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"Defending Against Speculative Attacks: Reputation,
Learning, and Coordination"

by

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Abstract

How does the central bank's incentive to build a reputation affect speculators' ability to coordinate and the likelihood of the devaluation outcome during speculative currency crises? What role does market information play in speculators' coordination and the central bank's reputation building? I address these questions in a dynamic regime change game that highlights the interaction between the central bank's reputation building and speculators' individual learning. On the one hand, the central bank has private information about its value from the currency peg and decides whether to maintain it. By defending against speculative attacks, it can build a reputation of defending, which may deter future attacks. On the other hand, speculators individually learn the central bank's value, and such learning may encourage speculators to coordinate an attack. I show that though learning makes the central bank's value approximate common knowledge over time, there is a unique equilibrium when learning is slow. In this equilibrium, no speculator attacks and the central bank sustains the currency peg forever, because the central bank obtains commitment power through the incentive to build a reputation. When learning is fast, there may be equilibria with attacks. In any equilibrium with attacks, the onset of the attack depends on the entire learning process. Once speculators attack, they attack frequently and infinitely often. Consequently, the central bank has no incentive to build a reputation and abandons the currency peg almost surely.

1 Introduction

This paper studies the interaction between the central bank's reputation building and speculators' individual learning in the context of currency attacks. A currency peg is an exchange rate regime that fixes the exchange rate of the domestic currency to another currency. If the domestic currency is overvalued under the peg, speculators will short the domestic currency (i.e., attack the currency peg), in the hope that the central bank will abandon the currency peg. In that event, speculators profitably buy back the domestic currency. The central bank defends the peg by selling foreign reserves to buy the domestic currency. Such a defensive measure is costly, and the cost of defending increases as more speculators join the attack. Therefore, the more speculators attack, the more likely it is that the central bank will abandon the peg.

By defending against an attack, the central bank signals its commitment to defend in the future. There are unobserved characteristics of the central bank that affect its current and future defending policies (Drazen, 2003). From a failed attack, speculators make inference about the unobserved characteristics: the central bank is more likely to have characteristics that induce strong incentives to defend. Thus, speculators believe that the central bank is more likely to defend in the future. Therefore, by defending against the ongoing attack, the central bank builds a reputation of defending against future attacks. In this aspect, the central bank behaves as the chain store in the reputation literature (Kreps and Wilson, 1982; Milgrom and Roberts, 1982; and Fudenburg and Levine, 1989). Then how does the central bank's incentive to build a reputation affect speculators' ability to coordinate and the likelihood of the devaluation outcome?

On the other hand, because speculators have the option to attack again after a failed attack, a currency crisis is intrinsically a dynamic phenomenon. In such a dynamic process, new information about the central bank's unobserved characteristics usually arrives, and speculators may individually learn the central bank's characteristics (Angeletos, Hellwig, and Pavan, 2007). Then what role does individual learning play in the dynamics of coordination among speculators and the central bank's reputation building?

I address these questions in a dynamic regime change game, in which the central bank privately knows its value from the currency peg and decides whether to maintain it, and any speculator's accumulated private information about the value is asymptotically perfect. Specifically, the central bank's value from the currency peg can be either high or low. If the value is high, the central bank is willing to sustain the currency peg at any cost. Therefore, by defending an attack, the central bank with a low value makes speculators believe it is more likely to have a high value. There is a continuum of speculators. Each speculator receives one private signal about the central bank's value in every period, and the central

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bank's value becomes approximate common knowledge over time.

Based on equilibrium properties in a complete information game, one may intuitively conjecture answers to the above questions. If it is common knowledge that the central bank has a low value, multiple equilibria exist because of the self-fulfilling nature of beliefs (Obstfeld, 1996). In an equilibrium of the complete information game, speculators attack, and the central bank may abandon the currency peg. In my model, individual learning leads the central bank's value to approximate common knowledge; as a consequence, it seems that individual learning is sufficient for speculators to attack in some equilibria. Specifically, one seems to be able to construct an equilibrium with attacks following the algorithm suggested by Cripps, Ely, Mailath, and Samuelson (2008). That is, speculators first just receive private signals without attacking until their knowledge about the central bank's value is sufficiently close to common knowledge, then some speculators attack. If the attack fails, they repeat this "learning then attacking" process.

This intuition, however, ignores the effect of the central bank's incentive to build a reputation on speculators' ability to coordinate. I show that if speculators learn slowly from their own private information, the model has a unique equilibrium, in which no speculator attacks, and the central bank sustains the currency peg forever. When speculators make decisions, they rely more on public information than their own private signals due to the coordination feature (Morris and Shin, 2002). The central bank's reputation serves as the public information. Once an attack fails, the central bank's reputation increases. Consequently, a new attack occurs only after learning offsets the increase in reputation. This can take a long time if learning is sufficiently slow. However, speculators must attack again within a bounded number of periods; otherwise, the central bank defends for sure in the previous "attacking" period. (This is the reason why the algorithm suggested by Cripps, Ely, Mailath, and Samuelson (2008) cannot be used to construct an equilibrium in this model.) Therefore, there is no equilibrium with attacks. Here, the equilibrium uniqueness is not due to the coordination failure among speculators, but to the full commitment power the central bank obtains from the incentive to build a reputation.

If speculators' learning from their own private information is sufficiently fast, then infinitely many equilibria with attacks exist. In any equilibrium with attacks, the central bank with a low value will abandon the currency peg almost surely. Though the equilibrium devaluation outcome is the same as that in some equilibria in a dynamic regime change game without a defender, the dynamics of attacking are significantly different. First, the earliest possible first attacking period may not be the first time speculators form a common belief. Suppose speculators form a common belief that the central bank's value is low in some period for the first time, but their learning speed in a large number of subsequent periods is slow. If speculators attack in an equilibrium in the period when they form a common belief for

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the first time, in the equilibrium, they will attack again at least once within a uniformly bounded number of periods if the attack fails. Then there is a period, such that, if the attack in that period fails, speculators cannot form a common belief and thus cannot attack within a uniformly bounded number of periods. As a consequence, the central bank has strict incentives to sustain the currency peg and build a reputation in previous "attacking" periods. Hence, the onset of a currency attack depends on the entire learning process. Second, once speculators attack, they attack frequently and infinitely often. Otherwise, the central bank defends in some "attacking" period for sure.

Besides the above theoretical contributions, this paper provides potential explanations for some empirical facts in currency crises. First, some currency pegs were believed to be unsustainable by speculators before the onset of the attack. For example, the European Exchange Rate Mechanism was "ripe" for attack at least two years before the attack occurred (Eichengreen and Wyplosz, 1993). In my model, a common belief that the central bank has a low value in some period is necessary but not sufficient for attacks in an equilibrium. In particular, even though in that period there is a common belief among speculators that the central bank has a low value, they cannot attack in any equilibrium if the learning speed is slow in subsequent periods. This is due to the central bank's strict incentive to defend. Second, in many currency crises, the central bank first defends the currency peg for a period of time and then abandons it. For example, in 1992, the Bank of Itay first defended the lira but eventually gave up. Previous works (Kurlat, 2010; Goldstein, Ozdenoren, and Yuan, 2011) explain this phenomenon by assuming that the central bank is facing uncertainties. In my model, the central bank is perfectly informed, and it will randomize when facing attacks on the equilibrium path. Therefore, this empirical fact is just an outcome of an equilibrium with attacks. Third, the policy measure to curb speculative attacks by "throwing sand into the excessively well-oiled wheels" of international finance (Eichengreen, Tobin, and Wyplosz, 1995) is supported by the result of my paper. An increase in the transaction cost in the foreign exchange market not only decreases the speculators' returns from a devaluation outcome, but also increases the lower bound of the central bank's reputation building speed. As a result, given the learning speed, it is more likely that the reputation building effect dominates the learning effect. Consequently, an increase in the transaction cost can ease the speculative pressure.

Though this paper focuses on defending against speculative currency attacks, the model can be easily applied to other environments. In real markets, the gold standard (Henderson and Salant, 1978; Salant, 1983), the unallocated cumulative catch quotas in fisheries (Gaudet, Moreaux, and Salant, 2002), and the unallocated "stock quotas" on autos and H1B visas (Gaudet and Salant, 2003) have been documented as targets of speculators. In political economy (for example, Edmond (2011)), a dictator and revolutionaries play the roles of

the central bank and speculators in the model, respectively. Recently, there has been an interest in potential speculative attacks on the European central bank's bailout policy to help countries in the euro zone with their sovereign debt crises. The results in this paper suggest that the interaction between the European central bank's reputation building and speculators' learning plays a critical role in determining whether attacks on the bailout policy occur and whether this policy is sustainable.

1.1 Previous Works on Currency Attacks

The first generation currency crisis models (Krugman, 1979; Flood and Garber, 1984; and Broner, 2008) treat a currency attack as a run on the central bank's foreign reserves. Though these models have nice features, they have difficulty in explaining the timing of attacks and jumps in the exchange rate. More important, they assume that the central bank takes no steps to alleviate the currency crisis.

The problems of the first generation currency crisis models led to the development of second generation models. Obstfeld (1996) analyzes a complete information model, in which multiple equilibria exist because of the coordination motive among speculators. Morris and Shin (1998) apply global games (introduced by Carlsson and van Damme (1993); see also Morris and Shin (2003)) to currency attacks, in which speculators observe the relevant fundamentals with small noises and show that there exists a unique equilibrium as the noise diminishes. Angeletos, Hellwig, and Pavan (2006), and Angeletos and Pavan (2011) demonstrate that the signaling effects of preemptive instruments lead to multiple equilibria. Goldstein, Ozdenoren, and Yuan (2011) uncover the informational complementarity among speculators, because the central bank is uncertain about its benefit from the fixed exchange rate and thus learns from the market. These models highlight the central bank's limited commitment to sustaining the currency peg and the coordination feature among speculators. However, these works all employ static environments, so they cannot help to analyze the interaction between the central bank's reputation building and speculators' individual learning.

Angeletos, Hellwig, and Pavan (2007) analyze the effects of individual learning on the dynamics of coordination in a dynamic regime change game, in which the central bank behaves myopically.[1] By abstracting away the central bank's incentive to build a reputation, they show that speculators' individual learning is sufficient for multiple equilibria with

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[1]Dynamic regime change games are applications of dynamic global games. Dasgupta, Steiner, and Stewart (2010) analyze the individual learning effect in a dynamic global game with asynchronous coordination. Huang (2011) studies the social learning effect on dynamics of coordination in a dynamic global game. Other papers contributing to this growing literature include Giannitsarou and Toxvaerd (2007) and Heidhues and Melissas (2006).

attacks at some prior beliefs.[2] Because the central bank is myopic in their model, they don't analyze the effects of the central bank's reputation building on speculators' ability to coordinate and the eventual devaluation outcome.

1.2 Other Related Literature

This paper contributes to the reputation literature. Wiseman (2009) studies the reputation bound of an informed player with uninformed players exogenously learning in a repeated chain store game. He establishes a lower bound of the chain store's equilibrium payoff when the precision of exogenous signals is small, which is strictly smaller than the Stackelberg payoff. Though he does not show the tightness of the established lower bound, many equilibria with the informed player's payoff lower than the Stackelberg payoff can be constructed. My model differs from Wiseman (2009) by highlighting the coordination motives among speculators. Hence, in my model, when learning is slow, the reputation bound equals the Stackelberg payoff. Furthermore, this reputation bound is established without requiring the central bank to be arbitrarily patient.

Because of the coordination motive among speculators, a common belief among speculators is necessary for attacks. The common belief concept is introduced by Monderer and Samet (1989) and generalized by Morris and Shin (2007). Since speculators learn to form common beliefs, this paper is related to Cripps, Ely, Mailath, and Samuelson (2008), who provide general conditions for common learning. All these theoretical works focus on economies with a finite number of players. And without the central bank's incentive to build a reputation, a common belief is sufficient for attacks in these papers. Complementing these works, I define and apply the common belief concept and the common learning concept among a continuum of speculators. I show that a common belief is not sufficient when there is a central bank that has an incentive to build a reputation.

The remainder of the paper is organized as follows. In Section 2, I present the model of defending against speculative attacks. I then first analyze the equilibrium behaviors of the policy maker in Section 3. Given candidate equilibrium actions of the policy maker, speculators play a dynamic regime change game with an exogenous regime change rule. Section 4 is devoted to the analysis of such an "induced" game. In Section 5, I characterize the equilibrium of the model of defending against speculative attacks and show how the interaction between reputation and learning determines the outcome of the model. In Section 6, I discuss some related issues and how my model differs from closely related papers. Section

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[2]In Angeletos, Hellwig, and Pavan (2007), from the second period on, speculators play a dynamic coordination game similar to the induced game of my model in Section 4. But the results in their model differ from those in my induced game. The key reason is that in their model, the state space is a continuum, so individual learning is not strong enough for speculators to coordinate at some prior beliefs. See Subsection 6.3 for a detailed discussion.

7 concludes. All omitted proofs are presented in the Appendix.

2 Defending a Regime Against Attacks

Time is discrete and is indexed by $t \in \{1, 2, \dots\}$. The game starts with the status quo in place.[3] There is a continuum of long-lived speculators of measure 1, indexed by i and uniformly distributed over $[0, 1]$. In any period t, speculator i ($i \in [0, 1]$) chooses between attacking the status quo or not. Denote by $a_{it} = 1$ that speculator i attacks in period t and by $a_{it} = 0$ otherwise. The size of attacks in period t is defined as the measure of speculators attacking. Let A_t denote the size of attacks in period t, then $A_t = \int_0^1 a_{it} di$. In every period, after observing the size of attacks in that period, a policy maker decides whether to sustain the status quo or abandon it. The game continues as long as the status quo is in place and ends once the policy maker abandons the status quo.

2.1 Payoffs

In period t, any speculator's flow payoff depends on his own action and the regime change outcome in that period. The flow payoff from not attacking is normalized to be 0, whether the status quo is abandoned or not. If speculator i attacks in period t, he receives $1 - c$ if the status quo is abandoned in period t and $-c$ otherwise. Here, $c \in (0, 1)$ is the transaction cost of attacking.

The policy maker receives a period benefit from maintaining the status quo, θ. But in order to maintain the status quo in period t, the policy maker needs to pay a cost A_t, which is just the size of attacks in period t.[4] So the net period t payoff of the policy maker from maintaining the status quo is $\theta - A_t$. If the policy maker abandons the status quo in period t, her period t payoff is 0.

Assume that all agents in this model share a common discount factor $\delta \in (0, 1)$. Then the average discounted payoff of a speculator i is:

$$
v_i = \begin{cases} (1 - \delta) \left[\sum_{t=1}^{T-1} \delta^{t-1}(-c)a_{it} + \delta^{T-1}(1 - c)a_{iT} \right], & \text{if the regime changes in period } T; \\ (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1}a_{it}(-c), & \text{if the regime never changes,} \end{cases}
$$

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[3]Because the model has various applications, I use general terminologies of regime change games. In currency attacks, the currency peg is the status quo in the model. So the event in which the central bank abandons the currency peg is called a "regime change."

[4]Generalizing the cost of defending to be a function of the size of the attacks $C(A_t)$ with $0 \le C(A_t) \le C(1) < H$ would not change the results in this paper.

and the average discounted payoff of the policy maker is:

$$
u = \begin{cases} (1-\delta) \sum_{t=1}^{T-1} \delta^{t-1}(\theta - A_t), & \text{if the regime changes in period } T; \\ (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1}(\theta - A_t), & \text{if the regime never changes.} \end{cases}
$$

2.2 Information

The policy maker's period benefit from maintaining the status quo, θ, is drawn from the set $\Theta \equiv \{L, H\}$ at the beginning of the game, where $0 < L < 1 < H$. All agents share a common prior belief about $\theta = L$, denoted by $\mu_1 = \Pr(\theta = L)$. Once picked, θ is fixed. The policy maker knows the picked θ as her private information. No speculator knows θ.

In any period t, before making his decision, speculator i observes a private signal $z_{it} = \theta + \xi_{it}$ about θ. Assume $\xi_{it} \sim \mathcal{N}(0, 1/\eta_t)$ is independent of θ, independent and identically distributed across i, and serially uncorrelated. That is, speculators observe conditionally independent signals in every period. Let z_i^t denote speculator i's private signals up to period t. Then in period t, all speculators simultaneously make decisions after observing their own private signals up to period t. The policy maker, observing the size of attacks A_t, then chooses to maintain the status quo or abandon it. Neither individual nor aggregate actions are observed by speculators; hence the only public information at the beginning of period t is that the status quo is still in place.

2.3 Equilibrium

The policy maker decides whether to maintain the status quo in period t based on past and current sizes of attacks and her type. Hence the policy maker's strategy is a mapping from her type and the attacking history to a real number in $[0, 1]$. So $s_2(\theta, \{A_\tau\}_{\tau=1}^{t})$ is the probability that the type θ policy maker maintains the status quo in period t, given the attacking history $\{A_\tau\}_{\tau=1}^{t}$.

The private history of a speculator i consists of his own private signals and past actions. Denote a typical history that any speculator observes before he makes the decision in period t by $h^t \in \mathbb{R}^t \times \{0, 1\}^{t-1}$ ($h^1 \in \mathbb{R}$ is just a speculator's private signal in the first period). Let $H = \cup_{t=1}^{\infty} h^t$ be the set of all relevant histories. Then any speculator i's strategy is defined as $s_i : H \to [0, 1]$, that is, $s_i(h^t)$ is the probability that speculator i attacks in period t, given his private history h^t.

The solution concept of this game is perfect Bayesian equilibrium (PBE). Some special features of this game simplify the definition of a PBE. So it is helpful to first analyze these features to get a simplified definition of a PBE for this game. First, because there is a continuum of speculators, any individual speculator is so "small" that his action cannot

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affect the current and future sizes of attacks. Hence, given the policy maker's strategy and other speculators' strategies, any individual speculator's action does not affect the time when the regime changes. As a result, a strategy of speculator i is part of a PBE, if and only if it prescribes an action after any history h_i^t to maximize his period t flow payoff. That is, in a PBE, any speculator behaves "myopically."

Second, define speculator i's *private belief* about $\theta = L$ in period t to be the belief formed after the history h_i^t. Besides the private history h_i^t, speculator i also makes inferences from the fact that the status quo is in place at the beginning of period t. In particular, since there is a continuum of speculators, fix a strategy profile, conditional on θ, the size of attacks in any period t is a deterministic number, $A_t(\theta)$. Then based on the policy maker's strategy, speculators update their beliefs. Since speculators share a common prior, in a PBE, their updated beliefs based just on the public history must be the same. Call this belief the *public belief*, and denote the period t public belief about $\theta = L$ by μ_t. Then in a PBE, any speculator i's private belief in period t has a common component and a private component. The common component is μ_t, the public belief about $\theta = L$. And the private component is his private history h_i^t. Denote speculator i's private belief about $\theta = L$ in period t by $\rho^{\mu_t}(h_i^t)$. Because speculators behave myopically in a PBE, their period t equilibrium actions depend only on the public belief about L in period t and their own private history up to period t.

Third, the policy maker is sequentially rational in a PBE. That is, after any attacking history $\{A_\tau\}_{\tau=1}^t$, the policy maker's equilibrium action has to maximize her continuation average discounted payoff. But given a public belief μ_t, the past attacks $\{A_\tau\}_{\tau=1}^{t-1}$ do not affect future plays. Hence, the policy maker's equilibrium continuation strategy in any period t depends only on θ (her type), μ_t (the public belief), and A_t (the size of attacks in period t).

Finally in a PBE, given the associated public belief μ_t, no regime change in period t is always on the equilibrium path unless the policy maker chooses to abandon the status quo in period t for all θ. But $H > 1$ and the largest possible cost incurred in sustaining the status quo is 1 (because the total measure of speculators is 1), so always maintaining the status quo is the unique dominant strategy of the policy maker with $\theta = H$. This implies that abandoning the status quo for all θ in period t is not a part of a PBE. As a result, in a PBE no speculator has information sets off the equilibrium path.

Definition 1 *A strategy profile $s = (s_i)_{i \in [0,1] \cup \{2\}}$ and a public belief system $\{\mu_t\}_t$ constitute a perfect Bayesian equilibrium if*

1. *given $(s_i)_{i \in [0,1]}$ and $\{\mu_t\}_t$, $s_2(\theta)$ prescribes a strategy after any attacking history with associated (μ_t, A_t) to maximize the type θ policy maker's continuation average discounted payoff, $\forall \theta \in \Theta$;*

2. *Given s_2 and other speculators' strategies, in any period t with associated μ_t, $s_i(h_i^t)$ solves the following maximization problem for any h_i^t:*

$$\max_{a \in [0,1]} \left\{ \left[(1 - s_2(L, \mu_t, A_t(L)))\rho^{\mu_t}(h_i^t) + (1 - s_2(H, \mu_t, A_t(H)))(1 - \rho^{\mu_t}(h_i^t)) - c \right] a \right\};$$

3. *Given s, $\{\mu_t\}_t$ is calculated by Bayes' rule on the path of play.*

3   The Policy Maker's Reputation

Because always maintaining the status quo is the unique dominant strategy for the policy maker when $\theta = H$, the model is like a reputation model in which $\theta = H$ is the commitment type. Therefore, we say that the policy maker is of a "strong" type if $\theta = H$ and is of a "weak" type if $\theta = L$. In an equilibrium, the strong policy maker always defends the status quo against any speculative attacks. Then how about the weak policy maker? In any equilibrium, if the size of attacks is smaller than L in any period t, the weak policy maker would like to maintain the status quo. Because by sustaining the status quo, the weak policy maker receives a positive flow payoff in period t (since $L > A_t(L)$) and non-negative continuation payoffs (since she can always abandon the status quo in period $t+1$), maintaining the status quo in period t with $A_t(L) < L$ dominates abandoning it.

The interesting case is when $A_t(L) \geq L$. Since the cost incurred in sustaining the status quo outweighs the benefit from the status quo, it is optimal for a myopic weak policy maker to abandon the status quo. But in the model, the policy maker would take into account her future payoffs when making the current decision. The following lemma shows that in any equilibrium, if $A_t(L) \geq L$ on the equilibrium path, the weak policy maker will randomize, provided that she is sufficiently patient.

Lemma 1 *Fix any $\delta \in (1 - L, 1)$. In any equilibrium, the weak policy maker maintains the status quo with probability $q_t \in (0, 1 - c)$ in period t after $A_t(L) \geq L$ on the equilibrium path.*

The intuition about the randomization of the weak policy maker when $A_t(L) \geq L$ in any equilibrium (provided that δ is sufficiently large) follows the argument in the reputation literature (Fudenberg and Levine, 1989). On the one hand, in an equilibrium, if the probability that the weak policy maker maintains the status quo is high, the expected payoff of any speculator from attacking would be less than the attacking cost. Therefore, the size of attacks is 0 (smaller than L). On the other hand, if the weak policy maker abandons the status quo for sure when $A_t(L) \geq L$, by deviating to maintain the status quo, she can quickly signal herself as a strong policy maker, so that she can deter all future attacks. When the policy maker is patient enough ($\delta > 1 - L$), this deviation is profitable.

When $A_t(L) \geq L$, Lemma 1 not only describes the weak policy maker's behavior on the equilibrium path, but also helps to pin down the continuation payoff of the weak policy maker in period t. Because abandoning the status quo brings the weak policy maker a 0 average discounted payoff, the fact that the weak policy maker randomizes on the equilibrium path when $A_t(L) \geq L$ implies that her equilibrium average discounted payoff in period t is 0. Then fix the continuation strategy profile, after any $A'_t > A_t(L)$, the sequential rationality requires the weak policy maker to abandon the status quo in period t, because she will receive a negative average discounted payoff by maintaining the status quo. Similarly, for all $A'_t < A_t(L)$, the weak policy maker will maintain the status quo for sure.[5] In another case of $A_t(L) < L$ on the equilibrium path, the weak policy maker maintains the status quo for sure in the equilibrium. Her action after any off-equilibrium size of attacks A'_t is also pinned down by the continuation strategy profile. In this case, the policy maker's decision rule in period t takes one of the following two forms: (i) if sustaining the status quo brings a positive average discounted payoff when the size of attacks is 1, the policy maker will maintain the status quo for sure for any A'_t; (ii) if there is $\hat{A}_t$ such that maintaining the status quo after $\hat{A}_t$ brings a zero average discounted payoff, the weak policy maker maintains the status quo for sure for all $A'_t < \hat{A}_t$, maintains the status quo with probability $q_t \in [0,1]$ when $A'_t = \hat{A}_t$, and abandons the status quo for sure for all $A'_t > \hat{A}_t$.

To sum up, fix the continuation strategy profile, the weak policy maker's equilibrium strategy is in the following form: assume the weak policy maker's average discounted payoff from maintaining the status quo after $\hat{A}_t(L)$ is 0, then

$$
s_2(L, \mu_t, A'_t) = \begin{cases} 1, & \text{if } A'_t < \hat{A}_t(L); \\ q_t, & \text{if } A'_t = \hat{A}_t(L); \\ 0, & \text{if } A'_t > \hat{A}_t(L). \end{cases}
$$

In addition, if $\hat{A}_t$ is the equilibrium size of attacks in period t, $q_t \in (0, 1-c)$ after $A'_t = \hat{A}_t$. Otherwise, $q_t \in [0,1]$.

Lemma 1 also implies that if the weak policy maker maintains the status quo in period t when facing attacks with size $A_t(L) \geq L$, the public belief updates according to the Bayes' rule:

$$
\mu_{t+1} = \frac{\mu_t q_t}{\mu_t q_t + (1 - \mu_t)}.
$$

Since $q_t < 1 - c$, $\mu_{t+1} < \mu_t$. Define the policy maker's *reputation* as the public belief about $\theta = H$, then the weak policy maker can build her reputation by defending the status

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[5]The size of attacks $A'_t \neq A_t$ cannot be reached by any individual speculator's unilateral deviation, but the weak policy maker needs to play optimally after A'_t, given the continuation strategy profile. Since the aggregate actions of speculators are not observable to speculators, they will play as if the size of attacks in period t is A_t. Therefore, the continuation payoff from maintaining the status quo is strictly positive if $A'_t < A_t$ and is strictly negative if $A'_t > A_t$.

quo against attacks. The higher the policy maker's reputation is, the less likely it is that speculators will attack in the future. As a result, the weak policy maker has an incentive to mimic the strong policy maker, which provides the weak policy maker some commitment power.

4   Common Learning Among Speculators

In this section, I analyze speculators' equilibrium behaviors. Suppose a strategy profile of speculators is part of an equilibrium, and A_t is the equilibrium size of attacks in period t. Then the corresponding equilibrium action of the weak policy maker is

$$s_2(L, \mu_t, A_t(L)) = \begin{cases} 1, & \text{if} \quad A_t(L) < L; \\ q_t \in (0, 1-c), & \text{if} \quad A_t(L) \geq L. \end{cases}$$

That is, given a relevant candidate equilibrium strategy of the policy maker, speculators know that if the size of attacks is less than the benefit θ, the regime does not change; if the size of attacks is greater than or equal to the benefit (this is true only if $\theta = L$), the regime changes with probability q_t. Because if $A_t = 0$ in period t, q_t could be any number in $[0, 1]$ for a possible size of attacks greater than or equal to L, let's first consider a general sequence $\{q_t\}_t$, with $q_t \in (0, 1]$ for all t. Then the regime change rule induced by the candidate equilibrium action of the policy maker is as follows (denote by $\hat{\tau}$ the time at which the regime changes):

$$\Pr(\hat{\tau} = t | \hat{\tau} \geq t) = \begin{cases} 0, & \text{if} \quad A_t < \theta; \\ 1 - q_t, & \text{if} \quad A_t \geq \theta. \end{cases}$$

Taking this regime change rule as exogenously given, the game played by speculators is called the "induced" game.

Consider the strategy profile in which no speculator attacks, no matter what the private history is. Because $A_t = 0$ for all t on the path of play, according to the regime change rule, the status quo will be in place forever. Hence, it is best for any speculator not to attack. Therefore, the strategy profile without attacks is an equilibrium (call it the *no attack equilibrium*). This pure coordination failure equilibrium directly follows from the continuum speculators assumption. Then, is there any equilibrium with attacks in the induced game? If so, when do attacks happen? How about the regime change outcome in such an equilibrium?

4.1   Conditions of Attacking

Any speculator i's equilibrium choice in any period t depends on both his private signals and his past actions. Such a dependence on private histories makes equilibrium strategies rather complicated. However, the following Lemma 2 shows that in any equilibrium, speculators'

strategies have a simple form. Denote the weighted average mean of speculator i's private signals up to period t by x_{it}, then the sequence $\{x_{it}\}_t$ is defined in the following two steps. First, let $\beta_t = \sum_{\tau=1}^{t} \eta_\tau$, so β_t parameterizes the precision of any speculator's private information accumulated up to period t. Then let $x_{i1} = z_{i1}$ and recursively define $x_{it+1} = \frac{\beta_t}{\beta_{t+1}} x_{it} + \frac{\eta_{t+1}}{\beta_{t+1}} z_{it+1}$. From the standard Gaussian updating formula, conditional on θ, $x_{it} \sim \mathcal{N}(\theta, 1/\beta_t)$. And x_{it} is the sufficient statistic of z_i^t about θ.

Lemma 2 *In any equilibrium, speculators employ symmetric cutoff rules in every period. In particular, any equilibrium is characterized by a sequence $\{x_t^*\}_{t=1}^{\infty}$ with $x_t^* \in \mathbb{R} \cup \{-\infty\}$, and any speculator attacks in period t if and only if $x_{it} \leq x_t^*$.*

So in any equilibrium, any speculator i's decision depends only on the sufficient statistic of his private signals. This is so both because private actions are not informative about θ and because the sufficient statistic leads to the same private belief as private signals do. Hence, in any equilibrium with the associated public belief system $\{\mu_t\}_t$, $\rho^{\mu_t}(h_i^t) = \rho^{\mu_t}(x_{it})$ ($\forall i$ and $\forall t$). In the no attack equilibrium, $x_t^* = -\infty$ for all t. Therefore, the question whether there is an equilibrium with attacks in the induced game can be formulated as the problem of whether there is a sequence $\{x_t^*\}_t$ such that $x_t^* \in \mathbb{R}$ for some t and x_t^* is the speculators' equilibrium threshold point in period t for all t.

Denote by $\tilde{x}_t$ the L-quantile of the distribution of x_t, conditional on $\theta = L$. Then

$$\Pr(x_t \leq \tilde{x}_t | \theta = L) = \Phi(\sqrt{\beta_t}(\tilde{x}_t - L)) = L,$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution. So conditional on $\theta = L$, in period t, the measure of speculators who have the sufficient statistic lower than or equal to $\tilde{x}_t$ is exactly L.

Lemma 3 *If the induced game has an equilibrium in which the associated public belief in period t is μ_t and conditional on $\hat{\tau} \geq t$, a positive measure of speculators attack in period t, then*

$$\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1 - q_t}. \tag{1}$$

Define the cutoff speculator in period t to be the speculator whose private statistic is x when other speculators are employing the symmetric cutoff strategy with threshold point x. Then the cutoff speculator's expected payoff from attacking in period t with associated public belief μ_t is

$$g(x, \mu_t) = \rho^{\mu_t}(x)\chi(\Pr(x_t \leq x | \theta = L) \geq L)(1 - q_t) - c,$$

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where $\chi(\cdot)$ is the indicator function. To interpret this, first note that the regime changes only if $\theta = L$ on which the cutoff speculator has private belief $\rho^{\mu_t}(x)$. Conditional on $\theta = L$, the regime changes with probability $1 - q_t$ if and only if there are at least L measure speculators attacking. Because any other speculator attacks if and only if his private statistic is less than or equal to x, the measure of speculators attacking in period t conditional on $\theta = L$ is $\Pr(x_t \le x | \theta = L)$.

Then the intuition of Lemma 3 is illustrated in Figure 1. In the induced game, if there exists an equilibrium with an attack in period t, the cutoff speculator is the marginal speculator, who is indifferent between attacking and not attacking in period t. Because refraining from attacking brings a speculator 0 payoff, the cutoff speculator's expected payoff from attacking is 0 in an equilibrium with an attack in period t. That is, in an induced game, if there exists an equilibrium with an attack in period t, $g(x, \mu_t) = 0$ must have a real root. If $x < \tilde{x}_t$, less than L measure speculators attack, so $\chi(\Pr(x_t \le x | \theta = L) \ge L) = 0$, which in turn implies that $g(x, \mu_t) = -c < 0$ independent of the public belief. When $x \ge \tilde{x}_t$, at least L measure speculators attack, so $\chi(\Pr(x_t \le x | \theta = L) \ge L)(1-q_t)\rho^{\mu_t}(x) - c = (1-q_t)\rho^{\mu_t}(x) - c$. Because $\rho^{\mu_t}(x)$ is continuous and strictly decreasing in x, and $\lim_{x \to \infty} \rho^{\mu_t}(x) = 0$, the necessary condition for the existence of a solution to $g(x, \mu_t) = 0$ is $\max_{x \ge \tilde{x}_t} \rho^{\mu_t}(x) = \rho^{\mu_t}(\tilde{x}_t) \ge \frac{c}{1-q_t}$. Figure 1 illustrates the cutoff speculator's expected payoff from attacking in period t with three different public beliefs $\mu_t > \mu'_t > \mu''_t$. They share a common part when $x < \tilde{x}_t$. When $x \ge \tilde{x}_t$, whether the curve $g(x, \mu)$ intersects the x-axis depends on the public belief. Because the private belief is strictly increasing in the public belief, when $\mu = \mu_t$ or $\mu = \mu'_t$, $g(x, \mu) = 0$ has a solution; when $\mu = \mu''_t$, there is no solution to $g(x, \mu) = 0$.
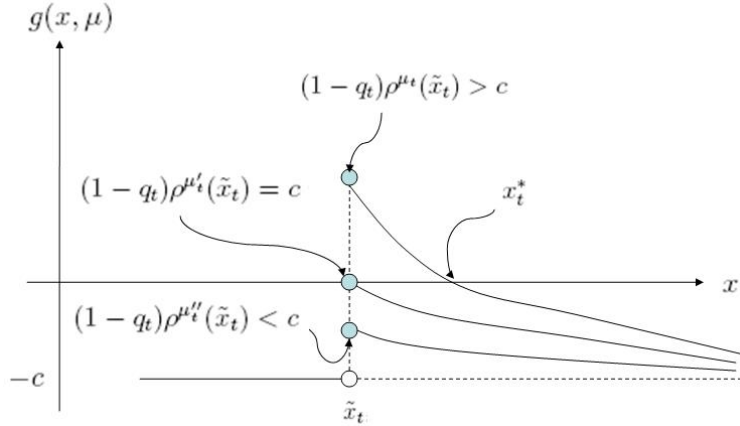

Figure 1: Cutoff speculator's expected payoff from attacking ($\mu_t > \mu'_t > \mu''_t$).

Figure 1 also suggests a sufficient condition for the existence of an equilibrium in which a positive measure of speculators attack in period t.

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Lemma 4 *If $\rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1-q_t}$, the induced game has an equilibrium in which a positive measure of speculators attack in period t.*

I prove this lemma in the appendix by construction. The key point of the construction is that if no speculator chooses to attack in any period τ, the regime does not change for sure, no matter whether $\theta = L$ or $\theta = H$. Therefore, the public belief does not change. So if speculators do not attack until period t, the public belief in period t equals μ_1. Then the condition of this lemma guarantees the possibility of attacks in period t.

4.2 Common Beliefs and Common Learning

Given the exogenous regime change rule, the game played by speculators features coordination motives in every period. However, because the regime does not change when $\theta = H$, speculators are uncertain about the coordination result. As a result, in an equilibrium in the induced game, if a speculator attacks in period t, his private belief about $\theta = L$ is at least $\frac{c}{1-q_t}$. But because of the coordination motive, this speculator needs to form a belief about other speculators' beliefs, form a belief about other speculators' beliefs about other speculators' beliefs, and so on. This infinite hierarchy of beliefs is called *common belief* (Monderer and Samet, 1989). In this subsection, I follow Morris and Shin (2007) to define a version of common belief in the model and apply this concept to the analysis of speculators' behaviors.

Consider the conditions for speculator i to attack. Because speculator i behaves "myopically," he attacks if and only if he believes that the regime changes with a probability of at least c. If $\theta = H$, the regime does not change. And conditional on $\theta = L$, the regime changes only if the size of attacks is greater than or equal to L. Conditional on the joint event $\theta = L$ and $A_t(L) \geq L$, the regime changes with probability $1 - q_t$. Therefore, if speculator i attacks, his private belief about the joint event $\theta = L$ and $A_t(L) \geq L$ is at least $\frac{c}{1-q_t}$. Because $A_t(L) \geq L$ only if at least L measure speculators $\frac{c}{1-q_t}$-believe the joint event, at least L measure speculators $\frac{c}{1-q_t}$-believe that at least L measure speculators $\frac{c}{1-q_t}$-believe the joint event, and so on. Therefore, speculator i attacks only if he $\frac{c}{1-q_t}$-believes the entire list of following events:[6]

1. $\theta = L$;

2. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe $\theta = L$;

3. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe statement (2);

4. when $\theta = L$, at least L measure speculators $\frac{c}{1-q_t}$-believe statement (3);

---

[6]As in Monderer and Samet (1989), a player p-believes an event if his posterior belief about the event is at least p.

5. ...

If when $\theta = L$, there are at least L measure speculators $\frac{c}{1-q_t}$-believe the above entire list of events, there is a *common* $(L, \frac{c}{1-q_t})$-*belief about* L among speculators.

Proposition 1 shows that if attacks happen in period t in an equilibrium, then there is a common $(L, \frac{c}{1-q_t})$-belief among speculators.

Proposition 1 *If the induced game has an equilibrium in which the associated public belief in period t is μ_t and conditional on $\hat{\tau} \geq t$, a positive measure of speculators attack in period t, then there is a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$ among speculators in period t.*

Proof. Because there is a continuum of speculators, a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$ is equivalent to the event that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$ when $\theta = L$. To see this, consider any speculator i's private belief in period t about the whole list of events:

$$\Pr(\theta = L, \text{statement 2, statement 3}, \ldots | x_{it})$$
$$= \Pr(\text{statement 3}, \ldots | \text{statement 2}, \theta = L, x_{it}) \Pr(\text{statement 2} | \theta = L, x_{it}) \Pr(\theta = L | x_{it}).$$

Obviously, $\Pr(\text{statement 3}, \ldots | \text{statement 2}, \theta = L, x_{it}) = 1$. Since sufficient statistics are conditionally independent, if there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$, $\Pr(\text{statement 2} | \theta = L, x_{it}) = \Pr(\text{statement 2} | \theta = L) = 1$. So if there are L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$, there is a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$. Conversely, if there are less than L measure of speculators $\frac{c}{1-q_t}$-believing $\theta = L$, any speculator i's private belief in period t about the whole list of events above is 0. So there is not a common $(L, \frac{c}{1-q_t})$-belief about L.

Then, from Lemma 3, I only need to show that the inequality (1) is equivalent to the event that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$. First, suppose $\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q}$. Because $\rho^{\mu_t}(x_t)$ is strictly decreasing in x_t, any speculator i with private sufficient statistic $x_{it} \leq \tilde{x}_t$ has a posterior belief greater than or equal to $\frac{c}{1-q_t}$. Then the definition of $\tilde{x}_t$ implies that there are at least L measure speculators $\frac{c}{1-q_t}$-believing $\theta = L$.

Now, suppose $\rho^{\mu_t}(\tilde{x}_t) < \frac{c}{1-q_t}$. Since $\rho^{\mu_t}(x_t)$ is continuous and strictly decreasing in x_t, $\exists\, \epsilon > 0$ such that $\rho^{\mu_t}(\tilde{x}_t - \epsilon) < \frac{c}{1-q_t}$. In addition, conditional on $\theta = L$, only speculators with private sufficient statistic less than $\tilde{x}_t - \epsilon$ have private beliefs about $\theta = L$ at least $\frac{c}{1-q_t}$. Then since $\Pr(x_t < \tilde{x}_t - \epsilon | \theta = L) < \Pr(x_t < \tilde{x}_t | \theta = L) = L$, a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ cannot be formed. ∎

Suppose $q_t < 1 - c$, whether there is a common $(L, \frac{c}{1-q})$-belief among speculators in period t depends on both the public belief (μ_t) and the precision of the accumulated private signals (β_t). Figure 2 shows that the $(\mu, 1/\beta)$-space is divided into a "no common
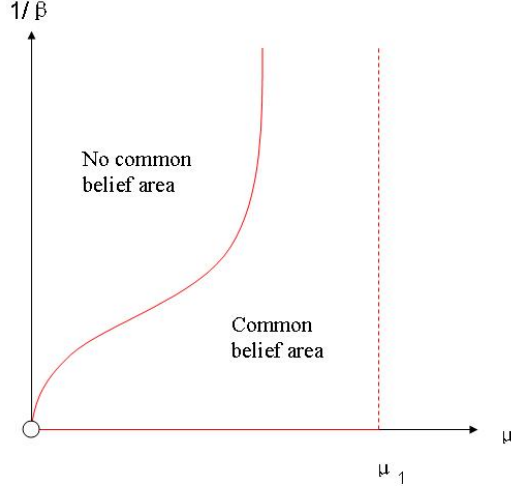
Figure 2: The $(\mu,\ 1/\beta)$-space.

belief half" and a "common belief half." On the one hand, fix β, because μ is the common component of speculators' private beliefs, the larger the μ is, the easier a common belief is formed. On the other hand, fix μ, as β increases, speculators know that their accumulated private information has a higher quality, they know that all other speculators know the accumulated private information has a higher quality, and so on. Because private statistics are conditionally independent, as β increases, it is easier for a common belief to form. In the northwest half of Figure 2, μ is small or β is small, so there is no common belief among speculators. Hence, if the $(\mu,\ \beta)$ combination is in the northwest half, speculators do not attack. In the southeast half, there is a common belief among speculators, so the necessary condition for an attack is satisfied.

Because a common $(L, \frac{c}{1-q_t})$-belief about $\theta = L$ is necessary for attacks in period t in the equilibrium, it is natural to ask how $\theta = L$ is commonly $(L, \frac{c}{1-q_t})$-believed. The answer to this question is "by learning." Each speculator collects one private signal in every period, hence his sufficient statistic is increasingly accurate. Given $p \in (0,1)$ and $\mu_1 \in (0,1)$, define $B_{it}^p(\theta) = \{x_{it} : \Pr^{\mu_1}(\theta|x_{it}) \geq p\}$ for any period t and any speculator i. Then $B_{it}^p(\theta)$ is the event that speculator i p-believes θ. Note that the public belief used by any speculator i to calculate his private beliefs in all periods is μ_1. That is, to form the private belief $\Pr^{\mu_1}(\theta|x_{it})$ about θ, speculator i ignores the information revealed from the public history $\hat{\tau} \geq t$.

Definition 2 *Speculator i learns $\theta \in \Theta$ individually, if for each $p \in (0,1)$, there is T such that for all $t > T$, $\Pr(B_{it}^p(\theta)|\theta) \geq p$. Speculator i learns Θ if he learns each $\theta \in \Theta$.*

The definition of *individual learning* is the same as that in Cripps, Ely, Mailath and Samuelson (2008). The following Lemma 5 provides a sufficient and necessary condition for any speculator to learn Θ.

Lemma 5 *Any speculator learns Θ individually, if and only if*

$$\lim_{t\to\infty} \beta_t = +\infty.$$

But individual learning about Θ may not be sufficient for attacks in an equilibrium, because the coordination motive requires common $(L, \frac{c}{1-q})$-belief about $\theta = L$. Intuitively, the notion of common learning in an economy consisting of a continuum of agents of measure 1 should be that after some period T, there is a common (p, p)-belief among speculators in every period for any $p \in (0, 1)$. However, this notion is too strong to be necessary for attacks in an equilibrium, because as shown in Lemma 4 that an attack happens in period t in an equilibrium if $\theta = L$ is common $(L, \frac{c}{1-q})$-belief in period t. So the common learning concept, which is defined as follows, is weaker than that in Cripps, Ely, Mailath and Samuelson (2008).

Definition 3 *Speculators $(L, \frac{c}{1-q_t})$-commonly learn $\theta = L$ in period t, if fixing the public belief at μ_1, there is a common $(L, \frac{c}{1-q_t})$-belief among speculators in period t.*

If speculators cannot individually learn Θ, that is, the sequence $\{\beta_t\}_t$ is bounded above by $\bar{\beta} < +\infty$, fixing $q_t = q$ sufficiently close to $1 - c$, speculators cannot $(L, \frac{c}{1-q})$-commonly learn L in any period t. Then the no attack equilibrium will be the unique equilibrium. So for the possibility of attacks in some equilibrium, I assume that speculators individually learn Θ, that is, $\lim_{t\to\infty} \beta_t = +\infty$. Lemma 6 below, together with Lemma 4, shows that if there is a subsequence of $\{q_t\}_t$ bounded above by some $\tilde{q} < 1 - c$, and speculators individually learn Θ, then there is an equilibrium in which attacks happen in some period T.

Lemma 6 *If there is $\tilde{q} < 1 - c$ such that $\{q_t\}_t$ has a subsequence bounded above by $\tilde{q}$, then individual learning implies common learning in some period T.*

4.3 Equilibrium of the Induced Game

The no attack equilibrium always exists in the induced game. According to the exogenous regime change rule, the regime does not change in the no attack equilibrium. Then what are conditions for the existence of an equilibrium with attacks? Suppose an equilibrium with attacks exists, what are the dynamics of attacks? Will the regime change when $\theta = L$?

Because of the flexibility of the sequence $\{q_t\}_t$, it is hard to get interesting conclusions in the induced game. Therefore, I focus on the case that the sequence $\{q_t\}_t$ has a subsequence bounded above by $\tilde{q} < 1 - c$.

Proposition 2 *Fix any $\mu_1 \in (0, 1)$, any sequence $\{q_t\}_t$ with a subsequence bounded above by $\tilde{q} < 1 - c$, and any strictly increasing and unbounded sequence $\{\beta_t\}_t$. With the exogenous regime change rule, multiple equilibria exist in the induced game:*

1. *the no attack equilibrium exists;*

2. *there exists an equilibrium with attacks in which there is T such that no attacks happen after period T if the status quo is in place at the end of period T;*

3. *there exists an equilibrium with attacks in which there is $t > T$ such that attacks happen in period t, for any T.*

The proof of Proposition 2 is illustrated in Figure 3 below. Suppose $q_t = \tilde{q} < 1 - c$, and β_1 is sufficiently large so that $\tilde{x}_1 \leq \frac{H+L}{2}$. Fixing an equilibrium, arrows indicate directions in which points move. The left graph of Figure 3 shows the no attack equilibrium. Since speculators do not attack, the public belief does not change. And as they accumulate private signals, the variance of the sufficient statistic goes to 0. Therefore, all arrows are going down in this graph. The middle graph of Figure 3 illustrates an equilibrium, in which speculators attack once and if the policy maker maintains the status quo when facing attacks, no speculator ever attacks again. Note attacks happen when the point $(\mu, 1/\beta)$ is in the "common belief half." And if an attack happens at some point $(\mu, 1/\beta)$, the arrow points to the southwest, because μ decreases and speculators keep learning. The right graph of Figure 3 illustrates an equilibrium in which speculators attack infinitely often. The key point here is the individual learning. If the initial point is in the "no common belief half," speculators cannot attack, so the public belief does not change. Then individual learning leads the path to cross the line $\tilde{\beta}(\mu)$, so attacks become possible.
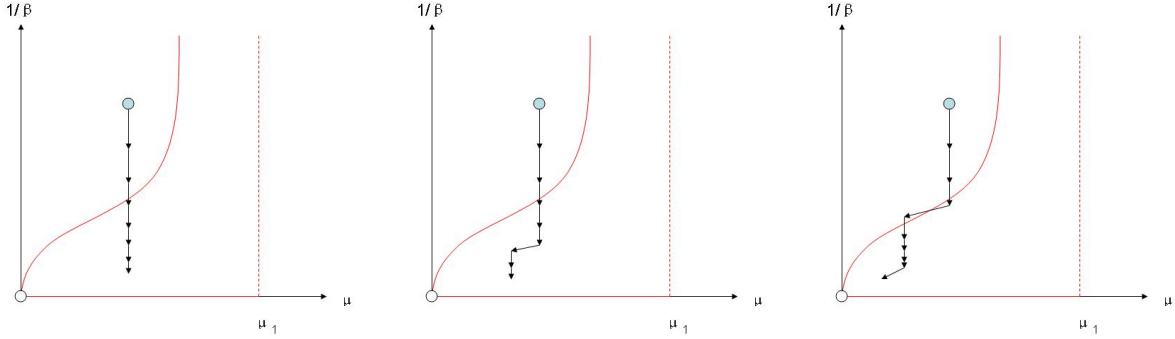

Figure 3: Multiple equilibria in the induced game.

In the no attack equilibrium and in any equilibrium in which speculators may terminate attacking in some finite period, speculators will learn the true state eventually. Hence, if $\theta = L$, they $(L, \frac{c}{1-\tilde{q}})$-commonly learn $\theta = L$ infinitely often. So no attack after some period is just due to the pure coordination failure. In these equilibria, even if $\theta = L$, with positive probability the regime does not change. Therefore, it is more interesting to analyze equilibria in which attacks happen infinitely often.

I first summarize three straightforward properties of an equilibrium in which specula-tors attack infinitely often. First, the exogenous regime change rule implies that conditional on $\theta = L$, if attacks happen in period t, $A_t(L) \geq L$. Otherwise, no speculator will choose to attack, because the regime changes with probability 0. Second, because speculators attack in-finitely often, if $\theta = L$, the regime changes with probability 1. Third, even if $\theta = H$, attacks happen infinitely often. This is so because fixing the public belief, speculators $(L, \frac{c}{1-\tilde{q}})$-commonly learn $\theta = L$ infinitely often even though the true state is H. In the following, I investigate two more equilibrium properties, which are significantly different from those of the model with the regime change outcomes endogenously determined by the policy maker.

Let $T_1(s)$ be the first period in which attacks happen in the equilibrium s in the induced game. Then $\min_s T_1(s)$ is pinned down by the first time there is a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ among speculators.

Corollary 1 *Suppose* $\lim_{t\to\infty} \beta_t = +\infty$, *then*

$$\min_s T_1(s) = \min_t \{t : \rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1 - q_t}\}.$$

Note for a fixed sequence $\{\beta_t\}_t$, Corollary 1 implies that $\min_s T_1(s)$ does not depend on how speculators learn after the period in which a common $(L, \frac{c}{1-q})$-belief about $\theta = L$ forms in the first time.

Fix an equilibrium s. Define $\mathcal{Q}(s) \subset \mathbb{N}$ such that in s, $A_t > 0$ if and only if $t \in \mathcal{Q}(s)$. That is, in the equilibrium s, $\mathcal{Q}(s)$ is the set of periods in which attacks happen.

Corollary 2 *Given any integer* $K \in \mathbb{N}$, *there is an equilibrium* s' *such that* $|T' - T| > K$ *for any* $T, T' \in \mathcal{Q}(s')$.

Suppose T and T' are two consecutive periods in which attacks happen. Then Corollary 2 implies that the number of periods between T and T' may be unbounded in an equilibrium. Call the periods between T and T' the *common learning phase*, since speculators only collect private signals and the public belief does not change. Note that for a fixed equilibrium, in some periods in the common learning phase, there is a common $(L, \frac{c}{1-q})$-belief about $\theta = L$, but speculators choose not to attack. An implication of this corollary is that the sequence of the sizes of attacks is not monotone in some equilibrium s'. For any three consecutive periods T, T', and T'' in $\mathcal{Q}(s')$, if $T' - T$ is sufficiently large and $T'' - T'$ is relative small, it is possible that $A_{T'} > A_T$ and $A_{T'} > A_{T''}$.

5   Reputation Versus Common Learning

Let's go back to the model where the regime change outcome is endogenously chosen by the policy maker. It is straightforward that the strategy profile in which the policy maker

always maintains the status quo and no speculator attacks is an equilibrium. Call this equilibrium the *no attack equilibrium.* In the no attack equilibrium, the type θ policy maker's average discounted payoff is θ, her largest feasible payoff (or the "Stackelberg payoff" in the reputation literature). Then are there equilibria with attacks? If so, what are the dynamics of attacking? What is the regime change outcome? And what is the lowest equilibrium payoff (the reputation bound) of the policy maker?

The analysis in section 3 shows that in any equilibrium with attacks, the policy maker's strategy must be in the following form: assume the weak policy maker's average discounted payoff from maintaining the status quo after $\hat{A}_t(L)$ is 0. Then

$$
s_2(L, \mu_t, A'_t) = \begin{cases} 1, & \text{if} \quad A'_t < \hat{A}_t(L); \\ q_t, & \text{if} \quad A'_t = \hat{A}_t(L); \\ 0, & \text{if} \quad A'_t > \hat{A}_t(L). \end{cases}
$$

In addition, if $\hat{A}_t$ is the equilibrium size of attacks in period t, $q_t \in (0, 1 - c)$ after $A'_t = \hat{A}_t$. Otherwise, $q_t \in [0, 1]$. Lemma 2 shows that in any equilibrium given the regime change rule induced by the policy maker's equilibrium actions, speculators employ the cutoff rule in every period and make decisions based only on the public belief and the private sufficient statistics. Furthermore, a slightly modified version of Proposition 1 shows that if attacks happen in period t in some equilibrium, then given the public belief μ_t, there must be a common (L, c)-belief about $\theta = L$ in period t. The requirement of a common (L, c)-belief about $\theta = L$ is due to the freedom of choosing $q_t \in (0, 1 - c)$. It seems that simply putting these two parts together, we can characterize all equilibria with attacks. Therefore, the equilibrium characterization should be very similar to Proposition 2, when speculators are assumed to be able to learn Θ individually. Is this generally true?

Let's first consider a strategy profile specifying (1) attacks happen, and (2) if the policy maker maintains the status quo in some period, no speculator ever attacks again.

Lemma 7 *Fix any $\delta \in (1 - L, 1)$. Consider a strategy profile with attacks. Suppose there is T such that speculators refrain from ever attacking again after period T, if the policy maker maintains the status quo at the end of period T. Then the strategy profile cannot be an equilibrium.*

Proof. Suppose there is an equilibrium s in which attacks happen and conditional on $\hat{\tau} > T$, no speculator attacks after period T. Without loss of generality, let $T = \max \mathcal{Q}(s)$, then T is the last period in which attacks happen. Because $A_t > 0$, $A_t(L) \geq L$. Therefore, the probability that the weak policy maker maintains the status quo in period T is $q_T < 1 - c$. Therefore, because abandoning the status quo brings the weak policy maker a 0 average discounted payoff in period T, the weak policy maker's average discounted payoff in period T is 0 on the equilibrium path.

Now consider the deviation of the policy maker in period T to $q_T = 1$. That is, the weak policy maker maintains the status quo for sure. By this deviation, $\hat{\tau} > T$. Since the deviation is not observable, no speculator chooses not to attack after period T. Then the weak policy maker's average discounted payoff in period T from this deviation is:

$$
\begin{aligned}
(1 - \delta)[(L - A_t) + L \sum_{\tau=1}^{\infty} \delta^\tau] \\
> \quad (1 - \delta)(L - 1) + \delta L \\
> \quad 0.
\end{aligned}
$$

Hence, this deviation is profitable. ∎

Lemma 7 implies that in any equilibrium, once attacks happen, speculators cannot terminate attacking. This is different from the second part of Proposition 2. In Proposition 2, the regime change rule is exogenous, so speculators do not attack after some period in some equilibrium. However, the regime change rule is endogenously chosen by the policy maker in the model. If speculators do not attack after some period T, the weak policy maker, who is sufficiently patient, will maintain the status quo for sure when facing attacks.

So whether there is an equilibrium with attacks is equivalent to whether there is an equilibrium in which speculators attack infinitely often. In any period t, if $A_t(L) \geq L$ and the policy maker maintains the status quo,

$$
\mu_{t+1} = \frac{\mu_t q_t}{\mu_t q_t + (1 - \mu_t)}.
$$

Since in any equilibrium with attacks, $q_t \in (0, 1 - c)$, so $\mu_{t+1} < \mu_t$. That is, if attacks happen in period t, and the status quo is in place at the end of period t, speculators believe that the policy maker is more likely to be strong. So by defending against attacks, the weak policy maker builds her reputation. On the other hand, however, speculators get more accurate information about the policy maker's type over time. This weakens the policy maker's incentive to build her reputation. An implicit assumption for this argument is that the common learning phase can be arbitrarily long. However, the following Lemma 8 shows that the number of periods in any common learning phase is uniformly bounded. Fix $\delta \in (1 - L, 1)$. Let $\bar{K}$ be the smallest integer such that

$$
(L - 1) + L \sum_{\tau=1}^{\bar{K}} \delta^\tau \geq 0.[7]
$$

---

[7]To see the existence of $\bar{K}$, note that $L - 1 < 0$ and that $(L - 1) + \frac{\delta L}{1 - \delta} > 0$. So $\bar{K} > 0$.

Lemma 8 *Fix any $\delta \in (1 - L, 1)$. Suppose there is an equilibrium s in which attacks happen in period t if and only if $t \in \mathcal{Q}(s)$. Then for any two consecutive periods T_n and T_{n+1} in $\mathcal{Q}(s)$, $T_{n+1} - T_n \leq \bar{K}$.*

Proof. Because attacks happen in period T_n and period T_{n+1} in the equilibrium s, the weak policy maker must randomize in these two periods. Furthermore, from Lemma 1, $q_{T_n} \in (0, 1 - c)$ and $q_{T_{n+1}} \in (0, 1 - c)$. Therefore, because abandoning the status quo always brings the policy maker 0 average discounted payoff, the weak policy maker's average discounted payoff is 0 in both period T_n and period T_{n+1}.

Now let's calculate the weak policy maker's average discounted payoff in period T_n from maintaining the status quo:

$$(1 - \delta)[(L - A_{T_n}(L)) + L \sum_{\tau=1}^{T_{n+1} - T_n - 1} \delta^\tau] + \delta^{T_{n+1} - T_n} 0 = 0.$$

Therefore,

$$
\begin{aligned}
0 &= (L - A_{T_n}(L)) + L \sum_{\tau=1}^{T_{n+1} - T_n - 1} \delta^\tau \\
&> (L - 1) + L \sum_{\tau=1}^{T_{n+1} - T_n - 1} \delta^\tau.
\end{aligned}
$$

So $\sum_{\tau=1}^{T_{n+1} - T_n - 1} \delta^\tau < \sum_{\tau=1}^{\bar{K}} \delta^\tau$ which implies that $T_{n+1} - T_n - 1 < \bar{K}$. Therefore, $T_{n+1} - T_n \leq \bar{K}$. Since this is true for all n, the claim is true. ∎

The fact that in any equilibrium common learning phases cannot be arbitrarily long is due to the weak policy maker's indifference between maintaining and abandoning the status quo when facing attacks. When speculators are in a common learning phase, while they acquire more accurate information about the policy maker's type, the policy maker is accumulating flow payoffs. Therefore, in order to make the policy maker indifferent at the beginning of a common learning phase, speculators have to attack again before the policy maker collects too many flow payoffs. This is different from Corollary 2, in which the regime change rule is exogenously given, so speculators do not need to make a policy maker randomize when she is facing attacks.

In any equilibrium with speculators attacking infinitely often, failed attacks decrease speculators' public belief about $\theta = L$. So considering the public history, the formed common (L, c)-belief about $\theta = L$ may be ruined. This happens especially when the incremental accuracy of private information cannot offset the discrete drop of the public belief due to the failed attacks. Therefore, speculators have to learn to form a common (L, c)-belief

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about $\theta = L$ again within a fixed number of periods. This suggests that the equilibrium characterization is determined by the comparison between the speed at which the public belief decreases and the speed at which speculators commonly learn, that is, the comparison between the policy maker's reputation building and speculators' common learning.

5.1 Slow Common Learning

The comparison between reputation and common learning is determined by three factors. First, attacks provide the policy maker chances to build her reputation. In addition, the more frequently attacks happen, the quicker the reputation is built. Once attacks begin, speculators have to attack again within $\bar{K}$ periods. So the reputation building speed is bounded below by $(1 - c)^{\frac{t - T_1}{\bar{K}}}$, because the probability of maintaining the status quo in any "attacking" period is bounded above by $(1 - c)$. Second, the accuracy of speculators' private sufficient statistics is strictly increasing. In a common learning phase, the policy maker cannot build her reputation, but speculators know more and more about her type. The learning speed is captured by the increasing rate of β, that is, the accuracy of new private signals. Third, β_{T_1} is the "stock" accuracy of speculators' private information, while the increments of β's are the "flow" accuracy. Though speculators can freely choose the "stock" accuracy before attacks happen (by coordinating not to attack until some T_1), the "flow" accuracy is given exogenously.
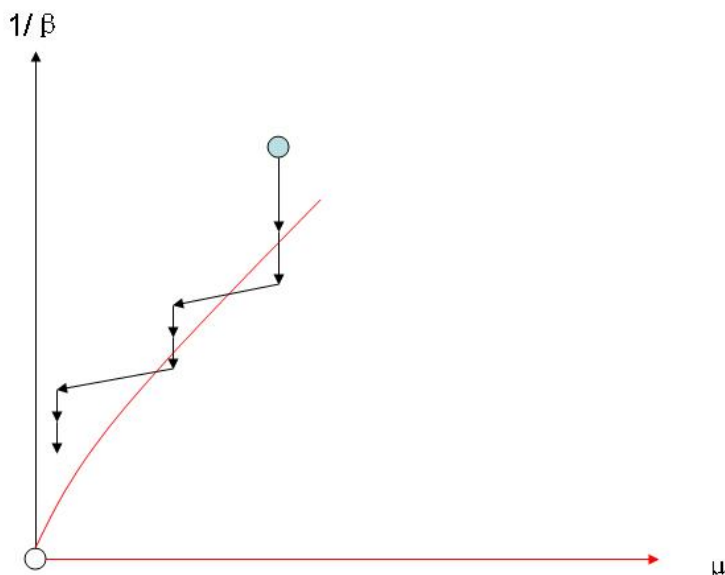

Figure 4: Reputation v.s. Common Learning.

Figure 4 shows the possibility that if speculators' learning speed is relatively slow, a strategy profile specifying attacks infinitely often cannot be an equilibrium. No matter what "stock" accuracy speculators choose (the initial point in the graph), it is possible

that the condition will move to the "no common belief half" in the $(\mu,\ 1/\beta)$-space. Then the slow learning speed implies that within $\bar{K}$ periods, the condition cannot move back to the "common belief half." That is, after an attack in some period T, it is impossible for speculators to attack before period $T + \bar{K}$. So the policy maker will deviate to maintain the status quo for sure in period T.

Proposition 3 below formalizes this argument and provides a sufficient condition for the uniqueness of the equilibrium. In particular, condition (2) below captures the above three factors: the numerator is the lower bound of the policy maker's reputation building speed, the denominator is speculators' common learning speed, and it is independent of the first period in which an attack happens.

Proposition 3 *Fix any $\delta \in (1-L,1)$. Suppose*

$$\lim_{t \to \infty} \frac{(1-c)^{\frac{t}{K}}}{\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}]} = 0. \tag{2}$$

There is no equilibrium in which attacks happen. In this case, the type θ policy maker's lowest equilibrium average discounted payoff is θ.

Proof. Lemma 7 implies that if attacks happen in an equilibrium s, $\mathcal{Q}(s)$ is unbounded. Let $T_1 \in \mathcal{Q}(s)$ be the first period in which attacks happen. Recall that $\Pr(x_t \leq \tilde{x}_t | \theta = L) = \Phi[\sqrt{\beta_t}(\tilde{x}_t - L)] = L$. So

$$\tilde{x}_t = \frac{\Phi^{-1}(L)}{\sqrt{\beta_t}} + L.$$

Define $\tilde{\mu}_t$ such that when the public belief is $\tilde{\mu}_t$, the speculator who has the private sufficient statistic $\tilde{x}_t$ forms the posterior belief $\rho^{\tilde{\mu}_t}(\tilde{x}_t) = c$. Then

$$\begin{aligned}
&\rho^{\tilde{\mu}_t}(\tilde{x}_t)\\
&= \frac{\tilde{\mu}_t \phi[\sqrt{\beta_t}(\tilde{x}_t - L)]}{\tilde{\mu}_t \phi[\sqrt{\beta_t}(\tilde{x}_t - L)] + (1-\tilde{\mu}_t)\phi[\sqrt{\beta_t}(\tilde{x}_t - H)]}\\
&= \frac{\tilde{\mu}_t \phi[\Phi^{-1}(L)]}{\tilde{\mu}_t \phi[\Phi^{-1}(L)] + (1-\tilde{\mu}_t)\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}]}\\
&= c.
\end{aligned}$$

Therefore, by rearranging terms, we have

$$\begin{aligned}
\tilde{\kappa}_t &\equiv \frac{\tilde{\mu}_t}{1 - \tilde{\mu}_t}\\
&= \frac{c}{(1-c)\phi[\Phi^{-1}(L)]}\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}].
\end{aligned}$$

Now define $\kappa_t \equiv \frac{\mu_t}{1-\mu_t}$, then $\mu_t \geq \tilde{\mu}_t$ if and only if $\kappa_t \geq \tilde{\kappa}_t$. For $t \leq T_1$, $\kappa_t = \kappa_1$. But by the beginning of any period $t > T_1$, Lemma 8 implies that attacks have happened at least Q

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times. Here Q is the smallest integer that is larger than or equal to $\frac{t-T_1}{K}$. Therefore,

$$
\begin{aligned}
\kappa_t &= \frac{\mu_t}{1-\mu_t} \\
&= \left(\frac{\mu_1}{1-\mu_1}\right) \prod_{\tau \in \mathcal{Q}(s) \cap \{\tau:\tau<t\}} q_\tau \\
&< \left(\frac{\mu_1}{1-\mu_1}\right)(1-c)^Q \\
&\leq \left(\frac{\mu_1}{1-\mu_1}\right)(1-c)^{\frac{t-T_1}{K}}.
\end{aligned}
$$

Suppose condition (2) holds. Then for any $\epsilon > 0$, there exists T such that for all $t > T$,

$$
\frac{(1-c)^{\frac{t}{K}}}{\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}]} < \epsilon.
$$

Take

$$
\epsilon < \frac{c(1-c)^{\frac{T_1}{K}-1}}{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]},
$$

then for all $t > T$,

$$
\begin{aligned}
\frac{\kappa_t}{\tilde{\kappa}_t} \\
&< \left[\frac{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]}{c(1-c)^{\frac{T_1}{K}-1}}\right] \frac{(1-c)^{\frac{t}{K}}}{\phi[\Phi^{-1}(L) - (H-L)\sqrt{\beta_t}]} \\
&< \left[\frac{\left(\frac{\mu_1}{1-\mu_1}\right)\phi[\Phi^{-1}(L)]}{c(1-c)^{\frac{T_1}{K}-1}}\right] \epsilon \\
&< 1.
\end{aligned}
$$

This inequality holds because the term in the square bracket is independent of t. Though T_1 depends on the specific equilibrium candidate (so I have to take a different ϵ for a different equilibrium candidate), the fact that we can find T is independent of the equilibrium candidate.

Therefore, for the strategy profile s, $\kappa_t < \tilde{\kappa}_t$ is equivalent to $\mu_t < \tilde{\mu}_t$. So for all $t > T$, $\rho^{\mu_t}(\tilde{x}_t) < c$ which implies that there is no common (L,c)-belief in period t. If s is an equilibrium in which attacks happen infinitely often, then Proposition 1 says that for any T, there is $t > T$ such that there is a common (L,c)-belief about $\theta = L$ among speculators. These lead to the contradiction. ∎

Three remarks about Proposition 3 are worth emphasizing. First, the equilibrium uniqueness is due to the commitment power brought about by the policy maker's incentive

to build her reputation. Because the strong policy maker behaves as a commitment type who always maintains the status quo, the weak policy maker wants to mimic the strong one so that she builds a reputation for being the strong type. This reputation incentive gives the weak policy maker a commitment power. If the status quo is in place, any speculator attacking will get a negative payoff, so no speculator wants to attack. Second, for any T, how speculators learn in the first T periods does not affect the equilibrium characterization. Because speculators will learn slowly after period T, the weak policy maker will want to build her reputation in the tail. The reputation incentive in the tail results in no attacks after some period T. Because once speculators attack, they attack infinitely often, speculators will never start attacking. Third, the policy maker does not need to be very patient. As long as $\delta > 1 - L$, the weak policy maker has the reputation incentive (that is, it is valuable for her to build a reputation to deter future attacks). Because $\bar{K}$ is non-increasing in δ, the reputation building speed is non-decreasing in δ.[8]

There are two intuitive comparative static analyses. First, when the attacking cost c becomes large, the numerator converges to 0 faster. So equation (2) is easier to hold. That is, the higher the attacking cost, the less likely the status quo is attacked. This implies that the policy measure of increasing the transaction cost in the foreign exchange rate market works in two ways. On the one hand, a higher transaction cost leads to a lower expected return of speculators from attacking, so speculators have less incentive to attack. On the other hand, a higher transaction cost means that the maximum probability that the policy maker defends against an ongoing attack is lower. Hence, by maintaining the status quo, the lower bound increase in the policy maker's reputation building is larger. So it is more likely that the policy maker's reputation building dominates speculators' common learning. Second, if the benefit L increases, the policy maker needs fewer periods to collect flow payoffs to make her average discounted payoff 0. That is, $\bar{K}$ is an increasing function of L. Hence, the larger the flow payoff is, the less likely it is that speculators attack. Note that an increase in H does not have any effect on the equilibrium characterization, because no matter how large H is, the strong policy maker's equilibrium behavior does not change.

Comparing Proposition 3 with Proposition 2, it is easy to see the role of the policy maker's reputation. In Proposition 2, there is a sequence $\{q_t\}_t$ such that for all possible prior beliefs and all strictly increasing and unbounded sequences of $\{\beta\}_t$, there are infinitely many equilibria with attacks. This conclusion is under the assumption that the regime change rule is exogenously given. But in Proposition 3, the policy maker decides whether to maintain or abandon the status quo, so she may deviate to maintain it, which leads to the difference between Proposition 3 and Proposition 2.

---

[8]Because $\bar{K}$ is defined to be an integer, there is $\epsilon > 0$ such that for all $\delta \in (1 - \epsilon, 1)$, $\bar{K}$ reaches its minimum, so the reputation building speed reaches its maximum.

5.2 Fast Common Learning

When common learning is fast, there may exist equilibria with attacks. A sequence of $\{\beta_t\}_t$ for an equilibrium with attacks can be identified by the method of "reverse engineering." Suppose we want an equilibrium in which attacks happen in period t if and only if $t \in \mathcal{Q} \subset \mathbb{N}$. Then a strictly increasing sequence of $\{\beta_t\}_t$ can be found by the following algorithm:

1. Arrange elements in $\mathcal{Q}$ to be $\{T_1, T_2, \dots\}$ such that $T_{n+1} > T_n$;

2. Find $A_{T_1}(L) = L \sum_{\tau=0}^{T_2 - T_1 - 1} \delta^\tau$;

3. Choose β_{T_1} such that

$$\frac{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))] + (1 - \mu_{T_1})\phi[\Phi^{-1}(A_{T_1}(L)) - (H - L)\sqrt{\beta_{T_1}}]} > c;$$

4. Given β_{T_1}, calculate $x_{T_1}^*$ and q_{T_1} such that

$$\Phi[\sqrt{\beta_{T_1}}(x_{T_1}^* - L)] = A_{T_1}(L)$$

   and

$$(1 - q_{T_1})\frac{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1}\phi[\Phi^{-1}(A_{T_1}(L))] + (1 - \mu_{T_1})\phi[\Phi^{-1}(A_{T_1}(L)) - (H - L)\sqrt{\beta_{T_1}}]} = c;$$

5. From Bayes' rule, calculate

$$\mu_{T_2} = \frac{\mu_{T_1} q_{T_1}}{\mu_{T_1} q_{T_1} + (1 - \mu_{T_1})};$$

6. In any $T_n \in \mathcal{Q}$ with μ_{T_n}, determine $A_{T_n}(L)$. Then similar to the case in T_1, calculate β_{T_n}, $x_{T_n}^*$, q_{T_n} and $\mu_{T_{n+1}}$. Note, $\beta_{T_{n+1}} > \beta_{T_n}$ for all $T_n, T_{n+1} \in \mathcal{Q}$;

7. Given $T_n, T_{n+1} \in \mathcal{Q}$, in any t such that $T_{n+1} > t > T_n$, pick any $\beta_t \in (\beta_{T_n}, \beta_{T_{n+1}})$ and $\beta_{t+1} > \beta_t$ if $t + 1 < T_{n+1}$. Then in any $t \notin \mathcal{Q}$, set $q_t = 1$ and $x_t^* = -\infty$.

Proposition 4 *Fix any $\delta \in (1 - L, 1)$. The sequence $\{\beta_t\}_t$ constructed in the above algorithm exists and leads to an equilibrium in which speculators attack in period $t \in \mathcal{Q}$ if and only if $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$.*

Suppose given the sequence of $\{\beta_t\}_t$, there is an equilibrium with attacks. Then, it is straightforward to show that there are infinitely many equilibria with attacks. So it is interesting to compare the properties of these equilibria with those of equilibria in the induced game. First, given any equilibrium with attacks, speculators never terminate attacking (see

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Lemma 7). This is due to the policy maker's incentive to maintain the status quo for future payoffs. But with an exogenous regime change rule, there are equilibria with attacks in which speculators stop attacking if the regime does not change at the end of some finite period.

Second, recall that $T_1(s)$ is the first period in which attacks happen in the equilibrium s. Then the first possible "attacking" period $\min_s T_1(s)$ depends on the entire sequence $\{\beta_t\}_t$. The reason why there may be no attack in period T by which there is a common (L,c)-belief among speculators is due to the policy maker's incentive to build her reputation. For example, speculators (L,c)-commonly learn $\theta = L$ by period T, but after period T, the accuracy of private sufficient statistics increases very slowly for a long time and then increases quickly. Because the "stock" of the accuracy of the private sufficient statistic in period T is bounded, if the slow increasing phase is very long, a common (L,c)-belief about $\theta = L$ cannot be formed at the end of such a phase. Then the policy maker has a strict incentive to maintain the status quo in period T no matter how aggressive the attack is. So no speculator chooses to attack in period T in any equilibrium. This is different from the conclusion in Corollary 1, where the weak status quo is abandoned exogenously with positive probability if the size of attacks is larger than or equal to L.

6 Discussions

6.1 Borrowing Constraints

Traditional models, such as the first generation models of currency attacks, explain speculative attacks as a run on the capacity of the policy maker to maintain the policy regime. I show in this subsection that imposing a reasonable borrowing constraint on the policy maker will not change the result of the model. Consider the currency attacks example. Suppose the central bank holds a credit line on an outside borrower, so that it can borrow at most 1 unit of the foreign reserve. Therefore, the central bank can borrow again if and only if its outstanding balance has been fully repaid.

Let T_n be the nth attacking period in an equilibrium. Because the borrowing constraint is not binding at the beginning of period T_1, as the same argument in Lemma 1, the policy maker maintains the status quo with probability $q_{T_1} \in (0, 1-c)$ if $A_{T_1}(L) \geq L$ in the equilibrium. So the policy maker's average discounted payoff from maintaining the status quo in period T_1 is 0. If the policy maker cannot repay her outstanding balance by period T_2, then the discounted benefits she collects until period T_2 cannot cover the defending cost in period T_1. Since she has to abandon the status quo in period T_2, her average discounted payoff in period T_2 is 0. This implies that the policy maker's average discounted payoff in period T_1 is negative, which leads to the contradiction. Therefore, the borrowing constraint is not binding at the beginning of period T_2. Then by induction, it can be shown that

imposing the borrowing constraint does not change the results of this paper.

6.2   The Myopic Policy Maker

When $\delta = 0$, the policy maker is myopic. So the equilibrium of the model prescribes an equilibrium in the one-shot game in every period with associated public belief μ_t. Since the policy maker does not value future payoffs, in period t, if $A_t(L) > L$, the policy maker will abandon the status quo for sure. When $A_t(L) = L$, the policy maker may randomize. Given a candidate equilibrium strategy of the policy maker, the induced game in period t could be solved by Figure 1. Suppose when $A_t(L) \geq L$, the policy maker abandons the status quo for sure. If and only if $\rho^{\mu_t}(\tilde{x}_t) \geq c$, the equation $g(x, \mu_t) = 0$ has a solution $x_t^* \in \mathbb{R}$. Therefore, there exists an equilibrium with attacks in the one-shot game in period t if and only if there is a common (L, c)-belief about $\theta = L$. In such an equilibrium, the policy maker maintains the status quo if and only if $A_t(L) < L$, and speculator i attacks if and only if $x_{it} \leq x_t^*$. Given this equilibrium in period t, if the status quo is in place at the beginning of period $t+1$, $\mu_{t+1} = 0$.

Given the policy maker's equilibrium strategy, the induced game among speculators in the one-shot game in any period differs from the model in Morris and Shin (1998). Because the policy maker maintains the status quo for sure if $A_t(L) < L$, attacking is not a dominant strategy for any private signal. Hence, this induced game is not a global game, and it has either a unique equilibrium in which no speculator attacks or multiple equilibria.

6.3   Continuum State Space

Angeletos, Hellwig, and Pavan (2007) (AHP) analyze a dynamic regime change game, in which θ is drawn from the real line, and in period t the regime changes if and only if $A_t(\theta) \geq \theta$. From the second period on, their model is very similar to the induced model in my paper, provided that $q_t = \tilde{q} < 1 - c$ for all t. However, the outcome of the induced model is different from the model in AHP. In the induced game, given any prior belief $\mu_1 \in (0, 1)$, individual learning results in infinitely many equilibria. In AHP, denote the infimum of the state surviving the attacks in the first period by $\underline{\theta}$. If $\underline{\theta}$ is sufficiently close to 1 (due to the extremely aggressive attacks in the first period), there exists a unique equilibrium, in which no attack can ever happen again.

This difference relies on the different common belief requirements. In the induced game, as long as there is a common $(L, \frac{c}{1-\tilde{q}})$-belief in any period t, there is an equilibrium in which some speculators attack in period t. Here, L and $\tilde{q}$ is fixed. So the learning effects will overturn any public belief. But in AHP, given a $\theta' < 1$, attacks happen only if there is a common $(\theta', \frac{c}{\Pr(\theta \leq \theta' | \theta > \underline{\theta})})$-belief about $\theta \leq \theta'$. But this common belief cannot be formed as

$\underline{\theta}$ is sufficiently close to 1.

6.4   Exogenous Public Information

The reputation bound of an informed player when uninformed short-lived players are learning about the informed player's type has been studied by Wiseman (2009) in a repeated chain store game. Though Wiseman (2009) does not show the tightness of the established reputation bound, one can construct an equilibrium with the chain store's payoff strictly lower than the Stackelberg payoff, no matter how slow the learning speed is. This is different from Proposition 3. Two assumptions of my model lead to this difference. First, speculators' private information is idiosyncratic. Because of the coordination feature, speculators put more weight on the public information when making decisions. The public information in my model is the policy maker's reputation; hence, the policy maker has a stronger incentive to build her reputation. Second, the game ends once the policy maker abandons the status quo. Because the continuation payoff from abandoning the status quo is the policy maker's minmax value, the policy maker has a strong incentive to maintain the status quo.

Now suppose besides private signals, in every period t speculators observe an exogenous public signal $y_t = \theta + \vartheta_t$, where $\vartheta_t \sim \mathcal{N}(0, 1/\alpha_t)$. Assume $\sum_{t=1}^{\infty} \alpha_t = +\infty$. Then conditional on $\theta = L$, in the limit, the sufficient statistic of the public signal is extremely precise. So the public signals will overturn the public belief formed from the fact that the status quo is in place. That is, the reputation built is easily ruined by the public signals. Therefore, a common (L, c)-belief can be formed frequently. Hence, an equilibrium with attacks exists.

7   Conclusion

In this paper, I analyze the interaction between the policy maker's reputation building and speculators' individual learning in the context of currency crises. I show that the policy maker's reputation has significant effects on speculators' abilities to coordinate. In particular, when speculators learning speed is slow, the reputation effect will dominate the learning effect. As a result, the model has a unique equilibrium in which no speculator attacks and the policy maker maintains the policy regime forever. Therefore, when the learning speed is slow, the policy maker effectively defends speculative attacks, because the incentive of building a reputation gives the policy maker the commitment power. In the case of a fast learning speed, equilibria with attacks may exist. In any equilibrium with attacks, the first attacking period depends on the entire learning process, the time interval between two consecutive attacking periods is uniformly bounded, and the weak policy maker abandons the status quo almost surely.

From a theoretical perspective, I study the informed player's reputation bound when uninformed players have coordination motives and exogenously learn the informed player's type. I show that the reputation bound equals the Stackelberg payoff when the uninformed players' learning speed is slow in the tail. This reputation bound does not require the informed player to be extremely patient. Besides, I complement the common belief and common learning literature by defining and applying the common belief concept and the common learning concept in an economy consisting of a continuum of players. In particular, the emergence of a counterpart with a reputation incentive has significant effects on speculators' abilities to coordinate: first, a common belief is not sufficient for speculators to coordinate, and second, individual learning, although it leads to common learning, is not sufficient for the existence of an equilibrium with coordination.

From an applied perspective, the policy measure of increasing the transaction cost in the foreign exchange market can help to ease speculative pressures from two channels. First, as a traditional channel, an increase in the cost of attacking reduces speculators' incentives to attack because of the lower expected payoff from attacking. Second, an increase in the cost of attacking elevates the lower bound of the central bank's reputation building speed, making it more likely that reputation building dominates speculators' learning.

A   Omitted Proofs

This section includes proofs of Propositions and Lemmas, which are stated in the text but not proved.

Proof of Lemma 1:

Suppose first, in the equilibrium, $A_t(L) \geq L$ implies $q_t \geq 1 - c$. Then any speculator i's payoff from attacking in period t is

$$(1 - q_t)\rho^{\mu_t}(h_i^t) - c < c - c = 0.$$

The strict inequality is due to the common support assumption of private signals with respect to θ. Therefore, any speculator who is attacking would like to deviate to not attack. This implies $A_t = 0$, which leads to a contradiction.

Now suppose $q_t = 0$, that is, the weak policy maker abandons the status quo for sure when the defending cost is larger than or equal to the flow payoff. Consider a deviation to maintain the status quo for sure in all periods $\tau \geq t$. Because the strong policy maker defends against any attack, no regime change in period t implies $\mu_{t+1} = 0$. That is, since this deviation is not observable by speculators, the public belief about $\theta = H$ shifts to 1. Then no speculator wants to attack in any period $\tau > t$. Therefore, the weak policy maker's average discounted payoff in period t is (note if the public belief about $\theta = H$ is 1, no speculator wants to attack ever again):

$$
\begin{aligned}
& (1 - \delta)[(L - A_t(L)) + \sum_{\tau=1}^{\infty} \delta^{\tau} L] \\
> \ & (1 - \delta)[\frac{L}{1 - \delta} - 1] \\
= \ & L - (1 - \delta) \\
> \ & 0.
\end{aligned}
$$

So this deviation is profitable, which implies that $q_t = 0$ when $A_t(L) \geq L$ is not part of an equilibrium.

Q.E.D.

Proof of Lemma 2:

Let's first show that in any equilibrium, speculators employ symmetric strategies. In any period t with the public belief μ_t, given all other speculators' strategies, conditional on θ, the total measure of attack in period t is a deterministic number. Therefore, the probability of the regime change is $\chi(A_t(L) \geq L)(1 - q)$, which is exogenously given to all speculators.

If $\chi(A_t(L) \geq L)(1-q) \leq c$, no matter what the private history is, a speculator will choose not to attack. If $\chi(A_t(L) \geq L)(1-q) > c$, any speculator i chooses to attack if and only if $\rho^{\mu_t}(h_i^t) \geq \frac{c}{1-q}$. Since two speculators with the same private history will form the same posterior belief, they will make the same choice. As a result, in any equilibrium, speculators employ symmetric strategies.

Now, in an equilibrium, because any individual actions cannot publicly be observed, a speculator's past actions are not informative about θ. Hence, a speculator forms his posterior belief based only on his private signals. By the standard Gaussian updating formula, for any speculator i, in any period t given μ_t, z_i^t and x_{it} lead to the same posterior belief. Therefore, $\rho^{\mu_t}(h_i^t) = \rho^{\mu_t}(x_{it})$. Because

$$\rho^{\mu_1}(x_{i1}) = \frac{\mu_1 \phi(\sqrt{\beta_1}(x_{i1} - L))}{\mu_1 \phi(\sqrt{\beta_1}(x_{i1} - L)) + (1-\mu_1)\phi(\sqrt{\beta_1}(x_{i1} - H))},$$

the monotone likelihood ratio property implies that $\rho^{\mu_t}(x_{it})$ is strictly decreasing in x_{it}. As a result, if $\chi(A_t(L) \geq L)(1-q) \leq c$ which is equivalent to $\chi(A_t(L) < L)$ because $1 - q > c$, all speculators will choose not to attack. That is, any speculators attack if and only if $x_{it} \leq x_t^* = -\infty$. If $\chi(A_t(L) \geq L)(1-q) > c$, there is an $x_t^* \in \mathbb{R}$ such that any speculator i attacks if and only $x_{it} \leq x_t^*$.

$$Q.E.D.$$

Proof of Lemma 3:

Suppose there is an equilibrium in which the public belief in period t is μ_t and conditional on $\hat{\tau} \geq t$, $A_t > 0$. According to the exogenous regime change rule, conditional on θ, if $A_t(\theta) < \theta$, $\Pr(\hat{\tau} = t|\hat{\tau} \geq t) = 0$. Hence, any speculator i's problem in period t is

$$\max_{a \in [0,1]} \quad [\chi(A_t(L) \geq L)(1-q_t)\rho^{\mu_t}(x_{it}) - c]a.$$

If $A_t(L) < L$, speculator i will choose not to attack, no matter what his private sufficient statistic is. Therefore, if $A_t(L) < L$, $A_t(L) = 0$. Equivalently, $A_t(L) > 0$ implies $A_t(L) \geq L$. From Lemma 2, any speculator i attacks in period t if and only if $x_{it} \leq x_t^*$. Hence, the speculator with private sufficient statistic x_t^* will receive 0 expected payoff from attacking. That is, $\rho^{\mu_t}(x_t^*) = \frac{c}{1-q_t}$. So for $A_t(L) \geq L$, $\Pr(x_t \leq x_t^*|\theta = L) \geq L = \Pr(x_t \leq \tilde{x}_t|\theta = L)$. So $x_t^* \geq \tilde{x}_t$. Because $\rho^{\mu_t}(x_t)$ is a strictly decreasing function of x_t, $\rho^{\mu_t}(\tilde{x}_t) \geq \frac{c}{1-q_t}$.

$$Q.E.D.$$

Proof of Lemma 4:

I prove this lemma by construction. Let's consider the strategy profile in which no speculator chooses to attack until period t. Because $A_\tau(L) = A_\tau(H) = 0$ for all $\tau < t$, $\hat{\tau} \geq t$

no matter whether $\theta = H$ or $\theta = L$. Then $\mu_t = \mu_1$. So $\rho^{\mu_t}(\tilde{x}_t) = \rho^{\mu_1}(\tilde{x}_t) \geq \frac{c}{1-q_t}$. Since $\rho^{\mu_t}(x)$ is continuous in x and $\lim_{x \to \infty} \rho^{\mu_t}(x) = 0$, $\exists x_t^* \in [\tilde{x}_t, \infty)$ such that $\rho^{\mu_t}(x_t^*) = \frac{c}{1-q_t}$. Then in period t, any speculator i attacks if and only if $x_{it} \leq \tilde{x}_t$. If attacks in period t fail, no speculator ever attacks again.

Let's verify that the constructed strategy profile is an equilibrium. In all periods $\tau \neq t$, since $A_\tau = 0$, $\Pr(\hat{\tau} = t | \hat{\tau} \geq t) = 0$. Therefore, refraining from attacking is the best response of any speculator. In period t, consider any speculator i. Since other speculators use the cutoff rule with the threshold point $x_t^* \in [\tilde{x}_t, \infty)$, $\Pr(x_t \leq x_t^* | \theta = L) \geq \Pr(x_t \leq \tilde{x}_t | \theta = L) = L$. So $\chi(A_t(L) \geq L) = 1$. In addition, $\rho^{\mu_t}(x_{it}) \geq \rho^{\mu_t}(x_t^*) = \frac{c}{1-q}$ if and only if $x_{it} \leq x_t^*$. Hence, speculator i attacks if and only if $x_{it} \leq x_t^*$. Therefore, the constructed strategy profile is an equilibrium.

$$Q.E.D.$$

Proof of Lemma 5:

Without loss of generality, I prove that $\theta = L$ is individually learned if and only if $\lim_{t \to \infty} \beta_t = +\infty$. First, suppose $\lim_{t \to \infty} \beta_t = \bar{\beta} < +\infty$. Note $\rho^{\mu_1}(x_{it}) \geq p$ is equivalent to

$$x_{it} \leq \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}.$$

Fix any p sufficiently close to 1,

$$
\begin{aligned}
&\Pr(B_{it}^p(L)|L) \\
=\ &\Pr(\{x_{it} : \rho^{\mu_1}(x_{it}) \geq p\}|L) \\
=\ &\Pr\left(\left\{x_{it} : \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}\right\}|L\right) \\
=\ &\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right] \\
<\ &\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\bar{\beta}}(H-L)} + \sqrt{\bar{\beta}}\frac{H-L}{2}\right] \\
<\ &p.
\end{aligned}
$$

That is, for p sufficiently close to 1, $\Pr(B_{it}^p(L)|L) < p$ for all t. As a result, no speculator can individually learn $\theta = L$. Put differently, if speculators can individually learn $\theta = L$, $\lim_{t \to \infty} \beta_t = +\infty$.

Now suppose $\lim_{t \to \infty} \beta_t = +\infty$. Fix any $p \in (0,1)$,

$$\Pr(B_{it}^p(L)|L)$$

$$= \Pr(\{x_{it} : \rho^{\mu_1}(x_{it}) \geq p\}|L)$$

$$= \Pr\left(\left\{x_{it} : \frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\beta_t(H-L)} + \frac{H+L}{2}\right\}|L\right)$$

$$= \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right].$$

Because $\lim_{t \to \infty}\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right] = +\infty$, there is T such that

$$\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_T}(H-L)} + \sqrt{\beta_T}\frac{H-L}{2}\right] > p.$$

. Since $\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right]$ is increasing in β_t, for all $t > T$,

$$\Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_t}(H-L)} + \sqrt{\beta_t}\frac{H-L}{2}\right]$$

$$> \Phi\left[\frac{\ln[(1-p)\mu_1] - \ln[(1-\mu_1)p]}{\sqrt{\beta_T}(H-L)} + \sqrt{\beta_T}\frac{H-L}{2}\right]$$

$$> p.$$

So if $\lim_{t \to \infty} \beta_t = +\infty$, speculators individually learn $\theta = L$.

$$Q.E.D.$$

Proof of Lemma 6:

In Proposition 1, I show that a common $(L, \frac{c}{1-q_T})$-belief among speculators in period T is equivalent to the inequality $(1 - q_T)\rho^{\mu_T}(\tilde{x}_T) \geq c$. Therefore, fix public belief $\mu_t = \mu_1$ for all t, then

$$(1 - q_t)\rho^{\mu_1}(\tilde{x}_t)$$

$$= (1 - q_t)\frac{\mu_1\phi[\Phi^{-1}(L)]}{\mu_1\phi[\Phi^{-1}(L)] + (1 - \mu_1)\phi[\Phi^{-1}(L) - (H - L)\sqrt{\beta_t}]}.$$

Because speculators individually learn Θ, $\lim_{t \to \infty} \beta_t = +\infty$, which implies that $\lim_{t \to \infty} \rho^{\mu_1}(\tilde{x}_t) = 1$. Since there is a subsequence of $\{q_t\}_t$ which is bounded above by $\tilde{q} < 1 - c$, there is T such that $(1 - q_T)\rho^{\mu_1}(\tilde{x}_T) \geq c$.

$$Q.E.D.$$

Proof of Proposition 2:

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The first part is trivial and directly follows from the continuum speculators assumption and the exogenous regime change rule.

For the second part, Lemma 6 shows that individual learning is sufficient for common learning in some period t. Therefore, there is an equilibrium with attacks. But after some period T, speculators just choose the pure "not attack" strategy, which leads to no attack after period T.

For the third part, because individual learning implies common learning in some period for any prior belief $\mu_1 \in (0, 1)$, there is an equilibrium in which attacks happen. If attacks fail in some period T, $\mu_T < \mu_1$, but $\mu_T \in (0, 1)$. Therefore, speculators (L, c)-commonly learn $\theta = L$ by some period $t > T$. Therefore, attacks can happen again in or after period t.

<div align="right">*Q.E.D.*</div>

Proof of Proposition 4:

Suppose a sequence of $\{\beta_t\}_t$ is constructed according to the algorithm and leads to an equilibrium consisting of sequences $\{\mu_t\}_t$, $\{x_t^*\}$, and $\{q_t\}_t$. Then Lemma 8 implies the necessity of $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$.

Now suppose that $T_{n+1} - T_n \leq \bar{K}$ for all $T_n, T_{n+1} \in \mathcal{Q}$, we want to show that the constructed sequence of $\{\beta_t\}_t$ exists, and that the associated sequences $\{\mu_t\}_t$, $\{x_t^*\}$, and $\{q_t\}_t$ constitute an equilibrium. Since $T_2 - T_1 \leq \bar{K}$ and $\delta \in (1 - L, 1)$, A_{T_1} is well defined. Because $\mu_{T_1} = \mu_1 \in (0, 1)$, and

$$\lim_{\beta \to \infty} \frac{\mu_{T_1} \phi[\Phi^{-1}(A_{T_1}(L))]}{\mu_{T_1} \phi[\Phi^{-1}(A_{T_1}(L))] + (1 - \mu_{T_1}) \phi[\Phi^{-1}(A_{T_1}(L)) - (H - L)\sqrt{\beta}]} = 1 > c,$$

β_{T_1} is well defined. Then $x_{T_1}^* \in \mathbb{R}$ and $q_{T_1} \in (0, 1 - c)$ are uniquely determined. Therefore, μ_{T_2} can be calculated from Bayes' rule. The rest of the proof follows from the induction.

<div align="right">*Q.E.D.*</div>

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