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PIER Working Paper 13-029

"Signaling Effects of Monetary Policy

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http://ssrn.com/abstract=2284167

Signaling Effects of Monetary Policy

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March 2013

Abstract

We develop a DSGE model in which the policy rate signals to price setters the central bank's view about macroeconomic developments. The model is estimated with likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* as a measure of price setters' inflation expectations. We find that the model fits the data better than a prototypical New Keynesian DSGE model because the signaling effects of monetary policy help the model account for the run-up in inflation expectations in the 1970s. The estimated model with signaling effects delivers large and persistent real effects of monetary disturbances even though the average duration of price contracts is fairly short. While the signaling effects do not substantially alter the transmission of technology shocks, they bring about deflationary pressures in the aftermath of positive demand shocks. The signaling effects of monetary policy have contributed (*i*) to heightening inflation expectations in the 1970s, (*ii*) to raising inflation and to exacerbating the recession during the first years of Volcker's monetary tightening, and (*iii*) to subduing inflation and to stimulating economic activity from 1991 through 2007.

Keywords: Bayesian econometrics; price puzzle; persistent real effects of nominal shocks; imperfect common knowledge; public signal; heterogeneous beliefs.

JEL classification: E52, C11, C52, D83.

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†I wish to thank Francesco Bianchi, Larry Christiano, Harald Cole, Marco Del Negro, Martin Eichenbaum, Jesus Fernandez-Villaverde, Cristina Fuentes Albero, Dirk Krueger, Alejandro Justiniano, Bartosz Maćkowiak, James Nason, Kristoffer Nimark, Luigi Paciello, Giorgio Primiceri, Ricardo Reis, Lucrezia Reichlin, Helene Rey, Frank Schorfheide, Paolo Surico, and Mirko Wiederholt for their very helpful comments and discussion. I thank seminar participants at the ECB-Bundesbank-University of Frankfurt's joint seminars, the LBS workshop on "The Macroeconomics of Incomplete Information: Empirical and Policy Perspectives," the CEPR-CREI workshop on "Information, Beliefs and Expectations in Macroeconomics," the Bank of England, the SED meetings 2011, the NBER/Philadelphia Fed Conference on "Methods and Applications for DSGE Models," the Northwestern University, the Chicago Fed, and the Conference on "Monetary Policy, Inflation, and International Linkages" organized by the Bundesbank for their helpful comments. Todd Messer provided excellent research assistance. An earlier version of this paper was circulated under the title "Public's Inflation Expectations and Monetary Policy." The views in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Federal Reserve Bank of Chicago or any other person associated with the Federal Reserve System. Correspondence to: Leonardo Melosi, Federal reserve Bank of Chicago, 230 S. La Salle street, Chicago, IL, 60604.

1 Introduction

An important feature of economic systems is that information is dispersed across market participants and policymakers. Dispersed information implies that publicly observable policy actions transfer information to market participants. An important example is the monetary policy rate, which conveys information about the central bank's view on macroeconomic developments. Such an information transfer may strongly influence the transmission of monetary impulses and the central bank's ability to stabilize the economy. Consider the case in which a central bank expects that a disturbance will shrink economic activity in the next few quarters. On the one hand, as predicted by standard macroeconomic models, cutting the policy rate has the effect of mitigating the contractionary effects of the shock. On the other hand, lowering the policy rate might accelerate the recession if this action convinces market participants that a contractionary shock is about to hit the economy. While the first type of effect has been intensively investigated in the empirical literature, the signaling effects of monetary policy have received far less attention. This paper develops a dynamic stochastic general equilibrium (DSGE) model to study the empirical relevance of the signaling effects of monetary policy and their implications for the propagation of policy and non-policy disturbances.

We develop a DSGE model in which price-setting firms face nominal rigidities and incomplete information. Firms observe their own specific technology conveying noisy private information about aggregate technology shocks that influence the future dynamics of firms' nominal marginal costs. Price setters also observe a noisy private signal about demand shocks and they observe the policy rate set by the central bank according to a Taylor-type reaction function. The policy signal provides public information about the central bank's view on current inflation and the output gap to firms.

The model features two channels of monetary transmission. The first channel emerges because the central bank can affect the real interest rate due to both nominal rigidities, as in standard New Keynesian models, and incomplete information. Changes in the real interest rate induce households to adjust their consumption. The second channel arises because the policy rate signals non-redundant information to price setters and hence influences their beliefs about macroeconomic developments. We label this second channel *the signaling channel* of monetary transmission. The signaling effects of monetary policy on the propagation of shocks critically rely on how price setters interpret the change in the policy rate. Raising the policy rate can be interpreted by price-setting firms in two ways. First, a monetary tightening might imply that the central bank is responding to a contractionary monetary shock, leading the central bank to deviate from its monetary rule. Second, a higher interest rate may also be interpreted as the response of the central bank to inflationary non-policy shocks, which, in the model, are an adverse aggregate technology shock or a positive demand shock. If the first interpretation prevails among price setters, tightening (easing) monetary policy curbs (fosters) firms' inflation expectations and hence inflation. If the second interpretation prevails, raising (cutting) the policy rate induces firms to expect higher (lower) inflation, and hence, inflation tends to increase (decrease).

The model is estimated through likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. The data range includes the 1970s, which were characterized by one of the most notorious episodes of heightened inflation and inflation expectations in the recent U.S. economic history. The estimated model features a fairly sluggish response of inflation to monetary disturbances and, at the same time, a short average duration of price contracts (i.e., four months). The signaling channel is found to have sizeable effects on the transmission of shocks. In the estimated model, raising the policy rate signals to firms that the central bank is likely to be responding to either a positive demand shock or a contractionary monetary shock. Firms, however, do not critically change their expectations about the aggregate technology shock from observing the policy rate as they hold fairly precise private information about this type of shock. A number of important implications follow. First, the signaling channel magnifies the real effects of monetary shocks. Second, inflation expectations respond positively to monetary shocks. These two features occur because price setters interpret – to some extent – a rise in the policy rate as the central bank's response to a positive demand shock that pushes up inflation expectations. Third, the inflationary effects of the signaling channel are negligible after an aggregate technology shock. When the Federal Reserve raises the policy rate to counter a negative technology shock, firms are induced to believe that the central bank may be reacting to either a positive demand shock or a contractionary monetary shock that have conflicting effects on firms' inflation expectations. These two effects cancel each other out. Fourth, the signaling effects of monetary policy bring about deflationary pressures in the aftermath of a positive demand shock. When the Federal Reserve raises the interest rate in response to a positive demand shock, firms attach some probability that a contractionary monetary shock might have occurred. Expecting a contractionary monetary shock tends to lower inflation expectations.

This dispersed information model (DIM) featuring signaling effects of monetary policy fits the data better than a model in which price setters have perfect information (i.e., the perfect information model or PIM). Quite interestingly, if the two models were estimated using a narrower data set that does not include the SPF, we could not have concluded that the DIM fits the data better than the PIM. In fact, we find that the PIM fits this narrower data set slightly better than the DIM. This finding suggests that the advantage of the DIM stems from its ability to fit the observed inflation expectations. In particular, the DIM is found to better fit the exceptionally high inflation expectations in the period ranging from 1975 through 1981.

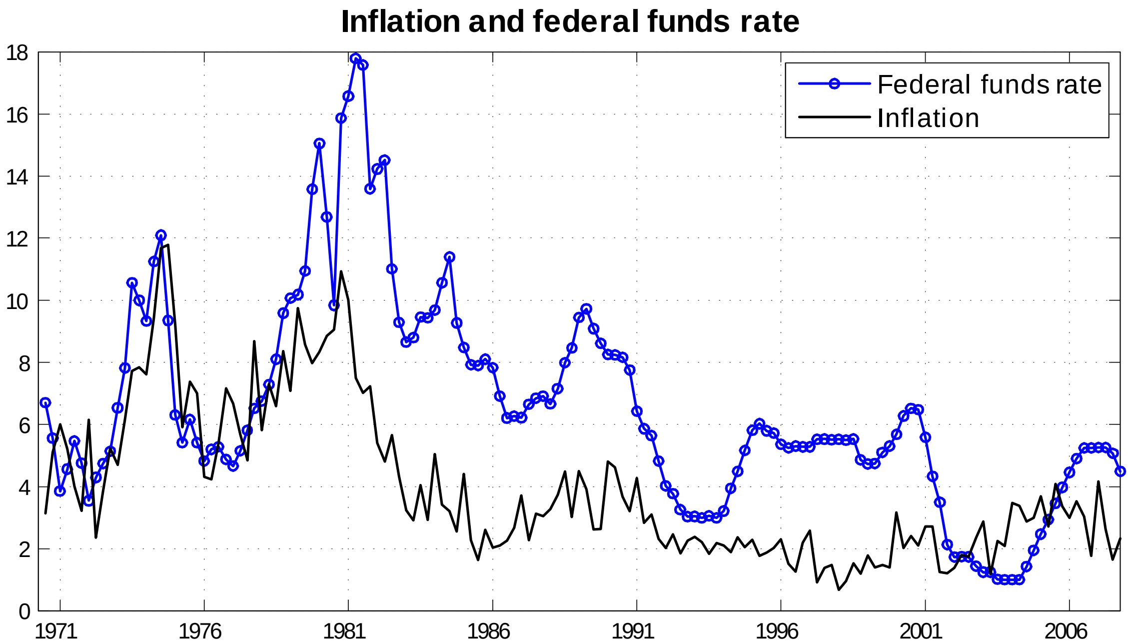

Figure 1: The inflation rate is obtained from the GDP deflator (*GDPDEF*) computed by the U.S. Bureau of Economic Analysis (BEA). The federal funds rate is the average of daily figures of the effective federal funds rate (*FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED). The vertical axis measures units of percentage points of annualized rates.

Using a Bayesian counterfactual experiment, we show that the signaling channel played a critical role in igniting inflation expectations in that period, critically helping the DIM fit the data.

Furthermore, the paper shows how to use *Bayesian counterfactuals* to shed light on the signaling effects of monetary policy on GDP, inflation, and inflation expectations. Our counterfactual analysis also reveals that the signaling effects of monetary policy are important in explaining why inflation did not suddenly fall as the Federal Reserve tightened monetary policy in the late 1970s, as shown in Figure 1. Furthermore, the signaling effects strongly exacerbated the Volcker's recession. The signaling effects associated with the low-frequency decline in the federal funds rate are found to have subdued the dynamics of inflation expectations from 1982 to 2004. The signaling effects of monetary policy had been rather stimulative throughout the 1970s but they had contributed to the slowdown in economic growth in the 1980s. In the first half of the 1990s and from 2001 through 2007, the signaling effects of monetary policy contributed to stimulating economic activity and lower the rate of inflation.

This paper also makes a methodological contribution by providing an algorithm to solve DSGE models in which agents find it optimal to forecast the forecasts of other agents. The solution routine proposed in the paper turns out to be sufficiently fast and reliable to allow likelihood-based estimation. The proposed algorithm belongs to the general solution methods developed by Nimark (2011). The proposed algorithm improves upon the one used in Nimark

(2008) as it does not require solving a system of nonlinear equations. Furthermore, the paper shows how to quantify information flows conveyed by the policy signal to private agents in the model.

The model studied in this paper is built on Nimark (2008). A particularly useful feature of Nimark's model is that the supply side of the model economy can be analytically worked out and characterized by an equation that nests the standard New Keynesian Phillips curve. The model studied in this paper shares this feature. Nonetheless, in Nimark (2008) the signaling channel does not arise because assumptions on the Taylor-rule specification imply that the policy rate conveys only redundant information to price setters.

This paper is also related to a quickly growing empirical literature that uses the SPF to study the response of public expectations to monetary policy decisions. Del Negro and Eusepi (2011) perform an econometric evaluation of the extent to which the inflation expectations generated by DSGE models are in line with the observed inflation expectations. There are two main differences between that paper and this one. First, in our settings, price setters have heterogeneous and dispersed higher-order expectations (HOE) as they observe private signals. Second, this paper fits the model to a data set that includes the 1970s, whereas Del Negro and Eusepi (2011) use a data set starting from the early 1980s. Coibion and Gorodnichenko (2011) find that the Federal Reserve raises the policy rate more gradually if the private sector's inflation expectations are lower than the Federal Reserve's forecasts of inflation. This empirical evidence can be rationalized in a model in which monetary policy has signaling effects and the central bank acts strategically to stabilize public inflation expectations.

The idea that the monetary authority sends public signals to an economy in which agents have dispersed information was pioneered by Morris and Shin (2003a). Morris and Shin (2003b), Angeletos and Pavan (2004, 2007), and Hellwig (2005) focus on the welfare effects of disclosing public information in models with dispersed information and complementarities. Angeletos, Hellwig, and Pavan (2006) study the signaling effects of policy decisions in a coordination game. Walsh (2010) shows that the (perceived or actual) signaling effects of monetary policy alter the central bank's decisions, resulting in a bias (i.e., an opacity bias) that distorts the central bank's optimal response to shocks. Unlike this paper, Walsh's study is based on a model that does not feature dispersed information. Baeriswyl and Cornand (2010) study optimal monetary policy in a DSGE model in which the central bank can use its policy instrument to disclose information about its assessment of the fundamentals. Price setters face two sources of information limitation: sticky information à la Mankiw and Reis (2002) and dispersed information of a type that is very similar to this paper. That contribution is mostly theoretical, whereas this paper carries out a full-fledged likelihood estimation of a model in which monetary policy has signaling effects.

There is a vast and quickly growing literature that studies models with dispersed information. Woodford (2002) and Maćkowiak and Wiederholt (2009) calibrate a price-setting model in which shocks are not common knowledge among firms to match a few stylized facts about long-lasting real effects of monetary disturbances. Maćkowiak and Wiederholt (2009) endogeneize the information structure, allowing firms to optimally allocate their limited attention. Maćkowiak and Wiederholt (2010) develop and calibrate a dynamic general equilibrium model in which both households and firms optimally allocate their limited attention to shocks. Angeletos and La'O (2009) develop a stylized monetary model to study the distinct role of higher-order beliefs in shaping the response of prices to shocks. Lorenzoni (2009) studies an island economy in which aggregate fluctuations are driven by the private sector's uncertainty about aggregate fundamentals. Adam (2009) shows that discretionary conduct of monetary stabilization policy can increase macroeconomic volatility if firms pay limited attention to aggregate developments. Gorodnichenko (2008) studies a model in which firms make state-dependent decisions on both pricing and acquisition of information. Hachem and Wu (2012) develop a model in which firms update their heterogeneous inflation expectations through social dynamics to study the effects of central bank communication.

This paper also belongs to a quite thin literature that carries out likelihood-based analyses on models with dispersed information. Nimark (2012) estimates an island model built on Lorenzoni (2009) and augmented with man-bites-dog signals, which are signals that are more likely to be observed when unusual events occur. Maćkowiak, Moench, and Wiederholt (2009) use a dynamic factor model to estimate impulse responses of sectorial price indexes to aggregate shocks and to sector-specific shocks for a number of models, including a rational inattention model. Melosi (2013) conducts an econometric analysis of a stylized DSGE model with dispersed information à la Woodford (2002).

Another empirical study related to this paper is Bianchi (forthcoming), who studies how agents' beliefs react to shifts in the monetary policy regime and the associated implications for the transmission mechanism of monetary policy. Bianchi and Melosi (2012) develop a DSGE model that features waves of agents' pessimism about how aggressively the central bank will react to future changes in inflation to study the welfare implications of monetary policy communication. Trabandt (2007) analyzes the empirical properties of a state-of-the-art sticky-information DSGE model à la Mankiw and Reis (2002) and compares them with those of a state-of-the-art DSGE model with sticky prices à la Calvo.

The paper is organized as follows. Section 2 describes the dispersed information model, in which monetary policy has signaling effects, as well as a model in which firms have complete information. The latter model will be used as a benchmark to evaluate the empirical performance of the dispersed information model. In Section 3, we perform some numerical experiment to show the macroeconomic propagation of monetary disturbances through the signaling channel. Section 4 deals with the empirical analysis of the paper. In Section 5, we conclude.

2 Models

Section 2.1 introduces the model with dispersed information and signaling effects of monetary policy. In Section 2.2, we present the time protocol of the model. Section 2.3 presents the problem of households. Section 2.4 presents firms' price-setting problem. In Section 2.5, the central bank's behavior and government's behavior are modeled. Section 2.6 deals with the log-linearization and the solution of the dispersed information model. Finally, Section 2.7 presents the perfect information model, which will turn out to be useful for evaluating the empirical significance of the dispersed information model.

2.1 The Dispersed Information Model (DIM)

The economy is populated by a continuum $(0, 1)$ of households, a continuum $(0, 1)$ of monopolistically competitive firms, a central bank (or monetary authority), and a government (or fiscal authority). A Calvo lottery establishes which firms are allowed to reoptimize their prices in any given period t (Calvo 1983). Households consume the goods produced by firms, demand government bonds, pay taxes to or receive transfers from the fiscal authority, and supply labor to the firms in a perfectly competitive labor market. Firms sell differentiated goods to households. The fiscal authority has to finance maturing government bonds. The fiscal authority can issue new government bonds and can either collect lump-sum taxes from households or pay transfers to households. The central bank sets the nominal interest rate at which the government's bonds pay out their return.

Aggregate and idiosyncratic shocks hit the model economy. The aggregate shocks are a technology shock, a monetary policy shock, and a demand shock. All of these shocks are orthogonal to each other at all leads and lags. Idiosyncratic shocks include a firm-specific technology shock and the outcome of the Calvo lottery for price optimization.

2.2 The Time Protocol

Any period t is divided into three stages. All actions that are taken in any given stage are simultaneous. At stage 0, shocks are realized and the central bank sets the interest rate for the current period t. At stage 1, firms update their information set by observing (i) their idiosyncratic technology, (ii) a private signal about the demand shocks, and (iii) the interest rate set by the central bank. Given these observations, firms set their prices at stage 1. At stage 2, households learn about the realization of all the shocks in the economy and therefore become perfectly informed. Households then decide their consumption, C_t; their demand for (one-period) nominal government bonds, B_t; and their labor supply, N_t. At this stage, firms hire labor and produce so as to deliver the demanded quantity at the price they have set at stage 1. The fiscal authority issues bonds and collects taxes from households or pay transfers to households. The markets for goods, labor, and bonds clear.

2.3 Households

Households have perfect information, and hence, we can use the representative household to solve their problem at stage 2 of every period t:

$$\max_{C_{t+s},B_{t+s},N_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} g_{t+s} \left[\ln C_{t+s} - \chi_n N_{t+s}\right],$$

where β is the deterministic discount factor and g_t denotes a preference shifter that scales up or down the period utility function. The logarithm of the preference shifter follows an autoregressive (AR) process: $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$ with Gaussian shocks $\varepsilon_{g,t} \backsim \mathcal{N}(0,1)$. We refer to g_t as demand conditions and to the innovation $\varepsilon_{g,t}$ as demand shock. Disutility from labor linearly enters the period utility function. Note that χ_n is a parameter that affects the marginal disutility of labor.

The flow budget constraint of the representative household in period t reads as

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t - T_t, \tag{1}$$

where P_t is the price level of the composite good consumed by households and W_t is the (competitive) nominal wage rate, R_t stands for the nominal (gross) interest rate, Π_t is the (equally shared) dividends paid out by the firms, and T_t stands for the lump-sum tax or transfers. Composite consumption in period t is given by the Dixit-Stiglitz aggregator $C_t = \left(\int_0^1 C_{j,t}^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}}$, where $C_{j,t}$ is consumption of the good produced by firm j in period t and ν is the elasticity of substitution between consumption goods.

At stage 2 of every period t, the representative household chooses its consumption of the good produced by firm j, labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum transfers/taxes, and the prices of all consumption goods. It can be shown that the demand for the good produced by firm j is:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\nu} C_t, \tag{2}$$

where the price level of the composite good is defined as $P_t = \left(\int (P_{j,t})^{1-\nu} di\right)^{\frac{1}{1-\nu}}$.

2.4 Firms' Price-Setting Problem

Firms are endowed with a linear technology $Y_{j,t} = A_{j,t}N_{j,t}$, where $Y_{j,t}$ is the output produced by the firm j at time t, $N_{j,t}$ is the amount of labor employed by firm j at time t, and $A_{j,t}$ is the firm-specific level of technology that can be decomposed into a level of aggregate technology (A_t) and a white-noise firm-specific component ($\varepsilon^a_{j,t}$). More specifically,

$$\ln A_{j,t} = \ln A_t + \widetilde{\sigma}_a \varepsilon^a_{j,t}, \tag{3}$$

where $\varepsilon^a_{j,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$ and $A_t = \gamma^t a_t$ with deterministic trend $\gamma > 1$ and a_t is the detrended level of aggregate technology that evolves according to the AR process $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_{a,t}$ with Gaussian shocks $\varepsilon_{a,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$.

Following Calvo (1983), we assume that a fraction θ of firms are not allowed to reoptimize their prices at stage 1 of any period. Those firms that are not allowed to reoptimize are assumed to index their prices to the steady-state inflation rate. Furthermore, we assume that firms have limited knowledge about the history of shocks that have hit the economy. More specifically, it is assumed that firms' information set includes the history of firm-specific technology $\ln A_{j,t}$, the history of a private signal $g_{j,t}$ on the demand conditions, the history of the nominal interest rate R_t set by the central bank, and the history of the price set by the firm. In terms of simbols, the information set $\mathcal{I}_{j,t}$ of firm j at time t is given by

$$\mathcal{I}_{j,t} \equiv \{\ln A_{j,\tau}, \ln g_{j,\tau}, R_\tau, P_{j,\tau} : \tau \leq t\}, \tag{4}$$

where $\ln g_{j,t}$ denotes the private signal concerning the demand conditions g_t. This signal is defined as follows:

$$\ln g_{j,t} = \ln g_t + \widetilde{\sigma}_g \varepsilon^g_{j,t}, \tag{5}$$

where $\varepsilon^g_{j,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. This signal is meant to capture the fact that arguably firms are used to carrying out market analyses to gather information about demand conditions g_t before setting their price. Firms receive the signals in $\mathcal{I}_{j,t}$ at stage 1 when they are called to set their price. Firms are assumed to know the model transition equations and their structural parameters. Furthermore, note that observing the history of their price $P_{j,t}$ conveys only redundant information to firms because their price is either adjusted to the steady-state inflation rate, which is known by firms, or a function of the history of the signals that have been already observed in the past. Thus, this signal does not play any role in the formation of firms' expectations and will be called the redundant signal. Henceforth, when we will refer to signals, we mean only the non-redundant signals (namely, $\ln A_{j,t}$, $\ln g_{j,t}$, and R_t). Finally, we assume that firms have received an infinitely long sequence of signals at any time t. This assumption substantially simplifies the task of solving the model by ensuring that the Kalman gain matrix is time invariant and is the same across firms.

Let us denote the (gross) steady-state inflation rate as π_*, the nominal marginal costs for firm j as $MC_{j,t} = W_t / A_{j,t}$, the time t value of one unit of the composite consumption good in period $t+s$ to the representative household as $\Xi_{t|t+s}$, and the expectation operator conditional on firm j's information set $\mathcal{I}_{j,t}$ as $\mathbb{E}_{j,t}$. At stage 1 of every period t, an arbitrary firm j that is allowed to reoptimize its price $P_{j,t}$ solves

$$\max_{P_{j,t}} \quad \mathbb{E}_{j,t} \left[\sum_{s=0}^{\infty} (\beta\theta)^s \, \Xi_{t|t+s} \left(\pi_*^s P_{j,t} - MC_{j,t+s} \right) Y_{j,t+s} \right],$$

subject to $Y_{j,t} = C_{j,t}$ (i.e., firms commit themselves to satisfying any demanded quantity that will arise at stage 2), to the firm's specific demand in equation (2), and to their production function. When solving the price-setting problem at stage 1, firms have to form expectations about the evolution of their nominal marginal costs, which will be realized in the next stage of the period (i.e., stage 2), using their information set $\mathcal{I}_{j,t}$. At stage 2, firms produce and deliver the quantity the representative household demands for their specific good at the price they set in the previous stage 1. It is important to recall that at stage 2, firms do not receive any further information or any additional signals to what they have already observed at stage 1.

Since firms find it optimal to set their prices in response to changes in their *nominal* marginal costs, they raise their prices, *ceteris paribus*, when they expect the price level to increase, that is, when they expect that the *other* price setters, on average, are raising their prices. Such a coordination motive in price setting and the availability of private information make it optimal for price setters to *forecast the forecasts* of other price setters (Townsend 1983a, 1983b). This feature of the price-setting problem raises challenges, which will be tackled in Section 2.6, when we solve the dispersed information model.

2.5 The Monetary and Fiscal Authority

There is a monetary authority and a fiscal authority. The monetary authority sets the nominal interest rate according to the reaction function:

$$R_t = (r_* \pi_*) \left(\frac{\pi_t}{\pi_*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_y} \eta_{r,t}, \tag{6}$$

where r_* is the steady-state real interest rate, π_t is the (gross) inflation rate, and Y_t^* is potential output. That is, the output level that would be realized if prices were perfectly flexible (i.e., $\theta = 0$). Note that $\eta_{r,t}$ is a random variable that affects the nominal interest rate in period t and is driven by the following process: $\ln \eta_{r,t} = \rho_r \ln \eta_{r,t-1} + \sigma_r \varepsilon_{r,t}$, with Gaussian shocks

$\varepsilon_{r,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. We will refer to the exogenous variable $\eta_{r,t}$ as the central bank's *deviation from the monetary rule* and to the innovation $\varepsilon_{r,t}$ as a monetary policy shock. The deviation from the monetary rule $\eta_{r,t}$ is intended to capture any exogenous deviation from the monetary policy, including central bank's errors in estimating the current rate of inflation and the current output gap or the central bank's willingness to smooth the dynamics of the policy rate R_t.

The flow budget constraint of the fiscal authority in period t reads as $R_{t-1}B_{t-1} - B_t = T_t$. The fiscal authority has to finance maturing government bonds. The fiscal authority can collect lump-sum taxes or issue new government bonds. Since there is neither capital accumulation nor government consumption, the resource constraint implies $Y_t = C_t$.

2.6 Log-linearization and Model Solution

First, we solve the firms' and households' problems, described in Sections 2.3 and 2.4, and obtain the consumption Euler equation and the price-setting equation. Second, we detrend the non-stationary variables before log-linearizing the model equations around their value at the nonstochastic steady-state equilibrium. Let us define the detrended real output as $y_t \equiv Y_t/\gamma^t$. We denote the log-deviation of an arbitrary (stationary) variable x_t from its steady-state value as $\widehat{x}_t$. As in Nimark (2008), we obtain the imperfect-common-knowledge Phillips curve that reads as[1]

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}_{t+1|t}^{(k)}, \tag{7}$$

where $\widehat{\pi}_{t+1|t}^{(k)}$ denotes the average k-th order expectations about the next period's inflation rate, $\widehat{\pi}_{t+1}$, that is, $\widehat{\pi}_{t+1|t}^{(k)} \equiv \underbrace{\int \mathbb{E}_{j,t}\ldots\int \mathbb{E}_{j,t}}_{k} \widehat{\pi}_{t+1}dj...dj$, any integer $k>1$. In addition, $\widehat{mc}_{t|t}^{(k)}$ denotes the average k-th order expectations about the real aggregate marginal costs $\widehat{mc}_t \equiv \int \widehat{mc}_{j,t}dj$, which evolve according to the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$, any integer $k>1$. Average higher-order expectations enter the specification of the Phillips curve because price setters find it optimal to *forecast the forecasts* of other price setters, as pointed out in Section 2.4.

The log-linearized IS equation is standard and reads as follows:

$$\widehat{g}_t - \widehat{y}_t = \mathbb{E}_t\widehat{g}_{t+1} - \mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{R}_t, \tag{8}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the complete information set, which includes the history of the three aggregate shocks. The central bank's reaction function

[1]See Appendix A for a detailed derivation of the Philips curve.

(6) can be written as the following:

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_y \left(\hat{y}_t - \widehat{y}_t^* \right) + \widehat{\eta}_{r,t}. \tag{9}$$

The demand conditions evolve according to $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \varepsilon_{g,t}$. The process for aggregate technology becomes $\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{a,t}$. The process leading the central bank's deviation from the monetary rule becomes $\widehat{\eta}_{r,t} = \rho_r \widehat{\eta}_{r,t-1} + \sigma_r \varepsilon_{r,t}$. We detrend and then log-linearize the signal equation concerning the level of aggregate technology (3) and obtain

$$\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a. \tag{10}$$

The signal equation concerning the demand conditions (5) is written as

$$\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g. \tag{11}$$

The signal about monetary policy is given by equation (9).

When firms solve their price-setting problem, they have to form expectations about the dynamics of their nominal marginal costs $MC_{j,t}$ by using their information set $\mathcal{I}_{j,t}$. To this end, firms solve a signal extraction problem using the log-linearized model equations, which are listed earlier, and the signal equations (9), (10), and (11).[2]

A detailed description of how we solve the model is provided in Appendix B. The proposed solution algorithm improves upon the one used in Nimark (2008) as our approach does not require solving a system of nonlinear equations.[3] When the model is solved, the law of motion of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t \right]'$ reads as follows:

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}, \tag{12}$$

where $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{\eta}_{r,t|t}^{(s)}, \widehat{g}_{t|t}^{(s)} : 0 \leq s \leq k \right]'$ is the vector of the average higher-order expectations (HOE) about the exogenous state variables (i.e., $\widehat{a}_t, \widehat{\eta}$, and $\widehat{g}_t$). The average s-th order expectations about the level of aggregate technology, $\widehat{a}_{t|t}^{(s)}$, are defined as the integral of firms' expectations about the average $(s-1)$-th order expectations across firms. In symbols,

[2] In the context of the log-linear model, observing R_t, as indicated in the information set (4), or $\widehat{R}_t$, as specified in the signal equation (9), does not change the price-setting decisions in that firms are assumed to know the model and, hence, the nonstochastic steady-state, including the nominal interest rate arising at the deterministic steady-state $\pi_* r_*$.

[3] Nimark (2009) introduces a method to improve the efficiency of these types of solution methods for dispersed information models in which agents (e.g., firms) use lagged endogenous variables to form their beliefs. An alternative solution algorithm based on rewriting the equilibrium dynamics partly as a moving average process and setting the lag with which the state is revealed to be a very large number is analyzed by Hellwig (2002) and Hellwig and Vankateswaran (2009).

$\widehat{a}_{t|t}^{(s)} = \int \mathbb{E}_{j,t}\left(\widehat{a}_{t|t}^{(s-1)}\right) dj$ for $1 \leq s \leq k$, where conventionally $\widehat{a}_{t|t}^{(0)} = \widehat{a}_t$. Analogously, the average HOE about the central bank's deviation from the monetary rule and demand conditions are given by $\widehat{\eta}_{r,t|t}^{(s)} = \int_{r,t|t}^{(s-1)} \widehat{\eta} dj$ for $1 \leq s \leq k$ and $\widehat{g}_{t|t}^{(s)} = \int \widehat{g}_{t|t}^{(s-1)} dj$ for $1 \leq s \leq k$, respectively. Note that we truncate the infinite hierarchy of average higher-order expectations, considering only orders smaller than or equal to the positive integer k. Henceforth, we set $k = 20$. The vector of average HOE is assumed to follow a Vector AutoRegressive (VAR) model of order 1[4]

$$X_{t|t}^{(0:k)} = \mathbf{M} X_{t-1|t-1}^{(0:k)} + \mathbf{N} \boldsymbol{\varepsilon}_t. \tag{13}$$

The parameter set of the log-linearized dispersed information model is given by the vector

$$\Theta_{DIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_a, \rho_g, \rho_r, \sigma_a, \widetilde{\sigma}_a, \sigma_g, \widetilde{\sigma}_g, \sigma_r, \gamma\right]'.$$

2.7 The Perfect Information Model (PIM)

If the noise variance of the private exogenous signals ($\widetilde{\sigma}_a$ and $\widetilde{\sigma}_g$) is equal to zero, higher-order uncertainty would fade away (i.e., $X_{t|t}^{(k)} = X_t$, for any integer k) and the linearized model would boil down to a prototypical (perfect information) three-equation New Keynesian DSGE model (e.g., Rotemberg and Woodford 1997; Lubik and Schorfheide 2004; and Rabanal and Rubio-Ramirez 2005). More specifically, the imperfect-common-knowledge Phillips curve (7) would become $\hat{\pi}_t = \kappa_{pc} \widehat{mc}_t + \beta \mathbb{E}_t \widehat{\pi}_{t+1}$, where $\kappa_{pc} \equiv (1-\theta)(1-\theta\beta)/\theta$ and the real marginal costs $\widehat{mc}_t = \hat{y}_t - \widehat{a}_t$. The IS equation and the Taylor rule would be the same as in the dispersed information model. In the perfect information model, the monetary shock propagates by affecting the intertemporal allocation the of consumption. The real effects of money solely emerge as a result of price stickiness as opposed to the sluggish adjustments of firms' expectations in the dispersed information model. We call this prototypical New Keynesian DSGE model the perfect information model (PIM). The parameter set of the log-linearized PIM is given by the vector

$$\Theta_{PIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_a, \rho_g, \rho_r, \sigma_a, \sigma_g, \sigma_r, \gamma\right]'.$$

3 The Signaling Channel of Monetary Transmission

A salient feature of the dispersed information model is that the policy rate R_t transfers information about the output gap and inflation to price setters. We call this transfer of information

[4] As is standard in the literature (e.g., Woodford 2002), we focus on equilibria where the higher-order expectations about the exogenous state variables follow a VAR model of order one. To solve the model we also assume common knowledge of rationality. See Nimark (2008, Assumption 1, p. 373) for a formal formulation of the assumption of common knowledge of rationality.

the *signaling channel of monetary transmission*. Price setters use the policy rate as a signal that helps them to track non-policy shocks (namely, technology shocks $\varepsilon_{a,t}$ and demand shocks $\varepsilon_{g,t}$) and, at the same time, to infer shocks to central bank's exogenous deviations from the monetary rule (i.e., monetary policy shocks $\varepsilon_{r,t}$).

This section is organized as follows. In Section 3.1, we define a set of measures to quantify the amount of information conveyed by the signals observed by firms. Measuring information flows will simplify the task of interpreting the macroeconomic implications of the signaling channel later on. In Section 3.2, we will shed light on the inflationary effects of the signaling channel.

3.1 Measuring Information Flows from the Signaling Channel

Following a standard practice in information theory (Cover and Thomas 1991), we use an entropy-based measure to assess how much information is provided by the signals firms observe in every period. The entropy measures the uncertainty about a random variable. For instance, the entropy associated with the level of aggregate technology $\widehat{a}_t$, which is normally distributed with (unconditional) covariance matrix $var(\widehat{a}_t)$, is defined as $H(\widehat{a}_t) \equiv 0.5 \log_2 [2\pi e \cdot var(\widehat{a}_t)]$.

We quantify the information flow conveyed by the signals as *the reduction of uncertainty* (i.e., entropy) at time t due to observing the signals in the information set $\mathcal{I}_{j,t}$. For instance, the information flow about aggregate technology conveyed by the signals in the information set $\mathcal{I}_{j,t}$ can be computed as $\mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t}) = H(\widehat{a}_t) - H(\widehat{a}_t | \mathcal{I}_{j,t})$, where the conditional entropy $H(\widehat{a}_t | \mathcal{I}_{j,t}) \equiv 0.5 \log_2 [2\pi e \cdot var(\widehat{a}_t | \mathcal{I}_{j,t})]$ and $var(\widehat{a}_t | \mathcal{I}_{j,t})$ denotes the variance of aggregate technology conditional on firms having observed the signals in their information set $\mathcal{I}_{j,t}$.[5] Note that having assumed that firms have received an infinitely long sequence of signals at any time t implies that the conditional covariance matrix $var(\widehat{a}_t | \mathcal{I}_{j,t})$ is time invariant and is the same across firms at any time. Hence, information flows do not vary over time or across firms and we can omit indexing the information flow $\mathcal{H}$ with j and t.

Analogously, define the entropy conditional on firms having observed *only* their private signals as $H(\widehat{a}_t | \mathcal{I}_{j,t} \diagup R^t) = 0.5 \log_2 [2\pi e \cdot var(\widehat{a}_t | \mathcal{I}_{j,t} \diagup R^t)]$, where $var(\widehat{a}_t | \mathcal{I}_{j,t} \diagup R^t)$ denotes the conditional variance of aggregate technology conditional on the history of private signals. We can hence measure the information flow that firms receive about aggregate technology from observing *solely the private signals* as $\mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t} \diagup R^t) \equiv H(\widehat{a}_t) - H(\widehat{a}_t | \mathcal{I}_{j,t} \diagup R^t)$. Let us define the entropy of aggregate technology conditional on firms having observed only the history of the policy signal as $H\left(\widehat{a}_t | \widehat{R}^t\right) \equiv 0.5 \log_2 \left[2\pi e \cdot var\left(\widehat{a}_t | \widehat{R}^t\right)\right]$, where $var\left(\widehat{a}_t | \widehat{R}^t\right)$ denotes the variance of aggregate technology conditional on firms having observed only the history of the

[5] The units of the measure $\mathcal{H}(\widehat{a}_t)$ are *bits* of information. The conditional variance can be pinned down by applying the Kalman-filter recursion, as shown in Appendix C.

policy signal R^t. We measure the information flow about aggregate technology only conveyed by the policy signal $\widehat{R}_t$ as $\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right)\equiv H\left(\widehat{a}_t\right)-H\left(\widehat{a}_t|\widehat{R}^t\right)$.

We compute the fraction of private information about the aggregate technology $\widehat{a}_t$ as the ratio of the private information flow to the information flow from all the signals in the information set $\mathcal{I}_{j,t}$; that is, $\vartheta_a\equiv\mathcal{H}\left(\widehat{a}_t;\mathcal{I}_{j,t}\diagup R^t\right)/\mathcal{H}\left(\widehat{a}_t;\mathcal{I}_{j,t}\right)$. It should be noted that $\vartheta_a\in[0,1]$. If ϑ_a is close to zero, then most of the information about aggregate technology stems from the policy signal. On the contrary, if ϑ_a is close to unity, then most of the information about aggregate technology stems from the private signal $\widehat{a}_{j,t}$.[6] Analogously, we can define the fraction of private information about the demand conditions $\widehat{g}_t$ as $\vartheta_g\equiv\mathcal{H}\left(\widehat{g}_t;\mathcal{I}_{j,t}\diagup R^t\right)/\mathcal{H}\left(\widehat{g}_t;\mathcal{I}_{j,t}\right)$.

Another useful statistic for assessing the macroeconomic effects of the signaling channel is the fraction of information about the non-policy exogenous state variables (i.e., the level of aggregate technology $\widehat{a}_t$ and the demand conditions $\widehat{g}_t$) conveyed by the policy signal. For instance, the fraction of information about the level of aggregate technology is computed as follows:

$$\Phi_a\equiv\frac{\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right)}{\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right)+\mathcal{H}\left(\widehat{\eta}_{r,t};\widehat{R}^t\right)+\mathcal{H}\left(\widehat{g}_t;\widehat{R}^t\right)}. \tag{14}$$

The numerator quantifies the information flow about the level of aggregate technology $\widehat{a}_t$ conveyed by the public signal. The denominator quantifies the information flow about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$) conveyed by the policy signal. This ratio Φ_a assumes values between zero and one. Note that while the ratio ϑ_a measures the accuracy of the private signal $\widehat{a}_{j,t}$ about level of aggregate technology $\widehat{a}_t$ *relative to that of the policy signal*, the ratio Φ_a evaluates the accuracy of the public signal *relative to the three exogenous state variables* (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$). Analogously, we can define the fraction of information about the deviation from the monetary rule conveyed by the policy signal Φ_m and the fraction of information about the demand conditions conveyed by the policy signal Φ_g. Note that $\Phi_a+\Phi_m+\Phi_g=1$.

Let us consider the following example. When the ratio ϑ_a is close to one, firms mostly rely on private information to learn about the aggregate technology. When ϑ_a is close to zero and Φ_a is sufficiently larger than zero, the quality of private information is rather poor relative to that of public information, and hence, firms mostly rely on the policy signal to learn about the level of aggregate technology. In this case, as we shall show in the next section, firms tend to interpret any changes in the policy rate as a response of the central bank to aggregate technology shocks and the inflationary effects of the signaling channel will be important. By the same token, the inflationary effects of the signaling channel depend on the relative precision of private information regarding the demand conditions relative to public information, as captured

[6] Note that the other private signal (i.e., $\widehat{g}_{j,t}$) does not convey any information about the level of aggregate technology because of the assumed orthogonality of structural shocks at all leads and lags.

by the ratios ϑ_g and Φ_g.

3.2 The Signaling Effects on the Propagation of Shocks

In this section, we analyze the signaling effects of monetary policy on *the propagation of structural shocks* (i.e., technology shocks, monetary policy shocks, and demand shocks) in the dispersed information model . It is illustrative to use equation (12) to decompose the effects of the average HOE about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$) on inflation, which leads us to the following:

$$\frac{\partial \pi_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{v}_a \cdot \frac{\partial X^a_{t+h}}{\partial \varepsilon_{i,t}} + \mathbf{v}_m \cdot \frac{\partial X^m_{t+h}}{\partial \varepsilon_{i,t}} + \mathbf{v}_g \cdot \frac{\partial X^g_{t+h}}{\partial \varepsilon_{i,t}}, \tag{15}$$

where $i \in \{a, r, g\}$ is the subscript that determines the shock of interest (i.e., technology, monetary, or demand shock) and the row vectors $\mathbf{v}_a, \mathbf{v}_m$, and $\mathbf{v}_g$ are subvectors of the second row of the matrix $\mathbf{v}_0$ in equation (12). Additionally, X^a_{t+h} is the column vector of h-step-ahead higher-order expectations (HOE) about the level of aggregate technology $\widehat{a}_t$, that is, $X^a_{t+h} \equiv \left[\widehat{a}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$.[7] Analogously, $X^m_{t+h} \equiv \left[\widehat{\eta}^{(s)}_{r,t+h|t} : 0 \leq s \leq k\right]$ and $X^g_{t+h} \equiv \left[\widehat{g}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$ are the column vectors of h-step-ahead HOE about the deviation from the monetary rule and the demand conditions, respectively.

It is important to emphasize that the signaling channel would be inactive if firms never observed the policy rate in the dispersed information model; that is, $R^t \notin \mathcal{I}_{j,t}$. Note that if the signaling channel were inactive, the impulse response vectors $\frac{\partial X^a_{t+h}}{\partial \varepsilon_{r,t}}$ and $\frac{\partial X^g_{t+h}}{\partial \varepsilon_{r,t}}$ would be equal to zero vectors at any time t and horizon h. This means that the average higher-order expectations about aggregate technology and demand conditions would not respond to monetary shocks because the policy rate is not observed and the private signals (i.e., $\widehat{a}_{j,t}$ and $\widehat{g}_{j,t}$) are orthogonal to monetary shocks. Furthermore, average HOE about central bank's deviations from the monetary rule X^m_{t+h} would not respond to non-policy shocks (i.e., $\varepsilon_{a,t}$ and $\varepsilon_{g,t}$) at any time t and horizon h if the signaling channel were inactive. More formally, when the signaling channel is inactive, the following zero restrictions hold: $\frac{\partial X^j_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{0}_{(k+1)\times 1}$ for all $i, j \in \{a, r, g\}^2$ and $i \neq j$. However, if the signaling channel is active, these zero restrictions do not necessarily apply because the policy signal is endogenous and conveys information about *all the aggregate shocks* that hit the economy. In other words, the policy signal is a *source of confusion* about the nature of shocks that have hit the economy, and therefore, the signaling channel relaxes the aforementioned zero restrictions. For instance, an increase in the policy rate can be interpreted by firms as the central bank's response to a contractionary monetary

[7] Conventionally, the average zero-order expectation about a random variable (say, the level of aggregate technology, $\widehat{a}_t$) is equal to the variable itself; that is, $\widehat{a}^{(0)}_{t+h|t} = \widehat{a}_{t+h}$, for any h.

shock, an adverse technology shock, or a positive demand shock.

The decomposition in equation (15) provides us with a powerful tool to interpret the signaling effects of monetary policy on the propagation of structural shocks. To shed light on the signaling effects of monetary policy on inflation expectations, we work out a decomposition for the average inflation expectations of the form (15), using the fact that the law of motion for the average first-order expectations about the endogenous variables $\mathbf{s}_t$ reads as $\mathbf{s}^{(1)}_{t+h|t} = \mathbf{v}_0 \mathbf{M}^h X^{(1:k+1)}_{t|t}$.[8]

Endowed with these tools, we conduct a simple numerical experiment, in which we set the Calvo parameter $\theta = 0.65$, the autoregressive parameters $\rho_a = 0.85$ and $\rho_r = 0.65$, and the variances $\sigma_a = 0.7$ and $\sigma_r = 0.5$. The central bank's response to inflation is determined by $\phi_\pi = 1.5$. For simplicity, we shut down the demand shocks (i.e., $\sigma_g = 0$) and the policy response to output gap (i.e., $\phi_y = 0$). Let us first consider the propagation of a contractionary monetary policy shock when the signal noise $\widetilde{\sigma}_a$ is set to be equal to the standard deviation of the aggregate technology shock, σ_a (i.e., the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a = 1$). In this numerical case, firms receive 92 percent of their overall information about the aggregate technology from the private signal (i.e., $\vartheta_a = 0.92$), suggesting that firms mostly rely on private information to learn about the non-policy shock. Firms will use the policy signal to mainly learn about monetary policy shocks. The bottom graphs of Figure 2 report the response of the average higher-order expectations about aggregate technology $\partial X^a_{t+h}/\partial \varepsilon_{r,t}$ (bottom left graph) and about the deviation from the monetary rule $\partial X^m_{t+h}/\partial \varepsilon_{r,t}$ (bottom right graph) from the first order up to the third order for h periods after the monetary shock.[9] Note that the average HOE about the deviation from the monetary rule are very close to the true value $\widehat{\eta}_{r,t+h}$, which is denoted by the red circles, showing that firms can rather easily figure out that the observed rise in the policy rate is due to a contractionary monetary policy shock.

The top left graph of Figure 2 shows that the response of inflation to the contractionary monetary shock is negative. The vertical bars in the top left graph are related to the decomposition (15) and isolate the effects of the change in the average HOE about aggregate technology on inflation h periods after the shock, $\mathbf{v}'_a \partial X^a_{t+h}/\partial \varepsilon_{r,t}$, from those about the deviation from the monetary rule, $\mathbf{v}'_m \cdot \partial X^m_{t+h}/\partial \varepsilon_{r,t}$. If the signaling channel were inactive (i.e., firms do not observe the policy rate), the gray bars would have zero length. It can be observed that the inflationary effects of HOE about aggregate technology (i.e., the gray vertical bars) are positive. This happens because the observed rise in the policy rate misleads firms, inducing them to believe − to some extent − that the central bank has raised the policy rate in response to a negative technology shock, as confirmed by the response of the average HOE about aggregate technology in the bottom left plot of Figure 2. However, note that these effects are very small

[8]For a formal proof of this result, see Appendix D, Proposition 2.

[9]Note that since we shut down the demand shock, $\partial X^g_{t+h}/\partial \varepsilon_{r,t} = \mathbf{0}_{(k+1)\times 1}$.

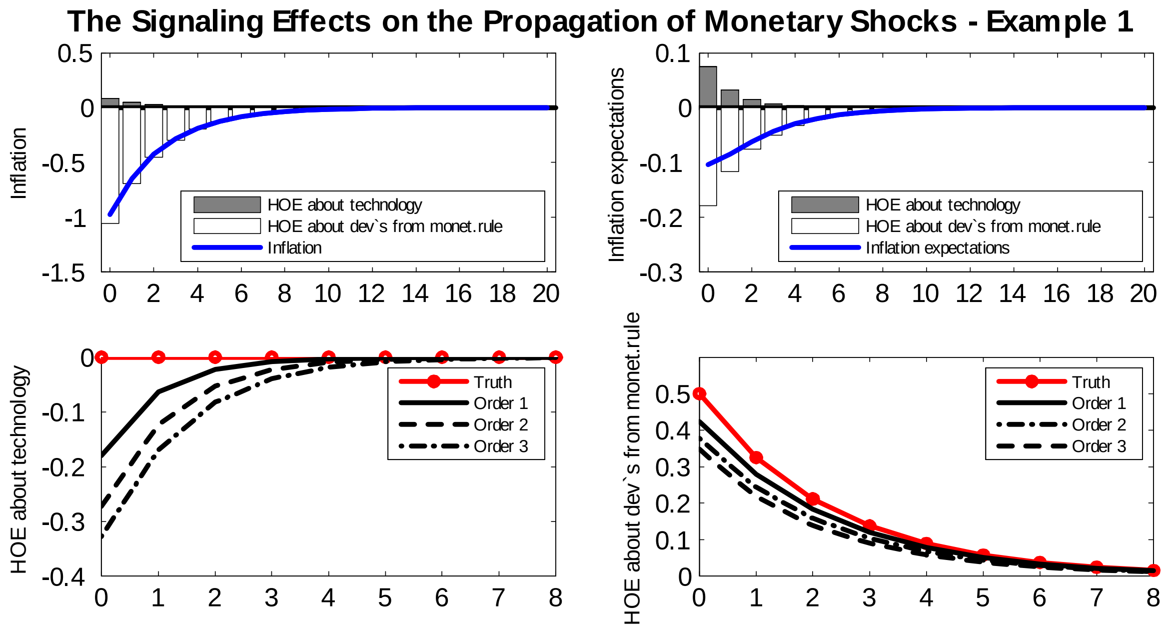

Figure 2: Impulse response functions to a one-standard deviation contractionary monetary shock: the case $\sigma_a/\widetilde{\sigma}_a = 1$, $\sigma_r = 0.5$, and $\phi_\pi = 1.5$. *Top plots*: Response of inflation (left) and inflation expectations (right). The gray (white) bars denote the effects of average higher-order expectations about the deviation from the monetary rule $\widehat{\eta}_{r,t}$ (about the level of aggregate technology $\widehat{a}_t$) on inflation or inflation expectations. *Bottom plots*: Response of average expectations about level of aggregate technology $\widehat{a}_t$ (left) and about the central bank's deviation from the monetary rule $\widehat{\eta}_{r,t}$ (right). The red line with circles marks the true value of the level of aggregate technology $\widehat{a}_t$ or the deviation from the monetary rule $\widehat{\eta}_{r,t}$. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in the top graphs represents units of percentage points of annualized rates.

quantitatively because firms acquire most of the information about aggregate technology from their private signals ($\vartheta_a = 0.92$). The effects of the signaling channel on inflation expectations are quantitatively more important, as shown in the top right graph of Figure 2.

Let us now turn our attention to the case in which (i) firms have imprecise private information (i.e., the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a$ is equal to 0.05) and (ii) the central bank's estimates about inflation and output gap are more accurate than those in the previous example (i.e., $\sigma_r = 0.1$). The following two features emerge. First, now firms acquire less private information about aggregate technology compared with the previous example as $\vartheta_a = 0.02 < 0.92$. Such a tiny ratio ϑ_a implies that firms have to almost entirely rely on the policy signal to learn about aggregate technology. Second, most of the information that firms receive from the policy signal is now about aggregate technology, since $\Phi_a = 0.84$. That the signaling channel conveys a lot of information about the non-policy shocks is confirmed by the bottom graphs of Figure 3. The response of the average HOE about the deviation from the monetary rule are quite far from the truth (i.e., the red line with circles), suggesting that these average HOE respond weakly to the observed rise in the policy rate due to the monetary shock. Conversely, the average

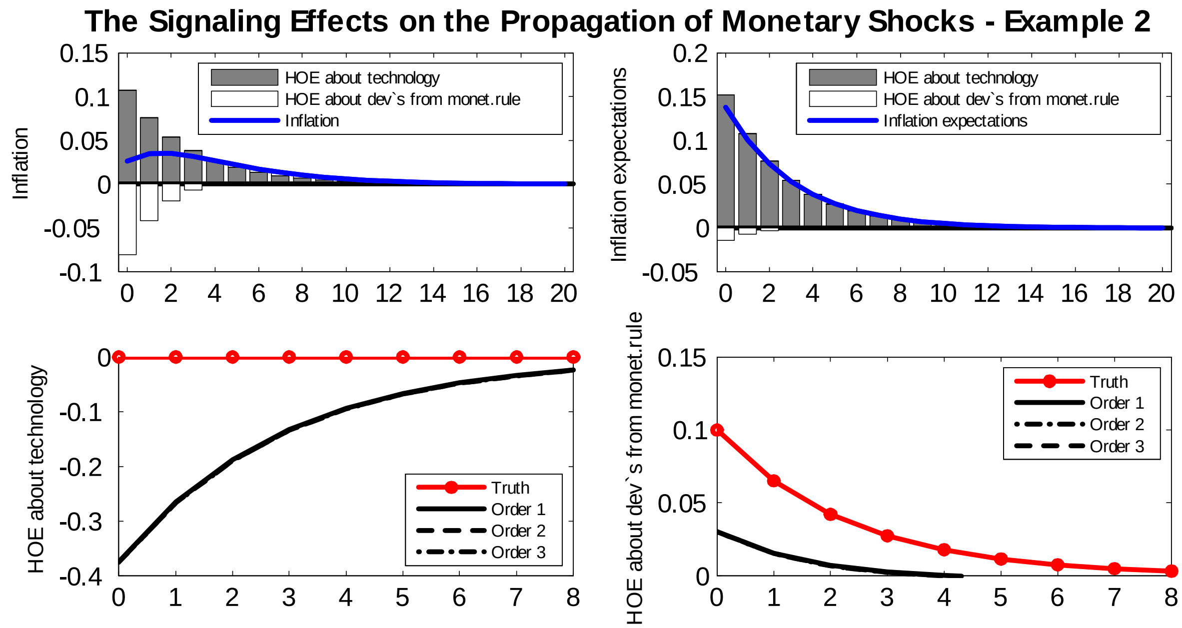

Figure 3: Impulse response functions to a one-standard deviation contractionary monetary shock: the case $\sigma_a/\widetilde{\sigma}_a = 0.05$, $\sigma_r = 0.1$, and $\phi_\pi = 1.5$. HOE means higher-order expectations.

HOE about aggregate technology respond more strongly to the monetary shocks, suggesting that firms mainly interpret the monetary tightening as the response of the central bank to an inflationary technology shock.

This is exactly the situation in which the inflationary effects of the signaling channel are *very strong*. The top graphs of Figure 3 show the effects of the signaling channel on inflation and inflation expectations as captured by the gray vertical bars. Signaling effects are so strong that inflation and inflation expectations actually rise in response to a contractionary monetary shock. Positive responses of the inflation rate to monetary shocks have been empirically documented in the VAR literature (Sims 1992; Uhlig 2005) and dubbed by Eichenbaum (1992) as a *price puzzle*. Having inflation respond positively to monetary shocks is not puzzling in this model with signaling effects of monetary policy, as rises (cuts) in the policy rate signals price setters about the occurrence of inflationary (deflationary) non-policy shocks.

We now focus on the case in which firms' private information is still quite imprecise (i.e., $\sigma_a/\widetilde{\sigma}_a = 0.05$), but the variance of the monetary shock is $\sigma_r = 0.5$ and hence bigger than that of the previous example. As in the previous case, firms do not rely on the private signal to learn about aggregate technology because this signal provides less information than the policy signal ($\vartheta_a = 0.17$). Nevertheless, unlike the previous case but similar to the first case, the policy signal is relatively less informative about aggregate technology than about the deviation from the monetary rule ($\Phi_a = 0.07$), since the central bank is less precise in estimating current inflation and output gap. Figure 4 shows that in this numerical example, the signaling channel

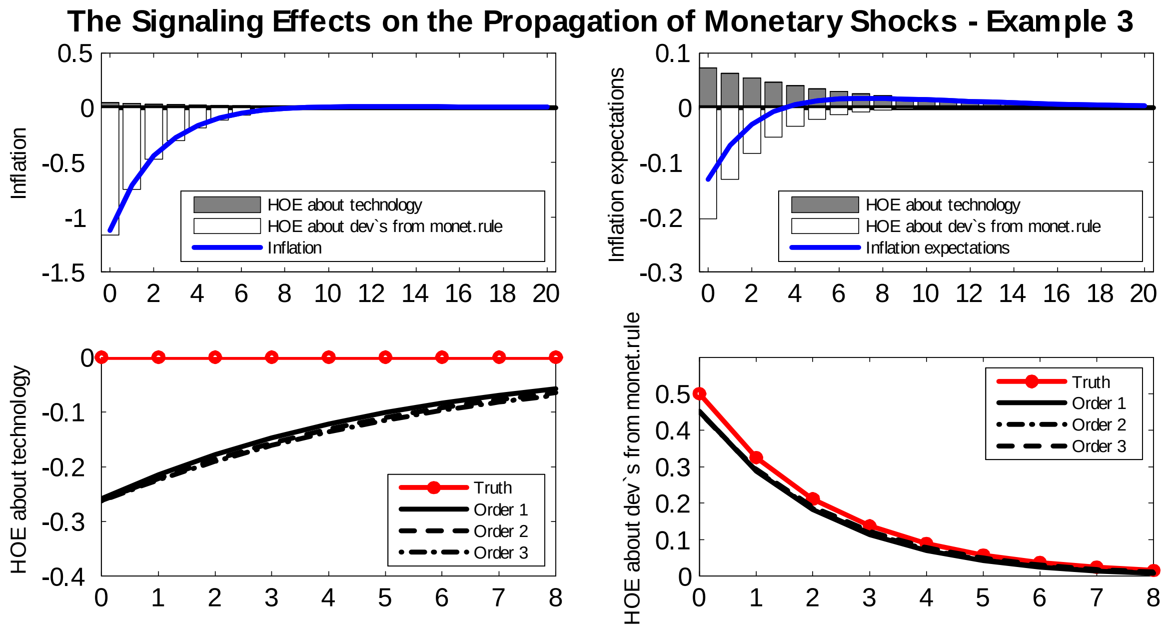

Figure 4: Impulse response functions to a one-standard deviation contractionary monetary shock: the case $\sigma_a/\widetilde{\sigma}_a = 0.05$, $\sigma_r = 0.5$, and $\phi_\pi = 1.5$. HOE means higher-order expectations.

has a quite *weak* inflationary effect and the price puzzle disappears, similar to the first numerical example depicted in Figure 2.

Finally, the inflationary consequences associated with non-policy shocks also influence the inflationary effects of the signaling channel. One parameter that clearly affects the inflationary consequences associated with technology shocks is the policy parameter ϕ_π. This parameter controls how forcefully the central bank adjusts the policy rate in response to inflation deviations from target. Suppose that the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a = 0.05$ and the standard deviation of the monetary shock $\sigma_r = 0.1$, exactly as in the second example where the inflationary effects of the signaling channel were so strong to give rise to the price puzzle (Figure 3). Unlike that example, though, the central bank adjusts the policy rate in a more aggressive manner: $\phi_\pi = 2.25 > 1.50$. Note that, as in the second example, private information still conveys less information about aggregate technology than the policy signal ($\vartheta_a = 0.01$) and the public signal is very informative about the non-policy shocks (i.e., $\Phi_a = 0.94$). Nonetheless, Figure 5 illustrates that a more aggressive monetary policy effectively limits the inflationary implications of the signaling channel. The reason is that while firms still fear that a change in the policy rate may correspond to an inflationary non-policy shock, they are also confident that the central bank will be able to shelter the economy from the inflationary consequences of such a shock. Such a confidence turns out to limit the inflationary consequences of the signaling channel.

Quite interestingly, recent empirical works (Castelnuovo and Surico 2010; Hanson 2004) have found that the price puzzle has become statistically insignificant after the 1970s $-$ that is,

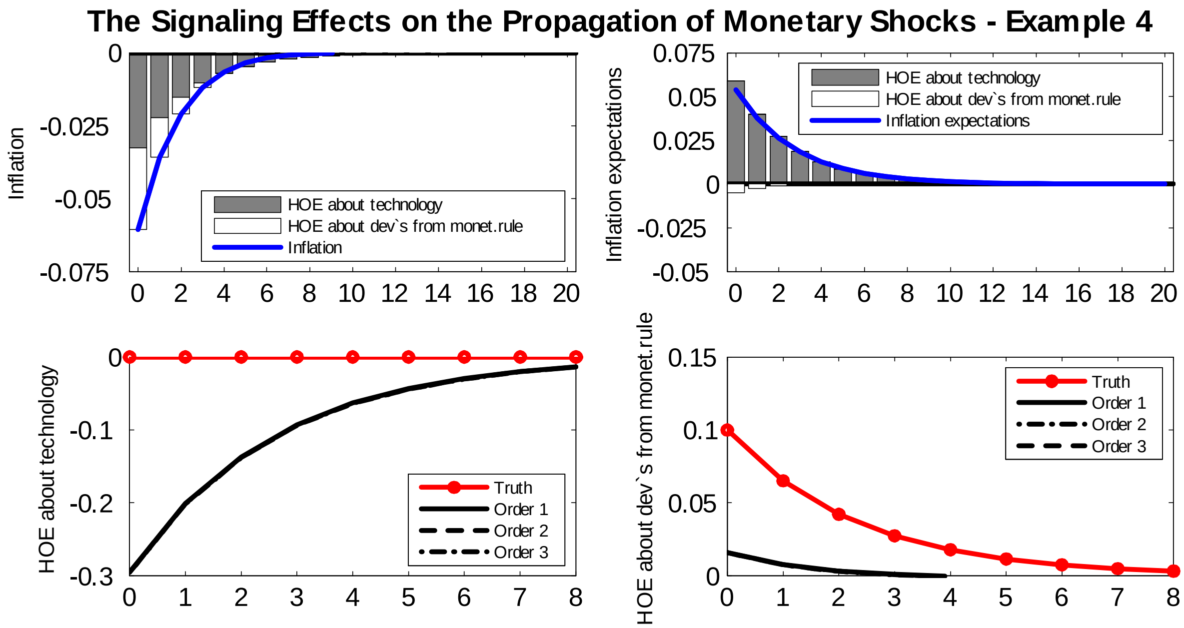

Figure 5: Impulse response functions to a one-standard deviation contractionary monetary shock: the case $\sigma_a/\widetilde{\sigma}_a = 0.05$, $\sigma_r = 0.1$, and $\phi_\pi = 2.25$. HOE means higher-order expectations.

exactly when U.S. monetary policy has arguably become more aggressive in stabilizing inflation.

Furthermore, note that the effects of HOE about aggregate technology are now deflationary as the gray bars in the top left graph lie in negative territory. This is a general-equilibrium effect that happens because a more aggressive monetary policy induces a larger drop of the average HOE about real marginal costs $\widehat{mc}_{t|t}^{(0:k)}$ in response to the fall in the average HOE about aggregate technology. This effect will be analyzed in more detail in Section 4.3. Finally, the response of inflation expectations is still positive but smaller than when the central bank is less aggressive in stabilizing the inflation rate, as reported in Figure 3. A negative response of inflation expectations can be obtained if the central bank followed a rule that requires adjusting the policy rate aggressively to changes in inflation expectations.

To sum up, we note that inflationary consequences of the signaling channel are strong when the following two conditions hold: (i) firms' private information is noisier than policy information and (ii) the policy rate is more informative about non-policy shocks than about monetary policy shocks. If the central bank adjusts its policy rate aggressively in response to inflation deviations from the target, the signaling effects of monetary policy on inflation and inflation expectations are reduced.

4 Empirical Analysis

This section contains the quantitative analysis of the model. Section 4.1 presents the data set and the state-space model for the econometrician. In Section 4.2, we discuss the prior and posterior distribution for the model parameters. In Section 4.3 we study the propagation of unanticipated monetary disturbances with particular emphasis on the functioning of the signaling channel. In Section 4.4, we deal with how the signaling channel affects the propagation of non-policy shocks, such as the technology shock and the demand shock. In Section 4.5, we formally assess the empirical performance of the dispersed information model relative to that of perfect information model introduced in Section 2.7. In Section 4.6, we evaluate the signaling effects of monetary policy on the observed dynamics of gross domestic product (GDP), the rate of inflation, and inflation expectations.

4.1 The State-Space Model for the Econometrician

The data set includes five observable variables: the U.S. GDP growth rate, U.S. inflation rate (from the GDP deflator), the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations. The data are quarterly and range from 1970:Q3 through 2007:Q4. Data on inflation expectations are obtained from the *Survey of Professional Forecasters* (SPF). The measurement equations are:

$$\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right) - \ln\left(\frac{GDP_{t-1}}{POP_{t-1}^{\geq 16}}\right) = \ln\gamma + \hat{y}_t - \hat{y}_{t-1},$$

$$\ln\left(\frac{PGDP_t}{PGDP_{t-1}}\right) = \ln\pi_* + \hat{\pi}_t,$$

$$FEDRATE_t = \ln R_* + \hat{R}_t,$$

$$\ln\left(\frac{PGDP3_t}{PGDP2_t}\right) = \hat{\pi}^{(1)}_{t+1|t} + \ln\pi_* + \sigma_{m_1}\varepsilon_t^{m_1},$$

$$\ln\left(\frac{PGDP6_t}{PGDP5_t}\right) = \hat{\pi}^{(1)}_{t+4|t} + \ln\pi_* + \sigma_{m_2}\varepsilon_t^{m_2},$$

where GDP_t is the gross domestic product computed by the U.S. Bureau of Economic Analysis (BEA) (i.e., *GDPC96*); $POP_t^{\geq 16}$ is the civilian non-institutional population aged 16 years old and over as computed by the U.S. Bureau of Labor Statistics (BLS) (i.e., *LNS10000000*); $PGDP_t$ is the GDP deflator computed by the BEA (i.e., *GDPDEF*); $FEDRATE$ is the average of daily figures of the effective federal funds rate (*FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED) database managed by the Federal Reserve Bank of St. Louis; and $PGDP2_t$, $PGDP3_t$, $PGDP5_t$, and $PGDP6_t$ are the SPF's mnemonics for the median forecasts

	DIM - Posterior			PIM - Posterior			Prior		
Name	Mean	5%	95%	Mean	5%	95%	Type	Mean	Std.
θ	0.2613	0.2450	0.2801	0.5796	0.5468	0.6114	$\mathcal{B}$	0.50	0.30
ϕ_π	1.0629	1.0451	1.0820	1.3234	1.2324	1.4200	$\mathcal{G}$	1.50	0.10
ϕ_y	0.3416	0.3212	0.3607	0.4356	0.1918	0.6560	$\mathcal{G}$	0.25	0.10
ρ_r	0.8613	0.8520	0.8713	0.4690	0.4163	0.5224	$\mathcal{B}$	0.50	0.20
ρ_a	0.9932	0.9911	0.9963	0.9751	0.9667	0.9832	$\mathcal{B}$	0.50	0.20
ρ_g	0.8505	0.8408	0.8597	0.8192	0.7949	0.8435	$\mathcal{B}$	0.50	0.20
$100\sigma_a$	0.7569	0.6440	0.8516	0.9961	0.8973	1.0957	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_a$	1.6048	1.3517	1.8332	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_g$	2.7843	2.6976	2.8610	0.8169	0.6908	0.9421	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_g$	34.277	30.789	38.068	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_r$	0.6372	0.6267	0.6429	0.6832	0.5717	0.7947	$\mathcal{IG}$	0.80	1.50
$100\sigma_{m_1}$	0.1291	0.1145	0.1452	0.1753	0.1585	0.1923	$\mathcal{IG}$	0.10	0.08
$100\sigma_{m_2}$	0.1222	0.1087	0.1381	0.1727	0.1565	0.1892	$\mathcal{IG}$	0.10	0.08
$100\ln\gamma$	0.4889	0.3786	0.5927	0.3302	0.3030	0.3556	$\mathcal{N}$	0.62	0.10
$100\ln\pi_*$	0.8327	0.7181	0.9514	0.7374	0.6124	0.8655	$\mathcal{N}$	0.65	0.10

Table 1: Prior and Posterior Statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM)

about the current, one-quarter-ahead, three-quarters-ahead, and four-quarters-ahead GDP price indexes, respectively. We relate these statistics with the first moment of the distribution of firms' expectations implied by the model. To avoid stochastic singularity, we introduce two independently and identically distributed (i.i.d.) Gaussian measurement errors $\varepsilon_t^{m_1}$ and $\varepsilon_t^{m_2}$.

4.2 Bayesian Estimation

At the deterministic steady state, the discount factor β depends on the linear trend of real output γ and the steady-state real interest rate R_*/π_*. Hence, we fix the value for this parameter so that the steady-state nominal interest rate R_*, the inflation rate π_*, and the growth rate γ match their sample mean. The prior and posterior statistics for the model parameters are reported in Table 1. The prior distribution for the Calvo parameter θ is chosen so as to put probability mass to values that are consistent with studies on price setting at micro levels (Bils and Klenow 2004; Klenow and Kryvtsov 2008; Nakamura and Steinsson 2008; Klenow and Malin 2010). The priors for the autoregressive parameters ρ_a, ρ_r, and ρ_g are broad enough to accommodate a wide range of persistence degrees for the three exogenous processes. The prior for the monetary policy parameters is set to be fairly tight because the inflation parameter ϕ_π seems to be only weakly identified in the perfect information model.[10] The volatility of the

[10]The posterior statistics for the DIM parameters reported in Table 1 and all the results in the subsequent sections would be substantially unchanged if one estimates the models using a broader prior for the policy parameters ϕ_π and ϕ_y.

monetary policy shock (σ_r) and that of the demand shock (σ_g) are informally taken according to the rule proposed by Del Negro and Schorfheide (2008) that the overall variance of GDP growth rate, inflation, and policy rate is roughly close to that observed in the presample, which ranges from 1960:Q1 through 1970:Q2. The variance of the idiosyncratic technology ($\widetilde{\sigma}_a$) and that for the private signal concerning the demand conditions ($\widetilde{\sigma}_g$) are crucial for the macroeconomic implications of the signaling channel, as shown in Section 3.2. To avoid conjecturing *a priori* that the signaling channel strongly influences firms' beliefs about non-policy shocks, we set a loose uniform prior over these parameters. It can be shown that the signaling channel implied by the prior has quantitatively weak inflationary effects.[11] The implied 95 percent prior credible set for the fraction of public information about the three exogenous state variables (i.e., Φ_a, Φ_m, and Φ_g) spans the spectrum of admissible values $(0, 1)$. Finally, the prior mean for the measurement errors (i.e., σ_{m1}, σ_{m2}) is set so as to match the variance of inflation expectations reported in the *Livingston Survey*.

We combine a prior distribution for the parameter set of the two models (i.e., the DIM and the PIM) with their likelihood function and conduct Bayesian inference. As explained in Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007), a closed-form expression for the posterior distribution is not available, but we can approximate the moments of the posterior distributions via the Metropolis-Hastings algorithm. We obtain 250,000 posterior draws for the dispersed information model and 1,000,000 draws for the perfect information model. The posterior statistics for the parameters of the perfect information model are fairly standard. As far as the DIM is concerned, the posterior mean for the Calvo parameter θ implies very flexible price contracts, whose implied duration is roughly four months. Such highly flexible prices imply that those firms that are allowed to reoptimize are primarily concerned with the average higher-order expectations about *current* real marginal costs, $\widehat{mc}_{t|t}^{(k)}$. See the imperfect-common-knowlegde Phillips curve (7).

The posterior mean for the inflation coefficient of the Taylor rule (ϕ_π) is substantially smaller than its prior mean. As discussed in Section 3, an accommodative monetary policy raises the inflationary consequences of shocks, and hence *ceteris paribus*, strengthens the inflationary effects of the signaling channel. The posterior mean for the variance of the monetary shock (σ_r) is slightly smaller than what is conjectured in the prior. As discussed in Section 3, a small variance of monetary shocks makes the policy signal $\widehat{R}_t$ more informative about non-policy shocks (i.e., demand and aggregate technology shocks) and, hence, tends to strengthen the inflationary effects of the signaling channel.

The posterior mean for the variance of the firm-specific technology shock $\widetilde{\sigma}_a$ implies that the posterior mean of the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a$ is 0.47. The posterior mean for the signal-to-noise ratio $\sigma_g/\widetilde{\sigma}_g$ is extremely small, suggesting that firms are less informed about demand

[11] See Appendix E.

shocks than about aggregate technology shocks. These estimates suggest that, *ceteris paribus*, firms will rely more on the policy signal to learn about demand shocks than to learn about aggregate technology shocks. This intuition is confirmed by looking at the statistics introduced in Section 3.1. The posterior distribution implies that the policy signal is mainly informative about aggregate technology as roughly $\Phi_a = 80$ percent of the public information flow is about aggregate technology. Nevertheless, firms almost entirely learn about the aggregate technology from their private signal (i.e., the level of their idiosyncratic technology): the 95-percent posterior credible set for the fraction of private information about the aggregate technology ϑ_a ranges from 0.9972 to 0.9983. These numbers suggest that the accuracy of private information about the level of aggregate technology is much higher than that of public information. Conversely, firms have to rely on the policy signal $\widehat{R}_t$ to learn about the demand conditions $\widehat{g}_t$, since the private signal conveys only $\vartheta_g = 0.10$ of the overall information firms gathered about this exogenous state variable. It is important to notice that the remaining $1 - \Phi_a = 20$ percent of information conveyed by the policy signal is equally split between information about the deviation from the monetary rule $\widehat{\eta}_{r,t}$ and demand conditions $\widehat{g}_t$, as the posterior means for Φ_m and Φ_g are roughly equal to 0.10. As we shall see, this feature will make it hard for firms to tell whether the observed changes in the policy rate are due to monetary shocks or to demand shocks.

To sum up, we find that firms receive most of their information about aggregate technology from their private signal but have to rely on the policy signal to learn about monetary shocks and demand shocks. The policy rate is found to be equally informative about these two types of shocks. Moreover, the central bank responds fairly weakly to inflation. In such an environment, the inflationary effects of the signaling channel are expected to be important, since firms will tend to interpret changes in the policy rate as the central bank's responses to demand shocks. We will analyze the implication of this feature in the next two sections.

4.3 Propagation of Monetary Shocks

Figure 6 shows the impulse response functions (and their 95 percent posterior credible sets in gray) of GDP growth rate, the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a monetary shock that raises the interest rate by 25 basis points. Two features of these impulse response functions have to be emphasized. First, inflation expectations respond positively to a monetary policy shock. Second, inflation and inflation expectations seem to react fairly sluggishly even though the estimated average duration of the price contract is only four months.

To shed light on the origins of such a high price rigidity, we plot the vertical bars in the top left graphs of Figure 7 that capture the decomposition (15) and measure the inflationary effects

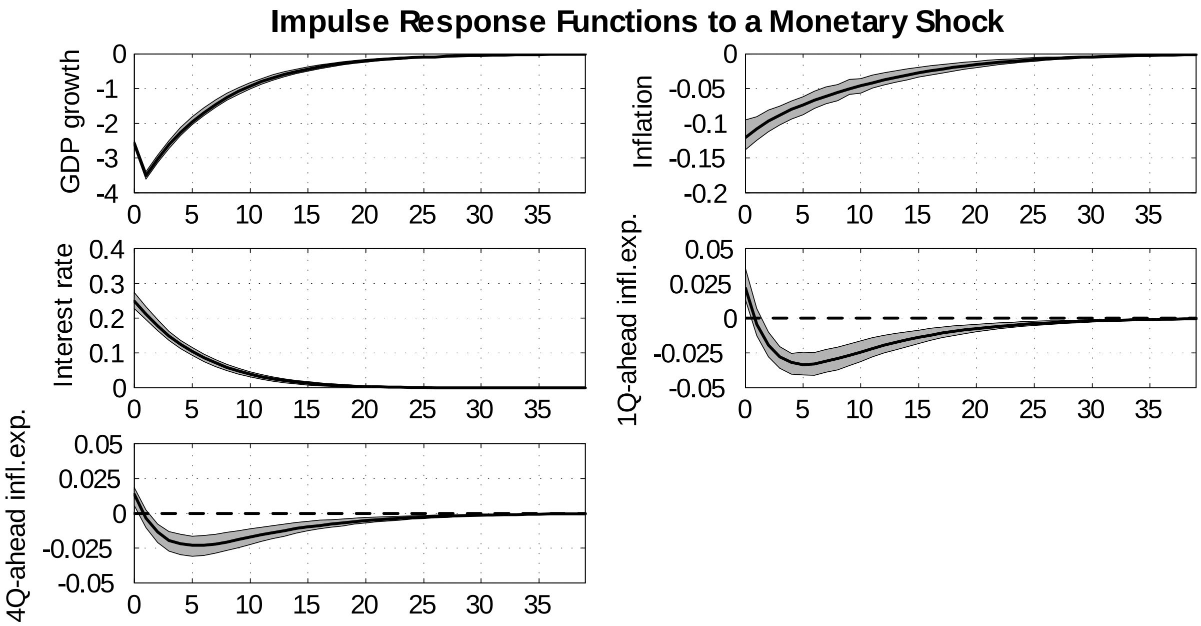

Figure 6: Impulse response function of the observable variables to a monetary shock that raises the interest rate by 25 bps. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

of the change in the average higher-order expectations (HOE) about technology $\mathbf{v}_a' \partial X_{t+h}^a / \partial \varepsilon_{r,t}$ (the gray bars), the deviation from the monetary rule $\mathbf{v}_m' \cdot \partial X_{t+h}^m / \partial \varepsilon_{r,t}$ (the white bars), and demand conditions $\mathbf{v}_g' \cdot \partial X_{t+h}^g / \partial \varepsilon_{r,t}$ (the black bars) for h periods after the monetary shock. The sum of the three vertical bars in the top graphs delivers the response of inflation and inflation expectations (i.e., the solid lines in the upper graphs). We observe a large effect of the average HOE about the demand conditions (i.e., the black bars), which can be interpreted as a situation in which price setters believe that the federal funds rate has been raised in response to a positive demand shock (see the lower right graph). Such an interpretation gives rise to inflationary pressures (captured by the black bars), which dampen the deflationary consequences (i.e., the white bars) associated with the contractionary monetary shock.

The three bottom graphs of Figure 7 report the response of the average HOE about the three exogenous state variables. The average HOE about aggregate technology fall right after the shock and then go back to zero rather quickly. Average HOE about the demand conditions rise substantially and remain misaligned with the truth for quite a long time (see the lower right graph). The average HOE about the deviation from the monetary rule are quite far from the truth and also remain so for a long period of time (see the lower middle graph). These responses of the average HOE about the three exogenous state variables confirm that firms mostly interpret the rise in the interest rate as the central bank's response to an inflationary demand shock. This triggers persistently high black bars in the top graphs, delivering a fairly

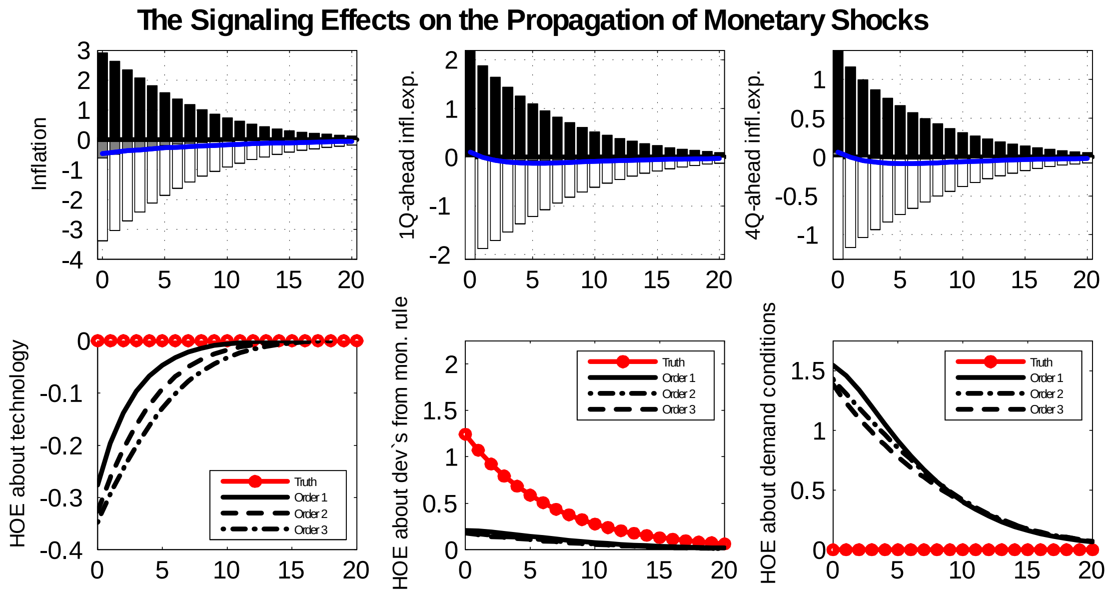

Figure 7: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady state equilibrium.

high degree of price rigidity.

To understand why firms interpret the rise in the policy rate as the central bank's response to a demand shock, recall from Section 4.2 that firms receive poor private information about the demand conditions ($\vartheta_g = 0.10$) and hence have to rely on the policy signal. The policy signal is equally informative about the demand conditions and the deviation from the monetary rule with the ratio Φ_g and Φ_m being roughly equal.

The fact that firms interpret a change in the policy rate as a signal that the central bank is responding to a demand shock also affects the response of inflation expectations to monetary shocks. The upper middle and right graphs show that inflation expectations react weakly to monetary shocks. They initially respond positively because of very strong inflationary effects of the HOE about the demand conditions. They turn negative later as the inflationary effects of the average HOE about the demand conditions and the aggregate technology decay faster than those associated with the average HOE about the deviation from the monetary rule.

Finally, note that the effects of the HOE about aggregate technology (i.e., the gray bars in the upper left graph) are deflationary – albeit quantitatively fairly small – as the gray vertical

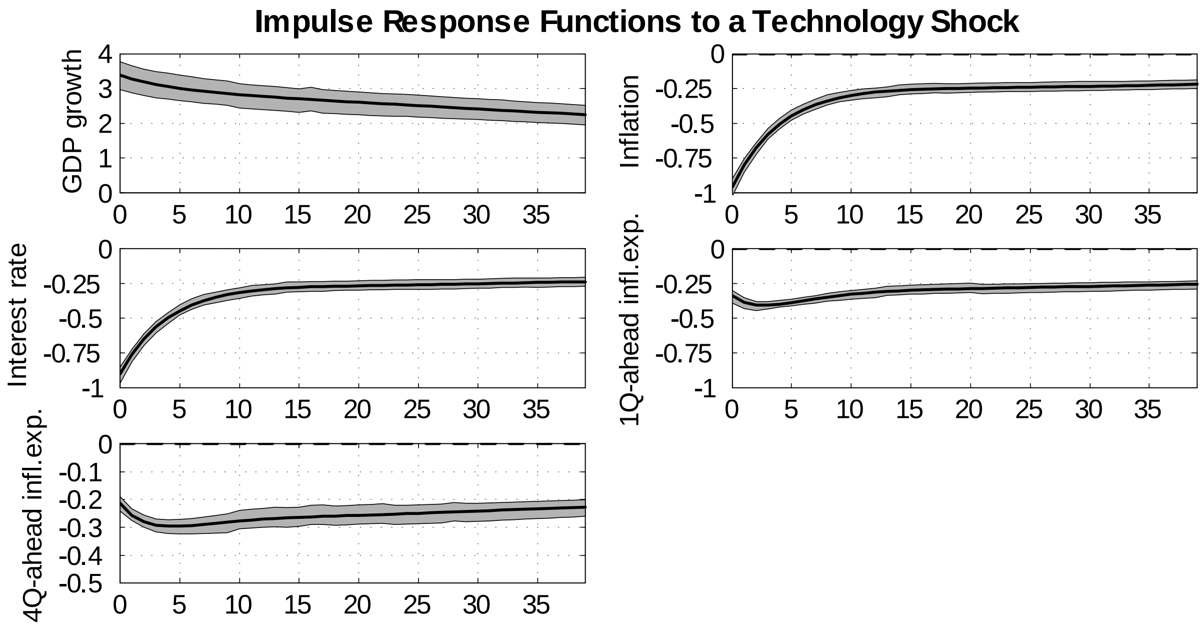

Figure 8: Impulse response function of the observable variables to a one-standard-deviation technology shock. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

bars in the top left graph lie in negative territory. This is a general-equilibrium result that is due to the sharp contraction of the average HOE about real marginal costs in response to the monetary shock. Since this is a general-equilibrium effect, it is hard to find one exact cause. However, the substantial price rigidity due to the presence of the signaling channel plays some role in exacerbating the negative elasticity of the average HOE about real marginal costs to monetary shocks (i.e., $\frac{\partial \widehat{y}^{(l)}_{t|t}}{\partial \widehat{a}^{(0:k)}_{t|t}} \frac{\partial \widehat{a}^{(0:k)}_{t|t}}{\partial \varepsilon_{r,t}} < 0$, for any order $0 \leq l \leq k$).

To sum up, the signaling channel turns out to enhance price rigidity and consequently the real effects of monetary disturbances. Furthermore, the signaling effects of monetary policy cause inflation expectations to respond positively to monetary shocks. The reason is that in the aftermath of a monetary shock firms tend to attach a non-negligible probability that the central bank has adjusted the policy rate to react to a demand shock.

4.4 Propagation of Non-policy Shocks

Figure 8 shows the response of GDP, the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a one-standard-deviation positive technology shock. Since the technology shock is almost unit root, the response of variables exhibits a high degree of persistence. In the aftermath of a positive technology shock, the GDP growth rate becomes positive, while both inflation and inflation expectations

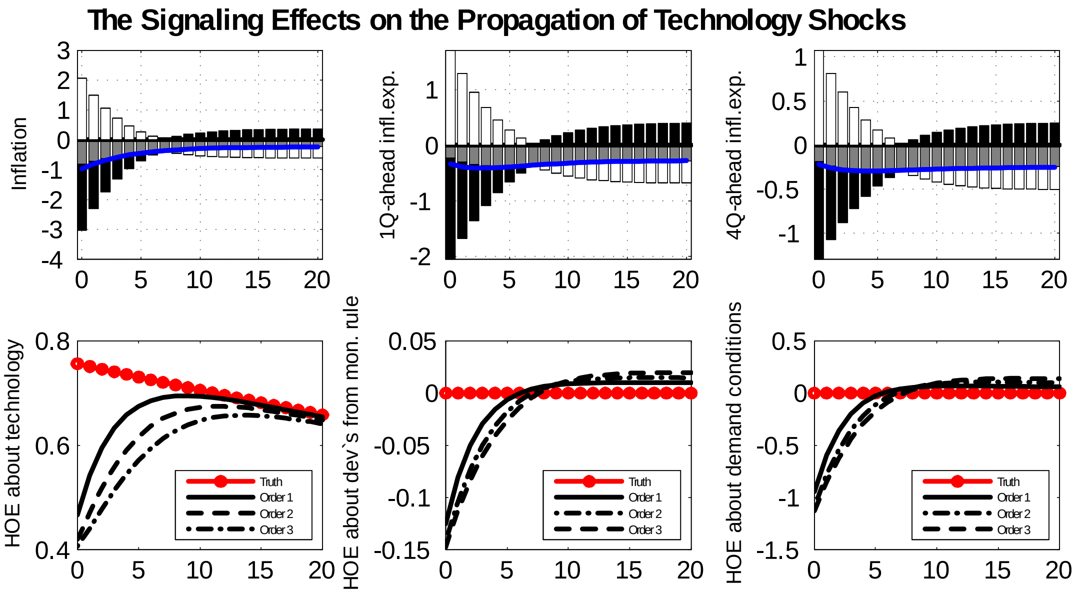

Figure 9: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a positive aggregate technology shock. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady state equilibrium.

fall.

Figure 9 graphs the decomposition of the response of inflation and inflation expectations to a negative technology shock (upper graphs) and the responses of average HOE about the three exogenous state variables $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$ (lower graphs). Let us focus, first, on the response of the average HOE in the lower graphs. A drop in the policy rate owing to a positive technology shock induces firms to believe that the central bank is responding to either an expansionary monetary shock or to a negative demand shock. If firms are persuaded that an expansionary monetary shock has occurred, then the confusion generated by the signaling channel would limit the response of inflation to the technology shock (See the white bars in the top graphs). However, if monetary easing leads firms to believe that a negative demand shock has hit the economy, the opposite effect on inflation would prevail. Firms' inflation expectations would further decrease, and inflation would go further down. This effect is captured by the black bars in the top graphs.

Two important features of Figure 9 deserve to be emphasized. Most importantly, the top left graph shows that the response of average expectations about the deviation from the monetary

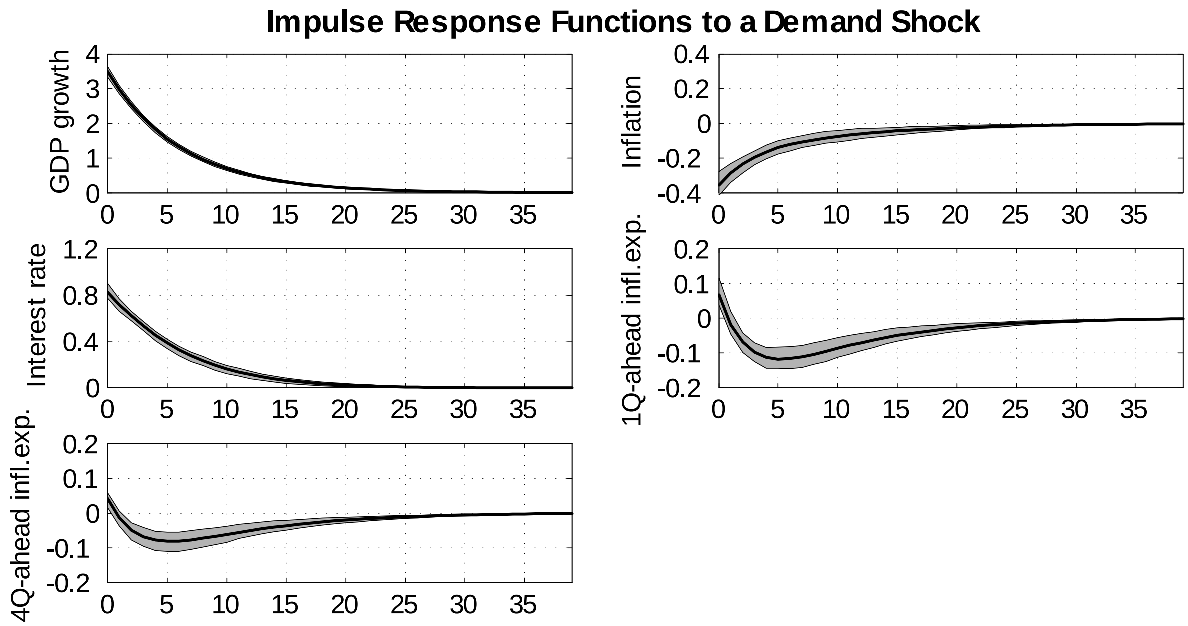

Figure 10: Impulse response function of the observable variables to a one-standard-deviation demand shock. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

rule (i.e., the white bars) and that about the demand conditions (i.e., the black bars) contribute to the adjustment of inflation by similar amounts in every quarter after the technology shock. Thus, the two effects of the confusion caused by the signaling channel on inflation cancel each other out. This implies that the signaling channel has small inflationary effects following technology shocks. This result squares with the previous finding that the signaling channel provides firms with virtually the same amount of information about the deviation from the monetary rule (Φ_m) and the demand conditions (Φ_g), as noticed in Section 4.2. Furthermore, notice that six quarters after the shock firms change their minds about the deviation from the monetary rule and the demand conditions. While firms initially expect the response of these state variables to be negative, later they expect that it becomes positive. This leads to a reversion of the effects of the HOE about these state variables on inflation and inflation expectations in the top graphs.

The propagation of a one-standard-deviation positive demand shock is described in Figure 10. It is important to emphasize that inflation responds *negatively* to demand shocks, while the GDP growth rate responds positively. Inflation expectations initially respond positively, but their response turns negative a few quarters after the shock. Note that the central bank raises its policy rate in the aftermath of a positive demand shock.

As reported in Figure 11, the signaling channel critically affects the propagation of demand shocks, causing inflation to respond negatively to demand shocks. There are two main effects

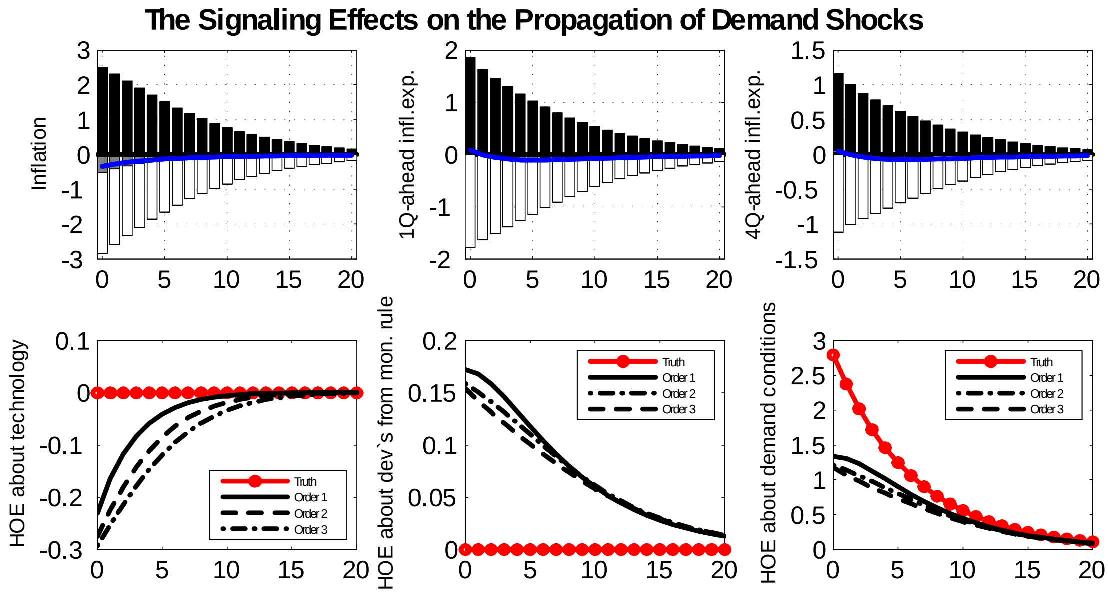

Figure 11: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a positive demand shock. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady state equilibrium.

that play an important role. First, the signaling channel confuses firms, inducing them to believe that a contractionary monetary shock has prompted the central bank to raise the policy rate (see the white bars). Second, the signaling channel also leads firms to believe that a negative technology shock might be the reason behind the observed rise in the federal funds rate (see the gray bars). Both effects push inflation down,[12] countering the rise in inflation due to the occurrence of a positive demand shock. Note also that while the second effect (captured by the gray bars in the top graphs) has quantitatively a fairly small impact on inflation and inflation expectations, the first effect (captured by the white bars) appears to substantially contribute to pushing inflation and inflation expectations down. This result seems to be in line with the fact that firms do not rely on public information to learn about aggregate technology ($\vartheta_a = 0.997$), while they observe the policy signal to learn about policy shocks.

[12]The deflationary effects associated with expecting an adverse technology shocks are due to general-equilibrium effects, which are discussed in Section 4.3.

4.5 Model Evaluation

In this section, we compare which of the two competing models (i.e., the dispersed information model, DIM, and the perfect information model, PIM) fits the data better. In Bayesian econometrics, this comparison is based on computing the posterior probability of the two candidate models. The marginal likelihood is the appropriate density for updating researcher's prior probabilities over a set of models. Since the marginal likelihood penalizes for the number of model parameters (An and Schorfheide 2007), it can be applied to gauge the relative fit of models that feature different numbers of parameters, including the DIM and the PIM.

Denote the data set used for estimation and presented in Section 4.1 as Y. The marginal likelihood associated with the DIM is defined as $P(Y|\mathcal{M}_{DIM}) = \int \mathcal{L}(Y|\Theta_{DIM})\, p(\Theta_{DIM})\, d\Theta_{DIM}$, where $\mathcal{L}(Y|\Theta_{DIM})$ denotes the likelihood function derived from the model and $p(\Theta_{DIM})$ is the prior density, whose moments are reported in Table 1.

A Bayesian test of the null hypothesis that the DIM is at odds with the data can be performed by comparing the marginal likelihood associated with the DIM ($\mathcal{M}_{DIM}$) and the PIM ($\mathcal{M}_{PIM}$). Under a 0-1 loss function, the test rejects the null if the DIM has a larger posterior probability than the PIM (Schorfheide 2000). The posterior probability of model $\mathcal{M}_s$, where $s \in \{DIM, PIM\}$, is given by:

$$\pi_{T,\mathcal{M}_s} = \frac{\pi_{0,\mathcal{M}_s} \cdot P(Y|\mathcal{M}_s)}{\sum_{s \in \{DIM,PIM\}} \pi_{0,\mathcal{M}_s} \cdot P(Y|\mathcal{M}_s)}, \tag{16}$$

where $\pi_{0,\mathcal{M}_s}$ stands for the prior probability of the model $\mathcal{M}_s$. Also, note that $P(Y|\mathcal{M}_s)$ is the marginal likelihood associated with the model $\mathcal{M}_s$. If the prior probabilities are equal across models (i.e., $\pi_{0,\mathcal{M}_{DIM}} = \pi_{0,\mathcal{M}_{PIM}} = 0.50$), then the model with the highest marginal likelihood is the one that attains the largest posterior probability.

The column labeled *Full Data Set* in Table 2 shows that the DIM attains the largest marginal likelihood, and hence, adopting equal prior probabilities across models leads us to reject the null. Note that the posterior probability in favor of the DIM is larger than that in favor of the PIM unless the prior probability in favor of the former (i.e., $\pi_{0,\mathcal{M}_{DIM}}$) is as small as $9.75E-8$. Such a low prior probability suggests that only if one has extremely strong *a priori* information against the DIM, one can favor the PIM over the DIM.

We check if there is any specific observable variable that the DIM fits particularly better than the PIM. To this end, we estimate the two models using a narrower data set that does not include the SPF as a measure of inflation expectations.[13] The log-marginal likelihood for the two competing models estimated using the narrower data set is reported in Table 2 under the column labeled *Excluding SPF*. It can be observed that now the two models fit the data

[13]The posterior statistics of this estimation based on a narrower data set are reported in Appendix F.

Log-Marginal Likelihood			
Full Data Set		Excluding SPF	
DIM	PIM	DIM	PIM
-212.4445	-228.5888	-306.4532	-304.87466

Table 2: The table reports the log-marginal likelihood for the dispersed information model (DIM) and the perfect information model (PIM) based on the full data set described in Section 4.1 (Full Sample) and on a narrower data set that does not include the Survey of Professional Forecasters (Excluding SPF). We use Geweke's harmonic mean estimator (Geweke 1999) to estimate the marginal likelihood for the two competing models.

similarly well, with the PIM having actually a little edge over the DIM. Therefore, the observed inflation expectations seem to play an important role in selecting the DIM as the best-fitting model. If we had estimated the two models using the data set that does not include the SPF, we could not have concluded that the DIM fits the data better than the PIM.

We showed that including the SPF to the data set raises the posterior probability in favor of the DIM. This suggests that the DIM is better than the PIM at fitting the observed inflation expectations. To further investigate this point, we obtain Figure 12, which reports the actual four-quarters-ahead inflation expectations (i.e., the red solid line) and the 95 percent posterior credible set for their model's predicted value (i.e., the gray areas) implied by the DIM (top graph) and the PIM (bottom graph), when the two models are estimated using the full data set that includes the SPF. It can be observed that the DIM does a better job than the PIM at fitting the run-up of inflation expectations observed from 1975 through 1981. The PIM systematically underpredicts inflation expectations in that period. Given the high variability of the data of the second-half of the 1970s, the PIM's inability to fit inflation expectations at that time explains the relatively poor score of this model in terms of marginal likelihood (Table 2). In Section 4.6, we will show that the signaling channel is key for the DIM to account for heightened inflation expectations during the period 1975−1981. Furthermore, we find that the DIM and the PIM fit the one-quarter-ahead inflation expectations similarly well.

Figure 12 shows that the DIM overpredicts the dynamics of inflation expectations in the last five years of the sample. This overprediction is associated with the rise in the federal funds rate observed in those years and its associated signaling effects on inflation expectations. The empirical performance of the DIM was not penalized too much in terms of marginal likelihood, since the last five years of the sample have been characterized by low macroeconomic volatility. Potential explanations for why the DIM overpredicts the four-quarters-ahead inflation expectations in this period include parameter instability (e.g., structural changes in the central bank's

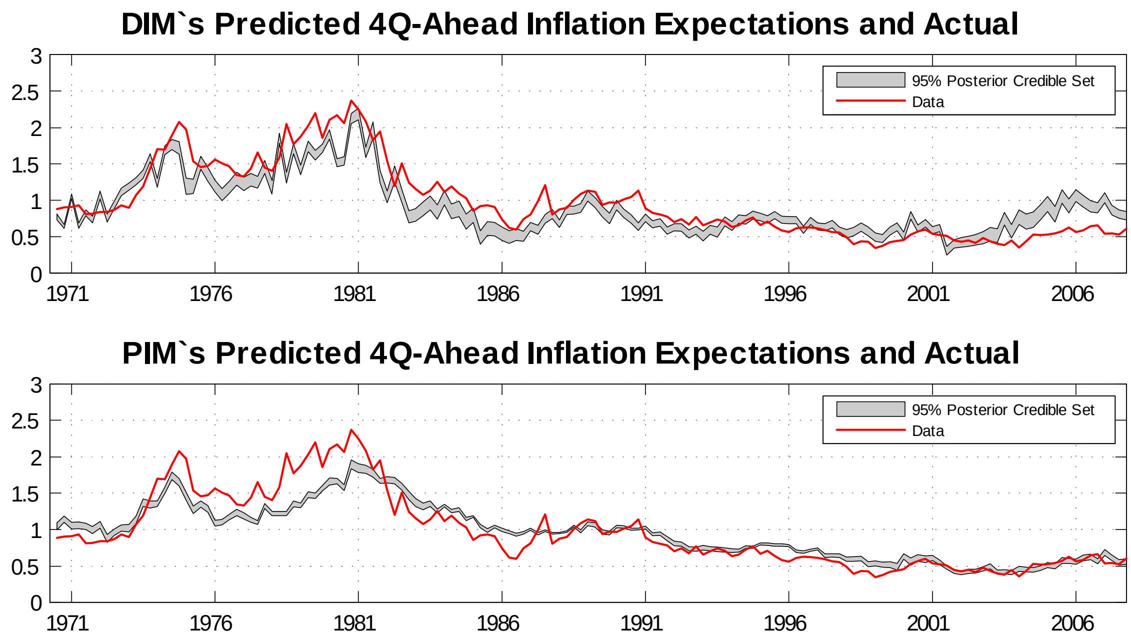

Figure 12: The 95 percent posterior credible set for the two-sided filtered series of four-quarters-ahead inflation expectations implied by the estimated dispersed information model (*Top graph*) and the estimated perfect information model (*Bottom graph*) and the actual series. The vertical axis in all graphs measures units of percentage points of quarterly rates.

attitude toward inflation stabilization)[14] and changes in the firms' private information structure (e.g., the introduction of more-advanced forms of communication and more-efficient ways of processing information). A scrupulous investigation of this interesting issue goes beyond the scope of the paper.

4.6 Bayesian Evaluation of the Signaling Channel

In this section, we want to use the DIM to empirically assess the signaling effects of monetary policy on observed GDP, inflation, and inflation expectations. To this end, we run a Bayesian counterfactual experiment using an algorithm that can be described as follows. In Step 1, for every posterior draw of the DIM parameters, we obtain the model's predicted series for the three structural shocks (aggregate technology shock $\varepsilon_{a,t}$, monetary shock $\varepsilon_{r,t}$, and the demand shock, $\varepsilon_{g,t}$) using the two-sided Kalman filter. In Step 2, for every posterior draw the associated filtered series of shocks obtained in Step 1 are used for simulating real output, the rate of inflation, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations from the following two models: (i) the DIM and (ii) the DIM in which monetary policy has no signaling effects. The latter model is obtained from the DIM by assuming that the history of the policy

[14]Bianchi (2010) estimates a Markov-switching DSGE model to U.S. data and finds that monetary policy has become dovish in those years.

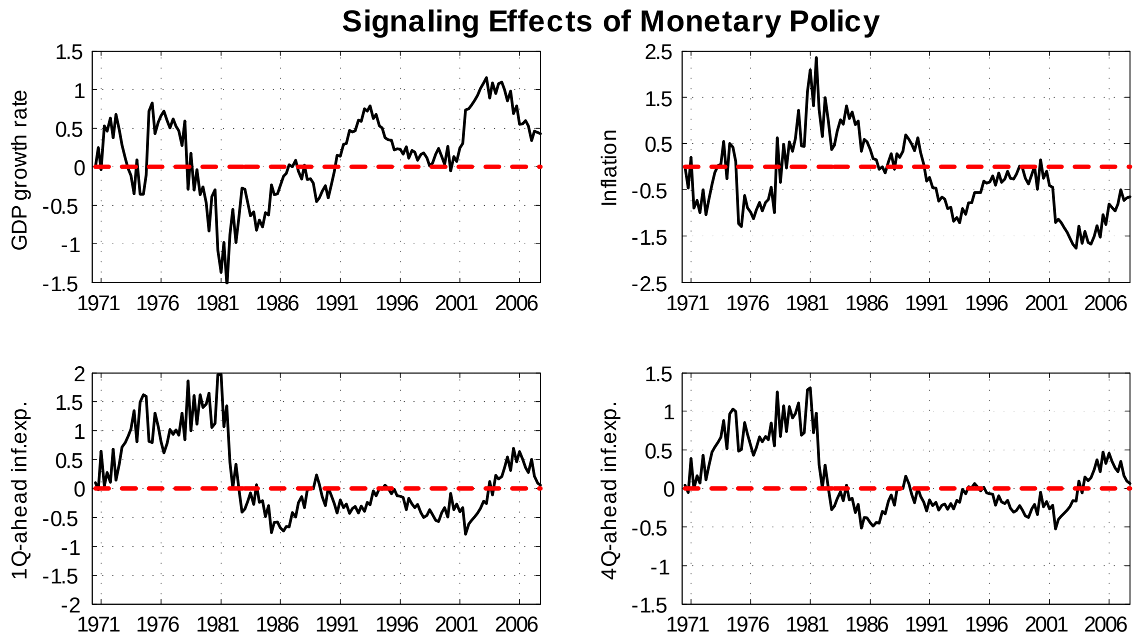

Figure 13: Signaling effects of monetary policy on gross domestic product (GDP), inflation, and inflation expectations. The solid black line is obtained by subtracting the across-posterior-draws mean of the series obtained by simulating the dispersed information model (DIM) without signaling channel from that of the series obtained by simulating the DIM, in which monetary policy has signaling effects. The two-sided filtered shocks of the estimated DIM are used to simulate the models. The vertical axis in all graphs measures units of percentage points of quarterly rates.

rate does not belong to firms' information set (i.e., $R^t \notin \mathcal{I}_{j,t}$, for all periods t and firms j). As discussed in Section 3.2, this assumption implies that firms form their expectations by only using their private information (i.e., the history of the signals $\widehat{a}_{j,t}$ and $\widehat{g}_{j,t}$). In Step 3, we compute the mean of the simulated series across posterior draws for the two models. Furthermore, in order to construct the series that capture the signaling effects of monetary policy on GDP, inflation, and inflation expectations, we subtract the mean of the series simulated using the DIM in which the signaling channel has been shut down from that of the series simulated using the DIM in which monetary policy has signaling effects.

The solid line in Figure 13 marks the signaling effects of monetary policy on GDP (top left graph), inflation (top right graph), one-quarter-ahead inflation expectations (bottom left plot), and four-quarters-ahead inflation expectations (bottom right plot), taken by subtracting the mean of the simulated series of the DIM without signaling effects from the DIM with signaling effects. Let us focus first on the dynamics of inflation expectations (bottom graphs). During the 1970s, the line lies in positive territory, implying that signaling effects have played a critical role in accounting for the heightened inflation expectations of the 1970s. The bottom graph of Figure 14 reports the simulated series of four-quarters-ahead inflation expectations from the DIM with signaling effects and the counterfactual (i.e., DIM in which the signaling

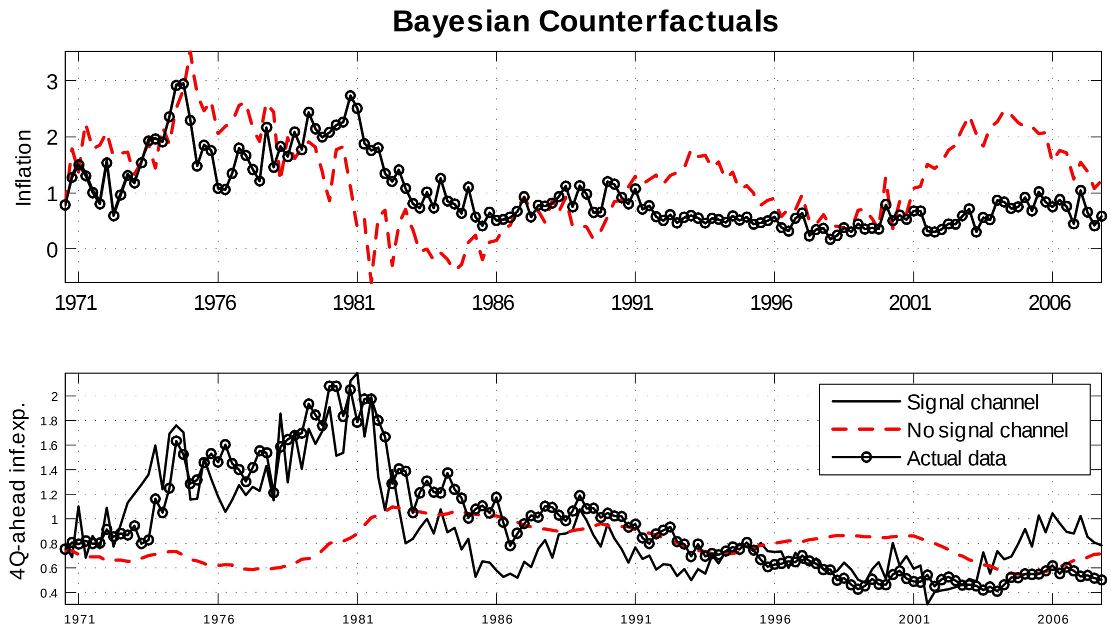

Figure 14: The graphs compare the inflation rate (top graph) and four-quarters-ahead inflation expectations (bottom graph) simulated from the estimated dispersed information model (DIM) (solid black line) with those simulated from the DIM in which the signaling channel is shut down (red dashed line). Model simulation is based on two-sided filtered shocks from the estimated DIM. The black line with circles denotes actual data. The vertical axis in all graphs measures units of percentage points of quarterly rates.

channel is shut down).[15] If the signaling channel was inactive (i.e., the red dashed line), inflation expectations would have been much lower in the 1970s. As discussed in Section 4.5, the fact that the DIM fits inflation expectations in 1970s better than the PIM is crucial for the former model to attain a larger posterior probability. This suggests that the signaling channel of monetary policy plays a critical role in improving the fit of the DIM relative to the PIM. Furthermore, after Volcker's disinflation period, the solid black lines of the bottom graphs of Figure 13 mostly lie in negative territory, suggesting that the signaling effects associated with the low-frequency decline of the federal funds rates have contributed to lowering inflation expectations until 2003.

The top right graph of Figure 13 shows that the signaling effects of monetary policy account for why inflation initially rose during the monetary tightening of the late 1970s. The top graph of Figure 14 shows that if the signaling effects were inactive, inflation would not have risen as observed in the data and would have actually declined rather quickly as the Federal Reserve began tightening monetary policy at the end of the 1970s. See the red dashed line in the top graph of Figure 14. Therefore, the signaling effects of monetary policy provide an explanation for why the inflation rate fell rather sluggishly during the monetary tightening of the late 1970s. Regarding the rise of inflation in the 1970s, the top right graph of Figure 13 shows that the

[15] Note that in the bottom graph of Figure 14 the dynamics of the actual data do not overlap that of the simulated data from the DIM because we have estimated this model adding the measurement error $\varepsilon_{2,t}$.

signaling channel had the effects of actually curbing the development in prices during this period of heightened inflation. This is quite the opposite of what we observed for the signaling effects on inflation expectations in the bottom graphs of Figure 13. Inflationary pressures have been reduced by the signaling channel in the 1990s and in the 2000s as the solid line in Figure 13 almost always lies in negative territory.

Finally, the top left graph of Figure 13 shows that the signaling effects of monetary policy on economic activity contributed to exacerbating Volcker's recession. While signaling effects of monetary policy have been contractionary for most of the 1980s, they have been stimulative for most of the 1990s (especially their first half) and for the 2000s.

To sum up, we find that inflationary effects from the signaling channel help the DIM to fit well the heightened inflation expectations of the 1970s, leading this model to attain a larger posterior probability than the perfect information model, as shown in Section 4.5. Furthermore, we show that signaling effects played an important role in raising inflation during the first years of the Volcker's monetary tightening and in exacerbating the associated recession. Signaling effects have also contributed to subduing inflation expectations from the 1980s to 2003 and the rate of inflation in the 1990s and 2000s. During the 1990s and 2000s, the signaling effects of monetary policy seem to have been quite stimulative with respect to economic activity; the contribution has reached up to 100 basis points in the first half of 2000s.

5 Concluding Remarks

This paper studies a DSGE model in which information is dispersed across price setters and the interest rate set by the central bank has signaling effects. In this model, monetary impulses propagate through two channels: (i) the traditional New Keynesian channel based on price stickiness and (ii) the signaling channel. The latter arises because changing the policy rate conveys information about the central bank's assessment on inflation and the output gap to price setters. This paper fits the model to a data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. This paper performs a formal econometric evaluation of the model with signaling effects of monetary policy. We find that this model fits the data better than its perfect information counterpart, mainly because of its ability to fit well the rise in inflation expectations from 1975 through 1981. Our likelihood analysis suggests that signaling effects accounted for the high inflation expectations of the 1970s. More precisely, the timid monetary contractions carried out by the Federal Reserve during the 1970s ended up boosting rather than curbing inflation expectations. While we find a very limited role of the signaling channel in explaining the surge of inflation in the first half of the 1970s, this channel accounts for why inflation did not suddenly fall as the Federal Reserve vigorously tightened monetary policy in the second half of 1970s. The signaling channel is also found to have exacerbated Volcker's recession. From the 1990s on, the signaling effects of monetary policy seem to have substantially contributed to subduing inflation and to stimulating economic activity. Furthermore, while the likelihood selects a very short average duration for price contracts, the signaling channel causes the real effects of monetary shocks to be very sizeable and persistent. The signaling channel is found to generate deflationary pressure in the aftermath of positive demand shocks, while it does not substantially alter the transmission of technology shocks.

We make a number of assumptions to keep the model sufficiently tractable to allow likelihood analysis. The relaxation of these assumptions is material for future research. First, households are perfectly informed. Relaxing this assumption is likely to affect the inflationary consequences of the signaling channel. For instance, if a monetary shock is partly perceived as a negative productivity shock, there will be at least two additional effects: (i) households will lower their expectations about the next period's real rate of interest and hence increase consumption; (ii) households will expect to be less productive in the future and therefore consume less. Note that a change in consumption will affect marginal costs in the model, and hence, depending on which effect dominates, the signaling effects on inflation could either be stronger or weaker than what is observed in the current setup. Second, another convenient shortcut, which should be abandoned in future research, is the assumption that firms use the linearized version of the true model to solve their signal extraction problem. This allows us to use the convenient characterization of the law of motion of the average expectations provided by the Kalman filter. Third, studying a dispersed information model with capital accumulation would be a fascinating extension to the current setup, since this is a standard feature of the perfect information DSGE models that are estimated (e.g., Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007). Nevertheless, this addition would be very challenging computationally at this stage and is likely to prevent the likelihood evaluation of the dispersed information model and the signaling channel.

Finally, in the dispersed information model the central bank communicates with firms only by setting the policy rate. However, it seems that market participants react to the central bank's announcements in practice. Empirically assessing the inflationary effects of monetary policy communication in a DSGE model with dispersed information is an interesting avenue for future research.

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Appendices

The Appendices are organized as follows. In Appendix A, we derive of the imperfect-common-knowledge Phillips curve (7). Appendix B details an algorithm to solve the dispersed information model. In Appendix C, we characterize the transition equations for the average higher-order expectations about the exogenous state variables – that is, equation (13). In Appendix D, we characterize the laws of motion for the three endogenous state variables (i.e., inflation $\hat{\pi}_t$, real output $\hat{y}_t$ and the interest rate $\hat{R}_t$). In Appendix E, we discuss the impulse response function of inflation and inflation expectations implied the prior for the model parameters. This is to verify that the main findings of the paper do not stem from our prior choice, but are actually driven by the data. Appendix F reports the posterior statistics for the parameters of the dispersed information model and the perfect information model estimated to a narrower data set that does not include the SPF as a measure of firms' inflation expectations.

A The Imperfect-Common-Knowledge Phillips Curve

The log-linear approximation of the labor supply can be shown to be given by $\widehat{c}_t = \widehat{w}_t$. Recalling that the resource constraint implies that $\hat{y}_t = \hat{c}_t$, we can then rewrite the labor supply as follows:

$$\hat{y}_t = \hat{w}_t. \tag{17}$$

Log-linearizing the equation for the real marginal costs yields

$$\widehat{mc}_{j,t} = \hat{w}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

Recall that $(\ln A_{j,t} - \ln \gamma \cdot t) \in \mathcal{I}_{j,t}$ and write

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{w}_{j,t} \underbrace{-\widehat{a}_t - \varepsilon^a_{j,t}}_{\ln A_{j,t} - \ln \gamma \cdot t},$$

where $\mathbb{E}_{j,t}$ is expectations conditioned on firm j's information set at time t ($\mathcal{I}_{j,t}$) defined in (4). Using equation (17) for replacing $\hat{w}_t$ yields

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{y}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

By integrating across firms, we obtain the average expectations on marginal costs:

$$\widehat{mc}^{(1)}_{t|t} = \hat{y}^{(1)}_{t|t} - \widehat{a}_t.$$

The linearized price index can be written as

$$\int \hat{p}^*_{j,t} dj = \frac{\theta}{1-\theta}\hat{\pi}_t.$$

Recall that we defined $\hat{p}^*_{j,t} = \ln P^*_{j,t} - \ln P_t$ and $\hat{\pi}_t = \ln P_t - \ln P_{t-1} - \ln \pi_*$. After some algebraic manipulation, we write

$$\ln P_t = \theta \left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta) \int \left(\ln P^*_{j,t}\right) dj. \tag{18}$$

The price-setting problem leads to the following first-order conditions:

$$\mathbb{E}\left[\sum_{s=0}^{\infty} (\beta\theta)^s \frac{\Xi_{j,t+s}}{P_{t+s}} \left[(1-\nu)\pi_*^s + \nu \frac{MC_{j,t+s}}{P^*_{j,t}}\right] Y_{j,t+s} | \mathcal{I}_{j,t}\right] = 0.$$

We define the stationary variables:

$$\begin{aligned} y_t &= \frac{Y_t}{\gamma^t},\ c_t = \frac{C_t}{\gamma^t};\ p^*_{j,t} = \frac{P^*_{j,t}}{P_t},\ y_{j,t} = \frac{Y_{j,t}}{\gamma^t}, \\ w_t &= \frac{W_t}{\gamma^t P_t},\ a_t = \frac{A_t}{\gamma^t}, R_t = \frac{R_t}{R_*},\ mc_{j,t} = \frac{MC_{j,t}}{P_t}, \\ \xi_{j,t} &= \gamma^t \Xi_{j,t}. \end{aligned}$$

And then we write

$$\mathbb{E}\left\{\xi_{j,t}\left[1-\nu+\nu\frac{mc_{j,t}}{p^*_{j,t}}\right] y_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \xi_{j,t+s} \left[(1-\nu)\pi_*^s + \nu\frac{mc_{j,t+s}}{p^*_{j,t}}\left(\Pi_{\tau=1}^s \pi_{t+\tau}\right)\right] y_{j,t+s} | \mathcal{I}_{j,t}\right\} = 0. \tag{19}$$

First realize that the square brackets are equal to zero at the steady state, and hence, we do not care about the terms outside them. We can write

$$\mathbb{E}\left[\left[1-\nu+\nu mc_{j,*} e^{\widehat{mc}_{j,t}-\widehat{p}^*_{j,t}}\right] + \sum_{s=1}^{\infty} (\beta\theta)^s \left[(1-\nu)\pi_*^s + \nu mc_{j,*} e^{\widehat{mc}_{j,t+s}-\widehat{p}^*_{j,t}+\sum_{\tau=1}^s \hat{\pi}_{t+\tau}}\right] |\mathcal{I}_{j,t}\right] = 0.$$

Taking the derivatives yields

$$\mathbb{E}\left[\widehat{mc}_{j,t} - \widehat{p}^*_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \left[\left(\widehat{mc}_{j,t+s} - \widehat{p}^*_{j,t} + \sum_{\tau=1}^{s} \hat{\pi}_{t+\tau}\right)\right] |\mathcal{I}_{j,t}\right] = 0.$$

We can take the term $\widehat{p}_{j,t}$ out of the sum operator in the second term and gather the common term to obtain

$$\mathbb{E}\left[\widehat{mc}_{j,t}-\frac{1}{1-\beta\theta}\widehat{p}^{*}_{j,t}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]=0.$$

Recall that $\widehat{p}^{*}_{j,t}\equiv\ln P^{*}_{j,t}-\ln P_{t}$ and cannot be taken out of the expectation operator. We write

$$\ln P^{*}_{j,t}=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}+\frac{1}{1-\beta\theta}\ln P_{t}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]. \tag{20}$$

Rolling this equation one step ahead yields

$$\ln P^{*}_{j,t+1}=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1}+\frac{1}{1-\beta\theta}\ln P_{t+1}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s+1}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t+1}\right].$$

Taking firm j's conditional expectation at time t on both sides and applying the law of iterated expectations, we obtain the following:

$$\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1}+\frac{1}{1-\beta\theta}\ln P_{t+1}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s+1}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t}\right].$$

We can take $\widehat{mc}_{j,t+1}$ inside the sum operator and write

$$\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)=(1-\beta\theta)\,\mathbb{E}\left[\frac{1}{1-\beta\theta}\ln P_{t+1}+\frac{1}{\beta\theta}\sum_{s=1}^{\infty}(\beta\theta)^{s}\,\widehat{mc}_{j,t+s}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].$$

Therefore,

$$\sum_{s=1}^{\infty}(\beta\theta)^{s}\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]=\frac{\beta\theta}{1-\beta\theta}\left[\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-\beta\theta\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]. \tag{21}$$

Hence, the equation (20) can be rewritten as:

$$\begin{aligned}\ln P^{*}_{j,t} \;=\;& (1-\beta\theta)\left\{\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]+\sum_{s=1}^{\infty}(\beta\theta)^{s}\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]\right\}\\ &+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].\end{aligned}$$

By substituting the result in equation (21), we obtain

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]\right] \\
&+\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

We consider the last term and write

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+(1-\beta\theta)\sum_{s=2}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] \\
&= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+ \\
&\quad+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\left(\sum_{\tau=1}^{s}\left[\left(\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\right)\right]+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\right).
\end{aligned}
$$

It then follows that

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&+(1-\beta\theta)\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

Because $\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)=\frac{(\beta\theta)^{2}}{1-\beta\theta}$, then after simplifying, we can write that

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= \beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

We substitute this result into the original equation to get the following expression:

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right] \\
&+\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right].
\end{aligned}
\tag{22}
$$

Note that by definition $\hat{\pi}_{t+1} \equiv \ln P_{t+1} - \ln P_t - \ln \pi_*$. Hence, we can write

$$\begin{aligned} \ln P_{j,t}^* &= (1-\beta\theta) \cdot \mathbb{E}\left[\widehat{mc}_{j,t} | \mathcal{I}_{j,t}\right] + (1-\beta\theta)\, \mathbb{E}\left[\ln P_t | \mathcal{I}_{j,t}\right] \\ &\quad + \beta\theta \cdot \mathbb{E}\left(\ln P_{j,t+1}^* | \mathcal{I}_{j,t}\right) - \beta\theta \ln \pi_*. \end{aligned} \tag{23}$$

We denote firm j's average k-th order expectation about an arbitrary variable $\hat{x}_t$ as

$$\mathbb{E}^{(k)}\left(\hat{x}_t | \mathcal{I}_{j,t}\right) \equiv \int \mathbb{E}\left(\int \mathbb{E}\left(\ldots\left(\int \mathbb{E}\left(\hat{x}_t | \mathcal{I}_{j,t}\right) dj\right) \ldots | \mathcal{I}_{j,t}\right) dj | \mathcal{I}_{j,t}\right) dj,$$

where expectations and integration across firms are taken k times.

Let us denote the average reset price as $\ln P_t^* = \int \ln P_{j,t}^* dj$. Note that we can rewrite equation (18) as follows

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta) \ln P_t^*. \tag{24}$$

Furthermore, we can integrate equation (23) across firms to obtain an equation for the average reset price:

$$\begin{aligned} \ln P_t^* &= (1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta) \ln P_{t|t}^{(1)} \\ &\quad + \beta\theta \ln P_{t+1|t}^{*(1)} - \beta\theta \ln \pi_*, \end{aligned} \tag{25}$$

where $x_{t|t}^{(1)}$ denotes the average first-order expectations about an arbitrary variable x_t of the model (e.g., the real marginal costs).

Let us plug equation (25) into equation (24) as follows:

$$\begin{aligned} \ln P_t &= \theta \ln P_{t-1} + (\theta - (1-\theta)\beta\theta) \ln \pi_* \\ &\quad + (1-\theta)\left[(1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta) \ln P_{t|t}^{(1)} + \beta\theta \ln P_{t+1|t}^{*(1)}\right]. \end{aligned} \tag{26}$$

From the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_*$ and from the price index (18) we get the following:[16]

$$\ln P_{t+1}^{*}=\frac{\widehat{\pi}_{t+1}}{1-\theta}+\ln P_{t}+\ln\pi_{*}.$$

Furthermore, the following fact is easy to establish:

$$\ln P_{t+1}=\widehat{\pi}_{t+1}+\ln P_{t}+\ln\pi_{*}.$$

Applying these three results to equation (26) yields

$$\begin{aligned}\widehat{\pi}_{t}+\ln P_{t-1}+\ln\pi_{*} &= \theta\ln P_{t-1}+\left(\theta-\left(1-\theta\right)\beta\theta\right)\ln\pi_{*} \\ &\quad +\left(1-\theta\right)\left[\left(1-\beta\theta\right)\cdot\widehat{mc}_{t|t}^{(1)}+\left(1-\beta\theta\right)\ln P_{t|t}^{(1)}+\beta\theta\left(\frac{\hat{\pi}_{t+1|t}^{(1)}}{1-\theta}+\ln P_{t|t}^{(1)}+\ln\pi_{*}\right)\right].\end{aligned} \tag{27}$$

And after simplifying, we get the following:

$$\widehat{\pi}_{t}=\left(1-\theta\right)\left(1-\beta\theta\right)\cdot\widehat{mc}_{t|t}^{(1)}+\left(1-\theta\right)\widehat{\pi}_{t|t}^{(1)}+\beta\theta\left(\hat{\pi}_{t+1|t}^{(1)}\right). \tag{28}$$

By repeatedly taking firm j's expectation and average the resulting equation across firms, we get

$$\widehat{\pi}_{t|t}^{(k)}=\left(1-\theta\right)\left(1-\beta\theta\right)\cdot\widehat{mc}_{t|t}^{(k+1)}+\left(1-\theta\right)\widehat{\pi}_{t|t}^{(k+1)}+\beta\theta\left(\hat{\pi}_{t+1|t}^{(k+1)}\right).$$

Repeatedly substituting these equations for $k\geq 1$ back to equation (28) yields the imperfect-common-knowledge Phillips curve:

$$\hat{\pi}_{t}=\left(1-\theta\right)\left(1-\beta\theta\right)\sum_{k=1}^{\infty}\left(1-\theta\right)^{k-1}\widehat{mc}_{t|t}^{(k)}+\beta\theta\sum_{k=1}^{\infty}\left(1-\theta\right)^{k-1}\widehat{\pi}_{t+1|t}^{(k)}.$$

B Solving the Dispersed Information Model

We solve the model assuming common knowledge of rationality (Nimark 2008) and focusing on equilibria where the higher-order expectations about the exogenous state variables; that is,

[16] This last result comes from observing that

$$\ln P_{t}=\theta\left(\ln P_{t-1}+\ln\pi_{*}\right)+\left(1-\theta\right)\ln P_{t}^{*}.$$

By using the fact that $\ln P_{t}=\widehat{\pi}_{t}+\ln P_{t-1}+\ln\pi_{*}$:

$$\widehat{\pi}_{t}+\ln P_{t-1}+\ln\pi_{*}=\theta\left(\ln P_{t-1}+\ln\pi_{*}\right)+\left(1-\theta\right)\ln P_{t}^{*}.$$

Rolling one period forward, we get

$$\widehat{\pi}_{t+1}=\left(\theta-1\right)\left(\ln P_{t}+\ln\pi_{*}\right)+\left(1-\theta\right)\ln P_{t+1}^{*}.$$

And finally, by rearranging, we get the result in the text.

$X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{\eta}_{r,t|t}^{(s)}, \widehat{g}_{t|t}^{(s)} : 0 \leq s \leq k\right]'$, follow the VAR(1) process in equation (13). Note that we truncate the order of the average expectations at $k < \infty$. Furthermore, we guess the matrix $\mathbf{v}_0$ that determines the dynamics of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]$ in equation (12). As shown in Appendix D, the structural equations of the model can be written in the following linear form:

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{29}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on a complete information set (i.e., an information set that includes the history of all structural shocks).

For a given parameter set Θ_{DIM}, take the following steps:

Step 0 Set $i = 1$ and guess the matrices $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$, and $\mathbf{v}_0^{(i)}$.

Step 1 Set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$ and solve the model given by equation (13) and equation (29) through a standard linear rational expectations model solver (e.g., Blanchard and Kahn 1980; Sims 2002). The solver delivers the matrix $\mathbf{v}_0^{(i+1)}$, such that $\mathbf{s}_t = \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$. As we will show in Appendix D, the matrices Γ_0, Γ_1, and Γ_2 in equation (29) are functions of the model parameter Θ_{DIM} as well as the guessed matrices $\mathbf{M}^{(i)}$ and $\mathbf{v}_0^{(i)}$.

Step 2 Given the law of motion (13) for $X_{t|t}^{(0:k)}$, in which we set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$, equation (10) for the signal concerning the aggregate technology, equation (11) for the signal concerning the demand conditions, and the equation

$$\widehat{R}_t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)},$$

for the endogenous policy signal $\widehat{R}_t \in \mathbf{s}_t$ solve the firms' signal extraction problem through the Kalman filter and determine the matrices $\mathbf{M}^{(i+1)}$ and $\mathbf{N}^{(i+1)}$. Appendix C provides a detailed explanation of how we characterize these matrices.

Step 3 If $\left\|\mathbf{M}^{(i)} - \mathbf{M}^{(i+1)}\right\| < \varepsilon_m$, $\left\|\mathbf{N}^{(i)} - \mathbf{N}^{(i+1)}\right\| < \varepsilon_n$, and $\left\|\mathbf{v}_0^{(i)} - \mathbf{v}_0^{(i+1)}\right\| < \varepsilon_v$ for any $\varepsilon_m > 0$, $\varepsilon_n > 0$, and $\varepsilon_v > 0$ and small, STOP or else set i=i+1 and go to STEP 1.

Given equation (13) and equation $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$ obtained in step 1, the law of motion of the model variables is as follows:

$$\begin{bmatrix} X_{t|t}^{(0:k)} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{(i+1)} & \mathbf{0} \\ \mathbf{v}_0^{(i+1)} \mathbf{M}^{(i+1)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1}^{(0:k)} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(i+1)} \\ \mathbf{v}_0^{(i+1)} \mathbf{N}^{(i+1)} \end{bmatrix} \boldsymbol{\varepsilon}_t. \tag{30}$$

C Transition Equation of High-Order Expectations

In this section, we show how to derive the law of motion of the average higher-order expectations of the exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, $\widehat{g}_t$,) for given parameter values and the matrix of coefficients $\mathbf{v}_0$. We focus on equilibria where the HOE evolve according to

$$X_{t|t}^{(0:k)} = \mathbf{M}X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{31}$$

where $\boldsymbol{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_{a,t} & \eta_{r,t} & \varepsilon_{g,t} \end{bmatrix}'$. Denote $\mathbf{X}_t \equiv X_{t|t}^{(0:k)}$, for notational convenience. Firms' reduced-form state-space model can be concisely cast as follows:

$$\mathbf{X}_t = \mathbf{M}\mathbf{X}_{t-1} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{32}$$

$$\mathbf{Z}_t = \mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t}, \tag{33}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1' & \mathbf{d}_2' & \left(\mathbf{1}_3^T\mathbf{v}_0\right)' \end{bmatrix}',$$

with $\mathbf{d}_1' = \left[1, \mathbf{0}_{1\times3(k+1)-1}\right]$, $\mathbf{d}_2' = [\mathbf{0}_{1\times2}, 1, \mathbf{0}_{1\times3k}]$, $\mathbf{1}_3^T = [0, 0, 1]$, and $e_{j,t} = \left[\varepsilon_{j,t}^a, \varepsilon_{j,t}^g\right]'$ and

$$\mathbf{Q} = \begin{bmatrix} \widetilde{\sigma}_a & 0 \\ 0 & \widetilde{\sigma}_g \\ 0 & 0 \end{bmatrix}.$$

Solving the firms' signal extraction problem requires applying the Kalman filter. The Kalman equation pins down firm j's first-order expectations about the model's state variables $\mathbf{X}_{t|t}(j)$ and the associated conditional covariance matrix $\mathbf{P}_{t|t}$:

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\left[\mathbf{Z}_t - \mathbf{Z}_{t|t-1}(j)\right], \tag{34}$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}_{t|t-1}', \tag{35}$$

where

$$\mathbf{P}_{t|t-1} = \mathbf{M}\mathbf{P}_{t-1|t-1}\mathbf{M}' + \mathbf{N}\mathbf{N}', \tag{36}$$

and the matrix $\mathbf{F}_{t|t-1} \equiv E\left[\mathbf{Z}_t\mathbf{Z}_t'|\mathbf{Z}^{t-1}\right]$, which can be shown to be

$$\mathbf{F}_{t|t-1} = \mathbf{D}\mathbf{P}_{t|t-1}\mathbf{D}' + \mathbf{Q}\mathbf{Q}'. \tag{37}$$

Therefore, combining equation (35) with equation (36) yields

$$\mathbf{P}_{t+1|t} = \mathbf{M}\left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}'_{t|t-1}\right]\mathbf{M}' + \mathbf{N}\mathbf{N}'. \tag{38}$$

Denote the Kalman-gain matrix as $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}$. Write the law of motion of firm j's first-order beliefs about $\mathbf{X}_t$ as

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{K}_t\left[\mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t} - \mathbf{D}\mathbf{X}_{t|t-1}(j)\right],$$

where we have combined equations (34) and (33). By recalling that $\mathbf{X}_{t|t-1}(j) = \mathbf{M}\mathbf{X}_{t-1|t-1}(j)$, we obtain

$$\mathbf{X}_{t|t}(j) = (\mathbf{M} - \mathbf{KDM})\,\mathbf{X}_{t-1|t-1}(j) + \mathbf{K}\left[\mathbf{DM}\cdot\mathbf{X}_{t-1} + \mathbf{DN}\cdot\boldsymbol{\varepsilon}_t + \mathbf{Q}e_{j,t}\right]. \tag{39}$$

The vector $\mathbf{X}_{t|t}(j)$ contains firm j's first-order expectations about the model's state variables. Integrating across firms yields the law of motion of the average expectation about $\mathbf{X}_{t|t}^{(1)}$:

$$\mathbf{X}_{t|t}^{(1)} = (\mathbf{M} - \mathbf{KDM})\,\mathbf{X}_{t-1|t-1}^{(1)} + \mathbf{K}\left[\mathbf{DM}\cdot\mathbf{X}_{t-1} + \mathbf{DN}\cdot\boldsymbol{\varepsilon}_t\right].$$

Note that $X_{t|t}^{(0:\infty)} = \left[X_t, X_{t|t}^{(1:\infty)}\right]'$ and that

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & \mathbf{0} \\ 0 & \rho_r & 0 & \mathbf{0} \\ 0 & 0 & \rho_g & \mathbf{0} \end{bmatrix}}_{\mathbf{R}_1} X_{t-1|t-1}^{(0:k)} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_g \end{bmatrix}}_{\mathbf{R}_2} \cdot \boldsymbol{\varepsilon}_t.$$

So by using the assumption of common knowledge in rationality, we can fully characterize the matrices $\mathbf{M}$ and $\mathbf{N}$:

$$\mathbf{M} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3k} \\ \mathbf{0}_{3k\times 3} & (\mathbf{M} - \mathbf{KDM})\,|_{(1:3k,1:3k)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}\,(\mathbf{DM})\,|_{(1:3k,1:3(k+1))} \end{bmatrix}, \tag{40}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{KDN}|_{(1:3k,1:3)} \end{bmatrix}, \tag{41}$$

where $\cdot|_{(n_1:n_2,m_1:m_2)}$ denotes the submatrix obtained by taking the elements lying between the n_1-th row and the n_2-th row and between the m_1-th column and the m_2-th column. Note that $\mathbf{K}$ in equation (40) and equation (41) denotes the steady-state Kalman gain matrix, which is obtained by iterating the equations (36) and (38) until convergence.

D The Laws of Motion for the Endogenous State Variables

In this section we introduce some useful results and characterize the law of motion (29) for the endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$.

D.1 Preliminaries

The *assumption of common knowledge in rationality* ensures that agents use the actual law of motion of higher-order expectations to forecast the dynamics of the higher-order expectations. The following propositions turn out to be useful for what follows:

Proposition 1 $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$. Then common knowledge in rationality implies that $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$. ■

Since we truncate beliefs after the k-th order, we define the matrix $\mathbf{T}^{(s)}$ as follows:

$$\mathbf{T}^{(s)} \equiv \begin{bmatrix} \mathbf{0}_{3(k-s+1)\times 3s} & \mathbf{I}_{3(k-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times(k+1-s)3} \end{bmatrix},$$

and we approximate the law of motion for $\mathbf{s}_{t|t}^{(s)}$ as $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

Proposition 2 *The following holds true:* $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_{t+h} = \mathbf{v}_0 X_{t+h|h+1}^{(0:k)}$. Given equation (13), it follows that $\mathbf{s}_{t+h} = \mathbf{v}_0 \left(\mathbf{M}^h X_{t|t}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_{t+1} \right)$. Common knowledge in rationality implies that repeatedly taking firms' expectations and then averaging across firms leads to an expression for the law of motion of the average higher-order expectations: $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ for any s. ■

Since we truncate beliefs after the k-th order, we can approximate the law of motion for the average higher-order expectations as $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

D.2 The Laws of Motion of the Endogenous State Variables

The laws of motion of the three endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$, are given by the IS equation (8), the Phillips curve (7), and the Taylor Rule (9). We want to write this system of linear equations as

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{42}$$

where $\mathbf{s}_t \equiv \left[\hat{\pi}_t, \hat{y}_t, \hat{R}_t\right]'$. It is obvious how to write equations (8) and (9) in the form (42). However, how to write the Phillips curve (7) in the form (42) requires a bit of work. First, note that given Propositions 1−2 and the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$, the imperfect-common-knowledge Phillips curve (7) can be rewritten as follows:

$$
\begin{aligned}
\mathbf{a}_0 X_{t|t}^{(0:k)} &= (1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_2^T\left[\mathbf{v}_0\mathbf{T}^{(s+1)}X_{t|t}^{(0:k)}\right] + \\
&-(1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s\left[\boldsymbol{\gamma}_a^{(s)\prime}X_{t|t}^{(0:k)}\right] \\
&+\beta\theta\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_1^T\left[\mathbf{v}_0\mathbf{M}\mathbf{T}^{(s+1)}X_{t|t}^{(0:k)}\right],
\end{aligned}
$$

where $\mathbf{1}_1^T = [1,0,0]$, $\mathbf{1}_2^T = [0,1,0]$, and $\boldsymbol{\gamma}_a^{(s)} = \left[\mathbf{0}_{1\times 3s}, (1,0,0), \mathbf{0}_{1\times 3(k-s)}\right]'$. The following restrictions upon vectors of coefficients $\mathbf{a}_0$ and $\mathbf{a}_1$ can be derived from the rewritten Phillips curve:

$$
\widehat{\pi}_t = \left[(1-\theta)(1-\beta\theta)\left[\boldsymbol{\nu}\mathbf{m}_1 - \left(\sum_{s=0}^{k-1}(1-\theta)^s\boldsymbol{\gamma}_a^{(s)\prime}\right)\right] + \beta\theta\boldsymbol{\nu}\mathbf{m}_2\right] X_{t|t}^{(0:k)}, \tag{43}
$$

where we define:

$$
\begin{aligned}
\mathbf{m}_1 &\equiv \begin{bmatrix} \mathbf{1}_2^T\mathbf{v}_0\mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_2^T\mathbf{v}_0\mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k\left[\mathbf{1}_2^T\mathbf{v}_0\mathbf{T}^{(k)}\right] \end{bmatrix}, \ \mathbf{m}_2 \equiv \begin{bmatrix} \mathbf{1}_1^T\mathbf{v}_0\mathbf{M}\mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_1^T\mathbf{v}_0\mathbf{M}\mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k\left[\mathbf{1}_1^T\mathbf{v}_0\mathbf{M}\mathbf{T}^{(k)}\right] \end{bmatrix}, \\
\boldsymbol{\nu} &\equiv \mathbf{1}_{1\times k}.
\end{aligned}
$$

E Signaling Effects Implied by the Prior

The aim of this section is to assess the extent to which the likelihood favors the idea that signaling effects associated with monetary policymaking are important. In other words, we want to rule out the possibility that this finding totally follows from the choice of the prior. Figure 15 reports the response of inflation (top left graph), inflation expectations (top middle and right graphs), and the average HOE about the three exogenous state variables (bottom graphs) to a monetary shock that raises the policy rate by 25 basis points when the DIM parameters are set to equal the *prior means* reported in Table 1. The vertical bars in the upper graphs are related to the decomposition (15) and isolate the inflationary effects of the change in

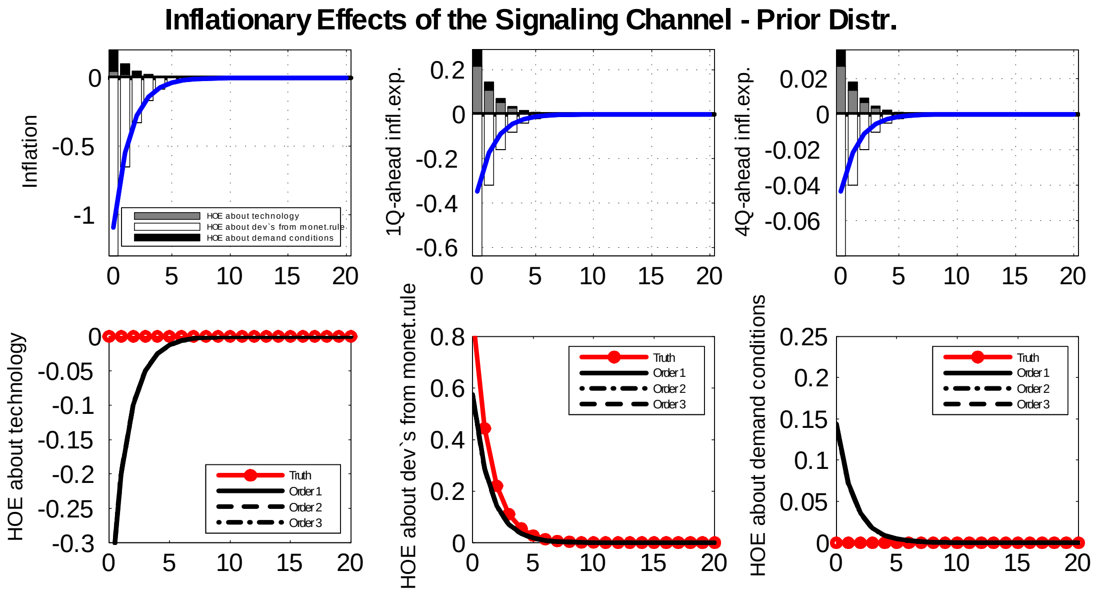

Figure 15: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the prior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady state equilibrium.

the average higher-order expectations (HOE) about technology $\mathbf{v}_a' \partial X^a_{t+h}/\partial \varepsilon_{r,t}$ (the gray bars), those about the deviation from the monetary rule $\mathbf{v}_m' \cdot \partial X^m_{t+h}/\partial \varepsilon_{r,t}$ (the white bars), and those about the demand conditions $\mathbf{v}_g' \cdot \partial X^g_{t+h}/\partial \varepsilon_{r,t}$ (the black bars) for h periods after the monetary shock.

Note that the sum of the gray vertical bars and the black vertical bars captures the effect of the signaling channel on inflation and inflation expectations. In Figure 15, these effects appear to be fairly small, especially for the response of inflation that is almost entirely explained by the shock and the average HOE about the deviation from the monetary rule on inflation (i.e., the white bars). More importantly, comparing Figure 15 with Figure 7, which is based on using the posterior mean to calibrate the DIM parameters, reveals that the Bayesian updating boosts the inflationary effects of the average HOE about the demand conditions (i.e., the black bars). Recall that high black bars imply that firms mostly interpret a rise in the policy rate as the central bank's response to a positive demand shock. As shown before, this result gives rise to important inflationary effects of the signaling channel in the estimated model. Such signaling effects of monetary policy dampen the response of inflation to monetary shocks and hence boost the real effects of monetary disturbances. The comparison of Figure 15 with Figure 7 reveals that high and persistent real effects of monetary shocks are not originated by the prior and actually appear to be mainly driven by the likelihood through the Bayesian updating.

F DIM and PIM Estimated with a Narrower Data Set

	DIM - Posterior			PIM - Posterior			Prior		
Name	Mean	5%	95%	Mean	5%	95%	Type	Mean	Std.
θ	0.3387	0.2615	0.4508	0.5297	0.4687	0.5907	$\mathcal{B}$	0.50	0.30
ϕ_π	1.0289	1.0001	1.1261	1.6471	1.4820	1.7939	$\mathcal{G}$	1.50	0.10
ϕ_y	0.1951	0.1199	0.2648	0.2710	0.0973	0.4377	$\mathcal{G}$	0.25	0.10
ρ_r	0.6643	0.6211	0.7149	0.4765	0.4111	0.5446	$\mathcal{B}$	0.50	0.20
ρ_a	0.9138	0.8491	0.9680	0.9927	0.9868	0.9991	$\mathcal{B}$	0.50	0.20
ρ_g	0.7291	0.6814	0.7737	0.9116	0.8697	0.9538	$\mathcal{B}$	0.50	0.20
$100\sigma_a$	0.3652	0.2169	0.6590	0.9808	0.8738	1.0837	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_a$	0.2035	0.0010	1.4803	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_g$	1.5582	1.4281	1.7191	1.4953	0.9122	2.0525	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_g$	35.3479	34.8243	35.8393	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_r$	0.4797	0.4372	0.5464	0.6858	0.5884	0.7841	$\mathcal{IG}$	0.80	1.50
$100\sigma_{m_1}$	0.1224	0.0583	0.2317	0.2806	0.0412	0.9287	$\mathcal{IG}$	0.10	0.08
$100\sigma_{m_2}$	0.1321	0.0498	0.2687	0.1282	0.0548	0.2077	$\mathcal{IG}$	0.10	0.08
$100\ln\gamma$	0.5087	0.3906	0.6214	0.4098	0.3308	0.4880	$\mathcal{N}$	0.62	0.10
$100\ln\pi_*$	0.7131	0.5994	0.8868	0.7847	0.6511	0.9127	$\mathcal{N}$	0.65	0.10

Table 3: Posterior statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM) using a data set that does not include inflation expectations from the Survey of Professional Forecasters

Table 3 reports the posterior statistics for the DIM and the PIM parameters when the two models are estimated using a narrower data set that does not include the SPF as a measure of price setters' inflation expectations. For the estimation, the same prior as the one detailed in Table 1 is adopted.