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"Shopping Externalities and Self-Fulfilling Unemployment Fluctuations "

by

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Shopping Externalities and Self-Fulfilling Unemployment Fluctuations*

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Abstract

We propose a novel theory of self-fulfilling fluctuations in the labor market. A firm employing an additional worker generates positive externalities on other firms, because employed workers have more income to spend and have less time to shop for low prices than unemployed workers. We quantify these shopping externalities and show that they are sufficiently strong to create strategic complementarities in the employment decisions of different firms and to generate multiple rational expectations equilibria. Equilibria differ with respect to the agents' (rational) expectations about future unemployment. We show that negative shocks to agents' expectations lead to fluctuations in vacancies, unemployment, labor productivity and the stock market that closely resemble those observed in the US during the Great Recession.

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1 Introduction

We propose a novel theory of self-fulfilling fluctuations in the labor market based on the presence of shopping externalities in the product market. A firm hiring an additional worker creates positive externalities on other firms, because buyers have more income to spend and less time to search for low prices when they are employed than when they are unemployed. If these externalities are sufficiently strong, the employment decisions of different firms become strategic complements: when one firm increases employment, other firms want to increase their employment as well, in order to take advantage of higher demand and higher prices in the product market. The strategic complementarity leads to multiple rational expectations equilibria. Equilibria differ with respect to the agents' expectations about future unemployment. When agents are optimistic about future unemployment, the value to a firm from finding an additional worker is higher, more vacancies are opened, unemployment begins to fall, and the agents' optimistic expectations are fulfilled. When the agents' expectations about future unemployment become pessimistic, the economy enters a recession featuring an immediate decline in the stock market, a rapid decline in labor market tightness and a progressive raise in unemployment. Moreover, these fluctuations may take place without any concurring change in technology.

The theory is motivated by three empirical differences between the shopping behavior of employed and unemployed people. First, unemployed people spend more time shopping than employed people. For example, using the American Time Use Survey (ATUS), we find that unemployed individuals spend approximately 15 and 20 percent more time shopping than employed individuals. Second, unemployed people pay lower prices than employed people. Using the Kielts-Nielsen Consumer Panel Data (KNCPD), we find that households with at least a non-employed head pay, for exactly the same goods, between 1.5 and 5 percent less than households with all heads employed. Third, unemployed people spend less than employed people. For example, using the Panel Study of Income Dynamics (PSID), Stephens (2001) finds that households reduce their food expenditures by approximately 15 percent when entering unemployment because of a mass layoff.

We use search theory to build a model that captures the empirical differences in the shopping behavior of employed and unemployed people. We consider an economy populated by workers and firms who exchange labor in a search market modeled as in Mortensen and Pissarides (1994), and who exchange output in a search market modeled as in Burdett and Judd (1983). In the labor market, vacant firms and unemployed workers come together through a constant return to scale matching process. In equilibrium, there is unemployment—because the matching process is frictional—and there are income differences between employed and unemployed workers—because workers are able to capture part of the labor market's gains from trade. In the product market, active firms (sellers) and workers (buyers) also come together through a constant return to scale matching process. In equilibrium, the distribution of prices is non-degenerate because—as in Butters (1977) and Burdett and Judd (1983)—some buyers contact only one seller and some buyers contact multiple sellers. In equilibrium, unemployed buyers tend to pay lower prices because, on average, they are able to contact more sellers.

We first prove that the model may admit multiple steady-state equilibria. This result is easy to understand. The employment decision of a firm generates two types of externalities on other firms. On the one hand, when a firm employs an additional worker, it congests the labor market and, hence, it increases the other firms' cost of hiring an additional worker. We refer to this as the *congestion externality* of employment. On the other hand, when a firm employs an additional worker, it increases the fraction of employed buyers in the product market. In turn, this change in the composition of the population of buyers increases the other firms' benefit from hiring a worker because employed buyers spend more and pay higher prices than unemployed buyers. We refer to these as the *shopping externalities* of employment. If the shopping externalities dominate the congestion externality, the employment decisions of different firms are strategic complements and the model admits multiple steady-state equilibria. Higher unemployment steady-states are associated with a lower value of a worker to the firm and with a lower distribution of prices in the product market. Intuitively, when steady-state unemployment is higher, buyers search more intensely in the product market and the equilibrium price distribution is pushed down towards the competitive price. Hence, when unemployment is higher, the value to a firm from employing an additional worker and producing additional output is lower.

We then characterize the entire set of equilibria, both stationary and non-stationary. We find that, when there are multiple steady states, the model admits multiple rational expectation equilibria for some initial values of unemployment. Equilibria differ with respect to the agents' expectations about future unemployment. Yet, all equilibria have rational expectations, in the sense the agents' behavior is such that the realized path of unemployment coincides with the expected path of unemployment. More importantly, we find that, for some initial values of unemployment, there exist equilibria that converge to different steady states. Hence, in our model economy, the effect of expectations about future unemployment may be so strong that it determines the long-run outcomes of the economy and not simply the path that the economy follows in order to reach a given long-run outcome.

In order to understand whether multiplicity is empirically relevant, we calibrate our model. We choose the parameter values so that the model reproduces the differences in the shopping behavior of employed and unemployed workers that we observe in the data, as well as the empirical transition rates between employment and unemployment. Given the calibrated parameter values, the model admits three types of rational expectation equilibria leading to three different steady-states. First, there is a unique rational expectation equilibrium that converges to the steady state with the lowest unemployment rate. Second, there is a unique rational expectation equilibrium that converges to the steady state with the highest unemployment rate. Third, there is a continuum of rational expectation equilibria—lying in between the first two types of equilibria—that converge to the intermediate steady state. The model generates multiple equilibria because, given the calibrated parameter values, the shopping externalities dominate the congestion externality. Moreover, given the calibrated parameter values, we find that the shopping externality that is caused by the difference in the search intensity of employed and unemployed workers is twice as large as the shopping externality that is caused by the difference in expenditures of employed and unemployed workers.

The results of our calibration suggest that the US economy may be subject to sentiment shocks, i.e. shocks that affect neither technology, preferences nor other fundamentals, but shocks that affect the agents' expectations about future unemployment. We formalize the notion of sentiment shocks by introducing a regime-switching process into the calibrated model. The process alternates between an optimistic regime, in which agents expect to reach a steady state with a relatively low unemployment rate, and a pessimistic regime, in which agents expect to reach a steady state with a relatively high unemployment rate.

We then use the regime-switching version of the calibrated model to assess the hypothesis that the Great Recession (i.e., the recession that took place in the US between December 2007 and June 2009) was caused by a negative sentiment shock. We find that, in response to a negative sentiment shock, the model predicts a pattern for unemployment, vacancies, labor productivity and stock market value that resembles quite closely the empirical behavior of these variables during the Great Recession and its aftermath. First, the model correctly predicts a large and persistent increase in unemployment. Second, the model correctly predicts that the increase in unemployment is ushered by a large and persistent decline in the value of the stock market. Third, the model correctly predicts that the increase in unemployment would occur despite any significant changes in the measured productivity of labor.

The first contribution of the paper is to identify and quantify a novel set of externalities—the shopping externalities—that can lead to strategic complementarities in the employment decision of different firms and, in turn, to multiple rational expectation equilibria. The first shopping externality is a standard *demand externality*, i.e. when a firm increases its workforce, it increases the demand facing other firms' because employed workers spend more than unemployed workers. The second shopping externality is a *market power externality*, i.e. when a firm increases its workforce, it lowers the extent of competition among other firms because employed workers spend less time searching for low prices than unemployed workers. Theoretically, presence of demand externalities has long been recognized as a possible source of multiplicity (see, e.g., Heller 1986, Roberts 1987, Blanchard and Kiyotaki 1988, Cooper and John 1988, and Gali 1996). Quantitatively, though, we find that the demand externality alone is not sufficient to generate multiplicity because of the small empirical differences between the expenditures of employed and unemployed workers. In contrast, we find that the combination of the demand externality and the market power externality are strong enough to generate multiple equilibria.

Several papers generate strategic complementarities in the employment decision of different firms and, in turn, generate multiple rational expectation equilibria by assuming increasing returns to scale in either matching or production. For example, Diamond (1982), Diamond and Fudenberg (1989) and Boldrin, Kiyotaki and Wright (1993) generate multiplicity by assuming increasing returns to scale in the product market matching function. Similarly, Benhabib and Farmer (1994), Farmer and Guo (1994), Christiano and Harrison (1999) and Mortensen (1999) generate multiplicity by assuming increasing returns to scale in the production function. In contrast, in our model, both the production and the matching technologies have constant returns to scale. Moreover, while there is no compelling empirical evidence of increasing returns to scale in either production or matching, we find that the empirical differences in the shopping behavior of employed and unemployed workers are strong enough to generate multiple equilibria.

The type of multiplicity obtained in our model—i.e. multiple rational expectations equilibria leading to different steady states—is different from the one obtained in Benhabib and Farmer (1994) and Farmer and Guo (1994)—i.e. multiple rational expectations leading to the same steady-state equilibrium. Similarly, the type of non-fundamental shocks that are introduced in our model are shocks to the agents' expectations about future long-run outcomes, while non-fundamental shocks in Benhabib and Farmer (1994) and Farmer and Guo (1994) are shocks to the agents' expectations about the path that the economy might follow in order to reach the unique long-run outcome. The difference is empirically important because, in calibrated models of search unemployment, the economy reaches its steady state rather quickly (see, e.g., Shimer 2005). The type of multiplicity and the type of global dynamics generated by our model are very similar to those obtained by Diamond (1982), Diamond and Fudenberg (1989), Boldrin, Kiyotaki and Wright (1993) and Mortensen (1999). Hence, one view of our paper is that it provides an alternative, empirically grounded micro-foundation for the macrobehavior first described by Diamond (1982). Moreover, unlike in this earlier literature, our paper formally introduces non-fundamental shocks in the model and quantitatively evaluates their effect on the economy.

The second contribution of the paper is to provide a coherent explanation for the joint behavior of the labor and the stock markets during the Great Recession and its aftermath. According to our model, the stock market crash that took place in 2007 occurred because agents' in the economy became pessimistic about future unemployment and, hence, about the future value of productive activities. The large and persistent increase in unemployment that took place between 2008 and 2009 occurred because the decline in the expected value of future productive activities led to a decline in vacancies, hiring and, in turn, to the materialization of the expected increase in unemployment. And the deterioration of the stock and the labor market took place without a large or persistent decline in labor productivity because the cause of the recession was not technological but rooted in the agents' expectations.

In contrast, the events that unfolded during the Great Recession are hard to reconcile with the view that the recession has been caused by technology shocks. In the context of the Diamond-Mortensen-Pissarides (DMP) framework, it is hard to make sense of the fact that, since 2009, labor productivity has return to its long-run trend, but unemployment has remained much higher than its pre-recession level. In the context of the Real Business Cycle (RBC) framework, it is hard to make sense of a large and persistent decline in employment in the face of a large negative wealth shock and a relatively small and transitory decline in productivity. For this reason, much recent research has been devoted to propose and evaluate alternative causes of the Great Recession. Several papers have argued that the cause of the recession was a tightening of credit constraints. Other papers have argued that the cause of the recession was a secular reallocation of labor from manufacturing to services that had been masked before the recession by the housing boom (see, e.g., Charles, Hurst and Notowidigdo 2012 and Jaimovich and Siu 2012). We think that our paper provides a worthwhile alternative to these two theories, since neither of them offers a completely satisfactory explanation of the events. For example, the credit crunch view is at odds with the evidence brought forward that firm's financial distress increases only temporarily during 2007 and 2008 (see Atkeson, Eisfeldt and Weill 2012). And the structural transformation view does not explain the large movements in the stock market than have accompanied the increase in unemployment.

Farmer (2011) was the first to propose an explanation of the Great Recession based on non-fundamental shocks. In Farmer's model, wages are pinned down by neither competitive forces nor bargaining forces. Rather, wages are determined by sentiments. The Great Recession can be explained as the consequence of an increase in real wages that leads to an increase in the unemployment rate, to a decline in the labor-to-capital ratio and, ultimately, to a stock market crash. Despite the obvious similarities, there are important differences between Farmer's paper and ours, both theoretically and empirically. From the theoretical point of view, wages are indeterminate in Farmer, while in our model they are uniquely pinned down by the process of bargaining between individual firms and individual workers. From the empirical point of view, Farmer's model predicts that real labor productivity and real wages should have increased during the Great Recession, while our model predicts that real labor productivity would not have changed and that real wages should have declined. While the evidence on wages is hard to interpret in the context of search models of the labor market, the evidence on labor productivity seems more supportive of our model.

2 Environment and Equilibrium Conditions

We develop a model economy with search frictions in both the labor and the product markets. We model the product market as in Burdett and Judd (1983). In this market, search frictions generate an equilibrium distribution of prices for identical goods and unemployed workers, having more time to search, end up paying lower prices. Moreover, the competitiveness of this market depends on the intensity of buyers' search which, in turn, depends on the unemployment rate. We model the labor market as in Mortensen and Pissarides (1994). In this market, search frictions generate equilibrium unemployment and income differences between employed and unemployed workers. Our model economy is simple enough to afford an analytic characterization of the equilibrium set, and it is rich enough to afford a quantitative evaluation of its implications.

2.1 Environment

The economy is populated by two types of agents—workers and firms—who exchange three goods—labor and two consumption goods. Labor is traded in a decentralized and frictional market modeled as in Mortensen and Pissarides (1994). The first consumption good is traded in a decentralized and frictional market modeled as in Burdett and Judd (1983). We shall refer to this good as the Burdett-Judd (BJ) good. The second consumption good is traded in a centralized and frictionless product market. We shall refer to this good as the Arrow-Debreu (AD) good.

The measure of workers is normalized to one. Each worker is endowed with one indivisible unit of labor. Each worker has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho)^{-t}u_w(x_t, y_t)$, where $1/(1+\rho) \in (0,1)$ is the discount factor and $u_w(x, y)$ is a periodical utility function defined over consumption of the BJ good, x, and consumption of the AD good, y. We assume that $u_w(x, y)$ is of the Cobb-Douglas form $x^{\alpha}y^{1-\alpha}$, where $\alpha \in (0,1)$. When unemployed, workers home produce $y_u > 0$ units of the AD good. When employed, workers earn w units of the AD good as wages. Moreover, all workers (both employed and unemployed) have access to a technology that allows them to transform the AD good into the BJ good at the rate of r to 1, with $r > 0$.

The measure of firms is positive. Each firm has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho)^{-t}u_f(x_t, y_t)$, where $u_f(x_t, y_t)$ is a periodical utility function. We assume that $u_f(x, y) = y$. That is, we assume that firms only care about consumption of the AD good. Each firm operates a constant return to scale technology that turns one unit of labor into x units of the BJ good and y units of the AD good, where x and y are such that $cx + y = y_e$, with $c \in (0, r)$ and $y_e > 0$. The parameter y_e describes the productivity of labor, measured in units of the AD good. The parameter c describes the rate at which firm-worker matches can implicitly transform the AD good into the BJ good.[1]

Markets open sequentially. The first market to open is the Mortensen-Pissarides (MP) labor market. In this market, firms create vacancies at the disutility cost $k > 0$. Then unemployed workers, u, and vacant jobs, v, come together through a constant return to scale matching function $M(u, v)$. The probability that an unemployed worker matches with a vacancy is $\lambda(\theta) \equiv M(1, \theta)$, where θ denotes the tightness of the labor market, v/u, and $\lambda : \mathbb{R}_+ \to [0,1]$ is a strictly increasing and concave function with boundary conditions $\lambda(0) = 0$ and $\lambda(\infty) = 1$. Similarly, the probability that a vacant job matches with an unemployed worker is $\eta(\theta) \equiv M(1/\theta, 1)$, where $\eta : \mathbb{R}_+ \to [0,1]$ is a strictly decreasing function with boundary conditions $\eta(0) = 1$ and $\eta(\infty) = 0$. When an unemployed worker and a vacant job match, they bargain over the current wage w and produce the two consumption goods according to the technology $x + cy = y_e$. While vacant jobs and unemployed workers search for each other in the MP market, existing firm-worker matches are destroyed with probability $\delta \in (0,1)$.

The second market to open is the BJ product market. In this market, sellers post the

[1] The reader may find it helpful to interpret the production technology as follows. The firm has to allocate a unit of the worker's time between producing the AD good and the BJ good. Producing each unit of the AD good requires $1/y_e$ units of time and producing each unit of the BJ good requires c/y_e units of time. According to this interpretation, ye is the highest quantity of the AD good that the worker can produce and c is the opportunity cost of allocating the worker's time to producing an extra unit of the BJ good rather than to producing the AD good.

unit price p. Then, buyers (workers) and sellers (firms) come together through a constant return to scale matching function $N(b, s)$, where b denotes the measure of workers' searches and s denotes the measure of active firms. In particular, we assume that an unemployed worker makes one search with probability $1-\psi_u$, and two searches with probability ψ_u, where $\psi_u \in [0, 1]$. Similarly, an employed worker makes one search with probability $1 - \psi_e$ and two searches with probability ψ_e, where $\psi_e \in [0, 1]$. We assume that ψ_u is greater than ψ_e in order to capture the idea that unemployed workers have—on average—more time to search in the product market than employed workers.[2] Then, given an unemployment rate of u, the measure of workers' searches is $b(u) \equiv 1 + \psi_e + u(\psi_u - \psi_e)$ and the measure of active sellers is $s(u) \equiv 1 - u$. The probability that a seller meets a buyer is $\mu(\sigma(u)) \equiv N(1/\sigma(u), 1)$, where $\sigma(u)$ denotes the tightness of the product market, $s(u)/b(u)$, and $\mu : \mathbb{R}_+ \to [0, 1]$ is a decreasing function. Similarly, the probability that a worker's search is successful is $\nu(\sigma(u)) \equiv N(1, \sigma(u))$, where $\nu : \mathbb{R}_+ \to [0, 1]$ is an increasing function. When a buyer meets a seller, it observes the seller's price and, then, decides whether and how much of the BJ good to purchase.[3]

The last market to open is the AD product market. In this market, workers choose how much of the AD good to buy and sellers choose how much of the AD good to sell. The price of the AD good is set to clear the market and is normalized to 1. That is, the AD good is the unit of account in our economy.

Several remarks about the environment are in order. First, notice that the economy displays constant return to scale. The production technology for goods has constant returns, the production technology for vacancies has constant returns, and the matching functions in both the MP and the BJ markets have constant returns. Hence, the multiplicity of equilibria that we obtain does not originate from increasing returns in production as in Kiyotaki (1988), Benhabib and Farmer (1994), Farmer and Guo (1994), Christiano and Harrison (1999) and

[2] In this paper, we assume that the average number of searches of employed and unemployed buyers is exogenous. Thus, it is legitimate to wonder what would happen if we were to endogenize the search intensity of the buyer. In general, unemployment would have two countervailing effects on search intensity. On the one hand, an unemployed buyer has more time and, hence, faces a lower cost of searching. On the other hand, an unemployed buyer has lower consumption and, hence, faces a lower return to searching. Thus, in principle, an unemployed buyer could choose to search more or less than an employed one. Empirically, though, we find that unemployed buyers spend 20 to 30 percent more time shopping than employed buyers and, in the quantitative part of the paper, we use this information to discipline the choice of the exogenous parameters ψ_e and ψ_u.

[3] We do not interpret the search process in the BJ market as a process of discovery of prices. Rather, we interpret it as a constraint on the number and location of stores a buyer can visit in a given interval of time. On some day, the buyer may be busy tending to his kids and he is able to shop only at the local convenience store. On some other days, the buyer may be relatively free and he is able to shop both at the supermarket in the suburbs and at the local convenience store.

Mortensen (1999), nor it does originate from increasing returns in matching as in Diamond (1982) and Diamond and Fudenberg (1989). Second, notice that we assume that workers make either one or two searches in the BJ market. As in Burdett and Judd (1983) and Butters (1977), the product market is competitive when all workers match with two sellers, monopolistic when all workers match with at most one seller, and has an average price between the competitive and the monopoly prices when a positive fraction of workers matches with one seller and a positive fraction of workers matches with two sellers. Third, notice that workers cannot access credit markets. In the quantitative section of the paper, we address this feature of the model by making sure that the decline in expenditures experienced by a worker who becomes unemployed, rather than the decline in income, is consistent with the data.

Finally, notice that the environment nests several special cases of interest. For $\alpha = 0$, the only active product market (the AD market) is perfectly competitive and the environment is the same as in Mortensen and Pissarides (1994). For $\psi_e = \psi_u = 0$, workers meet at most one seller at a time and the model is equivalent to a version of Mortensen and Pissarides in which the product market is a pure monopoly. For $\psi_e = \psi_u = \psi$, employed and unemployed buyers only differ with respect to their expenditures and not their search intensity. In either of these cases, only the demand externality is active, but not the market power externality. Conversely, if we set the worker's bargaining power to zero, employed and unemployed workers only differ with respect to their search intensity and not their expenditures. In this case, only the market power externality is active, but not the demand externality.

2.2 Equilibrium Conditions

We begin by deriving the equilibrium conditions for the Burdett-Judd product market. First, consider a buyer who enters the BJ market with z units of the AD good and who finds a lowest price of p. If $p > r$, the buyer does not purchase any of the BJ good. If $p \leq r$, the buyer purchases x units of the BJ good and y units of the AD good so as to maximize his periodical utility, $x^\alpha y^{1-\alpha}$, subject to the budget constraint $px + y = z$. That is, the buyer solves the problem

$$\begin{aligned} &\max_{x,y} x^\alpha y^{1-\alpha}, \\ &\text{s.t. } px + y = z. \end{aligned} \tag{1}$$

The solution to the above problem is

$$px = \alpha z,\ y = (1-\alpha)z. \tag{2}$$

The buyer finds it optimal to spend a fraction α of his income z on the BJ good and a fraction $1-\alpha$ on the AD good.

Next, consider a seller who posts the price p in the BJ market and denote as $F_t(p)$ the cumulative distribution of prices posted by the other sellers. If $p>r$, the seller's expected gains from trading in the BJ market are zero. If $p \leq r$, the seller's expected gains from trade are given by

$$\begin{aligned} S_t(p) &= \mu(\sigma(u_t))\frac{u_t(1+\psi_u)}{b(u_t)}\left[1-\frac{2\psi_u\nu(\sigma(u_t))F_t(p)}{1+\psi_u}\right]\frac{\alpha y_u(p-c)}{p} \\ &+\mu(\sigma(u_t))\frac{(1-u_t)(1+\psi_e)}{b(u_t)}\left[1-\frac{2\psi_e\nu(\sigma(u_t))F_t(p)}{1+\psi_e}\right]\frac{\alpha w(p-c)}{p}. \end{aligned} \tag{3}$$

The expression above can be understood as follows. The probability that a seller meets a buyer is $\mu(\sigma(u_t))$. Conditional on the seller meeting a buyer, the probability that the buyer is unemployed is $u_t(1+\psi_u)/b(u_t)$. Conditional on the seller meeting an unemployed buyer, the probability that the buyer is willing to purchase at the price p is $1-2\psi_u\nu(\sigma(u_t))F_t(p)/(1+\psi_u)$, where $2\psi_u\nu(\sigma(u_t))F_t(p)/(1+\psi_u)$ is the probability that the buyer has contacted a second seller and the second seller charges a price lower than p. As established in (2), the quantity of the BJ good purchased by an unemployed buyer is $\alpha y_u/p$ and the seller's gains from trade on each unit sold are $p-c$. Hence, the first line on the right-hand side of (3) represents the seller's expected gains from meeting an unemployed buyer. Similarly, the second line on the right-hand side of (3) represents the seller's expected gains from meeting an employed buyer.

The price distribution in the BJ market is consistent with the seller's optimal pricing behavior if and only if any price p on the support of F_t maximizes the seller's gains from trade. That is,

$$S_t(p) = S_t^* \equiv \max_{p_0} S_t(p_0), \text{ all } p \in \text{supp}F_t. \tag{4}$$

The following lemma characterizes the unique price distribution F_t that satisfies (4). The proof of this lemma follows arguments similar to those in Burdett and Judd (1983) and Head, Liu, Menzio and Wright (2012).

Lemma 1 (Equilibrium Price Distribution): *The unique price distribution consistent with (4) is*

$$\begin{aligned} F_t(p) = \ & \left\{u_t(1+\psi_u)\left[1-\frac{2\psi_u\nu(\sigma(u_t))}{1+\psi_u}\frac{(r-c)p}{(p-c)r}\right]y_u \right. \\ & \left. +(1-u_t)(1+\psi_e)\left[1-\frac{2\psi_e\nu(\sigma(u_t))}{1+\psi_e}\frac{(r-c)p}{(p-c)r}\right]w_t\right\} \Big/ \\ & 2\nu(\sigma(u_t))\left\{u_t\psi_u y_u+(1-u_t)\psi_e w_t\right\} \end{aligned}$$

with support $[\underline{p}_t, \overline{p}_t]$*, where* $c < \underline{p}_t < \overline{p}_t = r$*.*

Proof: See Appendix A.

The price distribution F_t is continuous. In fact, if F_t had a mass point at some $p_0 > c$, a seller posting p_0 could increase its gains from trade by charging $p_0 - \epsilon$. This deviation would increase the probability of making a sale by a discrete amount, but it would leave the gains from trade on each unit sold approximately constant.[4] The support of F_t is connected. In fact, if the support of F_t had a gap between p_0 and p_1, the seller's gains from trade would be strictly higher at p_1 than p_0, as the probability of making a sale is the same at p_0 and p_1 but the gains from trade on each unit sold are strictly greater at p_1. For the same reason, the highest price on the support of F_t is the buyer's reservation price r.

From Lemma 1, it follows that the equilibrium gains from trade, S_t^*, are equal to the gains from trade for a seller who charges the price r and sells only to buyers who have not met any other firm in the BJ market. That is, S_t^* is given by

$$\begin{aligned} S_t^* &= \mu(\sigma(u_t))\frac{u_t(1+\psi_u)}{b(u_t)}\left[1 - \frac{2\psi_u \nu(\sigma(u_t))}{1+\psi_u}\right]\frac{\alpha y_u(r-c)}{r} \\ &+ \mu(\sigma(u_t))\frac{(1-u_t)(1+\psi_e)}{b(u_t)}\left[1 - \frac{2\psi_e \nu(\sigma(u_t))}{1+\psi_e}\right]\frac{\alpha w(r-c)}{r}. \end{aligned} \tag{5}$$

Next we derive the equilibrium conditions for the Mortensen-Pissarides labor market. The cost to a firm from opening a vacancy is k. The benefit is $\eta(\theta_t)J_t$, where $\eta(\theta_t)$ is the probability that the firm fills the vacancy and J_t is the value of a worker to the firm. If $k > \eta(\theta_t)J_t$, firms do not want to open any vacancies and the tightness of the labor market, θ_t, must be equal to zero. If $k = \eta(\theta_t)J_t$, firms are indifferent between opening and not opening vacancies and the tightness of the labor market, θ_t, may be positive. Overall, θ_t is consistent with the firms' incentive to open vacancies if and only if

$$k \geq \eta(\theta_t)J_t, \tag{6}$$

and $\theta_t \geq 0$ with complementary slackness.

The value of a worker to the firm, J_t, satisfies the following Bellman Equation

$$J_t = S_t^* + y_e - w_t + \frac{1-\delta}{1+\rho}J_{t+1}. \tag{7}$$

In the current period, the firm's expected profits from employing the worker are $S_t^* + y_e - w_t$, where $S_t^* + y_e$ are the expected revenues generated by the worker and w_t is the wage paid

[4] The price p_0 cannot be equal to c because the equilibrium gains from trade are always strictly positive.

to the worker. In the next period, the worker becomes unemployed with probability δ and remains matched with the firm with probability $1-\delta$. In the first case, the continuation value of the worker to the firm is zero. In the second case, the continuation value of the worker to the firm is J_{t+1}.

The firm and the worker bargain over the current wage w_t. We assume that the bargaining outcome is such that

$$\begin{aligned} w_t &= y_u + \gamma\,(S_t^* + y_e - y_u), \\ S_t^* + y_e - y_u &= (1-\gamma)\,(S_t^* + y_e - y_u). \end{aligned} \tag{8}$$

In words, the bargaining outcome is such that the surplus of the match in the current period, $S_t^* + y_e - y_u$, is shared by the firm and the worker according to the fractions γ and $1-\gamma$.[5] This bargaining outcome coincides with the Generalized Nash Bargaining Solution given that the worker's and firm's outside options are as follows. The outside option of the worker is to produce yu units of the AD good, to make one search in the BJ market with probability $1-\psi_e$ and two searches with probability ψ_e, and to enter next period's MP market matched with the firm. The outside option of the firm is to remain idle in the current period and to enter next period's MP market matched with the worker.[6]

The final equilibrium condition is the law of motion for unemployment. At the opening of the BJ market in the current period, there are u_t unemployed workers and $1-u_t$ employed workers. During next period's MP market, each unemployed worker faces a probability $\lambda(\theta_{t+1})$ of becoming employed and each employed worker faces a probability δ of becoming unemployed. Hence, the measure of unemployed workers at the opening of next period's BJ market is

$$u_{t+1} = u_t\,(1-\lambda(\theta_{t+1})) + (1-u_t)\delta. \tag{9}$$

2.3 Rational Expectation Equilibrium

The equilibrium conditions derived in the previous section can be reduced to a system of two difference equations in the value of a worker to the firm, J_t, and unemployment, u_t. The

[5] Since employed and unemployed workers pay different prices in the BJ market, the wage bargaining outcome (8) does not guarantee that a worker is better off employed than unemployed. In the theoretical part of the paper, we proceed under the assumption that employed workers are always better off. In the quantitative part of the paper, we verify that, for the calibrated version of the model, employed workers are better off than unemployed workers in all rational expectation equilibria.

[6] The outside options here may be more or less realistic than the outside options in Pissarides (1985), Mortensen and Pissarides (1994) and many subsequent papers. However, they certainly simplify the analysis. The assumption that, in case of disagreement, the firm and the worker do not lose contact with each other simplifies the analysis by making w_t only a function of current variables. And the assumption that, in case of disagreement, the worker searches with the same intensity as an employed buyer simplifies the analysis by making w_t independent of the price distribution F_t.

first difference equation is the Bellman Equation for the value of a worker to the firm, which can be rewritten as

$$J_t = (1-\gamma)\left(S(u_t) + y_e - y_u\right) + \frac{1-\delta}{1+\rho} J_{t+1}, \tag{10}$$

where $S(u)$ denotes the firm's equilibrium gains from trade in the BJ market given that the unemployment rate is u.and is defined as

$$\begin{aligned} S(u) = \mu(\sigma(u)) \frac{u(1+\psi_u)}{b(u)} \left[1 - \frac{2\psi_u \nu(\sigma(u))}{1+\psi_u}\right] \frac{(r-c)}{r} \alpha y_u \\ + \mu(\sigma(u)) \frac{(1-u)(1+\psi_e)}{b(u)} \left[1 - \frac{2\psi_e \nu(\sigma(u))}{1+\psi_e}\right] \frac{(r-c)}{r} \alpha \left[(1-\gamma) y_u + \gamma \left(S(u) + y_e\right)\right], \end{aligned} \tag{11}$$

Equation (10) is obtained by substituting the equilibrium condition (8) for the wage w_t into (7). Equation (11) is obtained by substituting (8) into (5). Notice that the firm's gains from trade in the BJ market are only a function of unemployment because the probability with which the firm trades with different types of buyers and the quantity sold by the firm to different types of buyers are only functions of unemployment. Also, notice that $S(u)$ is bounded and, hence, the value to a worker to the firm must be bounded.

The second difference equation is the law of motion for unemployment, which can be rewritten as

$$u_{t+1} = u_t \left(1 - \lambda(\theta_{t+1})\right) + (1-u_t)\delta, \tag{12}$$

where $\theta(J)$ denotes the equilibrium tightness of the labor market when the value of a firm is J and is defined as

$$\theta(J) = \eta^{-1}\left(\min\left\{\frac{k}{J}, 1\right\}\right). \tag{13}$$

Equation (13) is obtained by noting that, since $\eta(\theta)$ is a strictly decreasing function of theta with boundary conditions $\eta(0) = 1$ and $\eta(\infty) = 1$, the equilibrium condition (6) is equivalent to $\theta_{t+1} = \eta^{-1}(\min\{k/J_{t+1}, 1\})$. Equation (12) is derived from the equilibrium condition (9) after substituting the market tightness θ_{t+1} with its equilibrium value.

The above observations motivate the following definition of equilibrium.

Definition 1: *A discrete-time Rational Expectation Equilibrium is a sequence* $\{J_t, u_t\}$ *such that:* (i) *For* $t = 0, 1, 2, ...,$ J_t *satisfies the Bellman Equation (10);* (ii) *For* $t = 0, 1, 2, ...,$ u_t *satisfies the law of motion (12);* (iii) $\lim_{t\to\infty} J_t$ *is finite and* u_{-1} *is given.*

Notice that condition (iii) is stronger than the transversality condition $\lim_{t\to\infty}(1+\rho)^{-t}J_t = 0$. Yet, condition (iii) does not rule out any additional equilibria because, since the return function of the firm $S(u) + y_e - y_u$, is bounded, the value of a worker to the firm, J_t, must be bounded too.

In this section, which was mainly devoted to describing the environment and the equilibrium conditions, it was natural to make the assumption of discrete time. In the remainder of the paper, which is mainly devoted to characterizing the set of rational expectation equilibria, it is more convenient to work in continuous time. We formally derive a continuous-time version of our discrete-time model in Appendix B. There, we assume that, over a period of length Δ, the technology parameters are $k\Delta$, $\delta\Delta$, $y_e\Delta$ and $y_u\Delta$, the preference parameter is $\rho\Delta$ and the matching function is $M(u,v)\Delta$. We then take the limit as Δ goes to zero and obtain the continuous-time equivalent to the equilibrium conditions (10) and (12). This leads to the following definition of equilibrium for the continuous-time version of the model.

Definition 2: *A continuous-time Rational Expectation Equilibrium is a path $\{u_t, J_t\}$ such that:*

(i) *For all $t \geq 0$, J_t satisfies the Bellman Equation*

$$(\rho+\delta)\, J_t = (1-\gamma)\,(S(u_t) + y_e - y_u) + \mathring{J}_t; \tag{14}$$

(ii) *For all $t \geq 0$, u_t satisfies the law of motion*

$$\mathring{u}_t = -u_t \lambda(\theta(J_t)) + (1-u_t)\delta; \tag{15}$$

(iii) $\lim_{t\to\infty} J_t$ *is finite and u_0 is given.*

3 Characterization: Multiplicity and Cycles

In this section, we characterize the set of rational expectation equilibria for the model economy described in Section 2. We accomplish this task in three steps. In the first step, we identify necessary and sufficient conditions under which the model admits multiple stationary equilibria. In the second step, we characterize the set of non-stationary equilibria in a neighborhood of the steady states by studying the properties of a linearized version of the dynamical system (14)-(15). In the last step, we characterize the entire set of rational expectation equilibria by studying the global properties of the dynamical system (14)-(15). We find that, for some initial conditions on the unemployment rate, the model admits multiple rational expectation equilibria which differ with respect to the agents' (self-fulfilling) beliefs about future unemployment. Moreover, we find that some of these equilibria lead to different steady states. Hence, agents' expectations about future unemployment can have such a strong impact on individual behavior so as to affect long-run outcomes in the economy. The fundamental reason behind the multiplicity of equilibria is the feed-back between agents' beliefs about future unemployment and the current value of hiring a worker and, hence, to create vacancies.

3.1 Steady-State Equilibria

The set of steady-state equilibria is the set of points (u, J) such that the unemployment rate and the value of a worker to a firm are stationary. In order to characterize the set of steady-state equilibria, we use equation (14) to find the locus of points where the unemployment rate is stationary (henceforth, the u-nullcline), we use equation (15) to find the locus of points where the value of a worker to a firm is stationary (henceforth, the J-nullcline), and then we look for the intersection between the two loci.

The unemployment rate is stationary when

$$u = \frac{\delta}{\delta + \lambda(\theta(J))}. \tag{16}$$

For $J < k$, the stationary unemployment is equal to $\overline{u} \equiv 1$. Intuitively, when $J < k$, the cost of opening a vacancy is greater than the value of filling a vacancy and, hence, the labor market tightness and the worker's job-finding rate are zero. For $J > k$, the stationary unemployment is greater than $1/(1+\delta)$ and smaller than 1 and it is strictly decreasing in J. Intuitively, as the value of filling a vacancy increases, the labor market tightness increases and so does the worker's job-finding rate. For $J \to \infty$, the stationary unemployment converges to $\underline{u} \equiv 1/(1+\delta)$. This happens because the labor market tightness goes to infinity and the worker's job-finding rate converges to 1. While the u-nullcline is always decreasing in u, its exact shape depends on the vacancy cost k and on the labor market matching function M.

The value of the firm is stationary when

$$J = \frac{(1-\gamma)\,(S(u) + y_e - y_u)}{\rho + \delta}. \tag{17}$$

For $u \in [\underline{u}, \overline{u}]$, the stationary value of the firm is bounded and continuous in u. Thus, there exists at least one steady-state equilibrium. If the stationary value of the firm is everywhere non-decreasing in u, then there exists only one steady-state equilibrium. If, on the other hand, the stationary value of the firm is decreasing in u for some $u \in [\underline{u}, \overline{u}]$, then there may exist multiple steady-state equilibria.

Whether the stationary value of the firm is increasing or decreasing in u, depends on whether the gains from trade in the BJ market are increasing or decreasing in u. Assuming that each seller has a probability $A \in [0, 1]$ of meeting a buyer in each period, i.e. $N(b, s) =$

As, the derivative of S with respect to u is given by

$$\begin{aligned}
S'(u) = \quad & A\Bigg\{-\frac{(1+\psi_u)(1+\psi_e)}{b(u)^2}\left[\left(\frac{2\psi_u}{1+\psi_u}\frac{A(1-u)}{b(u)}-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right)\alpha y_u\right] \\
& -\frac{(1+\psi_u)(1+\psi_e)}{b(u)^2}\left[\left(1-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right)\alpha\left(w-y_u\right)\right] \\
& +2A\frac{1+\psi_u}{b(u)^2}\left[\left(\frac{(1+\psi_u)u}{b(u)}\frac{\psi_u}{1+\psi_u}\right)\alpha y_u+\left(\frac{(1+\psi_e)(1-u)}{b(u)}\frac{\psi_e}{1+\psi_e}\right)\alpha w\right] \\
& +\frac{(1+\psi_e)(1-u)}{b(u)}\left(1-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right)\gamma\alpha S'(u)\Bigg\}\frac{(r-c)}{r}.
\end{aligned} \tag{18}$$

An increase in unemployment has four effects on the firm's gains from trade in the BJ market. First, an increase in unemployment increases the probability that—conditional on the seller meeting a buyer—the buyer is unemployed. Since unemployed buyers are less likely to purchase the good at the reservation price r, the increase in the conditional probability of meeting an unemployed buyer lowers the seller's probability of making a sale. This negative effect is represented by the first line on the right-hand side of (18) and we will refer to it as the *market power effect* of unemployment. Second, since unemployed buyers have less income, the increase in the conditional probability of meeting an unemployed buyer also lowers the seller's average size of a sale. This negative effect is represented by the second line on the right-hand side of (18) and we will refer to it as the *demand effect* of unemployment. Third, an increase in unemployment increases the probability that—conditional on the seller meeting a buyer in a particular employment state (i.e. employed or unemployed)—the buyer is willing to purchase at the reservation price r. This positive effect is represented by the third line on the right-hand side of (18) and we will refer to it as the *captivity effect* of unemployment. Finally, an increase in unemployment has an effect on the wage and, hence, on the quantity of the BJ good purchased by employed buyers. This effect is measured in the last line on the right-hand side of (18) and acts as a multiplier on the first three effects. Thus, the sign of $S'(u)$ depends on the relative strength of the market power, demand and captivity effects of unemployment, which in turn depend on parameter values.

The following theorem identifies a set of necessary and sufficient condition for the existence of multiple steady-state equilibria.

Theorem 1 (Multiplicity of Steady States): (i) *If and only if* $S'(u) < 0$ *for some* $u \in [\underline{u}, \overline{u}]$, *there exist a vacancy cost* k *and a labor market matching function* M *such that the model admits multiple steady-state equilibria.* (ii) *For any* $u \in [\underline{u}, \overline{u}]$, *there is a* $\overline{y}_e(u) \geq y_u$ *such that* $S'(u) < 0$ *if and only if* $y_e > \overline{y}_e(u)$. (iii) *There exists a* $\tilde{u} > 0$ *such that, for any* $u \in [\underline{u}, \tilde{u}]$, *there is a* $\overline{\psi}_e(u) \leq \psi_u$ *such that* $S'(u) < 0$ *if and only if* $0 \leq \psi_e < \overline{\psi}_e(u)$.

Proof: See Appendix C.

The second part of Theorem 1 states that $S(u)$ is strictly decreasing in u when the productivity of labor in the market is sufficiently high relative to the productivity of labor at home. This result is intuitive. The higher is y_e relative to y_u, the larger is the difference between the income of employed and unemployed buyers and, hence, the stronger is the demand effect of unemployment. The third part of Theorem 1 states that $S(u)$ is strictly decreasing in u when the search intensity of employed buyers is low enough relative to the search intensity of unemployed buyers. This result is also intuitive. The lower is ψ_e relative to ψ_u, the larger is the difference between the probability the probability that an employed buyer and an unemployed buyer are willing to purchase at the reservation price r and, hence, the stronger is the market power effect of unemployment.

The first part of Theorem 1 implies that as long as $S(u)$ is strictly decreasing in u for some $u \in [\underline{u}, \overline{u}]$, one can find a vacancy cost k and a labor market matching function M such that the u-nullcline crosses the J-nullcline at multiple points. To understand why our model may admit multiple steady-state equilibria, note that, when a firm increases its workforce, it generates two types of externalities on other firms. First, by increasing its workforce, a firm increases the tightness of the labor market and, hence, it increases the cost of hiring an additional worker for other firms. We refer to this as the *congestion externality* of employment. The congestion externality is negative and its strength is measured by the slope of the u-nullcline, which, in turn, can be increased or decreased by varying the vacancy cost k and the shape of the matching function M. Second, by increasing its workforce, a firm lowers the unemployment rate, the other firms' gains from trading in the BJ market and, hence, their benefit from hiring an additional worker. We refer to this as the *shopping externality* of employment. The sign of the shopping externality is the opposite of the sign of $S'(u)$, which, in turn, depends on the relative strength of the demand, market power and captivity effects in (18). When the shopping externality is positive and dominates the congestion externality, the hiring decisions of different firms are strategic complements and multiple steady-state equilibria arise.[7]

While strategic complementarities in employment are not unique to our model, the source of strategic complementarity is. In Diamond (1982), Diamond and Fudenberg (1989) and Boldrin, Kyiotaki and Wright (1993), the strategic complementarity in employment is caused

[7] There is no clear welfare ranking among steady-state equilibria with different unemployment. Fundamentally, this is because unemployment tends to lower aggregate potential output, but it also tends to lower prices and, hence, to reduce monopoly distortions in the BJ market.

by a thick market externality in the product market. That is, the higher is aggregate employment, the easier it is for a seller to find a buyer. In contrast, in our model the probability that a seller meets a buyer is constant. In Benhabib and Farmer (1994), Farmer and Guo (1994), Christiano and Harrison (1999) and Mortensen (1999), the strategic complementarity in employment is caused by a production externality. That is, the higher is aggregate employment, the higher is the output of a worker. In contrast, in our model the production technology is constant. The models by Heller (1986), Roberts (1987), Cooper and John (1988) and Gali (1996) are closest to ours. In those models, the strategic complementarity in employment is caused by a demand externality. That is, the higher is aggregate employment, the higher is the demand faced by an individual seller and the higher are its profits. In our model, the strategic complementarity in employment is caused by the shopping externality, which is given by the sum of the external effect of employment on demand and on market power (net of the captivity effect). In our model, the higher is aggregate employment, the higher is an individual seller's demand and the higher is an individual seller's probability of trade at a given price. As we will demonstrate in Section 5, it is precisely the combination of the demand externality and the market power externality that allows us to generate multiplicity for a reasonably calibrated version of the model. However, from a theoretical point of view, each externality in isolation is sufficient to generate multiplicity.

When $\gamma = 0$, the income of employed and unemployed buyers is identical and, hence, aggregate employment does not generate a demand externality. In this case, the derivative of $S(u)$ with respect to u is given by

$$\begin{aligned} S'(u) = \quad & A\left\{\frac{(1+\psi_u)(1+\psi_e)}{b(u)^2}\left[\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)} - \frac{2\psi_u}{1+\psi_u}\frac{A(1-u)}{b(u)}\right]\alpha y_u\right. \\ & \left. +2A\frac{1+\psi_u}{b(u)^2}\left[\frac{(1+\psi_u)u}{b(u)}\frac{\psi_u}{1+\psi_u} + \frac{(1+\psi_e)(1-u)}{b(u)}\frac{\psi_e}{1+\psi_e}\right]\alpha y_u\right\}\frac{(r-c)}{r}. \end{aligned} \tag{19}$$

For $\psi_e = 0$ and $\psi_u = 1$, $S'(u)$ is negative at, e.g., $u = 0.25$. In light of Theorem 1, this implies that the model can generate multiple steady states even when aggregate employment generates a market power externality, but not a demand externality.

Conversely, when $\psi_e = \psi_u = \psi$, the search intensity of employed and unemployed buyers is identical and, hence, aggregate employment does not affect the competitiveness of the product market. In this case, the derivative of $S(u)$ with respect to u is given by

$$\begin{aligned} S'(u) = \quad & A\left\{\left(1-\frac{2\psi}{1+\psi}\frac{A(1-u)}{1+\psi}\right)\alpha\gamma\left(S(u)+y_e\right)\right. \\ & +\frac{2A\psi}{(1+\psi)^2}\left[\alpha y_u+(1-u)\alpha\gamma\left(S(u)+y_e\right)\right] \qquad (20) \\ & \left.+(1-u)\left(1-\frac{2\psi}{1+\psi}\frac{A(1-u)}{1+\psi}\right)\gamma\alpha S'(u)\right\}\frac{(r-c)}{r}. \end{aligned}$$

For y_e large enough, $S'(u)$ is negative. In light of Theorem 1, this means that the model can generate multiple steady states even when aggregate employment generates a demand externality, but does not a market power externality.

3.2 Local Dynamics

We now turn to the task of characterizing the entire set of rational expectations equilibria, both stationary and non-stationary. To accomplish this task, we first analyze the dynamics of the model in a neighborhood of each of the steady-state equilibria and then we study the global dynamics of the model.

Let $\{E_i\}_{i=1}^n$, with $E_i = (u_i^*, J_i^*)$ and $u_1^* < u_2^* < ... < u_n^*$, denote the set of steady-state equilibria. Abstracting from the knife-edge case in which the u-nullcline and the J-nullcline are tangent at some (u_i^*, J_i^*), the number of steady-state equilibria, n, is odd. The set of rational expectation equilibria in a neighborhood of a steady state E_i can be derived by analyzing the eigenvalues associated with the linearized version of the dynamical system (14)-(15) around (u_i^*, J_i^*), which is given by

$$\begin{pmatrix} \mathring{u}_t \\ \mathring{J}_t \end{pmatrix} = \mathcal{M}_i \begin{pmatrix} u_t - u_i^* \\ J_t - J_i^* \end{pmatrix}, \qquad (21)$$

where the 2×2 matrix $\mathcal{M}_i$ is defined as

$$\mathcal{M}_i = \begin{pmatrix} -\delta - \lambda(\theta(J_i^*)) & -\lambda'(\theta(J_i^*))\theta'(J_i^*)u_i^* \\ -(1-\gamma)S'(u_i^*) & \rho+\delta \end{pmatrix} \qquad (22)$$

The eigenvalues, the determinant and the trace of $\mathcal{M}_i$ are, respectively, given by

$$\begin{aligned} \mathrm{Eig}_i = \quad & \mathrm{Tr}_i \pm \sqrt{Tr_i^2 - 4Det_i}, \\ \mathrm{Det}_i = \quad & -\left(\delta + \lambda(\theta(J_i^*))\right)(\rho+\delta) - (1-\gamma)\lambda'(\theta(J_i^*))\theta'(J_i^*)u_i^* S'(u_i^*), \qquad (23) \\ \mathrm{Tr}_i = \quad & \rho - \lambda(\theta(J_i^*)). \end{aligned}$$

First, suppose that E_i is an odd steady-state equilibrium, i.e. $i = 1, 3, ..., n$. At this steady state, the slope of the u-nullcline is smaller than the slope of the J-nullcline, i.e.

$$\frac{(1-\gamma)S'(u_i^*)}{\rho+\delta} > -\frac{\delta+\lambda(\theta(J_i^*))}{\lambda'(\theta(J_i^*))\theta'(J_i^*)u_i^*}. \tag{24}$$

The above inequality implies that the determinant of $\mathcal{M}_i$ is strictly negative and, hence, $\mathcal{M}_i$ has one real strictly positive eigenvalue and one real strictly negative eigenvalue: E_i is a saddle. In turn, this implies that, for any initial unemployment rate u_0 in a neighborhood of u_i^*, there exists one and only one J_0 in a neighborhood of J_i^* such that the solution to the dynamical system (14)-(15) converges to E_i given the initial condition (u_0, J_0). Hence, in a neighborhood of E_i, there exists one and only one rational expectation equilibrium that converges to E_i.

Next, suppose that E_i is an even steady-state equilibrium, i.e. $i = 2, 4, ..., n-1$, with $u_i^* < \delta/(\rho+\delta)$. At this steady state, the slope of the u-nullcline is greater than the slope of the J-nullcline, i.e.

$$\frac{(1-\gamma)S'(u_i^*)}{\rho+\delta} < -\frac{\delta+\lambda(\theta(J_i^*))}{\lambda'(\theta(J_i^*))\theta'(J_i^*)u_i^*}. \tag{25}$$

The above inequality implies that the determinant of $\mathcal{M}_i$ is strictly positive. Moreover, the fact that u_i^* is smaller than $\delta/(\rho+\delta)$ implies that the trace of $\mathcal{M}_i$ is strictly negative. Therefore, $\mathcal{M}_i$ has two (real or complex conjugate) eigenvalues with a strictly negative real part: E_i is a sink. In turn, this implies that, for any initial unemployment rate u_0 in a neighborhood of u_i^*, there exists a continuum of values for J_0 in a neighborhood of J_i^* such that the solution to the dynamical system (14)-(15) converges to E_i given the initial condition (u_0, J_0). Hence, in a neighborhood of E_i, there exists a continuum of rational expectations equilibria that converge to E_i.

Finally, suppose that E_i is an even steady-state equilibrium with $u_i^* > \delta/(\rho+\delta)$. At this steady state, the slope of the u-nullcline is greater than the slope of the J-nullcline and, hence, the determinant of $\mathcal{M}_i$ is strictly positive. Moreover, at this steady state $u_i^* > \delta/(\rho+\delta)$ and, hence, the trace of $\mathcal{M}_i$ is strictly positive. Therefore, $\mathcal{M}_i$ has two (real or complex conjugate) eigenvalues with a strictly positive real part: E_i is a source. In turn, this implies that, for any unemployment rate u_0 different from u_i^*, there are no values of J_0 such that the solution to the dynamical system (14)-(15) converges to E_i given the initial condition (u_0, J_0). Thus, there are no rational expectation equilibria that lead to E_i.

The above observations are summarized in the following theorem.

Theorem 2 (Local Dynamics): (i) *If $i = 1, 3, ...n$, there exists a unique rational expectation equilibrium converging to E_i from any u_0 in a neighborhood of u_i^*.* (ii) *If $i = 2, 4, ...n-1$ and $u_i^* < \delta/(\rho+\delta)$, there exists a continuum of rational expectation equilibria converging to*

E_i for any u_0 in a neighborhood of u_i^*. (iii) *If $i = 2, 4, ...n-1$ and $u_i^* > \delta/(\rho+\delta)$, there are no rational expectation equilibria converging to E_i for any u_0.*

Theorem 2 implies that any steady-state equilibrium E_i with an unemployment rate u_i^* smaller than $\delta/(\rho+\delta)$ is such that there exists at least one rational expectation equilibrium that leads to E_i for any initial unemployment rate u_0 in a neighborhood of u_i^*. In this sense, Theorem 2 implies that any steady-state equilibrium E_i with $u_i^* < \delta/(\rho+\delta)$ is robust to local perturbations and, hence, economically meaningful. Further, Theorem 2 implies that the steady-state equilibria E_i with $u_i^* < \delta/(\rho+\delta)$ are alternatively saddles and sinks. Hence, the behavior of the economy in a neighborhood of the saddle steady states is the same as in standard neoclassical macroeconomics: there exists only one rational expectation equilibrium that leads to the steady-state. The behavior of the economy in a neighborhood of the sink steady states is the same as in Benhabib and Farmer (1994) and Farmer and Guo (1994): there exists a continuum of rational expectation equilibria that lead to the steady state.

3.3 Global Dynamics

In order to characterize the entire set of rational expectation equilibria, we need to analyze the global dynamics of the dynamical system (14)-(15). For the sake of concreteness, we will carry out this task under the assumption that there exist three steady-state equilibria, E_1, E_2 and E_3, and that the second one is a sink. We will also assume that the J-nullcline is first decreasing and then increasing in unemployment. The assumptions are satisfied by all reasonable parameterizations of the model.[8]

The qualitative features of the set of rational expectation equilibria depend on properties of the stable manifolds associated with the stationary equilibria E_1 and E_3. We use $J_1^S(u)$ to denote the set of J's such that (u, J) belongs to the stable manifold associated with E_1. Similarly, we use $J_3^S(u)$ to denote the set of J's such that (u, J) belongs to the stable manifold associated with E_3. With a slight abuse of language, we refer to J_1^S and J_3^S as the stable manifolds.

Figure 1 plots the u-nullcline, the J-nullcline and the direction of motion of the dynamical system (14)-(15) in the six regions defined by the intersection of the two nullclines. Given the direction of motion of the dynamical system and given that any trajectory must cross the

[8] Even though the underlying economic forces are quite different, the dynamics of our model are similar to the dynamics of the models studied by Diamond and Fudenberg (1989), Boldrin, Kyiotaki and Wright (1993) and Mortensen (1999).

u-nullcline vertically and the J-nullcline horizontally, it follows that the backward extension of the stable manifold J_1^S to the left of E_1 lies in region III and exists the domain at $\underline{u}$. The backward extension of J_1^S to the right of E_1 goes through region II and then it may either (i) exit the domain at $\overline{u}$, (ii) exit the domain at $\underline{u}$ after going through regions V and III, or (iii) not exit the domain, circling between regions V, III, IV and II.[9] Similarly, the backward extension of the stable manifold J_3^S to the right of E_3 lies in region VI and exits the domain at uh. The backward extension of J_3^S to the left of E_3 goes through region V and then it may (i) exit the domain at $\underline{u}$ after going through region III, (ii) exit the domain at $\overline{u}$ after going through regions III, IV and II, or (iii) not exit the domain circling between regions III, IV, II and V. After eliminating incompatible cases, the above classification of the stable manifolds J_1^S and J_3^S leaves us with five qualitatively different cases to analyze.

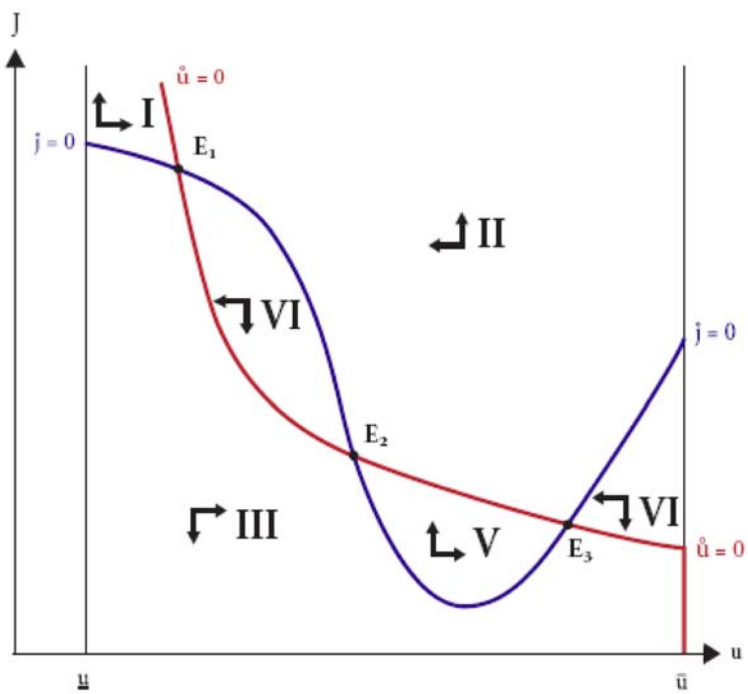


Figure 1: Phase diagram

Case 1: Figure 2 illustrates the case in which the right branch of J_1^S exists at $\overline{u}$ and the left branch of J_3^S exits at $\underline{u}$. In this case, there exist three types of rational expectation equilibria

[9]For the sake of brevity, the analysis abstracts from the knife-edge cases in which the stable manifolds are either homoclinic–i.e. the backward extension of the stable manifold associated with one saddle steady state converges to the same steady state–or heteroclinic–i.e. the backward extension of the stable manifold associated with one saddles steady state converges to the other saddle steady state.

for any initial unemployment $u_0 \in [\underline{u}, \overline{u}]$. First, there is a rational expectation equilibrium that starts at (u_0, J_0), with $J_0 = J_1^S(u_0)$, and then follows the stable manifold J_1^S to the low unemployment steady state E_1. Second, there is a rational expectation equilibrium that starts at (u_0, J_0), with $J_0 = J_3^S(u_0)$, and then follows the stable manifold J_3^S to the high unemployment steady-state E_3. Finally, there is a continuum of equilibria that start at (u_0, J_0), with $J_0 \in (J_3^S(u_0), J_1^S(u_0))$. Each one of these equilibria then follows a trajectory that remains inside the shaded area and converges to either E_2 or to a limit cycle around E_2. In contrast, all trajectories starting at (u_0, J_0), with $J_0 \notin [J_3^S(u_0), J_1^S(u_0)]$, are not rational expectation equilibria because they violate the transversality condition (iii) in Definition 2.

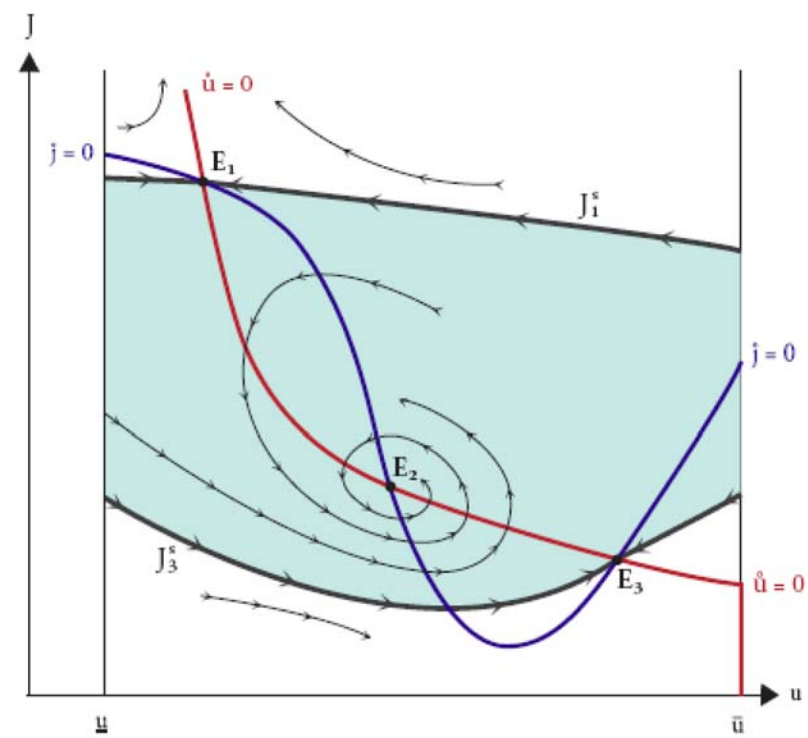
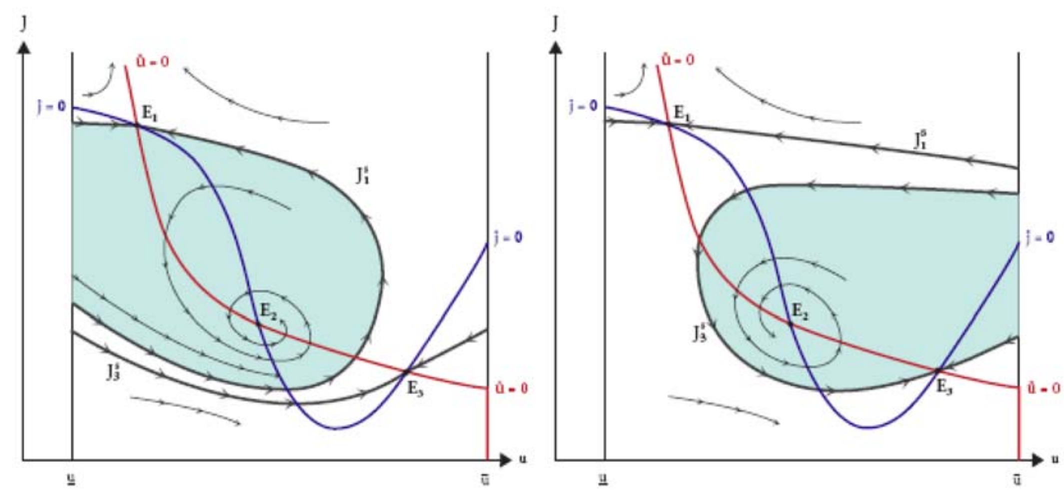


Figure 2: Global dynamics, case 1

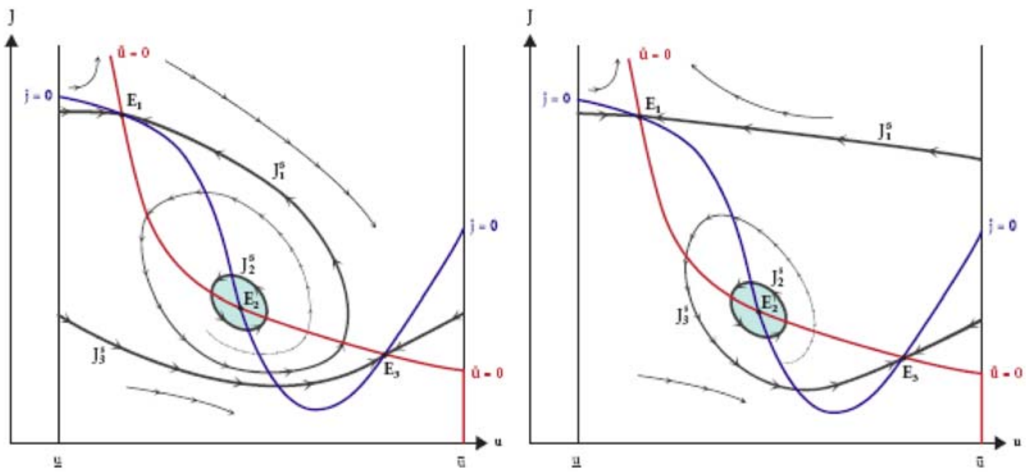
Cases 2 and 3: Figure 3(a) illustrates the case in which both the right branch of J_1^S and the left branch of J_3^S exit at $\underline{u}$. Figure 3(b) illustrates the case in which both the right branch of J_1^S and the left branch of J_3^S exit at $\overline{u}$. Since the two cases are specular, we only discuss the first. Let $\overline{u}_1$ denote the easternmost point on the stable manifold J_1^S. Then, for any initial unemployment $u_0 \in [\underline{u}, \overline{u}_1]$, there are three types of rational expectation equilibria. First, there are two equilibria that start at (u_0, J_0), with $J_0 = J_1^S(u_0)$, and then follow the stable manifold J_1^S to E_1. Second, there is one equilibrium that starts at (u_0, J_0), with $J_0 = J_3^S(u_0)$, and then follows the stable manifold J_3^S to E_3. Third, there is a continuum of equilibria that start at (u_0, J_0), with J_0 between the upper and the lower branches of J_1^S, and then follow a trajectory that remains in the shaded area and converges to either E_2 or to a limit cycle around E_2. For any initial unemployment $u_0 \in (\overline{u}_1, \overline{u}]$, the only rational expectation equilibrium is the stable manifold associated with E_3.



(a) Global dynamics, case 2(a) (b) Global dynamics, case 2(b)

Figure 3: Global dynamics, case 2 and 3

Cases 4 and 5: Figure 4(a) illustrates the case in which the right branch of J_1^S does not exit the domain $[\underline{u}, \overline{u}]$ and the left branch of J_3^S exit at $\underline{u}$. Figure 4(b) illustrates the case in which the right branch of J_1^S exits at $\overline{u}$ and the left branch of J_3^S does not exit the domain $[\underline{u}, \overline{u}]$. Since the two cases are specular to each other, let us focus on the first. In this case, one can prove (see Boldrin, Kyiotaki and Wright 1993, Proposition 5) that there exists a repellent limit cycle, J_2^C, around E_2. Let $\overline{u}_1$ denote the easternmost point on the stable manifold J_1^S, and let $\underline{u}_2$ and $\overline{u}_2$ denote the westernmost and the easternmost points on the limit cycle J_2^C. Then, for any initial unemployment $u_0 \in [\underline{u}, \underline{u}_2) \cup (\overline{u}_2, \overline{u}_1]$, there exist two types of equilibria: the stable manifold associated with E_1 and the stable manifold associated with E_3. For $u_0 \in [\underline{u}_2, \overline{u}_2]$, there are two additional types of equilibria. First, there are two equilibria that start at (u_0, J_0), with $J_0 = J_2^C(u_0)$, and then follow the limit cycle. Second, there is a continuum of equilibria that start at (u_0, J_0), with J_0 between the upper and the lower braches J_2^C, and then follow a trajectory that remains in the shaded area and either converges to E_2 or to an inner limit cycle. Finally, for $u_0 \in (\overline{u}_1, \overline{u}]$, the only equilibrium is the stable manifold associated with E_3.



(a) Global dynamics, case 3(a) (b) Global dynamics, case 3(b)

Figure 4: Global dynamics, case 3

The characterization of the set of non-stationary equilibria reveals several important features of the model. First, there exist initial values of the unemployment rate for which the model admits multiple equilibria. Different equilibria are associated with different expectations about future paths of unemployment. However, these expectations are always rational, in the sense that in any equilibrium the realized path of unemployment coincides with the one expected by the agents. The multiplicity of rational expectations equilibria arises because of the feed-back between the expectations about future unemployment and the current value of worker to a firm. When future unemployment is expected to be high, the current value of a worker to a firm is low, and, in turn, vacancies are low, which induces high unemployment in the future.

Second, there exist initial values of the unemployment rate for which the model admits equilibria that converge to different steady states. For example, in Figure 2, there are equilibria that converge to the low unemployment steady state, equilibria that converge to the high unemployment steady state, and equilibria that converge to the intermediate steady state. Hence differences in expectations about unemployment can be strong enough to affect the unemployment rate that the economy reaches in the long run, and not just the path that the economy follows to reach a particular steady state. Considering that the speed of convergence to the steady state is fairly high in calibrated search-theoretic models of the labor market (see, e.g., Shimer 2005), the result is necessary if we want expectations to have a quantitatively important effect on unemployment.

Third, there are cases in which the set of steady states that the economy might reach in the long run depends on the initial value of the unemployment rate. For example, in Figure 3, there are equilibria converging to E_1, E_2 and E_3 if the initial unemployment rate is less than $\overline{u}_1$, but there is only an equilibrium converging to E_3 if the initial unemployment rate is greater than $\overline{u}_1$. Hence differences in the economy's initial conditions may have dramatic effects on long-run outcomes.

Finally, there are cases in which there are periodic equilibria where unemployment and the value of the firm rotate counter-clockwise around the steady state E_2. Hence the model can generate truly endogenous business cycles, in which the fluctuations in u and J are caused by neither shocks to fundamentals nor shocks to expectations about future unemployment. While theoretically interesting, these endogenous business cycles do not emerge for reasonable parameterizations of the model.

4 Unemployment Sentiments

In the previous section, we showed that our model economy may follow different equilibrium paths depending on the agents' expectations about future unemployment, and that some of these paths may lead to different steady-state equilibria. These findings suggest that our economy may be subject to non-fundamental shocks—i.e. shocks not to current or future technology and preferences, but self-fulfilling shocks to the agents' expectations about future unemployment—and that non-fundamental shocks may have a persistent effect on labor market outcomes. In this section, we formalize the idea of these persistent non-fundamental shocks, which we shall refer to as *unemployment sentiments*. Section 4.1 introduces unemployment sentiments in the model and defines a Regime Switching Equilibrium (RSE). Section 4.2 illustrates the notion of RSE by means of a simple example. Section 5 quantitatively evaluates the hypothesis that the cause of the most recent US recession was a negative sentiment shock.

Regime Switching Equilibria are a class of rational expectations equilibria in which the economy switches between an optimistic and a pessimistic regime. In the optimistic regime, agents expect the economy to reach a steady state with a relatively low unemployment rate (conditional on remaining in the optimistic regime forever). In the pessimistic regime, agents expect the economy to reach a steady state with a relatively high unemployment rate (again, conditional on remaining in the pessimistic regime forever). Importantly, all agents understand that regime switches can occur, know the probability with which these switches happen, and observe when the economy is hit with such a shock. The agents take these switches into account when making their decisions. A RSE is conditioned on the parameters that govern the underlying exogenous stochastic process for the regime switches. Given initial conditions, and a realization of this stochastic process, the model predicts a unique outcome for the endogenous variables. In this sense an RSE is amenable to empirical analysis to the same extent as models with unique rational expectations equilibria and exogenous shocks to fundamentals (e.g. Real Business Cycle models).

4.1 Definition of Regime Switching Equilibria

In order to formally define a Regime Switching Equilibrium, it is necessary to introduce some additional notation. We denote as G the optimistic or "good" regime and as B the pessimistic or "bad" regime. We denote as $\pi_{GB}(u)$ and let $\pi_{BG}(u)$ the Poisson rates at which the economy switches from the optimistic to the pessimistic regime and from the pessimistic to the optimistic regime, given that the unemployment rate is u. We use $C_{GB}(u, J)$ to denote the jump in the value of the firm when the economy switches from the optimistic to the pessimistic regime, given that the unemployment rate is u and the value of the firm (an instant before the switch) is J. Similarly, we use $C_{BG}(u, J)$ to denote the jump in the value of the firm when the economy switches from the pessimistic to the optimistic regime, given that the unemployment rate is u and the value of the firm (an instant before the switch) is J. We let h denote a history of realizations of the switching process and $t_n(h)$ the n-th time at which the regime switches in history h.

Consider an arbitrary history h. For $t \in [t_n(h), t_{n+1}(h))$ with $t_n(h) = G$, the value of the firm J_t and the unemployment rate u_t satisfy the following differential equations

$$(\rho + \delta) J_t = (1 - \gamma) (S(u_t) + y_e - y_u) + \pi_{GB}(u_t) C_{GB}(u_t, J_t) + \mathring{J}_t, \tag{26}$$

$$\mathring{u}_t = -u_t \lambda(\theta(J_t)) + (1 - u_t)\delta. \tag{27}$$

The differential equation (27) is the usual law of motion for unemployment. The differential equation (26) is the Bellman Equation for the value of the firm in the presence of sentiment shocks. This equation is easy to understand. The current value of the firm, J_t, is given by the sum of three terms. The first term are the current profits of the firm, $(1-\gamma) (S(u_t) + y_e - y_u)$. The second term is the product between the rate at which the economy switches to the pessimistic regime, $\pi_{GB}(u_t)$, and the change in the value of the firm conditional on the switch, $C_{GB}(u_t, J_t)$. The last term, $\mathring{J}_t$, is the derivative of the value of the firm with respect to time if the economy remains in the optimistic regime.

Let $\{E_1^G, E_2^G, ...E_{n_G}^G\}$ denote the steady states of the system of differential equations (26)-(27) and let E_i^G denote the steady-state equilibrium that agents expect to reach in the optimistic regime. The agents' expectations are rational if and only if the initial condition for the system of differential equations (26)-(27) satisfies some requirements which depend on the nature of E_i^G. Suppose that E_i^G is a saddle point. In this case, the economy converges to E_i^G (and the agents' expectations are correct) if and only if the initial condition for (26)-(27) lies on the stable manifold associated with E_i^G. When E_i^G is a sink, the economy converges to E_i^G if and only if the initial condition for (26)-(27) belongs to the basin of attraction of E_i^G. Finally, when E_i^G is a source, the economy cannot converge to E_i^G, the agents' expectations are not rational and the conjectured RSE does not exist.

Next, we analyze the behavior of the economy in the pessimistic regime. For $t \in [t_n(h), t_{n+1}(h))$ with $t_n(h) = B$, the value of the firm J_t and the unemployment rate u_t satisfy the following differential equations

$$(\rho + \delta) J_t = (1 - \gamma) (S(u_t) + y_e - y_u) + \pi_{BG}(u_t) C_{BG}(u_t, J_t) + \mathring{J}_t, \tag{28}$$

$$\mathring{u}_t = -u_t \lambda(\theta(J_t)) + (1 - u_t)\delta. \tag{29}$$

We denote as $\{E_1^B, E_2^B, ...E_{n_B}^B\}$ the steady states of the system of differential equations (28)-(29) and with E_j^B, with $u_j^{*B} > u_i^{*G}$, the steady-state equilibrium that the agents expect to reach in the pessimistic regime. Again, the agents' expectations are rational if and only if the initial conditions for (28)-(29) satisfy the following requirements. When E_j^B is a saddle point, the agents' expectation is rational if and only if the initial condition for (28)-(29) lies on the stable manifold associated with E_j^B. When E_j^B is a sink, agents' expectation is rational if and only if the initial condition for (28)-(29) belongs to the basin of attraction of E_j^B. And, when E_j^B is a source, the economy cannot converge to E_j^B, the agents' expectations are not rational and the conjectured RSE does not exist.

The requirements that rational expectations impose on the initial conditions of the systems (26)-(27) and (28)-(29) are restrictions on the jump process for the value of a firm. In particular, if we denote with $J_{G,i}^S$ the stable manifold or the basin of attraction associated with E_i^G and with $J_{B,j}^S$ the stable manifold or the basin of attraction associated with E_j^B, the jump $C_{BG}(u, J)$ must be such that $J + C_{BG}(u, J)$ belongs to $J_{B,j}^S$ for all $J \in J_{G,i}^S$ and, similarly, the jump $C_{GB}(u, J)$ must be such that $J + C_{GB}(u, J)$ belongs to $J_{G,i}^S$ for all $J \in J_{B,j}^S$.

In words, the jumps must be such that—when the regime switches—the value of the firm lands on a point that converges to the steady-state equilibrium expected by the agents.

The above observations motivate the following definition of equilibrium.

Definition 3. *An i-j Regime Switching Equilibrium is given by switching rates,* $\pi_{GB}(u)$ *and* $\pi_{BG}(u)$*, jumps,* $C_{GB}(u, J)$ *and* $C_{BG}(u, J)$*, and history-dependent paths,* $\{u_t(h), J_t(h)\}$*, such that:* (i) *For any* h, $u_t(h)$ *and* $J_t(h)$ *satisfy the differential equations (26)-(29);* (ii) *For any* $u \in [\underline{u}, \overline{u}]$ *and* $J \in J^S_{G,i}(u)$, $J + C_{GB}(u, J) \in J^S_{B,j}(u)$*;* (iii) *For any* $u \in [\underline{u}, \overline{u}]$ *and* $J \in J^S_{G,i}(u)$, $J + C_{BG}(u, J) \in J^S_{Gi}(u)$*;* (iv) $J_0(h) \in J^S_{G,i}(u_0)$ *if* $h_0 = G$*, and* $J_0(h) \in J^S_{B,i}(u_0)$ *if* $h_0 = B$*.*

It is useful to distinguish between the notion of sentiments that is embedded in the above definition of RSE and the notion of sunspots that has been analyzed in most of the previous literature on multiple rational expectation equilibria (see, e.g., Benhabib and Farmer 1994 and Farmer and Guo 1994). Sentiments are shocks to the agents' expectations about the steady state that the economy is going to reach. Sunspots are shocks to the agents' expectations about the path that the economy is going to follow before reaching a given indeterminate steady state. The difference is important on several dimensions. From a substantive point of view, sentiment shocks have a potentially persistent effect on the outcomes of the economy, while sunspot shocks can only generate temporary fluctuations of the economy. From a technical point of view, a model with sentiment shocks must be analyzed using global techniques, while a model with sunspot shocks can be analyzed using linear approximations of the dynamical system around the steady state (see, e.g., Farmer 2000). To the best of our knowledge, ours and a recent paper by Farmer (2011) are the first papers to study the dynamics of a model with shocks to expectations about long-run outcomes. While in this paper we focus on sentiments, it is straightforward to extend our model to allow for sunspots. For example, if in the pessimistic regime the economy converges to a sink steady state, we can introduce sunspots by adding a mean-zero shock to the value of the firm. This sunspot shock would generate fluctuations in the path that the economy follows to reach the pessimistic steady state. The only additional restriction on the sunspot shocks would be that their support lies within the basin of attraction of E^B_j.

For the sake of clarity, it is also worth distinguishing between the notion of RSE in our model and the one used in monetary economics (see, e.g., Sims and Zha 2006 or Farmer et al. 2010). In our model, a regime switch is a self-fulfilling change in the agents' expectations about the value of an endogenous variable (unemployment). In the monetary economics literature, a regime switch is a persistent and large change in the value of exogenous technology or monetary policy parameters.

4.2 Regime Switching Equilibrium: An Example

Figure 5 illustrates a simple 1-2 Regime Switching Equilibrium. The blue solid line is the nullcline associated with the law of motion (26) for the value of a firm in the optimistic regime given a switching rate π_{GB} and a jump $C_{GB}(u, J) = \Delta$, $\Delta < 0$. In the optimistic regime there are three steady-state equilibria, E_1^G, E_2^G and E_3^G. In this regime agents expect to reach the low-unemployment saddle steady state, E_1^G. The solid black line through E_1^G is the stable manifold $J_{G,1}^S$. The dashed blue line is the nullcline associated with the law of motion (28) for the value of a firm in the pessimistic regime given a switching probability π_{BG} and a jump $C_{BG}(u, J) = J_{G,1}^S(u) - J$. In the pessimistic regime there are also three steady state equilibria, E_1^B, E_2^B and E_3^B. In this regime agents expect to reach the sink steady state, E_2^B. The shaded area between the stable manifolds associated with E_1^B and E_3^B describes the basin of attraction of E_2^B. The jump $C_{BG}(u, J)$ satisfies the equilibrium condition (iii) by construction. The jump $C_{GB}(u, J)$ satisfies the equilibrium condition (iii) because $J_{G,1}^S(u) + \Delta$.

Using Figure 5 we can recover the dynamics of the regime switching equilibrium. In the optimistic regime, the economy moves along the stable manifold $J_{G,1}^S$ and converges to the low unemployment steady state E_1^G. When agents become pessimistic, the value of a firm falls by Δ and the economy begins moving towards the high unemployment steady state E_2^B. When agents become optimistic again, the value of a firm jumps back to the stable manifold converging to E_1^G.

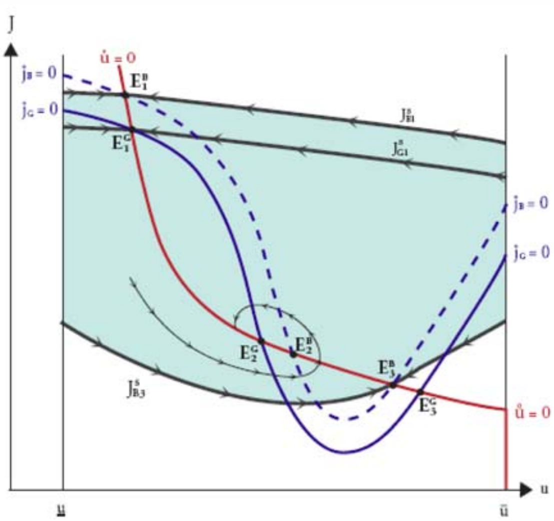


Figure 5: Example of a 1-2 Regime Switching Equilibrium

5 Sentiments and the Great Recession

Our goal in this section is to evaluate the hypothesis that the Great Recession—i.e., the recession experienced by the US economy between December 2007 and June 2009—was caused by a negative shock to expectations about long-run unemployment. We first calibrate the parameters of the model so as to match US data on the transitions of worker between employment and unemployment, on the expenditure and shopping behavior of employed and unemployed workers and on the dispersion of prices for identical goods. Second, we verify that, given the calibrated parameter values, the model admits a 1-2 Regime Switching Equilibrium, i.e. an equilibrium in which agents expect to reach the steady state with the lowest unemployment in the optimistic regime and the second lowest unemployment in the pessimistic regime.

We then derive the predictions of the model about unemployment, vacancies, labor productivity, prices and the stock market if, in the Fall of 2007, the US economy had been hit by a negative sentiment shock. We compare these predictions with the data. This exercise should be interpreted as an impulse response to a sentiment shock, where the size of the shock is disciplined by data on the drop in the stock market. We find that a negative sentiment shock is able to explain the three key facts about the Great Recession and the subsequent recovery: (i) unemployment doubled during the recession and has, since, remained above the pre-recession level, (ii) a stock market crash has led the increase in unemployment, and (iii) the movements of labor productivity and unemployment have been nearly uncorrelated.

5.1 Calibration Strategy

The first step in our quantitative analysis is to calibrate the parameters of the regime switching version of our model. The parameters describing preferences are the discount factor, ρ, and the exponent on the BJ good in the worker's utility function, α. Technology is described by the productivity of labor in the market, y_e, and at home, y_u, and by the rate of transformation of the AD and the BJ goods in the market, c, and at home, r. The search and bargaining frictions in the labor market are described by the vacancy cost, k, the destruction rate, δ, the bargaining power of workers, γ, and the matching function M which we assume to have the CES form $M(u,v) = uv(u^{\phi} + v^{\phi})^{-1/\phi}$. The search frictions in the product market are described by the probability that an unemployed worker searches twice, ψ_u, the probability that an employed worker searches twice, ψ_e, and by the matching function N, which we assume to be of the form $N(b,s) = s$. Finally, the evolution of sentiments is described by the switching probabilities, $\pi_{GB}(u)$ and $\pi_{BG}(u)$, and by the jump in the firm's value conditional on a switch from the good to the bad regime, $C_{GB}(u, J)$. For the sake of simplicity, we restrict attention to $\pi_{GB}(u)$ and $\pi_{BG}(u)$ that are independent of u, and let $C_{GB}(u, J) = \Delta$, $\Delta < 0$.

We calibrate the parameters of the model using US data from the years 1987-2007, which we interpret as a period during which the US economy was in the optimistic regime. We choose the cost of a vacancy, k, and the job destruction rate, δ, so that the average of the monthly unemployment to employment transition rate (henceforth, UE rate) and the average monthly employment to unemployment transition rate (EU rate) are the same in the data and in the optimistic regime of the model. We choose the parameter ϕ in the matching function M so that the elasticity of the UE rate to the vacancy-to-unemployment rate is the same in the data and in the model. This part of our calibration strategy is standard (see, e.g., Shimer 2005 or Menzio and Shi 2011). The parameter δ is equal to the EU rate. The parameter k can be calibrated using the UE rate because, in the model, the lower is k, the higher is the vacancy-to-unemployment ratio and, in turn, the higher is the worker's job-finding probability. Similarly, the parameter ϕ can be calibrated using the elasticity of the UE rate with respect to the vacancy-to-unemployment ratio because, in the model, this elasticity is a strictly increasing function of ϕ.

We normalize labor productivity, y_e, to 1 and we choose y_u and γ so that, in the model, the expenditures of unemployed workers relative to employed workers and the profit margin of firms are the same as in the data. Intuitively, one can use the ratio of expenditures for unemployed and employed workers to calibrate y_u because, in the model, the expenditure ratio y_u/w is a strictly increasing function of y_u. Similarly, one can use the profit margin of the firms to calibrate γ because, in the model, the profit margin $(S(u) + y_e - w)/\,(S(u)+y_e)$ is a strictly decreasing function of γ. We assume that the rate of transformation of the AD good into the BJ good in the market, c, is 1 and we choose r so that the model matches the (expenditure weighted) average of the ratio between the highest and the lowest price for identical goods. This is an appropriate target for r because, in the model, the ratio of the highest to the lowest price is strictly increasing in r.

Next, we choose the value of ψ_u and ψ_e so that, in the model, the amount of time spent shopping by unemployed workers relative to employed workers and the price paid for identical goods by unemployed workers relative to employed workers are the same as in the data. The calibration targets are intuitive. Under the assumption that the average number of searches is proportional to the time spent shopping, one can recover the difference between ψ_u and ψ_e from the amount of time spent shopping by different types of workers. Then, one can recover ψ_e from the price paid for identical goods by different types of workers because, in the model, the return of $\psi_u - \psi_e$ additional searches (measured by the decline in the average price paid) is strictly decreasing in ψ_e. Further, we choose α so that the model matches the (expenditure weighted) average of the standard deviation of log prices for identical goods. Intuitively, α determines the size of the BJ market (where there is price dispersion) and the size of the AD market (where there is no price dispersion) and, hence, it determined the average dispersion of prices.

We choose the value of π_{GB} so that, on average, the economy enters the pessimistic regime once every 50 years and we choose the value of π_{BG} so that, on average, the economy remains in the pessimistic regime for 10 years. Moreover, we choose the value of Δ so that, upon entering the pessimistic regime, the economy experiences a 20 percent decline in the value of the firm. While these calibration choices are somewhat arbitrary, they do capture our view that a negative sentiment shocks is a fairly rare and persistent event, which is ushered by a large decline in the value of the firms.

5.2 Data Sources and Target Values

Table 1 reports the value of the calibration targets, which we now motivate. We construct empirical measures of the workers' transition rates between employment and unemployment following the methodology developed by Shimer (2005). We find that, over the period 1987-2007, the average UE rate is 2.4 percent per month and the average EU rate is 43 percent per month. We also find that the elasticity of the UE rate with respect to the vacancy-to-unemployment rate is approximately 25 percent. As explained in Petrongolo and Pissarides (2000) and Menzio and Shi (2011), this elasticity is a biased estimate of the elasticity of M with respect to v because it abstracts from the fact that also employed workers search for jobs and, hence, they affect labor market tightness. For this reason, we target an elasticity of 65 percent, which is the value of the elasticity of M with respect to v estimated by Menzio and Shi (2011) after accounting for search on the job.

Table 1: Calibration targets

Labor Market Targets	
Monthly transition rate, UE	0.433
Monthly transition rate, EU	024
Elasticity UE rate wrt tightness	0.650
Product Market Targets	
Expenditures of U relative to E	0.85
Shopping time of U relative to E	1.25
St dev log prices	0.15
Max-min ratio	1.80
Price paid by U relative to E	0.98
Other Targets	
Profit margin	0.05
Real annual interest rate	0.035

We use existing estimates of the decline in expenditures experienced by households who transit from employment to unemployment in order to select a target for the expenditure difference between employed and unemployed workers. Bentolila and Ichino (2008) estimate the effect on food expenditures of transiting from employment to unemployment using the PSID. They find that a year of unemployment leads to a 19 percent decline in food expenditures. Stephens (2001) estimates the same effect to be around 14 percent when attention is restricted to individuals who enter unemployment as a result of either business closures or mass layoffs. Stephens (2004) obtains similar findings using data from the Health and Retirement Survey (HRS) in addition to the PSID. Based on these estimates, we choose to target a 15 percent expenditure difference between employed and unemployed workers.[10]

We use cross-sectional data from the American Time Use Survey (ATUS) to choose the target for the difference in the amount of time spent shopping by employed and unemployed workers. We restrict attention to individuals aged 22-55 and to the pre-recession years 2003-2005. We find that employed individuals spend between 24 and 33 percent less time shopping than non-employed individuals, and between 13 and 20 percent less than unemployed individuals.[11] Krueger and Mueller (2010) also measure differences in shopping time

[10] The elasticity of food expenditures with respect to income is likely to be low compared to other expenditures categories, such as luxury goods or semi-durable goods. Therefore, the estimated effect of moving into unemployment on food expenditures is likely to be low compared with the effect on overall expenditures.

[11] The estimation results vary depending on the definition of shopping time. We consider a broad definition of shopping time which includes time spent purchasing all goods and services plus related travel time, and a narrow definition of shopping time which includes time spent purchasing consumer goods and groceries plus related travel time.

between workers in different employment states. They find a difference in shopping time between employed and unemployed individuals of 29 percent in the US, 67 percent in Canada and Western Europe and 56 percent in Eastern Europe. On the basis of these findings, we choose to target a 25 percent difference in the amount of time spent shopping by employed and unemployed workers.

We use the Kielts-Nielsen Consumer Panel Data (KNCDP) to measure the extent of price dispersion, as well as the difference in prices paid by employed and non-employed households.[12] We restrict attention to individuals aged 22-55 and to the pre-recession years 2004-2007. We define a good at the barcode level, but we also consider broader definitions that allow for (i) brand substitution, (ii) size substitutions and (iii) brand and size substitution. We define a market as a Scantrack Market Area, which is the notion of market used by Nielsen. For each triple of product, market and quarter, we measure the distribution of transaction prices and we compute the standard deviation of log prices and various percentile ranges. We find that the median standard deviation of log prices is approximately 20 percent, which is similar to the findings in Sorensen (2000) and Moraga-Gonzales and Wildenbeest (2010). Furthermore, we find that the median 99-to-1 percentile ratio is 2.28, the median 98-to-2 percentile ratio is 2.07, and the median 95-to-5 percentile ratio is 1.80. Naturally, we find that all the measures of price dispersion increase as we consider broader product definitions. Given that some price dispersion may be caused by differences in store quality, we choose to target a standard deviation of log prices of 15 percent, and a max-to-min price ratio of 1.8.

We follow the methodology developed by Aguiar and Hurst (2007) to measure the difference in prices paid by employed and non-employed households. For each household in our sample, we construct a price index that is defined as the ratio of the household's actual expenditures to the counterfactual expenditures that the household would have incurred if it had purchased goods at their average price. We then regress the log of the household's price index on the household's employment status and on a number of other household's characteristics, as well as on an index of the estimated quality of the stores visited by the household. We find that the presence of an additional non-employed household head leads to a decline in the price index between 1 and 2.4 percent, depending on the employment status of the other household head and on the set of controls. The effect is substantially larger when we consider broader definitions of goods. For example, when we aggregate goods by

[12] The KNCPD is a panel dataset covering approximately 50,000 households over the period 2004-2009. Respondents use in-home UPC scanning devices to record information (price, quantity, outlet, etc. . .) about their purchases of grocery and non-grocery household items, which account for roughly 30 percent of total expenditures. This data is similar, although much broader in scope to that used by Aguiar and Hurst (2007) in their analysis of the shopping behavior of retired households.

size and brand, we find that the effect of an additional non-employed household head ranges between 1.4 and 5.2 percent. Based on these findings we choose to target a difference in prices paid by employed and unemployed workers of 2 percent.

5.3 Properties of the Calibrated Model

Table 2 reports the calibrated parameter values. Given these parameter values, there exists a 1-2 Regime Switching Equilibrium. The steady-state equilibrium associated with the optimistic regime is such that the unemployment rate is 5.25 percent and the value of a firm is approximately 10 times the monthly output of a worker. The steady-state equilibrium associated with the pessimistic steady-state is such that the unemployment rate is 8.5 percent and the value of a firm is 8 times the monthly output of a worker.

Table 2: Calibrated parameters

Preference Parameters		
ρ	Discount factor	0.003
α	BJ exponent in utility	1.00
Technology Parameters		
y_e	Market production, AD goods	1.00
y_u	Home production, AD goods	4.52
c	Market transformation, AD to BJ	1.00
r	Home transformation, AD to BJ	13.9
Labor Market Parameters		
κ	Vacancy cost	7.02
δ	Exogenous destruction rate	0.024
ϕ	MP matching function parameter	1.25
γ	Workers' bargaining power	0.75
Shopping Parameters		
$1+\psi_u$	Average searches by unemployed	1.28
$1+\psi_e$	Average searches by employed	1.02

The calibrated vacancy cost is $k = 7$ and the calibrated elasticity of substitution between unemployment and vacancies in the matching function is $\phi = 1.25$. These parameter values pin down the u-nullcline and, in turn, imply that the value of a firm must decline by approximately 20 percent as steady-state unemployment goes from 5.25 to 8.5 percent.

To understand why our calibrated model can generate such a large decline in the value of a firm in response to a relatively small change in the employment status of buyers, it is useful to analyze the derivative of $S(u)$ at $u = 0$:

$$S'(u) = \left\{ -\frac{1+\psi_u}{1+\psi_e}\left[\frac{2\psi_u}{(1+\psi_u)(1+\psi_e)} - \frac{2\psi_e}{(1+\psi_e)^2}\right]\alpha y_u \right.$$
$$-\frac{1+\psi_u}{1+\psi_e}\left[1 - \frac{2\psi_e}{(1+\psi_e)^2}\right]\alpha\left(w - y_u\right) \quad (30)$$
$$\left. +2\frac{1+\psi_u}{(1+\psi_e)^2}\frac{\psi_e}{1+\psi_e}\alpha w + \left(1 - \frac{2\psi_e}{(1+\psi_e)^2}\right)\gamma\alpha S'(u) \right\} \frac{(r-c)}{r}.$$

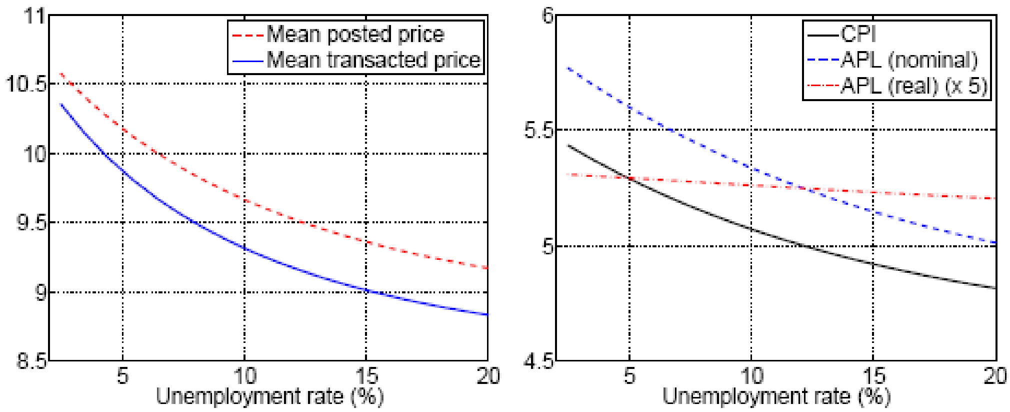
The first term on the right-hand side of (30) is the market power effect, i.e. the effect of the increase in the probability that a prospective buyer is unemployed on the firm's probability of making a sale at the reservation price r. The second term on the right-hand side of (30) is the demand effect of unemployment, i.e. the effect of the increase in the probability that a prospective buyer is unemployed on the amount of the BJ good that the firm sells at the reservation price r. The third term is the captivity effect of unemployment, i.e. the effect of the increase in unemployment on the probability that a worker in a given employment state is willing to purchase at the price r. The last term is the effect of an increase in unemployment on the quantity of the BJ good sold to employed buyers.

The calibration implies a value of ψ_e of 0.024, a value of ψ_u of 0.28, a value of α of 1, a value of r of 13.9, a value for the w-to-y_u ratio of 1.15 percent and a value of γ of 0.75. Given these parameter values, the market power effect is approximately equal to $0.45 \cdot y_u$. In fact, the increase in the probability that a prospective buyer is unemployed is equal to 1.25; the difference between the probability that an unemployed and an employed buyer is willing to purchase at the price r is approximately equal to 0.39; and $\alpha y_u(r-c)/r$ is approximately equal to $0.93 \cdot y_u$. The demand effect is approximately equal to $0.16 \cdot y_u$. In fact, the increase in the probability that a prospective buyer is unemployed is 1.25; the probability that an employed buyer is willing to purchase at the price r is approximately equal to 0.96; the difference between the expenditures of an unemployed and an employed buyer is equal to $w - y_u = 0.15 \cdot y_u$. And $\alpha(r-c)/r$ is approximately equal to 0.93. The captivity effect of an increase in unemployment is approximately equal to zero because $\psi_e = 0$. Finally, the last term on the right-hand side of (30) is approximately equal to $0.7 \cdot S'(0)$. Combining all of these effects, we find that $S'(0)$ is approximately equal to $(0.61/0.3) \cdot y_u \sim 2 \cdot y_u$. Since the calibrated value of $S(u) + y_e - y_u$ is approximately equal to $0.23 \cdot y_u$ at the optimistic steady state, an increase in unemployment from 5.25 to 8.5 percent generates a decline of the value of a firm, $(\rho+\delta)^{-1}(1-\gamma)(S(u) + y_e - y_u)$, of approximately 28 percent.

The above calculation reveal the importance of the market power externality, which is generated by the difference in search intensity between employed and unemployed buyers. If both employed and unemployed buyers searched with intensity $\psi_e \simeq 0$, all buyers would be willing to purchase at the reservation price r. In this case, $S'(0)$ would be proportional to the demand externality, which, in turn, is proportional to the difference in expenditures between employed and unemployed workers, $w - y_u = 1.15 \cdot y_u - y_u = 0.15 \cdot y_u$. However, unemployed buyers search with intensity ψ_u greater than ψ_e and are willing to purchase at price r only with probability 0.61. In this case, $S'(0)$ is proportional to the sum of the demand and market power externalities, which, in turn, is proportional to $1.15 \cdot y_u - 0.61 \cdot y_u = 0.54 \cdot y_u$. That is, the model with differences in search intensity between employed and unemployed buyers behaves like a simple monopolistic competition model a la Dixit-Stiglitz in which the difference between the expenditures of employed and unemployed buyers is 54 rather than 15 percent. The additional 39 percentage points represent the cost to a seller of the additional competitive pressure caused by an increase in the fraction of unemployed buyers.

For an individual seller posting a particular price p, the increase in the competitive pressure caused by an increase in unemployment manifests itself as a lower probability of trading. For sellers as a whole, the additional competitive pressure shows up as a decline in the average posted price and in the average transaction price (defined as posted prices weighted by quantity sold). As one can see in Figure 6(a), an increase in the unemployment rate from 5 to 9 percent leads to a 5.5 percent decline in the average posted price in the BJ market and to a 6 percent decline in the average transaction price. The decline in the average transaction price dominates the decline in expenditures and, hence, leads to an increase in the amount of the BJ good sold by the average seller. In particular, an increase in the unemployment rate from 5 to 9 percent generates a 1.3 percent increase in the quantity of the BJ good sold by the average seller.

The above observations on prices and quantities have important implications for the consumer price index, nominal labor productivity and real labor productivity. We define the consumer price index as $P(u) = Q^*_{BJ} P_{BJ}(u) + Q^*_{AD} P_{AD}(u)$, where Q^*_{BJ} and Q^*_{AD} are the quantities of BJ and AD goods sold in the low-unemployment steady-state and $P_{BJ}(u)$ and $P_{AD}(u)$ are the average transaction prices for BJ and AD goods when the unemployment rate is equal to u. Nominal labor productivity is $S(u) + y_e$. And real labor productivity is defined as $(S(u) + y_e)/P(u)$, i.e. nominal labor productivity divided by the consumer price index. As one can see in Figure 6(b), an increase in the unemployment rate from 5 to 9 percent lowers nominal labor productivity by 4 percent, it lowers the consumer price index by 3.3, and it leaves real labor productivity essentially unchanged. That is, higher unemployment lowers the firm's nominal revenues per worker, but since the decline occurs because of a decline in prices, measured real average labor productivity is almost unaffected.



(a) Prices in BJ market (b) Average labor productivity

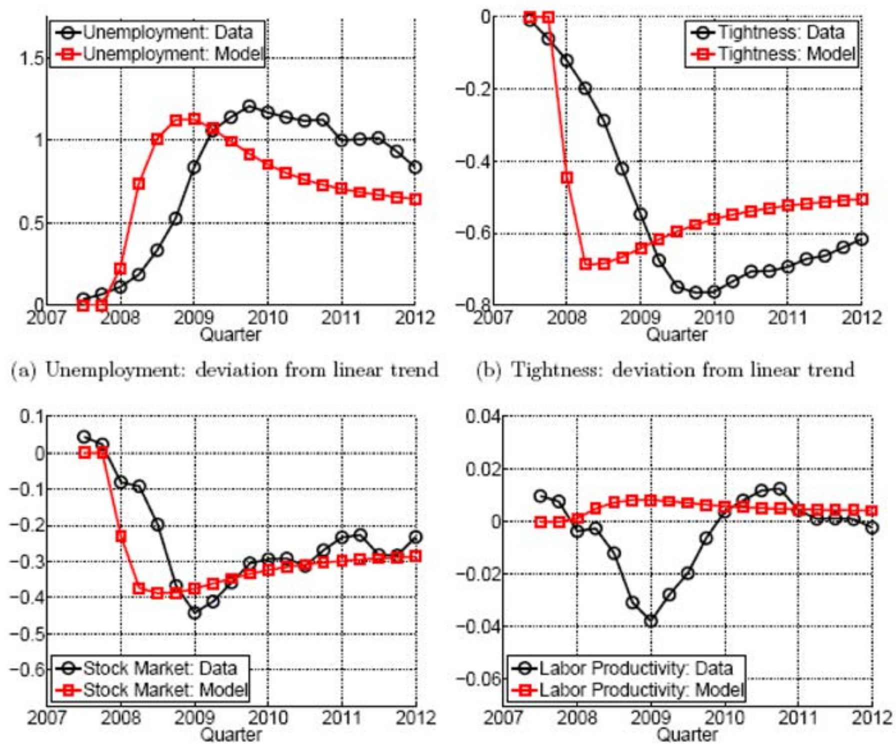
Figure 6: Prices and productivity by unemployment

5.4 Sentiments and the Great Recession

We now use the calibrated version of the model to ask what would have happened to unemployment, labor market tightness, labor productivity and firm value if the US economy had been hit by a negative sentiment shock in the Fall of 2007. More specifically, we assume that at the US economy was at the steady state associated with the optimistic regime when, in November 2007, the agents' expectations about long-run unemployment became pessimistic. We then follow the response of unemployment, labor market tightness, labor productivity and firm value to the sentiment shock.

Figure 7(a) plots the time series for the unemployment rate predicted by the model and observed in the data, measured as a percentage change relative to the first quarter of 2007. The model predicts that the unemployment rate would have increased by 110 percent between the last quarter of 2007 and the first quarter of 2009 and then it would have declined towards the value associated with the pessimistic steady state. In the data, the unemployment rate follows a similar pattern. It increases by 120 percent between the last quarter of 2007 and the second quarter of 2009 and then it slowly declines and settles at a level that is 80 percent higher than in the first quarter of 2007.

Figure 7(b) plots the time series for the vacancy-to-unemployment ratio predicted by the model and observed in the data, measured as a percentage change relative to the first quarter of 2007. The model predicts that the labor market tightness would have fallen by 70 percent in the first quarter of 2008 and, then, it would have remained approximately constant. In the data, the decline in the labor market tightness is more gradual, taking place between the last quarter of 2007 and the beginning of 2009, but the basic pattern is the same as in the model. In particular, the labor market tightness falls by 70 percent and persists at this lower level.



(a) Unemployment: deviation from linear trend (b) Tightness: deviation from linear trend

(c) Stock market: deviation from linear trend (d) Labor productivity: deviation from linear trend

Figure 7: Impulse response to a negative sentiment shock

Figure 7(c) plots the time series for the value of the stock market predicted by the model and observed in the data, measured as a percentage deviation from a linear trend. In the model, we construct the value of the stock market as follows. We assume that the value of a firm in the stock market, V, is equal to the value of the firm, J, net of the repayment of the firm's debt, D. Moreover, we assume that the firm's debt, D, is a constant fraction $d = 1/3$ of the value of the firm at the low unemployment steady state. Given these assumptions, a 1 percent change in the value of the firm, J, generates a $1/(1-d)$ percentage change in the stock market value, V. In the data, we measure the stock market using the Dow Jones

Index. We then compute the percentage deviation of the stock market from the value it would have had if, since the first quarter of 2007, it had grown at the same rate as in the average of the previous 30 years.

The model predicts that the stock market would have fallen by approximately 30 percent in the first quarter of 2008 and then it would have remained relatively constant, having reached the value associated with the pessimistic steady state. In the data, the decline in the stock market is slower and deeper than in the model, and it displays a weak recovery after 2009. However, like in the model, the stock market crashes during the recession and then remains well below its long-run trend. Moreover, if we look at the scatter plot of the unemployment rate and of the stock market (Figure 8), we observe a similar pattern in the model and in the data. In both the model and the data, the decline in the stock market precedes the bulk of the increase in the unemployment rate.

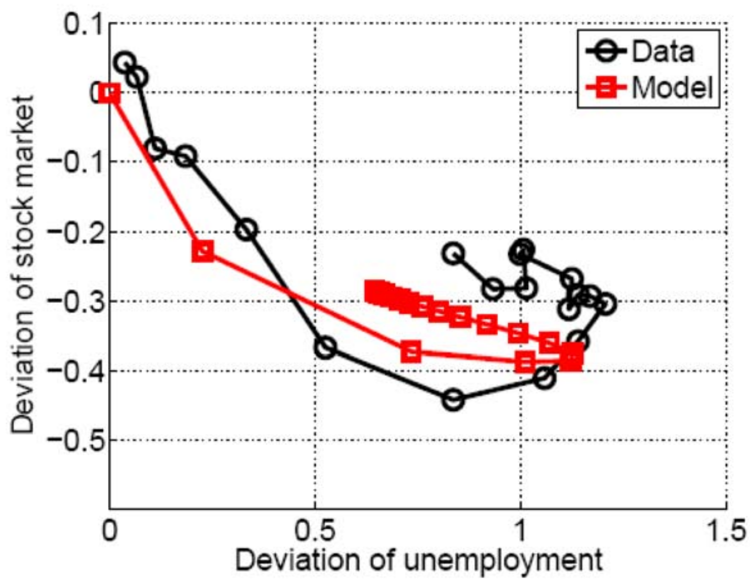


Figure 8: Unemployment rate and stock market: data vs model, deviations from a linear trend

Figure 7(d) plots the time series for the productivity of labor predicted by the model and observed in the data, measured as a percentage deviation from a linear trend. The model predicts that real labor productivity would have hardly changed during the Great Recession. Notice that the model predicts a large decline in the revenues generated by each worker, but this decline is due to a fall in the prices charged by firms in the BJ market and not by a fall in the quantity traded in the BJ market. Thus, after deflating the revenues generated by each worker by the consumer price index, the resulting real labor productivity does not fall (in fact, it experiences a small increase). In the data, real labor productivity did decline for a few quarters. In particular, there are five quarters (between the second quarter of 2008 and the second quarter of 2009), during which real labor productivity is 1 or more percentage points below trend. However, the magnitude of the decline in real labor productivity is small compared to the increase in unemployment and it is much more transitory. In fact, since the third quarter of 2009, real labor productivity has returned to trend, while the unemployment rate has remained 80 percent above its pre-recession level.

Overall, we think that our theory of sentiment shocks captures three key features of the Great Recession and of the subsequent weak recovery. First, the model captures the fact that the unemployment rate has increased substantially during the recession and has not recovered much since the end of the recession. From the perspective of our model, this happened because the agents' beliefs about future unemployment have remained pessimistic. Second, the model captures the fact that the Great Recession was ushered by a stock market crash. From the perspective of our model, this happened because the cause of the recession was precisely a change in the agents' expectations about the future profitability of firms (which is immediately reflected in the stock market) brought about by a change in the expectations about future unemployment (which took some time to materialize). Third, the model captures the fact that while unemployment has increased significantly and persistently, labor productivity has experienced a moderate and transitory decline. From the perspective of our model, this happened because the cause of the recession was not the decline in productivity but the change in sentiments about future unemployment.

In contrast, it is difficult to explain the recent behavior of the economy as the outcome of a technology shock. In the context of the Diamond Mortensen Pissarides framework, the small and temporary decline in labor productivity that we observed in the data should have caused a small and temporary increase in unemployment. Yet, in the data, unemployment has doubled between 2007 and 2009 and has remained well above its pre-recession level afterwards. In the context of the Real Business Cycle framework, the large negative wealth shock (which tends to increase labor supply) and a small decline in real wages (which should have a small negative effect on labor supply) that we observed in the data should have generated either an increase or a small decline in employment. Yet, in the data, non-employment has increased significantly.

Farmer (2011) was the first to advance the view that the Great Recession is the outcome of a non-fundamental shock. In Farmer's model, real wages are not pinned down by competitive forces or by bargaining forces. Rather, real wages are an exogenous variable that is determined by sentiments. An exogenous increase in wages leads to a decline in vacancies and, in turn, to an increase in the unemployment rate. Further, an exogenous increase in wages leads to a decline in the labor-to-capital ratio and, in turn, to a stock market crash. Despite the similarities between our model and Farmer's, there are important differences. First, wages are uniquely pinned down in our model. Hence, our model provides an explanation of the Great Recession that does not rely on a missing equation, but on the presence of strategic complementarities in the employment decision of different firms. Second, our model predicts that real labor productivity should have barely changed and that real wages should have declined during the Great Recession. In contrast, Farmer's model predicts that real labor productivity and real wages should have increased.

6 Conclusions

In this paper, we analyzed a model economy with search frictions in the labor market and in the product market. Search frictions in the labor market generate equilibrium unemployment and income differences between employed and unemployed workers. Search frictions in the product market lead to an equilibrium price distribution for identical goods and, because of differences in the amount of time available for shopping, to differences in the price paid by employed and unemployed workers. In this economy, a firm hiring an additional worker creates positive shopping externalities on other firms, because employed buyers have more income to spend and less time to search for low prices than unemployed buyers. We proved that, if these shopping externalities are strong enough, the employment decisions of different firms become strategic complements and multiple rational expectations equilibria emerge.

We calibrated the model and showed that the empirical differences in expenditure and shopping time between employed and unemployed workers are large enough to generate multiplicity and, hence, to open the door for non-fundamental shocks based on changes in agents' expectations about long-run unemployment. Finally, we formally introduced non-fundamental shocks into the model by defining a notion of Regime Swithcing Equilibrium. We showed that a negative shocks to the agents' expectations (i.e., a switch from an optimistic to a pessimistic regime) generates fluctuations in unemployment, vacancies, labor productivity and in the value of the stock market that look qualitatively and quantitatively similar to those observed in the US economy during the Great Recession.

Much work remains to be done. First, our paper is silent about the welfare properties of different equilibria. Equilibria with a higher vacancy rate may be better than equilibria with a lower vacancy rate because they lead to lower unemployment and more output. On the other hand, equilibria with a higher vacancy rate may be worse than equilibria with a lower vacancy rate because they are associated with higher vacancy costs in the labor market and with higher monopoly distortions in the product market. Therefore, unlike in most papers of multiplicity in macroeconomics (see, e.g., Diamond 1982, Roberts 1987, Cooper and John 1988, etc. . .), in our paper it is not true that the equilibrium with the highest level of economic activity is the most desirable. Instead, the best equilibrium is likely to depend on both parameter values and on initial conditions. Second, our paper is silent about optimal policy. Even the best rational expectation equilibrium is unlikely to be efficient because of the presence of externalities in both the labor and the product markets. Therefore, some government interventions (e.g., hiring subsidies/taxes, unemployment benefits, public spending) may be make the best equilibrium efficient and to eliminate suboptimal equilibria. Finally, our paper describes the macroeconomic effect of a shock to the agents' expectations about future unemployment, but it does not describe the cause of these expectations shocks, or why agents come to share the same expectations about future unemployment. Just like RBC is a theory of propagation of productivity shocks and not a theory of the origins of productivity shocks, our paper is a theory of propagation of non-fundamental shocks and not a theory of the origins of non-fundamental shocks.

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Appendix

A Proof of Lemma 1

Consider an arbitrary distribution of sellers over prices, $F_t(p)$, and let $\xi_t(p)$ denotes the measure of sellers posting the price p. Assume that a buyer who samples two sellers who post the same prices purchases the good from either seller with probability 1/2. Then, the gains from trade for a seller posting the price p are

$$\begin{aligned} S(p) \quad &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1-\frac{2\psi_u\nu(\sigma(u))\left[F(p)-\xi(p)/2\right]}{1+\psi_u}\right]\frac{\alpha y_u(p-c)}{p} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e\nu(\sigma(u))\left[F(p)-\xi(p)/2\right]}{1+\psi_e}\right]\frac{\alpha w(p-c)}{p}. \end{aligned} \tag{A1}$$

Notice that, for the sake of readability, we have dropped the time subscripts from S, F, u and w from the above expression and from the remainder of the proof.

Claim 1. For any $F(p)$, $S^* > 0$.

Proof. The gains from trade for a seller posting $p = r$ are

$$\begin{aligned} S(r) \quad &\geq \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\frac{1-\psi_u}{1+\psi_u}\frac{\alpha y_u(r-c)}{r}+ \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\frac{1-\psi_e}{1+\psi_e}\frac{\alpha w(r-c)}{r} > 0. \end{aligned}$$

Since $S^* \geq S(r)$ and $S(r) > 0$, we have $S^* > 0$. ■

Claim 2. If $F(p)$ satisfies (4), then $F(p)$ is continuous.

Proof. Suppose that there exists a price p_0 such that $\xi(p_0) > 0$. The gains from trade for a seller posting p_0 are

$$\begin{aligned} S(p_0) \quad &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1-\frac{2\psi_u\nu(\sigma(u))\left[F(p_0)-\xi(p_0)/2\right]}{1+\psi_u}\right]\frac{\alpha y_u(p_0-c)}{p_0} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e\nu(\sigma(u))\left[F(p_0)-\xi(p_0)/2\right]}{1+\psi_e}\right]\frac{\alpha w(p_0-c)}{p_0}. \end{aligned}$$

The gains from trade for a seller posting $p_0 - \epsilon$, $\epsilon > 0$, are

$$\begin{aligned} S(p_0-\epsilon) \quad &> \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1-\frac{2\psi_u\nu(\sigma(u))\left[F(p_0)-\xi(p_0)\right]}{1+\psi_u}\right]\frac{\alpha y_u(p_0-\epsilon-c)}{p_0-\epsilon} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e\nu(\sigma(u))\left[F(p_0)-\xi(p_0)\right]}{1+\psi_e}\right]\frac{\alpha w(p_0-\epsilon-c)}{p_0-\epsilon}, \end{aligned}$$

where the above inequality follows from (A1) and from $F(p_0-\epsilon) - \xi(p_0-\epsilon)/2 \leq F(p_0-\epsilon)$

and $F(p_0 - \epsilon) < F(p_0) - \xi(p_0)/2$. Since Claim 1 guarantees that $p_0 > c$, there exists an ϵ small enough that $S(p_0) < S(p_0 - \epsilon) \leq S^*$. Hence, if $F(p)$ has a mass point at p_0, it violates (4). ■

Claim 3. If $F(p)$ satisfies (4), then $\overline{p} = r$.

Proof. From Claim 1, it follows that $\overline{p} \leq r$. Suppose $\overline{p} < r$. Since Claim 2 guarantees that $F(p)$ has no mass points, the gains from trade for a seller posting $\overline{p}$ are

$$\begin{aligned} S(\overline{p}) &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1 - \frac{2\psi_u\nu(\sigma(u))}{1+\psi_u}\right]\frac{\alpha y_u(\overline{p}-c)}{\overline{p}} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1 - \frac{2\psi_e\nu(\sigma(u))}{1+\psi_e}\right]\frac{\alpha w(\overline{p}-c)}{\overline{p}}. \end{aligned}$$

The gains from trade for a seller posting r are

$$\begin{aligned} S(r) &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1 - \frac{2\psi_u\nu(\sigma(u))}{1+\psi_u}\right]\frac{\alpha y_u(r-c)}{r} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1 - \frac{2\psi_e\nu(\sigma(u))}{1+\psi_e}\right]\frac{\alpha w(r-c)}{r}. \end{aligned}$$

Clearly $S(\overline{p}) < S(r) \leq S^*$. Hence, if the highest price on the support of $F(p)$ is $\overline{p} < r$, $F(p)$ violates (4). ■

Claim 4. If $F(p)$ satisfies (4), then the support of $F_t(p)$ is connected.

Proof. Suppose there exist p_0, $p_1 \in \text{supp}F(p)$ such that $p_0 < p_1$ and $F(p_0) = F(p_1)$. Then, the gains from trade for a seller posting p_0 are

$$\begin{aligned} S(p_0) &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1 - \frac{2\psi_u\nu(\sigma(u))F(p_0)}{1+\psi_u}\right]\frac{\alpha y_u(p_0-c)}{p_0} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1 - \frac{2\psi_e\nu(\sigma(u))F(p_0)}{1+\psi_e}\right]\frac{\alpha w(p_0-c)}{p_0}. \end{aligned}$$

The gains from trade for a seller posting p_1 are

$$\begin{aligned} S(p_1) &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1 - \frac{2\psi_u\nu(\sigma(u))F(p_0)}{1+\psi_u}\right]\frac{\alpha y_u(p_1-c)}{p_1} \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1 - \frac{2\psi_e\nu(\sigma(u))F(p_0)}{1+\psi_e}\right]\frac{\alpha w(p_1-c)}{p_1}. \end{aligned}$$

Clearly $S(p_0) < S(p_1) \leq S^*$. Hence, if the support of $F(p)$ has a hole, $F(p)$ violates (4). ■

Claim 5. The only $F(p)$ that satisfies (4) is

$$F(p) = \left\{u(1+\psi_u)\left[1-\frac{2\psi_u\nu(\sigma(u))}{1+\psi_u}\frac{(r-c)p}{(p-c)r}\right]y_u + (1-u)(1+\psi_e)\left[1-\frac{2\psi_e\nu(\sigma(u))}{1+\psi_e}\frac{(r-c)p}{(p-c)r}\right]w\right\} \Big/ 2\nu(\sigma(u))\left\{u\psi_u y_u + (1-u)\psi_e w\right\} \tag{A3}$$

Proof. First, suppose $F(p)$ satisfies (4). From Claim 3, it follows that $S(r) = S^*$. From Claim 4, it follows that $S(p) = S^*$ for all $p \in [\underline{p}, r]$. Solving the equation $S(r) = S(p)$ for $F(p)$ leads to (A3). In turn, $\underline{p}$ can be found by solving $F(\underline{p}) = 0$. Next, suppose that $F(p)$ satisfies (A3). Then, it is easy to verify that $S(p) = S > 0$ for all $p \in [\underline{p}, r]$, $S(p) = 0$ for all $p > r$ and $S(p) < S$ for all $p < \underline{p}$. ■

B Continuous Time Limit

Let $\Delta \in (0, 1]$ denote the lenght of a period. Each worker has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho\Delta)^{-t}x_{\Delta t}^{\alpha}y_{\Delta t}^{1-\alpha}$, and each firm has preferences described by the utility function $\sum_{t=0}^{\infty}(1+\rho\Delta)^{-t}y_{\Delta t}$. In each period, an unemployed worker produces $y_u\Delta$ units of the AD good and can transform them into units of BJ good at the rate r. In each period, an employed worker can produce any combination of AD and BJ goods such that $x + cy = y_e\Delta$.

In the MP market, each firm pays a cost $k\Delta$ to maintain a vacancy for one period. Moreover, in the MP market, the number of matches between u unemployed workers and v vacancies is given by $M(u, v)\Delta$. This implies that an unemployed worker meets a vacancy with probability $\lambda(\theta)\Delta$ and a vacancy meets an unemployed worker with probability $\eta(\theta)\theta$. In each period, a firm-worker match faces a destruction probability $1 - e^{-\delta\Delta}$. In the BJ market, each unemployed worker makes 1 search with probability $1 - \psi_u$ and 2 searches with probability ψ_u. Similarly, each employed worker makes 1 search with probability $1 - \psi_e$ and 2 searches with probability ψ_e. Moreover, in the BJ market, the number of matches between s active firms and b buyers' searches is given by $N(b, s)$. This implies that a buyer's search is successful with probability $\nu(\sigma)$ and a seller meets a buyer with probability $\mu(\sigma)$.

The Bellman Equation for the value of a worker to a firm can be written as

$$J_t = (1-\gamma)\left(S(u_t, \Delta) + y_e\Delta - y_u\Delta\right) + \frac{1-\delta\Delta}{1+\rho\Delta}J_{t+\Delta}, \tag{B1}$$

where $S(u,\Delta)$ denotes the firm's periodical gains from trade in the BJ market and is given by

$$\begin{aligned} S(u,\Delta) &= \mu(\sigma(u))\frac{u(1+\psi_u)}{b(u)}\left[1-\frac{2\psi_u\nu(\sigma(u))}{1+\psi_u}\right]\frac{(r-c)}{r}\alpha y_u\Delta \\ &+\mu(\sigma(u))\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e\nu(\sigma(u))}{1+\psi_e}\right]\frac{(r-c)}{r}\alpha\left[(1-\gamma)y_u\Delta+\gamma\left(S(u,\Delta)+y_e\Delta\right)\right]. \end{aligned} \tag{B2}$$

After comparing (B2) and (11), it follows that

$$S(u,\Delta) = S(u)\Delta. \tag{B3}$$

The law of motion for unemployment can be written as

$$u_{t+\Delta} = u_t(1-\lambda(\theta(J_{t+\Delta},\Delta))\Delta) + (1-u_t)\delta\Delta, \tag{B4}$$

where $\theta(J,\Delta)$ denotes the equilibrium tightness of the labor market and is given by

$$\theta(J,\Delta) = \eta^{-1}\left(\min\left\{\frac{k\Delta}{J\Delta},1\right\}\right) = \theta(J). \tag{B5}$$

The limit for $\Delta \to 0$ of the difference equation (B1) is the differential equation (14). Similarly, the limit for $\Delta \to 0$ of the difference equation (B2) is the differential equation (15). ■

C Proof of Theorem 1

Proof of part (i): Suppose $S'(u) < 0$ for some $u \in [\underline{u}, \overline{u}]$. In this case, there exist u_0 and u_1 such that $\underline{u} < u_0 < u_1 < \overline{u}$ and $J_0 > J_1$, where $J_0 \equiv (1-\gamma)(S(u_0)+y_e-y_u)/(\rho+\delta)$ and $J_1 \equiv (1-\gamma)(S(u_1)+y_e-y_u)/(\rho+\delta)$. In what follows, we will find a vacancy cost k and a matching function M such that (u_0, J_0) and (u_1, J_1) are steady-state equilibria.

Define x_0 as $(1-u_0)\delta/u_0$ and x_1 as $(1-u_1)\delta/u_1$. From the inequalities $\underline{u} < u_0 < u_1 < \overline{u}$, it follows that $0 < x_1 < x_0 < 1$. Choose the vacancy cost k to be equal to $J_1-\epsilon$, where $\epsilon > 0$ and $\epsilon < \min\{x(J_0-J_1)/(x_0-x_1), J_1\}$. Such a choice for ϵ is always possible because $J_0 > J_1$, $x_0 > x_1$ and $J_1 > 0$. Choose the inverse of the job-finding probability, $\varphi(x) \equiv \lambda^{-1}(x)$, to be such that $\varphi(0) = 0$ and

$$\varphi'(x) = \begin{cases} 1+2\gamma_0 x, & \text{if } x \in [0,x_1] \\ 1+2\gamma_0 x_1 + 2\gamma_1(x-x_1), & \text{if } x \in [x_1,x_0] \\ 1+2\gamma_0 x_1 + 2\gamma_1(x_0-x_1) + \dfrac{(x-x_0)}{(1-x_0)(1-x)}, & \text{if } x \in [x_0,1], \end{cases} \tag{C1}$$

where the parameters γ_0 and γ_1 are

$$\gamma_0 = \frac{J_1 - k}{k x_1},$$

$$\gamma_1 = \frac{x_0 J_0 - x_1 J_1}{k(x_0 - x_1)^2} - \frac{1 + 2\gamma_0 x_1}{x_0 - x_1}.$$

First, notice that $\varphi(x)$ is strictly increasing and strictly convex for all $x \in [0, 1]$. In fact, $k < J_1$ implies $\gamma_0 > 0$ and $k > J_1 - x_0(J_0 - J_1)/(x_0 - x_1)$ implies $\gamma_1 > 0$. In turn, $\gamma_0 > 0$ and $\gamma_1 > 0$ imply that $\varphi'(x)$ is strictly positive and strictly increasing for all $x \in [0, 1]$. Second, notice that $\varphi(x)$ is such that $\varphi(0) = 0$ and $\varphi(1) = \infty$. Third, $\varphi(x)$ is such that

$$\begin{aligned} \varphi(x_1) &= \varphi(0) + \int_0^{x_1} (1 + 2\gamma_0 x) dx = \frac{J_1 x_1}{k}, \\ \varphi(x_2) &= \varphi(x_1) + \int_{x_1}^{x_0} (1 + 2\gamma_0 x_1 + 2\gamma_1 (x - x_1)) dx = \frac{J_0 x_0}{k}. \end{aligned} \tag{C2}$$

From the properties of $\varphi(x)$, it follows that the job-finding probability function $\lambda(\theta)$ is strictly increasing, strictly concave and such that $\lambda(0) = 0$, $\lambda(\infty) = 1$ and $\lambda'(0) = 1$. In turn, from the properties of $\lambda(\theta)$, it follows that the job-filling probability function $\eta(\theta) \equiv \lambda(\theta)/\theta$ is strictly decreasing and such that $\eta(0) = 1$ and $\eta(\infty) = 0$. Therefore, the function $\varphi(x)$ defined in (C1) implies a matching process $\lambda(\theta)$, $\eta(\theta)$, $M(u, v) = u\lambda(u, v)$ that satisfies all of the regularity assumptions made in Section 2. Moreover, since $\epsilon < J_1$, $k = J_1 - \epsilon > 0$. Therefore, the vacancy cost k the assumptions made in Section 2.

Now, notice that the function $\varphi(x)$ defined in (C1) implies that (u_0, J_0) and (u_1, J_1) are two steady-state equilibria. In fact, the definition of J_0 implies that

$$J_0 = \frac{(1 - \gamma)(S(u_0) + y_e - y_u)}{\rho + \delta},$$

and the first line in (C2) implies that

$$\begin{aligned} \frac{x_0}{\varphi(x_0)} = \frac{k}{J_0} \quad &\iff \quad \frac{\lambda(\varphi(x_0))}{\varphi(x_0)} = \frac{k}{J_0} \\ &\iff \quad x_0 = \lambda\left(\eta^{-1}\left(\frac{k}{J_0}\right)\right) \\ &\iff \quad u_0 = \frac{\delta}{\delta + \lambda(\theta(J_0))}. \end{aligned}$$

Similarly, the definition of J_1 and the second line in (C2) imply that

$$J_1 = \frac{(1-\gamma)\left(S(u_1)+y_e-y_u\right)}{\rho+\delta},$$
$$u_1 = \frac{\delta}{\delta+\lambda(\theta(J_1))}.$$

Now, suppose $S'(u) \geq 0$ for all $u \in [\underline{u}, \overline{u}]$. On the way to a contradiction, suppose that there exist two steady-state equilibria (u_0, J_0) and (u_1, J_1). From the stationarity condition (17) and the fact that $S'(u) \geq 0$ for all $u \in [\underline{u}, \overline{u}]$, it follows that

$$J_0 = \frac{(1-\gamma)\left(S(u_0)+y_e-y_u\right)}{\rho+\delta} \leq \frac{(1-\gamma)\left(S(u_0)+y_e-y_u\right)}{\rho+\delta} = J_1. \tag{C3}$$

From the stationarity condition (16), it follows that

$$u_0 = \frac{\delta}{\delta+\lambda(\theta(J_0))} < \frac{\delta}{\delta+\lambda(\theta(J_1))} = u_1. \tag{C4}$$

Since $J_0 \leq J_1$, $\lambda(\theta)$ is increasing in θ and $\theta(J)$ is increasing in J, $\lambda(\theta(J_0)) \leq \lambda(\theta(J_1))$, which contradicts the inequality in (C4). ■

Proof of parts (ii)-(iii): Given $u \in [\underline{u}, \overline{u}]$ and $y_e \geq y_u$, the seller's gains from trade in the BJ market are

$$\begin{aligned} S(u, y_e) = \; & \frac{\alpha(r-c)}{r}\Bigg\{A\frac{u(1+\psi_u)}{b(u)}\left[1-\frac{2\psi_u}{1+\psi_u}\frac{A(1-u)}{b(u)}\right]y_u \\ & +A\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right]\left[(1-\gamma)y_u+\gamma\left(S(u,y_e)+y_e\right)\right]\Bigg\}. \end{aligned}$$

The partial derivative of S with respect to y_e is

$$\frac{\partial S(u,y_e)}{\partial y_e} = \frac{\alpha(r-c)}{r}\left\{A\frac{(1-u)(1+\psi_e)}{b(u)}\left[1-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right]\gamma\left(\frac{\partial S(u,y_e)}{\partial y_e}+1\right)\right\}. \tag{C5}$$

The partial derivative of S with respect to u has the same sign as the function $G(u, y_e)$, which is given by

$$\begin{aligned} G(u, y_e) = \; & \frac{(1+\psi_u)(1+\psi_e)}{b(u)^2}\left[\left(1-\frac{2\psi_u}{1+\psi_u}\frac{A(1-u)}{b(u)}\right)y_u-\left(1-\frac{2\psi_e}{1+\psi_e}\frac{A(1-u)}{b(u)}\right)w\right] \\ & +2A\frac{1+\psi_u}{b(u)^2}\left[\frac{(1+\psi_u)u}{b(u)}\frac{\psi_u}{1+\psi_u}y_u+\frac{(1+\psi_e)(1-u)}{b(u)}\frac{\psi_e}{1+\psi_e}w\right]. \end{aligned}$$

The partial derivative of G with respect to y is

$$\frac{\partial G(u,y_e)}{\partial y_e} = -\frac{1+\psi_u}{b(u)^2}\left[1+\psi_e - \frac{4A(1-u)\psi_e}{b(u)}\right]\left[1+\frac{dS(u)}{dy_e}\right]\gamma. \tag{C6}$$

After substituting (C5) into (C6), we obtain

$$\frac{\partial G(u,y_e)}{\partial y_e} = \frac{-\frac{1+\psi_u}{b(u)^2}\left[1+\psi_e - \frac{4A(1-u)\psi_e}{b(u)}\right]\gamma}{1 - A\frac{1-u}{b(u)}\left[1+\psi_e - \frac{2A(1-u)\psi_e}{b(u)}\right]\frac{\alpha(r-c)}{r}\gamma}.$$

Let $\varphi(u)$ denote the partial derivative above because it depends on u but not on y_e. Then, we can write $H(u,y_e)$ as

$$H(u,y_e) = H(u,y_u) + \varphi(u)\left(y_e - y_u\right). \tag{C7}$$

For any $u \in [u,u]$, $H(u,y_u)$ is finite and $\varphi(u)$ is strictly negative. Therefore, there exists a $\overline{y}_e(u) \geq y_u$ such that $H(u,y_e) < 0$ for all $y_e \in [y_u, \overline{y}_e(u))$ and $H(u,y_e) < 0$ for all $y_e > \overline{y}_e(u)$. This completes the proof of part (ii). The proof of part (iii) is similar and it is omitted for the sake of brevity. ■