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"Premuneration Values and Investments in Matching Markets"

by

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Premuneration Values and Investments in Matching Markets*

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Abstract

We analyze a model in which agents make investments and then match into pairs to create a surplus. The agents can make transfers to reallocate their pretransfer ownership claims on the surplus. Mailath, Postlewaite, and Samuelson (2013) showed that when investments are unobservable, equilibrium investments are generally inefficient. In this paper we work with a more structured model that is sufficiently tractable to analyze the nature of the investment inefficiencies. We provide conditions under which investment is inefficiently high or low and conditions under which changes in the pretransfer ownership claims on the surplus will be Pareto improving, as well as examine how the degree of heterogeneity on either side of the market affects investment efficiency.

Keywords: Directed search, matching, premuneration value, prematch investments, search.

JEL codes: C78, D40, D41, D50, D83

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Premuneration Values and Investments in Matching Markets

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1 Introduction

1.1 Motivation

How are heterogeneous workers matched with heterogeneous firms? What determines the division of the resulting surplus? When will outcomes be efficient? We address these questions in the context of a market in which workers and firms first make productivity-enhancing investments, and then match into pairs to produce a surplus.

It is a familiar result that if workers and firms cannot contract prior to making their investments, then holdup problems may lead to inefficiencies. However, when there are many agents on each side of the market, Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) show that efficient two-sided investments are consistent with equilibrium. The presence of (close) substitutes for an agent's possible matches in the competitive matching market ensures that she is appropriately compensated for her investment, leading to the existence of an equilibrium with efficient investments.

The results of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) depend crucially on there being complete information about investments. Mailath, Postlewaite, and Samuelson (2013) study an economy similar to that in Cole, Mailath, and Postlewaite (2001), but with the significant difference that workers' investments are not observable when workers and firms match. The results are dramatically different: except in the extreme case that firms' pretransfer values from a match are independent of the worker with whom they match, investments will not be efficient.

It is important to understand not only that investments will be inefficient, but to understand the nature of the inefficiency. Will investments be inefficiently low, or can they be inefficiently high? How does the magnitude of ex ante heterogeneity of workers affect the inefficiency, and are there policy interventions that might ameliorate the inefficiencies? How does the allocation of property rights to the surplus affect investments? Mailath, Postlewaite, and Samuelson's (2013) model is too general to answer these questions. We address these questions here in the context of a more structured model.

1.2 Investment and Matching Markets

The agents in our analysis could be interpreted in many ways—we opened the paper by referring firms and workers, but we could just as well think of students and universities, men and women, lawyers and clients, and so on. For concreteness, we refer to them as laboratories and researchers.

We examine a market with a large set of laboratories of differing sophistication on one side and an analogous set of researchers of different abilities on the other side. Researchers first have the opportunity to invest in human capital, and then laboratories and researchers are matched into pairs.[1] Each pair produces a surplus, arising (for now) from the patents they create.

Depending on the relevant legal environment, the patents that arise out of a laboratory/researcher match may belong to the laboratory, or may belong to the researcher, or may be shared. A typical analysis takes the total surplus as the point of departure and focuses on how this surplus will be split, ignoring the question of whether this surplus initially belongs to the laboratory, the researcher, or partly to both. When there is no uncertainty about matching-relevant characteristics it makes no difference for most problems whether the patents belong to the laboratory or to the researcher. We expect laboratories to hire researchers in the former case and researchers to buy or rent laboratories in the latter case. In either case, a monetary payment from the party that owns the patents to the other party delivers the equilibrium division of the surplus. For any change in the distribution of patent ownership, there is an offsetting change in the equilibrium monetary transfer between agents preserving the equilibrium welfare distribution and investments.[2] In particular, outcomes with efficient investments exist no matter who owns the patents. If laboratories own the patents, for example, competition among laboratories to hire talented researchers will ensure that the latter capture the returns from their investments and hence face efficient investment incentives.

If researchers' match-relevant characteristics are unobservable, initial ownership will play a central role in the efficiency of investments and in the final welfare distribution.[3] Laboratories now cannot observe a researcher's

[1] In order to focus on the implications of unobservable researcher investments, we assume in much of the paper that laboratories' investments are fixed (in Section 4, we instead fix unobservable researcher investments, and allow laboratories to make investments).

[2] See Cole, Mailath, and Postlewaite (2001) for details.

[3] This is reminiscent of the Coase theorem (Coase, 1960): in the absence of bargaining frictions (such as asymmetric information), bargaining will result in an efficient allocation irrespective of the original allocation of property rights. On other hand, in the presence of asymmetric information, the possibility of reaching an efficient agreement depends on the

investment, precluding the enhanced competition that facilitates the researcher's capture of the returns on her investment when laboratories own the patents, and potentially leading to inefficient investments. More importantly, increasing the share of the patents owned by the researcher then provides incentives to invest more efficiently, giving rise to a link between initial ownership and investments that can have unexpected implications.

For example, an increase in the share of the surplus owned by the researcher does not necessarily harm laboratories. Increasing the share owned by researchers can lead to more investment, with the laboratories enjoying some of the fruits of that investment. In addition, there is competition among researchers for laboratories, and researchers who own more of the surplus find all laboratories more valuable. This intensifies the competition for laboratories, leading to higher market prices for laboratories. We show that when the share of the surplus owned by researchers is small, increasing this share increases laboratories' equilibrium payoff.

1.3 Premuneration Values

We have thus far proceeded as though the surplus created by a laboratory-researcher match arises entirely out of the resulting patents. It is straightforward to think of the patents as being owned by either the laboratory, the researcher, or shared, and to think of different legal structures assigning this ownership differently.

In general, the surplus generated by a match will be a composite of many different items, with the ownership of these various items split between the laboratory and researcher in different ways. The researcher's value of the match includes the value of the human capital she accumulates at the laboratory, as well as the value from contacts she makes at the laboratory. The researcher may also derive utility from laboratory parties and social opportunities, but may exert costly effort. The laboratory's value of the match may include the prestige of employing a noted researcher, as well as the accumulation of organizational capital that will be of use in other research endeavors, but may also include the costs of training the researcher. In addition, some of the value from the researcher's contacts may accrue to the firm, perhaps because they make it easier to hire additional researchers.

Rather than itemize all the elements that comprise the surplus in the match between the laboratory and researcher, we take as the primitive the

original allocation of property rights (see, for example, Cramton, Gibbons, and Klemperer (1987)). However, the similarity is superficial, since the Coase theorem ignores investments that may be taken before bargaining (Grossman and Hart, 1986).

aggregate match value to each of the agents in the absence of any transfers. Mailath, Postlewaite, and Samuelson (2013) call these values *premuneration values* (from *pre* plus the Latin *munerare*, to give or pay). The total surplus in a match is then simply the sum of the matched parties' premuneration values. The premuneration values determine the division of the surplus *in the absence of transfers*. In equilibrium, of course, there typically will be transfers. What is central to our problem is that any transfers that reallocate surplus are determined *after* investments have been made.

We find that premuneration values matter.[4] For example:

- When researchers do not own all the surplus from a match, they invest less than is efficient; their investments and payoffs increase as their premuneration value increases.

- Laboratories' equilibrium payoffs increase as researcher premuneration values increase if the latter are small, and then decrease. Thus, both sides can gain by having premuneration values allocate more of the surplus to researchers.

- When the heterogeneity of researchers' investments costs increases, investments become more efficient.

- When researchers' attributes are exogenously given, but unobservable, and laboratories invest, they generally invest more than is efficient.

- The increase of researcher investments and payoffs as their premuneration values increase depends on the fierceness of researcher competition for laboratories. If researchers are identical, competition will ensure that all surplus goes to laboratories, and in this case the premuneration values of the researchers is irrelevant. But heterogeneity among researchers will attenuate researchers' competition for laboratories and researchers will accordingly get positive surplus in equilibrium. Their equilibrium welfare then increases in their premuneration values and increases in the heterogeneity of their investment costs.

- When laboratories can become informed (at some cost), premuneration values determine which laboratories choose to become informed and the payoffs of all laboratories (both informed and uninformed) and

[4] Liu, Mailath, Postlewaite, and Samuelson (2013) examine a finite matching model with incomplete information but no investments in which premuneration values also play a central role.

researchers. Laboratories may gain by having premuneration values allocate more of the surplus to researchers. While some researchers gain from such a reallocation, the presence of some informed laboratories means that some researchers can lose from such a reallocation.

It is a familiar result that inefficiencies can arise when the characteristics of the agents on one side of the market cannot be observed. The important finding is that the equilibrium allocation depends upon premuneration values, sometimes counterintuitively. Premuneration values matter whenever there are unobserved investments or unobservable exogenous attributes. Investments in human capital are especially difficult to verify, bringing any market for skilled labor within the scope of our model.

The finding that premuneration values matter would be relatively innocuous if we could simply redesign them (perhaps via legislation stipulating property rights) to achieve efficiency, but such reallocation is often infeasible. For example, premuneration values often include future returns, requiring future costly actions and hence moral hazard problems that preclude reallocation.[5]

To illustrate the difficulties in redesigning premuneration values, consider a match between a student and a university. While at the university, the student acquires knowledge and skills that lead to higher lifetime earnings and a greater satisfaction in life after school. She may also make contacts that will be important in her career, and she may be a regular at campus parties and generally enjoy the social life of the university. Each of these increases the student's value of the match, and consequently the surplus in the match. The university may derive value from touting the student's background and her ability to play the saxophone as additions to its diverse and artistically rich community, as well as from claiming her as a graduate when she achieves fame and fortune. The university's value of these items also contributes to the surplus of the match. Each side owns some of these components, in the sense that the value of that component accrues to them. Some components might be owned by either side, depending on circumstances, but others are inextricably linked to a particular side. We might be able to reallocate the ownership of the student's future income stream, perhaps by financing her education with income-contingent loans, but there are obvious limits in the possible shifting. There is no obvious way to reallocate her utility from partying.

[5]See Mailath, Postlewaite, and Samuelson (2013, Sections 1.4 and 6.5) for a discussion of this.

1.4 Related Literature

Other papers have also investigated the relationship between the incentives for efficient investments and subsequent bargaining.[6] Acemoglu and Shimer (1999) analyze a worker-firm model in which firms (only) make ex ante investments. If wages are determined by post-match bargaining, a standard hold-up problem induces firms to underinvest. The hold-up problem disappears if workers have no bargaining power, but then there is excess entry on the part of firms. Acemoglu and Shimer show that efficient outcomes can be achieved if the bargaining process is replaced by wage-posting on the part of firms, followed by competitive search. de Meza and Lockwood (2010) examine an investment and matching model that gives rise to *excess* investment. Their overinvestment possibility rests on a discrete set of investment choices and the presence of bargaining power in a noncompetitive post-investment stage.

In contrast, the competitive post-investment markets of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) lead to efficient two-sided investments. Our analysis shows that this efficiency rests on both ex post competition and complete information, with the latter allowing prices to be conditioned on both worker and firm characteristics. Gall, Legros, and Newman (2006, 2009) and Bhaskar and Hopkins (2011) examine an alternative class of models in which information is complete and hence different prices can be set for different workers, but inefficiencies arise out of limitations on the ability to reallocate the surplus in a match via transfers, including limiting cases in which no transfers can be made. In contrast to these models, monetary transfers allow us to achieve any division of the surplus between a pair of matched agents.

Moving from complete-information to incomplete-information matching models typically gives rise to issues of either screening, as considered here, or signaling. Cole, Mailath, and Postlewaite (1995), Hopkins (2012), Hoppe, Moldovanu, and Sela (2009), and Rege (2008) analyze models incorporating signaling into matching models with investments.[7]

[6] Early literature suggesting that frictionless, competitive search might create investment incentives include Hosios (1990), Moen (1997), and Shi (2001). Eeckhout and Kircher (2010) provide an extension to asymmetric information, while Masters (2011) examines a model with two-sided investments.

[7] The inability to observe workers' characteristics forces a firm to offer the same payment to all workers. Firms setting the "impersonal prices" of Bulow and Levin (2006) similarly offer the same price to all workers, but Bulow and Levin offer a motivation in terms of institutional constraints rather than incomplete information, including the possibility that firms may be able and desirous of committing to such prices in order to secure a more

2 The Model

2.1 The Market

There is a unit measure of researchers whose types (names) are indexed by ρ and are distributed uniformly on $[0, 1]$, and a unit measure of laboratories whose types are indexed by λ and distributed uniformly on $[0, 1]$. For ease of reference, researchers are female and laboratories male.

At the first stage, each researcher chooses an attribute r. Each laboratory is characterized by an attribute ℓ, where for convenience we take the type of laboratory λ to be fixed at $\ell = \lambda$. Following the attribute choices, researchers and laboratories match, with each matched pair generating a surplus. Attributes are costly, but enhance the values generated in the second stage. The second-stage values depend only on the attributes of the researcher and laboratory, r and ℓ, and not on their underlying types.

The cost of attribute $r \in \mathbb{R}_+$ to researcher ρ is given by

$$c(r, \rho) = \frac{r^{2+k}}{(2+k)\rho^k}, \qquad k \in \mathbb{R}_+.$$

When $k = 0$, researchers are homogeneous in the sense that all have the same cost. When $k > 0$, higher ρ researchers have a lower cost of acquiring any level of the attribute. The parameter k is a measure of the heterogeneity of the researchers. Increases in k (and so in the heterogeneity of researchers) lead to greater differences in researcher investments and, consequently, to less competition among them for laboratories.

The cost function has the property that efficient researcher investments are independent of k. This allows us to study how the effects of changes in premuneration values vary with k, and how these changes affect the efficiency of investments.[8]

As we discuss in the introduction, premuneration values identify the ownership of the values generated by the match before any transfers are made. We assume that a match between a researcher of attribute r and a laboratory of attribute ℓ generates a researcher premuneration value of

$$\theta \ell r$$

lucrative equilibrium.

[8] The $2 + k$ term in the denominator is introduced to simplify the algebra. Appendix B summarizes the analysis for a more general cost function that allows for the possibility that costs do not become negligible as k gets large.

and a laboratory premuneration value of

$$(1-\theta)\ell r.$$

The total surplus from the match is

$$v(\ell, r) = \ell r.$$

The parameter θ describes the researcher's premuneration value share of the surplus, while $(1-\theta)$ describes the laboratory's share. We note that the values are increasing and supermodular—an increase in an agent's characteristic has a larger effect on premuneration values and surplus the larger the partner's characteristic.

2.2 Equilibrium

Matching takes place in a competitive market. Laboratories' attributes are observable and priced, with $p(\ell)$ denoting the price of a laboratory with attribute $\ell \in [0,1]$. Researchers' attributes are not observable to laboratories, hence the price of laboratory with attribute ℓ, $p(\ell)$, is the same to all researchers. Given a price function p, each researcher optimally chooses her attribute and the laboratory with whom she wishes to match. That is, researcher ρ solves

$$\max_{\ell,r} \ \theta\ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{1}$$

We denote by $r_R : [0,1] \to \mathbb{R}_+$ the function describing the attributes chosen by researchers and we let $\ell_R : [0,1] \to [0,1]$ be the function describing the laboratories chosen by researchers.

The function ℓ_R is *market-clearing* if it is one-to-one, onto, and every set of researchers $\mathcal{R}$, is mapped to a set of equal size of laboratories.[9] Given a price function p and researcher behavior r_R and ℓ_R (where ℓ_R is market clearing), the payoff to laboratory ℓ is $(1-\theta)\ell r_R(\ell_R^{-1}(\ell)) + p(\ell)$.

Definition 1 *A price function p and researcher choices (ℓ_R, r_R) constitute a* matching equilibrium *if*

[9]Formally, if μ is Lebesgue measure and $\mathcal{R}$ is a measurable set of researcher tyes, then $\mu(\mathcal{R}) = \mu\{\ell | \ell = \ell_R(\rho) \text{ for some } \rho \in \mathcal{R}\}$. Our assumption that laboratory attributes are exogenously and uniformly distributed on an interval (and our focus on a parametrized model) allows us to avoid various technical issues that arise with a continuum of agents in two-sided investment models; see Mailath, Postlewaite, and Samuelson (2013, Section 3.2) for a discussion.

1. *for every $\rho \in [0,1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves the researcher-optimization problem (1),*
2. *every researcher and laboratory earns nonnegative payoffs, and*
3. *ℓ_R is market-clearing.*

The second property of equilibrium is an *individual rationality* requirement, ensuring that all agents prefer participation to not participating.

We begin by identifying three useful properties of an equilibrium, the proofs of which are in Appendix A. The first is a direct implication of market clearing:

Lemma 1 *Every equilibrium price function p is strictly increasing and continuous.*

The researchers' cost function exhibits a single-crossing condition that gives the following lemma.

Lemma 2 *The equilibrium researcher attribute-choice function r_R is strictly increasing.*

The supermodularity of the surplus function ensures that matching is assortative.[10]

Lemma 3 *The equilibrium researcher laboratory-choice function ℓ_R is given by*

$$\ell_R(\rho) = \rho.$$

3 Investments and Payoffs

3.1 Efficient Investments

Efficient researcher attribute choices have a particularly simple form. First, strict supermodularity of the surplus function ℓr implies that, for any strictly increasing researcher attribute choice function, total surplus is maximized under assortative matching. Second, the cost function for researchers is decreasing in researcher index ρ, so for any researcher attribute distribution, the minimum cost of obtaining that distribution is for the attribute choice

[10] Legros and Newman (2007) offer general sufficient conditions for assortative matching when characteristics are observable but there are restrictions on the ability to divide the surplus.

function r_R to be (weakly) increasing. Thus, total net surplus is maximized when the matching on indices λ and ρ is positively assortative: laboratory λ will be matched with researcher $\rho = \lambda$. Total net surplus is thus maximized when the net surplus for each such matched pair is maximized. For the ρ-matched pair of laboratory and researcher, the surplus-maximization problem is (since laboratory $\lambda = \rho$ has attribute $\ell = \rho$)

$$\max_r \ \rho r - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{2}$$

The first-order condition is

$$\rho = \frac{r^{1+k}}{\rho^k},$$

immediately implying

$$r = \rho. \tag{3}$$

Hence, efficiency requires $r_R(\rho) = \rho$ and $\ell_R(\rho) = \rho$. As we indicated earlier, the efficient allocation does not depend on k, the degree of heterogeneity of the researchers.

3.2 Market Equilibrium

We turn to the structure of the market equilibrium. First, suppose that the equilibrium price of laboratories is differentiable, a supposition that will be validated by the equilibrium we construct.[11] Researcher ρ's problem is to choose ℓ and r to maximize

$$\theta \ell r - p(\ell) - c(r, \rho) \ = \ \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}.$$

The first order conditions are

$$\theta \ell = \frac{r^{1+k}}{\rho^k} \tag{4}$$

and

$$\theta r = p'(\ell). \tag{5}$$

In equilibrium, researcher ρ is matched with laboratory $\ell = \rho$, hence from (4) we have that in equilibrium

$$r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}. \tag{6}$$

[11] A standard revealed preference argument shows that in fact every equilibrium price function is differentiable, and so the equilibrium is unique.

For all $\theta \in (0,1)$, $\theta^{\frac{1}{1+k}} \in (0,1)$, and hence $r_R(\rho) < \rho$; for $\theta = 1$, $r_R(\rho) = \rho$. This immediately gives:

Proposition 1 *The researcher investment function given by (6) is a matching equilibrium investment function. Suppose $\theta < 1$, so that the laboratory premuneration value share is positive. Then in equilibrium, researchers invest less than the efficient level.*

For any given researcher ρ, $r_R(\rho)$ is increasing in both k and θ. As θ increases, the researcher has a larger share of the surplus, and hence has an increased incentive to invest; when k increases, less of a researcher's benefit is competed away, giving researchers further reason to increase their investment.

Combining the two first order conditions (5) and (6) gives

$$p'(\ell) = \ell \cdot \theta \cdot \theta^{\frac{1}{1+k}}$$

and hence

$$p(\ell) = \tfrac{1}{2}\ell^2 \cdot \theta^{\frac{2+k}{1+k}} \tag{7}$$

(the constant of integration is set so that $p(0) = 0$, as required by the individual-rationality requirement that payoffs be nonnegative).

3.3 Payoffs

Given the equilibrium choices, laboratory λ's payoff given θ and k is

$$\begin{aligned} u_L(\theta,k,\lambda) &\equiv (1-\theta)\ell r + p(\ell) \\ &= (1-\theta)\lambda(\lambda\theta^{\frac{1}{1+k}}) + p(\ell) \\ &= (1-\theta)\lambda^2\theta^{\frac{1}{1+k}} + \tfrac{1}{2}\lambda^2\theta^{\frac{2+k}{1+k}} \\ &= \tfrac{1}{2}\theta^{\frac{1}{1+k}}(2-\theta)\lambda^2. \end{aligned} \tag{8}$$

We are interested in identifying conditions under which the laboratory's payoff increases when the researcher's share of the surplus, θ, increases. From (8), the laboratory's payoff is increasing in its share of the surplus when $\frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) < 0$. This derivative is given by

$$\begin{aligned} \frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) &= \tfrac{1}{1+k}\theta^{\frac{-k}{1+k}}(2-\theta) - \theta^{\frac{1}{1+k}} \\ &= \theta^{\frac{-k}{1+k}}\left[\tfrac{1}{1+k}(2-\theta) - \theta\right]. \end{aligned}$$

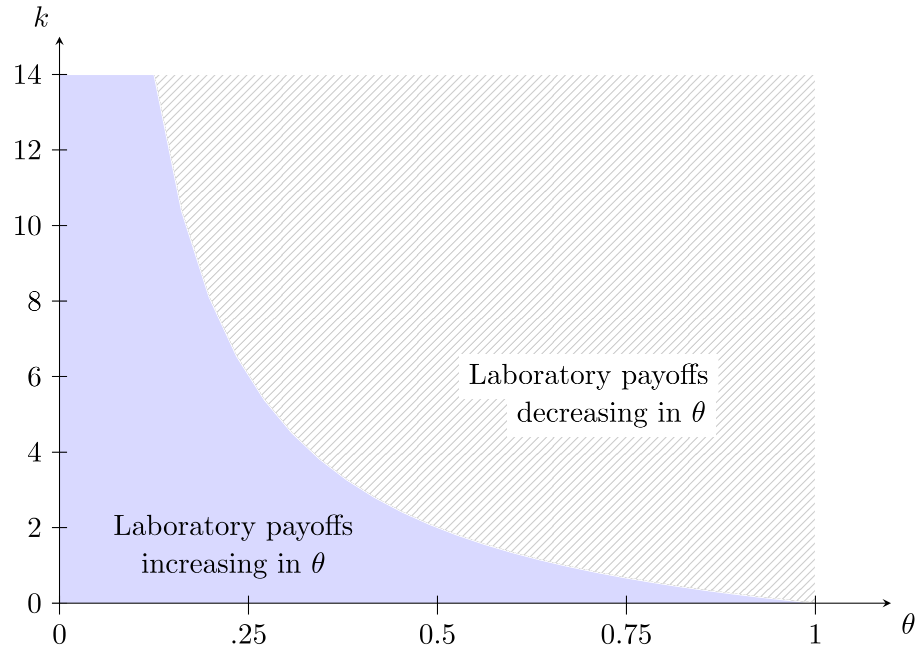

Figure 1: Parameter regions for which laboratory payoffs are increasing or decreasing in laboratory premuneration values.

Thus the sign of $du_L/d\theta$ is the same as the sign of $\frac{1}{1+k}(2-\theta)-\theta$, that is, of $2-(2+k)\theta$.

Figure 1 shows the region in which laboratories' payoffs increase as the researchers' premuneration values increase: (θ, k) combinations that are below and to the left of the curved line are situations in which the laboratories' payoff increases when the researchers' premuneration values increase.

Above the line, laboratories' payoff decrease as researchers' premuneration value increases. Hence, the line represents the optimal premuneration values from the laboratory's perspective. In summary:

Proposition 2 *Laboratories' equilibrium payoffs are first increasing in θ, the researchers' premuneration value share, are maximized at $2/(2+k)$, and then are decreasing in θ.*

For $k=0$, the laboratory's payoff is increasing for all θ, that is, laboratories' payoffs are maximized when premuneration values assign all the surplus to researchers. When $k=0$, researchers are identical and so the competition for laboratories is the most intense, with researchers bidding away all rents in the competition for higher attribute laboratories. Since laboratories ultimately capture all the surplus through market competition, they do best when total surplus is maximized, which is when $\theta = 1$.

For positive but small k, the laboratories' payoffs are maximized with θ near, but less than, 1. When $\theta < 1$, researchers' attribute choices will be less than the attribute choices that maximize total net surplus. This is nevertheless optimal for laboratories since they will not capture the entire surplus in the market given that competition among researchers is imperfect when $k > 0$. As k increases, competition among researchers decreases and the researcher share of the surplus that maximizes laboratory payoff decreases, approaching zero as k gets large.[12]

The researcher's payoff can be calculated as the total net surplus minus the laboratory's payoff. The total net surplus for a matched pair $\rho = \lambda$ is

$$\theta^{\frac{1}{1+k}}\rho^2 - \frac{(\theta^{\frac{1}{1+k}}\rho)^{2+k}}{(2+k)\rho^k} = \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{(2+k)}\theta\right].$$

From (8), the laboratory's payoff is $\rho^2\theta^{\frac{1}{1+k}}(1-\frac{1}{2}\theta)$, so the researcher's payoff is

$$\begin{aligned} u_R(\theta,k,\rho) &\equiv \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{2+k}\theta\right] - \rho^2\theta^{\frac{1}{1+k}}\left(1 - \tfrac{1}{2}\theta\right) \\ &= \theta^{\frac{1}{1+k}}\rho^2\left[1 - \tfrac{\theta}{2+k} - 1 + \tfrac{\theta}{2}\right] \\ &= \frac{k\theta^{\frac{2+k}{1+k}}}{2(2+k)}\rho^2. \end{aligned} \tag{9}$$

Proposition 3 *Researcher's equilibrium payoffs increase in θ, i.e., as researchers' premuneration value share increases.*

Thus, both researchers' and laboratories' payoffs increase, as the researcher's premuneration value increases, in the solid shaded region in Figure 1.

3.4 The Impact of Competition on Payoffs

We next investigate the effect of heterogeneity of researchers (via changes in k) on payoffs. The equilibrium payoffs of laboratories and researchers are given by (8) and (9). Figure 2 illustrates these payoffs as a function of k.

Researchers' payoffs increase in k, reflecting the enhanced investment incentives of reduced competition and reduced investment costs. Laboratories' payoffs increase with k. A larger value of k makes researchers more

[12] We note that as $k \to \infty$, $r(\rho) \to \rho$, i.e., investments become efficient.

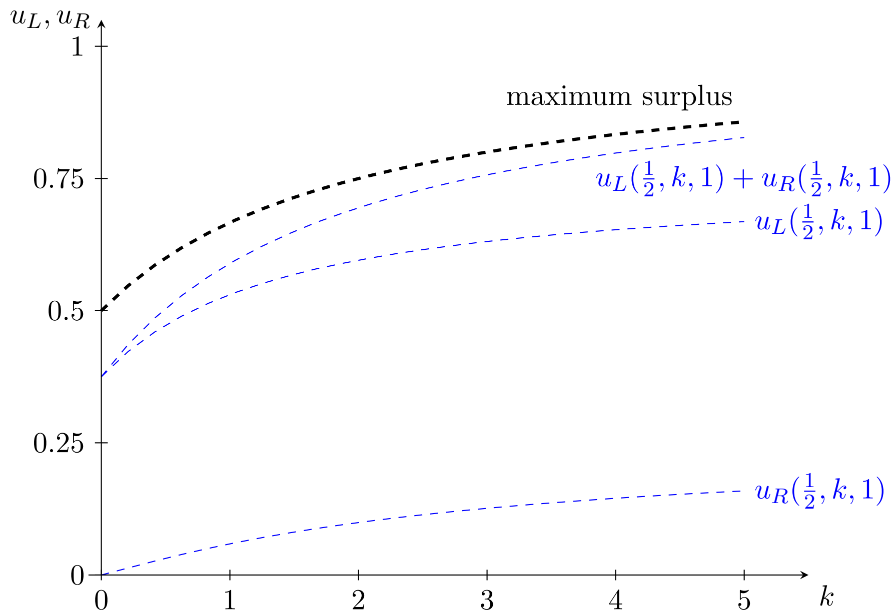

Figure 2: Payoffs for the scenario in which the researchers choose attributes for $\theta = \frac{1}{2}$ and $\rho = \lambda = 1$. Since researcher (respectively, laboratory) payoffs for index ρ (resp., λ) are proportional to ρ^2 (resp., λ^2), these also represent the proportionality factors for the other indices. The maximum surplus is $(k+1)/(k+2)$.

heterogeneous, and hence dampens their competition for laboratories, seemingly to the latter's deficit. However, this is outweighed by the enhanced researcher investment incentives of increasing k.

In the limit, when $k = 0$, all researchers are identical, giving rise to fierce competition that allows laboratories to capture all the surplus:

$$\begin{aligned} & \lim_{k \to 0} u_R(\theta, k, \rho) = 0 \\ \text{and} \quad & \lim_{k \to 0} u_L(\theta, k, \lambda) = \tfrac{\theta}{2}(2 - \theta)\lambda^2. \end{aligned}$$

At the other extreme, as $k \to \infty$, researchers have increasingly different values for any particular laboratory, dampening their competition. We then get efficient attribute choices, but the premuneration values still matter in terms of the division:

$$\lim_{k\to\infty} u_R(\theta, k, \rho) = \frac{\theta\rho^2}{2}$$

$$\text{and} \qquad \lim_{k\to 0} u_L(\theta, k, \lambda) = \left(1 - \frac{\theta}{2}\right)\lambda^2.$$

As k increases without bound, investments costs become insignificant. However, the convergence to an efficient outcome as $k \to \infty$ is not driven by the vanishing investment cost: Appendix B exhibits a cost function for which costs do not vanish as k gets large, but the equilibrium nonetheless approaches efficiency.

4 Laboratories Invest

Researchers' investments are inefficiently low when laboratory premuneration values are not degenerate. Does the inefficiency arise because researchers are choosing their unobserved attributes, or does it persist if their unobserved attributes are exogenous? To understand the source and nature of the inefficiency we next keep the information structure the same (laboratories attributes are commonly known but researchers' attributes are not), but have laboratories, rather than researchers, choose attributes.

We show that investments are again inefficient, though the forces behind this inefficiency are quite different from those in the case of researcher investments. Hence, it is the *unobservability* of the attributes that causes inefficient investments.

4.1 Market Equilibrium

As before, matching takes place in a competitive market, with laboratory attributes observable and priced. We use a superscript * to distinguish the prices, attribute choices and payoffs here from their analogs in Sections 2 and 3.

Researcher ρ has attribute $r = \rho$, so that researcher attributes are uniformly distributed on the unit interval. Given the price function p^*, researcher ρ chooses ℓ to maximize

$$\theta\ell\rho - p^*(\ell).$$

We denote by $\ell_R^* : [0,1] \to \mathbb{R}_+$ the function describing the laboratory attribute selected by researchers.

The cost of attribute $\ell \in \mathbb{R}_+$ to laboratory λ is

$$c(\ell, \lambda) = \frac{\ell^{2+k}}{(2+k)\lambda^k}, \qquad k \in \mathbb{R}_+.$$

Laboratories choose attributes given (p^*, r_L^*), where $r_L^* : \mathbb{R}_+ \to [0,1]$ is the *matching function* that specifies the attribute $r_L^*(\ell)$ of the researcher that the market matches to a laboratory with attribute ℓ. Laboratory λ chooses $\ell \in \mathbb{R}_+$ to maximize

$$(1-\theta)\ell r_L^*(\ell) + p^*(\ell) - c(\ell, \lambda).$$

We denote by $\ell_L^* : [0,1] \to \mathbb{R}_+$ the function describing the laboratories' attribute choices.[13]

Definition 2 *A price function p, matching function r_L^*, and strictly increasing laboratory attribute choices (ℓ_L^*, ℓ_R^*) constitute a* matching equilibrium *if*

1. *$\ell_R^*(\rho)$ is an optimal laboratory attribute for researcher ρ, for all $\rho \in [0,1]$,*
2. *$\ell_L^*(\lambda)$ is an optimal laboratory attribute for laboratory λ, for all $\lambda \in [0,1]$,*
3. *every researcher and laboratory earns nonnegative payoffs, and*
4. *markets clear: $r_L^*(\ell_R^*(\rho)) = \rho$ for all $\rho \in [0,1]$ and $\ell_R^*(\lambda) = \ell_L^*(\lambda)$ for all $\lambda \in [0,1]$.*

Before we describe the equilibrium, we note that since the laboratory cost function is the same functional form as the earlier researcher cost function, the efficient attribute choice for the laboratories is $\ell_L^*(\lambda) = \lambda$.

[13] As in the initial model, we are able to avoid many technical details. In particular, our notion of equilibrium assumes that ℓ_R^* and ℓ_L^* are strictly increasing; these properties can be deduced from the general model of Mailath, Postlewaite, and Samuelson (2013). Given these assumptions, market clearing requires $r_L^*(\ell_R^*(\rho)) = \rho$ and $\ell_R^*(\lambda) = \ell_L^*(\lambda)$.

In the equilibrium we analyze, the range of ℓ_R is an interval starting at 0, and so we need place no further restrictions on r_L^* (though setting $r_L^*(\ell) = 1$ for $\ell > \ell_R(1)$ would be natural). A central concern of Mailath, Postlewaite, and Samuelson (2013) is the appropriate treatment of matches when an attribute is chosen outside the range of putative equilibrium attributes and the set of such attributes does not form an interval.

Proposition 4 *A matching equilibrium is given by the collection* $(p^*, r_L^*, \ell_R^*, \ell_L^*)$, *where*

$$\begin{aligned} p^*(\ell) &= \frac{\theta\ell^2}{2\alpha}, \quad \textit{for } \ell \in \mathbb{R}_+, \\ r_L^*(\ell) &= \ell/\alpha, \quad \textit{for } \ell \in [0,\alpha], \\ \ell_L^*(\lambda) &= \ell_R^*(\lambda) = \alpha\lambda, \quad \textit{for } \lambda \in [0,1], \end{aligned}$$

where $\alpha := (2-\theta)^{\frac{1}{1+k}} \geq 1$. *Laboratory payoffs are given by*

$$u_L^*(\theta, k, \lambda) = \frac{k}{2(2+k)}(2-\theta)^{(2+k)/(1+k)}\lambda^2. \tag{10}$$

Researcher payoffs are given by

$$u_R^*(\theta, k, \rho) = \frac{1}{2}\theta(2-\theta)^{1/(1+k)}\rho^2. \tag{11}$$

We leave the proof to Appendix A as it is very similar to those in the model above.

If $\theta < 1$, then $\alpha > 1$ and laboratories overinvest relative to the efficient level. The private nature of researchers' attributes again distorts investment incentives, but for a very different reason when it is laboratories that invest. When researchers invest, they have an incentive to underinvest since they will not get the full return on their unobservable investment. Here, laboratories' investments *are* observable, and hence are not the source of the inefficiency.

Laboratories overinvest because of researchers' response to their investments. Consider laboratory λ's equilibrium investment. It is higher than the efficient level, so why doesn't the laboratory decrease its investment?

In the calculation of the efficient investment level, we know that an efficient outcome must match agents assortatively on index. If a laboratory's investment is too high, we can decrease the investment *keeping the matching fixed*, and thereby increase the surplus. In contrast, in the market equilibrium, a laboratory that decreased its investment level from the equilibrium level would find that the researcher's attribute that the laboratory is matched with decreases. It is this concern for the quality of the researcher (which it doesn't observe) with whom it is matched that makes it optimal for laboratories to invest more than the efficient level.

The intuition for laboratories' overinvestment is quite general, as long as laboratories' premuneration values increase with the attribute of their matched partner. Laboratories want higher-attribute researchers, and are willing to pay for them. But when they cannot directly observe researchers' attributes, they cannot simply pay for higher-attribute researchers by accepting lower prices to match, since that would be equally attractive to all researchers. But they can increase the attractiveness to matched partners by investing more. This makes a laboratory more attractive to all researchers, but more so for higher attribute researchers. Hence, a laboratory can combine an increase in their attribute with an increase in their price that will screen potential researchers so that only higher attribute researchers will find the combination attractive.

Unlike the researcher-investment case, though, the comparative statics of equilibrium payoffs with respect to the researchers' premuneration values are unremarkable: Researcher payoffs are increasing and laboratory payoffs are decreasing in θ.

4.2 The Impact of Competition on Payoffs

In Section 3.4, we discussed the impact of changes in researcher heterogeneity on competition and on payoffs. Here, we compare the effect of laboratory heterogeneity with that of researcher heterogeneity.

The equilibrium payoffs of laboratories and researchers are given by (8)–(9) (for the researcher-investment case) and (10)–(11) (for the laboratory-investment case). Figure 3 reproduces the payoffs from Figure 2 for the investing researchers case, adding the payoffs for the investing laboratories case as a function of k.

Researchers' payoffs increase in k when researchers make the investments, reflecting the enhanced investment incentives of reduced competition and reduced investment costs. Researchers' payoffs decrease in k when laboratories make the investments, as the reduced researcher competition leads to small laboratory investments.

Laboratories' payoffs under either scenario increase as k increases. A larger value of k makes researchers less homogeneous, and hence dampens their competition for laboratories, seemingly to the latter's deficit. However, this is outweighed by the enhanced researcher investment incentives when k increases (when researchers make investments), and the reduced cost of investments (when laboratories make investments).

In the limit, when $k = 0$, all agents on the endogenous-attribute side are identical, and so these agents are effectively perfectly competitive. As the endogenous-attribute side becomes perfectly homogeneous, the other side

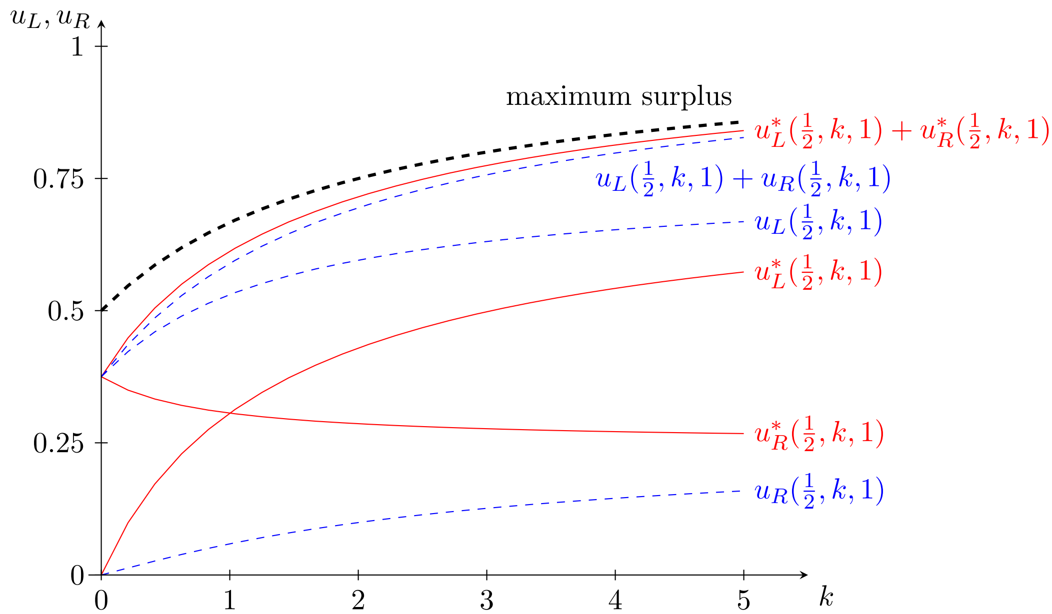

Figure 3: Payoffs for the scenario in which the researchers choose attributes (u_L and u_R) and when the laboratories choose attributes (u_L^* and u_R^*), for $\theta = \frac{1}{2}$ and $\rho = \lambda = 1$. Since researcher (respectively, laboratory) payoffs for index ρ (resp., λ) are proportional to ρ^2 (resp., λ^2), these also represent the proportionality factors for the other indices. The maximum surplus is $(k+1)/(k+2)$.

captures all the surplus:

$$\begin{aligned}
&\lim_{k\to 0} u_R(\theta, k, \rho) = 0, \\
&\lim_{k\to 0} u_L(\theta, k, \lambda) = \tfrac{\theta}{2}(2-\theta)\lambda^2, \\
&\lim_{k\to 0} u_R^*(\theta, k, \rho) = \tfrac{\theta}{2}(2-\theta)\rho^2, \\
\text{and} \qquad &\lim_{k\to 0} u_L^*(\theta, k, \lambda) = 0.
\end{aligned}$$

Interestingly, while the outcome is inefficient and the division of the surplus depends on the assignment of premuneration values, the extent of the inefficiency is independent of the side with endogenously determined attributes.

The other extreme is the limit as $k \to \infty$, i.e., when the endogenous attribute side becomes noncompetitive (and the cost of the attribute becomes negligible). In that case, we get efficient attribute choices in both scenarios, but the premuneration values still matter in terms of the division:

$$\lim_{k\to\infty} u_R(\theta,k,\rho) = \lim_{k\to 0} u_R^*(\theta,k,\rho) = \frac{\theta\rho^2}{2},$$

$$\text{and} \qquad \lim_{k\to 0} u_L(\theta,k,\lambda) = \lim_{k\to 0} u_L^*(\theta,k,\lambda) = \left(1-\frac{\theta}{2}\right)\lambda^2.$$

5 Endogenizing Information

Our analysis so far has assumed that laboratories could not learn researchers' attributes. This section returns to the researcher-investment case but allows laboratories to learn the attributes of researchers at a cost. We will see that changes in premuneration values can have surprising effects on which laboratories become informed and on the resulting division of the surplus.

We suppose that, by incurring a cost $\kappa > 0$, any given laboratory can acquire the ability to observe the attribute of each researcher. We can think of κ as the cost of hiring an agent who can test any applicant or the cost of installing a testing procedure. Assume that laboratories make their decisions of whether to become informed and researcher choose their investments simultaneously.[14]

If κ is sufficiently large, the gain in efficiency would not warrant a laboratory incurring the cost to become informed. On the other hand, for κ small, it is generally not an equilibrium for all laboratories to remain uninformed. To illustrate, suppose all laboratories are uninformed and that researchers choose attributes according to (6). If a laboratory deviates by becoming informed, it then can target any available researcher attribute, i.e., any attribute in the set $[0, r_R(1)] = [0,\ \theta^{1/(1+k)}]$. Suppose a laboratory $\lambda < \theta^{\frac{1}{1+k}}$ with (by assumption) $\ell = \lambda$ becomes informed and then offers a price p to the researcher with attribute $r = \ell$, i.e., to a researcher of type $\rho = \lambda\theta^{-1/(1+k)}$.[15] Since the price is simply a transfer between the two agents, such an offer is a profitable deviation if and only if the surplus generated by the resulting match, $\ell r - \kappa$ exceeds the ex post equilibrium

[14] This ensures that a laboratory cannot induce a change in researcher investment behavior by deciding to become informed.

[15] The bound $\lambda < \theta^{1/(1+k)}$ ensures that $r = \ell$ is feasible, i.e., $r < \theta^{1/(1+k)}$.

payoffs of the two agents (using (8)-(9) for the first equality):

$$\begin{aligned}
&u_L(\theta, k, \lambda) + u_R(\theta, k, \lambda\theta^{-1/(1+k)}) + c(\lambda, \lambda\theta^{-1/(1+k)}) \\
&\qquad = \left[\frac{1}{2}\theta^{1/(1+k)}(2-\theta) + \frac{k}{2(2+k)}\theta^{k/(1+k)} + \frac{1}{(2+k)}\theta^{k/(1+k)}\right]\lambda^2 \\
&\qquad = \frac{\theta^{1/(1+k)}}{2}\left[2-\theta+\theta^{(k-1)/(1+k)}\right] =: g(\theta)\lambda^2.
\end{aligned}$$

A straightforward calculation verifies the inequality $g(\theta) < 1$ for all interior θ (in particular, $g(1) = 1$, $g'(1) = 0$ and g is concave).

Thus for $\kappa > 0$ but not too large, in equilibrium, some laboratories will choose to become informed. However, it is clear that not *all* laboratories will choose to become informed, since laboratories with types near 0 cannot under any circumstance generate sufficient surplus to cover the cost κ.

A natural hypothesis is that for positive but not too large κ, there will be a *hybrid* equilibrium characterized by a threshold $\widetilde{\lambda}$ with laboratories $\lambda > \widetilde{\lambda}$ incurring the cost to become informed and laboratories with $\lambda < \widetilde{\lambda}$ not incurring the cost.

In such an equilibrium, informed laboratories are priced by a function $\hat{p} : [\widetilde{\lambda}, 1] \times \mathbb{R}_+ \to \mathbb{R}_+$, where $\hat{p}(\ell, r)$ is the price paid by researcher of attribute r to laboratory of attribute ℓ. We extend $\hat{p}$ to $[0,1] \times \mathbb{R}_+$ to cover uninformed laboratories by requiring $\hat{p}$ to be independent of r for $\ell < \tilde{\lambda}$. Researcher ρ maximizes

$$\max_{\ell, r} \theta\ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{12}$$

Definition 3 *A price function $\hat{p}$, cutoff $\tilde{\lambda} \in [0,1]$, and researcher choices (ℓ_R, r_R) constitute a* hybrid equilibrium *if*

1. *for every $\rho \in [0,1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves (12),*
2. *for every $\ell \in [0, \tilde{\lambda}]$, for all r and r', $\hat{p}(\ell, r) = \hat{p}(\ell, r')$,*
3. *no laboratory $\lambda \in [1, \tilde{\lambda})$ strictly prefers to be informed at a cost of κ,*
4. *no laboratory $\lambda \in [\tilde{\lambda}, 1]$ strictly prefers to be uninformed,*
5. *every researcher and laboratory earns nonnegative payoffs, and*
6. *ℓ_R is market-clearing.*

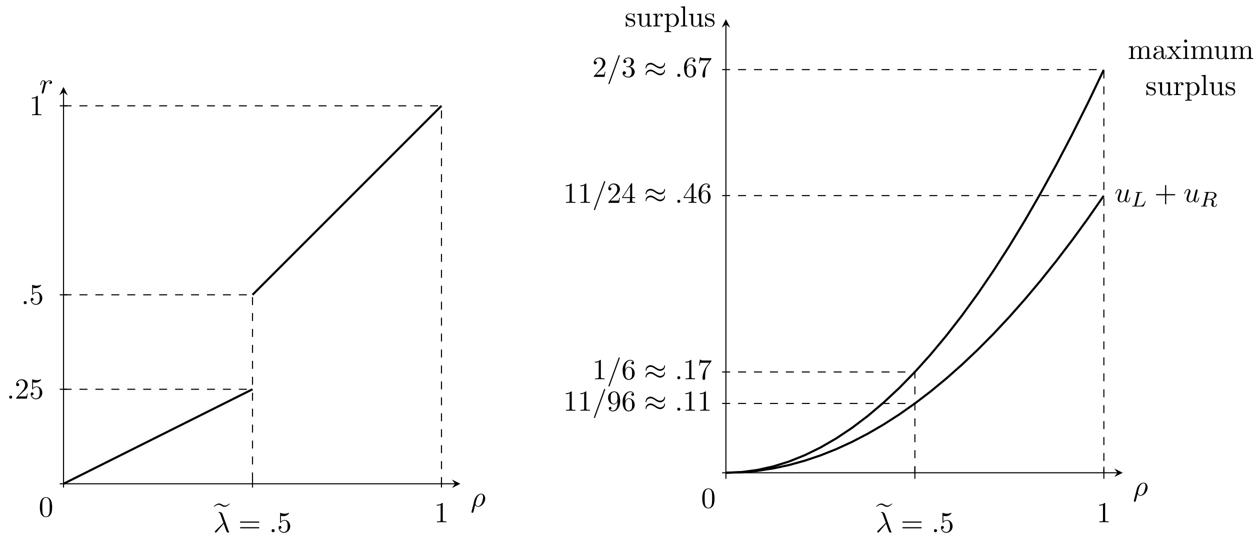

Figure 4: The researcher attribute choice function for the case $k = 1$, $\kappa = \frac{5}{96}$, $\theta = \frac{1}{4}$, and $\widetilde{\lambda} = \frac{1}{2}$ is illustrated on the left. At $\widetilde{\lambda} = \frac{1}{2}$, the efficiency gain from efficient investments equals $\frac{5}{96}$.

Intuitively, higher type researchers choose higher attributes, and match with higher attribute laboratories. Market clearing then implies that researchers $\rho \in [\tilde{\lambda}, 1]$ match with informed laboratories and so choose efficient investments.

We present here a hybrid equilibrium for the case in which $\kappa = \frac{5}{96}$, $\theta = \frac{1}{4}$ and $k = 1$ (thus $\theta^{\frac{1}{1+k}} = \frac{1}{2}$), and with switch point $\widetilde{\lambda} = \frac{1}{2}$, and then examine its comparative statics. See Appendix C for the analysis of general parameter values that underlies our discussion here.

The left panel of Figure 4 shows the researchers' investment levels, which jump at $\widetilde{\lambda}$ as researchers switch from the investments appropriate for matching with uninformed laboratories (described in (6)) to the efficient levels appropriate for matching with informed laboratories. Despite this discontinuity in investments, the payoffs of both researchers and laboratories must be continuous as their indices move across index $\widetilde{\lambda}$, since otherwise an agent just on the low-payoff side of $\widetilde{\lambda}$ would have an incentive to make the same investment as that of an agent just on the other side (high-payoff) of $\widetilde{\lambda}$. This joint indifference implies that at the switch point $\widetilde{\lambda}$ the gain in surplus equals the cost κ of becoming informed. Figure 4 (right panel) shows that the threshold pair $\tilde{\lambda} = \frac{1}{2}$ gives an efficiency gain of $\frac{5}{96}$, which equals the assumed value of κ.

The equilibrium price function is given by

$$\hat{p}(\ell, r) = \begin{cases} \frac{1}{16}\ell^2, & \text{if } \ell < \frac{1}{2}, \\ \frac{1}{2}\ell^2 - \frac{3}{4}r\ell + \frac{7}{192}, & \text{if } \ell \geq \frac{1}{2}. \end{cases}$$

For $\ell < \frac{1}{2}$, $\hat{p}(\ell, r)$ is given by (7), while for $\ell \geq \frac{1}{2}$, the price function is determined by the requirement that payoffs are continuous at $\frac{1}{2}$ and that efficient investments are optimal for the high index researchers. A researcher choosing an uninformed laboratory $\ell = \frac{1}{2}$ pays a price of $\frac{1}{64}$. The price paid by a researcher choosing $r = \frac{1}{2}$ to an informed laboratory $\ell = \frac{1}{2}$ is lower, taking the *negative* value of $-\frac{5}{192}$, compensating the researcher for the upward jump in investment from $\frac{1}{4}$ to $\frac{1}{2}$. Figure 5 illustrates the resulting payoff functions, which have a kink but not a discontinuity at $\frac{1}{2}$.

If the fixed cost of information κ decreased, the threshold $\widetilde{\lambda}$ that determines which laboratories decide to invest would decrease, until the net surplus increase that is a consequence of the threshold laboratory's becoming informed again equals κ.

More interesting is the role of premuneration values in determining who becomes informed, and the resulting payoffs. As θ decreases, researchers' investments decrease, and hence the inefficiency associated with any matched pair increases. The threshold for laboratories to become informed must then decrease, in order for the gain from becoming informed to be equal to κ. Hence, the extent of information acquisition increases as the researchers' premuneration value share decreases.

Not only does the threshold change in response to changes in θ, but the division of the surplus between laboratories and researchers is affected. If all laboratories are informed (such as would arise if $\kappa = 0$), investments are efficient and laboratory and researcher payoffs are *independent* of θ. In contrast, when $\kappa > 0$, as illustrated in Figure 5, the premuneration values affect the location of the threshold, and so affect all agents' payoffs, including those involving fully informed laboratories. For example, under the lower premuneration value share of $\theta = \frac{1}{9}$, all researchers matched with uninformed laboratories have a lower payoff than under $\theta = \frac{1}{4}$. However, all researchers matched with informed laboratories under $\theta = \frac{1}{4}$ are strictly better off under the lower premuneration value share of $\theta = \frac{1}{9}$. Moreover, all laboratories prefer the scenario of the higher researcher premuneration value share of $\frac{1}{4}$.

Finally, hybrid equilibria do not exist for all parameters, and in particular do not exist if researchers' premuneration values are too large. If we fix κ,

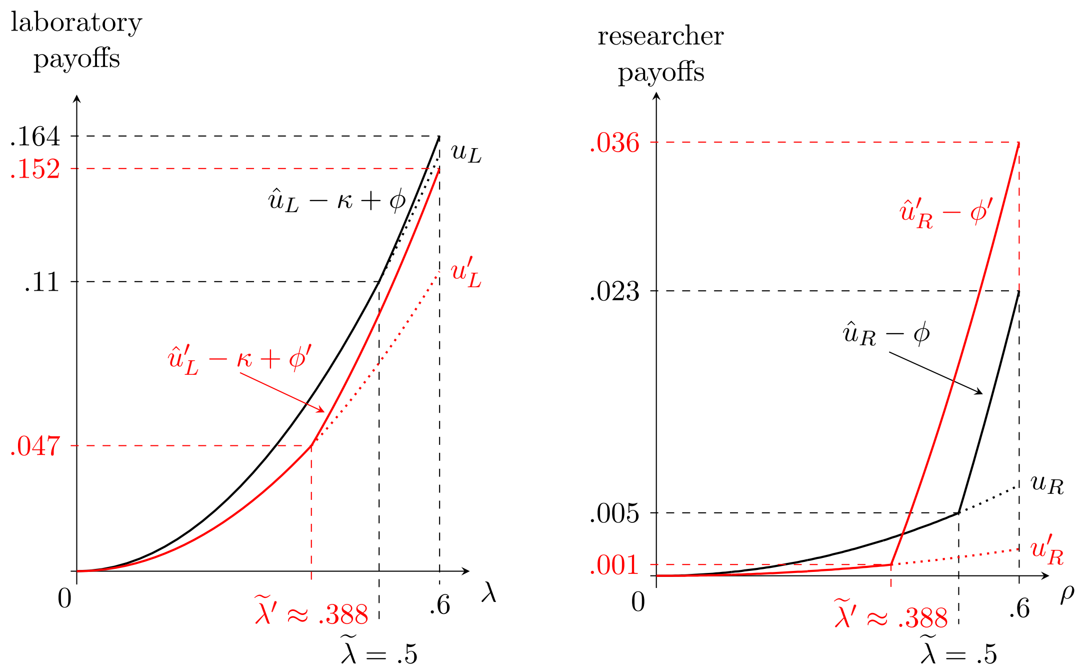

Figure 5: Payoffs in the hybrid equilibrium for the case $k = 1$, for $\lambda \leq .6$. The cost of becoming informed is $\kappa = \frac{1}{6} - \frac{11}{96} = \frac{5}{96}$. Two values of θ are illustrated, $\theta = \frac{1}{4}$ (which implies $\widetilde{\lambda} = \frac{1}{2}$) and $\theta = \frac{1}{9}$ (which implies $\widetilde{\lambda}' \approx .388$). The expressions for $\theta = \frac{1}{9}$ are indicated by a prime. For λ below $\widetilde{\lambda}$, laboratory payoffs are given by u_L, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_L - \kappa + \phi$. For ρ below $\widetilde{\lambda}$, researcher payoffs are given by u_R, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_R - \phi$. The constant in the price function (C.3) is $\phi = \frac{7}{192}$ for $\theta = \frac{1}{4}$, and $\phi' = \frac{65}{2688} \approx .024$ for $\theta = \frac{1}{9}$.

laboratories close to $\lambda = 1$ will have vanishingly small possible gains from acquiring information as θ goes to 1, and hence will choose *not* to become informed.

A Appendix: Proofs

Proof of Lemma 1. En route to a contradiction, suppose p is not strictly increasing. Then there are two laboratories $\ell' < \ell$ satisfying $p(\ell') \geq p(\ell)$. But then no researcher will choose laboratory ℓ' — why pay just as much or more for an inferior laboratory? Hence, ℓ_R cannot be market clearing. Continuity similarly follows from the observation that if the function p takes an upward jump at ℓ', then there will be an interval of laboratories $(\ell', \ell' + \varepsilon)$ that will be unchosen by researchers, again contradicting our assumption that ℓ_R is market clearing. ■

Proof of Lemma 2. We first argue that r_R is weakly increasing. Suppose not, so that there exist researchers $\hat{\rho} > \rho$ such that $\hat{r} = r_R(\hat{\rho}) < r_R(\rho) = r$. Since researchers are optimizing in their attribute and laboratory choices,

$$\theta \ell_R(\rho) r - p(\ell_R(\rho)) - \frac{r^{2+k}}{(2+k)\rho^k} \;\geq\; \theta \ell_R(\hat{\rho}) \hat{r} - p(\ell_R(\hat{\rho})) - \frac{\hat{r}^{2+k}}{(2+k)\rho^k}$$

and

$$\theta \ell_R(\hat{\rho}) \hat{r} - p(\ell_R(\hat{\rho})) - \frac{\hat{r}^{2+k}}{(2+k)\hat{\rho}^k} \;\geq\; \theta \ell_R(\rho) r - p(\ell_R(\rho)) - \frac{r^{2+k}}{(2+k)\hat{\rho}^k},$$

which can be added to give

$$\frac{r^{2+k}}{(2+k)\rho^k} + \frac{\hat{r}^{2+k}}{(2+k)\hat{\rho}^k} \;\leq\; \frac{\hat{r}^{2+k}}{(2+k)\rho^k} + \frac{r^{2+k}}{(2+k)\hat{\rho}^k},$$

a contradiction.

We now argue that r_R is strictly increasing. If r_R is not strictly increasing, there exist $\hat{\rho} > \rho$ such that $\ell_R(\hat{\rho}) = \hat{\ell} > 0$ and $\hat{r} = r_R(\hat{\rho}) = r = r_R(\rho)$, and so $\hat{\ell}$ is an optimal choice for both $\hat{\rho}$ and ρ at r. Since ℓ_R is market clearing, we can assume $\ell_R(\hat{\rho}) = \hat{\ell} > 0$. But this implies that ρ's choice of $r = \hat{r}$ must be optimal given $\hat{\ell}$. But this is impossible (since the marginal cost of attributes is strictly decreasing in ρ). ■

Proof of Lemma 3. We first argue that in equilibrium, the researcher laboratory-choice function is strictly increasing. Let $\hat{\rho} > \rho$ and hence, from

Lemma 2, $\hat{r} = r_R(\hat{\rho}) > r_R(\rho) = r$. We need to show that $\hat{\ell} = \ell_R(\hat{\rho}) > \ell_R(\rho) = \ell$. Suppose this is not the case. Then since researchers with attributes r and $\hat{r}$ are optimizing in their choice of laboratories, we have

$$\begin{aligned} \theta\ell r - p(\ell) &\geq \theta\hat{\ell} r - p(\hat{\ell}) \\ \text{and} \qquad \theta\hat{\ell}\hat{r} - p(\hat{\ell}) &\geq \theta\ell\hat{r} - p(\ell), \end{aligned}$$

which can be added to give

$$\ell r + \hat{\ell}\hat{r} \geq \hat{\ell} r + \ell\hat{r},$$

which is impossible if $\hat{\ell} \leq \ell$.

The conclusion of the lemma then follows from equilibrium ℓ_R being a strictly increasing and measure-preserving map from $[0,1]$ onto $[0,1]$: Fixing $\rho \in [0,1]$, and recalling footnote 9, we have $\mu\{\ell | \ell = \ell_R(\hat{\rho}) \text{ for } \hat{\rho} \in [0,\rho]\} = \mu([0,\ell_R(\rho)]) = \ell_R(\rho)$, and so $\rho = \mu([0,\rho]) = \ell_R(\rho)$. ■

Proof of Proposition 4. We first note that, under the specified price function, the researcher chooses ℓ to maximize

$$\theta\ell\rho - \frac{\theta\ell^2}{2\alpha},$$

so that

$$\ell = \alpha\rho,$$

which is the hypothesized form of ℓ_R^*.

Market clearing is immediate. It remains to confirm the optimality of laboratory behavior. Laboratory λ chooses ℓ to maximize

$$(1-\theta)\frac{\ell^2}{\alpha} + \frac{\theta\ell^2}{2\alpha} - \frac{\ell^{2+k}}{(2+k)\lambda^k}.$$

The first order condition is

$$2(1-\theta)\frac{\ell}{\alpha} + \frac{\theta\ell}{\alpha} - \frac{\ell^{1+k}}{\lambda^k} = 0,$$

implying

$$\ell = \left(\frac{2-\theta}{\alpha}\right)^{1/k}\lambda,$$

which equals $\alpha\lambda$ when $\alpha = (2-\theta)^{\frac{1}{1+k}}$. ■

B Appendix: A More General Cost Function

B.1 The Cost Function

We showed in the text that for the researcher-investment case, investments converged to efficient levels when $k \to \infty$, that is, when competition among researchers vanishes. For the cost function we employ for our main analysis, investment costs become insignificant, and one might think that it is the vanishing investment cost that underlies the convergence to efficiency. We demonstrate in this section that it is not necessary for investment costs to vanish for convergence to efficient investments. We do this by examining a more general class of cost functions, and show that investments converge to efficient levels as $k \to \infty$ even when costs do not vanish.

Consider the family of cost functions given by

$$c(a, \sigma) = \frac{\beta a^{2+k}}{\sigma^k},$$

where a is either r or ℓ and σ is correspondingly either ρ or λ, and $\beta \in \mathbb{R}_+$ may be a function of k. For the model in the main body of the paper,

$$\beta = (2 + k)^{-1}.$$

We now examine below a subfamily of this general class of cost functions with

$$\beta = \gamma^{-(1+k)}.$$

We first note that for the general class of cost functions with $c(a, \sigma) = \frac{\beta a^{2+k}}{\sigma^k}$, maximizing the net surplus requires, as in (2), matching indices and then, for each σ, choosing a to maximize

$$\sigma a - \frac{\beta a^{2+k}}{\sigma^k}.$$

Letting

$$\eta = [\beta(2 + k)]^{-1/(1+k)},$$

the maximizing value of a is $\eta\sigma$, and the maximized value of the surplus is $(1 + k)\eta\sigma^2/(2 + k)$.

If $\beta = (2 + k)^{-1}$, as in the cost functions in the body of the paper, then $\eta = 1$ for all k. and we recover the efficient investments (cf. (3)). If instead $\beta = \gamma^{-(1+k)}$, as the class of cost function we examine below,

$$\begin{aligned}
\eta &= [\beta(2 + k)]^{-1/(1+k)} \\
&= \gamma(2 + k)^{-1/(1+k)} \\
&\to \gamma \quad \text{as} \quad k \to \infty,
\end{aligned}$$

since

$$\lim_{k\to\infty}(2+k)^{-1/(1+k)} = \exp\left\{\lim_{k\to\infty}\tfrac{-1}{1+k}\log(2+k)\right\} = e^0 = 1.$$

B.2 Equilibrium

We now calculate the equilibrium of the researcher-invests model for this class of cost functions. Lemmas 1–3 hold for this class of cost functions (the same proofs are valid), and we need only determine the equilibrium price function p and researcher attribute function r_R. It is straightforward to verify that the following describes a matching equilibrium:

$$p(\ell) = \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\ell^2$$

and

$$r_R(\rho) = \eta\theta^{1/(1+k)}\rho.$$

Payoffs are given by

$$\begin{aligned} u_R(\theta,k,\rho) &= \theta\eta\theta^{1/(1+k)}\rho^2 - \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2 - \frac{\beta(\eta\theta^{1/(1+k)}\rho)^{2+k}}{\rho^k} \\ &= \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2\left\{1-2\beta\eta^{1+k}\right\} \\ &= \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2\left\{1-\frac{2}{2+k}\right\} \\ &= \frac{k\eta\theta^{(2+k)/(1+k)}\rho^2}{2(2+k)} \end{aligned} \tag{B.1}$$

and

$$\begin{aligned} u_L(\theta,k,\lambda) &= (1-\theta)\frac{\eta\theta^{(2+k)/(1+k)}\lambda^2}{\theta} + \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\lambda^2 \\ &= \frac{1}{2}\eta\theta^{1/(1+k)}\lambda^2(2-2\theta+\theta) \\ &= \frac{1}{2}\eta\theta^{1/(1+k)}\lambda^2(2-\theta). \end{aligned} \tag{B.2}$$

For the case in which laboratories invest, a straightforward argument analogous to the proof of Proposition 4 shows that the functions given in that proposition, namely

$$p^*(\ell) = \frac{\theta\ell^2}{2\alpha}, \quad \text{for } \ell \in \mathbb{R}_+, \tag{B.3}$$

$$r_L^*(\ell) = \ell/\alpha, \quad \text{for } \ell \in [0,\alpha], \tag{B.4}$$

$$\ell_L^*(\lambda) = \ell_R^*(\lambda) = \alpha\lambda, \quad \text{for } \lambda \in [0,1], \tag{B.5}$$

constitute an equilibrium once the definition of α is changed to

$$\alpha = \eta(2-\theta)^{1/(1+k)} = \left(\frac{2-\theta}{\beta(2+k)}\right)^{1/(1+k)}.$$

B.3 The Effects of Competitiveness

The following proposition shows that the limit efficiency of the two scenarios (as k becomes large) is *not* due to the negligibility of costs in the limit.

Proposition B.1 *Suppose the cost to agent $\sigma \in (0,1]$ of choosing attribute $a \in \mathbb{R}_+$ is given by*

$$c(a,\sigma) = \frac{a^{2+k}}{\gamma^{1+k}\sigma^k},$$

where $\gamma \in (0,1)$. The limit (as $k \to \infty$) maximum surplus is $\gamma\sigma^2$. Then,

$$\lim_{k\to\infty} u_R(\theta,k,\rho) = \lim_{k\to 0} u_R^*(\theta,k,\rho) = \frac{\gamma\theta\rho^2}{2},$$

$$\textit{and} \qquad \lim_{k\to 0} u_L(\theta,k,\lambda) = \lim_{k\to 0} u_L^*(\theta,k,\lambda) = \gamma\left(1-\frac{\theta}{2}\right)\lambda^2.$$

The results for the case in which researchers invest follow from (B.1)–(B.2), while those for laboratory investments can be calculated from (B.3)–(B.5).

C Appendix: Endogenizing Information

This section presents the calculations behind the hybrid equilibrium of Section 5. Let

$$\bar{\kappa}(\theta) := \frac{1}{(2+k)}\left[1+k-(2+k)\theta^{1/(1+k)} + \theta^{(2+k)/(1+k)}\right].$$

Since $\bar{\kappa}(1) = 0$ and $\kappa'(\theta) < 0$, we have $\bar{\kappa}(\theta) > 0$ for all $\theta \in [0,1)$.

Proposition C.1 *Suppose*

$$2(2-\theta)\bar{\kappa}(\theta) > \theta^{1/(1+k)}(1-\theta)^2. \tag{C.1}$$

For any $\kappa \in (\theta^2\bar{\kappa}(\theta), \bar{\kappa}(\theta))$*, satisfying*

$$\frac{1}{2} \geq \kappa\left[1 - \frac{1}{2\bar{\kappa}(\theta)}\right] + \sqrt{\frac{\kappa}{\bar{\kappa}(\theta)}}, \tag{C.2}$$

there exists a hybrid equilibrium with switch point

$$\widetilde{\lambda} = \sqrt{\frac{\kappa}{\bar{\kappa}(\theta)}}.$$

For all θ, the condition (C.1) fails for sufficiently large k. Since, $\lim_{k\to\infty}\bar{\kappa}(\theta) = 0$, as researcher heterogeneity becomes large (and so researcher choices become efficient), the critical cost of becoming informed must converge to zero. If $k=1$, then condition (C.1) simplifies to

$$f(\theta) := 8 - 4\theta + \theta^{1/2}\left\{16\theta - 15 - 5\theta^2\right\} > 0.$$

The function f has one root $\tilde{\theta} \approx 0.629$ in the open interval $(0,1)$, with $f(\theta) > 0$ for $\theta < \tilde{\theta}$ and $f(\theta) < 0$ for $\theta > \tilde{\theta}$.

For $k = 1$ and $\theta = \frac{1}{4}$, $\bar{\kappa}(\frac{1}{4}) = \frac{5}{24}$, and so $\kappa = \frac{5}{96}$ is in the interval $(\theta^2\bar{\kappa}(\theta), \bar{\kappa}(\theta))$ and implies $\widetilde{\lambda} = \frac{1}{2}$. These parameter values satisfy $\widetilde{\lambda} > \theta$, (C.1) and (C.2).

C.1 Informed Laboratories

This subsection characterizes the behavior and payoffs for the laboratories that are informed and the researchers with whom they match. We are interested in the case in which laboratories with indices in the interval $[\widetilde{\lambda}, 1]$ are informed, and (in equilibrium) match with researchers with the same set of indices. In this subsection, we accordingly suppose that researcher and laboratory indices are uniformly distributed on the interval $[\widetilde{\lambda}, 1]$.

For appropriate values of ϕ, the price function

$$\hat{p}(\ell, r) = \phi + \frac{\ell^2}{2} - (1-\theta)\ell r \tag{C.3}$$

will clear markets with researcher ρ choosing the efficient $\ell = \rho$ and $r = \rho$. In particular, researcher ρ's payoff from ℓ and r is

$$\theta\ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k} = \ell r - \phi - \frac{1}{2}\ell^2 - \frac{r^{2+k}}{(2+k)\rho^k}.$$

Maximizing the payoff yields $\ell = \rho$ and $r = \rho$ (the efficient choices), and a payoff value of

$$\frac{k}{2(2+k)}\rho^2 - \phi =: \hat{u}_R(\theta, k, \rho) - \phi.$$

Laboratory payoffs under $\hat{p}$ are given by

$$\frac{1}{2}\lambda^2 + \phi =: \hat{u}_L(\theta, k, \lambda) + \phi.$$

Note that the payoff functions $\hat{u}_R$ and $\hat{u}_L$ are defined to exclude the ϕ surplus reallocation. Moreover,

$$\hat{u}_R(\theta, k, \rho) + \hat{u}_L(\theta, k, \rho) - (u_R(\theta, k, \rho) + u_L(\theta, k, \rho)) = \bar{\kappa}(\theta)\rho^2. \tag{C.4}$$

If $\widetilde{\lambda} = 0$, then individual rationality implies $\phi = 0$, and the equilibrium is unique. If $\widetilde{\lambda} > 0$, there is a one parameter family of equilibrium price functions, indexed by

$$\phi \in [-\hat{u}_L(\theta, k, \widetilde{\lambda}),\ \hat{u}_R(\theta, k, \widetilde{\lambda})] = \left[\frac{-\widetilde{\lambda}^2}{2},\ \frac{k\widetilde{\lambda}^2}{2(2+k)}\right].$$

All these price functions induce the same efficient attribute choices, but imply different divisions of the surplus.

The total net surplus of the pair with index ρ is

$$\frac{k}{2(2+k)}\rho^2 + \frac{1}{2}\rho^2 = \frac{(1+k)}{(2+k)}\rho^2,$$

which is the maximum (i.e., efficient) value of the ρ-match surplus.

C.2 Equilibrium

Fix $\widetilde{\lambda} \in [0, 1]$ and consider a putative equilibrium in which laboratories with $\lambda \leq \widetilde{\lambda}$ are uninformed and laboratories with $\lambda > \widetilde{\lambda}$ choose to be informed. Within each region, researchers will be choosing attributes increasing in index, and researcher ρ will be matched with laboratory $\lambda = \rho$. Thus, researcher $\rho \leq \widetilde{\lambda}$ will choose $r = \theta^{1/(1+k)}\rho$ and be matched with the uninformed laboratory $\lambda = \rho$. Researcher $\rho > \widetilde{\lambda}$ will choose $r = \rho$ and be matched with the informed laboratory $\lambda = \rho$.

In this putative equilibrium, the set of chosen researcher attributes is $[0,\ \widetilde{r}] \cup (\widetilde{\lambda},\ 1]$, where $\widetilde{r} = \theta^{1/(1+k)}\widetilde{\lambda} < \widetilde{\lambda}$.

Recalling (C.4), for $\kappa < \bar{\kappa}(\theta)$, we choose $\widetilde{\lambda} \in (0, 1)$ so that the ex ante efficient surplus from the match of researcher $\widetilde{\lambda}$ and laboratory $\widetilde{\lambda}$ exactly exceeds the uninformed laboratory equilibrium match surplus by κ:

$$\kappa = \hat{u}_R(\theta, k, \widetilde{\lambda}) + \hat{u}_L(\theta, k, \widetilde{\lambda}) - \ u_R(\theta, k, \widetilde{\lambda}) - u_L(\theta, k, \widetilde{\lambda}) = \bar{\kappa}(\theta)\widetilde{\lambda}^2. \quad \text{(C.5)}$$

The pricing constant

$$\phi := \frac{k}{2(2+k)} \left[1 - \theta^{(2+k)/(1+k)}\right] \widetilde{\lambda}^2$$

in (C.3) makes laboratory $\widetilde{\lambda}$ indifferent between being informed and not:

$$\hat{u}_L(\theta, k, \widetilde{\lambda}) + \phi - \kappa = u_L(\theta, k, \widetilde{\lambda}).$$

This immediately implies that researcher $\widetilde{\lambda}$ is also indifferent *ex ante* between being matched with an informed or uninformed laboratory. We should then be able to simply "paste" the informed laboratory equilibrium for $\lambda \geq \widetilde{\lambda}$ to the uninformed laboratory equilibrium for $\lambda < \widetilde{\lambda}$.

C.3 Researcher Incentives to Deviate

For the researchers, we need only verify that researchers with indices below (respectively, above) $\widetilde{\lambda}$ prefer to be matched with uninformed (respectively, informed) laboratories rather than choosing a sufficiently high (respectively, low) attribute to be matched with an informed (respectively, uninformed) laboratory. But this follows from the single crossing property on the cost function together with the implied indifference for researcher $\widetilde{\lambda}$.

C.4 Laboratory Incentives to Deviate

Turning to the laboratories, there are two potentially profitable types of deviations. The first is that a laboratory with index $\lambda < \widetilde{\lambda}$ may find it profitable to be informed. The second is that a laboratory with index $\lambda > \widetilde{\lambda}$ may find it profitable to be uninformed.

C.4.1 Do Uninformed Laboratories Wish to be Informed?

Consider first a deviation by a laboratory $\lambda \leq \widetilde{\lambda}$ to becoming informed and targeting a researcher with attribute $r \leq \widetilde{r}$. The attribute r is chosen by researcher $\rho = \theta^{-1/(1+k)}r$, and matches with $\lambda = \rho = \theta^{-1/(1+k)}r$, paying a price of $\rho^2\theta^{(2+k)/(1+k)}/2 = \theta^{k/(1+k)}r^2/2$. The resulting ex post payoff is the researcher's share of the surplus less the price, $\theta \times \theta^{-1/(1+k)}r^2 - \theta^{k/(1+k)}r^2/2 = \theta^{k/(1+k)}r^2/2$. An offer of a price p satisfying

$$\theta^{k/(1+k)}r^2/2 < \theta\lambda r - p$$

will induce the researcher to accept the deviating offer. Such an offer is profitable for the laboratory if

$$u_L(\theta, k, \lambda) < (1-\theta)\lambda r + p - \kappa.$$

Thus, there is a p for which the deviation by the laboratory is strictly profitable if, and only if,

$$\begin{aligned}\kappa &< \lambda r - \theta^{k/(1+k)}r^2/2 - u_L(\theta, k, \lambda) \\ &= \lambda r - \theta^{k/(1+k)}r^2/2 - \frac{1}{2}\theta^{1/(1+k)}(2-\theta)\lambda^2 =: \Delta(\lambda, r).\end{aligned}$$

For $\lambda \le \theta\widetilde{\lambda}$, $\Delta(\lambda, \cdot)$ is maximized at $r = \theta^{-k/(1+k)}\lambda \le \widetilde{r}$, and has value $\theta^{-k/(1+k)}\lambda^2(1-\theta)^2/2$. Note that $\theta^{-k/(1+k)}\theta\widetilde{\lambda} = \widetilde{r}$. Moreover, for $\lambda \in [\theta\widetilde{\lambda}, \widetilde{\lambda}]$, $\Delta(\lambda, \widetilde{r})$ is uniquely maximized at $\lambda = \widetilde{\lambda}/(2-\theta)$. This implies that the maximum of $\Delta(\lambda, r)$ over $(\lambda, r) \in [0, \widetilde{\lambda}] \times [0, \widetilde{r}]$ is achieved at $(\widetilde{\lambda}/(2-\theta), \widetilde{r})$. Thus, there is no strictly profitable deviation if

$$\kappa \ge \Delta(\widetilde{\lambda}/(2-\theta), \widetilde{r}) = \frac{\theta^{1/(1+k)}\widetilde{\lambda}^2(1-\theta)^2}{2(2-\theta)}.$$

Substituting for $\widetilde{\lambda}$ from (C.5) and canceling κ yields (C.1).

C.4.2 Do Informed Laboratories Wish to be Uninformed?

If laboratory $\lambda \ge \widetilde{\lambda}$ deviates to being uninformed, then by posting a price p, the laboratory attracts all researchers who find matching with laboratory λ at that price attractive. The laboratory must have beliefs over the researchers attracted by such a deviation. We assume pessimistic beliefs: the laboratory assumes that the lowest attribute researcher will match.

We begin by considering $\lambda = 1$, and suppose this laboratory chooses to be uninformed. If it were to charge $p = \phi - \frac{1}{2} + \theta$, the equilibrium price paid by researcher $\rho = 1$ to match with laboratory $\lambda = 1$, researcher $\rho = 1$ incentives are unchanged. But that match is no longer relevant (given our assumption on beliefs), since lower attribute researchers are willing to pay that price. The most profitable deviation is to charge a higher price in attempt to screen out lower attribute researchers.[16]

We now argue that if $\theta < \widetilde{\lambda}$, the most profitable deviation by laboratory $\lambda = 1$ is to charge such a high price that $\rho = \widetilde{\lambda}$ is indifferent, and that such a deviation is not profitable. Researcher $\rho \geq \widetilde{\lambda}$ has chosen attribute ρ and has payoffs gross of costs of

$$\frac{\rho^2}{6} - \phi + \frac{\rho^2}{3} = \frac{\rho^2}{2} - \phi,$$

and is willing to match with the deviating laboratory $\lambda = 1$ at a price p if $\theta\rho - p \geq \rho^2/2 - \phi$, i.e., if $\theta\rho - \rho^2/2 + \phi \geq p$. The laboratory's goal is to maximize the lowest ρ satisfying this inequality through his choice of p. The quadratic on the left of the inequality is maximized at $\rho = \theta$ and is monotonically decreasing for larger ρ. This implies that if $\theta < \widetilde{\lambda}$, the optimal choice of p makes researcher $\widetilde{\lambda}$ just indifferent ($p = \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi$); no researcher is willing to match at a larger p.

The laboratory does not find this deviation profitable if

$$\begin{aligned} &\frac{1}{2} + \phi - \kappa \geq (1-\theta)\widetilde{\lambda} + \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi \\ \iff\ &\frac{1}{2} - \kappa \geq \widetilde{\lambda} - \widetilde{\lambda}^2/2. \end{aligned}$$

Using (C.5) to eliminate $\widetilde{\lambda}$ in the inequality and rearranging, one obtains condition (C.2).

Lower index informed laboratories also have no incentive to become uninformed, though for some this deterrence involves a concern that the researcher will have an attribute less than $\theta^{1/2}\widetilde{\lambda}$, rather than $\widetilde{\lambda}$. Lower informed laboratories may find it optimal to become informed if they could guarantee no researcher with an attribute below $\widetilde{\lambda}$ would find the price attractive. However, this is impossible: By becoming uninformed, laboratory $\rho = \widetilde{\lambda}$ cannot deter lower attribute researchers without deterring all researchers. A (loose) upper bound on the payoff from deviating is obtained by assuming that at the price p which makes the researcher with attribute $\widetilde{\lambda}$ just indifferent, the laboratory is guaranteed that the only additional researcher attribute attracted is $\widetilde{r} = \theta^{1/2}\widetilde{\lambda}$. It can be verified that even with such a payoff, the deviation is not profitable.

[16] At higher prices the highest attribute researcher prefers to match with laboratory $1-\varepsilon$, for ε small. But since the laboratory believes he will match with the lowest attracted attribute, this is irrelevant.

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