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"The Role of Quality in Service Markets Organized as Multi-Attribute Auctions"

by

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The Role of Quality in Service Markets Organized as Multi-Attribute Auctions*

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Abstract

We develop an empirical methodology to study markets for services. These markets are typically organized as multi-attribute auctions in which buyers take into account seller's price as well as various characteristics, including quality. Our identification and estimation strategies exploit observed buyers' and sellers' decisions to recover the distribution of sellers' qualities, the distribution of seller's costs conditional on quality, and the distribution of buyers' tastes. Our empirical results from the on-line market for programming services confirm that quality plays an important role. We use our estimates to study the effect of licensing restrictions and to assess the loss of value from using standard rather than multi-attribute auctions as is common in public procurement.

Keywords: quality, services, licensing, procurement, multi-attribute auctions, identification, unobserved heterogeneity, unobserved buyers' tastes, participation in auctions

JEL Classification: C14, C18, D22, D44, D82, L15, L86.

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1 Introduction

Recall the last time you hired someone to paint your house. Several painters evaluated the job and submitted their price quotes (bids). Since the candidates potentially differed in quality, you interviewed each of them to assess their professionalism and the likelihood that the job would be completed on time and according to your expectations. When submitting their bids, painters might have been uncertain about your idiosyncratic willingness to pay for quality and were most likely uninformed about the identity and costs of their competitors. You selected the painter taking both the price and his quality into consideration. You probably went through a similar process when hiring a nanny for your kids, a piano tuner, or a hairdresser, or when choosing a brokerage. In each of these examples, the price was important, but quality considerations also played an important role in your ultimate choice. Formally, you conducted a multi-attribute auction, where the price and the quality dimensions of the bidder were relevant to your selection.

The transactions described above are not rare. Services account for around 80% of the US gross domestic product, and many service markets, including private business sector procurement, are organized in the form of multi-attribute auctions. This is suggestive of the fact that, in addition to price, other bidder attributes, such as quality, play a role in choosing the provider. How important is that role? What is the distribution of buyer preferences for quality? How large is the dispersion of the quality of suppliers? How well do the observable (to an econometrician) supplier characteristics proxy for their quality? What is the distribution of supplier costs conditional on quality? Unfortunately, despite the manifest importance of these questions, the literature provides limited answers.

Answering these questions is important not only for understanding the efficiency and functioning of service markets but also for the evaluation of various policies and regulations. For example, the rise of the share of services in the US economy has been associated with a sharp increase in the prevalence of occupational licensing that aims to restrict the participation of low quality providers in the market.[1] As we will show below, the economic consequences of these laws crucially depend on the ability of market participants to assess the quality of service providers and on the joint distributions of qualities and the costs of providing the service at that level of quality. Yet, the methodology for recovering these objects in the data is not available in the literature. As another example, note that public service procurement is mostly organized in the form of a standard rather than multi-attribute auction. This means that the selection of a public service provider is determined mostly on price, whereas other important considerations such as quality of service are handled outside of the pricing mechanism through certification or other participation restrictions. This approach is often motivated by consideration of fairness or to prevent (the appearance of) corruption. Measuring the joint distributions of qualities and costs of service providers is essential for assessing the costs to the government of adopting alternative auction designs in service procurement.

The primary constraint on the analysis of service markets has been the limited nature and availability of the data. These markets are decentralized, with many small players providing similar but not identical services. Fortunately, recently such markets have become increasingly organized on the Internet, opening an avenue for their study. Yet, even with the wider availability of centralized data, the characteristics of the service providers observed in these data likely provide a relatively poor measure of quality. For example, while the observed qualifications

[1] Indeed, Krueger and Kleiner (2013) report that while in the 1950s less than 5% of the US workforce was in occupations covered by licensing laws at the state level, by 2008 29% of workers were required to have a government-issued license to do their job.

and experience of the painter mattered for your selection, the photographs of completed jobs probably provided considerably more information about quality. Similarly, you probably got more information about the quality of the nanny by observing her interaction with your kids than from knowing her age and education. Thus, a buyer is likely to have richer information on the quality of a service provider than what is recorded in the data set. The objective of this paper is to develop a methodology that supplements quality-related characteristics recorded in the data with information inferred from the observed economic choices made by buyers.

Our modeling choices are motivated by the data from an online market for programming services[2] that is explicitly structured as a multi-attribute auction and resembles an off-line service market in many ways. This market, like many other service markets, lacks a uniform and objective record of quality. We document, however, that the buyers often do not award the contract to the lowest bidder and that the premium the buyers agree to pay is not well accounted for by the difference in other observed seller characteristics. In other words, buyers are willing to pay a premium for desirable seller characteristics that are not recorded in the data.

In our model, each project advertised by a buyer is associated with a set of active bidders who submit price quotes for buyer consideration. Bidders' idiosyncratic costs for completing the project are private information. The buyer's utility from a specific seller's services is a function of the seller's characteristics with buyer-specific weights. It also includes a seller-specific component that reflects the value of the match as perceived by the buyer. One characteristic considered by the buyer captures the seller's permanent quality, which remains the same across projects. Buyers assess bidders' quality through private communications during the bidding process.

We begin by developing a strategy to identify the key objects of interest: the distribution of sellers' quality, the distribution of the buyers' tastes, and the distribution of the sellers' costs for completing the projects conditional on quality. While our environment is unique, it shares both the features of discrete choice and auction settings. Similar to other auction markets, sellers costs and prices in our environment are determined at the project level. At the same time, the winner is chosen in a way which is typical of a discrete choice (or differentiated products) setting where buyers make choices based on their idiosyncratic preferences taking into account price as well as other characteristics such as quality. Our identification strategy builds on the insights from these two literatures.

The discrete choice literature deals with the issue of recovering the unobserved bidder quality as well as the distribution of the buyers' tastes for quality and other sellers' characteristics. However, the existing identification arguments rely on a data structure where researchers observe decisions of a large number of consumers operating in the same market and, therefore, choosing from the same set of products. Thus, product market shares are given in the data and product qualities can be solved for from their relationship with the market shares. In our setting individual sellers' market shares conditional on the choice set cannot be precisely estimated from the data since very few buyers are choosing from exactly the same set of alternatives. Thus, if we were to pursue an identification strategy similar to that used in the discrete choice literature, we would have to use market shares defined by aggregating over many different choice sets. Existing results do not apply in such case: additional conditions are needed for individual sellers' qualities to be identified. In this paper, we pursue an alternative identification strategy that deviates from extending results from the discrete choice literature in order to address additional challenges which arise in our environment due to the presence of a large number of "transitory"

[2]Professional services such as computer services account for half of the 20% growth in GDP share of services over the last fifty years (Herrendorf, Rogerson, and Valentinyi (2009)).

sellers who enter the market only for a short time.

The presence of transitory participants is typical of many service markets that are often characterized by high turnover of sellers. For short-term sellers only a distribution of unobserved quality rather than seller-specific value of quality could be recovered. The buyers' choices and transitory sellers' pricing strategies depend on these sellers' qualities since buyers have an opportunity to discover it through commucation that precedes an auction. Thus, transitory sellers' qualities as well as the relationship between their prices and qualities have to be integrated out in estimation. The traditional approach which calls for parametrization of the distribution of transitory bidders qualities and solving for transitory bidders' pricing strategies conditional on quality, is computationally infeasible in our setting since it calls for solving a large number of asymmetric auctions where bidders' asymmetries depend on parameter values. Further, the distribution of transitory bidders' qualities may plausibly depend on his other characteristics that enter buyers' utilities separately from quality. To our knowledge the methodology for estimating mixture models where the index determining the outcome and the mixing probability distribution both depend on the same set of variables has not yet been developed in the literature.

The solution we propose is to aggregate sellers into the groups that share the same observable and unobservable characteristics so that a product in this setting can be defined not as an individual seller but rather as a group of sellers. Such a definition allows for easy mapping between the populations of permanent and transitory sellers and makes the methodological problems outlined above more tractable. To implement this strategy, we rely on the long-run performance of permanent bidders to classify them into groups of equal qualities conditional on observable characteristics. In particular, we construct a pairwise index that reflects one seller's probability of winning in the auctions where both sellers are among the potential bidders.[3] We show that between two sellers, the seller with the higher quality has a higher value of this index. This step effectively identifies (up to a monotone transformation of the support) the distribution of quality for permanent bidders as a function of other covariates while bypassing the issues of unobserved buyers' tastes, the unavailability of information on market shares, and the presence of transitory bidders. Due to private cost variation in our data, the prices of permanent sellers vary exogenously across auctions even after conditioning on all relevant factors. This variation in prices and the variation in the composition of the sets of active permanent sellers defined in terms of seller groups identifies the distribution of buyers' tastes, the support of the quality distribution as well as the joint distribution of prices and qualities of transitory sellers.

Next, we turn to the identification of the distribution of sellers' costs conditional on sellers' quality and other observable characteristics. Since in our market the award rule is not known,[4]

[3]Since the sellers often do not fully know their actual competitors when they decide on their bids, the set of potential bidders captures the relevant competitive environment.

[4]Note that the uncertainty about the weights a specific buyer would assign to different characteristics (and thus randomness of buyers' tastes) is an important feature of our environment. This unstructured nature of the auction format distinguishes the market we study from those studied in the previous auction literature, including the recent literature on "non-standard auction formats," which assumes that the decision rule is known to the bidders. These include standard auctions with discrimination or preferential treatment for particular types of participants in Marion (2007), Krasnokutskaya and Seim (2011), and Swinkels (2009) and scoring auctions where the award is based on a scoring rule that aggregates several bid components, e.g., Athey and Levin (2001), Asker and Cantillon (2010), Asker and Cantillon (2008), and Bajari and Lewis (2011). Scoring auctions additionally differ from our environment by treating bidders' characteristics as auction-level choice variables. While a multi-attribute auction format is prevalent in industry procurement, we found only two papers that study it. Greenstein (1993) and Greenstein (1995) analyze IBM's procurement and documents different weights that IBM through its choices was revealed to assign to different attributes of bidders. The analysis in these papers assumes that all

a mark-up over costs charged by bidders is not linked directly to the distribution of competitors' prices. Instead, it also depends on the distribution of buyers' tastes, the quality levels of a seller's competitors as well as his own quality level. Therefore, we can proceed to identify the distribution of bidders' costs only after we have identified the distributions of buyers' tastes and all relevant seller characteristics (including quality). We accomplish this by relying on the inversion method first proposed by Guerre, Perrigne, and Vuong (2000) and later applied in various environments by Li, Perrigne, and Vuong (2000), Jofre-Bonet and Pesendorfer (2003), Li, Perrigne, and Vuong (2002), Krasnokutskaya (2011), Athey and Haile (2002).

Having established the identification of the model, we develop an econometric methodology. First, we propose a nonparametric classification procedure that implements the first step of the identification strategy. It uses the index developed in the identification section to rank bidders with respect to quality, i.e., to effectively sub-divide them into groups of equal quality. Two practical issues may potentially arise in small samples. First, the transitivity of the quality ranking could be violated. Second, one may face indeterminacy with multiple compatible classifications. We solve these issues by proposing a method that relies on our index to jointly estimate the whole group structure instead of recovering individual pairwise rankings of bidders. Our algorithm also chooses the classification that is most strongly supported by the data.

Through our classification method, we consistently estimate a quality group structure. However, it provides no evidence on quality levels and therefore does not permit comparison across covariate values. In the next step, we estimate quality levels and the distribution of buyers' tastes by the Method of Moments. Since the joint distribution of prices and qualities of transitory bidders is non-parametrically identified from the data we do not need to solve for bidding strategies of transitory bidders inside the estimation routine. Instead, we estimate them jointly with the distribution of buyers' tastes and quality levels.

The proposed method has a number of attractive features in addition to overcoming the methodological difficulties we have outlined above. For example, the recovered distribution of qualities is not restricted to be orthogonal to other relevant seller characteristics such as the seller's country affiliation or measures of performance recorded in the data. This allows us to explore the economic importance of these relationships. Due to the variation in private costs present in our setting, we are able to estimate buyers' price sensitivity relative to buyers' tastes for other characteristics including (unobserved) quality, without using instruments as is usually done in a traditional differentiated products setting. The instruments typically used are the characteristics of competing products. These instruments are not available in environments such as ours, where all observable seller characteristics are potentially endogenous (e.g., reputation score or the number of projects completed). While our methodology is tailored to a specific service market, we believe it can be modified in a number of ways to apply to a wide range of settings with similar data structures.

Applying our identification and inference strategies to the data from the online market for programming services, we find that quality plays an economically significant role. The model with quality explains 70% of buyers' choices, whereas the model without quality explains only 25%. There is also clear evidence of nontrivial quality heterogeneity among the sellers. The buyers exhibit a strong preference for quality: an average buyer is willing to pay a 50% premium to move from the lowest to the highest quality level. Thus, the variations in quality levels across the sellers and in buyers' willingness to pay for quality account for more of the variation in the data than all other covariates combined. We estimate that the buyers participating in the

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the relevant bidder characteristics are observed in the data.

market were able to improve their utility over the outside option by 53% on average. Since outside option in our market most likely represents hiring somebody locally this number reflects the gain in utility generated by access to larger markets and better information and search technology enabled by the Internet.

We also estimate a statistically significant relationship between transitory sellers' bids and their qualities. This result demonstrates the strength of our identification strategy and the fit of our model, since the relationship between a transitory seller's quality and price is not directly observed in the data. While unobserved quality is commonly believed to be important in the service markets, our findings contribute to the literature by offering the first formal assessment of its role. Moreover, the presence of large heterogeneity in quality provides a plausible explanation for why the market we study, and service markets more generally, tend to be organized as multi-attribute auctions.

Our empirical results also contribute to a better understanding of the information and enforcement issues studied in the literature on Internet auctions. Main questions in this literature are whether consumers are able to obtain credible information about the product sold and whether the reputation scores used in these markets serve as an informative device that counteracts adverse selection problems or as the incentive mechanism that prevents moral hazard. Our empirical results indicate that the latter role is probably more relevant.

The estimated bidding strategies show expected regularities: they are increasing in costs, and the prices of the low-quality bidders are noticeably lower than those of the higher-quality ones. A particularly interesting insight is that when we combine bid distribution with bid strategies to recover the distribution of costs conditional on a seller's characteristics including quality, we find that a tight (low variance) cost distribution can rationalize the large variation in prices observed in this market. Intuitively, in a standard auction, the sensitivity of the winning probability to the increase in prices provides a high-cost bidder with an incentive to lower his mark-up. In contrast, our model allows for a seller-specific match component of buyers' tastes that is unobserved (and therefore purely stochastic) from the seller's point of view. The high-cost seller's probability of winning depends to a large degree on a favorable realization of this stochastic component. This gives the seller an incentive to drive the price up and gamble on a lucky outcome.

Finally, having obtained estimates of the primitives of the model, we assess the impact of various policies on the market we study. We first consider the effects of introducing occupation licensing such that only programmers with the quality above a certain threshold are allowed to participate in the market. We find that this results in a substantial loss of utility to buyers as well as the loss of total surplus. This effect arises due to the price increase as well as because many buyers have to buy more expensive quality or leave the market. We also find that the market share of high-quality sellers is affected by licensing only slightly. In fact, our model can generate a decline in the average purchased quality associated with licensing under the parameter values that are quite similar to the estimated parameter values. This is consistent with the consequences of occupational licensing documented in the existing empirical literature for other markets while our analysis suggests a mechanism that may generate such effect. Second, we assess the loss of surplus (and utility to government/buyer) from procuring programming services through a standard as opposed to a multi-attribute auction, possibly with restrictions on the qualities of programmers allowed to submit bids. We find that such policy is costly. It results in 19.5% reduction in the average quality purchased in this market which despite the decrease in price leads to 6.5% loss in government "utility" and 9.2% loss in total surplus. Imposing restrictions on participation increases the average quality purchased by 12% but it remains lower than the average quality purchased under multi-attribute mechanism.

The paper is organized as follows. Section 2 describes our market and the main features of the data that motivate our modeling choices. The model is described in Section 3. Section 4 outlines the identification and estimation strategies. Section 5 provides technical details of our classification methodology while Sections 6 and 7 present the results of empirical and of counterfactual analysis. Section 8 summarizes the findings and outlines directions for further research.

2 Market Description and Some Features of the Data

2.1 Market Description

We study a market mediated by an online platform that serves as a match-maker between the demand and supply for services of computer programming. This company provides an environment that allows buyers (the demand side) to post job announcements. At the same time it maintains the registry of potential sellers (the supply side). The registry provides limited information on verifiable "outside" credentials as well as information about the on-site performance of the seller. The latter includes reputation scores or ratings, buyers' numerical feedback about working with a given seller, as well as instances of delays and disputes. In the case of a dispute, the company provides professional arbitration services that ensure that a seller is paid if only if the completed job satisfies industry standards.

This intermediary company allocates jobs through multi-attribute auctions. Under the rules of such an auction, a buyer is allowed to take into account multiple seller characteristics in addition to the price quote. As a result, the selected seller is not necessarily the one who submits the lowest quote. An important feature of this mechanism is that the award rule is not announced and thus remains unknown to other market participants.

Suppliers can communicate with buyers before posting price quotes. Such an exchange of messages is very common. On average, each seller submits three messages per auction in our data. In some cases, a seller can attach an example of his work or a sketch of the proposed code. The number and the content of these communications are not observed by the other sellers. Hence, while the buyer has an opportunity to form an opinion about each sellers' quality, competing sellers have much more limited knowledge of their competitors' quality. In principle, competitors can infer a seller's quality from his long-run rate of winning in a way similar to that proposed in this paper.

When a seller contacts a buyer for any reason, his code name appears on the project webpage. Therefore, at any point in time, a visitor to the page can see the list of sellers who contacted the buyer before this point. This list generally does not coincide with the set of sellers who eventually submit price quotes, since a few sellers may ask the buyer a question without submitting a quote. Therefore, a prospective seller does not observe the set of his competitors.[5] Thus, price quotes are likely to reflect potential rather than actual competition in an auction.

[5]Another indication that the set of actual competitors is not observed is that bids are generally submitted throughout the time period allocated for the auction and, once submitted, are rarely revised. It is possible, however, to form an opinion about the potential competition by observing participation activity across similar auctions.

2.2 Some Data Regularities

We have access to the data from the starting date of our online programming market and for the subsequent 6 years of this company's operation. The majority of buyers over these six years are one-time participants. Less than 2% of buyers return with multiple projects. In addition, repeat buyers do not return with the same type of project. As a result, they very rarely work with the same seller repeatedly.

The multi-attribute feature of the auction is strongly supported in the data. Indeed, in our sample, 58% of the projects are awarded to a seller who quotes a price above the lowest price submitted in the auction. Table 1 documents the share of such projects as well as an average mark-up over the smallest bid for some project types.

Table 1: Projects Awarded at a Price That Exceeds Lowest Price in Auction

Type of Work	Project's Share	Price Mark-up
Database	64%	41.2%
Platforms	52%	37.9%
Graphics	71%	38.4%
Web-related	52%	41.2%

Note: The results in this table are based on a full sample that includes 600,000 projects. The "Project's Share" column reports the fraction of the projects that have been awarded to bidders with price quotes that exceed the lowest price quote for the respective project. "Price Mark-up" summarizes the average normalized difference between the winning price and the lowest price quote across projects that are awarded at a price that exceeds the lowest price quote. The differences in prices are normalized by the lowest price quote.

These results indicate that buyers consider seller characteristics other than price when choosing a winner. Thus, a demand model that takes sellers' heterogeneity into account is required to study this environment.

We explore the buyers' choices using a logit model with random coefficients (without choice-specific fixed effects). In this analysis, we set the dependent variable, Y_{li}, to be a project award dummy that is equal to one if the seller i won the project l and zero otherwise. The award depends on the buyer's utility from a specific alternative (seller), which is modeled as a linear function of seller characteristics, X_{ki}, (the number of ratings (experience), average score, delays, arbitration), seller location dummies, $\mu_{c(i)}$, and a seller-specific price quote, B_{li}:

$$Y_{li} = X_i\alpha_l + \gamma_l B_{li} + \mu_{c(i)} + \epsilon_{li} \tag{1}$$

Table 2 reports the results of this analysis. As can be seen from this table, the mean of the price coefficient is estimated to be positive and statistically significant. This result suggests an omitted variable bias since, in most markets, buyers dislike paying higher prices, other things equal. This means that some additional characteristic, not recorded in the data, affects buyers' choice in conjunction with the price, location and performance measures. Such an omitted variable should be positively aligned with the price and is, therefore, some vertical characteristic such as quality. Thus, a model that describes this setting should allow for an unobserved quality-like seller's attribute.

Another important feature of our data is summarized in Table 3. Let us define seller's tenure

Table 2: Logit Model with Random Coefficients

Variable	Coefficient	Std.Error
Normalized Price (mean)	6.461	0.743

Note: The results in this table are based on a full sample that includes 600,000 projects.

as the length of time that elapses between the date when he submits his last bid and the date when he submits his first bid. The share of sellers with short tenure is larger in the beginning years but settles down, so that the distribution of tenure is almost constant over the last three years. In these years, 30% of the sellers stayed in the market for more than a year, whereas 65% of the sellers left the market in less than three months. Substantial seller turnover is an important feature of our market as well as many other markets for services.

In contrast to other online markets, the sellers' performance does not appear to be related to their propensity to stay in the market. To see this, we define *permanent sellers* as the sellers with a tenure longer than one year and *transitory sellers* as the sellers who left after less than one year. Table 3 documents no significant differences between permanent and transitory sellers in the number of bids submitted before the first success (conditional on achieving at least one success), as well as in the distribution of reputation scores received by these bidders for their first or last projects. We obtain similar results when transitory sellers are redefined to be those who left after six or three months.

An interesting regularity emerges concerning the number of bids before the first success. When we compute the distribution of this variable for all transitory bidders (including those that did not win any projects), the time to the first success for transitory sellers appears to be substantially shorter than that for permanent sellers. This suggests that many transitory bidders do not wait for success, and that sorting into permanent or transitory groups is likely driven by sellers' outside opportunities rather than quality or performance differences among the sellers. Thus, the assumption that the distribution of quality is the same in permanent and transitory seller populations appears reasonable in this market.

In general, transitory sellers appear to be quite successful: their rate of winning is comparable to that of permanent sellers, and they often beat permanent sellers at comparable prices. Given that extensive communication between buyers and sellers is present, this suggests that buyers may be able to assess the quality of transitory sellers as accurately as they assess the quality of permanent sellers.

On the other hand, very little information about transitory sellers is publicly available. Indeed, public information is released when a seller completes a project, and transitory sellers usually complete one or two projects and leave the market. It is plausible, therefore, that competing sellers are not informed about transitory sellers' qualities. The situation is different for permanent sellers. The market may infer their quality from the long-run rate of their successes through reasoning similar to that we use later in this paper.

To summarize, the preliminary analysis of our data indicates that (a) a non-trivial model of demand should be considered; (b) the model should allow for the presence of an unobserved quality-like sellers' attribute; (c) it is important to account for the presence of a large number of transitory sellers; (d) buyers most likely observe the qualities of participating transitory sellers; and (e) the distribution of seller's quality does not depend on the seller's tenure.

Table 3: Analysis of Permanent vs. Transitory Sellers

	Tenure Distribution			
	$\leq$ 1m	$\leq$ 3m	$\leq$ 12m	$\leq$ 24m
overall	65%	75%	80%	90%
annual (last 3 years)	45%	65%	70%	75%
	Number of Bids Before First Success			
	$\leq$ 10%	$\leq$ 25%	$\leq$ 50%	$\leq$ 75%
tenure$\geq$ 12m	5	9	17	42
tenure$\leq$ 12m (success$\geq$ 1)	3	7	15	36
tenure$\leq$ 12m (all)	1	2	3	12
	First Reputation Score			
	8	9	10	
tenure$\geq$ 12m	5%	10%	85%	
tenure$\leq$ 12m	5%	9%	86%	
	Last Reputation Score			
	8	9	10	
tenure$\geq$ 12m	2%	30%	68%	
tenure$\leq$ 12m	2%	28%	70%	

Note: The results in this table are based on a full sample that includes 600,000 projects. In these calculations the "Tenure" variable reflects the total length of time a seller is observed to be active in the market, i.e., "Tenure" equals the length of time between the date of the last bid recorded in the data and the date of the first bid. Panel 1 records the cumulative distribution function of the "Tenure" variable, panel 2 records the inverse of the cumulative distribution function of the variable "Number of Bids Before First Success," and panels 3 and 4 record the probability distributions of the variables "First Reputation Score" and "Last Reputation Score." This table indicates that permanent and transitory sellers appear to be very similar in their performance.

3 The Model

3.1 Overview of Main Features

Our environment combines the features of discrete choice and auction environments. As in a discrete choice setting, a buyer procuring services for a specific project faces a choice among a finite set of alternatives. In our setting, alternative choices are services provided by different sellers who are heterogeneous in quality and other characteristics.

Similar to a discrete choice setting, buyers have heterogeneous tastes and therefore use different award rules. Since the award rules are not observed in the data we represent the weights a buyer assigns to different seller characteristics as random coefficients.

On the other hand, our environment possesses features that are typically observed in auction environments: the costs of participants are their private information that varies stochastically across sellers and across projects. This is because the service is not provided in a manufacturing setting, where all units are the exact replica of each other. In this market, the projects that deliver similar outputs may differ in many subtle ways. Similarly, sellers who are working on the same project may be restricted by different circumstances and may use somewhat different approaches.

Also, the buyers in our market are not just presented with a fixed set of alternatives. Rather, each of them has to choose from a set of sellers who submitted bids for their specific project. If participation is costly, sellers may strategically decide whether to participate in a given auction. Their probability of participation depends on the project and the seller's characteristics as well as the seller's beliefs about possible competition. These feature have fundamental implications for modeling sellers' pricing decisions as well as methodological implications for how buyers' tastes and sellers' costs could be recovered from the data.

In our setting the sellers are of two types: *permanent* and *transitory*. Permanent sellers participate in the market many times and may be assumed to stay in the market indefinitely, whereas transitory sellers appear only in a small number of auctions and are observed to leave the market after that. Presence of transitory sellers is an important feature of service markets emphasized both by empirical and theoretical studies. We explain below why this distinction is necessary from a methodological point of view. In our market, permanent and transitory sellers further differ in the availability of information about their quality: it is very likely that all market participants can obtain information about a permanent seller's quality, whereas a transitory seller's quality is likely to be known only to the seller himself and to the buyers of the projects for which he submits bids. The methodology we propose below is tailored to this feature of our market but it can be extended to allow for other informational settings.

In the interest of transparency, we first outline our method in the context of a simplified model that leaves out several empirically relevant features. In particular, it ignores observable project and seller heterogeneity that is present in our data. It also assumes (again for simplicity) that sellers select projects completely at random, ignoring strategic considerations. Later in a separate subsection, we will explain how our model and methodology could be adjusted to incorporate the observable heterogeneity among the buyers and sellers and the sellers' strategic entry decisions.

3.2 The Basic Model

Let S denote the set of sellers who operate in our service market. The set of sellers is divided into the set of permanent sellers (denoted by S^p) and the set of transitory sellers (denoted by S^t). Each seller i is characterized by some scalar quality level, $q_i \in \{q^1, \cdots, q^K\}$ where $q^1 < q^2 < \cdots < q^K$. The quality level of seller i, denoted by q_i, is fixed and does *not* vary across projects. Both S^p and S^t are partitioned into K quality groups, i.e., $S^r = \cup_{k \leq K} S^{r,k}$, $r \in \{p, t\}$ so that $S^{r,k}$ represents the set of sellers of type r (either permanent or transitory) who are characterized by quality level q^k. The fraction of sellers of type r who belong to quality group k is given by $\pi_{r,k}$ such that $\pi_{r,k} = \frac{|S^{r,k}|}{|S^r|}$. The frequency distributions across quality groups, $\{q_k, \pi_{r,k}\}_{k=1}^K$ with $r \in \{p, t\}$, are primitives of the model.

The quality of each permanent seller is common knowledge among the market participants, while the quality of each transitory seller is his private information.[6] The quality of a given transitory seller j is unobserved by the econometrician or other sellers. From their point of view, it is summarized by the random variable Q_j that is distributed according to a multinomial distribution $(K, (\pi_{r,k})_{k=1}^K)$ with support $\{q^1, \cdots, q^K\}$.

We use $A_l \subset S$ to denote the set of sellers who submit a bid for project l and refer to

[6] We believe that this assumption correctly reflects informational structure of service markets in general and of on-line market we study in particular. However, the methodology presented below could be extended to several other cases.

such sellers as *active* bidders.[7] For simplicity, we first assume that a decision to become an active bidder is non-strategic, i.e. a seller $i \in S^{r,k}$ becomes active for project l at random, with exogenous probability $\lambda_{r,k}$. These probabilities are common knowledge among all market participants. However, an active bidder does not observe who else is active in the same project. The events of being active are independent across the projects and the sellers.

Upon becoming active, each seller privately observes his cost $C_{i,l} \in \mathbb{R}_+$ for completing the project, quotes a bid/price $B_{i,l}$ and reveals his quality q_i to the buyer. Costs are independent across sellers and across projects for a given seller. The costs of seller i from quality group $S^{r,k}$ for $r \in \{p, t\}$ are distributed according to $F_k(\cdot)$. The distributions of costs are common knowledge among all sellers.

The demand side of the market consists of one-time buyers. Buyers observe all the relevant sellers' characteristics including (and restricted to in this section) quality levels. They procure services using multi-attribute auctions. In particular, a buyer l evaluates each active bidder i by the utility he would derive from this seller's services:

$$U_{i,l} = \alpha_l q_i - B_{i,l} + \epsilon_{i,l}. \tag{2}$$

Here α_l denotes the buyer's taste for quality and $\epsilon_{i,l}$ reflects the seller's idiosyncratic suitability for a given project as perceived by the buyer. We refer to $\epsilon_{i,l}$ as a seller-specific match component in the future. Each buyer also has an outside option that delivers $U_{0,l}$ if chosen. We let $\epsilon_l = \{\epsilon_{1,l}, ...\epsilon_{|A|,l}\}$ and refer to $(\alpha_l, \epsilon_l, U_{0,l})$ as the vector of buyers' tastes.

The buyer awards the project to seller i if and only if:

$$U_{i,l} \geq U_{j,l}, \text{ for all } j \in A_l \backslash \{i\} \text{ and } U_{i,l} \geq U_{0,l}.$$

In keeping with the definition of a multi-attribute auction, sellers do not observe the tastes or outside option of a specific buyer, and consider it as a random draw from some joint distribution of buyers' tastes and outside options in the population. We allow buyer's taste for quality, α, to be correlated with his outside option but require that (α, U_0) should be independent of the vector of match components, ϵ_l. We believe that this is a plausible assumption since the vector of match components effectively characterizes the realized set of entrants that is independent of other components of buyer's tastes.

In line with the existing empirical auction literature, we assume that the observed outcomes are from a type-symmetric pure strategy Bayesian Nash equilibrium (psBNE). In such an equilibrium, participants who are *ex ante* identical (i.e. those who belong to the same $S^{r,k}$) adopt the same strategy. Thus the bidding strategy for seller i who belongs to the group $S^{r,k}$ is denoted $\sigma^{r,k} : [\underline{c_k}, \overline{c_k}] \to \mathbb{R}_+$, and entrant i's expected profit from bidding b is given by

$$\Pi^{r,k}(b, c;\, \sigma^{-i}) \equiv (b - c) \Pr(i \text{ wins} \,|\, b,\, (r, k); \sigma^{-i}),$$

where σ^{-i} denotes a profile of other sellers' strategies that they would use should they become active in a given project, and $\Pr(i \text{ wins} \,|\, b,\, (r, k); \sigma^{-i})$ the probability of seller i winning the auction by bidding b when the other sellers' bids are consistent with the strategies σ^{-i}. Then a psBNE is a profile of strategies $\{\sigma^{r,k}\}_{r \in \{p,t\}, k \in \{1,...,K\}}$ such that $\sigma^{r,k}(c) = \arg\max_b \Pi^{r,k}(b, c;\, \sigma^{-i})$

[7]The set of active bidders is also partitioned according to quality groups, $A_l = \cup_{k \leq K} A_l^k$.

for all c. We prove the existence of such a psBNE[8] in Appendix B.

3.3 Discussion: Methodological Challenges

The primitives of the basic model outlined in the previous section are the joint distribution of buyers' tastes and outside option, the distribution of sellers' qualities, and the distributions of sellers' costs conditional on quality. These components are not directly observed in the data. Instead, our data summarize, for a large number of auctions, the set of active participants, their bids, and auctions' outcomes. We now explain the methodological issues associated with our setting and how our approach helps to overcome them.

In the simplest case, when only permanent sellers are present in the market our environment can be modified[9] to resemble a discrete choice setting with alternative-specific fixed effects and random coefficients. Such a setting is analyzed in McFadden (1989) and Berry, Levinsohn, and Pakes (2004). The estimation methodologies proposed in these papers rely on the fact that the data are organized by markets (choice sets) and, in each market, a large number of consumers faces a fixed set of alternatives so that the probability of choosing each alternative conditional on the choice set could be precisely estimated from the data and thus is assumed to be known for the purpose of identification. The choice-specific fixed effects are then identified by inverting the choice probabilities. The distribution of random coefficients is identified through the variation in the choice sets across markets or, in the case of individual choice data, through the variation in product characteristics if they vary across buyers. This identification mechanism is formally discussed in Berry and Haile (2011).[10]

Unfortunately, the structure of our data is different. Due to a large number of buyers and sellers, and stochastic participation of sellers, only a negligibly small set of buyers chooses from exactly the same set of alternatives in our data. If we were to follow a route analogous to discrete choice methodologies, we would have to rely on the choice probabilities defined at the level where observations are pooled across a large number of choice sets. As a result, the system of equations we would use to pin down fixed effects would have a more complicated structure than the one considered in Berry, Levinsohn, and Pakes (1995) and Berry and Haile (2011). The invertibility of such a system and thus the identification of our model do not follow from existing results and has to be independently established. In other words, the fixed effects methodology does not apply automatically and is valid only under some additional conditions such as (among others) that the probability of any two sellers directly competing against each other should be observable in the data (or could be precisely estimated). The method we propose below relies on similar but less data-demanding conditions. In this paper, we pursue an identification strategy that deviates from extending the results in Berry and Haile (2011) in order to address the additional challenges that arise in our environment due to the presence of a large number of transitory sellers.

Notice that in contrast to permanent sellers, even the unconditional (on the choice set) probability of winning[11] for transitory sellers cannot be consistently or precisely estimated from the data. This is a fundamental feature of our environment rather than the issue associated

[8]We extend this model for allow for strategic entry in section 4.4 and prove the existence of a psBNE for such an extended model in Appendix B.

[9]This would entail dividing the buyer's utility function by α_l. For details see section 6.3.1

[10]Other papers that study identification of discrete choice models are Fox and Gandhi (2012), Bajari, Fox, Kim, and Ryan (2012).

[11]We use "choice probability" and "probability of winning" interchangeably in the paper.

with an insufficient number of observations, since the number of observations associated with an individual transitory seller would not change even if we collected ten times more data than we already have. Further, as we explain above, a buyer making a choice is likely to observe the qualities of all bidders (including transitory ones). Therefore, the probability of winning for even a permanent seller depends on the quality of transitory competitors if they enter the buyer's choice set. Observability of quality (to the buyer) also implies that prices submitted by transitory sellers likely depend on their qualities. Thus, we have to account for the qualities of transitory sellers in the estimation, and we have to allow the prices of transitory sellers to depend on the qualities, and these qualities have to be characterized at the level of a distribution.

To express a permanent seller's probability of winning from our model, we have to integrate out the relationship between the transitory bidders' prices and qualities. This could be done within the standard framework by parameterizing the distribution of transitory sellers' qualities, solving for the relationship between the qualities and prices of transitory bidders from the model and then integrating this relationship out in the expression for the permanent bidders' probability of winning. In our setting, such a traditional approach would be computationally prohibitive, as one needs to solve a large number of auctions with asymmetric bidders where the degree of bidder asymmetry depends on parameter values. Moreover, the distribution of sellers' qualities likely depends on some other seller characteristics (such as country of origin or performance measures) that may separately enter the buyer's utility. Thus, the presence of transitory sellers transforms our setting into a mixture model, where both the index determining an outcome and the mixing probability depend on the same set of variables. To our knowledge, the method for estimating such models is not available in the literature.

The approach we propose is designed to overcome the issues we summarized above. It relies on the discrete nature of sellers' characteristics in our environment (in the general case, it calls for the discretization of characteristics similar to the methodologies in Ciliberto and Tamer (2009) or Chiappori and Salanie (2001)). Our idea is to aggregate sellers into groups such that sellers within the group share the same (observable and unobservable) characteristics and sellers' identities represent i.i.d draws from the corresponding group. Thus within this framework, a product is effectively defined not as an individual seller but rather as a group of sellers. This allows us to make transitory sellers "comparable" to permanent sellers and thus to link transitory bidders' distribution of qualities to the permanent bidders' distribution of qualities,[12] a device which greatly improves tractability. In order to pursue this strategy, we have to identify sellers with similar levels of unobserved characteristic (quality). We develop a classification algorithm that groups permanent sellers according to (discrete) levels of quality conditional on sellers' observable characteristics. We show that the quality group structure is nonparametrically identified from the data. In this procedure transitory active sellers are effectively integrated out. That is, it allows to circumvent the difficulties associated with identification of permanent sellers' qualities (fixed effects) in the presence of transitory sellers.

Due to private cost variation in our data, the price of a given "product" varies across auctions in exogenous way even after controlling for relevant factors. The variation in the prices (submitted by permanent sellers), and the variation in the composition of the sets of active permanent sellers defined in terms of seller groups identifies buyers' preferences in the presence of active transitory bidders. It also traces out the joint distribution of prices and qualities conditional on observable characteristics for transitory bidders.[13] The first result resolves the joint identifi-

[12] In particular, we would like to link their supports.

[13] This result relies on the assumption that the support of the quality distribution for transitory sellers is

cation of the index and mixing distribution issue, whereas the second result helps to overcome computational challenges. Specifically, the second result allows recovering the joint distribution of transitory bidders' prices and qualities from the data instead of re-solving multiple asymmetric auctions during the course of estimation. We elaborate on this point in section 4.3.2 of our paper.

4 Identification and Estimation of Structural Elements

This section discusses how we identify and estimate the structural elements of the model, which are the distributions of quality within the sets of permanent and transitory sellers, the joint distribution of buyers' tastes (α, ϵ, U_0), and the distribution of sellers' private costs for completing the jobs. Throughout the section, we take the set of sellers S as given and fixed, and suppress it from notations.

4.1 Recovering Quality Group Structure

Our first step is to recover the quality group structure of the set of permanent sellers, $\{S^{p,k}\}_{k=1,\ldots,K}$. We do this by exploiting differences in probability of winning across sellers. To formulate the result we need the the following two assumptions:

(A1) *Sellers' private costs $C_{i,l}$ are independent across all $i \in S$ and across l. For each seller i with $q_i = q^k$, his cost in each auction is an independent draw from some continuous distribution F_k with a density positive over support $[\underline{c_k}, \overline{c_k}]$.*[14]

(A2) *The three random vectors $(\alpha_l, U_{0,l})$, ϵ_l and C_l are mutually independent; match components $\epsilon_{i,l}$ are i.i.d. across i's; and $\epsilon_{i,l}$ and $(\alpha_l, U_{0,l})$ are continuously distributed with a density positive over $[\underline{\varepsilon}, \overline{\varepsilon}]$ and over $[0, \overline{\alpha}] \times [\underline{u_0}, \overline{u_0}]$ respectively.*[15]

For the remainder of this section we drop subscripts l (index for auctions/buyers) to simplify notations. Further, let $\mathcal{B}_i$ denote the support of prices submitted by a seller i in a PSBNE. For any $b \in \mathcal{B}_i \cap \mathcal{B}_j$, define a pair-specific index:

$$r_{i,j}(b) \equiv \Pr(i \,\text{wins} \mid B_i = b,\ i \in A,\ j \notin A).^{16} \tag{3}$$

contained in the support of the distribution of qualities for permanent sellers.

[14]Notice that this assumption does not allow for a persistent unobserved seller-specific cost component in addition to quality. This excludes, for example, differences in opportunity costs associated with sellers' location (e.g. urban vs. rural) if it is not observed in the data. It might be possible to separately account for this type of unobserved seller heterogeneity since our current strategy identifies unobserved quality from buyers' choices whereas unobserved cost persistence might be identified from the additional correlation in prices (unaccounted for by quality) over time. We leave this extension to future research.

[15]Notice that we require that ϵ_l is orthogonal to $(\alpha_l, U_{0,l})$, whereas α_l and $U_{0,l}$ are allowed to be dependent. Such restriction appears to be plausible since we can think of α_l and $U_{0,l}$ as of buyers' permanent tastes whereas ϵ_l characterizes active sellers' idiosyncratic suitabilities for the project which should not be related to buyer's outside option.

[16]Note that in (3) A denotes a random set of active participants. Therefore, in index formulation we are not conditioning on a specific set of active participants but rather on the event that "the set of active participants includes i but not j." In practice, we construct this index by pooling all the observations that satisfy this restriction.

Then,

Proposition 1 *Under (A1)-(A2),*

$$sign(r_{i,j}(b) - r_{j,i}(b)) = sign(q_i - q_j)$$

for any pair of permanent sellers i, j *and all* b *in the interior of* $\mathcal{B}_i \cap \mathcal{B}_j$.[17]

The intuition underlying this result is as follows. Given a set of sellers, uncertainty in competition that i faces when "$i \in A$ and $j \notin A$" is identical to that j faces when "$j \in A$ and $i \notin A$". If bidders i and j submit the same bid, the difference in winning probabilities must be solely due to the difference in their quality levels.[18,19]

Since quality ordering is transitive according to our model this result allows us to arrange all sellers in the order of (weakly) increasing quality which should also identify the sets of bidders who have equal quality.

The main issue that we need to overcome in translating this identification strategy into a viable estimation strategy is that, while qualities are transitive in our model, the estimation based on pairwise comparisons may result in estimates that violate transitivity in small samples, even when the number of the quality groups is only two. We need to have a method in which we determine the whole quality group structure in a way that satisfies transitivity. Below, we provide a heuristic argument on how we achieve this in a simple case of two quality (high and low) groups. We defer a formal exposition of the classification method until Section 5.

The idea is to divide the set of sellers into two groups such that sellers within each group are "closer" to each other than to sellers from the other group according to some metric which is based on index $r_{i,j}$. Such a division can be implemented in many different ways, as well as may result in several compatible classifications. Our algorithm proposes a mechanism which selects a classification which is best supported by data.

More specifically, we proceed according to the steps below. For each seller i, we first divide the other sellers into two groups, one with sellers more likely to have higher quality than i and the other with sellers more likely to have lower quality than i. This division is implemented by comparing p-values from a pairwise bootstrap test of the inequality restrictions $r_{i,j}(b) \geq r_{j,i}(b)$ for all b. Next, we check whether seller i is more likely to belong to the first group or to the second group through the pairwise inequality test results. Thus for each seller i, we have estimated a group structure. We choose one that has most empirical support (in terms of average p-values) This is our estimated quality group structure. As we shall explain later, the procedure can be extended to a more general case of multiple quality groups.

Since the true number of the quality groups is usually unknown this paper develops a method of estimating the number of quality groups from the data set. We show that, when the sample size is large, misspecifying the number of quality groups to be smaller than the true number of groups introduces an unsupportable restriction on the quality group estimation which is manifested through the weak empirical support of seller homogeneity for some of the estimated groups. On the other hand, when the number of quality groups is misspecified to be larger than or equal to

[17]Here $sign(x) \equiv 1\{x > 0\} - 1\{x < 0\}$ for all $x \in \mathbf{R}$.

[18]The index with restriction $\{i, j \in A\}$ is not monotone in bidders' quality. In fact, under such restriction the ranking of $r_{i,j}(b)$ and $r_{j,i}(b)$ depends on the distributions of buyers' tastes.

[19]Proposition 1 also holds if we relax (A2) to allow dependence between α and ϵ and only require ϵ_i to be i.i.d. conditional on α.

the true number of groups, the group estimation does not show any sign of misspecification bias. Utilizing the discrepancy between the two cases, we devise a consistent group number selection procedure.[20,21]

Note that once the quality group affiliations of sellers are known their identities are no longer important and can be treated as i.i.d draws from a respective quality group. This is because in a simple model[22] quality is the only seller's characteristic that enters buyer's utility function and all other seller-related variables are i.i.d draws conditional on quality group.

4.2 Identification of Quality Levels and Distribution of Buyers' Tastes

We now use the recovered quality group structure of permanent sellers to identify the quality levels and the joint distribution of buyer's tastes and outside option. We begin by outlining our argument heuristically in a simple example when the set of active bidders consists only of permanent sellers.

4.2.1 Simple Case: Permanent Bidders only

Let us consider auctions that attract only two active (permanent) bidders, i, j. The probability that i wins conditional on $B_i = b_i$ and $B_j = b_j$ and $A = \{i, j\}$ is given by:

$$\Pr(i \text{ wins}|B_i = b_i, B_j = b_j, A = \{i,j\}) = \Pr\left\{ \begin{array}{c} \alpha\Delta q_{j,i} + \Delta\epsilon_{j,i} \leq b_j - b_i \text{ and} \\ U_0 - \alpha q_i - \epsilon_i \leq -b_i \end{array} \right\}. \tag{4}$$

The equality is due to the assumed independence of private costs (and hence bids) from U_0, α, and ϵ. In the subsequent exposition we denote the function on the right hand side by $\varphi(b_i, b_j; q_i, q_j)$.

The conditional probability $\Pr(i \text{ wins}|B_i = b_i, B_j = b_j, A = \{i,j\})$ and thus the function $\varphi(B_i, B_j; q_i, q_j)$ can be recovered from the data for pairs of bidders with various quality group configurations. Since B_j and B_i are independent of U_0, α and ϵ, evaluating $\varphi(\cdot, \cdot; q_i, q_j)$ at different realizations of B_i, B_j amounts to evaluating the joint distribution of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}, U_0 - \alpha q_i - \epsilon_i)$ at different points on the support. Thus, provided the support of prices is large relative to the supports of our unobservables, this joint distribution is fully recoverable from data. In derivations below, we apply a well-known deconvolution result (Kotlarski's Theorem[23]) to iden-

[20]The fact that the number of groups is recovered from data ensures that the assumption of quality discreteness is not overly restrictive. Indeed, any continuous distribution can be approximated by a sequence of discrete distributions with finite supports. Therefore, one can obtain as good approximation of the continuous distribution of qualities by a discrete random variable as information in the data would allow if the support of the discrete random variable is not restricted. Note that modeling unobserved heterogeneity using a discrete distribution is common in empirical studies. For examples, see Heckman and Singer (1984), Keane and Wolpin (1997), and Crawford and Shum (2005) to name just a few.

[21]Later in the paper we explain how the result in Proposition 1 can be modified to account for project and seller's observed heterogeneity so that classification is implemented "everything else equal". However, Proposition 1 does not allow for unobserved project heterogeneity to be present. We have some preliminary results that allow extending methodology in this directions and will pursue this issue in the future research.

[22]We explain in Section 4.4 how the simple model can be extended to allow for observable seller characteristics so that identities of sellers can still be viewed as independent draws from a specific quality group.

[23]From Kotlarski (1967) and Theorem 2.1.1 in Rao (1992): Let X_1, X_2, and X_3 be three independent real-valued random variables. Define $Y_1 = X_1 + X_3$ and $Y_2 = X_2 + X_3$. If the characteristic function of (Y_1, Y_2) does not vanish, then the joint distribution of (Y_1, Y_2) determines the distributions of X_1, X_2, and X_3 up to a change

tify the marginal distributions of ϵ_i, α, $U_0|\alpha$ and the quality levels $(q^1, ..., q^K)$ subject to a few normalizations.

We begin by discussing the identification of the distribution of match components, ϵ_i. For this we consider auctions with two active participants who belong to the same quality group. Then, as discussed above the variation in b_i and b_j identifies the joint distribution of $(\Delta\epsilon_{j,i}, U_0 - \alpha q - \epsilon_i)$ which is summarized by $\varphi(b_i, b_j; q, q)$. Applying Kotlarski's theorem to $\varphi(b_i, b_j; q, q)$ identifies the distributions of ϵ_i and $U_0 - \alpha q$ subject to normalization that $E(\epsilon_i) = 0$.[24,25]

Next, to identify the distribution of the buyer's tastes for quality, α, the quality levels $(q^1, ..., q^K)$, and the conditional distribution of outside option, $U_0|\alpha$, we use conditional winning probability in (4) that corresponds to auctions with two active participants who belong to different quality groups, e.g., $q_i = q^{k_1}$ and $q_j = q^{k_2}$ such that $q^{k_1} > q^{k_2}$. As before, the joint distribution of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}, U_0 - \alpha q_i - \epsilon_i)$ is identified through variation in prices. This implies that the marginal distributions of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i})$ and $(U_0 - \alpha q_i - \epsilon_i)$ are identified. The distributions of ϵ_i's and thus of $\Delta\epsilon_{j,i}$ were identified previously, and therefore, could be integrated out from the marginal distribution of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i})$ to obtain the distribution of $\alpha\Delta q_{j,i}$ since ϵ_i, ϵ_j are independent from α. From this, the parameters $\Delta q_{j,i}$ are identified by considering $E[\alpha\Delta q_{j,i}] = \Delta q_{j,i}E[\alpha]$ and imposing a scale normalization $E[\alpha] = 1$. Thus, the quality levels $\{q^1, \cdots, q^K\}$ are identified given a location normalization (e.g. $q^1 = 0$). The marginal distribution of α can be recovered from the distribution of $\alpha\Delta q_{j,i}$ since $F_\alpha(y) = F_{\alpha\Delta q_{j,i}}(\Delta q_{j,i} y)$.

We are now ready to identify the conditional distribution of outside option given buyers' tastes for quality, $U_0|\alpha$. Again, ϵ_i's and $\Delta\epsilon_{j,i}$ are integrated out from the joint distribution of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}, U_0 - \alpha q_i - \epsilon_i)$ to obtain the joint distribution of $(\alpha\Delta q_{j,i}, U_0 - \alpha q_i)$. From the last distribution through a change of variables we can recover the joint distribution of $(\alpha q_j, U_0)$ and therefore of (α, U_0), and $U_0|\alpha$.

The argument in this section is reminiscent of Berry and Haile (2011), in that the distribution of buyers' tastes is identified through the variation in exogenous variable (price) that enters buyers' utility with a coefficient that does not change sign. Next, we present the formal extension of arguments in this section to the case with transitory sellers. The setting with transitory sellers differs substantially from the setting in Berry and Haile (2011) and requires new insights. In particular, the argument for the identification of $U_0|\alpha$ changes non-trivially since this distribution has to be separated from the distribution of transitory bidders' qualities conditional on their prices, $Q_k|b_k$.

4.2.2 Identification with Transitory Sellers

In this section we formally state identification result and outline its proof for the simple case when the set of active bidders consists of two permanent bidders i, j and a transitory bidder k. We provide the general result for the arbitrary numbers of permanent and transitory bidders and the proof in Appendix A.

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of the location.

[24]In this argument, we set $X_1 = \epsilon_j$, $X_2 = U_0 - \alpha q$, $X_3 = \epsilon_i$, $Y_1 = \epsilon_j - \epsilon_i$, $Y_2 = U_0 - \alpha q - \epsilon_i$ and apply Kotlarski's theorem as in previous footnote.

[25]Notice, that this argument identifies the distributions of ϵ_i even if they depend on q.

As before, we consider

$$\Pr(i\,\text{wins}|A^p = \{i,j\},\, A^t = \{k\},\, B_i = b_i,\, B_j = b_j,\, B_k = b_k)$$
$$= \Pr\left(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i} \leq b_i - b_j \,\text{and}\, Y_{i,k} - \epsilon_i \leq -b_i\right)$$

where

$$Y_{i,k} \equiv \max\{U_0 - \alpha q_i \ , \ \alpha(Q_k - q_i) - b_k + \epsilon_k\}. \tag{5}$$

The equality follows from the assumptions that (U_0, α, ϵ) and the C_i's (and therefore the B_i's) are independent; and that participation outcomes are orthogonal to $(U_0, \alpha, \epsilon, C)$.

In order to state the formal result we need to formulate an additional assumption.

(A3) *There exist i, $j \in S^p$ and some b_k such that (i) the characteristic functions of the joint distribution of $\epsilon_j - \epsilon_i$ and $Y_{i,k} - \epsilon_i$, do not vanish; and (ii) the joint support of $(B_j - B_i, -B_i)$ includes the joint support of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}, Y_{i,k} - \epsilon_i)$ conditional on $B_k = b_k$.*

Condition (i) is required for application of Kotlarski's Theorem. Condition (ii) is the higher level support conditions. We demonstrate how this condition could be rationalized by our model in Supplemental Appendix. In principle this condition can also be verified using observable distribution. That is, it holds if there exist extreme values of b_i, b_j in the data-generating process such that the conditional probability reaches 0 or 1.

Proposition 2 *Suppose (A1), (A2) hold. (i) If (A3) holds for some i, j with $q_i = q_j$ and some b_k, then the marginal distributions of ϵ_i and $Y_{i,k}$ given b_k are identified up to a location normalization (e.g. $E(\epsilon_i) = 0$). (ii) If in addition (A3) holds for some i', j' with $q_{i'} \neq q_{j'}$, then the quality differences $\Delta q_{j',i'}$ and the marginal distribution of α are identified up to a scale normalization (e.g. $E(\alpha) = 1$). Further, the conditional distribution of $U_0|\alpha$, and the conditional distribution of $Q_k|b_k$ are jointly identified if $q^1 = 0$.*

The identification of the marginal distribution of ϵ_i, $Y_{i,k}$, α and the constant parameters $q^1, ..., q^K$ follows the steps described in the previous section.

The identification of the conditional distribution of $U_0|\alpha$ and the conditional distribution of $Q_k|b_k$ are somewhat more involved. We now summarize the heuristic arguments for achieving this. To begin with, note that with F_ϵ, $q_{i'}$ and $q_{j'}$ identified and considered known, we can recover the conditional distribution of $Y_{i',k} + \alpha q_{i'}|\alpha$ conditional on given $B_k = b_k$ from the joint distribution of $(\alpha\Delta q_{j',i'} + \Delta\epsilon_{j',i'}, Y_{i',k} - \epsilon_{i'})$ using the arguments outlined in the previous section. By definition,

$$Y_{i',k} + \alpha q_{i'} = \max\{U_0,\, \alpha Q_k - B_k + \epsilon_k\}$$

Let y denote a generic point on the support of $Y_{i',k} + \alpha q_{i'}|\alpha$ given $B_k = b_k$. Under (A2), the three random vectors (U_0, α), ϵ and C are mutually independent (while correlation between α and U_0 is allowed). Hence, the identified conditional distribution of $Y_{i',k} + \alpha q_{i'}|\alpha$ can be summarized by function $\psi(y, \alpha, b_k)$ as follows:

$$\psi(y, \alpha, b_k) = \Pr(Y_{i',k} + \alpha q_{i'} \leq y \,|\, B_k = b_k, \alpha) = \Pr(U_0 \leq y|\alpha) \sum\nolimits_m \tilde{\pi}_m(b_k) F_{\epsilon_k}(b_k + y - \alpha q^m)$$

where $\tilde{\pi}_m(b) \equiv \Pr(Q_k = q^m | B_k = b_k)$. Note that the equality holds due to independence in (A2) and due to the Law of Total Probability.

Next, we can construct a similar equation for the same α, y but $b'_k \neq b_k$. Taking the ratio and re-arranging terms, we get

$$\psi(y, \alpha, b'_k) \sum\nolimits_m \tilde{\pi}_m(b_k) F_{\epsilon_k}(b_k + y - \alpha q^m) = \psi(y, \alpha, b_k) \sum\nolimits_m \tilde{\pi}_m(b'_k) F_{\epsilon_k}(b'_k + y - \alpha q^m), \qquad (6)$$

which is a linear equation in $2K$ unknown weights $\{\tilde{\pi}_m(b_k)\}_{m=1,..,K}$ and $\{\tilde{\pi}_m(b'_k)\}_{m=1,..,K}$.

Evaluating (6) at the same pairs of (b_k, b'_k) but different pairs of (α, y) gives us a linear system of equations in the $2M$ unknown weights. Also included into the linear system are two natural constraints on the vectors of weights: $\sum_m \tilde{\pi}_m(b_k) = 1$ and $\sum_m \tilde{\pi}_m(b'_k) = 1$. The $2M$ weights are then identified, provided the matrix of coefficients in the linear system formed as above has full rank at $2M$. The conditional distribution $F_{U_0|\alpha}$ is then identified from (6).

To summarize, in the auctions where permanent sellers from different groups participate and a permanent seller wins, the changes in the permanent bidder's probability of winning in response to the variation in the prices of permanent bidders identifies the quality levels corresponding to different groups and the marginal distribution of the taste for quality. In addition, the changes in the relationship between the probability of winning and the prices of permanent bidders across different values of the prices of transitory bidders identifies the distribution of transitory bidders' qualities conditional on price as well as the distribution of outside option conditional on buyer's taste for quality. Identification mechanism requires conditioning on the sets of active and potential bidders. Notice that we do not condition on the identities of bidders but rather on the quality group composition of the sets of active and potential bidders.

It only remains to recover the distribution of bidders' private costs. With identification results above, this can be done via an argument similar to that in Guerre, Perrigne and Vuong (2000). We provide the details in the Supplemental Appendix of this paper.

4.3 Estimation of Quality Levels and Distribution of Buyers' Tastes

The primitives of our model are nonparametrically identified as shown in the previous section. However, the complexity of our model makes full nonparametric estimation impractical. Instead, we implement only the classification step nonparametrically and then make parametric assumptions about the distribution of buyers' tastes, (α, U_0), β and ϵ, and proceed to estimate the parameters of these distributions using Generalized Method of Moments (GMM). Our "working horse" moment condition stems from the conditional winning probability of a permanent seller given the sets of active and potential bidders. We start by deriving a convenient representation of this winning probability, discuss computational feasibility of different approaches, and describe moment conditions used in estimation.

4.3.1 Probability of Winning the Auction

We begin by modifying the expression for the probability that a permanent bidder from a particular quality group wins the auction conditional on auction-level competitive structure observable in the data. We summarize competitive structure of an auction by a vector $\mathbf{I}_{A,N}$ which includes the numbers of permanent potential bidders and permanent active bidders by quality group, and the numbers of transitory potential bidders and transitory active bidders. Notice that in our setting this effectively summarizes all relevant information available to the researcher about the sets of potential and active participants.

The relationship between the bid and the quality of transitory sellers is not observable in the data and therefore has to be integrated out. More specifically, for each permanent bidder i from some quality group q^k at each auction l, we have

$$P\{i \text{ wins}|q_i = q^k,\, \mathbf{B}, \mathbf{I}_{A,N}\} = \sum\nolimits_{\bar{q}} \Pr\{i \,\text{wins}\,|q_i = q^k,\, Q_{A^t} = \bar{q}, \mathbf{B}, \mathbf{I}_{A,N}\} P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}, \tag{7}$$

where $\mathbf{B}$ denotes the vector of bids submitted at auction l, and Q_{A^t} denotes the random vector of the transitory bidders' qualities. The summation $\sum_{\bar{q}}$ is over the possible values of Q_{A^t}.[26]

The conditional winning probability $P\{i \text{ wins}|q_i = q^k,\, Q_{A^t} = \bar{q}, \mathbf{B}, \mathbf{I}_{A,N}\}$ is determined by the distribution of buyers' tastes and of buyers' outside option. Here we assume that $\epsilon_{i,l}$ and (α, β, U_0) are distributed according to $F(\epsilon|\theta_1)$ and $F(\alpha, \beta, U_0; \theta_2)$, distributions known up to a set of parameters $(\theta_1,\, \theta_2)$.

Further, $P\{Q_{A_t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}$ is the probability of active transitory bidders' qualities being equal to $\bar{q}$ given the bids and $\mathbf{I}_{A,N}$. We have shown in the preceding sections that the conditional probabilities $P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}$ are non-parametrically identified.[27] For parametric estimation, however, it is convenient to write $P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}$ in terms of objects which can be more easily related to our model (this provides us with better intuition for their parametrization). To this end, note that by Bayes' rule

$$P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\} = \frac{f(\mathbf{B}|Q_{A^t} = \bar{q}, \mathbf{I}_{A,N}) P\{Q_{A^t} = \bar{q}|\, \mathbf{I}_{A,N}\}}{f(\mathbf{B}|\mathbf{I}_{A,N})},$$

where $f(\mathbf{B}|Q^t = \bar{q}, \mathbf{I}_{A,N})$ and $f(\mathbf{B}|\mathbf{I}_{A,N})$ are joint conditional density functions of $\mathbf{B}$ given $(Q_{A^t} = \bar{q},\, \mathbf{I}_{A,N})$ and given $\mathbf{I}_{A,N}$. We utilize the independence of bidders' strategies conditional on $\mathbf{I}_{A,N}$ and apply the Law of Total probability to the denominator to obtain that

$$P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\} = \frac{\left(\prod_{j \in A^t} f(B^t_j|Q_{A^t,j} = \bar{q}_j, \mathbf{I}_N)\right) P\{Q_{A^t} = \bar{q}|\, \mathbf{I}_{A,N}\}}{\sum_q \left(\prod_{j \in A^t} f(B^t_j|Q_{A^t,j} = q_j, \mathbf{I}_N)\right) P\{Q_{A^t} = q|\, \mathbf{I}_{A,N}\}}, \tag{8}$$

where B^t_j and $Q_{A^t,j}$ are the bid and the quality of transitory bidder j. Notice that the distribution of bids of permanent bidders cancels out from the numerator and denominator because this distribution does not depend on the particular realization of the vector qualities of transitory bidders since these qualities remain unknown to permanent bidders. In addition, the conditional distribution of bids for transitory bidder j depends only on his own quality since he does not observe the quality of his transitory competitors.

We can express $P\{Q_{A^t} = \bar{q}|\, \mathbf{I}_{A,N}\}$ further in terms of primitives of the model. To see this, let us consider a simple case of non-strategic entry, i.e. probabilities of becoming an active bidder

[26]Notice that $P\{Q_{A^t} = \bar{q}|q_i = q^k,\, \mathbf{B}, \mathbf{I}_{A,N}\} = P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}$ since $\mathbf{I}_{A,N}$ summarizes all the necessary information about the set of competitors.

[27]Notice that $P\{Q_{A_t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\} = P\{Q_{A_t} = \bar{q}|\mathbf{B}^t, \mathbf{I}_{A,N}\}$. The equality holds because permanent sellers do not observe qualities of transitory sellers – we expand more on this further in the section. Also, from previous section $\tilde{\pi}_{\bar{q}}(\mathbf{B}^t) = P\{Q_{A_t} = \bar{q}|\mathbf{B}^t, \mathbf{I}_{A,N}\}$.

does not depend on the competitive structure of the auction. Then, (assuming also no other permanent active bidders except the winner, and no observed heterogeneity) $\mathbf{I}_{A,N}$ is given by the number of active transitory bidders in auction l, i.e., $\mathbf{I}_{A,N} = |A^t|$. As for $P\{Q_{A^t} = \bar{q}|\,\mathbf{I}_{A,N}\}$, first note that $\{Q_{A^t} = \bar{q}\}$ summarizes the event that the bidders in some set A^t are active and that the vector of their qualities, Q_{A^t}, is realized to be $\bar{q}$. Thus, we can write for each integer m:

$$P\left\{Q_{A^t} = \bar{q}||A^t| = m\right\} = \sum_{a:|a|=m} P\left\{Q_{a,j} = \bar{q}_j \text{ and } D_j = 1,\ \forall j \in a||A^t| = m\right\}.$$

Recall that $D_j = 1$ if player j enters auction l and $D_j = 0$ otherwise. In the expression above $\bar{q}_j$ is the j-th component of $\bar{q}$. Since entry decisions D_j are exogenous and independent across the players, we can rewrite the right hand side sum as

$$\begin{aligned} & \sum_{a:|a|=m} \prod_{j\in a} P\left\{Q_{a,j} = \bar{q}_j \text{ and } D_j = 1||A^t| = m\right\} \qquad (9) \\ = & \sum_{a:|a|=m} \prod_{j\in a} P\{D_j = 1|Q_{a,j} = \bar{q}_j, |A^t| = m\} P\left\{Q_{a,j} = \bar{q}_j||A^t| = m\right\} \\ = & \sum_{a:|a|=m} \prod_{j\in a} P\{D_j = 1|Q_{a,j} = \bar{q}_j\} P\left\{Q_{a,j} = \bar{q}_j\right\} \\ = & \ |\{a : |a| = m\}| \prod_{j\in a_0} P\{D_j = 1|Q_{a,j} = \bar{q}_j\} P\left\{Q_{a,j}\right\}. \end{aligned}$$

The last equality holds because the terms $P\{D_j = 1|Q_{a,j} = \bar{q}_j\}P\left\{Q_{a,j} = \bar{q}_j\right\}$ under the product and summation do not depend on a specific realization of A^t and a_0 is one of the possible such realizations. Further, $P\left\{Q_{a,j} = \bar{q}_j\right\}$ represents the proportion of transitory bidders with quality level of $q^{k(j)}$, such that $\bar{q}_j = q^{k(j)}$, that we earlier denoted by $\pi_{t,k(j)}$. Under non-strategic entry, the probabilities $P\{D_j = 1|Q_{A^t,j} = \bar{q}_j\} = \lambda_{t,k(j)}$ and $P\left\{Q_{A^t,j} = \bar{q}_j\right\} = \pi_{t,k(j)}$ are the primitives of the model.

When the event of becoming active reflects bidders' strategic decision, more work is required to link $P\{Q_{A^t} = \bar{q}|\mathbf{B}, \mathbf{I}_{A,N}\}$ to the bidders' participation strategies. Substituting (9) into (8) we arrive at the result that is stated below for a general case. We provide its full derivation in Appendix C.

Proposition 3 *Under (A1′), (A2′), and (A3),*

$$P\{Q_{A^t} = \bar{q}|\,\mathbf{B}, \mathbf{I}_{A,N}\,\} = \frac{g_{\bar{q}}(\mathbf{B}^t, \mathbf{I}_{A,N})}{\sum_q g_q(\mathbf{B}^t, \mathbf{I}_{A,N})}. \qquad (10)$$

where

$$g_q(\mathbf{B}^t, \mathbf{I}_{A,N}) \equiv \prod_{j\in A^t} \left(\pi_{t,k(j)} f(B^t_j|Q^t_j = q_j,\, \mathbf{I}_N) P\left\{j \textit{ is active}|\, Q^t_j = \bar{q}_j, \mathbf{I}_N\right\}\right).$$

and $\mathbf{B}^t = (B^t_1, ..., B^t_{|A^t|})$.

The probability $P\{Q_{A^t} = \bar{q}|\,\mathbf{B}, \mathbf{I}_{A,N}\,\}$ is now written in terms of the distribution of the sellers' equilibrium bidding and participation strategies as well as the distribution of the primitives such as the transitory sellers' qualities. This probability, together with the parametric specification of the distribution of the buyers' tastes, can be used to express the conditional winning probability in (7).

4.3.2 Computational Feasibility

We construct moment conditions using the expression for the conditional winning probabilities derived above, and estimate the structural parameters using GMM. A conventional approach suggests computing the equilibrium bidding and participation strategies of transitory bidders in (7) for each parameter guess (including a guess at $\{\pi_{t,k}\}_{k=1,\ldots,K}$ when minimizing the GMM objective function. In our setting, however, such an approach is computationally intractable, since it involves solving a large number of auctions with asymmetric bidders where the degree of bidder asymmetry depends on current guess at parameter value. Instead, we choose an approach where the distribution of transitory sellers' bids and the participation strategies conditional on the transitory seller qualities are jointly estimated with the parameters of the distributions of the buyers' tastes, outside option, and the quality levels.

Our argument for non-parametric identification of $P\{Q_{A^t} = \bar{q}|\, \mathbf{B}^t,\, \mathbf{I}_{A,N}\}$ can be extended in a straightforward way to show that the function $g_q(\cdot, \mathbf{I}_{A,N})$ on the right hand side of (10) is also nonparametrcially identified from the data. A similar argument can be applied to establish that data contain enough information to identify $g_q(\cdot, \mathbf{I}_{A,N})$ nonparametrically when the distribution of buyers' tastes and outside option is parametrically specified.

While we adopt a parametric specification of $g_q(\cdot, \mathbf{I}_{A,N})$ for our empirical study, one may, instead, use a semiparametric sieve approach similar to Ai and Chen (2003) that would allow recovering this function nonparametrically. Details in this direction are explained in the Supplemental Appendix.

Under our parametric approach we need to recover parametrized $P\left\{j \text{ is active}|Q_j^t = q_j,\, \mathbf{I}_N\right\}$, $f\left(B_j^t|Q_j^t = q_j, \mathbf{I}_N\right)$ and $\pi_{t,k}$. Working with these objects allows us to make use of further restrictions such as

$$\begin{aligned}\sum_q P\left\{j \text{ is active}|Q_{A^t,j} = q_j,\, \mathbf{I}_N\right\} P\left\{Q_{A^t} = q|\mathbf{I}_N\right\} &= P\left\{j \text{ is active}|\mathbf{I}_N\right\} \text{ or} \\ \sum_q f\left(B_j^t|Q_{A^t,j} = q_j, \mathbf{I}_N\right) P\left\{Q_{A^t} = q|\mathbf{I}_N\right\} &= f(B_j^t|\mathbf{I}_N). \end{aligned} \tag{11}$$

However, we also need additional conditions to separate $f\left(B_j^t|Q_{A^t,j} = q_j, \mathbf{I}_N\right)$ from $P\left\{j \text{ is active}|Q_{A^t,j} = q_j,\, \mathbf{I}_N\right\}$ and $P\{Q^t_{A^t,j} = \bar{q}_j\} = \pi_{t,k(j)}$ within $g_q(\mathbf{B}^t, \mathbf{I}_{A,N})$ since our identification strategy identifies only $g_q(\mathbf{B}^t, \mathbf{I}_{A,N})$ but not its separate components.

The weights $P\{Q_j^t = \bar{q}_j\} = \pi_{t,k(j)}$ can be estimated as parameters of the model. However, we choose to impose the restriction $\pi_{t,k(j)} = \pi_{p,k(j)}$ in estimation since it appears to be plausible in our environment. Notice that $\pi_{p,k},\ k = 1,..,K$, be nonparametrically recovered from the data once the quality group structure is estimated. Imposing this restriction helps us to reduce the burden of separately recovering different components inside $g_q(\mathbf{B}^t, \mathbf{I}_{A,N})$ - now we need to separate only $f\left(B_j^t|Q_{A^t,j} = q_j, \mathbf{I}_N\right)$ from $P\left\{j \text{ is active}|Q_{A^t,j} = q_j,\, \mathbf{I}_N\right\}$ - which arises under fully parametric approach. For this, one could use the expected profit condition that summarizes the optimal participation behavior of different groups of transitory potential bidders. Or one could alternatively use some exclusion restrictions if they are plausible. We include further discussions of this issue in the empirical part of our paper and Appendix E.

4.3.3 Generalized Method of Moments Estimation

We consider two sets of moment conditions. The moments in the first set are based on the expressions from (7) and Proposition 3 and are related to the probability that a permanent seller from the quality group q^k wins the auction under various configurations of the sets of active and potential bidders and for every possible value of q^k. This set of moments additionally includes the expected values of the winning price, of the price differences and squared price differences between the winner and some other permanent bidder, the product of such price difference and the winning price, the product of such price difference and the price of an active transitory bidder, the product of such price difference, the winning price and the price of an active transitory bidder, the product of the winning price and the price of an active transitory bidder, similar moments for characteristics other than price, as well as cross-products of permanent bidders' prices and non-price characteristics.

The second set of moments imposes restrictions implied by (11) on the first and second moment of the respective bid distributions as well as on the probabilities of participation. This set also includes moments associated with the optimality of participation behavior unless exclusion restrictions are used. In some specifications we also use a set of moments related to the probability that project is not allocated. More detailed information about the moments used in estimation is provided in Appendix E.

Under standard regularity conditions, the GMM estimator we use is asymptotically normal with a positive definite covariance matrix. Note that the estimation error due to using the estimated quality groups and the estimated $\{\pi_{p,k}\}_{k=1,\ldots,K}$ does not affect the asymptotic variance matrix because it has a convergence rate that is arbitrarily fast as the number of the auctions increases to infinity due to the finite number of quality groups. The formal definition of our estimator could be found in Supplemental Appendix.

4.4 Extensions

In this section we explain how our simple model can be enriched to allow for strategic auction participation and observable project and seller heterogeneity. All the identification and estimation results can be easily extended to such more general setting.

4.4.1 Endogenous Entry

In this section we extend our model to allow for strategic(endogenous) entry decisions by sellers. Let N_l denote the set of potential bidders for a given auction l. As in previous subsections, we abstract away from auction- and seller-level heterogeneities observed in the data. We explain how such heterogeneity can be introduced into the model and our methodology in the next section. In the empirical analysis we assume that the set of potential bidders is determined by the project characteristics such as type of work, the date of an auction, etc.

A set of potential bidders is partitioned into a set of potential permanent bidders, N_l^p, and potential transitory bidders, N_l^t. Recall, that the qualities of permanent sellers are known to all market participants and considered unknown parameters from a researcher's point of view. The quality of a transitory seller is only known to himself and to the buyer (if this seller decides to enter the auction by submitting a bid). For the researcher and all other market participants, the qualities of transitory potential bidders are summarized by a random vector $Q_{N^t} = \{Q_j : j \in N^t\}$, whose coordinates are i.i.d. draws from some multinomial distribution F_Q with support $\{q^1, ..., q^K\}$ and probability function $\Pr(Q_j = q^k) = \pi_{t,k}$ for $j \in N^t$.

During an auction for project l each potential bidder $i \in N_l^p \cup N_l^t$ observes some private signal, or entry costs, $E_{i,l}$, drawn from distribution F_E and is aware of N_l. More specifically, seller i's information set consists of $E_{i,l}$ and $I_{N,l}$, where the later contains information on the numbers of potential permanent bidders by quality group, and the total number of potential transitory bidders. Given this information set, potential bidder i decides whether to participate in the auction or not. His entry strategy σ_i^E is a mapping from the supports of $E_{i,l}$ and $I_{N,l}$ into $\{0, 1\}$. We denote the entry outcomes by $D_{i,l}$ ($D_{i,l} = 1$ if enters and $D_{i,l} = 0$ otherwise).

Upon entry, an active bidder observes a private cost $C_{i,l}$ for completing the project. As in the basic model, each active bidder i does not observe participation decisions of other potential bidders, and is thus unaware of the composition of the set of active bidders. He then submit a price $B_{i,l}$ based on his information set.

The potential bidders' strategies and PSBNE in this environment can be defined in a usual way. We focus on type-symmetric equilibria in which any pair of participants i, j who are *ex ante* identical (i.e. either "$i, j \in N_l^p$ and $q_i = q_j$" or "$i, j \in N_l^t$") adopt the same strategies. Appendix B provides further details and the proof of the equilibrium existence. It also argues that identification strategies described above remain applicable.

4.4.2 Project and Seller Heterogeneity

We now discuss how to extend the main methodology in Sections 4.1-4.3 to accommodate observable project and seller heterogeneity. In our application, projects differ in several observable dimensions such as the date of posting, the nature of the work, and other specification details. All the project characteristics in our setting are discrete. We, therefore, perform the analysis conditional on observable project characteristics (and consequently also on the set of potential bidders associated with these characteristics).

Further, the sellers differ by their country of origin as well as their recorded performance measures such as reputation scores, delays or instances of conflict. These performance measures may reflect market's information about seller's quality or may be indicative of other service dimensions that are valued by buyers. In any case, all observable seller characteristics (including country) may plausibly be correlated with seller's quality. Therefore, in contrast to a standard differentiated product environment the characteristics of competing sellers cannot be used as instruments in our setting.

Formally, we assume that each seller i is characterized by a vector of non-quality characteristics X_i (reported in data[28]) and a scalar measure of quality q_i. The support of the distribution of qualities among sellers with $X_i = x$ is given by $\{q^k(x) : 1 \leq k \leq K_x\}$ where K_x is the cardinality of the support given x. The proportion of various quality levels among permanent and transitory sellers with $X_i = x$ is $\{\pi_{r,k}(x) : r \in \{p, t\}, 1 \leq k \leq K_x\}$. Finally, we assume that buyers' utility from services provided by (x, q)- seller in an auction indexed by l is:

$$U_{i,l} = \alpha_l q_i + x_i \beta_l - B_i + \epsilon_{i,l}, \tag{12}$$

[28]All sellers' characteristics are discrete.

where β_l reflects buyer's tastes for observable seller characteristics.

Previous arguments hold once conditioning on the vector of non-quality characteristics of potential bidders, provided the required assumptions are satisfied once conditional on this vector. In particular, our classification algorithm is implemented within the subpopulation of sellers characterized by $X_i = x$. The argument for identification of the distribution of β is quite standard and is presented in Supplemental Appendix.

5 Details of Classification Procedure

This section translates the identification ideas presented in the preceding sections into an estimation methodology. While in population the qualities are transitive, the estimated quality group may not be so in small samples, if we use only individually pairwise tests to assign each seller to a quality group. This means that we need to estimate the whole group structure simultaneously. On the other hand, with a reasonably large number of sellers and with more than two quality groups, estimation of a group structure would involve too many pairwise comparisons. This paper suggests a feasible algorithm that classifies the sellers sequentially so that the transitivity of quality groups is maintained and the computational cost is substantially reduced.

5.1 Estimation of Classifications with Known Number of Groups

We consider the case where X_i takes values from a finite set $(x_1, \cdots, x_\Lambda)$. This implies that the set of the permanent sellers can be partitioned into Λ groups, $N_1, \cdots, N_\Lambda$, such that for each $\lambda = 1, \cdots, \Lambda$, and any $i, j \in N_\lambda$, $x_i = x_j$. The analysis in this section is performed conditional on x. For brevity, we omit conditioning on x in the exposition below, so that we simply write N instead of N_λ. Define K_0 to be the number of distinct quality levels among the permanent sellers.

For ease of exposition, we first present the case with the number of quality levels K_0 equal to 2 so that $q_i \in \{\bar{q}_h, \bar{q}_l\}$ for a pair of unknown numbers $\bar{q}_h$ and $\bar{q}_l$. We explain how the algorithm generalizes to the case with $K_0 > 2$ later. Let $N_h \subset N$ be the collection of high quality sellers within group λ and $N_l \subset N$ be the collection of low quality sellers within group λ. We estimate an ordered partition (N_h, N_l) of N in three steps. First, for each $i \in N$, we estimate two ordered partitions: one partition consists of the group of the sellers with higher or equal quality than that of i (denoted by $N_1(i)$) and the rest (denoted by $N \backslash N_1(i)$), and the other partition consists of the group of sellers with lower or equal quality than that of i (denoted by $N_2(i)$) and the rest. Second, among the two ordered partitions, we choose the one that is mostly likely to coincide with (N_h, N_l). Third, we choose i such that the estimated partition associated with this i is most strongly supported by the data.

Our method relies on the estimates of winning probabilities in Proposition 1. Let $W_{i,l}^p \in \{0, 1\}$ be an indicator taking 1 if the i-th seller that is permanent wins at auction l and 0 otherwise. Define $\hat{\delta}_{ij}(b) \equiv \hat{r}_{ij}(b) - \hat{r}_{ji}(b)$, where

$$\hat{r}_{ij}(b) \equiv \frac{\sum_{l=1}^{L} W_{i,l}^p K_h(B_{i,l} - b) 1\{j \notin A_l\}}{\sum_{l=1}^{L} K_h(B_{i,l} - b) 1\{j \notin A_l\}},$$

where $K_h(v) = K(v/h)/h$ for a univariate kernel function K. Then we construct test statistics:

$\hat{\tau}_{ij}^{+} = \int \max\{\hat{\delta}_{ij}(b), 0\}db$, $\hat{\tau}_{ij}^{-} = \int \max\{-\hat{\delta}_{ij}(b), 0\}db$, and $\hat{\tau}_{ij}^{0} = \int |\hat{\delta}_{ij}(b)|db$. We confine the integral domains to the intersection of bids submitted by i and j, and this restriction is omitted from the notation.

We use a bootstrap method to estimate the finite sample distributions of the test statistics. We first construct $\hat{r}_{ij,s}^{*}(b)$ and $\hat{\delta}_{ij,s}^{*}(b)$ using the s-th bootstrap sample, $s = 1, \cdots, B$, and consider the re-centered bootstrap test statistics:

$$\begin{aligned}
\hat{\tau}_{ij,s}^{*+} &= \int \max\{\hat{\delta}_{ij,s}^{*}(b) - \hat{\delta}_{ij}(b), 0\}db \\
\hat{\tau}_{ij,s}^{*-} &= \int \max\{-\hat{\delta}_{ij,s}^{*}(b) + \hat{\delta}_{ij}(b), 0\}db, \text{ and} \\
\hat{\tau}_{ij,s}^{*0} &= \int |\hat{\delta}_{ij,s}^{*}(b) - \hat{\delta}_{ij}(b)|db.
\end{aligned}$$

Using the bootstrap test statistics, we define the bootstrap p-values as follows:

$$p_z^*(i,j) = \frac{1}{B}\sum_{s=1}^{B} 1\left\{\hat{\tau}_{ij,s}^{*z} > \hat{\tau}_{ij}^{z}\right\} \text{ with } z \in \{+, -, 0\}.$$

We proceed in three steps as outlined above.

Step 1: Define

$$\begin{aligned}
\hat{N}_1(i) &= \{j \in N\backslash\{i\} : p_+^*(i,j) \leq p_-^*(i,j)\} \text{ and} \\
\hat{N}_2(i) &= \{j \in N\backslash\{i\} : p_+^*(i,j) > p_-^*(i,j)\}.
\end{aligned}$$

Step 2: We determine now whether seller i has the same quality as those of $N_1(i)$ or $N_2(i)$, i.e., low quality or high quality. If both $\hat{N}_1(i)$ and $\hat{N}_2(i)$ are non-empty,[29] we classify seller i as follows:

$$\begin{aligned}
\text{Take } \hat{N}_l(i) &= \hat{N}_1(i) \text{ and } \hat{N}_h(i) = \hat{N}_2(i) \cup \{i\} \text{ if } \min_{j \in \hat{N}_1(i)} \log p_0^*(i,j) < \min_{j \in \hat{N}_2(i)} \log p_0^*(i,j) \\
\text{Take } \hat{N}_l(i) &= \hat{N}_1(i) \cup \{i\} \text{ and } \hat{N}_h(i) = \hat{N}_2(i) \text{ if } \min_{j \in \hat{N}_1(i)} \log p_0^*(i,j) \geq \min_{j \in \hat{N}_2(i)} \log p_0^*(i,j).
\end{aligned}$$

That is, we put seller i into the high-quality group, if the evidence against the hypothesis that seller i has the same quality as the sellers in group $\hat{N}_1(i)$ is stronger than the evidence against the hypothesis that seller i has the same quality as the sellers in group $\hat{N}_2(i)$.

[29] If $\hat{I}_{1,\lambda}(i)$ is empty, we pick some level $\alpha = 0.05$:

$$\begin{aligned}
\text{Take } \hat{I}_{l,\lambda}(i) &= \varnothing \text{ and } \hat{I}_{h,\lambda}(i) = \hat{I}_{2,\lambda}(i) \cup \{i\} \text{ if } \min_{j \in \hat{I}_{2,\lambda}(i)} p_0^*(i,j) \geq \alpha. \\
\text{Take } \hat{I}_{l,\lambda}(i) &= \{i\} \text{ and } \hat{I}_{h,\lambda}(i) = \hat{I}_{2,\lambda}(i) \text{ if } \min_{j \in \hat{I}_{2,\lambda}(i)} p_0^*(i,j) < \alpha.
\end{aligned}$$

We proceed in a similar way if $\hat{I}_{2,\lambda}(i)$ is empty.

Step 3: For each $i \in N$, we compute the following index:

$$s^*(i) = \begin{cases} \frac{1}{|\hat{N}_l(i)|} \sum_{j \in \hat{N}_l(i)} \log p_+^*(i,j) \text{ if } i \in \hat{N}_h(i) \\ \frac{1}{|\hat{N}_h(i)|} \sum_{j \in \hat{N}_h(i)} \log p_-^*(i,j) \text{ if } i \in \hat{N}_l(i) \end{cases},$$

where $|\hat{N}_l(i)|$ denotes the cardinality of the set $\hat{N}_l(i)$. The quantity $s^*(i)$ indicates the weakness of the likelihood that i is classified into her right quality group. Then choose i^* that minimizes $s^*(i)$ over $i \in N$, and let $\hat{N}_h = \hat{N}_h(i^*)$ and $\hat{N}_l = \hat{N}_l(i^*)$. We take $\widehat{\mathcal{C}} = (\hat{N}_h(i^*), \hat{N}_l(i^*))$ as an estimated classification of players into two quality groups.[30]

The constructed classification can be justified as follows. We introduce the metric of classification discrepancy: for any two different classifications $\mathcal{C}_A = (N_h^A, N_l^A)$ and $\mathcal{C}_B = (N_h^B, N_l^B)$, $N = N_h^A \cup N_l^A = N_h^B \cup N_l^B$, we define

$$\Gamma(\mathcal{C}_A, \mathcal{C}_B) = \max \left\{ \#(N_h^A \Delta N_h^B), \#(N_l^A \Delta N_l^B) \right\},$$

where $N_h^A \Delta N_h^B$ denotes the set difference between N_l^A and N_l^B.

Let $N_h = \{i \in N : q_i = \bar{q}_h\}$ and $N_l = \{i \in N : q_i = \bar{q}_l\}$ and write $\mathcal{C} = (N_h, N_l)$. Also, we write $\widehat{\mathcal{C}} = (\hat{N}_h, \hat{N}_l)$. We show that the estimated classification $\widehat{\mathcal{C}}$ is *consistent* under regularity conditions.

THEOREM 1: *As* $L \to \infty$,

$$P\left\{\Gamma(\widehat{\mathcal{C}}, \mathcal{C}) \geq 1\right\} \to 0.$$

The generalization of the procedure to the case of $K_0 > 2$ with K_0 known can proceed as follows. First, we split N into $\hat{N}_h$ and $\hat{N}_l$ using the algorithm for $K_0 = 2$. Then we find a minimum value (denoted by $\hat{p}_h$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$, and $i, j \in \hat{N}_h$, and a minimum value (denoted by $\hat{p}_l$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$, and $i, j \in \hat{N}_l$. If $\hat{p}_h < \hat{p}_l$, we split $\hat{N}_h$ into $\hat{N}_{hh}$ and $\hat{N}_{hl}$ using the same algorithm for $K_0 = 2$, and otherwise, we split $\hat{N}_l$ into $\hat{N}_{lh}$ and $\hat{N}_{ll}$ using the same algorithm for $K_0 = 2$. We repeat the procedure. For example, suppose that we have classifications $\hat{N}_1, \cdots, \hat{N}_{k-1}$ obtained. For each $r = 1, \cdots, k-1$, we compute the minimum value (say, $\hat{p}_r$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$ and $i, j \in \hat{N}_r$, and then select its minimum (say, $\hat{p}_{r^*}$) over $r = 1, \cdots, k-1$. We split $\hat{N}_{r^*}$ into $\hat{N}_{r^*h}$ and $\hat{N}_{r^*l}$ using the algorithm for $K_0 = 2$ to obtain a classification of N into k groups. We continue until the groups become as many as K_0.

5.2 Consistent Selection of the Number of Groups

The methodology outlined above assumes that we know the exact number of groups. To accommodate the situation with real life data without knowledge of the number of the groups, we offer

[30] There may be alternative ways to obtain estimators of the quality partition. One way is to fix i and apply hypothesis testing to the null hypothesis that $q_i \leq q_j$. This essentially boils down to comparing the p-value $p_z^*(i,j)$ with a certain level of the test. One unattractive feature of this alternative procedure is that the result can be different depending on whether the null hypothesis is taken to be $q_i \leq q_j$ or $q_i \geq q_j$. This is because the conventional hypothesis testing procedure treats the null hypothesis and the alternative hypothesis asymmetrically. Hence in this paper, instead of comparing a p-value with a fixed level of the test, we compare a p-value with an alternative p-value, to capture the symmetry of the comparison.

a method of consistent selection of the number of groups. We suggest that the number of groups should be selected to minimize the criterion function that balances a measure of goodness-of-fit that captures a misspecification bias versus a penalty term that penalizes overfitting. The goodness-of-fit measure is based on the variance test approach.

Given an estimated classification $N = \cup_{k=1}^{K} \hat{N}_k$ with K groups, let

$$\hat{V}_k(K) = \left| \min_{i,j \in \hat{N}_k} \log p_0^*(i,j) \right|,$$

for each $k = 1, \cdots, K$.

Suppose that K_0 is the true number of groups. Let $g(L) \to \infty$ be such that $g(L)/\sqrt{L} \to 0$ as $L \to \infty$. Then, define

$$\hat{Q}(K) \equiv \frac{1}{K} \sum_{k=1}^{K} \hat{V}_k(K) + K g(L).$$

We select K as follows:

$$\hat{K} = \mathrm{argmin}_{1 \le K \le N} \hat{Q}(K).$$

The following theorem shows that this selection procedure is a consistent one.

Theorem 2: $P\{\hat{K} = K_0\} \to 1$ as $L \to \infty$.

The consistency result stems from two auxiliary facts. First, $\frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K) = O_P(1)$, as $L \to \infty$, if $K \ge K_0$. This measures the asymptotic behavior of the goodness-of-fit measure when the classification is weakly finer than the true classification. Since $Kg(L) \to \infty$ as $L \to \infty$, the minimization of $\hat{Q}(K)$ over K leans toward a lower choice of K that is closer to K_0. On the other hand, if the classification is strictly coarser than the true classification, i.e., $K < K_0$, the quantity $\frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K)$ diverges at a rate faster than $g(L)$, as $L \to \infty$. In this case, the minimization of $\hat{Q}(K)$ over K excludes K such that $K < K_0$ for large samples. Thus we obtain the consistency of $\hat{K}$.

We describe methods of constructing confidence sets for the estimated classifications as well as report the results of Monte Carlo study investigating the algorithm's small sample performance in Supplemental Appendix.

6 Empirical Results

6.1 Data Description

Our data include information on close to 600,000 projects that involve participation from around 50,000 different sellers. For every project, we observe the type of work, the approximate size of the project, the time requirements, and the location of the buyer. We also observe all bids submitted, the identity of the winner, and measures of the winner's subsequent performance.

The projects fall into several broad classes such as platform programming, databases, graphics programming and website design. The work is then further divided into finer categories within these classes. For example, one of the recurrent requirements is the specification that a particular programming language should be used. We focus on graphics-related programming projects in our analysis. The projects in this set involve programming computer games, computer-generated animation, and media-related programming. Our decision was mostly motivated by sample size considerations. However, this is also a highly specialized segment of the market. The related work is very sophisticated and is done exclusively by hard-core professionals. This, therefore, is an environment where the seller's quality is likely to matter. On the other hand, this environment perhaps would be characterized by lower variation in provider qualities as opposed to the less skilled types of projects.

Table 4 provides some descriptive statistics for projects in our data set. Each row of the table summarizes a marginal distribution of the correspondent variable. The table shows that a sizable number of the projects are very small (below $100). On the other hand, some of the projects are quite big (above $1000). We focus on the medium to medium-large size projects (between $100 to $700).[31] The projects are fairly short: the deadline for the majority of the projects is between one to three weeks. Median number of sellers submitting bids for a project is six while median number of permanent bidders is three. However, about 10% of projects receive more than 18 bids (10 from permanent bidders). The projects with a large number of bids tend to be small.

Table 4: Data Summary Statistics

	25%	50%	75%	90%
Project Characteristics				
Size	$150	$250	$500	$1000
Duration	5	10	14	21
Number of Bidders	4	6	11	18
Number of Permanent Bidders	1	3	6	10
Permanent Sellers' Characteristics				
Experience	75	100	150	250
Average Score	9.7	9.87	9.95	10
Arbitrations	0	0	0	1
Delays	0	0	0	1
Number of Projects	32,679			

The results in this table are based on a sample of projects with graphics-related programming. Duration of project is measured in days. Each row summarizes inverse cumulative function of the corresponding variable. Experience is defined as the number of completed projects.

The table also summarizes the characteristics of permanent sellers. It shows that a median permanent seller has completed 100 projects, while 10% of sellers completed 250 or more projects. The distribution of the average reputation scores appears to be quite tight. A median permanent

[31] The following anecdotal insight may help to put the size into the right perspective. One of the authors used this market to procure programming services: the project that costs $200 in this on-line market was quoted at $800 in the off-line programming market in Philadelphia.

seller has an average score of 9.87, while less than 25% have an average score below 9.7 or above 9.95. Similarly, a median permanent seller was never involved in an arbitration or had a delay. However, less than 10% of permanent sellers were involved in at least one arbitration or had at least one delay.

We group sellers into country groups by geographic proximity and similarity of language and economic conditions. We end up with seven country groups: North America (USA and Canada), Latin America, Western Europe, Eastern Europe, Middle East and Africa, South and East Asia, Australia (grouped with New Zealand). In our data North America, Eastern Europe and South or East Asia account for the majority of submitted bids.

We now turn to the discussion of the estimation results. We first summarize estimates from our classification procedure, then we discuss the parametric estimates of the buyers' tastes and sellers' quality levels as well as the bidding and participation strategies of transitory bidders.

6.2 Empirical Results: Classification

In this section we summarize the results of the group structure estimation. Classification algorithm is applied to the set of permanent participants specializing in graphics-related programming. For each seller we discard the first year of his tenure and only use observations that correspond to the later years of his career with on-line market.

We assume that the buyer cares about the seller's quality, price, and the seller's covariates such as reliability and country affiliation. We specifically distinguish between the seller's quality and reliability. In our environment quality reflects the seller's ability to handle complex and not fully specified jobs and his ability to deliver a product of superior quality. In contrast, reliability measures the likelihood that he completes the job if engaged, that he is on-time, maintains regular communication with the buyer, is responsive to buyer's requests, etc. We believe that the number and value of reputation scores reflects seller's reliability.[32,33] The seller's country affiliation can proxy for things such as convenience of working with a given seller related to time difference, the likelihood of language proficiency, and work culture.

All permanent sellers have a high number of ratings, therefore, we assume that the exact number of ratings is not important. We divide all the sellers into three cells according to the average reputation score: (cell 1) average reputation score less than 9.7, (cell 2) average reputation score above 9.7 and below 9.9, (cell 3) average reputation score above 9.9. This results approximately in an allocation of 30%, 30%, and 40% across cells.

The classification index is constructed for the pair of sellers on the basis of projects where they both belong to the set of potential bidders. Our data do not contain information on the set of potential sellers for a specific project. In our analysis we assume that the set of potential sellers for project l consists of all sellers who were active in the market (i.e., submitted bids or sent messages to buyers) during the week when project l was posted and who are qualified for the type of work indicated for project l (i.e., they bid for similar projects in the past). The pair-wise nature of our index does not have strong implications for our sample. We are able to compute an index for each pair of sellers within each of our cells. We have also experimented

[32] The number of scores is highly correlated with the number of projects completed. It, therefore, may also reflect seller's experience. On the other hand, the value of the reputation score may include information about the quality of work but also reflects whether buyer was satisfied with experience of working with this seller.

[33] We have also verified robustness of our results by repeating the analysis while including the number of arbitrations and delays as additional measures of reliability. The results remain virtually unchanged.

with alternative definitions of the sets of potential sellers. The results of the classification remain stable even with different definitions.

We follow the steps described in the Section 5. That is, we start by estimating a group structure for a range of the number of groups. We then apply a criterion function to select the structure with the number of groups most supported by the data. For this structure we then compute confidence sets. Table 8 in Supplemental Appendix demonstrates steps 1 and 2 for the group of Eastern European sellers with a medium level of average reputation score.

Table 5 reports the estimated group structures with corresponding confidence sets for cells of North American, Eastern European and East Asian sellers. We estimate multiple quality groups in each cell and the confidence sets associated with each group structure are quite tight. It is difficult to draw any substantive conclusions about the quality distribution on the basis of these results since classification into groups is ordinary and does not allow for comparison of levels across countries or reputation scores. We note here that even the cells that correspond to a very narrow range of reputation scores (such as medium or high reputation scores) allow for a non-trivial number of quality groups. Also, mass allocation between quality groups differs across cells. We defer the more interesting substantive inference to the section on the results of the parametric estimation.

6.3 Empirical Results: Parametric Estimation

In this section we present the results of the parametric analysis. We begin by summarizing our specification and the exact set of moments used in estimation. Next, we discuss the estimates of the objects of interest: parameters of buyers' tastes distribution, quality distributions for a range of covariate values, as well as sellers' bidding strategies and recovered cost distributions.

6.3.1 Parametric Specifications

As we stated in the previous section, we assume that buyers' utility from selecting a specific seller depends on the seller's quality, price quote, and country group affiliation as well as performance-related indicators such as the number of scores, and the average reputation score. We modify the utility specification for the purpose of estimation. More specifically, we divide the expression for the utility function by the quality coefficient α. This obtains a utility function specification that is often used in the estimation of differentiated product models:[34]

$$\tilde{u}_{li} = q_i(x) + x_i\tilde{\beta}_l - \tilde{\alpha}_l b_{li} + \tilde{\epsilon}_{li}.$$

Here $q_i(x)$ plays the role of a product-level unobservable that was first introduced into the differentiated products studies by Berry, Levinsohn, and Pakes (1995), and Nevo (2001). Further, we assume that utility errors, $\tilde{\epsilon}$, follow the Extreme Value Type I distribution with standard error σ_ϵ, while taste parameters α and β, and buyer's outside option are assumed to be distributed according to the normal distributions $N(\mu_{\alpha,U_0}, \Sigma_{\alpha,U_0})$, and $N(\beta_0, \Sigma_\beta)$, respectively.[35] We impose

[34]We could be worried about such re-parameterization in the case when zero belongs to the support of α. However, this would only mean that infinity belongs to the supports of $\tilde{\alpha} = \frac{1}{\alpha}$, $\tilde{\beta} = \frac{\beta}{\alpha}$, and $\tilde{\epsilon} = \frac{\epsilon}{\alpha}$, the case that can be easily accommodated.

[35]Strictly speaking, the distribution of α should have been chosen to have a non-negative support. However, we estimate the standard error of this distribution to be quite small so that this assumption does not make any

Table 5: Estimated Quality Groups by Supplier Covariates

Country Group	Average Score	Total Number of Suppliers	$Q = L$	$Q = M$	$Q = H$
North America	low	12	4	8	
			(6)	(10)	
North America	medium	13	4	9	
			(6)	(11)	
North America	high	17	12	5	
			(13)	(6)	
Eastern Europe	low	18	6	12	
			(8)	(14)	
Eastern Europe	medium	52	33	12	7
			(37)	(14)	(9)
Eastern Europe	high	83	6	65	12
			(7)	(69)	(15)
East Asia	low	91	62	18	11
			(68)	(22)	(13)
East Asia	medium	66	6	53	7
			(8)	(57)	(9)
East Asia	high	58	50	8	
			(53)	(11)	

This table shows the estimated group structure and a consistently selected number of groups for each cell determined by covariate values. Column 3 indicates the total number of the suppliers in the cell. Columns 4-6 report the size of the estimated quality group. The size of the corresponding confidence set with 90% coverage is reported in parenthesis. Note that the confidence set with the level (1 - α) for a given quality group is defined to be a random set whose probability of containing this quality group is ensured to be asymptotically bounded from below by (1-α).

the normalization assumptions implied by our identification argument. That is, we normalize the expected value of ϵ to be equal to zero, the expected value of α to be equal to one, and one of the quality levels (quality level 1 of the low average score group, the South and East Asian country group) to be equal to zero. We, therefore, aim to estimate the vector of parameters $\theta = \{\sigma_\epsilon, \sigma_\alpha, \beta_0, \Sigma_\beta, \{q_x\}\}$ where $\{q_x\}$ is the vector of quality levels that correspond to the quality groups recovered in the previous section.

We assume that transitory and permanents sellers' bid distributions are well approximated by normal distributions $N(\mu_{B^t}, \sigma^2_{B^t})$ and $N(\mu_{B^p}, \sigma^2_{B^p})$,[36] respectively. The means of the bid distribution depend on the seller's quality group, number of reputation scores (projects completed), and average reputation score, and on the number of potential competitors by quality group. We allow the effect of the average reputation score to vary flexibly with the number of scores. Notice that the bid distribution of transitory sellers depends both on the current average score and the long-run average score through the group structure. This is because the long-run average score

practical difference. The same comment applies to our assumption on the distribution of bids below.

[36]See the comment for the distribution of α above.

correctly reflects the seller's true reliability. However, it is not observed in the data for transitory sellers. Therefore, the buyer has to base his expectation of the long-run average reputation score on contemporaneously available measures when awarding the project. This, in turn, implies that transitory bidders would incorporate their current average scores into their bids. We explain how we link the transitory seller's long-run average score group to his current performance below.

Similarly, we approximate permanent and transitory bidders' respective probabilities of participation by normal distribution functions that depend on linear indices of the seller's quality group, number of reputation scores (projects completed), and average reputation score, and on the number of potential competitors by quality group. As in the case with the bid distribution, we allow the effect of the average reputation score to vary flexibly with the number of scores.

We have estimated the quality groups for permanent sellers conditional on country affiliation and long-run average reputation scores. The majority of transitory sellers complete only one or two projects. Hence, their long-run average reputation scores are not observed in the data. We assume that buyers use public information to form beliefs about the probability that a beginning seller with a given number and sum of scores and a given quality level belongs to a particular long-run average score group. We recover these beliefs non-parametrically using beginning of career and long-run data on permanent bidders. We use these beliefs to form the expected utility that the buyer derives from transitory sellers.

The detailed list of moment conditions used in estimation and the discussion of exclusion restrictions can be found in Supplemental Appendix.

6.3.2 Quality and Other Attributes as Determinants of Buyer's Choice

Table 6 reports the estimated coefficients of the buyers' utility function and quality levels. All estimates have the expected signs and small standard errors. In the estimation the prices are normalized by the project size; therefore, all coefficients represent the percentage mark-up over the project size that an average buyer would be willing to pay for the unitary increase in the corresponding covariate.

Notice that the estimated variance of ϵ is quite small, which indicates that the seller' covariates indeed play an important role in our environment in comparison to stochastic or unexplained factors. The price coefficient α has a comparable variance. Nevertheless, the price component plays a more important role than ϵ since it is additionally multiplied by price.

The last panel of the table reports the estimated quality levels across covariate cells. The estimated levels have the expected sign and are increasing according to group ranking. The differences across quality levels are substantial in magnitude. In addition, the model with quality is capable of explaining 70% of buyers' choices in comparison to the 25% that the model without quality could explain. These things indicate that quality plays an important role in our environment.

Next, we observe that the quality levels are consistent across covariate cells. There appears to be roughly three quality levels present in this market, with the lowest normalized to be around zero, the medium quality level estimated to be somewhere in the range 0.1-0.3, and the highest quality level is between 0.45-0.68. The exact levels differ across country groups with Eastern Europe characterized by the highest values for each quality level and North America characterized by the lowest "high" quality levels.

Having established that the quality levels are very similar across covariate groups, we can conclude based on the results from the previous section that there exist important differences in the distribution of quality mass across covariate levels. In particular, North America is missing a middle quality level, whereas the lowest average score cell for Middle Europe and the highest average score cell for South and East Asia are missing the lowest quality levels. Similarly, the medium score cell for Eastern Europe allocates the most mass to the lowest and medium quality levels, whereas the highest score cell allocates the most mass to the medium and high quality levels. We observe similar regularities in the case of South and East Asia. Hence, the distribution of qualities varies significantly with covariate values. That finding underscores the importance of using our methodology, which allows for such dependence, as opposed to a mixture methodology that would have to impose the restriction that the distribution of unobserved heterogeneity is orthogonal to other variables that enter utility function.

Country and long-run average reputation score appear to have independent effects on the buyer's utility. These effects, however, are rather small relative to the differences in quality levels. For example, an average buyer would be willing to pay almost 9% more of the project size, $(0.507-0.413 = 0.094)$, to obtain the service of a high-quality North American seller with a high reputation score rather than a high-quality North American seller with a low reputation score. Similarly, an average buyer would be willing to pay 14% more of the project size, $(0.668-0.544) = 0.124$, to hire a medium score, high-quality supplier from Eastern Europe rather than a medium score, high-quality supplier from South or East Asia.

We estimate that the number of reputation scores and an average reputation score matter for beginning or transitory bidders in a statistically significant way. For example, at any quality level, having no reputation scores bears a negative premium of close to 2%. On the other hand, having a positive but small number of scores erodes this negative premium to zero. The average reputation score does not appear to be important when the number of scores is really small. However, the difference between 9 points and 10 is rewarded with a 5% premium if the number of scores is moderate. This is comparable to the 7% premium documented above for the case of a long-run average reputation score that corresponds to the large number of scores.

Additionally, we estimate that buyers participating in this market gain about 53% improvement in utility relative to their outside option on average. This reflects the gain in utility generated by access to larger markets and better information and search technology enabled by the Internet.

6.3.3 The Role of Reputation Scores

Our empirical results contribute to a better understanding of the information and enforcement issues studied in the literature on Internet auctions. Main questions in this literature are whether consumers are able to obtain credible information about the product sold and what mechanism incentivizes sellers to exert higher effort. One possibility emphasized in the literature is that reputation score proxies for the quality of the seller, or, in other words, it serves as an informative device that counteracts adverse selection problems. On the other hand, it may represent an enforcement mechanism that eliminates moral hazard issues. The emerging consensus in the literature seems to indicate that the latter role is probably more relevant (e.g., Cabral and Hortacsu (2010)). Our empirical results appear to reinforce this consensus, as we explain below.

First, we estimate a non-trivial quality heterogeneity within a narrow band of reputation scores, which suggests that the reputation score predicts seller's quality very imperfectly. Second, we estimate that the bid distribution of transitory bidders significantly depends on the bidder quality in addition to all the covariates. Recall that this object is recovered from the variation in buyers choices since no direct link between transitory seller's bids and his quality is observed in the data. This suggests that the buyer must be informed about quality either directly or

Table 6: Buyers' Tastes and Quality levels

Variable			Coefficient	Std.Error
$\log(\sigma_\epsilon)$			-0.315**	0.04
$\log(\sigma_\alpha)$			-0.298**	0.01
$\log(\sigma_{U_0})$			-0.179**	0.076
σ_{α,U_0}			0.142	0.093
no ratings			-0.022**	0.007
$0 < \text{ratings} \leq 3$			-0.008**	0.002
$3 < \text{ratings} \leq 10$			-0.003	0.005
Average Score*$D_{0<ratings\leq 3}$			-0.004	0.006
Average Score*$D_{3<ratings\leq 10}$			0.033**	0.011
North America,	low score,	Q=1	-0.016**	0.007
North America,	low score,	Q=2	0.413**	0.009
North America,	medium score,	Q=1	-0.016**	0.008
North America,	medium score,	Q=2	0.433**	0.008
North America,	high score,	Q=1	-0.016**	0.003
North America,	high score,	Q=2	0.507**	0.004
Eastern Europe,	low score,	Q=1	0.263**	0.003
Eastern Europe,	low score,	Q=2	0.625**	0.005
Eastern Europe,	medium score,	Q=1	-0.103**	0.005
Eastern Europe,	medium score,	Q=2	0.255**	0.003
Eastern Europe,	medium score,	Q=3	0.672**	0.009
Eastern Europe,	high score,	Q=1	-0.107**	0.006
Eastern Europe,	high score,	Q=2	0.263**	0.005
Eastern Europe,	high score,	Q=3	0.668**	0.004
South and East Asia,	low score,	Q=1	0.000	
South and East Asia,	low score,	Q=2	0.089**	0.008
South and East Asia,	low score,	Q=3	0.449**	0.008
South and East Asia,	medium score,	Q=1	-0.019**	0.003
South and East Asia,	medium score,	Q=2	0.105**	0.007
South and East Asia,	medium score,	Q=3	0.544**	0.006
South and East Asia,	high score,	Q=1	0.105**	0.004
South and East Asia,	high score,	Q=2	0.556**	0.007

The results are based on the dataset consisting of $11,300$ projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

indirectly through signaling. It appears thus that information problems play only a limited role, at least in the segment of the market we study.[37] The reputation scores are nonetheless valued by the buyer, which must indicate that they fulfill their literal role, namely, that of creating reputation. In summary, evidence from the data appear to point more toward the role of reputation scores as addressing moral hazards rather than adverse selections.

6.3.4 Pricing Strategies, Cost Distributions and Quality Heterogeneity

Tables 9, and 10 in Supplemental Appendix report the estimated coefficients for bid distributions and participation probabilities. These coefficients are difficult to interpret without the context of a pricing game. We use them to comment on the performance of our estimation procedure and model fit.

Our estimates indicate a statistically significant relationship between the transitory sellers' mean bids, participation probabilities and quality levels. Further, the estimated coefficients for the permanent sellers' bid distributions and participation probabilities are very similar in sign and magnitude to the coefficients from the transitory sellers' bid distribution and participation probabilities. Recall that the transitory sellers' bid distribution is estimated jointly with the utility function parameters from observed buyers' choices via the set of moments that exploit the structure of our model and proposed identification strategy. In particular, there is no direct link between the transitory seller's bids and his quality level. Our estimates, therefore, support our assumption that the quality of the transitory bidder is observable to buyers.

We rely on first order condition from the permanent bidder optimization problem to compute the inverse bid functions, $\xi(b|(q,x))$, for sellers with various affiliations. More specifically, we compute inverse bid function as

$$\xi(b_i|(q,x)_i) = b_i - \frac{P(i\,\text{wins}\mid b_i;\sigma^E_{-i},\sigma^B_{-i})}{\frac{\partial}{\partial b}P(i\,\text{wins}\mid b;\sigma^E_{-i},\sigma^B_{-i})|_{b=b_i}}.$$

The details can be found in Supplemental Appendix. The distributions of the seller's costs are then recovered by combining the bid distributions and the inverse bid functions:

$$F_C(c|(q,x)) = F_B(\xi^{-1}(c|(q,x))|(q,x)).$$

Figure 2 in Supplemental Appendix depicts the estimated permanent sellers' bidding functions for North America, Eastern Europe, and South and East Asia respectively. Similarly, figure 3 in Supplemental Appendix shows the estimated densities of the cost distributions across country groups and across average reputation score levels. The estimated bid functions are increasing in costs, which is consistent with the theoretical predictions for the environment with private values. The graphs show that the mark-up over sellers' cost changes very little with cost level and, in fact, for some groups increases as costs reach the upper end of the support. This feature arises because the buyer's choice is based in part on a purely stochastic (from the seller's point of view) component, ϵ. As the seller's costs increase and therefore his ability to compete on price decreases, his probability of winning increasingly depends on the realization of the ϵ component, which in turn makes his bidding strategy less aggressive. As should be expected, this "gambling effect" appears to be most pronounced in the pricing strategies of lower quality

[37]Relatedly, Lewis (2011) finds little evidence of adverse selection in the context of the e-Bay auto market.

levels. In general, stochasticity plays an important role in our environment: sellers are uncertain about buyers' tastes as well as their actual competition. This accounts for the relatively large mark-ups we document in our environment.

The depicted cost distributions are based on the estimated bid distributions and inverse bid functions for permanent bidders. The estimated project cost distributions are typically "increasing" in sellers' quality. More specifically, the cost distribution of the high-quality group is always shifted to the right relative to the distribution of the medium-quality group. However, the low-quality group often has costs that are comparable to or even higher than the costs of the high-quality group. This indicates substantial costs heterogeneity unrelated to quality that characterize the participants in this market.

Notice further that the estimated project cost distributions appear to have substantially lower variances relative to the variance of the bid distributions. Thus, our model is capable of rationalizing the highly variable pricing environment through reasonably tight cost distributions. The "gambling" property of the bid functions described above explains this effect. Indeed, convexity or increasing mark-up near the end of the support induces high variance in sellers' prices and also explains the presence of really high bids in this environment. Thus, again our modeling choice for buyers' preferences appear to work well in this environment.

We assess the magnitude of the entry costs using a simple model of entry such that (a) entry cost constitutes the seller's private information, (b) entry cost is orthogonal to the seller's cost of completing the project, (c) the cost of completing the project is not observed at participation decision.[38] Under this model, the observed probability of participation satisfies the equation

$$F_S(E[\pi(q,x)]) = \Pr(i \in A(x,q)),$$

where $F_S(.)$ denotes the distribution of the entry costs and $\pi(q,x)$ is an ex-ante expected profit.

We estimate the mean and standard deviation of entry costs distribution by fitting the truncated normal distribution (truncated at 0) to the set of points implied by the ex-ante expected profit and the probability of participation values for various covariate cells and quality groups. The estimated value for the mean and standard deviation of the entry costs are 0.032 and 0.077 respectively. That is, entry costs roughly correspond to 7% of the project cost on average. This number is slightly higher than documented in other markets.[39] The relatively large entry costs estimated in this market may reflect the fact that active bidding for a project involves substantial interaction with the buyer and possibly preparation of supplementary materials.

7 Analysis of Economic Policies and Regulations

In this section we use the estimates from the on-line market for programming services to assess the impact of various policies and regulations. In particular, we first study the effects of occupational licensing, a policy used in many markets for professional services. Second, we analyze the costs to the government from using the standard as opposed to multi-attribute auctions in service procurement. Due to space constraints, this analysis is necessarily somewhat stylized. It clearly illustrates, however, that knowledge of the structural model parameters is essential

[38]The details of similar models can be found in Krasnokutskaya ans Seim (2011) and Li and Zheng (2009).

[39]Studies of the US highway procurement market have estimated entry costs to be around $2-5\%$ of the engineer's estimate.

for understanding the effects of different policies. It also yields new insights into possible forces underlying sometimes puzzling effects observed in the data.

7.1 The Effects of Occupational Licensing

The stated rationale for occupational licensing is to ensure that the quality of services purchased by consumers exceeds a chosen lower bound.[40] Accordingly, we model licensing requirements as the ability of the licensing authority to restrict participation in the market to sellers whose quality is above a set threshold.[41]

The results of this experiment are summarized in Table 12 in Supplemental Appendix. Column (1) presents various statistics from the market without licensing. In Column (2) the results correspond to the licensing regime that restricts market participation to medium- and high-quality sellers. In Column (3) licensing restrictions are assumed to be so severe that only high-quality sellers are allowed to participate in the market. Comparing Columns (1) and (2), we observe that the introduction of licensing leads to an increase in average price paid in the market (by about 5%) which is associated with the decline in the expected buyers' utility (on average by 8.5%). This effect reflects buyer's disutility from paying higher price per unit of quality (which maybe greater than the dollar amount increase if buyers are very price-sensitive). The expected utility of all types of buyers declines whereas the buyers with low preference for quality are hurt the most and many of them are now priced out of the market. In addition, the increase in the profits of participating sellers is not sufficient to compensate for the decline in buyers' utility and loss of profit to low-quality sellers, so overall surplus generated in this market declines as well (by about 4%). Interestingly, the average number of sellers does not change very much so these effects are driven mostly by the reduction in the variety rather than by the pure "total number of participants" effect.

Another interesting feature of the results is that the market share of high-quality sellers is barely affected. In fact, if we compute market shares conditional on the realized set of active bidders containing maximum number of quality levels (three before and two after the introduction of licensing) we observe that the share of high-quality sellers in such auctions actually declines (is re-allocated to medium-quality sellers) by about 10%. Thus, the average quality purchased in such auctions may decline as a result of licensing. Small positive change in the aggregate market share of high-quality sellers is explained by the increase in the share of the auctions where only high-quality bidders participate and thus win. However, a slight variations in the numbers and costs of potential bidders across quality levels can induce overall decline even in the aggregate market share of high-quality bidders. In short, licensing may result in marginalization of high quality and our model is capable of generating such an effect for the realistic values of structural parameters.

The descriptive empirical studies have long documented the ambiguous effects of licensing on

[40]The desirability of this restriction is motivated by the assumption that consumers are not well informed about the quality of service providers. We find no evidence that this is the case in our market. Our findings accord well with the evidence from other online markets. Even looking more broadly at all service markets in the US, Krueger and Kleiner (2013) find that workers do not earn significantly more conditional on their characteristics even after they are certified to be of high quality by the government. This is suggestive of the fact that government certification does not reveal new information to market participants.

[41]In this analysis we abstract away from characteristics other than quality and focus on Eastern European permanent sellers with medium reputation scores as potential auction participants. We set the number and the composition of the set of potential bidders to match these statistics for a typical auction.

quality with many of them finding a significant negative impact.[42] However, the literature so far has found it difficult to rationalize these effects theoretically. A possible explanation offered for this effect refers back to a decline in the number of market participants which leads to a decline in their effort if moral hazard is present. Our analysis suggests that a different mechanism may be able to produce this effect.

Notice that the results of this analysis depend importantly on the fact that sellers' costs vary across projects and that this variation constitutes sellers' private information.[43] An important property of our environment which is responsible for the documented outcomes is that the range of private costs is somewhat larger than the quality differences between the high- and the low-quality levels. As a result, both the high-quality and the medium-quality bidders directly compete with the low-quality bidders for a large range of the buyers' tastes (as opposed to deterministic costs oligopoly model where high-quality sellers would compete only with medium-quality sellers and competition is localized around the marginal buyers). The presence of low-quality sellers imposes double discipline on the prices of high-quality sellers: if high-quality bidders were to increase the price they would lose market share to both medium- and low-quality sellers. Also, any price response by medium-quality sellers to price increase by high-quality sellers would be substantially mitigated by the competitive pressure from low-quality sellers. The latter effect would contribute to re-allocation of market share from high-quality to medium-quality sellers. Thus, the presence of low-quality sellers prevents high-quality from choosing to provide a "boutique" service, that is, to serve a very small market at very high prices.

7.2 Using Standard as Opposed to Multi-Attribute Auctions

To avoid (the perception of) arbitrariness in the award of service procurement contracts, governments rely on standard as opposed to multi-attribute auctions. Our estimates allow us to assess the costs to the government from adopting this procurement mechanism. Column (1) of Table 13 in Supplemental Appendix contains the result of the experiment in which the government procures a programming services in our market using a standard auction mechanism. In Columns (2) and (3), in addition to using a standard auction mechanism, the government imposes a pre-certification requirement that ensures that either only medium- and high-quality bidders or only high-quality bidders are allowed to participate in these auctions.[44]

We find that using a standard instead of a multi-attribute auction mechanism results in a

[42]See, for example, the large-scale studies by the Canadian Competition Bureau (2007) and the U.K. Office of Fair Trading (2001).

[43]In our setting high- and low- quality sellers tend to have the costs that are substantially higher on average than the costs of medium-quality bidders. In a standard oligopoly game with differentiated products and deterministic costs the equilibrium with three quality levels would not exist given such relationship between the costs of different quality levels. Further, under all parameter combinations that we have found to produce an equilibrium for such a game, the licensing restriction results in both high-quality and medium-quality sellers increasing their prices and their market shares.

[44]In this analysis we impose a secret reserve price which is distributed according to the same distribution as the 1.3×(the costs of low-quality sellers). This is a somewhat arbitrary assumption that is imposed to ensure equilibrium existence. The reserve price is imposed consistently across columns and thus should not impact the comparison across different formats of the standard auction. As for the comparison with the multi-attribute auction, our assumption is more generous than the reserve price required by the US federal regulations (10% above the estimated costs) and hence should still allow for a reasonable comparison between the standard and multi-attribute formats (if anything, the results should be biased against us).

substantial loss of value to the government. While the prices paid by the government decrease by 5.8% on average, this reduction is also associated with 19.5% reduction in the average purchased quality which in turn entails 6.5% reduction in the value (expected utility) and 9.2% reduction in the overall surplus. The reasons for these effects are clear. Our findings indicate that the provision of high-quality service is substantially more expensive that the provision of medium-quality service. This makes high-quality bidders relatively uncompetitive on price. Thus, they win a small fraction of standard auctions and the government ends up mostly procuring the services of medium-quality sellers. The standard auction mechanism also does not allow to take into account seller's idiosyncratic suitability for a given project (match component). Both effects contribute to the loss of utility. The total surplus is further reduced by lower profits earned in this market. Pre-certification requirement improves the average purchased quality by 12%. However, the average quality under standard auction mechanism is still lower by 10% than the average quality purchased under the multi-attribute mechanism with licensing restriction. These results importantly depend on the features of our environment. For example, in settings where low-quality sellers are more competitive the effects are likely to be even stronger.

The new insights on the mechanisms determining the consequences of various policies that we obtained in this section stem from our modeling of service markets as multi-attribute auctions. They underscore the importance of using appropriate distributions of quality and costs as well as the distribution of consumer preferences when evaluating the effects of policies for specific environments. While the analysis of different service markets may call for richer models, we hope that the empirical methodology developed in this paper can be extended to accommodate features of many settings to allow recovering the key objects necessary for policy evaluation.

8 Conclusion

In this paper we proposed an empirical methodology that could be used to study many markets for services. It is applicable in the environment where heterogeneous buyers take into account seller characteristics (foremost their quality) in addition to price when choosing service providers, while sellers are small, distinct and characterized by a high turnover rate. Our methodology overcomes one of the most important hurdles that have inhibited the study of these markets in the existing literature – the lack of reliable data on sellers' quality.

The environment we consider combines features of discrete choice (differentiated products) and auction settings. We build on the insights offered by these two literatures to develop a novel identification strategy as well as an implementable econometric procedure that recovers the distribution of buyers' tastes, the distribution of sellers' qualities conditional on other seller characteristics, as well as the distribution of sellers' costs conditional on quality and other attributes.

Our empirical findings confirm the economic significance of quality differences in our market. In fact, these differences dominate other types of seller heterogeneity. Allowing for the variation in sellers' quality and buyers' tastes for quality significantly improves the fit of the model. Recovering the distribution of qualities conditional on sellers' performance-related characteristics provides interesting insights into the availability of information in the online markets as well as the role of performance measures, such as the "reputation scores" collected in these markets. The recovered distribution of costs conditional on sellers' characteristics including quality builds a foundation for a better understanding of the composition of participants attracted to online markets as well as the cost of delivering quality services. Finally, we use our estimates to study the effect of licensing restrictions and to assess the loss of surplus from using standard rather than multi-attribute auctions as is common in public procurement.

To the best of our knowledge, this paper marks the first effort to structurally analyze multi-attribute auctions with unobserved seller qualities and buyer tastes. Consequently, it has made simplifying assumptions which we expect to refine in future research. We expect the basic insights of our identification and implementation strategies to carry over to those environments.

First, we assume that sellers are uninformed about the realization of buyers' tastes (including the outside option) in each auction. That is, there is no auction- (or buyer-) level heterogeneity that is known to sellers but unobserved by the researcher. Such an assumption may be too strong in some settings. We have some preliminary results that indicate that this assumption could be relaxed under certain conditions.

Further, we assume the buyer is perfectly informed about the seller's quality. We realize that a number of alternative informational assumptions may appear to be applicable in similar environments. We carefully thought about this aspect of our analysis, and we believe that the assumption we currently use in the paper is the most appropriate in our market. This assumption is supported by our empirical results, which we discuss below. However, we also have some insights into how the model could be identified under alternative informational assumptions. We plan to pursue these issues in future research.

Finally, empirical results indicate that the reputation scores and to some degree the number of scores (or accumulated experience) are valued by the buyer. This potentially introduces dynamic considerations into the pricing and participation behavior of the sellers who are new in the market. We do not investigate these issues in our analysis. This does not impact the validity of our results. We exploit sellers' optimal decision making only in the last step when we use the seller's problem to recover the distribution of costs. In our analysis we rely on the problem of permanent sellers, who we believe are less concerned with the dynamic implications of their behavior. However, the reputation-building issues are of independent interest as an enforcement mechanism used in many Internet markets. We hope that future research will explore these issues in more detail.

In summary, we believe that our methodology opens the possibility of analyzing various aspects of service markets: from optimal pricing and optimal procurement to product design and studying the market mechanism that eliminates moral hazard concerns in this environment. Given the importance of service markets in modern economies, this seems to be an important research agenda.

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Appendix A: Proofs of Identification Results

Appendix A1: Proof of Proposition 1

Fix a set of sellers S. Let the set of entrants A be partitioned into those who are preferred to the outside option.(denoted by $A^1 \equiv \{i \in A : U_i \geq U_0\}$) and those who are not (denoted by $A^0 \equiv A \backslash A^1$). For any pair of permanent sellers i, j, let $\mathcal{A}_{i,j}$ denote the support of such a partition for entrants excluding i, j. That is, $\mathcal{A}_{i,j} \equiv \{(a, a') : a \cap a' = \varnothing \text{ and } a \cup a' \subseteq S \backslash \{i, j\}\}$. For any $(a, a') \in \mathcal{A}_{i,j}$, define:

$$\mathcal{P}_i(b; a, a') \equiv P(i \,\text{wins} \mid B_i = b,\ A^1 \backslash \{i\} = a,\ A^0 = a')$$

for any $b \in \mathcal{B}_i$.

LEMMA A1:*Suppose A1 and A2 hold. Consider any i, j with $\mathcal{B}_i \cap \mathcal{B}_j \neq \varnothing$. (a) For any $b \in \mathcal{B}_i \cap \mathcal{B}_j$ and $(a, a') \in \mathcal{A}_{i,j}$,*

$$q_i \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} q_j \Rightarrow \mathcal{P}_i(b; a, a') \left\{ \begin{array}{c} \geq \\ = \\ \leq \end{array} \right\} \mathcal{P}_j(b; a, a'). \tag{13}$$

(b) If $a^* \subset S \backslash \{i, j\}$ is such that either "$q_k = q_i$ or $q_k = q_j$" for all $k \in a^*$, then

$$sign(q_i - q_j) = sign(\mathcal{P}_i(b; a^*, a') - \mathcal{P}_j(b; a^*, a'))$$

for all $b \in \mathcal{B}_i \cap \mathcal{B}_j$ and any a' with $(a^*, a') \in \mathcal{A}_{i,j}$.

Proof of Lemma A1.. *Part (a).* Recall entry decisions are i.i.d. binary variables with success probability $\lambda_{r,k}$ for any $i \in S^{r,k}$. In equilibrium, sellers' bidding strategies are functions of private costs alone and are orthogonal to (α, ϵ, U_0). Given any pair of disjoint sets a, a' such that $(a \cup a') \subseteq N \backslash \{i, j\}$, let $\mathcal{E}(a, a')$ be a shorthand for the event "$\max_{s \in a'} U_s < U_0 \leq \min_{k \in a} U_k$". Then:

$$\begin{aligned} \mathcal{P}_i(b; a, a') &\equiv \Pr\left\{U_i \geq \max\nolimits_{k \in a} U_k \text{ and } U_i \geq U_0 \mid B_i = b, A^1 \backslash \{i\} = a, A^0 = a'\right\} \\ &= \int \Pr\left(\begin{array}{c} \Delta\epsilon_{k,i} - B_k \leq \alpha \Delta q_{i,k} - b \ \forall k \in a; \\ \text{and } U_0 - \epsilon_i \leq \alpha q_i - b \end{array} \middle| \alpha, \mathcal{E}(a, a') \right) dF(\alpha | \mathcal{E}(a, a')), \end{aligned} \tag{14}$$

where the equality follows from the Law of Total Probability and the facts that entry decisions are independent from realizations of α, ϵ, C, U_0; and that sellers' private costs are independent across each other as well as from α, ϵ, U_0. By similar arguments, $\mathcal{P}_j(b; a, a')$ takes a form that is almost identical to $\mathcal{P}_i$ in (14), except with all indices i therein replaced by j. By A1,2, the distribution of $(\Delta\epsilon_{k,i}, B_k)_{k \in a}$ is identical to that of $(\Delta\epsilon_{k,j}, B_k)_{k \in a}$ once conditioning on α and $\mathcal{E}(a, a')$. It then follows that (13) holds for all $b \in \mathcal{B}_i \cap \mathcal{B}_j$ and any $(a, a') \in \mathcal{A}_{i,j}$.

Part (b). It is sufficient to show that weak inequalities in (13) hold strictly for all $b \in \mathcal{B}_i \cap \mathcal{B}_j$ and any a^* that satisfies the conditions in part (b). By definition of a^*,

$$\begin{aligned} & \mathcal{P}_i(b; a^*, a') - \mathcal{P}_j(b; a^*, a') \\ = & \int \left(\begin{array}{l} \Pr\left(\begin{array}{c} \Delta\epsilon_{k,i} - B_k + b \leq \alpha \Delta q_{i,k} \ \forall k \in a^* \\ \text{and } U_0 - \epsilon_i + b \leq \alpha q_i \end{array} \middle| \alpha, \mathcal{E}(a^*, a')) \right) \\ - \Pr\left(\begin{array}{c} \Delta\epsilon_{k,j} - B_k + b \leq \alpha \Delta q_{j,k} \ \forall k \in a^* \\ \text{and } U_0 - \epsilon_j + b \leq \alpha q_j \end{array} \middle| \alpha, \mathcal{E}(a^*, a') \right) \end{array} \right) dF(\alpha | \mathcal{E}(a^*, a')) \end{aligned}$$

for all $b \in \mathcal{B}_i \cap \mathcal{B}_j$ and a' with $(a^*, a') \in \mathcal{A}_{i,j}$. Under A1,2, $(B_i)_{i \in a^*}$ are independent from $(\epsilon_i)_{i \in a^*}$ and α for any given set a^*. Under A1,2, $(\Delta\epsilon_{k,i})_{k \in a^*}$ is continuously distributed with positive densities conditional on α. Thus support of $(\Delta\epsilon_{k,i})_{k \in a^*}$ is $[\underline{\varepsilon} - \overline{\varepsilon}, \overline{\varepsilon} - \underline{\varepsilon}]^{\#\{a^*\}}$, which contains the zero vector in its interior. Likewise for $(\Delta\epsilon_{k,j})_{k \in a^*}$. For any b in the interior of $\mathcal{B}_i \cap \mathcal{B}_j$ there is positive probability that $(B_k - b)_{k \in a^*}$ is close enough to 0 and α is small enough so that $\mathcal{P}_i(b; a^*, a') >$ (and $=, <$) $\mathcal{P}_j(b; a^*, a')$ for all a' with $(a^*, a') \in \mathcal{A}_{i,j}$ whenever $\Delta q_{i,j} >$ (and $=, <$ respectively) 0. *Q.E.D.*

Proof of Proposition 1. By definition and an application of the Law of Total Probability, we can write $r_{i,j}(b)$ as:

$$\sum_{(a,a')\in\mathcal{A}_{i,j}} \Pr(i \text{ wins} \,|\, B_i = b, A^1 = i\cup a, A^0 = a')\Pr(A^1 = i\cup a,\, A^0 = a' | \, B_i = b, i\in A,\, j\notin A).$$

It follows from Lemma A1 that $\Pr(i\,\text{wins} \mid B_i = b, A^1 = i\cup a, A^0 = a') \geq$ (or $=$, $\leq$) $\Pr(j\,\text{wins} \mid B_j = b,\ A^1 = j\cup a,\ A^0 = a')$ whenever $\Delta q_{i,j} > 0$ (or $= 0$, < 0 respectively) for all $(a,a')\in\mathcal{A}_{i,j}$. Weak inequalities hold strictly for any $(a,a')\in\mathcal{A}_{i,j}$ such that "either $q_k = q_i$ or $q_k = q_j$" for all $k\in a$. Such a pair $(a,\ a')$ exists in $\mathcal{A}_{i,j}$ and occurs with positive probability even after conditioning on $B_i = b$ and $i\in A, j\notin A$. This is because entry decisions are exogenous and independent from private costs, as each potential bidder i entering with probability $\lambda_{r,k}$ if $i\in S^{r,k}, r\in\{t,p\}$. The same argument applies as we switch the role of i and j in the above sentence. Furthermore, under A1,2, $\Pr(A^1 = i\cup a,\ A^0 = a' | \, B_i = b, i\in A,\ j\notin A)$ is identical to $\Pr(A^1 = j\cup a,\ A^0 = a' | \, B_j = b,\ j\in A,\ i\notin A)$ for all $(a,a')\in\mathcal{A}_{i,j}$. It then follows that $sign(r_{i,j}(b) - r_{j,i}(b)) = sign(q_i - q_j)$. *Q.E.D.*

Appendix A2: Proposition 2

In this subsection we present a proof of Proposition 2 in the general case with multiple permanent and transitory bidders.

(A3′) *There exists a set $a = a^p\cup a^t$, a pair $i,j\in a^p$ and a vector of bids $b^t\equiv\{b_k\}_{k\in a^t}$ such that (i) the characteristic function of the joint distribution of $(\alpha\Delta q_{j,i} + \epsilon_j - \epsilon_i\ ,\ Y_{i,j}(b^t;a) - \epsilon_i)$ does not vanish; and (ii) the joint support of $(B_j - B_i, -B_i)$ includes the joint support of $(\alpha\Delta q_{j,i} + \epsilon_j - \epsilon_i,\ Y_{i,j}(b^t;a) - \epsilon_i)$ conditional on b^t.*

Proposition 2′ *Suppose (A1), (A2) hold. (i) If (A3′) holds for some i,j with $q_i = q_j$ and some (a,b^t), then the marginal distribution of ϵ_i and the distribution of $Y_{i,j}$ given (a,b^t) are jointly identified up to a location normalization (e.g. $E(\epsilon_i) = 0$). (ii) If in addition (A3′) also holds for some i',j' with $q_{i'}\neq q_{j'}$ and some $(a',b^{t\prime})$, then the quality difference $\Delta q_{i',j'}$ and the distribution of α are jointly identified up to a scale normalization (e.g. $E(\alpha) = 1$). Besides, the joint distribution of $(\alpha\Delta q_{j',i'}, Y_{i',j'})$ given $(a',b^{t'})$ is also identified.*

Proof of Proposition 2′. *Part (i).* Consider i,j and a^t,a^p,b^t satisfying Assumption (A3′) and $q_i = q_j$. Under Assumptions (A1) and (A2), $\epsilon_i,\epsilon_j,Y_{i,j}$ are mutually independent given any (b^t,a). Hence, for any $b_i,b_j\in\mathbb{R}$, the probability that i wins conditional on $B_i = b_i$, $B_j = b_j$ and a,b^t equals

$$\varphi(b_i,b_j;q_i,q_j)\equiv\Pr\left\{\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}\leq b_j - b_i\ \text{ and } Y_{i,j}(b^t;a) - \epsilon_i\leq -b_i\right\} \tag{15}$$

where $\Delta q_{j,i}\equiv q_j - q_i$ and likewise for $\Delta\epsilon_{j,i}$. Note the equality uses independence of private costs (and hence bids) from U_0,α, and ϵ. With B_j and B_i being independent of (U_0,α,ϵ), evaluating $\varphi(.,.;q_i,q_j)$ at different realizations of B_i,B_j only amounts to evaluating the same joint distribution of $(\epsilon_j - \epsilon_i,\ Y_{i,j}(b^t;a) - \epsilon_i)$ (with b^t,a fixed) at different points on the support. Thus, under Assumption A3-(ii), i.e. the large support condition, this joint distribution is identified. Mutual independence between ϵ_i,ϵ_j and $Y_{i,j}(b^t;a)$ given (b^t,a) implies their marginal distributions are identified up to some location normalizations (e.g. $\mathbb{E}(\epsilon)=0$) by an application of the Kotlarski's Theorem, or Theorem 2.1.1 in Rao (1992).

Part (ii). Without loss of generality, suppose conditions for part (ii) hold for $q_{i'} > q_{j'}$. Replicating arguments in part (i) with $i', j', a^{t\prime}, b^{t\prime}$ satisfying Assumption A3 and $q_{i'} > q_{j'}$ shows that the joint distribution of $(\alpha\Delta q_{j',i'} + \Delta\epsilon_{j',i'},\ Y_{i',j'}(b^{t\prime}; a') - \epsilon_{i'})$ given $(b^{t\prime}, a')$ is identified. This implies the marginal distribution of $\alpha\Delta q_{j',i'} + \Delta\epsilon_{j',i'}$ and the distribution of $Y_{i',j'}(b^{t\prime}; a') - \epsilon_{i'}$ given $(b^{t\prime}, a')$ are also recovered. With the marginal distribution of match components already identified from part (i), so is the distribution of $\Delta\epsilon_{j',i'}$. With $\alpha\Delta q_{j',i'}$ being independent from $\Delta\epsilon_{j',i'}$, this means the distribution of $\alpha\Delta q_{j',i'}$ can be identified as long as the characteristic-function of $\Delta\epsilon_{j',i'}$ is non-vanishing. It then follows that $\Delta q_{j',i'}$ and the distribution of α are identified up to some scale normalization (such as $\mathbb{E}[\alpha]=1$). Finally, note the joint distribution $(\alpha\Delta q_{j',i'}, Y_{i',j'}(b^{t\prime}; a'))$ given $(b^{t\prime}; a')$ can be recovered from knowledge of the distributions of $(\Delta\epsilon_{j',i'}, -\epsilon_i)$ and the distribution of $(\alpha\Delta q_{j',i'} + \Delta\epsilon_{j',i'}, Y_{i',j'}(b^{t\prime}; a') - \epsilon_{i'})$ given $(b^{t\prime}; a')$, as long as the characteristic function for $(\Delta\epsilon_{j',i'}, -\epsilon_i)$ is non-vanishing. This is because Assumptions A1 and A2 implies $(\Delta\epsilon_{j',i'}, -\epsilon_i)$ is independent from $(\alpha\Delta q_{j',i'}, Y_{i',j'}(b^{t\prime}; a'))$ with $(b^{t\prime}; a')$ fixed.

The proof of identification of the distribution of outside option and the distribution of qualities of transitory bidders conditional on bids is as presented in the text. *Q.E.D.*

Appendix B

Appendix B1: Details of the Model with Strategic Entry

This section supplements Section 4.4.1 by providing further details of the model with strategic entry. Potential bidders' strategies and psBNE are defined and the arguments for the extension of identification strategy are included. Next section provides the proof of equilibrium existence.

We assume that an analog of assumptions $(A1)$ and $(A2)$ hold for a set of potential bidders.

(A1′) *The private signal $E_{i,l}$ and the private cost $C_{i,l}$ are independent from each other, and the random vectors $(E_{i,l}, C_{i,l})$ are independent across all $i \in N_l$ and across auctions.*[45] *For each i with $q_i = q^k$, these costs are independent draws from the continuous distributions F^E_k and F^C_k with a density positive over supports $[\underline{e_k}, \overline{e_k}]$ and $[\underline{c_k}, \overline{c_k}]$ respectively.*

(A2′) *The three random vectors $(\alpha_l, U_{0,l})$, ϵ_l and $(E_{i,l}, C_{i,l})_{i\in N_l}$ are mutually independent; match components $\epsilon_{i,l}$ are i.i.d. across i's; $\epsilon_{i,l}$ and $(\alpha_l, V_{0,l})$ are continuously distributed with a density positive over $[\underline{\varepsilon}, \overline{\varepsilon}]$ and over $[0, \overline{\alpha}] \times [\underline{u}_0, \overline{u}_0]$ respectively.*

We focus on type-symmetric equilibria in which any pair of participants i, j who are *ex ante* identical (i.e. either "$i, j \in N^p_l$ and $q_i = q_j$" or "$i, j \in N^t_l$") adopt the same strategies.

The strategy of seller i from the quality group q^k, $\sigma^{r,k}$, consists of a participation strategy $\sigma^{r,k}_E$ (as defined above) and a bidding strategy, $\sigma^{r,k}_B : [\underline{c_k}, \overline{c_k}] \times Supp(I_{N,l}) \to \mathbb{R}_+$. Conditional on participation, a seller i's expected profit from bidding b is given by

$$\Pi^{r,k}(b, c_i, I_{N,l}; \sigma^{-i}_E, \sigma^{-i}_B) \equiv (b - c_i)\Pr(i \,\text{wins} \mid b,\ (r,k),\ I_{N,l}; \sigma^{-i}_E, \sigma^{-i}_B),$$

[45]Our identification results could be extended to the case when $E_{i,l}$ and $C_{i,l}$ are correlated for each seller i in an auction, but the random vectors $(E_{i,l}, C_{i,l})$ are independent across $i \in N_l$ and across auctions.

where $(\sigma_E^{-i}, \sigma_B^{-i})$ is the profile of strategies of the other participants.

A psBNE is a profile of strategies $\{(\sigma_E^{r,k}, \sigma_B^{r,k})\}_{r\in\{p,t\},\, k\in\{1,\ldots,K\}}$[46] such that $\sigma_B^{r,k}(c_{i,l}, I_{N,l}) = \arg\max_b \Pi^{r,k}(b, c_{i,l}, I_{N,l}; \sigma_E^{-i}, \sigma_B^{-i})$ and $\mathbb{E}[\Pi^{r,k}(\sigma_B^{r,k}(C_{i,l},\, I_{N,l}),\, C_i, I_{N,l})|E_{i,l} = e_{i,l}] - e_{i,l} \geq 0$ if and only if $\sigma_E^{r,k}(e_{i,l},\, I_{N,l}) = 1$.

We show that psBNE in monotone strategies exists (Proof is included in Appendix B). Given orthogonality between $E_{i,l}$ and $C_{i,l}$ and independence of $(E_{i,l}, C_{i,l})$ across bidders, the equilibrium participation strategies are monotone and are characterized by a threshold rule. That is, for any $I_{N,l}$, there exists $e^*_{p,k}$ and $e^*_{t,k}$ such that for a permanent potential bidder i with quality level q^k, $\sigma_E^{r,k}(e_{i,l},\, I_{N,l}) = 1$ iff $e_{i,l} \leq e^*_{p,k})$ whereas for each transitory sellers with quality level q_k, $\sigma_E^{r,k}(e_{i,l},\, I_{N,l}) = 1$ iff $e_{i,l} \leq e^*_{t,k}$.

It is easy to establish that under Assumptions (A1′), (A2′) and (A3) and for any given $I_{N,l}$ and in any pure-strategy Bayesian Nash equilibrium, the buyer's tastes $\{\alpha, \epsilon, U_0\}$ are independent from $\{D_{i,l}\}_{i\in N_l}$ (and therefore A_l); participation decisions $\{D_{i,l}\}_{i\in N}$ are mutually independent across all $i \in N_l$; and bids are mutually independent within each realized set of active bidders.

Extending Identification Argument

The reasoning behind pairwise comparisons in Proposition 1 remains unchanged in the presence of strategic participation. Since participation decisions are independent across potential sellers, for given i and j the probabilities for various sets of competitors to realize conditional on $i \in A$ and $j \notin A$ are still identical to those conditional on $j \in A$ and $i \notin A$. Thus, the proof of Proposition 1 which relies on this property remains unchanged.[47]

Identification of quality differences and the distribution of buyers' tastes also follow from the same arguments as before. In particular, since entry decisions and bid distributions are orthogonal to (α, ϵ, U_0) the variation in bids by permanent sellers is still a valid source of exogenous variation for identification. Further, these results also rely on the independence of entry and bidding strategies across active bidders (and in particular on independence of permanent bidders' strategies from those of transitory bidders) which continue to hold under endogenous entry as mentioned above.

Also note that, with $C_{i,l}$ being independent from $E_{i,l}$, the standard arguments used for backing out the distribution of private costs remain valid in the presence of endogenous entry.

Appendix B2: Existence of psBNE

In this section, we show how existence of psBNE naturally follow in our model. To simplify the notation and exposition, we abstract away from any auction or seller characteristics that are reported in data. Our arguments can be extended to accommodate these observed heterogeneities by conditioning the game on them.

PROPOSITION B2:*Suppose that the following conditions hold.*

(i) *The bids are commonly chosen from a closed interval of a real line.*

[46] We assume that $\sigma_B^{r,k} = \infty$ if $\sigma_E^{r,k} = 0$.

[47] Notice that in the model with strategic entry the pairwise index also depends on the set or rather on relevant information about the set of potential bidders, $\mathbf{I}_N$. In practice, the index is computed by pooling all the observations that are characterized by the same $\mathbf{I}_N$ which may entail using auctions with distinct (in terms of identities) sets of potential bidders.

(ii) *The conditional CDF of* ϵ *given* $(\alpha, U_0, Q) = (\bar{\alpha}, \bar{u}_0, \bar{q})$ *are uniformly continuous for all* $(\bar{\alpha}, \bar{u}_0, \bar{q})$ *in the support of* (α, U_0, Q).
(iii) *The project costs* C_i *are private, independent and identically distributed, and have a density function bounded uniformly away from zero.*
(iv) *The entry costs* E_i *are private, independent, and identically distributed, and have a density function bounded uniformly away from zero.*
(v) *The entry costs and the private costs are independent from each other.*

Then there exists a psBNE in the game.

Proof: We first consider a subgame in the bidding stage where a subset $A \subset N$ of bidders have already decided to enter. We first show that in each subgame with A, there exists a PSBNE. For this, we invoke Corollary 2.1 of Athey (2001) by which it suffices to show that the payoff faced by each bidder is continuous in the bid profile, and the expected payoff satisfies single crossing of incremental returns.

Given the other players' strategies $b_{-i} = (b_j)_{j \in A \setminus \{i\}}$, the interim expected payoff for bidder i is given by

$$U_i(b_i, c_i; b_{-i}) \equiv \int u_i(b_i, b_{-i}(c_{-i}); c_i) dF(c_{-i}|c_i),$$

where $u_i(b; c_i)$ is the payoff for bidder i conditional on the vector of bids b and private cost c_i, and independence among the private costs are used. In our model, the payoff $u_i(b; c_i)$ is given as follows:

$$u_i(b; c_i) \equiv (b_i - c_i) \Pr\{i \ wins|b\},$$

where, with $F(\bar{\varepsilon}|\bar{\alpha}, \bar{u}_0, \bar{q})$ denoting the joint CDF of ϵ given $(\alpha, U_0, Q) = (\bar{\alpha}, \bar{u}_0, \bar{q})$,

$$\Pr\{i \ wins|b\} = \int h(b; \bar{\alpha}, \bar{u}_0, \bar{q}) dF(\bar{\alpha}, \bar{u}_0, \bar{q}),$$

and $h(b; \bar{\alpha}, \bar{u}_0, \bar{q}) \equiv \int_{H_i(b; \bar{\alpha}, \bar{u}_0, \bar{q})} dF(\bar{\varepsilon}|\bar{\alpha}, \bar{u}_0, \bar{q})$ and

$$H_i(b; \bar{\alpha}, \bar{u}_0, \bar{q}) = \left\{ \bar{\varepsilon} \in \mathbf{R}^{|A|} : \begin{array}{c} \bar{\alpha} \Delta q_{j,i} + \Delta \bar{\varepsilon}_{j,i} \leq b_j - b_i \text{ and } \bar{u}_0 - \bar{\alpha} q_i - \bar{\varepsilon}_i \leq -b_i, \ \forall j \in N^p \\ \alpha(\bar{q}_k - q_i) + \Delta \bar{\varepsilon}_{k,i} \leq b_k - b_i, \ \forall k \in N^t \end{array} \right\}.$$

Since $F(\cdot|\bar{\alpha}, \bar{u}_0, \bar{q})$ is uniformly continuous for all $(\bar{\alpha}, \bar{u}_0, \bar{q})$ in the support of (α, U_0, Q), we find that $h(\cdot; \bar{\alpha}, \bar{u}_0, \bar{q})$ is continuous. By the Bounded Convergence Theorem, $\Pr\{i \ wins|b\}$ is continuous in b.

As for single crossing of incremental returns, we consider the following. For each bidder i, for $b_i \geq b_i'$ and $c_i \geq c_i'$

$$\begin{aligned} & u_i(b_i, b_{-i}, c_i) - u_i(b_i', b_{-i}, c_i) \\ = \ & (b_i - c_i) \Pr\{i \ wins|b_i, b_{-i}\} - (b_i' - c_i) \Pr\{i \ wins|b_i', b_{-i}\} \\ = \ & (b_i - b_i') \Pr\{i \ wins|b_i, b_{-i}\} + (b_i' - c_i) \left(\Pr\{i \ wins|b_i, b_{-i}\} - \Pr\{i \ wins|b_i', b_{-i}\} \right). \end{aligned}$$

The last term is bounded from below by

$$(b_i' - c_i') \left(\Pr\{i \ wins|b_i, b_{-i}\} - \Pr\{i \ wins|b_i', b_{-i}\} \right)$$

because

$$\Pr\{i \ wins|b_i, b_{-i}\} \leq \Pr\{i \ wins|b_i', b_{-i}\}.$$

Therefore, the payoff $u_i(b_i, b_{-i}, c_i)$ satisfies nondecreasing differences in (b_i, c_i). This implies that its expected payoff $U_i(\cdot, \cdot; b_{-i})$ satisfies single crossing of incremental returns. Hence by Corollary 2.1 of Athey (2001), each subgame with given A has a psBNE.

We now consider a psBNE in the entry stage. For this, we use Theorem 2 of Athey (2001). The action space is $\{0, 1\}$ with action denoted by d. We set $d = 0$ to denote participation and $d = 1$ to denote nonparticipation. The private type for each bidder i is entry cost E_i. The interim expected payoff function for each bidder i with entry cost e_i is given by

$$\begin{aligned} & (1 - d_i) \cdot \left(\mathbf{E}\left[U_i(b_i(C_i), C_i; (b_j)_{j \in A(d(e)) \setminus \{i\}})|E_i = e_i\right] - e_i\right) \\ = \ & (1 - d_i) \cdot \left(\mathbf{E}\left[U_i(b_i(C_i), C_i; (b_j)_{j \in A(d(e)) \setminus \{i\}})\right] - e_i\right) \end{aligned}$$

where $A(d) = \{i \in N : d_i = 0\}$, the bid strategy profile $b(c)$ are given by the subgame psBNE whose existence is previously established, and $d(e) = (d_i(e_i))_{i \in N}$. (Recall that the private costs C_i are revealed only after the bidder decides to enter and are independent of E_i.) Certainly Assumption A1 and single crossing of incremental returns in Theorem 2 of Athey (2001) are satisfied. Thus, the existence proof of PSBNE follows from the theorem. Q.E.D.

Appendix C: Proof of Results in Sections 4.3 and 5

Appendix C1: Proof of Theorem 1

Let $\tau_{ij}^+ = \int \max\{\delta_{ij}(b), 0\}db$, $\tau_{ij}^- = \int \max\{-\delta_{ij}(b), 0\}db$, and $\tau_{ij}^0 = \int |\delta_{ij}(b)|db$, where $\delta_{ij}(b) = r_i(b) - r_j(b)$. We confine the integral domains to $\mathcal{B}_i \cap \mathcal{B}_j$, and this restriction is omitted from the notation. From Theorem 2 of Lee, Song, and Whang (2013), one can show that under regularity conditions there exist fixed positive numbers σ_{ij}^+, σ_{ij}^-, and σ_{ij}^0 such that whenver $\delta_{ij}(b) = 0$ for all $b \in \mathbf{R}$ (i.e., under the least favorable configuration),

$$\frac{\sqrt{L}\{\hat{\tau}_{ij}^z - \mathbf{E}\hat{\tau}_{ij}^z\}}{\sigma_{ij}^z} \to_d \quad N(0, 1), \tag{16}$$

as $L \to \infty$ for $z \in \{+, -, 0\}$. Here L denotes the number of projects used for comparison.

As for the regularity conditions, we require two high-level conditions. First, the convergence in distribution (16) is satisfied under the least favorable configuration. The second condition is that the bootstrap tests are consistent against fixed alternatives. Detailed low level conditions for the convergence in distribution in (16) can be found in Lee, Song, and Whang (2013).

It is worth noting that the form of the test statistic is slightly different from that in Lee, Song, and Whang (2012) because $\hat{\delta}_{ij}(b)$ is the difference between the two kernel estimators. However, the observations pertaining to the i-th seller and the observations pertaining to the j-th seller are independent, if i and j are different. Using this particular property, we can apply the Poissonization method as in Lee, Song, and Whang (2013) by focusing on the Poissonized statistic and representing it as an independent sum of Poisson random variables. We omit the details.

Proof of Theorem 1: First, observe that

$$\begin{aligned} \text{if } i &\in N_l,\ P\left\{\#\left(\hat{N}_1(i)\Delta N_1(i)\right) \geq 1\right\} \to 0 \text{ and} \\ \text{if } i &\in N_h,\ P\left\{\#\left(\hat{N}_2(i)\Delta N_2(i)\right) \geq 1\right\} \to 0. \end{aligned}$$

This is a simple consequence of the consistency of the bootstrap tests. Also, the consistency of the bootstrap test implies that for any $i \in N_h$, we have that for any $j \in \hat{N}_2(i)$, $p_0^*(i,j) \to_P 0$ as $L \to \infty$ but for any $j \in \hat{N}_1(i)$, $p_0^*(i,j)$ is stochastically bounded as $L \to \infty$. Therefore, i is classified in the high-quality group with probability approaching one. That is, $P\{i \in \hat{N}_h\} \to 1$. Hence for each $i \in N_h$,

$$\begin{aligned} & P\left\{\Gamma((\hat{N}_h(i), \hat{N}_l(i)), \mathcal{C}) \geq 1\right\} \\ = \ & P\left\{\Gamma((\hat{N}_2(i) \cup \{i\}, \hat{N}_1(i)), \mathcal{C}) \geq 1\right\} + o(1) \to 0. \end{aligned}$$

Also, similarly, for any $i \in N_l$,

$$P\left\{\Gamma((\hat{N}_h(i), \hat{N}_l(i)), \mathcal{C}) \geq 1\right\} \to 0.$$

Since N is a fixed finite set as $L \to \infty$, we have

$$P\left\{\Gamma((\hat{N}_h(i), \hat{N}_l(i)), \mathcal{C}) \geq 1 \text{ for some } i \in N\right\} \to 0.$$

Therefore,

$$\begin{aligned} P\left\{\Gamma(\widehat{\mathcal{C}}, \mathcal{C}) \geq 1\right\} &= P\left\{\Gamma((\hat{N}_h(i^*), \hat{N}_l(i^*)), \mathcal{C}) \geq 1\right\} \\ &\leq P\left\{\Gamma((\hat{N}_h(i), \hat{N}_l(i)), \mathcal{C}) \geq 1 \text{ for some } i \in N\right\} \to 0, \end{aligned}$$

giving us the desired result. ■

Lemma C1: (i) If $K \geq K_0$, $\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K) = O_P(1)$, as $L \to \infty$.
(ii) If $K < K_0$, for any $M > 0$, as $L \to \infty$,

$$P\left\{\frac{\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K)}{g(L)} > M\right\} \to 1.$$

Proof: (i) The first statement of Lemma A1 can be proved in three steps. First we reclassify the true group structure into a finer one with K groups. Let this new group structure be $\{N_{k,1} : k = 1, \cdots, K\}$. Second, invoking Theorem 1 above, we show that the quantity $\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K)$ is asymptotically equivalent to the same quantity (denoted by $\frac{1}{K}\sum_{k=1}^K \tilde{V}_k(K)$) only with $\hat{N}_k$ replaced by $N_{k,1}$. Finally, we show that $\frac{1}{K}\sum_{k=1}^K \tilde{V}_k(K) = O_P(1)$. To see the latter convergence rate, first, note that

$$\frac{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}}{\sigma_{ij}^0} \to_d \mathbb{Z}$$

under the least favorable configuration, where $\mathbb{Z}$ is a standard normal random variable. This can be proved as in the proof of Theorem 1 of Lee, Song, and Whang (2012). Hence using the asymptotic validity of the bootstrap test, we find that

$$\begin{aligned} |\log p_0^*(i,j)| &= \left|\log\left(1 - \Phi(\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0)\right)\right| + o_P(1) \\ &\leq |\log(1 - \Phi(\mathbb{Z}))| + o_P(1). \end{aligned}$$

The inequality becomes an equality when we are under the least favorable configuration.

Suppose that $K \geq K_0$ and $P\{i,j \in \hat{N}_k\} \to 1$. Then $i,j \in N_k$, i.e., i and j belong to the same quality group. Hence by Proposition 1 of this paper, and $\delta_{ij}(b) = 0$ for all b on the intersection of supports of bids submitted by i and j. Since we confine the integral domain to such an intersection, it follows that we are under the least favorable configuration. From the previous arguments, this yields the result that $|\log p_0^*(i,j)| = O_P(1)$.

(ii) Suppose that $K < K_0$. Then for some $k = 1, \cdots, K$, and for some $i,j \in N_k$, $\tau_{ij}^0 > 0$. By invoking the smoothness conditions for the winning probabilities and using the proof of Theorem 4 of Lee, Song, and Whang (2012), it can be shown that under the $\sqrt{L}$-converging local alternatives, there exists $c_{ij} > 0$, for any $m > 0$,

$$P\left\{\frac{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}}{\sigma_{ij}^0} > m\right\} = P\left\{\mathbb{Z} + c_{ij} > m\right\} + o(1). \tag{17}$$

Using similar arguments, we can show that under the fixed local alternatives,

$$P\left\{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0 > m\right\} = P\left\{\mathbb{Z} + \sqrt{L}c_{ij} > m\right\} + o(1).$$

Therefore, for any $M > 0$,

$$\begin{aligned} &P\left\{(1/g(L))\,|\log p_0^*(i,j)| > M\right\} \\ \geq\ &P\{(1/\sqrt{L})|\log(1 - \Phi(\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0))| > Mg(L)/\sqrt{L}\} + o(1) \\ \geq\ &P\{(1/\sqrt{L})|\log(1 - \Phi(\mathbb{Z} + \sqrt{L}c_{ij}))| > Mg(L)/\sqrt{L}\} + o_P(1) \end{aligned}$$

as $L \to \infty$. Since $g(L)/\sqrt{L} \to 0$, it follows from (17) that the last probability converges to 1. ■

Proof of Theorem 2: For all $K > K_0$, we have

$$\begin{aligned} \hat{Q}(K_0) - \hat{Q}(K) &= \frac{1}{K_0}\sum_{k=1}^{K_0}\hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) + (K_0 - K)g(L) \\ &\to -\infty, \end{aligned}$$

because $\frac{1}{K_0}\sum_{k=1}^{K_0}\hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) = O_P(1)$ as $L \to \infty$.

And for all $K < K_0$, we have

$$\hat{Q}(K_0) - \hat{Q}(K) = \frac{1}{K_0}\sum_{k=1}^{K_0} \hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K) + (K_0 - K)g(L) \to -\infty,$$

because $\frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K) \to \infty$ faster than the rate $(K_0 - K)g(L) \to \infty$ as $L \to \infty$.

Therefore, $P\left\{\hat{Q}(K_0) - \hat{Q}(K) < 0\right\} \to 1$. Hence we find that $P\{\hat{K} = K_0\} \to 1$ as $L \to \infty$. ■

Appendix C2: Proof of Proposition 3

We formulate the moment conditions which are primarily based on the probability that permanent seller wins conditional on the information available to the econometrician as summarized by the expression in (19) and in accordance with the identification argument in the previous subsection.

We introduce some notation. For each x in the common support $\mathcal{X}$ of x_i, let $\mathbb{Q}_x \equiv \{q_{1,x}, \cdots, q_{K,x}\}$ be the set of possible quality levels for a seller $i \in N$ with $x_i = x$. With each $(x, q) \in \mathcal{X} \times \mathbb{Q}_x$ are associated sets of sellers indices, $A^p_{x,q,l} \equiv \{i \in A^p_l : (x_i, q_i) = (x, q)\}$, $N^p_{x,q,l} \equiv \{i \in N^p_l : (x_i, q_i) = (x, q)\}$, $A^t_{x,l} \equiv \{i \in A^t_l : x_i = x\}$ and $N^t_{x,l} \equiv \{i \in N^t_l : x_i = x\}$. It is convenient for exposition to arrange observations in a certain order. More specifically, the observations for permanent and transitory sellers are allocated into separate vectors. We enumerate observations for actual entrants first then for non-entrants, and group the observations for permanent sellers according to (x, q)-characteristics, and those for transitory sellers according to x-characteristics. Thus we write $B^t_{j,l}$ to denote the j-th transitory seller's bid at auction l, $B^p_{j,l}$ the j-th permanent seller's bid at auction l, $Q^t_{j,l}$ the j-th transitory seller's quality at auction l, and $W^p_{j,l} \in \{1, 0\}$ taking the value of one if and only if the j-th permanent seller wins at the l-th auction. Similarly, we define $x^t_{j,l}$, $x^p_{j,l}$, and $q^p_{j,l}$. After the rearrangement, the competitive nature of auction l is summarized by

$$\mathbf{I}_l \equiv \bigcup_{x\in\mathcal{X}} \bigcup_{q\in\mathbb{Q}_x} \{|A^p_{x,q,l}|, |N^p_{x,q,l}|, |A^t_{x,l}|, |N^t_{x,l}|\},$$

where $|A|$ for any set A denotes its cardinality. For each auction l, we define $\mathbf{B}_l = [\mathbf{B}^{p\prime}_l, \mathbf{B}^{t\prime}_l]'$, where $\mathbf{B}^t_l$ and $\mathbf{B}^p_l$ are random vectors with their j-th entries given by $B^t_{j,l}$ and $B^p_{j,l}$ respectively. We also define $\mathbf{Q}^t_{N,l}$ and $\mathbf{Q}^t_{A,l}$ to be both random vectors of entries $Q_{j,l}$ with $j = 1, \cdots, |N^t_l|$ and with $j = 1, \cdots, |A^t_l|$ respectively. We denote the set of values for $\mathbf{Q}^t_{N,l}$ by $\{\bar{\mathbf{q}}_{N,1}, \cdots, \bar{\mathbf{q}}_{N,\bar{K}_{N,l}}\}$ with $\bar{q}_{N,k} = (q_{N,1,k} \cdots q_{N,|N^t_l|,k})$. Similarly, the set of values for $\mathbf{Q}^t_{A,l}$ is denoted by $\{\bar{\mathbf{q}}_{A,1}, \cdots, \bar{\mathbf{q}}_{A,\bar{K}_{A,l}}\}$ with $\bar{q}_{A,k} = (q_{A,1,k} \cdots q_{A,|A^t_l|,k})$. These sets change across auctions because the dimensions of $\mathbf{Q}^t_{N,l}$ and $\mathbf{Q}^t_{A,l}$ change.

In accordance with the parametric estimation approach, we assume that ϵ_{il} and (α, β) are distributed according to $F(\epsilon|\theta_1)$ and $F(\alpha, \beta; \theta_2)$, distributions known up to a set of parameters (θ_1, θ_2), so that the vector of parameters to be estimated is given by $\theta = (\theta_1, \theta_2, (\mathbb{Q}_x : x \in \mathcal{X}))$ along with the parameters involved in the parametrization of $f(B^t_{i,l}|\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A^t_{x,l}|\, \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$.

We begin by deriving a representation of permanent seller's winning probability conditional on the vector of bids and auction competitive structure as observed by the econometrician. Unlike the econometrician, a buyer observes all the relevant characteristics for all actual competitors. Let $e^p_{x,q,k,l}(\mathbf{B}_l, \mathbf{I}_l; \theta, j) \equiv P\{W^p_{j,l} = 1 | \mathbf{B}_l, \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k,l}, \mathbf{I}_l\}$ be the probability that seller j (with (x,q)-characteristics) wins conditional on a full competitive structure of the auction, including information on transitory actual bidders' vector of qualities. More specifically,

$$e^p_{x,q,k,l}(\mathbf{B}_l, \mathbf{I}_l; \theta, j) = \int P\left(\alpha q + \beta x - B_{j,l} \geq \alpha q_i + \beta x_i - B_{i,l} \, \forall \, i \neq j | \, \alpha, \beta\right) dF_{\alpha,\beta}(\alpha, \beta).$$

Also let $p_{k,l} = P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k,l} | \mathbf{B}_l, \mathbf{I}_l\}$ be the probability that the of transitory actual bidders' qualities is $\bar{\mathbf{q}}_{k,l}$ conditional on the vector of bids $\mathbf{B}_l$, and on information about the auction's competitive structure as summarized in $\mathbf{I}_l$. Hence using this notation, we can rewrite

$$P\{W^p_{j,l} = 1 | \mathbf{B}_l, \mathbf{I}_l\} = \sum\nolimits_{k=1}^{\bar{K}_{A,l}} e^p_{x,q,k,l}(\mathbf{B}_l, \mathbf{I}_l; \theta, j) \, P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k,l} | \mathbf{B}_l, \mathbf{I}_l\}. \tag{18}$$

Let

$$\begin{aligned} \mathbf{I}_{l,1} &\equiv \{|N^p_{x,q,l}|, |N^t_{x,l}| : (x,q) \in \mathcal{X} \times \mathbb{Q}_x\}, \\ \mathbf{I}^p_{l,2} &\equiv \{|A^p_{x,q,l}| : x \in \mathcal{X}, \, q \in \mathbb{Q}_x\} \text{ and } \mathbf{I}^t_{l,2} \equiv \{|A^t_{x,l}| : x \in \mathcal{X}\}, \end{aligned}$$

so that $\mathbf{I}_l = \mathbf{I}_{l,1} \cup \mathbf{I}^p_{l,2} \cup \mathbf{I}^t_{l,2}$. We also let

$$\omega_{k,l} \equiv \prod_{i \in \bar{A}^t_l} P(\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k} | \, \bar{\mathbf{x}}^t_{i,l}),$$

and

$$g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}) \equiv \omega_{k,l} \prod_{x \in \mathcal{X}} \prod_{i \in \bar{A}^t_{x,l}} f(B^t_{i,l} | \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1}) P(i \in A^t_{x,l} | \, \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \, \mathbf{I}_{l,1}).$$

We restate Proposition 3 using the notation made fully explicit.

Proposition 3: Under (A1′)-(A3′), for each $x \in \mathcal{X}$, $q \in \mathbb{Q}_x$, and for the j-th permanent seller with (x,q)-characteristic who participated in auction l, [48]

$$P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k,l} | \mathbf{B}_l, \mathbf{I}_l\} = \frac{g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})}{\sum_{d=1}^{\bar{K}_{A,l}} g_d(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})}. \tag{19}$$

The quantities $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$ involve $f(\cdot | \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$, i.e., the density of a transitory seller's bids in equilibrium conditional on this seller's quality, and $P(i \in A^t_{x,l} | \, \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \, \mathbf{I}_{l,1})$, i.e., the probability of transitory seller i's participation in the auction conditional on his quality. As mentioned earlier, we estimate these equilibrium objects jointly with the parameters

[48] In fact, $\mathbf{E}[W^p_{x,q,l} | \mathbf{B}_l, \mathbf{I}_l] = \mathbf{E}[W^p_{j,l} | \mathbf{B}_l, \mathbf{I}_l]$ for all j such that $j \in A^p_{x,q,l}$ by symmetry. This formulation facilitates its sample analogue when we replace the sample version of the moment conditions for estimation.

of buyer's taste distribution and quality levels. In doing so, we do not need to recover these objects separately. Since in our setting the distribution of signals is the same for permanent and transitory bidders, we can use permanent bidder's optimization problem, bid distribution, and participation frequency to recover the distributions of signals. This requires knowing only $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$ and not separately $f(B_{i,l}^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$.

Proof of Proposition 3: For the purpose of the derivations below it is convenient to introduce mapping $\pi(\cdot; N, A) : \{1, \cdots, |N|\} \to N$. This mapping plays the following role. Sometimes we need to consider a scenario where a subset of potential bidders different from the one realized in the data would choose to participate in the auction. In considering such a case we would re-arrange the observations in such a way that the observations for this hypothetical set of actual bidders are listed first and the observations for the remaining potential bidders would be listed after them. The mapping $\pi(j; N, A)$ reflects the original (data set) position of the observation that would be listed in position j under this re-arrangement. In our analysis the order in which observations are listed within the set of entering or non-entering bidders is not important. Therefore, when re-arranging observations we do not consider all possible permutations (orderings) of the hypothetical set of actual bidders. Instead, we re-allocate them to the front of the vector without changing the order in which they were listed originally.

We use $\bar{N}_{x,q,l}^p$, $\bar{A}_{x,q,l}^p$, $\bar{N}_{x,l}^t$, $\bar{A}_{x,l}^t$ to denote the realizations of respective random sets as they are recorded in the data. Notice that $\pi(j; \bar{N}_l^p, \bar{A}_l^p) = j$ and $\pi(j; \bar{N}_l^t, \bar{A}_l^t) = j$. For simplicity, we write $\pi_l^t(j) = \pi(j; A^t, N_l^t)$ and $\pi_l^p(j) = \pi(j; A^p, N_l^p)$ whenever it is clear which A and N sets are used.

Notice that we consider the probability of a two-part event: (1) that a given vector of qualities characterizes a subset of potential bidders, (2) potential bidders characterized by these qualities enter.

First, as for $p_{k,l}$, note that by the Bayes rule, we can write

$$\begin{aligned} p_{k,l} &= P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\mathbf{B}_l, \mathbf{I}_l\} = \frac{f(\mathbf{B}_l|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l)P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_l\}}{f(\mathbf{B}_l|\,\mathbf{I}_l)} \\ &= \frac{f(\mathbf{B}_l^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l)P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_l\}}{f(\mathbf{B}_l^t|\,\mathbf{I}_l)} \\ &= \frac{f(\mathbf{B}_l^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k},\, \mathbf{I}_{l,1})P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_l\}}{f(\mathbf{B}_l^t|\mathbf{I}_l)}. \end{aligned} \tag{20}$$

The first equality holds because the bids of permanent sellers are independent of bids of the transitory sellers and do not depend on the qualities of the transitory sellers, $f(\mathbf{B}_l^p|\,\mathbf{I}_l) = f(\mathbf{B}_l^p|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l)$.

We denote terms in this expression by

$$(I) = f(\mathbf{B}_l^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}),\ (II) = f(\mathbf{B}_l^t|\,\mathbf{I}_{l,1}),\ (III) = P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_l\}.$$

Next, we work with these terms one by one.

Term (I) Notice that $\mathbf{B}_l^t$ are independent conditional on $\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}$, and $\mathbf{I}_{l,1}$. Therefore

$$(I) = \prod_{x \in \mathcal{X}} \prod_{j \in A_{x,l}^t} f(B_{j,l}^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}) = \prod_{x \in \mathcal{X}} \prod_{j \in A_{x,l}^t} f(B_{j,l}^t|\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}). \tag{21}$$

The last equality holds because the transitory seller knows his quality but not the quality of his transitory competitors.

Term (II) Applying the rule of total probability we obtain

$$(II) = \sum_{d=1}^{\bar{K}_A} f(\mathbf{B}_l^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}, \mathbf{I}_l) P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}|\, \mathbf{I}_l\} \tag{22}$$

$$= \sum_{d=1}^{\bar{K}_A} f(\mathbf{B}_l^t|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}, \mathbf{I}_{l,1}) P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}|\, \mathbf{I}_l\}.$$

We will return to this expression after we tackle term (III).

Term (III) Our goal here is to relate an event in (III) to transitory bidders' participation (entry) decisions, and to express (III) in terms of the participation probabilities of the transitory bidders. First, we consider

$$(III) = P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\, \mathbf{I}_{l,1}, \mathbf{I}_{l,2} = \bar{I}_{l,2}),$$

where $\bar{I}_{l,2} = (m_{x,q}^p, m_x^t : x \in \mathcal{X}$ and $q \in \mathbb{Q}_x)$. Then observe that this conditional probability is equal to

$$\begin{aligned} P(\mathbf{Q}_{A,l}^t &= \bar{\mathbf{q}}_{A,k}|\, \mathbf{I}_{l,1},\, |A_{x,q,l}^p| = m_{x,q}^p,\, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q) \\ &= \frac{P(|A_{x,q,l}^p| = m_{x,q}^p,\, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q|\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k},\, \mathbf{I}_{l,1}) P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\mathbf{I}_{l,1})}{P(|A_{x,q,l}^p| = m_{x,q}^p,\, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q|\, \mathbf{I}_{l,1})} \\ &= \frac{P(|A_{x,l}^t| = m_x^t \text{ for all } x,\, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\, \mathbf{I}_{l,1})}{\sum_{d=1}^{\bar{K}_A} P(|A_{x,l}^t| = m_x^t \text{ for all } x,\, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}|\, \mathbf{I}_{l,1})}. \end{aligned} \tag{23}$$

The second equality holds because the events $|A_{x,q,l}^p| = m_{x,q}^p,$ for all $(x,\, q)$ and $|A_{x,l}^t| = m_x^t,$ for all x are independent conditional on $\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k},\, \mathbf{I}_{l,1}$, and the event $|A_{x,q,l}^p| = m_{x,q}^p,$ for all $(x,\, q)$ is independent of $\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}$ conditional on $\mathbf{I}_{l,1}$. We next work on the expression $P(|A_{x,l}^t| = m_x^t$ for all $x,\, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\bar{\mathbf{x}}_{A,l}^t, \mathbf{I}_{l,1})$ in the numerator of equation (23). We then return to equations (23) and (20) to conclude our derivation. We let $\mathbf{Q}_{N,l}^t = (Q_{N,j,l}^t)_{j \in N_l^t}$ and $\mathbb{Q}_l^N$ be the set of values that $\mathbf{Q}_{N,l}^t$ takes. Then

$$\begin{aligned} &P(|A_{x,l}^t| = m_x^t \text{ for all } x, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\, \mathbf{I}_{l,1}) = \\ &= \sum_{\tilde{\mathbf{q}} \in \mathbb{Q}_l^N} P(|A_{x,l}^t| = m_x^t \text{ for all } x, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k},\, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}|\, \bar{\mathbf{x}}_{A,l}^t, \mathbf{I}_{l,1}) \\ &= \sum_{\tilde{\mathbf{q}} \in \mathbb{Q}_l^N} \prod_{x \in \mathcal{X}} P\left\{|A_{x,l}^t| = m_x^t, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}},\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}|\, \mathbf{I}_{l,1}). \end{aligned} \tag{24}$$

Further notice that $P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}|\, \bar{\mathbf{x}}_l^t,\, \mathbf{I}_{l,1}) = P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}|\, \bar{\mathbf{x}}_l^t) = \prod_{j \in N_l^t} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}_{j,l}^t)$. The probability $P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}_{j,l}^t)$ is primitive in our environment, which characterizes the distribution of sellers' qualities within $x-$cell. We now show how the expression for

$$P\left\{|A_{x,l}^t| = m_x^t, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}},\, \mathbf{I}_{l,1}\right\}$$

can be modified and then return to equation (24). Recall that $\pi_l^t(j)$ links elements from some set $\Omega_x \subset \{1, ..., |N_l^t|\}$ to a vector $\{1, ..., |A_l^t|\}$. Then for a given $\bar{\mathbf{q}}_{A,k}$ and $\tilde{\mathbf{q}}$ we obtain

$$
\begin{aligned}
& \sum_{\Omega_x \subset \bar{N}_{x,l}^t} P\left\{ \begin{array}{c} j \in A_{x,l}^t,\ \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\ \text{for all } \pi_l^t(j) \in \Omega_x \text{ and} \\ j \in N_{x,l}^t - A_{x,l}^t \text{ for all } \pi_l^t(j) \in N_{x,l}^t - \Omega_x \end{array} \middle| \, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\ \bar{\mathbf{x}}_l^t,\ \mathbf{I}_{l,1} \right\} \\
= & \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j) \in \Omega_x} P\left\{ j \in A_{x,l}^t,\ \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k} |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\ \bar{\mathbf{x}}_l^t,\ \mathbf{I}_{l,1} \right\} \\
& \times \prod_{\pi_l^t(i) \in \bar{N}_{x,l}^t - \Omega_x} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\ \bar{\mathbf{x}}_l^t,\ \mathbf{I}_{l,1} \right\},
\end{aligned}
$$

where the sum over all sets Ω_x that are consistent with the restrictions imposed on the set of entrants, i.e, $\Omega_x \subset \bar{N}_{x,l}^t$ such that $|\Omega_x| = m_x^t$, $\tilde{q}_{\pi_l^t(j)} = \bar{\mathbf{q}}_{A,j,k}$ for all j such that $\pi_l^t(j) \in \Omega_x$. Next,

$$
\begin{aligned}
= & \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j) \in \Omega_x} P\left\{ j \in A_{x,l}^t,\ \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k} |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\ \mathbf{I}_{l,1} \right\} \\
& \times \prod_{\pi_l^t(i) \in \bar{N}_{x,l}^t - \Omega_x} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\ \mathbf{I}_{l,1} \right\} \\
= & \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j) \in \Omega_x} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\ \mathbf{I}_{l,1} \right\} \\
& \times \prod_{\pi_l^t(i) \in \bar{N}_{x,l}^t - \Omega_x} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\ \mathbf{I}_{l,1} \right\}
\end{aligned}
$$

Notice that for every Ω_x the set of qualities within Ω_x and $N_{x,l}^t - \Omega_x$ is the same. Therefore, the expression above can be written

$$
\begin{aligned}
& \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j) \in \Omega_x} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\ \mathbf{I}_{l,1} \right\} \\
& \times \prod_{\pi_l^t(i) \in \bar{N}_{x,l}^t - \Omega_x} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\ \mathbf{I}_{l,1} \right\} \\
= & \ |\Omega^x| \prod_{\pi_l^t(j) \in \Omega_x^0} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\ \bar{\mathbf{x}}_{\pi_l^t(j),l}^t,\ \mathbf{I}_{l,1} \right\} \\
& \times \prod_{\pi_l^t(i) \in \bar{N}_{x,l}^t - \Omega_x^0} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\ \mathbf{I}_{l,1} \right\}
\end{aligned}
\tag{25}
$$

Here, $|\Omega^x|$ denotes the cardinality of set $\Omega^x = \{\Omega_x : \Omega_x \subset \bar{N}_{x,l}^t$, such that $|\Omega_x| = m_x^t$ and $\tilde{q}_{\pi_l^t(j)} = \bar{\mathbf{q}}_{A,j,k}$, for all $\pi_l^t(j) \in \Omega_x\}$, with Ω_x^0 representing one specific member of Ω^x. For example, we can set $\Omega_x^0 = \bar{A}_{x,l}^t$.

Returning with expression (25) to equation (24) obtains

$$\begin{aligned}
&\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} P(|A^t_{x,l}| = m^t_x \text{ for all}\, x, \text{ and } \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}|\, \mathbf{Q}^t_{N,l} = \tilde{\mathbf{q}},\, \mathbf{I}_{l,1}) P(\mathbf{Q}^t_{N,l} = \tilde{\mathbf{q}}|\, \mathbf{I}_{l,1}) \\
= &\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{\pi_l^t(j)\in\Omega_x^0} P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} \\
&\times \prod_{\pi_l^t(i)\in\bar{N}^t_{x,l}-\Omega_x^0} P\left\{i \in N^t_{x,l} - A^t_{x,l}|\, \mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{N,l} = \tilde{\mathbf{q}}|\, \mathbf{I}_{l,1}) \\
= &\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{j\in\bar{A}^t_{x,l}} P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} \\
&\times \prod_{\pi_l^t(i)\in\bar{N}^t_{x,l}-\bar{A}^t_{x,l}} P\left\{i \in N^t_{x,l} - A^t_{x,l}|\, \mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1}\right\} \prod_{j\in N_l^t} P(\mathbf{Q}^t_{N,j,l} = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}^t_{j,l})
\end{aligned}$$

Note that in the last summation over $\tilde{\mathbf{q}} \in \mathbb{Q}_l^N$, part of the vectors in $\tilde{\mathbf{q}}$ such that $(\tilde{q}_j)_{j\in\bar{A}_l^t}$, and hence the summation is essentially over vectors in $\mathbb{Q}_l^{N-\bar{A}_l^t}$ which is the set of values for $\mathbf{Q}^t_{N-\bar{A}_l^t} \equiv (Q^t_{N_l^t-\bar{A}_l^t,j,l})_{j\in N_l^t-\bar{A}_l^t}$. Thus we write the last sum as

$$\begin{aligned}
&\sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{j\in\bar{A}^t_{x,l}} P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} \prod_{j\in\bar{A}_l^t} P(\mathbf{Q}^t_{N,j,l} = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}^t_{j,l}) \qquad (26)\\
&\times \prod_{i\in\bar{N}^t_{x,l}-\bar{A}^t_{x,l}} P\left\{i \in N^t_{x,l} - A^t_{x,l}|\, \mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1}\right\} \prod_{j\in\bar{N}_l^t-\bar{A}_l^t} P(\mathbf{Q}^t_{N,j,l} = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}^t_{j,l}) \\
= &\prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} \left\{P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{N,j,l} = \tilde{\mathbf{q}}_j|\, \bar{\mathbf{x}}^t_{j,l})\right\} \\
&\times \sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{i\in\bar{N}^t_{x,l}-\bar{A}^t_{x,l}} \left\{P\left\{i \in N^t_{x,l} - A^t_{x,l}|\, \mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_i|\, \bar{\mathbf{x}}^t_{i,l})\right\}.
\end{aligned}$$

The expression in (26) is derived for an arbitrary $\bar{\mathbf{q}}_{\mathbf{A},\mathbf{k}}$. Therefore, we substitute it into both the numerator and the denominator of (23). Notice that the dimensionalities of actual and potential bidders' x-sets are the same in the numerator and the denominator and that is why both expressions contain the common factor:

$$\sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{i\in\bar{N}^t_{x,l}-\bar{A}^t_{x,l}} \left\{P\left\{i \in N^t_{x,l} - A^t_{x,l}|\, \mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{N,i,l} = \tilde{\mathbf{q}}_i|\, \bar{\mathbf{x}}^t_{i,l})\right\}$$

Therefore, after canceling out this factor, the expression in (23) transforms into

$$\begin{aligned}
P(\mathbf{Q}^t_{A,l} &= \bar{\mathbf{q}}_{A,k}|\mathbf{I}_{l,1}, \mathbf{I}_{l,2} = \bar{I}_{l,2}) \\
&= \frac{\prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} \left\{P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}|\, \bar{\mathbf{x}}^t_{j,l})\right\}}{\sum_{d=1}^{\bar{K}_l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} \left\{P\left\{j \in A^t_{x,l}|\, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d}|\, \bar{\mathbf{x}}^t_{j,l})\right\}}.
\end{aligned}$$

Having obtained an expression for (III), we now return to (II).

Denote $\omega^t_{A,k,l} = \prod_{j\in\bar{A}^t_l} P(\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}|\,\bar{\mathbf{x}}^t_{j,l})$ and write

$$(II) = \sum_{k=1}^{\bar{K}_A} f(\mathbf{B}^t_l|\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}) P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_l\} \tag{27}$$

$$= \sum_{k=1}^{\bar{K}_A} \frac{\omega^t_{A,k,l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} f(\mathbf{B}_j|\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}) P\left\{j \in A^t_{x,l}|\,\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\}}{\sum_{d=1}^{\bar{K}_l} \omega^t_{A,d,l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} P\left\{j \in A^t_{x,l}|\,\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1}\right\}}$$

Finally, combining (I), (II), and (III) obtains

$$p_{k,l} = \frac{\omega^t_{A,k,l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} f(\mathbf{B}_j|\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}) P\left\{j \in A^t_{x,l}|\,\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\}}{\sum_{d=1}^{\bar{K}_A} \omega^t_{A,d,l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}^t_{x,l}} f(\mathbf{B}_j|\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d}, \mathbf{I}_{l,1}) P\left\{j \in A^t_{x,l}|\,\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1}\right\}}.$$

Appendix E: Empirical Analysis Section

Moments Used in Estimation

The estimation is based on three sets of moment conditions. The first set of moments relates the probability that a permanent seller wins under a variety of configurations of the sets of permanent actual and potential bidders. The second set concerns the probability that project is not allocated for various configurations of the set of active permanent bidders. The third set links transitory and permanent sellers' empirical distribution of bids and participation frequencies to their theoretical counterparts.

The first set of moments is further subdivided into three sub-subsets:

(1a) Moments that are based on the permanent seller's probability of winning conditional on two or more permanent bidders belonging to the same quality group. In these moment conditions, we compute expectations of the following functions: a constant (equal to one), the winning bid, the difference between the winning bid and another bid submitted by a bidder from the same quality group, or the squared difference between the winning bid and another bid submitted by a bidder from the same quality group respectively.

(1b) Moments that are based on the permanent seller's probability of winning conditional on this seller's quality group, and one or more permanent bidders belonging to a different quality groups. In these moment conditions, we compute expectations of the following functions: a constant (equal to one), the winning bid, the squared winning bid, the difference between the winning bid and a bid submitted by seller from a different quality group, the squared difference between the winning bid and a bid submitted by a seller from a different quality group, respectively. We include moments for all possible pairs of different quality groups.

(1c) Moments that are based on the permanent seller's probability of winning conditional on this seller's quality group, one or more permanent bidders belonging to a different quality group, and at least one transitory qualified bidder belonging to a specific country group. In these moment conditions, we compute expectations of the following functions: the winning bid, the product of transitory bidder's bid and the differences between the winning bid and the bid of a permanent seller from a different quality group, the product of transitory bidder's characteristics other than price and the differences between the winning bid and the bid of a permanent seller from a different quality group.

The second set of moments matches the following empirical moments:

(2a) the probability that project is not allocated;

(2b) the first and second order moments involving prices submitted by permanent active bidders from different (x, q) groups given that project is not allocated.

The moments in the first two sets are computed for five most frequent configurations of the sets of active and potential bidders.

The third set of moments matches the following empirical moments to their theoretical counterparts: the mean and variance of the permanent and transitory bid distributions, as well as the covariance between the bid and the seller's other characteristics, the frequencies of transitory and permanent bidders submitting bids as well as the expected value of the actual bidders' characteristics conditional on a set of permanent potential bidders, country group for transitory bidders, and country, reputation score, and quality group for permanent bidders. We include a separate moment for each of the five most frequent configurations of the set of permanent potential bidders.

We estimate the distribution of transitory sellers' bids and their probability of participation separately, instead of working with the composite function $g_k(.)$ described in Section 4.3. That is why we include the third set of moments in addition to the probability-of-winning and probability-of-not-allocating moments. We rely on exclusion restrictions in order to separate the product of bid density and participation probability into individual components. We condition moments from set two on the country group of transitional sellers while restricting the coefficient that captures the effect of the number of scores or current average reputation score to be constant across countries. Therefore, the differences in the moments across country groups reveal the dependence of bidding or participation strategies on the bidder's own quality, since the distributions of qualities differ across country groups.

An alternative identification strategy relies on the expected profit conditions that summarize the optimal participation decision of transitory bidders. These conditions impose the restriction that in equilibrium only potential bidders with entry costs below the ax-ante expected profit value should participate. In our setting, re-computing the expected profit values at each iteration is very costly. That is why we opted for the exclusion restriction channel of identification. However, this alternative estimation approach is also feasible. We were able to obtain a set of coefficients using such an alternative estimation strategy. They are very similar to the set of estimates we report in the paper.

Supplemental Appendix

Section A: Nonparametric Identification

Section A1: Buyers' taste for observed characteristics

To see how to recover the distribution of β (buyers' tastes for observed characteristics), consider an auction involving active bidders who are all permanent sellers. Let their quality levels be identified. Then

$$\begin{aligned} & \Pr\left(i \text{ wins } \middle| A = a, \, (b_i)_{i \in a}\right) \\ = \; & \Pr\begin{pmatrix} \Delta\epsilon_{j,i} + \alpha\Delta q_{j,i} + \Delta x_{j,i}\beta \leq b_j - b_i \;\forall j \in a\backslash\{i\} \\ \text{and} \\ U_0 - \alpha q_i - \epsilon_i \leq -b_i \end{pmatrix} \end{aligned}$$

where $\Delta x_{j,i} \equiv x_j - x_i$. We need to introduce the following condition for identification of the distribution of buyers' taste for observed heterogeneity.

(Iden-Beta) *There exist some i and a subset of permanent sellers a with $i \in a$ such that (i) the $(|a|-1)$-by-J matrix $(\Delta x_{j,i})_{j \in a\backslash\{i\}}$, with J being the number of coordinates in x_i, has full rank; (ii) the support of $((\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta)_{j \in a\backslash\{i\}}, U_0 - \alpha q_i - \epsilon_i)$ is a subset of the support of $((B_j - B_i)_{j \in a\backslash\{i\}}, -B_i)$; and (iii) $((\Delta\epsilon_{j,i} + \alpha\Delta q_{j,i})_{j \in a\backslash\{i\}}, U_0 - \alpha q_i)$ have non-vanishing characteristic functions.*

Both (i) and (iii) in (Iden-Beta) are mild and standard assumptions in nonparametric identification. The support conditions in (ii) can be structurally justified along similar lines as mentioned in the Supplemental Appendix of Krasnokutskaya, Song and Tang (2013).

Proposition A1 *Suppose the distribution of (α, ϵ, U_0) and the quality levels are identified. If the condition (Iden-Beta) holds, then the distribution of β is identified.*

A sketch of the proof is as follows. Provided the joint support of $(B_j - B_i)_{j \in a\backslash i}$ and $-B_i$ is large enough to cover the joint support of $((\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta)_{j \in a\backslash i}, \, U_0 - \alpha q_i - \epsilon_i)$, we can recover the joint distribution of the latter. With F_{ϵ_i} and F_α identified as above, and under our assumption that $(\epsilon_i)_{i \in a}$ are i.i.d. and jointly independent from α, the distribution of $\Delta\epsilon_{j,i} + \alpha\Delta q_{j,i}$ is known. With β assumed to be independent from α and $(\epsilon_i)_{i \in N}$, we can identify the distribution of $(\Delta x_{j,i}\beta)_{j \in a\backslash i}$ by taking the ratio of the characteristic functions of $(\Delta\epsilon_{j,i} + \alpha\Delta q_{j,i} + \Delta x_{j,i}\beta)_{j \in a\backslash i}$ and $(\Delta\epsilon_{j,i} + \alpha\Delta q_{j,i})_{j \in a\backslash i}$. As long as the matrix $(\Delta x_{j,i})_{j \in a\backslash i}$ is full-rank, then the joint density of β is identified as the product of the joint density of $(\Delta x_{j,i}\beta)_{j \in a\backslash i}$ and the absolute value of the determinant of the square matrix $(\Delta x_{j,i})_{j \in a\backslash i}$ under the standard change-of-variable techniques.

Section A2: Distribution of private costs

We discuss how the distribution of project's costs can be identified. We consider a simple case when bidders' entry costs are independent of the project's costs. The general result obtains by combining steps presented below with the identification strategy proposed by Li and Gentry (2012).

The identification of the distribution of project's costs. in a simple case of signals independence follows an argument similar to Guerre, Perrigne, and Vuong (2000). To see this, note that the quality levels for permanent sellers, the distribution of quality levels of transitory sellers, and buyer tastes can be considered known since they are identified in preceding sub-sections.

The inverse bidding strategy can be recovered as follows. The first-order condition for bidder i choosing price b_0 in equilibrium is:

$$(b_0 - c_i) \frac{\partial}{\partial b} \Pr\{\varpi(A) \leq -b\} |_{b=b_0} = \Pr\{\varpi(A) \leq -b_0\} \tag{28}$$

where $\varpi(A)$ denotes the maximum of $\max_{j\in A\backslash\{i\}} (\alpha Q_j - \alpha q_i + \Delta x_{j,i}\beta + \Delta\epsilon_{j,i} - B_j)$ and $U_0 - \alpha q_i$.

Thus, it suffices to show that the distribution on the right-hand side can be identified. It would imply that the derivative on the left-hand side would also be identified, in which case the inverse bidding strategy (and consequently the distribution of private cost C_i) would also be recovered for every subpopulation of sellers with (x, q).

Note the right-hand side of (28) is:

$$\sum\nolimits_{\{a:a\subseteq N\}} P\{\varpi(A) \leq -b_i|\ A = a\} \Pr\{A = a\}.$$

For those entrants in $A\backslash\{i\}$ who are transitory sellers, their qualities are multinomial random variables whose distributions are recoverable, with the distribution of qualities assumed to be the same across the two subpopulations of permanent sellers and transitory sellers (i.e. $\pi_{p,k} = \pi_{t,k}$). Finally note that, given any fixed a, the distribution of $\varpi(a)$ can be constructed from the joint distribution of $\alpha, \epsilon, \beta, U_0$ identified earlier and the distribution of submitted bids given all independences assumptions in (A1) and (A2).

Section A3: Discussion of Support Conditions

Our identification strategy relies on the condition that the supports of prices quoted by permanent sellers should be sufficiently large (condition (A3)-(ii)). In this section we provide a heuristic argument showing that our model is capable of generating the variation in prices necessary for this condition to hold. We do so in the context of a simple model which abstracts away from the differences in unobserved qualities, stochastic participation, and the presence of transitory bidders but allows for the allocation rule to include a stochastic match component. Adding these features complicates the algebra but does not require any additional insight.

In this simple model, the support condition for identifying the distribution of match components is reduced to:

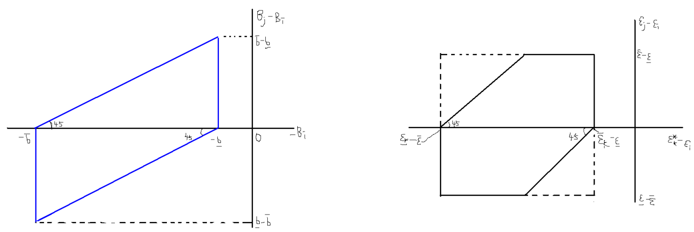
"There exist i, j with $q_i = q_j$ such that the support of $(B_j - B_i, -B_i)$ includes that of $(\epsilon_j - \epsilon_i, U_0 - \epsilon_i)$."

Note that, in a type-symmetric equilibrium which we consider here, the support of bids from i, j are identical, and denoted by $[\underline{b}_i, \overline{b}_i]$.

The respective supports of $(B_j - B_i, -B_i)$ and $(\epsilon_j - \epsilon_i, U_0 - \epsilon_i)$ are depicted in Figure 1.[49]

[49]In this Figure, $\underline{\varepsilon}$ and $\overline{\varepsilon}$ denote the infinum and supremum of the support of ϵ_i (and ϵ_j) and $\underline{u}_0, \overline{u}_0$ denote the infinum and supremum for the support of U_0.

Figure 1: Support Conditions



These figures show the respective supports of $(B_j - B_i, -B_i)$ and $(\epsilon_j - \epsilon_i, U_0 - \epsilon_i)$.

It is clear from these figures that a set of sufficient conditions for the support restrictions above is:

$$\text{“there exist } i, j \text{ with } q_i = q_j \text{ s.t. } \overline{b} > \overline{\varepsilon} - \underline{u}_0 \text{ and } \underline{b} < \underline{\varepsilon} - \overline{u}_0,\text{”}$$

which can be satisfied if (a) $\overline{b} > \overline{\varepsilon} - \underline{u}_0$ and (b) $\overline{b} - \underline{b} > (\overline{u}_0 - \underline{u}_0) + (\overline{\varepsilon} - \underline{\varepsilon})$. Condition (a) holds provided $\overline{b} \geq \overline{c_i} > \overline{\varepsilon} - \underline{u}_0$, where $\overline{c_i}$ is the supreme of the support of private costs for i and j. Condition (b) essentially requires the support of bids to be large relative to that of outside utility and match components. Intuitively, (b) also holds when the support of sellers' private costs is sufficiently large. We now provide a simple argument for this intuition.

The idea is to show that the bidding strategy is continuous in the length of the support of match component $\overline{\varepsilon} - \underline{\varepsilon}$ and the support of outside option.

Under type-symmetric PSBNE the bidders' strategies solve the maximization problem:

$$\sigma_i(c) \equiv \arg\max_b (b - c) \Pr\left(U_0 - \epsilon_i \leq -b\right) \Pr\left(\max_{j \in A\setminus\{i\}} \{-B_j + \epsilon_j\} - \epsilon_i \leq b\right) \tag{29}$$

The second probability in (29) represents i's belief, which is formed from i's knowledge of S, the distribution of private costs $\{F_k\}_{k \leq K}$, and the distribution of qualities in the population of sellers.

Suppose ϵ_i are i.i.d. uniform over $[\underline{\varepsilon}, \bar{\varepsilon}]$. Applying the Law of Total Probability, we can write the objective function for seller i with costs c as

$$(b - c) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{1 - F_{B_j}(b - \Delta\varepsilon_{i,j})}{\bar{\varepsilon} - \underline{\varepsilon}} d\varepsilon_j \right) \frac{F_{U_0}(-b + \varepsilon_i)}{\bar{\varepsilon} - \underline{\varepsilon}} \right] d\varepsilon_i. \tag{30}$$

Changing variables between ε_r and $\tau_r \equiv \frac{\varepsilon_r - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ for $r = i, j$, we can write (30) as

$$(b - c) \int_0^1 \left(\int_0^1 F_{U_0}(\underline{\varepsilon} - b + \delta\tau_i) \left[1 - F_{B_j}(b - \delta\Delta\tau_{i,j})\right] d\tau_j \right) d\tau_i \tag{31}$$

where $\delta \equiv \bar{\varepsilon} - \underline{\varepsilon}$ is the length of support of match components. Note (31) is continuous in both δ and the length of support for U_0. It then follows from an application of the Theorem of Maximum that the support of bids is continuous in the size of the support of match component and outside options.

Provided private costs vary sufficiently, the support of bids in a standard auction model with no match components (i.e. ϵ is degenerate at 0) and no outside option is an interval with non-degenerate interior. It then follows from the implication of the Theorem of Maximum that condition (b) holds whenever the support of ϵ and U_0 is small enough.

By the same token, we can provide similar structural justifications for the support conditions for recovering quality levels using i, j with $q_i \neq q_j$, as long as variation in private costs and quality differences are sufficiently large relative to that of buyers' tastes in (α, ϵ, U_0). In the current example, the presence of transitory sellers would not entail any qualitative different arguments for the structural justification of the support conditions.

Section B: Confidence Sets for Quality Classifications

Suppose that we have K_0 groups. Since we can estimate K_0 consistently, we assume we know it. We fix $k = 1, \cdots, K_0$ and construct a confidence set for the k-th quality group. In other words, we would like to construct a random set $\widehat{\mathcal{C}}_k \subset N$ such that

$$\text{liminf}_{L \to \infty} P\{N_k \subset \widehat{\mathcal{C}}_k\} \geq 1 - \alpha.$$

Thus this random set $\widehat{\mathcal{C}}_k$ is a confidence set for each quality group. In other words, the confidence set $\widehat{\mathcal{C}}_k$ for each k is a random set whose probability of containing the set of players with quality level q_k is ensured to be bounded from below by $1 - \alpha$ asymptotically.

We need to devise a way to approximate the finite sample probabilities like $P\{N_k \subset \widehat{\mathcal{C}}^k\}$. Since we do not know the cross-sectional dependence structure among the sellers, we use a bootstrap procedure that preserves this dependence structure from the original sample. We first estimate $\hat{N}_k$ as prescribed above and also obtain $\hat{\tau}^0_{ij}$. Given the estimate $\hat{N}_k$, we construct a sequence of sets as follows:

Step 1: Find $i_1 \in N \backslash \hat{N}_k$ that minimizes $\min_{j \in \hat{N}_k} \hat{\tau}^0_{i_1,j}$, and construct $\widehat{\mathcal{C}}_k(1) = \hat{N}_k \cup \{i_1\}$.

Step 2: Find $i_2 \in N \backslash \widehat{\mathcal{C}}_k(1)$ that minimizes $\min_{j \in \hat{C}_k(1)} \hat{\tau}^0_{i_2,j}$, and construct $\widehat{\mathcal{C}}_k(2) = \hat{C}_k(1) \cup \{i_2\}$.

Step m: Find $i_m \in N \backslash \widehat{\mathcal{C}}_k(m-1)$ that minimizes $\min_{j \in \hat{C}_k(m-1)} \hat{\tau}^0_{i_m,j}$ and construct $\widehat{\mathcal{C}}_k(m) = \widehat{\mathcal{C}}_k(m-1) \cup \{i_m\}$.

Repeat Step m up to $|N|$.

Now, for each bootstrap iteration $s = 1, \cdots, B$, we construct the sets $\hat{N}^*_{k,s}$ and $\{\widehat{\mathcal{C}}^*_{k,s}(m)\}$ following the steps described above but using the bootstrap sample. (Note that this bootstrap sample is independent of the bootstrap sample used to construct $p^*_0(i, j)$.)

Then, we compute the following:

$$\hat{\pi}^k(m) \equiv \frac{1}{B} \sum_{s=1}^{B} 1 \left\{ \hat{N}_k \subset \widehat{\mathcal{C}}^*_{k,s}(m) \right\}.$$

Note that the sequence of sets $\widehat{\mathcal{C}}^*_{k,s}(m)$ increases in m. Hence the number $\hat{\pi}^k(m)$ should also increase in m. An $(1-\alpha)\%$ level confidence set is given by $\widehat{\mathcal{C}}_k(m)$ with m, such that

$$\hat{\pi}^k(m-1) < 1-\alpha \leq \hat{\pi}^k(m).$$

If there exists no such $m \leq m_0$ that satisfies this inequality, then set $m = m_0$.

Appendix C: Monte Carlo Analysis

Here we explore properties of our classification algorithm in simulation analysis.

We choose the distributions of project and entry costs to be the same across quality levels and given by truncated normals with $N(1.5, 0.2^2)$ and $N(0.08, 0.02^2)$ correspondingly. Further, we assume that the distribution of the reserve price coincides with the bid distribution of high-quality bidders. The bidders are assumed to be heterogeneous with respect to quality only. Buyers' tastes, therefore, are represented by the distributions of α and ϵ. We fix the distribution of α to be truncated normal $N(0.4, 0.2^2)$ with support $[0, 1]$. The distribution of ϵ is also chosen to be truncated normal with mean 0 and variance σ_ϵ^2. We vary σ_ϵ in experiments below to explore the sensitivity of our methodology to the noise in buyers' tastes. We truncate the support of epsilon at $[-\sigma_\epsilon, \sigma_\epsilon]$. Finally, we assume that the set of suppliers consists of 30 programmers and is split equally between high- and low-quality suppliers.

We use the modified projection algorithm from Paarsch, Hubbard (2009) and Bajari (2000) to solve for participation and bidding strategies of our game. The data are generated through repetition of the following steps:

1. At each round, 10 randomly selected bidders from the set above are declared to be potential bidders.

2. For each potential bidder we draw an entry and project cost. We, then, apply participation and bidding strategies to these draws to determine whether a potential bidder enters the set of active participants and if he does what bids he submits.

3. Next, we take draws from the distributions of α, ϵ, and of the reserve price. The winner of the project is determined by evaluating submitted bids using the reserve price and buyer's tastes.

4. The data record the set of potential bidders with their qualities, outcomes of participation and entry decisions as well as the buyer's choice.

We use the simulated data to investigate the sensitivity of our methodology to the magnitude of the quality differences, the noise in buyer's preferences, and the number of available observations. For the first two experiments we tie the quality differences and the noise magnitude to the variance in the private project costs. That is, we consider (a) high-quality differences with $\Delta Q = 0.3(\bar{c} - \underline{c})$ and (b) low-quality differences with $\Delta Q = 0.3(\bar{c} - \underline{c})$. Similarly, we consider (c) low preference noise with $\sigma_\epsilon = 0.2\sigma_c$, (d) medium preference noise with $\sigma_\epsilon = 0.5\sigma_c$, (e) high preference noise with $\sigma_\epsilon = 1.2\sigma_c$. Finally, we explore how the performance of our procedure changes with sample size. Our procedure is performed at the individual level, therefore, we explore the performance of our procedure as a function of the average number of bids per supplier.

Table 7: Results of Simulation Study

		Probability of Correct Classification					
		number of bids=300		number of bids=200		number of bids=100	
d_Q	σ_ϵ	Q_H	Q_L	Q_H	Q_L	Q_H	Q_L
0.3	$0.2\sigma_c$	0.9773	0.9901	0.9613	0.9547	0.9314	0.9013
0.3	$0.5\sigma_c$	0.9645	0.9858	0.9477	0.9512	0.9223	0.8998
0.3	$1.5\sigma_c$	0.9619	0.9782	0.9457	0.9401	0.9207	0.8941
0.1	$0.2\sigma_c$	0.9632	0.9774	0.9329	0.9503	0.9164	0.8904
0.1	$0.5\sigma_c$	0.9551	0.9743	0.9263	0.9421	0.9034	0.8815
0.1	$1.5\sigma_c$	0.9518	0.9701	0.9227	0.9397	0.8927	0.8623

This table reports results of the simulation study of the sensitivity of the classification procedure to the quality differences, the magnitude of the preference noise, and the data set size. The latter is measured in the average number of bids per supplier. The difference in quality levels is measured relative to the project costs spread, i.e., $d_Q = \frac{Q_H - Q_L}{\bar{c} - \underline{c}}$. The variance of the preference noise is measured relative to the project cost variance, i.e., $\sigma_\epsilon = d_\epsilon \sigma_c$.

We run the simulation experiments as follows. For every set of parameters, we apply our procedure to 500 data sets simulated according to steps (1)-(4) described above. We then compute for every supplier the fraction of the data sets in which his type was correctly recovered. We report the average of these fractions across bidders of the same quality level in Table 7.

The results of the simulation analysis show that the classification procedure performs quite well.[50] In particular, it is not very sensitive to the magnitude of the preference noise. We would expect the preference noise to impede recovery of the quality level since it disguises the link between the probability of winning and the quality of participant. It would be natural to expect that the procedure should impose higher data requirements in the presence of more noise. However, the endogeneity of prices successfully compensates for the noise in buyers' preferences at least for moderate levels of noise. As the magnitude of the noise grows, bidding functions become flatter, thus ensuring that more observations fall in the neighborhood of a specific price level.

As expected, the performance of the procedure does depend on the importance of the quality differences. The estimation is more precise when quality differences are large and grows less precise as quality differences diminish. Finally, the procedure is sensitive to the size of the data set. As the number of bids drops from 300 bids per supplier to 100 bids per supplier the probability of correct classification drops from 0.96 to 0.92 for high-quality suppliers and from 0.98 to 0.89 for low-quality suppliers. The classification of low-quality suppliers is affected to a larger degree since due to the lower probability of participation, the number of bids they submit is substantially below the average.

[50]The procedure performs best when prices are scaled to lie in the [0,1] interval.

Section D: Details of GMM Estimation

Section D1: Moment conditions

We use the same notation in Appendix C of Krasnokutskay, Song, and Tang (2013). Using (18) and Proposition 3, we derive the moment conditions as follows. For any vector valued map $h^p_{x,q,1} : \mathbf{R}^{|A|} \to \mathbf{R}^{d_{h1}}$, and for each $x \in \mathcal{X}$, $q \in \mathbb{Q}_x$, we can write the moment condition as:

$$\begin{aligned} & \mathbf{E}\left[h^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l) W^p_{x,q,l} | \mathbf{I}_l\right] \\ = & \sum_{k=1}^{\bar{K}_{A,l}} \mathbf{E}\left[h^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l) e^p_{k,x,q,l}\left(\mathbf{B}_l, \mathbf{I}_l; \theta\right) \frac{\omega_{k,l} g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}; \theta)}{\sum_{d=1}^{\bar{K}_{A,l}} \omega_{d,l} g_d(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}; \theta)} | \mathbf{I}_l\right]. \end{aligned}$$

where $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}; \theta)$ is a parametric approximation of $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$,

$$\begin{aligned} W^p_{x,q,l} &= \frac{1}{|A^p_{x,q,l}|} \sum_{j \in A^p_{x,q,l}} W^p_{j,l} \text{ and} \\ e^p_{x,q,k,l}\left(\mathbf{B}_l; \theta\right) &= \frac{1}{|A^p_{x,q,l}|} \sum_{j \in A^p_{x,q,l}} e_{x,q,k,l}\left(\mathbf{B}_l; \theta, j\right). \end{aligned}$$

This can be further re-written as:

$$\begin{aligned} & \mathbf{E}\left[\mathbf{h}^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l)\left(W^p_{x,q,l} - e^p_{x,q,l}(\theta, g)\right) | \mathbf{I}_l\right] = 0 \text{ with} \\ & e^p_{x,q,l}(\theta, g) = \sum_{k=1}^{\bar{K}_l} e^p_{k,x,q,l}\left(\mathbf{B}_l; \theta\right) \frac{\omega_{k,l} g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}; \theta)}{\sum_{d=1}^{\bar{K}_{A,l}} \omega_{d,l} g_d(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}; \theta)}. \end{aligned} \tag{32}$$

As we discussed in the previous subsection, it may be useful, especially in the parametric setting, to separately specify and estimate $f(B^t_{i,l} | \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A^t_{x,l} | \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ functions. In such a case, restrictions associated with (a) transitory sellers' bid distribution, (b) the transitory sellers' probability of participation, as well as (c) restrictions summarizing optimal participation behavior may be additionally imposed. More details are given later.

To construct a sample version of the moment conditions, we first obtain a consistent estimator $\hat{\omega}_{k,l}$ of $\omega_{k,l}$ via the sample analog principle, and construct

$$\hat{\rho}_{x,q,l}(\theta) = \mathbf{h}^p_{x,q,1}(\mathbf{B}^p_l, \mathbf{I}_l)\left(W^p_{x,q,l} - \hat{e}^p_{x,q,l}(\theta)\right),$$

where $\hat{e}^p_{x,q,l}(\theta)$ is equal to $e^p_{x,q,l}(\theta)$ except that $\omega_{k,l}$ is replaced by $\hat{\omega}_{k,l}$ and $\hat{\omega}_{k,l}$ is the sample analogue of $\omega_{k,l}$. We define $\hat{\rho}_l(\theta)$ to be a column vector with $\hat{\rho}_{x,q,l}(\theta)$ stacked up with (x, q) running in $\mathcal{X} \times \mathbb{Q}_1$. Hence the dimension of $\hat{\rho}_l(\theta)$ is $d_{h,p} \times |\mathcal{X} \times \mathbb{Q}_1|$, where $|\mathcal{X} \times \mathbb{Q}_1|$ is the cardinality of the set $\mathcal{X} \times \mathbb{Q}_1$. Then define a general method of moment estimator as follows:

$$\hat{\theta}_{GMM} = \text{argmin}_{\theta \in \Theta} \hat{\mathbf{Q}}_{GMM}(\theta),$$

where

$$\hat{\mathbf{Q}}_{GMM}(\theta) = \left(\frac{1}{L}\sum_{l=1}^{L}\hat{\rho}_l(\theta)\right)\hat{\Sigma}^{-1}\left(\frac{1}{L}\sum_{l=1}^{L}\hat{\rho}_l(\theta)\right), \text{ and}$$

$$\hat{\Sigma} = \frac{1}{L}\sum_{l=1}^{L}\hat{\rho}_l(\bar{\theta})\hat{\rho}_l(\bar{\theta})',$$

and $\bar{\theta}$ is the first step estimator of θ_0 which is a minimizer of $\hat{\mathbf{Q}}_{GMM}(\theta)$ only with $\hat{\Sigma}^{-1}$ replaced by the identity matrix. Under regularity conditions, the estimator is known to be asymptotically normal with a positive definite covariance matrix. Note that the estimation error due to $\hat{\omega}_{k,l}$ does not affect the asymptotic variance matrix because the components $P\{\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}|\mathbf{I}_l\}$ of $\omega_{k,l}$ take values only from a finite set and hence has a convergence rate that is arbitrarily fast as $L \to \infty$.

Section D2: Semiparametric Estimation

The estimation method in the previous section employs parametrization of nonparametric functions g_k. The functions g_k involve the density of the transitory sellers' bids and participation probabilities. Since the bids and participations are equilibrium objects, one might prefer to use a more flexible specification for the nonparametric functions g_k. In this section, we explain how this extension can be done in practice.

Recall our definition $e^p_{x,q,l}(\theta, g)$ in (32) and define

$$\hat{\rho}_{x,q,s}(\theta, g) = \mathbf{h}^p_{x,q,1}(\mathbf{B}^p_l, \mathbf{I}_l)\left(W^p_{x,q,l} - \hat{e}^p_{x,q,l}(\theta, g)\right),$$

making its dependence on the nonparametric function g explicit. Let

$$\hat{\mathbf{m}}_{x,q,l}(\theta, g) = \frac{\sum_{s=1}^{L}\hat{\rho}_{x,q,s}(\theta, g)1\{\mathbf{I}_s = \mathbf{I}_l\}}{\sum_{s=1}^{L}1\{\mathbf{I}_s = \mathbf{I}_l\}}$$

and define $\hat{\mathbf{m}}_l(\theta, g)$ to be a column vector with $\hat{\mathbf{m}}_{x,q,l}(\theta, g)$ stacked up with (x, q) running in $\mathcal{X} \times \mathbb{Q}_1$. Hence the dimension of $\hat{\mathbf{m}}_l(\theta, g)$ is $d_{h,p} \times |\mathcal{X} \times \mathbb{Q}_1|$, where $|\mathcal{X} \times \mathbb{Q}_1|$ is the cardinality of the set $\mathcal{X} \times \mathbb{Q}_1$.

In the semiparametric estimation, we regard the nonparametric function g as an infinite dimensional nuisance parameter, and employ the sieve minimum distance estimation method of Ai and Chen (2003). As argued previously, the functions g are nonparametrically identified under our set-up. Now let $\mathcal{G}_L$ be the space of finite dimensional sieves whose dimension increases as the number of auctions L grows. Details about the appropriate sieves are found in the appendix. We construct the estimator as follows:

$$(\hat{\theta}_{CMD}, \hat{g}) = \text{argmin}_{(\theta,g)\in\Theta\times\mathcal{G}_L}\hat{\mathbf{Q}}_{CMD}(\theta, g),$$

where

$$\hat{\mathbf{Q}}_{CMD}(\theta, g) = \frac{1}{L}\sum_{l=1}^{L} \hat{\mathbf{m}}_l(\theta, g)\hat{\Sigma}_l^{-1}\hat{\mathbf{m}}_l(\theta, g),$$

and $\hat{\Sigma}_l$ is a consistent estimator of a nonsingular matrix Σ_l. Ai and Chen (2003) showed that under regularity conditions, we have

$$\sqrt{n}\left(\hat{\theta}_{CMD} - \theta_0\right) \to_d N(0, V),$$

where V is a nonsingular covariance matrix.

We turn to the form of the covariance matrix formula which follows Ai and Chen (2003). For each $i = 1, \cdots, N$, we let for $\alpha = (\theta, g) \in \mathcal{A}_L = \Theta \times \mathcal{G}_L$,

$$\begin{aligned} \mathbf{r}^{(1)}_{x,q,l}(\alpha) &= \mathbf{E}\left[\mathbf{h}^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l)\{W^p_{x,q,l} - e^p_{x,q,l}(\alpha)\}|\mathbf{I}_l\right] \\ \mathbf{r}^{(2)}_{x,q,l}(\alpha) &= \mathbf{E}\left[\mathbf{h}^t_{x,q,2}(\mathbf{B}_l, \mathbf{I}_l)\{W^t_{x,l} - e^t_{x,q,l}(\alpha)\}|\mathbf{I}_l\right] \end{aligned}$$

and $\mathbf{r}^{(1)}_l(\alpha) = [\mathbf{r}^{(1)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$ and $\mathbf{r}^{(2)}_l(\alpha) = [\mathbf{r}^{(2)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$. In other words, $\mathbf{r}^{(1)}_l(\alpha)$ is a column vector with vectors $\mathbf{r}^{(1)}_{x,q,l}(\alpha)$, $q \in \mathbb{Q}_x, x \in \mathcal{X}$, stacked up and $\mathbf{r}^{(2)}_l(\alpha) = [\mathbf{r}^{(2)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$ is a column vector with vectors $\mathbf{r}^{(2)}_{x,q,l}(\alpha)$, $q \in \mathbb{Q}_x, x \in \mathcal{X}$, stacked up. We define

$$\mathbf{m}_l(\alpha) = \begin{bmatrix} \mathbf{r}^{(1)}_l(\alpha) \\ \mathbf{r}^{(2)}_l(\alpha) \end{bmatrix}.$$

For each $j = 1, \cdots, d_\theta$, we define for $w \in \mathcal{G}$,

$$\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w - g_0] = \frac{\partial \mathbf{m}_l(\theta_0, \tau w + (1-\tau)g_0)}{\partial \tau}|_{\tau=0}.$$

The left-hand side term represents the pathwise derivative of $\mathbf{m}_l(\theta_0, g_0)$ along the direction $w - g_0$. Let w_j^* be the solution to the minimization problem:

$$\inf_{w_j\in\mathcal{G}}\left(\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta_j} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w_j - g_0]\right)' \Sigma_l \left(\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta_j} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w_j - g_0]\right),$$

where

$$\Sigma_l = \mathbf{E}\left[[\mathbf{r}^{(1)}_l(\alpha), \mathbf{r}^{(2)}_l(\alpha)][\mathbf{r}^{(1)}_l(\alpha)', \mathbf{r}^{(2)}_l(\alpha)']'|\mathbf{I}_l\right].$$

Then we take

$$D_{w^*,l} = \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta'} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}\left[w_1^* - g_0, \cdots, w_{d_\theta}^* - g_0\right].$$

The asymptotic covariance matrix formula becomes

$$V = \mathbf{E}\left[D_{w^*,l}\Sigma_l^{-1}D_{w^*,l}\right].$$

The estimation of the asymptotic covariance matrix formula can be done in a straightforward way. For example, we may follow Section 5 of Ai and Chen (2003), except that instead of using the series estimation to obtain the sample analogue of the conditional expectation $\mathbf{E}[\cdot|\mathbf{I}_l]$, we use the usual sample analogue of the conditional expectation with discrete conditional variables because $\mathbf{I}_l$ is a discrete random vector. Details are omitted.

Now let us consider the choice of the sieve space $\mathcal{G}_L$. Let $\bar{K} > 0$ be an integer such that for all $L \geq 1$, $\max_{l=1,\cdots,L} \bar{K}_l \leq \bar{K}$. Now let us discuss the construction of the sieve space $\mathcal{G}_L$. For each $k = 1, \cdots, \bar{K}$, and realized value of $(\mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$, the function $g_k(\cdot, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$ defined in (**??**) is of particular form. First we take $\mathcal{G}_L = \mathcal{G}_{1,L} \times \cdots \times \mathcal{G}_{\bar{K},L}$, where $\mathcal{G}_{k,L}$'s are constructed as follows. For $i = 1, \cdots, N$, L, and $k = 1, \cdots, \bar{K}$, let $\mathcal{G}_{k,L}$ and $\mathcal{F}_{k,L}$ be sieve spaces, where for each $g_{k,L} \in \mathcal{G}_{k,L}$, $g_{k,L} : \mathcal{I} \to [0,1]$ and for each $f_{k,L} \in \mathcal{F}_{k,L}$, $f_{k,L} : \mathbf{R} \times \mathcal{I} \to [0, \infty)$ and for each $\mathbf{I}_{l,1} \in \mathcal{I}$, $\int f_{k,L}(b, \mathbf{I}_{l,1}) db = 1$. Then we construct a sieve space $\mathcal{G}_{k,L}$ as the collection of maps $g_{k,L}(b, \mathbf{I}_{l,1})$, where

$$g_{k,L}(b, \mathbf{I}_{l,1}) = \prod_{x \in \mathcal{X}} \prod_{j=1}^{|\bar{A}_{x,l}^t|} f_{k,j,L}(b, \mathbf{I}_{l,1}) v_{k,j,L}^x(\mathbf{I}_{l,1}),$$

where $v_{k,j,L}^x \in \mathcal{V}_{k,j,L}^x$, and $f_{k,j,L} \in \mathcal{F}_{k,j,L}$. For $\mathcal{V}_{k,j,L}^x$, we choose a sieve space $\mathcal{M}_{k,j,L}^x$ of real-valued functions and define

$$\mathcal{V}_{k,j,L}^x = \left\{ \frac{\exp(m(\cdot))}{1 + \exp(m(\cdot))} : m \in \mathcal{M}_{k,j,L}^x \right\}.$$

As for $\mathcal{M}_{k,j,L}^x$, we can take polynomial series.

As for $\mathcal{F}_{k,L} = \times_{j=1}^{\bar{K}} \mathcal{F}_{k,j,L}$, we use a Hermite polynomial sieve, where

$$\mathcal{F}_{k,j,L} = \left\{ \begin{array}{c} f_{k,K_{L,k}}(x; \sigma_k, \varepsilon_{0,k}, r_{0,k}, \mathbf{a}_k) : \varepsilon_0 > 0, \sigma > 0, \; r_0, a_{s,k} \in \mathbf{R}, \; s = 1, \cdots, K_{L,k} \\ \int f_{k,K_{L,k}}(x; \sigma_k, \varepsilon_{0,k}, r_{0,k}, \mathbf{a}_k) dx = 1 \end{array} \right\},$$

and $f_{k,K_{L,k}}(x; \sigma_k, \varepsilon_{0,k}, r_{0,k}, \mathbf{a}_k)$ is defined to be

$$\frac{1}{\sqrt{2\pi}\sigma_k} \left(\varepsilon_{0,k} + \left\{ \sum_{s=1}^{K_{L,k}} a_{s,k} \left(\frac{x - r_{0,k}}{\sigma_k} \right)^s \right\}^2 \right) \exp\left(-\frac{(x - r_{0,k})^2}{2\sigma_k^2} \right).$$

Observe that

$$\int f_{k,K_{L,k}}(x; \sigma_k, \varepsilon_{0,k}, r_{0,k}, \mathbf{a}_k) dx = \varepsilon_{0,k} + \sum_{s=1}^{K_{L,k}} \sum_{t=1}^{K_{L,k}} a_{s,k} a_{t,k} \mathbf{E}\left(Z^{s+t}\right),$$

where Z is a standard normal random variable. The quantity $\mathbf{E}\left(Z^{s+t}\right)$ can be explicitly computed from the moment generating function.

Section E: Additional Empirical Results

Additional Restrictions Imposed in Estimation

(a) The restriction associated with transitory sellers' bid distribution:

$$f(\mathbf{B}_i^t|\,\mathbf{I}_l) = \sum_k f(\mathbf{B}_i^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1}) P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k,l}|\mathbf{I}_l) = \tag{33}$$
$$\sum_{k=1}^{\bar{K}_A} \frac{\omega_{A,k,l}^t \prod_{j\in\bar{A}_l^t}\{f(\mathbf{B}_j|\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}) P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1})\}}{\sum_{d=1}^{\bar{K}_l} \omega_{A,d,l}^t \prod_{j\in\bar{A}_l^t} P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1})}.$$

Moment conditions associated with this restriction would relate the empirical moments of the $f(\mathbf{B}_i^t|\,\mathbf{I}_l)$ distribution to the theoretical moments computed using (33).

(b) The restriction associated with the transitory sellers' probability of participation:

$$P(|A_{x,l}^t| = m_{x,l}|\,\mathbf{I}_{l,1}) = \sum_{k=1}^{\bar{K}_{A,l}} P(|A_{x,l}^t| = m_{x,l} \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_{l,1}) = \tag{34}$$
$$\sum_{k=1}^{\bar{K}_{A,l}} \prod_{j\in\bar{A}_l^t} \left\{P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}) P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j|\,\bar{\mathbf{x}}_{j,l}^t)\right\}$$
$$\times \sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} |\Omega| \prod_{i\in\bar{N}_l^t-\bar{A}_l^t} \left\{P(i \in N_l^t - A_l^t|\,\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1}) P(\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i|\,\bar{\mathbf{x}}_{i,l}^t)\right\}.$$

The derivation for this expression is provided in the proof of Proposition 3. Moment conditions associated with this restriction would relate the transitory sellers' empirical probability of participation and expected $x-$characteristics of entrants conditional on $\mathbf{I}_{l,1}$ to their theoretical counterparts using (34).

(c) The restriction related to the expected profit condition. This restriction summarizes optimal participation behavior. It is summarized by the threshold strategy where potential bidders with entry cost draws below the ex-ante expected profit participate in the auctions and those with higher draws stay out. This implies that in equilibrium

$$P(j \in A_{x,l}^t|\,\mathbf{Q}_{j,l}^t = q_{k,x},\, \mathbf{I}_{l,1}) = F_E(\mathbb{E}[\pi^t(j, \mathbf{Q}_{A,j,l}^t = q_{k,x}, \mathbf{I}_{l,1})]), \tag{35}$$

where $F_E(.)$ is the distribution function of entry costs E and $q_{k,x} \in \mathbb{Q}_x$.

Table 8: Estimated Quality Structure for a Given Number of Groups

	Number of Groups	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
	1	52	45	33	2	2	2
	2	0	7	12	31	26	24
	3	0	0	7	12	5	2
	4	0	0	0	7	12	5
	5	0	0	0	0	7	12
	6	0	0	0	0	0	7
$\bar{V}$		9.21	2.61	1.77	0.85	0.73	0.31
$Q(K)$		10.03	4.22	4.11	4.24	4.81	5.22

This table shows the estimated quality group structures for the various numbers of quality groups for Eastern European suppliers with the medium levels of average reputation score. Rows 1-6 record the number of suppliers estimated to belong to a respective group. Rows 7 and 8 record the value of the $p - value$ component of the criterion function and the value of the criterion function. The results are based on the penalty function $g(L) = log(log(L))$. Results indicate that the number of groups most supported by the data is equal to three.

Table 9: Participation Decision and Bid Distribution

	Score	Q	I(T)	II(T)	I(P)	II(P)
Mean						
Constant			0.552** (0.009)	-2.105** (0.019)	0.585** (0.052)	-2.173**(0.009)
No Ratings			-0.083** (0.025)	-0.33* (0.018)		
$0 < \text{Ratings} \leq 3$			0.033** (0.006)	0.005 (0.021)		
$3 < \text{Ratings} \leq 10$			0.041** (0.007)	0.005 (0.009)		
Number of Ratings					0.071 (0.063)	0.003 (0.007)
Average Score 1			-0.005 (0.005)	-0.004 (0.011)		
Average Score 2			0.009** (0.002)	0.011** (0.003)		
North America,	Low,	1	-0.218** (0.023)	0.231** (0.021)	-0.307** (0.046)	0.031 (0.027)
North America,	Low,	2	-0.173** (0.018)	-0.042 (0.022)	-0.271** (0.044)	-0.053** (0.025)
North America,	Medium,	1	-0.004 (0.043)	0.251** (0.012)	0.086** (0.043)	0.288** (0.023)
North America,	Medium,	2	-0.038** (0.022)	-0.139** (0.023)	-0.062 (0.046)	-0.105** (0.026)
North America,	High,	1	-0.173** (0.012)	0.134** (0.023)	-0.214** (0.041)	0.165** (0.017)
North America,	High,	2	-0.108** (0.021)	-0.265** (0.031)	-0.166** (0.039)	-0.193** (0.031)
Eastern Europe,	Low,	1	-0.026** (0.017)	0.232** (0.021)	-0.083* (0.043)	0.205** (0.017)
Eastern Europe,	Low,	2	-0.062* (0.022)	-0.194** (0.013)	-0.176** (0.038)	-0.108** (0.021)
Eastern Europe,	Medium,	1	-0.199** (0.035)	0.034** (0.015)	-0.246** (0.044)	0.013 (0.012)
Eastern Europe,	Medium,	2	-0.192** (0.024)	-0.245** (0.021)	-0.226** (0.048)	-0.198** (0.022)
Eastern Europe,	Medium,	3	-0.128** (0.032)	-0.257** (0.017)	-0.167** (0.051)	-0.232** (0.029)
Eastern Europe,	High,	1	-0.191** (0.024)	-0.131** (0.023)	-0.248** (0.038)	-0.257 (0.034)
Eastern Europe,	High,	2	-0.178** (0.043)	-0.091** (0.031)	-0.249** (0.051)	-0.099** (0.012)
Eastern Europe,	High,	3	-0.132** (0.022)	-0.221** (0.011)	-0.172** (0.038)	-0.206** (0.029)
South-East Asia,	Low,	2	-0.246** (0.034)	-0.231** (0.012)	-0.226** (0.041)	-0.204** (0.021)
South-East Asia,	Low,	3	-0.359** (0.044)	-0.075**(0.011)	-0.434** (0.043)	-0.051 (0.033)
South-East Asia,	Medium,	1	-0.108* (0.063)	-0.075** (0.021)	-0.196** (0.044)	-0.117** (0.017)
South-East Asia,	Medium,	2	-0.183** (0.028)	-0.071** (0.031)	-0.231** (0.038)	0.033** (0.010)
South-East Asia,	Medium,	3	-0.253** (0.035)	-0.074** (0.023)	-0.374** (0-.045	-0.059** (0.027)
South-East Asia,	High,	1	-0.112** (0.037)	-0.195** (0.011)	-0.178** (0.038)	-0.105** (0.012)
South-East Asia,	High,	2	-0.095** (0.024)	-0.299** (0.014)	-0.065** (0.053)	-0.281** (0.034)
Std Error			0.207** (0.009)		0.238** (0.011)	

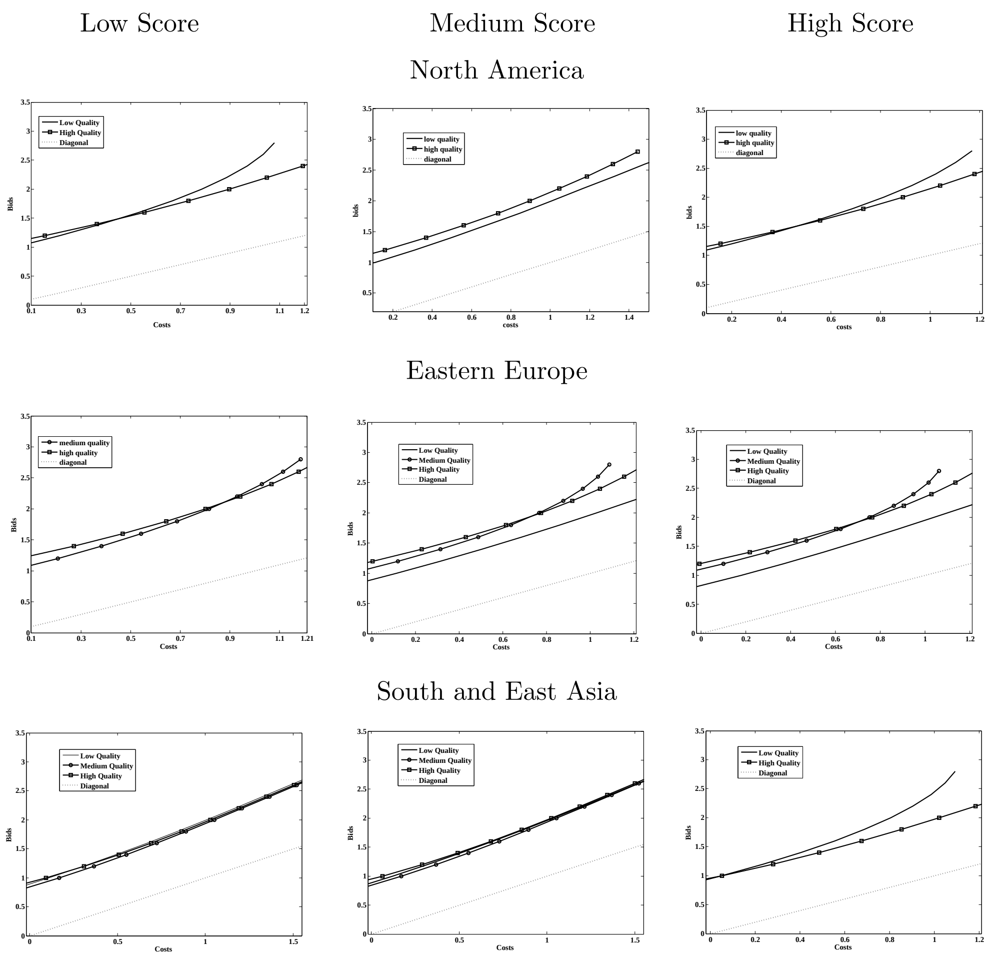
This table reports the effects of the covariates and the group premiums on sellers' bid distribution and participation decisions. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and probability of participation of the transitory and permanent sellers respectively. "Average Score 1" and "Average Score 2" denote interactions of the current average score variable with the indicators for $0 \leq Ratings \leq 3$ and $3 \leq Ratings \leq 10$. The results are based on the data set consisting $11,300$ projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Table 10: Participation Decision and Bid Distributions: Competitive Effects

			I(T)	II(T)	I(P)	II(P)
North America,	low,	1	0.021 (0.017)	0.021 (0.013)	0.003 (0.002)	0.011** (0.005)
North America,	low,	2	0.011 (0.012)	0.015 (0.012)	-0.005 (0.003)	0.011** (0.004)
North America,	medium,	1	-0.023** (0.011)	-0.089** (0.011)	-0.002 (0.002)	-0.004 (0.004)
North America,	medium,	2	-0.015* (0.008)	-0.031** (0.015)	-0.004** (0.002)	-0.007* (0.003)
North America,	high,	1	0.052 (0.031)	0.001 (0.0078)	-0.007** (0.002)	-0.012** (0.003)
North America,	high,	2	-0.026** (0.012)	-0.085** (0.021)	-0.008** (0.003)	-0.016** (0.004)
Eastern Europe,	low,	1	-0.005 (0.011)	0.001 (0.007)	-0.001 (0.002)	0.002 (0.005)
Eastern Europe,	low,	2	-0.007** (0.002)	-0.025** (0.012)	-0.005* (0.0026)	-0.009** (0.0026)
Eastern Europe,	medium,	1	-0.005 (0.003)	-0.007 (0.004)	-0.003* (0.001)	-0.007** (0.002)
Eastern Europe,	medium,	2	0.034 (0.031)	0.016 (0.012)	0.002 (0.004)	0.001 (0.003)
Eastern Europe,	medium,	3	-0.021** (0.011)	-0.016 (0.015)	0.001 (0.002)	-0.004 (0.004)
Eastern Europe,	high,	1	-0.011 (0.012)	-0.012 (0.017)	-0.002 (0.003)	-0.003 (0.005)
Eastern Europe,	high,	2	0.008 (0.005)	0.007 (0.004)	0.001 (0.005)	0.003 (0.002)
Eastern Europe,	high,	3	-0.007 (0.004)	-0.011 (0.019)	0.002 (0.003)	-0.0001 (0.003)
South-East Asia,	low,	1	0.001 (0.011)	-0.017* (0.003)	-0.002* (0.001)	-0.001 (0.001)
South-East Asia,	low,	2	-0.003 (0.004)	-0.004 (0.008)	-0.008** (0.003)	-0.019** (0.002)
South-East Asia,	low,	2	-0.023** (0.011)	-0.026** (0.011)	-0.001 (0.002)	0.001 (0.003)
South-East Asia,	medium,	1	-0.011 (0.012)	-0.013 (0.011)	-0.004 (0.003)	-0.001 (0.005)
South-East Asia,	medium,	2	-0.003 (0.011)	-0.002 (0.011)	-0.002 (0.002)	-0.004** (0.001)
South-East Asia,	medium,	3	0.023 (0.033)	0.015 (0.014)	-0.001 (0.002)	0.007 (0.005)
South-East Asia,	high,	1	-0.004 (0.003)	-0.007 (0.004)	-0.002* (0.001)	0.001 (0.001)
South-East Asia,	high,	2	-0.024** (0.012)	-0.039** (0.018)	-0.004* (0.002)	-0.007** (0.003)

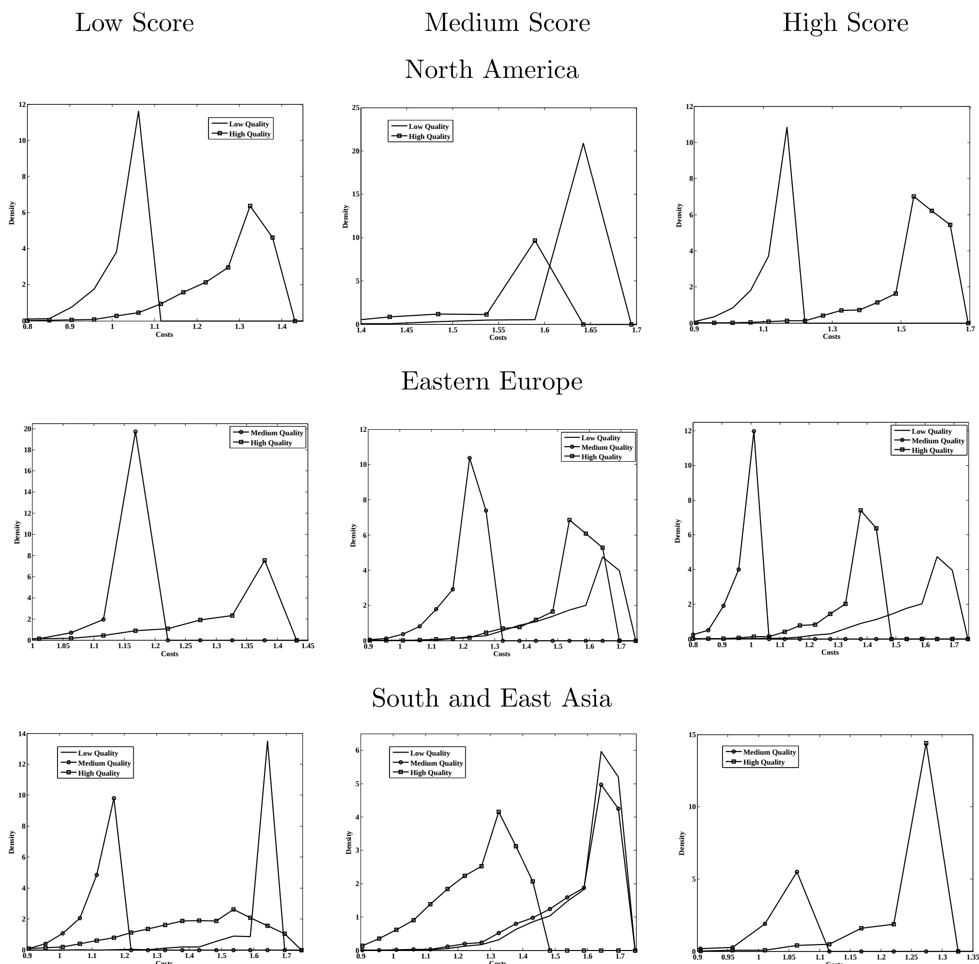
This table reports the coefficients summarizing the impact of the various potential competitors on sellers' bid distribution and participation decisions. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and the probability of participation of transitory and permanent sellers respectively. The results are based on the data set consisting 11,300 projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Figure 2: Bid Functions



The figure shows the equilibrium bidding strategies of permanent sellers recovered from the first order conditions of bidders' optimization program. The convexity at the upper end of the costs' support arises due to presence of stochastic component in buyers' tastes.

Figure 3: Density of Project's Cost Distribution



The figure shows the permanent sellers' distribution of costs recovered by combining estimated bidding strategies and bid distributions.

Table 11: Costs Distributions: Mean and Standard Deviation

Variable			Mean	Std.Deviation
Project Costs				
North America,	low score,	Q=1	1.039	0.053
North America,	low score,	Q=2	1.275	0.087
North America,	medium score,	Q=1	1.627	0.039
North America,	medium score,	Q=2	1.551	0.073
North America,	high score,	Q=1	1.135	0.053
North America,	high score,	Q=2	1.535	0.099
Eastern Europe,	low score,	Q=1	1.169	0.037
Eastern Europe,	low score,	Q=2	1.328	0.079
Eastern Europe,	medium score,	Q=1	1.576	0.119
Eastern Europe,	medium score,	Q=2	1.202	0.062
Eastern Europe,	medium score,	Q=3	1.531	0.101
Eastern Europe,	high score,	Q=1	1.575	0.119
Eastern Europe,	high score,	Q=2	0.981	0.048
Eastern Europe,	high score,	Q=3	1.354	0.089
South and East Asia,	low score,	Q=1	1.621	0.056
South and East Asia,	low score,	Q=2	1.119	0.051
South and East Asia,	low score,	Q=3	1.417	0.169
South and East Asia,	medium score,	Q=1	1.609	0.107
South and East Asia,	medium score,	Q=2	1.589	0.121
South and East Asia,	medium score,	Q=3	1.255	0.124
South and East Asia,	high score,	Q=1	1.074	0.036
South and East Asia,	high score,	Q=2	1.235	0.047
Entry Costs			0.082	0.081

This table summarizes means and standard errors of the estimated distributions of permanent sellers' project costs that are displayed in Figure 2.

Table 12: Multi-Attribute Auction with Licensing

	All Quality Levels (1)	Exclude Low Quality (2)	Exclude Low & Med. Quality (3)
Average Price	1.62	1.69	1.78
Average Expected Utility	1.16	1.05	0.91
Expected Utility, $\tau_{\alpha,90\%}$	2.21	2.10	2.14
Expected Utility, $\tau_{\alpha,70\%}$	1.57	1.46	1.47
Expected Utility, $\tau_{\alpha,50\%}$	1.01	0.94	0.92
Expected Utility, $\tau_{\alpha,30\%}$	0.56	0.43	0.36
Expected Utility, $\tau_{\alpha,10\%}$	-0.08	-0.21	-0.33
Expected Profit, Q=H	0.08	0.10	0.27
Expected Profit, Q=M	0.49	0.55	
Expected Profit, Q=L	0.03		
Expected Total Surplus	1.76	1.70	1.18
Composition of Entrants			
Average Number of Entrants	4	3.9	1.8
High Quality	0.9	1.1	1.8
Medium Quality	2.7	2.8	
Low Quality	0.4		
Market Shares			
High Quality	25%	25.5%	74%
Medium Quality	61%	64.5%	
Low Quality	7%		
Outside Option	7%	9%	26%

Note: In this table the expected utility and the total surplus are computed relative to the outside option.

Table 13: Standard Auction with Certification

	All Quality Levels	Exclude Low Quality	Exclude L & M Quality
Average Price	1.53	1.56	1.74
Average Expected Utility	1.09	0.99	0.87
Expected Utility, $\tau_{\alpha,90\%}$	2.16	2.11	2.10
Expected Utility, $\tau_{\alpha,70\%}$	1.51	1.48	1.41
Expected Utility, $\tau_{\alpha,50\%}$	0.97	0.94	0.87
Expected Utility, $\tau_{\alpha,30\%}$	0.58	0.52	0.38
Expected Utility, $\tau_{\alpha,10\%}$	-0.7	-0.19	-0.27
Expected Profit, Q=H	0.05	0.06	0.20
Expected Profit, Q=M	0.42	0.45	
Expected Profit, Q=L	0.05		
Expected Total Surplus	1.61	1.50	1.07
Composition of Entrants			
Average Number of Entrants	3.6	3.5	1.6
High Quality	0.4	0.45	1.6
Medium Quality	2.8	2.85	
Low Quality	0.4		
Market Shares			
High Quality	12%	15%	90%
Medium Quality	74%	79%	
Low Quality	10%		
Not Allocated	4%	6%	10%

Note: In this table the expected utility and the total surplus are computed relative to the outside option.