

Penn Institute for Economic Research Department of Economics University of Pennsylvania 3718 Locust Walk Philadelphia, PA 19104-6297 <u>pier@econ.upenn.edu</u> http://economics.sas.upenn.edu/pier

# PIER Working Paper 14-027

"The Network Effects of Air-Travel Demand"

by

Yanhao Wei

http://ssrn.com/abstract=2499570

## The Network Effects of Air-Travel Demand

By Yanhao Wei\*

As demand increases, airline carriers often increase flight frequencies to meet the larger flow of passengers in their networks, which reduces passengers' schedule delays and attracts more demand. Motivated by this, I study a structural model of the U.S. airline industry accounting for possible network effects of demand. Compared with previous studies, the model implies higher cost estimates, which seem more consistent with the unprofitability of the industry; below-marginal-cost pricing becomes possible and appears on many routes. I also study airline mergers and find that the network effects can be the main factor underlying their profitability. Keywords: Airlines, Network Effects, Flight Frequency, Merger, Networks.

<sup>\*</sup> Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104 (email: yanhao@sas.upenn.edu). The research has benefited from helpful conversations with Uli Doraszelski, Joe Harrington, Katja Seim, Holger Sieg and Xun Tang. Useful feedback was provided by seminar participants at the economics department and Wharton School of the University of Pennsylvania.

#### I. Introduction

Major airline carriers in the U.S. nowadays operate large-scale networks, as illustrated in Figure I.1. In a carrier's network, cities (nodes) are connected by direct services (links). Previous empirical studies on the airline industry have been very interested in the cost-side benefits of larger flow of passengers, i.e. more densely traveled links may have lower marginal costs, which are termed "economies of density" (see, for example, Caves et al. (1984), Brueckner and Spiller (1994), Berry, Carnall and Spiller, hereafter BCS (2006)). However, much of the previous studies have paid little attention to the possible demand-side benefits of larger flow, particularly the network effects of air-travel demand.<sup>1</sup>

There can be many mechanisms that produce network effects of demand.<sup>2</sup> The main mechanism that motivates my study is flight frequency. In the post-deregulation industry, airlines in the long term compete on network structure, which includes choosing the hub locations, and in the short term compete on price, aircraft size and flight frequency (see Alder (2001), Wei and Hansen (2007), Abdelghany and Abdelghany  $(2012)^3$ ). Within a network, flows of passengers vary across links and higher flows are mostly satisfied with higher flight frequencies; meanwhile, higher frequencies reduces passengers' schedule delays and implies better quality of service (see Wei and Hansen (2005), Givoni and Rietveld (2009)). This suggests that the value of a link, and more generally the value of the network, increases with the number of passengers.<sup>4</sup>

This paper studies such network effects with a structural model similar to those

<sup>&</sup>lt;sup>1</sup>Mayer and Sinai (2002) studied the network effect of hubbing: the number of routes created by adding one new link to a hub increases with the hub size. This is different from the network effects of demand that I study in this paper.

 $<sup>^{2}</sup>$ For example, carriers may have incentives to introduce new services such as wifi access on their busiest links first, because doing so creates more word-of-mouth. Carriers also respond to higher demand by using larger aircrafts, which are usually considered more comfortable and safer.

<sup>&</sup>lt;sup>3</sup>Alder (2001) assumes a two-stage game in which carriers choose hub-and-spoke networks in the 1st stage and compete on price, frequency and aircraft size in the 2nd stage. The framework has been common in the transportation research literature. Both Wei and Hansen (2007) and Abdelghany and Abdelghany (2012) have discussion on the difference between a carrier's strategic-level decisions such as choosing the network structure and planning-level decisions such as schedule development.

<sup>&</sup>lt;sup>4</sup>See Economides (1996) for a general discussion of network effects.

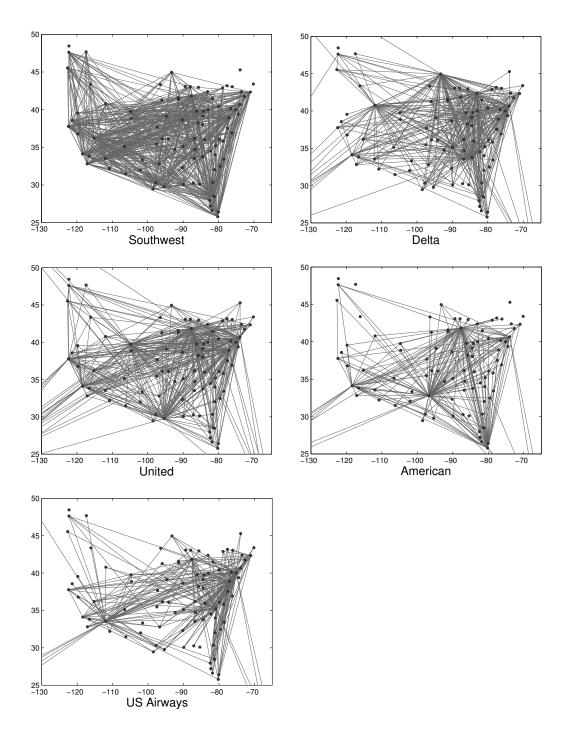


Figure I.1. Networks of five major carriers in 2012Q4-2013Q1, restricted to the 100 most-visited cities in the U.S. Links represent direct services between cities. Coordinates are longitude and latitude.

in recent discrete-choice studies of the airline industry (for example, BCS (2006), Peters (2006), Berry and Jia (2010)). Potential passengers choose which route to fly, where a route is constructed by one or more links (i.e. non-stop route and connection route). Motivated by the frequency mechanism, network effects are introduced by allowing the flow on any link to affect the demand on all the routes that utilize the link. For example, in Philadelphia, how frequently planes take off for Atlanta affects not only a passenger whose destination is Atlanta, but also a passenger that is going to make a connection in Atlanta. Meanwhile, all these passengers add to the flow between the two cities, which is a likely determinant of the frequency. The fact that one link usually serves multiple routes implies that there are "peer effects," where an increase in the demand on one route can have positive effects on the demand of all the routes that share links with it. The peer effects depend on the exact network structure of carrier: while a hub-and-spoke network is likely to have strong peer effects, a purely point-to-point network will have no such effects. Because the peer effects supplement the network effects of demand, the latter will also in general depend on the carrier's network structure.

I estimate the model with data from the Airline Origin and Destination Survey (DB1B), which is a 10% sample of airline tickets from the reporting carriers in the U.S. The identification could, in principle, face a problem similar to the "reflection" described in Manski (1993), which can be illustrated by the extreme case of a purely point-to-point network, where each route is isolated. In such a network, flow and demand coincide, thus the network effects would be the influence of the demand on itself. Similar to the use of social networks to resolve the reflection problem (see Bramoullé et al. (2009)), the identification of the network effects in this paper requires some asymmetry between the two ends of influence. Fortunately, the huband-spoke structure provides lots of such asymmetry, as the demand on a route is generally different from the flows on its links.

With network effects, the model implies much higher price elasticities of demand than a model without network effects. The intuition is clear: if a carrier increases prices, the demand will decrease, which would decrease the demand further if there are network effects. As a result, the model implies substantially higher marginalcost estimates compared with previous studies. I find these estimates consistent with the relative unprofitability of the U.S. airline industry, as indicated by the financial reports of the major carriers.

Apart from the overall level of marginal costs, network effects also have implications for other aspects of the cost structure. First, economies of density are found to be both greater and more prevalent than in a model without network effects. Second, due to peer effects, it is possible for carriers to find it optimal to price some routes below their marginal costs. Although the carrier cannot break even on a route with a price below marginal cost, the additional flow brought by the low price helps stimulate demand for the other routes in the network. My estimates suggest that the majority of connection routes<sup>5</sup> in the U.S. are priced below marginal cost. Interestingly, when Delta reduced the capacity at its Cincinnati hub in 2005, it claims that "connecting traffic is the least profitable for the airline."<sup>6</sup>

Since its deregulation, mergers have not been uncommon in the airline industry. When two carriers combine their networks, it is likely that the merged network will be both more densely flown and larger than either of the pre-merger networks. The increases in flow stimulate demand through network effects, and the larger network creates connection routes to serve more city-pair markets. Such considerations would be absent in a standard merger analysis that only considers the market power effect.<sup>7</sup> Through simulations, I find that it is possible for *both* price and demand to increase in a merger, allowing it to be fairly profitable. This result seems to fit what happened in the 2010 United-Continental merger, in which both the overall demand and price of United-Continental had increased relative to the industry averages. It also provides a possible explanation for the recurrent merger practices in

 $<sup>^5\</sup>mathrm{In}$  this paper connection routes only include one-stop non-codes hared routes. See the Model Section for details.

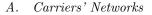
<sup>&</sup>lt;sup>6</sup>Business Courier, September 7, 2005.

<sup>&</sup>lt;sup>7</sup>For example, Peters (2006) focuses on the price changes caused by increases of market power in airline mergers.

the industry.

The paper is organized as follows. Section II introduces the model. Section III outlines the estimation method. Section IV gives a brief description of the data and presents the estimation results along with some of their implications. Section V uses the estimated model to study airline mergers. Section VI concludes.

#### II. Model



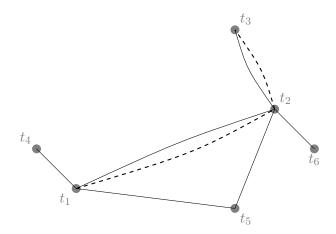


Figure II.1. Example of Networks. Carrier 1's (carrier 2's ) links are represented by the solid (dashed) lines.

There is a set of carriers and a set of cities. A city is characterized by its location, population, whether it is tourist destination, etc. We use N to denote the networks. If a carrier c offers direct flights from city t to t', we will say that *link* tt'c belongs to the networks, or  $tt'c \in N$ . Since it is very rare that a carrier serves only one direction between two cities (e.g. flights from Philadelphia to Los Angeles but not the reverse), tt'c and t'tc will be regarded as the same link. In other words, the networks are non-directed. We will also use s to denote a generic link more compactly. Figure.II.1 illustrates a simple case of two carriers. The links of carrier 1 are drawn in solid lines, while the links of carrier 2 are drawn in dashed lines. For example, we have  $t_1t_4c_1, t_4t_1c_1 \in N$  but  $t_1t_4c_2, t_4t_1c_2 \notin N$ .

A route carries a passenger from her origin city to her destination through a sequence of links. The links are commonly called the segments of the route. More formally, a nonstop route of carrier c that connects city t and t' is  $\{tt'c\}$ , provided that  $tt'c \in N$ ; while a one-stop route of carrier c that connects city t and t' is  $\{t\tilde{t}c, \tilde{t}t'c\}$  for some connection city  $\tilde{t}$ , provided that both the links are in N. We will also use j to denote a generic route more compactly, and  $s \in j$  to denote that one of the links of j is s. Since links have been defined as non-directed, so are the routes. In other words, the reverse of a route will be regarded as the same route.

In general, a route may have more than one stop, and it can also be code-shared, i.e. operated by different carriers on different segments. According to the Airline Origin and Destination Survey (DB1B), only about 1% of the passengers in the U.S. traveled on routes with more than one stop or routes that are code-shared. For simplicity, I will not consider these types of routes.

For the demand model which I will later describe in detail, a city-pair is regarded as a market, and the routes connecting the two cities are the "products" in the market.<sup>8</sup> Given the demand, i.e. the number of passengers traveling on each route, the *flow* on a link is simply the sum of the passengers across the routes which use the link. More formally, let  $q_j$  be the demand for j, then the flow on s is

(II.1) 
$$F_s = \sum_{j:s \in j} q_j$$

For example, in Figure II.1, in market  $\{t_1, t_2\}$  there are 3 products in total:  $\{t_1t_2c_1\}$ ,  $\{t_1t_2c_2\}$  and  $\{t_1t_5c_1, t_5t_2c_1\}$ . The flow on link  $t_1t_2c_2$  is the sum of the demand on

<sup>&</sup>lt;sup>8</sup>In many previous discrete-choice applications in the airline literature, markets have been defined as *ordered* city pairs and products are the directed routes or round-trip routes flying from one city to another. The main reason for such specification is that it allows the presence of a carrier at the origin to affect the demand on a route. While it is not conceptually difficult to apply these extensions, it will considerably increase the number of products and the computational burden when we want to treat frequency as endogenous.

two routes serving different markets:  $F(t_1t_2c_2) = q(\{t_1t_2c_2\}) + q(\{t_1t_2c_2, t_2t_3c_2\}).$ 

#### B. Demand

1. PASSENGER CHOICE. — For exposition, a city-pair market m will be fixed throughout this subsection. The "products" in the markets are the routes that connect the two cities. The products are differentiated by travel length, price, frequency, and so forth. The (indirect) utility of taking route j of individual i is

(II.2) 
$$u_{ij} = \beta' x_j - \alpha p_j + f_j + \xi_j + \varepsilon_{ij}$$

where  $x_j$  is a vector of observed characteristics of the route,  $p_j$  is the price,  $f_j$  is a measure of the network effects on route j which I will describe below, and  $\xi_j$  is a route-specific fixed-effect that captures unobserved (to us) characteristics of the route. As functions of the city characteristics and networks,  $x_j$  will include the carrier identity, the length of the route, a dummy for nonstop, etc.; the details are provided in the Data section.

In the frequency mechanism, the consumer demand for a route is affected by the flight frequencies, which adjust to the flows on segments of the route. Accordingly, I specify that for a non-stop route  $j = \{s\}$ ,

(II.3) 
$$f_j = \gamma_1 \log(F_s)$$

where  $F_s$ , as already defined, is the flow on link s. The logarithm specification is motivated by the diminishing reduction of delay as flight frequency increases.<sup>9</sup> A different motivation of the logarithm specification comes from treating a route as the nest of its flights, whose details are given in the appendix.

<sup>&</sup>lt;sup>9</sup>For instance, when there is only 1 flight per day between two cities, adding one more flight per day will reduce the average interval between flights by 12 hours. However, when there are already 10 flights per day, adding one more flight will not do much.

As to the network effects on a one-stop route, I will use a constant-elasticity-ofsubstitution (CES) function to account for the effects of the flows on both of the links. For  $j = \{s, s'\}$ ,

(II.4) 
$$f_j = \gamma_2 \log\left[\left(\frac{F_s^{\ \tau} + F_{s'}^{\ \tau}}{2}\right)^{1/\tau}\right]$$

It reduces to (II.3) when the two flows are equal. Note that CES is flexible in accommodating the differential effects of the larger and smaller of the two flows: when  $\tau > 1$ , what matters most is the larger of  $F_s$  and  $F_{s'}$ ; when  $\tau = 0$ , what matters is the sum; when  $\tau < 0$ , what matters most is the smaller flow. One probably would expect  $\tau < 0$  to be the case because, acting like a bottleneck of the route, it is usually the link with lower frequency that causes the delays.

Note that though the specifications are motivated by the frequency mechanism, they can capture other possible sources of network effects as well. For example, to meet increases in demand, carriers can either increase flight frequency or increase aircraft size. Givoni and Rietveld (2009) shows that, while the priority has been given to frequency, both means are being used in practice. Hence more dense links tend to fly larger aircrafts, which may give rise to network effects as well because they are usually considered safer and more comfortable.

To complete the demand model, I specify that the idiosyncratic preference  $\varepsilon_{ij}$  follows the distributional assumption necessary to generate the nested logit probabilities (McFadden (1981)) where all the routes in the market are nested against an "outside good" of not flying. With  $\lambda$  being the nesting parameter, the demand for route j is then

(II.5) 
$$q_j = M \frac{D^{\lambda}}{1+D^{\lambda}} \cdot \frac{e^{(\beta' x_j - \alpha p_j + f_j + \xi_j)/\lambda}}{D}$$

where M is the size of the market and

$$D \equiv \sum_{k:k \in m} e^{(\beta' x_k - \alpha p_k + f_k + \xi_k)/\lambda}$$

where the summation is across all the routes in market m. The "demand" of the outside good is

(II.6) 
$$q_0 = M \frac{1}{1 + D^{\lambda}}$$

Given the presence of network effects, demand is determined jointly by the discretechoice model and the flow-demand relation (II.1). More formally, fix the city characteristics, N and  $\xi$ . (II.2)-(II.5) describe a discrete-choice model that implies a vector of demand for any price p and flow F, where F can be either exogenously given or given by a vector of quantities q as in (II.1). In the later case, we may write the discrete-choice model as a function  $\Psi(p,q)$ . The demand predicted by our model is then a fixed point of  $\Psi(p, \cdot)$ .

In the appendix I show the uniqueness of fixed point under a restriction on  $\gamma$ . Unfortunately the uniqueness may not be guaranteed in general. However, I have not encountered multiplicity in my estimation and counterfactual analysis. It is also worth noting that the estimation, which will only rely on the local optimality of the carriers' pricing behaviors (i.e. first-order conditions), does not necessarily requires the uniqueness. This is explained in the supply subsection below.

2. NETWORK EFFECTS. — Network effects are embedded in the notion of fixed point. An increase in the demand (e.g. caused by a price drop) on route j will increase the flows on its links and thus  $f_j$ , which increases the mean utility of j. This is the case even when j is an isolated route. If route j is within a network, there are also the peer effects: an increase in the demand for j will increase the mean utility for any of the link-sharing routes, i.e.  $\{k \mid \exists s \text{ such that } s \in k \text{ and } s \in j\}$ . Of course, the subsequent increase in the demand on a "peer" route k will further affect k itself and the peers of k, which include j, and so forth.

In particular, we can see that a link used by more routes is likely to bear a much larger flow, not just because it sums up the demand on more routes, but also because the flow has positive effects on the demand on these routes. This offers an explanation to the substantial market-share differences among the carriers within a market. For example, in the market between Philadelphia and Los Angeles, there are five carriers offering nonstop routes. Among them, US Airways obtains the largest market share (about 40%). Indeed, Philadelphia is one of US Airways' hubs, and there are numerous one-stop routes in US Airways' network that utilize its Philadelphia-LA link.

On the other hand, network effects will not be fully captured if we let flow F be exogenously given. Such a model is most similar to previous discrete-choice applications, where exogenously-given flight frequencies are used as part of the utility specification.<sup>10</sup> <sup>11</sup> It is easy to see that the implied price elasticities of a demand system with network effects are much higher than that of a system without them: if a carrier increases prices, the demand will decrease, which would decrease the demand further if there are network effects.

## C. Supply

Given the nature of airline operations, costs should be incurred mostly at the link level. Nevertheless, there may be some costs at the route level. For example, checkin only needs to be done once whether the flight is nonstop or onestop, so the cost associated with it is not link-wise. Accordingly, I specify the marginal cost on route

<sup>&</sup>lt;sup>10</sup>For example, BCS (2006), Peters (2006), Berry and Jia (2010).

<sup>&</sup>lt;sup>11</sup>In general, to model frequency as an explicit decision variable in an empirical model is difficult. Frequency is not just one dimension of product quality that enters both utility and cost, but also a capacity constraint on the demand. It will be interesting for future research to demonstrate how this can be done.

j as

(II.7) 
$$mc_j = \sum_{s:s \in j} g_s + \mu' w_j + \omega_j$$

where  $g_s$  is the marginal cost on link s,  $w_j$  is a vector of route-level characteristics that are cost-relevant but not captured in  $g_s(.)$ , and  $\omega_j$  is a route-specific fixed-effect that captures unobservable determinants of marginal cost.<sup>12</sup>

We need to specify a functional form for  $g_s$ , which may exhibit both economies of density and economies of distance. It is important to note that there are two senses in which economies of distance may exist. First, regardless of the distance of flying, a proportion of the marginal cost is attributed to passenger check-in, ground service, taking-off and landing, etc. Second, an engineering argument suggests that larger planes tend to be more fuel efficient (Wei and Hansen (2003)), and large-size planes are usually used on long-distance links (Givoni and Rietveld (2009)).

To estimate both types of economies without imposing much structure, I will use a fairly flexible functional form from BCS (2006):

(II.8) 
$$g_s = \eta' w_s + h(d_s, F_s, \theta)$$

where  $w_s$  is a vector of link characteristics, and  $h(\cdot)$  is a polynomial (including the constant) of the link length  $d_s$  and the flow  $F_s$  up to degree 3, whose coefficients are collected in the vector  $\theta$ . If  $\partial h/\partial F_s < 0$  (or  $\partial h/\partial d_s < 0$ ), then the marginal cost on link s is decreasing in flow (or distance), indicating that economies of density (or economies of distance) are present.

The only part left to complete the model is the pricing behavior. I will assume that the carriers play a Nash-Bertrand equilibrium in price. The equilibrium can be characterized by a set of first-order conditions. Let q(p) be the demand for some

 $<sup>^{12}</sup>$ Here I note that a specification of "marginal cost", which is also standard in previous discrete-choice applications, implicitly assumes that flight frequency adjusts to flow. Putting one more passenger on an existing flight adds little costs, so most of the marginal cost should come from adding flights.

price p which is implicitly determined as fixed point of  $\Psi(p, \cdot)$ . We have for each j,

(II.9) 
$$q_j + \sum_{k:k \in c} (p_k - mc_k)(\partial q_k / \partial p_j) = 0$$

Note that because of the peer effects, the summation is across all the routes of carrier c, not just those in the same market as j. We will rely on the (II.9) for the estimation of the cost-side parameters.

First-order conditions require that a relatively small price change is not profitable for a carrier. Hence, (II.9) will only require that q(p) is locally unique, as long as we are willing to assume that small price changes will not entail jumps in the fixed-point demand. As an implicit function defined by  $\Psi$ , the local uniqueness of q(p) can be verified by checking numerically if  $\partial \Psi/\partial q$  is invertible, which is always the case in my experiences.

#### III. Estimation

#### A. Estimation Algorithm

My estimation adopts the framework of Berry, Levinsohn, and Pakes (1995) (hereafter BLP). It depends on the assumption that the unobservables  $\xi_j$  and  $\omega_j$  are mean independent of the exogenous variables, which includes the networks N and the city characteristics. In particular, instruments for  $p_j$ ,  $f_j$  and  $F_s$  are needed because both price and flow are endogenous. However, there are three differences from a standard application. First, the demand q is modeled as a fixed point. Second and related,  $\partial q/\partial p$  is computationally intensive, but needs to be evaluated for many trial parameter values if we want to estimate the demand and cost jointly. Third, the instrumental variable estimation on the cost side becomes inefficient as higher order terms are added to  $h(\cdot)$ , and I use a control function approach (Petrin and Train (2010)) to address this issue. This section explains these differences in detail. We start by writing  $\xi_j$  and  $\omega_j$  as functions of the parameters, given the observed demand and characteristics. First note the mean utility  $v_j \equiv \beta' x_j - \alpha p_j + f_j + \xi_j$  can be written as a function of  $\lambda$ , as it can be backed out from the observed demand using (II.5) and (II.6). So we have

$$\xi_j = v_j(\lambda) - \beta' x_j - f_j(\gamma, \tau) + \alpha p_j$$

Next, to back out  $\omega$ , we first need to find the marginal costs using the Bertrand-Nash assumption. The f.o.c. (II.9) in matrix form is

$$q + D(p - mc) = 0$$

where

$$D_{kj} = \begin{cases} \partial q_k / \partial p_j & \text{if } k, j \in c \text{ for some } c \\ 0 & \text{o.w.} \end{cases}$$

Recall that the demand q(p) is a fixed point of  $\Psi(p, \cdot)$ . Provided that  $\partial \Psi/\partial q$  is invertible, the implicit function theorem says that the demand function  $q(\cdot)$  is locally unique and

(III.1) 
$$\partial q'/\partial p = (\partial \Psi'/\partial p)(I - \partial \Psi'/\partial q)^{-1}$$

On the right hand side,  $\partial \Psi'_k / \partial p_j$  is the demand change on k after an incremental price change of j, holding the prices of the other routes and the effects of flow fixed.  $\partial \Psi'_k / \partial q_j$  is the demand change on k after an incremental change of the quantity of passengers on j, holding the quantities on the other routes and all the prices p fixed. This demand change is caused by network effects. It is not difficult to show that D is a function of the parameters  $\lambda$ ,  $\alpha$ ,  $\gamma$  and  $\tau$ . Now we can write the marginal costs implied by the first-order conditions as

$$mc = p + D(\lambda, \alpha, \gamma, \tau)^{-1}q$$

Once  $mc_j$  is computed, it is straightforward to find  $\omega_j$  with (II.7). Note that by its definition, D can be organized into a block diagonal matrix whose inversion can be carried out block by block.

There are two ways to compute the matrix D. The first way is direct numerical differentiation of q(p). This amounts to computing the demand q at a perturbed price, which is found by iterating  $\Psi$  for the fixed point at the perturbed price. The second method uses (III.1), where both  $\partial \Psi'/\partial p$  and  $\partial \Psi'/\partial q$  can be obtained by numerical differentiation. This method is faster for relatively small-size networks. However, due to the large number of routes in the data, inverting the equally large matrix  $(I - \partial \Psi'/\partial q)$ , which in general is not block diagonal, is very burdensome and prone to numerical errors. For this reason, I use the first method for the estimation.

The BLP estimator minimizes an objective function in which instruments interact with both  $\xi_j$  and  $\omega_j$ , and the demand-side and cost-side parameters are estimated jointly. In my application, this requires the matrix D to be computed for many trial parameter values. Due to the large size of the networks, one computation of D can take at least several hours. For this reason, I estimate the demand side and cost side sequentially.

More specifically, the demand-side parameters are first estimated by minimizing  $\|\sum z_j \xi_j\|$  where  $z_j$  is a vector of instruments, which includes  $x_j$ . Then, with D computed at the estimated demand-side parameters, the marginal costs can be computed, and the cost-side estimation amounts to a linear regression of the so-called markup equation, which is given by (II.7) and (II.8):

(III.2) 
$$mc_j = \sum_{s:s \in j} h(d_s, F_s, \theta) + \eta' \sum_{s:s \in j} w_s + \mu' w_j + \omega_j$$

The typical approach is again the instrumental variable method where the instruments are used to account for the endogeneity of  $F_s$ . However, I have found that the instrumental variable estimation produces large standard errors for the coefficients of the higher-order terms of  $F_s$ . On the contrary the OLS standard errors are much more acceptable. So it should be a weak instrument problem on the higher-order terms.<sup>13</sup>

For this reason, I take a control function approach, which is implemented through two stages. In the first stage flow F is regressed on its instruments and some exogenous variables, then in the second stage the markup equation (III.2) is estimated with OLS where the residuals of the first-stage regression are added on the right hand side. Generally speaking, the control function approach is less robust but more efficient (see, for example, Wooldridge (2007)).

Finally. it is important to note that term  $(I - \partial \Psi'/\partial q)^{-1}$  in (III.1) is where the notion of fixed point enters the estimation. If we ignore network effects, or more precisely, take flow as exogenously given, the term will not appear and instead of (III.1) we would have

(III.3) 
$$\partial q'/\partial p = \partial \Psi'/\partial p$$

In other words, whether we allow for network effects changes the way we compute the price elasticities, which will lead to different marginal cost estimates. In Section 4 I will display the marginal cost estimates without network effects as well.

#### B. Identification

The identification of most parameters is straightforward. Here, we focus on  $\lambda$ ,  $\gamma$  and  $\tau$ . The nesting parameter  $\lambda$  is identified from changes in the total market demand as the number of products varies across markets. In the extreme case of  $\lambda \to 0$ , the aggregate share of the routes remains fixed as the number of products vary. In other words, the market total demand is inelastic. As  $\lambda$  moves close to 1, the total demand in a market becomes more elastic.

Parameter  $\gamma$  measures the magnitude of network effects. It is identified from changes

 $<sup>^{13}</sup>$ BCS (2006) did not encounter much of the problem possibly because they included the actual flight frequencies in their instruments, treating them as exogenous.

of the demand as the flows vary across routes. Note this requires that some links are flown by multiple routes, otherwise we would have a "reflection" problem. This is illustrated by the example where N is a purely point-to-point network. When this is the case, for each route the flow and demand coincide and we would essentially be regressing demand on itself. However, this is an extreme case. Thanks to the hub-and-spoke, or multi-hub structure, demand on a route is generally different from the flows on its links. This is similar to the use of social networks to resolve the reflection problem in identifying social peer effects (Manski (1993) and Bramoullé et al. (2009)).

Finally, parameter  $\tau$  in (II.4) is identified through changes of the demand on onestop route as the larger/smaller of the two segment flows varies. For example, in the extreme case  $\tau \to -\infty$ , the variation of the larger flow has no effect on the demand. As  $\tau$  moves towards 0, the larger flow becomes more influential.

## C. Instruments

Instruments are required for both flow and price. Following Peters (2006) and Berry and Jia (2010), I treat the city characteristics and the networks N as exogenous, and derive instruments from them. In general, this is consistent with the idea that network structure is a long-term choice when compared with frequency and price.

I use two instruments for flow F, which can be then be used to construct the instruments for f. The first instrument,  $F^{IV1}$ , relies on the variation of "centrality" across links. Specifically,  $F^{IV1}(s) = \#\{j|s \in j\}$ , i.e. the number of routes that utilize link s. For example, in Figure II.1,  $F^{IV1}(t_1t_2c_1) = 6$  and  $F^{IV1}(t_1t_2c_2) = 2$ . If a link is flown by many routes, it is likely to have a large low of passengers. The other instrument makes use of the variation in population across nodes. Specifically,  $F^{IV2}(s)$  is the market size of the end cities of s. If the ends of a link are large cities, then it is likely that a lot of passengers will travel on it.

Standard instruments for price measure the market-level competitiveness. The BLP instruments are the sums of the characteristics of one's own products and the com-

petitors' products in the same market. Along this line, I include the number of nonstops, the sum of the lengths of the nonstops, the sum of the instruments for f of the nonstops, and the same sums but of the onestops, the competitors' nonstops and the competitors' onestops. Previous studies have also used the averages of these characteristics, e.g. the average length of the competitors' onestops, which I also include. Lastly, I also include the number of carriers in the market as instruments.

#### IV. Data and Results

#### A. Data

The Airline Origin and Destination Survey (DB1B) is a 10% sample of airline tickets from reporting carriers in the U.S. collected by the Bureau of Transportation Statistics. This paper uses the DB1B-coupon and the DB1B-market data, and covers the last quarter of 2012 and the first quarter of 2013. <sup>14</sup>

The networks for estimation include the 100 most-visited cities and the 12 largest carriers by passengers served. The selected sample includes roughly 90% of the tickets in the data. The flights of the contracting carriers, i.e. carriers that did not sell tickets but operate for the major carriers, are incorporated to the carriers for which they operate. For the link definition, I follow Berry (1992) where a link  $s \in N$ if no less than 90 passengers per quarter had been observed on that link in the data. It roughly corresponds to a medium-sized jet flying back and forth between the city pair once a week.

Price  $p_j$  is computed as the passenger-weighted average fare on the route. There is a non-negligible proportion (around 7%) of tickets with very low fares (e.g. \$5, \$15 per passenger), which might be purchased with frequent-flyer miles. While these tickets are included for the demand and flow, the associated fares are not included for the price. In total, there are 39,390 routes included in the estimation. Finally, following

 $<sup>^{14}</sup>$ My estimation restricts attention to the U.S. domestic market. However, a considerable proportion of the operations of some carriers are international. Many of the international travelers make connections at the domestic hubs and contribute to the flow on the networks of these carriers. Incorporating international routes to the analysis requires world-wide survey data on air-travel itineraries, which is hardly available.

BCS (2006), the market size M will be taken as proportional to the geometric mean of the city populations in the market for each quarter.<sup>15</sup>

|            | # of links | # of markets<br>served | maximum<br>degree | total flow (in<br>million) | total<br>passengers<br>(in million) |
|------------|------------|------------------------|-------------------|----------------------------|-------------------------------------|
| Southwest  | 666        | 1682                   | 55                | 53.6                       | 44.6                                |
| Delta      | 430        | 3643                   | 82                | 43.7                       | 32.2                                |
| United     | 462        | 3646                   | 86                | 34.3                       | 26.6                                |
| American   | 240        | 2877                   | 76                | 27.2                       | 21.0                                |
| US Airways | 291        | 2746                   | 69                | 28.0                       | 19.5                                |

TABLE 1—SUMMARY STATITICS OF THE NETWORKS OF THE FIVE LARGEST CARRIERS

Some summary statistics of the networks of the five largest carriers are displayed in Table 1. Southwest, while being the largest in terms of the number of links and the passengers carried, serves the fewest markets, thanks to its more point-to-point network structure. Table 2 provides the summary statistics for some of the variables that will enter the estimation.

Given the available data, product characteristics  $x_j$  will include a constant term, the carrier dummies, the market-level characteristics, and the route-level characteristics. At the market-level, it includes the distance of the market, the square of that distance, the numbers of cities in the market (0, 1, or 2) that fall into two categories of tourist destinations, and the number of cities (0, 1, or 2) with congested airports.<sup>16</sup> At the route-level, it includes a dummy for one-stop route, the length of the route, the square of that length, the product of the segment lengths, a dummy if any airport at the connection is congested, and finally the carrier's average presence at the

<sup>&</sup>lt;sup>15</sup>The definition of city in DB1B sometimes refer to a metropolitan statistical area (MSA). In such cases the MSA population is used. Otherwise the urban population is used. The populations of Hawaii are argumented by the tourists, whose number exceeds that of its residents.

<sup>&</sup>lt;sup>16</sup>First category of tourist destinations (large Cities) includes New York, LA, Washington DC, and San Francisco; second category (Vacation & Resort) includes Las Vegas, Atlantic City, Charlotte Amalie, and the cities in Florida and Hawaii.

Congested airports are the High-density traffic airports defined by FAA Regulations, Part 93-K. They are: Newark and LaGuardia in New York, National in Washington, and O'Hare in Chicago.

|   |                              | Mean | Standard<br>Deviation | Min  | Max  |
|---|------------------------------|------|-----------------------|------|------|
| j | Market distance (1000 miles) | 1.36 | 0.83                  | 0.00 | 0.00 |
|   | Tourism 1                    | 0.18 | 0.41                  | 0    | 2    |
|   | Tourism 2                    | 0.29 | 0.48                  | 0    | 2    |
|   | Congested ends               | 0.14 | 0.36                  | 0    | 2    |
|   | Congested connection         | 0.25 | 0.43                  | 0    | 1    |
|   | One-stop                     | 0.94 | 0.24                  | 0    | 1    |
|   | Route length (1000 miles)    | 1.67 | 0.89                  | 0.06 | 7.35 |
|   | Presence                     | 0.21 | 0.14                  | 0.00 | 0.79 |
| j | Price (\$100)                | 2.85 | 1.07                  | 0.21 | 15.8 |
| s | Congested ends               | 0.29 | 0.46                  | 0    | 2    |
| s | Distance (1000 miles)        | 1.01 | 0.70                  | 0.06 | 4.96 |
| s | Flow (million passengers)    | 0.09 | 0.13                  | 0.00 | 2.60 |

TABLE 2—Summary Statistics

Note: There are 39,390 observations for  $x_j$  and  $p_j$ . There are 2,568 observations for  $w_s, d_s$  and  $F_s$ 

two ends of the route. A carrier's presence at a city is measured by the percentage of links it serves at that city. The effect of airport presence on demand, i.e. "the hub dominance", was first introduced by Borenstein (1989, 1991).

On the cost side, the link characteristics  $w_s$  includes the carrier dummies and the number of congested ends of the link (0, 1 or 2); the route characteristics  $w_j$  is just a dummy capturing any cost that is not incurred link-wise. A more detailed specification of  $w_s$  may use the city dummies to control for fixed effects due to varying degrees of congestion, landing fees, gate rents, etc. I estimated the cost side with this alternative specification but have not found substantial changes to the estimates.

#### B. Parameter Estimates

1. DEMAND SIDE. — Column 2 in Table 3 presents the demand-side parameter estimates. The nesting parameter is estimated at 0.55, which is close to the estimates in previous discrete-choice studies of air-travel demand.<sup>17</sup> The rest of the parameters in the demand model are estimated with the expected signs, and all the standard errors are small. In particular, the price coefficient is estimated to be -0.52. The implied aggregate price elasticity (without considering network effects), which is the percentage change in total demand when all products' prices increase by 1 percent, is 1.43. Gillen et. al. (2003) conducted a survey that collected 85 demand elasticity estimates from cross-sectional studies. The elasticities ranged from 0.181 to 2.01, with a median of 1.33.<sup>18</sup> My estimate thus seems reasonable.

The effects of flow on demand are substantial. For nonstop routes,  $\gamma_1 = 0.39$ . This means that if we look at an isolated nonstop route in a large market, hypothetically "doubling the flow" will increase demand by roughly 30%. On the other hand, one-stop routes are even more flow-dependent ( $\gamma_2 = 0.52$ ), which could reflect that

 $<sup>^{17}{\</sup>rm For}$  example, The estimate in Peters (2006) is 0.595. The estimate in BCS is 0.605 for the single-type passenger configuration.

<sup>&</sup>lt;sup>18</sup>In the study, all 85 estimates were conducted between 1981 and 1986, which are slightly dated, and most of the estimates represent U.S. city-pair routes. The study is also used by Berry and Jia (2010) to compare with their estimated elasticity, which is 1.67 for 2006.

|          |                          | Demand<br>OLS | Demand<br>IV | Cost<br>OLS | Cost<br>CF<br>Fixed Flow | Cos<br>CI  |
|----------|--------------------------|---------------|--------------|-------------|--------------------------|------------|
| λ        | Nesting                  | 0.56 (.00)    | 0.55 (.00)   |             |                          |            |
| $\alpha$ | Price                    | -0.23 (.02)   | -0.52 (.02)  |             |                          |            |
| $\gamma$ | Nonstop                  | 0.56~(.01)    | 0.39(.02)    |             |                          |            |
|          | One-stop                 | 0.66~(.01)    | 0.52 (.01)   |             |                          |            |
| au       | CES                      | -0.21 (.02)   | -0.60 (.05)  |             |                          |            |
| β        | Constant                 | -3.18 (.04)   | -3.56 (.06)  |             |                          |            |
|          | Tourism 1                | 0.04(.01)     | 0.12(.01)    |             |                          |            |
|          | Tourism 2                | 0.34(.01)     | 0.34(.01)    |             |                          |            |
|          | Mkt. distance            | 1.70(.03)     | 1.34(.04)    |             |                          |            |
|          | Mkt. distance $^2$       | -0.13 (.01)   | -0.08 (.01)  |             |                          |            |
|          | Congested ends           | -0.16 (.01)   | -0.07 (.01)  |             |                          |            |
|          | Congested conn.          | -0.21 (.01)   | -0.22 (.01)  |             |                          |            |
|          | One-stop                 | -1.79 (.03)   | -1.42 (.06)  |             |                          |            |
|          | Route length             | -1.55(.05)    | -1.16 (.04)  |             |                          |            |
|          | Route length $^2$        | 0.11(.01)     | 0.10(.02)    |             |                          |            |
|          | Seg. length prod.        | -0.09 (.01)   | -0.09 (.01)  |             |                          |            |
|          | Presence                 | -0.37 (.04)   | 0.41 (.06)   |             |                          |            |
| μ        | Constant                 |               |              | 0.06(.03)   | -0.04 (.02)              | 0.06 (.03  |
| η        | Congested ends           |               |              | 0.02(.01)   | 0.00 (.00)               | 0.01 (.00  |
| Э        | Constant                 |               |              | 1.01 (.02)  | 0.22 (.02)               | 0.95 (.02  |
|          | Distance                 |               |              | 0.53 (.05)  | 0.53 (.05)               | 0.60 (.05  |
|          | Distance <sup>2</sup>    |               |              | 0.00(.03)   | 0.01 (.03)               | -0.02 (.03 |
|          | Distance <sup>3</sup>    |               |              | 0.01 (.00)  | 0.01 (.00)               | 0.02 (.00  |
|          | Flow                     |               |              | -1.18 (.13) | -0.57 (.13)              | -1.29 (.12 |
|          | Flow <sup>2</sup>        |               |              | 0.94~(.15)  | $1.10 \ (0.18)$          | 1.84 (.18  |
|          | Flow <sup>3</sup>        |               |              | -0.30 (.03) | -0.33 (.05)              | -0.51 (.05 |
|          | $Distance \cdot Flow$    |               |              | 0.03~(.23)  | 0.31 (.24)               | 0.32 (.24) |
|          | $Distance ^2 \cdot Flow$ |               |              | -0.14 (.09) | -0.25 (.09)              | -0.21 (.09 |
|          | $Distance \cdot Flow^2$  |               |              | 0.69(.16)   | 0.51(.20)                | -0.44 (.20 |

TABLE 3—PARAMETER ESTIMATES (EXCEPT FOR THE CARRIER DUMMIES)

Note: See the Data section for variable definitions, and see Table 2 for the summary statistics of the variables.

passengers care more about the frequencies on one-stops. This is reasonable because the frequencies on a one-stop also affect not only the delay between a passenger's desired departure time and the time of a flight, but also the the delay at the connection.

The substitution parameter  $\tau$  in the CES function is negative (-0.60), which means that on a one-stop route the smaller of the two flows has more influence on the demand. This is plausible because delays are most likely to be caused by the segment with lower frequency, which acts like a bottleneck of that route.

For comparison, Column 1 in Table 3 provides the estimates without instrumenting for price and frequency.<sup>19</sup> First, we see that the price coefficient  $\alpha$  is underestimated. This is expected because, for both pricing and cost reasons, the unobserved quality in  $\xi_j$  is likely to be positively correlated with  $p_j$ . Second, the network effects  $\gamma$ are overestimated. This is because  $\xi_j$ , positively correlated with the demand on j, is likely also positively correlated with the flows on j. Apart from these two important differences, all the parameters, with the only exception of the coefficient on the airport presence, are estimated to have the same signs as in Column 2. The negative sign on the airport presence can be caused by the biases in  $\alpha$  and  $\gamma$ , as presence is positively correlated with both price and flow.

Column 1 in Table 4 displays the carrier dummy estimates on the demand side. We see that the legacy carriers in general are preferable to the low-cost airlines (e.g. Southwest, AirTran).

2. SUPPLY SIDE. — Column 5 in Table 3 presents the cost-side parameter estimates using the control function approach. Note the relatively large standard errors on the higher-order terms of flow, which, as explained, would be even worse if an instrumental variable approach is used here.

<sup>&</sup>lt;sup>19</sup>This is a OLS regression of the utility equation (II.2), except that, to identify  $\lambda$ , an orthogonality condition between  $\xi_j$  and the number of routes in the market is added.

TABLE 4—CARRIER DUMMY ESTIMATES

|                |                 | Demand<br>IV | Cost<br>CF  |
|----------------|-----------------|--------------|-------------|
| $\beta / \eta$ | Southwest       | 0            | 0           |
|                | Delta           | 0.17 (.02)   | 0.16(.01)   |
|                | United          | 0.25 (.02)   | 0.17(.01)   |
|                | American        | 0.22 (.02)   | 0.17(.01)   |
|                | US Airways      | 0.30 (.02)   | 0.21 (.01)  |
|                | JetBlue         | 0.02 (.03)   | -0.09 (.02) |
|                | AirTran         | 0.13 (.03)   | -0.28 (.01) |
|                | Alaska          | 0.73(.04)    | 0.16(.03)   |
|                | Frontier        | -0.01 (.03)  | -0.29 (.01) |
|                | Hawaiian        | 1.20 (.11)   | -0.09 (.06) |
|                | Spirit          | -0.45 (.06)  | -0.60 (.02) |
|                | Virgin American | 0.11 (.07)   | -0.11 (.03) |

Note: The carrier constants of Southwest are set at zero.

Recall that parameter  $\mu$  measures the route-level marginal cost that is unrelated to the number of segments of the route. For example, it may capture the costs associated with passenger check-in. In Column 5 it is estimated to be 0.06, which translates into about 2-4% of the marginal cost of a typical flight. Parameter  $\eta$ captures the additional costs of using congested airports. Its estimate is small but positive, indicating that it is slightly more costly to fly between the cities with high-density traffic airports.

Parameter  $\theta$  is a vector of coefficients of the polynomial of flow and distance  $h(\cdot)$ . Given the flexible functional form of  $h(\cdot)$ , it is hard to directly interpret these coefficients. In the next subsection I discuss the implications of the estimates of  $\theta$ , and compare the estimates with those in Column 4, which ignores network effects.

To see the significance of the control function approach, Column 3 in Table 3 presents the same regression as that in Column 5, except that OLS is used. (III.2). Compared with Column 5, it overestimates economies of density. This should be expected as the unobserved cost  $\omega_j$  is likely to be negatively correlated with the flows on j. Finally, Column 2 in Table 4 displays the carrier dummy estimates on the cost side. We see that the legacy carriers in general incur higher marginal costs than the low-cost airlines.

### C. Implications of the Estimates

1. MARGINAL COSTS AND COMPARISON WITH ACCOUNTING DATA. — The first two columns in Table 5 display, for the five largest carriers, the implied costs per passenger miles (CPM) based on the marginal cost estimates in Column 4 and 5 in Table 3, respectively. CPM divides a carrier's variable cost by its total passenger miles, where the variable cost is found by integrating the estimated marginal costs. We see that accounting for network effects more or less doubles the estimated CPM. Recall this is because the implied price elasticities are much higher when network effects are present, which translate into smaller markups and higher marginal costs.

| CPM (cents) | Cost CF<br>Fixed Flow | Cost CF | 10-Q filing      |
|-------------|-----------------------|---------|------------------|
| All top 5   | 7.0                   | 14.8    | $10.4 \sim 16.3$ |
| Southwest   | 3.4                   | 12.8    | $7.4 \sim 13.9$  |
| Delta       | 7.7                   | 15.7    | $11.9\sim 17.2$  |
| United      | 9.4                   | 15.7    | $9.5 \sim 15.6$  |
| American    | 7.2                   | 13.9    | $10.5 \sim 16.6$ |
| US Airways  | 8.2                   | 16.7    | $13.9\sim19.1$   |

TABLE 5—COMPARISON WITH ACCOUNTING DATA: CPM

Note: "CPM" = cost per passenger miles. Last column provides a lower bound and a upper bound that are based on the 10-Q filings for the first quarter of 2013. See the main text for details.

I compare the cost estimates to the income statements in the five carriers' 10-Q filings for the first quarter of  $2013.^{20}$  It is noted that though my estimation has focused on

 $<sup>^{20}{\</sup>rm These}$  filings are available at the Securities and Exchange Commission website: www.sec.gov, or at Bloomberg Businessweek: investing.businessweek.com.

| Profit Margin (%) | Cost CF<br>Fixed Flow | Cost CF | 10-Q filing  |
|-------------------|-----------------------|---------|--------------|
| All top 5         | 138                   | 13      | $4 \sim 39$  |
| Southwest         | 380                   | 26      | $14 \sim 54$ |
| Delta             | 127                   | 12      | $-1 \sim 30$ |
| United            | 74                    | 5       | $3 \sim 42$  |
| American          | 111                   | 10      | $6 \sim 40$  |
| US Airways        | 128                   | 12      | $3 \sim 29$  |

TABLE 6—Comparison with Accounting Data: Profit Margins

*Note:* Last column is based on the 10-Q filings for the first quarter of 2013. Revenues are the reported passenger revenues (Cargo and Other revenues are not included); costs are calculated in the same way as in Table 5. The lower bound and the upper bound correspond to the bounds in the last column of Table 5.

the domestic market, among these five carriers, only Southwest's operations were domestic only. Unfortunately the other four carriers do not provide separate income statements for international and domestic operations. Given that the majority of their revenues were likely from the domestic market<sup>21</sup>, I believe that these income statements can still serve as useful benchmark. It is also noted that accounting practices generally are not geared toward reporting the economic notion of marginal cost. Nonetheless, as suggested by Einav and Levin (2010), the imperfectness of the accounting data should not prevent researchers from using them to cross-check their analyses, or even test their hypotheses (see e.g. Nevo (2001)).

The last column in Table 5 reports the CPM bounds that are based on the carriers' 10-Q filings. The upper bound is calculated from the reported operation costs, excluding the following items: Depreciation and Amortization, Profit Sharing, Other Expenses and Special Charges. Since salaries and rents can be relatively fixed, I provide a lower bound that further excludes the related items: Salaries and Benefits, Landing Fees and Airport Rents, and Other Rents. Note that this should be a fairly loose lower bound, because salaries are partly operation-dependent (e.g. pilots'

 $<sup>^{21}{\</sup>rm For}$  example, American Airlines states in the 10-Q that about 60% of its passenger revenues are derived from domestic operations.

earnings depend on the hours of flying) but have to be excluded as a whole, and landing fees, again operation-dependent, have to be excluded together with airport rents. For both bounds, total passenger miles are directly taken as reported in the filings.

Without network effects, I estimate the overall CPM for the top five carriers to be 7 cents. Berry and Jia (2010) finds a similar but slightly lower estimate of 6 cents for all the carriers. Compared with the accounting bounds, these estimates seem too small. Moreover, for each of the five carriers, the CPM estimate in Column 1 is below the accounting lower bound. On the other hand, the CPM estimates with network effects (Column 2) fall within the accounting bounds with only one exception (United).

The U.S. airline industry is relatively unprofitable.<sup>22</sup> The low cost estimates obtained without network effects seem to have difficulty in capturing this important feature. This observation is further confirmed in Table 6, which compares the estimated profit margins with the margins calculated from the 10-Q filings. We see that the estimates obtained with network effects, presented in Column 2, are more consistent with the accounting data.

2. ECONOMIES OF DISTANCE AND ECONOMIES OF DENSITY. — Recall that the polynomial  $h(\cdot, \theta)$  captures economies of distance and economies of density. As to economies of distance, the coefficients of the distance-squared and distance-cubed are both close-to-zero, as you can see in Column 4 and 5 of Table 3. In fact, the polynomial is mostly linear in distance within the range of the data points. However, the estimate for the polynomial constant in either column is positive, indicating that there is a "fixed" positive marginal cost even when the flight distance is close to zero. The "fixed" marginal cost captures the costs associated with airport rents, ground services, taking-off and landing, etc. that are incurred regardless of the distance of

<sup>&</sup>lt;sup>22</sup>As noted by David Barger, CEO of JetBlue: "The U.S. airline industry has not, in aggregate, made a single penny of profit in its 99 years of existence." - *Aviationweek*, Feb, 2013. Also see discussions in Berry and Jia (2010) for recent developments.

flying. Hence, the marginal cost per mile is decreasing, and it is in this sense that my estimates indicate economies of distance.

|  | Cost OLS   | Cost CF | Cost CF |  |
|--|------------|---------|---------|--|
|  | Fixed Flow |         |         |  |
| $\%$ of links with $\partial h/\partial F_s < 0$ | 99%        | 91%     | 97%     |  |
| Average $\partial h / \partial F_s$              | -1.13      | -0.40   | -0.97   |  |

TABLE 7—ESTIMATES OF ECONOMIES OF DENSITY

Note: The unit of the derivative is dollar per 10,000 passengers.

As to economies of density, Table 7 display the percentage of links on which economies of density are present (i.e.  $\partial h/\partial F_s < 0$ ), and the average derivative  $\partial h/\partial F_s$  across all the links, based on the estimates from Column 3, 4 and 5 in Table 3, respectively. With network effects, 97% of the links exhibit economies of density, and the marginal cost is reduced by \$0.97 for each 10,000 additional passengers on average. Without network effects, economies of density are estimated to be less prevalent and smaller (91% and \$0.40).

To understand why the network effects imply larger economies of density, note that when the flows on a route are large, it is often the case that there are many other link-sharing routes that contribute to the flows. On such a route, when there is a price drop on the route, the direct increase in demand will often correspond to only small percentage increase in the flows, which is unlikely to entail much further increases in demand. This means that when network effects are present, the price elasticities on routes with large flows tend to be smaller than those on other routes, so are the implied marginal costs.

3. ROUTE-LEVEL NEGATIVE MARGINAL PROFITS. — Based on the estimates in Column 5 of Table 3, negative marginal profit is present on 78% of the routes, accounting for 19% of the passengers in the data. In general, pricing below marginal cost is suboptimal because increasing the price lowers the demand, which by the negative marginal profit, would lead to an increase in profit. However, with the peer effects present, increasing the price will in addition have negative effects on the demand for other routes, on which the marginal profits may be positive. In other words, although the carrier cannot break even on a route with price below marginal cost, the additional flow brought by the low price helps stimulate demand for the other routes, especially the link-sharing routes, in the network.

Interestingly, the routes with negative marginal profits are all one-stop. This is mostly likely because a one-stop, with two links, generally has more link-sharing routes than a non-stop. As mentioned, the negative marginal profits on one-stop routes seems to match the claim that "connecting traffic is the least profitable for the airline," made by Delta when it reduced the capacity that mostly served connection passengers at its Cincinnati hub.

As a comparison, without network effects, the estimates in Column 4 of Table 3 imply positive marginal profits on all the routes, and in addition, roughly the same level of marginal profits on nonstops and one-stops.

More generally speaking, pricing below marginal cost in a certain market may be very well explained if we also take into account related markets. A good example can be found in Benkard (2004), where he studies the industry dynamics of widebodied aircrafts and finds that short-run negative marginal profits are possible as a way to speed up production and reduce future cost. Another example is Skype, who provides the instant message service for free possibly in hopes of creating more users for its Internet phone-calling service.

#### V. Merger

Mergers have not been uncommon in the U.S. airline industry. Given that the airline industry is relatively unprofitable, it begs the questions from the companies' perspective: whether and if so why a merger is a solution to the unprofitability. This section starts with a discussion on the factors that may affect merger outcomes in important ways, then conducts several simulation exercises to quantitatively understand the effects of these factors.

## A. Discussion

There are two possible scenarios after a consolidation. In the first scenario, the two carriers, while jointly maximizing profits, remain operating their respective networks separately. In this case, the merger is modeled as equivalent to a bilateral collusive arrangement between the two companies (a point noted by Baker and Bresnahan (1985)). This seems to be the case for the 2010 Southwest-AirTran merger. In the second scenario, a single consolidated carrier operates the combined networks, where the overlapping links are merged and the set of routes is re-generated. This is the case in, for example, the 2008 Delta-Northwest merger, the 2011 United-Continental merger, and the 2013 American-US Airways merger. Figure V.1 depicts the United-Continental networks before and after merger.

There are several factors to be considered in an airline merger evaluation. First is the market power. Antitrust regulators are often mostly concerned with the potential increases in price caused by reduction of competition, which has also been the focus of many merger analyses in the economics literature (for the airline industry, see Kim and Singal (1993), Peters (2006)).

Secondly, the network effects of demand can play important role in mergers. With network effects, the companies may need to be more careful about raising prices as the decreases in demand will have further negative effects on demand. More importantly, when networks are combined, the merged links will bear traffics from both of the two pre-merger networks, making the flows on the combined network likely more dense than either of the pre-merger networks. For the example depicted by Figure V.2, both the passengers between  $t_1$  and  $t_2$  and between  $t_2$  and  $t_4$  may benefit from a second-type merger, as the flow on the  $t_2t_3$  segment of their connection flights will increase. Consumer benefits as such are often emphasized by airlines to justify mergers. For example, for its ongoing merger with US Airways, American

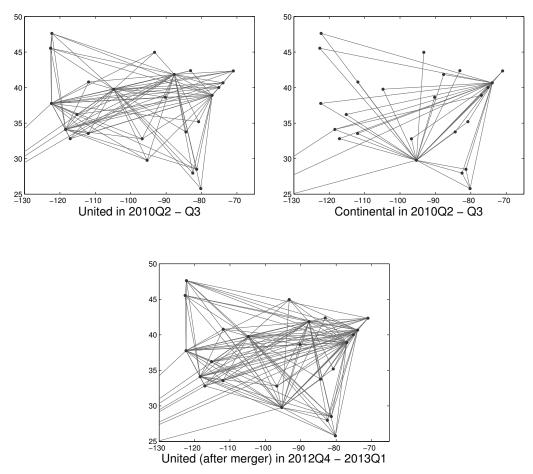


Figure V.1. Networks of United and Continental before and after merger, restricted to the 25 most-visited cities in the U.S. Coordinates are longitude and latitude.

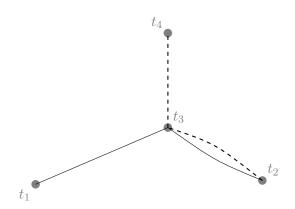


Figure V.2. Example of Merger. Carrier 1's (carrier 2's ) links are represented by the solid (dashed) lines.

Airlines claims that it will bring customers "a stronger airline that offers greater schedule options".<sup>23</sup>  $^{24}$ 

The third factor is the possible creation of more routes. Of course, this can only happen in a second-type merger, where routes are re-generated from the larger combined network. In the example in Figure V.2, a one-stop route between  $t_1$  and  $t_4$  will be created after the networks are combined, making it possible to fly between the two cities, which would be unserved otherwise. As a matter of fact, the other consumer benefit stated by the American Airlines for its merger is "access to more destinations".

The forth factor is also unique to the second-type merger. Combining the overlapping routes in the pre-merger networks indicates some loss of product differentiation. In the example in Figure V.2, the two nonstop routes between  $t_2$  and  $t_3$ , once combined, will be seen as a single product by the consumers. In our model, the degree of product differentiation is captured by the nesting parameter  $\lambda$ .<sup>25</sup>

 $<sup>^{23}{\</sup>rm By}$  Scott Kirby, President of American Airlines. See http://www.usairways.com/en-US/aboutus/pressroom/newamerican.html?cint=update\_21132.

 $<sup>^{24}</sup>$ Richard (2003) has been the only analysis that considers the frequency changes after a merger. However, it did not consider the peer effects.

<sup>&</sup>lt;sup>25</sup>Another consideration which may play a role in mergers but here I do not focus on, is economies of density. In general, they work in favor of the second-type merger where links are usually more densely flown. See Brueckner and Spiller (1991). Removing economies of density does not change the qualitative results of this section.

It is important to note that these factors do not act independently. For example, while new routes are created in a second-type merger, the additional demand brought by these routes will stimulate the demand on the existing ones through the network effects; even disregarding the new routes, a second-type merger is argued in favor of by the network effects and argued against by the loss of differentiation, it is hence unclear if combining networks is better in terms of profitability. As an attempt to quantitatively understand the effects of these factors, I conduct several simulation exercises.

#### B. Simulation Method

The method makes use of the estimated model, and applies the Bertrand-Nash equilibrium to simulate the outcomes.<sup>26</sup> The results will, of course, depend on the specific pre-merger network structures. Ideally, I would like to use the observed networks as large as those in the estimation to produce results most relevant to the industry. Unfortunately, as explained below, such large networks would render the computation virtually infeasible. Therefore I will use smaller networks that capture the multi-hub structure in the industry as much as possible. The purpose of the exercises is not to provide the most pertinent predictions, but to understand how each factor affects the profitability of either type of merger.

More specifically, for given sets of carriers and cities, the city characteristics, and the networks, one can compute a set of equilibrium prices under the estimated model<sup>27</sup>, by searching a solution to the first-order conditions (II.9). The equilibrium demand, flow and marginal costs can all be readily computed once the equilibrium prices are found. We will compare the equilibria before and after a merger, where the merger changes the set of products and ownerships according to the identities of the two consolidating carriers and the type of the merger.

 $<sup>^{26}</sup>$  Applications of this approach include Nevo (2000), Dubé (2004) and Peters (2006). The method is useful in evaluating the short-run post-merger industry outcome. In the medium to long-run, industry dynamics may become more relevant (see Benkard, et. al. (2010)).

 $<sup>^{27}</sup>$ More precisely, the estimated effects of route characteristics on demand and the effects of both route and link characteristics on marginal costs. The carrier fixed-effects and route-specific random fixed-effects are ignored.

The algorithm for equilibrium is presented in the appendix. Basically it starts with a guess of prices and iteratively updates it with (II.9) until a solution is found. There are two aspects of computational burden. First, as pointed out in Section III.A, the computation of  $\partial q/\partial p$  can be very expensive, and moreover, it needs to be computed many times as the algorithm searches for a solution. Second, the time needed for convergence towards a solution seems to increase with the magnitude of network effects. These make it very burdensome to compute equilibrium for large hub-and-spoke or multi-hub networks, because such networks typically have large-size  $\partial q/\partial p$  and considerable peer effects.

To be computationally feasible and to capture the multi-hub structure in the industry, I use the U.S. airline networks in 2010 right before United's acquisition of Continental, restricted to 5 major legacy carriers (i.e. Delta, United, American, US Airways, and Continental) and the 25 most-visited cities. Since the restriction is a considerable simplification of the large networks observed in the industry, from this point on I feel obligated to state these airline names with quotation marks. I simulate two mergers, one between "United" and "Continental", the other between "American" and "US Airways".

#### C. Results

Table 8 displays the results from the "Continental" - "United" merger simulation, including the percentage changes of total profits, total passenger miles (PM), revenue per passenger mile (RPM), cost per passenger mile (CPM), and the average flow across links. The numbers inside the parentheses are the corresponding percentage changes averaged across the other carriers. The last two rows of the table display the number of links and routes of the merged carrier. In the same manner, Table 9 displays the results from the "American" - "US Airways" merger simulation.

Column 2 in both tables displays the results for a first-type merger, where the carriers resume separate operations on their respective networks. We see here that the main force at work is the market power: as two carriers merge and jointly maximize their

|                | Separate<br>Fixed Flow | Separate   | Combined w/o<br>new routes | Combined    |
|----------------|------------------------|------------|----------------------------|-------------|
| Profits (%)    | 0.9  (0.6)             | 2.2(0.9)   | 10.8 (-1.1)                | 14.4 (-2.4) |
| PM (%)         | -3.7 (-0.1)            | -5.9(1.1)  | 1.9(0.3)                   | 6.0 (-0.6)  |
| RPM (%)        | 2.3 (0.5)              | 1.4 (-0.1) | 1.4 (-0.4)                 | 1.5(-0.4)   |
| CPM (%)        | 0.2(0.0)               | 0.6 (-0.1) | 0.4(-0.1)                  | 0.7 (-0.1)  |
| Flow dens. (%) | -3.3 (-0.2)            | -5.2(1.0)  | 14.0(0.1)                  | 19.9 (-0.8) |
| # Links        | 144                    | 144        | 131                        | 131         |
| # Routes       | 1244                   | 1244       | 1208                       | 1294        |

TABLE 8— "CONTINENTAL" - "UNITED" MERGER SIMULATION.

Note: PM is total passenger miles, RPM is revenue per passenger mile, and CPM is cost per passenger mile. The numbers outside the parentheses are for the merged airlines, while the numbers in the parentheses are the averages across the other carriers.

|                   | Separate<br>Fixed Flow | Separate    | Combined w/o<br>new routes | Combined    |
|-------------------|------------------------|-------------|----------------------------|-------------|
| Profits (%)       | 1.2(1.3)               | 3.6(2.6)    | 20.7 (-2.5)                | 25.6 (-4.5) |
| PM (%)            | -6.7(0.9)              | -8.4(2.5)   | 4.4 (-0.4)                 | 9.4 (-1.9)  |
| RPM (%)           | 4.8(0.2)               | 2.0 (-0.0)  | 3.8(-0.5)                  | 2.0 (-0.0)  |
| CPM (%)           | 0.2 (-0.0)             | 0.3 (-0.1)  | -0.3 (-0.1)                | 0.1 (-0.0)  |
| Flow dens. $(\%)$ | -6.6 (-0.7)            | -6.6 (-0.7) | 23.0 (-0.4)                | 29.8 (-2.0) |
| # Links           | 103                    | 103         | 175                        | 175         |
| # Routes          | 1656                   | 1656        | 1561                       | 1813        |

TABLE 9— "AMERICAN" - "US AIRWAYS" MERGER SIMULATION.

*Note:* See the notes for Table 8.

profits, they raise prices (RPM), which leads to decreases in demand (PM). Overall, the profit increases are relatively small (2.2% and 3.6%).

Column 4 in both tables displays the results for a second-type merger. We see that by combining their networks, carriers are able to achieve much higher profit increases (14.4% and 25.6%). Perhaps more importantly, in both simulations, even though the merged carrier raises prices, it is still able to see increases in demand. The qualitative result seems to fit what happened in the United-Continental merger: Comparing the second quarter of 2012 with the second quarter of 2010, the average price of United-Continental increased by 14.5%, about 1 percentage point higher than the average of the other major carriers; the total domestic passengers of United-Continental increased by 10%, about 7 percentage points higher than the average of the other major carriers.<sup>28</sup>

The large magnitude of increases in flow density in both second-type mergers (19.9% and 29.8%) indicate that network effects are an contributing factor to the profitability. Column 3 further confirms this by displaying the results from simulating the same mergers as in Column 4 except for that they only allow post-merger routes that overlap with those in the pre-merger networks, which removes the effect of route creation. Nevertheless, we see that the profit increases are much larger than those of the first-type mergers, suggesting that network effects are able to overcome the loss of product differentiation in a second-type merger.

As a final comparison, Column 1 displays the results of first-type mergers using the model without network effects.<sup>29</sup> We see that the profit increases are very small. Note that it can be a far-fetched exercise to use the model without network effects to simulate second-type mergers, because the model does not determine flow endogenously and thus has to assign flows to the combined network by some devised rules.

 $<sup>^{28}</sup>$  These statistics are estimated from the DB1B data. United acquired Continental in October, 2010. The integration of operations was completed in 2012.

 $<sup>^{29}</sup>$ More specifically, it uses the model where flow is exogenously given, and the corresponding estimates. The flows entering the utility are kept fixed throughout the merger. This is similar to the merger analyses where possible changes in flight frequencies are ignored.

#### VI. Conclusion

By accounting for the network effects of demand, this paper offers new insights into the structure and profitability of the airline industry. Compared with previous studies, the paper finds higher estimates of marginal costs, which seem more consistent with the relative unprofitability of the industry. With the peer effects present, below-marginal-cost pricing becomes possible and is found on many routes in the U.S. The paper also looks into the role of network effects in airline mergers, and finds that, in a merger, carriers can obtain higher profit increases by combining their networks rather than resuming separate operations.

Like other discrete-choice studies of the industry, the analysis in this paper, while focusing on the relation among price, flow, demand and cost structure, takes the networks as given. There have been some works in economics that explicitly study the network choices of airline carriers. Hendricks et al. (1999) studies carriers' network choice in a duopoly environment. Benkard et al. (2010) estimate the entry decisions of carriers in a dynamic setting. These two lines of research should be seen complementary, as the demand and cost structure are the building blocks of the preferences behind the carriers' choices. That being so, it would be interesting for future research to explore whether and how the presence of network effects has caused the hub-and-spoke to emerge as the dominant feature of airline networks.

#### VII. Appendix

1. LOGARITHM SPECIFICATION. — We focus on the within-nest (i.e. conditional on flying) choice as described in Section II.B, where a product has been defined as a route. Now suppose that product is defined as a flight. For example, if there are 10 flights per week flying between the city pair on route j, then the 10 flights are separate products in the market. Suppose that the mean utility of taking flight  $\ell$  on route j is  $w_j$ . This is, of course, a simplification as the flights on the same route may have different prices and unobserved characteristics. Using a nested logit model where the nests are the routes, the choice probability of nest j is

$$\frac{(\sum_{\ell \in j} e^{w_j/\gamma})^{\gamma}}{\sum_k (\sum_{\ell \in k} e^{w_k/\gamma})^{\gamma}} = \frac{e^{(\gamma \log(n_j) + w_j)}}{\sum_k e^{(\gamma \log(n_k) + w_k)}}$$

where  $n_j$  is the number of flights (or, the flight frequency) on route *j*. Hence it is equivalent to treating the mean utility of route *j* as  $(w_j + \gamma \log(n_j))$ . This motivates the logarithm specification in (II.3) and (II.4).

2. UNIQUENESS OF FIXED POINT. — I first present an easy-to-obtain bound on the parameters that guarantees uniqueness, then I offer a description of my computational experiences.

**Proposition** Let  $\overline{\gamma} = \max(\gamma_1, \gamma_2)$ . If parameters are such that  $\overline{\gamma}/\lambda \leq 1/2$ , the fixed point of  $\Psi(p, \cdot)$  is unique.

**Proof** Let  $q^* \gg 0$  be a fixed point of  $\Psi$ , and  $q \gg 0$  be some other demand. Define a "distance" measure:

$$\sigma(q,q^*) = \max_{j} \left\{ \max\left\{ \frac{q(j)}{q^*(j)}, \frac{q^*(j)}{q(j)} \right\} \right\}$$

Note that  $\sigma$  is always no less than 1, and  $\sigma = 1$  means  $q = q^*$ . Note that the distance is also defined for any two positive vectors with the same length. For any j, we have

$$\Psi_j(p,q) = \frac{\left(\sum_{k \in m} e^{w_k/\lambda + f_k/\lambda}\right)^{\lambda - 1} e^{w_j/\lambda + f_j/\lambda}}{1 + \left(\sum_{k \in m} e^{w_k/\lambda + f_k/\lambda}\right)^{\lambda}}$$

where  $w_j \equiv \beta' x_j - \alpha p_j + \xi_j$ , and  $f_j$  is specified as a function of q through (II.3), (II.4) and (II.1), which we write as  $f = \phi(q)$ . Let  $f^* = \phi(q^*)$ . It is not difficult to see that

$$\sigma\left(e^{f}, e^{f^{*}}\right) \leq \sigma(q, q^{*})^{\overline{\gamma}}$$

It is also not difficult to see that

$$\frac{\Psi_j(p,q)}{q_j^*} = \frac{\Psi_j(p,q)}{\Psi_j(p,q^*)} < \sigma \left(e^f, e^{f^*}\right)^{2/\lambda}$$

An easy loose bound of  $\sigma(\Psi(p,q),q^*)$  is then obtained:

$$\sigma(\Psi(p,q),q^*) < \sigma(q,q^*)^{2\overline{\gamma}/\lambda}$$

If  $\overline{\gamma}/\lambda \leq 1/2$ , then the above implies that q cannot be a fixed point. Since q is any demand, we conclude that  $q^*$  is the unique fixed point. Furthermore,  $\Psi$  acts like a contraction in terms of the "distance" defined by  $\sigma$ , and iteration of it converges to the fixed point.

The result above does not apply to the estimates of Table.3. But it shows that for a range of reasonable parameter values, the uniqueness of demand can be easily guaranteed. In computation, I iterate  $\Psi(p, \cdot)$  to find a fixed point. In my experiences, as long as  $\gamma < \lambda$ , different starting points always lead to the same limit. When  $\gamma > \lambda$ , the iteration typically diverges.

3. COMPUTATION OF EQUILIBRIUM. — With the network effects, or more precisely, the peer effects, prices in one market may affect the demand on another market. Hence it does not seem plausible to solve for equilibrium prices market by market. Instead, they need to be found all together. My algorithm is presented below. For small-size networks, it runs quickly and often converges easily. For larger networks, the computation becomes more burdensome and the convergence becomes much more difficult. It is worth noting that damping the update of price can be helpful for convergence.

- 1) Start with a initial price vector  $p^0$ , which contains the prices for all the routes in the networks.
- 2) Enter iteration L: Find the fixed-point demand q corresponding to  $p^L$  by iterating  $\Psi(p^L, \cdot)$ .
- 3) With price  $p^L$  and demand q given, compute the matrix D (see Sec.III.A) and the vector of marginal costs mc. Then update the price using the first-order conditions (II.9):  $p^{L+1} = mc D^{-1}q$ .
- 4) Exit if  $||p^L p^{L+1}||$  is adequately small, go to Step 2 otherwise.

#### REFERENCES

Adler, Nicole (2001), "Competition in A Deregulated Air Transportation Market." European Journal of Operational Research, 129: 337-345.

Abdelghany, Ahmed and Khaled Abdelghany (2010), Chapter 1 in "Modeling Applications in the Airline Industry." Ashgate Publishing.

Baker, Jonathan and Timothy Bresnahan (1985): "The Gains from Merger or Collusion in Product-Differentiated Industries." Journal of Industrial Economics, 33: 427–44.

Benkard, Lanier, Aaron Bodoh-Creed and John Lazarev (2010): "Simulating the Dynamic Effects of Horizontal Mergers: U.S. Airlines." Working Paper.

Benkard, Lanier (2004): "A Dynamic Analysis of the Market for Wide-Bodied Commercial Aircraft." Review of Economic Studies, 71(3): 581-611.

Berry, Steven (1992): "Estimation of a Model of Entry in the Airline Industry." Econometrica, 60, 889-917. Berry, Steven, Michael Carnall and Pablo T. Spiller (2006): "Airline Hubs: Costs, Markups and the Implications of Customer Heterogeneity." In Advances in Airline Economics: Competition Policy and Antitrust, Vol. 1, ed. Darin Lee, 183–214. Amsterdam: Elsevier.

Berry, Steven, James Levinsohn and Ariel Pakes (1995): "Automobile Prices in Market Equilibrium." Econometrica, 63(4): 841-90.

Berry, Steven, Panle Jia (2010): "Tracing the Woes: An Empirical Analysis of the Airline Industry." American Economic Journal: Microeconomics 2, Vol. 2(3), 1-43.

Borenstein, Severin (1989): "Hubs and High Fares: Dominance and Market Power in the U.S. Airline Industry." Rand Journal of Economics, 20, 344-365.

— (1991): "The Dominant-Firm Advantage in Multiproduct Industries: Evidence from the U.S. Airlines." Quarterly Journal of Economics, 106, 1237–66.

Bramoullé, Yann, Habiba Djebbari and Bernard Fortin (2009), "Identification of peer effects through social networks." Journal of Econometrics, 150(1): 41–55.

Brueckner, Jan K., and Pablo T. Spiller (1991): "Competition and Mergers in Airline Networks." International Journal of Industrial Organization, 9, 323-342.

— (1994): "Economics of Traffic Density in the Deregulated Airline Industry." Journal of Law and Economics, 37, 379-415.

Caves, Douglas, Laurits Christensen and Michael Tretheway (1984): "Economies of Density Versus Economies of Scale: Why Trunk and Local Service Airline Costs Differ." Rand Journal of Economics, 15, 471-489.

Crawford, G., O. Shcherbakov and M. Shum (2011): "The Welfare Effects of Monopoly Quality Choice: Evidence from Cable Television Markets." Working Paper.

Dubé, Jean-Pierre (2005): "Product Differentiation and Mergers in the Carbonated Soft Drink Industry." Journal of Economics and Management Strategy, 14(4).

Economides, Nicholas (1996): "The Economics of Networks." International Journal of Industrial Organization, vol. 14, no.6.

Einav, Liran and Jonathan Levin (2010): "Empirical Industrial Organization: A Progress Report.", Journal of Economic Perspectives, 24(2):145–162.

Gillen, David W., William G. Morrison, and Christopher Stewart. (2003): "Air Travel Demand Elasticities: Concepts, Issues and Measurement." Department of Finance Cananda Report.

Givoni, Moshe and Piet Rietveld (2009): "Airline's Choice of Aircraft Size - Explanations and Implications." Transportation Research Part A, 43, 500-510.

Hendricks, Ken, Michele Piccione, and Guofu Tan (1999): "Equilibria in Networks." Econometrica, 67(6), 1407-1434.

Kim, Han E. and Vijay Singal (1993): "Mergers and Market Power: Evidence from the Airline Industry", American Economic Review, 83, 549-569.

Manski, Charles F. (1993) "Identification of Endogenous Social Effects: The Reflection Problem." Review of Economic Studies, 60(3): 531-542.

Mayer, Christopher and Todd Sinai (2003): "Network Effects, Congestion Externalities, and Air Traffic Delays: Or Why Not All Delays Are Evil?" American Economic Reivew, 93(4), 1194-1125.

McFadden, Daniel. (1981): "Econometric Models of Probabilistic Choice." Structural Analysis of Discrete Data with Econometric Applications, 198–272, MIT Press.

Nevo, Aviv. (2000): "Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry." Rand Journal of Economics, 31, 395–421.

Peters, Craig (2006): "Evaluating the Performance of Merger Simulation: Evidence from the U.S. Airline Industry." Journal of Law and Economics, vol. XLIX., pp 627-649.

Petrin, Amil and Kenneth Train (2010): "A Control Function Approach to Endogeneity in Consumer Choice Models." Journal of Marketing Research, 47(1), pp 3–13.

Richard, Oliver (2003): "Flight Frequency and Mergers in Airline Markets." International Journal of Industrial Organization, Vol. 21, 907-922.

Wei, Wenbin and Mark Hansen (2003), "Cost economics of aircraft size." Journal of Transport Economics and Policy. 37(2), 279-296.

Wei, Wenbin and Mark Hansen (2005), "Impact of Aircraft Size and Seat Availability on Airlines' Demand and Market Share in Duopoly Markets." Transportation Research Part E, 41: 315–327.

Wei, Wenbin and Mark Hansen (2007), "Airlines' Competition in Aircraft Size and Service Frequency in Duopoly Markets." Transportation Research Part E, 43: 409–424.

Wooldridge, Jeffrey M., (2007): Lecture 6 of "What's New in Econometrics?", NBER Summer Institutes.