Working in Public and Private Firms[1]

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Summary

We develop a theoretical framework for comparing the style of work in public and private enterprises. We incorporate "socializing," an activity which yields utility for workers and affects a firm's output, into a simple multitask model of work organization. In contrast with previous models, we establish the two following results. First, the optimal workers' compensation policy displays a larger incentive intensity in the private firm than in the public firm. Second, labor productivity in the private firm may be higher or lower than in the public firm. Both results fit well with the findings of empirical work.

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1 Introduction

In political debates and in the popular press, privatization is typically praised as "increasing work incentives" and "increasing productivity." Behind those claims one often …nds a property-rights argument. Since, in the case of the public …rm, pro…ts belong to "no one," nobody cares about its e¢ciency. In fact, increasing the e¢ciency of a public …rm might even be construed as a disadvantage to its workers and managers.

On second re‡ection, though, it is not so clear why the public …rm has fewer reasons to use the instrument that raises e¢ciency in private …rms, namely, stricter incentives and, therefore, why one might expect to see any productivity di¤erences between public and private …rms. Is the e¢cient level of worker e¤ort not equally desirable in a public …rm? Might the public …rm not be able to deliver more to its workers, or to the customers of its product, or to taxpayers by raising productivity? Is the manager of the public …rm not subject to scrutiny by the popular press, by political authorities, or by the desire to be promoted to more prestigious positions, to the extent that he might not try to deliver productivity gains? In short, the fact that a …rm is publicly-owned is no hindrance to using the same instruments that have proved to raise productivity in private …rms.[1]

In fact, theoretical work conducted in agency settings with informational asymmetries shows the exact opposite of what proponents of the property-rights doctrine have claimed: A …rm's productivity is higher when, on top of maximizing pro…ts, the …rm tries to appease its workers and / or consumers of its product. In other words, the welfare-maximizing (public) …rm may use stricter incentives and be more productive than the pro…t-maximizing (private) one. Results in this spirit are reported by La¤ont and Tirole (1991) and Roemer and Silvestre (1992), in the context of regulation; Maskin (1992), in the context of auctions; and De Fraja (1993), in the context of managerial compensation.

What is the empirical evidence on factor productivity and incentives of public versus

[1] This view is challenged by Shleifer (1998), who argues that, because contracts are incomplete, private …rms cannot be mimicked by public …rms. See also Lülfesmann (1999).

private firms? On the one hand, empirical studies uniformly find that the incentive intensity, i.e., the extent to which pay is linked to measured performance (e.g., individual output), is stronger in private firms. For example, Martin and Parker (1997, chapter 9) argue that in the cases of British Steel, Rolls-Royce, British Airports Authority, British Airways, British Telecom and the National Freight Corporation, privatization led companies to link pay and promotion more closely to various employee performance measures. On the other hand, empirical studies find the effect of ownership on firm productivity to be ambiguous. Studies like Atkinson and Halvorsen (1986), Boardman and Vining (1989), Martin and Parker (1997, chapter 5) and Yarrow (1986) suggest that cases exist where privatization did not lead to productivity gains or was even detrimental to productivity. In sum, the data show that work incentives are stronger, but that factor productivity may or may not be higher under private than under public ownership.

This paper argues, using an agency-theoretic framework, that the link between ownership and productivity is not clear-cut from a theoretical point of view, either. More precisely, what we show is that incentives are (under certain conditions) stronger in the private firm as compared to the public firm. Productivity, on the other hand, may be higher or lower, depending on fundamentals of the model; namely, the production technology, workers' preferences and the distribution over workers' types. Therefore, in our model, the correlation between ownership and productivity is ambiguous. This result is consistent with empirical findings, as summarized above, but has not arisen in previous theoretical formulations.

The framework within which we reach this conclusion is as follows. We consider a multi-task firm, à la Holmstrom and Milgrom (1991), in which one task is labelled "individual," whereas the other task is labelled "cooperative." Workers allocate their time and effort between these two tasks (as well as choose total effort.) Both tasks exhibit positive marginal productivity, so the firm may want to encourage workers to allocate some effort to the cooperative task.

The cooperative task is understood as contributing to team production and, as such, has two attributes. The …rst attribute is that it is hard to assess the contribution that one particular worker makes. In that sense allocating e¤ort to the cooperative task is analogous to contributing towards public goods. The second attribute is that the worker may actually enjoy, or derive direct utility, from such contributions. The idea is that the worker "socializes," or maybe even acquires productive skills while working with others, which she can use elsewhere.

The presence of a cooperative task and the way we characterize it via the above two attributes should be familiar to everyone who works in pro…t or nonpro…t organizations. There is a vast social psychology literature that discusses how social interactions a¤ect job satisfaction, see Smith et al. (1983), and several empirical assessments of this phenomenon in the economics literature; see, for example, Clark (1996) and Drago and Gravey (1998). The premise that it is hard to directly monitor and remunerate the cooperative task seems also fairly plausible, and has been extensively discussed in the organizational behavior literature, see Deckop et al. (1999).

Assuming that these two attributes characterize the range of activities within a …rm, the choice of incentives in the …rm is subject to the following tradeo¤. An increase in the incentive intensity has two opposing e¤ects. On the one hand, and for the usual agency reasons, it raises a worker's total e¤ort. On the other hand, it lowers his cooperative e¤ort. This, again, is due to the usual agency reasons: The cooperative task is not observed and monetarily rewarded and, hence, when the incentive intensity is stronger, the opportunity cost of engaging in the cooperative task is higher (because time is diverted away from the individual task on which the worker earns credit), which results in less cooperative e¤ort.

The essence of our analysis is to compare how public versus private …rms choose incentives under this tradeo¤ and, as a result, how their productivities compare. In other words, we embed the two …rms in the same technological and informational environment and ask how incentives and productivity compare due to ownership structure. The way we model ownership structures is fairly traditional: The private …rm maximizes pro…ts, while the public …rm maximizes welfare, which includes pro…ts and workers' utility (in extensions of the model we also consider consumers' surplus.)[2]

Given this modeling approach, the logic underlying our results can be worded as follows. Since the e¤ort allocated to the cooperative task (and the socializing that comes with it) increases workers' utility, and since the public …rm incorporates the welfare of its workers into its own objective function, the public …rm chooses weaker incentives, which translates into more cooperative e¤ort and higher workers' welfare. Whether this takes away from or actually contributes to productivity depends on properties of the production function and the distribution over worker types. If the marginal productivity of the cooperative task is especially large or if the distribution over consumer types is su¢ciently dispersed (so that workers enjoy large "informational rents") the public …rm may very well exhibit higher productivity. It is worth stressing, though, that the private …rm is always more pro…table. This follows "by construction," since the private …rm is presumed to maximize pro…ts. Therefore, if the public …rm exhibits greater productivity, then it pays higher wages, and the extra wages it pays exceeds the extra productivity it derives from its workers. In that sense, the incentive choice of the publicly-owned …rm can be construed as "paying too much for productivity."

The paper proceeds as follows. In the next section we set up the model. In section 3 we solve the workers' maximization programs. In section 4 we solve the private and the public …rm's program and compare incentive intensities in the two …rms. In section 5 we present two examples illustrating our results. In section 6 we expand our model. Section 7 concludes.

2 The basic model

The …rm employs a continuum of workers, whose measure is 1. Each worker chooses how much e¤ort, x, to devote to an individual task, and how much e¤ort, y, to devote to a cooperative task. Let e denote the total e¤ort, $e \equiv x + y$.

[2] This assumption is standard in the literature. See Bös (1994) for a discussion of the objectives of public …rms. Furthermore, it employed in many of the theoretical papers we mentioned earlier, e.g., La¤ont and Tirole (1991).

The objective of each worker is to maximize her expected utility. Each worker is identified by a type-parameter, θ. The value of θ is private information to the worker, and indexes the utility she gets from cooperation. Let $g(y;\theta)$ denote the utility to a type-θ worker from exerting y units of cooperative effort. We assume that g is twice continuously differentiable and denote its partial derivatives by $g_y; g_{y\theta}$, etc. We assume that g satisfies $g(0;\theta) = 0$ for all θ. For strictly positive values of y we assume that both the utility, g, and the marginal utility, g_y, of cooperation are strictly increasing in θ: $g_\theta; g_{y\theta} > 0$. The function $g(\cdot;\cdot)$ also satisfies $g_y(0;\theta) = \infty$ and $g_y(\infty;\theta) = 0$ for all θ, and is strictly increasing and strictly concave in y: $g_y > 0 > g_{yy}$.

The (overall) utility function of a type-θ worker is:

$$u = w - c(e) + g(y;\theta); \tag{1}$$

where w is the wage, $c(\cdot)$ is the disutility of total effort, and $g(\cdot;\cdot)$ is, as noted, the utility from performing the cooperative task. The function $c(\cdot)$ is twice continuously differentiable, strictly increasing and strictly convex with $c(0) = c'(0) = 0$, and $c'(\infty) = \infty$.

The type-parameter θ is distributed across workers according to the c.d.f. $H(\theta)$. The support of H is $[\underline{\theta};\overline{\theta}]$ with $-\infty < \underline{\theta} < \overline{\theta} < \infty$. We denote the effort choices of a type-θ worker by $x(\theta)$ and $y(\theta)$, and the effort choices across workers by X and Y, where $X \equiv x(\cdot)$ and $Y \equiv y(\cdot)$, i.e., X is the function $x(\theta)$ as θ varies over $[\underline{\theta};\overline{\theta}]$.

The firm's production technology is $F(X;Y)$, and the firm's total output is Z:

$$Z = F(X;Y): \tag{2}$$

The firm is unable to monitor the cooperative effort of a worker, but can imperfectly monitor her individual effort. The idea is that it is easier to infer a worker's individual effort, knowing what the worker produced individually, than to infer a worker's cooperative effort, knowing what a group of workers produced jointly. To capture this idea, we assume that for each worker, θ, there exists a verifiable signal, s_θ, which is correlated with her individual effort according to:

$$s_\theta = x(\theta) + \epsilon; \tag{3}$$

where ϵ is a noise term with $E[\epsilon] = 0$. The ϵ's are i.i.d. random variables across workers.

The firm chooses a linear wage policy, $(a; b)$, meaning the wage it pays its workers consists of a base salary, b, and an effort-related bonus, as_θ:

$$w(\theta) = b + as_\theta; \tag{4}$$

where a is the piece rate, or, the "incentive intensity."

Restricting the attention to linear schemes may be justified on administrative or practical grounds, as discussed in detail by Chamley et al. (1989). Alternatively, Holmstrom and Milgrom (1987) have shown that the optimal incentive contract in an environment with risk aversion, noise, and dynamic production boils down to a linear contract of a reduced-form static model. Our analysis may be seen as being conducted in such a reduced-form setting.

Firms choose their wage policies to maximize their objective functions. The objective function of a private firm is its profit,

$$\Pi = Z - \int E_\epsilon[w(\theta)]dH(\theta), \tag{5}$$

where the unit price of output has been normalized to one. The objective function of a public firm is social welfare, S. We initially assume that social welfare is defined as the sum of the firm's profits and its workers' sum-of-utilities, namely:

$$S = \Pi + \int E_\epsilon[u(\theta)]dH(\theta). \tag{6}$$

This objective function can be rationalized in two ways, both of which are empirically relevant. First, the public firm may be a monopoly which ignores consumers' surplus because the firm is captured by its workforce and the governmental body entitled to its profits, e.g., the ministry of finance. Second, the public firm may be one of several suppliers interacting in a competitive market, say the world market, in which case it takes the price as given and (correctly) ignores the impact of its output decision on consumers' surplus.

3 Effort choices

When selecting its wage policy, (a, b), the firm has to take into account the effect the wage policy has on a worker's effort choice. A type-θ worker maximizes her expected utility:

$$\max_{x,y} \{b + ax - c(x + y) + g(y; \theta)\}. \tag{7}$$

By the restrictions on $c(\cdot)$ and $g(\cdot;\cdot)$ the solution is interior as long as $a > 0$, and is characterized by the first-order conditions:

$$c'(e^*) = a = g_y(y^*; \theta). \tag{8}$$

The LHS of (8) shows that e^* is the same for all θ's. Also, since $c(\cdot)$ is strictly convex, the LHS can be inverted to yield $e^*(a)$, which expresses the dependence of total effort on the incentive intensity, a. Likewise, since $g(y; \theta)$ is strictly concave in y, the RHS condition can be inverted to yield $y^*(a; \theta)$. The optimal level of individual effort is the residual $x^*(a; \theta) \equiv e^*(a) - y^*(a; \theta)$. We denote the optimal effort choices across workers by the functions $X^*(a), Y^*(a)$, i.e., $X^*(a) \equiv x^*(a; \cdot)$. By differentiation of the first order conditions, (8), we get the following comparative statics properties:

$$c''(e^*)\frac{\partial e^*}{\partial a} = 1 \Rightarrow \frac{\partial e^*}{\partial a} = \frac{1}{c''(e^*)} > 0.$$

$$g_{yy}\frac{\partial y^*}{\partial a} = 1 \Rightarrow \frac{\partial y^*}{\partial a} = \frac{1}{g_{yy}} < 0.$$

$$c''(e^*)\frac{\partial e^*}{\partial \theta} = 0 \Rightarrow \frac{\partial e^*}{\partial \theta} = 0.$$

$$g_{yy}\frac{\partial y^*}{\partial \theta} + g_{y\theta} = 0 \Rightarrow \frac{\partial y^*}{\partial \theta} = \frac{-g_{y\theta}}{g_{yy}} > 0.$$

$$u(\theta) \equiv b + ax^*(a; \theta) - c[x^*(a; \theta) + y^*(a; \theta)] + g(y^*(a; \theta); \theta) \Rightarrow$$

$$u'(\theta) = ax^*_\theta(a; \theta) - c'(e^*)[x^*_\theta(a; \theta) + y^*_\theta(a; \theta)] + g_y y^*_\theta(a; \theta) + g_\theta = g_\theta > 0;$$

where $u(\theta)$ is maximized expected utility of a type-θ worker and where, to conserve on notation, we suppress the dependence of u on $(a; b)$. The last line represents total differentiation of u with respect to θ (in fact, the last line is just an application of the envelope theorem.) Therefore, we have:

Lemma 1 (i) Higher incentive intensity, higher a, leads to higher total effort, higher individual effort and lower cooperative effort.

(ii) A higher value of θ increases cooperative effort and decreases individual effort. Total effort remains unchanged.

(iii) For given values of $(a; b)$, maximized utility is increasing in θ. ■

4 The main result

We now set up the firms' objective functions, and compare their optimal wage policies. We normalize the reservation utility of workers to be zero.[3] We assume that both the private and the public firm must deliver at least this reservation utility, i.e.,

$$E_{\theta}[u(\theta)] \geq 0; \theta \in [\underline{\theta}; \bar{\theta}]: \tag{9}$$

Since, by Lemma 1, u is increasing in θ, it suffices to require the participation constraint, (9), at $\theta = \underline{\theta}$. Also, since the objective of the private firm is to maximize its profit, it sets the base salary, b, at the lowest possible level which is consistent with $u(\underline{\theta}) = 0$: By (1), $u(\underline{\theta}) = 0$ is equivalent to:

$$b = c(e^*(a)) - g(y^*(a; \underline{\theta}); \underline{\theta}) - ax^*(a; \underline{\theta}):$$

Therefore, the private firm's wage bill is:

$$W(a) \equiv \int_{\underline{\theta}}^{\bar{\theta}} [b + ax^*(a; \theta)]dH(\theta) =$$

$$c(e^*(a)) - g(y^*(a; \underline{\theta}); \underline{\theta}) - a \int_{\underline{\theta}}^{\bar{\theta}} [x^*(a; \underline{\theta}) - x^*(a; \theta)]dH(\theta):$$

[3] The reservation utility can be interpreted as the value of being unemployed, in which case $x = y = w = 0$, so that $\bar{u} = 0$.

Let the firm's output, under the incentive intensity a, be G(a):

$$G(a) \equiv F(X^*(a); Y^*(a)):$$

Then, substituting into the firm's profit, (5), the objective of the private firm can be expressed as a function of the incentive intensity, a, alone:

$$\Pi(a) = G(a) - W(a) = \tag{10}$$
$$G(a) - c(e^*(a)) + g(y^*(a;\underline{\theta});\underline{\theta}) + a\int_{\underline{\theta}}^{\overline{\theta}} [x^*(a;\underline{\theta}) - x^*(a;\theta)]dH(\theta):$$

On the other hand, substituting (1) into (6), the objective of the public firm is:

$$S(a) = G(a) - c(e^*(a)) + \int_{\underline{\theta}}^{\overline{\theta}} g(y^*(a;\theta);\theta)dH(\theta): \tag{11}$$

The public firm maximizes this function, with respect to a and b, subject to the participation constraints:

$$u(\theta) = b + ax^*(a;\theta) - c(e^*(a)) + g(y^*(a;\theta);\theta) \geq 0,\ \theta \in [\underline{\theta};\overline{\theta}]: \tag{12}$$

Given (11) and (12), it is optimal for the public firm to select the incentive intensity, a, at the unconstrained optimum of (11) and then set the base salary, b, so that (12) are satisfied.

We can now state and prove our main Proposition.

Proposition 2 Assume $\Pi(a)$ and $S(a)$ are continuously differentiable, strictly concave in a, and admit maxima at a^{pr} and a^{pu} respectively. Then:

(i) The incentive intensity is higher under the optimal wage-policy of the private firm than under the optimal wage-policy of the public firm.

(ii) Total effort is higher in the private firm.

(iii) Cooperation among workers is higher in the public firm.

Proof. Using (10) and (11), the difference, Δ, between Π and S is expressed as follows.

$$\Delta(a) \equiv \Pi(a) - S(a); \tag{13}$$

where

$$\Delta(a) \equiv a \int_{\underline{\theta}}^{\bar{\theta}} [y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} [g(y^*(a;\theta);\theta) - g(y^*(a;\underline{\theta});\underline{\theta})]dH(\theta).$$

(In deriving the last expression we replaced x^* by y^*, using the identity $x^* + y^* = e^*$.)

The optimal incentive intensity in the private firm, a^{pr}, satisfies the first-order condition

$$\Pi'(a^{pr}) = S'(a^{pr}) + \Delta'(a^{pr}) = 0: \tag{14}$$

On the other hand, the optimal incentive intensity in the public firm, a^{pu}, satisfies

$$S'(a^{pu}) = 0: \tag{15}$$

In order to compare (14) and (15), we use (8) and the envelope theorem to show:

$$\Delta'(a) = \int_{\underline{\theta}}^{\bar{\theta}} [y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) > 0; \text{ all } a > 0. \tag{16}$$

Equations (13), (14) and (15), combined with the concavity of $\Pi(a)$ imply that $a^{pr} > a^{pu}$. Parts (ii) and (iii) follow now from Lemma 1, part (i). ■

The intuition behind the result is that the public firm takes workers' welfare and, in particular, workers' informational rents[4] into account while the private firm does not. Indeed, as we show in appendix A, the difference, Δ, between the private and the public firm's objectives is exactly equal to workers' informational rents.[5] Moreover, as the proof of proposition 2 shows, an increase in a has a first-order effect on workers' informational

[4] θ is private information to workers; hence workers with a higher θ get higher utility, i.e., they collect informational rents.

[5] The private firm also cares about its workers' welfare because of the participation constraints and because higher cooperative utility diminishes the need to make monetary compensation. Thus, workers' welfare figures into the private firm's maximization program as well. Nonetheless, these considerations affect the public firm, too, and, on top of that, workers' welfare appears directly in the public firm objective, but not in the private firm's objective.

rents and, hence, on the private …rm's pro…ts. So the private …rm chooses a bigger a to reduce informational rents. More intuitively, perhaps, the public …rm chooses a smaller a because that induces higher cooperative e¤orts, higher workers' utility and, by implication, higher public …rm's payo¤.

Nonetheless, although taking vs. not taking workers' welfare into account helps explain the di¤erence in incentive intensities, it is worth stressing that workers' welfare, per se, is not enough. In particular, the following elements are at play:

1. Heterogeneity of worker types. If the interval $[\underline{\mu}; \overline{\mu}]$ were degenerate, informational rents and $¢^{0}(a)$ would be 0, and a^{pr} would equal a^{pu}. In fact, in that case the objective functions of the private and the public …rms would be equal. The public …rm still cares about the utility of its workers; however, with one worker type and risk neutrality, the most e¢cient action is to maximize pro…t and then divide it between the …rm and the workers in whatever manner is desired. This is no longer the case when there are many worker types. In that case informational rents are not zero and ownership form / objective function matters.[6]

2. $g_{y\mu} > 0$. If $g_{y\mu}$ were 0, e.g., g independent of μ or $g = y + \mu$, $y^{¤}(a; \mu)$ would be constant in μ and the integrand in (16) would be 0. If $g_{y\mu} = 0$, the informational problem is non-existent since all workers make the same choice. Again we have $a^{pr} = a^{pu}$.

3. Related to the above two, if $g = 0$, but there is heterogeneity across workers with respect to the disutility from individual e¤ort, we get the opposite result. See appendix B.

4. On the other hand, the result does not hinge on the productivity of cooperative tasks. The result holds even if $F_y = 0$. Thus, this is a utility-based result not productivity-based.

5. Likewise the result does not depend on di¤erential monitoring ability. So far, we have assumed that workers' remuneration can be made contingent only on x. In appendix C we show that the result continues to hold (under certain conditions) if remuneration

[6] By extension, if $[\underline{\mu}; \overline{\mu}]$ is not degenerate, but is "small", i. e., little heterogeneity across workers, a^{pu} is close to a^{pr}.

can be made contingent on x and y.

Proposition 2 shows that $a^{pr} > a^{pu}$. What can be said about total output - $Z^{pr} = G(a^{pr})$ vs. $Z^{pu} = G(a^{pu})$ - in the private vs. the public firm?[7]

Proposition 3 If either $G(a)$ or $W(a)$ is increasing over $(a^{pu}; a^{pr})$, $Z^{pr} > Z^{pu}$.

Proof. (1) Assume G is increasing. Then the result follows by plugging a into G.

(2) Assume W is increasing, and assume to the contrary that $Z^{pu} \geq Z^{pr}$. Then the private firm can set $a = a^{pu}$ instead of a^{pr}. This will increase its output and decrease the wage bill, i.e., it will increase profits. ■

However, and unlike the situation in conventional one-dimensional private information models nothing can be said, in general, about the monotonicity of G or W. In fact, there is nothing to prevent G and W from being downward sloping - at least over some domain - and nothing to prevent a^{pu} and a^{pr} from occurring on this domain. We exhibit non-pathological examples in the next section.[8] More generally, the following computation shows that W is increasing only under certain conditions:

$$W(a) = c(e^*(a)) - g(y^*(a;\underline{\theta});\underline{\theta}) - a\int_{\underline{\theta}}^{\bar{\theta}} [x^*(a;\underline{\theta}) - x^*(a;\theta)]dH(\theta):$$

Therefore,

$$W'(a) = c'e^{*\prime} - g_y y^*_a(a;\underline{\theta}) - a x^*_a(a;\underline{\theta}) + a\int_{\underline{\theta}}^{\bar{\theta}} x^*_a(a;\theta)dH(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} [x^*(a;\underline{\theta}) - x^*(a;\theta)]dH(\theta):$$

The first three terms are zero because of the first order conditions, (8), and because $x^* + y^* = e^*$: So W is increasing if and only if $a\int_{\underline{\theta}}^{\bar{\theta}} x^*_a(a;\theta)dH(\theta) > \int_{\underline{\theta}}^{\bar{\theta}}[x^*(a;\underline{\theta}) - x^*(a;\theta)]dH(\theta)$. This inequality holds if workers are homogenous, or if x^* does not vary too much with θ. Otherwise, there is no reason for this inequality to hold.

The result that although the private firm employs stricter wage incentives, its productivity need not to be higher than in the case of the public firm distinguishes our

[7] The private firm is of course more profitable (by construction).

[8] Intuitively, in such cases $W'(a) < 0$ which occurs because informational rents decrease in a. So, in such cases, it pays the private firm to sacrifice productivity in return for (significantly) smaller informational rents.

formulation from previous formulations (where cooperative tasks and the attendant co-operative utility are not part of the formulation.) Further, this result is consistent with empirical findings discussed in the introduction. The empirical literature showed, using "before and after" comparisons, that once a public company is privatized it introduces performance-related pay, i.e., it tightens its incentives; yet, the effect on productivity is ambiguous. Likewise, the empirical literature showed that if we compare, at a given point in time, companies in the same sector, the privately-owned ones exhibit stronger incentives but not necessarily higher productivity than the publicly-owned ones.

5 Counterexamples

By way of two parametric examples we now demonstrate the possibility of higher productivity in the public firm, despite its lower incentive intensity. Both examples are worked out in detail within appendix D.

The first example specializes the model's functions and parameters as follows:

$$c(e) = \begin{cases} 0; & e < 1 \\ \frac{1}{2}(e - 1)^2; & e \geq 1 \end{cases};$$

$$g(y; \theta) = \theta y(1 - \frac{y}{2}),\ y \geq 0;$$

$$\theta \in \{\theta_1; \theta_2\} \text{ with } \theta_1 < \theta_2 \text{ and } \Pr(\theta_1) = \alpha:$$

$$Z = F(X; Y) = X(1 - \frac{X}{2}) + Y;$$

where:

$$X \equiv \alpha x(\theta_1) + (1 - \alpha)x(\theta_2);$$

$$Y \equiv \alpha y(\theta_1) + (1 - \alpha)y(\theta_2):$$

This example thus involves three parameters: $(\theta_1; \theta_2; \alpha)$. Suppose that they take the following values:

$$\theta_1 = \frac{1}{15}; \theta_2 = \frac{9}{10}; \alpha = \frac{1}{5}:$$

As shown in the appendix, the incentive intensities in the two firms are:

$$a^{pu} = 3.4734 \times 10^{-2};$$
$$a^{pr} = 5.6561 \times 10^{-2};$$

whereas productivities in the two firms are:

$$Z^{pu} = 1.0203;$$
$$Z^{pr} = 1.0183;$$

which shows that the public firm is more productive than the private one.

In the second example the functions and parameters are specialized as follows:

$$c(e) = \frac{e^2}{2} + 5;$$
$$g(y;\theta) = \frac{\theta[1 - \exp(-y)]}{10},$$
$$\theta \in \{\theta_1;\theta_2\} \text{ with } \theta_1 < \theta_2 \text{ and } \Pr(\theta_1) = \alpha.$$
$$Z = 1 + \frac{11}{10}(X + Y) - \alpha \exp[-2y(\theta_1)] - (1 - \alpha)\exp[-2y(\theta_2)];$$

where:

$$X \equiv \alpha x(\theta_1) + (1 - \alpha)x(\theta_2);$$
$$Y \equiv \alpha y(\theta_1) + (1 - \alpha)y(\theta_2).$$

Again, the example involves three parameters: $(\theta_1;\theta_2;\alpha)$. Suppose that they take the following values:

$$\theta_1 = 1;\theta_2 = 20;\alpha = 0.01.$$

As shown in the appendix, the incentive intensities in the two firms are:

$$a^{pu} = 2.9 \times 10^{-2};$$
$$a^{pr} = 84.9 \times 10^{-2}.$$

The corresponding productivity levels are:

$$Z^{pu} = 1.030;$$
$$Z^{pr} = 1.005.$$

Also in this case the public firm is more productive than the private one.

6 Extensions

6.1 Consumers' Surplus

The basic model developed above either portrays a firm selling its output to a competitive market or a monopoly, in which case the publicly-owned firm is assumed to put zero weight on consumers' surplus. We now consider a more general monopoly case, in which the public firm puts some (administratively or politically determined) weight on consumers' surplus.

Let $P(Z)$ be the inverse demand function and let $R(Z) \equiv ZP(Z)$ be the corresponding revenue function. Then $R(Z) - \int E_\epsilon[w(\theta)]dH(\theta)$ is the firm's profit. To conserve on notation we continue to call the objective of the private firm Π, and likewise for other functions in this section. The public firm's objective is now:

$$S = \Pi + \int E_\epsilon[u(\theta)]dH(\theta) + \gamma C(Z);$$

where $C(Z)$ is consumers' surplus, with $C'(Z) = P(Z)$, and γ is the weight attached to consumers' welfare.

The manipulations used in proving proposition 2 extend to the new scenario: The participation constraint of the lowest type will be binding for the private firm, whereas, for the public firm, a is chosen independently of these constraints, and b is adjusted to satisfy the constraints. The difference between the objectives of the firms is:

$$\Delta(a) = a \int_{\underline{\theta}}^{\bar{\theta}} [y^*(a;\theta);\theta) - y^*(a;\underline{\theta});\underline{\theta})]dH(\theta)$$

$$-\int_{\underline{\theta}}^{\bar{\theta}}[g(y^*(a;\theta);\theta) - g(y^*(a;\underline{\theta});\underline{\theta})]dH(\theta) - \gamma C(G(a)).$$

And, computing the derivative of $\Delta(\cdot)$, yields

$$\Delta'(a) = \int_{\underline{\theta}}^{\bar{\theta}}[y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) - \gamma P(Z)\int_{\underline{\theta}}^{\bar{\theta}}[F_x\frac{\partial x^*(a;\theta)}{\partial a} + F_y\frac{\partial y^*(a;\theta)}{\partial a}]dH(\theta); \quad (17)$$

where F_x is the $x(\theta)$-derivative of F, and F_y is the $y(\theta)$-derivative of F, both evaluated at $(x^*(a;\theta); y^*(a;\theta))$. Proposition 2 shows that the first term is positive; however, since, by Lemma 1, $\frac{\partial x^*(a;\theta)}{\partial a} > 0 > \frac{\partial y^*(a;\theta)}{\partial a}$, the second term cannot be signed - even if F_x, F_y are assumed to be positive. On the other hand, (17) suggests sufficient conditions under which proposition 2 continues to hold.

Proposition 4 Proposition 2 continues to hold provided: (i) Output is significantly more sensitive to cooperative effort than to individual effort: $F_y >> F_x > 0$. Or, (ii) The weight, γ, on consumer surplus is small. ■

Conversely, suppose that cooperative effort does not generate any productivity gain ($F_y = 0$) and that there is only one type of worker ($\bar{\theta} = \underline{\theta}$). By using Equation (17), it is easy to see that the public firm implements a higher total effort (and less cooperation) than the private firm - even if workers enjoy cooperating. This outcome corresponds to the one exhibited by previous principal-agent models of public and private firms.

6.2 Relaxing the manpower constraint

So far we have considered labor as a fixed input, e.g., because of large hiring and firing costs or employment regulation. We now examine the case where either firm faces the same unit-mass of potential employees, but firms are not obligated to employ all workers, i.e., each firm is able to choose how many workers to employ.

In analyzing this case we assume away consumers' surplus, i.e., we let $\gamma = 0$. It is well known that the effect of adding consumers' surplus (by itself) is to raise employment, see Roemer and Silvestre (1992). This is for the usual reason that with consumers' surplus it is desirable to have larger output. However, in this subsection we point out another reason for restricting employment in the private firm, which is that by doing so the firm alleviates the employees' participation constraint and thereby lowers informational rents. Thus, although the private firm lowers employment and output, its profits increases. In order to isolate this effect, we further assume that the incentive intensity is fixed at some level $a > 0$, the same for the public- and the private-firm, so that only the employment decision has to be determined.

Consider first the employment decision of the private firm. At any employment level, the firm chooses the base-salary b^{pr} so as to make the participation constraint binding for the marginal worker. That is, there is a worker type, θ^*, such that the set of employed workers is $[\theta^*; \bar{\theta}]$ and

$$u(\theta^*) = 0:$$

Hence one has

$$b^{pr} = c(e^*(a)) - g(y^*(a;\theta^*);\theta^*) - a x^*(a;\theta^*):$$

For a given incentive intensity, a, the private-firm's profit can be written as a function of its employment level or, equivalently, of the cutoff point, θ^*:

$$\begin{aligned} \Pi(\theta^*) &= F(X^*(a); Y^*(a)) - [1 - H(\theta^*)][c(e^*(a)) - g(y^*(a;\theta^*);\theta^*)] + \\ & \quad a \int_{\theta^*}^{\bar{\theta}} [x^*(a;\theta^*) - x^*(a;\theta)] dH(\theta): \end{aligned}$$

Consider now the public firm. Since workers' welfare enters its objective function, the public firm would like to hire those individuals with the highest valuation for the job (high θ). Although θ is private information to the worker, self-selection occurs, as in the private firm, by choosing b^{pu} so that the participation constraint of the cutoff individual θ^* is binding. Therefore, the public firm chooses employment by choosing the cutoff point θ^* which maximizes social welfare as given by:

$$S(\theta_*) = F(X^*(a), Y^*(a)) + \int_{\theta_*}^{\bar{\theta}} [g(y^*(a,\theta),\theta) - c(e^*(a))]dH(\theta).$$

Using the fact that $x^*(a,\theta) \equiv e^*(a) - y^*(a,\theta)$, the difference between Π and S can be written as:

$$\Delta(\theta_*) \equiv a \int_{\theta_*}^{\bar{\theta}} [y^*(a,\theta) - y^*(a,\theta_*)]dH(\theta) - \int_{\theta_*}^{\bar{\theta}} [g(y^*(a,\theta),\theta) - g(y^*(a,\theta_*),\theta_*)]dH(\theta). \quad (18)$$

By use of (8) and Lemma 1, the derivative of (18) is:

$$\Delta'(\theta_*) \equiv [1 - H(\theta_*)]g_\theta(y^*(a,\theta_*),\theta_*)h(\theta_*) > 0.$$

Therefore, employment is larger in the public firm, i.e., $\theta_*^{pr} > \theta_*^{pu}$. We summarize this as follows.

Proposition 5 Suppose: (i) the public firm's objective ignores consumers' surplus, and (ii) the private and the public firm use the same incentive intensity, a. Then, the employment level is higher in the public firm. ■

The intuition, again, is that the private firm tries to compress informational rents whereas the public firm does not, and that informational rents are reduced by firing workers. Case studies about privatization overwhelmingly support this model's prediction.[9]

7 Conclusion

In this paper we have developed a theoretical framework for comparing the workings of private and public enterprises. The main novelty is that we have incorporated workers' "socializing" in the management's problem of setting work incentives. Socializing has two faces. On the one hand, it is an activity which yields utility for the employees. On the

[9] For example, see the case-study by Martin and Parker (1997, ch. 8).

other hand, such activity can a¤ect the …rm's output to the extent that socializing brings about some cooperation among workmates. In a multitask model of work organization we have established two main results. First, the optimal workers' compensation policy displays a larger incentive intensity in the private …rm than in its public counterpart. Second, labor productivity in the private …rm may be higher or lower than in the public …rm. Both results …t well with the …ndings in empirical work. Moreover, there is - to our knowledge - no previous theoretical work on public vs. private …rms that can provide a uni…ed explanation of these empirical …ndings.

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APPENDIX

Appendix A: Informational Rents

The reservation utility of workers is zero. So, informational rents equal the maximized utility of workers facing the wage scheme $(a; b)$. The latter is:

$$\int_{\underline{\theta}}^{\bar{\theta}} E_{\varepsilon}[u(\theta)]dH(\theta) =$$

$$W(a) - c(e^*(a)) + \int_{\underline{\theta}}^{\bar{\theta}} g(y^*(a;\theta);\theta)dH(\theta) =$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \{[g(y^*(a;\theta);\theta) - g(y^*(a;\underline{\theta});\underline{\theta})] - a[y^*(a;\theta);\theta) - y^*(a;\underline{\theta});\underline{\theta})]\}dH(\theta)$$

$$= -\Delta(a):$$

Appendix B: One-Dimensional Effort

Here we retain heterogeneity but consider workers who derive no cooperative utility:

$$u = w - c(e;\theta):$$

We index workers so that $c_\theta < 0$ and $c_{e\theta} < 0$, i.e., the bigger is θ the more industrious the worker. Then, with linear incentives as above, type-θ worker chooses an $e^*(\theta)$ which satisfies $c_e(e;\theta) = a$.

The comparative statics of workers' decision goes as follows:

$$\begin{aligned}
0 &= c_{ee}\frac{de}{d\theta} + c_{e\theta} \\
\Rightarrow \frac{de}{d\theta} &= \frac{-c_{e\theta}}{c_{ee}} > 0: \\
1 &= c_{ee}\frac{de}{da} \\
\Rightarrow \frac{de}{da} &= \frac{1}{c_{ee}} > 0:
\end{aligned}$$

$$\begin{aligned}
u(\theta) &= b + ae^*(\theta) - c(e^*(\theta);\theta) \\
\Rightarrow u'(\theta) &= (a - c_e)\frac{de^*}{d\theta} - c_\theta > 0
\end{aligned}$$

Therefore, it suffices to require the participation constraint for the lowest type, $\underline{\theta}$, which implies

$$b = c(e^*(\underline{\theta})) - ae^*(\underline{\theta}).$$

Plugging this into the wage function, we get:

$$W(a) = c(e^*(\underline{\theta})) + a\int_{\underline{\theta}}^{\bar{\theta}}[e^*(\theta) - e^*(\underline{\theta})]dH(\theta).$$

Therefore,

$$W'(a) = (c_e - a)\frac{de^*}{da}(\underline{\theta}) + a\int_{\underline{\theta}}^{\bar{\theta}}\frac{de^*}{da}dH(\theta) + \int_{\underline{\theta}}^{\bar{\theta}}[e^*(\theta) - e^*(\underline{\theta})]dH(\theta)$$

$$= a\int_{\underline{\theta}}^{\bar{\theta}}\frac{de^*}{da}dH(\theta) + \int_{\underline{\theta}}^{\bar{\theta}}[e^*(\theta) - e^*(\underline{\theta})]dH(\theta) > 0;$$

because $\frac{de^*}{da}, \frac{de^*}{d\theta} > 0$, as per the comparative statics properties. Thus, $a^{pr} > a^{pu}$ in this case implies $Z^{pr} > Z^{pu}$ and $a^{pr} < a^{pu}$ implies $Z^{pr} < Z^{pu}$; a and Z move in the same direction.

The objective of the private firm is

$$\Pi(a) = G(a) - W(a) =$$
$$G(a) - c(e^*(\underline{\theta})) - a\int_{\underline{\theta}}^{\bar{\theta}}[e^*(\theta) - e^*(\underline{\theta})]dH(\theta).$$

On the other hand, the objective of the public firm is:

$$S(a) = G(a) - \int_{\underline{\theta}}^{\bar{\theta}}c(e^*(\theta))dH(\theta).$$

Thus,

$$\Delta(a) \equiv \Pi(a) - S(a) =$$
$$\int_{\underline{\theta}}^{\bar{\theta}}[c(e^*(\theta)) - c(e^*(\underline{\theta}))]dH(\theta) - a\int_{\underline{\theta}}^{\bar{\theta}}[e^*(\theta) - e^*(\underline{\theta})]dH(\theta);$$

And,

$$\Delta'(a) = -\int_{\underline{\theta}}^{\bar{\theta}} [e^*(\theta) - e^*(\underline{\theta})] dH(\theta) < 0.$$

So, in this instance, $\Pi(a)$ being concave implies $a^{pr} < a^{pu}$ and $Z^{pr} < Z^{pu}$, which is the opposite of proposition 2. The reason is that, in an attempt to reduce informational rents to high types, the private firm choose a lower incentive intensity and lower production. This is similar to inefficiency results in other models with private information, for instance, auctions or implicit contracts.

Appendix C: Differential monitoring ability

Imagine there were another signal, say $t_\theta = y(\theta) + \eta$, where η has zero mean and is i.i.d. across workers and independent of ε. Then workers' payment scheme can be made contingent on s and t: $w = b + a_1 s + a_2 t$.

Under such circumstances the first order conditions to the worker's problem are:

$$\begin{aligned} a_1 &= c'(x + y); \\ a_2 + g_y(y; \theta) &= c'(x + y). \end{aligned}$$

Plugging the first condition into the second we re-write the second condition as:

$$a_1 - a_2 = g_y(y; \theta).$$

This implies total effort, e^*, is still independent of θ and is dependent only on a_1, not a_2. Also, the comparative static result that y increases in θ continues to hold.

Now we can use the participation constraint to get rid of b, just like we did before. Then we can substitute this into the firm's profit and get an expression analogous to equation (10):

$$\Pi(a) = G(a) - c(e^*(a)) + g(y^*(a; \underline{\theta}); \underline{\theta}) + (a_1 - a_2) \int_{\underline{\theta}}^{\bar{\theta}} [x^*(a; \underline{\theta}) - x^*(a; \theta)] dH(\theta);$$

where a is now understood as the vector $(a_1; a_2)$.

Proposition 6 Assume Π is twice continuously differentiable and strictly concave in a, and that $\Pi_{a_1;a_2} < 0$. Then, the private firm chooses a higher individual incentive intensity, a_1, and a lower cooperative incentive intensity, a_2.

Proof. The objective function of the public firm, $S(a)$, is the same as before, (11):

$$S(a) = F(X^*(a); Y^*(a)) - c(e^*(a)) + \int_{\underline{\theta}}^{\bar{\theta}} g(y^*(a;\theta);\theta)dH(\theta):$$

Taking the difference between Π and S we obtain:

$$\Delta(a) \equiv (a_1 - a_2)\int_{\underline{\theta}}^{\bar{\theta}}[y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) - \int_{\underline{\theta}}^{\bar{\theta}}[g(y^*(a;\theta);\theta) - g(y^*(a;\underline{\theta});\underline{\theta})]dH(\theta):$$

Next we can take partial derivatives of Δ with respect to a_1 and a_2, using the first-order conditions of workers to simplify expressions. This gives us:

$$\Delta_{a_1} = \int_{\underline{\theta}}^{\bar{\theta}}[y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) > 0;$$
$$\Delta_{a_2} = -\int_{\underline{\theta}}^{\bar{\theta}}[y^*(a;\theta) - y^*(a;\underline{\theta})]dH(\theta) < 0:$$

The result now follows from the concavity of Π and from $\Pi_{a_1 a_2} < 0$. ■

Hence, the result that individual effort is subject to stricter incentives in the private firm does not depend on that type of effort being more easily monitored than cooperative effort.

Appendix D: Counterexamples

The first example specializes the model's functions and parameters as follows:

$$c(e) = \begin{cases} 0; & e < 1 \\ \frac{1}{2}(e - 1)^2; & e \geq 1 \end{cases};$$

$$g(y;\theta) = \theta y(1 - \frac{y}{2}),\ y \geq 0;$$

$$\theta \in \{\theta_1;\theta_2\} \text{ with } \theta_1 < \theta_2 \text{ and } \Pr(\theta_1) = \alpha:$$

$$Z = F(X;Y) = X(1 - \frac{X}{2}) + Y;$$

where:

$$X \equiv \alpha x(\theta_1) + (1 - \alpha)x(\theta_2);$$
$$Y \equiv \alpha y(\theta_1) + (1 - \alpha)y(\theta_2):$$

So this is a 3-parameter example: $(\alpha; \theta_1; \theta_2)$.

Computing the utility-maximizing effort levels yields:

$$e^*(a) = 1 + a;$$

$$y^*(a;\theta) = \begin{cases} 1 - \frac{a}{\theta} & \text{if } a < \theta \\ 0 & \text{if } a \geq \theta \end{cases};$$

and

$$x^*(a;\theta) = \begin{cases} (1 + \frac{1}{\theta})a & \text{if } a < \theta \\ 1 + a & \text{if } a \geq \theta \end{cases}:$$

Substituting this back into the data we get:

$$c(e^*(a)) = \frac{1}{2}(1 + a)^2 - (1 + a) + \frac{1}{2} = \frac{1}{2}a^2;$$

$$g(y^*(a;\theta);\theta) = \theta(1 - \frac{a}{\theta})(1 - \frac{1}{2} + \frac{a}{2\theta}) = \frac{\theta}{2}(1 - \frac{a^2}{\theta^2}) = \frac{\theta}{2} - \frac{a^2}{2\theta}; \quad (19)$$

$$\int_{\underline{\theta}}^{\bar{\theta}} [x^*(a;\theta_1) - x^*(a;\theta)]dH(\theta) = (1 - \alpha)(\frac{1}{\theta_1} - \frac{1}{\theta_2})a.$$

And this gives us:

$$\begin{aligned} W(a) &= c(e^*(a)) - g(y^*(a;\theta_1);\theta_1) - a\int_{\underline{\theta}}^{\bar{\theta}} [x^*(a;\theta_1) - x^*(a;\theta)]dH(\theta) \\ &= \frac{1}{2}a^2 - \frac{\theta_1}{2} + \frac{a^2}{2\theta_1} - (1 - \alpha)(\frac{1}{\theta_1} - \frac{1}{\theta_2})a^2 \\ &= -\frac{\theta_1}{2} + \frac{1}{2}[1 + \frac{1}{\theta_1} - 2(1 - \alpha)(\frac{1}{\theta_1} - \frac{1}{\theta_2})]a^2 \\ &= -\frac{\theta_1}{2} + \frac{1}{2}[1 + 2(1 - \alpha)\frac{1}{\theta_2} - (1 - 2\alpha)\frac{1}{\theta_1}]a^2. \end{aligned}$$

Let

$$R \equiv 1 + \frac{\alpha}{\theta_1} + \frac{1 - \alpha}{\theta_2};$$

and,

$$\delta \equiv (1-\alpha)\left(\frac{1}{\theta_1}-\frac{1}{\theta_2}\right).$$

Then:

$$W(a) = -\frac{\theta_1}{2}+\frac{1}{2}(R-\delta)a^2; \qquad (20)$$

$$X(a) = Ra,$$

and,

$$Y(a) = 1-(R-1)a.$$

Therefore,

$$\begin{aligned} G(a) &= 1+\left(1-\frac{Ra}{2}\right)Ra-(R-1)a = \\ &\quad 1+a-\frac{R^2}{2}a^2. \end{aligned} \qquad (21)$$

For future reference, let us note that the maximizer of $G(a)$ is:

$$a^{\star} \equiv \frac{1}{R^2},$$

and that F increases (decreases) below (above) $a^{\star}$. Given the expressions for $W(a)$ and $G(a)$ the objective of the private firm is:

$$\begin{aligned} \Pi(a) &= G(a)-W(a) = \\ &\quad 1+a-\frac{R^2}{2}a^2 \\ &\quad +\frac{\theta_1}{2}-\frac{1}{2}(R-\delta)a^2 \\ &= \frac{\theta_1}{2}+1+a-\frac{1}{2}(R^2+R-\delta)a^2. \end{aligned}$$

Assuming $R^2+R-\delta>0$, the maximum of Π is attained at

$$a^{pr} = \frac{1}{R^2+R-\delta}.$$

For the economics to make sense the following restrictions are required:

1. For Π to be concave the coefficient on a^2 must be negative, or, equivalently:

$$R^2 + R - \delta > 0. \tag{22}$$

2. For the worker's optimum to be interior we must have

$$a^{pr} < Min(\theta_1; \theta_2) = \theta_1,$$

which translates into:

$$R^2 + R - \delta > \frac{1}{\theta_1}. \tag{23}$$

Since (23) in conjunction with $\theta > 0$ implies (22), it suffices to require (23).

The objective function of the public firm is constructed as follows. From (19) we have:

$$\int g(y^*(a;\theta);\theta)dH(\theta) = \int (\frac{\theta}{2} - \frac{a^2}{2\theta})dH(\theta) =$$

$$\frac{1}{2}[\alpha\theta_1 + (1-\alpha)\theta_2 - (R-1)a^2].$$

Therefore, the objective of the public firm is

$$\begin{aligned} S(a) &= 1 + a - \frac{R^2}{2}a^2 - \\ &\quad \frac{1}{2}a^2 + \frac{1}{2}[\alpha\theta_1 + (1-\alpha)\theta_2 - (R-1)a^2] \\ &= 1 + \frac{1}{2}[\alpha\theta_1 + (1-\alpha)\theta_2] + a - \frac{1}{2}(R^2+R)a^2. \end{aligned}$$

So a^{pu} is given by:

$$a^{pu} = \frac{1}{R^2 + R}.$$

Since $R^2 + R > R^2 + R - \delta > 0$, we have $a^{pr} > a^{pu}$.

What can be said about productivity in the private vs. the public firm, i.e., is $Z^{pu} >$ or $< Z^{pr}$?

By proposition 3, if $W'(a) > 0$, then $Z^{pu} < Z^{pr}$. So a necessary condition for $Z^{pu} > Z^{pr}$ is $W'(a) < 0$.[10]

[10] a^{pr} must be such that $W'(a)$ and $G'(a) < 0$. It suffices to require $W'(a) < 0$ since the private firm maximizes $G(a) - W(a)$. So, at the optimum, $G'(a) = W'(a) < 0$.

3. Since W is quadratic, $W^0(a) < 0$ requires

$$\pm > R: \qquad (24)$$

4. Since F is symmetric around its maximum, $Z^{pu} > Z^{pr}$ if and only if

$$a^{pr} \text{ ¡ } a^{¤} > a^{¤} \text{ ¡ } a^{pu}; \qquad (25)$$

where $a^{¤}$ is the maximizer of $F(X(a); Y(a))$, i.e.,

$$a^{¤} \text{ ´ } \frac{1}{R^2}.$$

(25) is equivalent to:

$$\frac{1}{R^2} \text{ ¡ } \frac{1}{R^2+R} = \frac{1}{R^2(R+1)} < \frac{1}{R^2+R \text{ ¡ } \pm} \text{ ¡ } \frac{1}{R^2} = \frac{\pm \text{ ¡ } R}{R^2(R^2+R \text{ ¡ } \pm)} \quad , \qquad (26)$$
$$R^2 + R \text{ ¡ } \pm < (\pm \text{ ¡ } R)(R+1) = \pm R + \pm \text{ ¡ } R^2 \text{ ¡ } R \quad ,$$
$$\pm(R+2) > 2(R^2+R) \quad , \quad \pm > \frac{2(R^2+R)}{R+2}.$$

But $\frac{2(R^2+R)}{R+2} > R$ (because $2R^2 + 2R > R^2 + 2R$). So (26) implies (24). Thus, it su¢ces to require (26). On top of that we require (23).[11]

A concrete con…guration of parameters for which (23) and (26) are satis…ed is:

$$µ_1 = \frac{1}{15}; µ_2 = \frac{9}{10}; ® = 0:2:$$

Then:

$$R = 1 + (0:2)(15) + (0:8)(10)=(0:9) = 4 + 8=9 = 4:89$$
$$\pm = (0:8)(15 \text{ ¡ } \frac{10}{9}) = 11:11$$
$$\frac{2(R^2+R)}{R+2} = \frac{57:6}{6:89} = 8:35 < 11:11 = \pm$$
$$R^2 + R \text{ ¡ } \pm = 23:9 + 4:89 \text{ ¡ } 11:11 = 17:7 > 15 = \frac{1}{µ_1}.$$

[11] For the consistency of these two we must have

$$\frac{2(R^2+R)}{R+2} < R^2 + R \text{ ¡ } \frac{1}{µ_1}:$$

And,

$$\begin{aligned}
a^{pu} &= \frac{1}{R^2 + R} = \frac{1}{(44/9)^2 + (44/9)} = 3.4734 \times 10^{-2} \\
a^{pr} &= \frac{1}{R^2 + R - \delta} = \frac{1}{(44/9)^2 + (44/9) - 11.11} = 5.6561 \times 10^{-2} \\
G(a^{pu}) &= 1 + 3.4734 \times 10^{-2} - \frac{(44/9)^2}{2}(3.4734 \times 10^{-2})^2 = 1.0203 \\
G(a^{pr}) &= 1 + 5.6561 \times 10^{-2} - \frac{(44/9)^2}{2}(5.6561 \times 10^{-2})^2 = 1.0183
\end{aligned}$$

In the second example we let the variables indexing effort levels take both positive and negative values. The example specializes the functions and parameters as follows:

$$c(e) = \frac{e^2}{2} + 5,$$

$$g(y, \theta) = \frac{\theta[1 - \exp(-y)]}{10},$$

$$\theta \in \{\theta_1, \theta_2\} \text{ with } \theta_1 < \theta_2 \text{ and } \Pr(\theta_1) = \alpha.$$

$$Z = \frac{11}{10}(X + Y) - \alpha \exp[-2y(\theta_1)] - (1 - \alpha)\exp[-2y(\theta_2)] + 1,$$

where:

$$\begin{aligned}
X &\equiv \alpha x(\theta_1) + (1 - \alpha) x(\theta_2), \\
Y &\equiv \alpha y(\theta_1) + (1 - \alpha) y(\theta_2).
\end{aligned}$$

Computing the utility-maximizing effort levels yields:

$$y^*(a, \theta) = \ln\left(\frac{\theta}{10a}\right),$$

and

$$x^*(a, \theta) = a - \ln\left(\frac{\theta}{10a}\right).$$

Therefore,

$$e^*(a) = a.$$

Substituting this back we get:

$$c(e^*(a)) = \frac{1}{2}a^2 + 5;$$

and

$$g(y^*(a;\theta);\theta) = \frac{\theta}{10} - a.$$

Using the expression for the wage bill we obtain:

$$W(a) = \frac{1}{2}a^2 - a\left[(1-\alpha)\ln\left(\frac{\theta_2}{\theta_1}\right) - 1\right] + 5 - \frac{\theta_1}{10}.$$

Turning to the production function, we note that $X + Y = a$. Substituting back we derive how output relates to the incentive intensity:

$$G(a) = \frac{11}{10}a - \frac{a^2}{100}\left(\frac{\alpha}{\theta_1^2} + \frac{1-\alpha}{\theta_2^2}\right) + 1.$$

The objective function of the public firm $\Pi(a) = G(a) - W(a)$ can then be written as:

$$\Pi(a) = -\frac{1}{2}a^2\left[1 + \frac{1}{50}\left(\frac{\alpha}{\theta_1^2} + \frac{1-\alpha}{\theta_2^2}\right)\right] + a\left[(1-\alpha)\ln\left(\frac{\theta_2}{\theta_1}\right) + \frac{1}{10}\right] + \frac{\theta_1}{10} - 4.$$

The profit-maximizing incentive intensity is thus:

$$a^{pr} = \frac{(1-\alpha)\ln\left(\frac{\theta_2}{\theta_1}\right) + \frac{1}{10}}{1 + \frac{1}{50}\left(\frac{\alpha}{\theta_1^2} + \frac{1-\alpha}{\theta_2^2}\right)}.$$

Since social welfare is given by $S(a) = G(a) - c(e^*(a)) + \alpha g(y^*(a;\theta_1);\theta_1) + (1-\alpha)g(y^*(a;\theta_2);\theta_2)$, one can write the objective function of the public firm as:

$$S(a) = -\frac{1}{2}a^2\left[1 + \frac{1}{50}\left(\frac{\alpha}{\theta_1^2} + \frac{1-\alpha}{\theta_2^2}\right)\right] + \frac{a + \alpha\theta_1 + (1-\alpha)\theta_2}{10} - 4.$$

So a^{pu} is given by:

$$a^{pu} = \left[10 + \frac{1}{5}\left(\frac{\alpha}{\theta_1^2} + \frac{1-\alpha}{\theta_2^2}\right)\right]^{-1}.$$

A concrete configuration of parameters for which the public firm is more productive than the private firm is:

$$\mu_1 = 1; \mu_2 = 20; ® = 0:01:$$

In this case we have:

$$\begin{aligned} a^{pu} &= 2:9 \text{ £ } 10^{i\,2}; \\ a^{pr} &= 84:9 \text{ £ } 10^{i\,2}; \\ G(a^{pu}) &= 1:030; \\ G(a^{pr}) &= 1:005: \end{aligned}$$