Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://economics.sas.upenn.edu/pier

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"A Theory of Bargaining Deadlock"

by

Ilwoo Hwang

A Theory of Bargaining Deadlock*

Ilwoo Hwang†

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Abstract

I study a dynamic one-sided-offer bargaining model between a seller and a buyer under incomplete information. The seller knows the quality of his product while the buyer does not. During bargaining, the seller randomly receives an outside option, the value of which depends on the hidden quality. If the outside option is sufficiently important, there is an equilibrium in which the uninformed buyer fails to learn the quality and continues to make the same randomized offer throughout the bargaining process. As a result, the equilibrium behavior produces an outcome path that resembles the outcome of a *bargaining deadlock* and its resolution. The equilibrium with deadlock has inefficient outcomes such as a delay in reaching an agreement and a breakdown in negotiations. Bargaining inefficiencies do not vanish even with frequent offers, and they may exist when there is no static adverse selection problem. Under stronger parametric assumptions, the equilibrium with deadlock is unique under a monotonicity criterion, and all equilibria exhibit inefficient outcomes.

Keywords: bargaining game, asymmetric information, bargaining deadlock, delay, failure of learning, Coase conjecture.

JEL Classification number: C78, D82, D83.

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†University of Pennsylvania. e-mail:`ilhwang@sas.upenn.edu`

1 Introduction

There are many bargaining processes in which a bargainer may receive an outside offer during the process. Moreover, preferability of the outside offer often depends on the private information of the informed party. For example, consider an entrepreneur who negotiates to sell his company to an equity fund. The entrepreneur knows a company's fundamentals but is not able to verify them. During the bargaining process, a competitor might arrive and make an offer to buy the firm. The competitor is better informed about the fundamentals than the equity fund, so his offer is high if the fundamentals are good. For our purposes, the competitor's offer serves as an attractive outside option for the entrepreneur.[1]

In this example, when a bargainer is deciding whether or not to take the outside option, he must take into account the fact that choosing not to opt out may signal his private information. This paper analyzes the interplay of outside options and incomplete information in bargaining. Specifically, this paper analyzes the equilibrium effects of additional information provided by how bargainers respond to the outside option.

I study a model of an infinite-horizon bargaining game between a seller and a buyer. The seller privately knows the quality of his product. In each period, the buyer offers a price and the seller decides whether or not to accept the offer. After rejection, the seller's outside option randomly arrives. The value of the outside option is increasing in the product quality. If the seller does not receive an outside option or he chooses not to opt out, bargaining continues into the next period.

There are two sources of information which the buyer uses to update his belief about quality: the seller's decision to accept/reject the buyer's offer (acceptance behavior) and his decision about whether to take the outside option (opting-out behavior). Suppose the buyer proposes an offer that is rejected. Then the buyer believes that the quality is more likely to be high, since the high-quality seller has a higher reservation value. This informational effect of the acceptance behavior is common in the standard models of incomplete-information bargaining (Fudenberg, Levine, and Tirole (1985); Deneckere and Liang (2006)). There is no outside option

[1] In corporate finance, buyers of businesses are generally classified into two different categories: financial buyers and strategic buyers. Financial buyers are mostly equity funds interested in the return they can achieve by buying a business. Strategic buyers are typically a competitor or a company in the same industry, and they look for companies that will create a synergy with their existing businesses.

in their models, and so they only consider the effect of the acceptance behavior. As a result, the buyer's equilibrium belief moves only in one direction as the buyer becomes more confident that he is facing the high-quality seller.[2] This equilibrium dynamic of belief is known as the *skimming property* (Gul, Sonnenschein, and Wilson (1986)).

However, additional information is provided by the seller's opting-out behavior in this model and it has an opposite affect on belief updating. The buyer infers that the seller has not opted out by observing him still at the negotiation table. It might be that the seller has yet to receive an outside option, or he has received an outside option that he did not take. Since the value of the outside option is greater for the high-quality seller, he is more likely to opt out when the option arrives. Therefore, after observing that the seller has not opted out, the buyer adjusts his belief in the direction of low quality.

I show that when the outside option is sufficiently important, there is an equilibrium in which the two countervailing forces in belief updating exactly offset one another. As a result, the buyer's belief does not change over time and he continues to make the same randomized offer throughout the bargaining process. Since the buyer does not make more generous offers in response to continued rejections, and the seller's behavior does not change, the equilibrium behavior produces an outcome path that resembles an outcome of a *bargaining deadlock*. For simplicity, I refer to such an equilibrium as a deadlock equilibrium.

In the deadlock equilibrium, there is a threshold belief such that once the buyer's posterior belief reaches that point, it does not change until the bargaining ends. If the buyer is more confident that he faces a high-quality seller than the threshold, he offers a sufficiently high price that bargaining ends. If the buyer is less confident, he makes an agreement only with the low-quality seller by offering no more than the value of his outside option. In this case, in each period the buyer adjusts his belief in the direction of high quality. This equilibrium behavior lasts for a finite number of periods until the posterior reaches the threshold point.

If the buyer's posterior is equal to the threshold, the buyer uses a mixed strategy between offering the bargaining-ending price and the low-quality seller's outside option value. The mixing probability is determined to satisfy the low-quality seller's indifference condition, and as the time between periods becomes vanishingly small, the buyer offers the low price with a

[2]Similarly, in the context of a durable goods monopoly, the uninformed seller becomes more confident that the remaining buyers have low valuation.

probability close to one. In response to the buyer's low price offer, only the low-quality seller accepts it with a probability equal to the arrival probability of the outside option. The high-quality seller takes the outside option for sure if it arrives, while the low-quality seller does not. Since both types of sellers exit the game with the same probability, the posterior belief of the buyer remains the same in the next period, and the players continue to play in the same way.

If the buyer's prior expected quality is lower than the threshold belief, in equilibrium there is positive probability that an apparent bargaining deadlock arises: a sequence of the same low price offer is rejected by the seller, followed by a sudden resolution by either the buyer's high bargaining-ending offer, the low-quality seller's agreement on the low price, or the high-quality seller's opting out. Note that although the realization of the equilibrium outcome resembles a bargaining deadlock, the bargainers are not that uncompromising. Instead, the buyer and the low-quality seller are indifferent between a full compromise, and the bargaining ends with positive probability in each period. In this sense, the model provides an explanation of situations that look like bargaining deadlocks without the need to appeal to behavioral types.

I show that as the time between periods becomes vanishingly small, and if the buyer forms a prior belief such that the expected quality is lower than the threshold belief, then the bargaining reaches a deadlock phase almost immediately. Hence an outcome path of the equilibrium under frequent offers exhibits one of the following: either the bargaining ends immediately, or the aforementioned deadlock phase lasts for positive real time before a sudden resolution.

In the deadlock equilibrium, there are non-trivial bargaining inefficiencies. There is a bargaining delay in the deadlock equilibrium, and the expected length of delay is positive in the limit case of frequent offers. While the bargaining terminates (either by an agreement or an opt-out) incrementally over time, the failure of learning keeps the parties from reaching an agreement with certainty at any point in the bargaining process. Indeed, for any finite time, the bargaining continues beyond that point with positive probability. Moreover, the equilibrium exhibits the possibility of bargaining breakdown.

The inefficiencies found in the deadlock equilibrium have distinctive features compared to the ones in the standard model of incomplete-information bargaining. The standard model explains delay as a device by which the parties can credibly convey their genuine bargaining positions. Therefore, the adverse selection problem is alleviated over time as the uninformed party gradually learns private information. In this model, however, the adverse selection problem does not disappear because the buyer fails to learn the quality; hence the bargaining inefficiencies remain strong as long as the bargaining continues. Furthermore, a bargaining deadlock and real-time delay may exist even when there is no static adverse selection problem,[3] which contrasts with the result in the standard model (Deneckere and Liang (2006)).

In general, the model has multiple equilibria. There may exist an equilibrium where the informational effect of the acceptance behavior dominates that of the opting-out behavior, so that the equilibrium exhibits Coasian dynamics and so is approximately efficient when offers are frequent. But I show that under stronger parametric assumptions, the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion that requires that the buyer's equilibrium offer be nondecreasing in the posterior belief of expected quality. Moreover, I show that under the same condition, all equilibria exhibit similar characteristics, specifically the partial failure of learning and the inefficiency in the bargaining outcome, so neither source of information dominates each other.

The paper contributes to a rich literature on dynamic bargaining with incomplete information. Standard models of incomplete-information bargaining do not model outside options (See Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989) for a durable goods monopoly; Deneckere and Liang (2006) for bargaining with interdependent values; Cho (1990) for two-sided private information; Abreu and Gul (2000) for reputational bargaining), or they model them as an exogenous breakdown (See Sobel and Takahashi (1983); Spier (1992); Fuchs and Skrzypacz (2012) for breakdown after a finite-horizon bargaining; Fuchs and Skrzypacz (2010) for stochastic breakdown). Since the players do not have an opting-out decision, information is revealed only through the offer/response behavior. In the present paper, information is revealed via both the acceptance and opting-out behaviors, which is the main driving force of the bargaining deadlock.

A few papers have an equilibrium structure similar to the one studied here, although the underlying mechanism is different. Evans (1989) and Hörner and Vieille (2009) (public offer case) consider bargaining with interdependent values and show that the bargaining may result in an impasse when the buyer is too impatient (or short-lived) relative to the seller. On the

[3] A static adverse selection problem arises when the average value of the product is lower than the highest possible reservation value of the seller (Akerlof (1970)).

other hand, the present paper assumes a common discount factor, and a bargaining deadlock may exist even in the private value case. Abreu and Gul (2000) study a reputational bargaining game where each agent may be a behavioral type who demands a certain share of the pie and show that the equilibrium has a war of attrition structure exhibiting a deadlock. Even though each bargainer becomes less confident that the opponent is a normal type, they stick to imitating the behavioral type's behavior until a bargainer finally gives up. Compared to Abreu and Gul (2000), the present model does not assume behavioral types and a bargaining deadlock is associated with the uninformed buyer's failure of learning. Also it is known that introducing an outside option into their model may completely cancel out the deadlock and delay (explained in the next paragraph), while deadlock in this paper is a result of an interplay between the outside options and incomplete information.

There are papers in which some or all players can take an outside option that is available in every period. Compte and Jehiel (2002) (in the context of reputational bargaining) and Board and Pycia (2013) (in a durable goods monopoly) show that the introduction of an outside option may completely cancel out the impact of asymmetric information. In these papers, the players either agree with each other or opt out at the beginning of the game, so the equilibrium is efficient and information is revealed immediately. On the other hand, the stochastic arrival of outside options in this paper leads to non-trivial equilibrium dynamics.

Lee and Liu (2013) study a repeated bargaining game between a long-run player and a sequence of short-run players, where a stochastic disagreement outcome in each bargaining partially reveals private information of the long-run player. They focus on the incentive of the long-run player to build a reputation by choosing to gamble with the outside option, while the present paper analyzes the bargaining inefficiency caused by the informational effect of the outside options.[4,5]

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 constructs the deadlock equilibrium and describes the equilibrium dynamics and the outcome

[4]Compte and Jehiel (2004) raise an opposite question about bargaining dynamics and identify a source of gradualism in bargaining and contribution games.

[5]For other models that explain delay, Merlo and Wilson (1995) consider a complete information bargaining game where the bargaining surplus stochastically changes over time and derive an equilibrium delay. Yildiz (2004) considers a sequential bargaining model in which players are optimistic about their bargaining power and shows that there exists a uniquely predetermined settlement date as players learn over time.

path. In Section 4 I analyze the equilibrium behavior under the limit case of frequent offers and discuss real-time delay as well as other equilibrium characteristics. Section 5 finds sufficient conditions under which the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion, and under which all equilibria have similar characteristics. Section 6 discusses the role of assumptions and the robustness of the result under several extensions. Section 7 concludes. Some of the proofs are relegated to the Appendix.

2 Model

Consider an infinite-horizon, discrete-time bargaining game between a seller and a buyer. Periods are indexed by $n = 0, 1, 2, \ldots$. Let Δ be the length of the time interval between two successive periods, so period k occurs at time $k\Delta$.[6] Let $\delta = e^{-r\Delta}$ be a common discount factor, where $r > 0$ is a discount rate. Note that the discount factor becomes arbitrarily close to one as Δ converges to zero.

The seller holds an indivisible product that can be either high type (H) or low type (L). The type of the product is the seller's private information, and the buyer forms a prior belief $\pi_0 \in (0,1)$ that $\theta = H$. The buyer's value of the type-θ product is $u_\theta > 0 (u_H \geq u_L)$. For simplicity, assume that the seller has zero production cost.[7]

Each period consists of an offer stage and an outside option stage. In the offer stage, the buyer offers a price p to the seller. Then the seller decides either to accept or reject the offer. If he accepts the offer, the game ends, and the seller and the buyer obtain payoffs p and $u_\theta - p$, respectively. In the case of rejection, the game continues to the outside option stage where the outside option arrives to the seller with probability $\xi = 1 - e^{-\lambda\Delta}$.[8] I assume that the arrival of the outside option is private information to the seller. If the seller opts out, the game ends, and the seller and the buyer obtain payoffs of v_θ and zero, respectively. Assume that $v_H > v_L > 0$ and that the buyer's value of the product is no less than the seller's value from the outside option $(u_\theta \geq v_\theta)$. If either no outside option arrives or the option is rejected by the seller, the game continues into the next period. Figure 2.1 describes the timeline of the game.

[6] This is a common modeling scheme in the literature on bargaining theory. The literature mainly considers the case where Δ is arbitrarily small, so that the commitment power of the uninformed player disappears.

[7] The robustness of the result to the case of a positive production cost is discussed in Section 6.

[8] Note that $\lambda > 0$ represents a Poisson arrival rate of the outside options.

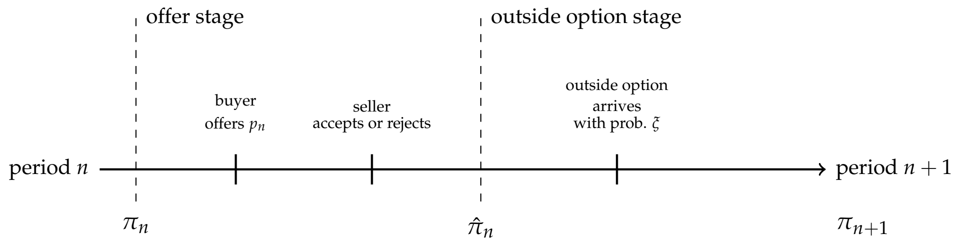


Figure 2.1: Timeline

Consider a seller's strategy in which he rejects any offer and opts out whenever the outside option arrives. Then the type-θ seller's expected payoff is

$$v_\theta^* \equiv \xi v_\theta + \delta(1-\xi)\xi v_\theta + \cdots = \frac{\xi}{1-\delta(1-\xi)} v_\theta.$$

Note that $v_\theta^* < v_\theta$, since the arrival of the outside option is delayed with positive probability.

It is clear that in any equilibrium of the game, the ex ante payoff of the type-θ seller must be no less than v_θ^*, and that the seller always rejects any offer below v_θ^*. Hereafter I call v_θ^* the *reservation price* of the type-θ seller. The following proposition says that in the case of complete information, v_θ^* is not only a lower bound but also the unique equilibrium payoff of the seller. The main intuition behind the proposition is similar to Diamond's paradox.

Proposition 1. *(Complete information) Suppose that the seller is type θ with probability one. Then there exists a unique subgame perfect equilibrium in which the buyer always offers v_θ^*, and the seller accepts any offer no less than v_θ^*.*

Proof. See the Appendix. □

A public history $h^n \in H^n$ is a sequence of rejected offers $\{p_k\}_{k=0}^{n-1}$ from period 0 to $n-1$. In addition to that, the seller privately knows the availability of outside options in the past. Let $o_k \in \{Y, N\}$ denote the availability of an outside option for the seller in period k. Then the seller's private history $h_S^n \in H_S^n$ at the offer stage is $h_S^n = (h^n, \{o_k\}_{k=0}^{n-1})$. I also define a public *interim* history $\hat{h}^n = (h^n, p_n) \in \hat{H}^n$ and private interim history $\hat{h}_S^n = (h^n, p_n, \{o_k\}_{k=0}^{n-1}) \in \hat{H}_S^n$ at the outside option stage.

The buyer's strategy is his offer $p_n : H^n \to \Delta(\mathbb{R}_+)$ at the offer stage. The type-θ seller's strategy consists of the acceptance probability $\sigma_{\theta n} : H_S^n \times \mathbb{R}_+ \to [0,1]$ at the offer stage, and the opting-out probability $c_{\theta n} : \hat{H}_S^n \times \{Y\} \to [0,1]$ at the outside option stage. Finally, define

$\pi_n = \Pr(\theta = H|h^n)$ and $\hat{\pi}_n = \Pr(\theta = H|\hat{h}^n)$ as a posterior belief and an interim belief of the buyer in period n, respectively.

We use the perfect Bayesian equilibrium (PBE) concept as defined in Fudenberg and Tirole (1991, Definition 8.2).[9] PBE implies that upon receiving an out-of-equilibrium offer, the continuation strategy of the seller is optimal.

As Δ goes to zero, the type-θ seller's reservation value converges to $\frac{\lambda}{r+\lambda}v_\theta$. Define $\eta = \frac{\lambda}{r+\lambda}$ as the seller's *effective discount rate*. Note that η can be any number between zero and one, depending on the ratio of the discount rate and the arrival rate of the outside option.

In this paper, I consider the case where outside options arrive frequently enough (relative to the discount rate), so that the outside options generate a sufficiently heterogeneous bargaining position of the seller according to his type. Specifically, I assume that the high type's reservation value is greater than the low type's payoff from the outside option.

Assumption. *(A1)*

$$\delta v_H^* > v_L + \frac{(1-\delta)(1-\xi)}{\xi} u_L.$$

Assumption 1 holds if (1) $\delta = e^{-r\Delta}$ is sufficiently large so that the interval between the periods is small enough, and (2) $\eta = \frac{\lambda}{r+\lambda}$ is sufficiently large so that the outside options arrive frequently enough that v_H^* is close to v_H. Note that Assumption 1 encompasses a case with private value ($u_H = u_L$).

The following lemma shows that in any (perfect Bayesian) equilibrium, the buyer's equilibrium offer is bounded above by the high type's reservation value. The intuition is similar to Proposition 1. This lemma and the following corollary helps in understanding the equilibrium structure of the game.

Lemma 2. *Suppose (A1) holds. Then in equilibrium, after any history h^n, the buyer never offers $p_n > v_H^*$.*

Proof. See the Appendix. □

Lemma 2 implies the following corollary:

Corollary 3. *Suppose (A1) holds. Then in equilibrium,*

[9] Formally speaking, Fudenberg and Tirole defined perfect Bayesian equilibria for finite games of incomplete information. The suitable generalization of their definition to infinite games is straightforward and is omitted.

(1) The high type accepts any $p \geq v_H^*$, rejects any $p < v_H^*$, and takes the outside option whenever the option arrives.

(2) The low type accepts any $p \geq \delta v_H^*$.

Note that the first part of Corollary 3 completely specifies the high type's equilibrium behavior after any history. So the equilibrium profile only needs to specify the behaviors of the low type and the buyer. Lemma 2 and Corollary 3 describe how the bargaining ends in any equilibrium. After any history, the buyer offers either $p_n = v_H^*$ or $p_n < v_H^*$. If he offers v_H^*, then both types of sellers accept it for sure, and the bargaining ends in period n with probability one. If $p_n < v_H^*$, then the high type rejects it for sure and takes the outside option if the option arrives. Therefore, the bargaining continues into the next period with positive probability, as the outside option does not arrive with probability one.

3 Deadlock Equilibrium

In this section I construct an equilibrium of interest. A heuristic argument for the equilibrium construction is provided here, while the complete description of the equilibrium (including behavior off the equilibrium path) is provided in the Appendix.

Definition. A perfect Bayesian equilibrium is called a *deadlock equilibrium* if the equilibrium behavior satisfies the following properties: there exists $\hat{p} < v_H^*$, $\pi^* \in (0,1)$ and $q \in (0,1)$ such that

1. If $\pi_n > \pi^*$,
   - the buyer offers v_H^* for sure; bargaining ends immediately.
2. If $\pi_n = \pi^*$,
   - the buyer offers either v_H^* or $\hat{p}$, or uses a mixed strategy between the two;
   - if $p_{n-1} = \hat{p}$, he offers v_H^* or $\hat{p}$ with probability q and $1-q$, respectively;
   - only the low type accepts $\hat{p}$ with probability ξ;
   - only the high type opts out for sure;
   - $\pi_{n+1} = \pi^*$.

3. If $\pi_n < \pi^*$,

    - the buyer offers some $p \leq \hat{p}$;
    - only the low type accepts p with positive probability;
    - $\pi_{n+1} \in (\pi_n, \pi^*]$.

In the deadlock equilibrium, there exists a cutoff belief π^* where the posterior, given that the bargaining continues, does not change once it reaches π^*. I call π^* a *deadlock belief* since the bargaining parties' behaviors do not change once the posterior reaches π^*; hence, the equilibrium behavior produces an outcome that resembles a bargaining deadlock.

The buyer's equilibrium offer sharply changes at the deadlock belief. If the posterior is greater than the deadlock belief, then the buyer offers v_H^* to end the bargaining process with both types of sellers. On the other hand, when the posterior is lower than π^* the buyer offers a much lower price and targets only the low type. Note that if the prior is less than π^*, the posterior is always less than or equal to π^* (unless bargaining ends) and the buyer never buys a high-type product.

I claim that the above profile is an equilibrium only if $\hat{p} = v_L$. Recall that by Corollary 3, if the buyer offers any price less than v_H^* the high type rejects the offer and opts out if the option is available, so he exits the game with probability ξ.

- Since the low-type seller accepts any $p \in [\delta v_H^*, v_H^*)$ with probability one (Corollary 3), $\hat{p}$ must be less than δv_H^*.
- Suppose that $\hat{p} \in (v_L, \delta v_H^*)$. Fix a history h^n with $\pi_n = \pi^*$. Let $\epsilon > 0$ be small that $\hat{p} - \epsilon > \max\{v_L, \delta \hat{p}\}$. Consider the buyer's deviation at h^n to offer $\hat{p} - \epsilon$.
    - I claim that in response to $\hat{p} - \epsilon$, the low type exits the game with probability ξ. If he exits with probability greater than ξ, the buyer's posterior becomes $\pi_{n+1} > \pi^*$. Hence the buyer offers v_H^* in period $n+1$. But then it is strictly optimal for the low type not to exit in period n, so his behavior is inconsistent with the belief. If he exits with probability less than ξ, then $\pi_{n+1} < \pi^*$, so the buyer offers $p_{n+1} \leq \hat{p}$ in period $n+1$. But then it is strictly optimal for the low type to accept p_n at period n.

- Then the low type must accept $\hat{p} - \epsilon$ with probability ξ and not take the outside option because $\hat{p} - \epsilon > v_L$. Hence offering $\hat{p} - \epsilon$ is a profitable deviation for the buyer, contradiction.

- Suppose that $\hat{p} < v_L$. Then it is suboptimal for the low type not to opt out when the posterior is π^* and the buyer offers $\hat{p}$, because the outside option's value is more than the buyer's offer.

Given that $\hat{p} = v_L$, the value of π^* and q is uniquely determined by the indifference conditions of the players at the deadlock belief. At $\pi_n = \pi^*$ the buyer must be indifferent between offering v_H^* and v_L. If the buyer offers v_H^*, then both types of sellers accept it for sure and the buyer obtains

$$U_F^* \equiv (1 - \pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*). \tag{3.1}$$

On the other hand, if $p_n = v_L$, the low type's response is $(\sigma_{Ln}, c_{Ln}) = (\xi, 0)$ and the buyer obtains

$$(1 - \pi^*)\xi(u_L - v_L) + \delta(1 - \xi)U_F^*. \tag{3.2}$$

Combining the above two formulas pins down the unique deadlock belief

$$\pi^* = \frac{(v_H^* - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - v_L)}{(u_H - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - v_L)}.$$

Now consider the seller's indifference condition. At the deadlock belief, the low type uses a mixed strategy between acceptance and rejection when the buyer offers v_L. So it must be the case that

$$v_L = \delta(qv_H^* + (1 - q)v_L), \tag{3.3}$$

which uniquely determines q.

But then why is the above profile an equlibrium? Can the buyer induce a higher acceptance probability by offering a higher price? For any $p < v_H^*$, if the low type accepts p with probability greater than ξ, then in the next period, the posterior becomes greater than π^* and the buyer offers v_H^*. So as long as the price is less than δv_H^*, the acceptance probability must be no greater than ξ. Therefore, if the buyer wants to increase the acceptance probability, he needs to raise the price at least to δv_H^*.

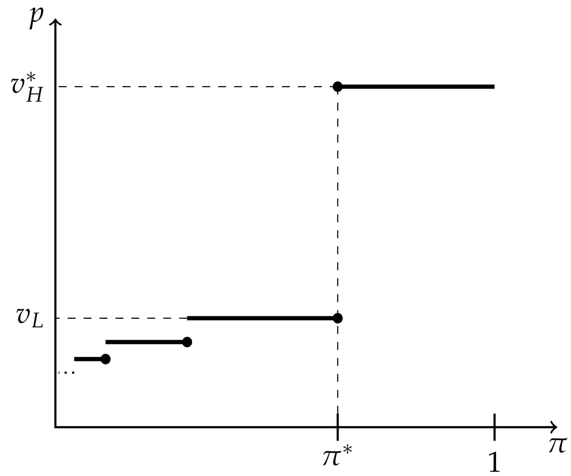


Figure 3.1: Buyer's equilibrium offer

What if the buyer offers δv_H^*? If the seller is the low type, he accepts the offer with probability one. However, if the seller is the high type, he rejects the offer and opts out if the option is available, and in that case, the buyer receives zero payoff. So if the outside option arrives with a high probability, the cost from the high type's opting out is greater than the benefit from trading with the low type. Assumption 1 necessitates such a high arrival rate of the outside option to guarantee the existence of the deadlock equilibrium. The following proposition summarizes the argument:

Proposition 4. *Suppose (A1) holds. Then the model generically has a unique deadlock equilibrium.*

Proof. See the Appendix. □

Figure 3.1 describes the buyer's equilibrium offer of the buyer as a function of the posterior belief. If the buyer's belief is greater than the cutoff belief π^*, he offers v_H^* and both types of sellers accept the offer for sure; hence the game ends immediately. When the belief is less than π^*, his offer is no more than the low type's value of the outside option (v_L), and the offer is nondecreasing in the belief. Later in this subsection I describe more details of the equilibrium offer when the belief is less than π^*.

Equilibrium behavior at the cutoff belief π^* is depicted in Figure 3.2. At the offer stage (described in the left panel), the buyer offers either v_H^* or v_L. If the buyer had offered v_L in the previous period, then he plays a mixed strategy between offering v_H^* and v_L, which satisfies the low type's indifferent condition (3.3). If the buyer offers v_H^*, then both types of sellers accept it

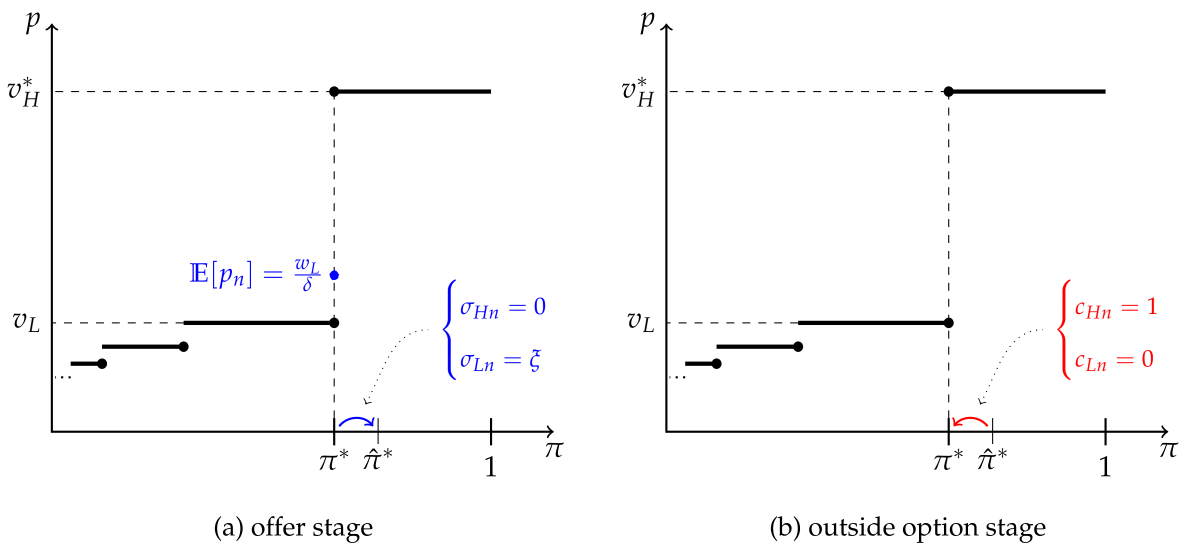


Figure 3.2: Equilibrium behavior at $\pi = \pi^*$

and hence the game ends. If the buyer offers v_L, then the high type rejects it, since it is lower than his reservation value. The low type accepts the offer with probability ξ. Therefore after the offer stage ends, the buyer's interim belief becomes $\hat{\pi}_n = \hat{\pi}^* \equiv \frac{\pi_n}{\pi_n + (1-\pi_n)(1-\xi)} > \pi^*$. At the outside option stage (right panel), only the high type exercises the outside option when it is available. Since the high type exits with probability ξ, the posterior belief π_{n+1} decreases back to π^*. From then on, the bargaining parties repeat the same behavior in each period: the buyer mixes between offering v_H^* and v_L; the low type accepts v_L with probability ξ while the high type rejects it; only the high type opts out. Note that the buyer's belief does not change unless bargaining ends, since the information from the seller's acceptance behavior and his opting-out behavior exactly offset one another.

What happens if the prior is lower than the deadlock belief? In the Appendix, I construct a sequence of prices $\{p_k^\dagger\}(p_0^\dagger = v_L, p_k^\dagger \in (v_L^*, v_L)$ for $k \geq 1)$ and a sequence of cutoff beliefs $\{\pi_k^\dagger\}(\pi_0^\dagger = \pi^*, \pi_k^\dagger \in (0, \pi^*)$ for $k \geq 1)$ that describe the equilibrium behavior when the belief is smaller than π^*. It is shown in the Appendix that both $\{p_k^\dagger\}$ and $\{\pi_k^\dagger\}$ are decreasing, and that for any prior $\pi_0 < \pi^*$, there exists $N \in \mathbb{N} \cup \{0\}$ such that $\pi_{N+1}^\dagger \leq \pi_0 < \pi_N^\dagger$. Here I consider the generic case that $\pi_{N+1}^\dagger < \pi_0$.

In the equilibrium, the buyer offers $p_N^\dagger$ in the first period. Then the low type accepts with positive probability such that the interim belief becomes $\pi_{N-1}^\dagger$. In the outside option stage, both types of sellers opt out if the outside option arrives, so the belief does not change at

$\pi^{\dagger}_{N-1}$. In the second period, the buyer increases his offer to $p^{\dagger}_{N-1}$ which will induce another mixed acceptance by the low type, and both types opt out when possible, and the posterior becomes $\pi^{\dagger}_{N-2}$. This behavior continues until the posterior reaches $\pi^{\dagger}_{0} = \pi^{*}$. Hence, it takes $\max\{N, 1\}$ periods for the posterior to reach the deadlock belief. Note that information about the seller's type comes only from his acceptance behavior, and the posterior strictly increases in each period. The left panel of Figure 3.3 describes the dynamics of belief on the equilibrium path when $N = 2$.

So if the prior is less than the deadlock belief, the equilibrium behavior produces an outcome path with the following characteristics:

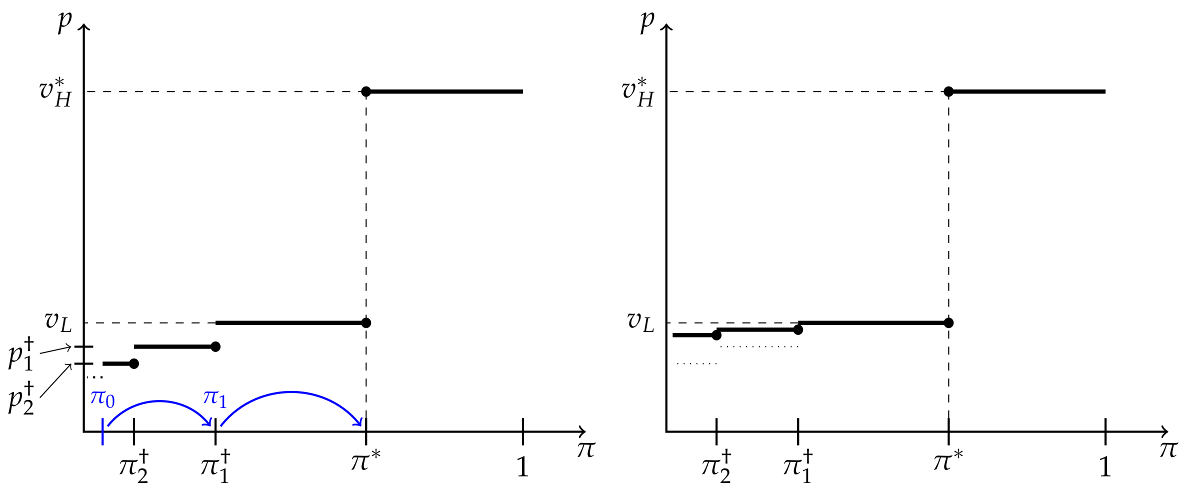
- Bargaining starts with a *pre-deadlock phase*. In this phase the buyer plays a pure offer strategy, and his offer is increasing over time so that the low type is indifferent between acceptance and rejection. Only the low type accepts the offer with positive probability, and both types of sellers opt out if possible. So an observed outcome in this phase has the following characteristics: the buyer offers a price less than v_L; the buyer's offer increases over time; bargaining might end with either acceptance of the buyer's offer (by the low type) or opting out (by both types).

- A *deadlock phase* begins once the buyer offers v_L. In this phase, the buyer continues to make the same randomized offer throughout the bargaining process. Only the low type accepts v_L with positive probability, and only the high type opts out if possible. Therefore, an outcome path features a sequence of the same offer of v_L being rejected repeatedly before bargaining ends.

- Bargaining ends with a sudden resolving behavior that is either 1) the buyer's bargaining-ending offer (v^{*}_{H}), 2) the low type's acceptance of v_L, or 3) the high type's opting out. Note that bargaining ends (either by an acceptance or an opt-out) in a finite number of periods with probability one.

4 Frequent Offers

Consider the limit case of frequent offers by letting the time between periods (denoted by Δ) converge to zero. Recall that $\eta = \frac{\lambda}{r+\lambda}$ is the effective discount factor.



(a) when $\pi_0 < \pi^*$

(b) offer function in the limit case

Figure 3.3: Equilibrium behavior

Proposition 5. *Suppose $\eta v_H > v_L + \frac{1-\eta}{\eta} u_L$. Then,*

1. *The deadlock equilibrium exists for sufficiently small Δ.*

2. *As Δ converges to zero, in the deadlock equilibrium,*

   - *the buyer's equilibrium offer when $\pi < \pi^*$ converges to v_L.*
   - *the length of the pre-deadlock phase (measured in real time) shrinks to zero.*
   - *the expected length of the deadlock phase does not shrink to zero.*

Proof.

1. (A1) is satisfied if $\eta v_H > v_L + \frac{1-\eta}{\eta} u_L$ and Δ is sufficiently small.

2. The proof is based on the construction of the deadlock equilibrium and is relegated to the Appendix.

□

As mentioned before, the deadlock equilibrium exists when the outside options are sufficiently important. In the limit case of frequent offers, this condition is represented by the effective discount factor being sufficiently high.

In the pre-deadlock phase, the equilibrium exhibits Coasian dynamics at a price v_L. Since the discount factor goes to one as Δ converges to zero, the difference between the buyer's successive offers vanishes as the buyer makes the low-type seller indifferent between acceptance and rejection. Moreover, the same force behind the Coase conjecture results in the pre-deadlock phase shrinking to zero. The right panel of Figure 3.3 describes the limit equilibrium offer by the buyer when Δ converges to zero.

However, the deadlock phase does not shrink in the limit case of frequent offers. More specifically, each resolution behavior of the deadlock phase (the buyer's bargaining-ending offer, the low type's acceptance of the low offer, and the high type's opt-out) converges to a Poisson arrival process. The indifference condition (3.3) implies that, as Δ converges to zero the probability of the buyer offering v_H^* converges to zero at the same rate. As a result, the buyer's equilibrium offer path (in real time) converges to the base offer of v_L with the endogenous Poisson arrival of v_H^*.[10] The low type's acceptance of offer v_L and the high type's opt-out occurs with probability $\xi = 1 - e^{-\lambda\Delta}$; hence, they converge to Poisson processes with parameter λ. Note that the Poisson arrivals of resolution behaviors are independent of each other.

Figure 4.1 summarizes the discussion above by depicting the limit distribution of the equilibrium outcome as $\Delta \to 0$. At any real time t', the height in the blue (red) area indicates the probability that the agreement (breakdown) happens anytime before t'. The height in the grey area is the probability that the bargaining continues beyond time t'. Note that for any finite t, bargaining will continue beyond time t with positive probability.

Real-Time Delay and Breakdown The outcome of the deadlock equilibrium exhibits various bargaining inefficiencies. Several key values, such as the expected length of delay and the probability of a breakdown, are derived in closed form.

The equilibrium behavior described in Section 3 implies that in the deadlock equilibrium, the bargaining is delayed with positive probability before it ends either in an agreement or in a breakdown. The following corollary states that the expected length of delay in real time is

[10] To see this, note that

$$\begin{aligned} q &= \frac{v_L/\delta - v_L}{v_H^* - v_L} \\ &= \frac{v_L}{v_H^* - v_L}(e^{r\Delta} - 1) = \frac{v_L r}{v_H^* - v_L}\Delta + o(\Delta), \end{aligned}$$

so as $\Delta \to 0$, the arrival of the buyer's offer $p = v_H^*$ converges to a Poisson process of rate $\frac{v_L r}{v_H^* - v_L}$.

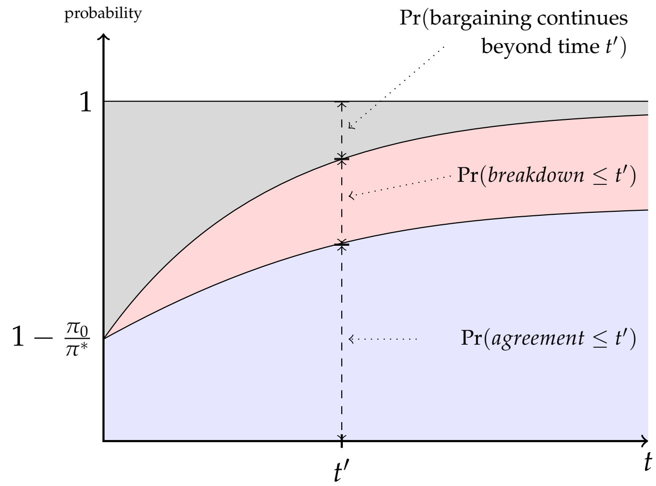


Figure 4.1: Limit distribution of the equilibrium outcome

positive even when the time between periods becomes arbitrarily small. So in the deadlock equilibrium, inefficiency does not disappear when offers are frequent. Let T_d be the (unconditional) expected length of delay, and let $\hat{T}_d$ be the expected length of delay conditional on deadlock.

Corollary 6. *In the deadlock equilibrium, the expected length of delay is positive if the prior is less than π^*. Moreover, as Δ converges to zero,*

$$\begin{aligned} \hat{T}_d &\to \frac{Z}{Z+\mu} \cdot \frac{1}{\lambda}, \\ T_d &\to \frac{\pi_0}{\pi^*}\hat{T}_d, \end{aligned}$$

where $Z = \frac{v_H^* - v_L}{v_L}$ and $\mu = \frac{r}{\lambda}$.

Proof. See the Appendix. □

Recall that Assumption 1 encompasses both the private and correlated value case, so that the real-time delay associated with the deadlock equilibrium can be found in both cases.

Another source of inefficiency in the deadlock equilibrium is the possibility of a breakdown resulting from the high type's opt-out. Let P_b be the ex ante probability of a breakdown, and $\hat{P}_b$ be the breakdown probability conditional on deadlock.

Corollary 7. *In the deadlock equilibrium, as Δ converges to zero,*

$$\begin{aligned} \hat{P}_b &\rightarrow \pi^* \frac{Z}{Z+\mu}, \\ P_b &\rightarrow \pi_0 \frac{Z}{Z+\mu}. \end{aligned}$$

Proof. See the Appendix. □

High Arrival Rate of the Outside Option The assumption in Proposition 5 implies that when the effective discount rate $\eta = \frac{\lambda}{r+\lambda}$ is arbitrarily close to one, the deadlock equilibrium exists in the limit of frequent offers. On the other hand, Corollary 6 implies that the expected length of the delay converges to zero as λ becomes arbitrarily high. So it is of interest to analyze the equilibrium behavior under sufficiently high λ.

Recall that at the deadlock belief, the low type accepts v_L with probability ξ. If ξ is close to one, almost every low-type seller accepts v_L and the interim belief after the offer stage becomes close to one. Then the high type exits the game with probability ξ and the posterior becomes π^*. Therefore, even though the equilibrium structure is preserved, the bargaining ends with a probability close to one.

Similar intuition can be applied to the limit case of frequent offers. Recall that as Δ goes to zero, each type of resolution behavior in the deadlock phase converges to a Poisson arrival process. As λ becomes arbitrarily high, the arrival rates of resolution behaviors also become arbitrarily high, and the (expected) length of the deadlock phase shrinks to zero.

What is the limit of π^* when λ becomes arbitrarily high? Fixing the discount rate r, as λ goes to infinity, the indifference condition of the buyer at π^* (from (3.1) and (3.2)) becomes

$$(1-\pi^*)(u_L - v_L) = \pi^*(u_H - v_H) + (1-\pi^*)(u_L - v_H).$$

Consider a static bargaining game where the buyer makes a take-it-or-leave-it offer to the seller and the seller has an outside option of v_θ. Then the left-hand side (right-hand side) of the above equation is the payoff to the buyer when he offers $v_L (v_H)$ to target low-type seller (both types of sellers). In other words, the buyer's optimal offer under arbitrarily high λ converges to one of static bargaining.

Interestingly, the limit distribution of the equilibrium outcome under high λ converges to the monopoly pricing equilibrium in Board and Pycia (2013), with the role of seller and buyer reversed. They consider a model of a seller-offer bargaining game where the buyer has private information about his valuation of the seller's good, and they assume that the buyer has an outside option available at any period. They show that there is a unique sequential equilibrium where the seller always offers an optimal monopoly price, and the buyer either accepts the offer or opts out immediately. So there is no bargaining delay in the equilibrium. If I switch the role of the seller and the buyer in Board and Pycia (2013), their equilibrium coincides to the limit distribution of the deadlock equilibirium with $\Delta \to 0$ and $\lambda \to \infty$.

5 Uniqueness

In general, there are multiple equilibria of this model. In particular, there may exist an equilibrium where the buyer uses an offer strategy similar to the 'Coasian' pricing (Fudenberg, Levine, and Tirole (1985); Gul, Sonnenschein, and Wilson (1986)). In this equilibrium, as the time between the periods becomes vanishingly small, the buyer's offer converges to v_H^* and the expected delay converges to zero, so the equilibrium outcome is approximately efficient. In the equilibrium with Coasian dynamics, although there are two sources of information, the information revealed by the seller's acceptance behavior dominates the information revealed by his opting-out behavior.[11]

Then the question is whether the deadlock equilibrium is one equilibrium of the model where two sources of information happen to offset one another. In this section, I show that under a stronger parametric assumption, the offsetting effect can be found in all PBE of the model. First, I present the parametric assumption stronger than (A1).

Assumption. *(A2)* $\frac{\xi}{1-\delta(1-\xi)}\delta v_H^* > u_L$.

A necessary condition for (A2) is $v_H^* > u_L$. Since $u_H \geq v_H$, the private value case $(u_H = u_L)$ does not satisfy (A2). More important, $v_H^* > u_L$ is a necessary condition for the existence of the static adverse selection problem. Suppose there is a static market where the buyer's value is u_θ and the seller's reservation value is v_θ^*. Then adverse selection in the trade exists if and only if $\mathrm{E}[u_\theta] < v_\theta^*$. Therefore, if $v_H^* > u_L$, the adverse selection problem arises for sufficiently low π_0.

Since (A2) implies (A1), (A2) guarantees the existence of the deadlock equilibrium. The following proposition shows that under (A2), the deadlock equilibrium is the only PBE satis-

[11] More discussion about Coasian equilibrium is in Section 6.

fying a monotonicity property. The property, called *nondecreasing offers*, requires that when the buyer's expected quality is higher, he tends to offer a higher price to the seller.

Definition. A strategy profile satisfies *nondecreasing offers* if for any history $h^n, h'^{n'}$ with $\pi_n < \pi_{n'}$, if the buyer offers p at h^n and p' at $h'^{n'}$, then $p \leq p'$.

Proposition 8. *Suppose (A2) holds. Then the deadlock equilibrium is the unique perfect Bayesian equilibrium that satisfies nondecreasing offers.*

Proof. See the Appendix. □

The following proposition states that under (A2), in every equilibrium neither source of information dominates the other, so the equilibrium has characteristics similar to those of the deadlock equilibrium.

Proposition 9. *Suppose (A2) holds. Then in every perfect Bayesian equilibrium of the game, if prior is low enough,*

(1) the posterior belief π_n never exceeds the deadlock belief π^* (defined in Section 3) conditional on the bargaining continues, and

(2) for any finite n, bargaining continues beyond period n with positive probability.

Proof. See the Appendix. □

6 Discussions

Random Arrival of Outside Options The random arrival of outside options assumed in this paper is a simple way of modeling a stochastic payoff of opt-out behavior. In principle, the bargaining parties can break the negotiation process at any point in time. However, the value of opting out typically changes over time. First, the value of the best available outside option may change over time. As in the example given in the introduction, a satisfactory outside offer often does not exist. Second, the cost of opting out may also change over time. Several external factors, such as the bargaining party's decision-making procedure and time-varying external environment, can affect the cost of taking the outside option. [12]

[12] In Section 7, I discuss a possible extension of the random value of the outside option.

Theoretically, the random arrival of outside options provides an alternative perspective on bargaining dynamics. Standard models of bargaining with an outside option assume that the option is available in every period to some or all of the bargaining parties. In bargaining with complete information, the outside option is either completely ineffective (when the value is low) or crucially effective (when the value is high) in determining an equilibrium behavior. In bargaining with incomplete information, an outside option may almost completely cancel out the impact of incomplete information, and the equilibrium features immediate termination of bargaining when the bargaining party either agrees or opts out depending on his private type.[13] In this paper the outside option is not available with positive probability. As a result, the bargaining continues into the next period with positive probability unless the buyer offers v_H^*; hence, the equilibrium shows non-trivial dynamics.

Production Cost and Heterogeneous Arrival Rate The result of this paper extends to the case where the seller has a positive cost of production. Suppose the type-θ seller has a production cost of $c_\theta > 0$. Recall that the seller's payoff is v_θ when he takes an outside option. Then the type-θ seller never accepts an offer if

$$p - c_\theta < \frac{\xi}{1-\delta(1-\xi)} v_\theta = v_\theta^*,$$

or $p < c_\theta + v_\theta^*$. To guarantee the existence of the deadlock equilibrium, a modified version of (A1) needs to be imposed:

Assumption. *(A1′)* $\delta(c_H + v_H^*) > v_L + \frac{(1-\delta)(1-\xi)}{\xi} u_L + (1 - \frac{1-\delta}{\xi}) c_L.$

Note that if $c_H = c_L = 0$, (A1) and (A1′) are equivalent.

Proposition 10. *Suppose (A1′) holds. Then there exists a deadlock equilibrium of the model.*

Proof. See the Appendix. □

Note that (A1′) encompasses a case where the value of the outside option is the same for both types ($v_H = v_L$). As long as the model's parameters induce the high type to have a stronger incentive to opt out, the deadlock equilibrium exists.

[13] See Compte and Jehiel (2002) for the effect of outside options on reputational bargaining, and Board and Pycia (2013) for the effect on a dynamic durable goods monopoly.

Similar intuition can be applied to check the robustness of the deadlock equilibrium when the arrival rate of the outside option is different across types. For example, the deadlock equilibrium may exist in the model where the outside option arrives only to the high-type seller.

Existence of Coasian Equilibrium When (A2) is not satisfied, the model has an equilibrium where Coasian dynamics lead to an approximately efficient outcome. In the Coasian equilibrium, the buyer plays a pure strategy at any history on the equilibrium path. The buyer gradually increases his offer over time. On the equilibrium path, the high type rejects the buyer's offer in all but the final period, and the low type uses a mixed acceptance strategy. If the initial offer is high enough, only the high type opts out in every period before the game ends. If the initial offer is low, then both types take the outside option until the offer exceeds some cutoff where it becomes suboptimal for the low type to opt out. As the time between periods becomes arbitrarily small, the initial offer converges to v_H^* and the equilibrium yields an approximately immediate trade.[14]

If the parameters satisfy (A2), then because of the static adverse selection problem, playing a Coasian strategy yields a negative payoff to the buyer. So the Coasian strategy profile does not hold as an equilibrium, and the deadlock equilibrium becomes a unique equilibrium under the monotonicity condition.

Permanent Outside Option Consider the case where the outside option does not disappear once it arrives. In the model of the permanent outside option, there exists a deadlock equilibrium that has the same equilibrium outcome as the one in the original model. The key reason is that both types of sellers have no incentive to keep the outside option. For the high type, since the buyer's maximum offer is strictly smaller than the payoff from the outside option $(v_H^* < v_H)$, he opts out once the option is available. The low type cannot signal that he has an outside option, since he always uses a mixed strategy on the equilibrium path. So he cannot use the outside option as a threat in future periods. Since the buyer's offer is increasing over time, the low type has no incentive to keep the outside option.

[14] A detailed description of the Coasian equilibrium in the presence of a stochastic outside option is available upon request. Hwang and Li (2013) construct a Coasian equilibrium in a model similar to the present paper, where the roles of the seller and the buyer are reversed.

Change in Timeline How robust is the deadlock equilibrium under different timelines of the game? Consider the case where the seller receives an outside option before the buyer offers a price. First, suppose that the seller's opting-out decision comes before the buyer offers a price, so the offer stage comes later than the outside option stage. Then a simple calculation shows that the equilibrium structure is unchanged. It is not surprising since the effect of switching the two stages only accounts for the discount factor.

What happens if the outside option arrives before the buyer makes an offer? In this case, there may exist multiple equilibria even if there is complete information about the quality. Under some range of parameters, the buyer's offer and the seller's opting-out decision have a self-fulfilling effect on each other. If the arrival rate of the outside option is low, there exists an equilibrium where the buyer makes an offer that is accepted only by the seller without the outside option, and the seller with the outside option rejects the offer and opts out. If the arrival rate of the outside option is high, there exists another type of equilibrium where the buyer makes an offer high enough so that the seller accepts it. And for the intermediate arrival rate, both equilibria may exist.

Two-Sided Incomplete Information It is generally known that in the model of a two-sided incomplete information bargaining game, severe multiplicity arises. The attempt to narrow down the equilibrium set results in either implausibility of the criterion or the non-existence of the equilibrium under certain parameters. I expect that in the model with the stochastic outside option, a similar multiplicity would arise.

Continuum of Types If the model assumes a continuum of seller's types, the main difficulty in the analysis is tracking the belief. Since the outside option does not arrive with probability one, the belief after an outside option stage has the same support as the one before the stage, but the belief about the high quality would decrease. Therefore, the posterior belief is not a truncation of the prior and therefore cannot be simplified to a state variable. So the equilibrium profile must describe the bargainer's behavior for any possible posterior belief.[15]

[15] Fuchs, Öry, and Skrzypacz (2012) analyzed an equilibrium where the posterior belief is an addition of multiple truncated beliefs. In their paper, such beliefs are formed when the future buyer cannot observe past offers, so the price history does not affect future buyers' beliefs and hence it does not affect their strategies. In this paper, there is a single buyer and he observes history of past offers. So an out-of-equilibrium offer affects the future belief of the

I conjecture that as in the two-types case, there are two countervailing forces in belief updating: the lower types tend to accept the buyer's offer and the higher types tend to opt out. However, it is unclear whether these countervailing forces would lead to a bargaining deadlock or to another equilibrium dynamic.

7 Concluding Remarks

One interesting extension is to assume a random value of the outside options instead of random availability. Consider the model where the type-θ seller receives an outside option in each period, and the value of the outside option is randomly drawn from distributions F_θ. Assume that F_H is first-order stochastic dominant over F_L. I conjecture that under some conditions on the distributions, there exists a deadlock equilibrium. In this case, neither type of seller plays a mixed strategy towards the outside option. Moreover, the low type would also opt out if he received a good enough outside option. But similar to the benchmark model, not taking the outside option conveys a bad signal about the quality of the product. It would be interesting to investigate whether and how a bargaining deadlock occurs in this extension.

Appendix

Proof of Proposition 1

It is sufficient to show that in any equilibrium, after any history the buyer never offers a price p greater than v_θ^*. First, observe that the buyer never makes an offer above u_θ, since his equilibrium payoff must be nonnegative. Given that, the seller's expected payoff after the rejection is no more than

$$z_1 \equiv \max\{\delta u_\theta, \xi v_\theta + \delta(1-\xi)u_\theta\} < u_\theta.$$

Note that the first (second) term in the bracket denotes the seller's maximum expected payoff when it is optimal for him to reject (accept) an outside option. So the seller accepts any offer $p > z_1$ after any history; hence, such offer is suboptimal for the buyer, since he can always make a lower offer $p - \epsilon > z_1$ and buy the product.

buyer, which makes the analysis difficult.

Proceeding with the same argument, given that the buyer's offer is bounded by z_m, the seller always accepts any offer above

$$z_{m+1} \equiv \max\{\delta z_m, \xi v_\theta + \delta(1-\xi)z_m\} < z_m,$$

so any offer greater than z_{m+1} is suboptimal for the buyer. Since $\{z_m\}$ is decreasing and converges to v_θ^*, for any $\epsilon > 0$ the buyer's offer $v_\theta^* + \epsilon$ is accepted by the seller, and hence is suboptimal.

Proof of Lemma 2

It is clear that $p \leq u_H$ after any history. Now I claim that if the buyer never offers more than $z_m > v_H^*$ in the equilibrium, both types surely accept any offer greater than

$$z_{m+1} = \max\{\delta z_m, \xi v_H + \delta(1-\xi)z_m\} < z_m.$$

The low type accepts p_n for sure, since his maximum payoff after the rejection is no more than $\max\{\delta z_m, \xi v_L + \delta(1-\xi)z_m\}$, which is less than z_{m+1}. Since $\{z_m\}$ is decreasing and converges to v_H^*, making an offer $v_H^* + \epsilon$ for any $\epsilon > 0$ is suboptimal for the buyer.

Proof of Proposition 4

Equilibrium Behavior at $\pi < \pi^*$

In this subsection, I construct sequences of prices $\{p_k^\dagger\}$ and cutoff beliefs $\{\pi_k^\dagger\}$ that describe the equilibrium behavior when the posterior is less than the deadlock belief π^*. In the deadlock equilibrium, at $\pi < \pi^*$ the buyer offers a price less than or equal to v_L and the low type uses a mixed acceptance strategy, and the bargaining reaches the deadlock phase in a finite number of periods.

Let $p_0^\dagger = v_L$, and let $p_k^\dagger$ be the equilibrium price when there are k periods until the bargaining reaches the deadlock phase. The low type is indifferent between accepting $p_k^\dagger$ and waiting k periods to accept $p_0^\dagger = v_L$. Since $p_k^\dagger$ must be strictly lower than v_L, opting out (when the option is available) is strictly optimal for the low-type seller in these k periods. Then the low type's indifference condition, which gives a recursive equation for $\{p_k^\dagger\}$, is given by

$$p_k^\dagger = \xi v_L + \delta(1-\xi)p_{k-1}^\dagger. \tag{7.1}$$

For the construction of $\{\pi_k^\dagger\}$, I define notions that make the analysis easier. Let $\beta(\pi, \pi')$ be the low-type seller's acceptance probability, which changes the posterior belief from π to π', given that both types of sellers opt out. That is, $\beta(\pi, \pi')$ satisfies

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1}{1-\beta(\pi,\pi')},$$

so $\beta(\pi, \pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}$. On the other hand, let $\tilde{\beta}(\pi, \pi')$ be the low type's acceptance probability, which changes the posterior belief from π to π', given that only the high type takes the outside option. So $\tilde{\beta}(\pi, \pi')$ satisfies

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1-\xi}{1-\tilde{\beta}(\pi,\pi')},$$

so $\tilde{\beta}(\pi, \pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}(1-\xi)$.

Let $\pi_0^\dagger = \pi^*$, and let $\pi_k^\dagger$ be the maximum belief where the buyer offers $p_k^\dagger$. That is, the buyer offers $p_k^\dagger$ if $\pi \in (\pi_{k+1}^\dagger, \pi_k^\dagger]$. Then when the belief is $\pi_1^\dagger$, the buyer is indifferent between offering $p_0^\dagger = v_L$ and $p_1^\dagger$. Either price leads to the posterior belief equal to $\pi_0^\dagger = \pi^*$, but the low type's acceptance probability is different. If the buyer offers $p_1^\dagger$, both types of sellers opt out, and the low type accepts with probability $\beta(\pi_1^\dagger, \pi^*)$. On the other hand, if the buyer offers $p_0^\dagger = v_L$, only the high type opts out, and the low type accepts with probability $\tilde{\beta}(\pi_1^\dagger, \pi^*)$.

Therefore, the payoff to the buyer when he offers $p_1^\dagger$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_1^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U_F^*,$$

where $U_F^* \equiv (1-\pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*)$ is defined in (3.1), and the payoff when he offers $p_0^\dagger = v_L$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U_F^*.$$

The indifference condition gives

$$(1 - \frac{\pi_1^\dagger}{\pi^*})(p_0^\dagger - p_1^\dagger) = \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger).$$

Note that the left-hand side of the equation above is the benefit of screening the low type at a lower price, while the right-hand side is the cost of the low type's opting-out. Hence $\pi_1^\dagger$ is given by

$$\frac{\pi_1^\dagger}{\pi^*} = \frac{(1-\delta)(1-\xi)v_L}{(1-\delta)(1-\xi)v_L + \xi(1-\pi^*)(u_L - v_L)}.$$

For $k > 1$, when the posterior is $\pi_k^\dagger$ the buyer is indifferent between offering $p_k^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-1}^\dagger)$, and offering $p_{k-1}^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-2}^\dagger)$. Let $W(\pi)$ be the buyer's expected payoff in the equilibrium when the posterior is π. Then

$$\begin{aligned} W(\pi_k^\dagger) &= (1-\gamma_k)(u_L - p_k^\dagger) + \delta\gamma_k(1-\xi)W(\pi_{k-1}^\dagger) && (7.2) \\ &= (1-\gamma_k)(u_L - p_{k-1}^\dagger) + \gamma_k W(\pi_{k-1}^\dagger), && (7.3) \end{aligned}$$

where $\gamma_k = \frac{\pi_k^\dagger}{\pi_{k-1}^\dagger}$. Note that (7.2) and (7.3) are the buyer's expected payoff when he offers $p_k^\dagger$ and $p_{k-1}^\dagger$, respectively. The indifference condition gives

$$\gamma_k W(\pi_{k-1}^\dagger) = (1-\gamma_k)(p_{k-1}^\dagger - v_L^*) \tag{7.4}$$

Then plugging (7.4) into (7.3) gives

$$W(\pi_k^\dagger) = (1-\gamma_k)(u_L - v_L^*).$$

Finally, plugging the equation above into (7.4) leads to

$$\frac{1}{\gamma_k} = 1 + (1-\gamma_{k-1})\frac{u_L - v_L^*}{p_{k-1}^\dagger - v_L^*}. \tag{7.5}$$

Note that since $\lim_{k\to\infty} p_k^\dagger = v_L$, γ_k converges to zero as k goes to infinity. Therefore, for any $\pi_0 \in (0, \pi^*)$, there exists $N \in \mathbb{N}$ such that $\pi_N^\dagger \le \pi_0 < \pi_{N-1}^\dagger$. Here I consider the generic case that $\pi_{N+1}^\dagger < \pi_0$.

Strategy Profile

- Buyer:

$$p_n(h^n) = p_n(\pi_n, p_{n-1}) = \begin{cases} v_H^* & \text{if } \pi_n > \pi^*, \\ q(p_{n-1}) \circ v_H^* + (1 - q(p_{n-1})) \circ v_L & \text{if } \pi_n = \pi^*, \\ p_{k-1}^\dagger & \text{if } \pi_n \in [\pi_k^\dagger, \pi_{k-1}^\dagger), \end{cases}$$

  where $\pi_0^\dagger = \pi^*$ and $q(p_{n-1}) = \max\{\frac{p_{n-1}/\delta - v_L}{v_H^* - v_L}, 0\}$.

- The low type:

$$\sigma_{Ln}(h_S^n) = \sigma_{Ln}(\pi_n, p_n) = \begin{cases} 1 & \text{if } p_n \geq \delta v_H^*, \\ \max\{0, \tilde{\beta}(\pi_n, \pi^*)\} & \text{if } p_n \in [v_L, \delta v_H^*), \\ \max\{0, \beta(\pi_n, \pi_k^\dagger)\} & \text{if } p_n \in [p_k^\dagger, p_{k-1}^\dagger), \\ 0 & \text{if } p_n < v_L^*, \end{cases}$$

$$c_{Ln}(\hat{h}_S^n) = c_{Ln}(\hat{\pi}_n) = \begin{cases} 1 & \text{if } \hat{\pi}_n < \hat{\pi}^*, \\ \max\{0, \tilde{\beta}(\hat{\pi}_n, \pi^*)/\xi\} & \text{if } \hat{\pi}_n \geq \hat{\pi}^*, \end{cases}$$

where $\hat{\pi}^* = \frac{\pi^*}{\pi^* + (1-\pi^*)(1-\xi)}$.

Optimality of Profile

The following lemma states that if (A1) holds, then offering any price $p \in [\delta v_H^*, v_H^*)$ at the deadlock belief is suboptimal for the buyer.

Lemma 11. *Suppose (A1) holds. Then at $\pi = \pi^*$, the buyer is better off by offering v_H^* than by offering a price $p \in [\delta v_H^*, v_H^*)$.*

Proof. Let $U_F(\pi) = (1-\pi)(u_L - v_H^*) + \pi(u_H - v_H^*)$ be the payoff to the buyer when he offers v_H^* to finish the bargaining at a posterior belief π. Then $U_F^* = U_F(\pi^*)$, where U_F^* is defined in (3.1).

Corollary 3 implies that offering a price $p \in (\delta v_H^*, v_H^*)$ is dominated by offering δv_H^*. If the buyer offers δv_H^*, the low type accepts it for sure. The high type opts out when the option is available, then the buyer offers v_H^* in the next period and bargaining ends. In this case the buyer's expected payoff is

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*).$$

If the buyer instead offers v_H^* and finishes the bargaining immediately, he obtains $U_F(\pi)$. Then the difference is

$$\underbrace{(1-\pi)(1-\delta)v_H^*}_{\text{benefit of screening}} - \underbrace{\pi\xi(u_H - v_H^*)}_{\text{cost of breakdown}} - \underbrace{\pi(1-\xi)(1-\delta)(u_H - v_H^*)}_{\text{cost of delay}}.$$

Therefore offering v_H^* yields greater payoff if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

A simple calculation shows that $\pi^* > \tilde{\pi}$ if and only if (A1) holds. $\square$

The optimality of each action is as follows:

1. c_{Ln} :

   (a) If $\hat{\pi}_n < \pi^*$, then for any value of $c_{Ln} \in [0,1]$, $\pi_{n+1} < \pi^*$ so $p_{n+1} \leq v_L$. Therefore the continuation payoff is no greater than v_L, so taking an outside option is optimal for the low type.

   (b) If $\hat{\pi}_n \in [\pi^*, \hat{\pi}^*]$, the only consistent strategy is to use a mixed strategy to induce $\pi_{n+1} = \pi^*$.

   (c) If $\hat{\pi}_n > \hat{\pi}^*$, then for any value of $c_{Ln} \in [0,1]$, $\pi_{n+1} > \pi^*$ so the buyer offers v_H^* in the next period. Therefore, the low type strictly prefers not to take an outside option.

2. σ_{Ln} :

   (a) By Corollary 3, the low type accepts any $p_n \geq \delta v_H^*$ for sure.

   (b) $p_n \in [v_L, \delta v_H^*)$: If $\pi_n \leq \hat{\pi}^*$, accepting the offer with probability $\sigma_{Ln} = \tilde{\beta}(\pi_n, \pi^*)$ that, combined with $c_{Ln} = \max\{0, \tilde{\beta}(\hat{\pi}_n, \pi^*)/\xi\}$, induces $\pi_{n+1} = \pi^*$ is the only consistent strategy of the low-type seller. if $\pi > \hat{\pi}^*$, the low type is strictly better off by rejecting p_n, not taking outside option and accepting $p_{n+1} = v_H^*$.

   (c) $p_n < v_L$: The construction of the sequences $\{(\pi_k^\dagger, p_k^\dagger)\}_{k=0}^\infty$ implies that the low type is indifferent between acceptance and rejection by following the above strategy profile.

3. p_n : Lemma 11 and the construction of the sequences $\{(\pi_k^\dagger, p_k^\dagger)\}_{k=0}^\infty$ imply that offer strategy p_n is the best response to the seller's strategy $(\sigma_{\theta t}, c_{\theta t})$.

Proof of Proposition 5

Let $\{\hat{p}_k^\dagger\}$ and $\{\hat{\pi}_k^\dagger\}$ be the limit of sequences $\{p_k^\dagger\}$ and $\{\pi_k^\dagger\}$ when $\Delta \to 0$. Then the recursive equation (7.1) implies that $\hat{p}_k^\dagger = v_L$ for any k. Therefore, the recursive equation (7.5) for $\gamma_k =$

$\frac{\pi_k^\dagger}{\pi_{k-1}^\dagger}$ becomes

$$\gamma_k = \frac{1}{1 + (1-\gamma_{k-1})\frac{u_L - \eta v_L}{v_L - \eta v_L}},$$

where $\eta = \frac{\lambda}{r+\lambda}$. Since a function $g(x) = \frac{1}{1+(1-x)\frac{u_L-\eta v_L}{v_L-\eta v_L}}$ is convex and has fixed points of one and $\frac{v_L-\eta v_L}{u_L-\eta v_L} < 1$, γ_k converges to $\frac{v_L-\eta v_L}{u_L-\eta v_L}$. Therefore, for any prior $\pi_0 \in (0, \pi^*)$ there exists a finite K such that $\hat{\pi}_K^\dagger \leq \pi_0$. Therefore, as Δ goes to zero, the equilibrium offer at π_0 converges to v_L, and the real-time length of the pre-deadlock phase, $K\Delta$, shrinks to zero.

In the deadlock phase, in each period the bargaining ends by 1) agreement at $p = v_H^*$ with probability q, 2) agreement at $p = v_L$ with probability $(1-q)(1-\pi^*)\xi$, and 3) opting-out with probability $(1-q)\pi^*\xi$. Therefore, the resolution period of the deadlock phase is a geometric distribution with parameter $q + (1-q)\xi$. As $\Delta \to 0$, the limit distribution becomes Poisson arrival process with a finite arrival rate, so the deadlock phase does not shrink.

Proof of Corollaries 6 and 7

The probability of agreement at $t = 0$ is $(1-\pi_0)\beta(\pi_0, \pi^*) = 1 - \frac{\pi_0}{\pi^*}$. The proof of Proposition 5 implies that $\hat{T}_d = \frac{\Delta}{q+(1-q)\xi}$, and letting $\Delta \to 0$ provides the desired result.

The probability of a breakdown conditional on the bargaining reaching the deadlock phase is

$$\frac{(1-q)\pi^*\xi}{q + (1-q)\xi},$$

so letting $\Delta \to 0$ provides the desired result..

Proof of Proposition 9

Suboptimality of Two-Period Screening

Lemma 12. *Suppose (A2) holds. Then in equilibrium, $p_n \in [\delta v_H^*, v_H^*)$ is never offered after any history.*

Proof. Recall from Lemma 11 that offering v_H^* yields a greater payoff than offering a sequence of prices $\delta v_H^*, v_H^*$ if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

On the other hand, from the inequality

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*) < 0,$$

the offer sequence $\delta v_H^*, v_H^*$ yields a negative payoff if and only if

$$\pi < \underline{\pi} \equiv \frac{\delta v_H^* - u_L}{\delta(1-\xi)(u_H - v_H^*) + (\delta v_H^* - u_L)}.$$

Suppose (A2) holds. Then a simple calculation shows that (A2) implies $\underline{\pi} > \tilde{\pi}$. Then for any $\pi \in [0,1]$, $p_n \in [\delta v_H^*, v_H^*)$ is not offered in equilibrium, since either $p = v_H^*$ or $p = 0$ is a profitable deviation. □

Upper Bound on the Equilibrium Posterior

Lemma 13. *In any equilibrium, there exists $\bar{\pi} \in (0,1)$ such that if $\pi_n > \bar{\pi}$ after any history, the buyer offers $p_n = v_H^*$.*

Proof. The maximum payoff of the buyer by screening the low type is

$$(1-\pi) \cdot (u_L - v_L^*) + \pi(1-\xi) \cdot \delta(u_H - v_H^*). \tag{7.6}$$

If instead the buyer offers v_H^*, then his payoff is $U_F(\pi)$. Therefore, if

$$\pi > \frac{v_H^* - v_L^*}{(1-\delta(1-\xi))(u_H - v_H^*) + v_H^* - v_L^*}$$

then the buyer strictly prefers to offer v_H^* regardless of the history. □

Lemma 14. *If $\pi_n \leq \bar{\pi}$ and $p_n < v_H^*$, $\pi_{n+1} \leq \bar{\pi}$.*

Proof. Suppose not; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n < v_H^*$, and $\sigma_{Ln} + (1-\sigma_{Ln})\xi c_{Ln} > \tilde{\beta}(\pi_n, \bar{\pi})$. By Lemma 12, $p_n < \delta v_H^*$. Moreover, by Lemma 13, $p_{n+1} = v_H^*$. Then it is optimal for the low type to reject both p_n and an outside option and wait for the next period offer, which leads to a contradiction. □

Lemma 15. *(1) If $\pi_n \leq \bar{\pi}$ and $p_n \in (v_L, \delta v_H^*)$, then $c_{Ln} = 0$.*

(2) If $\pi_n = \bar{\pi}$ and $p_n < v_L$, then $\sigma_{Ln} = 0$.

Proof. (1) Suppose not; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n \in (v_L, \delta v_H^*)$, and $c_{Ln} > 0$. Then opting-out must be at least as good as waiting, so $u_L(h^{n+1}) \leq v_L/\delta$. Then it is strictly optimal to accept p_n, contradicting Lemma 14.

(2) Suppose that there exists a history h^n where $\pi_n = \bar{\pi}$ and $p_n < v_L$, and $\sigma_{Ln} > 0$. Then by Lemma 14 $c_{Ln} < 1$, which implies $u_L(h^{n+1}) \geq v_L/\delta$. But then accepting p_n is suboptimal. Contradiction. □

Lemma 16. $\bar{\pi} \leq \pi^*$.

Proof. Suppose the contrary that there exists an equilibrium with $\bar{\pi} > \pi^*$. Then it suffices to show that for all history with a belief smaller than but sufficiently close to $\bar{\pi}$, offering v_H^* is optimal for the buyer.

Define

$$\tilde{U}(\bar{\pi}) = (1-\bar{\pi})\xi(u_L - v_L) + \delta(1-\xi)U_F(\bar{\pi}),$$

and let $\hat{U}(h^n)$ be the supremum of the buyer's expected payoff at h^n, given that the buyer offers $p < v_H^*$ at h^n. I claim that for any history h^n with a belief $\bar{\pi}$, $\hat{U}(h^n) < \tilde{U}(\bar{\pi})$. Suppose the bargaining ends after k periods. Then by Lemma 14, the probability of an agreement between the low-type seller before bargaining ends is no more than $\frac{1-\xi^k}{1-\xi}$. Since the low type never accepts any offer less than v_L,[16] making an agreement at v_L with the least delay yields the highest possible payoff to the buyer. Therefore the buyer's payoff is bounded by

$$\tilde{U}_k(\bar{\pi}) = (1-\bar{\pi})\frac{\xi(1-\delta^k(1-\xi)^k)}{1-\delta(1-\xi)}(u_L - v_L) + \delta^k(1-\xi)^k U_F(\bar{\pi}).$$

Since $\bar{\pi} > \pi^*$, $(1-\delta(1-\xi))U_F(\bar{\pi}) > (1-\bar{\pi})\xi(u_L - v_L)$, so $k = 1$ is optimal.

Now consider histories with beliefs less than $\bar{\pi}$. Then the continuity of the previous argument implies that for any $\beta > 0$, there exists $\epsilon > 0$ such that for any history h^n with a belief $\pi(h^n) \in (\bar{\pi} - \epsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, then $U(h^n) < \tilde{U}(\bar{\pi}) + \beta$.

Equations (3.1) and (3.2) imply that $\bar{\pi} > \pi^*$ if and only if $\tilde{U}(\bar{\pi}) < U_F(\bar{\pi})$. Then since $U_F(\pi)$ are continuous, for sufficiently small $\beta > 0$, there exists $\epsilon > 0$ such that for any history h^n with a belief $\pi(h^n) \in (\bar{\pi} - \epsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, $U(h^n) < \tilde{U}(\bar{\pi}) + \beta < U_F(\bar{\pi} - \epsilon)$. So the buyer's optimal offer is v_H^* for any history with $\pi \in (\bar{\pi} - \epsilon, \bar{\pi})$, which contradicts the definition of $\bar{\pi}$. □

Proof of Proposition 8

Fix a perfect Bayesian equilibrium that satisfies nondecreasing offers.

[16] Suppose not; that is, there exists a history $\hat{h}^m$ where the buyer offers a price less than v_L and the low type accepts it with positive probability. Then, the low type must take the outside option for sure (if it is available) at every history between h^n and $\hat{h}^m$. Then at $\hat{h}^m$ the posterior is $\bar{\pi}$, which contradicts Lemma 15.

Step 1 For any history h^n, $\pi_n \geq \pi_{n-1}$.

Proof. Suppose not; that is, there exists a history h^n such that $\pi_n < \pi_{n-1}$. Then in order to make the low type indifferent, the buyer's offer at h^n satisfies $\mathbb{E}[p_n] = p_{n-1}/\delta$. Therefore, the seller offers $p_n > p_{n-1}$ with positive probability, which violates the nondecreasing offers. □

In the proof of Proposition 9, I show that there exists $\bar{\pi} \leq \pi^*$ that bounds the posterior belief along the bargaining process. Then Step 1 implies that if $\pi_n = \bar{\pi}$ at some history h^n, then $\pi_{n+1} = \bar{\pi}$ after any $p_n < \delta v_H^*$, which implies that $\sigma_{Ln}(\bar{\pi}, x) = \xi$ for any $p \in (v_L, \delta v_H^*)$.

Step 2 At $\pi = \bar{\pi}$, the buyer's equilibrium offer is either v_L or v_H^*.

Proof. It is clear that offering $p < v_L$ is suboptimal for the buyer. Moreover, Lemma 12 says that any offer $p \in [\delta v_H^*, v_H^*)$ is suboptimal. Then it is sufficient to show that if any $p \in (\tilde{p}, v_H^*)$ is not offered, the same goes for any $p \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Suppose at some history h^n, the buyer offers $p_n \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Then in the next period, to make the low type indifferent, the buyer must use mixed offer between v_H^* and some (possibly multiple) $p \leq \tilde{p}$. Therefore the buyer's expected payoff at history h^n is

$$(1 - \bar{\pi})\xi(1 - p_n) + \delta(1 - \xi)(1 - v_H^*).$$

Now consider the deviation of the buyer to offer $p' = p_n - \epsilon$, where ϵ is small enough such that $p' > \max\{\delta\tilde{p}, v_L\}$. Then the buyer can make an agreement with the low type at a lower offer with the same probability and still use a mixed offer in the next period. So offering p' is a profitable deviation, which proves that any $p \in (v_L, v_H^*)$ cannot be offered in equilibrium. □

Step 3 $\bar{\pi} = \pi^*$.

Proof. Suppose $\bar{\pi} < \pi^*$. First, I claim that if $\pi_n = \bar{\pi}$, the buyer's offer must be v_H^*. Suppose v_L is offered in some history h^n. Then in the next period the buyer must use a mixed strategy between v_H^* and some $p \leq v_L$. Therefore, the buyer's expected payoff at history h^n is

$$(1 - \bar{\pi})\xi(1 - v_L) + \delta(1 - \xi)(1 - v_H^*),$$

which is greater than $1 - v_H^*$ since $\bar{\pi} < \pi^*$. So offering v_L at h^{n+1} is strictly better than v_H^*, contradictory to the fact that the buyer uses a mixed strategy.

Since the equilibrium satisfies nondecreasing offers, it must be that for all history h^n with $\pi_n < \bar{\pi}$, the buyer never offers v_H^*. Let $\bar{p}$ be a supremum of the buyer's offer at history h^n with $\pi_n < \bar{\pi}$. Then by Lemma 12, $\bar{p} \leq \delta v_H^*$. Fix ϵ sufficiently small that $\bar{p} - \epsilon > \delta \bar{p}$. Then there exists a history h^n with $\pi_n < \bar{\pi}$ where the buyer offers $p > \bar{p} - \epsilon$ with positive probability. Suppose that $\pi_{n+1} \geq \bar{\pi}$; then $p_{n+1} = v_H^*$ and the low type is strictly better off by rejecting p_n. If $\pi_{n+1} < \bar{\pi}$, then accepting p_n is a strict best response of the low type, violating consistency. □

Step 4 Behavior at $\pi \leq \pi^*$ is determined uniquely.

Proof. By step 1, the equilibrium belief is nondecreasing. Therefore, the backward induction method in the proof of Proposition 4 yields unique equilibrium behavior. □

Proof of Proposition 10

The buyer's indifference condition at the deadlock belief π^* is given by

$$(1-\delta(1-\xi))\{(1-\pi^*)u_L + \pi^* u_H - (c_H + v_H^*)\} = (1-\pi^*)\xi(u_L - (c_L + v_L)),$$

so

$$\pi^* = \frac{(c_H + v_H^* - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - (c_L + v_L))}{(u_H - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - (c_L + v_L))}.$$

The buyer can conduct a two-period screening by offering $(1-\delta)c_L + \delta(c_H - v_H^*)$ to the low type then offering $c_H + v_H^*$ to the remaining high type. In this case, his payoff is

$$(1-\pi)[u_L - (1-\delta)c_L + \delta(c_H - v_H^*)] + \pi\delta(1-\xi)[u_H - (c_H + v_H^*)].$$

Hence the two-period screening yields a higher payoff than offering $c_H + v_H^*$ if

$$\pi < \tilde{\pi} \equiv \frac{(1-\delta)[(c_H + v_H^*) - c_L]}{(1-\delta)[(c_H + v_H^*) - c_L] + (1-\delta(1-\xi))[u_H - (c_H + v_H^*)]}.$$

It can be shown that (A1') is satisfied if and only if $\pi^* > \tilde{\pi}$.

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