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"Should Courts Always Enforce What Contracting Parties Write?"

by

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Should Courts Always Enforce What Contracting Parties Write?*

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Abstract. We find an economic rationale for the common sense answer to the question in our title — courts should *not* always enforce what the contracting parties write.

We describe and analyze a contractual environment that allows a role for an active court. An active court can improve on the outcome that the parties would achieve without it. The institutional role of the court is to maximize the parties' welfare under a veil of ignorance.

We study a buyer-seller model with asymmetric information and ex-ante investments, in which some contingencies cannot be contracted on. The court must decide when to uphold a contract and when to void it.

The parties know their private information at the time of contracting, and this drives a wedge between ex-ante and interim-efficient contracts. In particular, some types pool in equilibrium. By voiding some contracts that the pooling types would like the court to enforce, the court is able to induce them to separate, and hence to improve ex-ante welfare.

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1. Introduction

It is self-evident that courts are active players in contractual relationships between economic agents. They routinely intervene in contractual disputes, excusing performance called for in the contract because of intervening events. Yet, in most of modern economic theory courts are treated (often not even modelled, but left in the background) as passive enforcers of the will of the parties embodied in their contractual agreements.

This simplistic view of the role of courts stems from the fact that in a world with complete contracts, to behave as a passive enforcer is clearly the best that a court that is interested in maximizing contracting parties' welfare can do. In the "classical" world of modern economic theory, contracts *are* complete.

In a world in which complete contracts are not feasible it is no longer obvious that a court should be a passive enforcer, and in fact it is no longer true. For example, the contracting parties may face some uninsurable risk and the court may improve their welfare if it is able to use some information available ex-post and excuse performance in some eventualities.[1]

Once the way for an active court is open, a host of related questions naturally arise. The aim of this paper is to address the following question. Suppose that the parties, at the time of contracting, were able to communicate effectively to the court *how* they would like the court to act in response to (ex-post) information that they are not able to take into account in their agreement. Is it then the case that the parties would ask the court to behave according to the same rules that would be chosen by a welfare-maximizing court?

The answer to the question above is "no" if the parties are asymmetrically informed at the time they contract and the court maximizes their ex-ante welfare, that is, their expected welfare *before* either party gets information not available to the other. Asymmetry in parties' information at the time they contract can lead to a "lemons-like" situation in which adverse selection leads to inefficient contracts.

[1] This is the case, for instance, in Anderlini, Felli, and Postlewaite (2003).

Courts that do not simply enforce contracts as they are written can sometimes ameliorate the inefficiency that results from asymmetric information.

We show in the paper that in a world where contracting parties are asymmetrically informed this is indeed the case. We also derive the optimal decision rule for a welfare-maximizing court. This rule implies that the court in equilibrium voids contracts that the contracting parties (at the contracting stage) would like the court to enforce. In other words, the conflict of interest between the parties and the court leads the court to actively intervene in the parties' contractual relationship and void contractual clauses explicitly included in the contract.

The potential benefit of a court's voiding explicit contractual clauses stems from asymmetry of information between the parties at the time they contract. We deal with the case in which one of the contracting parties may or may not have private information relevant to the other party; furthermore, the presence or absence of private information is itself private information. The possibility that one of the parties *may* have relevant private information will have a deleterious effect on contracting even when, in fact, that party *does not* have private information. In other words, the cases in which private information exists generates an externality on the cases in which there is no private information. This negative external effect can be ameliorated by providing incentives for a party with private information to disclose that information at the time of contracting. One way to provide such incentives is to void a contract when, ex-post, the belief is that there was such relevant private information. Voiding contracts in such cases will decrease the expected gain from withholding private information, thereby promoting disclosure. Such ex-post inferences will, of course, be imperfect; contracts between parties where there was *no* private information will sometimes be voided. There will be a net welfare gain when the improvement from the additional disclosure outweighs the inefficiency from voiding contracts when there was no private information.

The view that courts should maximize ex-ante welfare is a compelling one. If the parties were able to meet at the ex-ante stage (when they are symmetrically informed) agreements could be reached that circumvent inefficiencies that are unavoidable at the interim stage when the parties have private information. A court that maximizes ex-ante expected welfare will choose the same contingent rules of behavior as the parties would have chosen at that stage, were it possible. In other words, if the parties could meet at that point, they might *instruct* the court to void some contracts they might subsequently write. They will do this precisely because the parties will understand that while they may regret this in some circumstances, it may promote the disclosure of private information. This disclosure may increase the efficiency of contracting to an extent that more than outweighs any negative consequences of the court's intervention. The problem that the court is solving is that the parties are often *unable* to meet before the arrival of their private information. A court that maximizes ex-ante welfare acts as a *commitment* device that remedies the parties' inability to contract at the ex-ante stage.

1.1. The Role of Courts in Promoting Disclosure of Information

Courts have had an interest in promoting disclosure of information at least since the English case of Hadley vs. Baxendale in 1854.[2] The court held in that case that a defendant who breached a contract was liable only for damages that might reasonably have arisen given the known facts rather than the higher damages that were actually suffered because of circumstances known only to the plaintiff. As argued in Adler (1999), the limitation on damages implicit in the Hadley rule is a default that is often viewed as promoting disclosure: "A party who will suffer exceptional damages from breach need only communicate her situation in advance and gain assent to allowance so that the damages are unmistakably in the contemplation of both parties' at the time of contract."[3] The discussion of the role of courts in promoting information disclosure, to our knowledge, focusses primarily on the benefit of disclosure to the contracting parties. In the absence of disclosure, resources will be wasted in writing needless waiver clauses and inefficient precaution.

[2] 9 Exch. 341, 156 Eng. Rep. 145. (Court of Exchequer, 1854).

[3] See Ayres and Gertner (1989) and Bebchuk and Shavell (1991) for a discussion of the Hadley rule and it's role in promoting disclosure.

Courts will have an interest in promoting disclosure of information in our model, but for a very different reason, and with very different consequences. Courts will affect the amount of information that is revealed by informed parties through their treatment of contracts that reveal little information. While contracts may reveal little information simply because the parties *have* little information, courts will treat such contracts more harshly than they otherwise might because of the incentive effects such treatment will have on informed parties. Those with relevant information will reveal it in order that courts will more likely enforce the agreements that are made. Thus, courts are not examining a contract brought before them solely to uncover the parties' intent. They also take into consideration how the treatment of the contract will affect contracting parties different from the ones before them.

1.2. Related Literature

There is a growing literature that explicitly models the role of courts in contractual relationships. Bond (2003) and Usman (2002) model the agency problems (moral-hazard) that stem from hidden actions that the court itself can take. Bond (2003) analyzes optimal contracting between parties when judges can impose an outcome other than the contracted outcome in exchange for a bribe. Bond shows that in a simple agency model, this possibility will make the contracting parties less likely to employ high-powered contracts. Usman (2002) lays out a model in which contracts contain variables that are not observable to courts unless a rational and self-interested judge exerts costly effort. Usman analyzes contracting behavior and the incentive to breach when judges value the correct ruling but dislike effort.

The courts in these papers are governed by a judge who maximizes his or her personal utility. In contrast to these papers, there is a literature that analyzes courts that maximize the expected welfare of the contracting parties. Posner (1998) analyzes whether a court should consider information extrinsic to the contract in interpreting the contract. Closer to the current paper, Ayres and Gertner (1989) and Bebchuk and Shavell (1991) analyze the degree to which courts' interpretation of contracts affect incentives to reveal private information. The focus of this work is the effect of different court rules regarding damages for breach of contract on the incentives for parties to disclose information regarding the costs of breach at the time of contracting.

The model in the present paper is similar to that in Anderlini, Felli, and Postlewaite (2003). Anderlini, Felli and Postlewaite analyze the design of a welfare-maximizing court that actively determines circumstances in which a contract will be voided in order to provide insurance against unforeseen contingencies. Shavell (2003) presents a general examination of the role of courts in interpreting contracts. The primary focus of this paper is on how courts should interpret contracts that have specific terms compared to the interpretation of more general terms.

The present paper analyzes the role of a welfare-maximizing court that can affect the type of contracts that are written by excusing performance (voiding the contract) in some circumstances. The possibility of welfare improvements are a consequence of the effect of the court's rules for enforcing contracts on the parties' incentives to reveal private information. Unlike Ayres and Gertner (1989), Bebchuk and Shavell (1991) and Shavell (2003), our focus is on the externality that informed contracting parties may impose on uninformed contracting parties, which is absent from these papers.

The plan of the rest of the paper is as follows. We present the model in Section 2, and our results in Sections 3 and 4. Section 5 concludes the paper. For ease of exposition all proofs have been gathered in the Appendix.[4]

2. The Model

2.1. Uninformed Buyers

The uninformed buyers are risk-neutral and face two sources of uncertainty. The "state" can be either "risky" or "safe".[5] These two possibilities are embodied in

[4]In the numbering of Propositions, Lemmas, equations and so on, a prefix of "A" indicates that the relevant item can be found in the Appendix.

[5]Note that we are abusing the standard meaning of the word "state" as a "complete description" of all relevant uncertainty since the realization of a second random variable is involved in determining the complete description of the parties' uncertain environment.

the realization of a random variable θ that takes values in the set $\Theta = \{R, S\}$ with probabilities π_R, and $\pi_S = 1 - \pi_R$ respectively.

In the safe state, the value and cost of the widget are given by v_S and c_S respectively, with $v_S - c_S = \Delta > 0$.

The second source of uncertainty is that in the risky state, the cost and the value of the widget can be either "high" or "low." We represent this uncertainty by a random variable σ that takes values in the discrete set $\{H, L\}$ with probabilities q and $1 - q$ respectively. Critically, the uncertainty associated with θ is contractible, while the uncertainty associated with σ is not.

Given a realization of $\theta = R$ and $\sigma \in \{H, L\}$, the cost and value of the widget are denoted by c_σ and v_σ respectively.[6] For simplicity we assume that the gains from trade are constant, so $c_H > c_L$ and

$$v_S - c_S \ = \ v_H - c_H \ = \ v_L - c_L \ = \ \Delta \ > \ 0 \tag{1}$$

Of course, since $\Delta > 0$, this implies that it is always efficient to trade. This assumption is made for tractability. Our results would not qualitatively change if the costs and benefits were not perfectly correlated and the magnitude of the gains from trade were variable. Finally, we denote:

$$q\, c_H \ + \ (1-q)c_L \ = \ \overline{c} \quad \text{and} \quad q\, v_H \ + \ (1-q)v_L \ = \ \overline{v} \tag{2}$$

2.2. Informed Buyers

The informed buyers are also risk-neutral and are *fully* informed. For simplicity, we assume that *all* informed buyers observe the *same* realization of θ and σ — namely $\theta = R$ and $\sigma = H$. To put it another way, the informed buyers are all of the same *type*. They know that the cost and value are c_H and v_H respectively.[7]

[6] Notice that we are slightly abusing notation since we use θ and σ again to denote generic realized values of the two random variables.

[7] Thus it may be argued that our use of the labels "uninformed" and "informed" buyers is somewhat improper. The "uninformed" buyers *do* in fact know that they are uninformed about the

Notice that because of our assumption that θ is *contractible*, the informed buyers can *credibly* reveal that they are informed. They need not do so, however. As will be clear below, under some conditions, they may opt to pretend to be *uninformed*, and hence not to reveal their information about cost and value.

Our assumption that all informed buyers are of type (R, H) is obviously strong. However, we proceed in this way purely for the sake of (mostly notational) simplicity. The qualitative nature of our results would be largely unchanged if we allowed the informed buyers to observe with positive probability *all* possible combinations of the realizations of θ and σ.

2.3. Sellers

All sellers are uninformed. They face uncertainty about both θ and σ. Of course, the probabilities with which they evaluate the uncertainty they face depends on the equilibrium behavior of the informed buyers. In particular, the sellers will take into account whether the informed buyers pool with the uninformed by offering the same contract or whether they separate and reveal their type.

Sellers are risk-averse. They maximize the expected value of a strictly increasing and concave function $V : \mathbb{R} \to \mathbb{R}$ such that $V' > 0$ and $V'' < 0$.[8] Hence V must be unbounded below. We also assume that V is bounded above by $\overline{V}$.

2.4. Bargaining Power and Relationship-Specific Investment

We posit that the buyer has all the bargaining power ex-ante when a contract is proposed. In other words, the equilibrium contract is the result of a take-it-or-leave-it offer from the buyer to the seller. Ex-post, in some instances, renegotiation will take place. We assume that the seller has all the bargaining power in the ex-post

cost and value of the widget. We proceed in this way since we think that these terms facilitate the exposition.

[8]We let V be defined on negative values of wealth because we want to be able to consider contracts that, with some probability, give the seller a net loss. Defining V over the entire real line is a particularly simple way to accomplish this. Our results would be unchanged if we kept track of the seller's initial endowment, assumed that it is large enough to cover any losses, and defined V over the non-negative reals.

renegotiation: if renegotiation occurs, the seller makes a take-it-or-leave-it offer to the buyer.[9]

To summarize, the uninformed parties face a risk at the time they contract that the cost and value of the widget will be high or low at the time production and delivery are to take place. This risk can be avoided by not contracting ex-ante, and simply contracting after the state is realized. To provide a benefit to contracting ex-ante, we assume that the buyer (whether informed or uninformed) can undertake an ex-ante, non-contractible relationship-specific, investment $e \in [0,1]$ at a cost $\psi(e)$; we assume that ψ is convex, $\psi(0) = \psi'(0) = 0$ and $\lim_{e \to 1} \psi'(e) = +\infty$. A buyer's investment of e increases the value to him of the widget of an amount eY. Consequently, if the buyer chooses the level of relationship-specific investment e his value of the widget is $eY + \Delta + c_i$ with $i \in \{S, H, L\}$, depending on his type.

2.5. The Court

The court acts as a "Stackelberg leader" in the model. Before any uncertainty is realized and any contracts are drawn up, the court publicly announces the rules that it will follow to settle a possible dispute. courts do not in actuality commit to the rules by which disputes will be settled; instead, courts decide after they know the details of the dispute. Nevertheless, over time the accumulated decisions that courts have made in a broad array of cases creates a set of *precedents* that shape how future disputes will be settled. Rather than model this gradual evolution to the rule by which contractual disputes are resolved we treat the court as choosing, once and for all, a rule.

The court chooses the rule for settling possible disputes so as to maximize the parties' expected *ex-ante* welfare. In other words, the court maximizes welfare under

[9] The assumption that both ex-ante and ex-post, one or the other of the parties has all the bargaining power is primarily for expositional ease; none of our results depends on bargaining power being absolute for one or the other. Our results would *not* hold, however, if the buyer has all the bargaining power ex-post. In this case, clearly the incentive problem disappears in our model. Trading ex-post would always implement efficient risk-sharing and ex-ante investment. When the seller has any bargaining power ex-post, a trade-off between incentives and risk-sharing appears.

a veil of ignorance: it does not observe the parties' types and hence "averages" across all possibilities.

After all uncertainty is realized, each party to the contract can bring the other party to court, and the court will rule on the status of the contract. This ruling consists of the court's decision to *void* (alternatively, *excuse performance*) or to *enforce* (alternatively *uphold*) the contract, and is based on the information available to the court: the contract itself, and the value of θ. We assume that the court lacks the "detailed knowledge" to effectively use and manipulate price information. In particular we do not allow the court to force the parties to trade at a given price determined by the court herself. Moreover, we do not consider court rules that make the decision to void or uphold contingent on the actual price that a contract specifies, but only on the contract "type" itself.[10] Finally, we do not allow the court to utilize a message game in which the parties are required to report their information to the court, as in Maskin and Tirole (1999).[11]

If the court voids the existing contract, renegotiation takes place between the buyer and the seller. Recall that at this stage the seller has all the bargaining power.

2.6. Timeline

The timeline of decisions and events in our model is as follows. First, the court chooses a rule for voiding or upholding contracts. The rule chosen by the court is contingent on what it will observe when any contract is taken to court: the contract type,[12] and the realized value of θ.

[10]There are two contract "types" that the buyer can offer to the seller. One type of contract is the one offered by uninformed buyers and by those informed buyers who pretend to be uninformed, and another type is the one offered by informed buyers who declare to be informed.

[11]There are a variety of other factors on which one could imagine that the court may want to condition its decision to void or enforce; for instance the identity of the plaintiff — the party who takes the contract to the court.

In practice, allowing for court rules that can depend on prices or plaintiff identity would not alter the qualitative nature of our results. We proceed in this way because this simplifies the model and because we find it convincing to stipulate that the court rule must be to some extent "detail-free."

What is, of course, critical is that we are ruling out the possibility that going to court might trigger an ex-post extensive-form implementation game à la Moore and Repullo (1988).

[12]See footnote 10 above.

Next, there is a draw by Nature that determines the buyer's type, that is, whether the buyer is informed or uninformed. This draw is embodied in the random variable η that takes values in $\mathcal{T} = \{I, U\}$. The realization of η is I (for "informed") with probability γ and U (for "uninformed") with probability $1 - \gamma$.

If $\eta = I$, the cost and value of the widget are determined as c_H and v_H respectively. If $\eta = U$, then the cost and value of the widget are given by the realizations of θ and σ; the state can be either risky or safe and if it is risky the cost and value can be either high or low.

The buyer now observes the realization of η. So, the informed buyer knows that he is informed (and hence the cost and value — c_H and v_H), while the uninformed buyer knows that he is uninformed: he observes the realization $\eta = U$, but *not* the realized values of θ and σ. The buyer (informed or uninformed) makes a take-it-or-leave-it offer of a contract to the seller. Note that at this point the informed buyer chooses whether to reveal that he is informed to the seller. Since θ is "hard" and "contractible" information, an informed buyer *can* pretend to be uninformed, while an uninformed buyer *cannot* pretend to be informed.

The seller does not observe any of the realizations of Nature's draws, but knows the probabilities of each of the possible realizations. He accepts or rejects the contract offered by the buyer, calculating his expected utility using his beliefs (about Nature's draw) *updated* on the basis of the contract he has been offered. The seller accepts the contract if and only if the expected utility from it is at least as large as the expected utility he gets from rejecting and proceeding to the "renegotiation" stage below.

Once the contract-negotiation is over, the buyer chooses a level of relationship-specific investment $e \in [0, 1]$. This increases the value of the widget to the buyer, with the buyer incurring a cost as described in Subsection 2.4. At this stage the seller observes the level of relationship-specific investment selected by the buyer.

The realized θ ad σ are then observed by both seller and buyer (of course the informed buyer has nothing new to observe). Either party can take the contract to court; if either does, the court decides whether to void it or uphold it according to the announced rule.

If the court voids the contract, renegotiation occurs. As we mentioned above, renegotiation is modelled as a take-it-or-leave-it offer from the seller to the buyer of a price at which to trade. When renegotiation occurs, following the court's decision to void the contract, the parties' outside options are represented by the payoffs associated with no trade, which we normalize to zero.

Finally, trade occurs according to the terms of the original contract, if the court decides to enforce it, or according to the terms of the renegotiated agreement, if the court decides to void the original ex-ante contract.

3. A Candidate Equilibrium

We next describe a candidate equilibrium, which in the next section we show is the unique equilibrium of the model for an open set of parameter constellations. This equilibrium represents the case of interest, in which the court voids some contracts that the parties would like to see upheld.

Recall that there is never any risk associated with $\theta = S$, while the risk associated with $\theta = R$ is in some sense commensurate with the difference $c_H - c_L$. Given that the sellers are risk-averse, and that the risk associated with $\theta = R$ is *uninsurable*, it is intuitively plausible to focus on the case in which the court rule — denoted by $\mathcal{C}$ throughout the rest of the paper — would prescribe that a contract written by uninformed buyers should be voided if $\theta = R$ and upheld if $\theta = S$, while contracts written by informed buyers should be upheld.

Given this court rule, informed buyers will *separate* because if they pool with the uninformed buyers their contract will be voided with probability 1. Since the ex-post renegotiation gives all the bargaining power to the seller this leaves them worse off than if they separate, that is, write a contract that reveals their private information. By separating their contract is enforced, and they receive the net benefits of their relationship-specific investment. We next characterize the candidate equilibrium contracts when the court employs this rule and the informed buyers separate.

We take first the uninformed buyers. In general, the uninformed can specify two prices in an ex-ante contract: p_R and p_S. These are prices at which to trade in each of the $\{R, S\}$ states. They can also specify an ex-ante up-front transfer from the buyer to the seller t, and a "force majeure" clause that specifies a set of values of θ, denoted by $\mathcal{F}$, in which they stipulate that performance will be excused. [13] The term "force majeure" seems particularly apt in this context since the value of θ maps directly into how large the change in cost and value are. Recall that the ex-ante contract, by assumption, *cannot* be contingent on the realization of σ.

To sum up, an ex-ante contract for the uninformed buyers in our model can be taken to be an array $(\mathbf{p}, t, \mathcal{F}) = (p_R, p_S, t, \mathcal{F})$ that is interpreted as follows. The parties agree to an immediate transfer t from the buyer to the seller. They also agree to trade the widget at a price of p_θ, provided that the state is $\theta \notin \mathcal{F}$. Finally, the contract is void (the price p_θ is not binding on either the buyer or the seller) if $\theta \in \mathcal{F} \subseteq \{R, S\}$.[14]

Recall that in our candidate equilibrium the court rule $\mathcal{C}$ prescribes that for the uninformed contract, performance is excused whenever $\theta = R$. So, at this stage the relevant choice is whether to include S in $\mathcal{F}$ or not, and assuming $S \notin \mathcal{F}$, at what level to set p_S and t.

If the uninformed specify that $S \notin \mathcal{F}$, then p_S and t must be the solution to

$$\begin{array}{rl} \max\limits_{p_S, t, e_S} & \pi_S(e_S Y + \Delta + c_S - p_S) - t - \psi(e_S) \\ s.t. & \pi_S V(p_s - c_S + t) + \pi_R V(\Delta + e_S Y + t) \geq V(\Delta) \\ & \psi'(e_S) \;=\; \pi_S Y \end{array} \tag{3}$$

[13] If the parties' contract invokes a "force majeure" clause that makes it not binding, we assume that any trade is governed by ex-post renegotiation.

[14] Notice that, in the terminology of Maskin and Tirole (1990), the buyer in our model is an "informed principal in an environment of private values." Maskin and Tirole (1990) in a different setting show that the use of menus of contracts affect the set of possible equilibrium outcomes of the signalling game. In our model the buyer is signalling not only to the seller but also the court. Hence our model does not fall into the scope of the Maskin and Tirole (1990)'s analysis. For the sake of simplicity in our analysis we do not consider the case in which the buyer can offer a menu of contracts.

From the first order conditions, the solution to problem (3) is $t = -e_S Y$ and $p_S = c_S + \Delta + e_S Y$. The uninformed buyer's payoff is

$$\Pi^S = e_S Y - \psi(e_S) > 0 \tag{4}$$

If the uninformed specify $S \in \mathcal{F}$ their payoff will be zero, since in this case prices are always determined by ex-post renegotiation and the seller will appropriate all surplus. Hence the uninformed buyer will select a level of relationship-specific investment $e = 0$. The surplus will therefore be precisely Δ in every state. Hence to satisfy the seller's participation constraint, the buyer will be left with an expected profit of zero. Hence the uninformed buyer will write a contract specifying that $S \notin \mathcal{F}$, $t = -e_S Y$ and $p_S = c_S + \Delta + e_S Y$.

Notice that for the court rule that excuses performance in R for contracts written uninformed buyers it is irrelevant whether the contract specifies that $R \in \mathcal{F}$ or not. In the actual equilibrium of the model that we will focus on in Section 4 below, this will not be the case, however. When the parameters of the model are in the region of interest, the decision to specify $R \in \mathcal{F}$ will be shown to be *not robust* in the sense that it will not survive any trembles in the court's decision. This is because, while it will be optimal for the court to void the contract contingent on $\theta = R$, the uninformed would actually prefer (other things equal) the court to *uphold* it.

With this characterization of the contract for the uninformed buyers in our candidate equilibrium, it is straightforward to characterize the optimal behavior of the informed buyers. Suppose they separate as the candidate equilibrium specifies. The contract that separating informed buyers offer consists of a price at which to trade p_I, without the need to specify a separate up-front transfer.[15] Since the candidate

[15] If the court upholds the contract of the separating informed buyers then the contract is upheld with probability 1, hence there is no distinction (given no discounting) between money transferred up-front and money transferred at the time of trade. If the court voids the contract of the separating informed buyer, then the seller appropriates all the surplus ex-post. Hence the informed buyer will select a level of relationship-specific investment $e = 0$. The surplus will therefore be precisely Δ. Hence to satisfy the seller's participation constraint, the buyer will be left with an expected profit of zero. Therefore, the up-front transfer will be 0.

equilibrium specifies that their contract is upheld by the court, and no uncertainty remains at the time it is signed it will simply specify $p_I = c_H + \Delta$. Notice also that since the court always enforces the contract in this case, the buyer will select a ("first best") level of relationship-specific investment e^* satisfying

$$\psi'(e^*) \;=\; Y$$

Hence an informed buyer's profit when he separates is

$$\Pi^* \;=\; e^* Y \;-\; \psi(e^*). \tag{5}$$

If an informed buyer deviates from the candidate equilibrium and pools with the uninformed, offering the same contract as they do his profit is

$$\Pi^P \;=\; -t \;=\; e_S Y. \tag{6}$$

The contract will be voided by the court, and the buyer will keep the up-front transfer specified in the contract offered by the uninformed buyer. Ex-post the seller will appropriate the surplus, and hence the informed buyer will select $e = 0$.

Notice that, since e_S satisfies $\psi'(e_S) = \pi_S\, Y$, for π_S sufficiently small $\Pi^* > \Pi^P$. Therefore with the given court rule $\mathcal{C}$ the contract prescribed in the candidate equilibrium is optimal for the informed buyer.

For completeness we need to describe the beliefs of the seller if he were to observe a contract offer by a buyer who "declares to be uninformed" that differs from the solution to problem (3). [16] For now it is sufficient to assume that the seller's out-of-equilibrium beliefs are that the offer comes from an uninformed buyer with probability 1. We return to this issue in Subsection 4.2 below.

[16] Notice that there is no need to specify the beliefs of the seller if he observes another contract offer from a buyer who declares to be informed since θ cannot be "mis-reported" as it is hard information.

4. Results

We will show that for an open set of possible configurations of parameters all equilibria exhibit the same qualitative characteristics as the candidate equilibrium described in Section 3. Under these parametric conditions all equilibria of the model have the feature that the court *does not* always enforce what the contracting parties write. In particular, given that in equilibrium the informed and uninformed separate, the uninformed would prefer the court to enforce the contract they write in the risky state. The court, on the other hand, maximizes the parties' welfare behind a veil of ignorance that covers the informed and uninformed buyers. It finds it optimal to void the contract of the uninformed in the risky state, because this induces *separation* between the informed and uninformed buyers.

Going from the candidate equilibrium of Section 3 to substantiating the claims we have just made, one important step remains. In characterizing the equilibrium above, we took the court's rule as given. We must now show that the court's optimal rule is of the assumed form.

4.1. Preliminaries: The Extensive Form Game

Technically, our model is a three-player incomplete-information extensive-form game which we designate by Γ throughout the rest of the paper.

To ease the exposition, we confine a detailed description of Γ to the Appendix (Section A.1), while here we limit ourselves to a brief discussion of those elements of Γ that are essential to stating our results below.

A typical court rule will be denoted by $\mathcal{C}$. As we have already discussed a court rule is a map from the court's information (the contract type, and the realization of θ) into a decision to *void* or *uphold* the parties' contract.

Recall that a contract between an uninformed buyer and the seller is of the form $(\mathbf{p}, t, \mathcal{F}) = (p_R, p_S, t, \mathcal{F})$. Of course a contract between an informed buyer who declares to be uninformed is an object of exactly the same type. We let $\mathcal{A}_U$ be the set of all possible contracts of the form $(\mathbf{p}, t, \mathcal{F})$; a typical element of $\mathcal{A}_U$ will be denoted by

a_U. A contract between an informed buyer who declared to be informed, on the other hand is an object of the type (p_I, t_I), where p_I is the price of the widget, and t_I is an up-front transfer.[17] We denote $\mathcal{A}_I$ the set of all possible contracts of this type, with a_I a typical element of $\mathcal{A}_I$.

The payoffs of each player are straightforward. The buyer's objective is expected profit. Since contracting takes place after the buyer's type is realized, the informed buyer maximizes his profit, and the uninformed buyer maximizes his expected profit. The seller maximizes his expected utility, so he will accept a contract offer if an only if, given his updated beliefs, a contract guarantees an expected utility greater than or equal to $V(\Delta)$.[18] Finally, since the seller's equilibrium payoff is fixed at $V(\Delta)$,[19] the court maximizes the weighted sum of the expected profit of the uninformed and of the informed buyer, with weights $1-\gamma$ and γ respectively.

4.2. Preliminaries: Equilibrium

We are interested in the Perfect-Bayesian Equilibria of Γ (from now on PBE). As is common in models with asymmetric information, a large multiplicity of PBE can be "bootstrapped" using a variety of off-the-equilibrium-path beliefs (of the seller in our case). For example, suppose that the court follows the rule in which if enforces all contracts regardless of the state. Consider the contracting game Γ that the parties play after the court announces this rule. Suppose that the informed and the uninformed buyers pool and offer the seller a contract that leaves the seller with a positive expected surplus. This may be part of a PBE if the seller's beliefs are that any out-of-equilibrium offer is made by an informed buyer. A seller who held these

[17] In Section 3 we argued that when the court rule is to enforce this type of contract the distinction between p_I and t_I is redundant. This is of course still the case. However, in the general case, before we have determined the equilibrium behavior of the court, we must allow for the possibility that the court rule will be to void such contract. Hence, in general, we must allow for a separate up-front transfer t_I.

[18] The seller's reservation expected utility level is $V(\Delta)$ because if he rejects the ex-ante contract offered by the buyer trade will take place ex-post. In this case the buyer's investment will be 0 and the seller will appropriate the entire surplus Δ regardless of the realization of uncertainty.

[19] See Subsection 4.2 below.

beliefs could reject an offer at terms less favorable than the pooling offer, since the offer might generate an expected loss under those beliefs.

The seller can guarantee himself a surplus of Δ by rejecting all contracts, so that transactions are negotiated ex-post when it is assumed that he has all the bargaining power. Equilibria that leave the seller with expected surplus greater than Δ when the buyer is assumed to have all the bargaining power seem implausible. We follow the usual strategy of applying a refinement of the set of PBE equilibria that restricts the set of allowed beliefs in order to restrict attention to those that are more plausible. The refinement we use is *undefeated equilibria* (Mailath, Okuno-Fujiwara, and Postlewaite 1993). Consider a proposed sequential equilibrium and an offer by the buyer that is not part of the proposed equilibrium. Suppose there is an alternative equilibrium in which some non-empty set of buyer types make this offer and that that set is precisely the set of types who prefer the alternative equilibrium to the proposed equilibrium. The test requires the seller's beliefs at that offer in the original equilibrium to be consistent with this set. If the beliefs are not consistent, the second equilibrium *defeats* the proposed equilibrium. For any court rule $\mathcal{C}$, in any undefeated perfect Bayesian equilibrium, the contract offered to the seller will leave him with expected surplus equal to Δ.

From now on we will focus exclusively on the set of Undefeated PBE of Γ, which is denoted by UPBE.

4.3. Equilibrium Characterization

We state formally our main results next. The first pins down the equilibrium outcomes of our game under particular parametric restrictions. We postpone a discussion of the parametric restrictions and their interpretation until after the statement of our first proposition.

Proposition 1. *Equilibrium Outcomes: Consider the extensive form game Γ of Subsection 4.1 above.*

There exists a $\tilde{\pi}_S \in (0,1)$ *and a* $\tilde{q} \in (0,1)$, *and for every* $\underline{\gamma}, \overline{\gamma} \in (0,1)$ *with* $\underline{\gamma} < \overline{\gamma}$, *there exist a* $\tilde{c} > 0$ *such that whenever* $\pi_S < \tilde{\pi}_S$, $\gamma \in (\underline{\gamma}, \overline{\gamma})$, $c_H - c_L > \tilde{c}$, *and* $q < \tilde{q}$ *the following holds:*

Any UPBE of Γ *has the following properties.*

(i) *The informed and uninformed buyers separate, i.e.,* $a_U \neq a_I$*;*

(ii) *Trade between the informed buyer and the seller is governed by a contract which is enforced by the court and specifies* (p_I, t_I) *satisfying* $p_I + t_I = c_H + \Delta$*;*

(iii) *If* $\theta = S$, *trade between the uninformed buyer and the seller is governed by a contract in* $\mathcal{A}_U$ *which is enforced by the court and specifies* $S \notin \mathcal{F}$, $t = -e_S Y$ *and* $p_S = c_S + \Delta + e_S Y$*;*

(iv) *If* $\theta = R$, *the contract between the uninformed buyer and the seller is not binding and hence the terms of trade are left to ex-post renegotiation.*[20]

The first restriction of the proposition is that π_S should be below a particular upper bound $\tilde{\pi}_S$. The reason for this requirement is easy to explain in terms of the candidate equilibrium described in Section 3. Note that we are claiming that the informed buyers will find it optimal to separate from the uninformed. If we take as given that the court will uphold the contract, then the payoff of the informed buyer who separates is $\Pi^* = e^* Y - \psi(e^*)$, as in (5). If the informed buyer pretends to be uninformed, as we argued in Subsection 3, his profit will be $\Pi^P = e_S Y$, as in (6). Hence, since $e_S Y$ is an increasing function of π_S, which is 0 when $\pi_S = 0$ and equal to $e^* Y > e^* Y - \psi(e^*)$ when $\pi_S = 1$, we obtain $\Pi^* > \Pi^P$ if and only if π_S is below a given cut-off point.

The other parametric requirements of Proposition 1 are designed to capture the following class of contracting problems. We need to focus on cases in which the risky

[20]Notice that at this point we are not specifying *why* ex-post renegotiation will prevail in this case. Clearly, in general this could be either because the parties' contract contains a "force majeure" clause, or because the contract is voided by the court. In Proposition 4 we will argue that the latter is more plausible.

state R is "not very risky" when the informed separate, but at the same time becomes "very risky" when the informed pool with the uninformed.

Notice that the riskiness of $\theta = R$ is controlled by two parameters in our model. Other things being equal, it is high when $c_H - c_L$ is high and low when $c_H - c_L$ is low. Moreover, for a given level of $c_H - c_L$, the riskiness of state R is low when the probability that the cost is c_H is either close to 0 or close to 1. Conversely, for a given level of $c_H - c_L$, the riskiness of $\theta = R$ is high when the probability that the cost is c_H is "in the middle," towards 1/2.

When the informed buyer separates, conditional on $\theta = R$, the seller's updated probability that the cost is c_H is simply q. On the other hand, when the informed buyer pretends to be uninformed, conditional on $\theta = R$, the seller's updated probability that the cost is c_H is augmented by a factor that is increasing in γ.[21]

The rationale behind the parametric restrictions in Proposition 1 is now apparent. If we fix a value of $\gamma \in (0, 1)$ we can always find a q sufficiently low and a level of $c_H - c_L$ sufficiently high so that the risk that the seller faces if pooling obtains is high, but at the same time the risk he faces if the informed buyer separates is low.

So, why is the case of low risk with separation and high risk with pooling the interesting one? The answer is that this is what pins down the equilibrium outcomes described in Proposition 1, and the fact that the court *must*, in a sense to be discussed shortly, be voiding some of the contracts that the parties write.

Suppose for a moment that the court simply upheld *every* contract in $\mathcal{A}_U$, and that the uninformed buyer writes a contract without a force majeure clause ($\mathcal{F} = \emptyset$). Then it is intuitively clear that the informed buyer would want to *pool* with the uninformed because he knows that $\theta = R$ and $\sigma = H$. However the price p_R in this case will be "low" (in particular, lower than $c_H + \Delta$) to reflect the fact that, conditional on $\theta = R$, the seller's updated probability that the cost is c_L will be "high" because of the pooling behavior of the informed and uninformed buyers. If $c_H - c_L$ is high, this effect swamps other effects and pooling will be preferred.

[21]The updated beliefs for the case of pooling are computed in (A.42) below.

On the other hand in the case of pooling, conditional on $\theta = R$, the seller will be exposed to high risk, for which the uninformed buyer must compensate him. This makes this option unattractive for the court which maximizes the weighted sum of the informed and uninformed buyer's expected profit. Under the parametric restrictions of Proposition 1, the court wants to induce separation in equilibrium to shelter the uninformed buyer from the cost associated with the excessive risk that obtains under pooling.

How can the court achieve separation in equilibrium? Under the parametric restrictions of Proposition 1, the court can *only* ensure that in equilibrium the informed buyer separates from the uninformed by *voiding* contracts in $\mathcal{A}_U$ when $\theta = R$. To see this, consider any candidate separating equilibrium in which the court upholds *all* contracts in $\mathcal{A}_U$. Then, as we remarked above, the parametric conditions of Proposition 1 ensure that, conditional on $\theta = R$, any contract in $\mathcal{A}_U$ will expose the seller to *low* risk. It now follows that it is profitable for the uninformed buyer to propose a contract that stipulates a price (as opposed to a "force majeure" clause) p_R for state R. This will be profitable because the gain in incentives to undertake the relationship-specific investment e will outweigh the (low) additional risk associated with stipulating a price p_R that the court will uphold as binding on the parties' terms of trade when $\theta = R$. But once the uninformed buyer writes a contract that is binding when $\theta = R$ the informed buyer will want to pool with the uninformed to buy the widget at a low price.

Hence, if the court upholds contracts in $\mathcal{A}_U$ when $\theta = R$, separation *cannot* be achieved in equilibrium. Thus, it is clear that an optimal court rule will *not* always uphold what the contracting parties write.

In order to sharpen our focus on the court's intervention in the parties' contractual relationship we put forth three further results. The first sets a benchmark case of a rule $\mathcal{C}$ consistent with a UPBE of Γ. This UPBE is essentially an expansion of the candidate equilibrium discussed in Section 3 above.

Proposition 2. *Benchmark Equilibrium: Under the parametric restrictions of Pro-*

position 1, the following strategy profile constitutes a UPBE of the extensive-form game Γ.

The court upholds all contracts in $\mathcal{A}_I$ *and all contracts in* $\mathcal{A}_U$*, provided that* $\theta = S$*. If* $\theta = R$*, the court voids all contracts in* $\mathcal{A}_U$*.*

The informed buyer offers the seller the separating contract $t = 0$ *and* $p_I = c_H + \Delta$*, which the seller accepts. The informed buyer undertakes the "first-best" level of relationship-specific investment* e^**.*

The uninformed buyer offers the seller the following contract in $\mathcal{A}_U$*, which the seller accepts. There is no "force majeure" so that* $\mathcal{F} = \emptyset$*. Moreover,* $t = -e_S Y$*,* $p_S = c_S + \Delta + e_S Y$*, and* p_R *is a number to be defined below.*[22] *The uninformed buyer undertakes a level of relationship-specific investment equal to* e_S*.*

There are two features of this UPBE that we want to focus on. First, the court's rule $\mathcal{C}$ actually prescribes that some contracts will be voided. Second, the uninformed buyer will offer a contract that does *not* include a "force majeure" clause. Hence, the court will actually *intervene in equilibrium* to void a contract that would otherwise stand.

Our next two propositions clarify that these are necessary features of equilibria in our model. The first is in fact a feature of *any* UPBE. The second is a feature of any UPBE that is robust in a sense described below.

Proposition 3. *Necessity of Voiding by the Court: Under the parametric restrictions of Proposition 1, any court rule* $\mathcal{C}$ *that is part of a UPBE of* Γ *has the following property.*

Any contract in $\mathcal{A}_U$ *that has no "force majeure" clause for the state* R *is voided by the court when* $\theta = R$*.*

[22] In fact plugging in any value of p_R will yield a UPBE together with the rest of the strategy profile we have just described. It is useful to consider the case in which p_R is the solution to (7) below.

Therefore, the court *must* intervene in the contractual relationship between buyer and seller in our model. This is the only way to induce separation between informed and uninformed buyers. The court, in turn, wants to achieve separation to enhance ex-ante welfare.

Proposition 3 still leaves open the possibility that the parties, knowing that the court will void their contract will in fact include a "force majeure" clause for $\theta = R$ that replicates what the court would do in this case. However, it seems implausible that the parties would include in their contract a ("force majeure") clause that they prefer *not* to be enforced. In particular, if there were some chance that the court did not void the contract when $\theta = R$, the parties would not include the clause. We formalize this intuition next.

First we clarify what price p_R the uninformed will include in any contract without a "force majeure" clause. Consider the benchmark equilibrium of Proposition 2. The price p_R that "completes" this equilibrium, is not hard to compute. Recall that in this equilibrium we have that $p_S = c_S + \Delta + e_S Y$, and $t = -e_S Y$, so that the seller's payoff when $\theta = S$ is equal to Δ. It is then clear that the price p_R that the uninformed want to write in their contract "just in case" the court should enforce contracts in $\mathcal{A}_U$ when $\theta = R$ is the lowest that the seller will accept. This is easily computed as the p_R that solves

$$qV(p_R - e_S Y - c_H) + (1-q)V(p_R - e_S Y - c_L) = V(\Delta) \qquad (7)$$

Formally, we assume that the court may make a mistake in *implementing* the announcement it makes.[23] A contract that should be voided according to the court's rule will be upheld with (vanishingly small) positive probability. Formally, the court's strategy set is modified as follows. The court's information set remains the same, but the court is now constrained to voiding the contract with probability greater or equal to a small real number ε in every eventuality. For every $\varepsilon > 0$ this defines

[23] For simplicity we introduce noise only when the court voids contracts. The claims in Proposition 4 below remain true if we perturb the court's decision to uphold contracts as well.

a new incomplete-information extensive-form three-player game, $\Gamma(\varepsilon)$. The set of Undefeated PBE of $\Gamma(\varepsilon)$ is denoted by UPBE(ε). We then say that a UPBE of Γ is *robust* if it is the limit of any sequence of UPBE(ε) as ε tends to 0.

Our last formal claim shows that the court must intervene *in equilibrium* in *any* robust UPBE of the game Γ.

Proposition 4. *Robust Equilibria: Necessity of Court Intervention: Under the parametric restrictions of Proposition 3, any court rule $\mathcal{C}$ that is part of any robust UPBE of the extensive-form game Γ has the following properties.*

(i) *The uninformed buyer offers a contract that contains no "force majeure" clause ($\mathcal{F} = \emptyset$).*

(ii) *The court rule $\mathcal{C}$ prescribes that the equilibrium contract between the uninformed buyer and the seller is void when $\theta = R$.*

5. Discussion

5.1. Interpretation of Bayesian Equilibria

We discussed in the introduction a motivation for evaluating welfare at the ex-ante stage. At that point, the parties are symmetrically informed, and agreements can be reached that avoid inefficiencies that are unavoidable at the interim stage when the parties have private information. A court that maximizes the ex-ante expected surplus will choose the same rule as the parties would have chosen at that stage, were it possible. In other words, if the parties could meet at that point, they would instruct the court to void some contracts they might subsequently write. They will do this precisely because the parties will understand that while they may regret this in some circumstances, it will have a salutary incentive effect in other circumstances that more than outweighs any negative effect. It is in this sense that we say that the court is acting precisely as the parties would want.

There is, however, and alternate interpretation of Bayesian equilibria. From a formal viewpoint, the importance of modelling the buyer as having two types is to capture the fact that the seller is uncertain about what the buyer knows. The seller does not know whether or not the buyer is informed, but he knows that the buyer *does* know. While these two "types" of buyers – informed and uninformed – might in fact be thought of as a single agent who might or might not have received information, the alternative interpretation is that whether or not the buyer is informed or uninformed is exogenously given, and the probabilities reflect nothing more than the seller's inability to distinguish the two types. In other words, there may *never* have been a time at which the buyer did not have the relevant information, and consequently, never a time at which the parties would have chosen the rule. When a court maximizes ex-ante welfare, it is essentially averaging the benefits and losses of the *different* kinds of buyers. In other words, a court that maximizes expected welfare amounts to making interpersonal welfare judgements: it is setting a rule that will benefit some at the expense of others. The court in this case is essentially maximizing the welfare of the agents "behind the veil of ignorance".

5.2. Immutability of Court Rules

There are two distinct types of legal rules governing contracts: default rules that the parties can contract around by prior agreement and immutable rules that cannot be contracted around.[24] Immutable rules can be justified if society wants to protect contracting parties or if society wants to protect third parties. The first justification is based paternalism, and thus, irrelevant in a model with fully informed and fully rational actors. The protection of third parties can generally be thought of as rules to ameliorate externalities that contracting parties may impose on others.[25] Schwartz and Scott (2003) argue that in the absence of traditional externalities courts should not impose immutable rules on sophisticated parties (that is, parties that are fully informed and capable of making rational decisions). We have identified an externality qualitatively different from traditional externalities, namely an informational externality. Our contracting parties may be uninformed, but are certainly sophisticated.

[24]The following discussion draws heavily on Ayres and Gertner (1989).
[25]Price fixing is a prototypical example of a traditional externality.

They know what they do not know, and take this into account when they make decisions. Yet, if the court enforces all contracts between sellers and uninformed buyers, informed buyers have an incentive to masquerade as uninformed rather than disclose their private information and contract accordingly. A rule that sometimes voids contracts written by (seemingly) uninformed parties increases equilibrium information disclosure, and is consequently efficiency enhancing. The parties whose contracts are voided would contract around this rule if allowed, hence the rule must be immutable to be effective.

5.3. Common Law vs. Codified Law

We model the court as choosing the rules by which it interprets contracts at the initial stage, prior to the time parties contract. As we discussed above, this captures in a static model the dynamic process by which the court's rulings over time create a set of precedents that allows parties to forecast how contractual disputes will be resolved. One can think of common law as evolving over time in this way. Alternatively, the model can be interpreted as a model of codified law in which courts have no discretion in resolving disputes, and only apply the rules that have been set forth in statutes.

In our model there is no difference between the rules that would be optimal in a common law regime and those that would be optimal under codified law. A pair of potential contracting parties meet, with the buyer randomly being informed or not informed. We identify the ex-ante optimal choice of rule given the probabilities, and it makes no difference whether that rule is embodied in statutory law or evolves through the decisions that the court makes as it confronts repeated incarnations of the parties, at least in the long run.

The situation is quite different, however, if the set of contracting situations that arise are not repetitions of the same problem, varying only in the particular realizations of the random variables. When the court confronts a particular dispute, any investments the parties have made will be sunk, hence the decision the court makes will have no efficiency consequences for the parties in that dispute; any effect on investment incentives has to do only with contracts signed in the future. One of the parties in a contract may find performance onerous due to intervening circumstances, and ask the court to excuse performance. A court with discretion about interpreting contracts may be tempted to provide ex-post insurance to that party, and attempt to minimize the impact on future contracting parties by giving a very narrow argument for why the contract was not enforced. Of course, when there is an accumulation of a large number of highly specific circumstances under which contracts will not be enforced, the parties will understand that it is likely that the contract will not be enforced, even though they cannot forecast the courts' reasoning for excusing performance.[26]

Codified law provides a simple solution to this time inconsistency problem: it *commits* to a particular interpretation of contracts, at least to the extent that it limits judges' ability to intervene in contractual relationships after investments have been made. This advantage of codified law relative to common law comes at a cost however. The advantage of codified law is that the rules by which contracts are interpreted are set out prior to contracting. Subsequent to the choice of rules, but prior to the time at which performance is called for under a contract, circumstances may arise that were unforeseen at the time that the rules were codified. Moreover, some of the circumstances may be such that, had they been foreseen, the optimal codified rules would have excused performance when they arose. The advantage of common law is precisely that it allows ex-post adjustments to the rules governing contractual relationships.[27]

Appendix

A.1. The Extensive Form Game Γ

For completeness, in this Section we describe in full the three-player incomplete-information extensive-form game Γ played by the court, the Buyer and the Seller that we introduced in Subsection 4.1 above.

Recall from Subsection 4.1 that $\mathcal{A}_U$ denotes the set of all possible contracts of the form $(\mathbf{p}, t, \mathcal{F})$

[26] We thank Steve Morris for this point.

[27] Anderlini, Felli, and Postlewaite (2003) analyzes this case.

with typical element a_U. Also $\mathcal{A}_I$ denotes the set of all possible contracts of the type (p_I, t_I), with a_I a typical element.

The symbol $\mathcal{A}$ will denote $\mathcal{A}_U \times \mathcal{A}_I$, with typical element $a = (a_U, a_I)$.

The court can condition its decision to void or uphold on the realized value of θ and on the contract type.[28] We can obviously take the set of contract types to be $\mathcal{T} = \{U, I\}$. Hence the court's "information set" is $\mathcal{I} = \mathcal{T} \times \Theta$.[29]

In summary, the court observes the type of contract and whether the state was risky or not, and decides whether to uphold ($\mathcal{U}$) or void ($\mathcal{V}$) the parties' contract on the basis of its observation. Hence, the court's rule is a map $\mathcal{C} : \mathcal{I} \to \{\mathcal{V}, \mathcal{U}\}$. The set of all such possible rules is the court's strategy set, denoted by $\mathcal{S}_C$.

Given our notation it is clear that given a $\mathcal{C}$, $\mathcal{A}_U \times [0, 1]$ is the strategy set of the uninformed buyer, while $[\mathcal{A}_I \cup \mathcal{A}_U] \times [0, 1]$ is the strategy set of the informed buyer (who can pretend to be uninformed and offer a contract in $\mathcal{A}_U$).[30] We denote by a_U^e a typical element of $\mathcal{A}_U \times [0, 1]$ and by a_I^e a typical element of $[\mathcal{A}_I \cup \mathcal{A}_U] \times [0, 1]$. So, given a $\mathcal{C}$, a strategy for the buyer in the overall Bayesian game is a pair $a^e = (a_U^e, a_I^e)$ of contracts to offer and investment levels contingent on being informed or uninformed. The set of all such pairs will be denoted by $\mathcal{A}^e$. Since the buyer makes his take-it-or-leave-it contract offer to the seller knowing the court's rule $\mathcal{C}$, a buyer's strategy is a map $\mathcal{B} : \mathcal{S}_C \to \mathcal{A}^e$. The set of all such maps is the buyer's strategy set, denoted by $\mathcal{S}_B$.

The strategy set of the seller, who is always uninformed, is straightforward: he decides to accept ($\mathcal{Y}$) or reject ($\mathcal{N}$) the buyer's take-it-or-leave-it offer of a contract. So, given a rule $\mathcal{C}$ and an $a = (a_U, a_I)$, the seller's action set can be written as $\{\mathcal{Y}, \mathcal{N}\}$ and a seller's strategy is a map $\mathcal{S} : \mathcal{S}_C \times \mathcal{A} \to \{\mathcal{Y}, \mathcal{N}\}$.[31] The set of all such maps is the seller's strategy set, denoted by $\mathcal{S}_S$.[32]

[28]See footnote 10 above.

[29]Notice that some elements of $\mathcal{I}$ are in fact automatically ruled out. This is because only informed buyers can offer contracts of type I. Hence if the court observes I it will automatically know that $\theta = R$ (and $\sigma = H$).

[30]Recall that each type of buyer must select a level of relationship-specific investment in $[0, 1]$ as well as offering a contract to the seller.

[31]Notice that we are including "redundant information" in the seller's strategy set. This is because the seller does not actually observe a, but only the contract that the *realized* type of buyer offers. So, the decision of the seller is contingent only the component of a that refers to the actual realized value of $\eta \in \{U, I\}$. Taking this into account would only add to our notation. It is clear that perfection ensures that this is irrelevant for our results.

[32]Notice also that, purely for the sake of simplicity, we omit any variables relating to the re-negotiation stage (in case the contract is voided) from the description of the strategy sets of the buyer and of the seller. Since whenever re-negotiation occurs the seller appropriates all the surplus by assumption, this simplifies our notation and is of no consequence for the analysis.

Notice that the *beliefs* of the seller, in equilibrium will depend on the overall contract-offering strategy of the buyer a, and on the actual contract offer $o \in \mathcal{A}_U \cup \mathcal{A}_I$ that he actually observes. We will denote the *updated* beliefs of the seller concerning the realizations of θ and σ by $\hat{\pi}_R(o,a)$, $\hat{\pi}_S(o,a)$, and $\hat{q}(o,a)$.[33] We suppress the buyer's strategy from the notation when no confusion will result, and write the seller's updated beliefs as $\hat{\pi}_R$, $\hat{\pi}_S$, and $\hat{q}$.

This, together with the payoffs described in Subsection 4.1 above, completes the description of the extensive form game Γ.

A.2. A Benchmark Maximization Problem

In this section we characterize the solution to a benchmark maximization problem, as a function of some key parameters of our model. The results proved here will be used below to prove the claims made in the text.

Notice that the analysis in this section can be thought of as a characterization of the optimal contract between the uninformed buyer and the seller when the informed buyer separates, or alternatively when the proportion γ of informed buyers is 0.

Lemma A.1: *Consider the following problem:*[34]

$$\begin{array}{ll} \max\limits_{p_S,t,e_S} & \pi_S(e_S Y + \Delta + c_S - p_S) - t - \psi(e_S) \\ s.t. & \pi_S V(p_s - c_S + t) + \pi_R V(\Delta + e_S Y + t) \geq V(\Delta) \\ & \psi'(e_S) \; = \; \pi_S Y \end{array} \tag{A.1}$$

Then the solution is unique, and satisfies $t = -e_S Y$ *and* $p_S = v_S + e_S Y = c_S + \Delta + e_S Y$. *The corresponding maximized value of the objective function is* $\Pi^S = e_S Y - \psi(e_S) > 0$.

Proof: The solution is entirely characterized by the first order condition

$$V'(p_S - c_S + t) \; = \; \pi_S V'(p_S - c_S + t) + \pi_R V'(\Delta + e_S Y + t) \tag{A.2}$$

together with the first constraint of problem (A.1) holding as an equality. Since (A.2) implies $p_S - c_S + t = \Delta + e_S Y + t$, using the constraint we get that $p_S - c_S + t = \Delta + e_S Y + t = \Delta$. From this we obtain directly that $t = -e_S Y$ and $p_S = v_S + e_S Y = c_S + \Delta + e_S Y$, as required.

[33]When o is part of the profile of equilibrium contracts offered by the buyer, these beliefs, of course, are obtained using Bayes rule.

[34]Notice that problem (A.1) is in fact a re-statement of problem (3).

To see that the buyer's expected profit is as claimed we simply plug the solution back into the objective function of problem (A.1). This gives a value of $e_S Y - \psi(e_S)$, which is positive since e_S is the solution to $\max_{e_s \in [0,1]} \pi_S\, e_S\, Y - \psi(e_S)$. ■

Lemma A.2: *Consider the following problem*

$$\begin{array}{rl} \max\limits_{p_R, t, e_R} & \pi_R(e_R Y + \Delta + \bar{c} - p_R) - t - \psi(e_R) \\ s.t. & \pi_S V(\Delta + e_R Y + t) + \pi_R\left[qV(p_R + t - c_H) + (1-q)V(p_R + t - c_L)\right] \geq V(\Delta) \\ & \psi'(e_R) \;=\; \pi_R Y \end{array} \tag{A.3}$$

and let the maximized value of the objective function be Π^R.

Consider also the problem

$$\begin{array}{rl} \max\limits_{p_R, p_S, t, e^*} & e^* Y + \Delta - t - \psi(e^*) + \pi_S(c_S - p_S) + \pi_R(\bar{c} - p_R) \\ s.t. & \pi_S V(p_s - c_S + t) + \pi_R\left[qV(p_R + t - c_H) + (1-q)V(p_R + t - c_L)\right] \geq V(\Delta) \\ & \psi'(e^*) \;=\; Y \end{array} \tag{A.4}$$

and let the maximized value of the objective function be $\Pi^{S,R}$.

Then $\Pi^{S,R} > \Pi^R$.

Proof: Let $(\tilde{p}_R, \tilde{t}, e_R)$ be any solution to problem (A.3). Let $\tilde{p}_S = \Delta + e_R Y + c_S$. Notice next that the array $(\tilde{p}_R, \tilde{p}_S, \tilde{t}, e^*)$, because of the way we have set $\tilde{p}_S$ is feasible in problem (A.4). Substituting these values in the objective function of problem (A.4) we get

$$e^* Y + \Delta - \tilde{t} - \psi(e^*) - \pi_S(\Delta + e_R Y) + \pi_R(\bar{c} - \tilde{p}_R) \tag{A.5}$$

Since e^* is chosen to maximize $eY - \psi(e)$, it is clear that $e^* Y - \psi(e^*) > e_R Y - \psi(e_R)$. Hence the quantity in (A.5) is greater than

$$e_R Y + \Delta - \tilde{t} - \psi(e_R) - \pi_S(\Delta + e_R Y) + \pi_R(\bar{c} - \tilde{p}_R) \;=\; \pi_R(e_R Y + \Delta + \bar{c} - \tilde{p}_R) - \tilde{t} - \psi(e_R) \tag{A.6}$$

However the right-hand side of (A.6) is precisely Π^R, namely the maximized value of the objective function of problem (A.3). Therefore, we have found that the array $(\tilde{p}_R, \tilde{p}_S, \tilde{t}, e^*)$ is feasible in problem (A.4), and yields an expected profit for the buyer that is larger than Π^R. This is clearly enough to prove that $\Pi^{S,R} > \Pi^R$, as required. ■

Lemma A.3: *Consider again problem (A.4). Then the solution only determines the values of* $p_S + t$ *and* $p_R + t$*, and of course* e.

If we fix arbitrarily $t = 0$, then the solution to problem (A.4) is entirely characterized by $\psi'(e^*)$ $= Y$ and by the two equations

$$\begin{array}{c} V'(p_S^* - c_S) = qV'(p_R^* - c_H) + (1-q)V'(p_R^* - c_L) \\ \pi_S V(p_S^* - c_S) + \pi_R\left[qV(p_R^* - c_H) + (1-q)V(p_R^* - c_L)\right] = V(\Delta) \end{array} \tag{A.7}$$

that determine p_R^* and p_S^*.

Proof: The claim that e^* is determined as claimed requires no proof. To see that only $p_R + t$ and $p_S + t$ are determined notice that if (p_R, p_S, t, e^*) is feasible in problem (A.4), then any array (p'_R, p'_S, t', e') that satisfies $p'_R + t' = p_R + t$, $p'_S + t' = p_S + t$ and $e' = e^*$ is also feasible in problem (A.4), and gives the buyer the same expected profit.

To see that equations (A.7) determine the solution values for p_R and p_S when $t = 0$ simply notice that the first constraint in problem (A.4) must be binding, and that the first of equations (A.7) is a direct consequence of the Lagrangean first-order conditions associated with problem (A.4). ■

Lemma A.4: *Let $\Pi^{S,R}$ be as in Lemma A.2 and Π^S be as in Lemma A.1. Define also*

$$\Phi(Y, \pi_S) = [e^*Y - \psi(e^*)] \;-\; [e_S Y - \psi(e_S)] \tag{A.8}$$

Then

$$\Pi^{S,R} - \Pi^S \;=\; \Phi(Y, \pi_S) + \Delta + \pi_S[c_S - p_S^*(q, c_H, c_L, c_S)] + \pi_R[\overline{c} - p_R^*(q, c_H, c_L, c_S)] \tag{A.9}$$

where $\Phi(Y, \pi_S)$ is as in (A.8) and $p_R^*(q, c_H, c_L, c_S)$ and $p_S^*(q, c_H, c_L, c_S)$ are the solution to equations (A.7).

Proof: From Lemma A.1 we know that $\Pi^S = e_S Y \,-\, \psi(e_S)$. From Lemma A.3 we know that

$$\Pi^{S,R} \;=\; e^*Y + \Delta - \psi(e^*) + \pi_S[c_S - p_S^*(q, c_H, c_L, c_S)] + \pi_R[\overline{c} - p_R^*(q, c_H, c_L, c_S)] \tag{A.10}$$

The claim then follows directly from the definition of $\Phi(Y, \pi_S)$ as in (A.8). ■

Proposition A.1: *Let*

$$\widetilde{\Pi} \;=\; \max\{\Pi^S, \Pi^R, \Pi^{S,R}, 0\} \tag{A.11}$$

and denote again by $p_R^*(q, c_H, c_L, c_S)$ and $p_S^*(q, c_H, c_L, c_S)$ the solution to equations (A.7).

Then $\widetilde{\Pi} \neq \Pi^R$ and $\widetilde{\Pi} \neq 0$. Moreover, if

$$\Phi(R, \pi_S) + \Delta + \pi_S[c_S - p_S^*(q, c_H, c_L, c_S)] + \pi_R[\overline{c} - p_R^*(q, c_H, c_L, c_S)] \tag{A.12}$$

is less than zero, then $\widetilde{\Pi} = \Pi^S$*, if the quantity in (A.12) is greater than zero then* $\widetilde{\Pi} = \Pi^{S,R}$*, and if the quantity in (A.12) is equal to zero then* $\widetilde{\Pi} = \Pi^S = \Pi^{S,R}$*.*

Proof: From Lemma A.1 we know that $\Pi^S > 0$. Hence $\widetilde{\Pi} \neq 0$. From Lemma A.2 we know that $\Pi^{S,R} > \Pi^R$. Hence $\widetilde{\Pi} \neq \Pi^R$.

The claim that whether $\widetilde{\Pi} = \Pi^S$ or $\widetilde{\Pi} = \Pi^{S,R}$ or both depends on the sign of (A.12) is a direct consequence of Lemma A.4. ■

Lemma A.5*: Let two arrays of costs* (c'_S, c'_R, c'_L) *and* (c''_S, c''_R, c''_L) *be given and denote again by* $p^*_R(q, c_H, c_L, c_S)$ *and* $p^*_S(q, c_H, c_L, c_S)$ *the solution to equations (A.7). Assume that there exist two constants* x *and* y *for which* $c''_H = c'_H + x$*,* $c''_L = c'_L + x$ *and* $c''_S = c'_S + y$*.*

Then

$$\begin{aligned}\pi_S[c'_S - p^*_S(q, c'_H, c'_L, c'_S)] + \pi_R[\overline{c}' - p^*_R(q, c'_H, c'_L, c'_S)] \ = \\ \pi_S[c''_S - p^*_S(q, c''_H, c''_L, c''_S)] + \pi_R[\overline{c}'' - p^*_R(q, c''_H, c''_L, c''_S)]\end{aligned} \tag{A.13}$$

where $\overline{c}' = qc'_H + (1-q)c'_L$ *and* $\overline{c}'' = qc''_H + (1-q)c''_L$*.*

Proof: By inspection of equations (A.7) if (c'_S, c'_R, c'_L) and (c''_S, c''_R, c''_L) are such that $c''_H = c'_H + x$, $c''_L = c'_L + x$ and $c''_S = c'_S + y$ then $p^*_S(q, c''_H, c''_L, c''_S) = p^*_S(q, c'_H, c'_L, c'_S) + y$ and $p^*_R(q, c''_H, c''_L, c''_S) = p^*_R(q, c'_H, c'_L, c'_S) + x$. Plugging these equalities into (A.13) immediately proves the claim. ■

Lemma A.6*: Fix a* V *function and all other parameters of problem (A.4), except for* c_L *and* c_H*. Denote again by* $p^*_R(q, c_H, c_L, c_S)$ *and* $p^*_S(q, c_H, c_L, c_S)$ *the solution to equations (A.7).*

Then for any $B > 0$ *there exists a positive number* A *such that whenever* $c_H - c_L \geq A$

$$\pi_S[c_S - p^*_S(q, c_H, c_L, c_S)] + \pi_R[\overline{c} - p^*_R(q, c_H, c_L, c_S)] \ \leq \ -B \tag{A.14}$$

Proof: In view of Lemma A.5 we can restrict attention to the case in which $c_L = c_S = 0$ and c_H grows unboundedly large. Note that in this case $\overline{c} = qc_H$. Let $p^*_R(c_H)$ and $p^*_S(c_H)$ be the solution to

$$\begin{aligned}V'(p^*_S) = qV'(p^*_R - c_H) + (1-q)V'(p^*_R) \\ \pi_S V(p^*_S) + \pi_R\left[qV(p^*_R - c_H) + (1-q)V(p^*_R)\right] = V(\Delta)\end{aligned} \tag{A.15}$$

It is enough to show that

$$\pi_S[-p^*_S(c_H)] \ + \ \pi_R[qc_H - p^*_R(c_H)] \tag{A.16}$$

decreases without bound when c_H increases without bound.

From the first equation in (A.15), since V' is a decreasing function, we obtain $p_R^*(c_H) > p_S^*(c_H) > p_R^*(c_H) - c_H$. Using the second of these inequalities, we can assert that

$$\pi_S[c_H - p_R^*(c_H)] \; + \; \pi_R[qc_H - p_R^*(c_H)] \; \geq \; \pi_S[-p_S^*(c_H)] \; + \; \pi_R[qc_H - p_R^*(c_H)] \tag{A.17}$$

hence, rearranging the left-hand side of (A.17), it suffices to show that if we set $z = \pi_S + \pi_R q < 1$ then

$$p_R^*(c_H) \; - \; z\,c_H \tag{A.18}$$

grows without bound as c_H grows without bound.

Suppose now by way of contradiction that this is not the case. Let K be an upper bound for the quantity in (A.18). Then, for every $c_H > 0$

$$p_R^*(c_H) \; - \; c_H \; \leq \; K \; - \; (1-z)c_H \tag{A.19}$$

and hence we can conclude that our contradiction hypothesis implies that $p_R^*(c_H) - c_H$ decreases without bound as c_H increases without bound. However, since V is bounded above and unbounded below, for c_H sufficiently large this eventually contradicts the second equation in (A.15). ■

Lemma A.7: *Fix a V function and all parameters of problem (A.4), including a pair $c_H > c_L$, except for q.*

Consider now

$$f(q) \; \equiv \; \Delta + \pi_S[c_S - p_S^*(q, c_H, c_L, c_S)] + \pi_R[\bar{c} - p_R^*(q, c_H, c_L, c_S)] \tag{A.20}$$

Then $f(0) = f(1) = 0$ and f is continuous and quasi-convex over $[0, 1]$.

Proof: In view of Lemma A.5 we can restrict attention to the case in which $c_L = c_S = 0$ and $c_H > 0$. Note that in this case $\bar{c} = qc_H$. Let $p_R^*(q)$ and $p_S^*(q)$ be the solution to

$$\begin{gathered} V'(p_S^*) = qV'(p_R^* - c_H) + (1-q)V'(p_R^*) \\ \pi_S V(p_S^*) + \pi_R\left[qV(p_R^* - c_H) + (1-q)V(p_R^*)\right] = V(\Delta) \end{gathered} \tag{A.21}$$

It is enough to show that

$$f(q) \; \equiv \; \Delta + \pi_S[-p_S^*(q)] \; + \; \pi_R[qc_H - p_R^*(q)] \tag{A.22}$$

satisfies the required properties.

Notice that when $q = 0$ from (A.21) we know that $p_R^*(0) = p_S^*(0) = \Delta$. Hence $f(0) = 0$. Similarly, when $q = 1$ from (A.21) we know that $p_R^*(1) - c_H = p_S^*(1) = \Delta$. Hence $f(1) = 0$.

Continuity is obvious since $f(q)$ is in fact differentiable by the implicit function theorem.

It remains to show that f is quasi-convex over $[0, 1]$. Consider the problem

$$\begin{array}{ll} \max\limits_{p_R, p_S} & \pi_S[-p_S] \; + \; \pi_R[qc_H - p_R] \\ s.t. & \pi_S V(p_s) + \pi_R\left[qV(p_R - c_H) + (1-q)V(p_R)\right] \geq V(\Delta) \\ & q \in [0,1] \end{array} \tag{A.23}$$

It is enough to show that the maximized value of the objective function in (A.23) is quasi-convex in q. Let $g(\mathbf{p}, q)$ be the objective function of problem (A.23) and $\mathcal{O}(q)$ be the opportunity set. Let also $g^*(q) = \max_{\mathbf{p}\in\mathcal{O}(q)} g(\mathbf{p}, q)$ be the maximized value of the objective function of problem (A.23).

Let two values q' and q'' be given with $q' < q''$ and q', $q'' \in (0, 1)$. Let a number $\xi \in (0, 1)$ also be given and let $q = \xi q' + (1 - \xi)q''$.

We need to show that $g^*(q) \leq \max\{g^*(q'), g^*(q'')\}$.

Notice that given that the constraint in problem (A.23) is in fact linear in q we have that $\mathcal{O}(q) \subseteq \mathcal{O}(q') \cup \mathcal{O}(q'')$. Hence

$$g^*(q) \;=\; \max_{\mathbf{p}\in\mathcal{O}(q)} g(\mathbf{p}, q) \;\leq\; \max_{\mathbf{p}\in\mathcal{O}(q')\cup\mathcal{O}(q'')} g(\mathbf{p}, q) \;\leq\; \max\{g^*(q'), g^*(q'')\} \tag{A.24}$$

which is enough to prove the claim. ■

Proposition A.2: *As in Proposition A.1 let*

$$\widetilde{\Pi} \;=\; \max\{\Pi^S, \Pi^R, \Pi^{S,R}, 0\} \tag{A.25}$$

Fix a V function and all parameters of problem (A.4), except for c_H and c_L. Then there exists a positive number A such that whenever $c_H - c_L \geq A$ then $\widetilde{\Pi} = \Pi^S$.

Now fix a V, a pair c_H and c_L with $c_H - c_L$ sufficiently large, and all other parameters of problem (A.4), except for q. Then there exist two numbers $q' < q''$ in $(0, 1)$ such that when $q \in (q', q'')$ then $\widetilde{\Pi} = \Pi^S$, when $q \in [0, q') \cup (q'', 1]$ then $\widetilde{\Pi} = \Pi^{S,R}$, and when either $q = q'$ or $q = q''$ then $\widetilde{\Pi} = \Pi^S = \Pi^{S,R}$.

Proof: From proposition A.1 we know that $\widetilde{\Pi} \neq \Pi^R$ and $\widetilde{\Pi} \neq 0$.

From Lemma A.4 we know that $\Pi^{S,R} - \Pi^S = \Phi(Y, \pi_S) + \Delta + \pi_S[c_S - p^*_S(q, c_H, c_L, c_S)] + \pi_R[\bar{c} - p^*_R(q, c_H, c_L, c_S)]$.

Now fix a V and all parameters of problem (A.4) except for c_H and c_L. Since $\Phi(Y, \pi_S) + \Delta$ is independent of c_H and c_L, from Lemma A.6 and Proposition A.1 we know that there exists an A such that whenever $c_H - c_L \geq A$ we have $\Pi^{S,R} - \Pi^S < 0$. Hence $\widetilde{\Pi} = \Pi^S$. This is clearly enough to prove the first assertion in the statement of the proposition.

Now fix a V, all other parameters of problem (A.4) except for q, and fix a pair c_H and c_L such that for some $q \in [0, 1]$ we have that $\Pi^{S,R} - \Pi^S < 0$. Such pair must exist by the first assertion of the proposition we are proving.

Then by Lemma A.7 and Proposition A.1, since $\Phi(Y, \pi_S) > 0$ is independent of q, there must exist some q' and $q'' \in (0, 1)$ with $q' < q''$ such that $q \in (q', q'')$ implies $\Pi^{S,R} - \Pi^S < 0$, $q \in [0, q') \cup (q'', 1]$ implies $\Pi^{S,R} - \Pi^S > 0$, and q equal to either q' or q'' implies $\Pi^{S,R} - \Pi^S = 0$. This is clearly enough to prove the claim. ■

A.3. Preliminaries

In this section we prove a set of preliminary results that will help us rule out many possible cases in the proofs of our main results characterizing the set of UPBE of Γ below.[35]

Definition A.1: *Recall that in a UPBE $\mathcal{C}$ is endogenously determined as the court rule that maximizes the expected payoff of the court defined in Subsection 4.1. Let an arbitrary $\mathcal{C}$ be given. This clearly defines a two-player extensive-form game $\Gamma(\mathcal{C})$ between the buyer and the seller in an obvious way. In what follows we refer to a "UPBE given $\mathcal{C}$" for Γ as a PBE of $\Gamma(\mathcal{C})$ that gives the seller an equilibrium payoff of $V(\Delta)$.*

Lemma A.8: *There exists no UPBE of Γ in which $\mathcal{C}$ prescribes that all contracts are voided.*

Proof: If the court's rule prescribes that all contract are voided, the court's payoff is zero.

Consider now a $\mathcal{C}$ that prescribes upholding contracts in $\mathcal{A}_U$ if $\theta = S$, voiding contracts in $\mathcal{A}_U$ if $\theta = R$, and voiding contracts in $\mathcal{A}_I$.

Under this court rule, the informed buyer will select a level of relationship-specific investment that is equal to zero. This is because he knows that any contract (separating or pooling) he might offer the seller will in fact be voided.

[35] Recall from Subsection 4.2 that in any UPBE of Γ the seller's payoff is equal to $V(\Delta)$.

Fix any UPBE given $\mathcal{C}$. Now let $\hat{\pi}_S$ and $\hat{\pi}_R$ be the updated beliefs of the seller, conditional on observing the equilibrium contract offer of the uninformed buyer. Let also $z \in [0,1]$ be the seller's belief of having met a pooling informed buyer conditional on $\theta = R$ and on observing the equilibrium contract offer of the uninformed buyer. Then the equilibrium contract offer of the uninformed buyer, which the seller accepts, is the solution to

$$\begin{array}{ll} \max\limits_{p_S,t,e_S} & \pi_S(e_S Y + \Delta + c_S - p_S) - t - \psi(e_S) \\ s.t. & \hat{\pi}_S V(p_s - c_S + t) + \hat{\pi}_R[zV(\Delta + e_S Y + t) + (1-z)V(\Delta + t)] \geq V(\Delta) \\ & \psi'(e_S) \;=\; \pi_S Y \end{array} \tag{A.26}$$

Using the first order conditions of problem (A.26) it easily seen that the maximized value of its objective function is greater than zero. Hence in any UPBE of Γ given $\mathcal{C}$ the uninformed buyer's expected payoff is positive. Notice also that in any UPBE of Γ given $\mathcal{C}$, the informed buyer's payoff cannot be negative. This is simply because he can guarantee a payoff of zero by offering a separating contract in $\mathcal{A}_I$. Hence the court's payoff in any UPBE of Γ given $\mathcal{C}$ is positive. This is enough to prove the claim. ■

Lemma A.9: *Let a $\mathcal{C}$ be given prescribing that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, and that all contracts in $\mathcal{A}_U$ are upheld if $\theta = S$. Then in any UPBE of Γ given $\mathcal{C}$ the uninformed buyers do not include $\theta = S$ in $\mathcal{F}$.*

Proof: If the uninformed write a contract in $\mathcal{A}_U$ that specifies $S \in \mathcal{F}$, since the court voids when $\theta = R$, their trade will be left to ex-post renegotiation with probability one. Hence their payoff will be zero.

If they write a contract in $\mathcal{A}_U$ in which $S \notin \mathcal{F}$, since the court will uphold such contracts, their equilibrium contract offer, which the seller will accept, is the solution to problem (A.26). Since the maximized value of the objective function of this problem is positive, they will do so in preference to a contract in $\mathcal{A}_U$ with $S \in \mathcal{F}$. ■

Lemma A.10: *Let a $\mathcal{C}$ be given prescribing that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, all contracts in $\mathcal{A}_U$ are upheld if $\theta = S$, and all contracts in $\mathcal{A}_I$ are upheld. Then there exists a $\tilde{\pi}_S \in (0,1)$ such that whenever $\pi_S < \tilde{\pi}_S$ in every UPBE of Γ given $\mathcal{C}$ the informed buyers separate.*

Proof: By Lemma A.9 the uninformed buyers will write a contract in $\mathcal{A}_U$ that specifies $S \notin \mathcal{F}$. Separation yields the informed buyer a payoff of $e^*Y - \psi(e^*)$. By pooling, the informed buyer writes a contract that he knows will be voided by the court. Hence his payoff from pooling is $-t$, where t is the up-front transfer in the equilibrium contract of the uninformed buyers.

First, we check that there is a UPBE of Γ given $\mathcal{C}$ in which the informed buyers separate. If the informed buyers separate, then the uninformed buyers, in equilibrium, offer a contract to the seller, which the seller accepts, that is the solution to problem (A.1). Hence, from Lemma A.1, we know that the up-front transfer in the equilibrium contract of the uninformed buyers is $t = -e_S\, Y$. Since for π_S sufficiently small it must be that $e^*Y - \psi(e^*) > e_S\, Y$, this proves that the informed buyers do in fact prefer to separate. This is enough to verify that there is a UPBE of Γ in which the informed buyers separate.

We now need to argue that there are no UPBE of Γ given $\mathcal{C}$ in which a positive fraction of informed buyers pool with the uninformed. By way of contradiction, suppose that one exists. Notice that the informed buyers who pool with the uninformed select a level of relationship-specific investment equal to zero since they know that their contract will be voided by the court. Hence their payoff in any such UPBE is $-t$ where t is the up-front transfer in the equilibrium contract between the uninformed buyers and the seller. It also follows that in any such UPBE the equilibrium contract between the uninformed buyer and the seller solves problem (A.26). The solution to problem (A.26) is entirely characterized by the first-order condition

$$\frac{\hat{\pi}_S}{\pi_S} V'(p_S - c_S + t) \;=\; \hat{\pi}_S V'(p_S - c_S + t) + \hat{\pi}_R[zV'(\Delta + e_S Y + t) + (1-z)V'(\Delta + t)] \qquad \text{(A.27)}$$

plus the constraint of problem (A.26) holding as an equality.

Notice now that in any PBE of Γ we must have that $\hat{\pi}_S \leq \pi_S$. This is simply because, conditionally on being informed the probability that $\theta = S$ is in fact zero. Using this fact and the fact that V' is a decreasing function, we can use (A.27) to conclude that in any UPBE of Γ given $\mathcal{C}$ as hypothesized we must have that

$$V'(p_S - c_S + t) \;\geq\; \hat{\pi}_S V'(p_S - c_S + t) + \hat{\pi}_R V'(\Delta + e_S Y + t) \qquad \text{(A.28)}$$

From (A.28) it is immediate to conclude that $p_S - c_S + t \leq \Delta + e_S\, Y + t$. From this inequality, using the constraint of problem (A.26) holding as an equality, it is now immediate to conclude that in the solution to problem (A.26) we must have $t \geq -e_S Y$.

Therefore, when π_S is small enough to ensure that $e^*Y - \psi(e^*) > e_S\, Y$, the payoff to the informed buyers from pooling must be smaller than their payoff from separating. This implies that in the hypothesized UPBE the pooling uninformed buyers have a profitable deviation available.

Hence, this is enough to prove the claim. ■

Lemma A.11: *Let a $\mathcal{C}$ be given prescribing that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, all contracts in $\mathcal{A}_U$ are upheld if $\theta = S$, and all contracts in $\mathcal{A}_I$ are upheld. Then there exists a $\tilde{\pi}_S \in (0,1)$ such that whenever $\pi_S < \tilde{\pi}_S$ in every UPBE of Γ given $\mathcal{C}$ the court's payoff is given by*

$$\gamma \left[e^* Y - \psi(e^*)\right] + (1-\gamma)\left[e_S Y - \psi(e_S)\right] \tag{A.29}$$

Proof: Let $\tilde{\pi}_S$ be as in Lemma A.10. Then we know that in any UPBE of Γ given $\mathcal{C}$ the informed buyers separate. Hence their equilibrium payoff is $e^*Y - \psi(e^*)$.

By Lemma A.9 the uninformed will write a contract in $\mathcal{A}_U$ that specifies $S \notin \mathcal{F}$. Since the informed buyers separate, the equilibrium contract between the uninformed buyers and the sellers is the solution to problem (A.1). Hence, from Lemma A.1 we know that the equilibrium payoff to the uninformed buyers is $e_S Y - \psi(e_S)$. This is clearly enough to prove the claim. ■

Lemma A.12: *There exists a $\tilde{\pi}_S \in (0,1)$ such that whenever $\pi_S < \tilde{\pi}_S$ the following holds.*

If there exists a UPBE of Γ in which $\mathcal{C}$ prescribes that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, then it must also prescribe that all contracts in $\mathcal{A}_I$ are upheld.

Proof: Let $\tilde{\pi}_S$ be as in Lemma A.10. Recall that by Lemma A.8 there is no UPBE of Γ in which the court voids all contracts. Hence it is sufficient to show that there is no UPBE of Γ in which the court upholds contracts in $\mathcal{A}_U$ if $\theta = S$ and voids all other contracts. By contradiction, suppose that there exists one such UPBE.

Notice that in the hypothesized UPBE, the informed buyers select a level of relationship specific investment equal to zero. This is because they know that the court will void their contract (pooling or separating as it might be). By Lemma A.9 the uninformed will write a contract in $\mathcal{A}_U$ that specifies $S \notin \mathcal{F}$. Hence, in the hypothesized equilibrium, the uninformed buyers select a level of relationship-specific investment equal to e_S.

Hence, the total expected (across informed and uninformed buyers) surplus from trade is equal to

$$\gamma\,\Delta + (1-\gamma)\left[\Delta + e_S Y - \psi(e_S)\right] \tag{A.30}$$

Since in any UPBE the payoff to the (risk-averse) seller must be at least $V(\Delta)$, (A.30) immediately implies that the payoff to the court in the hypothesized UPBE must be less than $e_S Y - \psi(e_S)$.

However, we know from Lemma A.11 that the court's payoff from adopting a rule $\mathcal{C}$ that upholds contracts in $\mathcal{A}_I$, voids contracts in $\mathcal{A}_U$ if $\theta = R$ and upholds contracts in $\mathcal{A}_U$ if $\theta = S$ is as in (A.29).

Since the quantity in (A.29) is greater than $e_S Y - \psi(e_S)$ we now have a profitable deviation for the court from the hypothesized UPBE. This concludes the proof. ■

Lemma A.13*: There exists a $\tilde{\pi}_S \in (0, 1)$ such that whenever $\pi_S < \tilde{\pi}_S$ the following holds.*

If there exists a UPBE of Γ in which $\mathcal{C}$ prescribes that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, then in this UPBE it must also be the case that $S \notin \mathcal{F}$, and all contracts in $\mathcal{A}_I$ are upheld, and that all contracts in $\mathcal{A}_U$ are upheld if $\theta = S$.

Moreover, in any such UPBE the informed buyers separate and the court's payoff is given by (A.29).

Proof: Let $\tilde{\pi}_S$ be as in Lemma A.10. From Lemma A.12 it is sufficient to show that there is no UPBE of Γ in which $\mathcal{C}$ prescribes that all contracts in $\mathcal{A}_U$ are voided if $\theta = R$, all contracts in $\mathcal{A}_I$ are upheld, and all contracts in $\mathcal{A}_U$ are voided if $\theta = S$. By contradiction, suppose that there exists one such UPBE.

Notice that in any such UPBE, since all contracts in $\mathcal{A}_U$ are voided, regardless of θ, the uninformed buyers select a level of relationship-specific investment equal to zero. The same is true of any informed buyers who might pool. It then follows easily that the payoff to all uninformed buyers must be zero in such UPBE.

Since the informed buyers can get a payoff of $e^* - \psi(e^*) > 0$ by separating it follows that in any such UPBE all informed buyers must in fact separate.

To sum up, the court's payoff in the hypothesized UPBE would be $\gamma\ [e^* - \psi(e^*)]$. Since the latter is less than the quantity in (A.29), it now follows that the court has a profitable deviation available: announcing the $\mathcal{C}$ described in Lemma A.11. This proves that any UPBE must be of the form claimed.

The claim about the equilibrium payoff to the court is a direct consequence of Lemma A.11. Hence, the proof is now complete. ■

Lemma A.14*: There exists a $\tilde{\pi}_S \in (0, 1)$ such that whenever $\pi_S < \tilde{\pi}_S$ the following holds.*

If there exists a UPBE of Γ in which the uninformed write a contract with $R \in \mathcal{F}$, then such UPBE must also prescribe that $S \notin \mathcal{F}$ and that the court upholds all contracts in $\mathcal{A}_I$ and all contracts in $\mathcal{A}_U$ if $\theta = S$.

Moreover, in any such UPBE the informed buyers separate and the court's payoff is given by (A.29).

Proof: Since in the putative UPBE the uninformed buyers write a contract with $R \in \mathcal{F}$, their trade in this case is left to ex-post renegotiation, just as when the court voids contracts in $\mathcal{A}_U$ when $\theta = R$. Hence, exactly the same argument that proves Lemma A.13 applies to prove the claim. We do not repeat the details here. ■

Lemma A.15*: Let a $\mathcal{C}$ be given that prescribes that all contracts in $\mathcal{A}_U$ are upheld if $\theta = R$. Then in any UPBE given $\mathcal{C}$ the uninformed write a contract with $S \notin \mathcal{F}$.*

Proof: There are two cases to consider: the uninformed write a contract with $R \notin \mathcal{F}$ and $S \in \mathcal{F}$, and the uninformed write a contract with $R \in \mathcal{F}$ and $S \in \mathcal{F}$. In both cases we will show that the uninformed can gain by deviating and offering a contract that specifies $S \notin \mathcal{F}$.

The latter case can be ruled out using exactly the same argument as in the proof of Lemma A.14, and we do not repeat the details here.

Consider then a putative UPBE given $\mathcal{C}$ in which the uninformed write a contract with $R \notin \mathcal{F}$ and $S \in \mathcal{F}$. Then the equilibrium contract between the uninformed buyer and the seller, which the seller accepts, is the solution to

$$\begin{array}{rl} \max\limits_{p_R,t,e_R} & \pi_R(e_R Y + \Delta + \overline{c} - p_R) - t - \psi(e_R) \\ s.t. & \hat{\pi}_S V(\Delta + e_R Y + t) + \hat{\pi}_R[\hat{q} V(p_R + t - c_H) + (1-\hat{q})V(p_R + t - c_L)] \geq V(\Delta) \\ & \psi'(e_R) \; = \; \pi_R Y \end{array} \tag{A.31}$$

where $\hat{\pi}_S$, $\hat{\pi}_R$ and $\hat{q}$ are the updated beliefs of the seller, conditional on observing the equilibrium contract offer of the uninformed buyer.

Now, for given updated beliefs of the seller of course, consider a deviation from the part of the uninformed seller to offering a contract that specifies $S \notin \mathcal{F}$ that solves

$$\begin{array}{rl} \max\limits_{p_R,p_S,t,e^*} & e^* Y + \Delta - t - \psi(e^*) + \pi_S(c_S - p_S) + \pi_R(\overline{c} - p_R) \\ s.t. & \hat{\pi}_S V(p_s + t - c_S) + \hat{\pi}_R\left[\hat{q} V(p_R + t - c_H) + (1-\hat{q})V(p_R + t - c_L)\right] \geq V(\Delta) \\ & \psi'(e^*) \; = \; Y \end{array} \tag{A.32}$$

To conclude the argument, it is sufficient to show that the maximized value of the objective function of problem (A.32) exceeds that of problem (A.31).

Let $(\tilde{p}_R, \tilde{t}, e_R)$ be any solution to problem (A.31). Let $\tilde{p}_S = \Delta + e_R Y + c_S$. Notice next that the array $(\tilde{p}_R, \tilde{p}_S, \tilde{t}, e^*)$, because of the way we have set $\tilde{p}_S$ is feasible in problem (A.32). Substituting these values in the objective function of problem (A.32) we get

$$e^* Y + \Delta - \tilde{t} - \psi(e^*) - \pi_S(\Delta + e_R Y) + \pi_R(\overline{c} - \tilde{p}_R) \tag{A.33}$$

Since e^* is chosen to maximize $eY - \phi(e)$, it is clear that $e^* - \phi(e^*) > e_R Y - \psi(e_R)$. Hence the quantity in (A.33) is greater than

$$e_R Y + \Delta - \tilde{t} - \psi(e_R) - \pi_S(\Delta + e_R Y) + \pi_R(\bar{c} - \tilde{p}_R) \;=\; \pi_R(e_R Y + \Delta + \bar{c} - \tilde{p}_R) - \tilde{t} - \psi(e_R) \quad \text{(A.34)}$$

However the right-hand side of (A.34) is precisely the maximized value of the objective function of problem (A.31). Therefore, we have found that the array $(\tilde{p}_R, \tilde{p}_S, \tilde{t}, e^*)$ is feasible in problem (A.32), and yields an expected profit for the buyer that is larger than the maximized value of the objective function of problem (A.31). This is clearly enough to prove the claim. ■

Lemma A.16: *There exists a $\tilde{c}^P$ such that whenever $c_H - c_L > \tilde{c}^P$ the following is true. Let a $\mathcal{C}$ be given that prescribes that the court upholds all contracts in $\mathcal{A}_U$.*

Then, if there is a UPBE given $\mathcal{C}$ in which the uninformed write a contract with $R \notin \mathcal{F}$ in such UPBE the informed pool with the uninformed, and the uninformed write a contract with $S \notin \mathcal{F}$.

Proof: By Lemma A.15 we know that in any UPBE given $\mathcal{C}$, the uninformed write a contract with $S \notin \mathcal{F}$.

If the informed pool with the uninformed, the equilibrium contract between the uninformed and the seller is the solution to problem (A.32). Let (p_R, p_S, t, e^*) be a solution to problem (A.32). Notice that the sum $p_R + t$ is uniquely determined by the solution to the maximization problem. Recall the uninformed buyer knows that $\theta = R$ and $\sigma = H$, and hence that the value, net of proceeds from his relationship-specific investment is $\Delta + c_H$. The payoff to the informed buyer from pooling can therefore be unambiguously written as

$$e^* Y + \Delta + c_H - p_R - t - \psi(e^*) \quad \text{(A.35)}$$

The payoff to the uninformed buyer if he deviates from the putative pooling equilibrium and separates is zero if $\mathcal{C}$ prescribes that contracts in $\mathcal{A}_I$ are voided and if $\mathcal{C}$ prescribes that contracts in $\mathcal{A}_I$ are upheld is

$$e^* Y \;-\; \psi(e^*) \quad \text{(A.36)}$$

Therefore showing that the quantity in (A.36) is less than the quantity in (A.35) is enough to show that the informed do not want to deviate from the putative pooling equilibrium. So, we need to show that $\Delta + c_H - p_R - t > 0$.

Notice that the solution to problem (A.32) is entirely characterized by the first order condition

$$\frac{\hat{\pi}_S}{\pi_S} V'(p_S + t - c_S) \;=\; \hat{\pi}_S V'(p_S - c_S + t) + \hat{\pi}_R[\hat{q}V'(p_R + t - c_H) + (1 - \hat{q})V'(p_R + t - c_L)] \quad \text{(A.37)}$$

together with the first constraint of the maximization problem holding as an equality.

Suppose now that $\Delta + c_H - p_R - t \leq 0$. Then, using the fact the V' is decreasing using (A.37) we can conclude that

$$\frac{\hat{\pi}_S}{\pi_S} V'(p_S + t - c_S) \; < \; \hat{\pi}_S V'(p_S - c_S + t) + \hat{\pi}_R[\hat{q}V'(\Delta) + (1 - \hat{q})V'(\Delta + c_H - c_L)] \tag{A.38}$$

Since $0 < \hat{\pi}_S \leq \pi_S$ and $0 \leq \hat{q} < 1$, (A.38) implies that, as $c_H - c_L$ grows without bound it must be that $V'(p_S+t-c_S)$ approaches zero. Hence it must be that p_S+t-c_S approaches infinity. However, since $\Delta + c_H - p_R - t \leq 0$ this makes it impossible to satisfy the first constraint of problem (A.32) as an equality.

It now remains to rule out that in any UPBE under the given parametric conditions any positive fraction of informed buyers might separate in equilibrium. The computations are virtually identical to the ones we have just shown. In fact, when a positive fraction of informed buyers separate, the contract that the uninformed sign with the seller is still the solution to problem (A.32) (the updated beliefs need not be the same of course). Hence the argument we have just given is also sufficient to show that the informed buyers will in fact want to deviate and pool with the uninformed. ■

Lemma A.17: *There exists a $\tilde{c}^P$ such that whenever $c_H - c_L > \tilde{c}^P$ the following is true. Let a $\mathcal{C}$ be given that prescribes that the court upholds all contracts in $\mathcal{A}_U$.*

Then, if there is a UPBE given $\mathcal{C}$ in which the uninformed write a contract with $R \notin \mathcal{F}$ in such UPBE the court's payoff is no greater than the maximized value of the objective function of the following problem

$$\begin{array}{ll} \max\limits_{p_R, p_S, t, e^*} & e^*Y + \Delta - t - \psi(e^*) + (1-\gamma)[\pi_S(c_S - p_S) + \pi_R(\bar{c} - p_R)] + \gamma(c_H - p_R) \\ s.t. & (1-\gamma)\pi_S V(p_S + t - c_S) + \\ & \qquad (1-\gamma)\left\{\pi_R\left[qV(p_R + t - c_H) + (1-q)V(p_R + t - c_L)\right]\right\} + \\ & \qquad\qquad \gamma(p_R + t - c_H) \geq V(\Delta) \\ & \psi'(e^*) \; = \; Y \end{array} \tag{A.39}$$

Proof: Let $\tilde{c}^P$ be as in Lemma A.16. Using Lemma A.16 and Lemma A.15 we only need to consider a pooling equilibrium in which the uninformed buyer writes a contract with $\mathcal{F} = \emptyset$ and the court enforces all contracts in $\mathcal{A}_U$.

Hence in the putative UPBE both the informed and the uninformed buyer select a level of relationship-specific investment equal to e^*. It follows that if (p_S, p_R, t) are in fact the prices and transfer of the equilibrium contract written by the uninformed buyer with the seller, then plugging these values into the objective function of problem (A.39) yields the equilibrium payoff of the court.

The equilibrium contract between the uninformed buyer and the seller in the putative UPBE is given by any solution to the following problem.

$$\begin{array}{rl} \max\limits_{p_R, p_S, t, e^*} & e^* Y + \Delta - t - \psi(e^*) + \pi_S(c_S - p_S) + \pi_R(\overline{c} - p_R) \\ s.t. & (1-\gamma)\pi_S V(p_s + t - c_S) + \\ & \qquad (1-\gamma)\left\{\pi_R \left[qV(p_R + t - c_H) + (1-q)V(p_R + t - c_L)\right]\right\} + \\ & \qquad\qquad \gamma V(p_R + t - c_H) \geq V(\Delta) \\ & \psi'(e^*) \; = \; Y \end{array} \tag{A.40}$$

Notice that, again, the solution to problem (A.40) determines uniquely the sums $p_S + t$ and $p_R + t$, but not the actual prices and up-front transfer.

The solution to problem (A.40) is entirely determined by the first order condition

$$\begin{array}{l} (1-\gamma)(1-\pi_S)\; V'(p_S + t - c_S) \; = [(1-\gamma)\pi_R + \gamma] \cdot \\ \left\{ \dfrac{(1-\gamma)\pi_R q + \gamma}{(1-\gamma)\pi_R + \gamma} \; V'(p_R + t - c_H) + \left[1 - \dfrac{(1-\gamma)\pi_R q + \gamma}{(1-\gamma)\pi_R + \gamma}\right] V'(p_R + t - c_L) \right\} \end{array} \tag{A.41}$$

together with the first constraint of problem (A.40) holding as an equality.

Since the constraints in problems (A.39) and (A.40) are in fact the same, it now follows that the equilibrium payoff of the court in the putative UPBE is the solution to problem (A.39) with (A.41) imposed as an additional constraint.

Hence the equilibrium payoff of the court cannot be greater than the maximized value of the objective function in problem (A.39). ■

Lemma A.18: *For every $\underline{\gamma}$, $\overline{\gamma} \in (0,1)$ with $\underline{\gamma} < \overline{\gamma}$ and every $\tilde{q}^B \in (0,1)$ there exist a $\tilde{c}^B > 0$ such that whenever $\gamma \in (\underline{\gamma}, \overline{\gamma})$, $c_H - c_L > \tilde{c}^B$ and $q < \tilde{q}^B$, the maximized value of the objective function of problem (A.39) is less than $e_S Y - \psi(e_S)$, which in turn is less than the quantity in (A.29).*

Proof: Notice first of all that problem (A.39) is in fact a re-parameterized version of problem (A.4) in the following sense. If in problem (A.4) we replace π_S with $\hat{\pi}_S$, π_R with $\hat{\pi}_R$, and q with $\hat{q}$ where

$$\hat{\pi}_S \; = \; (1-\gamma)\pi_S \qquad \hat{\pi}_R \; = \; (1-\gamma)\pi_R + \gamma \qquad \hat{q} \; = \; \frac{(1-\gamma)\pi_R q + \gamma}{(1-\gamma)\pi_R + \gamma} \tag{A.42}$$

then we obtain problem (A.39). Note that $\hat{q}$ as in (A.42) is an increasing function of γ and of q.

Now fix the arbitrary $\underline{\gamma}$, $\overline{\gamma}$ and $\tilde{q}^B \in (0,1)$ of the statement of the Lemma. Notice that if we know that $q < \tilde{q}^B$ and $\gamma \in (\underline{\gamma}, \overline{\gamma})$, from (A.42) we immediately know that

$$\frac{\underline{\gamma}}{(1-\underline{\gamma})\pi_R + \underline{\gamma}} \leq \hat{q} \leq \frac{(1-\overline{\gamma})\pi_R \tilde{q}^B + \overline{\gamma}}{(1-\overline{\gamma})\pi_R + \overline{\gamma}} \tag{A.43}$$

and

$$(1-\overline{\gamma})\pi_S \leq \hat{\pi}_S \leq (1-\underline{\gamma})\pi_S \quad \text{and} \quad (1-\underline{\gamma})\pi_R + \underline{\gamma} \leq \hat{\pi}_R \leq (1-\overline{\gamma})\pi_R + \overline{\gamma} \tag{A.44}$$

Let $\mathcal{P}$ be the set of $(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$ that satisfies the restrictions in (A.43) and (A.44).

Using Proposition A.2, for every $\hat{\pi}_S$, $\hat{\pi}_R$ and $\hat{q}$ we can now choose an $A(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$ such that the maximized value of the objective function of problem (A.39) is less than $\Pi^S = e_S Y - \psi(e_S)$ if $c_H - c_L \geq A(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$.

The claim that $e_S Y - \psi(e_S)$ is less than the quantity in (A.29) requires no proof.

Hence if we let $\tilde{c}^B$ be equal to the supremum over $\mathcal{P}$ of $A(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$, the Lemma is proved. ■

Lemma A.19: *For every $\underline{\gamma}$, $\overline{\gamma} \in (0,1)$ with $\underline{\gamma} < \overline{\gamma}$ and every $\tilde{q}^B \in (0,1)$ there exist a $\tilde{c} > 0$ such that whenever $\gamma \in (\underline{\gamma}, \overline{\gamma})$, $c_H - c_L > \tilde{c}$ and $q < \tilde{q}^B$ the following is true. Let a $\mathcal{C}$ be given that prescribes that the court upholds all contracts in $\mathcal{A}_U$.*

Then in any UPBE given $\mathcal{C}$ the uninformed write a contract with $R \in \mathcal{F}$.

Proof: Fix the arbitrary $\underline{\gamma}$, $\overline{\gamma}$ and $\tilde{q}^B \in (0,1)$ of the statement of the Lemma.

Consider now $\tilde{c}^P$ as in Lemma A.16. As is clear from the proof of Lemma A.16 the value of $\tilde{c}^P$ depends on π_S, $\hat{\pi}_S$, $\hat{\pi}_R$ and $\hat{q}$ of equations (A.37) and (A.38). Keeping π_S fixed, let $\tilde{c}^P(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$ be the value of $\hat{c}^P$ yielded by Lemma A.16 as a function of the updated seller beliefs. Notice that $\tilde{c}^P(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$ is well defined provided that $0 < \hat{\pi}_S < 1$ and $0 \leq \hat{q} < 1$. Since in Lemma A.16 we have to allow for any fraction of informed buyers to pool in the beliefs of the seller, it follows from Bayes' rule that if $\gamma \in (\underline{\gamma}, \overline{\gamma})$ and $q < \tilde{q}^B$, then

$$0 \leq \hat{q} \leq \frac{(1-\overline{\gamma})\pi_R \tilde{q}^B + \overline{\gamma}}{(1-\overline{\gamma})\pi_R + \overline{\gamma}} \tag{A.45}$$

and

$$(1-\overline{\gamma})\pi_S \leq \hat{\pi}_S \leq \pi_S \quad \text{and} \quad \pi_R \leq \hat{\pi}_R \leq (1-\overline{\gamma})\pi_R + \overline{\gamma} \tag{A.46}$$

let $\mathcal{Q}$ be the set of $(\hat{\pi}_S, \hat{\pi}_R, \hat{q})$ that satisfy the restrictions in (A.45) and (A.46).

Now let the $\tilde{c}$ of the statement of the present Lemma be defined as

$$\tilde{c} \;=\; \max\left\{\tilde{c}^B, \sup_{(\hat{\pi}_S, \hat{\pi}_R, \hat{q}) \in \mathcal{Q}} \tilde{c}^P(\hat{\pi}_S, \hat{\pi}_R, \hat{q})\right\} \tag{A.47}$$

where $\tilde{c}^B$ is as in Lemma A.18.

Suppose that the claim is false and consider a putative UPBE in which the uninformed write a contract with $R \notin \mathcal{F}$. Since $\tilde{c}$ is as in (A.47), from Lemma A.16 we know that in the putative UPBE it must be that the informed pool with the uninformed, and the uninformed write a contract with $S \notin \mathcal{F}$.

Hence, the payoff to the uninformed in the putative UPBE is the maximized value of the objective function of problem (A.40), and the equilibrium contract of the uninformed is determined by the first order conditions (A.41) and the first constraint of problem (A.40) holding with equality.

Since in the putative UPBE the informed prefer to pool with the uninformed than to separate we know that their payoff from pooling is greater than their payoff from separating. Hence we get

$$e^* Y \,-\, \psi(e^*) \,+\, +\Delta \,-\, t \,+\, c_H \,-\, p_R \;>\; e^* Y \,-\, \psi(e^*) \,+\, +\Delta \,-\, t \tag{A.48}$$

Notice now that (A.48) together with the first constraint of problem (A.40) and the concavity of V implies that

$$\pi_S(c_S - p_S) + \pi_R(\overline{c} - p_R) \;<\; c_H - p_R \tag{A.49}$$

Inequality (A.49) implies that if we substitute the solution to problem (A.40) into the objective function of problem (A.39) we obtain a quantity that is greater than the maximized value of the objective function of problem (A.40). Hence since the constraints in the two problems are the same, we can now conclude that the maximized value of the objective function of problem (A.39) is greater than the maximized value of the objective function of problem (A.40).

Since the maximized value of the objective function of problem (A.40) is the payoff to the uninformed in the putative UPBE, using the way we have chosen $\tilde{c}$ in (A.47) and Lemma A.18, it now follows that the uninformed have a profitable deviation in writing a contract with $R \in \mathcal{F}$ which gives them a payoff of $e_S Y \,-\, \psi(e_S)$. This is clearly enough to prove the claim. ■

A.4. Proof of Proposition 1

Let $\tilde{\pi}_S$ be as in Lemma A.10. Fix an arbitrary pair $\underline{\gamma}$, $\overline{\gamma} \in (0, 1)$ with $\underline{\gamma} < \overline{\gamma}$, and an arbitrary $\tilde{q} \in (0, 1)$. Let $\tilde{c}$ be as in Lemma A.19.

We proceed by considering two mutually exclusive cases that exhaust all possibilities. First, a court rule $\mathcal{C}$ that prescribes that contracts in $\mathcal{A}_U$ are voided if $\theta = R$. Second, a court rule $\mathcal{C}$ that prescribes contracts in $\mathcal{A}_U$ are upheld when $\theta = R$.

In the first case, we know the following about any UPBE given $\mathcal{C}$. Using Lemma A.13, it must be that the uninformed buyers write a contract with $S \notin \mathcal{F}$, all contracts in $\mathcal{A}_U$ are upheld if $\theta = S$, all contracts in $\mathcal{A}_I$ are upheld, the uninformed buyers separate and the court's payoff is given by (A.29).

In the second case, we know the following about any UPBE given $\mathcal{C}$. Using Lemma A.19 and Lemma A.14, it must be that the uninformed write a contract with $R \in \mathcal{F}$ and $S \notin \mathcal{F}$. All contracts in $\mathcal{A}_U$ are upheld if $\theta = S$, all contracts in $\mathcal{A}_I$ are upheld, the uninformed buyers separate and hence the court's payoff is given by (A.29).

Notice that the court is therefore indifferent between a $\mathcal{C}$ as in the first or in the second case above. It may therefore want to randomize between two rules, one that upholds all contracts, and one that voids contracts in $\mathcal{A}_U$ if $\theta = R$ and upholds all other contracts. Since the contracting parties observe the realized $\mathcal{C}$ before contracting, depending on the realization of the court's draw, the UPBE in the two cases is still as described above.

To complete the argument, we check that in both cases the claims made in Proposition 1 hold both in the first and in the second case above. Clearly (i) is verified since in both cases the informed buyers separate. In both cases the court upholds the separating contract of the informed, and hence (ii) is true. Since in both cases the contract of the uninformed specifies $S \notin \mathcal{F}$ and is upheld by the court when $\theta = S$, claim (iii) follows from Lemmas A.13 and A.14. Claim (iv) follows directly from the fact that in the first case the court voids contracts in $\mathcal{A}_U$ when $\theta = R$, and in the second case the uninformed write a contract with $R \in \mathcal{F}$.

A.5. Proof of Proposition 2

Given the proof of Proposition 1 the claim is virtually immediate. If the court voids contracts in $\mathcal{A}_U$ when $\theta = R$, the uninformed can write either a contract with $R \in \mathcal{F}$ or one with $R \notin \mathcal{F}$ without altering any of the equilibrium payoffs.

A.6. Proof of Proposition 3

Suppose that the claim is false. Then, using Proposition 1, it suffices to rule out the following type of UPBE.

The court rule $\mathcal{C}$ enforces contracts in $\mathcal{A}_I$ and contracts in $\mathcal{A}_U$ if $\theta = S$. Moreover if $\theta = R$, $\mathcal{C}$ enforces contracts in $\mathcal{A}_U$ that have $\mathcal{F} = \emptyset$, and the uninformed, in equilibrium, write a contract with $R \in \mathcal{F}$.

Of course, in the putative UPBE, as required by Proposition A.4, the informed separate, and the payoff to the uninformed is $\Pi^S = e_S Y - \psi(e_S)$.

Consider now a deviation from the part of the uninformed to offering a contract with $\mathcal{F} = \emptyset$ that solves problem A.4. This contract (by the contradiction hypothesis) will be upheld by the court regardless of θ. Given the (population) beliefs of the seller it will be accepted, and hence will yield the uniformed buyer a payoff of $\Pi^{S,R}$ as in Lemma A.2. However, if $q > 0$ is sufficiently small we know from Proposition A.2 that $\Pi^{S,R} > \Pi^{S}$. Hence, what we have described is a profitable deviation from the putative UPBE from the part of the uninformed. Hence the proof is complete. ■

A.7. Proof Proposition 4

We start by observing that by standard arguments, the set of robust UPBE is (weakly) contained in the set of UPBE of Γ. This immediately implies that any robust UPBE of Γ must satisfy the requirements of Propositions 1 and 3. Hence, we only need to rule out a robust UPBE in which the uniformed offer a contract with $R \in \mathcal{F}$ and which is otherwise precisely the simple equilibrium described in Proposition 2. By way of contradiction, suppose that there is a robust UPBE of Γ as we have just described.

Now consider a deviation from the part of the uninformed buyer to offering a contract in $\mathcal{A}_U$ with $\mathcal{F} = \emptyset$. In particular, consider a contract that specifies a vector $(\hat{p}_S, \hat{p}_R, \hat{t})$ that solves the following maximization problem

$$\begin{aligned} \max_{p_S, p_R, t, \hat{e}} \quad & (\pi_S + \varepsilon\, \pi_R)(\hat{e}Y + \Delta) + \pi_S(c_S - p_S) + \varepsilon\, \pi_R\, (\overline{c} - p_R) - t - \psi(\hat{e}) \\ s.t. \quad & \pi_S V(p_S + t - c_S) \;+\; (1-\varepsilon)\, \pi_R V(\Delta + \hat{e}Y + t) \\ & + \varepsilon\, \pi_R\, [qV(p_R + t - c_H) \;+\; (1-q)V(p_R + t - c_L)] \;\geq\; V(\Delta) \\ & \psi'(\hat{e}) \;=\; (\pi_S + \varepsilon\, \pi_R)\, Y \end{aligned} \tag{A.50}$$

The contract $(\hat{p}_S, \hat{p}_R, \hat{t})$ is entirely characterized by the first order conditions

$$V'(\hat{p}_S - c_S + \hat{t}) \;=\; V'(\Delta + \hat{e}Y + \hat{t}) \;=\; q\, V'(\hat{p}_R + \hat{t} - c_H) + (1-q)\, V'(\hat{p}_R + \hat{t} - c_L) \tag{A.51}$$

together with the first constraint of problem (A.50) holding with equality.

For a given ε, consider the optimal contract the buyer offers the seller, following the deviation, when $q = 0$. The first order conditions (A.51) imply that $\hat{p}_R = \hat{e}\, Y + \Delta + c_L$, $\hat{p}_S = \hat{e}\, Y + \Delta + c_S$ and $\hat{t} = -\hat{e}\, Y$. Therefore, following the proposed deviation the buyer's payoff is

$$\Pi^{\varepsilon} = \hat{e}\, Y - \psi(\hat{e}) \tag{A.52}$$

From Proposition 1, if the uninformed buyer follows the prescription of the putative UPBE and offers the optimal contract with $R \in \mathcal{F}$ his payoff is $e_S\, Y - \psi(e_S)$. However, by inspection of the second constraint in problem (A.50) and of the second constraint in problem A.1, it is immediate that the quantity in (A.52) is greater than $e_S Y - \psi(e_S)$.

Hence, by continuity, if q and ε are sufficiently small, the uninformed buyer has a profitable deviation from the putative UPBE of $\Gamma(\varepsilon)$ to offering a contract in which $\mathcal{F} = \emptyset$. Hence the proof is now complete. ■

References

Adler, B. E. (1999): "The Questionable Ascent of Hadley v. Baxendale," University of California, Berkeley. Olin Program in Law & Economics. Working Paper No. 25.

Anderlini, L., L. Felli, and A. Postlewaite (2003): "Courts of Law and Unforeseen Contingencies," London School of Economics. S.T.I.C.E.R.D. Discussion Paper No. TE/03/447.

Ayres, I., and R. Gertner (1989): "Filling Gaps in Incomplete Contracts: An Economic Theory of Default Rules," *Yale Law Journal*, 87.

Bebchuk, L., and S. Shavell (1991): "Information and the Scope of Liability for Breach of Contract: The Rule of Hadley V. Baxendale," *Journal of Law, Economics & Organization*, 7, 284–312.

Bond, P. (2003): "Contracting in the Presence of Judicial Agency," Northwestern University. *Mimeo.*

Mailath, G. J., M. Okuno-Fujiwara, and A. Postlewaite (1993): "Belief based refinements in signaling games," *Journal of Economic Theory*, 60, 241–276.

Maskin, E., and J. Tirole (1990): "The Principal Agent Relationship with an Informed Principal, I: Private Values," *Econometrica*, 58, 379–409.

——— (1999): "Unforseen Contingencies and Incomplete Contracts," *Review of Economic Studies*, 66, 83–114.

Moore, J., and R. Repullo (1988): "Subgame Perfect Implementation," 56, 1191–1220.

POSNER, E. (1998): "The Parol Evidence Rule, The Plain Meaning Rule, and the Principles of Contractual Interpretation," *University of Pennsylvania Law Review*, 146, 533–577.

SCHWARTZ, A., AND R. E. SCOTT (2003): "Contract Theory and the Limits of Contract Law," *Yale Law Journal*, forthcoming.

SHAVELL, S. (2003): "On the Writing and Interpretation of Contracts," Harvard Law School. John M. Olin Center for Law, Economics, and Business. Discussion Paper No. 445.

USMAN, K. (2002): "Verifiability and Contract Enforcement: A Model with Judicial Moral Hazard," *Journal of Law, Economics & Organization*, 18, 67–94.