Product Durability and Innovations: The Duopoly Case

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April 22, 1997

This version: April 5, 2001

Abstract

We compare the rates of product innovation under rental versus sales when the product is durable and the market structure is one of duopoly. Our main conclusion is that sales induce a slower and more efficient rate of product introductions than rentals. The basic reason for this is that under rentals sellers are able to extract a higher surplus from buyers, and this higher surplus is dissipated away through excessive rate of product innovation. Since the exact opposite is true under monopoly market structure this highlights the role of market structure in determining the rate of product innovations when the product is durable.

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1 Introduction

Durable goods are usually sold rather than rented. What are the welfare consequences of a sales regime, as opposed to one in which products are rented? Coase (1972) showed that a rental regime facilitates the earning of monopoly rents by serving as a credible commitment against future price reductions. In contrast under a sale regime, the monopolist is forced to compete with previous units he has sold, lowering prices over time and approaching the outcome of perfect competition. Hence, in that context, welfare is higher under the sales regime (see also Bulow (1982)).

In the Coasian context the durable good is of fixed quality. In this paper we compare rentals versus sales when technological progress takes place, so the durable is of ever increasing quality. Natural examples are Personal Computers, Software, Cellular Phones, or Computer Games. In an earlier paper (Fishman and Rob, 2000), we considered this very question (i.e., compared rentals versus sales) in the context of a monopoly seller. We showed that when the monopoly sells its products, it innovates too slowly - from the perspective of social welfare. By contrast, if the monopoly is able to rent its products or, alternatively, to deliberately shorten the functional life of its product - planned obsolescence - it invests and innovates at the socially optimal level. Hence, in that case, the welfare properties of rental versus sales are the exact opposite to those of the classical Coasian framework.

In this paper we revisit these issues in a duopoly context. Interestingly, we find that the introduction of competition reverses our previous results: Innovations are too fast under rentals - compared with the social optimum - and the market outcome is less efficient when products are rented than if products are durable and sold.

The reason for this result is that when there is more than one potential innovator, competitive pressure to innovate ahead of competitors creates incentives for too frequent innovation - the preemption motive (see Fudenberg and Tirole (1985)). The importance of this effect depends on the profitability of innovation. When products are sold and designed to be durable, a consumer's willingness to pay for a new model is limited by the fact that the older model in her possession is still serviceable. Thus, product durability puts a brake on the preemption motive, slowing down investment to a more efficient level. By contrast, if products are rented, consumers have fewer options to fall back on when a new model is introduced and, hence, are willing to pay more for it. By making innovation more profitable, this intensifies the preemption motive and leads to an excessively high rate of product introductions. Thus, in a setting of recurrent innovation, the business practice of rentals reduces welfare.

Hence, the net result here is the same as Coase's result: The rental regime produces outcomes that are less efficient than the sales regime. However, both the context and the reasoning are different. In the Coasian context of fixed product quality, rentals lead to inefficiency because they reduce competition (from pre-sold units). On the other hand, in our context of continuous innovation, rentals lead to inefficiency because they create *too much* (or wasteful) competition.

We proceed as follows. In the next section we present the model. In section 3 we solve for the social optimum. In sections 4 and 5 we solve for duopoly equilibria under rentals and sales respectively. In section 6 we compare the two. In section 7 we briefly comment on planned obsolescence, and in section 7 we conclude.

2 The Model

There is a continuum of identical infinitely-lived consumers of measure 1. The utility a representative consumer derives from the product depends on its quality, denoted by q. A product of quality q delivers \$q worth of utility per period. Consumers' demand is discrete: They buy either zero or one unit. Time is continuous. The interest rate $r > 0$ is constant and the same for consumers and producers.

New and improved models are repeatedly introduced over time. The extent to which a new model improves upon its predecessor depends on the amount of time which has elapsed since the latter was introduced. Specifically, if the preceding model was of quality q, and was introduced t units of time ago, the new model is of quality $q+g(t)$. The g function is referred to as a **production function**, or a technological progress function, while t is referred to as a **gestation period**.

The quality of new models is cumulative. If two models are introduced in sequence, the second-generation model is of quality $q + g(t_1) + g(t_2)$, where t_1 and t_2 are the gestation periods of the first and second model, respectively. We normalize the initial quality to be 0.

The production of each new model requires the investment of a fixed cost, F, called the **implementation cost**. The idea is that the passage of time — the gestation period — leads to new scientific and theoretical knowledge, and F is the cost of translating the new knowledge into commercial applications.

The following assumption is imposed on the model's parameters:

Assumption A: $g(\cdot)$ is an increasing, concave, continuously differentiable, bounded function and $g(t) > rF$, for t sufficiently large.

We assume constant per-unit variable costs which are equal across different-generation products and normalized to zero. This represents the case where the bulk of the cost of a new product is the cost of creating a prototype. Once a prototype exists, the cost of replicating it is not much different from the cost of replicating its predecessor. For example, it costs hundreds of millions of dollars to develop a new micro-chip, but approximately \$150 to physically produce it, a cost which has not changed appreciably across different generation of micro-chips. As a result of the fixed cost of introductions, gestation periods must be positive; the incremental quality of a new product must be big enough to justify paying F. Therefore, introductions occur at discrete points in time.

All products are durable: They continue to give service after having been replaced by more advanced models. However, we assume that new products functionally supersede their predecessors; a consumer who owns a new model has no use for older models. Since consumers are identical, old products are "discarded" when a new model is introduced; there is no second-hand market.

We start out by deriving the socially optimal rate of innovations. This will serve as benchmark against which to compare the duopoly equilibria.

3 The Social Optimum

The social planner chooses gestation periods, $t_1, t_2, ...$, so that new models are introduced at $T_1 = t_1, T_2 = t_1 + t_2$, etc. Thus, T_i's are the calendar dates at which new models are introduced, while t_i's are the (inter-introductions) gestation periods.

Fix a list of gestation periods $(t_1, t_2, ...)$. Then, the model introduced at T_i is of quality $\sum_{j=1}^{i} g(t_j)$, and will deliver benefits over $[T_i, T_{i+1})$. Thus, as of date T_i, it generates a discounted benefit of

$\left[\sum\limits_{j=1}^{i} g(t_j)\right] \frac{1-e^{-rt_{i+1}}}{r}$, and, as of date 0, it generates a discounted benefit of $e^{-rT_i} \left[\sum\limits_{j=1}^{i} g(t_j)\right] \frac{1-e^{-rt_{i+1}}}{r}$. Consequently, the value of the welfare program at $(t_1, t_2, ...)$ is:

$$W(t_1, t_2, ...) = \sum_{i=1}^{\infty} e^{-rT_i} \left\{ -F + [\sum_{j=1}^{i} g(t_j)] \frac{1-e^{-rt_{i+1}}}{r} \right\}. \tag{1}$$

Consider the auxiliary optimization program of a social planner who is constrained to choose a constant level of t. That is, it must choose a t to maximize

$$G(t) \equiv W(t, t, ...) = \frac{e^{-rt}}{1-e^{-rt}} \left[-F + \frac{g(t)}{r} \right]. \tag{2}$$

By differentiating G it can be verified that G is single-peaked and that its peak is interior. Proposition 1 shows that the solution to the planner's unconstrained optimization problem, the maximization of W, is equivalent to the maximization of the constrained function, G.

Proposition 1: (i) There exists a unique solution to the planner's program for which the optimal gestation periods are constant, $t_1 = t_2 = ... = t^O \in (0, \infty)$, where t^O is the maximizer of G. t^O is characterized by the following condition.

$$g(t^O) = rF + g'(t^O) \frac{1-e^{-rt^O}}{r}. \tag{3}$$

Proof of Proposition 1: Let q denote the quality of the current product and $V(q)$ the value function for the planning problem right after the introduction of this product. Then V satisfies the following Bellman equation:

$$V(q) = \underset{t}{Max} \left\{ q\frac{1-e^{-rt}}{r} - e^{-rt}F + e^{-rt}V(q + g(t)) \right\}.$$

Subtracting q/r from both sides and with some manipulation:

$$V(q) - q/r = \underset{t}{Max} \left\{ \frac{g(t)e^{-rt}}{r} - e^{-rt}F + e^{-rt}[V(q + g(t)) - (q + g(t))/r] \right\}.$$

Define the function $\widetilde{V}(q) \equiv V(q) - q/r$. Then:

$$\widetilde{V}(q) = \underset{t}{Max} \left\{ \left[\frac{g(t)e^{-rt}}{r} - e^{-rt}F \right] + e^{-rt}\widetilde{V}(q + g(t)) \right\}.$$

Since the term in the square brackets does not depend on q, the value function, $\widetilde{V}$, is independent of q. It then follows that for a given t,

$$\widetilde{V}(q;t) = \frac{g(t)e^{-rt}}{r(1-e^{-rt})} - \frac{e^{-rt}F}{1-e^{-rt}} = \frac{e^{-rt}}{1-e^{-rt}}\left[-F + \frac{g(t)}{r}\right],$$

which is the value of the stationary planner's program, $G(t)$. Since G is uniquely maximized at some interior t^O and since (3) is the first-order condition for the maximization of G, the result follows. ■

4 The Duopoly Equilibrium in the Rental Regime

Now suppose there are two potential innovators, playing a noncooperative game of product introductions. In this section we assume that products are rented to consumers, rather than sold. Let q_i be the most advanced model — to date — produced by firm i, $i = 1, 2$, and let $q_m = max\{q_1, q_2\}$ be the **state-of-the-art technology** — the most advanced technology currently available. Then if t_m is the time which has elapsed since q_m was introduced, the highest quality that can be produced by any firm is $q_m + g(t_m)$, referred to as the **frontier technology**. We assume that at any date, either firm is able to produce the frontier technology by investing F, regardless of when it last innovated[1]. That is, the fact that a firm is closer to the frontier technology does not confer a technological advantage upon it, e.g., the fact that $q_1 > q_2$ (at some point in time) does not mean that firm 1 can introduce a higher-quality product than firm 2. However, it does confer a pricing advantage. Specifically, we assume Bertrand competition in the product market: If $q_i > q_j$, the equilibrium rental-fee of firm-i (for its most advanced model) is $q_i - q_j$, while the equilibrium rental-fee of firm-j is 0, or, in short, the rental fee of firm i is $p_i = Max(0, q_i - q_j)$.

Strategies. A strategy for a firm specifies, at any date, whether to introduce a new model, and if so, at what price to rent it. By the folk theorem — since the horizon is infinite — an infinite number of subgame perfect Nash equilibria exist. To reduce the equilibrium set, we restrict attention to **stationary** strategies. Such strategies allow firms to condition their choice on 'payoff-relevant'

[1] We allow both firms to simultaneously introduce the frontier technology, although, as will become clear below, this never happens in equilibrium.

variables only. This excludes bootstrapping equilibria in which strategies depend on otherwise 'irrelevant' history. In our context, the payoff-relevant variables are:

- q_1, q_2, i.e., the quality of the latest introductions of each firm.
- The time since the frontier technology was introduced, t_m.

We denote the triple (q_1, q_2, t_m) by s, and refer to it as the **state**.
s determines the frontier technology (which is $\max(q_1, q_2) + g(t_m)$.) Because of the Bertrand pricing assumption, both q_1 and q_2 are payoff-relevant as they determine the rental fee that the innovator can charge, as explained above.

We further reduce the class of equilibria under consideration by restricting attention to pure-strategy equilibria with the following properties:

(i) The equilibrium gestation period is a constant, $\bar{t}$.

(ii) The equilibrium strategies have the property that the probability with which a firm innovates does not decrease when innovation is more profitable for it and less profitable for its competitor. This property is referred to as **monotonicity**[2] and is formalized as follows.

Monotinicity: Consider two states s' and s'' and consider the highest rental-fees associated with them: p'_i, p'_j, p''_i and p''_j. Assume $p'_i \geq p''_i$ and $p'_j \leq p''_j$. Then strategies are monotonic if the probability with which firm-j innovates at s' does not exceed the probability with which it innovates at s'', and the probability with which firm-i innovates at s' is at least as great as at s''.

Proposition 2: Consider the rental regime. There is a unique, constant gestation-period equilibrium in monotonic strategies. The equilibrium gestation-period is t^R, which is the unique solution to:

$$g(t^R) = rF. \tag{4}$$

Proof: We prove the proposition via the following two Lemmas. Let $\bar{t}$ be the equilibrium gestation-period.

Lemma 1: Every model is introduced by the same firm

[2] While we restrict attention to equilibria with these properties, equilibrium strategies must be immune to *any* profitable deviation, monotonic or not.

Proof of Lemma 1: Suppose that firm 1 introduces the first n models, at calendar dates $\bar{t}, 2\bar{t}, ..., n\bar{t}, n \geq 1$ and let $T^2 = (n+1)\bar{t}$ be the first date at which firm 2 innovates. Observe that at any date z, $0 < z < \bar{t}$, $p_1 = p_2 = g(z)$. Thus the stationarity assumption implies that firm 2 innovates with probability zero when $p_1 = p_2 = g(z)$. Then, by monotonicity, firm 2 innovates with probability zero if $p_1 \geq g(z)$ and $p_2 \leq g(z), 0 < z < \bar{t}$.

Suppose all introductions after T^2 are by firm 2 (that is, firm 1 never again innovates after T^2). Consider the following deviation for firm 1: Beginning at $T^2 - \varepsilon$, introduce a new model every $\bar{t} - h$ periods; i.e. introduce at $\tau_1 = T^2 - \varepsilon$, $\tau_2 = T^2 - \varepsilon + (\bar{t} - h)$, $\tau_3 = T^2 - \varepsilon + 2(\bar{t} - h)$, and so on. If firm 1 adopts this deviation, then at any date $\tau_1 + z$, $0 < z < \bar{t} - h$, $p_1 > g(z)$ and $p_2 = g(z)$, which by the preceding implies that firm 2 will not introduce at any date prior to τ_2. By the same reasoning, as long as firm 1 continues to pursue the deviation strategy, firm 2 will not introduce at any date prior to τ_3 and so on. Thus, as long as firm 1 pursues the deviation strategy, it remains the lone innovator and is thus able to charge for the full surplus of each model. That is, the rental price of the model introduced by firm 1 at τ_i is $ng(\bar{t}) + ig(\bar{t} - h)$. Thus, firm 1's profit from the deviation, as evaluated at $T^2 - \varepsilon$, is

$$V^D = -F + [ng(\bar{t}) + g(\bar{t} - h)](\frac{1 - e^{-r(\bar{t} - h)}}{r}) + e^{-r(\bar{t} - h)}\{-F + [ng(\bar{t}) + 2g(\bar{t} - h)](\frac{1 - e^{-r(\bar{t} - h)}}{r})\} + ...$$

On the other hand, firm 2's profit along the proposed equilibrium path, evaluated at T^2, is $V^2 = -F + g(\bar{t})(\frac{1-e^{-r\bar{t}}}{r}) + e^{-r\bar{t}}\{-F + [2g(\bar{t})](\frac{1-e^{-r\bar{t}}}{r})\} + ... < V^D$ for sufficiently small ε and h. Since $V^2 \geq 0$ (or firm 2's equilibrium strategy could not be optimal), $V^D > 0$. Thus the deviation produces strictly positive discounted profit for firm 1 from date $T^2 - \varepsilon$, and onwards, while along the proposed equilibrium path, its future discounted profit at that date is zero. This proves that if firm 2 innovates at T^2, there must be a later date at which firm 1 once again innovates.

Let $T^3 > T^2$ be the first such date. Then, at T^3, since firm 2 was the most recent innovator, $p_1 = g(\bar{t})$, $p_2 \geq g(\bar{t})$. On the other hand, at T^2, $p_1 \geq g(\bar{t})$ and $p_2 = g(\bar{t})$. Thus, by monotonicity, it is not possible that firm 1 innovates at T^3 and firm 2 innovates at T^2. This contradiction proves that every model is introduced by the same firm. ■

Lemma 2: $\bar{t} = t^R$.

Proof of Lemma 2: Without loss of generality, suppose that firm 1 is the lone innovator. Then given the equilibrium gestation period, $\overline{t}$, its discounted profit as of date $\overline{t}$, is:

$$V(\overline{t}) = -F + g(\overline{t})(\frac{1-e^{-r\overline{t}}}{r}) + e^{-r\overline{t}}[-F + 2g(\overline{t})(\frac{1-e^{-r\overline{t}}}{r})] + ...$$
$$= \frac{1}{1-e^{-r\overline{t}}}[-F + \frac{g(\overline{t})}{r}].$$

Thus, by (4), $V(\overline{t}) = 0$ and, since $g(\cdot)$ is an increasing function, $V(\overline{t}) > 0$ for $\overline{t} > t^R$.

Suppose that $\overline{t} > t^R$. Since firm 1 does not innovate before date $\overline{t}$, its equilibrium strategy is to innovate with probability zero whenever $p_1 = p_2 = g(z)$, $z < \overline{t}$. Consider the following deviation for firm 2: introduce at dates $\overline{t} - \varepsilon$, $2\overline{t} - 2\varepsilon$, etc. As a result of this deviation , at every date $T > \overline{t} - \varepsilon$, $p_1 = g(\tau)$ while $p_2 \geq g(\tau)$, implying that firm 1 does not innovate as long as firm 2 pursues the deviation strategy. Thus firm 2's profit from the deviation, evaluated at $\overline{t} - \varepsilon$, is

$$\begin{aligned} V(\overline{t}-\varepsilon) &= -F + g(\overline{t}-\varepsilon)(\frac{1-e^{-r(\overline{t}-\varepsilon)}}{r}) + e^{-r(\overline{t}-\varepsilon)}[-F + 2g(\overline{t}-\varepsilon)(\frac{1-e^{-r(\overline{t}-\varepsilon)}}{r})] + ... \\ &= \frac{1}{1-e^{-r(\overline{t}-\varepsilon)}}[-F + g(\overline{t}-\varepsilon)/r] > 0, \end{aligned}$$

for ε sufficiently small. Thus $\overline{t} \leq t^R$. If $\overline{t} < t^R$ the innovator earns negative profit. Thus $\overline{t} = t^R$.

The preceding two lemmas have established that in any equilibrium one firm innovates every t^R periods. Existence is proved by construction.

Proof of Proposition 2: Let the equilibrium strategies be:

- If $t_m < t^R$, both firms introduce with zero probability.
- If $t_m = t^R$, firm-1 introduces the frontier technology if $q_1 \geq q_2$ and firm-2 does otherwise.
- If $t_m > t^R$, firm-i introduces the frontier technology with probability $(V_i - F)/V_i,\ i = 1, 2$, where $V_i = (q_m - q_i)\frac{1-e^{-rt^R}}{r}$.

It is straightforward to verify that these strategies lead to firm 1 introducing every t^R periods, that the strategies are stationary and monotonic, and are best responses to each other. ■

The duopoly equilibrium is characterized by a single innovator, which, having once innovated, maintains a competitive advantage that enables it to maintain its leadership position forever. The innovator rents the first model at $g(t^R)$, the second model at $2g(t^R)$ and so on, and each introduction costs F. The innovator's discounted profit, evaluated as of date zero, is zero. Therefore, the innovator's profit from the first few introductions is negative, whereas the profit from subsequent introductions is positive. In other words, the innovator is initially absorbing losses to achieve his leadership position, and then recovers its losses over time. Overall, its ex-ante profit is zero.

4.1 Comparing the Rate of Product Introductions under the Rental Equilibrium with the Social Optimum

Comparing (3) with (4) reveals that $t^O > t^R$. In the rental duopoly setting, the innovator must innovate frequently enough to make preemption by competitors unprofitable. This leads to over-investment in innovation, from the perspective of social welfare. We show that this problem is mitigated in the sales regime (see Proposition 4 below).

5 The Duopoly Equilibrium in the Sales Regime

Now suppose that all products are sold rather than rented. We again consider pure strategy, stationary equilibria with a constant gestation period, $\bar{t}$. The payoff-relevant variables in this instance are t_m, the time since the last introduction, and t^e, which is defined as consumers' expectation regarding how much time will elapse until the next introduction.[3] As we explain immediately below, t_m and t^e fully determine the payoff that can be collected from a new product introduction.

First, observe that in equilibrium, since consumers are identical, every consumer buys a unit of the new model when it is introduced — otherwise it would not be optimal for the innovator to introduce a new model at that time. Second, since products are durable, it follows that at the time the k-th model is introduced, each consumer owns a serviceable model of the $(k-1)$-st generation. Since old models continue to provide service, consumers are willing to pay only for the *incremental*

[3] Consumers' expectation plays no role in the rental regime.

quality of the new model. Furthermore, consumers are willing to pay for this increment only until such time as a better model yet is introduced (which is the reason for having to consider t^e).

More specifically, consider a specific point in time, and consider a consumer who at that point owns a model of quality q_{-1}. Imagine a new model of quality $q_0 = q_{-1} + g(t_m)$ is offered for sale (hence, q_{-1} was introduced t_m units of time ago), and assume the consumer expects the next model to occur t^e periods hence, at which point the consumer upgrades to the next generation model, q_{+1}.

By buying the model of quality q_0, the consumer gets a per-period utility increment of $q_0 - q_{-1} = g(t_m)$ over t^e periods, at which point she expects to discard it for a new model. Thus the discounted utility increment from adopting the new model is $g(t_m)\frac{1-e^{-rt^e}}{r}$, which is therefore the maximum that the consumer is willing to pay for the q_0 model. Obviously, this price is fully determined by t_m and t^e. And, unlike the rental regime, this price is independent of the identity of the previous innovator.

An equilibrium in the sales regime is one where, at each moment, (i) each firm is choosing a best-response to its rival given the values of t_m and t^e. And, (ii) consumers' expectation is correct, i.e., $t^e = \bar{t}$ for every model.

Proposition 3: (i) In the sales regime, the rational-expectations-equilibrium gestation-period is the unique solution to:

$$g(t^S)\frac{1-e^{-rt^S}}{r} = F. \tag{5}$$

(ii) $t^S > t^R$.

Proof of Proposition 3:

Necessity: Given the equilibrium gestation period, since the state is the same whenever a new model is introduced, the stationarity assumption implies that each model is introduced by the same firm. Suppose the lone innovator is firm 1. Then firm 1's equilibrium profit, as of date $\bar{t}$, is

$$[-F + g(\bar{t})\frac{1-e^{-r\bar{t}}}{r}][1 + e^{-r\bar{t}} + e^{-2r\bar{t}} + ...] = \frac{1}{1-e^{-r\bar{t}}}[-F + g(\bar{t})\frac{1-e^{r\bar{t}}}{r}].$$

Suppose $-F + g(\bar{t})\frac{1-e^{-r\bar{t}}}{r} > 0$. Then firm 1 earns positive discounted profit while firm 2's profit is zero. Since firm 1 innovates only at dates $\bar{t}, 2\bar{t}, ...$, its equilibrium strategy is to innovate with probability zero when $t_m < \bar{t}$. Since $-F + g(\bar{t})\frac{1-e^{-r\bar{t}}}{r} > 0$, there exists a $t' < \bar{t}$ so that

$-F + g(t')\frac{1-e^{r\bar{t}}}{r} > 0$. Consider the following deviation for firm 2: Innovate whenever the state is t'. Given firm 1's equilibrium strategy, firm 1 will never innovate as long as 2 conforms with this path. Thus the deviation generates positive profit for firm 2. This proves that $-F + g(\bar{t})\frac{1-e^{-r\bar{t}}}{r} = 0$, i.e., $\bar{t} = t^S$.

Sufficiency: Let the firms' strategies be as follows. If $t_m = t^S$, firm 1 innovates with probability 1 and firm 2 introduces with probability zero. If $t_m < t^S$, each firm introduces with probability zero. If $t_m > t^S$ (which is off the equilibrium path), each firm innovates with probability $\frac{g(t^S)(1-e^{-rt^S})-rF}{g(t^S)(1-e^{-rt^S})}$. It is straightforward to verify that these strategies lead to introductions by firm 1 every t^S periods, and that the strategies are stationary and best responses to each other. Thus, these strategies constitute an equilibrium.

(ii) Comparison of (4) and (5) shows that $t^S > t^R$. ■

While the preemption motive acts to speed up innovation under both the rental and sales regimes, the intensity of this effect is mitigated in the sales regime by consumers' reduced willingness to pay, which makes preemption less attractive. Hence, there is less innovation in the sales regime.

5.1 Comparing the Rate of Product Introductions under the Sales Equilibrium with the Social Optimum

We know $t^O > t^R$. What can be said about t^O versus t^S? t^O satisfies $g(t) = rF + g'(t)\frac{1-e^{-rt}}{r}$, or $g(t) - g'(t)\frac{1-e^{-rt}}{r} = rF$. On the other hand, t^S satisfies $g(t) - g(t)e^{-rt} = rF$. Since the LHS increase in t and since we can cancel $g(t)$, $t^S > t^O$ if $g(t)e^{-rt} > g'(t)\frac{1-e^{-rt}}{r}$, for all t. Or, equivalently, if $\frac{g(t)}{g'(t)} > \frac{e^{rt}-1}{r}$. If we let $g(t) = 1 - e^{-\lambda t}$, for some parameter $\lambda > 0$, $\frac{g(t)}{g'(t)} = \frac{1-e^{-\lambda t}}{\lambda e^{-\lambda t}} = \frac{e^{\lambda t}-1}{\lambda}$, and this is $> \frac{e^{rt}-1}{r}$ if $\lambda > r$. On the other hand, if $\lambda < r$, the exact opposite is true: $t^S < t^O$. So by a judicious choice of g we can make innovations too fast or too slow by comparison with the social optimum.

On the one hand, as observed above, competitive pressure to innovate before one's competitor leads to overinvestment just as in the rental regime. On the other hand, the fact that in the sales regime consumers pay only for the incremental value of each innovation, rather than for its full value, reduces the value of innovation to firms below its social value. This has the effect of slowing down the pace of innovation, relative to the social planner. Indeed, in the case of a monopoly innovator, considered by Fishman and Rob (2000), where the preemption effect is absent, this effect leads unambiguously to underinvestment. Hence, competition bring a new and opposing effect into play, and the net result of these opposing effects can go either way.

Nonetheless, as far as welfare is concerned we have the following result.

6 Social Welfare under Rental versus Sales

Proposition 4: Welfare is higher under the sales regime than under the rental regime.

Proof: Comparing (4) with (3) reveals that under the duopoly regime with planned obsolescence, welfare — the value of W — is zero: First, as discussed earlier, the innovating firm makes zero ex-ante profit. Second, the innovating firm extracts the full surplus from consumers. When models are infinitely durable, however, comparison of (5) with (3) reveals that welfare is positive: Ex-ante profits are still zero for the firms; however, consumers realize some surplus as a result of having to pay only for the incremental value of each innovation. Thus, welfare is higher in the sales regime. ■

In the rental regime, innovators extract the entire consumer surplus of each innovation. The fact that the innovator nevertheless earns zero discounted profit from the innovation process implies that the discounted utility from the stream of innovations equals the discounted value of the stream of implementation costs that must be invested. Thus, the net social benefit is zero. By contrast, under the sales regime, though the innovator still earns zero profit, part of the surplus is realized by consumers. Hence the social benefit from innovation exceeds its cost.

More intuitively, duopoly leads to overinvestment in innovation because the innovator must constantly preempt the incentives of potential competitors to usurp its position. Because prices, and, therefore, profit per unit of time, are higher under the rental regime than under the sales regime, overinvestment is a greater problem under rentals. Hence welfare is higher in the sales regime.

7 Planned Obsolescence

Even under the sales regime, innovators can extract more of the surplus from consumers by deliberately shortening the serviceable lives of their products. Specifically, suppose the innovator designs its products to last for t^R periods and then break down. In this case, the sales regime becomes essentially equivalent to the rental regime: If new models are introduced every t^R periods, consumers have no serviceable model in their possession when a new model is introduced and, hence, are willing to pay for its full surplus, just as in the rental regime. Hence in the competitive setting, planned obsolescence unambiguously reduces welfare.

8 Conclusion

We have shown that in a setting of continuous product innovation, competition leads to overinvestment in innovation. This problem is more acute when products are rented, or sold but designed to be short lived, in which case there is no second-hand market, than when products are sold and long-lived. In the latter case, the implicit existence of the second-hand market reduces the profitability of innovation and hence mitigates overinvestment.

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