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PIER Working Paper 13-023

"Efficient Learning and Job Turnover in the Labor Market"

by

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http://ssrn.com/abstract=1967503

Efficient Learning and Job Turnover in the Labor Market*

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April 19, 2013

Abstract

This paper studies the dynamics of workers' on-the-job search behavior and its consequences in an equilibrium labor market. In a model with both directed search and learning about the match quality of firm-worker pairs, I highlight the job search target effect of learning: as a worker updates the evaluation of his current job, he adjusts his on-the-job search target, which results in a different job finding rate. This model generates a non-monotonic relation between the employment-to-employment transition rate and tenure, which provides a new explanation of the hump-shaped separation rate-tenure profile.

Keywords: Learning, Directed Search, Job Turnover, On-the-job Search, Employment-to-employment Transition.

JEL Classification Codes: D83, J31

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*I am grateful to my advisor, George Mailath, for insightful instruction and encouragement. I thank Hanming Fang, Guido Menzio and Andrew Postlewaite for their invaluable guidance and excellent comments that improved the paper. I also thank Naoki Aizawa, Benjamin Lester, Kenneth Burdett, Chao Fu, Manolis Galenianos, Nils Gornemann, Kyungmin Kim, Ricardo Lagos, Giuseppe Moscarini, Moritz Ritter, Robert Shimer, Can Tian, Gabor Virag, Pierre-Olivier Weill, Xi Weng, and participants at the 2011 Royal Economic Society Conference, Search and the Matching Workshop at the Philadelphia Fed, the 2012 North American Winter Meeting of the Econometric Society, and the UPenn theory lunch for insightful comments. All remaining errors are mine.

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1 Introduction

An enormous number of employment-to-employment (EE) transitions take place in the U.S. labor market. Based on the estimation by Nagypal (2008), 2.2% of employed workers leave for a job in a different firm, and the flow of EE transitions accounts for 49% of all exits from employers, versus 20% of separations that are employment-to-unemployment (EU) transitions and 31% that are transitions from employment to being out of the labor force. In the labor search literature, EE transitions are generally accomplished through employed workers' on-the-job search (OJS) behavior. Numerous empirical studies show that workers' OJS behavior and its performance vary regarding tenure, the motivation for the OJS and other factors. In this paper, I develop an equilibrium search model to study the dynamics of workers' OJS behavior and its consequences.[1]

Specifically, I consider a directed search model in which firms post contracts and workers search for jobs. Once a worker and a firm meet, they form a one-to-one match. Their pair-specific match quality is initially unknown (with the same prior on both sides) and is revealed gradually over time. When the match is believed to be bad with very high probability, the firm destroys the job to avoid further loss and the worker becomes unemployed. Employed workers can search for a new job and so can unemployed workers, and the optimal search strategy depends on a worker's evaluation (belief) of his current match quality. Since a job with an extremely bad evaluation is going to be destroyed, an employed worker has an incentive to search on the job because (1) he is willing to find a new job with better pay and (2) he is afraid of losing his current job in the future.

Over time, a worker and his employer adjust their evaluation of the current job match quality based on the worker's past job performance. The diversity of individual histories results in ex post heterogeneity in the evaluation of the current match and therefore in the job search behavior of workers. This learning mechanism has two conflicting effects on the tenure-EE transitions profile. On the one hand, there is a standard *selection effect*, which was initially highlighted by Jovanovic (1979a,b and 1984). Over time, the match quality will be learned. Known good matches are kept and known bad matches are destroyed. Consequently, the proportion of good matches raises over time. For a particular worker, the longer his tenure, the higher the probability that his current match is good. Since good matches will not be destroyed, workers in good matches have less incentive to engage in OJS. Hence the selection effect suggests a negative relation between tenure and the EE transition rate. On the other hand, I show that there is a *job search target effect* of learning. Some workers believe their current job's match quality is good, and they don't need to worry about being fired in the near future, so they are attracted only by well-paying jobs. Other

[1] See, for examples, Mincer and Jovanovic (1982), Flinn (1986), Farber (1999), Fujita (2012) and Bjelland, Fallick, Haltiwanger and McEntarfer (2011).

workers believe their current job's match quality is not good enough, so they are afraid of losing their current job. As a result, they are less selective and target their search to jobs with lower pay. A matched worker with a long tenure but who has not revealed the type of his current match would be treated as being working in a bad match with high probability. Hence these workers are afraid of losing their job in the near future, and they have strong incentives to switch to a new job as soon as possible. In a frictional labor market, a worker with a low evaluation of his current job can adjust his OJS strategy to raise the probability of transition. This *job search target effect* would raise the possibility of EE transitions on average.

In general, this problem is hard to analyze in an equilibrium search framework. In a standard search model, as Burdett and Mortenson (1998) and Shi (2009) show, firms may post different wage schemes, which induces two dimensions of ex post heterogeneity among employed workers: (1) their evaluation of the current match quality, and (2) the wage scheme promised by their current employer. A worker's job search behavior depends on both of them, and therefore it is hard to analyze the worker's OJS dynamics in an equilibrium model. To obtain a tractable model, I follow Menzio and Shi (2011) and consider the socially efficient allocation, and implement the efficient solution by allowing agents to sign complete contracts. By focusing on a model with complete contracts, not only can I describe the interaction between the selection effect and the job search target effect and its empirical implications in a tractable model, but also I can separate their impact on labor markets from that of other mechanisms', such as the lack of agents' commitment and the particular form of wage formation, both of which have been well studied in the literature.

To characterize the socially efficient allocation, I start with the social planner's problem. A social planner decides (1) the separation rule of existing matches and (2) the search strategy for each worker. The efficient separation rule is given as a cutoff belief about match quality. When the belief about match quality is higher than the cutoff level, the planner keeps the underlying match. Otherwise, the planner destroys the match and naturally stops learning about its quality. Following the literature on directed search (Acemoglu and Shimer 1999, Moen 1997), I assume that there are numerous locations in the economy. A match forms only if a worker meets a firm at the same location. To workers, locations differ from each other in terms of the probability of finding a new job and promised pay. The efficient choice of a searching location is determined by the current state of a worker. If an employed worker is in a good match, he does not search for a new job. If an employed worker's current match quality is uncertain, he is sent to a specific location to find a new job, and the probability of getting a new job is non-increasing in the belief about his current match quality. An unemployed worker searches for a job at the location with the highest job-finding probability.

Under the efficient allocation, the interaction between job search and learning has a nontrivial impact on workers' job turnover. For matches with a short tenure, the job search target effect dominates the selection effect, so the EE transition rate is increasing in tenure. For matches with a long tenure, the selection effect dominates the job search effect, so the EE transition rate is decreasing in tenure. When the tenure is long enough, all uncertainty is resolved, and only good matches are kept and workers in those matches do not OJS anymore. As a result, the EE transition rate as a function of tenure first increases at low tenure levels, then decreases, and eventually becomes constant. Since the job separation rate is the sum of the EE transition rate and the EU transition rate, the separation rate-tenure profile also has a hump shape. These theoretical results are roughly consistent with a variety of stylized facts emerging from the data at both the micro and macro levels. For example, Farber (1994) find that the separation rate increases early in tenure and decreases later, but in the end, the separation rate becomes constant, and Menzio, Telyukova and Visschers (2012) find that the EE transition rate increases in tenure in the first four months, and decreases thereafter.

The current model also generates an implication for the relation between the OJS target of a worker and his motivation for OJS. When a worker has a high evaluation of his current match quality, he is not too afraid of losing his current job, so he looks only for promising new job if he searches on the job. Since promising jobs are also competitive, his job finding rate is low. On the other hand, when a worker has a low evaluation of his current match quality, he is afraid of being fired in the near future, so he wants to find a new job as soon as possible. As a result, his target job is less promising and also less competitive, and therefore, his job finding rate is high. This prediction is roughly consistent with a new empirical finding by Fujita (2012). He finds that (1) some workers engage in OJS because they fear losing their current job, while others search on the job because they are unsatisfied with their current job, and (2) the unsatisfied on-the-job searchers have a lower job finding rate and a higher wage growth due to the job transition.

My paper is closely related to Moscarini (2005), who nests Jovanovic's model (1984) into an equilibrium search model. There are several main differences. First, the economic intuitions of the separation-tenure relation are different in the two papers. In Moscarini (2005), the initial period in which the separation rate is increasing in tenure is called the wait-and-see phase, whose existence relies on the properties of the learning process. In particular, following Jovanovic (1984), Moscarini (2005) assumes that the production signal follows a diffusion process and therefore the sample path of the posterior of the match quality is continuous. Hence, an endogenous separation cannot be instantaneous but "kicks in" only after some time.[2] Thus, on average, the separation rate initially increases with tenure. However, the initial wait-and-see phase disappears if the learning process

[2] Mathematically, the separation rate is zero for a new match. Since the sample path of diffusion process is continuous and the separation rate is non-negative, the separation rate must increase for a while and then decrease.

has no continuous sample path.[3] In my model, the hump shape of the separation rate results from the combination of the job search target effect and the selection effect. Should the tenure-varying job search target effect be missing, both the EE transition rate and the separation rate would be decreasing in the beginning. Second, in Moscarini (2005), wage is determined over time by Nash bargaining between a worker and his employer, which is not efficient in general, while in my model, the learning and search allocation are efficient, which implies that the hump shape of the separation rate does not rely on the inefficiency of wage formation or workers' OJS behavior.

The rest of this paper is organized as follows. Section 2 presents the basic environment, individual payoff and learning process. I characterize the social planner's problem in Section 3. Section 4 considers a simple contract to implement the social planner's allocation in a frictional labor market. Section 5 discusses the empirical implications. A number of extensions are discussed in Section 6. All technical proofs can be found in the Appendix.

2 The Model

2.1 Physical Environment

Time is continuous. The economy is populated by a continuum of workers of measure one and by a continuum of firms of measure greater than 1. Each worker has the utility function $\int e^{-rT} C_T dT$, where $C_T \in \mathbb{R}$ is the worker's consumption at time T and r is his discount rate. Each firm has the payoff function$\int e^{-rT} \Pi_T dT$, where $\Pi_T \in R$ is the firm's profit at T. Each firm has one vacancy and can hire at most one worker. Vacant firms or unemployed workers are unproductive.

There is a continuum of locations indexed by a real number $l \in [0, 1]$. A vacant firm and a worker can match only if they are searching in the same location. In each period, both firms and workers decide which location to enter. A location is interpreted as a submarket if there are firms and workers there. Different submarkets can be indexed by the promised value to the worker, $x \in \mathbb{R}$, posted by firms in that market. I denote mapping $\Upsilon : [0, 1] \to \mathbb{R} \cup \varnothing$ as the submarket assignment function. In other words, $x = \Upsilon(l)$ is the promised value to the worker specified by the contract offered at location l, while $\Upsilon(l) = \varnothing$ means there is no submarket at location l. At location l with $\Upsilon(l) \neq \varnothing$, the ratio between the number of jobs that are vacant and the number of searching workers is denoted by $\tilde{\theta}(l) \in \mathbb{R}^+$. I refer to $\theta(x)$ as the tightness of the submarket at location l such that $x = \Upsilon(l)$. In other words, I do not distinguish between the two markets $l \neq l'$ with the same x.

[3] For example, the learning process is a Poisson process, as in my model.

All submarkets are subject to search frictions. In particular, workers and firms that are searching in the same location are brought into contact by a meeting technology with constant returns to scale that can be described in terms of the market tightness $\theta \in \mathbb{R}^+$. In particular, at any time a worker finds a vacant job with probability $p(\theta(l))$ at location l, where $\theta(l)$ is the market tightness at location l and function $p : \mathbb{R}^+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly increasing, strictly concave, which satisfies (i) $p(0) = 0$, (ii) $\lim\limits_{\theta \to 0} p'(\theta) = \infty$, and $\lim_{\theta \to \infty} p(\theta)$ is bounded by a finite number. Similarly, a vacancy meets a worker with rate $q(\theta(l))$ in location l where $q : \mathbb{R}^+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly decreasing function such that $q(\theta) = p(\theta)/\theta$ when $\theta > 0$, and $q(0)$ is bounded. When a firm and a worker meet, a new match is formed, and the worker's old match, if any, is destroyed.

Each firm chooses to enter at most one submarket by paying a maintenance flow cost k at any time and posts an employment contract x, which is the promised value to the worker. All workers, whether employed or unemployed, observe all available offers in the labor market and choose one submarket to enter and search for a new job. Different wage dynamics are allowed given the identical initial expected promise. In general, a worker's individual wage dynamics can depend on both the aggregate market variables and the match-specific payoff history.

The match between a firm and a worker is either good or bad. If the match is good, at any time, the matched firm receives 1 unit of payoff at a rate λ; if the match is bad, a matched firm receives nothing. Initially, a matched worker-firm pair shares symmetric information about the match quality with a common prior $\alpha_0 \in (0, 1)$ that the current match is good. They observe the outcomes and hold common posterior beliefs α_t throughout time, where α_t denotes the belief that they assign to the match being good at t, where t denotes the worker's tenure in his current job. For simplicity, no extra flow payoff is generated by a match. A match is destroyed exogenously at a rate δ at any time. An unemployed worker enjoys a flow payoff of $b > 0$, which can be interpreted as his home production. To avoid a trivial case where the endogenous separation is never optimal, assume $\alpha_0 \lambda > b > 0$, that is, a new match is better than no match, but no match is better than a bad match.

For a match with $\alpha = 1$, no belief adjustment happens regardless of its current period output. For a match with $\alpha \leq \alpha_0$, if the unit of payoff is received in tenure period t, α_{t^+} jumps to 1; otherwise, by standard Bayes' rule updating, the evolution of α_t follows

$$\dot{\alpha}_t = -\lambda(1 - \alpha_t)\alpha_t, \tag{1}$$

which is 0 if α_t equals either 0 or 1.

The worker's search strategy may depend on both the social state and his individual state. The former includes the unemployment rate and the distribution of the current match quality. The latter includes: (1) whether the worker is employed, and (2) the belief about the current match quality if he is employed. Formally, define $\Omega = [0, \alpha_0] \cup \{u\} \cup \{1\}$ as a worker's individual state space. A worker's state $\omega \in \Omega$ can be interpreted as follows. For an uncertain matched worker, his type $\omega \in [0, \alpha_0]$ is the belief about the current match quality. For a matched worker who has sent a good signal before, $\omega = 1$. For an unemployed worker, $\omega = u$. Denote the probability measure μ_T over Ω as the social state of the economy. Let $\Xi = \Delta(\Omega)$ denote the set to which μ_T belongs for all T. In this paper, I focus on the steady state, so $\mu_T = \mu^*$.

3 Efficient Allocation

To characterize the efficient allocation in the steady state, I solve the social planner's problem in the steady state first. Since I focus on the steady state, $\mu_T = \mu^*$, the planer's strategy depends on workers' individual states only.

For an unemployed worker, he enjoys a flow payoff, b. The planner sends him to search for a new job in submarket θ. In such a submarket, he finds that a new job arrives at a rate $p(\theta)$. To support the market tightness θ, the planner sends θ firms to this submarket for each unemployed worker. From the perspective of the planner, an unemployed worker's problem is governed by the following HJB function:

$$rS(u) = b + p(\theta(u))[S(\alpha_0) - S(u)] - k\theta(u), \tag{2}$$

where his efficient search strategy is pinned down by

$$k = p'(\theta(u))[S(\alpha_0) - S(u)]. \tag{3}$$

The left-hand side of (3) is the social marginal cost of vacancy creation, and the right-hand side is the social marginal benefit.

For an existing match, a good signal arrives at a rate of either λ or 0, which depends on the match quality, and an exogenous separation shock arrives at a rate δ. Given the belief of the current match quality, the planner chooses (1) the worker's on-the-job search strategy $\theta(\alpha)$, and (2) the separation strategy of the current match $z(\alpha) \in \{0, 1\}$. For any α, one can solve the associated value $S(\alpha)$ and policy function $\theta(\alpha)$, $z(\alpha)$.

Apparently, when $\alpha = 1$, it is inefficient to ask the worker to search on the job, so $\theta(\alpha) = \varnothing$, and $z(\alpha) = 0$. Thus the social value of a good match is pinned down by the following equation.

$$rS(1) = \lambda + \delta[S(u) - S(1)]. \tag{4}$$

For an uncertain match, $\alpha \in (0, \alpha_0]$. When $z(\alpha) = 0$, the social value of this match satisfies the following HJB function:

$$\begin{aligned} rS(\alpha) &= \alpha\lambda + \lambda\alpha(S(1) - S(\alpha)) - \lambda\alpha(1-\alpha)S'(\alpha) \\ &\quad + \delta(S(u) - S(\alpha)) + p(\theta(\alpha))[S(\alpha_0) - S(\alpha)] - k\theta(\alpha), \end{aligned} \tag{5}$$

and the optimal on-the-job search strategy is pinned down by

$$k = p'(\theta(\alpha))[S(\alpha_0) - S(\alpha)]; \tag{6}$$

When $z(\alpha) = 1$, I have $S(\alpha) = S(u)$. The following lemma shows that, given the social value of a unemployed worker, the socially optimal separation strategy $z(\alpha)$ can be characterized by a cutoff strategy.

Lemma 1. *Fixing $S(u)$, the constrained socially optimal problem of an employed worker satisfies the follows:*

1. *the optimal separation strategy is given by*

$$z(\alpha) = \begin{cases} 1 & \text{if } \alpha < \alpha_* \\ 0 & \text{otherwise} \end{cases}$$

2. *the cutoff belief α_* is the* largest *α such that*

$$S(\alpha_*) = S(u) \tag{7}$$

$$S'(\alpha_*) = 0 \tag{8}$$

and therefore $S(\alpha)$ is the solution of ODE (5) with the boundary conditions (7, 8).

3. *the on-the-job search is characterized by (6).*

The proof directly follows the exponential bandits literature, so it is omitted.[4] Given $S(\alpha)$, one can calculate the value of $S(u)$ by using equation (2). Hence, the efficient solution, $S(u), S(\alpha)$ can be solved as a fixed point of the system (2, 5) and satisfies (7, 8). The following proposition characterizes the properties of the efficient steady state. The stationary distribution μ^* is presented in the supplementary materials.

[4] See Keller, Rady and Cripps (2005). For an intuitive discussion on the boundary conditions (7) and (8), see chapter 4 of Dixit and Pindyck (1994).

Proposition 1. *The socially efficient allocation uniquely exists and it satisfies the following properties:*

1. $S(\alpha)$ *is convex for all* $\alpha \in [\alpha_*, \alpha_0]$.
2. $S(\alpha)$ *is strictly increasing for all* $\alpha \in [\alpha_*, \alpha_0]$,
3. $\theta(\alpha)$ *is strictly decreasing for all* $\alpha \in [\alpha_*, \alpha_0]$.
4. $\lim_{\alpha \to \alpha_*} \theta'(\alpha) = 0$ *and* $\lim_{\alpha \to \alpha_0} \theta(\alpha) = 0$.

The efficient job search strategy $\theta(\alpha)$ is strictly decreasing in α. The Bayes' rule of learning, equation (1), implies that α is decreasing in t, so the job finding rate of an employed worker with a current belief α_t, $p(\theta(\alpha_t))$ is increasing in his tenure t. When a worker starts to work at a firm, his evaluation of the current job is high, and he is encouraged to look only at promising jobs. Since promising jobs are also competitive, the job finding rate is low. When a worker stays at a firm for a long time and does not have a good record, his evaluation of the current match is low, so he is encouraged to look at less promising jobs to leave current position before it is destroyed. Since less promising jobs are less competitive, the job finding rate is high. As a result, I construct a theoretical link between a worker' evaluation of his current job and his efficient job finding rate.

Remark 1. *When* $p(\theta) = \min\{\theta, \bar{\theta}\}$ *and* $\bar{\theta}$ *is finite, there are no labor frictions in the market, and the on-the-job search decision problem is a linear programming problem with a corner solution:* $\theta(\alpha) = \bar{\theta}$ *if* $S(\alpha_0) - S(\alpha) > k$, $\theta = 0$ *otherwise.*

Remark 2. *When the match quality is known, only a good match is created, so* $\alpha_0 = 1$. *The absence of learning implies that matched workers' and firms' values are constant over time and on-the-job search is not efficient. Hence, there is only one active submarket in the social planner's solution.*

The intuition of the two remarks above is as follows. The social planner's fundamental trade-off is between the replacement premium of an existing match and the cost of creating a vacancy. When match quality is common knowledge, neither learning nor on-the-job search has value; hence, the optimal allocation is a corner solution. When the market is frictionless, the fundamental trade-off becomes a linear programming problem, and distinguishing a match with a different belief is not necessary. Hence, the optimal allocation is a corner solution as well. The two remarks imply that, in such an environment, under efficient allocation, nontrivial OJS dynamics can only result from the interaction between learning and search friction.

4 Decentralization

In this section, I consider the implementation of the social planner's solution. I assume that the contracts offered by firms to workers are bilaterally efficient in the sense that they maximize the joint value of the match, that is, the sum of the worker's expected lifetime utility and the firm's expected lifetime profits. I make this assumption because there are a variety of specifications of the contract space under which the contracts that maximize the firm's profits are, in fact, bilaterally efficient. As Menzio and Shi (2009) show in a similar environment, the profit-maximizing contracts are bilaterally efficient if the contract space is complete in the sense that a contract can specify the promised utility to the worker, x, the separation probability z and the worker's on-the-job search strategy θ. This result is intuitive. The firm maximizes its profits by choosing the contingencies z, x so as to maximize the joint value of the match and by choosing the contingencies for w so as to deliver the promised value x.[5]

In order to decentralize the social planner's optimal allocation, I first define the joint surplus of an uncertain existing match $M(\alpha)$. Note that $M(\alpha)$ is *not* the social surplus generated by the match since the matched worker and firm do not take into account the wage posting cost paid by the worker's potential new employer.

Labor market supply side. First, consider an employed worker at the beginning of the search and matching stage. Since the contract is bilaterally efficient, given the equilibrium market tightness function θ, the worker chooses to search in the submarket with promised value $x(\theta)$ to maximize the continuation value of his current match, which is given by

$$rM(1) = \lambda + \delta[V_u - M(1)] \tag{9}$$

$$\begin{aligned} rM(\alpha) &= \alpha\lambda + \lambda\alpha(M(1) - M(\alpha)) - \lambda\alpha(1-\alpha)M'(\alpha) \\ &\quad + \delta(V_u - M(\alpha)) + p(\theta(\alpha))[x(\theta) - M(\alpha)], \end{aligned} \tag{10}$$

and the optimal OJS strategy is

$$\theta(\omega) = \arg\max p(\theta)[x(\theta) - M(\omega)] \text{ for } \omega \in [0, \alpha_0] \cup \{1\} \tag{11}$$

[5]Moreover, one can prove that the profit-maximizing contracts are bilaterally efficient if they can specify the wage only as a function of tenure and productivity (while the separation and search decisions are made by the worker). This result is also intuitive. The firm maximizes its profits by choosing the wage when it meets a worker so as to deliver the promised value x and by choosing the wage as a function of the belief about the match so as to induce the worker to maximize the joint value of the match (by setting the wage equal to the product of the match). Alternatively, profit-maximizing contracts are bilaterally efficient if they can specify severance transfers that induce the worker to internalize the effect of his separation and search decisions on the firm's profits. See Moen and Rosen (2004), and Menzio and Shi (2009, 2011) for more examples.

By the same logic, an unemployed worker chooses to search in the submarket with tightness $\theta\left(x_u\right)$ and promised value x_u to maximize his value, which is given by

$$rV_u = b + p\left(\theta\left(u\right)\right)\left[x\left(\theta(u)\right) - V_u\right] \tag{12}$$

where the optimal search strategy is

$$\theta\left(u\right) = \arg\max p\left(\theta\right)\left[x\left(\theta\right) - V_u\right] \tag{13}$$

with a similar interpretation.

Labor market demand side. Firms without a match are on the demand side of the labor market. They choose whether to enter the labor market and which submarket to enter. The competition in the labor market implies that firms' expected discounted profit is zero, and there is no difference between any of the submarkets for any firm. A firm may form a new match at a rate $q(\theta)$, which depends on the tightness of the market the firm is in, and its expected profit is given by $M\left(\alpha_0\right) - x$.

By posting a new job, a firm needs to pay a flow cost k . Hence, the firm's free-entry condition is given by

$$q\left(\theta\right)\left[M\left(\alpha_0\right) - x\left(\theta\right)\right] = k. \tag{14}$$

Labor market equilibrium. The labor market equilibrium consists of (1) $M\left(\alpha\right), V_u, \theta\left(\alpha\right), \theta\left(u\right)$ and $x\left(\theta\right)$, which satisfy equations (9), (10), (11), (12), (13), and (14), and (2) a stationary distribution, μ^*, which is consistent with $\theta\left(\alpha\right)$ and $\theta\left(u\right)$. The firm's free-entry condition implies that $M\left(\alpha_0\right) - x = k/q\left(\theta\right)$ for any submarket. The fact that $q(\theta) = p(\theta)/\theta$ yields that

$$p\left(\theta\right)\left[M\left(\alpha_0\right) - x\right] = k\theta \tag{15}$$

Plugging (14) into (10) and (12) implies that

$$\begin{aligned} rM(\alpha) \quad = \quad & \alpha\lambda + \lambda\alpha(M(1) - M(\alpha)) - \lambda\alpha(1-\alpha)M'(\alpha) \\ & + \delta(V_u - M(\alpha)) + p(\theta(\alpha))[M\left(\alpha_0\right) - M(\alpha)] - k\theta, \end{aligned} \tag{16}$$

and

$$rV_u = b + p\left(\theta_u\right)\left[M\left(\alpha_0\right) - V_u\right] - k\theta_u \tag{17}$$

So in the equilibrium $M\left(\alpha\right) = S\left(\alpha\right)$, $V_u = S_u$, and therefore the equilibrium search strategy is efficient. Given the unique equilibrium strategy θ, one can uniquely pin down the individual's state transition and therefore the stationary distribution μ^*. The following proposition summarizes the analysis above.

Proposition 2. *A stationary equilibrium exists and it is efficient.*

5 Empirical Implications

Selection Effect and OJS Probability. In this model, learning about match quality generates a number of interesting empirical implications. First, as in Jovanovic (1984), there is a selection effect of learning. Specifically, over time, good matches should send signals with higher probability. Hence, the probability that the match quality is good and is known is increasing in the tenure. Since workers in good matches do not engage in OJS, the selection effect implies that the OJS probability of workers is decreasing in their tenure. Consider a randomly picked worker with tenure t. Without knowing his signal history, one does not know for sure whether this match is good or whether this worker is searching on the job. Nonetheless, it is possible to find the ex ante probability that a randomly chosen worker is searching on the job, as a function of t, which is

$$\sigma_t \equiv \begin{cases} \alpha_0 \Pr(\tau > t) + (1 - \alpha_0) & t < t^*, \\ 0 & t \geq t^*, \end{cases}$$

where τ is a random variable representing the time at which the good signal occurs, so the probability that it has not happened by t is $\Pr(\tau > t) = \exp(-\lambda t)$, which is decreasing in t. The critical cutoff time is defined as $t^* = \inf\{t > 0 | \alpha_0 - \alpha_* = \int_0^{t^*} \lambda \alpha_s (1 - \alpha_s) ds\}$, at which point the belief hits α_* and the firm optimally destroys the current uncertain match, so the worker becomes unemployed. Before this point, a match is bad with ex ante probability $1 - \alpha_0$, and the worker always searches for a new job. With complementary probability a_0, the match is good, and the worker searches only when a good signal has not arrived; hence, the quality remains uncertain. Therefore, the model predicts that the OJS probability is at first decreasing in workers' tenure but eventually becomes constant. This negative relation between the probability of OJS and tenure is supported by many empirical findings, for example, Pissarides and Wadsworth (1994).

OJS Target Effect. Departing from the standard learning literature, I highlight another effect of learning: the job search target effect. If a match has not generated good signal before, the belief about the current match decreases over time. As the separation time t^* approaches, the worker is afraid of being unemployed soon, so he adjusts his OJS strategy to raise the job finding rate. However, to raise the job finding rate, the worker must lower his OJS target because in an equilibrium labor market, only less promising jobs are less competitive and so have higher job finding rates. The following proposition formalizes the intuition above.

Proposition 3. *In the stationary equilibrium, the promised utility of the job targeted by a worker's OJS is increasing in his belief about his current match being good, and his job finding rate is decreasing in his belief.*

Owing to the lack of the data of workers' belief, it is hard to directly test the prediction in

Proposition 3. However, this prediction is indirectly supported by the recent empirical finding by Fujita (2012). Fujita (2012) distinguishes workers who are searching on the job based on their motivation: some of them are unsatisfied about their current job, and others are afraid of losing their current jobs. He finds that (1) the job finding rate of the former is higher than that of the latter, and (2) after job transitions, the former' wage increment is significantly higher than the latter's.

EE Transition. An EE transition takes place only if an employed worker actively searches for and gets a new job. At any moment, this tenure-dependent transition rate is defined by

$$\xi_t^{ee} \equiv \sigma_t p\left(\theta\left(\alpha_t\right)\right).$$

The EE transition happens only if a worker is looking for another job. Given his tenure, the probability that a worker is engaging in OJS is σ_t. Conditional on that, the probability that he actually finds a new job is $p(\theta(\alpha_t))$. The tenure effect on ξ_t^{ee} is driven by two forces, the *job search target effect* and the *selection effect* in opposite directions. The former implies that the probability of OJS declines over tenure, and so does the EE transition rate; while the latter implies that the job finding rate increases over tenure, and so does the EE transition rate. Hence, these two effects drive the EE transition rate in opposite directions over tenure. In the following proposition, I show that the job search target effect dominates in the early stage of a worker's tenure, while the selection effect dominates later. The intuition is that, in the early stage of a worker's tenure, his belief is high, so in his target market of OJS, market tightness θ is very small. By the Inada condition of the matching function, $p' \to \infty$ for small θ. As a result, a tiny change in the worker's OJS strategy, θ, has a huge impact on his job finding rate, and therefore the job search target effect dominates the selection effect in the beginning and the EE transition rate is increasing in tenure. When it comes to the worker with a long tenure, the job search target effect is diminished, so the EE transition rate is decreasing in tenure. When a worker's tenure is long enough, all uncertainty is resolved and only good matches are kept, so the match quality must be good and therefore the EE transition rate is zero.

Proposition 4. *There is $\underline{t} > 0$, $\bar{t} < t^*$ such that (1) $\bar{t} > \underline{t}$, and the EE transition rate ξ_t^{ee} is increasing in tenure for $t < \underline{t}$, decreasing for $t \in [\bar{t}, t^*]$, and zero for $t > t^*$.*

EU Transition. The EU transition rate as a function of tenure is $\xi_t^{eu} = \delta$ for any $t \neq t^*$, when the EU transition happens only as a result of exogenous separation. At $t = t^*$, in addition to exogenous separations, all matches that did not send a good signal will be endogenously destroyed, the measure of which is positive. The atom of the EU rate results from the assumption of a precise and uniform learning process. If the learning process is heterogeneous as a consequence of either different priors or noisy observations, such an atom can be eliminated. The mass point in the EU transition rate showing at a particular tenure point is considered empirically irrelevant. However, it fits the observation in the academic job market, where learning is based on relatively uniform and precise information on the quality and quantity of research publications.

Separation Rate. Separation of an existing match may result from either EE or EU transition; hence, the separation rate of an existing match with tenure t, ξ_t must be $\xi_t = \xi_t^{ee} + \xi_t^{eu}$. When $t \neq t^*$, the EU transition rate δ is tenure-free, the tenure effect on the separation rate is almost identical to that on the EE rate. At $t = t^*$, a positive measure of matches will be separated. Just as for that of the EU transition rate, this mass point of separation hazard at t^* is also empirically irrelevant.

Corollary 1. *Generically, the job separation rate is increasing in tenure for $t < \underline{t}$, decreasing for $t \in [\bar{t}, t^*]$, and equal to δ for $t > t^*$.*

The prediction on the separation rate-tenure profile is consistent with the previous empirical literature. For example, using weekly data, Farber (1994) finds that in the first six months, the separation rate is increasing in tenure. In the current paper, to focus on the dynamics of the EE transition rate on the job separation rate, I assume that the EU rate is constant for most t, and the hump-shaped job separation rate results from the similar shape of the EE transition rate. To the best of my knowledge, there is no empirical study using weekly data to estimate the EE and the EU transition rate-tenure profile. However, there is empirical evidence that supports the non-monotone EE transition rate-tenure profile in the analysis of monthly data sets. Using the U.S. Census' Survey of Income and Program Participation (SIPP), Menzio, Telyukova and Visschers (2012) find that (1) the EU transition rate is decreasing in tenure, and (2) the EE transition rate increases in tenure in the first four months (from 3% to 5%), and decreases thereafter. Since the EE transition makes up 49% of all separations, while the EU transition makes up only 20%, the shape of the EE transition rate should contribute more to the separation rate. Hence, it is reasonable to believe that the hump-shaped separation rate-tenure profile is mainly due to the similar shape of the EE-tenure profile.

6 Concluding Remark

I conclude the paper by discussing some possible extensions.

Bad News Cases. In this paper, I focus on the perfect good news learning process. This is perfect reasonable for some industries, for example, academia. However, an obvious and natural alternative is to study the perfect bad news learning process. Assume that a match generates an quality-independent flow payoff is $y > 0$, and a good match generates no extra loss but an bad one generates a 1-unit loss at a rate λ. Furthermore, as in the good news model, I assume that a good match is better than no match, and no match is better than a bad match to avoid a trivial case where endogenous separation is never optimal, i.e., $y - (1 - \alpha_0)\lambda > b > y - \lambda$. In this case, when bad news is realized, the firm learns that the match is bad and therefore fires the worker immediately. By observing a history with no bad news, a matched firm and worker become more and more optimistic about their match quality. In this economy, any existing match has a belief α higher than α_0 in equilibrium; thus on-the-job search is not valuable. The equilibrium has only one labor market with market tightness $\theta(u)$.

Imperfect Good News Cases. In the benchmark model, I assume that a bad match cannot generate any profit, which seems restrictive. What if it can generate one unit of reward at a lower rate $\lambda_b \in (0, \lambda)$? To avoid a trivial case, I assume $\lambda_b < b < \alpha_0\lambda + (1-\alpha_0)\lambda_b$. In other words, a new match is better than no match, but no match is better than a bad match. Then, given no reward arriving in $[t, t+dt)$, the belief at the end of that time period is $\alpha_{t+dt} = \frac{\alpha_t \exp(-\lambda dt)}{\alpha_t \exp(-\lambda dt) + (1-\alpha_t)\exp(-\lambda_b dt)}$ by Bayes' rule. Yet, if one reward is realized in $[t, t+dt)$, the belief about the match quality jumps up from α_t to

$$\alpha_{t+dt} = \frac{\alpha_t[1 - \exp(-\lambda dt)]}{\alpha_t[1 - \exp(-\lambda dt)] + (1 - \alpha_t)[1 - \exp(-\lambda_b dt)]},$$

by Bayes' rule. When dt goes to zero, the updating can be approximated by

$$\dot{\alpha}_t = \lim_{dt \to 0} \frac{\alpha_{t+dt} - \alpha_t}{dt} = \begin{cases} -(\lambda - \lambda_b)\alpha_t(1 - \alpha_t) & \text{no reward at } t, \\ \frac{\lambda\alpha_t}{\lambda\alpha_t + \lambda_b(1-\alpha_t)} & \text{one reward at } t, \end{cases}$$

and the probability that more than one reward is realized is $O(dt^2)$, which is negligible when dt is small. By the same logic, one can solve the social planner's optimal stopping belief and OJS strategy. Over time, good matches can survive with higher probability than bad ones due to the dynamics of endogenous separation driven by learning; thus, the empirical implications for job transitions still hold qualitively. However, the implications are slightly different from those in the benchmark model in the following sense: (1) No match is believed to be good for sure, and therefore, the endogenous separation will not disappear even for the match with long tenure. (2) The arrival of a reward can increase the belief about match quality; thus, it is possible that a belief $\alpha_t \in (\alpha_0, 1)$ appears in equilibrium. Clearly, it is inefficient to destroy a match with belief higher than α_0, and therefore, employed workers with belief $\alpha > \alpha_0$ will not search on-the-job under a bilaterally efficient contract. At the beginning of a match, the job search target effect works as well and dominates the selection effect. When the difference between λ and λ_b is sufficiently large, the learning process is similar to that in the perfect good news model. Hence, the non-monotonicity of the EE transition rate is preserved.

Informative Interview. In the benchmark model, the match is modeled as an experience good whose quality needs to be slowly learned over time. Yet, in some situations, the employer can extract non-trivial information about the match quality through an interview. Suppose a firm can draw an informative signal of the match quality and update its belief about the match quality through an interview before the match is formed. The signal is drawn from a match quality dependent distribution that satisfies the monotone likelihood ratio property (MLRP), and the updated posterior $\tilde{\alpha}_0 \in [\underline{\alpha}_0, \bar{\alpha}_0]$, where $0 < \underline{\alpha}_0 < \bar{\alpha}_0 < 1$. In this extension, the social planner will form a new match only if the updated posterior $\tilde{\alpha}_0$ is higher than a cutoff level that depends on the worker's current state. For an unemployed worker, this cutoff is the stopping time belief α_*. For an employed worker, this cutoff is the belief α_t about the worker's current match quality. Let $\Pr(\tilde{\alpha}_0 > \alpha_t)$ be the ex ante probability that the posterior is larger than the worker's current belief. Hence, the on-the-job search policy is determined by

$$\max_{\theta} p(\theta) \Pr(\tilde{\alpha}_0 > \alpha_t)\{\mathbb{E}[S(\tilde{\alpha}_0)|\tilde{\alpha}_0 > \alpha_t] - S(\alpha_t)\} - k\theta.$$

It is clear that both $\Pr(\tilde{\alpha}_0 > \alpha_t)$ and $\mathbb{E}[S(\tilde{\alpha}_0)|\tilde{\alpha}_0 > \alpha_t] - S(\alpha_t)$ are non-increasing in α_t; thus the optimal policy $\theta(\alpha)$ is non-increasing in α, which is similar to that in the benchmark model. Hence the empirical implications for workers' turnover predicted by the benchmark model are qualitively preserved.

Costly On-the-Job Search. Suppose workers' on-the-job search requires a flow cost ϵdt. To avoid a trivial case where OJS is always suboptimal, I assume that ϵ is small enough. Since the gain from on-the-job search $\max_{\theta}\{p(\theta)[S(\alpha_0) - S(\alpha)] - k\theta\}$ is increasing in the job replacement premium, $[S(\alpha_0) - S(\alpha)]$, for small enough ϵ, there exists a cut-off belief α^{ϵ} such that

$$\begin{aligned}
\max_{\theta} p(\theta)[S(\alpha_0) - S(\alpha)] - k\theta &\leq \epsilon \text{ if } \alpha \geq \alpha^{\epsilon}, \\
\max_{\theta} p(\theta)[S(\alpha_0) - S(\alpha)] - k\theta &> \epsilon \text{ if } \alpha < \alpha^{\epsilon}.
\end{aligned}$$

In other words, matched workers would search on-the-job only if they believed the match quality is low enough. When $\alpha < \alpha^{\epsilon}$, the social planner's problem is unchanged, and therefore, it is obvious that introducing costly OJS does not change the main result but reduces the social surplus $S(\alpha)$ for each α and therefore shortens the duration of experimentation.

Appendix A

Proof of Proposition 1

Existence and Uniqueness. The proof follows the standard contraction mapping fixed-point argument. Given any $S(u) \in [\frac{b}{r}, \frac{\lambda}{r}]$, from (2,4,5), there exists a unique solution $S(\alpha)$ such that

$$S(\alpha) = S(u) + \int_{\alpha_*}^{\alpha} h(x, S(x))dx, \tag{18}$$

where $h(\alpha, S(\alpha))$ is the right-hand side of (5) divided by r.

And $\theta(u) = \arg\max\{p(\theta)[S(\alpha_0) - S(\alpha)] - k\theta\}$. For an unemployed worker,

$$rS(u) = b + \max_{\theta}\{p(\theta)[S(\alpha_0) - S(u)] - k\theta\},$$

or

$$S(u) = \frac{b + p(\theta(u))S(\alpha_0) - k\theta(u)}{r + p(\theta(u))}. \tag{19}$$

The envelop theorem implies that

$$\frac{dS(u|S(\alpha_0))}{dS(\alpha_0)} = \frac{p(\theta(u|S(\alpha_0)))}{r + p(\theta(u|S(\alpha_0)))} \in (0, 1). \tag{20}$$

Combining (18) and (19) yields

$$S(\alpha) = \frac{b + p(\theta(u))S(\alpha_0) - k\theta(u)}{r + p(\theta(u))} + \int_{\alpha_*}^{\alpha} h(x, S(x))dx.$$

Define an operator $\boldsymbol{T}_p : C[0, \alpha_0] \to C[0, \alpha_0]$ where $S(\alpha) \in C[0, \alpha_0]$ is any bounded continuous differentiable function and $\boldsymbol{T}_p S = \frac{b - p(\theta(u))S(\alpha_0) - k\theta(u)}{r + p(\theta(u))} + \int_{\alpha_*}^{\alpha} h(x, S(x))dx$ where S is such that (18) and (19) hold. To prove the uniqueness of $S(\omega)$, one needs to verify whether $\boldsymbol{T}_p$ is a contraction mapping. For the second part of $\boldsymbol{T}_p S$, by the standard contraction mapping argument of the existence and uniqueness to the solution in the problem of an ordinary differential equation with initial value, $\boldsymbol{T}_p^2 S = \int_{\alpha_*}^{\alpha} h(x, S(x))dx$ satisfies the Blackwell sufficient condition. Moving to the first part, $\boldsymbol{T}_p^1$, one needs to check whether the Blackwell sufficient condition holds.

Let $S_1 > S_2$, then $S(u|S_1(\alpha_0)) > S(u|S_2(\alpha_0)$ following (20); thus, the monotonicity of $\boldsymbol{T}_p^1$ is proved. Move to the discounting property. Let $n \in \mathbb{R}^+$, and $S_3 = S_1 + n$. Following (20), $\frac{dS(u|S(\alpha_0))}{dS(\alpha_0)} < 1$ for all $S(\alpha_0) \in [S_1(\alpha_0), S_1(\alpha_0) + n]$; thus the discounting property of $\boldsymbol{T}_p^1$ holds. Hence, $\boldsymbol{T}_p = \boldsymbol{T}_p^1 + \boldsymbol{T}_p^2$ satisfies the Blackwell sufficient condition, and therefore it is a contraction mapping on a complete functional space, $C[0, \alpha_0]$. There exists a unique solution $S(\alpha)$ such that $S(\alpha) = \boldsymbol{T}_p S(\alpha)$, and $S(u)$ is also determined uniquely!

Convexity. Consider two α_1, α_2 such that (1) $\alpha_1, \alpha_2 \in (0, \alpha_0]$, and (2) $\alpha_* < \alpha_2 < \alpha_1 \le \alpha_0$. Let $\alpha^\zeta = \zeta\alpha_2 + (1-\zeta)\alpha_1$ where $\zeta \in (0,1)$. I want to show that for all α_1, α_2 and α^ζ, $S(\alpha^\zeta) \le \zeta S(\alpha_2) + (1-\zeta)S(\alpha_1)$. Denote $\theta^\zeta(\alpha)$ as the path of optimal on-the-job search starting from α^ζ during the current match. Denote by $S^g(\theta)$ the expected surplus from an arbitrary path of on-the-job search θ conditional on the true match quality being good and similarly for $S^b(\theta)$. Then $S(\alpha^\zeta) = \alpha^\zeta S^g(\theta^\zeta) + (1-\alpha^\zeta)S^b(\theta^\zeta)$. And it holds that

$$\begin{aligned} S(\alpha_1) &\ge \alpha_1 S^g(\theta^\zeta) + (1-\alpha_1)S^b(\theta^\zeta), \\ S(\alpha_2) &\ge \alpha_2 S^g(\theta^\zeta) + (1-\alpha_2)S^b(\theta^\zeta), \end{aligned}$$

since θ^ζ is a feasible price path. Hence

$$\begin{aligned} \zeta S(\alpha_2) + (1-\zeta)S(\alpha_1) &\ge \left\{ \begin{array}{c} \zeta[\alpha_1 S^g(\theta^\zeta) + (1-\alpha_1)S^b(\theta^\zeta)] \\ +(1-\zeta)[\alpha_2 S^g(\theta^\zeta) + (1-\alpha_2)S^b(\theta^\zeta)] \end{array} \right\} \\ &= (\zeta\alpha_1 + (1-\zeta)\alpha_2)S^g(\theta^\zeta) \\ &\quad +(1-\zeta\alpha_1-(1-\zeta)\alpha_2)S^b(\theta^\zeta) \\ &= \alpha S^g(\theta^\zeta) + (1-\alpha)S^b(\theta^\zeta) = S(\alpha), \end{aligned}$$

which contradicts the fact that the solution of the HJB function maximizes the planner's individual worker problem. This proves the claim.

Monotonicity of $S(\alpha)$. Since $S'' \ge 0$, and $S'(\alpha_*) = 0$, $S' \ge 0$ for all $\alpha \in [\alpha_*, \alpha_0]$. There are three cases. The first one is $S' = 0$ for all $\alpha \in [\alpha_*, \alpha_0]$, which implies that $\alpha_* = \alpha_0$. The second one is that $S' = 0$ for all $\alpha \in [\alpha_*, \hat{\alpha}]$ but $S' > 0$ for $\alpha \in (\hat{\alpha}, \alpha_0]$. But in this case, $S(\alpha) = S(u)$ and $S' = 0$ for $\alpha \in (\alpha_*, \hat{\alpha}]$, which contradicts the definition of α_*. The third one is that $S' > 0$ for all $\alpha \in (\alpha_*, \alpha_0]$.

Properties of $\theta(\alpha)$. The optimal on-the-job search decision satisfies $p'(\theta(\alpha))[S(\alpha_0) - S(\alpha)] = k$. When $\alpha \to \alpha_0$, $S(\alpha) \to S(\alpha_0)$, so $p'(\theta(\alpha)) \to 0$ and $\theta(\alpha) \to 0$. Differentiating $p'(\theta(\alpha))[S(\alpha_0) - S(\alpha)] = k$ yields

$$p''(\theta(\alpha))[S(\alpha_0) - S(\alpha)]\theta'(\alpha) = p'(\theta)S'(\alpha),$$

since $S(\alpha_0) - S(\alpha) > 0$ for any $\alpha < \alpha_0$, $p'' < 0$, I have $\theta'(\alpha) < 0$. Also, p', p'' and $S(\alpha_0) - S(\alpha)$ is bounded. When α goes to α_*, $S'(\alpha)$ goes to zero, and therefore θ' goes to zero. ■

Proof of Proposition 3

The optimality of workers' OJS search implies that $\theta(\alpha)$ must satisfy the following first-order condition: $p'(\theta)[x(\theta) - M(\alpha)] + p(\theta)x'(\theta) = 0$, which implies that

$$x'(\theta(\alpha)) = -\frac{p'(\theta(\alpha))[x(\theta(\alpha)) - M(\alpha)]}{p(\theta(\alpha))} < 0.$$

By Proposition 1, $\theta'(\alpha) < 0$; thus, I have $\frac{dx}{d\alpha} = x'(\theta(\alpha))\theta'(\alpha) > 0$. ■

Proof of Proposition 4

Differentiate ξ_t^{ee} yields

$$\dot{\xi}_t^{ee} = \underbrace{p'\theta'\dot{\alpha}\sigma_t}_{(+)} - \underbrace{\alpha_0\lambda\exp(-\lambda t)p(\theta(\alpha))}_{(+)}.$$

The first term of the right hand side is that conditional on the match not having sent a good signal before, the matched worker becomes more pessimistic over time, and therefore, his on-the-job search becomes more aggressive. Thus the probability of getting a new job becomes greater, and this raises the EE rate. The second one that lowers the EE rate is simply the decreasing probability of a good match not having sent a good signal.

When $t \to 0$, $\alpha_t \to \alpha_0$, so $\theta(\alpha) \to 0$ by Proposition 1. By the Inada condition of the matching function, $p'(\theta) \to \infty$, so the first force dominates the second. As t approaches t^*, $\alpha \to \alpha_*$. By Proposition 1, $\theta'(\alpha) \to 0$, which implies that the effect of the first force goes to zero. Hence, the second one becomes dominant. Yet, if a random match's tenure is greater than t^*, only the good match can survive, in which case the EE transition rate is zero. ■

7 Supplementary Materials: Stationary Distribution

In a large labor market with a continuum population of workers, by assuming "the law of large number" holds, the invariant distribution, if it exists, can be interpreted as the stationary cross-sectional distribution of workers' state. In particular, the following Proposition shows that the market equilibrium has the unique stationary wage distribution μ^* : There are two mass points at $\omega = 1$ and $\omega = u$. For $\omega \in [\alpha_*, \alpha_0]$, the probability density function is well defined. Denote the p.d.f. of stationary belief distribution as $\phi(\alpha)$ for $\alpha \in [\alpha_*, \alpha_0]$, $\beta = \mu^*(1)$ the probability mass at $\omega = 1$, and $\upsilon = \mu^*(u)$ at $\omega = u$.

Proposition 5. *The stationary distribution of workers' state, μ^*, is characterized by (υ, β, ϕ) where*

$$\upsilon = \mu^*(u), \beta = \mu^*(1), \phi(\alpha) \textit{ is the pdf of } \mu^* \textit{ for } \alpha \in [\alpha_*, \alpha_0],$$

the probability density function $\phi(\alpha)$ is given by

$$\phi(\alpha) = \kappa(\alpha)/A \textit{ for } \alpha \in (\alpha_*, \alpha_0], \tag{21}$$

$$\phi(\alpha_*) = 0, \tag{22}$$

and β, υ such that

$$\beta = \frac{1}{\delta A}\int_{\alpha_*}^{\alpha_0} \lambda\alpha\kappa(\alpha)d\alpha, \text{ and } \upsilon = \frac{\int_{\alpha_*}^{\alpha_0}\lambda\alpha\kappa(\alpha)d\alpha + \int_{\alpha_*}^{\alpha_0}\delta\kappa(\alpha)d\alpha + 1}{Ap(\theta(u))}, \tag{23}$$

where

$$A = \int_{\alpha_*}^{\alpha_0}\kappa(\alpha)d\alpha + \frac{\int_{\alpha_*}^{\alpha_0}(\lambda\alpha+\delta)\kappa(\alpha)+1}{\delta p(\theta(u))}\int_{\alpha_*}^{\alpha_0}\lambda\alpha\kappa(\alpha),$$

and

$$\kappa(\alpha) = \exp\left[\int_{\alpha_*}^{\alpha}\frac{\lambda s + \delta + p(\theta(s))}{\lambda s(1-s)}ds\right].$$

The stationary distribution can be calculated by making the inflow equal outflow at any $\omega \in \Omega^*$. The density at α_* is zero since the Markov process is right continuous with respect to calendar time. For interior point α, the only inflow comes from match with belief $\alpha' > \alpha$ that survives but has not sent an good signal, while the outflow is $\mu(\alpha)$. In the steady state, $\mu_{T_1}(\alpha) = \mu_{T_2}(\alpha) = \phi(\alpha)$ for any $T_1, T_2 \geq 0$.

$$\lambda\alpha(1-\alpha)\frac{d}{d\alpha}\phi(\alpha) = [\lambda\alpha + \delta + p(\theta(\alpha))]\phi(\alpha), \tag{24}$$

where $\phi(\alpha)$ is the probability density at α.

At α_0, the inflow comes from matched and unemployed workers who successfully find a new job; outflow is $\phi(\alpha_0)$, in the steady state, $\mu_{T_1}(\alpha_0) = \mu_{T_2}(\alpha_0)$ for any $T_1, T_2 \geq 0$, and thus I have

$$\int_{\alpha_*}^{\alpha_0} p(\theta(\alpha))\phi(\alpha)d\alpha + \upsilon p(\theta(u)) = \phi(\alpha_0), \tag{25}$$

where υ is the measure of u-workers. Both the left-hand side and right-hand side of (25) are finite, and therefore there is no mass point at α_0.

For unemployed workers, the inflow comes from the separation of an existing match, while the outflow is the measure of unemployed workers who find a job. Letting inflow equal outflow, I have

$$\nu p(\theta(u)) = \beta\delta + \int_{\alpha_*}^{\alpha_0}\delta\phi(\alpha)d\alpha + \phi_u, \tag{26}$$

where ϕ_u is the density of workers who have just been fired, β is the measure of 1-workers.

Combining (25) and (26) yields

$$\phi(\alpha_0) = \int_{\alpha_*}^{\alpha_0}[p(\theta(\alpha) + \delta]\phi(\alpha)d\alpha + \phi_u + \beta\delta. \tag{27}$$

For a successful match, the inflow comes from a new good signal sent by an existing uncertain match, and the outflow comes from the exogenous separation. Inflow equals outflow implies that

$$\beta = \frac{1}{\delta}\int_{\alpha_*}^{\alpha_0}\lambda\alpha\phi(\alpha)d\alpha. \tag{28}$$

Given the equilibrium $\theta(\alpha), \alpha_*$, and matching function $p(\cdot)$, one can obtain a general solution of the ODE (24), which is given by

$$\phi_{\tilde{A}}(\alpha) = \frac{1}{\tilde{A}} \exp[\int_{\alpha_*}^{\alpha} \frac{\lambda s + \delta + p(\theta(s))}{\lambda s(1-s)} ds], \tag{29}$$

where $1/\tilde{A}$ is a constant positive number to ensure $\phi > 0$. To fix A, one needs to use a boundary condition implied by the fact that ϕ is a density function and ν, β are the probability. The condition is given by

$$\int_{\alpha_*}^{\alpha_0} \phi(\alpha) d\alpha = 1 - \upsilon - \beta. \tag{30}$$

Plugging (26) and (28) into (30) yields

$$\int_{\alpha_*}^{\alpha_0} \phi(\alpha) d\alpha = 1 - \frac{\int_{\alpha_*}^{\alpha_0} (\lambda\alpha + \delta)\phi(\alpha) d\alpha + \phi(\alpha_*)}{p(\theta(u))} - \frac{1}{\delta} \int_{\alpha_*}^{\alpha_0} \lambda\alpha\phi(\alpha) d\alpha.$$

Let $\kappa(\alpha) = \exp[\int_{\alpha_*}^{\alpha} \frac{\lambda s + \delta + p(\theta(s))}{\lambda s(1-s)} ds]$, and $\phi_{\tilde{A}}(\alpha) = \kappa(\alpha)/\tilde{A}$. Then the $\tilde{A} = A$ satisfying the boundary condition (30) is given by

$$A = \int_{\alpha_*}^{\alpha_0} \kappa(\alpha) d\alpha + \frac{\int_{\alpha_*}^{\alpha_0} (\lambda\alpha + \delta)\kappa(\alpha) d\alpha + 1}{\delta p(\theta(u))} \int_{\alpha_*}^{\alpha_0} \lambda\alpha\kappa(\alpha) d\alpha.$$

Given the solution $\phi(\alpha)$, $\phi(\alpha_0)$, $\phi_u = \lim_{\alpha \to \alpha_*} \phi(\alpha) = 1/A$, and υ, β can be solved by (26) and (28). Since (21) and (23) uniquely pin down μ^*, the stationary distribution is unique. ■

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