability to match observed public debt dynamics.¹⁴

14. We solve the model following a similar backward-recursive strategy as in the theoretical analysis. First, taking as given a set of values $\{B_1, \gamma\}$, we solve for the equilibrium pricing and default functions by iterating on (q_0, b_1^i) and the default decision rule d_1 until the date-0 bond market clears when the date-1 default decision rule solves the government's optimal default problem (12). Then, in the second stage, we complete the solution of the equilibrium by finding the optimal choice of B_1 that solves the government's date-0 optimization problem (16). It is important to recall that, as explained earlier, for given values of B_1 and γ , an equilibrium with debt will not exist if either the government finds it optimal to default on B_1 for all realizations of g_1 or if at the given B_1 the consumption of L types is nonpositive. In these cases, there is no finite price that can clear the debt market.

4.1. Calibration

The model is calibrated to annual frequency, and most of the parameter values are set so that the model matches moments from European data. The calibrated parameter values are summarized in Table 2. The details of the calibration are available in Section A.5 of the Appendix. Note also that we assume a log-normal process for g_1 , so that $\ln(g_1) \sim N((1 - \rho_g)\ln(\mu_g) + \rho_g \ln(g_0), \sigma_e^2/(1 - \rho_g^2))$ and the cost of default takes the following functional form: $\varphi(g_1) = \varphi_0 + (\bar{g}_1 - g_1)/y$.

TABLE 2. Model Parameters

Parameter		Value
Discount Factor	β	0.96
Risk Aversion	σ	1.00
Avg. Income	y	0.79
Low Household Wealth	b_0^L	0.00
Avg. Gov. Consumption	μ_g	0.18
Autocorrel. G	ρ_g	0.88
Std Dev Error	σ_e	0.017
Initial Gov. Debt	B_0	0.35
Output Cost Default	φ_0	0.004

Note: Government expenditures, income, and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain, and Portugal.

We abstain from setting a calibrated value for γ and instead show results for $\gamma \in [0, 1]$. Data from the United States and Europe suggest that the empirically relevant range for γ is $[0.55, 0.85]$, and hence, when taking a stance on a particular value of γ is useful, we use $\gamma = 0.7$, which is the midpoint of the plausible range.¹⁵

15. In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds and 50.4% have retirement accounts (which are very likely to include government bonds). These figures would suggest values of γ ranging from 0.5 to 0.88.

4.2. Results

We examine the quantitative results in the same order in which the backward solution algorithm works. We start with the second period's utility of households under repayment and default. We then move to the first period and examine the equilibrium bond prices. Finally, we study the optimal government debt issuance B_1 for a range of values of γ .

4.2.1. Second Period Default Incentives for Given (B_1, g_1, γ) . Using the agents' optimal choice of bond holdings, we compute the equilibrium utility levels they attain at $t = 1$ under repayment versus default for different triples (B_1, g_1, γ) . Since we are looking at the last period of a two-period model, these compensating variations

In Europe, comparable statistics are not available for several countries, but Davies, Sandstrom, Shorrocks, and Wolff (2009) document that the wealth distribution is highly concentrated with Gini coefficients ranging between 0.55 and 0.85. In our model, since $b_0^L = 0$, the Gini coefficient of wealth is equal to γ .

reduce simply to the percent changes in consumption across the default and no-default states of each agent:¹⁶

$$\alpha^i(B_1, g_1, \gamma) = \frac{c_1^{i,d=1}(B_1, g_1, \gamma)}{c_1^{i,d=0}(B_1, g_1, \gamma)} - 1 = \frac{(1 - \varphi(g_1))y - g_1}{y - g_1 + b_1^i - B_1} - 1$$

A positive (negative) value of $\alpha^i(B_1, g_1, \gamma)$ implies that agent i prefers government default (repayment) by an amount equivalent to an increase (cut) of $\alpha^i(\cdot)$ percent in consumption.

The individual welfare gains of default are aggregated using γ to obtain the utilitarian representation of the social welfare gain of default:

$$\bar{\alpha}(B_1, g_1, \gamma) = \gamma \alpha^L(B_1, g_1, \gamma) + (1 - \gamma) \alpha^H(B_1, g_1, \gamma).$$

A positive value indicates that default induces a social welfare gain and a negative value a loss. The default decision is directly linked to the values of $\bar{\alpha}(B_1, g_1, \gamma)$. In particular, the repayment region of the default decision ($d(B_1, g_1, \gamma) = 0$) corresponds to $\bar{\alpha}(B_1, g_1, \gamma) < 0$ and the default region ($d(B_1, g_1, \gamma) = 1$) to $\bar{\alpha}(B_1, g_1, \gamma) > 0$.

16. These calculations are straightforward given that, in the equilibria, we solve for $b_1^L = 0$, and hence $b_1^H = B_1/(1 - \gamma)$. The same formula would apply, however, even if these conditions do not hold, using instead the policy functions $b_1^i(B_1, g_1, \gamma)$ that solve the households' problems for any given pair of functions $d_1(B_1, g_1, \gamma)$ and $q_0(B_1, \gamma)$, including the ones that are consistent with the government's default decision and equilibrium in the bond market.

FIGURE 2. Social Welfare Gain of Default $\bar{\alpha}(B_1, g_1, \gamma)$

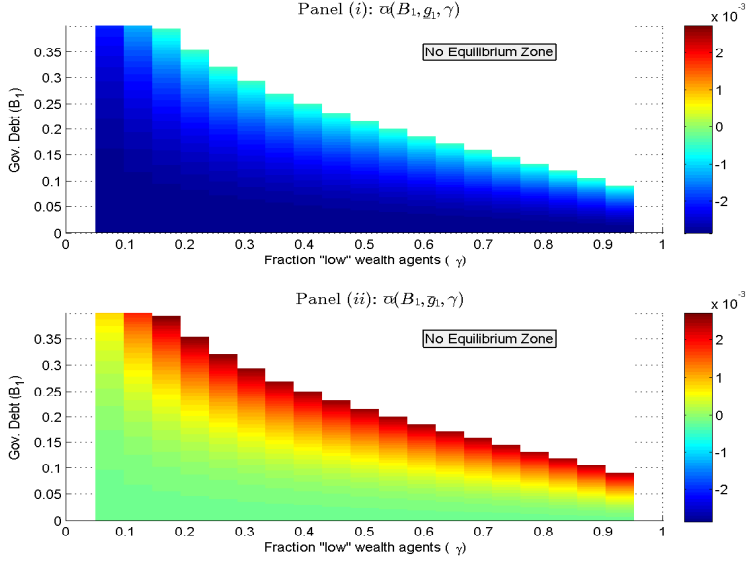


Figure 2 shows two intensity plots of the social welfare gain of default for the ranges of values of B_1 and γ in the vertical and horizontal axes, respectively. Panel (i) is for a low value of government purchases, \underline{g}_1 , set three standard deviations below μ_g , and panel (ii) is for a high value \bar{g}_1 set three standard deviations above μ_g . Figure A.2 in the Appendix shows the default decision rules that correspond to these two plots. The intensity of the color or shading in these plots indicates the magnitude of the welfare gain according to the legend shown to the right of each. The regions shown in white color and marked as “No Equilibrium Zone” represent values of (B_1, γ) for which the debt market collapses and no equilibrium exists. In

this zone, there is no equilibrium because, at the given γ , the government chooses to default on the given B_1 for all values of g_1 .¹⁷

The area in which the social welfare gains of default are well defined in these intensity plots illustrates two of the key mechanisms driving the government's distributional incentives to default: First, fixing γ , the welfare gain of default is higher at higher levels of debt, or conversely the gain of repayment is lower. Second, keeping B_1 constant, the welfare gain of default is also increasing in γ (i.e., higher concentration of debt ownership increases the welfare gain of default). This implies that lower concentration of debt ownership is sufficient to trigger default at higher levels of debt.¹⁸ For example, for a debt of 20% of GDP ($B_1 = 0.20$) and $g_1 = \bar{g}_1$, social welfare is higher under repayment if $0 \leq \gamma \leq 0.10$, but it becomes higher under default if $0.10 < \gamma \leq 0.6$, and for higher γ , there is no equilibrium because the government prefers default not only for $g_1 = \bar{g}_1$ but for all possible g_1 . If instead,

17. There is another potential “No Equilibrium Zone” that could arise if the given (B_1, γ) would yield $c_0^L \leq 0$ at the price that induces market clearing, and so the government would not supply that particular B_1 . This happens for low levels of B_1 relative to B_0 . To determine if $c_0^L \leq 0$ at some (B_1, γ) we need $q_0(B_1, \gamma)$, since combining the budget constraints of the L types and the government yields $c_0^L = y - g_0 - B_0 + q_0 B_1$. Hence, to evaluate this condition we take the given B_1 and use the H types Euler equation and the market clearing condition to solve for $q_0(B_1, \gamma)$, and then determine if $y - g_0 - B_0 + q_0 B_1 \leq 0$; if this is true, then (B_1, γ) is in the lower No Equilibrium Zone.

18. Note that the cross-sectional variance of initial debt holdings is given by $Var(b) = B^2(\gamma)/(1 - \gamma)$ when $b_0^L = 0$. This implies that the cross-sectional coefficient of variation is equal to $CV(b) = (\gamma)(1 - \gamma)$, which is increasing in γ for $\gamma \leq 1/2$.

the debt is 35% of GDP, then social welfare is higher under default for all the values of γ for which an equilibrium exists.

The two panels in Figure 2 differ in that panel (ii) displays a well-defined transition from a region in which repayment is socially optimal ($\bar{\alpha}(B_1, g_1, \gamma) < 0$) to one in which default is optimal ($\bar{\alpha}(B_1, g_1, \gamma) > 0$) but in panel (i) the social welfare gain of default is never positive, so repayment is always optimal. This reflects the fact that higher g_1 also weakens the incentives to repay.

4.2.2. *Bond Prices for Given (B_1, γ) .* Figure 3 shows $q_0(B_1, \gamma)$ as a function of γ for three values of B_1 ($B_L < B_M < B_H$) and a comparison with the prices from the model with the government committed to repay q^{RF} . The bond price functions are truncated when the equilibrium does not exist.

FIGURE 3. Equilibrium Bond Price

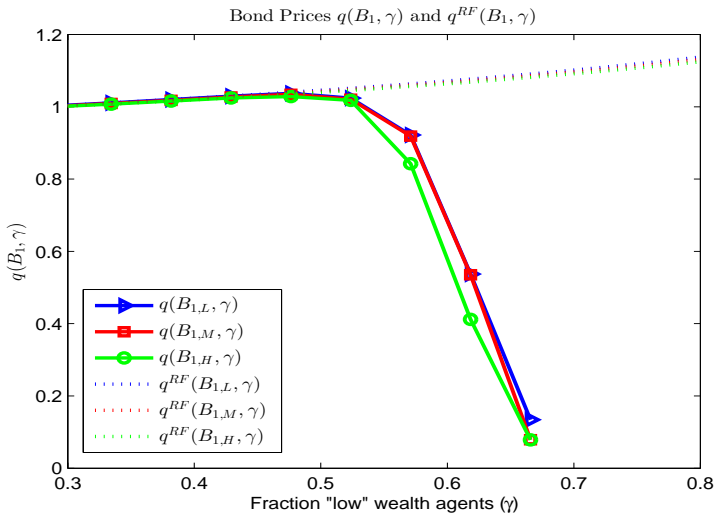


Figure 3 illustrates three key features of public debt prices discussed in Section 3:

(i) *The equilibrium price is decreasing in B_1 for given γ* (the pricing functions shift downward as B_1 rises). This follows from a standard demand-and-supply argument: For a given γ , as the government borrows more, the price at which households are willing to demand the additional debt falls and the interest rate rises. This effect is present even without uncertainty, but it is stronger in the presence of default risk.¹⁹

(ii) *Default risk reduces the price of bonds below the risk-free price and thus induces a risk premium.* Prices are either identical or nearly identical for the values shown for B_1 when $\gamma \leq 0.5$ since the probability of default is either zero or very close to zero. As γ increases above 0.5, however, the risk premium becomes nontrivial and bond prices subject to default risk fall sharply below the risk-free prices.

(iii) *Bond prices are a nonmonotonic function of γ :* When default risk is sufficiently low, bond prices are increasing in γ , but eventually they become a steep decreasing function of γ . Whether bond prices are increasing or decreasing in γ depends on the relative strength of a demand composition effect versus the effect of increasing γ on default incentives. The composition effect results from the fact that, as γ increases, H -type agents become a smaller fraction of the population and wealthier in percapita terms, and therefore, a higher $q_0(B_1, \gamma)$ is needed to clear the market. On the other

19. Sections A.3 and A.4 of the Appendix provide proofs showing that $q'(B_1, \gamma) < 0$ in the log-utility case. In Figure 3, the scale of the vertical axis is too wide to make the fall in $q(B_1, \gamma)$ as B_1 rises visible for $\gamma < 0.5$.

hand, higher concentration of debt ownership strengthens distributional incentives to default, which pushes for lower bond prices. This second effect starts to dominate for $\gamma > 0.5$, producing bond prices that fall sharply as γ rises, while for lower γ , the composition effect dominates and prices rise gradually with γ .²⁰

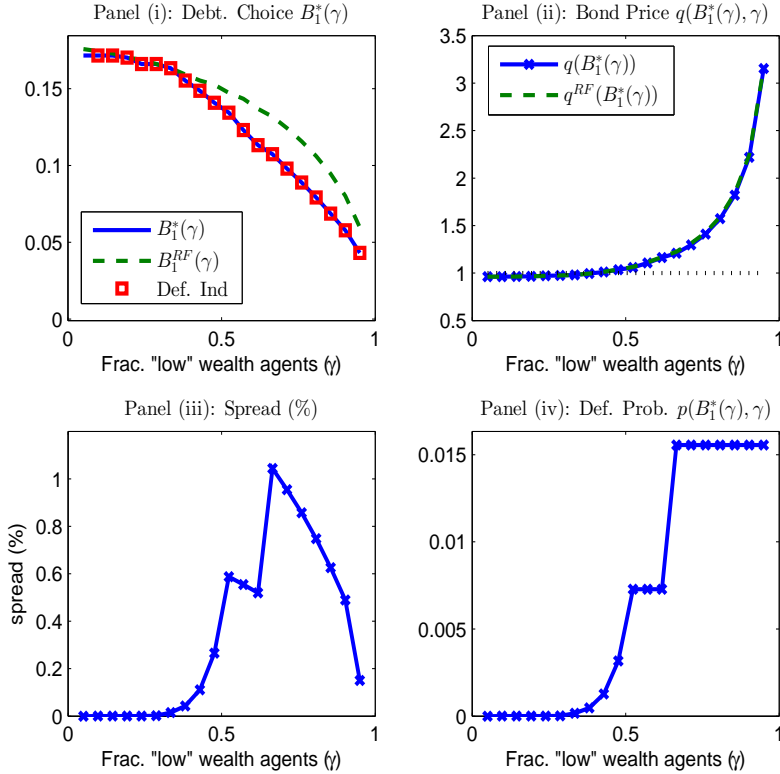
4.2.3. Optimal Debt Choice and Competitive Equilibrium. Given the solutions for household decision rules, tax policies, bond pricing function, and default decision rule, we finally solve for the government's optimal choice of debt issuance in the first period (i.e., the optimal B_1 that solves problem (16)) for a range of values of γ . Given this optimal debt, we can go back and identify the equilibrium values of the rest of the model's endogenous variables that are associated with the optimal debt choices.

Figure 4 shows the four main components of the equilibrium: Panel (i) plots the optimal first-period debt issuance in the model with default risk, $B_1^*(\gamma)$, and in the case when the government is committed to repay so that the debt is risk free, $B_1^{RF}(\gamma)$; panel (ii) shows the equilibrium debt prices that correspond to the optimal debt of the same two economies; panel (iii) shows the default spread (the difference in the inverses of the bond prices); and panel (iv) shows the probability of default. Since the government that has the option to default can still choose a debt level for which it prefers to repay in all realizations of g_1 , we identify with a square in panel

20. Sections A.3 and A.4 of the Appendix provide further details, including an analysis of the bond demand decision rules that validates the intuition provided here.

(i) the equilibria in which $B_1^*(\gamma)$ has a positive default probability. This is the case for all but the smallest value of γ considered ($\gamma = 0.05$), in which the government sets $B_1^*(\gamma)$ at 20% of GDP with zero default probability.

FIGURE 4. Competitive Equilibrium with Optimal Debt Policy



It is evident from panel (i) of Figure 4 that optimal debt falls as γ increases in both the economy with default risk and the economy with a government committed to repay. This occurs because in both cases the government seeks to reallocate consumption across agents and across periods by altering the product $q(B_1, \gamma)B_1$ optimally, and in doing this, the government internalizes the response of bond prices

to its choice of debt. As γ rises, this response is influenced by the stronger default incentives and demand composition effect. At equilibrium, the latter dominates in this quantitative experiment, because panel (ii) shows that the equilibrium bond prices rise with γ . Hence, the government internalizes that as γ rises the demand composition effect strengthens, the demand for bonds, pushing bond prices higher, and as a result, it can actually attain a higher $q(B_1, \gamma)B_1$ by choosing lower B_1 . This is a standard Laffer curve argument: In the upward sloping segment of this curve, increasing debt increases the amount of resources that the government acquires by borrowing in the first period.

Although the Laffer curve argument and the demand composition effect explain why both $B_1^*(\gamma)$ and $B_1^{RF}(\gamma)$ are decreasing in γ , default risk is not innocuous. As Panel (i) shows, the optimal B_1 choices of the government that cannot commit to repay are lower than those of the government that can. This reflects the fact that the government optimally chooses smaller debt levels once it internalizes the effect of default risk on the debt Laffer curve and its distributional implications. The negative relationship between B_1 and γ is in line with the empirical evidence on the negative relationship between public debt ratios and wealth Gini coefficients at relatively high levels of inequality noted in the Introduction and documented in Section A.1 of the Appendix.

Panels (iii) and (iv) show that, in contrast with standard models of external default, in this model, the default spread is neither similar to the probability of

default nor does it have a monotonic relationship with it.²¹ Both the spread and the default probability start at zero for $\gamma = 0.05$ because $B_1^*(0.05)$ has zero default probability. As γ increases up to 0.5, both the spread and the default probability of the optimal debt choice are similar in magnitude and increase together, but for the regions where the default probability is constant (for $\gamma > 0.5$), the spread falls with γ .²² These results are in line with the findings of the theoretical analysis in Section 3.

The determination of the optimal debt choice and the relationship among the four panels of Figure 4 can be illustrated further as follows. Define a default-threshold value of γ , $\hat{\gamma}(B_1, g_1)$, as the one such that the government is indifferent between defaulting and repaying for a given (B_1, g_1) . The government chooses to default if $\gamma \geq \hat{\gamma}$. Figure 5 shows the optimal debt choice $B_1^*(\gamma)$ together with curves representing $\hat{\gamma}(B_1, g_1)$ for several realizations of g_1 . The curves for the lowest (\underline{g}), highest (\bar{g}), and mean (μ_g) realizations are identified with labels.

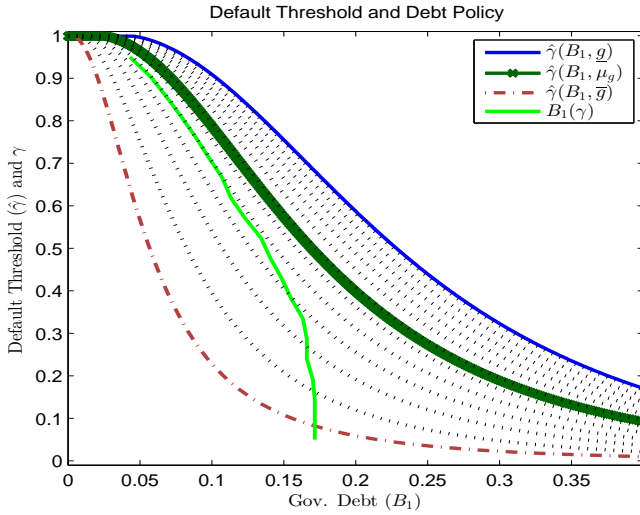
Figure 5 shows that, because of the stronger default incentives at higher γ and higher realizations of g_1 , the default-threshold curves are decreasing in B_1 and g_1 .

21. In the standard models, the two are similar and a monotonic function of each other because of the arbitrage condition of a representative risk-neutral lender.

22. As we explained before, this is derived from the composition effect that strengthens the demand for bonds and results in increasing prices (with default risk and without) as γ increases. This result disappears in the extension of the model that introduces foreign lenders (see Section 5).

There are, therefore, two key “border curves.” First, for pairs (B_1, γ) below $\hat{\gamma}(B_1, \bar{g}_1)$, repayment can be expected to occur for sure because the government will repay even if the highest realization of g_1 is observed. Second, for pairs (B_1, γ) above $\hat{\gamma}(B_1, \underline{g}_1)$, default can be expected to occur for sure because the government will choose default even if the lowest realization of g_1 is observed.

FIGURE 5. Default Threshold, Debt Policy and Equilibrium Default



Note: \underline{g} and \bar{g} are the smallest and largest possible realizations of g_1 in the Markov process of government expenditures, which are set to $-/+ 3$ standard deviations off the mean respectively. The dotted lines correspond to a set of selected thresholds for different values of g_1 .

It follows from the example above that, for equilibria with debt exposed to default risk to exist, the optimal debt choice $B_1^*(\gamma)$ must lie in between the two borders (if it is below $\hat{\gamma}(B_1, \bar{g}_1)$ the debt is issued at zero default risk, and if it is above $\hat{\gamma}(B_1, \underline{g}_1)$, there is no equilibrium). Moreover, the probability of default is implicitly determined as the cumulative probability of the value of g_1 corresponding to the highest debt-threshold curve that $B_1^*(\gamma)$ reaches. This explains why the default probability in

Panel (iv) of Figure 4 shows constant segments as γ rises above 0.5. As Figure 5 shows, for $\gamma \leq 0.5$, the optimal debt is relatively invariant to increases in γ , starting from a level that is actually in the region of risk-free debt and then moving into the region exposed to default risk. In this segment, the optimal debt falls slightly, and the probability of default rises gradually as γ rises. For γ from 0.5 to 0.6, the optimal debt falls but along the same default threshold curve (not shown in the plot), and hence the default probability remains constant at about 0.007. For $\gamma > 0.6$, the debt choice falls gradually but always along the default-threshold curve associated with a default probability of 0.015.

These findings suggest that the optimal debt is being chosen seeking to sell the “most debt” that can be issued while keeping default risk low. In turn, the most debt that is optimal to issue responds to the incentives to reallocate consumption across agents and across periods internalizing the dependence of the debt Laffer curve on the debt choice. In fact, for all values of $B_1^*(\gamma)$ that are exposed to nontrivial risk of default (those corresponding to $\gamma \geq 0.5$), $B_1^*(\gamma)$ coincides with the maximum point of the corresponding debt Laffer curve (see Figure A.6 of the Appendix). Hence, the optimal debt yields the maximum resources to the government that it can procure given its inability to commit to repay. Setting debt higher is suboptimal because default risk reduces bond prices sharply, resulting in a lower amount of resources, and setting it lower is also suboptimal because then default risk is low and extra borrowing generates more resources since bond prices fall little.

5. Extensions

This section summarizes the results of four important extensions of the model. First, a political bias case in which the social welfare function assigns weights to agents that deviate from the fraction of L and H types observed in the economy; second, an economy in which risk-neutral foreign investors can buy government debt; third, a case in which proportional distortionary taxes on consumption are used as an alternative tool for redistributive policy; and fourth, a case in which agents have access to a second asset as a vehicle for saving.

5.1. Biased Welfare Weights

Assume now that the weights of the government's payoff function differ from the utilitarian weights γ and $1 - \gamma$. This can be viewed as a situation in which, for political reasons, the government's welfare weights are biased in favor of one group of agents. The government's welfare weights on L - and H -type households are denoted ω and $(1 - \omega)$, respectively, and we refer to ω as the government's political bias.

The government's default decision at $t = 1$ is determined by the following optimization problem:

$$\max_{d \in \{0,1\}} \{W_1^{d=0}(B_1, g_1, \gamma, \omega), W_1^{d=1}(g_1)\}, \quad (18)$$

where $W_1^{d=0}(B_1, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ denote the government's payoffs in the cases of nodefault and default, respectively. Using the government budget constraints to

substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma, \omega) = \omega u(y - g_1 + b_1^L - B_1) + (1 - \omega)u(y - g_1 + b_1^H - B_1) \quad (19)$$

and

$$W_1^{d=1}(g_1) = u(y(1 - \varphi(g_1)) - g_1). \quad (20)$$

We can follow a similar approach as before to characterize the optimal default decision by comparing the allocations it supports with the first-best allocations. The parameter ε is used again to represent the dispersion of hypothetical decentralized consumption allocations under repayment: $c^L(\varepsilon) = y - g_1 - \varepsilon$ and $c^H(\gamma, \varepsilon) = y - g_1 + \varepsilon(\gamma)/(1 - \gamma)$. Under default the consumption allocations are again $c^L = c^H = y(1 - \varphi(g_1)) - g_1$. Recall that under repayment, the dispersion of consumption across agents increases with ε , and under default, there is zero consumption dispersion. The repayment government payoff can now be rewritten as:

$$W^{d=0}(\varepsilon, g_1, \gamma, \omega) = \omega u(y - g_1 + \varepsilon) + (1 - \omega)u\left(y - g_1 + \frac{\gamma}{1 - \gamma}\varepsilon\right).$$

The socially efficient planner chooses its optimal consumption dispersion ε^{SP} as the value of ε that maximizes the aforementioned expression. Since as of $t = 1$, the only instrument the government can use to manage consumption dispersion relative to what the decentralized allocations support is the default decision, it will repay only if doing so allows it to get closer to ε^{SP} than by defaulting.

The planner's optimality condition is now:

$$\frac{u'(c_1^H)}{u'(c_1^L)} = \frac{u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\varepsilon^{SP}\right)}{u'(y - g_1 - \varepsilon^{SP})} = \left(\frac{\omega}{\gamma}\right) \left(\frac{1-\gamma}{1-\omega}\right). \quad (21)$$

This condition implies that optimal consumption dispersion for the planner is zero only if $\omega = \gamma$. For $\omega > \gamma$, the planner likes consumption dispersion to favor L types so that $c_1^L > c_1^H$, and the opposite holds for $\omega < \gamma$.

The key difference of political bias versus the model with a utilitarian government is that the former can support equilibria with debt subject to default risk even without default costs. Assuming $\varphi(g_1) = 0$, there are two possible scenarios depending on the relative size of γ and ω . First, if $\omega \geq \gamma$, the planner again always chooses default as in the setup of Section 2. This is because for any decentralized consumption dispersion $\varepsilon > 0$, the consumption allocations feature $c^H > c^L$, while the planner's optimal consumption dispersion requires $c^H \leq c^L$, and hence, ε^{SP} cannot be implemented. Default brings the planner the closest it can get to the payoff associated with ε^{SP} , and hence, it is always chosen. In the second scenario, $\omega < \gamma$ (i.e., the political bias assigns more (less) weight to H (L) types than the fraction of each type of agents that actually exists). In this case, the model can support equilibria with debt even without default costs. In particular, there is a threshold consumption dispersion $\hat{\varepsilon}$ such that default is optimal for $\varepsilon \geq \hat{\varepsilon}$, where $\hat{\varepsilon}$ is the value of ε at which $W_1^{d=0}(\varepsilon, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ intersect. For $\varepsilon < \hat{\varepsilon}$, repayment is preferable because $W_1^{d=0}(\varepsilon, g_1, \gamma, \omega) > W_1^{d=0}(g_1)$. Thus, without default costs, equilibria for which repayment is optimal require two conditions: (a) that

the government's political bias favors bondholders ($\omega < \gamma$), and (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Figure 6 illustrates the main quantitative predictions of the model with political bias. The scenario with $\omega = \gamma$, shown in blue corresponds to the utilitarian case of Section 4, and the other two scenarios correspond to high and low values of ω ($\omega_L = 0.25$ and $\omega_H = 0.45$, respectively).²³

23. Note that along the blue curve of the utilitarian case both ω and γ effectively vary together because they are always equal to each other, while in the other two plots ω is fixed and γ varies. For this reason, the line corresponding to the ω_L case intersects the benchmark solution when $\gamma = 0.25$, and the one for ω_H intersects the benchmark when $\gamma = 0.45$.

FIGURE 6. Equilibrium of the Model with Political Bias for different values of ω

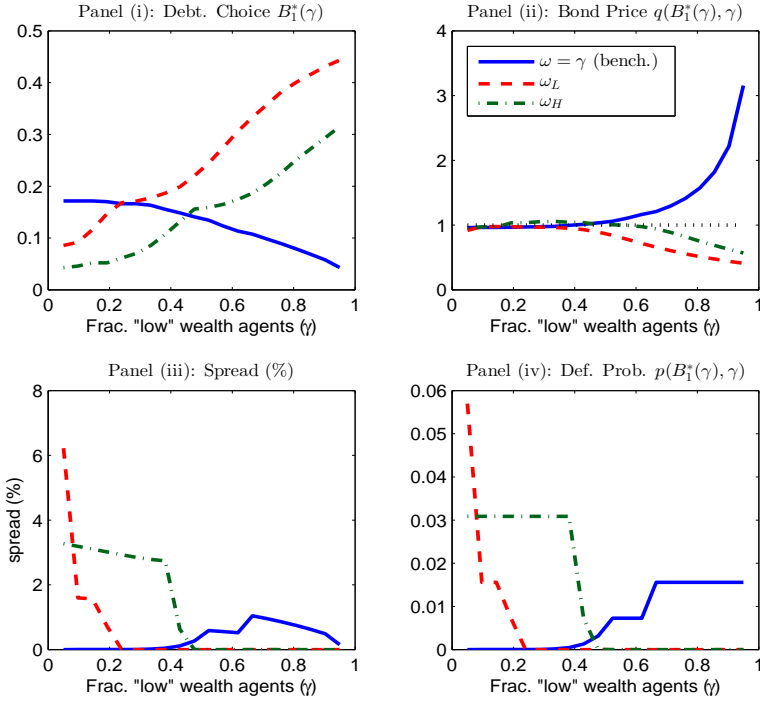


Figure 6 shows that the optimal debt level is increasing in γ . This is because the incentives to default grow weaker and the repayment zone widens as γ increases for a fixed value of ω . Moreover, the demand composition effect of higher γ is still present, so along with the lower default incentives, we still have the increasing per capita demand for bonds of H types. These two effects combined drive the increase in the optimal debt choice of the government. It is also interesting to note that in the ω_L and ω_H cases, the equilibrium exists for all values of γ (even those that are lower than ω). Without default costs, each curve would be truncated exactly where γ equals either ω_L or ω_H , but since these simulations retain the default costs used

in the utilitarian case, there can still be equilibria with debt for lower values of γ (as explained earlier).

In this model with political bias, the government is still aiming to optimize debt by focusing on the resources it can reallocate across periods and agents, which are still determined by the debt Laffer curve, and internalizing the response of bond prices to debt choices.²⁴ This relationship, however, behaves very differently than in the benchmark model because now higher optimal debt is carried at decreasing default probabilities, which leads the planner internalizing the price response to choose higher debt, whereas in the benchmark model, lower optimal debt was carried at increasing equilibrium default probabilities, which led the planner internalizing the price response to choose lower debt.

In the empirically relevant range of γ , and for values of ω lower than that range (since $\omega_L = 0.25$ and $\omega_H = 0.45$, while the relevant range of γ is $[0.55, 0.85]$), this model can sustain significantly higher debt ratios than the model with utilitarian payoff, and those ratios are close to the observed European median. At the lower end of that range of γ , a government with ω_H chooses a debt ratio of about 25%, while a government with ω_L chooses a debt ratio of about 35%.

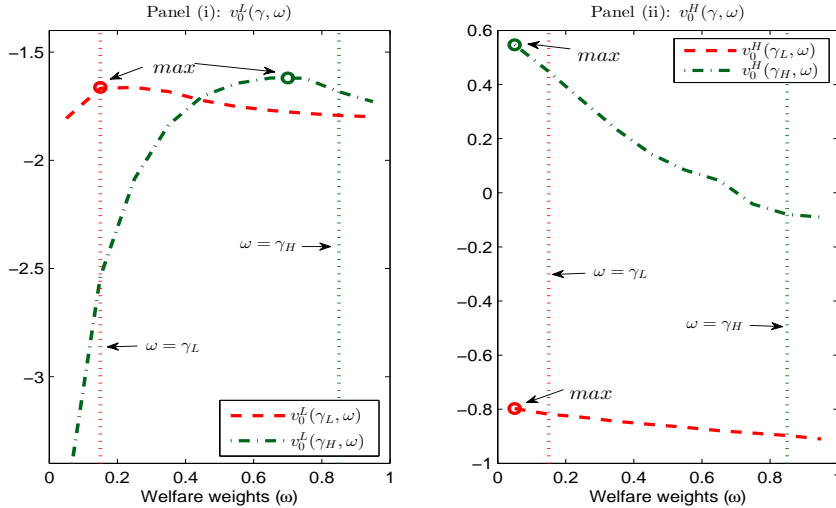
24. When choosing B_1 , the government takes into account that higher debt increases disposable income for L -type agents in the initial period but it also implies higher taxes in the second period (as long as default is not optimal). Thus, the government is willing to take on more debt when ω is lower.

The behavior of equilibrium bond prices (panel (ii)) with either $\omega_L = 0.25$ or $\omega_H = 0.45$ differs markedly from the utilitarian case. In particular, the prices no longer display an increasing, convex shape, instead they are (for most values of γ) a decreasing function of γ . This occurs because the higher supply of bonds that the government finds optimal to provide offsets the demand composition effect that increases individual demand for bonds as γ rises. At low values of γ , the government chooses lower debt levels (panel (i)) in part because the default probability is higher (panel (iv)), which also results in higher spreads (panel (iii)). But as γ rises and repayment incentives strengthen (because ω becomes relatively smaller than γ), the probability of default falls to zero, the spreads vanish, and debt levels increase. The price remains relatively flat because, again, the higher debt supply offsets the demand composition effect.

The political bias extension yields an additional interesting result: For a sufficiently concentrated distribution of bond holdings (high γ), L -type agents prefer that the government weighs the bondholders more than a utilitarian government (i.e., there are values of γ and ω for which, comparing equilibrium payoffs under a government with political bias versus a utilitarian government, $v^L(B_1, \omega, \gamma) > v^L(B_1, \gamma)$). To illustrate this result, Figure 7 plots the equilibrium payoffs in the political-bias model for the two types of agents as ω varies for two values of γ ($\gamma_L = 0.15$ and $\gamma_H = 0.85$). The payoffs for the L and H types are in Panels (i) and (ii), respectively. The vertical lines identify the payoffs that would be attained with

a utilitarian government (which by construction coincide with those under political bias when $\omega=\gamma$).

FIGURE 7. Welfare as a function of Political Bias for different values of γ



Panel (ii) shows that the payoff of H types is monotonically decreasing in ω because H types always prefer the higher debt levels attained by low ω governments, since debt enhances their ability to smooth consumption at a lower risk of default. In contrast, Panel (i) shows that the payoff of L types is nonmonotonic in ω and has a well-defined maximum. In the $\gamma = \gamma_L$ case, the maximum point is at $\omega = \gamma_L$, which corresponds to the equilibrium under the utilitarian government, but when $\gamma = \gamma_H$, the maximum point is at about $\omega = 0.75$, which is smaller than γ_H . Thus, in this case, ownership of public debt is sufficiently concentrated for the agents that do not hold it to prefer a government that chooses B_1 , weighting the welfare of bond holders by more than the utilitarian government. This occurs because with γ_H , the utilitarian government has strong incentives to default, and thus, the equilibrium

supports low debt, but L -type agents would be better off if the government could sustain more debt, which the government with $\omega = 0.75$ can do because it weights the welfare of bondholders more and thus has weaker default incentives. The L type agents desire more debt because they are liquidity-constrained (i.e., $\mu^L > 0$) and higher debt improves the smoothing of taxation and thus makes this constraint less tight.

These results yield an important political economy implication: Under a majority voting electoral system in which candidates are represented by values of ω , it can be the case that majorities of either L or H types elect governments with political bias $\omega < \gamma$. This possibility is captured in Figure 7. When the actual distribution of bond holdings is given by γ_H , the majority of voters are L types, and thus, it follows from panel (i) that the government represented by the ω at the maximum point (around 0.75) is elected. In this case, agents who do not hold government bonds vote for a government that favors bondholders (i.e., L types are weighed at 0.75 instead of 0.85 in the government's payoff function). When the distribution of bond holdings is given by γ_L , the majority of voters are H types and the electoral outcome is determined in Panel (ii). Since the payoff of H -type agents is decreasing in ω , they elect the government at the lower bound of ω . Hence, under both γ_L and γ_H , a candidate with political bias beats the utilitarian candidate. This result is not general, however, because we cannot rule out the possibility that there could be a $\gamma \geq 0.5$ such that the maximum point of the L -types payoff is where $\omega = \gamma$, and hence, the utilitarian government is elected under majority voting.

5.2. *International Investors*

While a large fraction of sovereign debt in Europe is in the hands of domestic households (for the countries in Table 1, 60% is the median and 75% the average), the fraction in the hands of foreign investors is not negligible. For this reason, we extend the benchmark model to incorporate foreign investors and move from a closed to an open economy. In particular, we assume that there is a pool of international investors modeled in the same way as in the Eaton-Gersovitz class of external default models: risk-neutral lenders with an opportunity cost of funds equal to an exogenous, world-determined real interest rate \bar{r} . As is common practice, we assume that these bonds are *pari passu* contracts, which rules out the possibility for the government to discriminate among borrowers when choosing to default. This also maintains the symmetry with the baseline model, in which the government was not allowed to default on a particular set of domestic households.

Since foreign lenders are the marginal investors of sovereign debt, in this model, the price of the bond is given by $q(B_1, \gamma) = (1 - p(B_1, \gamma))/(1 + \bar{r})$ where $p(B_1, \gamma)$ is the default probability. More precisely, $p(B_1, \gamma) = E_{g_1}[d(B_1, \gamma, g_1)]$. While the arbitrage condition is functionally identical to the one of the Eaton-Gersovitz models, they embody different mechanisms. The two are similar in indicating that, because of risk neutrality, risk premia are equal to default probabilities. But there is a critical difference in how these probabilities are determined. In Eaton-Gersovitz models, they follow from the values of continuation versus default of a representative agent, while in our model, they are determined by comparing those values for a utilitarian

social welfare function, which in turn depend on the dispersion of individual payoffs of default versus repayment (and on the welfare weights). Hence, concentration of debt ownership affects default probabilities via changes in the relative magnitudes of individual payoffs of default versus repayment.

We let the position of foreign investors be denoted B_t^f , which also defines the economy's net foreign asset position. We assume that a fraction φ^f of the initial stock of debt is in their hands.²⁵ That is, $B_0^f = \varphi^f B_0$. A fraction $(1 - \varphi^f)$ is distributed among domestic households according to γ . We denote the domestic demand by $B_t^d = \gamma b_t^L + (1 - \gamma)b_t^H$. The debt market clearing condition is

$$B_t^d + B_t^f = B_t. \tag{22}$$

We do not restrict the value of $[\gamma b_1^L + (1 - \gamma)b_1^H]$ to be less than or equal to B_1 so B_1^f could be positive or negative. When $B_1^f > 0$ the country is a net external borrower, because the bonds issued by the government are less than the domestic demand for them, and when $B_1^f < 0$, the country is a net external saver.

The problems of the agents and the government remain identical to those described in Section 3. Of course, agents understand that there is a new pricing equation and that market-clearing conditions incorporate the foreign demand. We solve the model numerically using the same parameter values of the benchmark

25. This assumption is made only to approximate the quantitative predictions of the model with the data but is not crucial for any of the results presented below.

model. We set $\varphi^f = 0.25$ to match the average fraction of foreign debt observed in our sample of European countries and \bar{r} to 2% to match the average real interest rate in Germany in the 2000/2007 period.

Figure 8 shows how the planner's welfare gain of default varies with γ and B_1 for different levels of government expenditures ($g_1 = \underline{g}_1$ and $g_1 = \bar{g}_1$). The no-equilibrium region, which exists for the same reasons as before, is shown in white. In line with the characteristics of default incentives of the benchmark model, within the region where the equilibrium is well defined, for a given γ , the planner's value of default increases monotonically with the level of debt B_1 . However, we observe that, contrary to the benchmark case, in the economy with foreign lenders, conditional on B_1 , the welfare gain of default has an inverted-U shape in the γ dimension. That is, for a given B_1 , the value of α decreases with γ for low γ reaches a minimum point and then increases with γ . This also determines a bell-shaped No Equilibrium Zone. The intuition for this result is simple and derives from the decision rules of domestic agents (see Figure A.12 of the Appendix for the corresponding plot). For a given level of B_1 , when γ is below ($\gamma < 0.25$ for $B_1 = B_M$ for example), the country is on average a foreign borrower (i.e., $B^f > 0$). That implies that a default generates a direct increase in domestic resources equal to the forgone debt payments to foreign lenders. In this region, both L -type and H -type agents are at the borrowing limit. As γ increases, the country becomes a net saver in foreign markets. Increases in γ are associated with an increasing portion of domestic debt in the hands of H -types. This reduces the benefit of a default on foreign lenders. However, as in the model

without foreign lenders, domestic consumption dispersion increases. For midrange γ 's, the first effect dominates the second and repayment is the preferred option. As γ increases even further, the dispersion in domestic consumption increases to points where default is again the optimal alternative for the government. In this region, the main driver of domestic default is redistribution among domestic agents as in our benchmark economy.

FIGURE 8. Planner's Welfare Gain of Default $\bar{\alpha}(B_1, g_1, \gamma)$

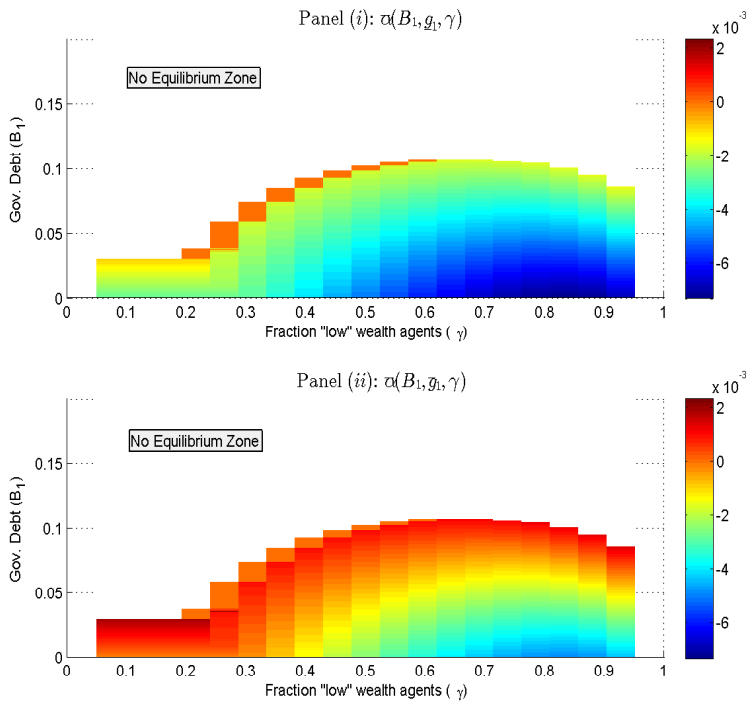
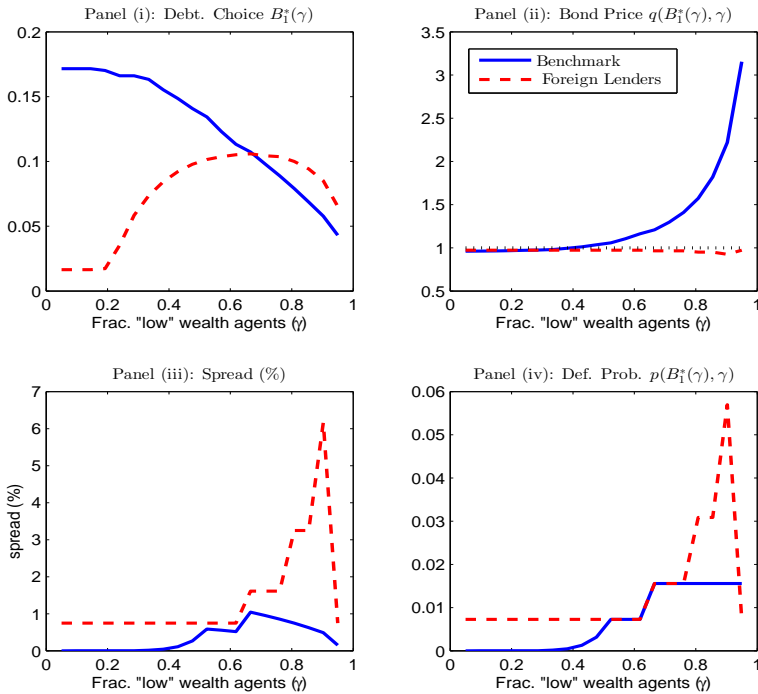


Figure 9 shows the comparison of the equilibrium functions in the economy with foreign lenders versus the benchmark economy. As we described before, the introduction of foreign lenders and the possibility of a “foreign” default constraint

debt values and result in lower debt levels than in the benchmark, for values of γ lower than 0.75 (see panel (i)). This is also reflected in higher default probabilities and spreads in the economy with foreign lenders than in the benchmark (see panels (iii) and (iv)). By construction, the upper bound on the price of the economy with foreign lenders is $(1+r)^{-1}$, so the distributive effect that negative real interest rates have in the benchmark economy dissipate (see panel (ii)). This induces the government to take on more risk and redistribute via debt issuance in the economy with foreign lenders than in the benchmark.

FIGURE 9. Equilibrium Domestic and Foreign Debt Holdings



5.3. *Redistributive Taxation (Partial Default)*

In the benchmark model, issuing debt in the first period and defaulting in the second are the only tools the government can use to reduce consumption dispersion. We now examine how the model's predictions change by adding an alternative tool for redistribution. In particular, we introduce a proportional consumption tax that is invariant across periods and realizations of g_1 .²⁶

With consumption taxes, the cost of purchasing goods on the left-hand-side of the agents' budget constraints becomes $(1 + \tau^c)c_t^i$ for $i = L, H$ and $t = 0, 1$. Since aggregate consumption in each period is still given by $y - g_t$, the government budget constraints simplify to the following expressions. At date 0, the constraint is:

$$\tau_0 + \tau^c y = g_0(1 + \tau^c) + B_0 - q_0 B_1. \quad (23)$$

In the second period, under repayment, the constraint is:

$$\tau_1^{d_1=0} + \tau^c y = g_1(1 + \tau^c) + B_1. \quad (24)$$

26. Using this tax as opposed to other taxes simplifies the solution of the model because aggregate consumption (as a function of y and g_t) is pinned down by the resource constraint, and thus, it is known before solving the agents' problem. As a result, the algorithm does not need to iterate on the level of transfers to close the government budget constraint, while introducing a wealth tax would require it. An income tax would require not only iterating on aggregate transfers but also extending the model to consider endogenous labor supply because the benchmark model assumes no income heterogeneity, which makes proportional income tax close to a lump-sum tax.

In the second period, under default, the budget constraint is:

$$\tau_1^{d_1=1} + \tau^c y(1 - \varphi(g_1)) = g_1(1 + \tau^c). \quad (25)$$

Using the budget constraints of agents and the government, we can proceed as with the benchmark model and derive expressions for the difference in consumption of the two agents in the two periods and default scenarios: $c_0^H - c_0^L = 1/(1 - \gamma)[(B_0 - q(B_1, \gamma)B_1)/(1 + \tau^c)]$, $c_1^{H,d=0} - c_1^{L,d=0} = (1/(1 - \gamma))(B_1/(1 + \tau^c))$, and $c_1^{H,d=1} - c_1^{L,d=1} = 0$. These expressions are similar to those presented in Section 4, except that the terms that include public debt in the date-0 difference and the date-1 difference under repayment are divided by $(1 + \tau^c)$. Consumption taxes reduce consumption dispersion in both instances. Intuitively, the consumption tax plays a role akin to inflation in reducing the real value of public debt. Hence, this tax always reduces consumption dispersion in the first period (as long as there is a primary deficit) and always reduces dispersion in the second period under repayment. In fact, for assuming repayment, this tax is a better mechanism for redistribution because debt can only reduce dispersion at date 0 at the expense of increasing it at date one. Moreover, the tax's ability to redistribute is not hampered by the debt Laffer curve that hampers the ability to redistribute with debt because of default risk. The tax acts in fact as a defacto partial default in both periods. As a result, if we allow it to be chosen optimally, letting τ^c go to infinity would be optimal because it completely removes consumption dispersion in all periods at no cost. The purpose of this analysis, however, is to see how the existence of an alternative redistribution

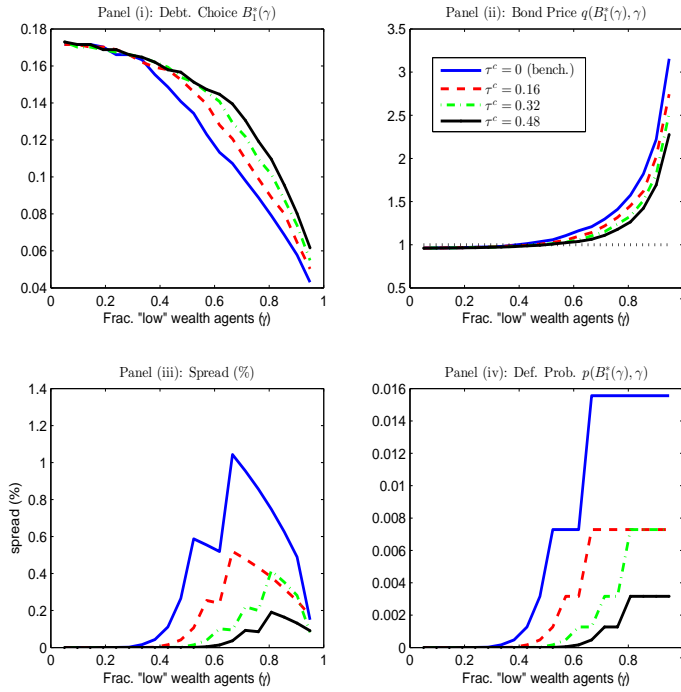
tool affects the results we have obtained for debt and default risk, rather than focus on the optimal use of the consumption tax.²⁷

We solve the model numerically using the parameters of the benchmark calibration. For the value of the consumption tax, we use $\tau^c = 0.16$ which is the value estimated for the Eurozone in Mendoza, Tesar, and Zhang (2014). We also solve the model for $\tau^c = 0.32$ and $\tau^c = 0.48$, which are two and three times larger than the data proxy.²⁸ Figure 10 presents the comparison of the equilibrium of the benchmark economy with the three economies with taxes. This figure shows that the main result of the paper (the ability to support the existence of public debt with positive default risk) is robust to incorporating distributive taxes.

27. The consumption tax as modeled lacks important tradeoffs that would make it suboptimal to set it infinitely large. In particular, it is nondistortionary because there is no labor supply choice, and because, since the tax is constant, it does not distort savings decisions. The tax is also assumed to be known with perfect foresight, whereas uncertainty about inflation would have an effect similar to a partial risk of default that would affect demand for bonds and bond prices.

28. In the tax rate estimates in Mendoza, Tesar, and Zhang (2014), the highest consumption tax is observed in Finland at 24%.

FIGURE 10. Comparison Equilibrium Benchmark vs Proportional Consumption Taxes



As consumption taxes increase, default incentives grow weaker (see lower default probabilities in panel (iv)), which is natural because the tax reduces consumption dispersion, and hence, it reduces the need for the government to use default to lower dispersion. As a result, the government can sustain higher debt levels(see panel (iv)), although the effect is not very large (debt increases by at most 5 percentage points for $\gamma = 0.6$ and a change in the consumption tax from 0% to 48%). This effect is negligible when default incentives are weak to start with, which occurs at low values of γ . The properties of the price function observed in the benchmark case (i.e., increasing in γ for a given level of default risk) are sustained in the economy with

consumption taxes (see panel (ii)). In line with the reduction in default probabilities, the model with consumption taxes also displays lower spreads than the benchmark (see panel (iii)).

The findings in this Section suggest that the existence of other tools for redistribution contributes to support higher debt levels at lower default frequencies. The results are also interesting as an illustration of what happens if we combine de jure outright default with partial, de facto default (the latter commonly takes place via inflation, but as noted earlier the consumption tax in this setup plays the same role as inflation in reducing the real debt burden).

5.4. A Second Asset

We now review the implications of adding a risk-free asset that agents can use as a vehicle of savings, in addition to public debt. In particular, agents have access to a nonstochastic production technology $y_t^i = z(k_t^i)^\theta$ with $0 < \theta < 1$, where y_t^i is total output and k_t^i is capital for agent of type i in period t , respectively.²⁹ The initial aggregate level of capital is denoted by K_0 . L -type agents are now endowed with b_0^L and k_0^L units of public debt and capital, respectively, while H -type agents have endowments given by $b_0^H = (B_0 - \gamma b_0^L)/(1 - \gamma)$ and $k_0^H = (K_0 - \gamma k_0^L)/(1 - \gamma)$. Capital depreciates at rate δ . At period 0, agents choose how much of their savings they want to allocate to public bonds $b_1^i \geq 0$ and capital $k_1^i \geq 0$.

29. The curvature in production allows us to obtain a well-defined portfolio choice of bonds and capital for private agents.

The full characterization of the equilibrium and the detailed quantitative results are discussed in Section A.10 of the Appendix. Here, we focus only on the main results. Intuitively, the existence of the second asset implies that agents arbitrage returns across public debt and risk-free capital, taking into account the risk of default on public debt. In particular, the optimality conditions for H -type agents imply:

$$\frac{q_0}{[\theta z(k_1^i)^{\theta-1} + 1 - \delta]^{-1}} = \frac{E_{g_1} [u'(c_1^{i,d=0})(1 - d_1)]}{E_{g_1} [u'(c_1^{i,d=0})(1 - d_1) + u'(c_1^{i,d=1})d_1]}. \quad (26)$$

Thus, the default spread of this model (i.e., the gap between the yield on government bonds and the marginal productivity of capital) is determined by default risk, weighted by marginal utility in each state of the world, since agents are risk-averse. From this perspective, the second asset has a similar effect as introducing the foreign investor because it introduces an opportunity cost of funds, but with the difference that this cost is now endogenous and falls as capital investment rises.

Adding the second asset also affects the government's debt and default choices. In particular, the government now has to consider the initial distributions of both bonds and capital across agents, and the effects of its choices on the individual capital and bond decisions $\{b_1^i, k_1^i\}$, which depend on B_1 and default risk. Default is more costly because via its effects on capital allocations it can cause higher consumption dispersion than before.

To make this case as close as possible to the benchmark model for the quantitative analysis, we assume $k_0^i = K_0$ (i.e., all initial heterogeneity is in initial bond holdings). Still, this results in heterogeneous bond and capital holdings for the second period.

We set z to normalize GDP to 1, $\theta = 0.33$ and $\delta = 0.10$ (standard values), and set K_0 so the capital-to-output ratio is equal to 2. All other parameters are the same as in the benchmark.

Agents in this setup have the option to switch from debt to capital as default risk rises, which adversely affects the capacity of the government to issue debt. On the other hand, the weaker default incentives because of the additional adverse consumption dispersion effects enhance the ability to issue debt. In line with these arguments, we found that for low enough γ , so that default is a zero-probability event, both models have nearly identical results. As γ rises up to $\gamma < 0.6$, the model with two assets sustains higher optimal debt than the benchmark (by at most 4 percentage points), but at higher values of γ the opposite is true (debt in the benchmark is at most 2 percentage points higher). Thus, optimal debt levels do not differ by wide margins, even though the effects of adding the second asset are noticeable.

The arbitrage condition connecting bond prices and capital returns also has interesting quantitative implications for spreads and bond prices. First, since reallocation of savings from debt to capital reduces the marginal product of capital, the default risk spread rises as γ rises, and this happens even though the probability of default, which rises with γ , is actually lower than in the benchmark. Second, bond prices rise at a much lower rate as γ rises than in the benchmark, and, in fact, are never higher than 1 even with the highest values of γ considered (i.e., bond yields are never negative). In this dimension, the quantitative results are very different in the setup with two assets, although they are qualitatively similar.

6. Conclusions

This paper proposes a framework in which domestic sovereign default and public debt subject to default risk emerge as an equilibrium outcome. In contrast to standard models of sovereign default on external debt, this model highlights the role of the domestic distribution of public debt ownership and the distributional effects of default across domestic agents in shaping the government's default incentives. These are features common to both the historical episodes of outright domestic default documented by Reinhart and Rogoff (2011) and the ongoing European debt crisis.

In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Distributional incentives alone cannot support equilibria with debt, because default is always optimal in the second period, and hence, the debt market cannot function in the first period. We also showed that equilibria with debt exposed to default risk can exist if we introduce either exogenous default costs or government preferences biased in favor of bond holders.

The main finding of the paper (i.e., that distributional incentives to default tempered by default costs can support equilibria with debt exposed to default risk) is robust to other three important extensions of the model: adding foreign investors who can hold a portion of the debt issued by the government, introducing consumption taxes as an alternative tool for redistributing resources across agents, and adding a second asset as a vehicle for saving.

This paper is a first attempt at developing a blueprint for research into models of domestic sovereign default driven by distributional incentives, and their interaction with agent heterogeneity and incomplete insurance markets. It has two main limitations, both byproducts of the two-period life horizon: First, it takes as given a distribution of bond ownership across agents. Second, it does not capture endogenous costs of default that would result from losing the benefits of public debt as a vehicle for self-insurance and liquidity-provision (see Aiyagari and McGrattan (1998)). In further work, D’Erasmus and Mendoza (2014) we are aiming to develop a framework that takes both issues into account.

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