A Model with Costly-State Verification

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- Elements:
  1. Information asymmetries between lenders and borrowers $\implies$ costly state verification (Townsend, 1979).
  2. Debt contracting in nominal terms: Fisher effect.
  3. Changing spreads.

- We will calibrate the model to reproduce some basic observations of the U.S. economy.
Flowchart of the Model
Households

- Representative household:

\[
E_0 \sum_{t=0}^{\infty} \beta^t e^d_t \left\{ u(c_t, l_t) + \nu \log \left( \frac{m_t}{p_t} \right) \right\}
\]

- \(d_t\) is an intertemporal preference shock with law of motion:

\[
d_t = \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}, \epsilon_{d,t} \sim \mathcal{N}(0, 1).
\]

- Why representative household? Heterogeneity?
Asset Structure

- The household saves on three assets:
  1. Money balances, \( m_t \).
  2. Deposits at the financial intermediary, \( a_t \), that pay an uncontingent nominal gross interest rate \( R_t \).
  3. Arrow securities (net zero supply in equilibrium).

Therefore, the household’s budget constraint is:

\[
c_t + \frac{a_t}{p_t} + \frac{m_t + 1}{p_t} = w_t l_t + R_{t-1} \frac{a_{t-1}}{p_t} + \frac{m_t}{p_t} + T_t + F_t + tre_t
\]

where:

\[
re_t = (1 - \gamma^e) n_t - w^e
\]
Optimality Conditions

- The first-order conditions for the household are:

\[ e^{d_t} u_1(t) = \lambda_t \]

\[ \lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right\} \]

\[ -u_2(t) = u_1(t) \nu_t \]

- Asset pricing kernel:

\[ SDF_t = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \]

and standard non-arbitrage conditions.
Competitive final producer with technology

\[ y_t = \left( \int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \]

Thus, the input demand functions are:

\[ y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t \quad \forall i, \]

Price level:

\[ p_t = \left( \int_0^1 p_{it}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}. \]
Intermediate Goods Producers

- Continuum of intermediate goods producers with market power.

Technology:

\[ y_{it} = e^{z_t} k_{it-1}^{\alpha} l_{it}^{1-\alpha} \]

where

\[ z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}, \epsilon_{z,t} \sim \mathcal{N}(0, 1) \]

Cost minimization implies:

\[
mc_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha \frac{w_t^{1-\alpha} r_t^\alpha}{e^{z_t}}
\]

\[
\frac{k_{t-1}}{l_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}
\]
Sticky Prices

- Calvo pricing: in each period, a fraction $1 - \theta$ of firms can change their prices while all other firms keep the previous price.

- Then, the relative reset price $\Pi_t^* = p_t^* / p_t$ satisfies:

\[
\begin{align*}
\varepsilon g_t^1 &= (\varepsilon - 1) g_t^2 \\
 g_t^1 &= \lambda_t m c_t y_t + \beta \theta E_t \Pi_{t+1}^{\varepsilon} g_{t+1}^1 \\
 g_t^2 &= \lambda_t \Pi_t^* y_t + \beta \theta E_t \Pi_{t+1}^{\varepsilon-1} \left( \frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2
\end{align*}
\]

- Given Calvo pricing, the price index evolves as:

\[
1 = \theta \Pi_t^{\varepsilon-1} + (1 - \theta) \Pi_t^{*1-\varepsilon}
\]
Capital Good Producers I

- Capital is produced by a perfectly competitive capital good producer.

- Why?

- It buys installed capital, $x_t$, and adds new investment, $i_t$, to generate new installed capital for the next period:

$$x_{t+1} = x_t + \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t$$

where $S[1] = 0$, $S'[1] = 0$, and $S''[\cdot] > 0$.

- Alternative:

1. Adjustment cost in capital.

2. Time to build.
Technology illiquidity.

Importance of irreversibilities?

The period profits of the firm are:

\[ q_t \left( x_t + \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t \right) - q_t x_t - i_t = q_t \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t - i_t \]

where \( q_t \) is the relative price of capital.
Capital Good Producers III

- Discounted profits:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left( q_t \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t - i_t \right)
\]

Since this objective function does not depend on \( x_t \), we can make it equal to \((1 - \delta) k_{t-1}\).

- First-order condition of this problem is:

\[
q_t \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) - S' \left[ \frac{i_t}{i_{t-1}} \right] \left[ \frac{i_t}{i_{t-1}} \right] + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S' \left[ \frac{i_{t+1}}{i_t} \right] \left( \frac{i_{t+1}}{i_t} \right)^2 = 1
\]

and the law of motion for capital is:

\[
k_t = (1 - \delta) k_{t-1} + \left( 1 - S \left[ \frac{i_t}{i_{t-1}} \right] \right) i_t
\]
Entrepreneurs use their (end-of-period) real wealth, \( n_t \), and a nominal bank loan \( b_t \), to purchase new installed capital \( k_t \):

\[
q_t k_t = n_t + \frac{b_t}{p_t}
\]

The purchased capital is shifted by a productivity shock \( \omega_{t+1} \):

1. Lognormally distributed with CDF \( F(\omega) \) and
2. Parameters \( \mu_{\omega,t} \) and \( \sigma_{\omega,t} \)
3. \( E_t \omega_{t+1} = 1 \) for all \( t \).

Therefore:

\[
E_t \omega_{t+1} = e^{\mu_{\omega,t+1} + \frac{1}{2} \sigma_{\omega,t+1}^2} = 1 \Rightarrow \mu_{\omega,t+1} = -\frac{1}{2} \sigma_{\omega,t+1}^2
\]

This productivity shock is a stand-in for more complicated processes such as changes in demand or the stochastic quality of projects.
The standard deviation of this productivity shock evolves:

$$\log \sigma_{\omega,t} = (1 - \rho_{\sigma}) \log \sigma_{\omega} + \rho_{\sigma} \log \sigma_{\omega,t-1} + \eta_{\sigma} \varepsilon_{\sigma,t}, \varepsilon_{\sigma,t} \sim \mathcal{N}(0, 1).$$

The shock $t + 1$ is revealed at the end of period $t$ right before investment decisions are made. Then:

$$\log \sigma_{\omega,t} - \log \sigma_{\omega} = \rho_{\sigma} (\log \sigma_{\omega,t-1} - \log \sigma_{\omega}) + \eta_{\sigma} \varepsilon_{\sigma,t}$$

$$\Rightarrow \hat{\sigma}_{\omega,t} = \rho_{\sigma} \hat{\sigma}_{\omega,t-1} + \eta_{\sigma} \varepsilon_{\sigma,t}$$

More general point: stochastic volatility.
The entrepreneur rents the capital to intermediate goods producers, who pay a rental price $r_{t+1}$.

Also, at the end of the period, the entrepreneur sells the undepreciated capital to the capital goods producer at price $q_{t+1}$.

Therefore, the average return of the entrepreneur per nominal unit invested in period $t$ is:

$$R^k_{t+1} = \frac{p_{t+1}}{p_t} \frac{r_{t+1}}{q_t} + \frac{q_{t+1} (1 - \delta)}{q_t}$$
Debt Contract

- Costly state verification framework.

- For every state with associated $R_{t+1}^k$, entrepreneurs have to either:
  1. Pay a state-contingent gross nominal interest rate $R_{t+1}^l$ on the loan.
  2. Or default.

- If the entrepreneur defaults, it gets nothing: the bank seizes its revenue, although a portion $\mu$ of that revenue is lost in bankruptcy.

- Hence, the entrepreneur will always pay if it $\omega_{t+1} \geq \bar{\omega}_{t+1}$ where:
  $$R_{t+1}^l b_t = \bar{\omega}_{t+1} R_{t+1}^k p_t q_t k_t$$

- If $\omega_{t+1} < \bar{\omega}_{t+1}$, the entrepreneur defaults, the bank monitors the entrepreneur and gets $(1 - \mu)$ of the entrepreneur’s revenue.
Zero Profit Condition

- The debt contract determines $R^l_{t+1}$ to be the return such that banks satisfy its zero profit condition in all states of the world:
  
  $$
  \left(1 - F(\overline{\omega}_{t+1}, \sigma_{\omega,t+1})\right) R^l_{t+1} b_t
  $$

  Revenue if loan pays

  $$
  + (1 - \mu) \int_{0}^{\overline{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1}) R^k_{t+1} p_t q_t k_t
  $$

  Revenue if loan defaults

  $$
  = s_t R_t b_t
  $$

  Cost of funds

- $s_t = 1 + e^{\tilde{s}+\tilde{s}_t}$ is a spread caused by the cost of intermediation such that:
  
  $$
  \tilde{s}_t = \rho_s \tilde{s}_{t-1} + \sigma_s \epsilon_{s,t}, \ \epsilon_{s,t} \sim \mathcal{N}(0,1).
  $$

- For simplicity, intermediation costs are rebated to the households in a lump-sum fashion.

- External finance premium.
Optimality of the Contract

- This debt contract is not necessarily optimal.

- However, it is a plausible representation for a number of nominal debt contracts that we observe in the data.

- Also, the nominal structure of the contract creates a Fisher effect through which changes in the price level have an impact on real investment decisions.

- Importance of working out the optimal contract.
Define share of entrepreneurial earnings accrued to the bank:

$$\Gamma(\omega_{t+1}, \sigma_{\omega,t+1}) = \omega_{t+1} (1 - F(\omega_{t+1}, \sigma_{\omega,t+1})) + G(\omega_{t+1}, \sigma_{\omega,t+1})$$

where:

$$G(\omega_{t+1}, \sigma_{\omega,t+1}) = \int_{0}^{\omega_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1})$$

Thus, we can rewrite the zero profit condition of the bank as:

$$\frac{R_{t+1}^k}{s_t R_t} \left[ \Gamma(\omega_{t+1}, \sigma_{\omega,t+1}) - \mu G(\omega_{t+1}, \sigma_{\omega,t+1}) \right] q_t k_t = \frac{b_t}{p_t}$$

which gives a schedule relating $R_{t+1}^k$ and $\omega_{t+1}$. 
Characterizing the Contract II

- Now, define the ratio of loan over wealth:

\[ \varrho_t = \frac{b_t}{p_t} = \frac{q_t k_t - n_t}{n_t} = \frac{q_t k_t}{n_t} - 1 \]

- and we get

\[ R_{k+1}^t \left[ \Gamma (\omega_{t+1}, \sigma_{\omega, t+1}) - \mu G (\omega_{t+1}, \sigma_{\omega, t+1}) \right] (1 + \varrho_t) = \varrho_t \]

that is, all the entrepreneurs, regardless of their level of wealth, will have the same leverage, \( \varrho_t \).

- A most convenient feature for aggregation.

- Balance sheet effects.
Problem of the Entrepreneur

- Maximize its expected net worth given the zero-profit condition of the bank:

\[
\max_{\varrho_t, \omega_{t+1}} \mathbb{E}_t \left\{ \frac{R^{k}_{t+1}}{R_t} (1 - \Gamma(\omega_{t+1}, \sigma_{\omega,t+1})) + \eta_t \left[ \frac{R^{k}_{t+1}}{s_t R_t} [\Gamma(\omega_{t+1}, \sigma_{\omega,t+1}) - \mu G(\omega_{t+1}, \sigma_{\omega,t+1})] - \frac{\eta_t}{1+\varrho_t} \right] \right\}
\]

- After a fair amount of algebra:

\[
\mathbb{E}_t \frac{R^{k}_{t+1}}{R_t} (1 - \Gamma(\omega_{t+1}, \sigma_{\omega,t+1})) = \mathbb{E}_t \eta_t \frac{n_t}{q_t k_t}
\]

where the Lagrangian multiplier is:

\[
\eta_t = \frac{s_t \Gamma_{\omega}(\omega_{t+1}, \sigma_{\omega,t+1})}{\Gamma_{\omega}(\omega_{t+1}, \sigma_{\omega,t+1}) - \mu G_{\omega}(\omega_{t+1}, \sigma_{\omega,t+1})}
\]

- This expression shows how changes in net wealth have an effect on the level of investment and output in the economy.
Death and Resurrection

- At the end of each period, a fraction $\gamma^e$ of entrepreneurs survive to the next period and the rest die and their capital is fully taxed.

- They are replaced by a new cohort of entrepreneurs that enter with initial real net wealth $w^e$ (a transfer that also goes to surviving entrepreneurs).

- Therefore, the average net wealth $n_t$ is:

$$ n_t = \gamma^e \frac{1}{\Pi_t} \left[ (1 - \mu G(\bar{\omega}_t, \sigma_{\omega,t})) R^k_t q_{t-1} k_{t-1} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right] + w^e $$

- The death process ensures that entrepreneurs do not accumulate enough wealth so as to make the financing problem irrelevant.
The Financial Intermediary

- A representative competitive financial intermediary.

- We can think of it as a bank but it may include other financial firms.

- Intermediates between households and entrepreneurs.

- The bank:
  1. Lends to entrepreneurs a nominal amount $b_t$ at rate $R^l_{t+1}$,
  2. But recovers only an (uncontingent) rate $R_t$ because of default and the (stochastic) intermediation costs.
  3. Thus, the bank pays interest $R_t$ to households.
The Monetary Authority Problem

- Conventional Taylor rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_{\Pi}(1-\gamma_R)} \left( \frac{y_t}{y} \right)^{\gamma_y(1-\gamma_R)} \exp(\sigma_m m_t)
\]

through open market operations that are financed through lump-sum transfers \( T_t \).

- The variable \( \Pi \) represents the target level of inflation (equal to inflation in the steady-state), \( y \) is the steady state level of output, and \( R = \frac{\Pi}{\beta} \) the steady state nominal gross return of capital.

- The term \( \varepsilon_{mt} \) is a random shock to monetary policy distributed according to \( \mathcal{N}(0, 1) \).
Using conventional arguments, we find expressions for aggregate demand and supply:

\[ y_t = c_t + i_t + \mu G(\omega_t, \sigma_{\omega, t}) (r_t + q_t (1 - \delta)) k_{t-1} \]

\[ y_t = \frac{1}{\nu_t} e^{z_t} k_{t-1}^\alpha l_t^{1-\alpha} \]

where \( \nu_t = \int_0^1 \left( \frac{p_{it}}{p_t} \right)^{\varepsilon} di \) is the inefficiency created by price dispersion.

By the properties of Calvo pricing, \( \nu_t \) evolves as:

\[ \nu_t = \theta \Pi_t^\varepsilon \nu_{t-1} + (1 - \theta) \Pi_t^{*\varepsilon} \]

We have steady state inflation \( \Pi \). Hence, \( \hat{\nu}_t \neq 0 \) and monetary policy has an impact on the level and evolution of measured productivity.
The first-order conditions of the household:

\[ e^{d_t} u_1(t) = \lambda_t \]

\[ \lambda_t = \beta \mathbb{E}_t \{ \lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \} \]

\[ -u_2(t) = u_1(t) \omega_t \]
The first-order conditions of the intermediate firms:

\[ \varepsilon g_t^1 = (\varepsilon - 1) g_t^2 \]
\[ g_t^1 = \lambda_t mc_t y_t + \beta \theta E_t \Pi_t^\varepsilon g_{t+1}^1 \]
\[ g_t^2 = \lambda_t \Pi_t^* y_t + \beta \theta E_t \Pi_t^{\varepsilon-1} \left( \frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2 \]
\[ k_{t-1} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t} l_t \]
\[ mc_t = \left( \frac{1}{1 - \alpha} \right)^{1 - \alpha} \left( \frac{1}{\alpha} \right)^\alpha \frac{w_t^{1-\alpha} r_t^\alpha}{e^{z_t}} \]
Equilibrium Conditions III

- Price index evolves:

\[ 1 = \theta \Pi_{t}^{\varepsilon-1} + (1 - \theta) \Pi_{t}^{*1-\varepsilon} \]

- Capital good producers:

\[
q_{t} \left(1 - S \left[ \frac{i_{t}}{i_{t-1}} \right] - S' \left[ \frac{i_{t}}{i_{t-1}} \right] \frac{i_{t}}{i_{t-1}} \right) \\
+ \beta \mathbb{E}_{t} \frac{\lambda_{t+1}}{\lambda_{t}} q_{t+1} S' \left[ \frac{i_{t+1}}{i_{t}} \right] \left( \frac{i_{t+1}}{i_{t}} \right)^{2} = 1 \\
k_{t} = (1 - \delta) k_{t-1} + \left(1 - S \left[ \frac{i_{t}}{i_{t-1}} \right] \right) i_{t}
\]
Equilibrium Conditions IV

- Entrepreneur problem:

\[
R_{t+1}^k = \Pi_{t+1} \frac{r_{t+1} + q_{t+1} (1 - \delta)}{q_t}
\]

\[
\frac{R_{t+1}^k}{s_t R_t} [\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})] = \frac{q_t k_t - n_t}{q_t k_t}
\]

\[
\mathbb{E}_t \frac{R_{t+1}^k}{R_t} (1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})) =
\]

\[
\left(\mathbb{E}_t s_t \frac{1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}{1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu \bar{\omega}_{t+1} F_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}\right) \frac{n_t}{q_t k_t}
\]

\[
R_{t+1} b_t = \bar{\omega}_{t+1} R_{t+1}^k p_t q_t k_t
\]

\[
q_t k_t = n_t + \frac{b_t}{p_t}
\]

\[
n_t = \gamma^e \frac{1}{\Pi_t} \left[ (1 - \mu G(\bar{\omega}_t, \sigma_{\omega,t})) R_t^k q_{t-1} k_{t-1} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right] + \omega^e
\]
The government follows its Taylor rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_{\Pi}(1-\gamma_R)} \left( \frac{y_t}{y} \right)^{\gamma_y(1-\gamma_R)} \exp(\sigma_m m_t)
\]

Market clearing

\[
y_t = c_t + i_t + \mu G(\overline{\omega}_t, \sigma_{\omega,t}) (r_t + q_t (1 - \delta)) k_{t-1}
\]

\[
y_t = \frac{1}{\nu_t} e^{z_t} k_{t-1}^{1-\alpha}
\]

\[
\nu_t = \theta \Pi_t^\varepsilon \nu_{t-1} + (1 - \theta) \Pi_t^{*-\varepsilon}
\]
Stochastic processes:

\[ d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{d,t} \]
\[ z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t} \]
\[ s_t = 1 + e^{\bar{s} + \tilde{s}_t} \]
\[ \tilde{s}_t = \rho_s \tilde{s}_{t-1} + \sigma_s \varepsilon_{s,t} \]
\[ \log \sigma_{\omega,t} = (1 - \rho_\sigma) \log \sigma_{\omega} + \rho_\sigma \log \sigma_{\omega,t-1} + \eta_\sigma \varepsilon_{\sigma,t} \]
Calibration

- Utility function:
  \[ u(c_t, l_t) = \log c_t - \psi \frac{l_t^{1+\theta}}{1+\theta} \]
  \(\psi\): households work one-third of their available time in the steady state and \(\theta = 0.5\), inverse of Frisch elasticity.

- Technology:

  \[
  \alpha \begin{bmatrix} 0.33 \\ 0.023 \\ 8.577 \\ 14.477 \end{bmatrix}
  \]

- Entrepreneur:

  \[
  \mu \begin{bmatrix} 0.15 \\ 2.528 \\ \frac{n}{n-k} \approx 2 \end{bmatrix} \bar{s} \approx 25 \text{bp.}
  \]

- For the Taylor rule, \(\Pi = 1.005\), \(\gamma_R = 0.95\), \(\gamma_\Pi = 1.5\), and \(\gamma_y = 0.1\) are conventional values.

- For the stochastic processes, all the autoregressive are 0.95.
We can find the deterministic steady state.

We linearize around this steady state.

We solve using standard procedures.

Alternatives:

1. Non-linear solutions.

2. Estimation using likelihood methods.
Figure 3.1: Shock to Preferences, 1

- $y$ vs. Quarters after shock
- $c$ vs. Quarters after shock
- $i$ vs. Quarters after shock
- $l$ vs. Quarters after shock
- $k$ vs. Quarters after shock
- $r$ vs. Quarters after shock
- $w$ vs. Quarters after shock
- $mc$ vs. Quarters after shock
- $ppi$ vs. Quarters after shock
Figure 3.2: Shock to Preferences, 2

- $R_n$ (top left)
- $R_k$ (top middle)
- spread (top right)
- $q$ (middle left)
- $n$ (middle middle)
- $b$ (middle right)
- $\omega$ (bottom left)
- $v$ (bottom right)
Figure 3.3: Shock to Productivity, 1

- **y**: Productivity
- **c**: Consumption
- **i**: Investment
- **l**: Labor
- **k**: Capital
- **r**: Real interest rate
- **w**: Wage
- **mc**: Marginal cost
- **ppi**: Price index
Figure 3.4: Shock to Productivity, 2

- **Rn**: The graph shows a function of time (quarters after shock) with values ranging from $-8 	imes 10^{-4}$ to $0$.
- **Rk**: The graph shows a function of time (quarters after shock) with values ranging from $-8 	imes 10^{-4}$ to $0$.
- **Spread**: The graph shows a function of time (quarters after shock) with values ranging from $0$ to $2$.
- **q**: The graph shows a function of time (quarters after shock) with values ranging from $-6$ to $2 	imes 10^{-3}$.
- **n**: The graph shows a function of time (quarters after shock) with values ranging from $-4$ to $4 	imes 10^{-3}$.
- **b**: The graph shows a function of time (quarters after shock) with values ranging from $-10$ to $0$.
- **oomega**: The graph shows a function of time (quarters after shock) with values ranging from $-4$ to $0$.
- **v**: The graph shows a function of time (quarters after shock) with values ranging from $-10$ to $0$. 

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Figure 3.5: Shock to Volatility, 1

Graphs showing the response of various economic variables to a shock over time.
Figure 3.6: Shock to Volatility, 2

- **Rn**: Graph showing the response of Rn to the shock over 20 quarters.
- **Rk**: Graph showing the response of Rk to the shock over 20 quarters.
- **spread**: Graph showing the response of spread to the shock over 20 quarters.
- **q**: Graph showing the response of q to the shock over 20 quarters.
- **n**: Graph showing the response of n to the shock over 20 quarters.
- **b**: Graph showing the response of b to the shock over 20 quarters.
- **oomega**: Graph showing the response of oomega to the shock over 20 quarters.
- **v**: Graph showing the response of v to the shock over 20 quarters.
- **mmuo_t**: Graph showing the response of mmuo_t to the shock over 20 quarters.
Figure 3.7: Shock to Spread, 1

Graphs showing the response of different economic variables to a shock, with axes labeled 'Quarters after shock' and various scales for the y-axis, such as $10^{-3}$ and $10^{-4}$. Variables include $y$, $c$, $i$, $l$, $k$, $r$, $w$, $MC$, and $PPi$.
Figure 3.8: Shock to Spread, 2

- $R_n$
- $R_k$
- Spread
- $q$
- $n$
- $b$
- $\omega$
- $\nu$
- $S$
Figure 3.9: Shock to Survival, 1
Figure 3.10: Shock to Survival, 2

- **$R_n$**: Blue line shows a positive trend over quarters after shock, with an initial decrease and then stabilization. The red dashed line indicates the baseline without a shock.

- **$R_k$**: Similar to $R_n$, the blue line shows a positive trend post-shock, with the red dashed line representing the baseline.

- **Spread**: Decreases over time, indicated by the blue line, and remains below the baseline (red dashed line).

- **$q$**: Shows a positive trend, with the blue line increasing over quarters after shock, and the red dashed line representing the baseline.

- **$n$**: Decreases over time, with the blue line indicating a negative trend post-shock, and the red dashed line showing the baseline.

- **$b$**: Increases over time, with the blue line showing a positive trend, and the red dashed line representing the baseline.

- **$\omega$**: Shows a positive trend, with the blue line increasing over quarters after shock, and the red dashed line indicating the baseline.

- **$\nu$**: Increases over time, with the blue line showing a positive trend, and the red dashed line representing the baseline.

- **$\gamma$**: Decreases over time, with the blue line indicating a negative trend post-shock, and the red dashed line showing the baseline.
Figure 3.12: Shock to Monetary Policy, 2
How Can We Use the Model?

- Christiano, Motto, and Rostagno (2003): Great depression.
Figure: IRFs of Output to Different Fiscal Policy Shocks