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PIER Working Paper 14-042

"What Can the Duration of Discovered Cartels Tell Us About the Duration of Cartels?"

by

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What Can the Duration of Discovered Cartels Tell Us About the Duration of Cartels?*

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July 26, 2014

Abstract

*We express our appreciation for the comments of seminar participants at Wharton-BEPP. The first author acknowledges the financial support of the National Science Foundation (SES-1148129).

Abstract: There are many data sets based on the population of discovered cartels and it is from this data that average cartel duration and the annual probability of cartel death are estimated. It is recognized, however, that these estimates could be biased because the population of discovered cartels may not be a representative sample of the population of cartels. This paper constructs a simple birth-death-discovery process to theoretically investigate what it is we can learn about cartels from data on discovered cartels.

Keywords: Cartel detection, Collusion, Antitrust. JEL Codes: L1, L4

1 Introduction

It is well understood that cartels are socially harmful. What is far less clear is the magnitude of this harm. The seriousness of the cartel problem in an economy depends on: 1) how many cartels there are; 2) how large is the overcharge (and the elasticity of market demand); and 3) how long cartels last. Currently, what we know about these issues comes from cartels which, to their disappointment, were discovered and convicted. Other than that the number of discovered cartels is a lower bound on the number of cartels, we know little about how many cartels there are.[1] On the issue of the overcharge, there is a fair amount of work estimating how much higher is the price charged by discovered cartels; see, for example, Connor and Bolotova (2006), Oxera (2009), Connor (2010), and Boyer and Kotchoni (2012). Regarding the last factor - how long cartels last - the consensus measure of average cartel duration is the average duration of discovered cartels which most studies find to be 5-7 years.[2]

Cartel duration is not only relevant to assessing the severity of the cartel problem but also to evaluating whether current levels of enforcement are sufficient to deter cartel formation. Some economists have expressed a lack of concern about cartels on the premise that cartels are inherently unstable and thus, even if they form, they will not be around for long. That is, of course, an empirical question. There is also an active debate regarding whether penalties in some jurisdictions are too low - so that there is under-deterrence of cartels - or too high - so that they are in excess of what is necessary to deter and may be creating social costs. For example, Connor and Lande (2012) argues that there is under-deterrence. In the case of the European Union, Boyer et al (2011) provides evidence against the under-deterrence

[1] While this does not pertain to the question of how many cartels there are now, there is research that addresses how many cartels there were when cartels were lawful. At various points in time, Austria, Denmark, Germany, Finland, Norway, and Sweden permitted cartels but required that they be registered with the government. For an analysis of the Finnish cartel registry, see Hyytinen, Steen, and Toivanen (2013, 2014).

[2] Levenstein and Suslow (2006) summarize the findings of many studies on cartel duration.

claim, though see Combe and Monnier (2011) for a different view.[3] An evaluation of the extent to which enforcement is either under- or over-deterring cartels depends on the incremental profits earned from collusion, penalties in the event of discovery and conviction, and the likelihood of discovery and conviction. Using data on the duration of discovered cartels, these and other studies use an estimate of 15% per year that a cartel is caught and convicted.[4]

In assessing the extent of the cartel problem and determining whether enforcement should be strengthened or weakened, estimates of the average duration of discovered cartels and the annual probability of discovery and conviction for discovered cartels are useful only to the extent that they are reasonable proxies for the average duration of all cartels (discovered and undiscovered) and the annual probability of discovery and conviction for all cartels (discovered and undiscovered). The objective of this paper is to explore to what extent they are reasonable proxies. Are estimates using data on discovered cartels biased? If so, what is the direction and possible magnitude of the bias? Is the estimate that a cartel has a 15% chance of being caught and convicted in a year biased and, if it is, is it too high or too low?

Contrary to the general perception (for which we later provide references), we show that average duration of discovered cartels need not be a biased measure of average cartel duration. In fact, if the empirical model used in previous studies is correctly specified then estimates are unbiased. However, under plausible assumptions, estimates of the cartel population based on data using discovered cartels are shown to be biased and we are able to characterize when, for example, average du-

[3] Also see Harrington (2014a) who argues that enforcement policy is more effective than is generally believed. For some reasons as to why we should not ease up on enforcement even if we think penalties are in the over-deterrence region, see Harrington (2014b).

[4] For 184 convictions by the Antitrust Division of the U.S. Department of Justice over 1961-88, Bryant and Eckard (1991) estimated the annual chances of of disovery and conviction to lie between 13 and 17%, while for 86 convictions by the European Commission over 1969-2007, Combe, Monnier, and Legal (2008) estimated it to be around 13%.

ration of discovered cartels is an over-estimate and when it is an under-estimate of average cartel duration.

2 Model

There are generally recognized to be two primary reasons why the duration of discovered cartels may be a biased measure of the duration of all cartels: measurement error and sample selection bias. Cartel duration is measured as the time between a cartel's birth and its death. Cartel death is typically well-documented, though there can be cases in which cartels remain active even after discovery.[5] Far more problematic is dating the birth of a cartel. In most data sets, a cartel's date of birth is either the earliest time for which there is evidence of a cartel or is the product of negotiation between the defendants and the competition authority. In either case, it is reasonable to presume that official cartel birth is no earlier than actual cartel birth and, therefore, any bias from measurement error is likely to result in the duration of discovered cartels' being an under-estimate.

In this paper, we will not address the issue of measurement error and instead presume that the duration of discovered cartels is accurately measured. Our focus is on characterizing selection bias associated with using the population of discovered cartels to draw inferences about the population of cartels. In order to produce some clean insight, the noise associated with finite samples is also not considered as it is assumed there is an infinite number of industries of which some fraction are cartelized out of which some fraction are discovered. We will then be characterizing what is referred to as asymptotic bias which is the bias present with infinite samples.[6]

[5] There are some well-documented cases of post-conviction collusion in government procurement auctions in Japan; see Kawai and Nakabayashi (2014).

[6] An assumption made throughout the analysis is that we can only observe whether a cartel was discovered but not whether, upon discovery, it was active or inactive. While that assumption fits most empirical studies, such information is available in some instances; see De (2010) and Levenstein

For a continuum of industries, assume there is a cartel birth process that is constant over time so that, in each period, a mass of cartels are created, which is normalized to one.[7] Once cartelized, a cartel can "die" for natural reasons (due to a change in market conditions or firm-specific factors so collusion is no longer stable) or it can die because it has been detected and convicted by the competition authority (which, for brevity, will hereon be referred to as "discovery"). A cartel can then have three possible terminal states: 1) it can die a natural death (which we refer to as "collapse") and not be discovered; 2) it can collapse and be discovered; and 3) it can be discovered (and thus die through conviction). The probability that a cartel collapses in the current period is λ, the probability of discovery conditional on the cartel being active in the current period is ρ, and the probability of discovery conditional on the cartel just having collapsed in the current period is β. There are a variety of reasons why the likelihood of discovery may depend on whether the cartel is still active or has collapsed. For example, if customer complaints are triggered by suspicious price movements then a sharp price decline associated with the cartel collapsing may trigger discovery.[8] However, the primary reason why we have allowed for this possible dependence of discovery on the state of the cartel is to take account of leniency programs. A deterrent to a member of an active cartel to applying for leniency is that it'll cause the cartel to collapse and, as a result, the firm will forego future collusive

and Suslow (2011). However, measurement issue is a concern. For example, suppose a customer complained and firms learned about it which then caused the cartel to collapse. Thus, at the time a formal investigation is opened up, the cartel would be inactive and one might be inclined to conclude that the cartel collapsed and was then discovered but the reality is that discovery - in the form of a customer complaint, not the formal investigation - caused the collapse and, therefore, the cartel was actually active at the time of discovery.

[7] Allowing the birth process to be time-dependent (perhaps sensitive to the business cycle) is a worthy extension but, at this stage, it would be counter-productive to gaining some initial insight into selection bias.

[8] The implications for cartel pricing of having discovery depend on past prices is explored in Harrington (2004, 2005).

profits. However, if the cartel is no longer active then that cost disappears in which case firms may then self-report.[9] This would provide a rationale for $\beta > \rho$ so that cartel collapse makes discovery more likely. Alternatively, it could be the case that $\beta = \rho$ so the probability of discovery is the same whether the cartel is alive or just died. At this stage, it is assumed: $\lambda \in (0,1)$, $\rho \in (0,1)$, $\beta \in [0,1]$.

Studies that estimate the annual probability of discovery for discovered cartels - such as Bryant and Eckard (1991) and Combe, Monnier, and Legal (2008) - specify a constant hazard rate model. This is a continuous time model for which there is a constant probability of being caught in any instant. The model of this paper is a discrete time analogue in that the probabilities of death and discovery are also constant over time. In addition, it is worth noting that results can be stated either in terms of average cartel duration or the probability of death in that the former equals the inverse of the latter. For example, an over-estimate of average cartel duration is equivalent to an under-estimate of the probability of death.

Before moving on, let us note that the possibility of selection bias associated with using data on discovered cartels is recognized, though different views have been expressed regarding the direction of bias. In the original study of Bryant and Eckard (1991, pp. 535-536), it was suggested (though not presumed) that "the life of a caught conspiracy is typically no longer than that of an uncaught conspiracy ... and [so] our probability [of death] estimate is an upper bound." This suggestion that the probability of death is over-estimated was picked up more recently in Connor and Lande (2012, pp. 462-463): "[a cartel's probability of death] p is computed from samples of discovered cartels. Founders of never-discovered cartels might rationally conjecture a lower p. Thus, computed sizes of p may well overstate the actual average p for all cartels." This direction to the bias is based on the presumption that if a cartel was not caught then it would have survived longer and, therefore, the duration of discov-

[9]The birth and death of cartels is modeled in Harrington and Chang (2013) and, under the assumption of full leniency, all dying cartels have firms racing for leniency which, for the current model, means $\beta = 1$.

ered cartels is less than that for undiscovered cartels. While those papers emphasize that cartel duration is under-estimated, Levenstein and Suslow (2011, p. 463) believe that cartel duration is over-estimated: "Because our sampling procedure relies on antitrust prosecutions, it is less likely to capture very short lived cartels. These may form and disappear without ever attracting the attention of the authorities. Thus, as with most other samples of cartels, our estimates of cartel duration may be biased upward relative to the universe of all cartels ever attempted."

In light of the disparity of views, the primary motivation for this paper is to rigorously examine the matter in order to inject some clarity. In Section 3, it is assumed that all cartels are produced by the same process; that is, all cartels have the same (λ, ρ, β). Thus, cartels are *ex ante* the same but are *ex post* different because they have different realizations of the death-discovery stochastic process. This is essentially the model that underlies studies like Bryant and Eckard (1991). We find, contrary to the various views just expressed, there is no bias: average cartel duration for discovered cartels equals average cartel duration. However, inferences drawn by previous studies regarding the probability of discovery are mistaken. In Section 4, the more reasonable assumption is made which is that cartels are governed by different processes; that is, cartels can have different values for (λ, ρ, β). Now there is bias and we investigate the determinants of the direction and size of the bias.

3 Cartels are Subject to the Same Death and Discovery Process

Suppose a mass 1 of cartels are born each period and that has been true for the infinite past; hence, the cartel population is in the steady-state. There is then a mass of cartels that are born in the current period out of which $(1-\lambda)\rho+\lambda$ die - a fraction λ collapse and a fraction $(1-\lambda)\rho$ do not collapse but are discovered and thus die - and $(1-\lambda)\rho+\lambda\beta$ are discovered - a fraction $\lambda\beta$ collapse and are discovered and a fraction $(1-\lambda)\rho$ do not collapse and are discovered. Given that a cartel that died with duration of one period must have experienced birth and death in the current period (born at the start and died at the end) then the mass of cartels with duration 1 is $(1-\lambda)\rho+\lambda$. Analogously, the mass of cartels discovered with duration 1 is $(1-\lambda)\rho+\lambda\beta$.

Cartels with a duration of two periods must have been born one period ago, survived for that period, and then died in the current period. With a mass 1 of cartels born a period ago, a mass $(1-\lambda)(1-\rho)$ of them survived to the current period - they neither collapsed nor were discovered - and, of those, a fraction $(1-\lambda)\rho+\lambda$ die this period which implies there is a mass $[(1-\lambda)\rho+\lambda](1-\lambda)(1-\rho)$ of cartels that lasted two periods and, analogously, a mass $[(1-\lambda)\rho+\lambda\beta](1-\lambda)(1-\rho)$ of cartels that are discovered after two periods.

These values are shown in Table 1 along with values for other durations. Given that all cartels eventually die, average cartel duration is just the sum of the numbers in the column "Mass of cartels of duration t that died in the current period" with each number weighted by the length of cartel duration:

$$\begin{aligned}\sum_{t=1}^{\infty} t\left[(1-\lambda)\rho+\lambda\right]\left[(1-\lambda)(1-\rho)\right]^{t-1} &= \left[(1-\lambda)\rho+\lambda\right]\sum_{t=1}^{\infty} t\left[(1-\lambda)(1-\rho)\right]^{t-1} \\ &= \left[(1-\lambda)\rho+\lambda\right]\left(\frac{1}{\left[(1-\lambda)\rho+\lambda\right]^2}\right) \\ &= \frac{1}{\lambda+\rho-\lambda\rho}.\end{aligned}$$

Average cartel duration is then $(\lambda+\rho-\lambda\rho)^{-1}$.

Next, let us turn to the average duration of discovered cartels which we will also refer to as average discovery time (conditional on being discovered). Recognizing that only a mass $\frac{(1-\lambda)\rho+\lambda\beta}{1-(1-\lambda)(1-\rho)}$ of all cartels are discovered, average time until discovery is the sum of the numbers in the column "Mass of cartels of duration t discovered in the current period" divided by the total mass of discovered cartels with each weighted by the length of duration:[10]

$$\sum_{t=1}^{\infty} t \left(\frac{\left[(1-\lambda)\rho + \lambda\beta \right] \left[(1-\lambda)(1-\rho) \right]^{t-1}}{\frac{(1-\lambda)\rho+\lambda\beta}{1-(1-\lambda)(1-\rho)}} \right) = \sum_{t=1}^{\infty} t \left(\frac{\left[(1-\lambda)(1-\rho) \right]^{t-1}}{\frac{1}{1-(1-\lambda)(1-\rho)}} \right)$$
$$= \sum_{t=1}^{\infty} t \left[(1-\lambda)\rho + \lambda \right] \left[(1-\lambda)(1-\rho) \right]^{t-1} = \frac{1}{1-(1-\lambda)(1-\rho)} = \frac{1}{\lambda+\rho-\lambda\rho}.$$

Average discovery time then equals average cartel duration.

Property 1: If all cartels have the same probabilities of collapse and discovery then average duration for discovered cartels (average discovery time) equals average duration for all cartels.

This property may seem counter-intuitive because those cartels that are discovered would have continued to survive if they had not been discovered in which case one might think that the average time until death would necessarily exceed the average time until discovery. But the issue is not whether discovery shortens cartel life - it does - but rather whether discovery delivers a representative sample of the population of cartels. Inspecting Table 1, note that the discovery process samples a fraction $(1-\lambda)\rho + \lambda\beta$ of all surviving cartels and, therefore, the population of discovered cartels is, indeed, a representative sample of the population of all cartels. Hence, the average age of cartels in the discovered population is the same as the average age of cartels in the entire population. Given that much of the paper will involve loosening the restrictive assumptions that underlie that result, the takeaway should not be that average duration for discovered cartels is a good proxy for average cartel duration. Rather the takeaway is that the common perception that average duration for discovered cartels is necessarily a biased estimate of average cartel duration is incorrect.

Though, under the assumption of a common birth-death-discovery process, we can derive an unbiased measure of cartel duration, it is not possible to measure the

[10] This calculation does presume $\rho > 0$ and/or $\beta > 0$ so that there is a positive mass of cartels that are discovered.

likelihood that a cartel is discovered. What we can measure is the cartel survival rate $(1-\lambda)(1-\rho)$; that is, the probability that an active cartel neither collapses nor is discovered. Inspecting Table 1, the survival rate equals the ratio of the mass of discovered cartels in period $t+1$ to the mass of discovered cartels in period t. Though the survival rate can be derived, the probability of collapse λ and the probability of discovery (conditional on being active) ρ cannot be separately identified, and nothing can be said about β (which is the probability of discovery conditional on having just collapsed).

The inability to disentangle λ and ρ is a point not well-appreciated. Assuming that the empirical model is correctly specified, the estimate of around .15 from studies such as Bryant and Eckard (1991) is an estimate of the probability of death, $1-(1-\lambda)(1-\rho)$, not the probability of discovery and conviction, ρ. If there is no model misspecification, it is then an upper bound on ρ (which is only achieved when $\lambda=0$ so cartels are perfectly stable). Yet, many studies appeal to this empirical work to justify assuming there is a 15% chance each year that a cartel is caught and convicted. For example, in showing that the socially optimal penalty is approximately 75% of the current formula used by the European Commission, Katsoulacos and Ulph (2013) assumes the annual probability a cartel pays penalties is .15. Hinloopen and Soetevent (2008) engage in experimental work to investigate the impact of leniency programs and assume that subjects who choose to communicate incur a penalty with probability .15. In discussing the role of customer damages, Sweeney (2006, p. 843) appeals to Bryant and Eckard (1991) to note that "only about 13-17 per cent of cartels in the U.S. are detected." And, until deriving this result, one of the authors of this paper was equally guilty of this error (Harrington, 2014a).

Another misconception is that these estimates allow one to infer how many cartels escape discovery. As a recent example, an *Amicus Curiae* Brief submitted to the U.S. Seventh Circuit Court refers to "estimates suggesting that more than two-thirds of conspiratorial activity goes undetected and unpunished."[11] If we use the expression in Table 1 (and, for simplicity, assume $\beta = \rho$), the fraction of cartels that are not discovered equals $\frac{\lambda(1-\rho)}{\lambda(1-\rho)+\rho}$. As already noted, λ and ρ are not separately identified so it would seem one could not infer how many cartels go undiscovered. If the estimated survival rate $s = (1-\lambda)(1-\rho)$ is substituted into this expression, the fraction of cartels that escape discovery can be expressed as $1 - \frac{\rho}{1-s}$. Given that $\rho \in (0, 1-s]$, the fraction of cartels that escape discovery can range from almost none (when ρ is close to $1-s$) to almost all (when ρ is close to zero). There is then no basis upon which to infer how many cartels go undiscovered.

To summarize the results of this section, if all cartels are subject to the same birth-death-discovery process then the population of discovered cartels is a representative sample of the population of cartels, which implies: 1) average duration of discovered cartels is an unbiased measure of average cartel duration; and 2) the estimated probability of death for discovered cartels is an unbiased measure of the probability of death for cartels. However, the probability of collapse and the probability of discovery cannot be separately identified and this has the implication that: 1) assuming there is no model misspecification, the estimated death rate in studies based on the duration of discovered cartels is an upper bound on the probability a cartel is caught and convicted; and 2) nothing can be inferred about the fraction of cartels that escape detection.

[11] "*Amicus Curiae* Brief of Economists and Professors in Support of Appellant's Petititon for Rehearing *En Banc*"- Motorola *Motorola Mobility LLC v. AU Optronics*, No. 14-8003, 2014 WL 1243797 (7th Cir. Mar. 27, 2014); p. 4. In making this statement, the Brief references Bryant and Eckard (1991) and Combe, Monnier, and Legal (2008).

4 Cartels are Subject to Different Death and Discovery Processes

4.1 General Properties

Given that it is clearly restrictive to assume that all cartels are subject to the same stochastic process determining death and detection, let us enrich the environment by allowing cartels to be generated by different processes. This is modeled by allowing the values for (λ, ρ, β) to vary across cartels. This heterogeneity could be due, for example, to industry traits or the characteristics of the cartel. For example, the probability of cartel collapse, λ, could depend on product substitutability and demand variability, both of which affect the gains to cheating and the losses from punishment; or on the ease with which firms can monitor compliance. The discovery parameters ρ and β could depend on the number of firms involved in the cartel as well as the type of consumers they face, where industrial customers are more likely to detect collusion than consumers in a retail market.

In each period, a unit mass of cartels is born and $k : [0,1]^3 \rightarrow R_+$ is the density function over cartel traits (λ, ρ, β) for those newly formed cartels. By the analysis in the preceding section, the average cartel duration for a type-(λ, ρ, β) cartel is $CD(\lambda, \rho) \equiv (\lambda + \rho - \lambda\rho)^{-1}$. The average cartel duration across cartels of all types is simply the weighted average of the average cartel duration for a type-(λ, ρ, β) cartel where the weight is $k(\lambda, \rho, \beta)$:

$$\int\int\int (\lambda + \rho - \lambda\rho)^{-1} k(\lambda, \rho, \beta)\, d\lambda d\rho d\beta.$$

In calculating the average duration of discovered cartels (or average discovery time), first note that the mass of discovered cartels with duration 1 is

$$\int\int\int [(1-\lambda)\rho + \lambda\beta]\, k(\lambda, \rho, \beta)\, d\lambda d\rho d\beta,$$

with duration 2 is

$$\int\int\int (1-\lambda)(1-\rho)\left[(1-\lambda)\rho+\lambda\beta\right] k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta,$$

and with duration T is

$$\int\int\int \left[(1-\lambda)(1-\rho)\right]^{T-1}\left[(1-\lambda)\rho+\lambda\beta\right] k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta.$$

The total mass of discovered cartels is then

$$\begin{aligned}\Gamma &\equiv \sum_{t=1}^{\infty}\int\int\int \left[(1-\lambda)(1-\rho)\right]^{t-1}\left[(1-\lambda)\rho+\lambda\beta\right] k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \\ &= \int\int\int \left(\frac{(1-\lambda)\rho+\lambda\beta}{\lambda+\rho-\lambda\rho}\right) k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta.\end{aligned}$$

Hence, average discovery time is

$$\begin{aligned}&\left(\frac{1}{\Gamma}\right)\sum_{t=1}^{\infty}\int\int\int t\left[(1-\lambda)(1-\rho)\right]^{t-1}\left[(1-\lambda)\rho+\lambda\beta\right] k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \\ =&\left(\frac{1}{\Gamma}\right)\int\int\int \left(\frac{1}{\left[1-(1-\lambda)(1-\rho)\right]^2}\right)\left[(1-\lambda)\rho+\lambda\beta\right] k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \\ =&\left(\frac{1}{\Gamma}\right)\int\int\int \left(\frac{1}{\lambda+\rho-\lambda\rho}\right)\left(\frac{(1-\lambda)\rho+\lambda\beta}{\lambda+\rho-\lambda\rho}\right) k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \\ =&\left(\frac{1}{\Gamma}\right)\int\int\int CD(\lambda,\rho)\, P(\lambda,\rho,\beta)\, CD(\lambda,\rho)\, k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta\end{aligned}$$

where $P(\lambda,\rho,\beta)\equiv(1-\lambda)\rho+\lambda\beta$ is the per period probability that a cartel is detected.

The task before us is to compare average duration for all cartels (denoted ACD for average cartel duration)

$$ACD \equiv \int\int\int CD(\lambda,\rho)\, k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \tag{1}$$

and average duration for discovered cartels (denoted ADT for average discovery time)

$$ADT \equiv \int\int\int \theta(\lambda,\rho,\beta)\, CD(\lambda,\rho)\, k(\lambda,\rho,\beta)\, d\lambda d\rho d\beta \tag{2}$$

where

$$\theta(\lambda,\rho,\beta) \tag{3}$$
$$\equiv \left(\int\int\int\left(\frac{(1-\lambda)\rho+\lambda\beta}{\lambda+\rho-\lambda\rho}\right)k(\lambda,\rho,\beta)\,d\lambda d\rho d\beta\right)^{-1}\left(\frac{(1-\lambda)\rho+\lambda\beta}{\lambda+\rho-\lambda\rho}\right)$$
$$= \left(\frac{1}{\Gamma}\right)P(\lambda,\rho,\beta)\,CD(\lambda,\rho)$$

In calculating average cartel duration, the average duration of a type-(λ,ρ,β) cartel is weighted by how many of them are in the population which is $k(\lambda,\rho,\beta)$. However, average discovery time weights the average duration of a type-(λ,ρ,β) cartel by $\theta(\lambda,\rho,\beta)\,k(\lambda,\rho,\beta)$. If $\theta(\lambda,\rho,\beta)$ is correlated with average duration then average discovery time is a biased measure of average duration.

In examining $\theta(\lambda,\rho,\beta)$, note that it is proportional to the probability that a type-(λ,ρ,β) cartel is discovered in a given period, $P(\lambda,\rho,\beta)$, multiplied by average cartel duration for a cartel of type-(λ,ρ,β), $CD(\lambda,\rho)$. If one thinks of discovery as a sampling process, $\theta(\lambda,\rho,\beta)$ has a natural interpretation. A cartel with a higher probability of discovery is more likely to be sampled and thus will be more heavily weighted in the calculation of average discovery time. A cartel with longer duration has more opportunities to be sampled because there are more periods for which it can be discovered; a cartel that is not discovered in the year of its death avoids being sampled. Thus, higher duration results in a higher value for $\theta(\lambda,\rho,\beta)$ and those cartels are more heavily weighted in the calculation of average discovery time.

Generally, we expect average discovery time to differ from average cartel duration and, in the ensuing analysis, we will seek to characterize how they differ. But before doing so, it is worth noting that the two measures are identical when $\beta = 1$.[12] Examining (3), $\beta = 1$ implies $\theta(\lambda,\rho,1) = 1$ for all (λ,ρ) and, therefore, $ADT = ACD$.

[12] A leniency program could possibly result in β being close to one. As shown in Harrington and Chang (2013), if leniency is full and firms do not anticipate colluding again then, upon collapse, firms will act to minimize expected penalties which implies it is a dominant strategy to apply for leniency. Hence, all dying cartels are discovered.

The intuition is immediate. If an active cartel is discovered then the act of discovery causes its death so its discovery time equals its duration. If a cartel that collapses is then discovered (as is the case when $\beta = 1$) then again its discovery time equals its duration. In other words, the population of discovered cartels is the same as the population of cartels in which case the "sample" of cartels derived from discovery is actually the universe of cartels. Clearly, there is no bias in that situation.

When instead $\beta < 1$, average discovery time is generally a biased measure of average cartel duration. The issue is what we can say about the direction and magnitude of that bias: Does average discovery time tend to over- or under-estimate average cartel duration? Given that ρ and β are apt to be positively related, let us suppose for the moment that $\beta = \varphi(\rho)$ where $\varphi'(\rho) \geq 0$. The weighting factor in ADT is then

$$\theta(\lambda, \rho, \varphi(\rho)) = \left(\frac{1}{\Gamma}\right)\left(\frac{(1-\lambda)\rho + \lambda\varphi(\rho)}{\lambda + \rho - \lambda\rho}\right),$$

and is increasing in the probability of discovery:

$$\frac{\partial\theta(\lambda, \rho, \varphi(\rho))}{\partial\rho} = \left(\frac{1}{\Gamma}\right)\left(\frac{(1-\lambda)\lambda(1-\varphi(\rho)) + \lambda\varphi'(\rho)\left[(1-\lambda)\rho + \lambda\right]}{(\lambda + \rho - \lambda\rho)^2}\right) > 0.$$

Unsurprisingly, cartels endowed with a higher discovery probability are more likely to be sampled (through discovery) and thus are given more weight in the calculation of ADT. Furthermore, holding λ fixed, a higher discovery probability results in shorter cartel duration: $(\lambda + \rho - \lambda\rho)^{-1}$ is decreasing in ρ. Thus, variation in ρ causes cartels with shorter duration to be weighted more, holding λ fixed. Cartels that are more likely to be discovered are more heavily represented in the population of discovered cartels and cartels that are more likely to be discovered have shorter duration. This effect causes average discovery time to *under-estimate* average cartel duration.

Next consider the effect of changing the probability of collapse:

$$\frac{\partial\theta(\lambda, \rho, \varphi(\rho))}{\partial\lambda} = -\left(\frac{1}{\Gamma}\right)\left(\frac{\rho(1-\varphi(\rho))}{(\lambda + \rho - \lambda\rho)^2}\right) < 0.$$

Cartels that are more likely to collapse are then under-represented in the population of discovered cartels because they are more likely to die and disappear before being detected. Thus, variation in λ causes cartels with shorter duration to be weighted less, holding ρ fixed. Cartels that are less likely to collapse are more heavily represented in the population of discovered cartels and cartels that are less likely to collapse have longer duration. This effect causes average discovery time to *over-estimate* average cartel duration.

Summing up to this point, average duration of discovered cartels could either be an over-estimate or an under-estimate of average duration of all cartels. Relatively unstable cartels (that is, those with a high value for λ) tend to be under-represented in the pool of discovered cartels because they are more likely to die before being discovered. Given that unstable cartels have shorter duration, this effect will cause average discovery time to over-estimate average cartel duration. Cartels that are relatively likely to be discovered (that is, those with a high value for ρ) tend to be over-represented in the pool of discovered cartels. Given that cartels more likely to be discovered have shorter duration then this effect will cause average discovery time to under-estimate average cartel duration. The following result is immediate.

Property 2: If there is variation in the discovery probability ρ but not in the collapse probability λ then average duration of discovered cartels is an under-estimate of average cartel duration: $ADT < ACD$. If there is variation in the collapse probability λ but not in the discovery probability ρ then average duration of discovered cartels is an over-estimate of average cartel duration: $ADT > ACD$.

The bias in using duration on discovered cartels to estimate average cartel duration (or the probability of cartel death) comes from inter-industry variation in the stochastic processes determining death and detection. If there is no variation - all cartels are subject to the same stochastic process - then there is no bias.

4.2 Numerical Analysis

The preceding section showed: 1) when there is variation across cartels in the probability of collapse and/or the probability of discovery then, generically, average discovery time is a biased measure of average cartel duration; 2) when there is only variation in the probability of collapse then average discovery time is an over-estimate of average cartel duration; and 3) when there is only variation in the probability of discovery then average discovery time is an under-estimate of average cartel duration. It is then variation in cartel collapse and discovery rates that creates bias. A reasonable conjecture is that more variation in collapse (discovery) probabilities ought to make average discovery more of an over-estimate (under-estimate) of average cartel duration. To assess the validity of that conjecture, we turn to numerical analysis.

To reduce the number of parameters, set $\beta = \rho$ so that the probability of discovery is the same whether the cartel is active or has just collapsed. The task is to specify distributions on the parameters, calculate average cartel duration and average discovery time, and measure bias which we'll define as the percentage difference between ADT and ACD:

$$Bias = 100\left(\frac{ADT - ACD}{ACD}\right).$$

Note that positive (negative) bias means that average discovery time over-estimates (under-estimates) average cartel duration.

Both λ and ρ are assumed to follow beta distributions.[13] A beta distribution is defined by two shape parameters and can be uniquely characterized by the mean μ and coefficient of variation c (which is the standard deviation divided by the mean). We first consider the case where λ and ρ are independently distributed with the following parameters:

$$(\mu_\lambda, \mu_\rho) \in \{0.05, 0.10, 0.15\}^2, \quad (c_\lambda, c_\rho) \in \{0.1, 0.4, 0.8\}^2.$$

[13] Numerical analysis was also conducted using uniform and triangular density functions and the results are qualitatively similar.

Thus, the average probability of collapse or of discovery ranges from .05 to .15 per period, which correspond to a per period death rate ranging from .10 to .28. The standard deviation can be small - only 10% of the mean - or large - 80% of the mean. In total, there are 81 parameterizations.

The results are displayed in Table 2. In reading the tables, consider Table 2a which reports average cartel duration for each parameterization. There are 9 rows and 9 columns. The top three rows correspond to $\mu_\lambda = .05$, the middle three rows to $\mu_\lambda = .1$, and the bottom three rows to $\mu_\lambda = .15$. Analogously, the three columns on the left correspond to $\mu_\rho = .05$, the middle three columns to $\mu_\rho = .1$, and the three columns on the right to $\mu_\rho = .15$. Within each block of three rows and three columns (corresponding to particular means for λ and ρ), there are three rows and columns associated with different values for (c_λ, c_ρ). For example, if $(\mu_\lambda, \mu_\rho) = (.1, .1)$ and $(c_\lambda, c_\rho) = (.4, .8)$ then the cartel duration is 6.35 which means on average a cartel survives for 6.35 periods.

As conjectured, more variation in λ results in a larger over-estimate, while more variation in ρ results in a larger under-estimate. To show the former result, hold fixed the distribution on ρ and the mean of λ. As the inter-industry variation in λ is increased (which occurs when the coefficient of variation increases holding the mean fixed), the bias increases. For example, if $(\mu_\lambda, \mu_\rho) = (.1, .1)$ and $c_\rho = .1$ (so there is little variation in ρ) then increasing variation in λ by raising c_λ from .1 to .4 to .8 causes the bias to go from 0.0% (bias is almost zero) to 3.1% (average discovery time slightly over-estimates average cartel duration) to 10.2% (average discovery time more significantly over-estimates average cartel duration). In all cases, as c_λ increases - holding μ_λ, μ_ρ, and c_ρ fixed - average discovery time increases relative to average cartel duration so the former is more of an over-estimate. Analogously, as c_ρ increases - holding μ_λ, μ_ρ, and c_λ fixed - average discovery time declines relative to average cartel duration so the former is more of an under-estimate.

Property 3: As the variation across cartels in the probability of collapse increases, the difference between average discovery time and average cartel duration increases so average discovery time is more of an over-estimate. As the variation across cartels in the probability of discovery increases, the difference between average discovery time and average cartel duration decreases so average discovery time is more of an under-estimate.

A second takeaway is that the extent of over- or under-estimation is not very large in magnitude. For most parameterizations, it is less than 20%. It is possible, however, for the absolute size of bias to be significant. This occurs when the mean and variance of the collapse rate is large and the mean and variance of the discovery rate is small. For example, if $(\mu_\lambda, \mu_\rho) = (.15, .05)$ and $(c_\lambda, c_\rho) = (.8, .1)$ then average discovery time over-estimates average cartel duration by about 34%.

Although the case when λ and ρ are independent is a useful benchmark, it could be more natural for λ and ρ to be positively correlated. For example, in an industry with more firms, the probability of collapse may tend to be higher because there is a larger incremental gain to cheating which makes the cartel less stable. In addition, more firms may raise the probability of discovery because there are more people involved so there is a greater chance that someone, either intentionally or inadvertently, would reveal information about the cartel to a customer or the authorities. Furthermore, a higher discovery probability will reduce the expected payoff from collusion and that could lead to a higher probability of collapse.

To assess the robustness of our results, let us then allow λ and ρ to be positively correlated. Using a system of bivariate distributions introduced by Plackett (1965), joint distributions between λ and ρ are constructed that have the same marginal distributions assumed for the independent case. Plackett's system is a general method to generate bivariate distribution given any pair of marginal distributions. The joint distribution is characterized by one parameter $\psi \geq 0$. As ψ goes from 0 to 1, the two random variables go from being highly negatively correlated to independent. As ψ goes from 1 to $+\infty$, the two random variables are increasingly positively correlated.

Hence one can numerically search over ψ for desired correlation. For illustration, Figure 1 plots the marginal densities when $(\mu_\lambda, \mu_\rho) = (.1, .1)$ and $(c_\lambda, c_\rho) = (.4, .8)$, and Figure 2 plots the corresponding joint densities for correlation being 0, .3 and .7, respectively.

Results are in Tables 3 and 4 and they are presented in the same manner as Table 2. Table 3 assumes λ and ρ have a moderate positive correlation of 0.30 ($\psi \simeq 2.8$), and Table 4 assumes λ and ρ have a more significant correlation of 0.70 ($\psi \simeq 16$). We find the relationship between the inter-industry variation of λ and ρ and bias is quantitatively similar to that in the independent case. However, when either λ or ρ have a large degree of variation, there can be violation to the pattern described in Property 3. For example, when $(\mu_\lambda, \mu_\rho) = (.1, .1)$ and $c_\rho = .8$ in Table 4, the bias *decreases* as the variation in λ is increased by raising c_λ from .1 to .4. However, these violations are all small in magnitude, and we believe that the main message of Property 3 is broadly relevant even when λ and ρ are positively correlated.

5 Concluding Remarks: What Do We Tell Policymakers?

Suppose a policymaker asked: "So what is your best guess about cartel duration and the probability of cartel survival based on data from discovered cartels?" In concluding this paper, we will try to answer that question though admittedly our answer is constructed on a blend of rigorously derived results and informed speculation. To begin, let us review our findings. If all cartels are subject to the same birth-death-discovery process then there is no selection bias. However, if the more plausible assumption is made that cartels are endowed with different probabilities of collapse and discovery then the population of discovered cartels is a biased sample of the population of cartels. If inter-industry variation in the probability of collapse is large (small) relative to variation in the probability of discovery then long-lived (short-lived) cartels are over-represented in the sample of discovered cartels and, therefore, average cartel duration is less (more) than measured duration for discovered cartels.

There would seem to be a stronger case to be made that inter-industry variation in the rates of internal collapse exceeds inter-industry variation in the rates of discovery. From the theory of collusion, the difficulty of colluding - and by extrapolation the likelihood of collapse - can vary significantly across markets due to market concentration, product substitutability, excess capacity, firm asymmetries, demand variability, price transparency, and other factors (Motta, 2004). While there are then many sources of inter-industry variation in collapse rates, there seem to be far fewer industry-specific traits influencing the likelihood that a cartel is discovered. Cartels with more members means there are more individuals involved and a greater chance that someone, either inadvertently or intentionally, would reveal information; and cartels in intermediate goods markets are probably more likely to be discovered than those in retail markets because of the sophistication of buyers. While these do provide some sources of inter-industry variation in discovery rates, it is difficult to think of many other ones and, more importantly, the primary source of discovery - enforcement policy - is not industry-specific as it is applicable to all cartels within a country.

If, as has just been suggested, inter-industry variation in the rates at which cartels collapse exceeds inter-industry variation in the rates at which they are discovered then it follows from the analysis of this paper that selection bias will cause estimates of cartel duration (based on discovered cartels) to exceed true duration and estimates of the death rate to be less than the true death rate. However, in terms of its magnitude, our numerical analysis suggests this bias is probably not particularly large. For a wide range of assumptions, it is less than 10-15% (though can exceed 30%). While selection bias is being argued to cause measured cartel duration to exceed true cartel duration, measurement error will operate in the opposite direction (as explained in Section 2). If cartel birth is mismeasured then it is surely because the cartel operated prior to the official date of birth used to measure duration. Unfortunately, we do not have estimates of the size of measurement error. As long as measurement error is neither substantively less than or substantively more than 10-20%, measures of duration and death rates based on the observed sample of discovered cartels are quite possibly reasonable approximations of duration and death rates of the latent population of cartels. At this time, that is the best answer we have to offer to inquistive policymakers.

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Table 1: Population of Cartels in the Current Period

t	Mass of cartels born t periods ago and are alive in the current period	Mass of cartels of duration t that died in the current period	Mass of cartels of duration t discovered in the current period
1	1	$(1-\lambda)\rho+\lambda$	$(1-\lambda)\rho+\lambda\beta$
2	$(1-\lambda)(1-\rho)$	$(1-\lambda)(1-\rho)[(1-\lambda)\rho+\lambda]$	$(1-\lambda)(1-\rho)[(1-\lambda)\rho+\lambda\beta]$
3	$[(1-\lambda)(1-\rho)]^2$	$[(1-\lambda)(1-\rho)]^2[(1-\lambda)\rho+\lambda]$	$[(1-\lambda)(1-\rho)]^2[(1-\lambda)\rho+\lambda\beta]$
$\vdots$			
T	$[(1-\lambda)(1-\rho)]^{T-1}$	$[(1-\lambda)(1-\rho)]^{T-1}[(1-\lambda)\rho+\lambda]$	$[(1-\lambda)(1-\rho)]^{T-1}[(1-\lambda)\rho+\lambda\beta]$
Sum		$[(1-\lambda)\rho+\lambda]\sum_{t=1}^{\infty}[(1-\lambda)(1-\rho)]^{t-1}$	$[(1-\lambda)\rho+\lambda\beta]\sum_{t=1}^{\infty}[(1-\lambda)(1-\rho)]^{t-1}$
		$=1$	$=\frac{(1-\lambda)\rho+\lambda\beta}{1-(1-\lambda)(1-\rho)}$

Table 2a: Average Cartel Durations When λ and ρ Are Independent ($\psi = 1$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	10.31	10.66	11.64	6.93	7.39	8.78	5.23	5.68	7.26
	$c_\lambda = .4$	10.66	11.11	12.42	7.03	7.54	9.2	5.26	5.75	7.57
	$c_\lambda = .8$	11.64	12.42	15.21	7.3	7.96	10.69	5.37	5.94	8.65
$\mu_\lambda = .1$	$c_\lambda = .1$	6.93	7.03	7.3	5.29	5.46	5.95	4.28	4.49	5.13
	$c_\lambda = .4$	7.39	7.54	7.96	5.46	5.69	6.35	4.36	4.61	5.42
	$c_\lambda = .8$	8.78	9.2	10.69	5.95	6.35	7.9	4.58	4.95	6.55
$\mu_\lambda = .15$	$c_\lambda = .1$	5.23	5.26	5.37	4.28	4.36	4.58	3.62	3.73	4.06
	$c_\lambda = .4$	5.68	5.75	5.94	4.49	4.61	4.95	3.73	3.88	4.33
	$c_\lambda = .8$	7.26	7.57	8.65	5.13	5.42	6.55	4.06	4.33	5.51

Table 2b: Average Discovery Time When λ and ρ Are Independent ($\psi = 1$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	10.3	10.26	9.98	6.92	7.1	7.28	5.22	5.47	5.88
	$c_\lambda = .4$	11.01	11.09	11.04	7.12	7.38	7.81	5.29	5.6	6.23
	$c_\lambda = .8$	12.85	13.45	15.11	7.6	8.12	9.75	5.49	5.95	7.54
$\mu_\lambda = .1$	$c_\lambda = .1$	6.95	6.83	6.49	5.29	5.26	5.07	4.27	4.31	4.25
	$c_\lambda = .4$	7.88	7.79	7.48	5.63	5.66	5.6	4.43	4.52	4.6
	$c_\lambda = .8$	10.71	11.06	11.99	6.56	6.87	7.79	4.85	5.14	6.05
$\mu_\lambda = .15$	$c_\lambda = .1$	5.25	5.15	4.9	4.28	4.21	4	3.62	3.59	3.44
	$c_\lambda = .4$	6.22	6.12	5.82	4.71	4.68	4.52	3.84	3.86	3.79
	$c_\lambda = .8$	9.71	9.99	10.8	5.97	6.21	6.96	4.46	4.68	5.39

Table 2c: Biases in Percentage When λ and ρ Are Independent ($\psi = 1$)

%		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	0.0	-3.7	-14.3	-0.1	-3.9	-17.1	-0.2	-3.7	-19.1
	$c_\lambda = .4$	3.3	-0.2	-11.1	1.2	-2.2	-15.2	0.6	-2.6	-17.7
	$c_\lambda = .8$	10.4	8.3	-0.6	4.2	2.1	-8.8	2.2	0.1	-12.9
$\mu_\lambda = .1$	$c_\lambda = .1$	0.2	-2.9	-11.1	0.0	-3.8	-14.8	-0.1	-4.1	-17.2
	$c_\lambda = .4$	6.7	3.4	-6.0	3.1	-0.4	-11.8	1.7	-1.9	-15.1
	$c_\lambda = .8$	22.0	20.2	12.2	10.2	8.2	-1.4	5.8	3.7	-7.6
$\mu_\lambda = .15$	$c_\lambda = .1$	0.4	-2.1	-8.8	0.1	-3.3	-12.7	0.0	-3.8	-15.2
	$c_\lambda = .4$	9.4	6.4	-2.1	5.0	1.6	-8.8	3.0	-0.6	-12.6
	$c_\lambda = .8$	33.7	32.1	24.9	16.4	14.7	6.2	9.9	8.0	-2.2

Table 3a: Average Cartel Durations When Correlation is 0.3 ($\psi \simeq 2.8$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	10.32	10.73	11.79	6.94	7.43	8.9	5.23	5.71	7.37
	$c_\lambda = .4$	10.73	11.41	13.14	7.07	7.72	9.79	5.29	5.88	8.07
	$c_\lambda = .8$	11.79	13.14	17.64	7.37	8.35	12.43	5.41	6.19	10.1
$\mu_\lambda = .1$	$c_\lambda = .1$	6.94	7.07	7.37	5.29	5.5	6.03	4.28	4.52	5.2
	$c_\lambda = .4$	7.43	7.72	8.35	5.5	5.84	6.73	4.38	4.73	5.77
	$c_\lambda = .8$	8.9	9.79	12.43	6.03	6.73	9.27	4.64	5.23	7.75
$\mu_\lambda = .15$	$c_\lambda = .1$	5.23	5.29	5.41	4.28	4.38	4.64	3.62	3.76	4.11
	$c_\lambda = .4$	5.71	5.88	6.19	4.52	4.73	5.23	3.76	3.99	4.6
	$c_\lambda = .8$	7.37	8.07	10.1	5.2	5.77	7.75	4.11	4.6	6.58

Table 3b: Average Discovery Time When Correlation is 0.3 ($\psi \simeq 2.8$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	10.32	10.32	10.05	6.94	7.14	7.36	5.23	5.5	5.95
	$c_\lambda = .4$	11.09	11.38	11.45	7.16	7.59	8.22	5.32	5.75	6.59
	$c_\lambda = .8$	13.05	14.38	17.48	7.7	8.61	11.34	5.54	6.26	8.8
$\mu_\lambda = .1$	$c_\lambda = .1$	6.95	6.84	6.49	5.29	5.28	5.1	4.28	4.33	4.29
	$c_\lambda = .4$	7.92	7.91	7.57	5.67	5.81	5.8	4.46	4.65	4.81
	$c_\lambda = .8$	10.9	11.88	13.87	6.66	7.35	9.09	4.92	5.47	7.1
$\mu_\lambda = .15$	$c_\lambda = .1$	5.25	5.15	4.88	4.28	4.23	4.01	3.62	3.61	3.46
	$c_\lambda = .4$	6.23	6.16	5.8	4.74	4.78	4.62	3.87	3.96	3.92
	$c_\lambda = .8$	9.89	10.78	12.63	6.07	6.68	8.18	4.53	5.01	6.37

Table 3c: Biases in Percentage When Correlation is 0.3 ($\psi \simeq 2.8$)

%		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
$\mu_\lambda = .05$	$c_\lambda = .1$	0.0	-3.8	-14.8	-0.1	-3.9	-17.4	-0.1	-3.6	-19.3
	$c_\lambda = .4$	3.3	-0.3	-12.8	1.3	-1.8	-16.1	0.7	-2.2	-18.4
	$c_\lambda = .8$	10.7	9.4	-0.9	4.5	3.2	-8.7	2.4	1.1	-12.9
$\mu_\lambda = .1$	$c_\lambda = .1$	0.1	-3.2	-12.0	0.0	-3.9	-15.3	-0.1	-4.1	-17.6
	$c_\lambda = .4$	6.5	2.5	-9.2	3.2	-0.5	-13.8	1.8	-1.7	-16.7
	$c_\lambda = .8$	22.4	21.3	11.6	10.4	9.2	-2.0	6.1	4.7	-8.4
$\mu_\lambda = .15$	$c_\lambda = .1$	0.3	-2.6	-9.9	0.1	-3.5	-13.4	-0.1	-4.0	-15.9
	$c_\lambda = .4$	9.1	4.8	-6.4	4.9	1.0	-11.6	3.0	-0.8	-14.9
	$c_\lambda = .8$	34.1	33.5	25.1	16.7	15.7	5.6	10.2	8.9	-3.1

Table 4a: Average Cartel Durations When Correlation is 0.7 ($\psi \simeq 16$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
	$c_\lambda = .1$	10.34	10.82	11.99	6.95	7.49	9.08	5.24	5.75	7.53
$\mu_\lambda = .05$	$c_\lambda = .4$	10.82	11.84	14.2	7.12	7.99	10.69	5.32	6.05	8.87
	$c_\lambda = .8$	11.99	14.2	22.09	7.47	8.89	15.67	5.47	6.53	12.87
	$c_\lambda = .1$	6.95	7.12	7.47	5.31	5.54	6.13	4.29	4.56	5.3
$\mu_\lambda = .1$	$c_\lambda = .4$	7.49	7.99	8.89	5.54	6.06	7.29	4.42	4.91	6.31
	$c_\lambda = .8$	9.08	10.69	15.67	6.13	7.29	11.92	4.7	5.62	10.12
	$c_\lambda = .1$	5.24	5.32	5.47	4.29	4.42	4.7	3.63	3.79	4.18
$\mu_\lambda = .15$	$c_\lambda = .4$	5.75	6.05	6.53	4.56	4.91	5.62	3.79	4.14	5
	$c_\lambda = .8$	7.53	8.87	12.87	5.3	6.31	10.12	4.18	5	8.76

Table 4b: Average Discovery Time When Correlation is 0.7 ($\psi \simeq 16$)

Periods		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
	$c_\lambda = .1$	10.34	10.39	10.12	6.95	7.21	7.45	5.24	5.55	6.04
$\mu_\lambda = .05$	$c_\lambda = .4$	11.19	11.8	11.93	7.23	7.89	8.78	5.36	5.96	7.11
	$c_\lambda = .8$	13.33	15.81	21.8	7.82	9.32	14.36	5.62	6.7	11.26
	$c_\lambda = .1$	6.96	6.85	6.48	5.3	5.32	5.14	4.29	4.37	4.33
$\mu_\lambda = .1$	$c_\lambda = .4$	7.97	8.07	7.58	5.72	6.02	6.02	4.5	4.84	5.07
	$c_\lambda = .8$	11.16	13.22	17.28	6.8	8.1	11.59	5.01	5.98	9.17
	$c_\lambda = .1$	5.24	5.14	4.84	4.29	4.24	4.02	3.63	3.63	3.48
$\mu_\lambda = .15$	$c_\lambda = .4$	6.25	6.19	5.64	4.78	4.91	4.69	3.9	4.1	4.06
	$c_\lambda = .8$	10.15	12.12	16.09	6.21	7.45	10.61	4.62	5.54	8.36

Table 4c: Biases in Percentage When Correlation is 0.7 ($\psi \simeq 16$)

%		$\mu_\rho = .05$			$\mu_\rho = .1$			$\mu_\rho = .15$		
		$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$	$c_\rho = .1$	$c_\rho = .4$	$c_\rho = .8$
	$c_\lambda = .1$	0.0	-4.0	-15.6	-0.1	-3.8	-17.9	-0.1	-3.5	-19.7
$\mu_\lambda = .05$	$c_\lambda = .4$	3.4	-0.3	-16.0	1.5	-1.2	-17.9	0.9	-1.5	-19.8
	$c_\lambda = .8$	11.1	11.4	-1.3	4.8	4.9	-8.4	2.7	2.6	-12.6
	$c_\lambda = .1$	0.0	-3.7	-13.2	0.0	-4.1	-16.2	-0.1	-4.1	-18.3
$\mu_\lambda = .1$	$c_\lambda = .4$	6.4	1.1	-14.7	3.3	-0.7	-17.4	2.0	-1.4	-19.6
	$c_\lambda = .8$	23.0	23.6	10.3	10.9	11.1	-2.8	6.4	6.4	-9.4
	$c_\lambda = .1$	0.1	-3.4	-11.5	0.0	-3.9	-14.5	-0.1	-4.2	-16.8
$\mu_\lambda = .15$	$c_\lambda = .4$	8.7	2.3	-13.5	4.9	0.1	-16.5	3.1	-1.1	-18.9
	$c_\lambda = .8$	34.9	36.7	25.0	17.2	18.0	4.9	10.6	10.8	-4.5

Figure 1: Marginal Densities. $\mu_\lambda = \mu_\rho = .1$, $c_\lambda = .4$, $c_\rho = .8$.

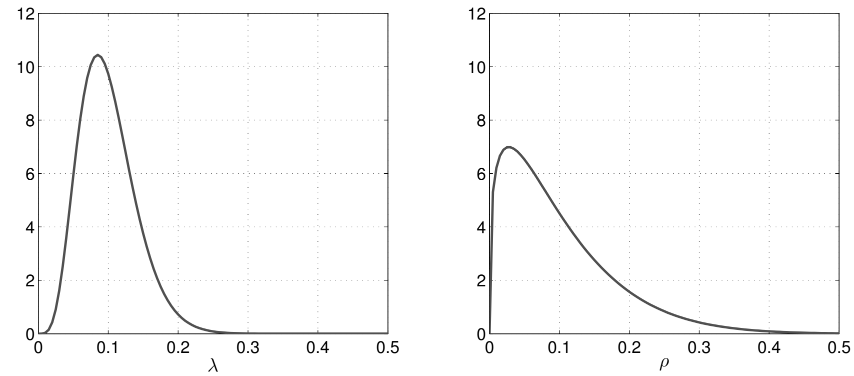

Figure 2: Joint Densities, Correlation being 0, .3 and .7, respectively.