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"Market Selection and Asymmetric Information"

by

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Abstract

We consider a dynamic general equilibrium asset pricing model with heterogeneous agents and asymmetric information. We show how agents' different methods of gathering information affect their chances of survival in the market depending upon the nature of the information and the level of noise in the economy.

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1. Introduction

A wide-spread concept in economics is the idea that agents will benefit from superior information and, therefore, informed agents will drive uninformed agents out of the market. Another idea is that agents with better beliefs will drive agents with worse beliefs out of the market. For example, in support of the efficient-markets hypothesis, Cootner [5] (see also Alchian [1] and Friedman [12]) makes the following statement:

"Given the uncertainty of the real world, the many actual and virtual investors will have many, perhaps equally many, forecasts...If any group of investors was consistently better than average in forecasting stock prices, they would accumulate wealth and give their forecasts greater and greater weight...If this process worked well enough, the present price would reflect the best information about the future..."[1]

In the quote above, good beliefs and good information seem to be similar concepts. However, in a dynamic model, heterogeneity of information has different implications from heterogeneity of beliefs. For example, if agent 1 is informed that event a has probability 0.3 and agent 2 is not informed, then prices may reveal this information to agent 2 when agent 1 has significant wealth, but not if 1 has small wealth because then prices will not be significantly affected by 1's decisions. So, agent 1's accumulation of wealth based on superior information may limit 1's future ability to profit from superior information. On the other hand, if agent 1 believes correctly that the probability of a is 0.3 and agent 2 believes that it is 0.5 then agent 1 may tend to accumulate more wealth than agent 2, regardless of agent 1's relative wealth.

The literature on survival has concentrated on the question of whether agents with correct beliefs will drive agents with incorrect beliefs out of the market and on the more general question of whether rational agents will drive irrational agents out of the market.[2] In this paper, we develop a dynamic general equilibrium model with heterogeneous agents and asymmetric information. Agents are fully

[1] Cootner [5, p. 80], quoted in Figlewski [10].

[2] See Blume and Easley [2] and [3], De Long, Shleifer, Summers and Waldman [6, 7, 8], Shleifer and Summers [18], Palomino [15], and Sandroni [17].

rational. Their beliefs are correct and they learn from prices. However, agents differ in their methods of gathering information.

Agents trade based on their private information and on the information revealed by prices. After trade takes place, payoff relevant states of nature are revealed. Then, the wealth distribution adjusts which, in turn, changes prices and the information revealed by them. Next period, trade takes place based on new private information and on new information revealed by new prices. After many periods, some agents may become wealthy while others may become poor. Agents who eventually have no wealth also have no impact on prices (do not survive). In the long run, prices will not reflect the (potentially relevant) information gathered by these agents. Our main objective is to determine which methods of gathering information are conducive to survival in the market.[3]

1.1. Informal description of the model

We consider a dynamic asset pricing model with long-lived agents, long-lived trees, as in Lucas' [13] model, and a risk-free asset in zero supply. Each tree pays a dividend that is either high or low. There is a signal about next period's dividend. The probability of high dividends is high when the signal is good and low when the signal is bad. The informativeness of the signal is increasing in a parameter, x. Each period, agents decide how much to consume, how much to save, and how to allocate their savings among the assets. All agents maximize expected discounted logarithmic utility.

There are 3 types of rational agents. Their beliefs are based on all the information available, which includes the information revealed by share prices and interest rates. In each period, the signal is revealed to a subset of agents in one of four possible scenarios. In scenario s_1, the signal is observed only by agent 1. In scenario s_{12}, the signal is observed only by agents 1 and 2. In scenario s_{23}, the signal is observed only by agents 2 and 3. In scenario s_3, the signal is observed only by agent 3. By assumption, agent 2 never obtains information exclusively. The probability distribution over the information scenarios is constant over time.

An additional agent, agent 4, is a noise trader whose belief is random in every period. The noise trader is introduced to ensure that the inference problem of the rational agents is non-trivial. The noise trader would eventually vanish if he received no other endowment, except for his initial shares. Then, prices would fully reveal all private information, and trade among rational agents would eventually stop. We avoid this uninteresting case by assuming that the noise

[3]Figlewski [10] is an early study of the related question of how wealth reallocations affect information aggregation through prices.

trader moves trees in and out of the economy keeping a constant fraction z of the aggregate wealth. We refer to z as the level of noise in the economy.

1.2. Informal description of the results

We say that agent j vanishes if agent j's relative wealth converges to zero as time goes to infinity. We also say that agent j survives if agent j does not vanish and that an agent dominates the economy if all other rational agents vanish. Any agent who vanishes has no long run effect on share prices and interest rates.

The survival of the rational agents depends on x and z. We divide these parameters into three cases. Given z, the signals can be weakly informative (x small), at an intermediary level, or strongly informative (x large). In particular, when x is close to one, the signals become an almost perfect predictor of next period's dividends. Alternatively, given x, the level of noise can be low (z small), intermediary, or high (z large).

For low levels of noise or, alternatively, for strongly informative signals, prices will eventually be fully revealing and all agents survive.

For intermediary levels of noise and informativeness of the signals, assume that the probability of s_{23} is zero and the probability of s_1 is strictly positive. So, when agent 2 observes the signal so does agent 1, but not conversely (under this assumption, we say that agent 2's method of gathering information is inferior to agent 1's method). Then, agents 1 and 3 survive and agent 2 vanishes *if and only if* agent 3 observes the signal with strictly positive probability. In particular, agent 2 may vanish and agent 3 may survive, even if agent 2 observes the signal with arbitrarily high probability and agent 3 observes the signal with arbitrarily small probability.

This result shows that pairwise comparisons between agents do not suffice to determine who vanishes in the economy. Agent 1 may not drive agent 2 out of the market even under the extreme assumption that agent 2's method of gathering information is inferior to agent 1's method. The survival of agent 2 depends on whether or not there exists another agent who is informed when 1 is not.[4] Moreover, it also shows that the ability of obtaining exclusive information may ensure survival even when another agent, who is informed arbitrarily more often, cannot survive.

The intuition behind these results is as follows: If the level of noise (or alternatively the level of informativeness of the signals) is intermediary, then there

[4]This result is in contrast to the results obtained by Blume and Easley [2] and Sandroni [17]. For the case of exogenously specified beliefs, it is possible to make pairwise comparisons between agents.

exists a threshold $\bar{w} \equiv \frac{z}{x}$ such that prices fully reveal information obtained by any agent with wealth above $\bar{w}$. So, if agent 2's method of gathering information is inferior to agent 1's, and these are the only two agents who obtain information, then the wealth of agent 1 is eventually above $\bar{w}$, prices become fully revealing, and all agents survive. However, if agent 1's wealth is above $\bar{w}$, and agent 3 also collects information, then agent 3 also infers the signal from prices when 1 informed. However, with strictly positive probability, agent 1 cannot infer the signal from prices when 3 is informed because it can be shown that agent 3's wealth is below $\bar{w}$. Hence, agent 1's wealth tends to go down when it is above $\bar{w}$. So, infinitely often, the wealth of agent 1 is below $\bar{w}$. At this point, with strictly positive probability, agent 2 cannot infer the signal from prices when 1 is informed. Since agent 1 is informed when 2 is informed, there is a persistent tendency for the wealth of agent 2 (relative to the wealth of agent 1) to go down and, hence, agent 2 vanishes. On the other hand, agent 3 must survive. Otherwise, given that agent 2 vanishes, the wealth of agent 1 would eventually stay above $\bar{w}$. Analogously, agent 1 must survive. If not, the wealth of agent 3 would eventually stay above $\bar{w}$, and the same argument applies.

The results are very different when the level of noise is high (or, alternatively, when the signals are weakly informative). In this case, we show that an agent k vanishes when there is agent j who observes the signal sufficiently more often than k does (excluding the scenarios in which both agents are informed or both uninformed).[5]

In contrast with the case of intermediary level of noise, when the level of noise is high, survival is assured by being informed frequently. For example, agent 3 vanishes when 3 is informed sufficiently less often than another agent even if agent 3 is informed exclusively. So, in the long run, prices may not reveal available information because the agent who obtains it vanishes from the economy. Moreover, regardless of the chances that agent 3 is informed, agent 2 vanishes if agent 2's method of gathering information is inferior to agent 1's method (because then 2 is informed infinitely less often that 1). However, if agent 2 is informed sufficiently more often than the other agents, then agent 2, who is never informed exclusively, dominates the economy, while agents 1 and 3, who are informed exclusively, vanish.[6] That is, agent 2 vanishes if a single agent is informed whenever 2 is informed, but agent 2 may dominate the economy when different agents are informed whenever 2 is informed.

The intuition behind this result is simply that if the level of noise is high

[5]The threshold depends on x and z.

[6]This is reminiscent of the example of Dekel and Scotchmer [9], who show, in an evolutionary context, that nonoptimizing behavior can survive as long as it does better than average.

(or, alternatively, if the signals are weakly informative) then prices do not reveal the signal, with positive probability, in all scenarios, even if the wealth of the informed agents is high. So, the relative wealth of agent k over agent j tends to go up when k is informed and j is uninformed and, conversely, tends to down when k is uninformed and j is informed. The overall effect depends upon the probability that k is informed and j uninformed and the probability that k is uninformed and j informed.

If the signal is uninformative (x is small), then the forecasts of informed and uninformed agents are similar. We show that if agent k is informed slightly less often agent j, then agent k vanishes, provided that x is small enough, but strictly positive. This result is similar to what would be obtained in the case of differences in beliefs. If an agent has correct beliefs, then an agent with incorrect, but arbitrarily close to the truth, beliefs will be driven out of the market (see Blume and Easley [2] and Sandroni [17]). The intuition behind this result is that if x is small (or z is high) then the prices do not reveal much information and it is the price revelation mechanism that increases the chances of survival of uninformed agents. So, any agent who is informed less frequently than another does not survive when x is small. This is not necessarily true when x is high because then prices are likely to reveal information, nor is it true when x is zero because then the signals reveal no information and the forecasts of informed and uninformed agents are identical.

2. The Model

We study a discrete-time infinite-horizon model. There are four types of long-lived agents, long-lived trees, a risk-free asset in zero supply, and a single consumption good. There is a good state of nature, h, in which each tree gives d_h units of the consumption good (the high dividend) and a bad state of nature, l, in which each tree gives $d_l < d_h$ units of consumption (the low dividend). The risk-free asset is denominated in units of the consumption good.

At each point in time, either a good signal, g, or a bad signal, b, is observed by some of the agents. If the signal g is observed, then state h occurs *next period* with probability $\rho^g \equiv 0.5(1+x)$, where $x \in (0,1)$. If the signal b is observed, then state h occurs next period with probability $\rho^b \equiv 0.5(1-x)$. These probabilities are fixed and, hence, do not depend on current or past outcomes of the economy. We also assume that the good and bad signals occur with equal probability independently of any current or past outcome. This implies that the unconditional probability of a high dividend is 0.5. The absolute value of the difference between the conditional (on the signal) and unconditional probability of high dividends is $x/2$. We refer to the parameter x as the *informativeness* of the signals.[7]

There are three types of rational agents; we denote their collection by $\Lambda \equiv \{1, 2, 3\}$. There are four possible information scenarios; we denote their collection by $\Theta \equiv \{s_1, s_{12}, s_{23}, s_3\}$. These scenarios determine which types of agents observe the signal. The signal is observed by only the type 1 agent in scenario s_1, by only types 1 and 2 agents in scenario s_{12}, by only types 2 and 3 agents in scenario s_{23}, and by only the type 3 agent in scenario s_3.[8] The realized scenario is public information. The probabilities of the scenarios are fixed and given by σ_k, $s_k \in \Theta$. Clearly, $\sum_{s_k \in \Theta} \sigma_k = 1$, although our results still hold if there are scenarios in which all or no agents observe the signal.

By assumption, agent 2 never observes the signal exclusively. Agent 1 (3) sometimes observes the signal exclusively if $\sigma_1 > 0$ ($\sigma_3 > 0$).

We are interested in rational-expectations equilibria in which prices may not fully reveal the private information of traders. So, as usual, we introduce a noise trader. Agent 4, the noise trader, in period t assigns probability ρ_t^n to the event of high dividends in period $t+1$. We assume ρ_t^n is a random variable independent of any other random variable in the economy, and uniformly distributed over $(0, 1)$. We will need to make an assumption on asset holdings to guarantee that the noise trader is not eliminated from the market. The role of the noise trader is simply to make the inference problem nontrivial. We will not be conducting any welfare or efficiency analysis.

Let $\Sigma = \{h, l\} \times \{g, b\} \times \Theta \times (0, 1)$ be the set of combinations of states of nature, signals about next period's state of nature, information scenarios, and the noise trader's probabilities of next period's state of nature.

Let N be the set of natural numbers, and $N_+ \equiv N \cup \{0\}$. The set of all t-histories is denoted Σ^t, for $t \in N \cup \{\infty\}$. Let $\Im_0 \subset \cdots \Im_t \subset \cdots \subset \Im$ be the filtration on Σ^∞, where $\Im_0$ is the trivial σ-algebra, $\Im_t$ is the algebra generated by all t-histories, and $\Im$ is the σ-algebra generated by the algebra $\Im^0 \equiv \bigcup_{t \in N_+} \Im_t$. Let $\{\Im_t^d,\ t \in N_+ \cup \{\infty\}\}$ be the subfiltration of $\{\Im_t,\ t \in N_+ \cup \{\infty\}\}$ which differentiates only the states of nature h and l.

Let d_t be the $\Im_t^d$-measurable random variable representing dividends per tree at period t. That is, $d_t = d_h$ (d_l) if state h (l) occurs in period t. Share prices and interest rates are given by the $\Im_t$-measurable random variables p_t and i_t,

[7] This terminology reflects the fact that when x is close to one then the signals are almost perfect predictors for dividends next period. On the other hand, if x close to zero, then rational predictions that take into account the signals are similar to those that ignore them.

[8] We treat each type of agent as a single agent for grammatical simplicity. Since each agent is a price taker, this is equivalent to a model in which there is a continuum of agents of each type.

respectively. Share prices and interest rates are not necessarily fully revealing because these random variables may also be measurable according to subfiltrations of $\{\Im_t,\ t \in N_+ \cup \{\infty\}\}$. In particular, let $\{\Im_t^j,\ t \in N_+ \cup \{\infty\}\}$, $j \in \Lambda$, be the subfiltrations of $\{\Im_t,\ t \in N_+ \cup \{\infty\}\}$ representing information available to agent j (including information revealed by share prices and interest rates). We also define $\Im_t^4 \equiv \Im_t$ and $\Im_{-1} \equiv \Im_0$.

Agents $j \in \Lambda$ are born with k_{-1}^j trees, as in Lucas' [13] model, and receive no other endowments. Agent 4 is born with k_{-1}^4 trees and may receive more (or, in unusual circumstances, lose) trees in the future. Agent $j \in \Lambda \cup \{4\}$ enters period t with share holdings k_{t-1}^j and bond holdings b_{t-1}^j. For $j \in \Lambda$, agent j's wealth, w_t^j, is the market value of j's assets before consumption and trade takes place, i.e., $w_t^j \equiv (p_t + d_t)k_{t-1}^j + b_{t-1}^j$. In each period, before consumption and trade take place, but after share prices and interest rates are observed, the noise trader's share holdings are adjusted so that he has a constant fraction $z \in (0,1)$ of aggregate wealth. Denoting $k_{t,t-1}^4$ for agent 4's share holdings after this adjustment, we have $w_t^4 \equiv (p_t + d_t)k_{t,t-1}^4 + b_{t-1}^4$. The aggregate wealth in the economy is given by $W_t \equiv (p_t + d_t)K_t$, where $K_t \equiv k_{t-1}^1 + k_{t-1}^2 + k_{t-1}^3 + k_{t,t-1}^4$ is the $\Im_t$-measurable random variable which measures the (real) number of trees available in the economy at period $t \in N_+$. At the end of each period, agents consume and obtain new shares and bond holdings. Agent j's consumption, share holdings and bond holdings, at period t, are given by the $\Im_t^j$-measurable functions c_t^j, k_t^j, and b_t^j, respectively.

Agent j's fraction of the aggregate wealth, held at period t, is given by $\alpha_t^j \equiv w_t^j / W_t$. We refer to α_t^j as agent j's relative wealth or wealth share. By assumption, $k_{t,t-1}^4$ solves $\alpha_t^4 = z$.[9] This assumption ensures that the noise trader will not be driven out of the economy. We refer to z as the *level of noise* in the economy.

When the market opens in period t, agent j observes the scenario, the signal if the scenario requires, current dividends d_t, and prices (p_t, i_t). The agent's choice of (c_t^j, k_t^j, b_t^j) satisfies the appropriate period t budget constraint,

$$c_t^j + p_t k_t^j + (b_t^j / i_t) = (p_t + d_t)k_{t-1}^j + b_{t-1}^j, \text{ for } j \in \Lambda,$$

or

$$c_t^4 + p_t k_t^4 + (b_t^4 / i_t) = (p_t + d_t)k_{t,t-1}^4 + b_{t-1}^4,$$

as well as $c_t^j \geqslant 0$ and $w_t^j \geqslant 0$. The latter restriction is the familiar restriction ruling out default.

[9] Thus, $k_{t,t-1}^4 = z\left(k_{t-1}^1 + k_{t-1}^2 + k_{t-1}^3\right) / (1-z) - b_{t-1}^4 / \left((p_t + d_t)(1-z)\right)$.

Markets clear if

$$c_t^1 + c_t^2 + c_t^3 + c_t^4 = d_t K_t, \quad (1)$$

$$b_t^1 + b_t^2 + b_t^3 + b_t^4 = 0, \text{ and} \quad (2)$$

$$k_t^1 + k_t^2 + k_t^3 + k_t^4 = K_t. \quad (3)$$

Let $\mathcal{P}$ and $\mathcal{P}^j$ be the probability measures on $(\Sigma^\infty, \Im)$ representing the true probability measure and agent j's belief about outcomes of the economy. Agents 1, 2, and 3 are rational and, therefore, $\mathcal{P}^j = \mathcal{P}$, $j \in \Lambda$.

In period t, agent j's expected discounted utility function is given by

$$\mathcal{E}^j \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \log c_{t+\tau}^j \middle| \Im_t^j \right\},$$

where $\beta < 1$ is the discount factor and $\mathcal{E}^j$ is the expectation operator associated with agent j's belief $\mathcal{P}^j$.

In equilibrium, agents maximize expected discounted utility subject to the sequence of budget constraints, and markets clear in every period.

2.1. Comments on some of the assumptions

We have assumed that all agents in our model have logarithmic utility functions. It is an open (and, we believe, hard) question whether or not our results generalize when agents have more general preferences. The intuition behind some of our results may still hold in more general settings. However, at present, we caution the reader to see these results as examples rather than a general theory.

The assumption that agents have a logarithmic utility function allows for explicit solution of the model. It also simplifies the comparison of our results with previous results regarding the survival of agents with incorrect beliefs because, as shown by Blume and Easley [2], agents with incorrect beliefs accumulate less wealth than agents with correct beliefs, even if markets are incomplete. Agents with incorrect beliefs would be driven out of the market by agents with correct beliefs, for all parameter values.

The noise trader does not expect the asset reallocations that occur each period, when maximizing his utility. This assumption simplifies the analysis, since the noise trader's behavior has the same parametric form as the other traders in our economy. As mentioned above, we introduce noise traders to make the inference problem non-trivial. The additional assumption that the level of noise z is constant simplifies the exposition of the results, but is not necessary. Our main results depend on the size of the level of noise. The cutoff points depend only on the exogenous variable x and not on endogenous variables.

Many of our results do not rely on the specific types of agents and scenarios assumed in this model.[10] However, they simplify the exposition and suffice for most of our applications.

3. Rational Expectations Equilibria

In this section we begin the description of the rational expectations equilibria of our economy. The gross return of a share of a tree, Φ_t, in period t is given by

$$\Phi_t \equiv \frac{p_t \; + \; d_t}{p_{t-1}}.$$

We refer to Φ_t as the market return (of a tree).

Agent j's savings ratio at period t is given by

$$\delta_t^j \equiv 1 - \frac{c_t^j}{w_t^j}.$$

This ration describes gross savings behavior, rather than current (or net) savings, since δ_t^j is the market value of agent j's assets after consumption takes place divided by the market value of agent j's assets before consumption takes place (it does not describe additions to current asset holdings).

The fraction of agent j's savings allocated to the risk-free asset, φ_t^j, and to tree's shares, $1 - \varphi_t^j$, are defined by

$$v_t^j \equiv (1 - \varphi_t^j, \varphi_t^j) \equiv \left(\frac{p_t k_t^j}{p_t k_t^j + (b_t^j / i_t)}, \frac{(b_t^j / i_t)}{p_t k_t^j + (b_t^j / i_t)} \right).$$

We refer to v_t^j as agent j's portfolio. Define $r_{t+1} \equiv (\Phi_{t+1}, i_t)$. It is well known that agents with log utility optimally save a constant fraction of their wealth. This constant is equal to their discount factor. Moreover, they optimally choose a portfolio according to a simple myopic rule. We state this as Lemma 1.[11]

[10] In particular, Propositions 4 and 5 could be easily extended to a model with finitely many agents with different methods of gathering information.

[11] Lemma 1 also applies to the noise trader because of our assumption that he does not expect the asset reallocations. This ensures that his prediction of the evolution of his wealth has the mutliplicative structure needed. The realized evolution of his wealth is not multiplicative.

Lemma 1. *The optimal savings ratio of agent j is $\delta_t^{j*} = \beta$. The optimal portfolio of agent j in period t, v_t^{j*}, solves*

$$\max_{v_t^j} \mathcal{E}^j \left\{ \log \left(v_t^j \cdot r_{t+1} \right) \middle| \Im_t^j \right\},$$

which implies the first order condition

$$\mathcal{E}^j \left\{ \frac{i_t - \Phi_{t+1}}{v_t^{j*} \cdot r_{t+1}} \middle| \Im_t^j \right\} = 0.$$

Proof - See Sandroni [16, proof of Lemma 1].

In period t agents know the interest rate i_t, so the only relevant uncertainty in determining optimal portfolios concerns Φ_{t+1}. Since agents differ in their information, it is possible that the market return of trees depends upon the distribution of wealth. This then raises the possibility that the noise trader's wealth adjustment (which keeps his wealth share at z) influences Φ_{t+1}. In that case, an informed agent, when maximizing the log of next period's wealth, would need a belief over the adjustment and how it influences Φ_{t+1}. Fortunately, this is not the case in this economy because the equilibrium determination of Φ_{t+1} is particularly simple.

By Lemma 1, in equilibrium, $c_t^j = (1-\beta) w_t^j$. The requirement that the market for current consumption clear, (1), then implies $d_t K_t = (1-\beta)(p_t + d_t) K_t$. Hence, in equilibrium,

$$\bar{p}_t \equiv \frac{\beta}{(1-\beta)} d_t \tag{4}$$

and so

$$\bar{\Phi}_{t+1} \equiv \frac{1}{\beta} \frac{d_{t+1}}{d_t}. \tag{5}$$

Share prices $\bar{p}_t$ thus depend only on current dividends and on the discount factor; they do not depend upon the signals. Therefore, only interest rates can reveal relevant information to the uninformed agents. We define

$$\hat{\Phi}_t(h) \equiv \frac{1}{\beta} \frac{d_h}{d_t} \text{ and } \hat{\Phi}_t(l) \equiv \frac{1}{\beta} \frac{d_l}{d_t} \tag{6}$$

as the equilibrium market returns in period $t+1$ when state h and when state l occurs, respectively. (The subscript t indicates that these two returns are $\Im_t$, but not $\Im_{t+1}$, measurable.)

Two assumptions play a crucial role in obtaining the simple structure on share prices: the utility from consumption is logarithmic and all agents have the same discount factor. When agents have log utilities, but differ in their discount factors, aggregate demand for consumption will depend upon the distribution of wealth.

The current wealth distribution does play a role in determining the equilibrium interest rate within a period. However, future interest rates are not relevant in maximizing the log of next period's wealth. Hence, agents' optimal savings and portfolio decisions depend upon current interest rates, but do not depend upon agents' beliefs over future wealth distributions and, in particular, do not depend upon agents' beliefs over the noise trader's wealth adjustment.

By Lemma 1, an agent's optimal portfolio and savings ratio do not depend upon his wealth. Accordingly, we can describe agents' behavior according to their informational characteristic: informed, uninformed, or noise trader (we say that agents who observe the signal are informed and those who do not are uninformed, regardless of what they infer from interest rates and share prices). Let v_t^ℓ denote the optimal portfolio of an agent with characteristic $\ell = i$ (informed), u (uninformed), and n (noise trader). Let ρ_t^ℓ be the probability an agent of characteristic ℓ assigns in period t to a high dividend in period $t+1$. That is, for $\ell = i, u$,

$$\mathcal{P}\left\{ \Phi_{t+1} = \hat{\Phi}_t(h) \middle| \Im_t^\ell \right\} = \rho_t^\ell,$$

and for $\ell = n$,

$$\mathcal{P}^4\left\{ \Phi_{t+1} = \hat{\Phi}_t(h) \middle| \Im_t \right\} = \rho_t^n.$$

Thus, $\rho_t^i = \rho^g$ if the signal is good and $\rho_t^i = \rho^b$ if the signal is bad, and ρ_t^u is the probability of state h conditional on interest rates.

From the first order condition in Lemma 1, the optimal fraction of agents' savings allocated to the risk-free asset is, for $\ell = i$, u, and n,

$$\varphi_t^{\ell *} \equiv \left((1-\rho_t^\ell) \frac{\hat{\Phi}_t(h)}{(\hat{\Phi}_t(h) - i_t)} - \rho_t^\ell \frac{\hat{\Phi}_t(l)}{(i_t - \hat{\Phi}_t(l))} \right). \tag{7}$$

Let α_t^ℓ be the fraction of the aggregate wealth held by agents of characteristic ℓ, for $\ell = i$ and u.

Since the risk-free asset is in zero net-supply,

$$\varphi_t^{u*}\alpha_t^u + \varphi_t^{i*}\alpha_t^i + \varphi_t^{n*} z = 0. \tag{8}$$

Substituting for $\varphi_t^{\ell *}$, we obtain

$$\frac{\left((1-\rho_t^i)\alpha_t^i + (1-\rho_t^u)\alpha_t^u + (1-\rho_t^n)z\right)\hat{\Phi}_t(h)}{\hat{\Phi}_t(h) - i_t} = \frac{\left(\rho_t^i\alpha_t^i + \rho_t^u\alpha_t^u + \rho_t^n z\right)\hat{\Phi}_t(l)}{i_t - \hat{\Phi}_t(l)} \tag{9}$$

Solving for interest rates, using $\alpha_t^u + \alpha_t^i + z = 1$ and (6), we have

$$\frac{1}{i_t} = f(\alpha_t^i, \rho_t^i, \rho_t^u, \rho_t^n, d_t),$$

where $f(\alpha_t^i, \rho_t^i, \rho_t^u, \rho_t^n, d_t)$ is given by

$$\beta d_t \left[\alpha_t^i \left((1-\rho_t^i)\frac{1}{d_l} + \rho_t^i \frac{1}{d_h} \right) + (1-\alpha_t^i - z) \left((1-\rho_t^u)\frac{1}{d_l} + \rho_t^u \frac{1}{d_h} \right) \right.$$
$$\left. + z \left((1-\rho_t^n)\frac{1}{d_l} + \rho_t^n \frac{1}{d_h} \right) \right].$$

Denote by f_b (f_g) the function describing the inverse of the interest rate if the bad (good) signal occurs *and* this information is revealed to the uninformed agents:

$$f_b(\alpha_t^i, \rho_t^n, d_t) \equiv f(\alpha_t^i, \rho^b, \rho^b, \rho_t^n, d_t)$$

and

$$f_g(\alpha_t^i, \rho_t^n, d_t) \equiv f(\alpha_t^i, \rho^g, \rho^g, \rho_t^n, d_t).$$

For completeness, we observe that, as long as the noise is not too large relative to the informativeness of the signal, there is a fully-revealing rational expectations equilibrium.

Proposition 1. *A fully-revealing rational expectations equilibrium exists if and only if* $z < x/(1+x)$.

Proof - A fully revealing rational expectations equilibrium exists if and only if

$$f_b(\alpha_t^i, \rho_t^n, d_t) > f_g(\alpha_t^i, \tilde{\rho}_t^n, d_t) \text{ for all } \rho_t^n, \tilde{\rho}_t^n \in (0,1).$$

Since f is decreasing in ρ_t^n, this is equivalent to

$$f_b(\alpha_t^i, 1, d_t) > f_g(\alpha_t^i, 0, d_t),$$

which is equivalent to

$$z < x/(1+x).$$

■

As usual, the existence of the fully-revealing equilibrium is independent of the wealth share of the informed agents. In principle, interest rates can reveal the signal even if there are no informed agents! While this extreme situation is ruled out by the restriction that prices cannot reveal information that agents (with positive wealth share) do not have, that restriction has no force when α_t^i is positive (even if arbitrarily small). As a result, we do not take the fully-revealing equilibrium to be a sensible description of behavior. Our focus is on the partially-revealing equilibrium (which we describe in the next section), in which the wealth share of the informed agents plays a central role. As will also become clear, the parameter region where the fully-revealing equilibrium does not exist is of some interest. The partially-revealing equilibrium allows us to make some revealing comparisons as the level noise increases from below the critical value, $x/(1+x)$, to above it.

4. The Partially-Revealing Rational Expectations Equilibrium

When the wealth share of the informed agents is small, we should expect prices not to reveal much of the informed agents' information. Indeed, much of our intuition is motivated by precisely that consideration. In this section, we describe the partially-revealing equilibrium that is the basis of our asymptotic analysis. In this equilibrium, when the wealth share of the informed agents is small, prices do not reveal the informed agents' information with high probability (so that, in an expected sense, prices do not reveal much information).

Denote by $f_{b,0.5}$ ($f_{g,0.5}$) the function describing the inverse of the interest rate if the bad (good) signal occurs, and the uninformed agents believe that states h and l have equal probability next period:

$$f_{b,0.5}(\alpha_t^i, \rho_t^n, d_t) \equiv f(\alpha_t^i, \rho^b, 0.5, \rho_t^n, d_t)$$

and

$$f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) \equiv f(\alpha_t^i, \rho^g, 0.5, \rho_t^n, d_t).$$

These functions are illustrated in Figure 1.

Let $\rho_t^n(b)$ and $\rho_t^n(g)$ be the noise trader beliefs that satisfy

$$f_{g,0.5}(\alpha_t^i, 0, d_t) = f_{b,0.5}(\alpha_t^i, \rho_t^n(b), d_t)$$

and

$$f_{g,0.5}(\alpha_t^i, \rho_t^n(g), d_t) = f_{b,0.5}(\alpha_t^i, 1, d_t).$$

That is, since $\rho^g = 0.5(1+x)$ and $\rho^b = 0.5(1-x)$,

$$\rho_t^n(b) = \frac{x}{z}\alpha_t^i \text{ and } \rho_t^n(g) = 1 - \frac{x}{z}\alpha_t^i.$$

Note that

$$\rho_t^n(b) \in (0,1) \text{ and } \rho_t^n(g) \in (0,1) \text{ if and only if } \alpha_t^i < \frac{z}{x}.$$

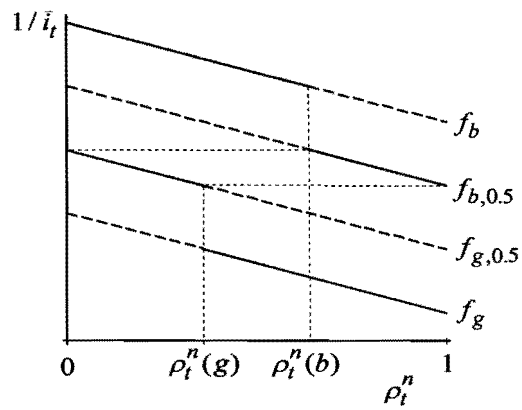

Figure 1: The equilibrium interest rate functions. The solid lines describe $1/\bar{\imath}_t$ for good and bad signals.

Define $\bar{\imath}_t$ as follows: If the signal is good,

$$\frac{1}{\bar{\imath}_t} \equiv \begin{cases} f_g(\alpha_t^i, \rho_t^n, d_t), & \text{if } \rho_t^n \geqslant \rho_t^n(g), \\ \\ f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t), & \text{if } \rho_t^n < \rho_t^n(g), \end{cases} \tag{10}$$

and if the signal is bad,

$$\frac{1}{\bar{\imath}_t} \equiv \begin{cases} f_{b,0.5}(\alpha_t^i, \rho_t^n, d_t), & \text{if } \rho_t^n > \rho_t^n(b), \\ \\ f_b(\alpha_t^i, \rho_t^n, d_t), & \text{if } \rho_t^n \leqslant \rho_t^n(b). \end{cases} \tag{11}$$

Proposition 2. *The pair* $(\bar{p}_t, \bar{\imath}_t)$ *are the share prices and interest rates in the partially-revealing rational expectations equilibrium.*

Proof - See Appendix.

The equations that define $\bar{\imath}_t$ are relatively simple. The functions f_b, $f_{b,0.5}$, $f_{g,0.5}$, and f_g are linear in ρ_t^n (see Figure 1). They all have the same negative slope and the intercepts decrease (in the order given).[12] Hence, it is easy to check

[12]Some simple comparative statics results follow directly from the equations defining interest rates. For example, not surprisingly, equilibrium interest rates $\bar{\imath}_t$ tend to be low when agents are patient (i.e., β is high), when current dividends are high, when the signal is bad or when the noise trader is pessimistic.

the condition under which interest rates are fully revealing. The noise trader beliefs $\rho_t^n(b)$ and $\rho_t^n(g)$ are the bounds on ρ_t^n that delineate when interest rates do and do not reveal the signal. That is, when the signal is bad, interest rates fully reveal the signal if and only if the noise trader is sufficiently pessimistic ($\rho_t^n \leqslant \rho_t^n(b)$). When the signal is good, interest rates fully reveals the signal if and only if the noise trader is sufficiently optimistic ($\rho_t^n \geqslant \rho_t^n(b)$).

In particular, if the relative wealth of the informed agents is greater than z/x, i.e., if $\alpha_t^i > z/x$, then, $\rho_t^n(b) > 1$ and $\rho_t^n(g) < 0$ and, therefore, interest rates fully reveal the signal, with probability one. In this particular case, $1/\bar{\imath}_t = f_b(\alpha_t^i, \rho_t^n, d_t)$ when the signal is bad, and $1/\bar{\imath}_t = f_g(\alpha_t^i, \rho_t^n, d_t)$ when the signal is good. So, for all beliefs of the noise traders, interest rates are lower when the signal is bad than when the signal is good, i.e.,

$$f_b(\alpha_t^i, \rho_t^n, d_t) > f_g(\alpha_t^i, \tilde{\rho}_t^n, d_t) \text{ for all } \rho_t^n, \tilde{\rho}_t^n \in (0,1).$$

On the other hand, if the relative wealth of the informed agents is smaller than z/x, i.e., $\alpha_t^i < z/x$, then, $\rho_t^n(b) < 1$ and $\rho_t^n(g) > 0$. With strictly positive probability, interest rates now need not reveal the signal. If the signal is bad and the belief of the noise trader, ρ_t^n, is sufficiently optimistic ($\rho_t^n > \rho_t^n(b)$), then there exists a belief $\tilde{\rho}_t^n \in (0,1)$ (a more pessimistic belief than ρ_t^n, i.e., $\tilde{\rho}_t^n \leqslant \rho_t^n$) such that $1/\bar{\imath}_t = f_{b,0.5}(\alpha_t^i, \rho_t^n, d_t) = f_{g,0.5}(\alpha_t^i, \tilde{\rho}_t^n, d_t)$. Analogously, if the signal is good and the belief of the noise trader ρ_t^n is sufficiently pessimistic ($\rho_t^n < \rho_t^n(g)$), then there exists a belief $\tilde{\rho}_t^n \in (0,1)$ (a more optimistic belief than ρ_t^n, i.e., $\tilde{\rho}_t^n \geqslant \rho_t^n$) such that $1/\bar{\imath}_t = f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) = f_{b,0.5}(\alpha_t^i, \tilde{\rho}_t^n, d_t)$. In both cases, the uninformed agents will be unsure if a given interest rate is from a good signal and a pessimistic noise trader or a bad signal and an optimistic noise trader.[13] So, if $\alpha_t^i < z/x$, the ex ante probability that the interest rate is not revealing is the probability that the signal is good and $\rho_t^n < \rho_t^n(g)$ or that the signal is bad and $\rho_t^n > \rho_t^n(b)$. In both cases, the probability that ρ_t^n falls in the required range is $1-(x/z)\,\alpha_t^i$. Since good and bad signals are equally likely, the ex ante probability the interest rate *is* revealing is $(x/z)\,\alpha_t^i$. Note that (in contrast to the fully-revealing equilibrium), this probability is, for $\alpha_t^i < z/x$, strictly increasing in α_t^i, converging to 0 as $\alpha_t^i \to 0$, and converging to 1 as $\alpha_t^i \to z/x$.

Equilibrium interest rates depend on the distribution of wealth. To continue the description of the equilibrium, we now describe how the distribution of wealth evolves. We say that dividends *confirm* the signal in period t if either dividends are high in period t and the signal was good in period $t-1$, or dividends are low in period t and the signal was bad in period $t-1$. Analogously, we say that

[13]In this case, the probability of high dividends conditional on interest rates is 0.5 ($\rho_t^u = 0.5$), which equals the unconditional probability of high dividends (0.5).

dividends *do not confirm* the signal in period t if either dividends are high in period t and the signal was bad in period $t-1$, or dividends are low in period t and the signal was good in period $t-1$.

We also say that j has an *effective informational advantage* over $\tilde{j}$ in period t, if in that period, j is informed, $\tilde{j}$ is uninformed, and interest rates do not reveal the signal. We will show that if in period t, agent j has an effective informational advantage over $\tilde{j}$, then the relative wealth of agent j over $\tilde{j}$ is increased by the factor $(1+x)$ when dividends confirm the signal, and decreased by the factor $(1-x)$ when dividends do not confirm the signal.

More precisely, let $\{\zeta_t,\ t \in N\}$ be the sequence of independent, identically distributed random variables defined by

$$\zeta_t \equiv \begin{cases} \log(1+x)\,, & \text{if dividends confirm the signal in period } t, \\ \log(1-x)\,, & \text{if dividends do not confirm the signal in period } t. \end{cases}$$

So, ζ_t is positive when dividends confirm the signal and negative when they do not. Moreover,

$$\bar{\zeta} \equiv \mathcal{E}\left\{\zeta_t \mid \Im_{t-1}\right\} = 0.5\left\{(1-x)\log(1-x) + (1+x)\log(1+x)\right\} > 0. \qquad (12)$$

The conditional expectation of ζ_t is strictly positive, reflecting the fact that when the signal is good (bad) it is more (less) likely that dividends, next period, will be high.

Let $\xi_t^{j\tilde{j}}$, $j, \tilde{j} \in \Lambda$, be the $\Im_t$-measurable random variable defined by

$$\xi_t^{j\tilde{j}} \equiv \begin{cases} \zeta_t, & \text{if, in period } t-1,\ j \text{ has an effective informational advantage over } \tilde{j}, \\ -\zeta_t, & \text{if, in period } t-1,\ \tilde{j} \text{ has an effective informational advantage over } j, \\ 0, & \text{otherwise.} \end{cases}$$

Let $\alpha_t^{j\tilde{j}} \equiv \alpha_t^j / \alpha_t^{\tilde{j}}$, $j, \tilde{j} \in \Lambda$, be the wealth of agent j relative to agent $\tilde{j}$, in period t.

Lemma 2. *In equilibrium, for* $j, \tilde{j} \in \Lambda$,

$$\log \alpha_{t+1}^{j\tilde{j}} = \log \alpha_t^{j\tilde{j}} + \xi_{t+1}^{j\tilde{j}}.$$

Proof - See Appendix.

Lemma 2 shows that the relative wealth of two rational agents follows a simple rule. If two rational agents are either informed or uninformed or if interest reveal the signal then there is no trade between them.[14] They will choose the same savings and portfolios and so their relative wealth will not change. However, if one agent has an effective informational advantage over another then there will be trade between them. They will choose different portfolios and the wealth of the informed agent (relative to the uninformed agent) will go up if and only if dividends confirm the signal observed in the previous period. It is more likely that the relative wealth of the informed agent (relative to the uninformed agent) will go up than down because it is more likely that the signals will confirm the dividends than that they will not.

The distribution of wealth has *no* effect on how much the relative wealth of two rational agents goes up or down when they trade (it depends only on x). However, it affects the probability of trade. If the wealth is concentrated in the hands of informed agents then interest rates tend to reveal the signal which reduces the probability that an agent would have an effective informational advantage over another. Conversely, if the wealth is concentrated in the hands of uninformed agents then the chances of trade among rational agents increase.

5. Asymptotic Properties of the Equilibrium

In this section, we define survival, and show that survival of a particular agent depends on how informative the signals are, the level of noise in the economy, and the probability of the scenarios.

Definition 1. *Agent $j \in \Lambda$* vanishes *if the fraction of aggregate wealth held by agent j, α_t^j, converges to zero, $\mathcal{P}$-almost surely, as $t \to \infty$.*

Definition 2. *Agent $j \in \Lambda$* survives *if the fraction of aggregate wealth held by agent j, α_t^j, does not converge to zero, $\mathcal{P}$-almost surely, as $t \to \infty$.*

Definition 3. *Agent $j \in \Lambda$* dominates the economy *if, $\mathcal{P}$-almost surely, the fraction of aggregate wealth held by agent j converges to $1 - z$.*

Definition 4. *Agent $j \in \Lambda$* never dominates the economy *if, $\mathcal{P}$-almost surely, the fraction of aggregate wealth held by agent j does not converge to $1 - z$.*

[14] We say that two rational agents trade if one is informed, the other is uniformed, and interest rates do not reveal the signal. In this case, the two agents choose different portfolios, and so their relative wealth is affected by dividend and signal realizations. Part of the wealth gain (or loss) of an agent may be at the loss (or gain) of the noise trader.

Definition 5. *The economy is* never dominated by a single agent *if, $\mathcal{P}$-almost surely, there is no agent whose fraction of aggregate wealth converges to* $1-z$, *that is, for $\mathcal{P}$-almost all s, there is no $j \in \Lambda$ such that $\alpha_t^j(s) \to 1-z$.*

The last definition is equivalent to the statement that $\mathcal{P}$-almost surely, the wealth of at least two rational agents (where their identities can depend on the sample path) does not approach zero.

The criteria for survival is based on relative wealth because only those agents with positive relative wealth have any influence on share prices and interest rates. In particular, if agent j were alone in the economy (with the noise trader) then equilibrium share prices and interest rates would be given by equations (4), (10), and (11), with the additional restriction that α_t^i would be $1-z$ if j is informed and zero if j is uninformed. The functions f_g, $f_{g,0.5}$, $f_{b,0.5}, f_b$ are continuous in α_t^i. So, if j dominates the economy then equilibrium share prices and interest rates eventually become close to equilibrium share prices and interest rates of the economy in which agent j is alone in the economy (with the noise trader).

Definition 6. *Agent 2's method of gathering information is* inferior to *agent 1's method if $\sigma_1 > 0$, and $\sigma_{32} = 0$.*

The same definition applies if we replace 1 by agent 3. By assumption, agent 2 never observes the signal exclusively. Whenever agent 2 is informed either agent 3 or agent 1 is also informed. If $\sigma_{32} = 0$ then whenever agent 2 is informed, agent 1 is informed. If $\sigma_1 > 0$ then agent 1 is informed alone with strictly positive probability. So, if agent 2's method of gathering information is inferior to agent 1's, then agent 1 observes the signal whenever agent 2 does, but not conversely.

It is convenient to divide the parameters z (level of noise) and x (degree of informativeness of the signals) into three cases.

Case A: $z < \frac{x}{2+x}$ or, equivalently, $x > \frac{2z}{1-z}$. In this case, we say that the level of noise is low and the signals are strongly informative.

Case B: $\frac{x}{2+x} < z < \frac{x}{1+x}$ or, equivalently, $\frac{z}{1-z} < x < \frac{2z}{1-z}$. In this case, we say that the level of noise and informativeness of the signals are intermediary.

Case C: $z > \frac{x}{1+x}$ or, equivalently, $x < \frac{z}{1-z}$. In this case, we say that the level of noise is high and the signals are weakly informative.

Figure 2 shows the three cases.

In case A, there are initial endowments $(k_{-1}^1, k_{-1}^2, k_{-1}^3)$ such that the equilibrium will be fully revealing with probability one and no trade among the rational agents will take place. This claim can be easily verified because if $z < x/(2+x)$ then $(1-z)/2 > z/x$. Choose $\varepsilon > 0$ small enough such that $(1-z-\varepsilon)/2 > z/x$ and $(k_{-1}^1, k_{-1}^2, k_{-1}^3)$ so that $\alpha_0^1 = \alpha_0^3 = \frac{1-z-\varepsilon}{2}$ and $\alpha_0^2 = \varepsilon$. Then, regardless of the

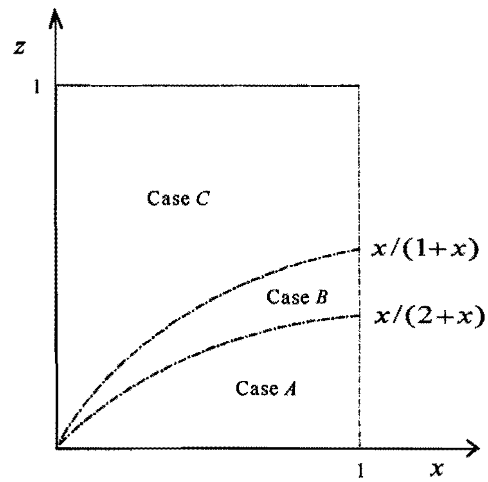

Figure 2: The parameter regions.

scenario, the fraction of the wealth held by informed agents satisfies $\alpha_0^i > z/x$. Hence, the signal is revealed to the uninformed agent, and there is no trade among rational agents. In the next period, the fraction of the wealth held by each agent will be the same as in the previous period. By induction, the fraction of the wealth held by each agent remains fixed.

It can also be shown that, in case A, even if the initial endowments are such that trade among rational agents is possible, the relative wealth will eventually be such that trade (among rational agents) will stop. This follows from the properties of the relative wealth of rational agents (shown in Lemma 2). We do not make a formal demonstration of this result in this paper. Instead, we focus on the more interesting cases, B and C.

In case B, interest rates may or may not reveal the signal with probability one, but trade among rational agents occurs infinitely often. The maximum fraction of aggregate wealth that the informed agents may have is $1 - z$. In case B, $1 - z$ is greater than $\bar{w} \equiv \frac{z}{x}$. So, when the fraction of the aggregate wealth held by informed agents is greater than $\bar{w}$, interest rates are fully revealing with probability one. If this fraction is smaller than $\bar{w}$, interest rates are not revealing with positive probability. However, either the fraction of the aggregate wealth held by agent 1 or the fraction of the aggregate wealth held by agent 3 is strictly smaller than $\bar{w}$. Otherwise the fraction of the aggregate wealth held by agents 1 and 3 is between $2z/x$ and $1 - z$ which implies that $z \leqslant \frac{x}{2+x}$. Hence, there is at least one scenario such that interest rates does not reveal the signal, with positive probability. Thus, as long as scenarios s_1 and s_3 occur with strictly positive probability ($\sigma_1 > 0$ and $\sigma_3 > 0$), trade among the rational agents takes place infinitely often.

In case C, interest rates do not reveal the signal, with positive probability, in all periods. This claim is easy to verify because the fraction of aggregate wealth held by informed agents is smaller $(1 - z)$ which, in case C, is smaller than $\frac{z}{x}$.

Proposition 3. *Assume that the level of noise and informativeness of the signals is intermediary (case B). Also assume that agent 2's method of gathering information is inferior to agent 1's method. Then,*

1. *if $\sigma_3 > 0$ then agent 2 vanishes;*
2. *if $\sigma_3 = 0$ then agent 2 survives (in fact, all agents survive).*

Proof - See Appendix.

Proposition 3 also holds if we exchange the roles of agents 3 and 1. Proposition 3 shows that, in case B, if agent 2 is informed whenever agent 1 is informed (and agent 1 may also be informed alone), then agent 2 vanishes when agent 3 is informed exclusively and survives otherwise.

Proposition 3 shows that, in general, it is not possible to determine whether or not an agent vanishes by "pairwise comparisons between agents." That is, in case B, if agent 2's method of gathering information is inferior to agent 1's, then whether agent 2 vanishes or survives does not depend on, for example, how frequently agent 1 observes the signal when agent 2 does not (i.e., the probability of scenario s_1). It depends on whether or not there exists another agent who is informed when agent 1 is not informed.

The intuition behind this result is as follows: when the relative wealth of agent 1 is above $\bar{w}$, interest rates reveal the signal with probability one in scenarios s_1 and s_{12}, while in scenario s_3 (when the relative wealth of the informed agent 3 is below $\bar{w}$) interest rates do not reveal the signal, with strictly positive probability. Moreover, agents 1 and 3 only trade in scenario s_3 when 3 has an effective informational advantage over 1. Then, as demonstrated in Lemma 2, each time agents 1 and 3 trade, the wealth of agent 1 relative to agent 3 may go up or down (depending on whether dividends are confirming) but, on average, it will tend to go down. So, the relative wealth of agent 1 will tend to go down until it is below $\bar{w}$. At this point, with positive probability, there will be trade between agents 1 and 2 in scenario s_1. Therefore, infinitely often, 1 will have an effective informational advantage over 2. By assumption, 2 never has an effective informational advantage over 1. So, the wealth of agent 2 relative to agent 1 will tend to go down. Eventually, the wealth of agent 2 relative to agent 1 approaches zero. Therefore, the relative wealth of agent 2 (with respect to the aggregate wealth) must go to zero and agent 2 vanishes.

On the other hand, if $\sigma_3 = 0$, agent 1 trades with the other rational agents only when he has an informational advantage over them. So, the relative wealth of agent 1 tends to go up and will eventually be above $\bar{w}$. At this point, interest rates will reveal the signal, with probability one, in scenarios s_1 and s_{12}. There is no further trade among rational agents and all survive.

Proposition 4. *Assume that the level of noise and informativeness of the signals are intermediary (case B). Then, the economy is never dominated by a single agent.*

Proof - See Appendix.

The intuition behind Proposition 4 is similar to that of Proposition 3. In case B, if agent j dominates the economy then, eventually, the relative wealth of j must be greater than $\bar{w}$. At this point, if j is informed then interest rates reveal the signal, with probability one. So, agent j trades only with agents who have an informational advantage over j. Then, the relative wealth of agent j will tend to go down and, therefore, cannot converge to $1 - z$.

Corollary 1. *Assume that the level of noise and informativeness of the signals is intermediary (case B). Also assume that agent 2's method of gathering information is inferior to agent 1's method (or agent 3's). Then, agents 1 and 3 survive.*

If agent 2's method of gathering information is inferior to agent 1's method then, by Proposition 3 , either all agents survive or agent 2 vanishes. By Proposition 4, at least two agents must survive. Therefore, both agents 1 and 3 survive independently of the probabilities that they are informed. Furthermore, agent 3 will not eventually have arbitrarily small relative wealth even if agent 3 observes the signal with arbitrarily small probability, because the relative wealth of agent 1 will be infinitely often below $\bar{w}$.

Assume that agents 1 and 2 are informed together with probability $1 - 2\varepsilon$ (i.e., $\sigma_{23} = 1 - 2\varepsilon$). Moreover, assume that agents 1 and 3 observe the signal individually with probability ε ($\sigma_1 = \sigma_3 = \varepsilon$). It follows from Propositions 3 and 4 that agent 2 vanishes and agent 3 survives. This example shows that it is possible that an agent will vanish from the economy whereas another survives even though the agent who survives (agent 3) observes the signal arbitrarily less frequently (as ε approaches zero) than the agent who vanishes (agent 2).

Let $\sigma^{j/k}$ be the probability that agent $j \in \Lambda$ observes the signal and $k \in \Lambda$ does not observe the signal. Define $\gamma(x,z) \equiv 1 - \frac{x}{z}(1-z)$. In case C, $\gamma(x,z) \in (0,1)$ because, by definition, $x < \frac{z}{1-z}$.

Proposition 5. *Assume that the level of noise is high and the signals are weakly informative (case C). Then, given any two agents $j, k \in \Lambda$,*

1. *if $\gamma(x,z)\sigma^{j/k} > \sigma^{k/j}$ then agent k vanishes, and*
2. *if $\gamma(x,z)\sigma^{j/k} < \sigma^{k/j}$ then agent j never dominates the economy.*

Proof - See Appendix.

In Proposition 5, we compare the frequencies that two agents are informed excluding the events in which they are both informed or both uninformed. Part 1 shows that, in case C, if agent j is informed $\gamma(x,z)$ times more often than agent k then agent k vanishes. In particular, if agent j is informed more often than all other agents (by a factor of $\gamma(x,z)$) then agent j dominates the economy, $\mathcal{P}$ almost surely. Conversely, part 2 shows that if agent j is not informed more often than all other agents (by a factor of $\gamma(x,z)$) then agent j never dominates the economy.

Proposition 5 is in sharp contrast to Propositions 3 and 4. When the level of noise is high, an agent vanishes if there is another agent who acquires information $\gamma(x,z)$ times more often. This is not true when the level of noise is intermediary because an agent may survive when another agent is informed arbitrarily more often. Moreover, when the level of noise in intermediary, no single agent dominates the economy. However, if the level of noise is high, an agent who is informed $\gamma(x,z)$ times more often than all other agents dominates the economy.

The key difference between cases B and C is that in case B there is a threshold $\bar{w}$ such that if the relative wealth of an agent is above $\bar{w}$ then any information acquired by this agent will be fully revealed by interest rates. However, when the level of noise is high (signals are weakly informative) then interest rates do not reveal the signal, with positive probability, in any scenario. Therefore, if agent j is informed and k is uninformed, then, with positive probability, the agents will trade and so, the relative wealth of the informed agent will tend to go up. In the next period, the probability that agent j will have an information advantage over k is smaller than in the previous period if the relative wealth of j over k

went up. However, if j has sufficiently higher probability than k of observing the signal then the probability that j will have an informational advantage over k is still greater than the chances that k will have an informational advantage over j. In particular, agent k vanishes if agent j is informed sufficiently more often than agent k, and agent j can only dominate the economy by being informed sufficiently more often than any other agent.

Corollary 2. *Assume that the level of noise is high and the signals are weakly informative (case C). Also assume that agent 2's method of gathering information is inferior to agent 1's method. Then,*

1. *agent 2 vanishes,*
2. *if $\gamma(x,z)(\sigma_1+\sigma_{12})>\sigma_3$ then agent 3 vanishes and agent 1 dominates the economy, and*
3. *if $\gamma(x,z)(\sigma_1+\sigma_{12})<\sigma_3$ then agent 3 survives.*

Proof - If agent 2's method of gathering information is inferior to agent 1's method, then

$$\gamma(x,z)\sigma^{1/2}>\sigma^{2/1}=0.$$

By Proposition 5, agent 2 vanishes.

Since $\sigma_{23}=0$,

$$\sigma^{1/3}=\sigma_1+\sigma_{12} \text{ and } \sigma^{3/1}=\sigma_3,$$

and the rest follows from Proposition 5: If agent 3 vanishes, then since agent 2 also vanishes, agent 1 dominates the economy. On the other hand, if agent 1 never dominates the economy and agent 2 vanishes, agent 3 survives.

■

Corollary 2 shows that, when the level of noise is high, if agent 2's method of gathering information is inferior to agent 1's method then agent 2 vanishes. This holds true even if agent 3 is never informed (as opposed to the case of intermediary level of noise). Moreover, Corollary 2 also shows that, when the level of noise is high, agent 3 does not necessarily survive. It is necessary and sufficient that agent 3 is informed sufficiently often.

We have shown that if agent 1 is always informed when agent 2 is informed (but not conversely), then agent 2 vanishes. The next corollary shows that this result changes dramatically when it is assumed that agent 2, while never informed exclusively, has a method of gathering information that is not inferior to any other agent's.

Corollary 3. *Assume that the level of noise is high and the signals are weakly informative (case C). If $\gamma(x,z)\sigma_{23} > \sigma_1$ and $\gamma(x,z)\sigma_{12} > \sigma_3$ then agent 2 dominates the economy.*

Corollary 3 shows that, in case C, agent 2 dominates the economy whenever agent 2 is informed sufficiently more often than agents 1 and 3. Agent 2 is informed either with agent 3 or with agent 1, but never exclusively. Therefore, if agent 2 is informed sometimes with agent 3 and sometimes with agent 1, then agent 2 may dominate the economy (and agents 1 and 3 may vanish). This corollary shows that the ability to obtain exclusive information is not necessary for survival or for market dominance.

When the signals become less informative (or when the level of noise increases) we move from region A, where all agents survive, to region B, where at least two agents survive, to region C, where perhaps only one agent will survive. In this sense, survival seems to become "more difficult" when the signal become less informative or when the noise level increases. Accordingly, the critical factor $\gamma(x,z)$ is increasing in z and decreasing in x. So, in region C, it is "more difficult" for infrequently informed agents to survive when the signals are weak or when the level of noise is high. This result is intuitive because uninformed agents obtain information from prices. When the signals are weak (or when the level of noise is high) then prices tend not to reveal information. This increases the likelihood that informed agents have effective advantage over uninformed agents.

Assume that there is no noise in the economy ($z = 0$). Then, as in the Milgrom-Stokey [14] no-trade theorem, interest rates reveal the signal, trade among rational agents eventually stops, and all agents survive. By contrast, assume that the level of noise is high (z close to one). Then, $\gamma(x,z)$ is close to one and, therefore, by Proposition 5, any agent who acquires information less frequently than another vanishes.

It is interesting to notice, however, that the same results applies if the signals are not very informative (x is close to zero). Then, $\gamma(x,z)$ is also close to one and again by Proposition 5, any agent who acquires information less frequently than another vanishes.

Notice that if x is close to zero then the beliefs of informed and uninformed agents are similar. So, the information advantage itself may be small, but it persists because prices tend not to not reveal information.

Consider the following example: Assume that, for $\varepsilon > 0$,

$$\sigma_{23} = \sigma_{12} = 0.25 + \varepsilon \text{ and } \sigma_1 = \sigma_3 = 0.25 - \varepsilon.$$

That is, agent 2 is informed slightly more often than 1 and 3. However,

$$\gamma(x,z) \longrightarrow 1 \text{ as } x \longrightarrow 0.$$

So, there exists x small enough such that

$$\gamma(x,z)\sigma_{23} > \sigma_1 \text{ and } \gamma(x,z)\sigma_{12} > \sigma_3.$$

By Corollary 3, agent 2 dominates the economy. This example shows that for any level of noise z, if agents 1 and 3 acquire information arbitrarily less often that agent 2 then there exists x small enough such that agent 2 dominates the economy. However, when $x = 0$ then the signals reveal no information because the probability of high dividends do not change with the signals. Therefore, there is no trade between rational agents and all survive.

If the signals are not very informative (x is small) then, with small amounts of noise, the economy will be far from environments covered by the no-trade theorem. Speculative trade persists and agents will permanently gain from private information. If the signals become more informative (x increases) then it takes more noise to achieve the same results (i.e., to reach the same γ).

A. Appendix

A.1. Proof of Proposition 2

By construction, we just need to check the consistency of the probabilities of the states of nature with the beliefs of the uninformed agents.

Assume that the signal is good and $\rho_t^n > \rho_t^n(g)$. Then, $1/\bar{\imath}_t$ is smaller than $f_g(\alpha_t^i, \rho_t^n(g), d_t)$ because f_g is a decreasing function of ρ_t^n. However, the lowest value that $1/\bar{\imath}_t$ can assume if the signal is bad is $f_{b,0.5}(\alpha_t^i, 1, d_t)$. But, by definition, $f_{b,0.5}(\alpha_t^i, 1, d_t)$ is equal to $f_{g,0.5}(\alpha_t^i, \rho_t^n(g), d_t)$, and $f_{g,0.5}(\alpha_t^i, \rho_t^n(g), d_t)$ is greater than $f_g(\alpha_t^i, \rho_t^n(g), d_t)$. Hence, $1/\bar{\imath}_t$ is smaller than the lowest value that $1/\bar{\imath}_t$ can assume if the signal were bad. Thus, the uninformed agents know that the signal is good. This is consistent with the definition of f_g.

Similarly, if the signal is bad and $\rho_t^n < \rho_t^n(b)$, then again the interest rate is fully revealing.

Assume now that the signal is good and $0 < \rho_t^n < \rho_t^n(g)$. Then,

$$\frac{1}{\bar{\imath}_t} = f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t).$$

Let $\hat{\rho}_t^n$ be such that $f_{b,0.5}(\alpha_t^i, \hat{\rho}_t^n, d_t) = f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t)$. Clearly, $\hat{\rho}_t^n \leqslant 1$ because

$$f_{b,0.5}(\alpha_t^i, \hat{\rho}_t^n, d_t) = f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) \geqslant f_{g,0.5}(\alpha_t^i, \rho_t^n(g), d_t) = f_{b,0.5}(\alpha_t^i, 1, d_t).$$

Moreover, $\hat{\rho}_t^n \geqslant \rho_t^n(b)$ because

$$f_{b,0.5}(\alpha_t^i, \hat{\rho}_t^n, d_t) = f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) \leqslant f_{g,0.5}(\alpha_t^i, 0, d_t) = f_{b,0.5}(\alpha_t^i, \rho_t^n(b), d_t).$$

So, uninformed agents do not know if the signal is good and the noise trader has belief ρ_t^n or if the signal is bad and the noise trader has belief $\hat{\rho}_t^n$. Note, however, that the unconditional probability that the signal is good or bad are identical. Moreover, the probability that the inverse of the interest rates is in an interval

$$\left[f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) - y,\ f_{g,0.5}(\alpha_t^i, \rho_t^n, d_t) + y\right]$$

conditional that the signal is good is equal to $2yd_l d_h / \left[\beta d_t z(d_h - d_l)\right]$ (for small $y > 0$), and the probability that the inverse of the interest rates is in an interval

$$\left[f_{b,0.5}(\alpha_t^i, \rho_t^n, d_t) - y,\ f_{b,0.5}(\alpha_t^i, \rho_t^n, d_t) + y\right]$$

conditional that the signal is bad is also equal to $2yd_l d_h / \left[\beta d_t z(d_h - d_l)\right]$ (again, for small $y > 0$). By Bayes' rule, the probability that the signal is bad (and the probability the signal is good) conditional on interest rates is equal to 0.5. Hence, the uninformed agents believe that the state h will occur next period with probability

$$0.5\rho^b + 0.5\rho^g = 0.5(0.5 + x/2) + 0.5(0.5 - x/2) = 0.5.$$

This is consistent with the definition of $f_{g,0.5}$.

An analogous argument applies for the case when the signal is bad and $\rho_t^n(b) < \rho_t^n < 1$.

■

A.2. Proof of Lemma 2

By definition,

$$\log \alpha_{t+1}^{j\tilde{j}} = \log \alpha_t^{j\tilde{j}} + \log\left(\frac{v_t^j r_{t+1}}{v_t^{\tilde{j}} r_{t+1}}\right).$$

Therefore, if j has an effective informational advantage over $\tilde{j}$,

$$\log \alpha_{t+1}^{j\tilde{j}} = \log \alpha_t^{j\tilde{j}} + \log\left(\frac{v_t^i r_{t+1}}{v_t^u r_{t+1}}\right),$$

while if $\tilde{j}$ has an effective informational advantage over j,

$$\log \alpha_{t+1}^{j\tilde{j}} = \log \alpha_t^{j\tilde{j}} - \log\left(\frac{v_t^i r_{t+1}}{v_t^u r_{t+1}}\right).$$

Otherwise,

$$\log \alpha_{t+1}^{j\tilde{j}} = \log \alpha_t^{j\tilde{j}}.$$

The result now follows from the following:

Lemma A.1. *Assume that $\rho_t^u = \frac{1}{2}$ (i.e., interest rates do not reveal the signal), then*

$$\log\left(\frac{\upsilon_t^{i*} r_{t+1}}{\upsilon_t^{u*} r_{t+1}}\right) = \zeta_{t+1}.$$

Proof - By Lemma 1,

$$\varphi_t^{\ell *} = \left((1-\rho_t^\ell)\frac{\hat{\Phi}_t(h)}{\hat{\Phi}_t(h) - i_t} - \rho_t^\ell \frac{\hat{\Phi}_t(l)}{i_t - \hat{\Phi}_t(l)}\right);\ \ell = i,\ u. \tag{A.1}$$

Hence, for $\ell = i$, u, when dividends at $t+1$ are high,[15]

$$\begin{aligned}\log(\upsilon_t^\ell r_{t+1}) &= \log\left(\left(1 - (1-\rho_t^\ell)\frac{\hat{\Phi}_t(h)}{\hat{\Phi}_t(h) - i_t} + \rho_t^\ell \frac{\hat{\Phi}_t(l)}{i_t - \hat{\Phi}_t(l)}\right)\hat{\Phi}_t(h)\right.\\ &\quad \left. + \left((1-\rho_t^\ell)\frac{\hat{\Phi}_t(h)}{\hat{\Phi}_t(h) - i_t} - \rho_t^\ell \frac{\hat{\Phi}_t(l)}{i_t - \hat{\Phi}_t(l)}\right) i_t\right)\\ &= \log\left(\frac{\rho_t^\ell i_t\left(\hat{\Phi}_t(h) - \hat{\Phi}_t(l)\right)}{i_t - \hat{\Phi}_t(l)}\right).\end{aligned}$$

Similarly, when dividends at $t+1$ are low,

$$\log(\upsilon_t^\ell r_{t+1}) = \log\left(\frac{\left(1-\rho_t^\ell\right) i_t\left(\hat{\Phi}_t(h) - \hat{\Phi}_t(l)\right)}{\hat{\Phi}_t(h) - i_t}\right).$$

If $\rho_t^u = 0.5$, then, when dividends at $t+1$ are high,

$$\log\left(\frac{\upsilon_t^i r_{t+1}}{\upsilon_t^u r_{t+1}}\right) = \log\left(\frac{\rho_t^i}{0.5}\right),$$

and when dividends at $t+1$ are low,

$$\log\left(\frac{\upsilon_t^i r_{t+1}}{\upsilon_t^u r_{t+1}}\right) = \log\left(\frac{(1-\rho_t^i)}{0.5}\right).$$

By definition, $\rho_t^i = 0.5(1+x)$ if the signal, in period t, is good and $\rho_t^i = 0.5(1-x)$ if the signal, in period t, is bad. Therefore, if dividends confirm the signal in period $t+1$,

$$\log\left(\frac{\upsilon_t^i r_{t+1}}{\upsilon_t^u r_{t+1}}\right) = \log(1+x),$$

[15] It is worth noting here $\rho_t^i = \mathcal{P}\{d_t = d_h | \Im_t\}$.

while if dividends do not confirm the signal in period $t+1$,

$$\log\left(\frac{v_t^i r_{t+1}}{v_t^u r_{t+1}}\right) = \log(1-x).$$

■

A.3. Proof of Proposition 3

We first need a series of intermediate results:

Lemma A.2. *Assume that agent 2's method of gathering information is inferior to agent 1. Then, the ratio α_t^2/α_t^1 converges $\mathcal{P}$-almost surely. Moreover, the limit is finite.*

Proof - By Lemma 1, if agent 1 is informed then

$$\mathcal{E}^1\left\{\frac{i_t-\Phi_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t^1\right\} = \mathcal{E}\left\{\frac{i_t-\Phi_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} = 0,$$

and so

$$\mathcal{E}\left\{\frac{i_t}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} = \mathcal{E}\left\{\frac{\Phi_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} \equiv \pi_t^1.$$

Hence, given any portfolio $v=(1-\varphi,\varphi)$,

$$\mathcal{E}\left\{\frac{vr_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} = (1-\varphi)\mathcal{E}\left\{\frac{\Phi_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} + \varphi\mathcal{E}\left\{\frac{i_t}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} = \pi_t^1.$$

Since $v=v_t^{1*}$ is one feasible choice for v, $\pi_t^1=1$. Then, by setting $v=v_t^{2*}$, we also have

$$\mathcal{E}\left\{\frac{v_t^{2*}r_{t+1}}{v_t^{1*}r_{t+1}}\middle|\Im_t\right\} = 1.$$

If agent 1 is not informed, then agent 2 is also not informed. In this case, $v_t^{2*}=v_t^{1*}$. Hence, the equation above always holds.

However, $\alpha_t^2/\alpha_t^1 = \prod_{\tau=1}^{t-1}\left(v_\tau^{2*}r_{\tau+1}/v_\tau^{1*}r_{\tau+1}\right)$, and so

$$\alpha_{t+1}^2/\alpha_{t+1}^1 = \left(\alpha_t^2/\alpha_t^1\right)\left(v_t^{2*}r_{t+1}/v_t^{1*}r_{t+1}\right).$$

Hence,

$$\mathcal{E}\left\{\frac{\alpha_{t+1}^2}{\alpha_{t+1}^1}\middle|\Im_t\right\} = \frac{\alpha_t^2}{\alpha_t^1}.$$

Thus, $\left\{\alpha_t^2/\alpha_t^1\right\}$ is a positive martingale and, hence, converges $\mathcal{P}$-almost surely.

■

Lemma A.3. *Let $\{\vartheta_t\}$ be a sequence of independent random variables that only assume the values* 0 *and* 1, *and satisfy $\mathcal{P}(\vartheta_{t+1} = 1) = q > 0$. Let $\{\phi_t\}$ be a sequence of (possibly dependent) random variables that also only assume the values* 0 *and* 1, *with ϕ_t independent of ϑ_t for all t. Define $A \equiv \{\phi_t = 1$ infinitely often$\}$ and $B \equiv \{\phi_t \vartheta_t = 1$ infinitely often$\}$. Then,*

$$\mathcal{P}(B) = \mathcal{P}(A).$$

Proof - If $\mathcal{P}(A) = 0$ then $\mathcal{P}(B) = 0$ because $B \subset A$. If $\mathcal{P}(A) > 0$ then $\mathcal{P}(B) = \mathcal{P}(B \mid A).\mathcal{P}(A)$. However, by the Borel-Cantelli lemma, $\mathcal{P}(B \mid A) = 1$. ■

Lemma A.4. *Let $\{z_t\}$ be a sequence of uniformly bounded random variables such that for every $t \geqslant 1$ and $l \geqslant 1$, $\mathcal{E}\left(z_{t+l} | \Im_t\right) = 0$. Then,*

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} z_t = 0 \;\; \mathcal{P}\text{-a.s.}$$

Proof - The random variables $\{z_t\}$ are uncorrelated because for every $t \geqslant 1$ and $l \geqslant 1$,

$$\mathcal{E}\left(z_t\right) = \mathcal{E}(\mathcal{E}\left(z_t | \Im_{t-1}\right)) = 0$$

and

$$\mathcal{E}\left(z_t z_{t+l}\right) = \mathcal{E}\left(\mathcal{E}\left(z_t z_{t+l} | \Im_t\right)\right) = \mathcal{E}\left(z_t \mathcal{E}\left(z_{t+l} | \Im_t\right)\right) = 0.$$

Thus, $cov(z_t z_{t+l}) = 0$. The conclusion follows from the law of large numbers for uncorrelated random variables (see Chung [4, Thm 5.1.2]). ■

Lemma A.5. *Assume that agent* 2*'s method of gathering information is inferior to agent* 1*'s. Suppose $\sigma_1 > 0$. Assume that either the level of noise and informativeness of the signals is intermediary (case B) and $\sigma_3 > 0$ or assume that the level of noise is high and the signals are weakly informative (case C). Then, interest rates do not reveal the signal in scenario s_1, infinitely often, $\mathcal{P}$-almost surely.*

Proof - In case C, interest rates do not reveal the signal in scenario s_1 infinitely often, because interest rates are not revealing, in any scenario, with probability bounded away from zero.

Consider now case B and $\sigma_3 > 0$. We will show that the wealth of agent 1 is below $\frac{z}{x}$ infinitely often and, therefore, by Lemma A.3, interest rates will not reveal the signal in scenario s_1, infinitely often.

Let $D \in \Im$ be the set of outcomes such that, at most finitely many times, interest rates do not reveal the signal in scenario s_1. Assume, by contradiction, that $\mathcal{P}(D) > 0$. Let $D_t \in \Im$ be the set of outcomes such that interest rates always reveal the signal, in scenario s_1, from period t on. Clearly, $D_t \subset D$ and there exists $\bar{t}$ such that $\mathcal{P}(D_{\bar{t}}) > 0$.

Define

$$\bar{k} \equiv \min_{s \in D_{\bar{t}}} \frac{\alpha_{\bar{t}}^2(s)}{\alpha_{\bar{t}}^1(s)}, \; \varepsilon \equiv \frac{z}{x}\frac{\bar{k}}{2+\bar{k}} > 0, \text{ and } \bar{\alpha}^u \equiv 1 - z - (\frac{z}{x} - \varepsilon) > 0.$$

Let $\bar{\rho}^b \in (0,1)$ and $\bar{\rho}^g \in (0,1)$ be given by

$$\bar{\rho}^b \equiv \frac{x}{z}(1 - z - \bar{\alpha}^u) \text{ and } \bar{\rho}^g \equiv 1 - \frac{x}{z}(1 - z - \bar{\alpha}^u).$$

Let ϑ_t be the sequence of random variables defined by

$$\vartheta_t = \begin{cases} 1, & \text{if in period } t \text{ the scenario is } s_1\text{, and either} \\ & \quad \text{the signal is good and } \rho_t^n \leqslant \bar{\rho}^g\text{, or} \\ & \quad \text{the signal is bad and } \rho_t^n \geqslant \bar{\rho}^b\text{,} \\ 0, & \text{otherwise.} \end{cases}$$

Let $E \in \Im$ be the set of outcomes such that ϑ_t equals 1 and $\alpha_t^1 \leqslant \frac{z}{x} - \varepsilon$ at most finitely many times.

Interest rates do not reveal the signal in scenario s_1 when $\alpha_t^1 \leqslant \frac{z}{x} - \varepsilon$, and either the good signal and $\rho_t^n \leqslant \bar{\rho}^g$ occurs or the bad signal and $\rho_t^n \geqslant \bar{\rho}^b$ occurs. Hence, $D \subset E$.

Let ϕ_t^1 be the sequence of random variables defined by

$$\phi_t^1 = \begin{cases} 1, & \text{if } \alpha_t^1 \leqslant \frac{z}{x} - \varepsilon, \\ 0, & \text{otherwise.} \end{cases}.$$

Let $F \in \Im$ be the set of outcomes such that $\alpha_t^1 \leqslant \frac{z}{x} - \varepsilon$ occurs finitely often. By Lemma A.3, $\mathcal{P}(F) = \mathcal{P}(E)$, and since $F \subset E$, $\mathcal{P}(D \cap F) = \mathcal{P}(D \cap E) = \mathcal{P}(D)$. Hence, $\mathcal{P}(D_{\bar{t}} \cap F) > 0$.

The portfolios of agents 1 and 2 differ only in scenario s_1, and then only when interest rates do not reveal the signal. Hence, for every $s \in D_{\bar{t}}$, $\alpha_t^1/\alpha_t^2 = \alpha_{\bar{t}}^1(s)/\alpha_{\bar{t}}^2(s)$ for $t \geq \bar{t}$. Moreover, by the definition of ε, for every $s \in D_{\bar{t}}$, if $\alpha_{\bar{t}}^1(s) \geqslant \frac{z}{x} - \varepsilon$ then $\alpha_{\bar{t}}^1(s) + \alpha_{\bar{t}}^2(s) \geqslant \frac{z}{x} + \varepsilon$.

Consider a path $s \in D_{\bar{t}} \cap F$. There exists $t(s)$ such that if $t \geqslant \max\{\bar{t}, t(s)\}$, then $\alpha_t^1 + \alpha_t^2 \geqslant \frac{z}{x} + \varepsilon$. Hence, since $z \geq \frac{x}{2+x}$,

$$\alpha_t^3(s) \leqslant 1 - z - \left(\frac{z}{x} + \varepsilon\right) \leq \frac{z}{x} - \varepsilon.$$

Let H be the set of outcomes in which scenario s_3 and, either the good signal and $\rho_t^n \leqslant \bar{\rho}^g$, or, the bad signal and $\rho_t^n \geqslant \bar{\rho}^b$, occurs with strictly positive frequency. By the law of large numbers, $\mathcal{P}(H) = 1$. Hence,

$$\mathcal{P}(D_{\bar{t}} \cap F \cap H) > 0.$$

Interest rates are not revealing if scenario s_3 occurs, $\alpha_t^3 \leq \frac{z}{x} - \varepsilon$, and either the good signal and $\rho_t^n \leqslant \bar{\rho}^g$ occurs, or the bad signal and $\rho_t^n \geqslant \bar{\rho}^b$ occurs. Hence, interest rates are not revealing in scenario s_3, with strictly positive frequency, on $D_{\bar{t}} \cap F \cap H$.

By Lemma A.1,

$$\mathcal{E}\left\{\log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) \middle| \Im_t\right\} = \bar{\zeta} > 0,$$

in scenario s_3, if interest rates do not reveal the signal, and

$$\mathcal{E}\left\{\log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) \middle| \Im_t\right\} = 0,$$

in scenario s_3, if interest rates reveal the signal.

By definition, in all paths of $D_{\bar{t}} \cap F$, if $t \geqslant \bar{t}$ then

$$\mathcal{E}\left\{\log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) \middle| \Im_t\right\} = 0$$

in scenarios s_1 and s_{12} because interest rates reveal the signal to agent 3. Therefore,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mathcal{E}\left\{\log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) \middle| \Im_t\right\} > 0 \text{ on } D_{\bar{t}} \cap F \cap H.$$

Define the sequence of random variables $\{z_t\}$ by

$$z_{t+1} \equiv \log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) - \mathcal{E}\left\{\log\left(\frac{v_t^{3*} r_{t+1}}{v_t^{1*} r_{t+1}}\right) \middle| \Im_t\right\}.$$

This sequence is uniformly bounded because $\log\left(\frac{v_t^{3*}r_{t+1}}{v_t^{1*}r_{t+1}}\right)$ assumes only the values $\pm log(1+x)$, $\pm log(1-x)$, and zero. By construction, for all $t \geqslant 1$ and $l \geqslant 1$, $E\{z_{t+l}|\Im_t\} = 0$. By Lemma A.4,

$$\lim_{t\to\infty}\frac{1}{t}\log\left(\frac{\alpha_t^3}{\alpha_t^1}\right) = \lim_{t\to\infty}\frac{1}{t}\sum_{\tau=1}^{t-1}\log\left(\frac{v_\tau^{3*}r_{\tau+1}}{v_\tau^{1*}r_{\tau+1}}\right) > 0,\ \mathcal{P}\text{-a.s., on } D_{\bar{t}}\cap F\cap H.$$

However, $\alpha_t^3 \leqslant 1$. Hence,

$$\lim_{t\to\infty}\ \alpha_t^1 = 0,\ \mathcal{P}\text{-a.s., on } D_{\bar{t}}\cap F\cap H.$$

But F is the set of outcomes for which $\alpha_t^1 \leq \frac{z}{x} - \varepsilon$ only occurred a finite number of times, a contradiction.

■

Lemma A.6 (Freedman [11, Prop 4.5]). *Let $\{z_t\}$ be a sequence of uniformly bounded random variables such that for every $t \geqslant 1$, $\mathcal{E}(z_{t+1}|\Im_t) = 0$. Let $V_t \equiv VAR(z_{t+1}|\Im_t)$ where VAR is the variance operator associated with $\mathcal{P}$. Then,*

$$\sum_{t=1}^{n} z_t \text{ converges to a finite limit as } n\to\infty,\ \mathcal{P}\text{-a.s., on } \left\{\sum_{t=1}^{\infty} V_t < \infty\right\}$$

and

$$\sup_n \sum_{t=1}^{n} z_t = \infty \text{ and } \inf_n \sum_{t=1}^{n} z_t = -\infty,\ \mathcal{P}\text{-a.s., on } \left\{\sum_{t=1}^{\infty} V_t = \infty\right\}.$$

Proof of Proposition 3 - (Part 1). By Lemmas A.1 and A.5,

$$\mathcal{E}\left\{\log\left(\frac{v_t^{1*}r_{t+1}}{v_t^{2*}r_{t+1}}\right)\middle|\Im_t\right\} = \bar{\zeta} > 0 \text{ infinitely often.}$$

However,

$$\mathcal{E}\left\{\log\left(\frac{v_t^{1*}r_{t+1}}{v_t^{2*}r_{t+1}}\right)\middle|\Im_t\right\} \geqslant 0$$

because agent 1 is informed whenever agent 2 is informed. Therefore, $\mathcal{P}$-almost surely,

$$\sum_{t=1}^{\infty}\mathcal{E}\left\{\log\left(\frac{v_t^{1*}r_{t+1}}{v_t^{2*}r_{t+1}}\right)\middle|\Im_t\right\} = \infty.$$

By Lemma A.2, $\mathcal{P}$-almost surely,

$$\log\left(\frac{\alpha_t^1}{\alpha_t^2}\right) = \sum_{\tau=1}^{t-1} \log\left(\frac{v_\tau^{1*} r_{\tau+1}}{v_\tau^{2*} r_{\tau+1}}\right) \text{ converges.}$$

We wish to show that $\log\left(\frac{\alpha_t^1}{\alpha_t^2}\right)$ converges to positive infinity, $\mathcal{P}$-almost surely. So, suppose not. That is, suppose $\log\left(\frac{\alpha_t^1}{\alpha_t^2}\right)$ converges, with positive probability, to minus infinity or to a finite number.

Let

$$z_{t+1} \equiv \log\left(\frac{v_t^{1*} r_{t+1}}{v_t^{2*} r_{t+1}}\right) - \mathcal{E}\left\{\log\left(\frac{v_t^{1*} r_{t+1}}{v_t^{2*} r_{t+1}}\right) \middle| \Im_t\right\}.$$

Then, $\{z_t\}$ is a sequence of uniformly bounded random variables such that $\mathcal{E}\{z_{t+1}|\Im_t\} = 0$. Moreover, $\sum_t z_t$ converges, with positive probability, to minus infinity. This contradicts Lemma A.6. Hence, $\log\left(\frac{\alpha_t^1}{\alpha_t^2}\right)$ converges to positive infinity, $\mathcal{P}$-almost surely. But, $\alpha_t^1 \leqslant 1$. Therefore, α_t^2 converges to zero, P-almost surely.

(Part 2). Assume, by contradiction, that, with strictly positive probability, interest rates do not reveal the signal infinitely often. Then, by the argument given above, both agents 2 and 3 (who only observe the signal when 1 does) vanish. So, agent 1 dominates the economy. Therefore, the relative wealth of agent 1 will eventually be above $\frac{z}{x}$, a contradiction. Therefore, with probability one, interest reveal the signal only finitely often and all agents survive.

■

A.4. Proof of Proposition 4

Assume, by contradiction, that α_t^j converges to $1-z$ on a set $A \in \Im$ such that $\mathcal{P}(A) > 0$. Then, α_t^k converges to 0, on A for all rational $k \neq j$. Thus,

$$\log\left(\frac{\alpha_t^j}{\alpha_t^k}\right) = \sum_{\tau=1}^{t-1} \log\left(\frac{v_\tau^{j*} r_{\tau+1}}{v_\tau^{k*} r_{\tau+1}}\right) \text{ converges to positive infinity on } A.$$

Since $1 - z > \frac{z}{x}$, α_t^j is eventually greater than $\frac{z}{x}$ on A. Hence, except for at most finitely many times,

$$\mathcal{E}\left\{\log\left(\frac{v_t^{j*} r_{t+1}}{v_t^{k*} r_{t+1}}\right) \middle| \Im_t\right\} = 0, \text{ whenever } j \text{ is informed, on } A.$$

By Lemma A.1,

$$\mathcal{E}\left\{\log\left(\frac{v_t^{j*}r_{t+1}}{v_t^{k*}r_{t+1}}\right)\middle|\Im_t\right\}\leqslant 0,\ \text{whenever } j \text{ is uninformed.}$$

Let $\{z_t\}$ be the sequence of uniformly bounded random variables defined by

$$z_{t+1}\equiv\log\left(\frac{v_t^{j*}r_{t+1}}{v_t^{k*}r_{t+1}}\right)-\mathcal{E}\left\{\log\left(\frac{v_t^{j*}r_{t+1}}{v_t^{k*}r_{t+1}}\right)\middle|\Im_t\right\}.$$

Then, $\mathcal{E}\{z_{t+1}|\Im_t\}=0$ and $\sum_t z_t$ converges, with positive probability, to infinity. This contradicts Lemma A.6.

■

A.5. Proof of Proposition 5

(Part 1) Let $\bar{\rho}^b\in(0,1)$ and $\bar{\rho}^g\in(0,1)$ be given by

$$\bar{\rho}^b\equiv\frac{x}{z}(1-z)\ \text{and}\ \bar{\rho}^g\equiv 1-\frac{x}{z}(1-z).$$

Let $\{\eta_t\}$ be the sequence of random variables defined by

$$\eta_t=\begin{cases}\bar{\zeta}, & \text{when } j \text{ is informed but } k \text{ is not, and either}\\ & \quad\text{the signal is good and } \rho_t^n\leqslant\bar{\rho}^g \text{ occurs, or}\\ & \quad\text{the signal is bad and } \rho_t^n\geqslant\bar{\rho}^b \text{ occurs,}\\ -\bar{\zeta}, & \text{when } k \text{ is informed but } j \text{ is not,}\\ 0, & \text{otherwise,}\end{cases}$$

where $\bar{\zeta}$ is defined in (12). We also define

$$\varsigma_t\equiv\mathcal{E}\left\{\log\left(\frac{v_t^{j*}r_{t+1}}{v_t^{k*}r_{t+1}}\right)\middle|\Im_t\right\}.$$

Since $\left(1-\frac{x}{z}(1-z)\right)\sigma^{j/k}>\sigma^{k/j}$, $\{\eta_\tau\}$ is a sequence of independent, identically distributed random variables such that

$$E(\eta_\tau)>0.$$

Moreover,

$$\varsigma_t\geqslant\eta_t,$$

because $\alpha_t^u \geqslant 0$, $\rho_t^n(b) \leqslant \bar{\rho}^b$ and $\rho_t^n(g) \geq \bar{\rho}^g$.

By the law of large numbers, $\mathcal{P}$-almost surely,

$$\liminf \frac{1}{t}\sum_{\tau=1}^{t} \varsigma_\tau \geqslant \liminf \frac{1}{t}\sum_{\tau=1}^{t} \eta_t > 0.$$

By Lemma A.4, $\mathcal{P}$-almost surely,

$$\liminf \frac{1}{t}\log\left(\frac{\alpha_t^j}{\alpha_t^k}\right) = \liminf \frac{1}{t}\sum_{\tau=1}^{t-1}\log\left(\frac{v_t^{j*} r_{t+1}}{v_t^{k*} r_{t+1}}\right) = \liminf \frac{1}{t}\sum_{\tau=1}^{t-1}\varsigma_\tau > 0.$$

However, $\alpha_t^j \leqslant 1$. Therefore, $\mathcal{P}$-almost surely,

$$\liminf \log\left(\frac{\alpha_t^j}{\alpha_t^k}\right) = \infty \Rightarrow \lim\left(\frac{\alpha_t^j}{\alpha_t^k}\right) = \infty \Rightarrow \alpha_t^k \to 0.$$

(Part 2) Since $\left(1 - \frac{x}{z}(1-z)\right)\sigma^{j/k} < \sigma^{k/j}$, $\{\eta_t\}$ is a sequence of independent, identically distributed random variables such that

$$\mathcal{E}(\eta_t) < 0.$$

Assume, by contradiction, that α_t^j converges to $1-z$ on a set $A \in \Im$ such that $\mathcal{P}(A) > 0$. Then, α_t^k converges to 0, on A. Hence, in scenarios that j is informed, $\rho_t^n(b)$ converges to $\bar{\rho}^b$ and $\rho_t^n(g)$ converges to $\bar{\rho}^g$ on A. However, in the scenarios that k is informed, $\rho_t^n(b)$ converges to 0 and $\rho_t^n(g)$ converges to 1 on A. Thus,

$$\mathcal{E}\left\{(\varsigma_t - \eta_t)\middle|\,\Im_{t-1}\right\} \text{ converges to zero on } A.$$

So,

$$\frac{1}{t}\sum_{\tau=1}^{t}\mathcal{E}\left\{(\varsigma_\tau - \eta_\tau)\middle|\,\Im_{t-1}\right\} \to 0 \text{ on } A.$$

By Lemma A.4, and the law of large numbers,

$$\lim \frac{1}{t}\sum_{\tau=1}^{t}\varsigma_\tau = \lim \frac{1}{t}\sum_{\tau=1}^{t}\eta_\tau < 0 \text{ on } A.$$

By Lemma A.4,

$$\frac{1}{t}\sum_{\tau=1}^{t}\log\left(\frac{v_t^{j*} r_{t+1}}{v_t^{k*} r_{t+1}}\right) < 0 \text{ on } A.$$

So,

$$\log\left(\frac{\alpha_t^j}{\alpha_t^k}\right) = \sum_{\tau=1}^{t-1} \log\left(\frac{v_t^{j*} r_{t+1}}{v_t^{k*} r_{t+1}}\right)$$

converges to minus infinity on A. However, $\alpha_t^k \leqslant 1$. Hence, α_t^j converges to zero on A, contradiction.

■

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