CARESS Working Paper 00-02
The Drawbacks of Electoral Competition

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September 1999. This version, March 2000.

Abstract

According to the conventional view, in politics, just as in economic markets, competition between politicians is a force that pushes towards efficiency. We provide a model that challenges this view. In the model, candidates can promise to provide a public good or to engage in redistributive politics. We show that the more intense is competition (measured by an increase in the number of candidates) the greater the inefficiency. This is because the tendency to focus on policies that provide particularistic benefits increases with the number of candidates at the expense of policies that benefit the population at large.

We also examine the impact of voters' ideology, participation, and information on the efficiency of the electoral process, by allowing for heterogeneity in voters' responsiveness to electoral promises. The larger the fraction of non-responsive voters, the less efficient the political process. This is because electoral competition focuses on swing voters, increasing the value of policies with targetable benefits.

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1 Introduction

Electoral competition is typically viewed as a force pushing towards the efficiency of political outcomes. For instance, drawing an explicit parallel between economic and political markets, Becker (1958) defines an *ideal political democracy* as a system in which "individuals endeavor to acquire political office through perfectly free competition for the votes of a broadly based electorate," and argues that an ideal political democracy would be "perfectly responsive to the 'will' of people." In Becker's view, the shortcomings of real-world democratic systems are due to some measure of monopoly power of existing candidates. This suggests that removing the barriers to entry in the political process would reduce the inefficiencies by increasing the extent of electoral competition.[1]

Even those who identify further discrepancies between real-world democracies and the "ideal political democracy" (candidates may lack the ability to commit to platforms, voters may be poorly informed)[2] do not challenge the view that, by providing a check on self-interested politicians, electoral competition is beneficial.[3]

We challenge the view that electoral competition is necessarily beneficial. In our model, the more intense is electoral competition (as measured by the number of candidates), the greater the inefficiency. The idea is the following. The nature of elections is such that candidates do not, in general, obtain unanimous approval; rather, they receive support from a fraction of the electorate. Therefore, particularistic (or pork-barrel) projects that can be targeted to subsets of the electorate can be more appealing to politicians, than policies that benefit the population at large. This fundamental distortion results in inefficiencies, the severity of which depends on the number of parties. The larger the number of parties, the smaller the fraction of the electorate whose votes a candidate will win in equilibrium, and hence the greater the appeal of particularistic policies relative to policies with diffuse benefits. This tradeoff between the targetability of particularistic policies and the efficiency of policies that benefit the population at large, worsens with the number of parties.

This argument does not require that candidates should lack the ability to commit

[1] The similarity between electoral competition and economic competition has also been explored by Buchanan (1968), Stigler (1972), and Wittman (1989).

[2] See Osborne and Slivinski (1996) and Besley and Coate (1998), and Coate and Morris (1995), respectively. For a survey of models dealing with the inefficiency of democracy see Persson and Tabellini (2000) .

[3] See Barro (1973) for an early account of the electoral process as a mechanism to move the private preferences of the officeholder towards those of the electorate.

to a platform, or that they be better informed than voters. In fact, we study the effects of electoral competition in a model that is very close in spirit to the "ideal political democracy." However, our conclusions regarding the efficiency of electoral competition are markedly different from Becker's.

In our model, there are N candidates who compete for office by making a binding electoral promise to each voter concerning the allocation of a budget if elected. Candidates maximize their expected vote share. Voters are homogeneous: each voter will vote for the candidate promising him the most utility. Candidates have a budget of one dollar per voter and are faced with two possible choices: promise to invest all the money in the public good in which case all voters get utility G (in money units), or promise to redistribute the money. If $G > 1$ efficiency requires that the public good be provided. Money (local public goods, pork-barrel projects), however, can be targeted to subsets of voters. Even when efficiency requires that the public good be provided, it may not be an equilibrium for candidates to do so. Suppose that all candidates offer the public good. Then, they receive an equal share of $1/N$ of the vote. Now, if $G < N$, offering transfers of more than G to more than $1/N$ of the voters is feasible, and is a superior strategy. Thus, it is not an equilibrium for all candidates to offer the public good, and hence the equilibrium is inefficient for $1 < G < N$.

In this model, the inefficiency takes the form of under-provision of the public good. This is in contrast with the classic analysis of the inefficiency of provision of public goods in democracies (see for instance Stiglitz (1988)) which displays over- or under-provision, depending on the difference between the policy preferred by the median voter and the Samuelsonian optimum. This median voter model relies on exogenous restrictions on the dimension of the policy space, such as linear taxes, and therefore (1) cannot capture the incentives toward tactical redistribution which underlie our results, and (2) does not yield clear implications about the effect of the number of parties on the efficiency of the outcome.

Efficiency in our model is given by the probability that the public good is provided. The range of values of G that give rise to an inefficient equilibrium grows with N. This suggests that also the size of the inefficiency should increase with the number of candidates. Indeed, we show that, for any given value of G, the probability that the public good is provided is decreasing in the number of candidates. Intuitively, this is because the value of the targetability of transfers becomes more important as the number of candidates increases. As the number of candidates goes to infinity, the probability that the public good is provided converges to zero.

This inefficiency result is in contrast with the familiar efficiency result obtained in the Cournot model of market competition, where increasing the number of firms increases the efficiency of the equilibrium. Thus, in political markets, contrary to economic markets, it may be beneficial to restrict the number of competing candidates. We show that one could view our result as buttressing majoritarian electoral systems, if one takes "Duverger's Law" as implying that majoritarian electoral rules result in equilibria with a small number of parties (two). Even if one does not accept this interpretation of Duverger's Law, our model suggests an interesting testable implication, that the fraction of resources spent on public goods should be a decreasing function of the number of political parties in a democracy.

We then introduce heterogeneity among voters by allowing them to differ in their degree of responsiveness to electoral promises. This can be interpreted in several ways. One interpretation is that there is an ideological dimension in voters' preferences that makes some voters more likely ex-ante to favor a specific candidate. An alternative interpretation is that some groups of voters are less likely to participate in the election. Finally, some voters might be less informed than others, and hence less responsive to differences in the offers made by the different candidates. We call voters who are less responsive partisans and the others swing voters. We study the effects of this heterogeneity on the efficiency of the electoral process. Candidates compete less intensely for voters who are less responsive to electoral promises.[4] This has a consequence for efficiency because, when electoral competition focuses on swing voters, candidates have an even stronger incentive to provide pliable policies targeted to these voters, instead of policies whose services benefit swing and partisan voters equally. This leads to lower provision of the public good, relative to the case where voters have no ideological bias.

Thus, in our model, it is not lack of responsiveness by voters per se that creates inefficiencies. Rather, it is the differential responsiveness by different groups of voters. This may be relevant for thinking about policies to increase voters' participation. We also use this insight to discuss the role of campaign spending.

1.1 Related Literature

Inefficiency in Legislatures

Most of the literature on the inefficiency of democracy focuses on decision making in legislatures (see Persson and Tabellini (2000) for a comprehensive survey of the

[4]This feature of our model parallels a finding by Lindbeck and Weibull (1987), and has been documented empirically by Stromberg (1999). See Section 7.4 for more on the comparison between our model and other models of redistributive politics.

literature). For instance, Baron (1991) models the legislative process via a sequential bargaining model. These models analyze specific, very structured legislative processes; they do not address candidates competing in large elections.

In Chari *et al.* (1997), policy is determined through bargaining between a president and local representatives. This gives rise to a common pool problem whereby an excessive number of local public projects are financed from general taxation (see also Weingast *et al.* (1981)). Similarly, Persson *et al.* (1997) compare congressional and parliamentary systems. In their model, the politician choosing the level of public good provision has the option of foregoing the public good and appropriating the money for his own district; again, a common pool problem arises. Our setup differs from these models in the following essential way. Increasing the number of districts does not increase the extent of competition, as legislators represents individual districts and do not compete in the same arena. In contrast, we focus on national candidates who do not represent any specific district: when proposing to increase transfers to a district, our candidates take into account the loss of another district. Thus, the common pool problem is logically different from the incentives generated by electoral competition. The common pool inefficiency arises only because legislators are assigned to districts.

Inefficiency in Elections

Lizzeri and Persico (1998) compare winner-take-all with proportional systems, and consider the effects of the magnitude of districts, in terms of public good provision. That paper only considers two-candidate elections. The main point of that paper is that, even with two candidates these systems lead to different outcomes. The current paper focuses on one electoral system only to deal with the issue of the role of electoral competition.

Persson and Tabellini (1999) discuss a two-candidates model related to Lizzeri and Persico (1998) and also provide some empirical evidence on the provision of public goods in different political systems.

Coate and Morris (1995) focus on voters' imperfect information on the effects of government policy. Besley and Coate (1998) discuss the efficiency of representative democracy where candidates are citizens who, if elected, implement their favorite policies. Theirs is a model of repeated elections, and the inefficiency depends on the dynamic nature of the model.

Number of Candidates

Myerson (1993) compares electoral systems in terms of the inequality of redistribution in a model with N candidates. Our paper borrows the model of redistribution from Myerson, and adds a public good. The presence of the public good introduces the tradeoff between efficiency and targetability which is the focus of our analysis, and affords efficiency results which are absent in Myerson's paper.

The effect of the number of candidates on political competition has been explored in the setting of spatial competition (see, for instance, Palfrey (1984), Austen-Smith and Banks (1988)). The spatial model is not naturally suited to discuss efficiency (no two points in the policy space are Pareto-comparable); see Shepsle (1991) for a review of this strand of the literature. Besley and Coate (1997) and Osborne and Slivinski (1996) provide a different model of multi-candidate elections with endogenous entry by citizen-candidates who have policy preferences and do not commit to electoral platforms.

Heterogeneity in the Responsiveness of Voters

Lindbeck and Weibull (1987) and Dixit and Londregan (1995, 1998) are models of redistributive politics with ideological voters. For a detailed discussion of the relationship between our model and this strand of the literature, see Section 7.4.

Alesina, Baqir, and Easterly (1999) discuss the provision of public goods in relation to ethnic divisions. Their analysis of U.S. local public goods shows that the shares of spending on public goods is inversely related to ethnic fragmentation. To explain this phenomenon, they provide a model of two-stage budgeting procedure, where first voting takes place on the size of the budget, and in a second stage on its composition. Our model of Section 6 provides an alternative explanation for this phenomenon.

1.2 Outline of the Paper

Section 2 describes the model. Section 3 presents the key technical step that allows us to solve for the equilibria in the paper. This is a necessary condition of linearity of the payoff of candidates, that must hold in equilibrium regardless of parameters such as the number of candidates, or the degree of ideological heterogeneity of voters. This condition is applied later in a variety of settings, to yield a complete characterization of equilibrium. Section 4 is the central part of the paper; it describes the equilibrium with equal (ex-ante) treatment of voters, and provides the comparative statics result with respect to the number of candidates. Section 6 introduces ideology. Section 7 discusses a number of implications of the model. Section 8 concludes.

2 Model

2.1 Economy and Agents

There are N candidates. There is a continuum of voters with measure one. The set of voters is denoted by V. There are two goods, money and a public good. The public good can only be produced by using all the money in the economy.[5]

Each voter has an endowment of one unit of money. The public good yields a utility of G to each voter. Voters have no a priori preference for either candidate, and have linear utility over goods.[6]

Candidates make binding promises to each voter concerning the policy if elected. A candidate can offer to provide the public good (to all voters); alternatively, he can offer different taxes and transfers to different voters. Each voter votes for the candidate who promises her the greatest utility. Candidates maximize their expected vote share. This assumption on candidates' objective can be interpreted as describing a *proportional system* where the spoils of office (seats in an assembly) are divided proportionally to the share of the vote.

2.2 Game

A pure strategy for a candidate specifies whether he chooses to offer the public good or transfers (he cannot offer both). In the event he chooses transfers, a pure strategy specifies a promise of a transfer to each voter. Formally, a pure strategy is a function $\Phi : V \to [0, +\infty)$, where $\Phi(v)$ represents the consumption promised to voter v. The function Φ satisfies one of two conditions: either $\Phi(v) = G$ for all v's, signifying that the candidate offers the public good; or, $\int_V \Phi(v)dv = 1$, which is the balanced budget condition when a candidate offers transfers. In the latter case, $\Phi(v) - 1$ represents the transfer (or tax, if negative) promised to voter v.

There are two stages of the game:

Stage 1 Candidates choose offers to voters simultaneously and independently.

Stage 2 Each voter v gets offers $(\Phi_1(v), \ldots, \Phi_N(v))$ from candidates. After observing the offers, voter v votes for candidate i if, for all $j \neq i$, $\Phi_i(v) > \Phi_j(v)$. Ties are

[5] This assumption can be relaxed. See Lizzeri and Persico (1998) for one way to do so in the case of two candidates.

[6] This assumption is made solely to simplify the notation and presentation of the results; all the results hold for any increasing utility function. Thus, in Section 5 we will extend our analysis to the case in which voters are risk averse.

resolved by randomizing with equal probability.

Voters' behavior can be made fully consistent with the usual rationality assumptions (strategic voting). Assume that the probability that the policy proposed by any given candidate is implemented is equal to this candidate's share of the vote; then, it is a dominant strategy for a voter to vote for the candidate who proposes the policy that is best for her. This interpretation of policy implementation is the one that we will adopt when computing the probability of provision of the public good.

For values of G smaller than N there is no equilibrium in pure strategies. To see this, consider for instance the case of two candidates. Suppose candidate 1's strategy was Φ_1. If $\Phi_1(v) = G$ for all v, i.e. candidate 1 promises each voter the public good, then candidate 2 can choose to promise more than G to more than 50 percent of the voters and obtain more than 50 percent of the votes. This is impossible in equilibrium. Suppose then that candidate 1 chooses to offer money. Now candidate 2 could take a set of voters V_1 with small positive measure such that $\Phi_1(v) > 0$ for $v \in V_1$, offer zero to these voters and use the saved money to finance offers of $\Phi_1(v) + \epsilon$ to all other voters. The set V_1 and the ϵ can be chosen so that candidate 2 wins with a share of the vote arbitrarily close to one hundred percent. Thus, at equilibrium both candidates will be employing mixed strategies.

A mixed strategy in this game could be a very complicated object, since the space of pure strategies is large. We discuss the case where $\Phi_i(v)$, the offer made by candidate i to voter v, is a realization from a random variable with c.d.f. $F_i^v : \Re_+ \to [0, 1]$.[7] Note that, even when candidates use mixed strategies, each voter observes her realized promises before voting, not random variables.

We concentrate on the case of *equal treatment*, where, in equilibrium, candidates treat all voters identically ex-ante. It turns out that we only need to look at "simple" strategies of the following form: candidate i chooses to promise the public good with probability β_i and promise money with probability $1 - \beta_i$. When candidates redistribute, they draw promises to all voters from the same F_i (notice the absence of the superscript v).[8] The empirical distribution of transfers by candidate i to voters is F_i; hence, $F_i(x)$ is the fraction of supporters who receive promises below x from candidate i. By manipulating F_i, candidate i is able to target transfers to sections of the electorate. We want to stress that there is a natural interpretation for these mixed strategies: choosing F_i can be thought of as choosing the Lorenz curve in the

[7] We rule out correlation between offers to different voters.

[8] Of course, the fact that offers are realizations of the same random variable does not mean that each voter gets the same offer.

population.[9]

3 The Linearity of Returns to Transfers

In this section we characterize the function $H^*(x)$, the equilibrium probability of winning a voter with an offer of x. This function summarizes all the information about the opponents' behavior that is relevant for a candidate. Regardless of the number of candidates and of the ideological preferences of voters, $H^*(x)$ turns out to be piecewise linear. This crucial property is used in the next sections to obtain a complete characterization of the equilibrium strategies, i.e., the probability of provision of the public good and the distribution of transfers.

Let F^* denote the equilibrium distribution of transfers. We now show that $H^*(x) = x/N$ whenever x belongs to the support of F^* (and $x \neq G$), and $H^*(x) \leq x/N$ otherwise.[10] Candidate 1's problem is to choose a c.d.f. F to maximize his expected vote share subject to the budget constraint. If the candidate offers x to a voter, the probability of winning his vote is $H^*(x)$; if the candidate chooses offers according to F, his expected vote share is $\int H^*(x)\, dF(x)$. Below, we will prove that the min of the support of H^* is zero, and the max is N. So, the candidate solves

$$\max_F \int_0^N H^*(x)\, dF(x) \text{ s.t. } \int_0^N x dF(x) \leq 1.$$

The associated Lagrangean is

$$\mathcal{L} = \int_0^N \left[H^*(x) + \lambda(1-x)\right] dF(x).$$

Let A denote the support of F^*, and λ^* be the optimal value of the Lagrange multiplier. Since F^* maximizes the Lagrangean, it must be that $H^*(x) - \lambda^* x$ is maximal, and constant, on A. Since 0 is the inf of A, it must be that $const = H^*(0) - \lambda^* \cdot 0 = 0$. Since N is the sup of A, we have $0 = H^*(N) - \lambda^* \cdot N = 1 - N\lambda^*$. This shows that $\lambda^* = 1/N$. Since $H^*(x) - \lambda^* x$ is maximal, and equal to zero, on A, we have $H^*(x) = x/N$ for x in A, and $H^*(x) - x/N \leq 0$ outside of A.

[9] Given a poulation with mean income μ and income distribution described by a c.d.f. $F(x)$, the Lorenz curve is the graph of the fraction of total income owned by the poorest u-th fraction of the population, and is given by $L(u) = (1/\mu) \cdot \int_0^u F^{-1}(s)\, ds$.

[10] Notice that $H^*(x)$ is discontinuous at $x = G$ whenever the public good is offered with positive probability.

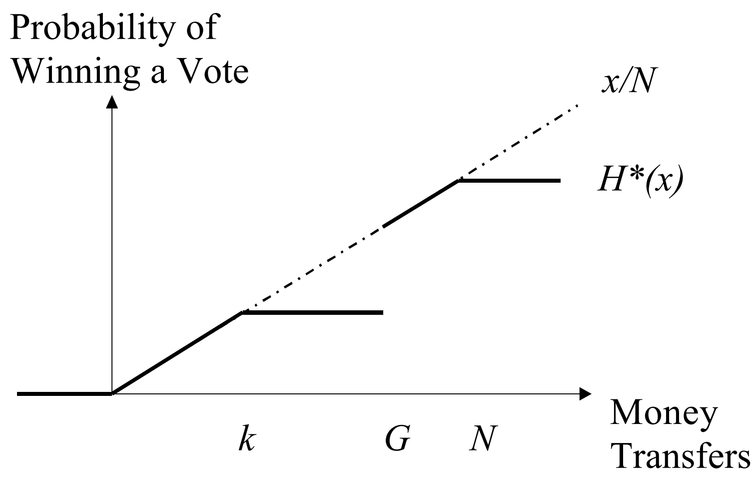

Figure 1: Returns to transfers are linear in equilibrium.

We now verify that the minimum of the support of H^* is zero, and the maximum is N. First, F^* is continuous.[11] Suppose then that the minimum of A is $m > 0$. Because of the budget constraint, $m < G$; therefore, because F^* is continuous, the probability of winning a vote by offering m equals zero. But then a candidate is paying m to obtain zero votes, which cannot be optimal.

Denote with M the maximum of A. At equilibrium, M has to equal N. Indeed, suppose it was smaller than N. Then, a player could deviate and give $N - \varepsilon$ to more than $1/N$ of the voters. This guarantees a vote share larger than $1/N$, contradicting equilibrium. Suppose then that M was larger than N. Because zero and M are in the support of F, one optimal strategy is to offer close to M to $1/M$ of the electorate, and zero to the rest. However, this strategy yields a vote share smaller than $1/N$ if M is greater than N. Thus, $M = N$.

Finally, we show that the support of H^* is the union of two intervals, $[0, k]$ and $[G, N]$, for some positive $k < G$. In other words, we show that the only interval where F can be flat is $[k, G]$. Suppose that there is an interval (a, b) such that $H^*(b) - H^*(a) = 0$, $H^*(G) > H^*(b)$, and such that a and b are in the support of H^*. This means that

[11] Suppose F is discontinuous at $\widehat{x}$. Then, a fraction $p > 0$ of voters are offered $\widehat{x}$. But this cannot be optimal. Candidate 1 gains by offering slightly more than $\widehat{x}$ to $p - \varepsilon$ of these voters, financing this deviation by offering zero to the remaining ε. This deviation profitably breaks the tie at $\widehat{x}$.

the probability of obtaining a vote by offering b is the same as by offering a. However, offering b is more costly. Thus, offering b cannot be optimal. A similar logic applies to the interval $[G, N]$.

There is a simple intuition behind the linearity of $H^*(x)$. $H^*(x)$ represents the expected benefit from spending x. At an optimum, the benefit to a candidate has to equal the shadow cost of x. The cost of spending x is the shadow cost of the budget constraint, i.e., the opportunity cost of funds for a candidate. This opportunity cost increases linearly with x. Therefore, $H^*(x)$ must be linear in x.

4 Equilibrium

Let us first consider the game of pure redistribution. Myerson (1993) provides an analysis of the game where there is no public good. We adapt the following result from his paper.[12]

Theorem 1 *(Myerson) If $G < 1$ there is a unique equilibrium with equal treatment. The public good is not provided, and candidates choose transfers according to the distribution Q_N given by $Q_N(x) = \left(\frac{x}{N}\right)^{\frac{1}{N-1}}$ for x in $[0, N]$.*

To see why this is an equilibrium, observe that the probability of winning a vote with an offer of x is $H^*(x) = [Q_N(x)]^{N-1}$. We showed in Section 3 that $H^*(x) = x/N$. Solving for $Q_N(x)$ concludes the argument.

We now consider the case where $G > 1$. In this case the public good must be provided with positive probability in equilibrium. To see this, suppose that candidates $2, ..., N$ behave according to the equilibrium described in Theorem 1. Then, if candidate 1 offers the public good, he receives a share of the vote of $G/N > 1/N$ for $G > 1$.

Furthermore, if $G < N$ there is no equilibrium where the public good is provided with probability one; if all candidates offer the public good, offering transfers above G to more than one Nth of the voters is feasible and is a profitable deviation. Thus, in the region where $1 < G < N$, candidates randomize between promising transfers and offering the public good.

Theorem 2 *There is a unique equilibrium with equal treatment. For $1 < G < N$, the equilibrium is characterized by a probability β of providing the public good and a distribution of transfers F^* with support $[0, k] \cup [G, N]$. The number k is the unique solution in*

[12] Myerson proves a more general result that treats any rank-scoring rule.

$(0,1)$ to the equation $\left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right)=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right)$, and $\beta=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k}{N}\right)^{\frac{1}{N-1}}$. The probability β is increasing in G. For $G>N$ the public good is provided with probability one.

Proof: In view of the discussion in Section 3, in equilibrium it must be that $H^*(x)=x/N$ on $[0,k]\cup[G,N]$. When H^* has this form, the payoff of a candidate who redistributes according to F is

$$\begin{aligned}&\int_0^\infty H^*(x)\,dF(x)\\ \leq\ &\int_0^\infty \frac{x}{N}dF(x)=\frac{1}{N},\end{aligned}$$

and the strict inequality holds only if F has a larger support than H^*.

Now, let us turn our knowledge of H^* into a characterization of F^*. The probability of winning a vote with an offer of x in $[0,k]$ is

$$H^*(x)=\left[(1-\beta)\,F^*(x)\right]^{N-1}.$$

Equating to x/N and solving for F^* yields

$$F^*(x)=\frac{1}{1-\beta}\left(\frac{x}{N}\right)^{\frac{1}{N-1}}\ \text{ for } x\in(0,k)\,. \tag{1}$$

The probability of winning a vote with an offer of x in $(G,N]$ is

$$H^*(x)=\left[(1-\beta)\,F^*(x)+\beta\right]^{N-1}.$$

Equating to x/N and solving for F^* yields

$$F^*(x)=\frac{1}{1-\beta}\left[\left(\frac{x}{N}\right)^{\frac{1}{N-1}}-\beta\right]\ \text{ for } x\in(G,N)\,. \tag{2}$$

To complete the characterization of F^* we look for conditions to pin down β and k. The first condition is given by the continuity of F^*, which requires $F^*(k)=F^*(G)$. Substituting from (1) and (2), we get

$$\beta=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k}{N}\right)^{\frac{1}{N-1}}. \tag{3}$$

The second condition is given by the budget constraint, i.e., $\int_0^k xf(x)\,dx + \int_G^N xf(x)\,dx = 1$. To compute the budget constraint, observe that, on $[0,k]\cup[G,N]$,

$$f^*(x) = \frac{1}{1-\beta}\left(\frac{1}{N}\right)^{\frac{1}{N-1}}\frac{1}{N-1}x^{\frac{2-N}{N-1}},$$

and therefore

$$\int_a^b xf^*(x)\,dx = \frac{1}{1-\beta}\left(\frac{1}{N}\right)^{\frac{1}{N-1}}\frac{1}{N-1}\frac{x^{\frac{N}{N-1}}}{N}(N-1)\Bigg|_a^b = \frac{1}{1-\beta}\left(\frac{x}{N}\right)^{\frac{N}{N-1}}\Bigg|_a^b.$$

Using this equation, the budget constraint can be expressed as

$$\frac{1}{1-\beta}\left[\left(\frac{k}{N}\right)^{\frac{N}{N-1}}+1-\left(\frac{G}{N}\right)^{\frac{N}{N-1}}\right]=1,$$

or

$$\beta = \left(\frac{G}{N}\right)^{\frac{N}{N-1}}-\left(\frac{k}{N}\right)^{\frac{N}{N-1}}. \tag{4}$$

Equations (3) and (4) form a system of two equations in the unknowns k and β.

It is possible to solve for k. Indeed, from (3) and (4), k must solve

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k}{N}\right)^{\frac{1}{N-1}}=\left(\frac{G}{N}\right)^{\frac{N}{N-1}}-\left(\frac{k}{N}\right)^{\frac{N}{N-1}},$$

or

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right)=\left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right). \tag{5}$$

The function $h(z) = \left(\frac{z}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{z}{N}\right)$ is single-peaked on $[0,\infty)$, has a maximum at $z=1$, and has value zero at $z=0$ and $z=N$. Because h is single peaked, equation (5) only has two solutions: one is $k=G$; the other is $k^*(G,N)=h^{-1}(G)$, where h^{-1} denote the inverse of h on the interval $[0,1]$. Only the second solution can be part of an equilibrium, since the first solution requires that β equals zero, and this is impossible when $1<G<N$. Solving for β is accomplished by substituting $k^*(G)$ into equation (3), to obtain $\beta^*(G,N)$.

Observe that $k^*(G,N)$ is decreasing in G since $h(z)$ is increasing on $(0,1)$ and decreasing on $(1,N)$. Therefore, $\beta^*(G,N)=(G/N)^{\frac{1}{N-1}}-(k^*(G,N)/N)^{\frac{1}{N-1}}$ is increasing in G.

To verify that the pair $\beta^* (G, N)$ and $k^* (G, N)$ indeed forms an equilibrium, we need to check that the expected vote share from offering the public good, S_G, is equal to $1/N$,

$$S_G = \sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} \left[(1-\beta) F^* (G)\right]^{N-1-j} \beta^j = \frac{1}{N}.$$

Using the fact that $\sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} q^{N-1-j} p^j = \left[(q+p)^N - q^N\right] /pN$, we rewrite the above equation as

$$\frac{\left[(1-\beta) F^* (G) + \beta\right]^N - \left[(1-\beta) F^* (G)\right]^N}{\beta N} = \frac{1}{N}. \tag{6}$$

Since $F^* (G) = F^* (k)$ we can rewrite this as

$$\frac{\left[(1-\beta) F^* (G) + \beta\right]^N - \left[(1-\beta) F^* (k)\right]^N}{\beta N} = \frac{1}{N},$$

and after substituting from (1) and (2), we obtain

$$\frac{\left[\frac{G}{N}\right]^{\frac{N}{N-1}} - \left[\frac{k}{N}\right]^{\frac{N}{N-1}}}{\beta N} = \frac{1}{N}.$$

This equation is the same as equation (4). Thus, when $\beta = \beta^* (G, N), k = k^* (G, N)$, it is indeed the case that $S_G = 1/N$. This shows that the pair $\beta^* (G, N), k^* (G, N)$ forms an equilibrium.

Uniqueness follows from the discussion in Section 3 together with the uniqueness of the solutions of β and k. ∎

The analysis of Theorem 2 reduces the characterization of the equilibrium to finding two parameters k and β. These parameters can be easily obtained by first finding the unique $k \neq G$ that solves the equation $\left(\frac{k}{N}\right)^{\frac{1}{N-1}} \left(1 - \frac{k}{N}\right) = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} \left(1 - \frac{G}{N}\right)$. Given k, β is easily computed from $\beta = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k}{N}\right)^{\frac{1}{N-1}}$. It is thus straightforward to compute these parameters and verify numerically that the probability of provision of the public good declines with the number of candidates for every value of G. In the Appendix we present the equilibrium values of β and k for different pairs G, N. For given value of G, $\beta^* (G, N)$ declines with N. Figure 2 below reports the functions $\beta^* (G, N)$ for $N = 2, ..., 6$.

In addition to the numerical results, it is possible to show analytically that as the number of candidates converges to infinity the probability that the public good is provided converges to zero.

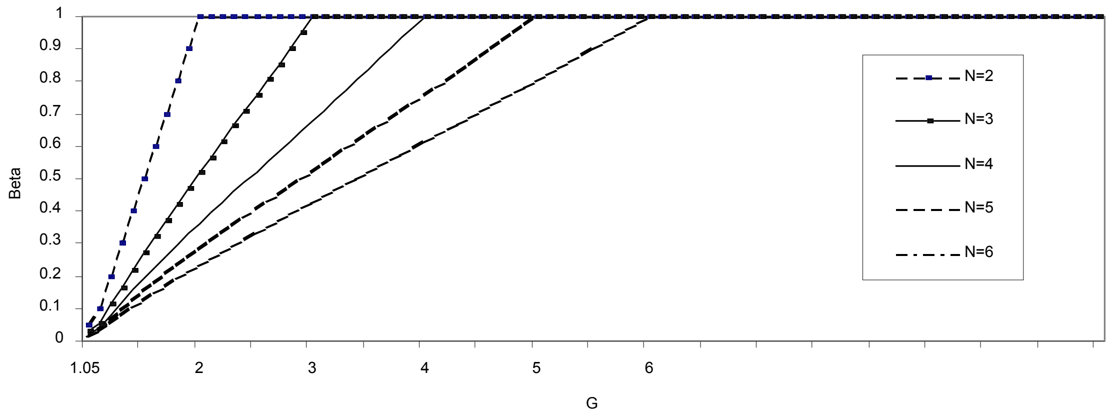

Figure 2: Probability of public good provision with N candidates.

Corollary 1 *For any given $G > 1$, the probability $\beta^*(G, N)$ that the public good is provided converges to zero as the number of candidates N converges to infinity.*

Proof: First, recall from the proof of Theorem 2 that $k^*(G,N) \in (0,1)$, and therefore $\left(1 - \frac{k^*(G,N)}{N}\right) \to 1$. Since $k^*(G,N)$ solves $\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right) = \left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right)$ and the left hand side converges to 1 as $N \to \infty$, it must be that $\left(\frac{k^*(G,N)}{N}\right)^{\frac{1}{N-1}} \to 1$. Noticing that $\beta^*(G,N) = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k^*(G,N)}{N}\right)^{\frac{1}{N-1}}$ and $\left(\frac{G}{N}\right)^{\frac{1}{N-1}} \to 1$ yields the conclusion. ∎

Suppose now that the winning candidate implements redistribution according to the equilibrium strategy, the c.d.f. $F^*(x)$. This can be interpreted as the Lorenz curve (empirical distribution of transfers) in the population. As the number of candidates increases this distribution tends to become more unequal. This is because the support of the distribution is $[0, k^*(G,N)] \bigcup [G, N]$. As N increases, the value of the highest transfers made to voters goes up. Furthermore, the value of $k^*(G,N)$ was shown numerically to decline with N. A more formal discussion of the way the overall distribution of consumption changes with the number of candidates is the topic of the next section.

5 Efficiency

If voters are risk neutral, the only relevant measure of efficiency is the probability that the public good is provided in equilibrium. The effect of the number of candidates on this probability has been discussed above. Here, we consider the welfare effects of the number of candidates in the general case where voters may be risk averse. In this case, voters care not only about the probability of public good provision, but also about the distribution of transfers at equilibrium. We show that any risk averse voter prefers a smaller number of candidates.

It is important to distinguish between ex ante and ex post Pareto efficiency. This is because candidates' strategies involve randomization, and thus there are two notions of allocation, one before and one after the uncertainty is realized. We consider the perspective of a voter ex-ante, before the resolution of the uncertainty generated by candidates' randomized strategies. Before electoral promises are made, allocations are lotteries over consumption. Ex ante Pareto efficiency ranks such allocations, taking the point of view of a voter who considers the expected utility of the outcome of the election before receiving an electoral promise.

We believe that the ex-ante perspective is more appropriate for evaluating the effects of competition and for thinking about issues of constitutional design. This corresponds to evaluating the alternatives behind a Rawlsian veil of ignorance. An alternative viewpoint is the ex-post one, which compares final allocations (after the uncertainty is resolved). After the winning policy is implemented, an allocation is a consumption vector that specifies how much each voter consumes. Ex post Pareto efficiency ranks such allocations, from the viewpoint of a voter who has already received this consumption. In the context of redistributive politics almost all outcomes are bound to be ex-post Pareto efficient, and thus in our context this concept seems less interesting.

Voter v' preferences are now represented by a monotonic, concave utility function $u_v(x)$ defined over consumption. Thus, when the public good is provided this voter experiences a utility $u_v(G)$, and when he receives a consumption (endowment plus transfers) of x, he experiences a utility $u_v(x)$.

When the public good is provided, denote the distribution of promises in consumption terms by $\mathbb{I}_{(-\infty,G)}(x)$ (the right-continuous step function that is zero when $x < G$ and is one when $x \geq G$). Then, the expected c.d.f. of electoral promises (in consumption terms) is

$$J_N(x) = \beta \mathbb{I}_{(-\infty,G)}(x) + (1-\beta) F^*(x).$$

Since in equilibrium all candidates have the same probability of winning, J_N is the expected distribution of electoral promises made by any candidate.

Substituting for $F^*(x)$ from (1) and (2) yields

$$J_N(x) = \begin{cases} \left(\frac{x}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [0, k^*(G,N)) \\ \left(\frac{k^*(G,N)}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [k^*(G,N), G) \\ \left(\frac{x}{N}\right)^{\frac{1}{N-1}} & \text{for } x \in [G, N] \end{cases}$$

Denote

$$Q_N(x) = \left(\frac{x}{N}\right)^{\frac{1}{N-1}}.$$

Then, $J_N(x) = Q_N(x)$ on $[0, k^*(G,N)) \cup [G, N]$. We now show that the functions $Q_N(x)$ and $Q_{N+1}(x)$ only cross once on $(0, N+1)$, and they cross at a point greater than 1.

Lemma 1 *Given any $N \geq 2$ there is a unique $\widetilde{x}_N \in (1, N)$ with the property that $Q_N(x) > Q_{N+1}(x)$ if and only if $x > \widetilde{x}_N$.*

Proof:

$$\begin{aligned} Q_N(x) - Q_{N+1}(x) &= \left(\frac{x}{N}\right)^{\frac{1}{N-1}} - \left(\frac{x}{N+1}\right)^{\frac{1}{N}} \\ &= x^{\frac{1}{N}}\left[x^{\frac{1}{N(N-1)}}\left(\frac{1}{N}\right)^{\frac{1}{N-1}} - \left(\frac{1}{N+1}\right)^{\frac{1}{N}}\right] \end{aligned}$$

The expression in brackets is increasing in x, is negative for values of x close to zero and positive for $x = N$. Therefore, there exists a unique $\widetilde{x}_N$ at which $Q_N(x) - Q_{N+1}(x)$ equals zero. To verify that $\widetilde{x}_N > 1$, observe that when $x = 1$ the expression in brackets equals $\left(\frac{1}{N}\right)^{\frac{1}{N-1}} - \left(\frac{1}{N+1}\right)^{\frac{1}{N}}$ which is negative because $\left(\frac{1}{N}\right)^{\frac{1}{N-1}}$ is an increasing function of N. ∎

Corollary 2 *Given any $N \geq 2$ and $G < 1$, all risk averse voters prefer an election with N candidates to an election with $N+1$ candidates.*

Proof: By Theorem 1, $Q_N(x)$ is the equilibrium distribution of transfers when $G < 1$. Lemma 1 shows that $Q_N(x)$ dominates $Q_{N+1}(x)$ in the sense of second-order stochastic dominance. ∎

We now show that all voters prefer elections with fewer candidates. The proof uses the property that $\beta(G, N) < \beta(G, N+1)$. In the appendix we show numerically that this assumption holds for N between 2 and 10.

Proposition 1 *Given any $N \geq 2$ and G such that $1 < G < N+1$, all risk averse voters with monotonic preferences prefer an election with N candidates to an election with $N+1$ candidates.*

Proof: We show that $J_N(x)$ dominates $J_{N+1}(x)$ in the sense of second-order stochastic monotonic dominance (see Huang and Litzenberger (1988)). This requires proving that: (i) the expected consumption (transfers or public good) is no smaller under $J_N(x)$ than under $J_{N+1}(x)$; and (ii) that $S(z) := \int_0^z [J_N(x) - J_{N+1}(x)]\, dx \leq 0$ for all $z \geq 0$. Part (i) is straightforward given the fact that $\beta^*(G, N)$ is decreasing in N. We prove part (ii) by showing that $J_N(x)$ crosses $J_{N+1}(x)$ only once, and from below, for $x \in (0, N+1)$. We divide the proof into two subcases.

1. $G < \widetilde{x}_N$.

   In this case, in view of Lemma 1, $J_N(x)$ crosses $J_{N+1}(x)$ exactly once, and from below, on $x \in (G, N+1)$. Therefore, we must check that, for $x \in (0, G)$ $J_N(x)$ is smaller than $J_{N+1}(x)$. For any $n \geq 2$, the function $J_n(x)$ is constant and equal to $Q_n(G) - \beta^*(G, n)$ on $(k^*(G, n), G)$. Since by assumption $G < \widetilde{x}_N$, we have $Q_N(G) < Q_{N+1}(G)$. Recalling that $\beta^*(G, N) > \beta^*(G, N+1)$, we conclude that $J_N(x) < J_{N+1}(x)$ for values of x between $\max\{k^*(G, N), k^*(G, N+1)\}$ and G.

   For values of x smaller than $\max\{k^*(G, N), k^*(G, N+1)\}$ it is easily verified that $J_N(x) < J_{N+1}(x)$ because $Q_N(x) < Q_{N+1}(x)$ (see Figure 3).

2. $G > \widetilde{x}_N$.

   In this case, in view of Lemma 1, $J_N(x)$ is always greater than $J_{N+1}(x)$ on $x \in (G, N+1)$. So, it suffices to check that $J_N(x)$ crosses $J_{N+1}(x)$ at most once, and then from below, on $(0, G)$. To this end, observe that if $J_N(x)$ crosses $J_{N+1}(x)$ on $(0, G)$, any crossing must happen on $(0, 1)$ because for all n, $J_n(x)$ is constant on $(k^*(G, n), G)$ and $k^*(G, n) < 1$. Now, take the smallest point $\widehat{x}$ in $(0, 1)$ at which $J_N(x)$ crosses $J_{N+1}(x)$. On $(0, 1)$, we have $Q_{N+1}(x) > Q_N(x)$ (indeed, $\widetilde{x}_n > 1$ for all n by Lemma 1). Therefore, $\widehat{x}$ must be such that $J_{N+1}(\widehat{x}) < Q_{N+1}(\widehat{x})$, that is, $J_{N+1}(x)$ is constant for x between $\widehat{x}$ and G. But then there can be no further crossings between $J_N(x)$ and $J_{N+1}(x)$ on $(\widehat{x}, G)$, which proves

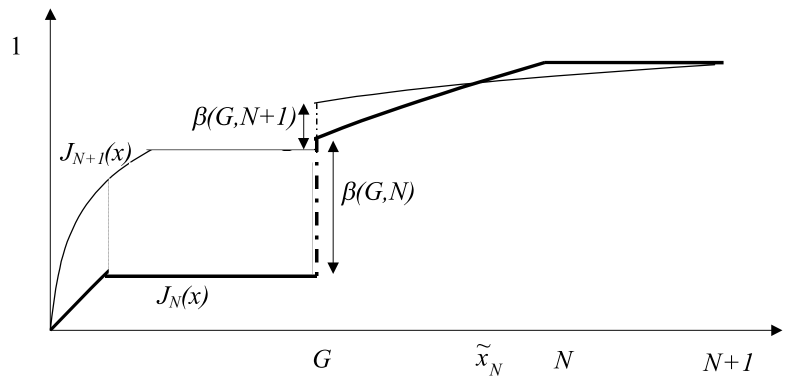

Figure 3: Case $G < \widetilde{x}_N$.

uniqueness of the crossing. To show that the crossing is from below, notice that $\widehat{x}$ is, by construction, the smallest point at which the two functions cross, and, for x small enough, $J_{N+1}(x) = Q_{N+1}(x) > Q_N(x) = J_N(x)$.

∎

6 The Effect of Differential Responsiveness

We now allow voters to differ in their degree of responsiveness to electoral promises. This heterogeneity can be interpreted in several ways. One interpretation is that there is an ideological dimension in voters' preferences that makes some voters more likely ex-ante to favor a specific candidate. An alternative interpretation is that some groups of voters are less likely to participate in the election. Finally, some voters might be less informed than others, and hence less responsive to differences in the offers made by the different candidates.

First, we address redistributional issues: we show that less responsive voters tend to be disadvantaged in the process of redistributive politics (Theorem 3). This is because competition for those voters is less intense.

Next, we show that this heterogeneity in responsiveness is a source of inefficiency in our model (Theorem 4). The fact that electoral competition focuses on swing voters, creates an even stronger incentive for candidates to offer redistributive platforms targeted to these voters, instead of a public good whose services benefit responsive an non responsive voters equally. This leads to lower provision of the public good, relative to the case where voters are homogeneous.

We first discuss the case where voters may vote ideologically. Then we show that the model can also be interpreted as one of differential participation or information.

There are $N+1$ groups of voters: $V_S, V_1, ..., V_N$, where V_S is of size $1-\varphi$, and each V_i is of size φ/N for $i=1,...,N$. Voters in V_i are *partisan* for candidate i: with probability p they *vote ideologically*, i.e., they ignore electoral promises and vote for candidate i; with probability $1-p$ they vote for the candidate who makes the higher promise. Voters in V_S are *swing voters*: they have no ideological preference, they always vote for the candidate making the higher promise.

This way of modeling ideology allows for the possibility that there is another dimension of the policy space (such as war and peace, abortion rights ...) that matters to some voters more than to others, and on which candidates have distinct positions. The model can be readily extended to treat several degrees of partisanship by allowing for groups with different probabilities p_i of voting ideologically.

Theorem 3 *Assume that, for each of the N candidates, there is a fraction φ/N of partisan voters who with probability p vote ideologically; the remaining $(1-\varphi)$ are swing voters. Assume further that $G(1-\varphi)<1$. Then, the following is an equilibrium. Each candidate chooses transfers according to a pair of distributions: offers to partisan voters are drawn from the distribution $F_P^*(x)=\left(x\frac{1-\varphi p}{N(1-p)}\right)^{\frac{1}{N-1}}$ on $\left[0,\frac{N(1-p)}{1-\varphi p}\right]$; offers to swing voters are drawn from the distribution $F_S^*(x)=\left(x\frac{1-\varphi p}{N}\right)^{\frac{1}{N-1}}$ on $\left[0,\frac{N}{1-\varphi p}\right]$.*

Proof: Denote by K_S and K_P the maximum of the support of the transfers to swing and partisan voters, respectively. In equilibrium, $[F_S^*(x)]^{N-1}$ and $[F_P^*(x)]^{N-1}$ must be uniform on $[0,K_S]$ and $[0,K_P]$, respectively, in view of the discussion in Section 3. We use the equation $[F_S^*(x)]^{N-1}=x/K_S$ to find $F_S^*(x)$,

$$F_S^*(x)=\left(\frac{x}{K_S}\right)^{\frac{1}{N-1}} \text{ on } [0,K_S]. \tag{7}$$

Analogously,

$$F_P^*(x)=\left(\frac{x}{K_P}\right)^{\frac{1}{N-1}} \text{ on } [0,K_P]. \tag{8}$$

It remains to solve for K_S and K_P. To this end, we first note that candidates must receive the same expected return from offering x to swing and partisan voters, hence

$$[F_S^*(x)]^{N-1}=(1-p)[F_P^*(x)]^{N-1}.$$

Substituting from equations (7) and (8) we rewrite the above equation as

$$\frac{K_P}{K_S} = (1 - p) . \tag{9}$$

Next, we note that the budget constraint requires that

$$(1 - \varphi) \int_0^{K_S} x dF_S^* (x) + \varphi \int_0^{K_P} x dF_P^* (x) = 1,$$

so substituting from (7) and (8) we have

$$(1 - \varphi) \int_0^{K_S} \frac{1}{(N - 1) (K_S)^{\frac{1}{N-1}}} x^{\frac{2-N}{N-1}} x dx + \varphi \int_0^{K_P} \frac{1}{(N - 1) (K_P)^{\frac{1}{N-1}}} x^{\frac{2-N}{N-1}} x dx = 1$$

Solving the integrals and simplifying we get

$$\frac{1}{N} (1 - \varphi) K_S + \frac{1}{N} \varphi K_P = 1. \tag{10}$$

Solving equations (9) and (10) for K_S and K_P yields $K_S = \frac{N}{1-\varphi p}, K_P = N \frac{1-p}{1-\varphi p}$. ∎

Theorem 3 shows that less responsive voters are disadvantaged in the process of redistributive politics. It also shows that in equilibrium all partisan voters are treated equally by all candidates. It does not matter whether a voter is partisan in favor of candidate i or j. All that matters is the candidate's degree of responsiveness to electoral promises. Thus, the analysis would be the same if we collected all partisan voters into one group of "unresponsive voters." The result would still hold if with probability p these voters did not hear the promises, i.e. they were less informed. Analogously, it would still hold if with probability p these voters did not vote.

In the next theorem we show that the inefficiency in our model is proportional to the fraction of unresponsive voters in the electorate. To this end, we make the simplifying assumption that these voters are completely unresponsive to electoral promises, i.e., $p = 1$. We are then able to show that, when a fraction φ of the electorate is partisan, the probability of public good provision corresponds to the probability in a model where no voter is partisan, and the public good has value $G (1 - \varphi)$. Thus, the probability that the public good is provided is decreasing in the fraction of partisan voters.

The intuition is the following: when there are φ partisan voters and $p = 1$, in equilibrium partisans receive zero transfers, thus freeing up resources for candidates to compete for swing voters. Then, candidates can allocate transfers of $1/ (1 - \varphi)$ per swing voter. This is more money per capita than was available in the case of no partisans. Normalizing the per capita money to 1 requires normalizing the value of the public good to $G(1-\varphi)$. After this normalization, the equilibrium can be derived from Theorem 2.

Theorem 4 *Assume that, for each of the N candidates, there is a fraction φ/N of partisan voters who with probability $p = 1$ vote ideologically for that candidate; the remaining $(1-\varphi)$ are swing voters. Then, the following is an equilibrium. Partisan voters never receive any trasnfers. For $1 < G(1-\varphi) < N$, the equilibrium is characterized by a probability β of providing the public good and a distribution of transfers F_S^* with support $[0, w] \cup [G, N/(1-\varphi)]$, where w solves $\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G(1-\varphi)}{N}\right) = \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{w(1-\varphi)}{N}\right)$ and $\beta = \left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}$. For $G(1-\varphi) > N$ the public good is provided with probability one.*

Proof: See Appendix. ∎

7 Discussion

7.1 Contrast With Two-Candidates Winner-Take-All

In winner-take-all systems, candidates maximize the probability of winning the election and not the expected vote share. These are system with a strong majoritarian bias, where most of the powers of policy setting are delegated to the party with a majority of votes, and where minority parties do not have much influence. A winner-take-all system of electoral incentives also prevails in the direct election of a premier, where the candidate with the highest number of votes receives all the spoils of office.

Duverger's Law states that winner-take-all electoral systems are associated with only two competing parties, while proportional systems are associated with a larger number of parties. A "strong version" of Duverger's Law is that winner-take-all electoral rules result in equilibria with only two parties, because voters coordinate on the two parties most likely to be elected, and avoid wasting their vote on other parties. A "weak version" of Duverger's Law simply records the empirical regularity that electoral systems with a higher "degree of proportionality" tend to be associated with a larger number of parties (see, e.g., Cox (1997)). Whichever version of Duverger's Law we believe in, it is interesting to compare multi-candidate elections under the proportional system with two-candidate elections under a majoritarian system.

Accordingly, we compare the results in Section 4 to the equilibrium in a two-candidates election with a winner-take-all rule. In this, we follow Austen-Smith and Banks (1988), who compare the equilibrium in a proportional system with three candidates with the equilibrium of a winner-take-all system with two candidates.[13] In our model, the comparison rests on the following theorem, proved in Lizzeri and Persico (1998).

Theorem 5 *Suppose $N = 2$, and $1 < G < 2$. Under the winner-take-all system, in the unique equilibrium with equal treatment both candidates offer the public good with probability $1/2$ for $G \in (1, 2)$. Candidates offer the public good with probability zero if $G < 1$ and with probability one if $G > 2$.*

Comparing this equilibrium with the analysis in Section 4 shows that the proportional system is generally more inefficient than a winner-take-all system. The probability of public good provision is larger in the two-candidates winner-take-all election relative to the equilibrium with N candidates in the proportional system, unless $N = 2$ and $G < 3/2$, or $N = 3$ and G is between 1.963 and 2.

This comparison suggests that two-candidate winner-take-all elections lead to more efficient outcomes than elections under the proportional system with more than 2 candidates. If we take Duverger's Law as implying that winner-take-all systems are incompatible with more than two parties, then our findings could be constructed as an argument in favor of winner-take-all systems over proportional systems.

7.2 Campaign Spending

We now build on the analysis of Section 6 to discuss the role of campaign spending. Our model lends itself to a discussion of the efficiency consequences of campaign spending. The main idea is the following. As was shown in Section 6, the presence of partisan voters has negative consequences for the provision of public goods. One possible view of campaign spending is that it creates partisans out of non-committed voters. This view parallels the idea, familiar in industrial economics, of advertising to relax price competition (see Tirole (1988)). If this is the outcome of campaign spending, then our model suggests that campaign spending has negative welfare effects.[14]

[13] Austen-Smith and Banks (1988) analyze proportional representation in a three-candidate model of spatial competition that integrates the electoral and legislative processes. The possibility of using their analysis for a discussion of efficiency is limited by their use of the spatial model.

[14] Notice that in our model, campaign spending distorts policy outcomes, thus, the negative effect of campaign spending is present even before taking into account the resource cost of this spending.

An alternative view is that campaign spending has an informative role. According to this view the number of partisan voters is reduced by advertising, as voters become more responsive to campaign promises, and less partisan. This also has a parallel in industrial economics, in the idea of informative advertising. In this case, campaign spending will have positive welfare effects.

We now present a very stylized model of advertising that captures this dual role of advertising.

Assume that there are only two candidates and four groups of voters: just as in Section 6, the first one is the set of initial partisans for candidate 1, it is denoted by P_1^0, and has mass $\varphi/2$. The second is the set of initial partisans for candidate 2 is denoted by P_2^0. Members of P_i^0 are predisposed in favor of candidate i in a sense that will be made precise below. The third and fourth sets of voters are initial swing voters S_i^0 and each has mass $(1-\varphi)/2$. The reason we split the set of swing voters in two will become clear.

There are two periods. At the beginning of period 1, each candidate is endowed with a budget B. Candidate 1 can use this budget in two ways: spend it on members of S_1^0 to create more partisans in favor of himself, or spend it on members of P_2^0 and make them into swing voters. Candidate two faces the same problem. Suppose that the first kind of persuasion costs C_S per voter, and the second costs C_P per voter.

Denote by P_i^1 the set of voters who are partisans in favor of candidate i at the end of period one. These voters vote for candidate i irrespective of electoral promises. Denote by S^1 the set of swing voters at the end period 1. These voters vote for the candidate who offers the higher utility.

In period 2 candidates observe the number of partisan voters and their allegiance, and the game described in Section 6 takes place.

The following fact is immediate.

Fact 1 If $C_S > C_P$, in equilibrium candidate i only advertises to members of P_j^0. If $C_S < C_P$, in equilibrium candidate i only advertises to members of S_i^0.

The important implication of Fact 1 is that, if $C_S > C_P$, advertising reduces the number of swing voters relative to the initial situation. In light of Theorem 4, in this case, advertising has negative consequences for the provision of the public good. In contrast, when $C_S < C_P$, advertising has positive consequences for efficiency.

7.3 The Size of Government

As the number of candidates increases, the size of government is affected by two contrasting forces: (i) the probability that the public good is provided decreases, implying a smaller size of government; but also (ii) the distribution of transfers becomes more unequal and the extent of redistribution increases, increasing the size of government. We now show that in our model the combined effect of these two forces is ambiguous. This implies that there is no direct relation between the size of government and the efficiency of the equilibrium outcome. This is of interest in relation to the literature dealing with the determinants of the size of government (see Persson and Tabellini (2000) for a review). In Milesi-Ferretti *et al.* (2000), the effective number of parties appears to have ambiguous effect (sometimes positive and sometimes negative, but never statistically significant) on the total size of government expenditure.

We define the size of government as the aggregate amount of taxes levied by the winning candidate in equilibrium. Recall that our model applies to an economy where voters have an endowment of one unit of money, and candidates need to tax voters if they want to provide the public good or to offer transfers to other voters. If the public good is provided, the size of government is maximal, because voters are taxed the full amount. If transfers of $\Phi(v)$ are provided to voter v, the size of the tax on that voter is $\max\{1-\Phi(v), 0\}$. When transfers are distributed to the electorate according to $F^*(x)$, the total size of government is given by $\int_0^1 (1-x)\, dF^*(x)$.

As we saw in Section 4, as the number of candidates increases, the probability that the public good is provided decreases. This effect is in the direction of lower size of government as the number of candidates increases. When G is large relative to N, this effect implies that the size of government decreases with the number of candidates. For example, fix G at 2.5. Then, if $N = 2$ the public good is provided for sure and the size of goverment is maximal, while if $N = 3$ the winning candidate offers transfers with probability 0.24, resulting in a smaller expected size of government.

On the other hand, the distribution of transfers tends to become more unequal as the number of candidates increases. Thus, more resources are used for redistribution, increasing the size of government. To see this, consider the case where $G < 1$. In this case, the public good is never provided and $F^*(x)$ is equal to $Q_N(x) = \left(\frac{x}{N}\right)^{\frac{1}{N-1}}$ by Theorem 1, hence the size of government $\int_0^1 (1-x)\, dF^*(x)$ can be computed to be $\left(\frac{1}{N}\right)^{\frac{1}{N-1}} \left(1-\frac{1}{N}\right)$, which is increasing in N. A continuity argument shows that the expected size of government is also increasing in N for G bigger than, but close to, 1.

7.4 Other models of Redistributive Politics

Lindbeck and Weibull's (1987) influential paper provides the first model of redistributive politics based on utility-maximizing voters. Lindbeck and Weibull show that voters who are more responsive to electoral promises are treated favorably by the political process. That model is hard to extend beyond the two-candidates case; in contrast, our model lends itself to treating the multi-candidates case. Nonetheless, the way we model heterogeneous responsiveness is similar in spirit to Lindbeck and Weibull's ideology, and the flavor of our Theorem 3 is also close.

The two models differ in their predictions concerning efficiency. If we introduce a public good in the Lindbeck-Weibull model, there is an efficient equilibrium if groups are homogeneous.[15] This is a consequence of assumptions on the interplay between a stochastic ideology component and the curvature of voters' utility functions, which are imposed to ensure concavity of the candidates' payoff functions, and hence the existence of a pure strategy equilibrium. As a result of these assumptions, the incentives to target voters are purely a function of ideological or utility differences between groups. Thus, when groups are homogeneous, candidates lose the incentive to target subgroups of the population; in our terminology, they do not gain by offering more than G to more than $1/N$-th of the electorate. Therefore, the tradeoff between targetability and efficiency disappears and the public good is provided efficiently.

The propensity towards efficiency of the Lindbeck-Weibull model results from the assumptions on the distributions of ideology and on the curvature of the utility function. It seems desirable to logically separate the effects of ideology from the incentive to target sub-groups. Our paper shows that these incentives have important and rich efficiency consequences, independently of the effect of ideological differences across groups which are the focus of Lindbeck and Weibull, and which we address in Section 6.

8 Conclusion

The idea that economic competition is beneficial has solid theoretical underpinnings, for instance in the analysis of the Cournot model, where the outcome becomes more efficient with the number of firms N and approaches the perfectly competitive outcome as $N \to \infty$. In contrast, the view that electoral competition is beneficial has been

[15] See Lizzeri and Persico (2000).

argued only informally, through the analogy between competition in markets and in elections.

We have provided a tractable model of N-candidates electoral competition where candidates choose whether to offer to provide a public good or to target transfers to sub-groups of the population. We have shown that the incentive to offer particularistic platforms increases with the number of candidates, and that, therefore, in equilibrium the probability that the public good is provided decreases with N. In fact, as $N \to \infty$, the public good is never provided, no matter how efficient it is. Voters standing behind a Rawlsian "veil of ignorance" prefer elections with smaller numbers of candidates.

If we subscribe to a "strong version" of Duverger's law, i.e., that electoral systems with a strong majoritarian (winner-take-all) component lead to two-party competition, while proportional systems are consistent with a larger number of parties, then our analysis can be interpreted as supporting winner-take-all systems over proportional systems.

An empirical implication of the analysis is that the fraction of the public sector budget devoted to redistribution should be larger in democracies with a large number of parties. However, the implications concerning the size of government (total size of the budget) are ambiguous. Milesi-Ferretti, Perotti, and Rostagno (2000) present some evidence broadly consistent with these predictions.[16]

We then discussed the role of the responsiveness of voters to electoral promises, and showed that the larger the group of pre-committed voters, the less efficient the political process in terms of provision of public goods. One way to interpret pre-commitment is as ideology: voters who are ideologically biased towards one candidate are less likely to be responsive to electoral promises. In this sense, the degree of ideological polarisation of an election is negatively related to efficiency. Interestingly, Alesina, Baqir, and Easterly (1999) show that the level of provision of public goods in local jurisdiction in the US is inversely related to their ethnic fragmentation. If we believe that the degree of ethnic fragmentation is positively related to the degree of ideological bias of voters in a district, our model provides an alternative explanation for their empirical findings.

Another way to interpret responsiveness is as "information:" voters who are not well-informed about platforms are less responsive to electoral promises. Thus, the larger the fraction of uninformed voters in the population, the larger the inefficiency. This is interesting in relation to the role of electoral advertising in our model. Depend-

[16] Their analysis focusses on the effects of the degree of proportionality of the electoral system on the composition of public spending. However, the number of parties has a (weak) positive effect on the fraction of transfers.

ing on its nature, advertising can be a source of efficiency (if it is informative) or of inefficiency (if it increases the fraction of ideologically commited voters).

The drawback of electoral competition identified in this paper results from a force inherent to the electoral mechanism. We believe that this force should be taken into account in any discussion of the merits of electoral competition. Of course our model is stylized, and abstracts from some important issues. One of these is corruption. Electoral competition may have the effect of reducing corruption.[17] A complete evaluation of the role of electoral competition should take into account the consequences of corruption as well as those discussed in the present paper. Another element that our analysis has abstracted from is the role of government coalitions. It would be desirable to have a richer model that incorporates post-electoral coalitional bargaining. We believe that the main elements of our analysis would be robust to such considerations.

[17] See Myerson (1993) for a model of multicandidate electoral competition that studies the performance of electoral systems in the presence of corrupt candidates.

Appendix

A Proof of Theorem 4.

Proof: Since partisan voters get no money in equilibrium, we now focus on swing voters. By the same argument as in the proof of Theorem 2, equilibrium requires that if a candidate offers transfers, he cannot do better than by choosing F^*. This requires that $H^*(x) = x(1-\varphi)/N$ on $[0,w] \cup [G, N/(1-\varphi)]$.

The probability of winning a vote with an offer of x in $[0,w]$ is

$$H^*(x) = [(1-\beta) F^*(x)]^{N-1}.$$

Equating to $x(1-\varphi)/N$ and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta}\left(\frac{x(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \text{ for } x \in (0,w). \tag{11}$$

The probability of winning a vote with an offer of x in $(G, N/(1-\varphi)]$ is

$$\begin{aligned} H^*(x) &= \sum_{j=0}^{N-1} \binom{N-1}{j} ((1-\beta) F^*(x))^j \beta^{N-1-j} \\ &= [(1-\beta) F^*(x) + \beta]^{N-1}. \end{aligned}$$

Equating to $x(1-\varphi)/N$ and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta}\left[\left(\frac{x(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \beta\right] \text{ for } x \in (G, N/(1-\varphi)). \tag{12}$$

To complete the characterization of F^* we look for conditions to pin down β and w. The first condition is given by the continuity of F^*, which requires $F^*(w) = F^*(G)$. Substituting from (11) and (12), we get

$$\beta = \left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}. \tag{13}$$

The second condition is given by the budget constraint, i.e., $\int_0^w xf(x)\,dx + \int_G^{\frac{N}{(1-\varphi)}} xf(x)\,dx = 1/(1-\varphi)$. To compute the budget constraint, observe that, on $[0,w] \cup [G, N/(1-\varphi)]$,

$$f^*(x) = \frac{1}{1-\beta}\left(\frac{(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1} x^{\frac{2-N}{N-1}},$$

and therefore

$$\int_a^b x f^*(x)\,dx = \frac{1}{1-\beta}\left(\frac{(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1}\frac{x^{\frac{N}{N-1}}}{N}(N-1)\Bigg|_a^b = \frac{(1-\varphi)^{\frac{1}{N-1}}}{1-\beta}\left(\frac{x}{N}\right)^{\frac{N}{N-1}}\Bigg|_a^b .$$

Using this equation, the budget constraint can be expressed as

$$\frac{(1-\varphi)^{\frac{1}{N-1}}}{1-\beta}\left[\left(\frac{w}{N}\right)^{\frac{N}{N-1}} + \frac{1}{(1-\varphi)^{\frac{N}{N-1}}} - \left(\frac{G}{N}\right)^{\frac{N}{N-1}}\right] = \frac{1}{(1-\varphi)},$$

or

$$\beta = \left(\frac{G(1-\varphi)}{N}\right)^{\frac{N}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{N}{N-1}} . \tag{14}$$

Equations (13) and (14) form a system of two equations in the unknowns w and β.

It is possible to solve for w. Indeed, from (13) and (14), w must solve

$$\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}} = \left(\frac{G(1-\varphi)}{N}\right)^{\frac{N}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{N}{N-1}} ,$$

or

$$\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G(1-\varphi)}{N}\right) = \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{w(1-\varphi)}{N}\right). \tag{15}$$

The function $h(z) = \left(\frac{z}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{z}{N}\right)$ is single-peaked on $[0,\infty)$, has a maximum at $z=1$, and has value zero at $z=0$ and $z=N$. Because h is single peaked, equation (15) only has two solutions: one is $w=G$; the other is $w^*(G) = h^{-1}(G)$, where h^{-1} denote the inverse of h on the interval $[0,1]$. Only the second solution can be part of an equilibrium, since the second solution requires that β equals zero, and this is impossible when $1 < G(1-\varphi) < N$. Solving for β is accomplished by substituting $w^*(G)$ into equation (13), to obtain $\beta^*(G)$. Observe that $w^*(G)$ is decreasing in G since h is increasing on $(0,1)$; therefore, $\beta^*(G) = (G(1-\varphi)/N)^{\frac{1}{N-1}} - ((1-\varphi)w^*(G)/N)^{\frac{1}{N-1}}$ is increasing in G.

To verify that the pair $\beta^*(G)$ and $w^*(G)$ indeed forms an equilibrium, we need to check that the expected vote share from offering the public good, S_G, is equal to $1/N$,

$$S_G = \sum_{j=0}^{N-1}\frac{1}{j+1}\binom{N-1}{j}(1-\beta)^{N-1-j}\beta^j(F^*(G))^{N-1-j} = \frac{1}{N}.$$

Using the fact that $\sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} q^{N-1-j} p^j = \left[(q+p)^N - q^N \right] / pN$, we rewrite the above equation as

$$\frac{\left[(1-\beta) F^*(G) + \beta\right]^N - \left[(1-\beta) F^*(G)\right]^N}{\beta N} = \frac{1}{N}.$$

Since $F^*(G) = F^*(w)$ we can rewrite this as

$$\frac{\left[(1-\beta) F^*(G) + \beta\right]^N - \left[(1-\beta) F^*(w)\right]^N}{\beta N} = \frac{1}{N},$$

and after substituting from (11) and (12), we obtain

$$\frac{\left[\frac{G(1-\varphi)}{N}\right]^{\frac{N}{N-1}} - \left[\frac{w(1-\varphi)}{N}\right]^{\frac{N}{N-1}}}{\beta N} = \frac{1}{N}.$$

This equation is the same as equation (14). Thus, when $\beta = \beta^*(G), w = w^*(G)$, it is indeed the case that $S_G = 1/N$. This shows that the pair $\beta^*(G), w^*(G)$ forms an equilibrium.

Uniqueness follows from the discussion in Section 3 together with the uniqueness of the solutions of β and w. ∎

B Numerical Solution of β and k

In this appendix we report the solutions to the system of equations

$$\begin{cases} \left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{k}{N}\right)=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G}{N}\right) \\ \beta=\left(\frac{G}{N}\right)^{\frac{1}{N-1}}-\left(\frac{k}{N}\right)^{\frac{1}{N-1}} \end{cases}$$

that characterize β and k (see Theorem 2). The following two tables report the values of β and k respectively, as a function of G (rows) and N (columns).

beta	2	3	4	5	6	7	8	9	10
1.05	0.05	0.028633766	0.020772272	0.016515846	0.013795895	0.011888136	0.010467323	0.009363756	0.0084794
1.1	0.1	0.056825788	0.041117691	0.032650077	0.027251877	0.023471223	0.020658422	0.01847529	0.0167269
1.2	0.2	0.11202327	0.080666404	0.063899504	0.053257273	0.045824483	0.040304957	0.036026923	0.0326044
1.3	0.3	0.16583705	0.11888724	0.093966172	0.078211577	0.067235931	0.059099641	0.052801392	0.0477675
1.4	0.4	0.2184693	0.15597508	0.12302452	0.10227039	0.087845376	0.077169195	0.068914501	0.0623228
1.5	0.5	0.27009076	0.19209051	0.15121687	0.12555997	0.10776588	0.094615865	0.084459528	0.0763566
1.6	0.6	0.32084845	0.22736812	0.17866123	0.14818443	0.12709038	0.11152349	0.099512836	0.0899374
1.7	0.7	0.37087143	0.26192245	0.2054569	0.17023088	0.1458963	0.12796178	0.11413781	0.103124
1.8	0.8	0.42027516	0.29585232	0.23168849	0.19177307	0.16424903	0.14398943	0.12838772	0.115967
1.9	0.9	0.4691648	0.32924413	0.25742894	0.21287417	0.18220429	0.15965636	0.14230781	0.128506
2	**1**	**0.51763809**	**0.36217427**	**0.28274181**	**0.23358881**	**0.19981011**	**0.17500548**	**0.15593687**	**0.140776**
2.1	1	0.5657876	0.39471109	0.30768295	0.25396465	0.21710822	0.19007395	0.16930848	0.152809
2.2	1	0.61370286	0.42691636	0.33230194	0.27404363	0.23413515	0.20489424	0.18245187	0.164631
2.3	1	0.66147231	0.45884649	0.35664308	0.29386292	0.25092312	0.21949488	0.19539271	0.176266
2.4	1	0.70918525	0.49055343	0.38074628	0.31345565	0.26750074	0.23390112	0.20815364	0.187733
2.5	1	0.75693396	0.52208551	0.40464771	0.33285156	0.2838935	0.24813542	0.22075479	0.19905
2.6	1	0.80481603	0.55348798	0.42838036	0.35207748	0.30012431	0.26221785	0.23321409	0.210237
2.7	1	0.85293723	0.58480355	0.45197444	0.37115771	0.31621378	0.27616646	0.24554762	0.221305
2.8	1	0.90141512	0.61607274	0.47545779	0.39011436	0.33218056	0.28999748	0.25776983	0.232268
2.9	1	0.9503837	0.64733421	0.49885611	0.40896762	0.34804159	0.30372562	0.26989378	0.243138
3	**1**	**1**	**0.67862496**	**0.52219322**	**0.42773597**	**0.36381227**	**0.31736422**	**0.28193127**	**0.253925**
3.1	1	1	0.7099804	0.54549124	0.44643638	0.37950666	0.3309254	0.29389303	0.264640
3.2	1	1	0.74143443	0.56877073	0.46508444	0.39513762	0.34442021	0.3057888	0.275290
3.3	1	1	0.77301934	0.59205076	0.48369446	0.4107169	0.35785874	0.31762747	0.285883
3.4	1	1	0.80476563	0.61534907	0.50227965	0.42625526	0.3712502	0.32941714	0.296428
3.5	1	1	0.83670166	0.63868202	0.52085213	0.44176259	0.38460302	0.34116521	0.306931
3.6	1	1	0.86885324	0.66206471	0.539423	0.45724791	0.39792489	0.35287843	0.317397
3.7	1	1	0.90124295	0.68551095	0.55800247	0.47271948	0.41122286	0.36456299	0.327832
3.8	1	1	0.93388935	0.70903324	0.57659982	0.48818486	0.42450337	0.37622453	0.338242
3.9	1	1	0.96680596	0.73264283	0.5952235	0.50365092	0.43777228	0.38786819	0.348630
4	**1**	**1**	**1**	**0.75634961**	**0.61388113**	**0.51912392**	**0.45103494**	**0.39949868**	**0.359002**
4.1	1	1	1	0.78016217	0.63257956	0.53460952	0.46429621	0.41112028	0.369360
4.2	1	1	1	0.80408774	0.65132485	0.55011281	0.47756049	0.42273688	0.379709
4.3	1	1	1	0.82813219	0.67012238	0.56563837	0.49083177	0.43435202	0.390052
4.4	1	1	1	0.85230007	0.68897677	0.58119027	0.50411363	0.44596888	0.400391
4.5	1	1	1	0.8765946	0.707892	0.59677211	0.51740928	0.45759036	0.410729
4.6	1	1	1	0.90101778	0.7268714	0.61238705	0.53072157	0.46921904	0.42106
4.7	1	1	1	0.92557045	0.74591767	0.62803781	0.54405305	0.48085725	0.43141
4.8	1	1	1	0.95025245	0.76503296	0.64372675	0.55740594	0.49250705	0.441760
4.9	1	1	1	0.97506281	0.78421887	0.65945583	0.57078218	0.50417028	0.452116
5	**1**	**1**	**1**	**0.99999946**	**0.80347652**	**0.67522669**	**0.58418346**	**0.51584854**	**0.462480**
5.1	1	1	1	1	0.82280659	0.69104064	0.5976112	0.52754326	0.472855
5.2	1	1	1	1	0.84220937	0.7068987	0.61106661	0.53925566	0.483241
5.3	1	1	1	1	0.86168483	0.72280165	0.6245507	0.55098679	0.49363
5.4	1	1	1	1	0.88123264	0.73875002	0.63806427	0.56273754	0.504051
5.5	1	1	1	1	0.90085228	0.75474413	0.65160796	0.57450867	0.514476
5.6	1	1	1	1	0.92054303	0.77078412	0.66518226	0.58630079	0.524916
5.7	1	1	1	1	0.94030408	0.78686998	0.67878751	0.59811441	0.535372
5.8	1	1	1	1	0.96013454	0.80300156	0.69242395	0.60994991	0.545843
5.9	1	1	1	1	0.98003348	0.81917861	0.70609167	0.62180759	0.55633
6	**1**	**1**	**1**	**1**	**0.99999192**	**0.8354008**	**0.71979072**	**0.63368766**	**0.566833**
6.1	1	1	1	1	1	0.8516677	0.73352103	0.64559026	0.577354
6.2	1	1	1	1	1	0.86797886	0.74728249	0.65751545	0.587890
6.3	1	1	1	1	1	0.88433379	0.76107492	0.66946325	0.598444
6.4	1	1	1	1	1	0.90073196	0.77489808	0.68143362	0.609015
6.5	1	1	1	1	1	0.91717287	0.78875174	0.69342649	0.619603
6.6	1	1	1	1	1	0.93365597	0.8026356	0.70544172	0.630208
6.7	1	1	1	1	1	0.95018074	0.81654935	0.71747919	0.640830
6.8	1	1	1	1	1	0.96674667	0.83049267	0.72953873	0.651469
6.9	1	1	1	1	1	0.98335325	0.84446524	0.74162014	0.662124
7	**1**	**1**	**1**	**1**	**1**	**1**	**0.85846672**	**0.75372323**	**0.672797**
7.1	1	1	1	1	1	1	0.87249678	0.76584778	0.683486
7.2	1	1	1	1	1	1	0.88655508	0.77799358	0.694191
7.3	1	1	1	1	1	1	0.90064129	0.7901604	0.704913
7.4	1	1	1	1	1	1	0.91475509	0.80234802	0.715651
7.5	1	1	1	1	1	1	0.92889617	0.8145562	0.72640
7.6	1	1	1	1	1	1	0.94306421	0.82678473	0.737174
7.7	1	1	1	1	1	1	0.95730608	0.83903336	0.747960
7.8	1	1	1	1	1	1	0.97147996	0.85130189	0.75876
7.9	1	1	1	1	1	1	0.98572709	0.86359009	0.76957
8	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0.87589775**	**0.780408**
8.1	1	1	1	1	1	1	1	0.88822465	0.791255
8.2	1	1	1	1	1	1	1	0.90057059	0.802116
8.3	1	1	1	1	1	1	1	0.91293536	0.812992
8.4	1	1	1	1	1	1	1	0.92531877	0.823883
8.5	1	1	1	1	1	1	1	0.93772062	0.834788
8.6	1	1	1	1	1	1	1	0.95014073	0.845708
8.7	1	1	1	1	1	1	1	0.9625789	0.856641
8.8	1	1	1	1	1	1	1	0.97503495	0.867589
8.9	1	1	1	1	1	1	1	0.98750871	0.878550
9	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0.889525**
9.1	1	1	1	1	1	1	1	1	0.900513
9.2	1	1	1	1	1	1	1	1	0.911515
9.3	1	1	1	1	1	1	1	1	0.922531
9.4	1	1	1	1	1	1	1	1	0.933559
9.5	1	1	1	1	1	1	1	1	0.944601
9.6	1	1	1	1	1	1	1	1	0.955655
9.7	1	1	1	1	1	1	1	1	0.966722
9.8	1	1	1	1	1	1	1	1	0.977802
9.9	1	1	1	1	1	1	1	1	0.988895
10	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0.998487**

k	2	3	4	5	6	7	8	9	10
1.05	0.95	0.95081989	0.95108718	0.95121973	0.95129891	0.9513516	0.95138909	0.9514172	0.9514390
1.1	0.9	0.90322917	0.9042587	0.90476515	0.90506642	0.9052662	0.90540837	0.90551472	0.9055972
1.2	0.8	0.81254921	0.81637438	0.81822708	0.81932035	0.8200417	0.82055338	0.82093518	0.8212309
1.3	0.7	0.72750193	0.73551018	0.73933182	0.74157003	0.7430401	0.74407963	0.74485357	0.7454521
1.4	0.6	0.64772884	0.66099012	0.66722973	0.67085853	0.673232	0.67490548	0.67614893	0.6771092
1.5	0.5	0.57294902	0.59225865	0.60122321	0.60640306	0.6097779	0.6121514	0.61391173	0.6152693
1.6	0.4	0.50294373	0.52885237	0.54073027	0.54755284	0.5519825	0.55509062	0.55739203	0.5591647
1.7	0.3	0.43754562	0.47037951	0.48525845	0.4937592	0.4992613	0.50311404	0.50596268	0.5081545
1.8	0.2	0.37663121	0.41650488	0.43438573	0.44455363	0.451117	0.45570483	0.45909274	0.4616971
1.9	0.1	0.32011561	0.36693845	0.38774625	0.39953159	0.4071218	0.41241961	0.41632778	0.4193298
2	**0**	**0.26794919**	**0.32142649**	**0.3450194**	**0.35834014**	**0.3669039**	**0.37287426**	**0.377275**	**0.3806534**
2.1	0	0.22011561	0.2797447	0.3059214	0.32066843	0.3301371	0.33673299	0.34159195	0.345320
2.2	0	0.17663121	0.24169256	0.27019867	0.28624032	0.2965332	0.30369993	0.30897763	0.3130264
2.3	0	0.13754562	0.20708878	0.23762254	0.2548085	0.2658352	0.27351242	0.27916563	0.2835023
2.4	0	0.10294373	0.17576738	0.20798497	0.22614979	0.2378127	0.24593575	0.25191858	0.2565087
2.5	0	0.072949017	0.14757439	0.18109506	0.20006137	0.2122574	0.22075889	0.22702371	0.2318319
2.6	0	0.047728836	0.12236483	0.15677621	0.17635772	0.1889803	0.19779105	0.20428932	0.2092796
2.7	0	0.027501926	0.1	0.1348637	0.15486806	0.1678087	0.1768588	0.18354178	0.1886782
2.8	0	0.012549213	0.080344879	0.11520267	0.13543429	0.1485839	0.15780382	0.16462316	0.1698702
2.9	0	0.00322917	0.063265612	0.0976465	0.11790925	0.1311598	0.14048095	0.14738922	0.1527120
3	**0**	**2.06795E-25**	**0.048626964**	**0.082055307**	**0.10215529**	**0.1154009**	**0.12475655**	**0.13170773**	**0.1370727**
3.1	0	0	0.036289711	0.068294805	0.088043113	0.1011815	0.11050723	0.1174571	0.1228320
3.2	0	0	0.026107887	0.056235275	0.075450751	0.0883841	0.097618639	0.10452516	0.109879
3.3	0	0	0.017925861	0.04575075	0.064262768	0.0768989	0.085984607	0.092808201	0.09811273
3.4	0	0	0.011575198	0.036718351	0.054369597	0.0666231	0.075506277	0.082210114	0.08743819
3.5	0	0	0.006871288	0.029017784	0.045667001	0.05746	0.066091455	0.072641671	0.07776856
3.6	0	0	0.003609764	0.022530996	0.038055648	0.0493187	0.057654024	0.064019907	0.0690231
3.7	0	0	0.001562768	0.017141984	0.031440784	0.0421135	0.050113461	0.056267589	0.06112689
3.8	0	0	0.000475169	0.012736773	0.025731997	0.0357637	0.043394413	0.049312758	0.05401032
3.9	0	0	6.09403E-05	0.009203562	0.020843052	0.0301933	0.03742634	0.043088332	0.04760867
4	**0**	**0**	**0**	**0.006433043**	**0.016691797**	**0.0253306**	**0.032143214**	**0.037531765**	**0.041861**
4.1	0	0	0	0.004318919	0.013200128	0.0211083	0.027483246	0.032584739	0.03671372
4.2	0	0	0	0.002758595	0.010294003	0.017463	0.023388666	0.028192901	0.03211242
4.3	0	0	0	0.001654076	0.007903493	0.0143352	0.019805518	0.024305629	0.02800950
4.4	0	0	0	0.000913051	0.005962871	0.0116695	0.016683484	0.020875818	0.02436004
4.5	0	0	0	0.000450162	0.00441072	0.0094139	0.013975728	0.017859691	0.02112231
4.6	0	0	0	0.000188444	0.003190043	0.0075203	0.011638751	0.015216626	0.01825763
4.7	0	0	0	6.0915E-05	0.002248393	0.0059442	0.009632257	0.012908994	0.01573018
4.8	0	0	0	1.22881E-05	0.00153797	0.0046443	0.00791903	0.010902012	0.01350685
4.9	0	0	0	7.84E-07	0.001015713	0.0035831	0.006464809	0.009163601	0.01155706
5	**0**	**0**	**0**	**4.1359E-25**	**0.000643349**	**0.0027263**	**0.005238176**	**0.007664253**	**0.00985264**
5.1	0	0	0	0	0.000387406	0.002043	0.004210432	0.006376899	0.00836771
5.2	0	0	0	0	0.000219168	0.0015053	0.003355483	0.005276789	0.00707851
5.3	0	0	0	0	0.000114565	0.0010884	0.002649716	0.004341361	0.0059633
5.4	0	0	0	0	5.40024E-05	0.0007706	0.002071881	0.003550132	0.00500230
5.5	0	0	0	0	2.21037E-05	0.0005327	0.001602958	0.002884568	0.00417742
5.6	0	0	0	0	7.37453E-06	0.0003585	0.001226035	0.002327974	0.00347231
5.7	0	0	0	0	1.78125E-06	0.0002339	0.000926172	0.001865376	0.00287219
5.8	0	0	0	0	2.38683E-07	0.0001472	0.00069027	0.001483407	0.00236371
5.9	0	0	0	0	7.58745E-09	8.88E-05	0.000506932	0.001170192	0.0019349
6	**0**	**0**	**0**	**0**	**2.06795E-25**	**5.10E-05**	**0.000366328**	**0.000915239**	**0.00157509**
6.1	0	0	0	0	0	2.76E-05	0.00026006	0.000709329	0.00127470
6.2	0	0	0	0	0	1.38E-05	0.000181025	0.00054441	0.00102528
6.3	0	0	0	0	0	6.30E-06	0.000123282	0.000413495	0.00081936
6.4	0	0	0	0	0	2.54E-06	8.19259E-05	0.000310561	0.00065036
6.5	0	0	0	0	0	8.63E-07	5.29593E-05	0.000230453	0.00051253
6.6	0	0	0	0	0	2.30E-07	3.31759E-05	0.000168795	0.00040087
6.7	0	0	0	0	0	4.15E-08	2.00473E-05	0.0001219	0.00031104
6.8	0	0	0	0	0	3.70E-09	1.16186E-05	8.66929E-05	0.00023930
6.9	0	0	0	0	0	5.86E-11	6.41166E-06	6.06299E-05	0.00018246
7	**0**	**0**	**0**	**0**	**0**	**2.00E-33**	**3.33787E-06**	**4.16307E-05**	**0.00013779**
7.1	0	0	0	0	0	0	1.6193E-06	2.80129E-05	0.0001030
7.2	0	0	0	0	0	0	7.2E-07	1.84323E-05	7.61701E-0
7.3	0	0	0	0	0	0	2.86668E-07	1.18298E-05	5.56697E-0
7.4	0	0	0	0	0	0	9.87781E-08	7.38337E-06	4.01798E-0
7.5	0	0	0	0	0	0	2.79397E-08	4.46532E-06	2.8611E-05
7.6	0	0	0	0	0	0	5.9375E-09	2.60555E-06	2.00781E-0
7.7	0	0	0	0	0	0	7.95907E-10	1.45914E-06	1.3869E-05
7.8	0	0	0	0	0	0	4.76074E-11	7.79121E-07	9.41678E-0
7.9	0	0	0	0	0	0	3.76701E-13	3.93395E-07	6.27485E-0
8	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1.85845E-07**	**4.09602E-0**
8.1	0	0	0	0	0	0	0	8.1E-08	2.61378E-0
8.2	0	0	0	0	0	0	0	3.1959E-08	1.62655E-0
8.3	0	0	0	0	0	0	0	1.11153E-08	9.8428E-07
8.4	0	0	0	0	0	0	0	3.27755E-09	5.77244E-0
8.5	0	0	0	0	0	0	0	7.71328E-10	3.26769E-0
8.6	0	0	0	0	0	0	0	1.3093E-10	1.77685E-0
8.7	0	0	0	0	0	0	0	1.83929E-10	9.22592E-0
8.8	0	0	0	0	0	0	0	5.23338E-13	4.54061E-0
8.9	0	0	0	0	0	0	0	2.06752E-15	2.09857E-0
9	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**9E-09**
9.1	0	0	0	0	0	0	0	0	3.52553E-0
9.2	0	0	0	0	0	0	0	0	1.2348E-09
9.3	0	0	0	0	0	0	0	0	3.75289E-1
9.4	0	0	0	0	0	0	0	0	9.47303E-1
9.5	0	0	0	0	0	0	0	0	1.85547E-1
9.6	0	0	0	0	0	0	0	0	2.51658E-1
9.7	0	0	0	0	0	0	0	0	1.90925E-1
9.8	0	0	0	0	0	0	0	0	5.0176E-15
9.9	0	0	0	0	0	0	0	0	9.9E-18
10	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**4.1359E-25**

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