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"Premuneration Values and Investments in Matching Markets"

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Premuneration Values and Investments in Matching Markets[*]

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Abstract

We examine markets in which agents make investments and then match into pairs, creating surpluses that depend on their investments and that can be split between the matched agents. In general, each of the matched agents would "own"part of the surplus in the absence of interagent transfers. Most of the work in the large bargaining-and-matching literature ignores this initial ownership of the surplus. We show that when investments are not observable to potential partners, initial ownership affects the efficiency of equilibrium investments and affects the agents' payoffs. In particular, it is possible that reallocating initial ownership could increase welfare on both sides of the match.

1 Introduction

1.1 Investment and Matching Markets

This paper considers markets in which agents can match to produce a surplus, which they can then divide between them. For example, we will often speak in terms of a large set of laboratories of different sophistication and an analogous set of researchers. Each researcher can combine with a laboratory to create valuable patents, giving rise to a surplus to be split between the laboratory and researcher. Such markets have been addressed by a large matching-and-bargaining literature, with a typical analysis identifying an equilibrium matching of researchers to laboratories and specifying how the value of the patents generated by the matched pair is allocated.

We are interested in cases in which the researchers must invest in human capital before the market that matches researchers to laboratories opens. Perhaps with the hold-up literature in mind, we might then expect investments to be inefficient, since researchers will equate the marginal cost of their investment to the marginal private benefit, rather than the potentially higher marginal social benefit. However, sufficient competition can ensure that private and social incentives are aligned. In particular, Cole, Mailath, and Postlewaite (2001) show quite generally that equilibrium investments will be efficient when there are many agents on both sides of the market and there is no uncertainty about the parties' investment at the time of matching. When a researcher increases her investment, she becomes more valuable to all laboratories, and competition among them ensures that the researcher captures the increase in the value of patents resulting from that investment.

Depending on the relevant legal environment, the patents that arise out of a laboratory/researcher match may belong to either the laboratory or the researcher. Analyses of investment and matching markets typically pay no attention to the specification of the initial ownership of the match surplus. When there is no uncertainty about matching-relevant characteristics, this omission is innocuous. It makes no difference whether the patents belong to the laboratory, which then hires a researcher, or the patents belong to the researcher, who may then buy (or rent) a laboratory. In either case, a monetary payment from the party that owns the patents to the other party delivers the equilibrium division of the surplus. For any change in the distribution of patent ownership, there is an offsetting change in the equilibrium monetary transfer between agents preserving the equilibrium welfare distribution and investments.

The point of this paper is that things change dramatically with the incorporation of asymmetric information. If researchers' match-relevant characteristics are unobservable, initial ownership can play a central role in the efficiency of investments and in the final welfare distribution.[1] Laboratories now cannot observe a researcher's investment, precluding the enhanced competition that facilitates the researcher's capture of the returns on this investment and potentially leading to inefficient investments. More importantly, increasing the share of the patents owned by the researcher then provides incentives to invest more efficiently, giving rise to a link between initial ownership and investments.

Increasing the share of the surplus owned by the researcher does not necessarily harm laboratories. There is competition among researchers for laboratories, and researchers who own more of the surplus find all laboratories more valuable. This intensifies the competition for laboratories, leading to higher market prices for laboratories. We show below that as the share of the surplus owned by researchers increases from zero, laboratories' equilibrium welfare can increase.

Researchers' welfare will generally increase as well when the share of the surplus they own increases. How much it increases depends on how fierce the competition is among researchers for laboratories. If researchers are identical, competition will drive market prices for laboratories to the point that in equilibrium, all surplus goes to laboratories, and in this case the share of the surplus owned by researchers is irrelevant. But when researchers differ in their investment costs, they will differ at the matching stage. Heterogeneity among researchers will attenuate researchers' competition for laboratories and researchers will accordingly get positive surplus in equilibrium. Their equilibrium welfare increases in the share of the surplus they own and increases in the heterogeneity of their investment costs.

1.2   Premuneration Values

The above discussion of researcher/laboratory matches focussed on the value of patents that stem from a match. In essence, this is a question of the property rights to the patents. It is relatively straightforward to think of

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[1]This is reminiscent of the Coase theorem (Coase, 1960): in the absence of bargaining frictions (such as asymmetric information), bargaining will result in an efficient allocation irrespective of the original allocation of property rights. On other hand, in the presence of asymmetric information, the possibility of reaching an efficient agreement depends on the the original allocation of property rights (see, for example, Cramton, Gibbons, and Klemperer (1987)). However, the similarity is superficial, since the Coase theorem ignores investments that may be taken before bargaining (Grossman and Hart, 1986).

the patents as being owned by either the laboratory or the researcher, and to think of different legal structures assigning this ownership differently.

In general, the surplus generated by a match will be a composite of many different items. It is less familiar to talk about the ownership or property rights to some of these components, and clear that in some cases we cannot reasonably talk about reallocating this ownership. For our leading example of laboratories and researchers, while much of the surplus arises out of the value of the resulting patents, there are other significant sources of value. There is the value to the researcher of human capital she accumulates at the laboratory, as well as the value from contacts she makes at the laboratory. She may derive utility from laboratory parties and social opportunities. The laboratory owns the prestige of employing a noted researcher, and also accumulates organizational capital that will be of use in other research endeavors. The laboratory will receive value from the researcher's existing contacts, perhaps finding it easier to hire additional researchers. Moreover, the ownership of these components of the value is difficult to alter.

Rather than itemize all the elements that comprise the surplus in the match between the laboratory and researcher, we take as the primitive the aggregate match value to each of the agents in the absence of any transfers. We call these values *premuneration values* (from *pre* plus the Latin *munerare*, to give or pay). The total surplus in a match is then simply the sum of the matched parties' premuneration values. The premuneration values determine the division of the surplus *in the absence of transfers*. In equilibrium, of course, there typically will be transfers. What is central to our problem is that any transfers that reallocate surplus are determined *after* investments have been made.

When investments are observable, premuneration values are irrelevant since the transfers can effectively sterilize their effect on investment incentives and on payoffs. The central message of this paper is that when investments are *unobservable*, premuneration values do matter because they will affect equilibrium investments. Continuing with our laboratory/researcher interpretation, we find:

- Premuneration values affect investment incentives. Investments will be inefficient unless premuneration values assign all of the surplus to researchers (more generally, to the side whose investments cannot be observed).

- Laboratories' equilibrium payoffs increase as researcher premuneration values increase if the latter are small, and then decrease. Researchers'

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payoffs increase as do their premuneration values. In particular, both sides can gain by having premuneration values allocate more of the surplus to researchers.

- When laboratories can become informed (at some cost), premuneration values determine which laboratories choose to become informed and the payoffs of all laboratories (both informed and uninformed) and researchers. Laboratories may gain by having premuneration values allocate more of the surplus to researchers. While some researchers gain from such a reallocation, the presence of some informed laboratories means that some researchers can lose from such a reallocation.

Premuneration values will matter whenever there are unobserved investments. We expect investments in human capital to be especially difficult to verify, bringing any market for skilled labor within the purview of the analysis.

The finding that premuneration values matter would be relatively innocuous if we could simply redesign them (perhaps via legislation stipulating property rights) to achieve efficiency, but such reallocation is often infeasible. For example, premuneration values often include future returns, requiring future costly actions and hence moral hazard problems that preclude reallocation.

As another example, consider a match between a student and a university. While at the university, the student will acquire knowledge and skills that will lead to higher lifetime earnings and a greater satisfaction in life after school. She may also make contacts that will be important in her career, and she may be a regular at campus parties and generally enjoy the social life of the university. Each of these increases the student's value of the match, and consequently the surplus in the match. On the other side, the university may derive value from touting the student's background and her ability to play the saxophone as additions to its diverse and artistically rich community, as well as from claiming her as a graduate when she achieves future fame and fortune. The university's value of these items also contributes to the surplus of the match. Each side owns some of these components, in the sense that the value of that component accrues to them. Some components might be owned by either side, depending on circumstances, but others are inextricably linked to a particular side. We might be able to reallocate the ownership of the student's future income stream, perhaps by financing her education with income-contingent loans, but there are obvious limits in the possible shifting. There is no obvious way to reallocate her utility from partying.

1.3 Related Literature

Our focus on the relationship between the incentives for efficient investments and subsequent bargaining is shared by a number of other papers.[2] Acemoglu and Shimer (1999) analyze a worker-firm model in which firms (only) make ex ante investments. If wages are determined by post-match bargaining, then the resulting effective power gives rise to a standard hold-up problem inducing firms to underinvest. The hold-up problem disappears if workers have no bargaining power, but then there is excess entry on the part of firms. Acemoglu and Shimer show that efficient outcomes can be achieved if the bargaining process is replaced by wage posting on the part of firms, followed by competitive search. de Meza and Lockwood (2009) examine an investment and matching model that gives rise to *excess* investment. Their overinvestment possibility rests on a discrete set of investment choices and the presence of bargaining power in a noncompetitive post-investment stage.

In contrast, the competitive post-investment markets of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) lead to efficient two-sided investments.

Moving from complete-information to incomplete-information matching models typically gives rise to issues of either screening, as considered here, or signaling. Cole, Mailath, and Postlewaite (1995), Hopkins (2011), Hoppe, Moldovanu, and Sela (2009), and Rege (2008) analyze models incorporating signaling into matching models with investments.

To keep the intuition in the forefront, we refer to the match partners as researchers and laboratories, though the analysis applies to any matching context. In order to focus on the investment behavior when investments are not observable, we assume that laboratories' investments are fixed, and we assume a particular investment cost function. Mailath, Postlewaite, and Samuelson (2012) analyze a similar problem in which both sides of a match make investments and only mild assumptions on cost functions are assumed. As in this paper, premuneration values are central in the analysis. Mailath, Postlewaite, and Samuelson (2012) establish conditions for the existence and efficiency of equilibrium. We view those results as the counterparts of existence and the first welfare theorem for competitive equilibrium. However, as in many general equilibrium settings, the generality of the setting pre-

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[2]Early indications that frictionless, competitive search might create investment incentives appear in Hosios (1990), Moen (1997) and Shapley (2001). Eeckhout and Kircher (2010) provide an extension to asymmetric information, while Masters (2009) examines a model with two-sided investments.

cludes sharp comparative static results. This paper adds enough structure to investigate the implications of premuneration values.

2 The Model

2.1 The Market

There is a unit measure of researchers whose types (names) are indexed by ρ and are distributed uniformly on $[0, 1]$, and a unit measure of laboratories whose types are indexed by λ and distributed uniformly on $[0, 1]$. For ease of reference, researchers are female and laboratories male.

At the first stage, each researcher chooses an attribute r. Each laboratory is characterized by an attribute ℓ, where for convenience we take the type of laboratory λ to be fixed at $\ell = \lambda$. Following the attribute choices, researchers and laboratories match, with each matched pair generating a surplus. Attributes are costly, but enhance the values generated in the second stage. The second-stage values depend only on the attributes of the researcher and laboratory, r and ℓ, and not on their underlying types.

The cost of attribute $r \in \mathbb{R}_+$ to researcher ρ is given by

$$c(r, \rho) = \frac{r^{2+k}}{(2+k)\rho^k}, \qquad k \in \mathbb{R}_+.$$

When $k = 0$, researchers are homogeneous, in the sense that all have the same cost. When $k > 0$, higher ρ researchers have a lower cost of acquiring any level of the attribute. We will see that as k increases, and hence researchers become more heterogeneous, their investments differ more, and this will lead to less competition among them for laboratories.

We choose this cost function not only for tractability, but because with it the efficient allocation of resources is independent of k. We will find that the effects of premuneration values vary with k, and (as we noted) that premuneration values have an effect on the efficiency of investments. This cost function allows us to study these effects while holding fixed the efficient allocation.[3]

Before any transfers are made, premuneration values identify the ownership of the values generated by the match of a researcher and laboratory.

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[3]The $2 + k$ term in the denominator is introduced to simplify the algebra. Appendix A summarizes the analysis for a more general cost function that allows for the possibility that costs do not become negligible as k gets large. Our results continue to hold for the more general cost function.

If the prevailing law is that patents arising from the researcher's discoveries are the property of the laboratory, that portion of the value belongs to the laboratory. Presumably the laboratory then must pay a researcher to induce a match. Alternatively, it may be that given the prevailing law, patents are the property of the researcher, hence their value is included in the researcher's premuneration value. In this case we would expect that researchers would have to rent laboratories to carry out their work. In general, the parties' premuneration values will be an amalgam of many components, including some of which are impervious to legal meddling.

We assume that a match between a researcher of attribute r and a laboratory of attribute ℓ generates a researcher premuneration value of

$$\theta \ell r$$

and a laboratory premuneration value of

$$(1 - \theta)\ell r.$$

The total surplus from the match is

$$v(\ell, r) = \ell r.$$

The parameter θ describes the researcher's premuneration value share of the surplus, while $(1 - \theta)$ describes the laboratory's share . We note that the values are increasing and supermodular—an increase in an agent's characteristic has a larger effect on premuneration values and surplus the larger the partner's characteristic.

2.2   Equilibrium

Matching takes place in a competitive market. Laboratories' attributes are observable and priced, with $p(\ell)$ denoting the price of a laboratory with attribute $\ell \in [0, 1]$. Researchers' attributes are not observable to laboratories, hence the price of laboratory with attribute ℓ, $p(\ell)$, is the same to all researchers. Given a price function p, each researcher optimally chooses her attribute and the laboratory with whom she wishes to match. That is, researcher ρ solves

$$\max_{\ell, r} \ \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2 + k)\rho^k}. \tag{1}$$

We denote by $r_R : [0, 1] \to \mathbb{R}_+$ the function describing the attributes chosen by researchers and we let $\ell_R : [0, 1] \to [0, 1]$ be the function describing the laboratories chosen by researchers.

The function ℓ_R is *market-clearing* if it is one-to-one, onto, and every set of researchers $\mathcal{R}$, is mapped to a set of equal size of laboratories.[4]

Definition 1 *A price function p and researcher choices (ℓ_R, r_R) constitute a* matching equilibrium *if,*

1. *for every $\rho \in [0,1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves the researcher-optimization problem (1),*

2. *every researcher and laboratory earns nonnegative payoffs, and*

3. *ℓ_R is market-clearing.*

The second property of equilibrium is an *individual rationality* requirement, ensuring that all agents prefer participation to not participating.

We first identify three useful properties of an equilibrium. The first is a direct implication of market clearing:

Lemma 1 *Every equilibrium price function p is strictly increasing and continuous.*

Proof. En route to a contradiction, suppose p is not strictly increasing. Then there are two laboratories $\ell' < \ell$ satisfying $p(\ell') \geq p(\ell)$. But then no researcher will choose laboratory ℓ' — why pay just as much or more for an inferior laboratory? Hence, ℓ_R cannot be market clearing. Continuity similarly follows from the observation that if the function p takes an upward jump at ℓ', then there will be an interval of laboratories $(\ell', \ell' + \varepsilon)$ that will be unchosen by researchers, again contradicting our assumption that ℓ_R is market clearing. ∎

The researchers' cost function exhibits a single-crossing condition that gives the following:

Lemma 2 *The equilibrium researcher attribute-choice function r_R is strictly increasing.*

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[4]Formally, if μ is Lebesgue measure and $\mathcal{R}$ is a measurable set of researcher tyes, then $\mu(\mathcal{R}) = \mu\{\ell | \ell = \ell_R(\rho) \text{ for some } \rho \in \mathcal{R}\}$. Our assumption that laboratory attributes are exogenously and uniformly distributed on an interval (and our focus on a parametrized model) allows us to finesse various technical issues that arise with a continuum of agents in two-sided investment models; see Mailath, Postlewaite, and Samuelson (2012, Section 3.2) for a discussion.

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Proof. We first argue that r_R is weakly increasing. Suppose not, so that there exist researchers $\hat{\rho} > \rho$ such that $\hat{r} = r_R(\hat{\rho}) < r_R(\rho) = r$. Since researchers are optimizing in their attribute and laboratory choices,

$$\theta \ell_R(\rho) r - p(\ell_R(\rho)) - \frac{r^{2+k}}{(2+k)\rho^k} \geq \theta \ell_R(\hat{\rho}) \hat{r} - p(\ell_R(\hat{\rho})) - \frac{\hat{r}^{2+k}}{(2+k)\rho^k}$$

and

$$\theta \ell_R(\hat{\rho}) \hat{r} - p(\ell_R(\hat{\rho})) - \frac{\hat{r}^{2+k}}{(2+k)\hat{\rho}^k} \geq \theta \ell_R(\rho) r - p(\ell_R(\rho)) - \frac{r^{2+k}}{(2+k)\hat{\rho}^k},$$

which can be added to give

$$\frac{r^{2+k}}{(2+k)\rho^k} + \frac{\hat{r}^{2+k}}{(2+k)\hat{\rho}^k} \leq \frac{\hat{r}^{2+k}}{(2+k)\rho^k} + \frac{r^{2+k}}{(2+k)\hat{\rho}^k},$$

a contradiction.

We now argue that r_R is strictly increasing. Fix $\hat{\rho} > \rho$. Since ℓ_R is market clearing, we can assume $\ell_R(\hat{\rho}) = \hat{\ell} > 0$. If r_R is not strictly increasing, $\hat{r} = r_R(\hat{\rho}) = r = r_R(\rho)$, and so $\hat{\ell}$ is an optimal choice for both $\hat{\rho}$ and ρ at r. But this implies that ρ's choice of $r = \hat{r}$ must be optimal given $\hat{\ell}$. But this is impossible (since the marginal cost of attributes is strictly decreasing in ρ). $\blacksquare$

The supermodularity of the surplus function ensures that matching is assortative:

Lemma 3 *The equilibrium researcher laboratory-choice function ℓ_R is given by*

$$\ell_R(\rho) = \rho.$$

Proof. We first argue that in equilibrium, the researcher laboratory-choice function is strictly increasing. Let $\hat{\rho} > \rho$ and hence, from Lemma 2, $\hat{r} = r_R(\hat{\rho}) > r_R(\rho) = r$. We need to show that $\hat{\ell} = \ell_R(\hat{\rho}) > \ell_R(\rho) = \ell$. Suppose this is not the case. Then since researchers with attribute r and $\hat{r}$ are optimizing in their choice of laboratories, we have

$$\begin{aligned}
\theta \ell r - p(\ell) &\geq \theta \hat{\ell} r - p(\hat{\ell}) \\
\text{and} \qquad \theta \hat{\ell} \hat{r} - p(\hat{\ell}) &\geq \theta \ell \hat{r} - p(\ell),
\end{aligned}$$

which can be added to give

$$\ell r + \hat{\ell} \hat{r} \geq \hat{\ell} r + \ell \hat{r},$$

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which is a contradiction.

The conclusion of the lemma then follows from equilibrium ℓ_R being a strictly increasing and measure-preserving map from $[0,1]$ onto $[0,1]$: Fixing $\rho \in [0,1]$, and recalling footnote 4, we have $\mu\{\ell|\ell = \ell_R(\hat{\rho}) \text{ for } \hat{\rho} \in [0,\rho]\} = \mu([0,\ell_R(\rho)]) = \ell_R(\rho)$, and so $\rho = \mu([0,\rho]) = \ell_R(\rho)$. ■

3 Investments and Payoffs

3.1 Efficient Investments

We next show that the efficient researcher attribute choices have a particularly simple form.

The strict supermodularity of the surplus function ℓr implies that, for any strictly increasing researcher attribute choice function, total surplus is maximized under assortative matching. In addition, the cost function for researchers is decreasing in researcher index ρ, so for any researcher attribute distribution, the minimum cost of obtaining that distribution is for the attribute choice function r_R to be (weakly) increasing. Thus, total net surplus is maximized when the matching on indices λ and ρ is positively assortative: laboratory λ will be matched with researcher $\rho = \lambda$.

Total net surplus is then maximized when the net surplus for each such matched pair is maximized. For the ρ-matched pair of laboratory and researcher, the surplus-maximization problem is (since laboratory $\lambda = \rho$ has attribute $\ell = \rho$)

$$\max_r \; \rho r - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{2}$$

The first-order condition is

$$\rho = \frac{r^{1+k}}{\rho^k},$$

immediately implying

$$r = \rho. \tag{3}$$

Hence, efficiency requires $r_R(\rho) = \rho$ and $\ell_R(\rho) = \rho$. Notice that the efficient allocation does not depend on k, the competitiveness of the researchers.

3.2 Market Equilibrium

We turn to the structure of the market equilibrium. First, suppose that the equilibrium price of laboratories is differentiable, a supposition that will be

validated by the equilibrium we construct.[5] Researcher ρ's problem is to choose ℓ and r to maximize

$$\theta\ell r - p(\ell) - c(r,\rho) \;=\; \theta\ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}.$$

The first order conditions are

$$\theta\ell = \frac{r^{1+k}}{\rho^k} \qquad (4)$$

and

$$\theta r = p'(\ell). \qquad (5)$$

In equilibrium, researcher ρ is matched with laboratory $\ell = \rho$, hence from (4) we have that in equilibrium

$$r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}. \qquad (6)$$

For all $\theta \in (0,1)$, $\theta^{\frac{1}{1+k}} \in (0,1)$, and hence $r_R(\rho) < \rho$; for $\theta = 1$, $r_R(\rho) = \rho$. This immediately gives:

Proposition 1 *The researcher investment function given by (6) is a matching equilibrium investment function. Suppose $\theta < 1$, so that the laboratory premuneration value share is positive. In equilibrium, researchers invest less than the efficient level.*

For any given researcher ρ, $r_R(\rho)$ is increasing in both k and θ. The intuition is that as θ increases, the researcher has a larger share of the surplus, and hence has an increased incentive to invest; when k increases, less of a researcher's benefit is competed away, again giving researchers greater incentive to invest.

Combining the two first order conditions (5) and (6), it is straightforward to derive the price function. We have

$$p'(\ell) = \ell \cdot \theta \cdot \theta^{\frac{1}{1+k}}$$

and hence

$$p(\ell) = \tfrac{1}{2}\ell^2 \cdot \theta^{\frac{2+k}{1+k}} \qquad (7)$$

(the constant of integration is set so that $p(0) = 0$, as required by the individual-rationality requirement that payoffs be nonnegative).

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[5]A standard revealed preference argument shows that in fact every equilibrium price function is differentiable, and so the equilibrium is unique.

3.3 Laboratory Payoffs

Given the equilibrium choices, laboratory λ's payoff given θ and k is

$$
\begin{aligned}
u_L(\theta, k, \lambda) &\equiv (1-\theta)\ell r + p(\ell) \\
&= (1-\theta)\lambda(\theta^{\frac{1}{1+k}}\lambda) + p(\ell) \\
&= (1-\theta)\lambda^2\theta^{\frac{1}{1+k}} + \tfrac{1}{2}\lambda^2\theta^{\frac{2+k}{1+k}} \\
&= \tfrac{1}{2}\theta^{\frac{1}{1+k}}(2-\theta)\lambda^2.
\end{aligned}
\tag{8}
$$

We are interested in identifying conditions under which the laboratory's payoff increases when the researcher's share of the surplus, θ, increases. From (8), we see that the laboratory's payoff is increasing its share of the surplus when $\frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) < 0$. This derivative is given by

$$
\begin{aligned}
\frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) &= \tfrac{1}{1+k}\theta^{\frac{-k}{1+k}}(2-\theta) - \theta^{\frac{1}{1+k}} \\
&= \theta^{\frac{-k}{1+k}}\left[\tfrac{1}{1+k}(2-\theta) - \theta\right].
\end{aligned}
$$

Thus the sign of $du_L/d\theta$ is the same as the sign of $\frac{1}{1+k}(2-\theta) - \theta$, which is the same as that of $2 - (2+k)\theta$.

Figure 1 shows the region in which laboratories' payoffs increase as the researchers' premuneration value increases: (θ, k) combinations that are below and to the left of the curved line are situations in which the laboratories' payoff increases when the researchers' premuneration value increase. Above the line, laboratories' payoff decrease as researchers' premuneration value increases. Hence, the line represents the optimal premuneration values from the laboratory's perspective. In summary:

Proposition 2 *Laboratories' equilibrium payoffs are first increasing and then decreasing in θ (i.e., in the researchers' premuneration value share). The smaller is k, the larger is the value of θ that maximizes the laboratories' payoff.*

For $k = 0$, the laboratory's payoff is increasing for all θ, that is, laboratories' payoffs are maximized when premuneration values assign all the surplus to researchers. It may seem at first strange that laboratories prefer that researchers own all the surplus, but upon reflection this is not surprising. When $k = 0$, researchers are identical and the competition for laboratories is essentially as in Bertrand, with researchers bidding away all rents in the competition for higher attribute laboratories. Since laboratories
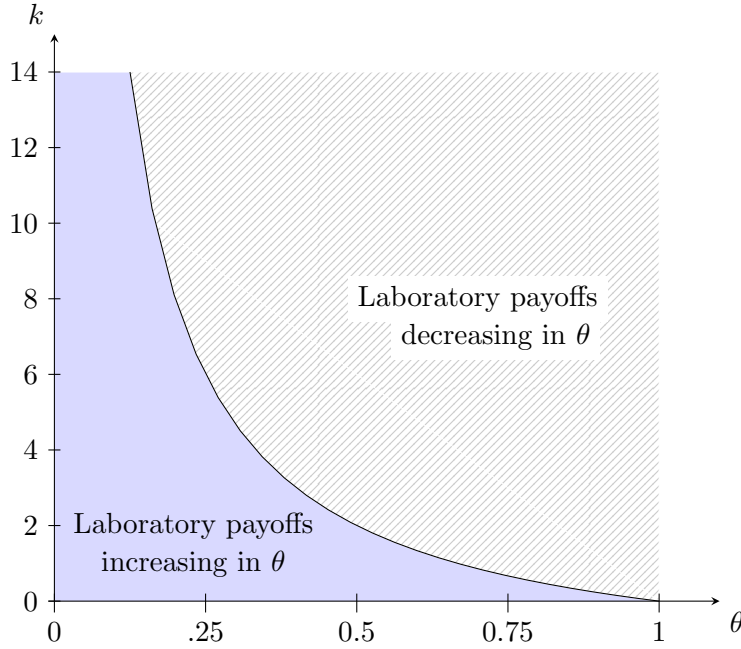
Figure 1: Parameter regions for which laboratory payoffs are increasing or decreasing in laboratory premuneration values.

will ultimately capture all the surplus through market competition, they do best when total surplus is maximized, which is when $\theta = 1$.

For positive but small k, the laboratories' payoffs are maximized with θ near, but less than, 1. When $\theta < 1$, researchers' attribute choices will be less than the attribute choices that maximize total net surplus. This is nevertheless optimal for laboratories since they will not capture the entire surplus in the market given that competition among researchers is imperfect when $k > 0$. As k increases, competition among researchers decreases and the researcher share of the surplus that maximizes laboratory payoff decreases, approaching zero as k gets large.[6]

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[6]We note that as $k \to \infty$, $r(\rho) \to \rho$ and the gross surplus for the match with $\rho = \lambda$ is ρ^2.

3.4 Researcher Payoffs

The researcher's payoff can be calculated as the total net surplus minus the laboratory's payoff. The total net surplus for a matched pair $\rho = \lambda$ is

$$\theta^{\frac{1}{1+k}}\rho^2 - \frac{(\theta^{\frac{1}{1+k}}\rho)^{2+k}}{(2+k)\rho^k} = \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{(2+k)}\theta\right].$$

From (8), the laboratory's payoff is $\rho^2\theta^{\frac{1}{1+k}}(1-\tfrac{1}{2}\theta)$, so the researcher's payoff is

$$
\begin{aligned}
u_R(\theta, k, \rho) &\equiv \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{2+k}\theta\right] - \rho^2\theta^{\frac{1}{1+k}}\left(1 - \tfrac{1}{2}\theta\right) \\
&= \theta^{\frac{1}{1+k}}\rho^2\left[1 - \tfrac{\theta}{2+k} - 1 + \tfrac{\theta}{2}\right] \\
&= \frac{k\theta^{\frac{2+k}{1+k}}}{2(2+k)}\rho^2.
\end{aligned}
\tag{9}
$$

Proposition 3 *Researcher's equilibrium payoffs increase in θ (i.e., as researchers' premuneration value share increases).*

Thus, both researchers' and laboratories' payoffs increase, as the researcher's premuneration value increases, in the solid shaded region in Figure 1.

4 Endogenizing Information

We now allow the laboratories to, at some cost, learn the attributes of researchers. Typically, we expect it would be costly for the uninformed party to learn the attribute of a potential partner. Consider, for example, the matching of students to colleges. Estimates from 11 highly selective liberal arts colleges indicate that they spent about $3,000 on admissions (i.e., ascertaining students' attributes) per matriculating student in 2004.[7] The cost for simply identifying whether a foreign high school diploma comes from a legitimate high school is $100.[8] We suppose that, by incurring a cost $\kappa > 0$, any given laboratory can acquire the ability to observe the attribute of each researcher, and that laboratories choose whether to become informed *after* researchers' investments have been made.[9]

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[7] Memorandum, Office of Institutional Research and Analysis, University of Pennsylvania, July 2004. We thank Barnie Lentz for his help with these data.

[8] "Vetting Those Foreign College Applications," *New York Times*, September 29, 2004, page A21.

[9] This implies that a laboratory cannot induce a change in researcher investment behavior by deciding to become informed.

If κ is too large, the gain in efficiency would not be sufficient to warrant a laboratory paying to become informed. On the other hand, for κ small, it is generally not an equilibrium for all laboratories to remain uninformed. To illustrate this, set $k = 1$ and suppose all laboratories are uninformed and that researchers choose attributes according to (6). If a laboratory deviates by becoming informed, it then can target any available researcher attribute, i.e., any attribute in the set $[0, r_R(1)] = [0, \theta^{1/2}]$. Suppose a laboratory $\lambda < \theta^{\frac{1}{2}}$ with (by assumption) $\ell = \lambda$ becomes informed and then offers some price p to the researcher with attribute $r = \ell$, i.e., to a researcher of type $\rho = \lambda\theta^{-\frac{1}{2}}$.[10] Since the price is simply a transfer between the two agents, such an offer is a profitable deviation if and only if the net surplus generated by the resulting match, $\ell r - \kappa$, exceeds (from (8)-(9))

$$u_L(\theta, 1, \lambda) + u_R(\theta, 1, \lambda\theta^{-\frac{1}{2}}) = \left[\frac{1}{2}\theta^{\frac{1}{2}}(2 - \theta) + \frac{1}{6}\theta^{\frac{3}{2}}\theta^{-1}\right]\lambda^2 \equiv g(\theta)\lambda^2. \quad (10)$$

A straightforward calculation verifies the inequality $g(\theta) < 1$ and hence this deviation is profitable for small κ.

Thus for $\kappa > 0$ but not too large, some laboratories will choose to become informed. However, it is clear that not *all* laboratories will choose to become informed, since laboratories with types near 0 cannot under any circumstance generate sufficient surplus to cover the cost κ.

A natural hypothesis is that for positive but not too large κ, there will be a *hybrid* equilibrium characterized by a threshold $\widetilde{\lambda}$ with laboratories $\lambda > \widetilde{\lambda}$ incurring the cost to become informed and laboratories with $\lambda < \widetilde{\lambda}$ not incurring the cost.

4.1 Informed Laboratories

The first step in investigating a hybrid equilibrium is to characterize the behavior and payoffs for the laboratories that are informed and the researchers with whom they match. We are interested in the case in which laboratories with indices in the interval $[\widetilde{\lambda}, 1]$ are informed, and (in equilibrium) match with researchers with the same set of indices. We accordingly suppose that researcher and laboratory indices are uniformly distributed on the interval $[\widetilde{\lambda}, 1]$. One obviously interesting special case will be that in which $\widetilde{\lambda} = 0$, so that all laboratories are informed.

Each informed laboratory can potentially set different prices for different researchers, and so there will be a price for each possible matched pair of

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[10]The bound $\lambda < \theta^{\frac{1}{2}}$ ensures that $r = \ell$ is feasible, i.e., $r < \theta^{1/2}$.

attributes. For appropriate values of ϕ, the price function

$$\hat{p}(\ell, r) = \phi + \frac{\ell^2}{2} - (1 - \theta)\ell r \qquad (11)$$

will clear markets with researcher ρ choosing the efficient $\ell = \rho$ and $r = \rho$. In particular, researcher ρ's payoff from ℓ and r is

$$\theta \ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k} = \ell r - \phi - \frac{1}{2}\ell^2 - \frac{r^{2+k}}{(2+k)\rho^k}.$$

Maximizing the payoff yields $\ell = \rho$ and $r = \rho$ (the efficient choices), and a payoff value of

$$\frac{k}{2(2+k)}\rho^2 - \phi \equiv \hat{u}_R(\theta, k, \rho) - \phi.$$

Laboratory payoffs under $\hat{p}$ are given by

$$\frac{1}{2}\lambda^2 + \phi \equiv \hat{u}_L(\theta, k, \lambda) + \phi.$$

Note that the payoff functions $\hat{u}_R$ and $\hat{u}_L$ are defined to exclude the ϕ surplus reallocation.

If $\widetilde{\lambda} = 0$, then individual rationality implies $\kappa = 0$, and the equilibrium is unique. If $\widetilde{\lambda} > 0$, there is a one parameter family of equilibrium price functions, indexed by

$$\phi \in [-\hat{u}_L(\theta, k, \widetilde{\lambda}), \ \hat{u}_R(\theta, k, \widetilde{\lambda})] = \left[\frac{-\widetilde{\lambda}^2}{2}, \ \frac{k\widetilde{\lambda}^2}{2(2+k)}\right];$$

all these price functions induce the same efficient attribute choices, but imply different divisions of the surplus.

The total net surplus of the pair with index ρ is

$$\frac{k}{2(2+k)}\rho^2 + \frac{1}{2}\rho^2 = \frac{(1+k)}{(2+k)}\rho^2, \qquad (12)$$

which is the maximum (i.e., efficient) value of the ρ-match surplus.

4.2   Becoming Informed

We construct a hybrid equilibrium for the case in which $\kappa = \frac{5}{96}$, $\theta = \frac{1}{4}$ and $k = 1$; thus $\theta^{\frac{1}{1+k}} = \frac{1}{2}$. The analysis for general parameters is in Appendix
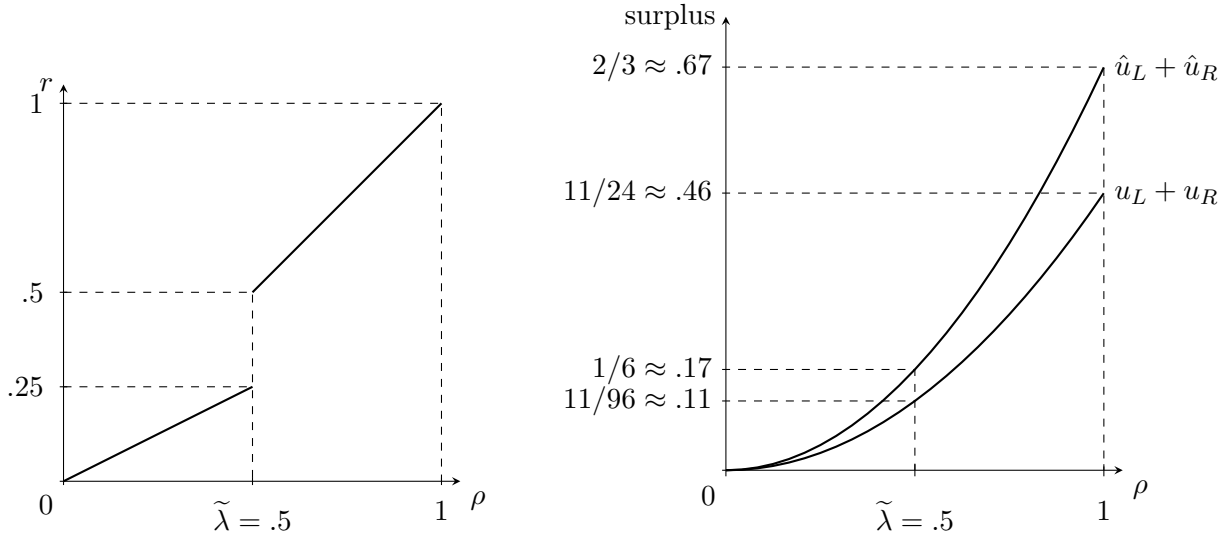
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Figure 2: The researcher attribute choice function for the case $k = 1$, $\theta = \frac{1}{4}$, and $\widetilde{\lambda} = \frac{1}{2}$ is illustrated on the left. The determination of $\widetilde{\lambda}$ for $\kappa = \frac{5}{96}$ is illustrated on the right, giving $\widetilde{\lambda} = \frac{1}{2}$.

B, which derives constraints on θ and k that are necessary for the existence of such an equilibrium.

The first step is to determine $\widetilde{\lambda}$. From (8)–(9), if a laboratory and researcher with indices $\lambda = \rho$ match and the laboratory is uninformed, the total net surplus generated is

$$\frac{1}{48}\rho^2 + \frac{7}{16}\rho^2 = \frac{11}{24}\rho^2. \tag{13}$$

Using (12), we see that there is an efficiency gain of $\frac{2}{3}\widetilde{\lambda}^2 - \frac{11}{24}\widetilde{\lambda}^2 = \frac{5}{24}\widetilde{\lambda}^2$ for agents of type $\rho = \lambda = \widetilde{\lambda}$ if higher-index laboratories become informed.

The left panel of Figure 2 shows that the researchers' investments levels take a jump at $\widetilde{\lambda}$, as researchers switch from the investments appropriate for matching with uninformed laboratories to those appropriate for matching with informed laboratories. Despite this discontinuity in investments, the payoffs of both researchers and laboratories must be continuous as we move across index $\widetilde{\lambda}$, since otherwise an agent just on the low-payoff side of $\widetilde{\lambda}$ would have an incentive to mimic an agent just on the other side (high-payoff) of $\widetilde{\lambda}$. This joint indifference implies that the switching point $\widetilde{\lambda}$ must be set so that the gain in surplus equals the cost κ of becoming informed.

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Figure 2 (right panel) shows that the threshold pair $\tilde{\lambda} = \frac{1}{2}$ gives a gain in surplus of $\kappa = \frac{5}{96}$.

We ensure that payoffs are continuous at $\tilde{\lambda}$ by setting appropriate prices. We have no control over prices for laboratories below $\tilde{\lambda}$, whose prices are fixed by (7), the equilibrium price function with uninformed laboratories. However, we can affect the prices of laboratories above $\tilde{\lambda}$ by choosing the value of ϕ. This initial value is set so that the price takes a downward jump at $\tilde{\lambda}$, compensating the researcher for the upward jump in investment. In particular, we set $\phi = \frac{7}{192}$, and hence informed laboratories set the price function $p(r, l) = \frac{1}{2}l^2 - \frac{3}{4}rl + \frac{7}{192}$ for the matching $l_R(\rho) = \rho$. Figure 3 illustrates the resulting payoff functions, which have a kink but not a discontinuity at $\tilde{\lambda}$.

To confirm that this is an equilibrium, we need to verify some incentive constraints. First, for the researchers, we need only verify that those with indices below (respectively, above) $\tilde{\lambda}$ prefer to be matched with uninformed (respectively, informed) laboratories rather than choosing a sufficiently high (respectively, low) attribute to be matched with an informed (respectively, uninformed) laboratory. This follows from the fact that researcher $\tilde{\lambda}$ is indifferent between the two options, and the single-crossing property on the researchers' cost functions.

The laboratories' incentive constraints are somewhat more involved. Appendix B.2 establishes the two key conditions, namely that a laboratory with type less than $\tilde{\lambda}$ does not prefer to become informed, and a laboratory with type above $\tilde{\lambda}$ does prefer to acquire information.

If the fixed cost of information κ decreased, the threshold $\tilde{\lambda}$ that determines which laboratories decide to invest would decrease, until the net surplus increase that is a consequence of the threshold laboratory's becoming informed again equals κ.

More interesting is the role of premuneration values in determining who becomes informed, and the resulting payoffs. As θ decreases, researchers' investments decrease, and hence the inefficiency associated with any matched pair increases. The threshold for laboratories to become informed must then decrease, in order for the gain from becoming informed to be equal to κ. Hence, the extent of information acquisition increases as the researchers' premuneration value share decreases.

Not only does the threshold change in response to changes in θ, but the division of the surplus between laboratories and researchers is affected. Note first that in the fully informed benchmark of $\tilde{\lambda} = 0$ (arising from, for example, $\kappa = 0$) in Section 4.1, the constant ϕ equals zero and not only are
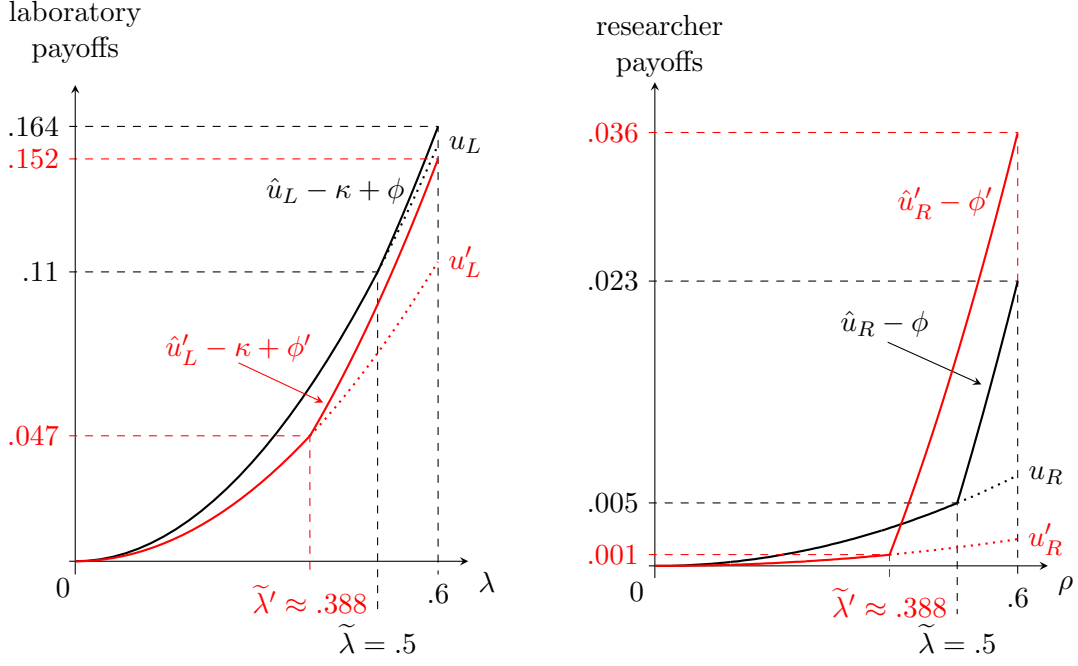
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Figure 3: Payoffs in the hybrid equilibrium for the case $k = 1$, for $\lambda \leq .6$. The cost of becoming informed is $\kappa = \frac{1}{6} - \frac{11}{96} = \frac{5}{96}$. Two values of θ are illustrated, $\theta = \frac{1}{4}$ (which implies $\widetilde{\lambda} = \frac{1}{2}$) and $\theta = \frac{1}{9}$ (which implies $\widetilde{\lambda}' \approx .388$). The expressions for $\theta = \frac{1}{9}$ are indicated by a prime. For λ below $\widetilde{\lambda}$, laboratory payoffs are given by u_L, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_L - \kappa + \phi$. For ρ below $\widetilde{\lambda}$, researcher payoffs are given by u_R, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_R - \phi$. The constant in the price function (11) is $\phi = \frac{7}{192}$ for $\theta = \frac{1}{4}$, and $\phi' = \frac{65}{2688} \approx .024$ for $\theta = \frac{1}{9}$.

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investments efficient, but laboratory and researcher payoffs are *independent* of θ. In contrast, when $\kappa > 0$, as illustrated in Figure 3, the premuneration values affect the location of the threshold, and so affects all agents' payoffs, including those involving fully informed laboratories. For example, under the lower premuneration value share of $\theta = \frac{1}{9}$, all researchers matched with uninformed laboratories have a lower payoff than under $\theta = \frac{1}{4}$. However, all researchers matched with informed laboratories under $\theta = \frac{1}{4}$ are strictly better off under the lower premuneration value share of $\theta = \frac{1}{9}$. Moreover, all laboratories prefer the scenario of the higher researcher premuneration value share of $\frac{1}{4}$.

Finally, hybrid equilibria do not exist for all parameters, and in particular do not exist if researchers' premuneration values are too large. If we fix κ, laboratories close to $\lambda = 1$ will have vanishingly small possible gains from acquiring information as θ goes to 1, and hence will choose *not* to become informed.

5   Laboratories Invest

Researchers' investments are inefficiently low in the model of Section 2 when laboratory premuneration values are not degenerate. It isn't obvious whether inefficiency stems from the asymmetry of information about researchers' investments or, more simply, from asymmetry about researchers' attributes. To understand the source and nature of the inefficiency we next keep the information structure the same (laboratories attributes are commonly known but researchers' attributes are not), but have laboratories, rather than researchers, choose attributes.

We will show that investments here are again inefficient, though the forces behind this inefficiency are quite different from those in the case of researcher investments. Hence, it is the *unobservability* of the attributes that causes inefficient investments.

Assume that researcher ρ has attribute $r = \rho$, so that researcher attributes are uniformly distributed on the unit interval. The cost of attribute $\ell \in \mathbb{R}_+$ to laboratory λ is

$$c(\ell, \lambda) = \frac{\ell^{2+k}}{(2+k)\lambda^k}, \qquad k \in \mathbb{R}_+.$$

As before, matching takes place in a competitive market, with laboratory attributes observable and priced. We use a superscript * to distinguish the prices, attribute choices and payoffs here from their analogs in Sections 2

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and 3. Given the price function p^*, research ρ chooses ℓ to maximize

$$\theta\ell\rho - p^*(\ell).$$

We denote by $\ell_R^* : [0,1] \to \mathbb{R}_+$ the function describing the laboratory attribute selected by researchers.

Laboratories choose attributes given the price function p^* and a *matching function* $r_L^* : \mathbb{R}_+ \to [0,1]$ that specifies the attribute $r_L^*(\ell)$ of the researcher that the market matches to a laboratory with attribute ℓ. Laboratory λ chooses $\ell \in \mathbb{R}_+$ to maximize

$$(1 - \theta)\ell r_L^*(\ell) + p^*(\ell) - c(\ell, \lambda).$$

We denote by $\ell_L^* : [0,1] \to \mathbb{R}_+$ the function describing the laboratories' attribute choices.[11]

Definition 2 *A price function p, matching function r_L^*, and strictly increasing laboratory attribute choices (ℓ_L^*, ℓ_R^*) constitute a* matching equilibrium *if*

1. *$\ell_R^*(\rho)$ is an optimal laboratory attribute for researcher ρ, for all $\rho \in [0,1]$,*

2. *$\ell_L^*(\lambda)$ is an optimal laboratory attribute for laboratory λ, for all $\lambda \in [0,1]$,*

3. *every researcher and laboratory earns nonnegative payoffs, and*

4. *markets clear: $r_L^*(\ell_R^*(\rho)) = \rho$ for all $\rho \in [0,1]$ and $\ell_R^*(\lambda) = \ell_L^*(\lambda)$ for all $\lambda \in [0,1]$.*

Before we describe the equilibrium, we note that since the laboratory cost function is the same functional form as the earlier researcher cost function, the efficient attribute choice for the laboratories is $\ell_L^*(\lambda) = \lambda$.

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[11] As in the earlier model, we are able to finesse many technical details. In particular, our notion of equilibrium assumes that ℓ_R^* and ℓ_L^* are strictly increasing; these properties can be deduced from the general model of Mailath, Postlewaite, and Samuelson (2012). Given these assumptions, market clearing requires $r_L^*(\ell_R^*(\rho)) = \rho$ and $\ell_R^*(\lambda) = \ell_L^*(\lambda)$.

In the equilibrium we analyze, the range of ℓ_R is an interval starting at 0, and so we need place no further restrictions on r_L^* (though setting $r_L^*(\ell) = 1$ for $\ell > \ell_R(1)$ would be natural). A central concern of Mailath, Postlewaite, and Samuelson (2012) is the appropriate treatment of matches when an attribute is chosen outside the range of putative equilibrium attributes and the set of such attributes does not form an interval.

Proposition 4 *A Matching equilibrium is given by the vector $(p^*, r_L^*, \ell_R^*, \ell_L^*)$, where*

$$p^*(\ell) = \frac{\theta \ell^2}{2\alpha}, \quad \text{for } \ell \in \mathbb{R}_+,$$
$$r_L^*(\ell) = \ell/\alpha, \quad \text{for } \ell \in [0, \alpha],$$
$$\ell_L^*(\lambda) = \ell_R^*(\lambda) = \alpha\lambda, \quad \text{for } \lambda \in [0, 1],$$

for $\alpha = (2 - \theta)^{\frac{1}{k+1}}$. Laboratory payoffs are given by

$$u_L^*(\theta, k, \lambda) = \frac{k}{2(k+2)}(2-\theta)^{(k+2)/(k+1)}\lambda^2. \tag{14}$$

Researcher payoffs are given by

$$u_R^*(\theta, k, \rho) = \frac{1}{2}\theta(2-\theta)^{1/(k+1)}\rho^2. \tag{15}$$

We leave the proof to Appendix C as it is very similar to those in the model above.

If $\theta < 1$, then $\alpha > 1$ and laboratories overinvest relative to the efficient level. The private nature of researchers' attributes again distorts investment incentives, but for a very different reason when it is laboratories that invest. When researchers invest, they have an incentive to underinvest since they will not get the full return on their unobservable investment. Here, laboratories' investments *are* observable, hence not the source of the inefficiency.

Laboratories overinvest because of researchers' response to their investments. Consider laboratory λ's equilibrium investment. It is higher than the efficient level, so why doesn't the laboratory decrease it's investment?

In the calculation of the efficient investment level, we know that an efficient outcome must match agents assortatively on index. If a laboratory's investment is too high, we can decrease the investment *keeping the matching fixed*, and thereby increase the surplus. In contrast, in the market equilibrium, a laboratory that decreased its investment level from the equilibrium would find that the researcher's attribute that the laboratory is matched with decreases. It is this concern for the quality of the researcher (which it doesn't observe) with whom it is matched that makes it optimal for laboratories to invest more than the efficient level.

The intuition for laboratories' overinvestment is quite general, as long as laboratories' premuneration values increase with the attribute of their

matched partner. Laboratories want higher-attribute researchers, and are willing to pay for them. But when they cannot directly observe researchers' attributes, they cannot simply pay for higher-attribute researchers by accepting lower prices to match, since that would be equally attractive to all researchers. But they can increase the attractiveness to matched partners by investing more. This makes a laboratory more attractive to all researchers, but more so for higher attribute researchers. Hence, a laboratory can combine an increase in their attribute with an increase in their price that will screen potential researchers so that only higher attribute researchers will find the combination attractive.

Unlike the previous case, though, the comparative statics of equilibrium payoffs with respect to the researchers' premuneration values is unremarkable: Researcher payoffs are increasing and laboratory payoffs are decreasing in θ.

6 Discussion

6.1 Summary

We have worked with particular cost and surplus functions, but the intuition behind our results is quite general. When researchers' investments cannot be observed by laboratories, researchers' investments are inefficiently low because part of the return to those investment accrues to laboratories. Shifting some of the premuneration value of the laboratory to the researcher will mitigate the inefficiency, sometimes enough to more than compensate the laboratory for the decrease in premuneration value. When laboratories make investments, the laboratories' competition for high attribute researchers is mediated not only by price, but also by their investments. This leads laboratories to couple inefficiently high investments with high prices, in an effort to screen out researchers whose unobservable attributes are undesirably low.

We assumed that investment was "one-sided" (either laboratories' attributes or researchers' attributes were exogenously given) and that private information was one-sided (laboratories' attributes, whether exogenous or endogenous, were commonly known). Mailath, Postlewaite, and Samuelson (2012) analyze a general model in which investment is two-sided, but private information remains one-sided. That paper shows, among other things, that investments will be efficient if and only if the premuneration values of the side which cannot observe the attributes of the other side are independent of those attributes. In general, when researchers' investments are unobservable, the researchers' inability to capture all of their investment

returns pushes researchers toward underinvestment. But when laboratories also make investments, the tendency for laboratory overinvestment gives researchers incentives to *increase* their investment. There are similar opposing forces in laboratory incentives to invest. What forces predominate typically will depend on the specific cost and premuneration functions.

6.2   The Role of Competition

As k increases, researchers become less homogeneous, dampening their competition for laboratories. How does this affect payoffs?

The equilibrium payoffs of laboratories and researchers are given by (8)–(9) (for the case in which researchers are investing) and (14)–(15) (for the case in which laboratories are investing). Figure 4 illustrates these payoffs as a function of k.

Researchers' payoffs increase in k when researchers make the investments, reflecting the enhanced investment incentives of reduced competition and reduced investment costs. Researcher's payoffs decrease in k when laboratories make the investments, as the reduced researcher competition leads to small laboratory investments.

Laboratories' payoffs under either scenario increase as does k. A larger value of k makes researchers less homogeneous, and hence dampens their competition for laboratories, seemingly to the latter's deficit. However, this is overwhelmed by the enhanced researcher investment incentives of increasing k (when researchers make investments), and the reduced cost of investments (when laboratories make investments).

In the limit, when $k = 0$, all agents on the endogenous-attribute side are identical, and so these agents are effectively perfectly competitive. As the endogenous-attribute side becomes perfectly homogeneous, the other side captures all the surplus:

$$\lim_{k \to 0} u_R(\theta, k, \rho) = 0,$$
$$\lim_{k \to 0} u_L(\theta, k, \lambda) = \tfrac{\theta}{2}(2 - \theta)\lambda^2,$$
$$\lim_{k \to 0} u_R^*(\theta, k, \rho) = \tfrac{\theta}{2}(2 - \theta)\rho^2,$$
$$\text{and} \quad \lim_{k \to 0} u_L^*(\theta, k, \lambda) = 0.$$

Interestingly, while the outcome is inefficient and the division of the surplus depends on the assignment of premuneration values, the extent of the inefficiency is independent of the side with endogenously determined attributes.
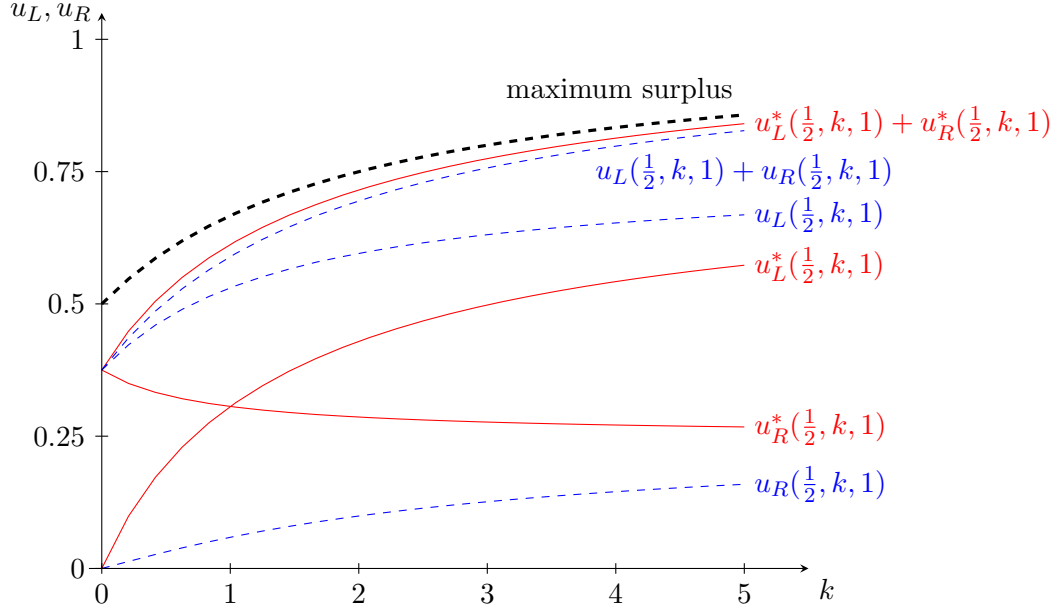
Figure 4: Payoffs for the scenario when the researchers choose attributes (u_L and u_R) and when the laboratories choose attributes (u_L^* and u_R^*), for $\theta = \frac{1}{2}$ and $\rho = \lambda = 1$. Since researcher (respectively, laboratory) payoffs for index ρ (resp., λ) are proportional to ρ^2 (resp., λ^2), these also represent the proportionality factors for the other indices. The maximum surplus is $(k+1)/(k+2)$.

The other extreme is the limit as $k \to \infty$, i.e., when the endogenous attribute side becomes noncompetitive (and the cost of the attribute becomes negligible). In that case, we get efficient attribute choices in both scenarios, but the premuneration values still matter in terms of the division:

$$\lim_{k \to \infty} u_R(\theta, k, \rho) = \lim_{k \to 0} u_R^*(\theta, k, \rho) = \frac{\theta \rho^2}{2},$$

and $\qquad \lim_{k \to 0} u_L(\theta, k, \lambda) = \lim_{k \to 0} u_L^*(\theta, k, \lambda) = \left(1 - \frac{\theta}{2}\right) \lambda^2.$

As k increases without bound, investments costs become insignificant, and one might suspect that this lies behind the convergence to an efficient outcome as $k \to \infty$. Appendix A.3 shows that this is not the case, exhibiting a cost function for which costs do not vanish as k gets large, but the equilibrium nonetheless approaches efficiency.

6.3 Laboratory Price-Taking

We have modeled the matching between researchers and laboratories as a competitive market in which agents take prices as given. The equilibrium prices can be supported in a model in which laboratories set prices. It is easy to see from the equilibrium that a given laboratory cannot do better by raising the price assigned to it. In the equilibrium, there are laboratories whose price and quality are arbitrarily close to any given laboratory. Hence for any discrete increase in price, no researcher would choose a laboratory which offered itself at a price higher than the equilibrium price. The more interesting possibility is that of a laboratory deviating to a price less than the equilibrium price. Such a deviation would attract new researchers, some of lower quality than the laboratory's equilibrium match and some of higher quality. A laboratory that offered a lower price would then have to form a belief about the quality of a researcher that approached it. There are many beliefs the laboratory could hold that would make the expected outcome worse than the equilibrium outcome, including, for example, the belief that it is the researcher whom it would have attracted in the equilibrium.

6.3.1 Revealing Information

Suppose researchers could certify their quality, perhaps by taking exams or completing internships that demonstrate their skills. If there is a fixed cost of such certification, we have a problem analogous to the case in which laboratories could become informed at a cost κ. As in that case, if κ is too large, no researcher will pay to certify her attribute, and for any $\kappa > 0$, researchers near the bottom will choose not to reveal. For κ not too large we may have hybrid equilibria characterized by a threshold $\widetilde{\rho} \in (0,1)$, with researchers below the threshold choosing not to certify and those above choosing to certify. We will illustrate the analog of the hybrid equilibria when laboratories could become informed for the case of researcher certification.

Assume that $\theta = \frac{1}{4}$ and $k = 1$; thus $\theta^{\frac{1}{1+k}} = \frac{1}{2}$. Assume also that the cost of certification is $\kappa = \frac{5}{96}$, as in the hybrid equilibrium above. Then it is straightforward to verify that a price $p(l) = \frac{1}{8}l^2$ in the case that researchers have not certified attributes and price $p(r,l) = \frac{l^2}{2} - \frac{3}{4}rl + \frac{3}{192}$ for certified researchers will induce researchers with $\rho < \frac{1}{2}$ to choose not to certify and researchers with $\rho > \frac{1}{2}$ to certify, and that markets will clear.

It is not surprising that there is a hybrid equilibrium when researchers can pay to reveal that has the same threshold as in the hybrid equilibrium when laboratories can pay to become informed ($\tilde{\rho} = \tilde{\lambda} = \frac{1}{2}$), when the cost

of certifying is the same as the cost of becoming informed ($\kappa = \frac{5}{96}$). The increase in the net surplus when researchers' attributes are known over the surplus when they are unknown is independent of *how* the attributes came to be known. Whether the cost of making attributes is borne by researchers or by laboratories, it will be worthwhile for the cost to be borne when the efficiency gain equals that cost; the price for the matched pairs above the threshold can be adjusted up or down so that the gain accrues to the side of the market that is bearing the cost.

One might suspect that if the cost to researchers of certifying their quality is not too high, the uncertainty would "unravel": high-quality researchers would reveal themselves, making it optimal for the highest-attribute researchers in the remaining pool to reveal themselves, and so on until most researchers' qualities are known.[12] Indeed, to avoid such unraveling, Harvard Business School students have successfully lobbied for policies that prohibit students' divulging their grades to potential employers, while the Wharton student government adopted a policy banning the release of grades.[13] These efforts to suppress information may be counterproductive. Suppose that if information is not suppressed, then we have a hybrid equilibrium in which high-type researchers reveal their investments. The hybrid equilibrium pastes together the equilibrium for unknown researcher attributes at the bottom with the equilibrium for known researcher attributes at the top. Making the attributes of the higher attribute researchers known increases the payoffs to those researchers and leaves unchanged the payoffs to the lower attribute researchers. In the limit as the cost of revelation approaches zero, all researchers will reveal their information and all will be better off than in information were completely suppressed.

6.4 Modifying Premuneration Values

One might hope to ameliorate the inefficiency of prematching investments by reallocating premuneration values. For example, in a match between a researcher and a laboratory, one might want to allocate all of the surplus to the researcher. However, part of the surplus may consist of laboratory's value of the enhanced prestige that comes from employing an eminent researcher. How does one modify the researcher's premuneration value so that it includes the laboratory's incremental prestige?

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[12]See Grossman (1981), Milgrom (1981), or Okuno-Fujiwara, Postlewaite, and Suzumura (1990) for analyses of such unraveling.

[13]Ostrovsky and Schwarz (2010) investigate the optimal amount of information to disclose from the students' perspective.

There are ways that premuneration values can be structured so that the value of human capital is shared. In a match between a student and a university, the student's premuneration value typically includes all the value of a her human capital. However, it is possible to divert some of this value to others. For example, income-contingent loans in a number of countries (including Australia, Sweden and New Zealand) effectively give the lender a share of students' future income (Johnstone, 2001).[14]

While it is possible to attain some modification of premuneration values, there are obvious problems in doing so. Should the laboratory in the discussion above own half of all future discoveries by the researcher, or only those discoveries that can directly be linked to the human capital acquired by the researcher while matched with the laboratory? And if the latter, how can one ascertain the link between discoveries and human capital acquired? These questions are not abstract issues, but arise in situations in which precisely such property rights were central. The University of New Mexico sued a former researcher for rights to patents that he applied for four years after he had left the university, arguing that the patents stemmed from research that he had done before leaving.[15]

A   Appendix: A More General Cost Function

A.1   The Cost Function

This section examines a more general attribute cost function, given by

$$c(a, \sigma) = \frac{\beta a^{2+k}}{\sigma^k},$$

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[14]In the summer of 2010, the UK debated the possibility of partially funding higher education though a "graduate tax" levied on college graduates' income (http://www.bbc.co.uk/news/education-10649459). Basketball star Yao Ming (Houston Rockets) has a contract with the China Basketball Association calling for 30% of his NBA earnings to be paid to the Chinese Basketball Association (in which he played prior to joining the Rockets), while another 20% will go to the Chinese government. Similar arrangements hold for Wang Zhizhi (Dallas Mavericks) and Menk Bateer (Denver Nuggets and San Antonio Spurs). (See the *Detroit News*, April 26, 2002, http://www.detnews.com/2002/pistons/0204/27/sports-475199.htm/.) We can view the initial match between Yao Ming and his Chinese team as producing a surplus that includes the enhanced value of his earnings as a result of developing his basketball skills, and the contract as specifying a split of this surplus.

[15] "Universities Try to Keep Inventions From Going 'Out the Back Door,' " *Chronicle of Higher Education*, May 17, 2002.

where a is either r or ℓ and σ is correspondingly either ρ or λ, and $\beta \in \mathbb{R}_+$ may be a function of k. In our leading example,

$$\beta = (2+k)^{-1},$$

while in Proposition A.1 below, we assume

$$\beta = \gamma^{-(1+k)}.$$

We first note that maximizing the net surplus requires, as in (2), matching indices and then, for each σ, choosing a to maximize

$$\sigma a - \frac{\beta a^{2+k}}{\sigma^k}.$$

Letting

$$\eta = [\beta(2+k)]^{-1/(1+k)},$$

the maximizing value of a is $\eta\sigma$, and the maximized value of the surplus is $(1+k)\eta\sigma^2/(2+k)$.

If $\beta = (2+k)^{-1}$, as in the body of the paper, then $\eta = 1$ for all k. and we recover the efficient investments (cf. (3)). If instead $\beta = \gamma^{-(1+k)}$, as in Proposition A.1, then,

$$\eta = [\beta(2+k)]^{-1/(1+k)}$$
$$= \gamma(2+k)^{-1/(1+k)}$$
$$\to \gamma \quad \text{as} \quad k \to \infty,$$

since

$$\lim_{k\to\infty} (2+k)^{-1/(1+k)} = \exp\left\{ \lim_{k\to\infty} \tfrac{-1}{1+k} \log(2+k) \right\}$$
$$= e^0 = 1.$$

A.2 Equilibrium

We now calculate the equilibrium for this more general cost function. Since Lemmas 1–3 are true with this cost function (the same proofs are valid), we need only determine the equilibrium price function p and researcher attribute function r_R. It is straightforward to verify that the following describes a matching equilibrium:

$$p(\ell) = \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\ell^2$$

and
$$r_R(\rho) = \eta\theta^{1/(1+k)}\rho.$$

Payoffs are given by

$$
\begin{aligned}
u_R(\theta, k, \rho) &= \theta\eta\theta^{1/(1+k)}\rho^2 - \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2 - \frac{\beta(\eta\theta^{1/(1+k)}\rho)^{2+k}}{\rho^k} \\
&= \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2\left\{1 - 2\beta\eta^{1+k}\right\} \\
&= \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\rho^2\left\{1 - \frac{2}{2+k}\right\} \\
&= \frac{k\eta\theta^{(2+k)/(1+k)}\rho^2}{2(2+k)}
\end{aligned}
\tag{A.1}
$$

and

$$
\begin{aligned}
u_L(\theta, k, \lambda) &= (1-\theta)\frac{\eta\theta^{(2+k)/(1+k)}\lambda^2}{\theta} + \frac{1}{2}\eta\theta^{(2+k)/(1+k)}\lambda^2 \\
&= \frac{1}{2}\eta\theta^{1/(1+k)}\lambda^2(2 - 2\theta + \theta) \\
&= \frac{1}{2}\eta\theta^{1/(1+k)}\lambda^2(2 - \theta).
\end{aligned}
\tag{A.2}
$$

For the case in which laboratories invest, a straightforward argument analogous to the proof of Proposition 4 shows that the functions given in that proposition, namely

$$p^*(\ell) = \frac{\theta\ell^2}{2\alpha}, \quad \text{for } \ell \in \mathbb{R}_+, \tag{A.3}$$

$$r_L^*(\ell) = \ell/\alpha, \quad \text{for } \ell \in [0, \alpha], \tag{A.4}$$

$$\ell_L^*(\lambda) = \ell_R^*(\lambda) = \alpha\lambda, \quad \text{for } \lambda \in [0, 1], \tag{A.5}$$

constitute an equilibrium once the definition of α is changed to

$$\alpha = \eta(2-\theta)^{1/(1+k)} = \left(\frac{2-\theta}{\beta(2+k)}\right)^{1/(1+k)}.$$

A.3 The Effects of Competitiveness

The following proposition shows that the limit efficiency of the two scenarios (as k becomes large) is *not* due to the negligibility of costs in the limit.

30

Proposition A.1 *Suppose the cost to agent $\sigma \in (0, 1]$ of choosing attribute $a \in \mathbb{R}_+$ is given by*

$$c(a, \sigma) = \frac{a^{2+k}}{\gamma^{1+k}\sigma^k},$$

where $\gamma \in (0, 1)$. The limit (as $k \to \infty$) maximum surplus is $\gamma\sigma^2$. Then,

$$\lim_{k\to\infty} u_R(\theta, k, \rho) = \lim_{k\to 0} u_R^*(\theta, k, \rho) = \frac{\gamma\theta\rho^2}{2},$$

$$and \quad \lim_{k\to 0} u_L(\theta, k, \lambda) = \lim_{k\to 0} u_L^*(\theta, k, \lambda) = \gamma\left(1 - \frac{\theta}{2}\right)\lambda^2.$$

The results for the case in which researchers invest follow from (A.1)–(A.2), while those for laboratory investments can be calculated from (A.3)–(A.5).

B   Appendix: Endogenizing Information

B.1   Becoming Informed

Fix $\widetilde{\lambda} \in [0, 1]$ and consider a putative equilibrium in which laboratories with $\lambda \leq \widetilde{\lambda}$ are uninformed and laboratories with $\lambda > \widetilde{\lambda}$ choose to be informed. If $\widetilde{\lambda} = 1$, all laboratories are uninformed. Within each region, researchers will be choosing attributes increasing in index, and researcher ρ will be matched with laboratory $\lambda = \rho$. Thus, researcher $\rho \leq \widetilde{\lambda}$ will choose $r = \theta^{1/(1+k)}\rho$ and be matched with the uninformed laboratory $\lambda = \rho$. Researcher $\rho > \widetilde{\lambda}$ will choose $r = \rho$ and be matched with the informed laboratory $\lambda = \rho$.

In this putative equilibrium, the set of chosen researcher attributes is $[0, \widetilde{r}] \cup (\widetilde{\lambda}, 1]$, where $\widetilde{r} = \theta^{1/(1+k)}\widetilde{\lambda} < \widetilde{\lambda}$.

Given a (not too large) cost of becoming informed κ,[16] we choose $\widetilde{\lambda} \in (0, 1)$ so that the ex ante efficient surplus from the match of researcher $\widetilde{\lambda}$ and laboratory $\widetilde{\lambda}$ exactly exceeds the uninformed laboratory equilibrium match surplus by κ:

$$\hat{u}_R(\theta, k, \widetilde{\lambda}) + \hat{u}_L(\theta, k, \widetilde{\lambda}) - \kappa = u_R(\theta, k, \widetilde{\lambda}) + u_L(\theta, k, \widetilde{\lambda}). \tag{B.1}$$

The pricing constant ϕ in (11) is chosen so that laboratory $\widetilde{\lambda}$ is indifferent between being informed and not:[17]

$$\hat{u}_L(\theta, k, \widetilde{\lambda}) + \phi - \kappa = u_L(\theta, k, \widetilde{\lambda}). \tag{B.2}$$

---

[16]Specifically, $\kappa < (2 - 3\theta^{1/2} + \theta^{3/2})/3$.
[17]Specifically, set $\phi = [(1 - \theta^{3/2})\widetilde{\lambda}^2]/6$.

This immediately implies that researcher $\widetilde{\lambda}$ is also indifferent *ex ante* between being matched with an informed or uninformed laboratory. We should then be able to simply "paste" the informed laboratory equilibrium for $\lambda \geq \widetilde{\lambda}$ to the uninformed laboratory equilibrium for $\lambda < \widetilde{\lambda}$.

For the researchers, we need only verify that researchers with indices below (respectively, above) $\widetilde{\lambda}$ prefer to be matched with uninformed (respectively, informed) laboratories rather than choosing a sufficiently high (respectively, low) attribute to be matched with an informed (respectively, uninformed) laboratory. But this follows from the single crossing property on the cost function together with the implied indifference for researcher $\widetilde{\lambda}$.

B.2 Laboratory Incentives to Deviate

Turning to the laboratories, there are two potentially profitable types of deviations. The first is that a laboratory with index $\lambda < \widetilde{\lambda}$ may find it profitable to be informed. The second is that a laboratory with index $\lambda > \widetilde{\lambda}$ may find it profitable to be uninformed.

B.2.1 Do uninformed laboratories wish to be informed?

Consider first a deviation by a laboratory $\lambda \leq \widetilde{\lambda}$ to becoming informed and targeting a researcher with attribute $r \leq \widetilde{r}$. The attribute r is chosen by researcher $\rho = \theta^{-1/(1+k)}r$, and matches with $\lambda = \rho = \theta^{-1/(1+k)}r$, paying a price of $\rho^2 \theta^{(2+k)/(1+k)}/2 = \theta^{k/(1+k)}r^2/2$. The resulting ex post payoff is the researcher's share of the surplus less the price, $\theta \times \theta^{-1/(1+k)}r^2 - \theta^{k/(1+k)}r^2/2 = \theta^{k/(1+k)}r^2/2$. An offer of a price p satisfying

$$\theta^{k/(1+k)}r^2/2 < \theta\lambda r - p$$

will induce the researcher to accept the deviating offer. Such an offer is profitable for the laboratory if

$$u_L(\theta, k, \lambda) < (1-\theta)\lambda r + p - \kappa.$$

Thus, there is a p for which the deviation by the laboratory is strictly profitable if, and only if,

$$\kappa < \lambda r - \theta^{k/(1+k)}r^2/2 - u_L(\theta, k, \lambda)$$
$$= \lambda r - \theta^{k/(1+k)}r^2/2 - \frac{1}{2}\theta^{1/(1+k)}(2-\theta)\lambda^2 = \Delta(\lambda, r).$$

For $\lambda \leq \theta\widetilde{\lambda}$, $\Delta(\lambda, \cdot)$ is maximized at $r = \theta^{-k/(1+k)}\lambda \leq \widetilde{r}$, and has value $\theta^{-k/(1+k)}\lambda^2(1-\theta)^2/2$. Note that $\theta^{-k/(1+k)}\theta\widetilde{\lambda} = \widetilde{r}$. Moreover, for $\lambda \in [\theta\widetilde{\lambda}, \widetilde{\lambda}]$,

$\Delta(\lambda, \widetilde{r})$ is uniquely maximized at $\lambda = \widetilde{\lambda}/(2 - \theta)$. This implies that the maximum of $\Delta(\lambda, r)$ over $(\lambda, r) \in [0, \widetilde{\lambda}] \times [0, \widetilde{r}]$ is achieved at $(\widetilde{\lambda}/(2 - \theta), \widetilde{r})$. Thus, there is no strictly profitable deviation if

$$\kappa \geq \Delta(\widetilde{\lambda}/(2 - \theta), \widetilde{r}) = \frac{\theta^{1/(1+k)}\widetilde{\lambda}^2(1 - \theta)^2}{2(2 - \theta)}. \tag{B.3}$$

This gives us a joint restriction on κ, k and θ that must hold if the hybrid equilibrium is to exist.

Suppose, for example, that κ is sufficiently small that there is a $\widetilde{\lambda} \in (0, 1)$ satisfying

$$\hat{u}_R(\theta, k, \widetilde{\lambda}) + \hat{u}_L(\theta, k, \widetilde{\lambda}) - \kappa = u_R(\theta, k, \widetilde{\lambda}) + u_L(\theta, k, \widetilde{\lambda}). \tag{B.4}$$

It not true in general that for $\widetilde{\lambda}$ satisfying (B.4), inequality (B.3) is satisfied. For example, for $k = 1$, (B.4) implies

$$\kappa = \frac{\widetilde{\lambda}^2}{3}\left\{2 - 3\theta^{1/2} + \theta^{3/2}\right\}, \tag{B.5}$$

so that (B.3) requires

$$\frac{\widetilde{\lambda}^2}{3}\left\{2 - 3\theta^{1/2} + \theta^{3/2}\right\} \geq \frac{\theta^{1/2}\widetilde{\lambda}^2(1 - \theta)^2}{2(2 - \theta)}.$$

Simplifying, we obtain the inequality

$$f(\theta) = 8 - 4\theta + \theta^{1/2}\{16\theta - 15 - 5\theta^2\} \geq 0.$$

This function has one root $\widetilde{\theta} \approx 0.629$ in the open interval $(0, 1)$, with $f(\theta) > 0$ for $\theta < \widetilde{\theta}$ and $f(\theta) < 0$ for $\theta > \widetilde{\theta}$.

B.2.2 Do informed laboratories wish to be uninformed?

The second type of deviation is less easily dealt with. If laboratory $\lambda \geq \widetilde{\lambda}$ deviates to being uninformed, then by posting a price p, the laboratory attracts all researchers who find matching with laboratory λ at that price attractive. The laboratory must have beliefs over the researchers attracted by such a deviation. We assume pessimistic beliefs: the laboratory assumes that the lowest attribute researcher will match.

We begin by considering $\lambda = 1$, and suppose this laboratory chooses to be uninformed. If it were to charge $p = \phi + \frac{1}{2} + \theta$, the equilibrium price paid by researcher $\rho = 1$ to match with laboratory $\lambda = 1$, researcher $\rho = 1$

incentives are unchanged. But that match is no longer relevant (given our assumption on beliefs), since lower attribute researchers are willing to pay that price. The most profitable deviation is to charge a higher price in attempt to screen out lower attribute researchers.[18]

We now argue that if $\theta < \widetilde{\lambda}$, the most profitable deviation by laboratory $\lambda = 1$ is to charge such a high price that $\rho = \widetilde{\lambda}$ is indifferent, and that such a deviation is not profitable. Researcher $\rho \geq \widetilde{\lambda}$ has chosen attribute ρ and has payoffs gross of costs of

$$\frac{\rho^2}{6} - \phi + \frac{\rho^2}{3} = \frac{\rho^2}{2} - \phi,$$

and is willing to match with the deviating laboratory $\lambda = 1$ at a price p if $\theta\rho - p \geq \rho^2/2 - \phi$, i.e., if $\theta\rho - \rho^2/2 + \phi \geq p$. The laboratory's goal is to maximize the lowest ρ satisfying this inequality through his choice of p. The quadratic on the left of the inequality is maximized at $\rho = \theta$ and is monotonically decreasing for larger ρ. This implies that if $\theta < \widetilde{\lambda}$, the optimal choice of p makes researcher $\widetilde{\lambda}$ just indifferent ($p = \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi$); no researcher is willing to match at a larger p.

The laboratory does not find this deviation profitable if

$$\frac{1}{2} + \phi - \kappa \geq (1 - \theta)\widetilde{\lambda} + \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi$$

$$\Longleftrightarrow \frac{1}{2} - \kappa \geq \widetilde{\lambda} - \widetilde{\lambda}^2/2.$$

Using (B.5) to eliminate κ in the inequality and rearranging, one obtains

$$3 \geq 6\widetilde{\lambda} - \widetilde{\lambda}^2\{7 - 6\theta^{1/2} + 2\theta^{3/2}\},$$

and since the right side is maximized at $\theta = 1$, the inequality is implied by $3 \geq 6\widetilde{\lambda} - 3\widetilde{\lambda}^2$, which is always true.

Lower index informed laboratories also have no incentive to become uninformed, though for some this deterrence involves a concern that the researcher will have an attribute less than $\theta^{1/2}\widetilde{\lambda}$, rather than $\widetilde{\lambda}$. Lower informed laboratories may find it optimal to become informed if they could guarantee no researcher with an attribute below $\widetilde{\lambda}$ would find the price attractive. However, this is impossible: By becoming uninformed, laboratory $\rho = \widetilde{\lambda}$ cannot deter lower attribute researchers without deterring all researchers. A (loose) upper bound on the payoff from deviating is obtained

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[18]At higher prices the highest attribute researcher prefers to match with laboratory $1 - \varepsilon$, for ε small. But since the laboratory believes he will attract the lowest available attribute, this is irrelevant.

by assuming that at the price p which makes the researcher with attribute $\widetilde{\lambda}$ just indifferent, the laboratory is guaranteed that the only additional researcher attribute attracted is $\widetilde{r} = \theta^{1/2}\widetilde{\lambda}$. It can be verified that even with such a payoff, the deviation is not profitable.

C Appendix: Proof of Proposition 4

We first note that, under the specified price function, the researcher chooses ℓ to maximize
$$\theta\ell\rho - \frac{\theta\ell^2}{2\alpha},$$
so that
$$\ell = \alpha\rho,$$
which is the hypothesized form of ℓ_R^*.

Market clearing is immediate. It remains to confirm the optimality of laboratory behavior. Laboratory λ chooses ℓ to maximize
$$(1-\theta)\frac{\ell^2}{\alpha} + \frac{\theta\ell^2}{2\alpha} - \frac{\ell^{2+k}}{(2+k)\lambda^k}.$$

The first order condition is
$$2(1-\theta)\frac{\ell}{\alpha} + \frac{\theta\ell}{\alpha} - \frac{\ell^{1+k}}{\lambda^k} = 0,$$
implying
$$\ell = \left(\frac{2-\theta}{\alpha}\right)^{1/k}\lambda,$$
which equals $\alpha\lambda$ when $\alpha = (2-\theta)^{\frac{1}{k+1}}$.

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