Efficient Job Allocation[*]

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Abstract

This paper considers equilibrium directed search with a finite number of heterogeneous workers and firms, where firms compete in direct mechanisms. Unlike previous findings, Nash equilibrium here does solve the problem of coordination failure. Restricting the match value function to be supermodular, and that firms use truthful strategies also imply positive assortative matching and decentralized trading prices which are consistent with the stable (cooperative equilibrium) outcome. The equilibrium mechanism is not an auction. Instead, to attract better skilled workers, firms post a fixed wage rule and hire the most skilled applicant who applies.

Keywords. Matching. Directed Search. Direct Mechanisms. Supermodularity. Truthful Equilibrium.

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1 Introduction

We consider decentralized trade in a frictional labor market with differentiated firms and workers. As in the directed search literature [e.g. Montgomery (1991), Peters (1991), Acemoglu and Shimer (1999a,b), Moen (1997), Lagos (2000), Burdett, Shi and Wright (2000)], firms advertise their vacancies in some central location - say "help wanted" ads in newspapers - and all job seekers observe those job advertisements and choose which firm to apply to. But an important question here is what should a firm's advertisement state? For example the firm might advertise a wage, realizing that a higher wage will attract a better pool of job applicants. But suppose several workers apply. How will the firm then choose the successful applicant? The directed search approach typically assumes the firm will randomly select one worker from the pool of applicants. But with heterogeneous workers this rule is obviously inefficient. Alternatively, it might be more profitable to advertise a job auction with some reserve price. Not only would such an auction ensure the most profitable [quality adjusted] worker is hired, it would also maximize firm surplus ex-post [see Shimer (1999) for example]. However in the world considered here, where firm and worker types are complementary inputs (i.e. the match value function is supermodular), each firm wishes to attract a more highly skilled worker. This creates a tension between maximizing ex-post surplus and attracting a pool of highly skilled job applicants. It turns out that ex-ante job auctions are too aggressive. Instead, in an effort to attract more highly skilled job applicants, an optimal equilibrium advertisement essentially implies a fixed wage rule (that attracts sufficient high skilled workers) and a promise that the firm will select the most skilled worker from the pool of applicants.

A second question is how the decentralized market can allocate workers to firms? The directed search literature (with identical workers) finds there is a coordination problem where several workers might apply for the same job, while some firms receive no applicants. Conversely, with differentiated firms and workers and complementary inputs, the literature on assignment problems establishes that positive assortative matching [that the best firm matches with the best worker, and so on] is the socially efficient outcome [see Roth and Sotomayor (1990) for a useful survey]. But the so-called assignment game does not explain how trading prices might be determined in a decentralized, frictional market environment.[1]

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[1]The notable exception being Shimer and Smith (2000) who consider a search model with two-sided

This paper shows that in this latter case, where firms and workers are fully differentiated [the distribution of types being common knowledge], and where firms compete in direct mechanisms, equilibrium implies both positive assortative matching and trading prices which are consistent with a 'stable outcome'. The underlying insight being that worker heterogeneity and competition in direct mechanisms solves the 'coordination problem' described in the directed search framework.

But the equilibrium advertisement is not a job auction. Unlike the competing auction literature where match values are idiosyncratic and considered as independent random draws from some exogenous distribution,[2] here workers and firms are ex-ante heterogeneous, where the distribution of types is common knowledge. Most importantly, with complementary inputs, firms wish to attract relatively highly skilled workers. Of course, an efficient mechanism will ensure that the most skilled worker that applies gets the job, but the equilibrium terms of trade are not determined by the bid of the next best qualified worker in the pool of job applicants. The following example demonstrates this point intuitively. The paper proves it formally assuming firms compete for job applicants using direct mechanisms.

1.1   Why Auctions are not Optimal : An Example

Consider a labor market where there is a unit mass of heterogeneous firms, whose type y is uniformly distributed over the interval [0,1], and a unit mass of workers with type x uniformly distributed over [0,1]. Assume that if a firm y matches with a worker x, the value of their match is $Q(x, y) = xy$, and that a firm or a worker obtain a zero payoff if they fail to match. If $u^*(x), \pi^*(y)$ describe their respective equilibrium payoffs in the assignment game, a stable matching allocation requires

$$\pi^*(y) = \max_x [Q(x, y) - u^*(x)]$$

which says that given the (reservation) equilibrium payoff of workers, firm y hires that worker which maximises firm surplus. As we have strictly complementary inputs [or supermodularity, i.e $Q_{xy} > 0$], it is well known that the unique 'stable outcome' implies

heterogeneity and Nash bargaining. They derive conditions under which assortative matching arises.

[2]See McAfee (1993), Peters (1991), Peters and Severinov (1997), Burguet and Sakovics (1999) for example, where related work in a common values environment includes Biais et al. (1999)

positive assortative matching, which in this case implies firm y hires worker $x = y$. Further, the equation above and the Envelope Theorem imply

$$\frac{d\pi^*}{dy} = \frac{\partial Q}{\partial y}(x, y).$$

As $x = y$ in the equilibrium allocation, this now implies the differential equation $\frac{d\pi^*}{dy} = y$. Further $Q(0,0) = 0$ and non-negative payoffs imply the unique solution $\pi^*(y) = \frac{1}{2}y^2$. It now follows that the unique equilibrium is that firm y hires worker $x = y$ at wage $u^*(x) = \frac{1}{2}x^2$.

However suppose now that the terms of trade are determined non-cooperatively. In particular, when firms act strategically, profit maximization suggests each firm will not offer more than the value of their employee's outside option. So what is a worker's outside option? Suppose positive assortative matching $y = x$ describes the equilibrium allocation, but that worker x' deviates from the equilibrium allocation by applying to firm $y \neq x'$? To evaluate the worker's payoff in this state, we need to know what a firm does when two workers apply for the same job. The natural guess is that the firm should use an (efficient) job auction. In that case, should worker x' deviate by visiting a more productive firm, $y > x'$, she loses the job auction to the other job candidate $x = y$ (who is more productive) and so obtains a payoff of zero. Conversely, if worker x' visits a less productive firm, $y < x'$, she will win the job auction against the other job applicant $x = y$, but the terms of trade are determined by the bid of that competing worker. As the workers' outside option is to remain unemployed (given the friction that within the same period no worker can visit another firm), then the less productive worker $x = y$ will offer full surplus $Q(y, y)$ as his equilibrium bid [the value of his outside option being zero]. Hence worker x''s optimal winning bid implies a net payoff of $w = Q(x', y) - Q(y, y)$. Should firms compete using job auctions, the presumed equilibrium allocation $x = y$ implies the value of each worker's outside option is therefore

$$u_0(x) = \max_{y \leq x}[Q(x, y) - Q(y, y)].$$

For the case $Q = xy$, this implies $u_0(x) = \frac{1}{4}x^2$. Anticipating that firms will only match the value of a worker's outside option in equilibrium, suppose $u_0(x)$ describes the equilibrium payoff of workers. Then firm y would like to hire worker x' where

$$x' = \arg\max_x[Q(x, y) - u_0(x)].$$

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Given $Q = xy$ and $u_0(x) = \frac{1}{4}x^2$, then any firm $y < \frac{1}{2}$ is best off attracting worker $x' = 2y$ at wage $w = u_0(x') = y^2$.

The point is that if the terms of trade are determined by job auctions, each firm will attempt to poach a more productive worker, rather than hire their 'designated' worker $x = y$ under the efficient allocation.[3] A firm can do so because of the friction: once workers have applied to a firm, they cannot visit another firm. And so what is the a strategy firm y may use which will attract worker $x' = 2y$ at wage y^2 [assuming all other firms use job auctions]? Clearly advertising a job auction potentially creates too much wage competition from a lesser skilled job applicant. Instead, the firm might advertise a fixed wage $w = y^2$, and state that in case several applicants apply, it will allocate the job at that wage to the highest skilled worker. As workers with skill $x > 2y$ will not apply [the value of their outside option $u_0(x) > y^2$], worker $x' = 2y$ anticipates successfully applying for this job which matches his current payoff $u_0(x')$. In particular, by precommiting not to maximize ex-post surplus, the firm is potentially able to attract this more productive worker and so increase profit.[4] Considering equilibrium trade with a finite number of traders where firms compete in direct mechanisms, this paper formally establishes that the above insight is appropriate: firms do not use job auctions and instead precommit to some fixed wage rule. The conclusion provides an intuitive interpretation for the final equilibrium outcome.

Most of the paper focusses on the two firm, two worker case. Section 2 outlines the basic model structure and section 3 describes optimal direct mechanisms. Section 4 considers a Nash equilibrium when workers are heterogeneous but firms are identical. It shows that multiple equilibria are possible, but once attention is restricted to truthful strategies [in the sense of Bernheim and Whinston (1986a,b)] equilibrium implies a unique set of prices. Section 5 then supposes both workers and firms are heterogeneous and shows that if the match value function is strictly supermodular, the (unique) truthful equilibrium implies positive assortative matching and payoffs are consistent with a stable outcome. Section 6 then generalises this last result to the N firm, N worker case.

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[3]This statement holds for any Q which is strictly supermodular.

[4]Though in this example, it is assumed that worker type is observed, we show that this holds even under anonymity. In equilibrium, sellers have the incentive to use ex ante mechanisms which are ex-post less aggressive than pure auctions.

2   The Model

Consider a labor market with two firms and two workers. Each firm has one vacancy and each worker holds one unit of indivisible labour. The workers are heterogeneous and indexed by a type $x \in \mathcal{X} = \{H, L\}$. The section that follows assumes firms are identical and for now are indexed by $y \in \mathcal{Y} = \{1, 2\}$. For any matched pair $(x, y) \in \mathcal{X} \times \mathcal{Y}$, the output generated is described by a production function $Q : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}^+$. Given identical firms, let $Q(H, y) = Q^H$, $Q(L, y) = Q^L$ where $Q^H > Q^L > 0$. $\mathcal{X}, \mathcal{Y}$ and Q are common knowledge, but the workers are anonymous.

Matching is determined by a two-stage game where firms compete in direct mechanisms. In the first stage of the game, the firms simultaneously post advertisements which describe their job (i.e. describe their type y) and a wage/job allocation mechanism. These mechanisms are described further below. However, as in the directed search literature, these posted mechanisms are assumed to be enforceable. For example, if a job advertisement states that the worker hired will be paid a certain wage, then any worker who is hired must be able to enforce that wage in a court of law. Otherwise such advertisements have no content and wages should then be determined by ex-post bargaining.

Both workers costlessly observe the posted vacancies and given the advertised 'mechanisms', each worker chooses simultaneously which firm to visit. Given those decisions, and prior to the mechanism being played, each worker observes whether the other worker has made the same choice or not. At this stage a worker can choose to walk away and so realize a payoff of zero, but cannot visit the other firm. Given any workers who remain, the firm's advertised mechanism is then played and determines the final payoffs; i.e. the job is allocated and all sidepayments are made according to the advertised mechanism.

All agents are risk neutral, expected utility maximizers. If worker x gets job y and receives a wage w, the worker obtains utility w and the firm obtains profits $Q(x, y) - w$ from the transaction. The firm obtains zero profits in case the vacancy remains unfilled.

3   An Optimal Direct Mechanism

Clearly, if a worker's type were verifiable, the firm's optimal advertisement would describe who it would hire and at what wage, depending on the worker type that applies for the

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job. However, to preserve anonymity we shall assume such information is not verifiable.[5] It turns out that this assumption is not important - in equilibrium, the advertised mechanisms will imply the workers' visit decisions fully reveal their type. We shall return to this issue in the conclusion.

Given workers are anonymous and that their type is not verifiable, each firm is restricted to posting a direct mechanism. Such a mechanism invites any workers who apply to state simultaneously their type $m \in \{L, H\}$. Depending on those "messages", the mechanism then describes what contracts are offered. Of course allowing workers to walk away prior to the mechanism being played implies no worker obtains a negative (expected) payoff.

We assume that the number of workers who apply for the job is verifiable, and so the mechanism can condition on that number. Again, in equilibrium it turns out that this assumption plays no important role but helps simplify the analysis. We now consider the optimal mechanism for each case.

3.1   The Optimal Mechanism if only one worker applies.

In this case, the (single) applicant x reports a type $m \in \{L, H\}$ and conditional on m, the mechanism states which contract this applicant is offered. As we have only two types of workers, we can focus on just two types of possible contracts. Given m, the firm either offers

(a) a flat wage contract; the worker is paid a fixed wage w and the firm's payoff is then $Q^x - w$, or

(b) a pure profit contract; the worker pays the firm some fixed amount π and retains the residual value of output $Q^x - \pi$,

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[5]Note that the anonymity assumption is not specific to the case of heterogeneous workers studied in this paper. Implicit in the literature on directed search (Montgomery (1991), Lagos (2000), Burdett, Shi and Wright (2000),...) is the assumption that firms cannot condition the allocation procedure on the *name* of the applicant, which in the case of identical agents necessarily results in the firm randomly allocating across applicants.

where an intermediate case would be a profit sharing contract.[6] By choosing w, π appropriately, it is straightforward to design a fully revealing mechanism. For example, suppose a worker reporting $m = L$ is offered the flat wage contract w, while a worker reporting $m = H$ is offered the pure profit contract with fixed fee π. Self-selection is guaranteed if (w, π) satisfy

$$Q^L - \pi < w < Q^H - \pi,$$

as applicant x then obtains greatest utility by reporting $m = x$.

Of course rather than using a separating mechanism, it may be optimal to offer a pooling mechanism, say offer only a pure profit contract with fixed fee π. In fact, we shall find that equilibrium implies worker search is always perfectly directed, where each firm obtains exactly one applicant. Furthermore, as the workers' search decisions are fully revealing, this will imply that each firm 'knows' the worker's type when it receives one applicant. Hence in equilibrium there will be no efficiency gain to using a separating mechanism when only one worker applies. Anticipating that result, we simplify the exposition by assuming each firm offers a 'pooling' mechanism. In particular, with only a small loss of generality, assume a firm offers a pure profit contract with fixed fee π when only one worker applies. Note that at this stage, this is consistent with a procurement auction, where π might be interpreted as the firm's reserve price.

3.2 The Optimal Mechanism if two workers apply.

Again, a direct mechanism requires that each applicant x reports a type $m \in \mathcal{X} = \{L, H\}$. This time with two applicants, a direct mechanism specifies a message game $\Gamma(\mathcal{X} \times \mathcal{X}, u(\cdot))$, where each participant simultaneously announces a message $m \in \mathcal{X}$, and conditional on those messages, an allocation rule and sidepayments imply an outcome function $u(\cdot) : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+ \times \mathbb{R}^+$.

Now fix arbitrary values $u_L, u_H \geq 0$. Suppose the firm wishes to construct a direct mechanism which implies the workers obtain expected payoffs u_L, u_H respectively should both apply for the job [where the assumption that either worker can walk away before the mechanism is played restricts these payoffs to being non-negative]. Obviously such a

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[6]More generally, a worker sending message m might be given a profit sharing contract (α_m, β_m) where a worker of type x who reports message m obtains contracted payoff $\alpha_m + \beta_m Q^x$.

mechanism needs to be incentive compatible. But we should be most interested in an efficient direct mechanism - one that ensures the more productive worker gets the job. If such a mechanism also implements the (given) expected utilities $u_L, u_H \geq 0$, then it must be optimal: it not only maximises joint rents, it also extracts maximal surplus [given the chosen (advertised) payoffs u_L, u_H]. The following establishes that such a direct mechanism exists for any $u_L, u_H \geq 0$. Given that, the advertising game will then determine u_L, u_H.

(A1)-(A3) describe a set of (anonymous) allocation rules which induce truth-telling as an iterated dominant strategy equilibrium *and* implement any [given] payoffs $u_L, u_H \geq 0$.

(A1) If both workers report $m = H$, the firm chooses one worker with equal probability and offers that worker a pure profit contract with fee $\pi = \frac{1}{2}\left(Q^L + Q^H\right) - 2u_L$. In this event, worker H obtains expected payoff $a = u_L + \frac{1}{4}[Q^H - Q^L] > u_L$ and worker L obtains expected payoff $b = u_L - \frac{1}{4}[Q^H - Q^L] < u_L$.

(A2) If both workers report message $m = L$, the job is not filled and both workers obtain a payoff of zero.

(A3) If one worker reports H, the other L, the worker reporting $m = L$ is given sidepayment u_L, while the worker reporting H is given the job and a pure profit contract with fee $\pi = Q^H - u_H$.

These rules imply a message game with the following normal form [where worker H plays rows (and receives the first number in the pay-off pair) and worker L plays columns].

$$m_L$$

		H	L
m_H	H	(a, b)	(u_H, u_L)
	L	$(u_L, Q^L - [Q^H - u_H])$	$(0, 0)$

Although these allocation rules respect anonymity, the use of pure profit contracts implies payoffs are not symmetric. Indeed, as allocation rule (A1) implies $a > u_L$, then for $u_H > 0$, worker $H's$ strict dominant strategy is to report $m_H = H$.[7] Further, as (A1) also implies $b < u_L$, worker $L's$ (iterated) dominant strategy is to report $m_L = L$. Hence

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[7]All equilibria described below imply $u_H > 0$. Of course, $u_H = 0$ implies only weak dominance.

allocation rules (A1)-(A3) imply that truth telling is incentive compatible. Further as the equilibrium message strategies imply truth-telling, rule (A3) then allocates the job to the most productive worker *and* implements the (advertised) payoffs $u_H, u_L \geq 0$. Hence this mechanism maximises the firm's surplus [given u_H, u_L] and so is optimal.

More intuitively, note that these rules induce truth-telling by giving a flat sidepayment to any worker reporting $m = L$, and randomly allocates a pure profit contract to any worker reporting $m = H$, where the job fee depends on how many report $m = H$. By efficiently separating the workers in this way, the firm obtains expected payoff $Q^H - u_H - u_L$.

For the remainder of the paper, attention is restricted to direct mechanisms of this form and as a result, the only payoff relevant variables when two workers apply are the firm's advertised choice of $u_L, u_H \geq 0$. In reduced form, the firm's advertisement is considered as a triple (π, u_L, u_H) where π is the firm's fee in a pure profit contract should one worker apply, while should both apply the workers obtain (u_L, u_H) respectively and the firm obtains payoff $Q^H - u_L - u_H$.

4   A Nash Equilibrium in Direct Mechanisms

The previous section implies each firm's optimal direct mechanism reduces to a triple (π, u_H, u_L). This section now characterises a (perfect) Nash equilibrium to the two stage game described above. We shall let (π', u'_L, u'_H) denote the job advert posted by firm 1, and (π, u_L, u_H) denote the advert posted by firm 2. Throughout we shall only consider equilibria where firms play pure strategies. Given those posted adverts, let $\sigma_x, x \in \{L, H\}$, denote the probability that worker x visits firm 1. We can now define an equilibrium.

Definition 1 *A (perfect) Nash equilibrium is a sextuple* $(\pi, u_L, u_H, \pi', u'_L, u'_H)$ *and a pair* (σ_H, σ_L) *where*

(a) *given* $(\pi, u_L, u_H, \pi', u'_L, u'_H)$, *then* (σ_H, σ_L) *describes a Nash equilibrium in visit strategies, and*

(b) *given those subgame visit strategies* (σ_H, σ_L), (π', u'_L, u'_H) *and* (π, u_L, u_H) *describe a Nash equilibrium in pure advertising strategies.*

Before characterising such equilibria, it is worth quickly explaining the qualitative difference implied by this mechanism approach. Typically the directed search approach assumes workers are equally productive $[Q^H = Q^L = Q]$ and that firms post a single price, a wage w. This is a special case of the above with $\pi = Q - w$ and $u_H = u_L = \frac{1}{2}w$. As such a 'mechanism' treats the workers equally, there is a co-ordination problem where each worker does not know which firm the other worker will visit. A symmetric equilibrium finds that workers randomise on their visit strategies.

But with heterogeneous workers and direct mechanisms, firms can post a price triple (π, u_L, u_H) which treats the differently skilled workers unequally [even though the job allocation rules respect anonymity]. Somewhat surprisingly, this richer pricing strategy and equilibrium results in perfectly directed search despite anonymity; there is no coordination problem with heterogeneous workers.

To see this, suppose for simplicity that both firms announce $\pi, \pi' \le Q^L$. Standard algebra[8] shows that in any subgame with randomised visiting strategies $\sigma_L, \sigma_H \in (0, 1)$:

$$\sigma_H = \frac{u_L + \pi' - Q^L}{u'_L + u_L + \pi' + \pi - 2Q^L} \tag{1}$$

$$\sigma_L = \frac{u_H + \pi' - Q^H}{u'_H + u_H + \pi' + \pi - 2Q^H}. \tag{2}$$

Also note that firm 1's expected payoff is

$$\Pi_1 = [\sigma_L(1 - \sigma_H) + \sigma_H(1 - \sigma_L)]\,\pi' + \sigma_L\sigma_H\left[Q^H - u'_H - u'_L\right],$$

where $\sigma_L(1 - \sigma_H) + \sigma_H(1 - \sigma_L)$ is the probability that exactly one worker applies for the job (and the firm obtains payoff $\pi' \le Q^L$) and $\sigma_L\sigma_H$ is the probability that both apply.

Now consider firm 1's best response, given firm 2 uses some (pure) price strategy (π, u_L, u_H). In particular, suppose that the best response of firm 1 implies both workers randomise. Given that, we can use (1) and (2) to substitute out u'_L and u'_H in the above equation for

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[8]Given σ_L, worker H is indifferent to visiting firms 1 and 2 if and only if

$$(1 - \sigma_L)\left(Q^H - \pi'\right) + \sigma_L u'_H = \sigma_L\left(Q^H - \pi\right) + (1 - \sigma_L)u_H$$

where the LHS is the expected payoff to visiting firm 1, and the RHS is the expected payoff to visiting firm 2. Re-arranging implies (2). (1) is obtained in the same way [and is symmetric].

Π_1. Simplifying implies the reduced form profit function for firm 1 is

$$\widetilde{\Pi}_1(\sigma_L, \sigma_H) = \sigma_L \left[Q^L - u_L\right] + \sigma_H \left[Q^H - u_H\right] - \sigma_L \sigma_H \left[Q^H + 2Q^L - u_H - u_L - 2\pi\right] \quad (3)$$

Given firm 2's price strategy (π, u_L, u_H), this reduced form profit function describes firm 1's payoff as a function of the visit strategies (σ_H, σ_L), where those probabilities are determined by firm 1's choice of prices and (1), (2).

Lemma 2 *Given $(\sigma_L, \sigma_H) \in [0,1] \times [0,1]$, then $\widetilde{\Pi}_1$ defined by (3) is a maximum at one of the corners where $\sigma_L, \sigma_H \in \{0,1\}$.*

Proof. Given any $\sigma^H \in [0,1]$, notice that $\widetilde{\Pi}_1$ is linear in σ^L. Hence for any such σ^H, $\widetilde{\Pi}_1$ is a maximum at $\sigma^L = 0$ or 1. Now fix $\sigma^L = 0$ or 1. $\widetilde{\Pi}_1$ is now a linear function of σ^H and hence a maximum occurs at $\sigma^H = 0$ or 1. This completes the proof of the lemma. ∎

Lemma 1 implies that an equilibrium does not exist where [along the equilibrium path] both workers randomise on which firm to visit. In particular given firm 2's price strategy, a best response by firm 1 implies σ_x is either zero or one; i.e. neither worker randomises in the subgame. The following shows how firm 1's choice of (π', u'_L, u'_H) does this. But this result is strikingly different to the coordination failure considered in the standard directed search approach. There with identical workers and $\sigma_H = \sigma_L = \sigma$, the above reduced form profit function $\widetilde{\Pi}_1$ becomes a quadratic in σ. Further an equilibrium firm 2 price strategy then implies $\widetilde{\Pi}_1$ is concave in σ [and a symmetric equilibrium implies $\sigma = \frac{1}{2}$ is payoff maximising]. However worker heterogeneity implies $\widetilde{\Pi}_1$ is never concave in σ_L, σ_H. Each firms' best response implies a corner solution where worker search is perfectly directed. The issue is how do the firms do this?

For ease of exposition, we restrict attention to equilibria where both firms make strictly positive expected profit.[9] Anticipating that the visit decisions of the two workers are polarized in equilibrium, suppose in equilibrium the high productivity worker visits firm 1 and the low productivity worker visits firm 2; i.e. $\sigma_H = 1$ and $\sigma_L = 0$. A strictly positive

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[9]Lemma 2 implies a possible equilibrium is $\sigma_x \in \{0,1\}$ and the other $\sigma_{-x} \in (0,1)$ [i.e. at most one worker mixes]. But (3) implies this outcome is a (weak) best response for both firms only if one firm makes zero profit [the one which worker x never visits]. Restricting attention to strictly positive profit equilibria implies only perfectly polarised equilibria exist; i.e. (σ_H, σ_L) is either $(1,0)$ or $(0,1)$.

profit equilibrium then requires $\pi' \in (0, Q^H]$ and $\pi \in (0, Q^L]$ (otherwise the respective workers walk away).

Given firm 2's price strategy (π, u_L, u_H), it can be shown there is a continuum of best responses (π', u'_L, u'_H) for firm 1, but that set of best responses imply the same [subgame] visit probabilities σ_H and σ_L.[10] The following Lemma identifies necessary and sufficient conditions on (π, u_L, u_H) so that a best response by firm 1 implies $(\sigma_H, \sigma_L) = (1, 0)$ in the resulting subgame.

Lemma 3 *Given $\pi \in (0, Q^L]$ and $u_L, u_H \geq 0$, necessary and sufficient conditions on (π, u_L, u_H) so that (i) $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm 1, and (ii) firm 1 makes strictly positive profits, are*
(R1) $Q^H - u_H > 0$,
(R2) $u_H - u_L \leq Q^H - Q^L$,
(R3) $2\pi \leq Q^H + Q^L - u_H$.
Further, given $(R1) - (R3)$, firm 1's best response implies

$$\pi' = Q^H - u_H, \ u'_L \leq Q^L - \pi,$$

and generates expected payoff $Q^H - u_H$.

Proof. In Appendix.

Given firm 2's strategy satisfies $(R1) - (R3)$, firm 1's best response induces equilibrium visit strategies $(\sigma_H, \sigma_L) = (1, 0)$. Firm 1 can do this by announcing

$$\pi' = Q^H - u_H - \varepsilon, \ u'_L \leq Q^L - \pi - \varepsilon, u'_H \text{ large}$$

where $\varepsilon > 0$ (but small). This price strategy sets a sufficiently low job fee π' and sufficiently large sideoffer u'_H that worker $H's$ strictly dominant strategy is to visit firm 1. Sideoffer $u'_L < Q^L - \pi$ then implies worker $L's$ optimal strategy is to visit firm 2 [and

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[10]See Coles and Eeckhout (1999) for a complete discussion. Given (π, u_L, u_H), then for any strategy (π', u'_L, u'_H) which implies both $\sigma_L, \sigma_H \in (0, 1)$, firm 1 can increase both u'_L, u'_H by some amount ε, and then increase its fee π' by some compensating amount $\theta\varepsilon$ so that the worker's expected payoffs are unchanged. This variation does not change the equilibrium visit strategies, and risk neutrality implies the firm makes the same expected profit. Hence there is a continuum of best responses which can be indexed by ε. However each best response implies the same visit probabilities σ_L, σ_H.

gets the job at price $\pi \leq Q^L$]. In this way, firm 1 obtains payoff $\pi' = Q^H - u_H - \varepsilon$ and worker H obtains payoff $Q^H - \pi' = u_H + \varepsilon$. Note that this strategy undercuts 'by a penny' worker $H's$ outside option, which is to visit firm 2 and obtain u_H. Of course existence of an equilibrium will need a suitable tie-breaking assumption so that firm 1 can set $\varepsilon = 0$ and worker H still visits firm 1 [and worker L visits firm 2].

$(R3)$ is a *competition condition*. If $(R3)$ does not hold, firm 1's best response is to poach both workers by offering $u'_H = Q^H - \pi$ and $u'_L = Q^L - \pi$.[11] That strategy would give firm 1 a payoff of $2\pi - Q^L$ and $(R3)$ is necessary to ensure this does not exceed $Q^H - u_H$. Of course in equilibrium, firm 2 will set π sufficiently small [satisfying $(R3)$] to deter firm 1 from poaching both workers. In this sense $(R3)$ describes price competition, where each firm wishes to attract at least one worker.

$(R2)$ is a *coordination condition*. It guarantees that firm 1 is better off attracting worker H (with payoff $Q^H - u_H$) rather than worker L (with payoff $Q^L - u_L$). $(R2)$ determines which worker firm 1 will choose to attract. When firm 2 announces $u_H - u_L < Q^H - Q^L$, we shall refer to this strategy as being '**weak**' - it invites firm 1 to attract the high productivity worker. Conversely announcing $u_H - u_L > Q^H - Q^L$ is called '**tough**' - it invites firm 1 to attract the low productivity worker.

We now repeat the analysis to obtain conditions where firm 2's best response is to attract the low productivity worker only.

Lemma 4 *Given $\pi' \in (0, Q^H]$ and $u'_L, u'_H \geq 0$, necessary and sufficient conditions on (π', u'_L, u'_H) so that (i) $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm 2, and (ii) firm 2 makes strictly positive profits, are*
$(R1')$ $Q^L - u'_L > 0$,
$(R2')$ $u'_H - u'_L \geq Q^H - Q^L$,
$(R3')$ $2\pi' \leq 2Q^L - u'_L$.
Given $(R1') - (R3')$, firm 2's best response implies

$$\pi = Q^L - u'_L \text{ and } u_H \leq Q^H - \pi'$$

which generates payoff $Q^L - u_L$.

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[11]By also setting $\pi' < Q^L - u_L$, $\sigma_L = 1$ is a dominant strategy for worker L, and $\sigma_H = 1$ then describes a Nash equilibrium in visit strategies.

Proof. In Appendix.

If firm 1 plays tough [i.e. $(R2')$ holds] and announces a sufficiently low job fee π' [satisfying $(R3')$] then firm 2's best response is to attract worker L using the above price strategy [with u_L large to ensure $\sigma_L = 0$ is a dominant strategy].

Using Lemmas 3 and 4, it is now straightforward to describe (perfect) Nash equilibria where $(\sigma_H, \sigma_L) = (1, 0)$ in the resulting subgame. Formally, existence requires the tie-breaking assumptions that if indifferent to visiting a firm, worker H visits firm 1 and worker L visits firm 2. The main feature however is that there is a continuum of such equilibria. We illustrate this with an example.

An equilibrium with $(\sigma_H, \sigma_L) = (1, 0)$ exists where firm 1 posts

$$\pi' = \frac{1}{2} Q^L, u'_L = 0.9 Q^L \text{ and any } u'_H > Q^H - 0.1 Q^L.$$

This offer is competitive [i.e. satisfies (R3') - π' is sufficiently small that firm 2's best response is not to attract both workers] and is 'tough' [i.e. satisfies (R2') and so firm 2's best response is to attract worker L]. It also makes a rather extravagant sideoffer $u'_L = 0.9 Q^L$ to worker L.

As this price strategy satisfies $(R1') - (R3')$, firm 2's best response implies $\pi = 0.1 Q^L$ (by Lemma 4). Note that the extravagant sideoffer u'_L of firm 1 forces firm 2 to set a very low job fee. Of course equilibrium requires that firm 1's strategy has to be a best response to firm 2's strategy. Lemma 3 therefore requires firm 2 posts $u_H = Q^H - \frac{1}{2} Q^L$. As long as u_L is large enough (i.e. firm 2 plays "weak"), then the above strategy for firm 1 and

$$\pi = 0.1 Q^L, u_H = Q^H - \frac{1}{2} Q^L \text{ and } u_L \text{ large enough,}$$

describe a perfect Nash equilibrium with $(\sigma_H, \sigma_L) = (1, 0)$. Note firm 1 obtains equilibrium payoff $\frac{1}{2} Q^L$ and firm 2 makes profit $0.1 Q^L$.[12]

This example reveals the source of multiplicity - the sideoffers u'_L, u_H determine the equilibrium job fees π, π', but the sideoffers themselves are not uniquely determined. In particular given $(R1) - (R3)$, firm 1's best response implies

$$\pi' = \frac{1}{2} Q^L, u'_L \leq 0.9 Q^L,$$

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[12]Note that though sellers are identical, they do not necessarily receive the same profits as we have not imposed symmetric pricing strategies.

where the fee π' is well determined, but the choice of u'_L only requires that it is sufficiently low that it does not attract worker L. However equilibrium requires that firm 1 offers the highest possible value of u'_L consistent with not attracting worker L. Furthermore, firm 1 must then post an even higher sideoffer u'_H to guarantee 'tough'. It is not clear why firm 1 should do this. Given firm 2's strategy satisfies $(R3)$, firm 1 does not wish to attract both workers and such large sideoffers u'_L, u'_H appear unreasonable. Indeed if worker L were to deviate and visit firm 1, firm 1 would realise a very large loss.

Such generous sideoffers are weakly dominated by posting less generous ones. This source of multiplicity is well known in the game theory literature and the concept of trembling hand perfection is typically used as the appropriate equilibrium refinement.[13] In particular, if there is a small (vanishing) probability that worker L will deviate (i.e. a tremble is possible), then such generous sideoffers are strictly dominated and the equilibrium described above will not survive this refinement. However, as Bernheim and Whinston (1986a) argue, adopting that approach in this context imposes severe difficulties. We therefore adopt the refinement used in the common agency literature, that of *truthful equilibrium*. This requires that if a firm makes sideoffers $u'_L, u'_H > 0$ then the firm's profit, should it succeed in attracting both workers, must be no lower than equilibrium profit.

Definition 5 *(Truthful Nash Equilibrium)*
A configuration $\{(\pi', u'_L, u'_H), (\pi, u_L, u_H), (\sigma_H, \sigma_L) = (1, 0)\}$ *is a Truthful Nash Equilibrium if and only if it is a Nash equilibrium and satisfies:*
(T1) $Q^H - u'_L - u'_H \geq \pi'$,
(T2) $Q^H - u_L - u_H \geq \pi$.

The restrictions (T1),(T2) imply that when it is privately optimal to attract only one worker, the sideoffers (u_H, u_L) which bid for both workers cannot be so large that profits would fall should both workers apply.

Theorem 6 (Heterogeneous Buyers) *A Truthful Nash Equilibrium with* $(\sigma_H, \sigma_L) = (1, 0)$ *exists, is unique and implies* $(\pi', u'_L, u'_H) = (\pi, u_L, u_H) = (Q^L, 0, Q^H - Q^L)$.

Proof. In Appendix.

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[13]For example the Bertrand pricing game where firms have heterogeneous marginal costs.

A Truthful Nash Equilibrium in direct mechanisms is consistent with both firms posting second price sealed bid auctions with reserve price Q^L. If one worker applies, the worker is employed at job fee Q^L, while if both apply, an auction implies worker H gets the job but must pay job fee Q^L [which can be interpreted as the equilibrium bid of worker L, though see Section 6 below]. However, note this outcome does not satisfactorily resolve the coordination problem. Neither firm plays strictly tough nor strictly weak and it is the required tie-breaking assumption for existence of equilibrium which coordinates the workers strategies. Indeed, the same pricing strategies and the converse tie breaking assumption implies a Truthful Nash Equilibrium with $\sigma_L = 1, \sigma_H = 0$. We now show how firm heterogeneity resolves this coordination problem and so provide new insights into the assignment problem.

5   Heterogeneous Firms and Workers

Again use $x \in \{L, H\}$ to index the respective workers, but with heterogeneous firms it is now useful to index the firms by $y \in \{L, H\}$. We denote output $Q(x, y) = Q^{xy} > 0$, where the first superscript refers to the worker's type, the second to the firm's type. It is assumed that $Q^{HH} > Q^{LL}$ [so that we can refer to H types as being more productive] and that types are strict complements (i.e. $Q(x, y)$ is *supermodular*)

$$Q^{HH} + Q^{LL} > Q^{HL} + Q^{LH}. \tag{4}$$

Hence the socially efficient allocation is that worker x matches with firm $y = x$, an outcome typically referred to as positive assortative matching.

Given the results of the previous section, it should not be surprising that the firms' best responses again perfectly coordinate the worker's visit decisions. But this time [Truthful Nash] Equilibrium coordinates the firms' pricing strategies. As in Theorem 6, firm L still posts a job auction - a second price sealed bid auction with reserve price $\pi = Q^{LL}$. When both workers apply to firm L, their expected payoffs are $u_H = Q^{HL} - Q^{LL}, u_L = 0$. But (4) implies $u_H - u_L < Q^{HH} - Q^{LH}$; firm $L's$ job auction is strictly weak and firm H's strict best response is to attract worker H. Equilibrium guarantees positive assortative matching.

The following establishes this insight formally. As the argument is the same as before, we only sketch details. Let σ_H, σ_L denote the workers' visit strategies, where σ_x is the probability that worker x visits firm H. Let (π', u'_L, u'_H) denote the mechanism posted by firm H, and (π, u_L, u_H) denote the mechanism posted by firm L. As before only consider strictly positive profit equilibria, where a little work establishes that each firm's best response implies $\sigma_H, \sigma_L \in \{0, 1\}$ in the subgame. The intuition is as before - each firm's best response coordinates the visit strategies of each worker. Also, only consider Truthful Nash Equilibria which now require

(T3) $Q^{HH} - u'_L - u'_H \geq \pi'$

(T4) $Q^{HL} - u_L - u_H \geq \pi$,

where the interpretation is the same as before.

Theorem 7 *Given (4), a Truthful Nash Equilibrium with $\sigma_H = 1$, $\sigma_L = 0$ exists, is unique and implies*

$$\pi = Q^{LL}$$
$$\pi' = Q^{LL} + [Q^{HH} - Q^{HL}],$$

$u'_H = u_H = Q^{HL} - Q^{LL}$, $u'_L = u_L = 0$

Proof. In Appendix.

As described above, firm L competes for the high productivity worker by offering surplus $u_H = Q^{HL} - Q^{LL}$ should both workers apply. This can be interpreted as a job auction with reserve price Q^{LL}. Such competition forces firm H to lower its job fee π' [as described in the Theorem] to attract worker H. However, unlike Theorem 6, the firm strategies described in Theorem 7 are strictly coordinated. The relevant coordination condition (R2) which implies a weak strategy by firm L is

$$(R2) \ u_H - u_L \leq Q^{HH} - Q^{LH}.$$

As firm L sets $u_H - u_L = Q^{HL} - Q^{LL}$, (4) implies that (R2) holds with strict inequality, i.e. firm L plays strictly weak. Hence firm H strictly prefers to attract worker H. Indeed, note that unlike Theorem (6), $(\sigma_H, \sigma_L) = (1, 0)$ is a dominant strategy equilibrium should both firms shave their reserve prices π', π by a 'penny'.

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Theorem 8 *Given (4), a Truthful Nash equilibrium with $\sigma_H = 0$, $\sigma_L = 1$ does not exist.*

Proof. In Appendix.

Together Theorems 7 and 8 imply that (4) and the restriction to Truthful Nash Equilibria implies the firm's pricing strategies are perfectly coordinated. The low productivity firm plays strictly weak and attracts the low productivity worker. That firm also competes for the high productivity worker by posting a job auction, but firm H matches the implied sideoffer to attract the high productivity worker. These price strategies then perfectly direct the workers' search strategies and guarantees positive assortative matching. Given these insights we can now generalise to the N firm, N worker case.

6 The $N \times N$ Case

We now consider the general assignment problem with N differentiated firms and workers, where in the first stage of the game each firm simultaneously posts a direct mechanism, and in the second stage each worker chooses simultaneously which firm to visit. Clearly this case is much more complicated than the 2×2 case. Although we believe the equilibrium described below is the unique Truthful Nash Equilibrium, we can only establish that it describes such an equilibrium. Nevertheless this approach confirms how positive assortative matching arises as the outcome of a Truthful Nash Equilibrium with competing direct mechanisms. The underlying insight is that each firm essentially advertises a single price - a reservation job fee - and then constructs a mechanism which implies the most qualified worker who applies gets the job, and that worker retains the entire surplus [output less the firm's stated job fee]. Firms do not advertise job auctions.

Suppose there are N workers indexed by type i with productivities x_i satisfying $x_1 < x_2 < x_3..... < x_N$ and N firms indexed by type j and productivities y_j satisfying $y_1 < y_2..... < y_N$. Production between worker i and firm j is denoted $Q(x_i, y_j)$ where $Q(x_1, y_1) > 0$, is strictly increasing in both arguments and strictly supermodular:

$$Q(x_i, y_i) + Q(x_j, y_j) > Q(x_i, y_j) + Q(x_j, y_i) \text{ for all } i \text{ and } j \neq i.$$

Each firm j simultaneously advertises a direct mechanism. Given those advertisements, each worker simultaneously chooses which firm to visit. Let σ_{ij} denote the probability that worker i visits firm j. As before we wish to find a Truthful Nash equilibrium.

Let u_i^* denote worker i's expected payoff, and π_j^* be firm j's expected payoff in a Truthful Nash equilibrium. We only consider equilibria where positive assortative matching describes the final equilibrium allocation; that $\sigma_{ii} = 1$ for all i, and σ_{ij} for all $i \neq j$. This implies that equilibrium payoffs satisfy

$$\pi_i^* = Q(x_i, y_i) - u_i^*, \text{ for all } i. \tag{5}$$

The 2×2 case suggests that payoffs might be determined by competition in second price sealed bid auctions; that is

$$u_i^* = Q(x_i, y_{i-1}) - Q(x_{i-1}, y_{i-1}),$$

where worker i's outside option is to visit firm $i - 1$ and obtain the job according to a second price job auction. That result does not extend to the NxN case. The equations above would imply $u_{i+1}^* + \pi_i^* = Q(x_{i+1}, y_i) - u_i^*$, and as $u_i^* > 0$ for $i > 1$ we would then obtain $u_{i+1}^* + \pi_i^* < Q(x_{i+1}, y_i)$ for such i. Hence competition in job auctions would imply a gain to trade exists between each worker $i + 1$ and firm i (for $i > 1$). Not surprisingly, competition in direct mechanisms bids such surpluses away. The appropriate insight is that a Truthful Nash Equilibrium with competition in direct mechanisms implies Bertrand competition.

A Truthful Nash Equilibrium will find that each firm j's direct mechanism bids competitively for worker $j + 1$. The restriction to truthful strategies constrains firm j's outside offer "u_H" for worker $j + 1$ to the point where firm j is just indifferent to attracting that worker. As firm $j + 1$ matches that offer in equilibrium, worker $j + 1$'s equilibrium payoff satisfies

$$u_{j+1}^* = Q(x_{j+1}, y_j) - \pi_j^*. \tag{6}$$

For existence of equilibrium we shall need the formal tie breaking assumption that if worker $j + 1$ is indifferent to visiting firm $j + 1$ or j then the worker chooses to visit firm $j + 1$.

The (candidate) equilibrium payoffs are now defined recursively by (5) and (6), with starting value $u_1^* = 0$ [where truthful strategies imply worker 1 receives no outside bids and so firm 1 extracts full surplus]. Note that given u_i^*, (5) determines firm i's equilibrium profit π_i^* and (6) then determines u_{i+1}^* and so on. Most importantly, these payoffs satisfy what

is defined as a "stable outcome" in the literature on the assignment game [see Koopmans and Beckmann (1957), Shapley and Shubik (1972), Roth and Sotomayor (1990)]. Strict supermodularity implies uniqueness. The firms' profits satisfy

$$\pi_j^* = \max_i [Q(x_i, y_j) - u_i^*]$$

and there is no gain to trade between firm j and any other worker $i \neq j$, and of course

$$u_i^* = \max_j [Q(x_i, y_j) - \pi_j^*].$$

Given these (candidate) equilibrium payoffs, we now construct the equilibrium direct mechanisms. As positive assortative matching implies each firm j receives exactly one applicant in equilibrium, then this candidate equilibrium requires each firm j specifies the fee $\pi = \pi_j^*$ if one worker shows.

Now consider the equilibrium direct mechanism should 2 workers apply. As firm j will use this mechanism to compete for worker $j + 1$, suppose that if two workers apply, firm j believes they are workers j and $j + 1$. An equilibrium direct mechanism corresponds to the one previously defined by rules (A1)-(A3), where each applicant is asked to report $m \in \{L, H\}$, and allocations and prices are determined using $Q^H \equiv Q(x_{j+1}, y_j)$, $Q^L \equiv Q(x_j, y_j)$, $u_L = 0$ and $u_H = u_{j+1}^*$; i.e. firm j bids u_H equal to worker $j + 1's$ equilibrium payoff. Note that if the message pair is (L, H), allocation rule (A3) and (6) imply the job is allocated to the worker reporting $m = H$ at fee π_j^*; i.e. the firm continues to extract its equilibrium payoff [and so this mechanism is truthful]. Also note that if the message pair is (H, H) the job is randomly allocated at a fee $\frac{1}{2}[Q(x_{j+1}, y_j) + Q(x_j, y_j)]$ which is strictly greater than π_j^*. Hence the job fee is increasing in the number of workers who report $m = H$, and so whenever two workers apply, the job is either filled with a fee no lower than π_j^*, or is not filled at all. The aim now is to show that this set of mechanisms and positive assortative matching describes a Truthful Nash Equilibrium.

Theorem 9 *A Truthful Nash equilibrium exists where each firm j uses the direct mechanism described above, and there is positive assortative matching.*

Proof.   There are two steps. The first step is to show that given these mechanisms, $\sigma_{ii} = 1$ for all i describes an equilibrium in visit strategies. The second step is to show

that given all firms post the above mechanisms, no firm j can increase profit by posting a different one.

Step 1: Suppose all workers $i \neq k$ choose $\sigma_{ii} = 1$ and consider the optimal strategy of worker k given those strategies and the above mechanisms. By visiting any other firm $j \neq k$, the direct mechanism [given two workers apply] implies she can get the job at a fee no lower than π_j^*. As these prices are consistent with a stable outcome, $\sigma_{kk} = 1$ is privately optimal for all k.

Step 2 : Suppose all firms $j \neq k$ post the above mechanisms and suppose firm k deviates by posting $\pi > \pi_k^*$; she raises her job fee in the event of only one worker applying. Then regardless of whatever else she specifies in her mechanism, the corresponding subgame implies the workers' equilibrium visit strategies are (a) $\sigma_{ii} = 1$ for $i > k$, (b) $\sigma_{i,i-1} = 1$ for $i \leq k$ and $i > 1$, (c) worker 1 walks away.[14]

To see why, note that these strategies imply the deviating firm k does not get an applicant $[\sigma_{ik} = 0$ for all $i]$ while all other firms attract one worker. Given this set of visit strategies, we now show that no worker has an incentive to deviate, and so (a)-(c) describes a [subgame] equilibrium.

First consider (a) - those workers $i > k$. By visiting firm i they obtain payoff u_i^*. By deviating and visiting any firm $j \neq i$, the posted mechanisms imply worker i expects to pay a fee no lower than π_j^* for the job at that firm. As the set of equilibrium payoffs describe a stable outcome, it follows that $\sigma_{ii} = 1$ is privately optimal for these workers. Hence (a). Now consider worker $i = k$. As firm k has raised her job fee $\pi > \pi_k^*$, worker k now visits firm $k - 1$. As visit strategies (a)-(c) imply she is the only applicant, she obtains the job at fee π_{k-1}^* and by (6), obtains her original equilibrium payoff u_k^*. Worker $i = k - 1$ now realises that worker k will apply for the job at firm $k - 1$. Further, worker k's (iterated) dominant strategy is to report $m = H$ in firm k's 2-person message game. As allocation rule (A1) implies getting the job at firm $k - 1$ would now involve paying a job fee which exceeds π_{k-1}^*, worker $k - 1$'s optimal strategy is instead to apply to firm $k - 2$ and so obtain her original equilibrium payoff u_{k-1}^*. The argument continues in the

[14]Note if worker 1 visits any firm, her [iterated] dominant strategy is to report $m = L$, i.e. this worker automatically realises a zero payoff. It is more convenient to assume this worker does not participate at all, and simply walks away.

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same way where all workers $i < k$ apply to firm $i - 1$, and each has an iterated dominant strategy to report $m = H$ in firm $i - 1$'s 2-person message game. Of course worker $i = 1$ has nowhere else to go. Hence $(a) - (c)$ describes an equilibrium in visit strategies. The critical feature is that all workers obtain their original expected payoffs u_i^*.

The central feature is that these mechanisms perfectly coordinate the workers' visit strategies. In particular, given this (coordinated) best response in the subgame by workers, each worker i is guaranteed a payoff of at least u_i^*. Hence to attract at least one worker with positive probability, firm k must set job fee $\pi \le \pi_k^*$ and a mechanism which offers an expected payoff of at least u_i^* for some worker i. But as these payoffs describe a stable outcome, an optimal mechanism is to attract worker k with a job fee $\pi = \pi_k^*$. Hence the stated mechanism is an optimal strategy and we have identified a Truthful Nash equilibrium. ■

7 Conclusion

The paper has shown that when workers are heterogeneous, and firms compete in direct mechanisms, worker search is perfectly directed. Similar in spirit to Moen (1997), equilibrium implies different types do not crowd out each other's matching probabilities. However unlike Moen (1997), here such coordination does not depend on competing market-makers. Instead, firms post reasonably rich price mechanisms and equilibrium implies worker search is perfectly coordinated. Of course, coordination problems will re-appear should there exist several workers of the same type. While the directed search model is a useful way of thinking about market frictions within groups of identical workers, our contribution is to establish that across heterogeneous agents, applicants will sort themselves into those firms that correspond to the efficient allocation. Our theory predicts that pools of applicants will be homogeneous in their ability.

With complementary inputs, and when attention is restricted to truthful strategies, equilibrium implements the first best allocation - there is positive assortative matching and prices are consistent with a stable outcome. The equilibrium terms of trade are akin to Bertrand competition where each firm's mechanism competes for the next more productive worker [relative to the one they actually succeed in hiring]. In equilibrium and in reduced form, firm y posts a single job fee $\pi^*(y)$ and promises to hire the most productive

worker that applies with payoff $w = Q(x, y) - \pi^*(y)$. Of course, to retain anonymity it was assumed that the equilibrium mechanism could not condition on the worker's type. But if skills are fully verifiable, this mechanism reduces to advertising a wage schedule $w_y(x) \equiv Q(x, y) - \pi^*(y)$ and a proviso that the most productive worker who applies will be hired. For the example in section 1.1, each firm y might post a linear wage schedule $w_y(x) = xy - 0.5y^2$, where a more productive firm posts a steeper wage schedule and a smaller intercept. Such advertisements then perfectly direct worker search; each worker x applies to firm $y = x$. Worker x does not apply to firm $y' > x$ as she won't get the job, and does not apply to firm $y' < x$ as the offered wage $w_{y'}(x)$ is lower than $w_y(x)$.

It has also been shown that in equilibrium, the only relevant wage-bids made by each firm y, are those made for the equilibrium applicant $x = y$, and for the next more productive worker $x' > x$. Note that the continuum case considered earlier implies $x' - x = 0$. In that case an even simpler equilibrium mechanism exists: each firm y simply announces a fixed wage $w_y = Q(y, y) - \pi^*(y)$ and promises to hire the most productive worker that applies. As before, these advertisements induce positive assortative matching and the arguments presented in section 6 imply these advertisements are privately optimal.

8 Appendix

Given $\pi \leq Q^L$ and $u_L, u_H, u'_L, u'_H \geq 0$, a Nash equilibrium in visiting strategies $\sigma_L, \sigma_H \in [0, 1]$ implies

$$\text{if } (1 - \sigma_L) \max[Q^H - \pi', 0] + \sigma_L u'_H \; > \; \sigma_L[Q^H - \pi] + (1 - \sigma_L)u_H \text{ then } \sigma_H = 1$$
$$\text{if } (1 - \sigma_L) \max[Q^H - \pi', 0] + \sigma_L u'_H \; = \; \sigma_L[Q^H - \pi] + (1 - \sigma_L)u_H \text{ then } \sigma_H \in [0, 1]$$
$$\text{if } (1 - \sigma_L) \max[Q^H - \pi', 0] + \sigma_L u'_H \; < \; \sigma_L[Q^H - \pi] + (1 - \sigma_L)u_H \text{ then } \sigma_H = 0$$

$$\text{if } (1 - \sigma_H) \max[Q^L - \pi', 0] + \sigma_H u'_L \; > \; \sigma_H[Q^L - \pi] + (1 - \sigma_H)u_L \text{ then } \sigma_L = 1$$
$$\text{if } (1 - \sigma_H) \max[Q^L - \pi', 0] + \sigma_H u'_L \; = \; \sigma_H[Q^L - \pi] + (1 - \sigma_H)u_L \text{ then } \sigma_L \in [0, 1]$$
$$\text{if } (1 - \sigma_H) \max[Q^L - \pi', 0] + \sigma_H u'_L \; < \; \sigma_H[Q^L - \pi] + (1 - \sigma_H)u_L \text{ then } \sigma_L = 0$$

where if only one worker visits firm 1, that worker x walks away and obtains a zero payoff should the job fee π' exceed Q^x [and note $\pi \leq Q^L$].

Proof of Lemma 3

Fix (π, u_L, u_H) satisfying $\pi \leq Q^L$ and $u_L, u_H \geq 0$. We prove $(R1) - (R3)$ are necessary and sufficient conditions in turn.

(i) $(R1) - (R3)$ **are necessary**.

First consider all those strategies (π', u'_L, u'_H) which imply $(\sigma_H, \sigma_L) = (1, 0)$ is a Nash equilibrium. Given $\sigma_L = 0$, then $\sigma_H = 1$ requires $\max[Q^H - \pi', 0] \geq u_H$. Given $\sigma_H = 1$, then $\sigma_L = 0$ requires $u'_L \leq [Q^L - \pi]$. This equilibrium outcome and strictly positive profit also requires $\pi' \leq Q^H$ [otherwise worker H walks away]. As $u_H \geq 0$, it now follows that any best response by firm 1 which implies $(\sigma_H, \sigma_L) = (1, 0)$ and which also generates strictly positive profit implies $\pi' \leq Q^H - u_H$, $u'_L \leq Q^L - \pi$ and generates firm 1 profit $\pi' \leq Q^H - u_H$. We now consider (R1)-(R3) in turn.

(R1) is necessary, otherwise if a best response exists which generates $(\sigma_H, \sigma_L) = (1, 0)$, it implies profit $\pi' \leq Q^H - u_H \leq 0$ which contradicts strictly positive profit.

(R2) is necessary, otherwise firm 1 might post

$$\pi' = Q^L - u_L - \varepsilon, u'_H \leq Q^H - \pi, u'_L \text{ large}$$

where $\varepsilon > 0$. This strategy implies $(\sigma_H, \sigma_L) = (0, 1)$ is an iterated dominant strategy equilibrium and generates profit $Q^L - u_L - \varepsilon$. If $(R2)$ does not hold, this payoff strictly dominates $Q^H - u_H$ for ε small enough, which contradicts $(\sigma_H, \sigma_L) = (1, 0)$ being consistent with a best response by firm 1.

$(R3)$ is necessary, otherwise firm 1 might post

$$\pi' \text{ small enough, } u'_L = Q^L - \pi - \varepsilon, \ u'_H = Q^H - \pi - \varepsilon$$

where $\varepsilon > 0$. This strategy implies $(\sigma_H, \sigma_L) = (1, 1)$ is a dominant strategy equilibrium and generates profit $Q^H - u'_L - u'_H = 2\pi - Q^L - 2\varepsilon$. If $(R3)$ does not hold, this payoff strictly dominates $Q^H - u_H$ for ε small enough, which contradicts $(\sigma_H, \sigma_L) = (1, 0)$ being consistent with a best response by firm 1.

(ii) $(R1) - (R3)$ **are sufficient.**

Consider the conditions determining σ_H in a Nash equilibrium [described above]. Multiplying both sides by σ_H, these conditions imply $\sigma_H \in [0, 1]$ and

$$\sigma_H \sigma_L u'_H \geq \sigma_H \left[\sigma_L [Q^H - \pi] + (1 - \sigma_L) u_H - (1 - \sigma_L) \max[Q^H - \pi', 0] \right] \quad (7)$$

Similarly, the conditions determining σ_L imply $\sigma_L \in [0, 1]$ and

$$\sigma_L \sigma_H u'_L \geq \sigma_L \left[\sigma_H [Q^L - \pi] + (1 - \sigma_H) u_L - (1 - \sigma_H) \max[Q^L - \pi', 0] \right] \quad (8)$$

Given (R1)-(R3) we now show that $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm 1.

First note that the strategy described in the lemma implies $(\sigma_H, \sigma_L) = (1, 0)$ is a Nash equilibrium and generates profit $Q^H - u_H > 0$. We now show this strategy is profit maximising by considering all other possible strategies. We do this by considering three separate cases.

(a) Consider all those strategies satisfying $\pi' \leq Q^L$. In this case, both workers will accept employment if only one worker shows, and firm 1's expected payoff is then

$$\Pi_1 = \left[\sigma_L (1 - \sigma_H) + \sigma_H (1 - \sigma_L) \right] \pi' + \sigma_L \sigma_H \left[Q^H - u'_H - u'_L \right].$$

Using (7),(8) to substitute out u'_L, u'_H and rearranging implies

$$\begin{aligned}
\Pi_1 \leq \ & [Q^H - u_H] - [1 - \sigma_H - \sigma_L + \sigma_L \sigma_H][Q^H - u_H] \\
& - \sigma_L \sigma_H [Q^L + Q^H - u_H - 2\pi] \\
& - \sigma_L (1 - \sigma_H)[Q^H - u_H - Q^L + u_L].
\end{aligned}$$

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Also note that $1 - \sigma_H - \sigma_L + \sigma_L \sigma_H \geq 0$ for all $\sigma_H, \sigma_L \in [0,1]$. Hence $(R1) - (R3)$ and $\sigma_H, \sigma_L \in [0,1]$ imply $\Pi_1 \leq Q^H - u_H$, and so all strategies with $\pi' \leq Q^L$ are dominated by the one described in the lemma.

(b) Consider all those strategies satisfying $Q^L < \pi' \leq Q^H$. In this case, worker L walks away if the sole applicant, and firm 1's expected payoff is

$$\Pi_1 = \sigma_H \left[(1 - \sigma_L)\pi' + \sigma_L(Q^H - u'_H - u'_L) \right].$$

Using (7),(8) to substitute out u'_L, u'_H and rearranging implies

$$\Pi_1 \leq \sigma_H[Q^H - u_H] - \sigma_L \sigma_H \left[Q^H + Q^L - u_H - 2\pi \right] - \sigma_L(1 - \sigma_H)u_L$$

Hence $(R1) - (R3)$, $\sigma_H, \sigma_L \in [0,1]$ and $u_L \geq 0$ imply $\Pi_1 \leq [Q^H - u_H]$ as required.

(c) Consider all those strategies satisfying $\pi' > Q^H$. This time firm 1's expected payoff is

$$\Pi_1 = \sigma_L \sigma_H (Q^H - u'_H - u'_L).$$

Using (7),(8) to substitute out u'_L, u'_H and rearranging implies

$$\Pi_1 \leq \sigma_L \sigma_H[Q^H - u_H] - \sigma_L \sigma_H \left[Q^H + Q^L - u_H - 2\pi \right] - \sigma_H(1 - \sigma_L)u_H - \sigma_L(1 - \sigma_H)u_L$$

Hence (R1)-(R3), $\sigma_H, \sigma_L \in [0,1]$ and $u_L, u_H \geq 0$ imply $\Pi_1 \leq [Q^H - u_H]$ as required.

Hence (R1)-(R3) are sufficient to imply the strategy described in the lemma is a best response by firm 1 and implies $(\sigma_H, \sigma_L) = (1,0)$ is a Nash equilibrium in the resulting subgame.

This completes the proof of Lemma 3.∎

Proof of Lemma 4

Note that strictly positive profit for firm 2, and a best response by firm 2 which is consistent with $(\sigma_H, \sigma_L) = (1,0)$, requires $\pi \leq Q^L$ [otherwise worker L walks away]. Given $\pi \leq Q^L$, the conditions stated at the start of the Appendix describe any Nash equilibrium in visit strategies (σ_H, σ_L).

Now fix (π', u'_L, u'_H) with $\pi' \leq Q^H$ and $u'_L, u'_H \geq 0$. Proving the lemma requires 2 steps.

Step 1 : $\pi' \leq Q^L$ is necessary.

Proof of Step 1: Using the same argument demonstrated in the proof of lemma 3, it follows that any strategy which implies $(\sigma_H, \sigma_L) = (1, 0)$ is a Nash equilibrium in visit strategies and generates strictly positive profit, implies firm 2 payoff $\pi \leq Q^L - u'_L$. Now suppose $Q^L < \pi' \leq Q^H$. By posting

$$\pi \text{ small enough}, u_L = \varepsilon, u_H = Q^H - \pi' + \varepsilon$$

where $\varepsilon > 0$, this strategy implies $(\sigma_H, \sigma_L) = (1, 1)$ is a dominant strategy equilibrium and generates payoff of $\pi' - 2\varepsilon$. But $\pi' > Q^L$ and ε small enough implies this payoff exceeds $Q^L - u'_L$, which contradicts that $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm 2.

Step 2 : As $\pi' \leq Q^L$ is necessary for $(\sigma_H, \sigma_L) = (1, 0)$ being consistent with a best response by firm 2, the proof which established lemma 3 is now easily adapted to prove lemma 4.∎

Proof of Theorem 6

Any strictly positive profit Nash equilibrium with $(\sigma_H, \sigma_L) = (1, 0)$ in the subgame requires that this outcome is consistent with both firms playing best responses. By lemmas 3,4 those best responses require $(R1) - (R3), (R1') - (R3')$ and imply

$$\pi' = Q^H - u_H, \ \pi = Q^L - u'_L. \tag{9}$$

Using (9) to substitute out π, π' in $(R1) - (R3), (R1') - (R3')$, we obtain the equilibrium constraints:

$$(R1) : u_H < Q^H, (R2) : u_H - u_L \leq Q^H - Q^L, (R3) : u_H - 2u'_L \leq Q^H - Q^L$$

$$(R1') : u'_L < Q^L, (R2') : u'_H - u'_L \geq Q^H - Q^L, (R3') : 2u_H - u'_L \geq 2[Q^H - Q^L].$$

where $u'_H, u'_L, u_H, u_L \geq 0$. Also use (9) to substitute out π, π' in $(T1), (T2)$ to obtain the truthfulness conditions:

$$u'_H - u_H \leq -u'_L \tag{10}$$

$$u_L + u_H - u'_L \leq Q^H - Q^L. \tag{11}$$

The problem now is to solve these conditions for u'_H, u'_L, u_H, u_L.

Note (R2) and (R2′) imply $u'_H - u_H \geq u'_L - u_L$. With (10) this implies $u'_L - u_L \leq -u'_L$, and so $u_L \geq 2u'_L$.

Subtracting (R3) from (R3′) implies $u_H + u'_L \geq Q^H - Q^L$. With (11) this implies $u_L + u_H - u'_L \leq u_H + u'_L$, and so $u_L \leq 2u'_L$. Hence $u_L = 2u'_L$.

We can now substitute out u_L. (R2) becomes $u_H - 2u'_L \leq Q^H - Q^L$, and (11) becomes $u_H + u'_L \leq Q^H - Q^L$. Adding these two inequalities implies $2u_H - u'_L \leq 2[Q^H - Q^L]$, and with $(R3')$ this now implies

$$2u_H = u'_L + 2[Q^H - Q^L].$$

Using these solutions to substitute out u_L, u_H in (11) now implies $u'_L \leq 0$. Hence $u'_L = 0$ and so $u_L = 0$ and $u_H = Q^H - Q^L$. (10) and $(R2')$ now imply $u'_H = Q^H - Q^L$. Direct inspection shows that these values satisfy all the above conditions, which completes the proof of the Theorem.■

Proof of Theorem 7

As the structure of the proof is identical to the proof of Theorem 6 we only sketch details. First we must obtain the conditions analogous to lemmas 3 and 4.

Lemma A2 : Given $\pi \in (0, Q^{LL}]$, necessary and sufficient conditions on (π, u_L, u_H) so that (i) $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm H, and (ii) firm H makes strictly positive profits, are:

$(R1)$ $Q^{HH} - u_H > 0$

$(R2)$ $u_H - u_L \leq Q^{HH} - Q^{LH}$

$(R3)$ $2\pi \leq Q^{HL} + Q^{LL} - u_H$

Firm H's best response implies

$$\pi' = Q^{HH} - u_H \text{ and } u'_L \leq Q^{LL} - \pi.$$

Proof. The argument used to prove lemma 3 applies directly. Note (R3) arises as firm H can poach both workers by offering $u'_L = Q^{LL} - \pi, u'_H = Q^{HL} - \pi$, and the corresponding profit $Q^{HH} - u'_H - u'_L$ cannot exceed $Q^{HH} - u_H$. ■

Lemma A3 : Given $\pi' \in (0, Q^{HH}]$, necessary and sufficient conditions on (π', u'_L, u'_H) so that (i) $(\sigma_H, \sigma_L) = (1, 0)$ is consistent with a best response by firm L, and (ii) firm L makes strictly positive profits, are:

(R1') $Q^{LL} - u'_L > 0$

(R2') $u'_H - u'_L \geq Q^{HL} - Q^{LL}$,

(R3') $\pi' - \max[Q^{LH} - \pi', 0] + u'_L \leq Q^{HH} + Q^{LL} - Q^{HL}$.

Firm $L's$ best response implies

$$\pi = Q^{LL} - u'_L \text{ and } u_H \leq Q^{HH} - \pi'.$$

Proof. Is straightforward by adapting the proof of lemmas 3,4. $(R3')$ arises because firm L can poach both workers by posting $u_H = Q^{HH} - \pi'$ and $u_L = \max[Q^{LH} - \pi', 0]$ (where worker L walks away from firm H if $\pi' > Q^{LH}$) and the corresponding profit $Q^{HL} - u_H - u_L$ cannot exceed $Q^{LL} - u'_L$.∎

Any strictly positive profit Nash equilibrium with $(\sigma_H, \sigma_L) = (1, 0)$ in the subgame requires that this outcome is consistent with both firms playing best responses. Lemmas A2,A3 imply

$$\pi' = Q^{HH} - u_H, \ \pi = Q^{LL} - u'_L. \tag{12}$$

Using (12) to substitute out π, π' in $(R1) - (R3), (R1') - (R3')$ in lemmas A2,A3 implies equilibrium constraints

$$(R1) \ u_H < Q^{HH}, (R2) \ u_H - u_L \leq Q^{HH} - Q^{LH}, (R3) \ u_H - 2u'_L \leq Q^{HL} - Q^{LL}$$

$$(R1') \ u'_L < Q^{LL}, (R2') \ u'_H - u'_L \geq Q^{HL} - Q^{LL},$$

$$(R3') \ u_H + \max[u_H - Q^{HH} + Q^{LH}, 0] - u'_L \geq Q^{HL} - Q^{LL},$$

and (12) in $(T3), (T4)$ implies truthfulness conditions

$$u_H - u'_L - u'_H \geq 0. \tag{13}$$

$$u_L + u_H - u'_L \leq Q^{HL} - Q^{LL}. \tag{14}$$

Again the problem is to solve these constraints for $u'_H, u'_L, u_H, u_L \geq 0$.

Lemma A4 : A solution does not exist with $u_H \geq Q^{HH} - Q^{LH}$.

Proof : By contradiction. Suppose $u_H \geq Q^{HH} - Q^{LH}$, and so $(R3')$ reduces to

$$(R3') : 2u_H - u'_L \geq Q^{HH} + Q^{HL} - Q^{LH} - Q^{LL}. \qquad (15)$$

Subtracting $(R2')$ from $(R2)$ and using (13) implies $2u'_L - u_L \leq Q^{HH} - Q^{LH} - Q^{HL} + Q^{LL}$. Subtracting $(R3)$ from (15) implies $u_H + u'_L \geq Q^{HH} - Q^{LH}$. With (14) it follows that $2u'_L - u_L \geq Q^{HH} - Q^{LH} - Q^{HL} + Q^{LL}$. Hence

$$u_L = 2u'_L - [Q^{HH} - Q^{LH} - Q^{HL} + Q^{LL}].$$

Adding $(R2)$ and (14) gives $2u_H - u'_L \leq Q^{HH} + Q^{HL} - Q^{LH} - Q^{LL}$. (15) now implies $2u_H - u'_L = Q^{HH} + Q^{HL} - Q^{LH} - Q^{LL}$.

Substituting out u_L, u_H in (14) implies $u'_L \leq \frac{1}{3}[Q^{HH} - Q^{HL} - Q^{LH} - Q^{LL}]$. But the above solution for u_L now implies $u_L \leq -\frac{1}{3}[Q^{HH} - Q^{HL} - Q^{LH} + Q^{LL}]$ and (4) implies the required contradiction. This completes the proof of Lemma A4.∎

Hence if a solution exists, it implies $u_H < Q^{HH} - Q^{LH}$ and $(R3')$ becomes $u_H - u'_L \geq Q^{HL} - Q^{LL}$. (14) immediately implies $u_L = 0$ and also that $u_H = u'_L + Q^{HL} - Q^{LL}$. Adding $(R2')$ and (13) imply $u_H - 2u'_L \geq Q^{HL} - Q^{LL}$. Substituting out u_H, using the solution given, now implies $u'_L \leq 0$. Hence $u'_L = 0$. Finally (13) and $(R2')$ now imply $u'_H = Q^{HL} - Q^{LL}$. Given (4), direct inspection shows that these values satisfy all of the above conditions, which completes the proof of the Theorem.∎

Proof of Theorem 8

The methodology is identical to the proofs of Theorems 6 and 7. We sketch the essential points.

Assuming an equilibrium with $(\sigma_H, \sigma_L) = (0, 1)$ exists, then the usual argument implies this is consistent as a best response for firm H if and only if : $(R1) : Q^{LH} - u_L > 0$, $(R2) : u_H - u_L \geq Q^{HH} - Q^{LH}$, and $(R3) : Q^{HH} - \max[Q^{HL} - \pi, 0] - \max[Q^{LL} - \pi, 0] \leq Q^{LH} - u_L$. In that case firm H posts $\pi' = Q^{LH} - u_L$.

This outcome is also a best response for firm L if and only if $(R1')$: $Q^{HL} - u'_H > 0$, $(R2') : u'_H - u'_L \leq Q^{HL} - Q^{LL}$, and $(R3')$: $Q^{HL} - \max[Q^{HH} - \pi', 0] - \max[Q^{LH} - \pi', 0] \leq Q^{HL} - u'_H$. In that case, firm L posts $\pi = Q^{HL} - u'_H$.

Given $\pi = Q^{HL} - u'_H$, a contradiction argument using $(R3)$ and (4) implies $u'_H \geq Q^{HL} - Q^{LL}$. Substituting out π in $(R3)$ now implies

$$2u'_H - u_L \geq Q^{HH} + Q^{HL} - Q^{LH} - Q^{LL}. \tag{16}$$

Similarly, substituting out $\pi' = Q^{LH} - u_L$ in $(R3')$ implies

$$u'_H - 2u_L \leq Q^{HH} - Q^{LH}. \tag{17}$$

Also truthful strategies requires

$$u'_H + u'_L - u_L \leq Q^{HH} - Q^{LH}, \tag{18}$$

$$u_H + u_L - u'_H \leq 0. \tag{19}$$

Again the problem is to solve these constraints for $u'_H, u'_L, u_H, u_L \geq 0$. The Theorem is established by proving no solution exists.

Subtracting (17) from (16) implies

$$u'_H + u_L \geq Q^{HL} - Q^{LL} \tag{20}$$

and subtracting (20) from (18) gives

$$u'_L - 2u_L \leq Q^{HH} - Q^{LH} - Q^{HL} + Q^{LL}. \tag{21}$$

Now $(R2), (R2')$ imply

$$u_H - u_L - u'_H + u'_L \geq Q^{HH} - Q^{HL} - Q^{LH} + Q^{LL}. \tag{22}$$

and subtracting (19) from (22) gives

$$u'_L - 2u_L \geq Q^{HH} - Q^{HL} - Q^{LH} + Q^{LL} \tag{23}$$

Hence (21) and (23) imply

$$u'_L = 2u_L + Q^{HH} - Q^{HL} - Q^{LH} + Q^{LL} \tag{24}$$

Substituting out u'_L in (22) using (24) gives

$$u_H + u_L - u'_H \geq 0 \tag{25}$$

Hence (25) and (19) imply

$$u'_H = u_H + u_L \tag{26}$$

Substituting out u'_H in (20) using (26) gives $u_H + 2u_L \geq Q^{HL} - Q^{LL}$ while substituting out u'_L, u'_H in (18) using (24) and (26) implies $u_H + 2u_L \leq Q^{HL} - Q^{LL}$. Hence $u_H + 2u_L = Q^{HL} - Q^{LL}$. Using (26) to substitute out u'_H, and this latter condition to substitute out u_H, (16) now implies

$$u_L \leq -\frac{1}{3}[Q^{HH} - Q^{LH} - Q^{HL} + Q^{LL}]$$

and (4) implies a solution with $u_L \geq 0$ cannot exist. This completes the proof of Theorem 8.∎

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