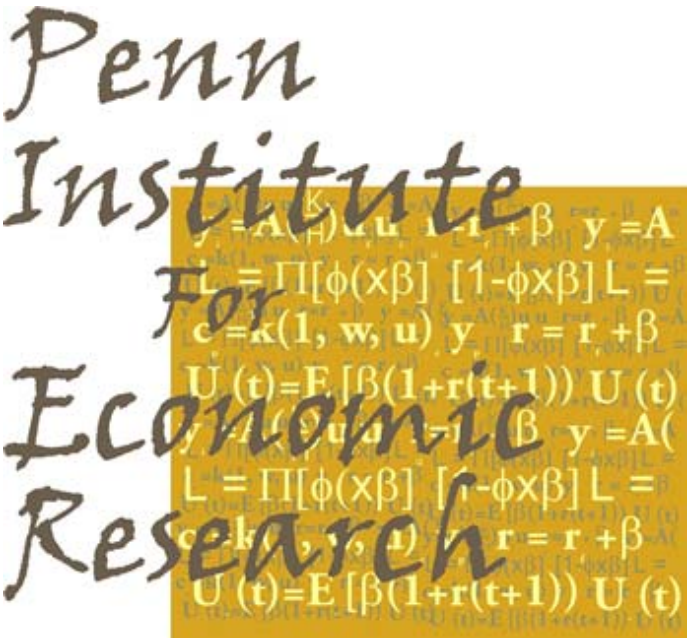

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"Directed Search and Job Rotation"

by

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Directed Search and Job Rotation*

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Abstract

We consider the impact of job rotation in a directed search model in which firm sizes are endogenously determined and match quality is initially unknown. A large firm benefits from the opportunity of rotating workers so as to partially overcome loss of mismatch. As a result, in the unique symmetric equilibrium, large firms have higher labor productivity and lower separation rates. In contrast to the standard directed search model with multi-vacancy firms, this model can generate a positive correlation between firm size and wage without introducing any ex ante productivity differences or imposing any non-concave production function assumption.

Keywords: Directed Search, Job Rotation, Firm Size and Wage, Firm Size and Labor Productivity

JEL Classification Codes: L11; J31; J64

1 Introduction

The practice of job rotation is commonly observed in large firms. In the literature, it is well known that a job rotation policy mainly results from learning of pair-wise match quality between workers and jobs. However, little work has been done to address the impact of job rotation within firms on the labor market. One reason is that the study of job rotation requires a framework that

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simultaneously considers the internal labor market of a firm and the external labor market. Yet, in the canonical job search model, labor economists' favorite work horse, a firm is treated as a single job vacancy, and therefore it is impossible to distinguish between the internal and external labor market. Recently, many job search papers, including Hawkins (2011), Kaas and Kircher (2011), Lester (2010) and Tan (2012) have shed light on the endogenous determination of firm size, which has the potential to study the interaction between a firm's internal and external labor market.

In this paper, we employ a directed search model with multi-vacancy firms to examine the role of job rotation in the labor market. In particular, we assume that a firm can choose its size by determining the number of job vacancies. A large firm can hire more workers, which requires a higher fixed cost. All workers are ex ante identical, but they may be good at different jobs, which is initially unknown. The match quality between a worker and a job is uncertain when the worker is hired but can be learned afterwards through a process of job rotation. Firms can reassign workers to different positions to partially overcome the loss of mismatch, and larger firms have a higher degree of freedom of reallocation and, therefore, can expect higher revenue per match.

Our main result highlights the impact of job rotation on the labor market. In the unique symmetric equilibrium, we obtain a *positive* correlation between firm size, labor productivity and wage, which is consistent with stylized facts summarized by Oi and Idson (1999). In addition, in line with recent empirical findings by Papageorgiou (2011), the model sucessfully implies a *negative* correlation between firm size and the separation rate. Without the opportunity of job rotation, however, the correlation between firm size, labor productivity and wage is negative for all parameters, which is the result of a standard directed search model with multi-vacancy.

Our paper is related to the literature in two ways. First, Meyer (1994) and Ortega (2001) point out the learning role of job rotation in firms. They provide a justification of job rotation, but both authors narrow their studies within the boundary of a single firm. As a step further, we apply their insight in a competitive labor market model to study the effect of within-firm job rotation on the external labor market. Papageorgiou (2011) is the only paper that studies the impact of job rotation on the labor market but with a different focus. He pays more attention to the interaction between tenure effect and job reallocation within a firm, while, in contrast, we focus on how the internal labor market in the presence of job rotation affects job allocation in the external labor market. In his model, firm sizes are exogenous rather than endogenously determined as in ours. In addition, he uses a Pissarides-Mortensen model and introduces heterogeneous firms, so the pricing mechanism in his paper is Nash bargaining instead of wage posting, and the search is random rather than directed.

Second, the directed search model we employe follows Montgomery (1991), Peters (1991), Burdett, Shi and Wright (2001), and their later extension by Lester (2010) to the multi-vacancy

case. Kaas and Kircher (2011) also study a directed search model with multi-vacancy firms. However, none of these papers can generate a relationship between firm size, wage and labor productivity that is in line with observations without introducing *ex ante* exogenously dispersed random productivity[1]. In our model, the presence of learning and job rotation creates an *ex post* heterogeneity among firms and, therefore, generates a positive relation between wage and firm size. Shi (2002) introduces a frictional product market to overcome this problem. In his paper, large firms have more incentives to attract workers since they have a bigger share in the product market and are anxious to produce enough output. Tan (2012) allows for local convexity in the production function to generate a positive size-wage differential. Yet, in our model, the production function is concave.

The rest of this paper is organized as follows. We first set up the model and characterize the unique symmetric equilibrium. Next, we derive the implications of our model and discuss the result and compare them to the empirical evidence.

2 The Model

2.1 Setup

There are N workers and M firms on the market, both of which are ex ante identical. Denote $\lambda = M/N$ as the ratio of firms to workers. Note that λ does not represent the labor market tightness since the number of vacancies is endogenous in this model. Following the literature, we first consider the individual decision problem given N, M as finite numbers, then we fix λ and take N, M to infinity to approximate the equilibrium in a large labor market.

A match of a worker-job pair is good with probability $\rho \in (0, 1]$ and bad with a complementary probability. If the match is good, we say the quality is 1, meaning that the worker-job match can produce 1 unit of revenue; otherwise, it is 0. The match quality is initially unknown and learned later. We assume the match quality is independent across jobs and workers, even within a multi-job firm.

The game has four stages: offer posting stage (I), job searching stage (II), learning and rotation stage (III), and production stage (IV). At Stage I, the job posting stage, each firm decides how many vacancies to post, k, and at what wage level, w, where w is potentially a function of k. For simplicity, we assume that they can create $k \in \{1, 2\}$ vacancies with cost $C(k)$, thus the market tightness, defined as the ratio of vacancies to workers, is $\theta \in [\lambda, 2\lambda]$. Without loss of generality,

[1] In both Lester (2010) and Kaas and Kircher (2011), if firms have homogenous productivity, the relation between wage and firm size is negative.

we assume a convex cost function with $C(1) = 0$, $C(2) = C$, $0 < C < \rho$. We assume that wage, $w \in [0, 1]$, does not depend on any further information such as the realized number of applicants and revealed match quality in Stage III. We assume a firm can commit to the verifiable wage it posts, and the firing strategy, which may depend on the result of learning.[2] Consequently, firms pay the first round of wages to all employees at Stage III and pay the second round only to the remaining ones at Stage IV.

At Stage II, the job searching stage, each worker observes (k, w_k) of every firm and applies for the firms that offer the highest expected payoff. We assume that workers can only apply to a firm, instead of a specific position in that firm. If the number of workers that apply for a particular firm exceeds the number of vacancies posted, the firm randomly hires just enough workers; otherwise the firm hires all applicants. Then the firm assigns job positions randomly to employees. Hence, a worker's expected payoff from applying to a firm is determined jointly by both the posted wage and the probability of getting a job.

At Stage III, the learning and rotation stage, a firm randomly assigns hired worker(s) to its position(s) and pays the first round of wage. If possible, the firm learns match qualities of all job-worker pairs by switching workers to different working positions.[3] In particular, a firm with k jobs and h employees, $1 \leq h \leq k$, learns about the match qualities of all $P_h^k = k!/(k-h)!$ possible worker-job pairs, which have $2^{P_h^k}$ possible realizations, and assigns workers to job positions to deliver the highest revenue. A large firm with $k = 2$ has the freedom to assign jobs to employee(s) to derive the highest revenue, which creates a potential benefit margin compared to a small firm ($k = 1$). For example, if a firm posts 2 jobs, A and B, and hires 2 workers I and II, it can observe the match qualities of pairs $\{(I, A), (I, B), (II, A), (II, B)\}$, with the value of, say, $\{1, 0, 0, 1\}$. In this specific case, clearly the firm shall let I do job A and II goes to B to earn 2 as the total revenue, provided that the firm pays $2w_2$ to workers. The job reallocation benefit can be fully described as follows. From the point of view of an employee hired by a two-job firm, his match quality state is $s \in \{AB, \bar{A}\bar{B}, A\bar{B}, \bar{A}B\}$, where AB means his match quality is 1 with both job A and B, and $\bar{A}\bar{B}$ means 0 with each, and both $A\bar{B}, \bar{A}B$ can be interpreted in a similar way. When

[2] The contract specifies the wage at Stage III and Stage IV, conditional on workers being employed, and the firing rule contingent on revealed match qualities of all employed workers in the same firm. Without loss of generality, we focus on the contract space in which (1) the separation and job rotation rule is ex post incentive compatible, and (2) wage is time-invarying. Specifically, a firm commits to firing unqualified and/or redundant worker(s), and to paying all of its hired worker(s) the posted wage at Stage III, and paying all remaining worker(s) the same wage at Stage IV. Since workers and firms are risk neutral, the optimal contract in this particular form is also optimal in a larger feasible contract set where firms can pay time-varying wages, and need not fire unqualified and redundant workers with positive probability.

[3] We assume that the rotating and learning process serves only to reveal the match qualities but does not generate any production.

two workers' states are $\left(\bar{A}B, A\bar{B}\right)$, the firm can match between I and job B and II and job A. Hence, the probability of overcoming one or two mismatch can generate extra revenue for a large firm. Tables 1 to 3 summarize all possible cases for $k \in \{1, 2\}$.

Prob(s)	ρ	$1-\rho$
Employee's s	A	$\bar{\text{A}}$
Total Payoff	1	0

Table 1: $(k, h) = (1, 1)$

Prob(s)	ρ^2	$(1-\rho)^2$	$\rho(1-\rho)$	$\rho(1-\rho)$
Employee's s	AB	$\bar{\text{A}}\bar{\text{B}}$	$\text{A}\bar{\text{B}}$	$\bar{\text{A}}\text{B}$
Total Payoff	1	0	1	1

Table 2: $(k, h) = (2, 1)$

		Employee I			
	Prob(s_I)	ρ^2	$(1-\rho)^2$	$\rho(1-\rho)$	$\rho(1-\rho)$
	Prob(s_{II})	$s_{II} \backslash s_I$	AB	$\bar{\text{A}}\bar{\text{B}}$	$\text{A}\bar{\text{B}}$	$\bar{\text{A}}\text{B}$
	ρ^2	AB	2	1	2	2
Employee II	$(1-\rho)^2$	$\bar{\text{A}}\bar{\text{B}}$	1	0	1	1
	$\rho(1-\rho)$	$\text{A}\bar{\text{B}}$	2	1	1	2
	$\rho(1-\rho)$	$\bar{\text{A}}\text{B}$	2	1	2	1

Table 3: $(k, h) = (2, 2)$

At Stage IV, the production stage, a firm is given the option of firing its employee(s), and then production takes place depending on the match quality of each worker-job pair. A firm with k jobs pays every remaining employee another w_k. By separating the unproductive pairs and firing associated workers, firms can avoid paying extra wages.

2.2 Analysis

The solution concept we adopt is a symmetric rational expectations equilibrium (henceforth, equilibrium), in which each firm chooses to be a large one with the same probability and posts the same contracts, and each worker applies to a large firm with the same probability. We focus on this equilibrium selection because it delivers a limiting matching technology that has all of the properties required by the competitive model. However, as pointed out by Peters (2000), the selection is interesting for its own sake. One implication of this restriction is that it forces mixed strategy continuation equilibria, since in Stage I (II), firms (workers) cannot predict exactly where the other firms (workers) are going in a mixed strategy equilibrium, which is compelling in the large markets considered here. The idea that firms (workers) should not be able to predict the behavior of other firms (workers) is much more convincing. The symmetric equilibria have the

nice property that they can be interpreted as equilibria in which firms (workers) choose the best replies to the average behavior of the other firms (workers) in the market (and in which firms (workers) guess this average correctly). In this sense they have the nice informational properties of anonymous equilibria in the sense that workers can compute their best replies from aggregate information about the market. We will solve the game backwards.

Stage IV: Production Stage. At the last stage, firms fire workers when necessary. Specifically, the optimal firing choice of a firm is to fire a worker in one of the two following situations: (1) the worker is unqualified for any position in the firm, or (2) two workers are qualified for the same position and only one worker is enough to deliver the highest payoff, 1. In the latter case, the firm will randomly fire one of the two workers. A small firm will keep its only employee and pay the wage if the match quality is 1, and fire the employee otherwise, so the probability of separation is simply $1-\rho$. Alternatively, a large firm with only one employee decides to keep her if she is good at either one or two jobs, and to fire her if her match quality turns out to be 0 on both jobs. In this case, the probability of a worker getting fired is $(1-\rho)^2$. A large firm with two employees keeps both of them and pays wages if the total revenue of 2 can be received, and fire the one(s) with 0 quality at both jobs. If the two workers are good at the same job and bad at the other, one will be randomly selected and fired. Combined, the overall probability for either one of the two workers losing her job is $(1-\rho)^2+\rho^2(1-\rho)^2$. Given any history of Stage II the job searching stage, which will be defined later, a firm learns about match qualities of all possible worker-job pairs in Stage III, and then, if possible, it assigns jobs to workers to yield the highest revenue. Then we step back to Stage II and characterize the equilibrium in this subgame for any given history in which firms play symmetric strategies. Then, we will characterize each firm's offer posting strategy given the strategies of workers.

Stage III: Learning and Rotation Stage. A firm with k jobs and h employees, $1 \leq h \leq k$, learns about the match qualities of all possible worker-job pairs in this stage through the practice of job rotation, and pays the promised wage w to employees regardless of the learning results. In particular, for a small firm with one job A and one employee, there are only two possibilities: the employee is either good at the job or not. This employee receives the promised wage w at this stage for sure, and she will receive the same w at the next stage if the match quality turns out to be 1, or she will be fired and receive nothing. For the worker, her expected payoff at the beginning of this stage is

$$V_1(\rho; w) = (1+\rho)\, w,$$

where $V_1(\rho; w)$ denotes the expected payoff to a worker in a small firm with wage level w. Define $F_{kh}(\rho)$ the expected total revenue to a firm with k vacancies and h employees, so

$$F_{11}(\rho) = \rho.$$

A small firm then takes away the rest, that is, $F_{11}(\rho) - V_1(\rho; w)$.

The only worker in a large firm gets promised wage w in this stage, and she takes advantage of job rotation, and in the next stage, she gets fired only when she is bad at both positions with probability $(1-\rho)^2 < 1-\rho$, so her expected payoff is

$$V_{21}(\rho; w) = \left(1 + 2\rho - \rho^2\right) w$$

and the expected total revenue is also higher than that of a small firm,

$$F_{21}(\rho) = 2\rho - \rho^2 > F_{11}(\rho),$$

and the firm takes away $F_{21}(\rho) - V_{21}(\rho; w)$. If a large firm has two employees, then the expected total revenue is even higher,

$$F_{22}(\rho) = -2\rho^4 + 4\rho^3 - 4\rho^2 + 4\rho > 2F_{11}(\rho).$$

Observe that $F_{22}(\rho) < 2F_{21}(\rho)$, so the marginal labor productivity in a large firm is decreasing in the number of employees. Now for the two workers, they will get fired for sure if they are bad at both jobs, or with equal probability if they are only good at the same job. The payoff to each worker is then

$$V_{22}(\rho; w) = \left(1 + 2\rho - 2\rho^2 + 2\rho^3 - \rho^4\right) w,$$

and the firm gains an expected profit $F_{22}(\rho) - 2V_{22}(\rho; w)$ now that there are two workers.

Given the ex post incentive compatible separation and job rotation rule, and since there is no strategic interaction at Stages III and IV, matched workers' and firms' payoffs are uniquely pinned down by the contracts they signed. Hence, an equilibrium in our four-stage game is consistent with an equilibrium in a reduced-form two-stage game that includes Stages I and II in the original game, and the payoff is specified as follows: in a small firm with wage w, the worker's payoff is $V_1(\rho; w)$, and the firm's is $F_{11}(\rho) - V_1(\rho; w)$; in a large firm with wage w and one worker, the worker's payoff is $V_{21}(\rho; w)$, and the firm's is $F_{21}(\rho) - V_{21}(\rho; w)$; in a large firm with wage w and two workers, both workers' payoffs are equally given by $V_{22}(\rho; w)$, and the firm's payoff is $F_{22}(\rho) - 2V_{22}(\rho; w)$. In the rest of this paper, we directly solve equilibria of this reduced-form game as those of the whole game.

Stage II: Job Searching Stage. The realization of firms' job posting at Stage I can be summarized by a history vector $H = ((k^j, w^j)_{j=1}^M)$ listing the number of vacancies and the wages of all M firms. Let $\mathcal{H}$ be the set of all possible H's. In principle, a worker's strategy is defined as $\gamma : \mathcal{H} \to [0,1]^M$. Given a history H, a worker chooses a vector γ such that (1) γ^j is the probability that he applies to firm $j \in \{1, 2, ..M\}$ and (2) $\sum_{j=1}^M \gamma^j = 1$.

Consider the problem of worker i who is deciding whether and to which firm to apply. Firm j posts k^j positions and wage w^j, for $j \in \{1, 2, ..M\}$. If $k^j = 1$, firm j promises its prospective worker the expected payoff $V_1(\rho; w^j) = (1 + F_{11}(\rho))\, w^j$; if $k^j = 2$, the expected payoff depends on how many workers firm j eventually gets, and it is either $V_{21}(\rho; w^j) = (1 + F_{21}(\rho))\, w^j$ or $V_{22}(\rho; w^j) = (1 + F_{22}(\rho)/2)\, w^j$. When the rest $N-1$ workers play identical strategies γ, this worker i chooses strategy $\hat{\gamma}$ to maximize her expected utility

$$\left\{ \begin{array}{c} \sum_{j \text{ s.t. } k^j=1} \hat{\gamma}^j \Omega_1(\gamma^j) V_1(\rho; w^j) \\ + \sum_{j \text{ s.t. } k^j=2} \hat{\gamma}^j \left[\Omega_{21}(\gamma^j) V_{21}(\rho; w^j) + \Omega_{22}(\gamma^j) V_{22}(\rho; w^j) \right] \end{array} \right\} \tag{1}$$

where $\Omega_1(\gamma^j)$ stands for the probability that this worker is hired if she applies to firm j which posts $k^j = 1$ positions, that is,

$$\begin{aligned} \Omega_1(\gamma^j) &= \left(1 - \gamma^j\right)^{N-1} + \sum_{n=1}^{N-1} \left[\frac{(n-1)!}{n!(N-1-n)!} \right] \left(\gamma^j\right)^n (1 - \gamma^j)^{N-1-n} \frac{1}{n+1} \\ &= \frac{1}{N\gamma^j} \left[1 - (1 - \gamma^j)^N \right], \end{aligned} \tag{2}$$

if she is the only applicant, she gets the job for sure; otherwise all applicants get the job with equal probability. The number of applicants at firm j has a binomial distribution. Similarly, $\Omega_{21}(\gamma^j)$ is the probability that this worker is the only applicant at the large firm j and gets a job for sure,

$$\Omega_{21}(\gamma^j) = (1 - \gamma^j)^{N-1}, \tag{3}$$

and $\Omega_{21}(\gamma^j)$ is the probability that this worker needs to work with someone else in the large firm j,

$$\begin{aligned} \Omega_{22}(\gamma^j) &= \sum_{n=1}^{N-1} \left[\frac{(n-1)!}{n!(N-1-n)!} \right] \left(\gamma^j\right)^n (1 - \gamma^j)^{N-1-n} \frac{2}{n+1} \\ &= \frac{2}{N\gamma^j} [1 - (1 - \gamma^j)^N] - 2(1 - \gamma^j)^{N-1}. \end{aligned} \tag{4}$$

A symmetric equilibrium at this stage is such that every worker chooses the same application probability vector γ, and moreover, a worker applies to firms of the same size and wage with equal probabilities. Given any history $H = \left((k^j, w^j)_{j=1}^M \right)$, $\gamma^*(H)$ is the symmetric solution if $\gamma^*(H)$ is a solution to (1) and $\gamma^{*j}(H) = \gamma^{*l}(H)$ if $(k^j, w^j) = \left(k^l, w^l\right)$, $j \neq l$. As mentioned before, we require symmetry across all workers' behavior to ensure an equilibrium that consists of only mixed strategies. In a large market, it is impossible for an individual worker to be fully informed about other workers' job application choices; therefore, modeling it by a mixed-strategy equilibrium is

more plausible. More important, we assume that a worker applies to firms with identical (k, w) to ensure the anonymity of firms in that workers distinguish between firms only by their sizes and posted wages instead of their names, j. This plays the role of search friction in our model. The symmetry is preserved when we take M and N to infinity.

To model a large market, we will follow the literature and let $M \to \infty$ and $N \to \infty$ such that $\lambda = M/N$ remains constant. Define $\mu(k, w) = \lim_{M\to\infty} \left(\sum_{j=1}^{M} 1_{\{(k^j, w^j)=(k,w)\}} \right) / M$. At the limit, a history is described by an offer distribution μ. Define the queue length at firm j as $q^j = \lim_{N\to\infty} \gamma^j N$. Using (2), (3) and (4), it is straightforward to establish the hiring probabilities as functions of queue lengths at the limit. If firm j posts one vacancy, then

$$\Omega_1(q^j) = \frac{1}{q^j}\left(1 - e^{-q^j}\right);$$

otherwise, firm j decides to become a large firm and posts two job openings,

$$\begin{aligned} \Omega_{21}(q^j) &= e^{-q^j}, \\ \Omega_{22}(q^j) &= \frac{2}{q^j}\left(1 - e^{-q^j} - q^j e^{-q^j}\right). \end{aligned}$$

In a symmetric equilibrium, given $\mu(k, w)$, all workers play an identical strategy and receive the same and highest utility level denoted as U. Specifically, a worker applies to a small firm j with positive probability only if

$$\Omega_1(q^j) V_1\left(\rho; w^j\right) = U; \tag{5}$$

similarly, a worker applies to a large firm j with positive probability only if

$$\Omega_{21}(q^j) V_{21}\left(\rho; w^j\right) + \Omega_{22}(q^j) V_{22}\left(\rho; w^j\right) = U. \tag{6}$$

Here, U is referred to as the market utility level in the literature. Solving these two equations gives q^j's as functions of w^j and U. Dropping ρ, define $Q_1(U, w^j)$ as the greater value between the unique q^j as the solution to (5) and zero; define $Q_2(U, w^j)$ by doing the same to (6). Combined, we have $Q_{k^j}(U, w^j)$, which determines the equilibrium queue length at firm j with (k^j, w^j), when the market utility is U.

Definition 1. *Given an offer distribution $\mu(k^j, w^j)$, a symmetric equilibrium of the Stage II game is characterized by (q^j, U) such that*

1. $q^j = Q_{k^j}(U, w^j)$ *for all* j, *and*
2. $\int Q_{k^j}(U, w^j)\, d\mu(k^j, w^j) = 1/\lambda$.

Hence, workers are indifferent between applying to any firm j as long as $q^j > 0$. At the same time, zero queue length implies that this firm cannot provide the market utility level to workers.

Stage I: Offer Posting Stage. Now take one step back and consider a firm's problem at the limit. Expecting the form of $Q_k(U,w)$ and U, firm j's strategy is to choose a probability distribution μ^j over $\{1,2\}\times\mathbb{R}^+$, where $\mu^j(k,w)$ is the probability that firm j posts k vacancies and a wage w. If the firm posts a single vacancy, it chooses w_1 to maximize the expected profit,

$$\pi_1^*(U) = \max_{w_1}\left\{\pi_1(U,w_1) = \Phi_1(Q_1(U,w_1))(F_{11}(\rho) - V_1(\rho;w_1))\right\}, \tag{7}$$

where $\Phi_1(q_1) = q_1\Omega_1(q_1) = 1-e^{-q_1}$ is the probability that a small firm successfully hires a worker at infinity, and $F_{11}(\rho) - V_1(\rho;w_1)$ is the expected profit to the firm. The market utility level U is taken as given, and the firm can attract applicants only if it can provide U level of expected utility to its potential worker(s). At the same time, the representative firm solves the problem associated with a large one,

$$\pi_2^*(U) = \max_{w_2}\left\{\pi_2(U,w_2) = \left[\begin{array}{c}\Phi_{21}(Q_2(U,w_2))[F_{21}(\rho) - V_{21}(\rho;w_2)] \\ +\Phi_{22}(Q_2(U,w_2))[F_{22}(\rho) - 2V_{22}(\rho;w_2)] - C\end{array}\right]\right\} \tag{8}$$

where $\Phi_{21}(q_2) = q_2\Omega_{21}(q_2) = q_2e^{-q_2}$ is the probability that a large firm gets only one applicant, and $\Phi_{22}(q_2) = (q_2/2)\Omega_{22}(q_2) = 1 - e^{-q_2} - q_2e^{-q_2}$ is the probability it gets at least two applicants and therefore two employees. Define

$$\pi^*(U) = \max\left\{\pi_1^*(U), \pi_2^*(U)\right\}. \tag{9}$$

Naturally, to get coexistence of both small and large firms, it requires that $\pi^* = \pi_1^* = \pi_2^*$, which is feasible in certain parameter subspaces.

Definition 2. *A symmetric equilibrium of the Stage I game consists of a distribution $\mu^*(k,w)$, a market utility level U^*, and queue lengths q^j, satisfying*

1. $\mu^j(k,w) = \mu^*(k,w)$,
2. $\pi_{k^j}(U^*,w^j) = \pi^*(U^*)$ *if* $d\mu^*(k^j,w^j) > 0$,
3. $\pi_{k^j}(U^*,w^j) \leq \pi^*(U^*)$ *if* $d\mu^*(k^j,w^j) = 0$,
4. (q^j,U^*) *is the equilibrium of the job application game.*

Equilibrium Characterization. In the following proposition, we show that in the unique equilibrium, the only realized history contains identical small firms and/or identical large ones: in a

small firm's contract, the proposed wage is w_1^*, in a large firm's contract, it is w_2^*, and the associated equilibrium queue lengths in small and large firms are q_1^* and q_2^*, respectively. Let ϕ^* be the equilibrium probability of becoming a small firm. As a result, the proportion of small firms is $\mu(1, w_1^*) = \phi^*$, and $\mu(2, w_2^*) = 1 - \phi^*$ for large ones. Since workers play a symmetric strategy, they will ignore firms' identity if they proposed the same contract. Hence, we can use σ^* as the probability of applying to the group of small firms, and $1 - \sigma^*$ to the large firms. Immediately, we have

$$\begin{aligned} \sigma^* &= \lambda \phi^* q_1^*, \\ 1 - \sigma^* &= \lambda (1 - \phi^*) q_2^*, \end{aligned}$$

where ϕ^* is the equilibrium probability that a firm becomes a small firm. Given the equilibrium queue lengths q_1^* and q_2^*, (ϕ^*, σ^*) can be uniquely pinned down. Combining all of the four stages, we can characterize the equilibrium in the following proposition.

Proposition 1. *There exists a list of functions: $\underline{c}(\rho) \in (0, \rho)$, $\underline{\lambda}(C, \rho) > 0$, and $\bar{\lambda}(C, \rho) > 0$. Fix a set of parameters (λ, C, ρ) such that $C \in (\underline{c}(\rho), \rho)$ and $\lambda \in \left(\underline{\lambda}(C, \rho), \bar{\lambda}(C, \rho)\right)$. There exists a unique equilibrium in which large firms and small ones coexist. The equilibrium can be characterized by a list of functions $(\phi^*, w_1^*, w_2^*, \sigma^*)$ such that: There exists a unique pair of (q_1^*, q_2^*), the queue lengths at a small firm and at a large one, and $(\phi^*, \sigma^*) \in (0, 1) \times (0, 1)$ such that*

$$\phi^* = \frac{q_2^* - 1/\lambda}{q_2^* - q_1^*}, \sigma^* = \lambda q_1^* \phi^* = \frac{\lambda q_1^* (q_2^* - 1/\lambda)}{q_2^* - q_1^*}, q_2^* > q_1^* > 0,$$

and the wages in small and large firm markets are given by

$$\begin{aligned} w_1^* &= \frac{F_{11}(\rho) q_1^* e^{-q_1^*}}{(1 + F_{11}(\rho))(1 - e^{-q_1^*})}, \\ w_2^* &= \frac{F_{21}(\rho) + q_2^* [F_{22}(\rho) - F_{21}(\rho)]}{1 + F_{21}(\rho) + (e^{q_2^*} - 1 - q_2^*)(F_{22}(\rho) + 2)/q_2^*}. \end{aligned}$$

If C, ρ and/or λ lie outside the specified region, which can be decomposed into three regions, there is no heterogeneity in realized firm sizes. The intuition behind these three situations is simple. If $C \in (\underline{c}(\rho), \rho)$ and λ is either too small or too large, firms are also the same size. When λ is too small, there are so few firms in the market relative to workers such that it is easy to hire two workers and to take advantage of job rotation. In equilibrium, no firm chooses to become a small one. Similarly, when λ is too large, there are so many firms and vacancies that it is not only costly to post an extra vacancy, but also hard to fill both of them in a large firm. In equilibrium, no firm wants to be a large one. The coexistence of small and large firms is only possible when

C is high enough compared to ρ, and $\lambda \in \left(\underline{\lambda}(C,\rho), \bar{\lambda}(C,\rho)\right)$. The region in which $C \leq \underline{c}(\rho)$ corresponds to the case of $U^* \geq \rho = F_{11}(\rho)$, and the market utility is so high that a small firm cannot earn a positive profit. As a result, in this region, all firms are the same size. There are two possible cases here: either all firms choose to randomize between being large and not entering by paying an unacceptable wage, or all firms choose to randomize between being small and not entering. The outcome relies on the value of λ. Neither of these two possibilities is of interest. In the following subsection, we focus on the coexistence case and characterize the impact of job rotation on labor market variables.

2.3 Implications

In this subsection, we look at the implications of the unique symmetric equilibrium. The model simultaneously gives predictions on relationships between firm size and productivity, separation rate, wage, which are roughly in line with empirical findings.

Size and Job Rotation Rate. In our model, the job rotation rate is trivially increasing in firm size. We can generalize our model one step further and allow firms to post $1, 2, .., K$ vacancies. Now that a larger firm can overcome the mismatch loss even more via reassignment of jobs, a higher rotation rate will appear. This is consistent with the empirical finding of Papageorgiou (2011). We will see how this higher job rotation benefit of larger firms affects the labor market.

Size and Labor Productivity. The average labor productivity of a small firm is simply $F_{11}(\rho) = \rho$, and that of a large firm is a convex combination $\Phi_{22}F(\rho;2,2)/2 + \Phi_{21}F(\rho;2,1)$, which is greater than ρ since $F(\rho;2,2) > 2\rho$ and $F(\rho;2,1) > \rho$ for any $\rho \in (0,1)$. As stated before, the marginal labor productivity of a large firm is decreasing in size measured as the number of employees, $F(\rho;2,2) < 2F(\rho;2,1)$, and therefore the production function of a large firm is concave in labor.

Size and Separation Rate. In a recent empirical work, Papageorgiou (2011) analyzes the Survey of Income and Program Participation data, and finds that workers in larger firms are less likely to be separated from their firms even conditional on workers' wages. In our paper, for tractability, we assume that after a firm learns the quality of all possible matches between its workers and positions, it has the option to fire incapable employees and create separations. Due to the job rotation feature, large firms have a lower overall separation rate than small firms in our model. In particular, given the specific form of contract, as discussed in the previous section, workers in small firms suffer a separation rate at $r_1^S = 1 - \rho$ in Stage IV, and those in large firms working without or with co-workers face the separation rate at $r_{21}^S = (1-\rho)^2$ or $r_{22}^S = (1-\rho)^2 + \rho^2(1-\rho)^2$. It is obvious that $r_{21}^S < r_{22}^S < r_1^S$ for any $\rho \in (0,1)$. Therefore, we have the following result established.

Proposition 2. *The separation rate in a large firm is smaller than that in a small firm.*

Size and Wage Differential. In standard directed search models with multi-vacancy firms, it is well known that small firms always post higher wage in the unique equilibrium[4]. However, this contradicts the observations on the labor market;[5] it is the large firms that pay higher wages to workers. In our model, large firms have the opportunity to reallocate workers over jobs and partially overcome the mismatch between workers and jobs. This job rotation feature creates two simultaneous forces that drive the size-wage differential in different directions. The first effect lies in the increased expected productivity of large firms. When their expected productivity is higher, large firms may be able and willing to pay higher wages to their workers, which makes their job offers more attractive to workers. The second effect is due to the reduced job separation rate in large firms. Lower unemployment risk in large firms works together with the first effect to pull up the expected utility that large firms promise to their applicants, that is, $V_2 = (\Omega_{21}V_{21} + \Omega_{22}V_{22}) / (\Omega_{21} + \Omega_{22}) > V_1$. However, the smaller separation rate can potentially push wages down. Taking both effects into consideration, we claim that, when the mismatch risk is high compared to the extra cost of becoming a large firm, large firms can provide higher promised utility; and when the mismatch risk is even higher so that the first effect dominates, large firms pay higher wages.

Result 1. *Large firms offer lower wages than small firms if there is no mismatch, $\rho = 1$. For any $\rho \in (0,1)$, there exists a $\bar{c}(\rho) \in (\underline{c}(\rho), \rho]$ such that for any $C \in (\underline{c}(\rho), \bar{c}(\rho))$, $V_2 > V_1$. Furthermore, when ρ and C are small enough, there exist a set of (ρ, C) such that $w_2^* > w_1^*$.*

We provide a numerical illustration of this result due to difficult derivation of an analytical proof. In Figure 1, we illustrate how w_1/w_2 and V_1/V_2 depend on C and ρ. When $\rho = 1$, we replicate the result of a standard directed search model with multi-vacancy firms, simply because there is no risk of mismatch. In this case, large firms offer lower wages for any positive C. When ρ is small, it is possible to obtain the wage premium of large firms. The intuition is as follows. Smaller ρ implies a higher probability of mismatch and, consequently, a greater job rotation benefit and a higher wage premium; thus the wage premium is decreasing in ρ. There are four relevant regions. Region I corresponds to the case of $C \leq \underline{c}(\rho)$, which is not of interest. In region II, C is relatively high so becoming a large firm is costly, and ρ is large and the advantage of rotation is limited; thus, small firms provide more promising offers in the equilibrium, $V_1 > V_2$. In region III, (ρ, C) is moderate and the advantage of rotation raises large firms' expected productivity so that their offer become more attractive than those of small firms', and $V_2 > V_1$. However, since workers in large firms face smaller unemployment risk, when (ρ, C) belongs to this region, to provide higher

[4] See the discussion in Shi (2002) and Tan (2012).

[5] For example, Brown and Medoff (1989) and Oi and Idson (1999) point out that there exists a positive size-wage differential in the labor market.

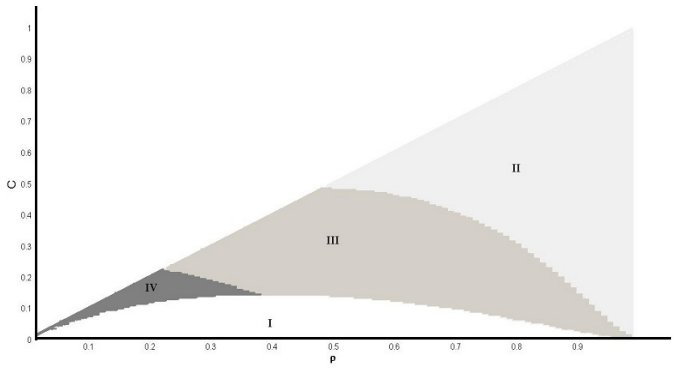

Figure 1: Decomposition of (ρ, C)-space. I: $C \leq \underline{c}(\rho)$, no co-existence of firms of two sizes. II $V_1 > V_2$ and $w_1 > w_2$. III $V_1 < V_2$ and $w_1 > w_2$. IV $V_1 < V_2$ and $w_1 < w_2$.

expected utility, large firms do not need to pay high wages, so $w_2^* < w_1^*$. In region IV, (ρ, C) is small enough, and the difference in unemployment risk is limited, hence $w_2^* > w_1^*$.

For standard directed search models to generate a positive correlation between firm size and wage, an exogenous productivity difference is required. In particular, Kaas and Kircher (2011) and Lester (2010) assume that firms randomly draw their productivity levels from a pre-determined distribution before they enter the labor market, and high productivity firms decide to be large and low productivity firms choose otherwise. If the ex ante distribution of productivity is dispersed enough, this technology difference can overcome the frictional effect of coordination failure and can generate a reasonable size-wage differential. In their models, large firm size and a wage premium are the consequence of high productivity. Our model suggests a somewhat reversed direction of such a relationship: even with ex ante homogeneity assumed, large firms may emerge, taking advantage of the opportunity of job rotation, which in turn induces high productivity and a wage premium.

Size and Vacancy Yield.[6] Let ν_k be the equilibrium vacancy yield of firms posting k vacancies, which is the probability of filling a position in these firms. In our benchmark model, we have $\nu_1 = \Phi_1(q_1^*)$, $2\nu_2(1-\nu_2) = \Phi_{21}(q_2^*)$, and $(\nu_2)^2 = \Phi_{22}(q_2^*)$. In other words, $\nu_2 = \Phi_{22}(q_2^*) + \Phi_{21}(q_2^*)/2$. Our simulation implies that the equilibrium vacancy yield is greater in large firms for any $\rho \in (0,1]$ and $C \in (\underline{c}(\rho), \rho)$. This is a typical result in directed search models, for example, Lester (2010), because wages play an allocative role in the workers' application decision. However, in comparison to a model without the opportunity of job rotation, $\rho = 1$, our model here predicts a greater disparity between the vacancy yields of firms with one vacancy and those with

[6] We thank an associate editor for encouraging us to investigate this issue in our framework.

multiple vacancies, i.e., the difference between ν_2 and ν_1 is amplified as ρ becomes smaller. This is inconsistent with the empirical relation between vacancy yield and firm size. We believe our implication on the relation between vacancy yield and firm size is inconsistent with the empirical evidence because of an important factor argued by Davis, Faberman, and Haltiwanger (2010), a heterogeneity in job recruiting standard across firms with different sizes, is missing in most directed search models including ours. In practice, firms are heterogeneous in terms of size before they make wage posting and hiring decisions, workers also have both unobservable and observable heterogeneities, and therefore, this two-sided ex ante heterogeneity may induce heterogeneous hiring standards across firms. Since our model is static and all vacancies and workers are assumed to be ex ante homogeneous, this preexisting heterogeneity cannot be captured. Introducing two-sided heterogeneity into a directed search model is not tractable. To avoid this intractability and to capture the idea of a heterogenous job recruiting standard, in the extension section, we consider the possibility that large firms have a different job recruiting standard from small firms, and show that a negative relation between firms size and vacancy yield can be generated.

3 Extensions and Discussions

Informative Interview. In our main model, we assume vacancies are ex ante homogeneous across firms. Now we extend our main model to investigate the possibility that large firms have a different job recruiting standard from small firms. Suppose a large firm, by paying an extra cost, can afford a more sophisticated human resources department and, therefore, can draw an informative but noisy signal about the match quality between potential employees and their positions. We introduce a heterogeneity of interview technology among firms of different sizes to capture the idea, proposed by Davis, Faberman, and Haltiwanger (2010), that large firms have higher job recruiting standards than small firms. To simplify the analysis, we focus on the following signal generating technology. If a worker is good at neither position, a bad signal is realized with probability $1-\delta$, where $\delta \in (0,1)$.[7] Hence, conditional on being matched with a large firm, the probability that a worker passes the interview is

$$\eta = 1-(1-\rho)^2(1-\delta) < 1,$$

[7] We assume that large firms cannot acquire workers' match quality information position by position, which implies that firms will randomly allocate a qualified employee over positions. Since our interest is not in studying the effect of interviews on firms' job assignment to new workers, we believe this assumption does not lose any generality.

which is close to zero when $\rho, \delta \to 0$. If a worker passes the interview, his posterior of being good at each position is given by

$$\bar{\rho} = \frac{\rho}{\rho + (1-\rho)(\delta + \rho(1-\delta))} \in (\rho, 1).$$

Similar analysis yields the equilibrium wages w_1^* in small firms and $w_2^*(\bar{\rho})$[8] in large ones, and vacancy yields in small and large firms are given by

$$\begin{aligned} \upsilon_1 &= \Phi_1, \\ \upsilon_2 &= \eta(\Phi_{22} + \Phi_{21}/2). \end{aligned}$$

When δ is small (the signal is precise), large firms are very selective, and therefore, the vacancy yield in large firms can be smaller than that in small firms. Figure 2 shows some numerical examples. For small ρ and C, when δ is small, $\upsilon_2 < \upsilon_1$, and $w_1^* < w_2^*$. Since a match is good with probability $\bar{\rho} > \rho$ in large firms, both the productivity difference and the separation rate difference between large firms and small firms are amplified. On the other hand, the interview effect will decrease the possibility of job rotation. However, in our model, since the job rotation rate in small firms is always zero, our prediction on the relation between job rotation rate and firm size still holds.

We assume that large firms can only draw signal from matched workers. What if they could draw signals from all applicants? The result will not change qualitatively. The reason is as follows. In equilibrium, a large firm faces finitely many applicants. Even though there are more than 2 applicants, the probability that the firm cannot hire enough workers is always positive if $\delta \in (0, 1)$. When both ρ and δ are small, the vacancy yield can be arbitrarily small. Hence, our prediction on the relation between vacancy yield and firm size still holds.

Contract Forms. In our model, we assume firms can only commit to a time-invarying wage. The time-invarying wage assumption is without loss of generality because (1) the goal of this exercise is to explain the cross-sectional empirical link between firm sizes and wages, instead of comparing the dynamics of wages paid by firms of various sizes, and (2) the optimal contract in this restricted domain is also an optimal contract in the contract space where wage can be time-varying.

Time Consuming Learning. In our model, firms immediately learn workers' match quality in all positions. The only cost of learning is the first period wage paid by firms. In practice, learning is time consuming, and the time for learning is increasing in the number of objects. As a result, it seems more reasonable to assume that a large firm needs more time to learn its employees' match

[8] The wages in large firms, $w_2^*(\bar{\rho})$ is obtained by replacing ρ by $\bar{\rho}$ in the expression of w_2^* in Proposition 1.

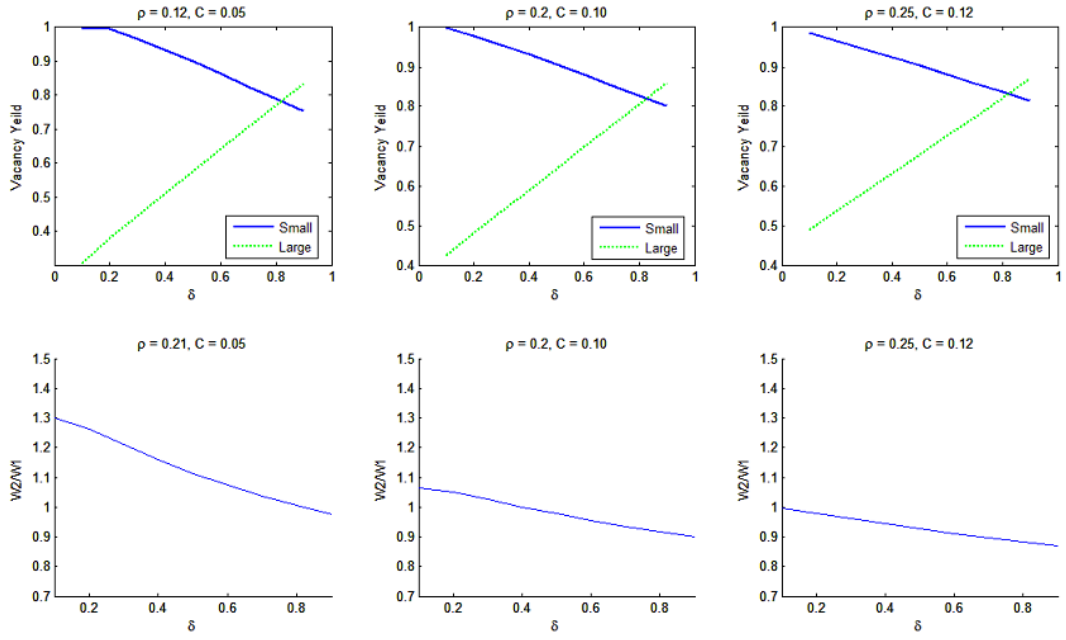

Figure 2: Vacancy Yield and Wage Ratio at Different ρ, C, and δ.

quality in both vacancies than a small firm, which will weaken the advantage of job rotation in large firms. However, when ρ is small, the job rotation effect is strong enough to overcome this heterogenous learning effect. Consequently, our results still hold.

4 Conclusion

We modified a standard directed search model to explain the size-wage differential observed in the labor market, highlighting the effect of the practice job rotation. However, in contrast to the standard directed search model with multi-vacancy firms, our modified model can generate a positive correlation between firm size and wage without introducing any ex ante exogenous productivity heterogeneity or imposing any non-concave production function assumptions. We assume ex ante homogeneous firms and workers and initially unknown match quality that determines labor productivity. Firm sizes are endogenously determined. By paying an extra cost, a large firm benefits from the opportunity of rotating workers so as to partially overcome the loss of mismatch. As a result, in the unique symmetric equilibrium, large firms have higher labor productivity and wages, and a lower separation rate. In future research, we would like to study the interaction between internal labor markets and external labor markets in a fully dynamic model.

Appendix

Proof of Proposition 1. By (5), we have

$$w_1 = \frac{q_1 U^*}{(1+\rho)\left(1-e^{-q_1}\right)} \text{ for } q_1 > 0,$$

and w_1 is not well-defined when $q_1 = 0$. So there is a one-to-one and negative relation between w_1 and q_1 when $q_1 > 0$. The maximization problem (7) is therefore equivalent to the following,

$$\pi_1^* = \max_{q_1>0} \left\{\rho \Phi_1(q_1) - q_1 U^*\right\} \tag{10}$$

Similarly, by (6), we have

$$w_2 = U^* \left[e^{-q_2}\left(F_{21}(\rho)+1\right) + \frac{1}{q_2}\left(1-e^{-q_2}-q_2 e^{-q_2}\right)\left(F_{22}(\rho)+2\right)\right]^{-1} \text{ for } q_2 > 0$$

So the problem of (8) can also be re-written so that q_2 is the control variable,

$$\pi_2^* = \max_{q_2>0} \left\{\Phi_{21}(q_2) F_{21}(\rho) + \Phi_{22}(q_2) F_{22}(\rho) - q_2 U^* - C\right\}. \tag{11}$$

The first-order conditions to (10) and (11) are

$$U^* \geq \rho e^{-q_1}, \tag{12}$$

$$U^* \geq e^{-q_2} F_{21}(\rho) + q_2 e^{-q_2}\left(F_{22}(\rho) - F_{21}(\rho)\right), \tag{13}$$

where the equalities hold when $q_1, q_2 > 0$. We focus on the situation where both small and large firms coexist, so we combine (12) and (13) at equalities and obtain the necessary condition for interior solutions (q_1^*, q_2^*),

$$q_1^* = q_2^* - \ln\left(\frac{1}{\rho}\left[F_{21}(\rho) + q_2^*\left(F_{22}(\rho) - F_{21}(\rho)\right)\right]\right), \text{ and } q_1^* > 0. \tag{14}$$

This also implies that $q_2^* > q_1^*$. Moreover, the necessary condition for coexistence requires $\pi^* = \pi_1^* = \pi_2^*$, which implies

$$\begin{aligned} & \rho\left(1-e^{-q_1^*}-q_1^* e^{-q_1^*}\right) \\ = \; & \left(1-e^{-q_2^*}-q_2^* e^{-q_2^*}\right) F_{22}(\rho) - \left(q_2^*\right)^2 e^{-q_2^*}\left(F_{22}(\rho) - F_{21}(\rho)\right) - C. \end{aligned} \tag{15}$$

These two equations give the unique solution (q_1^*, q_2^*) when it exists. Then (w_1^*, w_2^*) can be expressed as functions of (q_1^*, q_2^*) by using (5), (6), (12) and (13). *Q.E.D.*

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