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"Dynamic Education Signaling with Dropout", Second Version

by

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Dynamic Education Signaling with Dropout*

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Abstract

We study students' dropout behavior and its consequences in a dynamic signaling model. Workers pay an education cost per unit of time and cannot commit to a fixed education length. Workers face an exogenous dropout risk before graduation. Since low-productivity students' cost is high, pooling with early dropouts helps them to avoid a high education cost. In equilibrium, low-productivity students choose to endogenously drop out over time, so the productivity of students in college increases along the education process. We find that the maximum education length is decreasing in the prior about a student being highly productive. We characterize the joint dynamics of returns to education and the dropout rate and provide an explanation of the declining dropout rate over the time students spend in school. We also extend the baseline model by allowing human capital accumulation and show that the dynamics of the dropout rate are helpful in decomposing the returns to education into the signaling effect and the human capital accumulation effect.

Keywords: Dynamic Education Signaling, Dropout

JEL Classification Codes: D83, J31

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1 Introduction

The dropout rate observed in tertiary education in the US is high. In a recent survey paper, Bound and Turner (2011) report that only about half of those who begin first-level degree programs actually obtain their degrees. Also, the probability that a student will dropout is decreasing over the number of years they spend in college.[1] While some students drop out for exogenous reasons, such as financial constraints; others voluntarily choose to drop-out.[2]

To understand the presence and dynamics of students' dropout behavior, it is necessary to take a closer look at their incentives to drop out. In the labor market, employers may perceive early dropping out as a signal indicating low productivity, caused by (unobserved) bad habits, poor health, etc. So, students should take this signaling effect into account when making the decision to drop out. Our paper studies the interaction between students' dropout behavior and the signaling effect, and how the incentive to drop out varies over the number of years they spend in college.

To capture the dynamics of dropping out, we investigate a dynamic signaling model. The main innovation of this paper is that by introducing a simple and yet realistic modeling device (exogenous dropout risk), we provide a tractable framework to study the joint dynamics of returns to education and the dropout rate in the presence of signaling concerns. In particular, we consider a dynamic model where (1) a worker (student) privately knows his productivity, and (2) faces an exogenous dropout risk in each period. Once the shock arrives, the worker has to go on the job market. We interpret this exogenous dropout process as random shocks faced by workers, driven by exogenous problems such as financial constraints, family reasons and the arrival of utility shocks. Since whether and when the worker will be forced to drop out is not known with certainty by the worker at the beginning of his education, one can expect, under these features, that workers who drop out do not have any offers when they leave college.

Since a high-productivity worker leaves school with a positive probability, a low-productivity worker may have incentives to mimic him by voluntarily quitting school in order to save future education costs. Nevertheless, if in some period a low-productivity worker dropped out with probability one, while the high-productivity worker stayed with a positive probability, the next period's beliefs about the worker being a high-productivity worker would jump to one. If the corresponding jump in wages was large enough, the low-productivity worker would have incentives not to drop out in the current period, leading to a contradiction. On the other hand, if low-

[1] See Hendricks and Leukhina (2013) as an example.

[2] A study from the Bill & Melinda Gates Foundation (2009) shows that students drop out of college for many reasons. For example, 52% of dropouts mentioned that "I just couldn't afford the tuition and fees," 71% mentioned that "I needed to go to work and make money," and so on.

productivity workers did not endogenously drop out at some period, then learning would be slow, which makes education less attractive for them, incentivizing dropout in the current period. We show that, in order to balance these two forces, low-productivity workers randomize between dropping out and staying in school in almost all periods.

This paper has three main results. First, we characterize the joint dynamics of wages and dropout rates. By doing so, we show that a worker's drop out behavior varies over the number of years he has spent in school, and we provide a natural explanation of the dropout rate-grade profile based on information asymmetry. In equilibrium, to ensure the low-type worker's randomization, the wage increment in each period must equal the marginal cost of education for the low-type worker, which endogenously pins down the belief-updating process and the low-type worker's dropout behavior. Unfortunately, even though we can derive the equilibrium relation between workers' dropout rate and their years of education in the discrete time model, its dynamics are hard to analytically characterize. Hence, we examine the continuous time limit of our discrete time model. At the continuous time limit, we explicitly show that the dropout rate of a worker is decreasing in his year in school.

In addition, the model proposed generates an implication of the relation between maximum equilibrium education duration and the prior about a worker being highly productive. The maximum equilibrium education duration is decreasing in the prior. In particular, when the prior about the worker being highly productive approaches one, no wasteful education appears in any perfect Bayesian equilibrium. The reason is as follows. Because workers face an exogenous dropout risk in each period, when the prior is high enough, firms believe a dropout is a high-type worker who suffered from an exogenous shock with very high probability. Hence, firms are willing to pay him a high wage. Since education is costly, both the high type and the low type worker prefer dropping out to staying in school. Consequently, the equilibrium of the asymmetric information game converges to that of the symmetric information game, in which the worker is a high type with probability one, as the information asymmetry vanishes.

Last but not least, we extend the pure signaling model by allowing human capital accumulation. In this extension, we consider a model in which education not only separates high-type workers from low-types but also enhances all workers' productivity. As a result, both the signaling effect and the human capital accumulation effect contribute to the returns to education. We characterize the equilibrium passing D1 criteria, and we illustrate that one can quantitatively decompose the signaling effect and the human capital accumulation effect by using data on wages and the dropout rate.

Even though we present our model in an education signaling environment, our insights are also useful for understanding some other environments in which sending signals is not only costly but also time-consuming. For example, consider a firm owner who is trying to sell his firm. In order to signal the type of the firm, the owner may wait for some time. The opportunity cost of waiting is likely to be low if the quality of the firm is good. The risk of dropping out may be reinterpreted as liquidity shocks or hedging considerations that force the owner to sell the firm early. The observed dropout rate can be interpreted as transaction volume. Another example is given by central banks defending themselves from currency attacks. In this case, the cost of defending may depend on the fundamentals of the economy, known only by the central bank. As time passes, the posterior belief about the economy being healthy increases, so the size of the attacks decreases and the attacks eventually vanish. The exogenous shocks may result from random events in international markets, such as a devaluation of the foreign currency used to defend against attacks.

Related Literature

This paper is related to a growing literature studying dynamic signaling games with preemptive offers. To best of our knowledge, the literature springs from Weiss (1983) and Admati and Perry (1987). They argue that the static signaling model (Cho and Kreps, 1987) overlooks the dropout behavior of workers. Think of a two-type signaling model. If a separating equilibrium is supposed to be played as predicted by Cho and Kreps (1987), once a worker arrives on the first day of school, the separation has already happened, and firms believe that the worker has high productivity. Hence, the worker should drop out immediately. Cho and Kreps (1987) avoid this challenge by directly assuming that a worker can commit to his decision about the duration of his education. In practice, it is hard to see where the commitment power comes from.

Nöldeke and van Damme (1990) formulate an explicitly dynamic game-theoretic version of the signaling model. In their model, long-lived firms simultaneously make *public preemptive offers* to the worker in each period, and the worker decides to accept an offer or to continue his education. They focus on equilibria that satisfy the *never a weak best response* (NWBR) requirement provided by Kohlberg and Mertens (1986), and they find that equilibria outcome converges to the Riley outcome when the time interval between two education decision points goes to zero. Nonetheless, Swinkels (1999) argues that Nöldeke and Van Damme's result crucially depends on the fact that job offers are publicly made. Hence, he considers a model where two short-lived firms enter and simultaneously make *private preemptive offers* to the worker in each period before the worker decides on whether to continue his education. Swinkels finds that, when the interval between consecutive offers goes to zero, the unique sequential equilibrium in this game is a pooling one at no education. Our model is different from Nöldeke and van Damme (1990) and Swinkels (1999) in two respects. First, neither Nöldeke and van Damme (1990) nor Swinkels (1999) studies the interaction between dropout rate dynamics and the signaling effect. In the former, some workers do not go to school at all and the rest stay in school until the "graduation day." No worker drops out in between. In the latter, no one goes to school in the first place. Instead, in our model, the joint dynamics of the return to education and the dropout rate are one of the main implications. Second, instead of allowing firms to make preemptive offers, we assume that the informed party, the worker, moves first (going to the job market or not) and, consequently, conditional on being in the job market, the uninformed agents, firms, make him offers. Since dropouts cannot return to school, there is no need to distinguish between private offers and public offers in our model.

The closest paper to our research is Kremer and Skrzypacz (2007). They consider a finite horizontal model in which an informative (type-dependent) signal about the type of the worker is publicly announced.[3] In their model, the joint dynamics of the dropout rate and wages (trade probability and price in their language) are also characterized. Their characterizations depend on the presence of an extra signal at the deadline. We provide a different framework to analyze the dynamics of the return to education and the dropout rate in the absence of such a type-dependent extra signal by introducing *type-independent* dropout risk. In addition, we study the role of the observed dropout rate in a model with productive education: dropout data are helpful in distinguishing the human capital accumulation effect and the signaling effect on the return to education.

Our model is also related to the dynamic adverse selection literature. Janssen and Roy (2002) study a dynamic lemons market problem and show that each equilibrium involves a sequence of increasing prices and qualities traded over time. Trade is delayed and therefore inefficient, but all goods are sold out in finitely many periods. In their model, the time-on-the-market of a good is used to signal the quality of the good. Hörner and Vieille (2009) study a dynamic bargaining game in which a single seller faces a sequence of buyers and show that the observability of previously rejected prices can cause a bargaining impasse. Kim (2011) examines the roles of different pieces of information about sellers' past behavior in a dynamic decentralized lemons market. He suggests that market efficiency is not monotone in the amount of information available to buyers but depends crucially on what information is available under what market conditions. Camargo and Lester (2011) investigate a dynamic decentralized lemons market with one-time entry. They demonstrate how prices and the composition of assets evolve over time given an initial fraction of lemons. They find that the patterns of trade depend systematically on the initial fraction of lemons, which is similar to the structure of our result. However, they focus on the dynamics of trade and price.

The rest of this paper is organized as follows. In the next section we present the model with

[3]One example of such signals is students' GPA. See also Daley and Green (2012) and Dilme (2012) for infinite horizon models

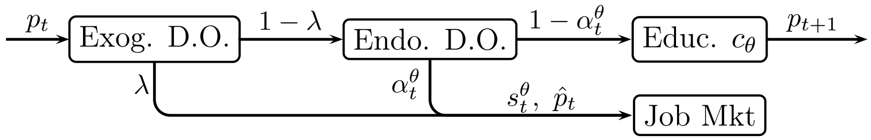

Figure 1: Schematic representation of the timing of the model (D.O. denotes dropout).

a type-independent dropout rate and characterize the set of equilibria. We consider a model with human capital accumulation in Section 3. In Section 4, we conclude. All omitted proofs are in Appendix A. In Appendix B, we study an extension in which workers' dropout risk is type-dependent. In Appendix C, we consider a multiple-type extension of the baseline model.

2 Model

Time is discrete, $t = 0, 1, 2, \ldots$. There is one worker who has a type $\theta \in \{H, L\}$, which is his private information with a common prior $p_0 = \Pr(\theta = H) \in (0, 1)$. The productivity of a type θ worker (henceforth, the θ-worker) is Y_θ. We normalize $Y_H = 1$ and $Y_L = 0$. In period 0 the worker decides whether to go to school or not. In the rest of the periods, if the worker continues going to school, he pays a type-contingent cost per unit of time, c_θ, where $0 < c_H < c_L$ and $c_H < 1$. The worker, regardless of his type, in each period is subject to an exogenous shock that results in the worker being forced to drop out of school with probability $\lambda \in (0, 1)$, regardless of his type. The exogenous dropout is interpreted as financial or utility shocks. In addition to this exogenous dropout, the worker may decide to endogenously drop out and go on the job market voluntarily.

The timing is summarized as follows. First, nature determines the type of the worker, choosing H with probability p_0. If the worker is still in school, in period t: (1) the worker exogenously drops out with probability λ and, if he does not exogenously drop out, decides whether to endogenously drop out. (2) If the worker decides not to drop out, he pays the education cost and goes to the next period. If the worker drops out, he goes on the job market. (3) Two firms enter the job market and simultaneously make job offers to the worker who has dropped out. (4) The worker can choose to take either offer or a zero-value outside option. Figure 1 schematically represents the timing of the model.

The utility of the θ-worker who has t periods of education and accepts a wage of w is $U(w, t) = w - c_\theta t$. The profit of a firm that employs a θ-worker at a wage w is given by $Y_\theta - w$. When a firm hires no worker, its profit is zero.

A dropout (behavior) strategy for the θ-worker is $\alpha^\theta : \{0, 1, \ldots\} \to [0, 1]$, the probability that the type θ worker chooses to drop out at t conditional on reaching its decision point. We use $s_t^\theta \equiv \lambda + (1-\lambda)\alpha_t^\theta$ to denote the total probability of the type $\theta \in \{L, H\}$ dropping out in period t. Finally, for each strategy profile, let $T^\theta \equiv \min\{t | s_t^\theta = 1\} \in \{0\} \cup \mathbb{N} \cup \infty$, which is the maximum number of education periods the type θ worker may receive under the given strategy profile.

Define p_t to be the posterior about a worker who reached period t being an H-worker, and let $\hat{p}_t$ be the same posterior about a worker who dropped out at t. When a worker goes on the job market in period t, two firms Bertrand-compete given their updated belief $\hat{p}_t$. We denote the sequence of wage offers by w. So, they both will offer $w_t = \hat{p}_t$. On the path of play, firms have correct beliefs about the dropout's type, $\hat{p}_t$; thus, they obtain zero expected profit. The worker will take the offer with the higher wage if it is positive. The solution concept we employ is a perfect Bayesian equilibrium:

Definition 1. *A **perfect Bayesian equilibrium (PBE)** is a strategy profile $\{(\alpha^\theta)_{\theta=L,H}, w\}$ and a belief sequence p such that:*

1. *the θ-worker chooses α^θ to maximize his expected payoff given w,*
2. *if a worker drops out with education t, firms offer $w_t = \hat{p}_t$, where*

$$\hat{p}_t = \frac{p_t s_t^H}{p_t s_t^H + (1-p_t) s_t^L} \ , \tag{1}$$

when it is well defined, and

3. *when it is well defined, p_t is updated following Bayes' rule*

$$p_{t+1} = \frac{p_t(1-s_t^H)}{p_t(1-s_t^H) + (1-p_t)(1-s_t^L)} \ . \tag{2}$$

The value function of the θ-worker in period t is

$$V_t^\theta = \lambda \hat{p}_t + (1-\lambda) W_t^\theta \ ,$$

where $\hat{p}_t$ is his payoff when he exogenously drops out, and $W_t^\theta \equiv \max\{\hat{p}_t, V_{t+1}^\theta - c_\theta\}$ is his continuation value in the complementary event. The worker will decide to endogenously drop out when $\hat{p}_t > V_{t+1}^\theta - c_\theta$, stay in school when $\hat{p}_t < V_{t+1}^\theta - c_\theta$, and potentially randomize when $\hat{p}_t = V_{t+1}^\theta - c_\theta$.

2.1 Preliminary Analysis

In our model, pooling at no education is always an equilibrium. The reason is, as usual, that off the path of play firms may consider that the type of a deviator is L, so no type has incentives to receive education. However, since our goal is to study the dynamics of workers' dropout behavior, we mainly focus on equilibria in which wasteful signaling is present. To construct such equilibria, we start with providing some necessary conditions for such equilibria to exist. Lemma 1 characterizes the behavior of the worker before the L-worker drops out for sure (graduation).

Lemma 1. *In any equilibrium where $T^L > 0$, in all periods $t < T^L$,*

1. *there is positive voluntary dropout by the L-worker, that is, $\alpha_t^L > 0$, and*
2. *there is no voluntary dropout by the H-worker, that is, $\alpha_t^H = 0$.*

Proof. The proof is in the appendix on page 21. □

Since, by Lemma 1, when $T^L > 0$, L-workers randomize in any PBE in every period before T^L, for all periods $t < T^L$,

$$\hat{p}_{t+1} - \hat{p}_t = c_L, \tag{3}$$

so the low-type worker is always indifferent between dropping out and staying in school except (possibly) in his last possible period T^L. This fact implies that the wage must increase linearly before T^L. Notice that the constant returns to education are driven by the following assumptions in our model: First, workers do not discount the future. Second, the marginal cost of education is time-invariying, and last, there are two types of workers.[4] Without any of these assumptions, the returns to education will be time-varying. However, in each case, the (discounted) returns to education are still equal to the marginal education cost of the worker who (1) is still in school with positive probability, and (2) has the lowest productivity among workers in school.

Lemma 2. *In any equilibrium, $T^H \in \{T^L, T^L + 1\}$.*

Proof. The proof is in the appendix on page 21. □

Lemma 2 shows how (exogenous) dropout disciplines the beliefs of the firms about early dropouts. If, for example, at some period all L-workers have already dropped out but not all H-workers, all (exogenous and endogenous) dropouts are H-workers. Therefore, there is no reason for them to wait, because staying in school longer is costly and does not provide any wage increment.

Remark 1. The result is implied by the presence of a dropout risk so that dropping out in each period before T^H is on the path of play. When $\lambda = 0$, we are back to Cho and Kreps (1987) in which Lemma 2 does not necessarily hold. The reason is that, off the path of play, firms can impose a belief threat to force H-workers to stay in school after $T^L + 1$. Also, this result depends on the fact that signaling is unproductive. In Section 3, we study a productive signaling model in which Lemma 2 is not true any more.

[4] As we will show in Appendix C, when there are more than two types, the returns to education are concave.

2.2 Equilibrium Analysis

The equilibrium prediction of the game varies regarding the prior p_0, so we first focus on the game in which the worker has high productivity with a probability which is almost one, and then consider the case where p_0 is small. Lemma 3 describes the equilibrium set when the prior p_0 is close to 1.

Lemma 3. *Set*

$$p_0^1 \equiv \frac{1 - c_H}{1 - (1 - \lambda) c_H} \ . \tag{4}$$

Then, if $p_0 > p_0^1$, the only equilibrium outcome is pooling at no education.

Proof. The proof is in the appendix on page 22. □

The intuition behind Lemma 3 is as follows. Because of the presence of the dropout risk, workers may drop out in the first period on the path of play. Firms' beliefs on the dropout being the high type are pinned down by equation (1). When the prior p_0 is high, firms' posterior is high as well, and therefore, they offer the dropout a high wage. Hence, when the prior is close to 1, the H-worker would voluntarily drop out to take the high wage offer instead of staying in school.

Remark 2. The result in Lemma 3 and the economic intuition above relies on the presence of the dropout risk. In a model where $\lambda = 0$, wasteful signaling can be supported even when the prior about the type being high (p_0) is very close to 1. The reason is that there are equilibria in which there is no dropping out in the first period on the path of play. Since, off the path of play, a belief threat can be imposed, early dropouts are punished with low wages so no worker has the incentive to drop out. In our model, since the dropout risk is $\lambda > 0$, on the path of play, workers may drop out in any period before T^H. Hence, the belief about the dropout being the high type is pinned down by the equilibrium requirement, $\frac{\lambda p_0}{\lambda p_0 + 1 - p_0}$, which is arbitrary close to one when p_0 is large enough.

Remark 3. Lemma 3 implies that there is no signaling waste when $p_0 \to 1$ in any equilibrium. Consequently, the equilibrium education length converges to that in the symmetric information model as p_0 goes to 1. The continuity result also depends on the positive dropout risk. When the dropout risk is zero, we are back to Cho and Kreps (1987), a Riley-outcome-like equilibrium always exists for any $p_0 < 1$. In other environments, one can avoid this discontinuity result by (1) imposing a belief-based refinement (see Mailath, Okuno-Fujiwara and Postlewaite, 1993), (2) assuming education is productive (Swinkels, 1999), and (3) introducing extra type-dependent signals (Daley and Green, 2011, 2013).

Now we consider the model when p_0 is smaller than p_0^1. First, suppose p_0 is slightly smaller than p_0^1. In such a model, the no education pooling equilibrium trivially exists. However, there is another (semi-separating) equilibrium in which

1. one-period education is supported on the path of play, and

2. the L-worker randomizes between no education and one-period education.

The intuition is as follows. If the L-worker endogenously drops out in period 0 with some positive probability but the H-worker does not, the market belief about the worker who did not drop out in period 0 is strictly greater than p_0, and therefore the wage in period 1 is higher than that in period 0. To ensure that the L-worker is indifferent between dropping out in period 0 and period 1, the wage difference between the two periods must be exactly equal to the L-worker's marginal cost of education, which will pin down the equilibrium belief updating and the L-worker's strategy. In fact, there is another cutoff $p_0^2 < p_0^1$ such that, for any $p_0 \in (p_0^2, p_0^1]$, there is an equilibrium where the L-worker randomizes between receiving no education and one-period education. Since p_0 is still large, after one-period belief updating, p_1 becomes greater than p_0^1, so in the continuation game, the only equilibrium is that all workers drop out immediately. As a result, the maximum equilibrium education is one period.

When p_0 is slightly smaller than p_0^2, repeating a similar argument, we can construct equilibria with (1) pooling at no education, (2) one-period education, and (3) two-period education. By using an induction argument, we can construct a sequence of cutoff values p_0^k where $k = 1, 2, 3, ...$ and $p_0^k > p_0^{k+1}$ such that when $p_0 \in (p_0^{k+1}, p_0^k]$, there exist equilibria with T periods education where $T \leq k$. The following theorem formalizes the intuition above and characterizes possible education lengths in the set of all equilibria:

Theorem 1. *Let* $T^* \equiv \lceil \frac{1-c_H}{c_L} \rceil$.[5] *There exists a unique strictly decreasing sequence* $\{p_0^k\}_{k=1}^{T^*} \in (0,1)$ *such that for any* $p_0 \in (p_0^{k+1}, p_0^k]$

1. *there is a pooling equilibrium at no education;*

2. *for any* $0 < T \leq k$*, there is a semi-separating equilibrium lasting* T *periods, and*

3. *for any* $T > k$*, there is no equilibrium lasting* T *periods.*

Proof. The proof is in the appendix on page 22. ☐

[5] $\lceil x \rceil = \min\{n \in \mathbb{Z} | n \geq x\}$ denotes the smallest integer no lower than x.

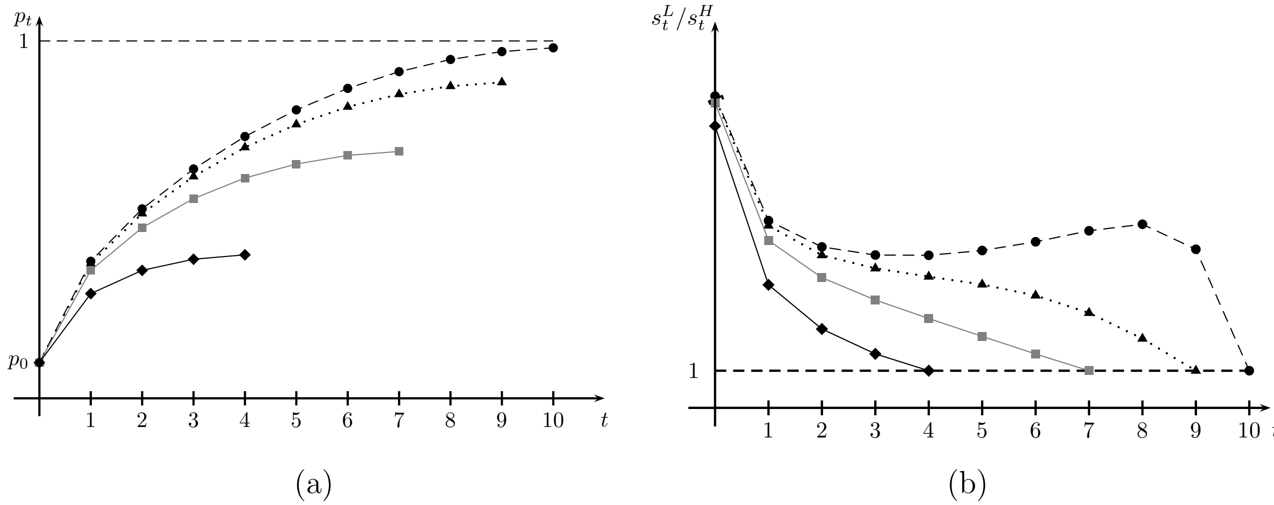

Figure 2: (a) p_t for different equilibria. (b) s_t^L/s_t^H for different equilibria. The dots with the same shape correspond to the same equilibrium, and they are linked with a straight line for visual clarity. The parameter values are $c_H = 0.032$, $c_L = 0.097$, $\lambda = 0.1$ and $p_0 = 0.1$.

Theorem 1 is one of the main results of this paper. It implies that the maximum duration of equilibrium education is non-increasing in the prior about the worker being high productivity. As the prior goes to zero, maximum education duration goes to its finite upper bound T^*.

In Lemma 3 we already discussed the case where p_0 is close to 1. Now, consider the case in which p_0 is not close to 1. As we have shown in Lemma 1, the low type endogenously drops out with positive probability and the high type does not voluntarily drop out; thus, $s_t^L > s_t^H$, which means that p_t is pushed up over time. The low-type indifference condition (3) implies that $\hat{p}_t$ is linear before T^L. These two observations imply that p_t and $\hat{p}_t$ will be high enough (close to 1) after finitely many periods. The smaller the prior p_0, the more periods of education can be supported in an equilibrium. This suggests that the maximum education duration supported by an equilibrium is non-increasing in p_0. In Figure 2, we plot some equilibrium belief sequences p_t and dropout rate ratio sequences s_t^L/s_t^H.[6] In each equilibrium, $T^L = T^H = T$ is the "graduation period." The high-type worker's dropout rate is $s_t^H = \lambda$ for all $t < T$ and $s_t^H = 1$ at $t = T$. The low-type worker's dropout rate satisfies $s_t^L \in (\lambda, 1)$ for $t < T$, and at $t = T$, $s_t^L = 1$. Note that it may not be monotone.

[6] We plot s_t^L/s_t^H in order to have a nice-looking graph. Indeed, for example, in all plotted equilibria $s_t^H = \lambda = 0.1$ for $t < T^H$ and $s_{T^H}^H = 1$, so s_t^H makes a big jump up at the end. Intuitively, s_t^θ for $t < T^H$ looks like a "flow probability" while $s_{T^H}^\theta$ looks like a "lump-sum probability" (See section 2.4 for the continuous time limit). Note also that, given p_t, only the ratio s_t^L/s_t^H determines $\hat{p}_t$.

Empirical Implications. Our model has two important empirical implications. First, fixing an equilibrium, we can calculate the education return sequence: $w_{t+1} - w_t = c_L$. It is equal to the L-worker's marginal cost of education. Second, the model can predict workers' equilibrium dropout behavior. Since workers' types are their private information, researchers can only observe the unconditional (or observed) dropout probablity, defined as follows

$$m_t \equiv p_t s_t^H + (1 - p_t) s_t^L \ .$$

In equilibrium, $s_t^H = \lambda$ for $t < T^L$, so $m_t \equiv p_t \lambda + (1 - p_t) s_t^L$. The dynamics of m_t are driven by two forces. First, the belief increases over time, which pushes the observed dropout rate down. Second, L-workers' drop out strategy varies over time. As we show in figure 2(b), s_t^L may not be monotone over time. When s_t^L increases, it pushes m_t up. Hence, the observed dropout rate may go up and down over time, which depends on the interaction between two forces. Unfortunately, in a discrete time model, we cannot analytically characterize the dynamics of workers' dropout rate. In Section 2.4, we analytically characterize the observed dropout dynamics at the continuous time limit of the original model.

2.3 Refinement

Without imposing any refinement, multiple equilibria exist for most p_0. The main reason we do not have equilibrium uniqueness is the arbitrariness of belief after T^H off the path of play, similar to that in Cho and Kreps (1987). Hence, we still have belief threats that push duration down.

By imposing an appropriate criterion on beliefs off the path of play, for example, D1 as defined by Banks and Sobel (1987), one can shrink the equilibrium set.[7] The spirit of these refinements is that, off the path of play, firms put a positive probability only on the type that is most likely to deviate. In our model, since the marginal cost of education of the high-type worker is strictly smaller than that of the low-type worker, any sequence of wages off the path of play (after T^H) that induces the low-type worker to deviate must also induce the high-type worker to deviate. As a result, off the path of play, firms put a positive belief only on the high-type worker, i.e., $p_t = \hat{p}_t = 1$ for any $t > T^H$. Given this belief sequence off the path of play, we will say a PBE is eliminated by D1 if $\hat{p}_T < 1 - c_H$, since otherwise the high-type worker would have incentives to stay in school for one more period. If an equilibrium is not eliminated by D1, we say that it

[7]Belief monotonicity is another refinement concept commonly used in the dynamic signaling literature (see Swinkels, 1999 or Daley and Green, 2012). However, it does not help here. The reason is that, given that λ is the same for all players, the increase in beliefs after a deviation can be arbitrarily slow, which prevents the H-worker from deviating.

passes D1. These concepts are not enough to select a unique equilibrium, similar to Nöldeke and van Damme (1990). The key reason for the multiplicity is that, in our model, the education choice is an integer instead of a real number. Consider the following case as an example.

Example 1. *Suppose $p_0 \in (1-c_H, p_1^0)$. It is easy to show that there is a PBE in which $s_0^H = \lambda$ and $s_0^L = 1$. Since, in this equilibrium, $p_1 = \hat{p}_1 = 1$ is on the path of play, it passes D1. However, there is another PBE consisting of pooling at no education, that is, $s_0^H = s_0^L = 1$, so $p_0 = \hat{p}_0 > 1 - c_H$. Hence, pooling at no education also passes D1.*

Nevertheless, as shown below, when the length of the interval is small, the D1 criterion is essentially unique, in the sense that the outcomes of all equilibria passing D1 become arbitrarily close to each other.

2.4 Frequent Dropout Decision

In this section we consider the limit where the length of the interval is arbitrarily small. This is in general interpreted as leaving the worker with no commitment power, since the worker has to get an education through making sequential, arbitrarily small investments. This limit allows us to establish the unique equilibrium passing D1 and have a cleaner equilibrium characterization. In particular, it allows us to easily characterize the relationship between the observed dropout rate and years of education, which is an important empirical implication of our model.

In the base model discussed in the previous section, we assume that the time length between two consecutive periods is 1. We now parameterize the length of the period as $\Delta > 0$. Since we will consider a family of models parameterized by Δ, we fix $\tilde{c}_L, \tilde{c}_H, \tilde{\lambda} \in \mathbb{R}_{++}$ in all this section and, for each Δ, $c_\theta(\Delta) \equiv \tilde{c}_\theta\Delta$ and $\lambda(\Delta) \equiv \tilde{\lambda}\Delta$, for $\theta \in \{L, H\}$.[8] The following lemma establishes that the maximum length of an equilibrium is a non-trivial function of p_0 when Δ gets small:

Lemma 4. *Consider any strictly decreasing sequence $\Delta_n \to 0$. Fix a $p_0 \in (0,1)$, and let $\kappa_n(p_0) \equiv \Delta_n T_n^*$ (T^* defined in Theorem 1) be the maximum real-time length of an equilibrium when the length of the period is Δ_n. Then, $\kappa(p_0) \equiv \lim_{n\to\infty} \kappa_n(p_0)$ exists, belongs to $(0, \frac{1}{\tilde{c}_L})$ and is strictly decreasing in p_0.*

Proof. The proof is in the appendix on page 27. ☐

Lemma 4 stands in sharp contrast to Swinkels (1999), in which the only equilibrium is pooling at no education when Δ is small enough. The contrast between this result and Swinkels' illustrates

[8]This limit corresponds to interpreting $\tilde{c}_\theta$ to be the flow cost for each $\theta \in \{L, H\}$, and interpreting $\tilde{\lambda}$ as the rate at which workers are exogenously forced to drop out.

the critical role of timing in the two models. In Swinkels(1999), firms can make preemptive offers to attract all the workers in school and end the game immediately, so that no wasteful education is present in equlibrium. In our model, firms cannot directly disturb the worker's signaling process by making an in-school offer, and therefore, semi-separating equilibria can survive.

As noted before, D1 selects PBE where $p_T \in [1 - \Delta\tilde{c}_H, 1]$. As Δ goes to zero, the last period equilibrium belief converges to 1. The following lemma establishes that, when Δ is small, only in equilibria with a real education length close to $\kappa(p_0)$ (the maximum length at p_0) is the last period equilibrium belief close to 1.

Lemma 5. *Consider a strictly decreasing sequence $\Delta_n \to 0$. Let $e(\Delta_n)$ be an equilibrium of the model with the length of the period Δ_n. Fix $p_0 \in (0,1)$ and any $\tau \in (0, \kappa(p_0))$. Let $p_{T_n}(\Delta_n)$ be the maximum last period beliefs of an equilibrium with $T_n \equiv \lceil \tau/\Delta_n \rceil$ periods of education for Δ_n. Then, $\lim_{n\to\infty} p_{T_n}(\Delta_n)$ exists and is strictly lower than 1. If, instead, $\tau = \kappa(p_0)$, then $\lim_{n\to\infty} p_{T_n}(\Delta_n) = 1$.*

Proof. The proof is in the appendix on page 28. □

Lemma 5 implies that equilibria passing D1 have a real duration of $\kappa(p_0) + O(\Delta_n)$. Indeed, otherwise the last period's beliefs are bounded away from 1 and hence are lower than $1 - \Delta\tilde{c}_H$. In the proof of Theorem 1 (see Lemma 11) we explicitly construct for each p_0 equilibria in which the last period's beliefs belong to $[1 - \Delta\tilde{c}_H, 1]$. So, for each p_0 and small $\Delta > 0$, there are equilibria passing D1, and their duration is close to $\kappa(p_0)$.

As we will show in Theorem 2, taking the continuous time limit of the original model allows us not only to characterize the equilibrium in a cleaner way but also to derive the relationship between the observed dropout rate and the number of years students spend in school. Since, when Δ_n is small, both s_t^H and s_t^L are $O(\Delta_n)$ (in all periods except maybe the last two), m_t is also $O(\Delta_n)$ (in all periods except maybe the last two). To study the dynamics of workers' dropout behavior at the continuous time limit, we define the *observed dropout rate* as follows. Given a sequence Δ_n and $m(t)$, define the associated observed dropout rate as $\tilde{m}_t^n \equiv \frac{m_t}{\Delta_n}$, and $\tilde{s}_t^{L,n} \equiv \frac{s_t^L}{\Delta_n}$. In the following theorem, we characterize the continuous time limit of equilibrium belief and the observed dropout rate.

Theorem 2. 1. *Consider a strictly decreasing sequence $\Delta_n \to 0$. For each Δ_n, let p_t^n, $\hat{p}_t^n$ and m_t^n be the beliefs, wage and total dropout sequences of an equilibrium passing D1 in the model with the length of the period Δ_n. Then, there exist functions $p, \hat{p}, \tilde{m} : (0, \kappa(p_0))$ such that $\lim_{n\to\infty} p^n_{\lceil \tau/\Delta_n \rceil} = p(\tau)$, $\lim_{n\to\infty} \hat{p}^n_{\lceil \tau/\Delta_n \rceil} = \hat{p}(\tau)$ and $\lim_{n\to\infty} \tilde{m}^n_{\lceil \tau/\Delta_n \rceil} = \tilde{m}(\tau)$ for all $\tau \in (0, \kappa(p_0))$.*

2. $\hat{p}(\tau) = 1 - (\kappa(p_0) - \tau)\tilde{c}_L$, $\tilde{m}(\tau) = \frac{p(\tau)}{\hat{p}(\tau)}\tilde{\lambda}$ *and* p *solves the following equation, with* $p(0) = p_0$*:*

$$\dot{p}(\tau) = \frac{\tilde{\lambda}p(\tau)(p(\tau) - \hat{p}(\tau))}{\hat{p}(\tau)} \ . \tag{5}$$

Proof. The proof is in the appendix on page 29. □

At the continuous time limit, we can easily characterize the dynamics of the observed dropout rate. Interestingly, even though the dropout rate of the L-workers need not be monotone (recall, for example, Figure 2), the *observed* dropout rate is decreasing, which is consistent with much of the empirical evidence, for example, Hendricks and Leukhina (2013).

Theorem 3. *At the continuous time limit of all equilibria, the observed dropout rate is decreasing over time.*

Proof. The proof is in the appendix on page 29. □

Since $\dot{\tilde{m}}(t) = -\dot{p}(t)(\tilde{s}^L(t) - \tilde{\lambda}) + (1 - p(t))\dot{\tilde{s}}^L(t)$, as time goes on, there are two effects on the observed dropout rate. First, there is a skimming effect: the proportion of L-workers becomes smaller over time, since the L-workers' dropout rate is higher than $\tilde{\lambda}$, the dropout rate of the H-worker's. Hence, this skimming effect, measured by $-\dot{p}(t)(\tilde{s}^L(t) - \tilde{\lambda}) < 0$, pushes the observed dropout rate down. Second, there is another effect: for L-workers still in school, their dropout rate s_t^L may go either up or down, which is measured by $(1 - p(t))\dot{\tilde{s}}^L(t)$. When it goes up, it pushes the observed dropout rate up as well. However, we can show that the second effect is always dominated by the first one, and thus the observed dropout rate is always declining over time.

2.5 Discussions

2.5.1 The Role of Dropout Risk

The equilibrium characterization crucially depends on the presence of dropout risk. What happens if the dropout risk is arbitrarily small? What if workers' dropout risk is type-dependent? We address these issues here.

First, we consider the limit case where λ goes to zero in the discrete time model. Figure 3 (a) plots $\{p_0^k\}_{k=1}^{T^*}$ for different values of λ. As we see, when $\lambda \to 0$, p_0^k for all k collapses to 1. This implies that, when λ is low, for almost all priors (1) H-workers' dropout risk is very small, so most of them graduate, (2) most of L-workers do not go to school, so p_1 is close to 1, but $\hat{p}_1$ remains low due to the low λ, and (3) the maximum length of an equilibrium is T^*. This is consistent with the canonical signaling model, where $\lambda = 0$. In the other limit, when $\lambda \to 1$, $p_0^k - p_0^{k+1} = c_L$ for all

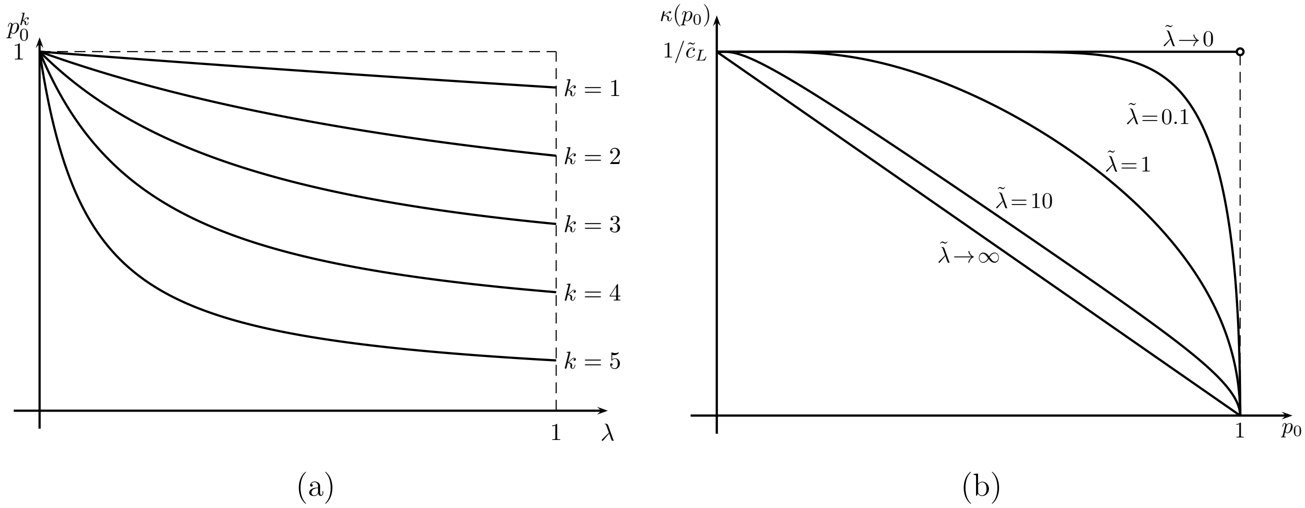

Figure 3: (a) The role of λ in the discrete time model: $\{p_0^k\}$ as a function of λ. (b) The role of λ in the continuous time limit: $\kappa(p_0)$ as a function of p_0, for different values of $\tilde{\lambda}$.

$k > 1$. This is a consequence of the fact that when λ is close to 1, so are s^L and s^H. Therefore, as we see in (1), $\hat{p}_t$ is close to p_t for all t. Since $\hat{p}_t$ increases linearly in any equilibrium, this imposes a nearly linear evolution on p_t and therefore also on p_0^k .

Second, we consider the continuous time limit when the dropout rate is small, that is, when $\tilde{\lambda}$ is small. From the equation that $\kappa(p_0)$ satisfies (equation (9) in the proof of Lemma 4), it is easy to see that $\lim_{\tilde{\lambda}\to 0}\kappa(p_0) = \frac{1}{\tilde{c}_L}$ for all $p_0 \in (0,1)$. Indeed, as we see in Figure 3 (b), as $\tilde{\lambda}$ gets small, $\tau(p_0)$ converges to $\frac{1}{\tilde{c}_L}$ for all $p_0 \in (0,1)$. Hence, the length of an equilibrium passing D1 gets close to $\frac{1}{\tilde{c}_L}$ when the interval gets short and $\tilde{\lambda}$ gets small. This is consistent with the finding of Cho and Kreps (1987) that the only equilibrium that passes D1 is the least costly separating equilibrium, found by Riley (1979), that requires an education length equal to $\frac{1}{\tilde{c}_L}$.

Finally, one may wonder whether it is restrictive to assume that H-workers and L-workers face the same exogenous dropout risk. Without a second thought, it seems that low-productivity workers should have a higher probability of dropping out than high-productivity workers, which seems to conflict with our assumption. However, this naive intuition is based on the total dropping-out behavior, s_t^L, which is driven both by workers' choices (that are related to their productivity) and by exogenous shocks (that may not be related to their productivity). As we have shown, on the equilibrium path, $s_t^L \geq s_t^H$ in each period. Yet, it is still useful to know whether our equilibrium characterization is robust by relaxing this homogeneous dropout risk assumption. In Appendix B, we consider perturbations of the baseline model by considering heterogeneous dropout risk. The equilibrium characterization is robust to such perturbation.

2.5.2 Preemptive Offers

In our model, we assume that firms cannot make preemptive offers. In a dynamic signaling model in which firms make preemptive offers and students face no exogenous dropout risk, Swinkels (1999) shows that the only equilibrium is pooling at no education when the time interval of a period is small enough. A natural question is what happens if firms can make preemptive offers in the presence of exogenous dropout. In our model, if preemptive offers are allowed, all non-pooling equilibria are destroyed as in Swinkels (1999). The idea is that, from the penultimate period to the last period, belief updating is slow. So firms can post an offer to attract both H-workers and L-workers and obtain non-negative profit. In the following, we show that, when firms can privately and frequently make preemptive offers, all of the semi-separating equilibria we constructed do not exist.

We illustrate the idea at the continuous time limit. Suppose there is a semi-separating equilibrium. In equilibrium, we have $p_T = \hat{p}_T$ where T is the graduation time. When Δ is small, the H-worker's value $V^H(p_{T-\Delta})$ can be approximated by

$$-\tilde{c}_H\Delta + \lambda\Delta\hat{p}_{T-\Delta} + (1-\lambda\Delta)V^H(p_T) \ .$$

Since $\hat{p}_{T-\Delta} = p_T - \tilde{c}_L\Delta$ and $V^H(p_T) = p_T$, we have

$$\dot{V}^H(p_T) = \lim_{\Delta\to 0} \frac{V^H(p_T) - V^H(p_T - \Delta)}{\Delta} = \tilde{c}_H \ .$$

By Theorem 2, $\dot{p}(t) \to 0$ as $t \to T$. Hence, there exists an $\epsilon > 0$ such that, for $t \in (T-\epsilon, T)$,

$$V^H(p_t) < p(t).$$

Notice that, in such a time instant, a firm can deviate by making an offer $w_t \in (V^H(p_t), p(t))$ such that

1. the offer will be accepted by both H-workers and L-workers, and

2. making such an offer is profitable.

Consequently, when firms can privately make preemptive offers, no semi-separating equilibrium exists.

3 Productive Education

In the baseline model, we assume that education serves as a pure information extraction mechanism and does not affect the worker's productivity. This is clearly a theoretical simplification. In reality, going to school is not only useful as a signaling device (where workers signal some innate abilities, for example) but also it enhances workers' productivity (human capital accumulation). Hence, the observed positive education-wage profile comes from both human capital accumulation and the signaling motive of dropping out.

In such a situation, the returns to education incorporate both the signaling effect and the human capital accumulation effect. In this section, we extend the baseline model by assuming that getting an education can enhance the worker's productivity. In our model, by using the data on wages and the dropout rate, one can decompose the returns to education into the human capital accumulation effect and the signaling effect.[9]

We assume that a θ-worker with t periods of education has productivity equaling $Y_\theta = a_\theta + h(t)$, where $\theta = \{H, L\}$, a_θ captures the intrinsic productivity, and $h(t)$ captures productivity accumulated through education. Again, we assume that a worker's intrinsic productivity is a_H with probability p_0 and $a_L$with probability $1 - p_0$, and we normalize $a_L = 0$ and $a_H = 1$. To illustrate the main idea, we focus on the simplest specification of the human capital accumulation function. We assume that there is some finite number $\hat{T} \in \mathbb{N}$ such that

$$h(t) = \begin{cases} ht & \text{if } t < \hat{T}, \\ h\hat{T} & \text{otherwise,} \end{cases}$$

where $h \in (c_H, c_L)$ is the marginal human capital accumulation coefficient until $\hat{T}$.[10] Notice that production for human capital is concave. Also, the socially efficient outcome is that the L-worker gets no education and the H-worker gets $\hat{T}$ periods of education. When $\hat{T} = 0$, getting an education does not enhance any worker's productivity at all, which is the case in our baseline model. In this section, we focus on the case where $\hat{T} > 0$. We show that the equilibria is similar to those in the benchmark model.

Given an equilibrium, in any period $t < \hat{T}$, the wage for dropouts is given by their expected productivity, that is

$$w_t = \hat{p}_t(1 + ht) + (1 - \hat{p}_t)\, ht = \hat{p}_t + ht,$$

so the return to education is given by

$$w_{t+1} - w_t = \underbrace{\hat{p}_{t+1} - \hat{p}_t}_{\text{(signaling)}} + \underbrace{h}_{\text{(human capital)}}$$

[9]There is a large literature empirically studying how to distinguish human capital accumulation theory and signaling theory, for example, Tyler, Murnane, and Willett (2000), Bedard (2001), Frazis (2002), etc. Fang (2006) estimates a static education choice model with both human capital accumulation and a signaling mechanism and claims that the signaling effect is at most about one-third of the actual college wage premium.

[10]More generally, most of our results would also apply if the return was time-varying with $h_t \in (c_H, c_L)$ for $t < \hat{T}$ and $h_t < c_H$ for $t \geq \hat{T}$. The equilibrium analysis is similar.

where $\hat{p}_{t+1} - \hat{p}_t$ is the contribution of the signaling effect and h is the contribution of the human capital accumulation effect.

Here Lemma 1 still holds; that is, in any equilibrium the L-worker randomizes between dropping out and continuing in school in the first several periods. Hence, we must have $w_{t+1} - w_t = c_L$, which implies that

$$\hat{p}_{t+1} - \hat{p}_t = c_L - h > 0.$$

We now focus on equilibria passing D1. First, we consider the case where $\hat{T}$ is large.

Theorem 4. *Fix a p_0, there is a $T^L(p_0) \in \mathbb{N}$ such that, when $\hat{T} > T^L(p_0)$, there is a unique equilibrium passing D1. In the equilibrium,*

1. *L-workers drop out in each period with probability s_t^L where $s_t^L \in (\lambda, 1)$ for $t < T^L(p_0)$ and $s_{T^L(p_0)}^L = 1$, and*
2. *H-workers do not voluntarily drop out before $\hat{T}$ and dropout for sure in period $\hat{T}$.*

Proof. The proof is in the appendix on page 30. □

In contrast to the baseline model, when $\hat{T}$ is large enough, there is a unique equilibrium passing D1. The intuition for this result is as follows. Since education until $\hat{T}$ is efficient for H-workers, they prefer to stay in school until $\hat{T}$. L-workers, instead, keep dropping out until some T^L. The dropout rate needs to be high enough to imply increases in $\hat{p}_t$ equal to $c_L - h$. Therefore, $T^H > T^L$. This uniquely pins down the dropout rate of the L-workers, so a unique equilibrium exists. In our baseline model, instead, we have different possible behaviors in the last two periods, so in general equilibria passing D1 with $T^L = T^H$ or $T^L = T^H - 1$ may exist.[11]

Theorem 4 implies that when $\hat{T}$ is large enough, in the unique equilibrium passing D1, there are two phases. In the first phase, the return to education is c_L, and both the signaling effect and the human capital accumulation effect contribute to it. The observed dropout rate $m(t)$ varies over time. In the second phase, the return to education is h, which purely comes from the human capital accumulation effect. The observed dropout rate is constant.

Remark 4. The specification allows us to distinguish the effect of human capital accumulation and dynamic signaling on return to education. First, from period T^L to $\hat{T}$, only the H-worker is in school, so the return to education is h and the observed dropout rate is λ. From period 0 to T^L, the return to education is c_L, which is different from h. As a result, one can directly estimate $\hat{T}, h, c_L$ and λ from the data on wages and the dropout rate. Second, after recovering the

[11]Here, a weaker refinement, belief monotonicity can select the unique equilibrium in our productive-signaling model as well.

parameters c_L and h one can also calculate the contribution of the signaling effect on the return to education, which is $(c_L - h)/c_L$ for $t < T^L$ and zero for $t \geq T^L$.[12]

Similar to the baseline model, we can examine the continuous time limit of the model and obtain a clean characterization of the equilibrium education returns and the dropout rate dynamics. For the limit, we use $h(\Delta_n) \equiv \Delta_n \tilde{h}$ and $\hat{T} = \hat{\tau}/\Delta_n$, with a constant $\tilde{h}$ and $\hat{\tau}$.

Corollary 1. *Consider a strictly decreasing sequence $\Delta_n \to 0$. For each Δ_n, let p_t^n, w_t^n and m_t^n be the beliefs, wage and total dropout sequences of equilibria passing D1 in the model with the length of the period Δ_n. Then, there exist some $\tau^L(p_0) > \tau$ and functions $p(\tau), \hat{p}(\tau), \tilde{m}(\tau)$ such that $\lim_{n\to\infty} p^n_{\lceil \tau/\Delta_n \rceil} = p(\tau)$, $\lim_{n\to\infty} \hat{p}^n_{\lceil \tau/\Delta_n \rceil} = \hat{p}(\tau)$, $\lim_{n\to\infty} \tilde{m}^n_{\lceil \tau/\Delta_n \rceil} = \tilde{m}(\tau)$ for all $\tau \in (0, \hat{\tau})$ and*

1. *when $t < \tau^L(p_0)$, $\dot{w}(t) = \tilde{c}_L$ and $\dot{\tilde{m}}(t) < 0$, and*
2. *when $t \in (\tau^L(p_0), \hat{\tau})$, $\dot{w}(t) = \tilde{h}$ and $m(t) = \tilde{\lambda}$.*

The proof is similar to that of the baseline model, so it is omitted here. Notice that the return to education is $\tilde{c}_L$ before $\hat{\tau}$ and becomes $\tilde{h}$ after $\hat{\tau}$, and the observed dropout rate is initially declining over time and then becomes constant after $\hat{\tau}$.

When $\hat{T}$ is small, there exists equilibria in which the game ends later then $\hat{T}$. In such equilibria, staying in school more than $\hat{T}$ period is socially inefficient and workers do so purely for signaling reasons, so only the signaling effect contributes to the return to education. In this case, similar to the baseline model, there are multiple equilibria passing D1. However, the return to education is c_L for every period, which is observably different from the case in which $\hat{T}$ is large. Since the equilibrium construction and characterization are similar, we only provide the characterization at the continuous time limit.

Corollary 2. *Consider a strictly decreasing sequence $\Delta_n \to 0$. For each Δ_n, let p_t^n, w_t^n and m_t^n be the beliefs, wage and total dropout sequences of equilibria passing D1 in the model with the length of the period Δ_n. Then if $\hat{\tau} < \tau^L(p_0)$ (defined in Corollary 1) there exist functions $p(\tau), \hat{p}(\tau), \tilde{m}(\tau)$ such that $\lim_{n\to\infty} p^n_{\lceil \tau/\Delta_n \rceil} = p(\tau)$, $\lim_{n\to\infty} \hat{p}^n_{\lceil \tau/\Delta_n \rceil} = \hat{p}(\tau)$, $\lim_{n\to\infty} \tilde{m}^n_{\lceil \tau/\Delta_n \rceil} = \tilde{m}(\tau)$ for all $\tau \in (0, \tau^L(p_0))$ and*

1. *for all $t < \tau^L(p_0)$, $\dot{w}(t) = \tilde{c}_L$ and $\dot{\tilde{m}}(t) < 0$, and*

[12] The simple identification strategy works because of the special specification of the human capital accumulation technology: (1) there are two parameters in the $h(t)$ functions, and (2) human capital accumulation does not depend on the worker's demographic characteristics. In general, one can allow more complicated technology by considering the general function forms of $h(t)$ and other realistic factors, for example, workers' races, IQ test scores, etc. However, fully exploring this issue and structurally estimating the model are beyond the scope of this paper.

2. at $t = \hat{\tau}$, all p, $\hat{p}$, and $\tilde{m}$ are continuous but non-differentiable.

The results in this section highlight the role of the observed dropout rate when estimating the social returns to education. Only when no type is willing to voluntarily drop out (so dropping out has no signaling value) will wages be determined by the increase in productivity due to education. In this case, the observed drop out rate does not vary over time, and the individual return to education is equal to the social return to education. If, instead, (some) workers voluntarily drop out, wages are determined by the education costs of this type. In this case, the observed dropout rate varies over time, and the individual return to education is greater than the social return to education.

4 Concluding Remarks

This paper presents a tractable dynamic-signaling model in which wasteful education takes place over several periods of time. Workers face an exogenous dropout risk and pay an education cost per unit of time. We make three contributions to the literature. We find that exogenous dropout induces endogenous dropout in most of the education periods. This disciplines the market's beliefs about dropouts and rationalizes the large observed dropout rates. In particular, we find that the maximum length of education is decreasing in the prior about the worker being productive. Second, we provide neat empirical implications for the return to education and the dropout rate. The equilibrium education return is equal to the low-type worker's marginal cost of education. At the continuous time limit of the original discrete time model, we derive the relationship between the observed dropout rate and the workers' grade. Third, we also extend the baseline model by allowing productive education. By doing so, we highlight the role of the data on the dropout rate in decomposing the individual return to education into the signaling effect and the human capital accumulation effect (the social return to education).

A Appendix: Omitted Proofs

A.1 The Proof of Lemma 1

Let's first prove a preliminary result:

Lemma 6. *(The L-worker does not beat the market) For all PBE and t,* $V_t^L \leq p_t$.

Proof of Lemma 6. Fix a PBE. Let τ be the time at which the game ends. Then,

$$p_t V_t^H + (1 - p_t) V_t^L \leq \mathbb{E}_t[w_\tau | \tau \geq t].$$

Note that, due to the education costs (i.e. signaling waste), there is a (weak) inequality, and it is strict if $t < T^L$. Also,

$$\mathbb{E}_t[w_\tau | \tau \geq t] = \sum_{\tau=t}^{\infty} \Pr(\tau, t) \hat{p}_\tau = \sum_{\tau=t}^{\infty} \Pr(\tau, t) p_t \frac{\Pr^H(\tau, t)}{\Pr(\tau, t)} = p_t \sum_{\tau=t}^{\infty} \Pr^H(\tau, t) = p_t \ .$$

where $\Pr(\tau, t)$ denotes the conditional probability in period t that the game ends in period τ, and $\Pr^H(\tau, t) = s_\tau^H \prod_{t'=t}^{\tau-1} (1 - s_{t'}^H)$ is further conditioning on the dropout being type H. The last equality holds because the high type has strictly positive dropout rate and therefore he drops out in finite time with probability one. Since $V_t^H \geq V_t^L$ (the H-worker can mimic the L-worker at a cheaper price) the result holds. □

Suppose there is no endogenous dropout by the L-worker in period t, then $p_{t+1} \leq p_t \leq \hat{p}_t$. But, $\hat{p}_t \leq W_t^L = V_{t+1}^L - c_L$ due to the fact that the L-worker does not voluntarily drop out. By Lemma 6, $V_{t+1}^L \leq p_{t+1} \leq \hat{p}_t$; thus $\hat{p}_t \leq \hat{p}_t - c_L$, which is a clear contradiction. So (1) is true. Therefore (2) is also true, since $W_t^H \geq V_{t+1}^H - c_H \geq \hat{p}_{t+1} - c_H$ by definition of W_t^H and V_t^H, and $\hat{p}_{t+1} - c_H = \hat{p}_t + c_L - c_H > \hat{p}_t$ by the indifference condition of the L-worker. *Q.E.D.*

A.2 The Proof of Lemma 2

Assume first $T^H > T^L + 1$. In this case, $p_{T^L+1} = 1$. Using equation (1) we know $\hat{p}_{T^L+1} = 1$. Since the payoff of the worker is bounded by 1, and waiting until next period is costly, the worker is better off dropping out at $T^L + 1$. This is a contradiction.

Lemma 1 implies that $S_{T^L}^H > 0$, and therefore $T^H \geq T^L$. *Q.E.D.*

A.3 The Proof of Lemma 3

The wage in period $t = 1$ is bounded above by 1. This implies that for the H-worker to be (weakly) willing to get one period of education, it must be the case that $w_0 \leq 1 - c_H$. This implies that

$$1 - c_H \geq \hat{p}_0 = \frac{p_0 s_0^H}{p_0 s_0^H + (1 - p_0) s_0^L} \geq \frac{p_0 \lambda}{p_0 \lambda + 1 - p_0} \ .$$

Solving for p_0 under the equality, we get that the threshold for the existence of an equilibrium with non zero education satisfies equation (4). *Q.E.D.*

A.4 The Proof of Theorem 1

The proof of Theorem 1 is divided into several steps. To make the proof clear to the reader, we note that we will be following this **road map**:

1. We begin defining and proving some properties of the "pull-back functions," which will be used to construct equilibria in the rest of the proof (lemmas 7 and 8).
2. In subsection A.4.1 we define some putative values for p_0^k, denoted $\tilde{p}_0^k$, and we prove by induction that, if $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, then there is no equilibrium with more than k periods of education.
3. Then, in subsection A.4.2 we show that, if $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, there exists an equilibrium where the L-worker is indifferent on dropping out or not for all periods except (maybe) the last for all $T \in \{0, ..., k-1\}$.
4. Finally, in subsection A.4.3 we show that, if $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, there exists an equilibrium with length k. Therefore, $p_0^k = \tilde{p}_0^k$.

We begin this proof by stating and proving two results that will simplify the rest of the proof and the proofs of other results in our paper. The first one states two properties of the "**pull-back functions**" $\mathcal{S}_\tau(\cdot, \cdot)$ and $\mathcal{M}_\tau(\cdot)$:

Lemma 7. *For any $\tau \in \mathbb{N}$, let $\mathcal{S}_\tau : [0,1]^2 \to [0,1]$ and $\mathcal{M}_\tau : [0,1] \to \mathbb{R}$ be the functions defined by*

$$\mathcal{S}_\tau(p, \hat{p}) \equiv \frac{\mathcal{S}_{\tau-1}(p, \hat{p}) \mathcal{M}_\tau(\hat{p})}{\mathcal{M}_\tau(\hat{p})(1 - \lambda) + \mathcal{S}_{\tau-1}(p, \hat{p})\lambda} \ , \tag{6}$$

$$\mathcal{M}_\tau(\hat{p}) \equiv \hat{p} - \tau c_L \ , \tag{7}$$

with $\mathcal{S}_0(p, \hat{p}) \equiv p$ and $\mathcal{M}_0(\hat{p}) = \hat{p}$. Then, if $\hat{p} > \tau c_L$, $\mathcal{S}_\tau(p, \hat{p})$ is continuous and strictly increasing in both arguments.

Proof of Lemma 7. It is obvious when $\tau = 1$, and it holds when $\tau > 1$ by induction argument. $\square$

The meaning of the pull-back functions is the following. Fix an equilibrium and some $t > 0$ where the L-worker is still present. Then, using equation (1), (2) and the indifference condition $\hat{p}_t = \hat{p}_{t-1} + c_L$, we can obtain p_{t-1} and $\hat{p}_{t-1}$ from p_t and p_{t-1}. These take the form, respectively, of $\mathcal{S}_\tau(p_t, \hat{p}_t)$ and $\mathcal{M}_\tau(\hat{p}_t)$. If we apply this iteratively, we can find $p_{t-\tau} = \mathcal{S}_\tau(p_t, \hat{p}_t)$ and $\hat{p}_{t-\tau} = \mathcal{M}_\tau(\hat{p}_t)$ for any $\tau \in \{1, ..., t\}$. So, since by Lemma 1 the L-worker is indifferent between dropping out or not in all periods except the last period, the pull-back functions give us the values of the belief sequences p and $\hat{p}$ for all periods prior to a given period. The following lemma formalizes this intuition:

Lemma 8. *For any equilibrium with $T > 1$ periods of education and any $T > \tau \geq \tau' \geq 0$ we have*

$$p_{\tau'} = \mathcal{S}_{\tau-\tau'}(p_\tau, \hat{p}_\tau) \quad \textit{and} \quad \hat{p}_{\tau'} = \mathcal{M}_{\tau-\tau'}(\hat{p}_\tau) \ .$$

Proof of Lemma 8. Note that, by Lemma 1, in all periods $t < T - 1$, the L-worker is indifferent between dropping out or not and $s_t^H = \lambda$. This implies that if $t < T - 1$, $\hat{p}_{t-1} = \hat{p}_t - c_L$. We can use equations (1) and (2), with $s_t^H = \lambda$, to express the posterior at time t in terms of the posterior of workers in education and in the market at time $t + 1$:

$$p_t = \frac{p_{t+1}\hat{p}_t}{\hat{p}_t(1-\lambda) + p_{t+1}\lambda} = \frac{p_{t+1}(\hat{p}_{t+1} - c_L)}{(\hat{p}_{t+1} - c_L)(1-\lambda) + p_{t+1}\lambda} = \mathcal{S}_1(p_{t+1}, \hat{p}_{t+1}) \ .$$

Using this formula recursively and the fact that $\mathcal{S}_\tau(p, \hat{p}) = \mathcal{S}_{\tau-1}(\mathcal{S}_1(p, \hat{p}), \mathcal{M}_1(\hat{p}))$ we obtain the desired result. $\square$

A.4.1 Constructing the Upper Bound on the Length

Define the sequence $\tilde{p}_0^k \equiv \mathcal{S}_{k-1}(p_0^1, 1 - c_H)$, where p_0^1 is defined in (4). Our goal is to show that $\tilde{p}_0^k$ has the same properties that p_0^k (stated in the statement of the theorem), so $p_0^k = \tilde{p}_0^k$. We are going to prove first, by induction, that if $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, then there is no equilibrium with more than k periods of education:

Step 1 (induction hypothesis): If $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, there is no equilibrium with more than k periods of education. If an equilibrium has k periods of education, then $\hat{p}_0 \leq \hat{p}_0^k \equiv \mathcal{M}_{k-1}(1 - c_H)$.[13]

[13]The second induction hypothesis is included in order to make the argument simple in the induction argument (step 4).

Step 2 (proof for $k = 0$ periods of education): By Lemma 3 there is no equilibrium with education for $p_0 > p_0^1$. Also, in the same proof, it is shown that for all equilibria in this region, $\hat{p}_0 = p_0 \geq p_0^1 > 1 - c_H = \mathcal{M}_0(1 - c_H)$.

Step 3 (proof for $k = 1$ period of education): Assume that p_0 is such that there is an equilibrium with 1 period of education. Then, $\hat{p}_0 \leq \hat{p}_0^1 = 1 - c_H$ (at least the H-worker has to be willing to wait). Using Bayes' update (equations (1) and (2)) we can express $\hat{p}_0 \equiv \hat{p}_0(p_0, s_0^L, s_0^H)$ and $p_1 = p_1(p_0, s_0^L, s_0^H)$. Therefore, using these equations, we can write p_0 in terms of $\hat{p}_0$, p_1 and s_0^H in the following way:

$$p_0 = p_{-1}(p_1, \hat{p}_0, s_0^H) \equiv \frac{p_1 \hat{p}_0}{\hat{p}_0(1 - s_0^H) + p_1 s_0^H} \ .$$

The RHS of the previous expression is maximized when $s_0^H = \lambda$. Therefore, if an equilibrium ends with a length of two periods, the initial prior is at most $p_0^1 \equiv \frac{1-c_H}{1-c_H(1-\lambda)}$.

Step 4 (induction argument for $k > 1$): Assume that the induction hypothesis is true for $k - 1$ where $k > 1$. We need to verify whether it is true for k.

Assume that p_0 is such that there exists some equilibrium with k periods of education. Denote the beliefs sequences for this equilibrium p and $\hat{p}$. Note that, by the induction hypothesis, $p_1 \leq p_0^{k-1}$ and $\hat{p}_1 \leq \hat{p}_0^{k-1}$, since the continuation play after 1 is itself an equilibrium with initial prior p_1. Since $k > 2$, by Lemma 1, the H-worker is strictly willing to wait in period 0, so $s_0^H = \lambda$, and the L-worker randomizes in period 0. Then, $\hat{p}_0 = \hat{p}_1 - c_L \leq \hat{p}_0^{k-1} - c_L = \hat{p}_0^k$. Therefore, by Lemma 8, $p_0 = \mathcal{S}_1(p_1, \hat{p}_1)$, and that this is increasing in both arguments. So, the maximum value it can take is $\tilde{p}_0^k \equiv \mathcal{S}_1(\tilde{p}_0^{k-1}, \hat{p}_0^{k-1})$.

Step 5 (T^* is the limit): Note that T^* is such that

$$\hat{p}_0^{T^*+1} \leq 0 < \hat{p}_0^{T^*} \ .$$

Then, since $\hat{p}_0^{T^*+1} \leq 0$, there is no equilibrium longer than T^* periods of education.

A graphical intuition of the proof can be found in Figure 4. It graphically represents both $\tilde{p}_0^T$ and $\hat{p}_0^T$ used in the proof.

A.4.2 Constructing L-equilibria

Now, we prove a result related to the set of equilibria where the L-worker is indifferent in all periods, which is similar to Theorem 1. For each $p_0 \in (0, 1)$, we use $\tilde{T}^L(p_0)$ to denote the maximum number of education periods of an equilibrium where the L-worker is indifferent to dropping out in

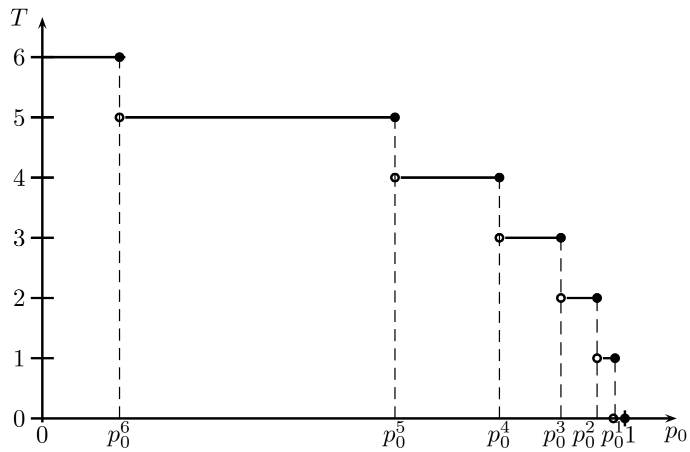

Figure 4: Maximum length of equilibria as a function of the prior p_0. As we see, this function is left continuous and decreasing.

all periods except (maybe) the last. We name these equilibria L-equilibria. The following lemma shows that, for any $p_0 \in (0,1)$, there is a finite integer k such that, for each $T = 0, 1, ..., k$ there is an L-equilibrium that lasts for T periods of education, and no L-equilibrium with a length more than k.

Lemma 9. *Let's define $T^{**} \equiv \lceil \frac{1-c_L}{c_L} \rceil$, $p_0^{L,k} \equiv \mathcal{S}_k(1,1)$ for $k = 0, ..., T^{**}$ and $p_0^{L,T^{**}+1} \equiv 0$. Then, if $p_0 \in (p_0^{L,k+1}, p_0^{L,k}]$ for some $k = 0, ..., T^{**}$, we have $\tilde{T}^L(p_0) = k$. Furthermore, for each $T \leq \tilde{T}^L(p_0)$, there is a unique L-equilibrium with T periods of education.*

Proof of Lemma 9. Fix some $p_0 \in (0,1)$. If $p_0 > \mathcal{S}_k(1,1)$ for some $k \leq T^{**}$ there is no L-equilibrium with k periods of education. Indeed, if there was one (ending at $p_k = \hat{p}_k$), then $p_0 = \mathcal{S}_k(p_k, p_k)$. But since $\mathcal{S}_k(p_k, p_k)$ is strictly increasing in p_k and $p_0 > \mathcal{S}_T(1,1)$, then $p_0 > \mathcal{S}_k(p,p)$ for all $p \in [0,1]$. This is clearly a contradiciton. Note also that, in an L-equilibrium with T periods of education, $\hat{p}_T - \hat{p}_0 = Tc_L \leq 1$. Since $(T^{**}+1)c_L > 1$, we have $\tilde{T}(p_0) < T^{**} + 1$,.

Fix $k < T^{**}$, $p_0 \in (p_0^{L,k+1}, p_0^{L,k}]$ and $T \leq k$. Note that $\mathcal{S}_T(p,p)$ is continuous and strictly increasing when $p > Tc_L$ for any $T \leq T^{**}$ and $\lim_{p \searrow Tc_L} \mathcal{S}_T(p,p) = 0$.[14] So, since $p_0 \leq \mathcal{S}_k(1,1) \leq \mathcal{S}_T(1,1)$, there exists a unique $p_T \in (Tc_L, 1)$ such that $p_0 = \mathcal{S}_T(p_T, p_T)$. Furthermore, there is an equilibrium with length T with $p_t = \mathcal{S}_{T-t}(p_T, p_T)$ and $\hat{p}_t = \mathcal{M}_{T-t}(p_T)$. The argument for $k = T^{**}$ is analogous. $\square$

[14]Note that, if $T \leq T^{*}*$ then $Tc_L < 1$, and, by definition, $\mathcal{M}_T(Tc_L) = 0$. Using the definition of $\mathcal{S}_\tau(\cdot,\cdot)$, we have that $\mathcal{S}_T(c_LT, c_LT) = 0$.

Lemma 10. *For any $k \le T^{**}$, we have $p_0^{L,k} \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k)$.*

Proof of Lemma 10. Note first that

$$\overbrace{\frac{1-c_H}{1-(1-\lambda)c_H}}^{p_0^1} > \overbrace{\frac{1-c_L}{1-(1-\lambda)c_L}}^{p_0^{1,L}} = \mathcal{S}_1(1,1) > \mathcal{S}_1(p_0^1, 1-c_H)$$

By definition, for $k > 1$, $p_0^k = \mathcal{S}_{k-1}(p_0^1, 1-c_H) = \mathcal{S}_1(p_0^{k-1}, \mathcal{M}_{k-2}(1-c_H))$ and $p_0^{k,L} = \mathcal{S}_{k-1}(p_0^{1,L}, 1-c_L) = \mathcal{S}_1(p_0^{k-1}, \mathcal{M}_{k-2}(1-c_L))$. Also, note that $\mathcal{M}_k(1-c_H) > \mathcal{M}_k(1-c_L) > \mathcal{M}_{k+1}(1-c_H)$. Therefore, since $\mathcal{S}_1(\cdot,\cdot)$ is strictly increasing in both arguments, we have $p_0^{L,k} \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k)$. □

A.4.3 Constructing H-equilibria

Lemma 9 implies that for any $p_0 \in (0,1)$, an L-equilibrium lasting for at most k periods can be constructed, where k satisfies that $p_0 \in (p_0^{L,k+1}, p_0^{L,k}]$. However, Lemma 10 shows that $p_0^{L,k} < \tilde{p}_0^k$. For $p_0 \in (p_0^{L,k}, \tilde{p}_0^k]$, there is no L-equilibrium lasting for k periods. The question now is whether there is any other equilibrium that lasts for k periods in this last region. Lemma 11 shows that the answer to this question is yes.

An equilibrium that lasts for $T > 0$ periods of education is an H-equilibrium if and only if, in equilibrium, the L-worker strictly prefers dropping out in period $T-1$. In other words, in an H-equilibrium $p_T = 1$. Note that each equilibrium is either an L-equilibrium or an H-equilibrium, and never both.

Lemma 11. *If $p_0 \in (p_0^{L,k}, \tilde{p}_0^k]$, there exists an H-equilibrium of length k, for $k \in \{1, ..., T^{**}\}$. If $p_0 \in (\tilde{p}_0^{k+1}, p_0^{L,k}]$, there exists an L-equilibrium of length k, for $k \in \{1, ..., T^* - 1\}$.*

Proof of Lemma 11. For $p_0 \in (\tilde{p}_0^{k+1}, p_0^{L,k}]$ the proof of the previous lemma tells us that there exists an L-equilibrium of length k. To prove the case $p_0 \in (p_0^{L,k}, \tilde{p}_0^k]$, we define the function $g : (p_0^{1,L}, p_0^1] \to (1-c_L, 1-c_H]$, denoting $\hat{p}_{T^*}$ if $p_{T^*} \in (1-c_L, 1-c_H]$. It is given by

$$g(p) \equiv \frac{\lambda p}{\lambda p + 1 - p} \ .$$

Then for all $p_0 \in (p_0^{L,k}, p_0^k]$ there exists a unique $f(p_0) \in (p_0^{1,L}, p_0^1]$ such that $p_0 \equiv \mathcal{S}_{k-1}(f(p_0), g(f(p_0)))$. Indeed, we have that $\lim_{p \searrow p_0^{1,L}} g(p) = 1 - c_L$ and $g(p_0^1) = 1 - c_H$. So, we have

$$\lim_{p \searrow p_0^{L,1}} \mathcal{S}_{k-1}(p, g(p)) = p_1^{L,k} \quad \text{and} \quad \mathcal{S}_{k-1}(p_0^1, g(p_0^1)) = \tilde{p}_0^k \ .$$

Since $\hat{p}(\cdot)$ is continuous and strictly increasing, $\mathcal{S}_{k-1}(\cdot,\cdot)$ is continuous in both arguments and strictly increasing, then there exists such $f(p_0)$, and is unique.

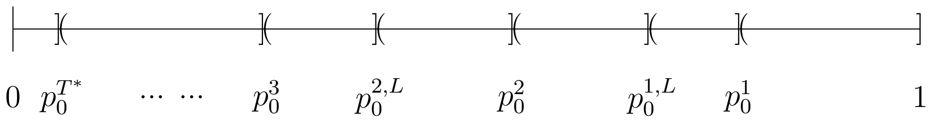

Figure 5: Partition construction

Let's construct one equilibrium with k education periods when $p_0 \in (p_0^{L,k}, \tilde{p}_0^k]$, for $k \leq T^* - 1$. Our claim is that it can be defined by $p_k = \hat{p}_k = 1$, $p_t = \mathcal{S}_{t-1}(f(p_0), g(f(p_0)))$ and $\hat{p}_t = g(f(p_0)) - c_L(k-t-1)$, for $t \in \{0, ..., k-1\}$. To prove that, we show that the corresponding strategies are well defined. Note that, if the L-worker is indifferent in period 0, we have

$$s_t^L = \frac{1}{1 + \frac{(1-\lambda)(1-p_t)\hat{p}_t}{\lambda p_t (1-\hat{p}_t)}} = \frac{\lambda}{1 - \frac{(1-\lambda)(p_t - \hat{p}_t)}{p_t(1-\hat{p}_t)}} \ .$$

The first equality shows that $s_t^L < 1$. The second equality shows that, if $p_t^1 > \hat{p}_t$, then $s_t^L > \lambda$, which is equivalent to $p_0^2 < p_0^1$, which is true as long as $\hat{p}_t > 0$. Since, when $k < T^*$, $\hat{p}_0 = g(f(p_0)) - c_L(k-1) > 0$, the result holds in this case.

Finally, there are two possible cases. If $T^{**} = T^*$, we know from the previous lemma that there exists an L-equilibrium with length T^{**} in $(0, p_0^{L,T^*})$. If $T^{**} = T^* - 1$ then there exists some $p \in (p_0^{1,L}, p_0^1]$ such that $g(p) = T^{**}c_L$. Indeed, in this case $1 \leq T^* c_L < 1 - c_H + c_L$, so $T^{**}c_L \in (1 - c_H, 1 - c_L]$. Therefore, we can use the same argument as for $p_0 \in (p_0^{L,k}, \tilde{p}_0^k]$, for $k \leq T^* - 1$. The idea of the partition construction can be summarized in Figure 5. $\square$

Finally, note that the set $\{\tilde{p}_0^k\}_{k=0}^{T^*+1}$ is such that $\tilde{p}_0^k > \tilde{p}_0^{k+1}$ for all k. Furthermore, for all $0 \leq k \leq T^*$ and $0 \leq T \leq k$, if $p_0 \in (\tilde{p}_0^{k+1}, \tilde{p}_0^k]$, there exists an equilibrium with T periods of education and no equilibrium with a length larger than k. So, $p_0^k \equiv \tilde{p}_0^k$, for $k = 0, ..., T^* + 1$, satisfies the statement of Theorem 1, and therefore its proof is complete. *Q.E.D.*

A.5 The Proof of Lemma 4

We will do the proof first fixing the maximum real time and solving for the corresponding p_0, and then showing that for all p_0 there exists a unique limit for the maximum real time. Fix $\bar{\kappa} \in (0, \frac{1}{\tilde{c}_L})$. In order to save notation, consider a strictly decreasing sequence Δ_n such that $\frac{\bar{\kappa}}{\Delta_n} \in \mathbb{N}$ for all $n \in \mathbb{N}$. Using the Bayes' rule, we have the following equation relating $p_0^{\bar{\kappa}/\Delta_n, L}$ and $p_0^{\bar{\kappa}/\Delta_n - 1, L}$

(defined in Lemma 9):[15]

$$\frac{1}{p_0^{\bar{\kappa}/\Delta_n,L}} = \frac{\tilde{\lambda}\Delta_n}{1-\tilde{c}_L\bar{\kappa}} + \frac{1-\tilde{\lambda}\Delta_n}{p_0^{\bar{\kappa}/\Delta_n-1,L}} = \sum_{m=0}^{\bar{\kappa}/\Delta_n} \frac{\tilde{\lambda}\Delta_n(1-\tilde{\lambda}\Delta_n)^m}{1-\tilde{c}_L(\bar{\kappa}-m\Delta_n)} + (1-\tilde{\lambda}\Delta_n)^{\bar{\kappa}/\Delta_n} \ . \tag{8}$$

When Δ_n is small, each term of the sum can be approximated as follows

$$\frac{\tilde{\lambda}\Delta_n(1-\tilde{\lambda}\Delta_n)^m}{1-\tilde{c}_L(\bar{\kappa}-m\Delta_n)} = \frac{\tilde{\lambda}e^{-\tilde{\lambda}s}}{1-\tilde{c}_L(\bar{\kappa}-s)}\Delta_n + O(\Delta_n^2)$$

where $s \equiv m\Delta_n$. The last term of the RHS of equation (8) satisfies $\lim_{n\to\infty}(1-\tilde{\lambda}\Delta_n)^{\bar{\kappa}/\Delta_n} = e^{-\tilde{\lambda}\bar{\kappa}}$. Since each term in the sum is a bounded function (note that s ranges from 0 to $\bar{\kappa}$) multiplied by Δ_n, at the limit $\Delta_n \searrow 0$ the sum converges to the integral, so we have

$$\frac{1}{\tilde{p}_0(\bar{\kappa})} \equiv \lim_{n\to\infty} \frac{1}{p_0^{\bar{\kappa}/\Delta_n,L}} = e^{-\tilde{\lambda}\bar{\kappa}} + \int_0^{\bar{\kappa}} \frac{e^{-\tilde{\lambda}s}\tilde{\lambda}}{1-\tilde{c}_L s} ds \ .$$

Note that the RHS of the previous expression is equal to 1 when $\bar{\kappa} = 0$. Differentiating it with respect to $\bar{\kappa}$ we find

$$\frac{d}{d\bar{\kappa}}\frac{1}{\tilde{p}_0(\bar{\kappa})} = -\tilde{\lambda}e^{-\tilde{\lambda}\bar{\kappa}} + \frac{e^{-\tilde{\lambda}\bar{\kappa}}\tilde{\lambda}}{1-\tilde{c}_L\bar{\kappa}} = \frac{e^{-\tilde{\lambda}\bar{\kappa}}\tilde{\lambda}\tilde{c}_L\bar{\kappa}}{1-\tilde{c}_L\bar{\kappa}} \geq 0 \ .$$

Therefore, $p_0^{\bar{\kappa}} \in (0,1)$ when $\bar{\kappa} \in (0, \frac{1}{\tilde{c}_L})$.

Note that, for each $p_0 \in (0,1)$, there exists a unique $\bar{\kappa}$ such that $\tilde{p}_0(\bar{\kappa}) = p_0$. Indeed, $\lim_{\bar{\kappa}\to 0}\tilde{p}_0(\bar{\kappa}) = 1$, $\lim_{\bar{\kappa}\to 1/\tilde{c}_L}\tilde{p}_0(\bar{\kappa}) = 0$ and $\tilde{p}_0(\cdot)$ is strictly increasing in $(0, \frac{1}{\tilde{c}_L})$. Therefore, for each p_0 there exists a unique $\kappa(p_0) \equiv \tilde{p}_0^{-1}(p_0)$ that satisfies the conditions of the lemma. It is given by the solution of

$$\frac{1}{p_0} = e^{-\tilde{\lambda}\kappa(p_0)} + \int_0^{\kappa(p_0)} \frac{e^{-\tilde{\lambda}s}\tilde{\lambda}}{1-\tilde{c}_L s} ds \ . \tag{9}$$

Q.E.D.

A.6 The Proof of Lemma 5

Proceeding similarly as in the proof of Lemma 4, we have that

$$\frac{1}{p_0} = \frac{e^{-\tilde{\lambda}\tau}}{p_{T_n}(\Delta_n)} + \int_0^{\tau} \frac{e^{-\tilde{\lambda}s}\tilde{\lambda}}{p_{T_n}(\Delta_n)-\tilde{c}_L s} ds + O(\Delta_n) \ .$$

[15] We use $p_0^{\bar{\kappa},L}$ defined in Lemma 9 instead of $p_0^{\bar{\kappa}}$ for simplicity. Lemma 10 and the fact that $p_0^{\bar{\kappa}/\Delta_n} - p_0^{t/\Delta_n-1} = O(\Delta_n)$ guarantee that $p_0^{\bar{\kappa}/\Delta_n,L}$ and $p_0^{\bar{\kappa}/\Delta_n}$ will be asymptotically equal.

Note that the RHS of the previous equation is decreasing in $p_{T_n}(\Delta_n)$. Furthermore, the RHS is lower than $\frac{1}{p_0}$ when $p_{T_n}(\Delta_n) = 1$, since it would be equal if $\tau = \kappa(p_0)$, but, by assumption, $\tau < \kappa(p_0)$. Also, when $p_{T_n}(\Delta_n) = p_0$, the RHS is larger than p_0. Indeed, it would be equal to p_0 if $\tau = 0$, but $\tau > 0$ and, as is shown in the proof of Lemma 4, the RHS is increasing in τ. Therefore, there exists a unique limit of $p_{T_n}(\Delta_n)$, and is strictly lower than 1. *Q.E.D.*

A.7 The Proof of Theorem 2

Note that the convergence of $\hat{p}$ is an immediate consequence of Lemma 5. Indeed, we have that $\lim_{n\to\infty} \hat{p}_{T_n} = \lim_{n\to\infty} p_{T_n} = 1$. Furthermore, since the H-worker has to be (weakly) willing to remain in education at $T_n - 1$, we have that $\hat{p}_{T_n-1} = \hat{p}_{T_n} + O(\Delta_n)$. So, by Lemma 1, we have $\hat{p}_t = 1 - (T_n - t)\bar{c}_L\Delta + O(\Delta)$ or, in real time, $p_\tau = 1 - (\kappa(p_0) - \tau)\bar{c}_L + O(\Delta)$.

To prove the convergence of p, we proceed similarly to Lemma 4. Using Bayes' rule, it is easy to prove that, for $t < T_n - 1$, we have

$$\frac{1}{p_t} = \frac{1 - \tilde{\lambda}\Delta_n}{p_{t+1}} + \frac{\tilde{\lambda}\Delta_n}{\hat{p}_t} = e^{-\tilde{\lambda}\Delta_n t} + \int_0^{t\Delta_n} \frac{e^{-\tilde{\lambda}s}\tilde{\lambda}}{1 - (\kappa(p_0) - s)\tilde{c}_L} ds + O(\Delta_n) \ .$$

If we differentiate each side of the equation with respect to $\tau \equiv \Delta t$ (or, alternatively, compute $\frac{p_{t+1} - p_t}{\Delta_n}$), we find the stated differential equation.

Finally, using Bayes' rule, it is easy to verify that $m_t = \frac{p_t\lambda}{\hat{p}_t}$ when $t < T_n$. Therefore, trivially, $\tilde{m}(\tau) = \frac{p(\tau)\tilde{\lambda}}{\hat{p}(\tau)}$ for all $\tau \in (0, \kappa(p_0))$. *Q.E.D.*

A.8 The Proof of Theorem 3

From Theorem 2 we can differentiate $\tilde{m}(\tau)$ and we get

$$\tilde{m}'(\tau) = \tilde{\lambda}\frac{p'(\tau)\hat{p}(\tau) - p(\tau)\hat{p}'(\tau)}{\hat{p}(\tau)^2} = -\tilde{\lambda}\frac{p(\tau)\big(\tilde{c}_L - (p(\tau) - \hat{p}(\tau))\tilde{\lambda}\big)}{\hat{p}(\tau)^2} \ .$$

Note that the last expression is negative only if $(p(\tau) - \hat{p}(\tau)) \leq \frac{\tilde{c}_L}{\tilde{\lambda}}$. If we differentiate this expression we have

$$\frac{d}{d\tau}\big(p(\tau) - \hat{p}(\tau)\big) = \tilde{\lambda}\frac{p(\tau)(p(\tau) - \hat{p}(\tau))\tilde{\lambda}}{\hat{p}(\tau)} - \tilde{c}_L \ .$$

First, assume that there is some τ^* such that $p'(\tau^*) - \hat{p}'(\tau^*) = 0$. In this case, the previous expression is negative, so this is a maximum, since $p'(\kappa(p_0)) = 0 < \tilde{c}_L = \hat{p}'(\kappa(p_0))$. It is easy to show that, $\tilde{m}'(\tau^*) < 0$, so $m(\tau)$ for all $\tau \in (0, \kappa(p_0))$. Otherwise, $p(\tau) - \hat{p}(\tau)$ is maximum when $\tau = 0$, and $p'(0) - \hat{p}'(0) \leq 0$, what implies that $\tilde{m}'(\tau) < 0$ for all $\tau \in (0, \kappa(p_0))$. *Q.E.D.*

A.9 The Proof of Theorem 4

First note that, in any equilibrium passing D1, the length of education (i.e. $\max\{T^L, T^H\}$) must be no lower than $\hat{T}$. Otherwise, if a worker deviates and drops out at $\max\{T^L, T^H\} + \varepsilon$, for some $\varepsilon > 0$ small, he should be considered an H-worker, and so should receive a wage offer of $1 + (\max\{T^L, T^H\} + \varepsilon)h$. Nevertheless, the wage at $\max\{T^L, T^H\}$ is no larger than $1 + \max\{T^L, T^H\}h$, so the deviation is profitable for the H-worker.

Let's assume $T^L < \hat{T}$.[16] In any equilibrium passing D1, H-workers do not voluntarily drop out in period t where $T^L \le t < \hat{T}$. The reason is that when $t > T^L$, $p_t = 1$, so for H-workers, the marginal return to education is h which is greater than the marginal cost c_H. In the period T^L, we have that $s^L_{T_L} = 1$, which implies

$$\hat{p}_{T^L} = \frac{p_{T^L}\lambda}{p_{T^L}\lambda + 1 - p_{T^L}} \quad \Rightarrow \quad p_{T_L} = \frac{\hat{p}_{T^L}}{\hat{p}_{T^L}(1-\lambda) + \lambda} \equiv f(\hat{p}_{T^L}) \ .$$

Note that $\lim_{\hat{p}_{T^L} \to 1} f(\hat{p}_{T^L}) = 1$. Furthermore, note that $\hat{p}_{T_L}$ needs to be such that the L-worker wants to drop out, so

$$\hat{p}_{T^L} + hT^L \ge 1 + h(T^L + 1) - c_L \quad \Rightarrow \quad \hat{p}_{T^L} \ge 1 + h - c_L \ .$$

So, using a technique similar to the one used in Theorem 1, for each $\hat{p} \in [1 + h - c_L, 1)$ we can construct a sequence of p and $\hat{p}$ using the pull-back functions defined in Lemma 7, now with $c_L - h$ instead of c_L in equation (7). Indeed, proceeding similarly, it is easy to show that for every $\hat{p} \in [1 + h - c_L, 1)$ the sequence $(p^{\hat{p}}_\tau \equiv \mathcal{S}_\tau(f(\hat{p}), \hat{p} - c_L + h))_\tau$ is such that for any $p^{\hat{p}}_\tau \ne p^{\hat{p}'}_{\tau'}$ for all $\hat{p} \ne \hat{p}' \in [1 + h - c_L, 1)$ and $\tau, \tau' \in \mathbb{N}$. Also, given that $\mathcal{S}_1(1, 1 - c_L + h) = f(1 - c_L + h)$, it is easy to show (proceeding similarly to Lemma 11) that for all $p_0 \in (0, 1)$ there exists a unique $\hat{p} \in [1 + h - c_L, 1)$ and $\tau \ge 0$ such that $p_0 = \mathcal{S}_\tau(f(\hat{p}), \hat{p} - c_L + h)$. *Q.E.D.*

B Appendix: Type-Dependent Dropout Risk (Not For Publication)

Here we consider a model in which a worker's dropout rate is correlated with his productivity. It turns out that our predictions in Section 2 are robust. There are three relevant cases: (1) $\lambda_H > \lambda_L \ge 0$, (2) $\lambda_L > \lambda_H > 0$, and (3) $\lambda_L \ge \lambda_H = 0$.

[16] This is true, for example, if $1 + \hat{T}h < \hat{T}c_L$, that is, if L-workers prefer to drop out at 0 than wait until $\hat{T}$.

B.1 $\lambda_H > \lambda_L \geq 0$ Case

The first case we consider is $\lambda_H > \lambda_L \geq 0$; that is, the high-type worker exogenously drops out at a higher rate than the low-type worker. The following lemma implies that the equilibrium set in this case coincides with the base model when $\lambda = \lambda_H$:

Lemma 12. *Assume that $\lambda_H > \lambda_L \geq 0$. Then, $(\alpha^L, \alpha^H, w, p, \hat{p})$ is a PBE if and only if it is also a PBE in the benchmark model with $\lambda = \lambda_H$.*

Proof. We first prove that Lemma 1 (which holds when $\lambda_H = \lambda_L$) is still valid when $\lambda_H \geq \lambda_L$. Consider T as the maximum periods lower than T^L where $s_t^L \leq s_t^H$. In this case

$$p_{T+1} \leq p_T \leq \hat{p}_T \ .$$

Furthermore, since the L-worker is voluntarily dropping out at time $T+1$, this implies $\hat{p}_T \leq \hat{p}_{T+1} - c_L$. Nevertheless, since $s_{T+1}^L \geq s_{T+1}^H$, we have $\hat{p}_{T+1} \geq p_{T+1}$, which is a contradiction, since

$$\hat{p}_{T+1} \leq p_{T+1} \leq p_T \leq \hat{p}_T \leq \hat{p}_{T+1} - c_L \ .$$

So, when $\lambda_H \geq \lambda_L$, it is still true that $s_t^L > s_t^H$ in all periods of all equilibria before T^L. Therefore, relaxing of the constraint $\lambda_L = \lambda_H = \lambda$ to $\lambda_L \leq \lambda_H = \lambda$ does not introduce new equilibria. Trivially, it does not destroy any equilibria, since in the model $\lambda_L = \lambda_H = \lambda$, in all equilibria, $s_t^L > \lambda$ for all equilibria and period $t \leq T^L$. ☐

The intuition behind this lemma is that, in our original model, by Lemma 1, the endogenous dropout rate of the low-type worker is positive in all periods before (maybe) the last. So, the constraint $s_t^L \geq \lambda$ was never binding in equilibrium. Therefore, all equilibria from the base model for $\lambda = \lambda_H$ are also equilibria for the case $\lambda_H > \lambda_L \geq 0$. On the other hand, for any equilibrium in the case where $\lambda_H > \lambda_L$, let $\tilde{\alpha}_t^L$ denote the low type's strategy. It must be true that $\tilde{\alpha}_t^L \geq \lambda_H - \lambda_L$. Define $\hat{\alpha}_t^L = \tilde{\alpha}_t^L - (\lambda_H - \lambda_L) \geq 0$. One can easily verify that $\hat{\alpha}_t^L$ can be supported in a PBE of the game with a symmetric exogenous dropout rate, $\lambda = \lambda_H$.

B.2 $\lambda_L > \lambda_H > 0$ Case

As we can see in Figure 6, s^L may be non-monotone. In particular, there are some equilibria where it is initially decreasing and then increasing and finally it goes down again. Now, s^L is restricted to be no lower than $\lambda_L > \lambda_H$. We may guess that this constraint will be potentially binding in two connected regions, one for large $\hat{p}$ and the other for intermediate values. In any equilibrium, when this constraint is binding, both types strictly prefer to wait. Different from

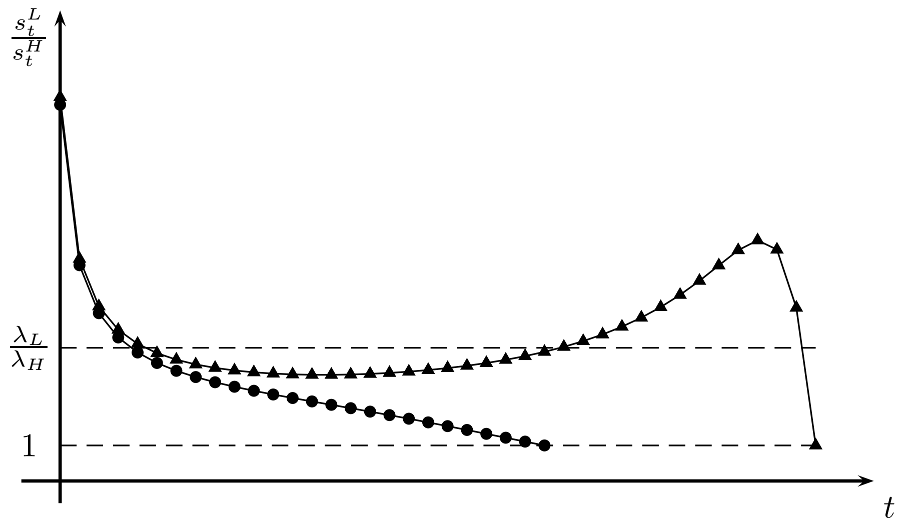

Figure 6: Endogenous dropout rate of the low-type worker

the benchmark model, the equilibrium belief p_t still goes up since $\lambda_L > \lambda_H$. After some periods, the constraint may become not binding anymore, and the low-type worker starts to play a mixed strategy again. However, the neat equilibrium characterization in the benchmark model can not survive for some parameters. Fortunately, the following theorem shows that the equilibrium characterization in the benchmark model still works when λ_L is not significantly larger than λ_H.

Theorem 5. *For any given set of parameters (λ, c_L, c_H, p_0) there exists $\varepsilon > 0$ such that if $\lambda_H = \lambda$ and $\lambda_L = (\lambda, \lambda + \varepsilon]$ then the set of PBE is the same.*

Proof. Note that Lemma 6 still holds (the H-worker can imitate the strategy of the L-worker). Now we try to prove a result analogous to Lemma 1. Assume that the L-worker is not voluntarily dropping out in period t, so his dropout rate is $\lambda + \varepsilon$. First assume that the dropout rate of the H-worker is larger than $\lambda + \varepsilon$. In this case, we can apply the exact same argument as in the proof of Lemma 1, so we obtain again a contradiction. Assume now that $s^H \in [\lambda, \lambda + \varepsilon)$. In this case $p_{t+1} = p_t + O(\varepsilon)$ and $\hat{p}_t = p_t + O(\varepsilon)$, so $\hat{p}_t - p_{t+1} = O(\varepsilon)$. Then, using the same logic as in the proof of Lemma 1, we have

$$\hat{p}_t \leq W_t^L \leq V_{t+1}^L - c_L \leq p_{t+1} - c_L \ .$$

Therefore, $\hat{p}_t - p_{t+1} \leq -c_L$. But this is inconsistent with $\hat{p}_t - p_{t+1} = O(\varepsilon)$. That proves that, if $\varepsilon > 0$ is small enough, the model with $\lambda_H = \lambda$ and $\lambda_L = \lambda + \varepsilon$ does not have more equilibria than for the case $\varepsilon = 0$.

Let's prove the reverse. Assume that there exists a sequence $\{\varepsilon_n > 0\}_{n\in\mathbb{N}}$ such that $\lim_{n\to\infty}\varepsilon_n = 0$ and, for each n, there exists an equilibrium in our original model and t_n is reached with positive probability on the path of play under this equilibrium such that $s^L_{t_n} \in [\lambda, \lambda + \varepsilon_n)$. This implies $p_{t_n+1} = p_{t_n} + O(\varepsilon_n)$ and $\hat{p}_{t_n} = p_{t_n} + O(\varepsilon_n)$, so $\hat{p}_{t_n} - p_{t_n+1} = O(\varepsilon_n)$.[17] So,

$$\hat{p}_{t_n} = W^L_{t_n} = V^L_{t_n+1} - c_L \leq p_{t_n+1} - c_L \ .$$

This, again, is a contradiction. □

B.3 $\lambda_L \geq \lambda_H = 0$ Case

In this case, there is no exogenous drop-out by the H-worker. Consider first $\lambda_L = 0$. In this case our model is equivalent to Cho and Kreps (1987), only corrected by the fact that the education choice is restricted to be discrete. The reason is that the worker decides about his education without interacting with the firms. Once the decision to drop out has been made, the worker cannot change the market's belief about his type. Furthermore, early dropping out may be off the path of play, so beliefs can be arbitrarily assigned in those events. Therefore, the equilibrium predictions of both models share the same characteristics.

Intuitively, when $\lambda_L > 0$, nothing essential changes. The reason is that the belief threats off the path of play when $\lambda_L = 0$ are replaced by the potentially exogenous dropping out of the L-worker, so now deviations from early dropout are still punished.

Note that our main mechanism in the benchmark model is not present here. Indeed, in our benchmark model, as is proven in Lemma 1, the L-worker uses the fact that the H-worker exogenously drops out to mimic him in order to save the high cost of education. Since the H-worker exogenously drops out, early dropout cannot be punished too much, constraining the belief threats by the firms. This is no longer true when $\lambda_H = 0$, so the set of equilibria is qualitatively different from the $\lambda_H > 0$ case.

C Appendix: Multiple Types (Not For Publication)

Now we consider the $N > 2$ types case in which $\theta \in \{1, 2, 3, ..., N\}$ with a prior p^θ_0, where $\sum_{\theta=1}^N p^\theta_0 = 1$. The type θ worker has a cost of waiting c^θ, $c^\theta > c^{\theta+1}$. The productivity of θ is Y^θ, $Y^\theta < Y^{\theta+1}$. All types exogenously drop out with probability λ.

[17] Using some abuse of notation, p_{t_n} and $\hat{p}_{t_n}$ denote the corresponding posteriors in the n-th equilibrium of the sequence.

The equilibrium concept is the same as in Definition 1 but adapted to the fact that now we have many types. Note that firms' offers depend only on the expected productivity and not on other moments of the productivity distribution. This fact helps us to keep our definition simple:

Definition 2. *A* ***perfect Bayesian equilibrium (PBE)*** *is a strategy profile* $\{(\alpha^\theta)_{\theta=1,\dots,N}, w\}$, *a belief sequences* p^θ *for all* $\theta \in \{1, ..., N\}$ *such that:*

1. *the* θ*-worker chooses* α^θ *to maximize her expected payoff given* w,
2. *if a worker drops out with education* t, *firms offer* $w_t = \sum_{\theta=1}^N \hat{p}_t^\theta Y^\theta$, *where* $\hat{p}_t^\theta$ *satisfies*
$$\hat{p}_t^\theta = \frac{p_t^\theta s_t^\theta}{\sum_{\theta'=1}^N p_t^{\theta'} s_t^{\theta'}} \ , \tag{10}$$
when it is well defined, and
3. *when it is well defined,* p_t^θ *is updated according to the Bayes' rule*
$$p_{t+1}^\theta = \frac{p_t^\theta (1 - s_t^\theta)}{\sum_{\theta'=1}^N p_t^{\theta'} (1 - s_t^{\theta'})} \ . \tag{11}$$

Let T^θ be the last time the θ-worker is in school. The following theorem shows that our insight into the binary-type model can be easily extended to a multiple-types model.

Theorem 6. *Under the previous assumptions, in any equilibrium:*

1. *in each period* t, *there is at most one type, indifferent to dropping out,*
2. *more productive types stay longer in education,* $T^\theta \le T^{\theta+1}$,
3. *there is positive voluntary dropout in all periods, and*
4. *the expected productivity of dropouts,* $\hat{Y}_t \equiv \sum_{\theta=1}^N \hat{p}_t^\theta Y^\theta$, *is concave in* t.

Proof. 1. Assume that, in period t, there are two types $\theta_1, \theta_2 \in \Theta$, with $c^{\theta_1} < c^{\theta_2}$, and both are indifferent between dropping out or not. Let τ_1 and τ_2 denote, respectively, the stopping times of the continuation strategies that make players indifferent on dropping out or not.[18] Then, we have

$$\hat{Y}_t = \mathbb{E}[w_{\tau_{\theta_2}} - c^{\theta_2}\tau_{\theta_2}] \ge \mathbb{E}[w_{\tau_{\theta_1}} - c^{\theta_2}\tau_{\theta_1}] > \mathbb{E}[w_{\tau_{\theta_1}} - c^{\theta_1}\tau_{\theta_1}] = \hat{Y}_t \ .$$

[18] For this proof, for a given strategy, it is convenient to use the random variable τ, which gives the duration of the game.

The first (weak) inequality is from the optimality of the θ_2-worker. The strong inequality is because $\mathbb{E}[\tau_{\theta_1}] > 0$ and $c^{\theta_1} < c^{\theta_2}$. The equalities come from the fact that i-workers with $i \in \{1, 2\}$ are indifferent between dropping out (and getting $\hat{Y}_t$) or staying and following τ_i. Therefore, we have a contradiction.

2. Assume otherwise; that is there exists $\theta_1, \theta_2 \in \Theta$ such that $\theta_1 < \theta_2$ and $T^{\theta_1} > T^{\theta_2}$. Let τ_{θ_1} be the stopping time of the continuation strategy after T^{θ_2}, given by the strategy of θ_1. Then, note that

$$\hat{Y}_{T^{\theta_2}} \geq \mathbb{E}[w_{\tau_{\theta_1}} - c^{\theta_2}\tau_{\theta_1}] > \mathbb{E}[w_{\tau_{\theta_1}} - c^{\theta_1}\tau_{\theta_1}] \geq \hat{Y}_{T^{\theta_2}} \ .$$

This is clearly a contradiction. The first inequality comes from the optimality of the θ_2-worker choosing to drop out at T^{θ_2} (since they could deviate to mimic the θ_1-worker). The second inequality is given by the fact that since $\theta_1 < \theta_2$, $c^{\theta_2} < c^{\theta_1}$ and since $T^{\theta_1} > T^{\theta_2}$, $\mathbb{E}[\tau_{\theta_1}] > 0$. The last inequality comes from the optimality of the θ_1-worker choosing to drop out at $T^{\theta_1} > T^{\theta_2}$ (since they could deviate to mimic the θ_2-worker).

3. Define $\Theta_t = \{\theta | T^\theta \geq t\}$ and $\theta_t = \min\{\Theta_t\}$. We proceed as in the proof of Lemma 6. Now we have

$$\begin{aligned}
\mathbb{E}_t[w_\tau | \tau \geq t] &= \sum_{\tau=t}^{\infty} \Pr(\tau, t)\hat{Y}_\tau = \sum_{\tau=t}^{\infty} \Pr(\tau, t) \frac{\sum_\theta Y^\theta s_\tau^\theta p_t^\theta \Pr^\theta(\tau, t)}{\Pr(\tau, t)} \\
&= \sum_\theta p_t^\theta Y^\theta \sum_{\tau=t}^{\infty} s_\tau^\theta \Pr^\theta(\tau, t) = \sum_\theta p_t^\theta Y^\theta = Y_t \ ,
\end{aligned}$$

where $\Pr(\tau, t)$ and $\Pr^\theta(\tau, t) = s_\tau^\theta \prod_{t'=t}^{\tau-1}(1 - s_{t'}^\theta)$ are defined as in the proof of Lemma 6.

Note that, by the previous result,

$$\sum_{\theta=\theta_t}^{N} p_t^\theta V_t^\theta = \mathbb{E}_t[w_\tau | \tau \geq t] - \sum_{\theta=\theta_t}^{N} p_t^\theta c^\theta \tau^\theta(t) < \mathbb{E}_t[w_\tau | \tau \geq t] \ ,$$

where $\tau^\theta(t)$ is the stopping time for the θ-worker conditional on reaching t. Since $V_t^\theta \leq V_t^{\theta+1}$ (since the $(\theta+1)$-worker can mimic the θ-worker at a lower cost), and $\sum_{\theta=\theta_t}^{N} p_t^\theta = 1$ we have that $V_t^{\theta_t} < Y_t$.

Assume that in period t there is no voluntary dropout. In this case, $\hat{Y}_t = Y_t$. Since we just showed $V_{\theta_t} < Y_t$, the θ_t-worker is willing to drop out, which is a contradiction.

4. Note that, by part 3 of this theorem, we have that $\hat{Y}_{t+1} - c^{\theta_t} \leq \hat{Y}_t$. Furthermore, $\hat{Y}_{t+1} - c^{\theta_{t+1}} \geq \hat{Y}_t$. This implies that $\hat{Y}_{t+1} - \hat{Y}_{t+1} \in [c^{\theta_{t+1}}, c^{\theta_t}]$. Since c^θ is decreasing in θ and, by part 2 of this theorem, the θ_t-worker is (weakly) increasing in t, $\hat{Y}_t$ is concave in t.

$\square$

Most features of the two-type model are preserved. However, note that under many types we have decreasing returns to education instead of linear ones, since lower types are skimmed out before higher types in equilibria. This pattern of decreasing returns to education is consistent with many empirical studies, for example, Frazis (2002), Habermalz (2003), Heckman *et al.* (2008) and Manoli (2008). The equilibrium construction in multiple-type models is almost identical to that in the two-type model and thus is omitted.

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