The Optimal Timing of Affirmative Action*

Hanming Fang† Roland G. Fryer, Jr.‡

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Abstract

When should the government intervene to fulfill affirmative action goals? This paper addresses this question in a dynamic model of education and employment. We compare the effects of implementing affirmative action in university admissions and labor market hiring. The informational advantage of selective universities over the firms regarding the quality of the students' grade schools is identified as an important consideration in favor of affirmative action in the university admission stage. The intuition is that selective universities can use the information advantage to sift through the previously rejected minority students and identify those who are most likely to succeed if admitted. This information advantage is inversely related to the difference between high and low quality grade schools. Another important determinant of the optimal timing is the differences in the firms' and the selective universities' misallocation costs.

Keywords: Statistical Discrimination, Affirmative Action.

JEL Classification Numbers: J71, J72.

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†Department of Economics, Yale University, P.O. Box 208264, New Haven, CT 06520-8264. Email: hanming.fang@yale.edu.

‡Harvard Society of Fellows and NBER, 1050 Massachusetts Avenue, Cambridge, MA 02138. Email: rgfryer@nber.org.
“But Freedom is not enough. You do not wipe away the scars of centuries by saying: Now you are free to go where you want, and do as you desire, and choose the leaders you please. You do not take a person who, for years, has been hobbled by chains and liberate him, bring him up to the starting line of a race and then say, ‘you are free to compete with all the others,’ and still believe that you have been justly fair.”


1 Introduction

In his 1965 commencement address at Howard University titled “To Fulfill These Rights,” President Lyndon Baines Johnson described the necessity of a governmental intervention aimed at eradicating the effects of past discrimination against African Americans. President Johnson argued that we cannot take a man who has been crippled by slavery and disenfranchised for centuries and expect him to compete with others who have not endured these physical and social shackles, and call this justice. Moreover, in 1965 we also forced the man to run on a “different track”, a track littered with social segregation, discrimination, financial hurdles and other obstacles, for a significant part of his life. Therefore, he argued, equal opportunity is not enough. Three months after his famous address, Johnson signed executive order 11246, which has come to be known as affirmative action.

Affirmative action is a costly public policy, both politically and fiscally. An immensely important question, but hitherto ignored, is where in the competition should the society optimally handicap those that have been historically disadvantaged. For example, does affirmative action need to be implemented in every stage of the race? Should we give a handicap as soon as the competition begins, and then allow them to continue running on their own relatively perilous path? Or, should we wait until the end of the contest and have a system that gives the prize to the disadvantaged contestant so long as he does not lose by “too much”?

This paper analyzes this query in a dynamic model of education and employment. A short synopsis of our model is as follows. There is a continuum of agents with heterogeneous skill investment costs (due to, for example, heterogeneous parental wealth or heterogeneous ability). We divide a typical agent’s life into two stages of education – grade schools (GS) and universities – and one stage of employment. We assume that agents are born into residential neighborhoods that have grade schools with either high or low quality. In grade school, students decide whether
to invest in skills that may prepare them for the university. Education in high quality GS is more productive than that in low quality ones in the sense that, with the same human capital investment, a student is more likely to be qualified in a high quality GS. There are also two types of universities: selective and non-selective. Selective universities have active admission policies while non-selective universities simply admit any student who applies. In college, students decide whether or not to invest in another set of skills that may make them qualified for high-productivity jobs in the firms. Similar to high quality GS, education in selective universities is more productive than that in non-selective ones in the sense that, with the same human capital investment, a student is more likely to be qualified in a selective university. That is, selective universities have a *developmental advantage* over non-selective ones. We assume that firms only hire university graduates; and they have a well-paying complex job and a low-paying simple job. The main friction in the model is that students’ qualifications are not perfectly observed: selective universities do not perfectly observe high school applicants’ qualification; and firms do not perfectly observe university graduates’ qualification; instead they observe noisy signals.

We first characterize the equilibria of the dynamic game without affirmative action. The results are quite intuitive. Students from low quality GS are more likely to attend non-selective universities and have worse labor market outcomes. Therefore, there is potential for disparate employment outcomes between individuals due to asymmetries in their initial conditions. In our model, the disparate outcomes between racial groups could occur both as a result of the groups’ different initial conditions, or when the groups are coordinated on different equilibria.\(^1\) We model affirmative action (AA) as a government mandate of increasing the share of “disadvantaged” racial group on the high-paying complex job. Our main result is that, under some intuitive conditions, it is optimal to place the burden of affirmative action on selective universities. The conditions can be described as follows. First, selective universities knows more about the quality of students’ grade schools than the employers, which gives them an *informational advantage* over the employers. We believe this is an empirically realistic assumption. An evidence in support of this assumption is Steinberg’s (2002) detailed account of the admission process at the selective Wesleyan University. Wesleyan assigns admission officers in charge of a set of high schools located in a particular region of the country. The admission officers can access the detailed statistics of every high school in their respective region, as well as additional information about students through contacts with the high school guidance

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\(^1\)Both are consistent with the economists’ notion of statistical discrimination. We choose to emphasize the disparate outcome as a result of different initial conditions.
counselors. Our second condition is that, the selective university’s misallocation cost, i.e., the cost on a selective university if it admits a student who fails, is (sufficiently) smaller than the firm’s misallocation cost, i.e., the cost on a firm if it employs an unqualified worker on the complex job. While we believe this to be a reasonable assumption, unfortunately we do not have direct evidence in support of such a hypothesis.

Given these assumptions, the intuition for our result is as follows. Suppose the burden of the affirmative action is placed on the firms, they would have to hire minority college graduates (who they deemed not good enough in the absence of the AA mandate) to the complex job. Because the employers do not have complete knowledge of students’ GS quality, the additionally-hired minority workers as a result of the AA requirement are a mixture of (former) low and high quality GS graduates. However, on average the low quality GS students who did not initially make it to the complex job have lower skill investment costs than high quality GS students in the same situation. This is the case because the inferior educational technology of the low quality GS provides a lower investment incentive for their students. Thus among students who did not qualify, those from low quality GS are actually on average “better.” When the AA burden is placed to the selective universities, they can use their informational advantage to precisely identify and admit the group of high-ability students from low quality GS. These students had either initially invested but failed anyway due to the inferior education technology, or were initially discouraged from investing at all. Put it differently, the selective universities could sift through the additional minority workers that employers would have hired on the complex job under AA mandate and only enroll those who are more likely to be qualified (namely those from low quality schools). This, together with the developmental advantage of the selective universities over the non-selective ones, ensures that the additional minorities that end up on the complex job when AA burden is placed on the selective universities will be more qualified than those if the burden is on the employers. However, in our model, the selective universities also endures a misallocation cost from having to admit students that they would have chosen not to admit in the absence of AA. The second condition mentioned above ensures that the benefits from informational and developmental advantages of the selective universities dominates the additional misallocation costs of the selective universities when they are burdened with the AA requirements.

There is an extensive economics literature on the pros and cons of affirmative action, both empirically and theoretically. For example, Freeman (1973), Butler and Heckman (1977), Leonard (1984), Smith and Welch (1984), Heckman and Paynor (1989), Welch (1989) and Donohue and
Heckman (1990) empirically estimated the magnitude of the black-white wage convergence since the 1960’s that can be attributed to affirmative action; and Holzer and Neumark (2000) provided an excellent review of the existing evidence from economics and other disciplines on the efficiency costs of affirmative action. On the other hand, Coate and Loury (1993), Mailath, Samuelson, and Shaked (2000), Chan and Eyster (2002), Epple, Romano and Sieg (2003), Loury, Fryer and Yuret (2003), and Moro and Norman (2003) studied the theoretical implications of affirmative action on skill investment, worker mismatch, long run stereotypes, college quality, and redistribution etc. Our paper builds on Coate and Loury (1993) and study the hitherto unstudied question of the optimal timing of the affirmative action.

The remainder of the paper is structured as follows. Section 2 presents a dynamic model of education and employment; Section 3 analyzes the equilibrium of the model; Section 4 introduces affirmative action and presents our main result about its optimal timing; and Section 5 concludes and discusses some interpretations of our results. Appendix A contains all the formulae omitted from the main text; and Appendix B contains the proofs of all the results.

2 A Dynamic Model of Education and Employment

We consider a model with interactions between three sorts of agents: students, universities and firms. For clarity, the model is presented with a single racial group. Several parameters in the model could plausibly be functions of race, and we highlight these throughout. We begin with a detailed description of the basic building blocks of the model.

2.1 The Basic Building Blocks

Students There is a continuum of students with unit measure. Students attend grade schools before they can be admitted to universities and eventually be employed in the labor market. As we will describe below, they make skill investment decisions both in grade school and in the university. Let $k$ denote a student’s skill investment cost, which is a random draw from a cumulative distribution function $G(\cdot) : \mathbb{R}_+ \rightarrow [0, 1]$. This cost, $k$, is her private information. For simplicity, we assume that investment costs in grade school and in college are the same.

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2Broadly interpreted, the skill investment cost $k$ may be affected by the student’s innate ability, the wealth of her family, or the neighborhood in which she is reared. Thus, it is possible that the cost distribution $G$ might depend on race or other social categories.
**Grade Schools** Grade schools (GS) can be either of low \((l)\) or high \((h)\) quality. High and low quality GS differ in their educational production technology: if a student in a high quality GS invests in skills (by incurring the investment cost \(k\)), she becomes qualified with probability \(\phi_h = 1\) whereas a student who invests in skills in a low quality GS becomes qualified with probability \(\phi_l < 1\) (see Table 1). The difference, \(1 - \phi_l\), measures the quality difference between the two grade schools.\(^3\) If a student does not invest in skills, we assume that she is unqualified regardless of the quality of her GS.

<table>
<thead>
<tr>
<th>Grade School Quality</th>
<th>Invest</th>
<th>Not Invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h)</td>
<td>(\phi_h = 1)</td>
<td>0</td>
</tr>
<tr>
<td>(l)</td>
<td>(\phi_l &lt; 1)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Grade School Educational Production Technologies.

The quality of GS is exogenously assigned to a student according to her residential location, and is orthogonal to her skill investment cost.\(^4\) Let \(\alpha \in (0, 1)\) and \(1 - \alpha\) denote the fraction of students assigned to low and high quality GS respectively. To the extent that different racial groups reside in different neighborhoods, one could also imagine that \(\alpha\) might depend on race.

**Universities** There are two types of universities dubbed as “selective” and “non-selective.” They differ in two dimensions. First, selective universities design their admission policy to optimally select students in order to maximize an objective function to be detailed below, while non-selective universities simply admit all applicants rejected by selective universities.\(^5, 6\) Second, they differ in their educational production technologies.

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\(^3\)The dramatic difference in the quality of grade schools is becoming an increasingly contentious public issue. Kozol (1991) provides astonishing accounts of what he called “savage inequalities” in the public school fundings in East St. Louis (IL), Chicago, New York City, Camden (NJ), Washington, D.C. and San Antonio (TX).

\(^4\)This is an empirically realistic assumption due to the heavy reliance of the school finance on local property taxes. See Wise (1967) for a review of the history and workings of the school finance system in the U.S.

\(^5\)According to Bowen and Bok (1998, p. 15), the vast majority of undergraduate institutions do have have enough applicants to be able to pick and choose among them. They estimate that only about 20 to 30 percent of almost 4,100 four-year colleges in the U.S. are selective.

\(^6\)According to National Center for Education Statistics (2002, Table 184), close to 40% of the high school completers in the United States did not enroll in any colleges in 2001. In our model, however, we can interpret non-enrollment in any college simply as enrolling in non-selective universities.
We summarize in Table 2 how the qualification of a university graduate is determined by her skill investment decisions in college, her GS qualification when entering the university, and the quality of the university itself. Consider a qualified GS student. If she continues to invest in skills in a selective university, then she will be qualified at graduation to successfully perform on the complex job in the firm with probability 1; if she does not invest in college, she will be qualified with probability $\zeta_{QN} \in (0, 1)$.\(^7\) Now consider an unqualified grade school student. If she invests in the university, she will be qualified at graduation with probability $\zeta_{UI} < 1$. If she does not invest, however, she will be qualified with probability 0.\(^8\) To capture the idea that investment in college is more important in producing skills useful for the labor market, we assume that $\zeta_{UI} > \zeta_{QN}$.

In contrast, non-selective universities are inferior to the selective universities in their education production technology captured by a proportional factor $\phi_c \in (0, 1)$, where $1 - \phi_c$ measures the quality difference between the two types of universities. It is important to emphasize that, once the GS qualification is controlled, we assume that the quality of GS does not affect the subsequent college educational production.

**Firms**

Firms hire university graduates when they enter the labor market. Each firm has two jobs, a complex job and a simple job. The simple job can be performed by all workers, but the complex job can be successfully performed only by qualified university graduates. Specifically, the firm’s net payoff, i.e., output net of wage payments, from a qualified university graduate on the complex job is denoted $\chi_f^q > 0$; the net payoff of assigning an unqualified university graduate to the complex job is $-\chi_f^u < 0$. The payoffs from the simple job are normalized to zero, independent of the workers’ qualification.

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\(^7\)In words, $\zeta_{QN}$ represents the degree to which investing in the university is a necessary condition for being qualified for a position in the firm.

\(^8\)In words, $\zeta_{UI}$ represents the degree to which investing in the university is a sufficient condition for becoming qualified for a position in the firm.
2.2 Sequence of Actions, Informational Structure, Payoffs and Strategies

Now that we have provided a description of the basic building blocks of our model, we present the sequencing of events, information structure and further details of the payoffs. The sequence of actions can be usefully divided into seven stages:

- **Stage 1:** A fraction $\alpha \in (0, 1)$ of the students are assigned to low quality GS; and Nature draws a skill investment cost $k$ from distribution $G$ for each student.

- **Stage 2:** Each student observes $k$ and the quality of her GS, and makes a dichotomous GS investment decision. A student’s skill investment choice affects her GS qualification according to Table 1. Neither a student’s skill investment decision nor her actual qualification are perfectly observable to universities.

- **Stage 3:** For each GS student, nature emits a noisy but informative signal, $\theta_s \in [0, 1]$, to selective universities as to whether or not they are qualified. We follow Coate and Loury (1993) in the specification of the test signal technology (see Figure 1 for a graphical illustration of the test signal distributions). The signal for a *qualified* GS student is distributed uniformly from $[\theta_q, 1]$, where $\theta_q < 1$; and the signal for an *unqualified* GS student is distributed uniformly from $[0, \theta_u]$ where $1 > \theta_u > \theta_q$.

In effect, the signal generates one of three outcomes: “pass” ($\theta > \theta_u$); “fail” ($\theta < \theta_q$); and “unclear” ($\theta \in [\theta_q, \theta_u]$). Universities are certain that a student is qualified (respectively,
unqualified) whenever \( \theta > \theta_u \) (respectively, \( \theta < \theta_q \)); and while the signal is ambiguous when \( \theta \in [\theta_q, \theta_u] \). Moreover, every \( \theta \in [\theta_q, \theta_u] \) provides the employer with the same information because the likelihood ratio \((1 - \theta_q)/\theta_u\) is constant in this range. For notational convenience, let \( \rho_q = (\theta_u - \theta_q)/(1 - \theta_q) \) (respectively, \( \rho_u = (\theta_u - \theta_q)/\theta_u \)) denote the probability that a qualified (respectively, unqualified) student generates an unclear signal.

- **Stage 4:** Selective universities observe each student’s GS quality and test signal \( \theta \), then determine whom to admit. We assume that universities care about two performance criterion. First, they derive utility \( \chi^q > 0 \) from producing qualified students, and disutility \( -\chi^u < 0 \) from producing unqualified students. Second, they care about their students’ placement in the following way: they derive utility \( R^q > 0 \) from placing a qualified graduate on the complex job; and disutility \( -R^u < 0 \) from placing an unqualified student on the complex job.\(^9\) The selective universities attempt to maximize their expected payoff in determining their admission policies.

- **Stage 5:** Students, after realizing which university they are admitted to (selective or non-selective), make another dichotomous investment decision. A university student’s skill investment decision, her GS qualification, and university quality all affect her likelihood of being qualified at college graduation as described in Table 2. Neither a student’s skill investment decision nor her actual qualification is perfectly observable to firms in the labor market.

- **Stage 6:** After students make their second investment decision, nature reveals another noisy but informative signal, \( \theta_c \in [0, 1] \), regarding whether or not she is qualified to employers. We assume that the test signal technology in this stage is identical to that in stage 3.

- **Stage 7:** Employers make hiring decisions. This hiring decision is simply to choose whether to assign each university graduate to the complex or the simple job. When making the job assignment decision, the firms observe the signal emitted by university students \( \theta_c \), as well as the selectivity of their universities, but does not perfectly observe her qualification for the complex job. The firms observe neither the signal the students sent in stage 3 to universities \( \theta_s \), nor the quality of their previously attended GS.\(^{10}\) This is a crucial assumption which we

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\(^9\)While the specifications of the selective university’s payoffs are ad hoc, they can be justified on reputational grounds. For example, these payoffs are quite natural if universities care about producing educated citizens on one hand and maximizing alumni giving and the likelihood of placing quality students in firms on the other.

\(^{10}\)When there is no confusion, we will hereafter drop the subscripts of \( s \) and \( c \) on the testing technologies to make
think are realistic. In fact it is illegal in many states for the employer to ask about the place
of birth where mostly likely a candidate would have attended her grade schools (see Steingold
1997).

Finally, we assume that if a student is assigned to the complex job, she will obtain a wage \( \omega > 0 \)
in stage 7; if she is assigned to the simple task, she will obtain a wage of 0. A student’s dynamic
payoff is simply the wage from the job she is assigned to in stage 7 net of her skill investment cost
\( k \), if incurred in Stage 2 and/or Stage 5.\(^{11}\)

Now we are ready to describe the strategies of the three types of active decision makers in our
model: the students, who have to make skill investment decisions in stages 2 and 5; the selective
universities, who make admission decisions in stage 4; and the employers, who make hiring decisions
in stage 7.

A strategy for a student is a pair of skill investment decisions respectively in stage 2 and 5,
knowing her private skill investment cost \( k \).

Selective universities decide whether or not to admit a student in stage 4 after observing her
noisy signal and her grade school quality. Due to the signal structure specified by Figure 1, selective
universities only need to specify grade school specific admission criterion for three classes of
students: passers, failers, and those who emit unclear signals. For the equilibrium we will focus on
below, we assume (and then verify) that selective universities will admit passers with probability
one; admit failers with probability zero; and admit those with unclear signals with a grade school
specific probability \( \mu^c_i \in [0, 1] \), where \( i \in \{l, h\} \). A selective university is said to be “liberal”
towards a grade school of quality \( i \) if they give them the “benefit of the doubt” (i.e., \( \mu^c_i = 1 \))
and “conservative” if \( \mu^c_i = 0 \).

Firms make hiring decisions in stage 7, observing the quality of each applicant’s university and
her noisy signal \( \theta \). It is obvious from the firms’ payoff structure that without affirmative action
pressures, independent of university quality, all students with a pass signal will be assigned by the
firms to the complex job with probability one, and all failers will be relegated to the simple job
with probability one. For expositional clarity, we will impose this equilibrium restriction. Thus,
we only need to determine the probability that a graduate with an unclear signal from a university
of quality \( j \in \{S, N\} \) will be assigned to the complex job. Let \( \mu^f_j \in [0, 1] \) denote this probability.

\( ^{11} \)Note that we are not allowing the wage rates to depend on the workers’ expected productivity. Endogenizing
the wages will surely complicate the analysis without changing our results.
As before, a firm is said to be liberal towards a quality $j$ university if $\mu_j^f = 1$ and conservative if $\mu_j^f = 0$.

3 Analyzing the Model

3.1 Best Responses

Before defining and characterizing the equilibrium, we describe the best responses of various decision makers in our model.

We first analyze the firms’ optimal job assignment decision in Stage 7. To maximize its expected profit, a firm needs to use its available information to forecast the probability that each college graduate is qualified for the complex task. Suppose $\pi_j^c, j \in \{S, N\}$ is the firm’s prior belief that a quality $j$ university student is qualified for the complex job; suppose that the firm also observes a signal $\theta$ from a quality $j$ college graduate. Given $\pi_j^c$ and $\theta$, the firm formulates a posterior probability $\psi(\pi_j^c, \theta)$ (using Bayes’ Rule) that the student is qualified:

$$\psi(\theta; \pi_j^c) = \begin{cases} 
1 & \text{if } \theta \geq \theta_u \text{ (pass)}, \\
\frac{\pi_j^c \rho_q}{\pi_j^c \rho_q + (1-\pi_j^c) \rho_u} & \text{if } \theta \in [\theta_q, \theta_u] \text{ (unclear)}, \\
0 & \text{if } \theta \leq \theta_q \text{ (fail)}. 
\end{cases}$$

(1)

The firm’s expected payoff from assigning this worker to the complex job is given by $\psi(\theta; \pi_j^c) \chi_q^f - \left[1 - \psi(\theta; \pi_j^c)\right] \chi_u^f$. As we mentioned earlier, the firms will optimally assign all workers with pass signal to the complex task with probability one, and those with fail signal to the simple task with probability one. The optimal task assignment rule for workers with unclear signals (i.e. when $\theta \in [\theta_q, \theta_u]$), denoted by $\mu_j^{f^*}$, depends on $\pi_j^c$. Exploiting the fact that $\psi(\theta; \pi_j^c)$ is monotonically increasing in $\pi_j^c$ when $\theta \in [\theta_q, \theta_u]$, we have:

Lemma 1 There exists a $\hat{\pi}^c \in (0, 1)$ such that

$$\mu_j^{f^*}(\pi_j^c) = \begin{cases} 
1 & \text{if } \pi_j^c > \hat{\pi}^c \\
[0, 1] & \text{if } \pi_j^c = \hat{\pi}^c \\
0 & \text{if } \pi_j^c < \hat{\pi}^c. 
\end{cases}$$

(2)

In words, Lemma 1 demonstrates that firms assign quality $j$ university graduates with unclear signals to the complex job if and only if they have a sufficiently optimistic prior about students from
these universities. Of course, in equilibrium firms’ priors $\pi^c_j$ must be consistent with the students’ skill investment behavior.

Now we describe the university students’ optimal skill investment decisions in Stage 5. First, it is easy to see that a student will invest in skills in universities if and only if her skill investment cost $k$ is less than a threshold; and this threshold is equal to her net benefit of investment which equals the increase in expected earnings if she invests in skills. The net benefit for a university student from investing in skills depends on the quality of her university, her pre-college qualification, as well as the employers’ subsequent hiring policy regarding students from her university with unclear signals. For $j \in \{S, N\}$, write $k_{SQ}^* (\mu_j^f)$ [respectively, $k_{SU}^* (\mu_j^f)$] as the investment thresholds for quality $j$ university students who are qualified [respectively, unqualified] entering the university, anticipating firms’ hiring policy $\mu_j^f$ toward unclear signals in Stage 7. To derive these thresholds, we simply need to compare the difference in expected payoffs if a student invests versus if she does not invest. As an example, we provide details for the derivation of the threshold $k_{SQ}^* (\mu_S^f)$. Consider a student in a selective university who was qualified in GS with investment cost $k$. If she invests, from the education production function specified in Table 2, she will become a qualified university graduate with probability one. Under the firms’ hiring policy $\mu_S^f$, she will be assigned to the complex task with probability $(1 - \rho_a) + \rho_q \mu_S^f$. Thus her expected continuation payoff is $\left[ (1 - \rho_q) + \rho_q \mu_S^f \right] \omega - k$. If she does not invest, she will be qualified with probability $\zeta_{QN}$. If she turns out to be qualified, she will be assigned to the complex task with probability $(1 - \rho_q) + \rho_q \mu_S^f$; otherwise, she will be assigned to the complex job with probability $\rho_q \mu_S^f$. Thus her expected payoff is $\zeta_{QN} \left[ (1 - \rho_q) + \rho_q \mu_S^f \right] \omega + (1 - \zeta_{QN}) \rho_q \mu_S^f \omega$. Thus she will invest if and only if

$$k \leq k_{SQ}^* (\mu_S^f) \equiv (1 - \zeta_{QN}) \left[ (1 - \rho_q) + (\rho_q - \rho_a) \mu_S^f \right] \omega. \quad (3)$$

The details of other thresholds are provided in Appendix A.1. The levels of these thresholds provide a measure of the investment incentives in universities. The following lemma provides conditions under which the thresholds can be intuitively ranked:

**Lemma 2** If $\theta_q + \theta_u \leq 1$, $\zeta_{UI} + \zeta_{QN} \leq 1$ and $\phi_c \leq \zeta_{UI} (1 - \rho_a) / \left[ (1 - \zeta_{QN}) (1 - \rho_q) \right]$, then under any firm hiring policy $\mu_S^f$ and $\mu_N^f$, we have

$$k_{SQ}^* (\mu_S^f) \geq k_{SU}^* (\mu_S^f) \geq k_{QN}^* (\mu_N^f) \geq k_{NU}^* (\mu_N^f).$$

The conditions for Lemma 2 are intuitive. The first, $\theta_q + \theta_u \leq 1$, is a standard assumption in all models of statistical discrimination. It directly implies that $\rho_q < \rho_u$, that is, unqualified agents...
are more likely than qualified ones to receive unclear signals. The second condition, \( \zeta_{UI} + \zeta_{QN} \leq 1 \), imposes a restriction on the university education production technologies. Recall that both \( \zeta_{UI} \) and \( \zeta_{QN} \) measure the value-added of skill investment in the university. To see the role of this condition, it is useful to consider the following extreme case. Suppose that \( \zeta_{QN} \) is quite close to one. Then there is virtually no incentive to invest in the universities for students who are qualified in GS. This will cause \( k_{SQ}^* \geq k_{SU}^* \) and \( k_{NQ}^* \geq k_{NU}^* \) to be violated. The third condition on \( \phi_c \) imposes a minimum quality difference between selective and non-selective universities (i.e. \( \phi_c \) sufficiently small). This condition is required for \( k_{SU}^* \geq k_{NQ}^* \) to hold. Lemma 2 will be used in the proof of Lemma 3.

It is useful to define the continuation value \( V_{jQ}(k; \mu_j^f) \) [respectively \( V_{jU}(k; \mu_j^f) \)] as the maximal payoff of a quality \( j \) university student with investment cost \( k \) who was qualified (respectively, unqualified) in grade school, anticipating the firms’ hiring policy \( \mu_j^f \). They directly follow from the definitions of university investment thresholds \( k_{jQ}^* \) \( \mu_j^f \) and \( k_{jU} \) \( \mu_j^f \) above. Take \( V_{SQ}(k; \mu_S^f) \) for example. If \( k \leq k_{SQ} \) \( \mu_S^f \), we know that she will continue skill investment in a selective university, and obtain an expected payoff of \( (1 - \rho_q) + \rho_q \mu_S^f \omega - k \). If instead, her cost is \( k > k_{SQ} \) \( \mu_S^f \), she will choose not to invest in college, in which case her expected payoff is \( \zeta_{QN}(1 - \rho_q) + \rho_q \mu_S^f \omega + (1 - \zeta_{QN}) \rho_u \mu_S^f \omega \). Thus the continuation value function \( V_{SQ}(k; \mu_S^f) \) is given by:

\[
V_{SQ}(k; \mu_S^f) = \begin{cases} 
\left[(1 - \rho_q) + \rho_q \mu_S^f \right] \omega - k & \text{if } k \leq k_{SQ}^* \mu_S^f \\
\zeta_{QN} \left[(1 - \rho_q) + \rho_q \mu_S^f \right] \omega + (1 - \zeta_{QN}) \rho_u \mu_S^f \omega & \text{if } k > k_{SQ}^* \mu_S^f 
\end{cases}
\]

(4)

The other continuation value functions, \( V_{NQ}(k; \mu_N^f) \), \( V_{SU}(k; \mu_S^f) \) and \( V_{NU}(k; \mu_N^f) \), can be analogously derived (see Appendix A.2 for details). An important feature of the continuation value functions is that they are all piece-wise linear and weakly decreasing in \( k \).

Now we are ready to describe the optimal skill investment decisions for GS students in Stage 2. As we mentioned earlier, we will focus on equilibria in which the selective universities will admit “clear pass” and reject “clear fail” GS students with probability one. Let \( \mu_{i}, i \in \{ h, l \} \), denote the probability that selective universities will admit quality \( i \) GS students with unclear signals. Anticipating particular levels of \( \left( \mu_i^h, \mu_i^l, \mu_i^N \right) \), a quality \( i \) GS student with cost \( k \) will invests if and only if

\[
\phi_i \left[(1 - \rho_q) + \rho_q \mu_i^h \right] V_{SQ}(k; \mu_S^f) + \left[\rho_q (1 - \mu_i^h) \right] V_{NQ}(k; \mu_N^f) - k \\
\geq \phi_i \left[\rho_u \mu_i^h \left[ V_{SU}(k; \mu_S^f) + (1 - \rho_u \mu_i^h) \right] V_{NU}(k; \mu_N^f) \right]
\]

(5)
where the left hand side is the expected payoff if she invests; and the right hand side is the expected payoff if she does not invest. Exploiting the piece-wise linearity of \( V_{jQ}(\cdot; \mu_j^f) \) and \( V_{jU}(\cdot; \mu_j^f) \) in \( k \) and the ordinal ranking of the university investment thresholds established in Lemma 2, we can show that, under the same conditions as those for Lemma 2, inequality (5) holds if and only if \( k \leq k^*_i \) where \( k^*_i \) solves

\[
k_i \in \phi_i \left\{ \frac{[1 - \rho_q] + \rho_q \mu_i^q}{V_{SQ}(k_i; \mu_S^f)} + \left[ \rho_q (1 - \mu_i^q) \right] \frac{V_{NQ}(k_i; \mu_N^f)}{V_{SU}(k_i; \mu_S^f)} - (1 - \rho_u \mu_i^u) \frac{V_{NU}(k_i; \mu_N^f)}{V_{NU}(k_i; \mu_N^f)} \right\}.
\]

(6)

**Lemma 3** Under the same conditions as those in Lemma 2, Eq. (6) has a unique solution \( k^*_i \); and quality \( i \) GS students invest if and only if \( k \leq k^*_i \).

When we need to signify that \( k^*_i \) depends on \( (\mu_i^q, \mu_S^f, \mu_N^f) \), we will write \( k_i^* \left( \mu_i^q, \mu_S^f, \mu_N^f \right) \). Lemma 3 plays an important role in the analysis of selective universities’ admission decisions; it is also useful in calculating the dynamic equilibrium of the model (see Section 3.4).

Finally, we will analyze the best response admission policies for selective universities. First, for each applicant, the selective universities need to use the noisy signal she received in Stage 3 together with her GS quality to infer whether she is qualified; second, they also need to forecast whether this particular student will continue skill investment in the university stage if admitted; third, given their concern about their students’ job placement, they also have to take into account the firms’ hiring policy in Stage 7. It is worth noting, however, that the selective universities care about the qualification of their enrolled GS students both directly and indirectly: directly because the GS qualification directly enters the university education production function (see Table 2); indirectly because the GS qualification helps predict the likelihood that the student will continue skill investment in college.

Let \( \pi_i^s \in [0, 1], i \in \{ h, l \} \), denote the selective universities’ prior belief that a quality \( i \) GS student is qualified (of course, in equilibrium, \( \pi_i^s \) needs to be consistent with the GS students’ skill investment behavior analyzed above). Together with a noisy signal \( \theta \) received in Stage 3, the selective universities with assign a posterior probability \( \psi(\theta; \pi_i^s) \) that this student from a quality \( i \) GS is qualified, where \( \psi(\cdot; \pi_i^s) \) is exactly the same Bayesian updating formula as (1). Now, the selective universities also need to update their belief about the student’s skill investment cost distribution. This is important for the simple reason: a qualified GS student will likely have a lower skill investment cost than an unqualified student as a result of Lemma 3.\(^{12}\) Such updating of the

\(^{12}\)Some unqualified low quality GS student may have low skill investment cost: they could be unqualified despite
cost distribution conditional on GS qualification is facilitated by our characterization of \( k^*_i \) given in Lemma 3. For expositional purposes, we leave the details to Appendix A.3. With these conditional cost distributions, and the subsequent skill investment incentives measured by \( k^*_{SQ} \) and \( k^*_{SU} \), we can calculate the selective universities’ expected payoff from admitting quality \( i \) GS students with unclear signals. The selective universities’ best response admission policy toward students with unclear signals is characterized by the following lemma:

**Lemma 4** Fix any firm hiring policy toward selective university students with unclear signals \( \mu^f_S \in [0, 1] \), and let selective universities students invest according to \( k^*_{SQ} (\mu^f_S) \) and \( k^*_{SU} (\mu^f_S) \). Then there exists \( \hat{\pi}^*_i (\mu^f_S) \in (0, 1) \) such that selective universities’ best response admission policy toward quality \( i \) GS students with unclear signals is

\[
\mu^*_i (\pi^*_i; \mu^f_S) = \begin{cases} 
1 & \text{if } \pi^*_i > \hat{\pi}^*_i (\mu^f_S) \\
[0, 1] & \text{if } \pi^*_i = \hat{\pi}^*_i (\mu^f_S) \\
0 & \text{if } \pi^*_i < \hat{\pi}^*_i (\mu^f_S) 
\end{cases}
\]  

(7)

In words, selective universities admit GS students with unclear signals for whom they are sufficiently optimistic about their group.

### 3.2 Equilibrium

As we already mentioned, we focus on equilibria in which selective universities admit clear passers and reject clear failers with probability one.\(^{13}\) To formally define our restricted equilibrium concept, we introduce the following mappings. First, define mappings \( \Phi_l : [0, 1]^3 \rightarrow [0, 1] \) and \( \Phi_h : [0, 1]^3 \rightarrow [0, 1] \) as follows:

\[
\Phi_h (\pi^h_i, \pi^c_N, \pi^c_S) = G \left[ k^*_{h} \left( \mu^c_h \left( \pi^h_i; \mu^f_S (\pi^c_N) \right), \mu^f_S (\pi^c_S), \mu^f_S (\pi^c_N) \right) \right],
\]  

(8)

\[
\Phi_l (\pi^l_i, \pi^c_N, \pi^c_S) = \phi_l G \left[ k^*_{l} \left( \mu^c_l \left( \pi^l_i; \mu^f_S (\pi^c_N) \right), \mu^f_S (\pi^c_S), \mu^f_S (\pi^c_N) \right) \right].
\]  

(9)

These mappings have simple interpretations. Suppose that the proportions of qualified students in quality \( i \) GS and universities are \( \pi^*_i, \pi^c_N \) and \( \pi^c_S \) respectively. Then from Lemma 1, we know that firms would optimally choose hiring policies \( \mu^f_S (\pi^c_N) \) and \( \mu^f_S (\pi^c_S) \). Given the firms’ hiring investing in skills as a result of the low quality education production technology as captured by \( \phi_l \).

\(^{13}\)This restriction can be easily relaxed at the cost of significantly complicated notation, with no substantial gain in intuition. The selective universities may rationally satisfy these restrictions in equilibria we discuss below.
policy and \( \pi^s_i \), we know from Lemma 4 that the selective universities would choose an admission policy \( \mu^c_i \left( \pi^s_i; \mu^f_S \left( \pi^c_N \right) \right) \). Given \( \mu^c_i \left( \pi^s_i; \mu^f_S \left( \pi^c_N \right) \right) , \mu^f_S \left( \pi^c_N \right) \), and \( \mu^f_N \left( \pi^c_N \right) \), we then follow from Lemma 3 that quality \( i \) grade school students will optimally invest in skills if \( k \) is lower than \( k^* \left( \mu^c_i \left( \pi^s_i; \mu^f_S \left( \pi^c_N \right) \right), \mu^f_S \left( \pi^c_N \right), \mu^f_N \left( \pi^c_N \right) \right) \). Thus, given a candidate profile \( \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \), the mapping \( \Phi_i \left( \pi^s_i, \pi^t_i, \pi^c_S, \pi^c_N \right) \) gives the fraction of qualified quality \( i \) GS students under the best responses of all involved decision makers.

Now we define mappings that measure the fraction of qualified graduates from quality \( j \) universities. Fix a profile \( \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \). We first define mappings \( Q_{j,i} : [0,1]^4 \rightarrow [0,1] \) and \( M_j : [0,1]^4 \rightarrow [0,1], j \in \{S,N\}, i \in \{h,l\} \) where \( Q_{j,i} \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \) is the total measure of qualified quality \( j \) university graduates originally from quality \( i \) GS, and \( M_j \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \) is the total measure of quality \( j \) university students under the best responses of all involved decision-makers. These expressions involve tedious but simple accounting, thus are detailed in Appendix A.4. Based on the mappings \( Q_{j,i} \) and \( M_j \), we can further define mappings \( \Phi_N : [0,1]^4 \rightarrow [0,1] \) and \( \Phi_S : [0,1]^4 \rightarrow [0,1] \) by

\[
\Phi_N \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) = \frac{Q_{N,h} \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) + Q_{N,l} \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right)}{M_N \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right)}
\]

(10)

\[
\Phi_S \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) = \frac{Q_{S,h} \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) + Q_{S,l} \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right)}{M_S \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right)}
\]

(11)

Given a candidate profile \( \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \), the mapping \( \Phi_j \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \) gives the fraction of qualified quality \( j \) university students under the best responses of all involved decision-makers.

An equilibrium of the dynamic model of education and employment is simply a fixed point of the mappings \( \left( \Phi_h, \Phi_l, \Phi_S, \Phi_N \right) : [0,1]^4 \rightarrow [0,1]^4 \). In words, an equilibrium of the model is a vector of beliefs \( \left( \pi^s_h, \pi^t_l, \pi^c_S, \pi^c_N \right) \) that are mutually consistent with the optimal behavior of all decision-makers. The existence of such (restricted) equilibrium follows directly from Kakutani’s fixed point theorem: first, the best responses of all decision makers, and subsequently the four mappings, are upper hemi-continuous; second, the mappings are defined on the compact and convex set \([0,1]^4 \). It is worth repeating that our equilibrium concept imposes the restriction that selective universities will admit clear passers and reject clear failers with probability one.

For any equilibrium belief profile \( \left( \pi^s_h, \pi^s_l, \pi^c_S, \pi^c_N \right) \), we can use the best response mappings \( \mu^c_i \left( \cdot; \cdot \right) \) and \( \mu^f_j \left( \cdot \right) \) defined in Lemma 4 and 1 to obtain the equilibrium admission policies by selective universities and hiring policies by firms, denoted with some abuse of notation by \( \left( \mu^c_h, \mu^c_l, \mu^f_S, \mu^f_N \right) \). These are called equilibrium policies. Generically, these equilibrium policies are pure, i.e. \( \left( \mu^c_h, \mu^c_l, \mu^f_S, \mu^f_N \right) \in \{0,1\}^4 \). It is possible to rule out some pure equilibrium policies. First, if \( \phi_i \) is sufficiently small,
specifically, if \( \phi_l \leq \phi_c \zeta_{UI} / \left[ 1 + (\rho_u - \zeta_{UI}) \phi_c \right] \), \( \mu_l^* \leq \mu_h^* \) in any restricted equilibrium of the model. To see this, suppose to the contrary that there is an equilibrium in which \( \mu_l^* = 1, \mu_h^* = 0 \). Then we can substitute them into Eq. (6) to obtain the implicit equations that determine the value of \( k_h^* \) and \( k_l^* \). After some simple calculations, we would obtain \( k_h^* > k_l^* \) under the conditions on \( \phi_l \), a contradiction to the selective universities optimal admission policy in Lemma 4. Second, we can also rule out equilibria in which \( \mu_f^* S < \mu_f^* N \) if the selective universities are sufficiently more productive than the non-selective ones. The intuition is very simple. The fact that selective universities are optimally selecting GS students implies that the students enrolled in selective universities are better than those in non-selective universities. If selective universities are moreover sufficiently more productive, then it implies that selective universities graduates are more likely to be qualified that non-selective universities graduates. Formally,

**Lemma 5** In any equilibrium of the model, firms’ hiring policy satisfies \( \mu_f^* S \geq \mu_f^* N \) in that equilibrium if \( \phi_c \leq (1 - \rho_u) / (1 - \rho_q) \).

Note that the conditions for Lemma 5 are weaker than those for Lemma 3.\(^{14}\) Taking into account the above restrictions on the possible pure equilibrium outcomes, we are left with nine possible equilibrium policy profiles.

### 3.2.1 Leading Case Equilibrium

Instead of belaboring on the conditions under which each equilibrium policy profile may arise, we will focus on a particularly interesting equilibrium, dubbed as our leading case equilibrium whose policies are given by \( \left( \mu_h^* = 1, \mu_l^* = 0, \mu_f^* S = 1, \mu_f^* N = 0 \right) \). In this equilibrium selective universities are liberal toward high quality GS students but conservative toward low quality GS students in admission; whereas firms are liberal toward selective university graduates and conservative toward non-selective university graduates in hiring.\(^{15}\) Our main results, though derived in the context of

---

\(^{14}\)To see this, recall that the conditions stated for Lemma 3 are: \( \zeta_{UI} + \zeta_{QN} \leq 1 \) and \( \phi_c \leq \zeta_{UI} (1 - \rho_u) / \left[ (1 - \zeta_{QN})(1 - \rho_q) \right] \). The two conditions jointly imply that \( \phi_c \leq (1 - \rho_u) / (1 - \rho_q) \).

\(^{15}\)We think these are natural equilibrium policies to focus on for the following reasons. Admission officers in selective universities often categorize their applicants into “Admit”, “Reject”, “Admit Minus” and “Reject Plus.” (See Steingold 1997). The “Admit” category corresponds to our “clear passers” and the “Reject” category to our “clear failers”; the last two categories corresponds to those with unclear signals in our model. When the “Admit Minus” and “Reject Plus” cases are finally decided, often the admission officers will look at information about the applicants’ extracurriculum activities, AP courses and leadership roles etc. in determining whether they should be
the leading case equilibrium, are robust to other equilibria. We now prove a simple lemma that will be used later when we analyze the effects of affirmative actions.

**Lemma 6** In the leading case equilibrium, if
\[
\phi_c \leq \left[ 1 - \rho_u (\zeta_{UI} - (1 - \zeta_{UI}) \rho_u ) \right] \left[ (2 - \rho_u) (1 - \zeta_{UI}) (1 - \rho_q) \right]
\]
and \( \phi_l \leq \phi_c \zeta_{UI} / \left[ 1 + (\rho_q - \zeta_{UI}) \phi_c \right] \), then \( k^*_h \geq k^*_NQ \geq k^*_NU \geq k^*_l \).

### 3.3 Introducing Groups

We have thus far analyzed a one-group model. In our setup, introducing observationally distinct racial groups does not generate inter-group interactions because the production technologies at the grade schools, universities and firm are all linear and moreover, neither selective universities nor firms have capacity constraints.\(^{16}\) Thus, when we introduce multiple groups into the model, we simply need to analyze the equilibrium of the groups separately. Now suppose there are two observationally distinct racial groups, say “blacks” and “whites.” We assume that a student’s race is observable to both selective universities and firms. The groups can differ in many different dimensions.

First, the probability of being assigned to low quality GS for blacks is higher than that of the whites. For example, Kozol (1991) documented that in many poorly funded, low quality grade schools, the students are predominantly blacks or Hispanics. Fryer and Levitt (2002) report that 35% of white kindergarten kids attend a school where there are no black children. Because where children attend GS is mainly determined by the residential locations of their parents, and school qualities are largely determined by the property tax bases of the residential community, we would expect that blacks are more likely in low quality GS than whites.

Second, blacks and whites may differ in skill investment cost distributions. A consistent empirical finding is that whites tend to grow up in households more conducive to educational achievement than blacks (see, for example, Brooks-Gunn and Duncan 1997, Mayer 1997, and Phillips et al. 1998). Thus, it is natural to assume that the skill investment cost distribution for the blacks first order stochastically dominates that of the whites.

Third, we can also argue that the quality of low quality GS for blacks and whites could be admitted. To the extent that high quality grade schools offer much more opportunities on these activities, we feel that their students are more likely to be admitted. On the firm hiring side, old-boy network of selective universities alumni may be a reason for the firms’ liberal policy toward selective university graduates.

\(^{16}\)In the discrimination literature, Mailath, Samuelson and Shaked (2000) and Moro and Norman (2003) emphasized intergroup interaction as the reason of the discriminatory equilibrium.
different. The poor predominantly white neighborhoods are still relatively more resourceful than
the poor predominantly black neighborhoods. Thus it is conceivable that the quality of education
for low quality GS attended by whites are still better than that for the blacks.

The recent developments in discrimination literature in economics have focused on discrimi-
natory outcomes in multiple equilibria models with ex ante identical groups (Arrow 1973, Coate
and Loury 1993). Such models are theoretically more interesting than models that explain group
differences in outcomes by appealing to differential in initial conditions. However, empirically we
can not deny that more than a century of institutionalized discrimination has lead to important
initial differences between blacks and whites; and these differences continue today. Moreover, there
is no clear evidence of whether the black-white labor market gaps are mainly due to differences
in fundamentals or in equilibrium selections. In our model, the black-white differences in labor
market gaps can arise both as a result of differences in initial conditions and as a result of differ-
ences in the selected equilibrium. In fact, differences in initial conditions between the racial groups
may lead them to be selected in different equilibrium policy profiles as we will see in the numerical
example in the next section.

3.4 A Numerical Example

In this section, we present a numerical example to illustrate the model. Consider the following
parameterization of the model:

- Skill investment costs are uniformly distributed on $[0, 6]$;
- The probability of attending low quality grade schools is $\alpha = 0.5$;
- The noisy test technology parameters are $\theta_u = 0.6, \theta_q = 0.3$;
- The educational production technology parameters are: $\phi_c = 0.45, \zeta_{UI} = 0.8, \zeta_{QN} = 0.1$. But
  we allow the parameter $\phi_1$, which measures the difference between the high and low quality
  grade schools, to vary at selected values.
- Selective university payoff parameters are: $\chi_q^c = 2.0, \chi_u^c = 0.5, R_q = 0.5, R_u = 0.2$;
- Firm’s technology and wage parameters: $\chi_q^f = 3, \chi_u^f = 1, \omega = 3.5$.

17Moro (2003) took an initial step in addressing this question by empirically implementing a stylized structural
model using data from the Current Population Survey. His result points to the importance of fundamentals.
Table 3 shows the equilibrium policy profiles under the above parametrization for selected values of $\phi_l$. The most noteworthy feature from Table 3 is that changes in $\phi_l$ affect equilibrium policy profiles and the labor market outcomes are greatly changed when $\phi_l$ is sufficiently large (in our simulation, we find that when $\phi_l = 0.58$, the selective universities start to follow a liberal policy toward low quality GS). An important lesson is that, if a government were to help the disadvantaged minorities by improving the quality of the low quality schools, the improvement needs to be sufficiently drastic in order to have noticeable effects on the labor market outcomes of the minorities; piece-meal improvement may have little effect.

<table>
<thead>
<tr>
<th>$\phi_l$</th>
<th>Equilibrium Policies</th>
<th>Prob. Admitted to Selective Univ.</th>
<th>Prob. Hired on Complex Job</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu^*_H$</td>
<td>$\mu^*_I$</td>
<td>$\mu^*_S$</td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.4</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>0</td>
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</tr>
<tr>
<td>0.6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.7</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium Outcomes For Selected Values of $\phi_l$.

4 Affirmative Action

4.1 Modelling Affirmative Action Mandates

In practice, affirmative action is an amalgam of legislations and court rulings than a single coherent policy; moreover, AA legislations and court rulings are often different in employment, education, and government contracting (Holzer and Neumark 2000). How should AA be modelled? In the existing literature (notably Coate and Loury 1993, Moro and Norman 2003), AA mandates are modelled as “strict quotas,” requiring firms to achieve the same probability of assignment to the complex job for both groups. While there has been considerable debate and uncertainty about the precise meaning of affirmative action, strict quotas have been struck down since the Supreme Court ruling on the landmark 1978 Bakke case (438 U.S. 265), at least in university admissions.

Instead of requiring complete racial equality, which may be a long-run goal, we take a more pragmatic, short-run view of AA: we require that pro-active steps be taken to erase the racial differences in the percentages hired on the higher-paying complex jobs. In other words, we suppose
that a regulatory authority (interchangeably, the government) wants to increase (relative to the status quo) by an additional $\Delta > 0$ percent more blacks to be employed on the higher-paying complex jobs.

There are several operandi for the government to achieve the AA goal. One possible intervention is to equalize the initial conditions for blacks and whites by, for example, lowering the probability of blacks being assigned to the low quality GS, increasing the quality of black schools, and equalizing the distribution of skill investment costs. If such equalization of initial conditions could be achieved, then racial parity would result as long as blacks and whites play the same equilibrium. However, we believe that such interventions are not practically feasible. Thirty-seven years after the Supreme Court decision in *Brown v. Board of Education* (1954), Kozol (1991) still found mind-boggling differences in schools attended mostly by whites and nonwhites. Many people interviewed in his study believe that equal schools would simply be impossible. In fact, such inequality in school qualities will persist as long as the public school financing system relying heavily on local property taxes remains unchanged. Furthermore, equalizing school finances is not enough to equalize school qualities: first, schools in poor neighborhoods in fact have higher needs for funding because they have been neglected for so long; second, poor neighborhoods will still lack community funding and parental involvement, all of which are integral variables in the production of school quality.

For the above reason, we will assume that any equalizing in initial conditions will at best be incomplete. Thus, the government may still want to explore interventions in later stages, namely in university admission and labor market hiring, to improve the percentages of minorities hired on the higher-paying complex jobs.

Affirmative action is implemented in the hiring stage if the government holds the firms responsible for achieving a $\Delta$ percentage increase of blacks on the complex jobs. Under this AA mandate, firms must take the actions of the universities and students as given, and change their hiring standards to increase black representation on the complex job by $\Delta$ percent.

Affirmative action is implemented in the university admissions stage if the government holds the selective universities responsible for the policy prescription. Under this AA mandate, the selective universities must take the actions of the students as given, anticipate the sequentially rational hiring policies of the firms, and change their admission policies to increase the black representation on the complex job by $\Delta$ percent.

We provide sufficient conditions under which it is optimal for the society to place the burden of affirmative action on the selective universities. The argument proceeds in two steps. First, we
analyze the costs of the above two AA mandates when $\Delta$ is sufficiently small. When $\Delta$ is small, we can approximately hold students’ investment incentives unchanged by the AA mandates. We show that under reasonable conditions the optimal timing of affirmative action is in the university admissions stage. Second, we show that when $\Delta$ may affect students’ investment incentives, implementing AA in the university admission stage in fact provides better investment incentives for the students. In what follows, we analyze the case that the blacks are playing the leading case equilibrium. But the intuition revealed in our analysis is general, thus we can suitably modify our arguments to obtain same qualitative insights when the blacks are in other possible equilibria.

4.2 The Effect of Small $\Delta$

We assume that the blacks are initially playing the leading case equilibrium with the equilibrium policies $\mu_{k}^{*} = 1, \mu_{l}^{*} = 0, \mu_{h}^{*} = 1, \mu_{l}^{*} = 0$. We begin by comparing the implementation of AA in the hiring stage versus the university admissions stage, assuming that $\Delta$ is small. When $\Delta$ is sufficiently small, we can assume that students’ investment incentives, i.e. their investment thresholds, are not affected by AA. We also assume that the conditions of Lemma 6 holds in our discussions.

4.2.1 Affirmative Action in the Hiring Stage

Consider AA in the hiring stage. Firms are now required to hire additional $\Delta$ percentage points of blacks on the complex jobs. Recall that, in the leading case equilibrium prior to the AA requirement, firms place all black students from selective universities with pass or unclear signals on the complex job, and only assign black students from non-selective universities with pass signals to the complex job. Therefore, faced with the AA mandate, firms will clearly favor hiring blacks with unclear signals from non-selective universities. The reason is simple: All blacks from selective universities initially rejected for the complex job have failing signals and thus are unqualified for sure, whereas blacks with unclear signals from non-selective universities are qualified for the complex job with strictly positive probability.

The social cost of implementing AA in the hiring stage is as follows. The newly hired blacks from non-selective universities with unclear signals would not on average be qualified enough for the firms (since they were not hired on the complex job in the absence of the AA mandate). Thus the firms will suffer an expected loss from hiring them under the AA mandate. The social cost of
implementing AA in the hiring stage, denoted by $C_f$, can be expressed as:

$$C_f = \Delta \left\{ \left[ 1 - \frac{\pi^e_N \rho_q}{\pi^e_N \rho_q + (1 - \pi^e_N) \rho_u} \right] \chi^f_u - \frac{\pi^e_N \rho_q}{\pi^e_N \rho_q + (1 - \pi^e_N) \rho_u} \chi^f_q \right\}. \quad (12)$$

### 4.2.2 Affirmative Action in the University Admission Stage

Now consider AA in the university admissions stage. To satisfy the AA mandate, selective universities must change their admission policy to ensure that $\Delta$ percent more blacks will be eventually employed by firms on the complex task.

The first important difference between AA in the hiring stage and the university admission stage is as follows. In the hiring stage, the firms can satisfy the AA mandate by hiring exactly $\Delta$ percent more blacks on the complex job. Selective universities, however, cannot simply admit $\Delta$ percentage more blacks and expect the AA mandate to be fulfilled because not every additional black student admitted by the selective university will be subsequently placed in the complex job by the firms. We call this the “slippage effect.” Moreover, the additional black students the selective universities admit (away from non-selective universities) also had positive probability of being hired on the complex job. Thus, the selective universities would have to account for the fact that additional blacks they admit and place on the complex job will lower the number of blacks from non-selective universities hired on the complex job. We call this the “relocation effect.” Both the slippage and relocation effects imply that the selective universities must admit more than $\Delta$ percent more blacks in order to eventually fulfill the AA mandate.

We first argue that, when needing to admit more black students, the selective universities will optimally admit black students from low quality GS with unclear signals. The reason is simple. In the leading case equilibrium, all black students from high quality GS initially rejected by the selective universities necessarily have failing signals and thus are unqualified with probability one. In contrast, black students from low quality GS with unclear signals are qualified with positive probability. Moreover, since $k^*_h > k^*_l$, students with unclear signals from low quality GS have, on average, lower skill investment costs. Relative to those with failing signals from high quality GS, the students with unclear signals from low quality GS are not only “better prepared” (i.e. they are more likely to be qualified), but also have higher “potential” (i.e. they are more likely to further invest in the selective colleges).

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18 More precisely, the skill investment cost distribution among black students with unclear signals from low quality grade schools are first order stochastically dominated by that of the students with “clear fail” signals from high quality grade schools.
To calculate the actual additional number of blacks the selective universities need to admit to satisfy the AA mandate, we now need to calculate the slippage and the relocation effects. To this end, we first compute the probability that a low quality GS graduate with an unclear signal, if admitted to the selective universities, will be qualified upon graduation. We denote this probability by $\tilde{\pi}$.

To calculate $\tilde{\pi}$, we note that the total measure of low quality GS students with unclear signals is given by

$$\phi_t G (k^*_t) \rho_q + \left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u,$$

where $k^*_t$ is the equilibrium investment incentive for low quality GS student which we assume will remain unchanged when $\Delta$ is small. The term $\phi_t G (k^*_t) \rho_q$ in the above expression represents the measure of qualified students who unfortunately receive an unclear signal. All the students included in this term will continue investing if admitted to the selective universities since their costs are smaller than $k^*_t$ (recall from Lemma 6 that $k^*_t < k_{SQ}^*$). The term $\left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u$ is the measure of unqualified students who receive an unclear signal. Among the students included in this term, only those with cost less than $k_{SU}^*$ will continue investing if admitted to the selective universities and they will become qualified with probability $\zeta_{UI}$. Thus, we have

$$\tilde{\pi} = \frac{\phi_t G (k^*_t) \rho_q + \left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u G_U (k_{SU}^*) \zeta_{UI}}{\phi_t G (k^*_t) \rho_q + \left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u}.$$ (13)

Since firms will continue to follow a liberal hiring policy toward selective university graduates, a randomly selected low quality GS student with an unclear signal, if enrolled in the selective universities, will eventually be assigned to the complex job with probability $\tilde{\pi} + (1 - \tilde{\pi}) \rho_u < 1$. The difference $1 - \left[\tilde{\pi} + (1 - \tilde{\pi}) \rho_u\right]$ is the magnitude of the “slippage effect.”

To quantify the “relocation” effect, we note that the additional black students admitted by the selective universities under the AA mandate would have gone to the non-selective universities, and they would have been qualified with probability $\tilde{\pi}$ given by

$$\tilde{\pi} = \frac{\phi_c \phi_t G (k^*_t) \rho_q + \left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u G_U (k_{SU}^*) \zeta_{UI}}{\phi_t G (k^*_t) \rho_q + \left[1 - G (k^*_t) + (1 - \phi_t) G (k^*_t)\right] \rho_u} < \phi_c \tilde{\pi},$$ (14)

where the inequality follows from our earlier result that $k_{SU}^* < k_{SU}^*$. Since the firms follow a conservative hiring policy toward non-selective university graduates, such a black student will be assigned to the complex job with probability $\tilde{\pi} \rho_q + (1 - \rho_q)$, $\tilde{\pi} \rho_q + (1 - \rho_q)$ is the magnitude of the “relocation effect.”
Taking into account both the slippage and the relocation effect, an additional low quality GS black student with unclear signals admitted to the selective universities will ultimately result in an additional hire on the complex job with probability \( \tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q) \). Thus to satisfy the AA mandate of \( \Delta \) percent additional blacks on the complex jobs, selective universities must admit additional

\[
\tilde{\Delta} = \frac{\Delta}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)}
\]  

percent blacks from low quality GS with unclear signals. The firms will subsequently hire \([\tilde{\pi} + (1 - \tilde{\pi}) \rho_u] \tilde{\Delta} \) more blacks from selective universities, but will lose \([\tilde{\pi} (1 - \rho_q)] \tilde{\Delta} \) blacks from non-selective universities as a result of the selective universities’ admission change in response to the AA mandate.

Now we are ready to calculate the social costs of AA in the university admission stage. There are two components. The first component is the firms’ misallocation cost. It is given by

\[
\begin{align*}
\text{Term 1} &= [\tilde{\pi} + (1 - \tilde{\pi}) \rho_u] \Delta \left[ \frac{(1 - \tilde{\pi}) \rho_u}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u} \chi_u^f - \frac{\tilde{\pi}}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u} \chi_q^f \right] + [\tilde{\pi} (1 - \rho_q)] \tilde{\Delta} \chi_q^f \\
\text{Term 2} &= \Delta \left\{ (1 - \tilde{\pi}) \rho_u \chi_u^f - \left[ \tilde{\pi} - \tilde{\pi} (1 - \rho_q) \right] \chi_q^f \right\}
\end{align*}
\]  

where Term 1 is the net cost from the additional black workers hired from the selective universities, and Term 2 is the lost profit from those qualified blacks from non-selective universities who are now admitted to the selective universities.

The second component is the cost on the selective universities for having to admit an additional \( \tilde{\Delta} \) percent blacks whom they would not have found optimal to admit in the absence of AA. Their loss is

\[
\tilde{\Delta} \left[ (1 - \tilde{\pi}) [\chi_u^c + \rho_u R_u] - \tilde{\pi} (\chi_q^c + R_q) \right].
\]  

(17)

To understand this expression, note that among the \( \tilde{\Delta} \) percent additional black student admitted by selective universities under the AA mandate, a fraction \( \tilde{\pi} \) of them will be qualified and hired on the complex task upon graduating. This provides a positive payoff of \( \tilde{\pi} (\chi_q^c + R_q) \). Similarly, a fraction \( 1 - \tilde{\pi} \) of them will be unqualified for which a proportion \( \rho_u \) of them will be mistakenly assigned to the complex job, yielding a negative payoff of \( (1 - \tilde{\pi}) [\chi_u^c + \rho_u R_u] \).

Putting (16) and (17) together, the total cost of AA in university admissions is then

\[
\begin{align*}
C_c &= \tilde{\Delta} \left\{ (1 - \tilde{\pi}) \rho_u \chi_u^f - \left[ \tilde{\pi} - \tilde{\pi} (1 - \rho_q) \right] \chi_q^f \right\} + \tilde{\Delta} \left[ (1 - \tilde{\pi}) [\chi_u^c + \rho_u R_u] - \tilde{\pi} (\chi_q^c + R_q) \right] \\
 &= \tilde{\Delta} \left\{ (1 - \tilde{\pi}) \rho_u \left[ \chi_u^c + \rho_u R_u + \chi_u^f \right] - \left[ \tilde{\pi} - \tilde{\pi} (1 - \rho_q) \right] \left( \chi_q^c + R_q + \chi_q^f \right) \right\} \\
&\quad - \tilde{\Delta} \tilde{\pi} (1 - \rho_q) (\chi_q^c + R_q).
\end{align*}
\]  

(18)
4.2.3 Comparison of Affirmative Actions in the Hiring and University Admissions

Now we can compare the social costs of affirmative actions in the hiring and university admissions stages. Using (12) and (18), we obtain

\[ C_f - C_c = \Delta \left\{ \frac{1 - \pi_c^N \rho_q}{\pi_c^N \rho_q + (1 - \pi_c^N) \rho_u} \chi_f^u - \frac{\pi_c^N \rho_q}{\pi_c^N \rho_q + (1 - \pi_c^N) \rho_u} \chi_f \right\} 
- \tilde{\Delta} \left\{ (1 - \tilde{\pi}) \rho_u \left( \frac{\chi_c^u + R_q + \chi_f}{\rho_u} \right) - \left[ \tilde{\pi} - \tilde{\pi} (1 - \rho_q) \right] \left( \chi_c + R_q + \chi_f \right) \right\} 
+ \tilde{\Delta} \left( 1 - \rho_q \right) \left( \chi_c + R_q \right) 
= \Delta \left\{ \frac{\tilde{\pi}}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} \left( \chi_c + R_q \right) - \frac{(1 - \tilde{\pi}) \rho_u}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} \left( \chi_c^u + R_q + \chi_f \right) \right\}, \tag{19} \right. 

\text{Term 1}

\text{Term 2}

\text{Term 1 captures the differences in the firms’ changes of profits under the two AA regimes. Intuitively the firms’ profits must be larger when AA is implemented in the university admission stage. This is a result of the information advantage of the selective universities, captured by } \Xi \text{ defined in (19). It measures the difference between the posterior probability of being qualified among the additional black workers hired on the complex job under the two AA regimes. We can prove an intuitive yet important lemma:}

\textbf{Lemma 7} \quad \Xi > 0.

\text{Lemma 7 states that if AA is implemented in the university admission stage, the additional black students that the firms assign to the complex task are more likely to be qualified than those whom would have been hired had firms been responsible for the AA target. When firms are held responsible for the AA, the additional workers they hire are from non-selective universities with unclear signals, who are a mixture of students from both types of GS. Among them, those who ended up in the non-selective universities from low quality GS are stochastically more likely to be qualified. It is beyond the firms’ capacity, however, to accurately distinguish the GS quality of these students. Selective universities, however, have finer information in that they know which GS each student attended. As a consequence, they can find “diamonds in the rough” by sifting}
through the previously rejected black students and identify those who are most likely to succeed if admitted. Moreover, by admitting them to the selective universities, these students will be more likely to get “back on track” due to the developmental advantage of the selective universities over the non-selective ones.

An immediately implication of Lemma 7 is that the information advantage of the selective universities would push the optimal timing of affirmative action toward the university admission stage especially when the firms’ misallocation cost measured by $\chi_q^f + \chi_u^f$ is large.

Now we show that, for a fixed level of $\chi_q^f + \chi_u^f$, AA in admission is less costly if the informational advantage of the selective universities over the firms are large. We first provide conditions under which that the selective universities’ information advantage, measured by $\Xi$, decreases with the quality of low quality GS $\phi_l$. Intuitively, the close $\phi_l$ is to 1, the less difference between the high and low quality GS. Thus, the selective universities’ informational advantage of knowing the quality of the GS would have a smaller impact.

Lemma 8 Suppose $\zeta_{UI}$ is sufficiently close to one and $\phi_c$ is sufficiently close to zero. Then $\Xi$ is strictly decreasing in $\phi_l$.

Now with Lemma 8, we prove our first main result:

**Proposition 1** Let $\Delta$ be small. Suppose $\zeta_{UI}$ is sufficiently close to one and $\phi_c$ is sufficiently close to zero, and fix $(\chi_u^c, R_u, \chi_q^c, R_q)$. Then there exists $Q > 0$ such that for every $\chi_q^f + \chi_u^f > Q$, we can find $\bar{\phi}_l > 0$ such that the social cost of AA is higher in the hiring stage for all $\phi_l < \bar{\phi}_l$.

### 4.3 Allowing Investment Incentives to be Endogenous

So far we assumed that the AA requirement is negligible so as not to affect the investment incentives. We now show that if AA is allowed to affect the effort incentives, implementing the AA in the university admission stage may improve investment incentives, whereas AA in the hiring stage hurts incentives.

**Proposition 2** Suppose that the equilibria of the model before and after the AA are both the leading case equilibrium.

(1) If AA is implemented in the hiring stage, then:

- $k_{SQ}^*$, $k_{SU}^*$ will be unchanged; $k_{NQ}^*$ and $k_{NU}^*$ will decrease as long as $\rho_q \neq \rho_u$ and remain unchanged if $\rho_q = \rho_u$;
• \( k_h^* \) and \( k_l^* \) will decrease.

(2) If AA is implemented in the admission stage, then:

• \( k_{SQ}^*, k_{SU}^*, k_{NU}^* \) and \( k_h^* \) will be unchanged;

• \( k_l^* \) will increase if \( \rho_q = \rho_u \); and for every \( \zeta_{UI} \), there exists values of \( \rho_q \) and \( \rho_u \) that are sufficiently close such that \( k_l^* \) is increased.

The proof is simply a comparative statics exercise involving the investment thresholds. To convey some intuitions, note that AA in the hiring stage will result in an increase in \( \mu_f^N \). This change in the hiring policy toward non-selective university students clearly will have no impact on selective university students' investment incentives. For non-selective university students, however, the incentives to invest decreases because it is less important to receive clear pass signals when \( \mu_f^N \) increases. Increase in \( \mu_f^N \) also improves the continuation value from attending non-selective universities, thus decreasing the incentives to invest in GS. In contrast, AA in the university admission stage will result in an increase in \( \mu_c^l \), while leaving the firms' hiring policies unchanged. Thus university students' investment incentives are unchanged, neither is that for the high quality GS students. When \( \rho_q \) and \( \rho_u \) are sufficiently close, qualified students will benefit from the increase in \( \mu_c^l \) as much as non-qualified students in entering the selective universities. Thus, the investment incentives may be improved when \( \mu_c^l \) increases.

5 Conclusions and Discussion

This paper presents a simple dynamic model of education and employment to investigate the optimal timing of affirmative action. When comparing the effects of implementing affirmative action in university admissions and labor market hiring, we identity the informational advantage of selective universities over the firms regarding the quality of the students' grade schools as an important consideration in favor of affirmative action in the university admission stage. The intuition is that selective universities can use the information advantage to sift through the previously rejected minority students and identify those who are most likely to succeed if admitted. This information advantage is inversely related to the difference between high and low quality grade schools. Another important determinant of the optimal timing is the differences in the firms’ and the selective universities’ misallocation costs.
Our results are derived in a very stylized model, thus should be interpreted cautiously. However, we do believe that the two crucial factors we highlighted in our analysis for the optimal timing of affirmative action are robust. In particular, our analysis points to the importance of the informational and developmental advantage of selective universities over other institutions. Such information and developmental advantages should be one of the main considerations in any public policy debate about the optimal timing of affirmative actions. A question we need to ask is what institutions are best equipped to identify high-potential, but maybe low-opportunity minorities and allow them to “get back on track.” We also highlighted the importance of the misallocation costs of the involved institutions. Are the firms hurt more by having to hire possibly under-qualified workers than the selective universities by having to admit possibly less-prepared students? However, this comparison is difficult to objectively undertake.

In Section 4.1 we motivated AA in the admission and/or hiring stage by arguing that equalizing initial conditions across racial groups is difficult to achieve. One may argue that large public programs such as Head Start is an important step toward de-linking residential location and schools which may be thought of as an early AA intervention in itself. Unfortunately, such early childhood education programs are often of short term, and participating students eventually have to attend public schools in their residential neighborhood. Gains from initial intervention are found to fade out over the first two years of school (Currie and Duncan 1995). Thus, if extending our model to more finely distinguish early childhood schools from high schools, we may find that AA in early childhood schools may not be optimal if “depreciation rate” on the effects of interventions is large. What is unique about affirmative action in university admission is that universities are typically the first stage in which one’s residential location is not important any more.

Our analysis also highlights the perils of AA in the university admission stage, namely, the extent of assistance to the minorities should be wider than the intended AA goal to account for the slippage and relocation effects. The magnitude of the slippage and relocation effects is an important factor in the comparison between AA in the hiring and admission stages. The smaller the slippage effect, the better to implement the AA in the admission stage.

Our analysis abstracts from considerations of education benefits from diversity. Such considerations will obviously lead to further support for AA in the university admissions. We also assumed wages on the complex and simple jobs to be exogenous. We believe that endogenizing the wages will not affect the main messages of this paper.
References


A Appendix: Omitted Details.

In this appendix, we formally define some of the more cumbersome expressions that were deleted from the text for expositional clarity.

A.1 Investment Thresholds for University Students

We provide the derivation of the investment thresholds for university students in Stage 5. We showed in the text that

\[ k^*_S \mu^f_S \equiv (1 - \zeta_{QN}) \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_S \right] \omega. \]

**Derivation of** \( k^*_S \mu^f_S \).

Consider a selective university student with cost \( k \) who was unqualified in grade school. Her expected payoff if she invests is given by \( \zeta_{UI} \left[ (1 - \rho_q) + \rho_q \mu^f_S \right] \omega + (1 - \zeta_{UI}) \rho_a \mu^f_S \omega - k \); and it is given by \( \rho_u \mu^f_S \omega \) if she does not invest. Thus she invests in the selective college if

\[ k \leq k^*_S \mu^f_S \equiv \zeta_{UI} \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_S \right] \omega. \quad (A1) \]

**Derivation of** \( k^*_N \mu^f_N \).

Consider a non-selective university student with cost \( k \) who was qualified in grade school. Her expected payoff is \( \phi_c \left[ (1 - \rho_q) + \rho_q \mu^f_N \right] \omega + (1 - \phi_c) \rho_u \mu^f_N \omega - k \) if she invests while in the non-selective university; and it is \( \phi_c \zeta_{QN} \left[ (1 - \rho_q) + \rho_q \mu^f_N \right] \omega + (1 - \phi_c \zeta_{QN}) \rho_u \mu^f_N \omega \) if she does not. Thus, she invests if and only if

\[ k \leq k^*_N \mu^f_N \equiv \phi_c \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right] \omega. \quad (A2) \]

**Derivation of** \( k^*_U \mu^f_U \).

Consider a non-selective university student with cost \( k \) who was unqualified in grade school. Her expected payoff is \( \phi_c \zeta_{UI} \left[ (1 - \rho_q) + \rho_q \mu^f_U \right] \omega + (1 - \phi_c \zeta_{UI}) \rho_u \mu^f_U \omega - k \) if she invests while in the non-selective university; and it is \( \rho_u \mu^f_U \omega \) if she does not invest. Thus she invests in the non-selective university if

\[ k \leq k^*_U \mu^f_U \equiv \phi_c \zeta_{UI} \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_U \right] \omega. \quad (A3) \]

A.2 Continuation Value Functions in Universities

Here we provided the omitted details of the continuation value functions in universities that we used to derive the grade school investment thresholds. Analogous to the derivation of \( V^{}_S^{} (k; \mu^f_S) \)
in the main text, they follow from the definition of the university investment thresholds in Appendix A.1.

\[ V_{SU} (k; \mu_S^f) = \begin{cases} \zeta_{UI} \left[ (1 - \rho_q) + \rho_q \mu_S^f \right] \omega + (1 - \zeta_{UI}) \rho_a \mu_{E}^f \omega - k & \text{if } k \leq k^*_S \left( \mu_S^f \right) \\ \rho_a \mu_S^f \omega & \text{if } k > k^*_S \left( \mu_S^f \right) \end{cases} \quad (A4) \]

\[ V_{NQ} (k; \mu_N^f) = \begin{cases} \phi_c \left[ (1 - \rho_q) \omega + \rho_q \mu_N^f \omega \right] - k & \text{if } k \leq k^*_N \left( \mu_N^f \right) \\ \phi_c \omega \left\{ \zeta_{QN} \left[ (1 - \rho_q) + \rho_q \mu_N^f \right] + (1 - \zeta_{QN}) \rho_a \mu_N^f \right\} & \text{if } k > k^*_N \left( \mu_N^f \right) \end{cases} \quad (A5) \]

\[ V_{NU} (k; \mu_N^f) = \begin{cases} \phi_c \omega \left\{ \zeta_{UI} \left[ (1 - \rho_q) + \rho_q \mu_N^f \right] \right\} - k & \text{if } k \leq k^*_N \left( \mu_N^f \right) \\ \phi_c \rho_a \mu_N^f \omega & \text{if } k > k^*_N \left( \mu_N^f \right). \end{cases} \quad (A6) \]

### A.3 Conditional Skill Investment Cost Distributions

Recall that \( G_iQ (\cdot) \) [respectively, \( G_iU (\cdot) \)] is the CDF of skill investment costs for quality \( i \) grade school students conditional on being qualified [respectively, unqualified]. From Lemma 3, we know that quality \( i \) grade school students will invest in skills if and only if \( k \leq k^*_i \), the investment threshold determined as the solution to Eq. (6).

**Details of \( G_iQ \).** Given the grade school educational production technology in Table 1, we know that a quality \( i \) grade school student could be qualified only if she invests, i.e., only if \( k \leq k^*_i \). Thus, the cost distribution conditional on being qualified is simply the initial distribution \( G (\cdot) \) truncated above at \( k^*_i \). Thus, for \( i \in \{ h, l \} \),

\[ G_iQ (k) = \begin{cases} 1 & \text{if } k > k^*_i \\ \frac{G(k)}{G(k^*_i)} & \text{if } k \leq k^*_i. \end{cases} \quad (A7) \]

**Details of \( G_hU \).** For a high quality grade school student, her investment \( k \) is for sure higher than \( k^*_h \) conditional on being unqualified. Thus,

\[ G_hU (k) = \begin{cases} 0 & \text{if } k < k^*_h \\ \frac{G(k) - G(k^*_h)}{1 - G(k^*_h)} & \text{if } k \geq k^*_h. \end{cases} \quad (A8) \]

**Details of \( G_lU \).** For a low quality grade school student, being unqualified does not necessarily imply that she did not invest (recall \( \phi_l < 1 \)). Taking this into account, we have

\[ G_lU (k) = \begin{cases} \frac{(1 - \phi_l)G(k) \left[ 1 - (1 - \phi_l)G(k^*_l) + [1 - G(k^*_l)] \right]}{(1 - \phi_l)G(k^*_l) + [1 - G(k^*_l)]} & \text{if } k < k^*_l \\ \\ \frac{(1 - \phi_l)G(k) \left[ 1 - (1 - \phi_l)G(k^*_l) + [1 - G(k^*_l)] \right]}{(1 - \phi_l)G(k^*_l) + [1 - G(k^*_l)]} & \text{if } k \geq k^*_l. \end{cases} \quad (A9) \]
A.4 Measures of University Students

In this section, we provide omitted details of the measures of university students we used to define the two mappings $\Phi_S$ and $\Phi_N$ in Section 3.2. Recall that all the mappings $Q_{j,i}, M_j, i \in \{h,l\}, j \in \{N,S\}$ are functions of $(\pi^{*}_{h}, \pi^{*}_{l}, \pi^{*}_{S}, \pi^{*}_{N})$ which enter these mappings through their effects on $k^{*}_{i}, \mu^{e*}_{i}$ and $k^{*}_{jQ}$. In the expressions below, we simplify the exposition by using $k^{*}_{jQ}$ as shorthand for $k^{*}_{jQ}\left(\mu^{f*}_{j}\left(\pi^{*}_{j}\right)\right)$, $k^{*}_{i}$ as shorthand for $k^{*}_{i}\left(\mu^{e*}_{i}\left(\pi^{*}_{i}; \mu^{f*}_{S}\left(\pi^{*}_{S}\right)\right), \mu^{f*}_{S}\left(\pi^{*}_{S}\right), \mu^{f*}_{N}\left(\pi^{*}_{N}\right)\right)$ and $\mu^{e*}_{i}$ as shorthand for $\mu^{e*}_{i}\left(\pi^{*}_{i}; \mu^{f*}_{S}\left(\pi^{*}_{S}\right)\right)$, where the mappings $k^{*}_{jQ}\left(\cdot\right), k^{*}_{i}\left(\cdot, \cdot, \cdot\right)$ and $\mu^{e*}\left(\cdot, \cdot\right)$ are defined respectively in Lemma 2 to 4.

**Derivation of $Q_{S,h}$**.  
Recall that $Q_{S,h}$ is the total measure of qualified selective university students originally from high quality grade schools:

$$Q_{S,h}(\pi^{*}_{h}, \pi^{*}_{l}, \pi^{*}_{S}, \pi^{*}_{N}) = (1 - \alpha) \left\{ \frac{\text{Term 1}}{G(k_{h}^{*}) \left[ (1 - \rho_{q}) + \rho_{q}\mu^{e*}_{h} \right] G_{hQ}(k_{S,Q}^{*})} + \frac{\text{Term 2}}{G(k_{h}^{*}) \left[ (1 - \rho_{q}) + \rho_{q}\mu^{e*}_{h} \right] \left[ 1 - G_{hQ}(k_{S,Q}^{*}) \right] \zeta_{QN}} + \frac{\text{Term 3}}{[1 - G(k_{h}^{*})] \rho_{q}\mu^{e*}_{h} G_{hU}(k_{S,U}^{*}) \zeta_{UI}} \right\}.$$

The above expression can be understood as follows. Term 1 is the fraction of high quality grade school students who make the skill investment in grade school ($k \leq k_{h}^{*}$), who are admitted to the selective university (captured by the probability term $(1 - \rho_{q}) + \rho_{q}\mu^{e*}_{h}$) and who continue investment in college ($k \leq k_{S,Q}^{*}$) which guarantees that they are all qualified with probability one. Term 2 is the fraction of high quality grade school students who make the skill investment in grade school ($k \leq k_{h}^{*}$), who are admitted to the selective university (captured by the probability term $(1 - \rho_{q}) + \rho_{q}\mu^{e*}_{h}$), but who do not invest in skills in college ($k \geq k_{S,Q}^{*}$). Such students are qualified when graduating from the selective universities with probability $\zeta_{QN}$. Term 3 is the fraction of high quality grade school students who do not invest in grade school ($k > k_{h}^{*}$), who get into selective university by luck (captured by the probability term $\rho_{q}\mu^{e*}_{h}$), who invest in skills in the selective university ($k < k_{S,U}^{*}$). Such students are qualified when graduating from the selective universities with probability $\zeta_{UI}$.

**Derivation of $Q_{S,l}$**.  
Recall that $Q_{S,l}$ is the total measure of qualified selective university
students originally from low quality grade schools. By similar accounting as above, it is given by

\[ Q_{S,l}(\pi^S, \pi^l, \pi^S, \pi^N) = \alpha \left\{ \phi_l G(k^*_l) \left[ (1 - \rho_q) + \rho_q \mu^c_l \right] G_{lQ}(k^*_Q) \right. \]
\[ + \left. \phi_l G(k^*_l) \left[ (1 - \rho_q) + \rho_q \mu^c_l \right] \right\} \zeta_{QN} \]
\[ + \left[ 1 - G(k^*_l) + (1 - \phi_l) G(k^*_l) \rho_u \mu^c_l G_{U}(k^*_U) \right\} \zeta_{U1} \} \]

**Derivation of** \( Q_{N,h} \). Recall that \( Q_{N,h} \) is the total measure of qualified non-selective university students originally from high quality grade schools.

\[ Q_{N,h}(\pi^S, \pi^l, \pi^S, \pi^N) = \phi_c (1 - \alpha) \left\{ G(k^*_h) \left[ \rho_q (1 - \mu^c_h) \right] G_{hQ}(k^*_Q) \right. \]
\[ + \left. G(k^*_h) \left[ \rho_q (1 - \mu^c_h) \right] \right\} \zeta_{QN} \]
\[ + \left[ 1 - G(k^*_h) + (1 - \phi_l) G(k^*_h) \right\} \zeta_{U1} \} \]

**Derivation of** \( Q_{N,l} \). Recall that \( Q_{N,l} \) is the total measure of qualified non-selective university students originally from low quality grade schools.

\[ Q_{N,l}(\pi^S, \pi^l, \pi^S, \pi^N) = \phi_c \alpha \left\{ G(k^*_l) \left[ \rho_q (1 - \mu^c_l) \right] G_{lQ}(k^*_Q) \right. \]
\[ + \left. G(k^*_l) \left[ \rho_q (1 - \mu^c_l) \right] \right\} \zeta_{QN} \]
\[ + \left[ 1 - G(k^*_l) + (1 - \phi_l) G(k^*_l) \right\} \zeta_{U1} \} \]

**Derivation of** \( M_S \) and \( M_N \). Recall that \( M_S \) and \( M_N \) are respectively the total measure of selective and non-selective university students. From simple accounting, they are given by

\[ M_S(\pi^S, \pi^l, \pi^S, \pi^N) = (1 - \alpha) \left\{ G(k^*_h) \left[ (1 - \rho_q) + \rho_q \mu^c_h \right] + [1 - G(k^*_h)] \rho_u \mu^c_h \right\} \]
\[ + \alpha \left\{ \phi_l G(k^*_l) \left[ (1 - \rho_q) + \rho_q \mu^c_l \right] + [1 - G(k^*_l) + (1 - \phi_l) G(k^*_l)] \rho_u \mu^c_l \right\} \]
\[ M_N(\pi^S, \pi^l, \pi^S, \pi^N) = (1 - \alpha) \left\{ G(k^*_l) \left[ \rho_q (1 - \mu^c_l) \right] + [1 - G(k^*_l)] (1 - \rho_u \mu^c_l) \right\} \]
\[ + \alpha \left\{ \phi_l G(k^*_l) \left[ \rho_q (1 - \mu^c_l) \right] + [1 - G(k^*_l) + (1 - \phi_l) G(k^*_l)] (1 - \rho_u \mu^c_l) \right\} \}

**B Appendix: Proofs.**

**Proof of Lemma 1:**

**Proof.** The expected profit of assigning a student from quality \( j \) universities with unclear signals to the complex job is

\[ \frac{\pi^c_j \rho_q}{\pi^c_j \rho_q + \left( 1 - \pi^c_j \right) \rho_u} \chi_J + \frac{\left( 1 - \pi^c_j \right) \rho_u}{\pi^c_j \rho_q + \left( 1 - \pi^c_j \right) \rho_u} \chi_J. \]
The firm’s profit from assigning her to the simple job is 0. Thus, the firm will optimally assign such a worker to the complex job, i.e., $\mu^f_j(\pi^c_j) = 1$, if and only if

$$\pi^c_j > \pi^c \equiv \frac{\rho_q \lambda^f_q}{\rho_q \lambda^f_q + \rho_u \lambda^f_u}. $$

**Proof of Lemma 2:**

Proof. First note that the condition $\theta_q + \theta_u \leq 1$ implies that $\rho_q \leq \rho_u$.

- Using the formulas $k^*_S (\mu^f_S)$ and $k^*_N (\mu^f_N)$ given by (3) and (A1), we have

$$k^*_S (\mu^f_S) - k^*_N (\mu^f_N) = (1 - \zeta_{QN}) \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_S \right] - \zeta_{UI} \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right]$$

Hence $k^*_S (\mu^f_S) - k^*_N (\mu^f_N) \geq 0$ for any $\mu^f_S \in [0,1]$ if and only if $\zeta_{UI} + \zeta_{QN} \leq 1$.

- Using the formulas $k^*_S (\mu^f_S)$ and $k^*_N (\mu^f_N)$ given by (A1) and (A2), we have

$$k^*_S (\mu^f_S) - k^*_N (\mu^f_N) = \zeta_{UI} \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right] - \phi (1 - \zeta_{QN}) \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right]$$

Note, $(1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \leq 1 - \rho_q$ since $\rho_q < \rho_u$; and $(\rho_q - \rho_u) \mu^f_S \geq \rho_q - \rho_u$ since $\mu^f_S \leq 1$, hence

$$k^*_S (\mu^f_S) - k^*_N (\mu^f_N) \geq \zeta_{UI} \omega (1 - \rho_u) - \phi (1 - \zeta_{QN}) \omega (1 - \rho_q)$$

which is non-negative if $\phi \leq \frac{\zeta_{UI} (1 - \rho_u)}{(1 - \zeta_{QN})(1 - \rho_q)}$.

- Finally, Using the formulas $k^*_N (\mu^f_N)$ and $k^*_N (\mu^f_N)$ given by (A2) and (A3), we have

$$k^*_N (\mu^f_N) - k^*_N (\mu^f_N) = \phi (1 - \zeta_{QN}) \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right] - \phi \zeta_{UI} \omega \left[ (1 - \rho_q) + (\rho_q - \rho_u) \mu^f_N \right]$$

which is non-negative if and only if $\zeta_{UI} + \zeta_{QN} \leq 1$. 

\[36\]
**Proof of Lemma 3:**

**Proof.** Since the left hand side of Eq. (6) is strictly increasing in \( k_i \), it suffices to show that its right hand side (LHS) is weakly decreasing in \( k_i \). To prove this, we first note that under the stated conditions, Lemma 2 tells us that \( k_{SQ}^* \left( \mu_S^f \right) \geq k_{SU}^* \left( \mu_S^f \right) \geq k_{NQ}^* \left( \mu_N^f \right) \geq k_{NU}^* \left( \mu_N^f \right) \). This ordering, together with the detailed formulas of \( V_{iQ} \left( \cdot; \mu_j^f \right) \), \( V_{iU} \left( \cdot; \mu_j^f \right) \) provided in Appendix A.2, implies that the slope of the LHS of Eq. (6) when \( k_i \leq k_{NU}^* \left( \mu_N^f \right) \) is given by

\[-\phi_i \left\{ \left[ \left( 1 - \rho_q \right) + \rho_q \mu_i^f \right] + \left[ \rho_q \left( 1 - \mu_i^c \right) \right] \right\} = 0.\]

Similarly, when \( k_i \in \left[ k_{NU}^* \left( \mu_N^f \right), k_{NQ}^* \left( \mu_N^f \right) \right] \), its slope is \(-\phi_i \left\{ \left[ \left( 1 - \rho_q \right) + \rho_q \mu_i^f \right] + \left[ \rho_q \left( 1 - \mu_i^c \right) \right] \right\} < 0; \) when \( k_i \in \left[ k_{NQ}^* \left( \mu_N^f \right), k_{SU}^* \left( \mu_S^f \right) \right] \), its slope is \(-\phi_i \left\{ \left[ \left( 1 - \rho_q \right) + \rho_q - \rho_u \mu_i^c \right] \right\} \leq \phi_i \left( 1 - \rho_u \right) < 0; \) when \( k_i \in \left[ k_{SU}^* \left( \mu_S^f \right), k_{SQ}^* \left( \mu_S^f \right) \right] \), its slope is \(-\phi_i \left\{ \left[ \left( 1 - \rho_q \right) + \rho_q \mu_i^f \right] \right\} < 0; \) and finally, when \( k_i > k_{SQ}^* \left( \mu_S^f \right) \), its slope is zero. Thus the LHS of Eq. (6) is weakly decreasing in the whole domain. Thus Eq. (6) has a unique solution. The threshold property also follows immediately. \( \square \)

**Proof of Lemma 4:**

**Proof.** Let \( \pi_i^s \in [0, 1], i \in \{h, l\}, \) be the selective universities’ prior that a quality \( i \) GS student is qualified. Suppose that they observe an unclear signal \( \theta \in [\theta_q, \theta_u] \). Then the posterior that such a student is qualified is given by \( \pi_i^s \rho_q / \left[ \pi_i^s \rho_q + \left( 1 - \pi_i^s \right) \rho_u \right] \). Given the selective universities’ prior belief \( \pi_i^t \), they also effectively formulate a belief about \( k^*_i = G^{-1} \left( \pi_i^s / \phi_i \right) \) where \( \phi_i = 1. \)

Together with the information about the firms’ hiring policy toward selective university graduates with unclear signals \( \mu_S^f \), the selective universities can forecast the probability that such a student will be a qualified college graduate, if admitted into the selective university. This probability is given by

\[
\Psi_i \left( \pi_i^s; \mu_S^f \right) = \frac{\pi_i^s \rho_q}{\pi_i^s \rho_q + \left( 1 - \pi_i^s \right) \rho_u} \left\{ G_{iQ} \left( k_{SQ}^* \left( \mu_S^f \right) \right) + \left[ 1 - G_{iQ} \left( k_{SQ}^* \left( \mu_S^f \right) \right) \right] \right\} \\
+ \frac{\left( 1 - \pi_i^s \right) \rho_u}{\pi_i^s \rho_q + \left( 1 - \pi_i^s \right) \rho_u} G_{iU} \left( k_{SU}^* \left( \mu_S^f \right) \right) \left[ \zeta_{QI} \right].
\]

To understand the first term, recall that \( \pi_i^s \rho_q / \left[ \pi_i^s \rho_q + \left( 1 - \pi_i^s \right) \rho_u \right] \) is the posterior probability that a student from a quality \( i \) GS with unclear test signal is qualified. The conditional skill investment cost distribution for a qualified quality \( i \) GS student is given by \( G_{iQ} \). From our characterization of the selective university students’ investment incentives, we know that if \( k \leq k_{SQ}^* \left( \mu_S^f \right) \) which occurs with probability \( G_{iQ} \left( k_{SQ}^* \left( \mu_S^f \right) \right) \), they will further invest in skills if admitted to
a selective university, in which case they will be qualified with probability one. Similarly, if \( k > k_{SQ}^* \left( \mu_S^f \right) \), which occurs with probability \( 1 - G_{iQ} \left( k_{SQ}^* \right) \), they will not further invest in universities, in which case they will be qualified with probability \( \zeta_{QN} \). Thus in expectation, a qualified quality \( i \) GS student will be a qualified graduate from the selective university with probability \( G_{iQ} \left( k_{SQ}^* \left( \mu_S^f \right) \right) + (1 - G_{iQ} \left( k_{SQ}^* \left( \mu_S^f \right) \right)) \zeta_{QN} \). Analogously, the second term represents the probability that an unqualified quality \( i \) GS will be qualified if admitted to the selective university. It could be easily verified, using the expressions of \( G_{iQ} \) and \( G_{iU} \) given in Appendix A.3 that \( \Psi_i \left( \pi_i^*; \mu_S^f \right) \) is increasing in \( \pi_i^* \) for a fixed \( \mu_S^f \).

Next, with \( \Psi_i \left( \pi_i^*; \mu_S^f \right) \) calculated above, together with firms hiring policy \( \mu_S^f \), the probabilities of a qualified and unqualified selective university graduate being assigned to the complex job are respectively given by

\[
P_Q \left( \mu_S^f \right) = (1 - \rho_q) + \rho_q \mu_S^f \quad \text{and} \quad P_U \left( \mu_S^f \right) = \rho_u \mu_S^f.
\]

Thus the selective university’s expected payoff can be written as:

\[
\Psi_i \left( \pi_i^*; \mu_S^f \right) \left[ \chi_q^c + P_Q \left( \mu_S^f \right) R_q \right] - \left[ 1 - \Psi_i \left( \pi_i^*; \mu_S^f \right) \right] \left[ \chi_u^c + P_U \left( \mu_S^f \right) R_u \right].
\]

Therefore, for a particular firm hiring policy \( \mu_S^f \), \( \mu_i^c \left( \pi_i^*; \mu_S^f \right) = 1 \) if and only if

\[
\Psi_i \left( \pi_i^*; \mu_S^f \right) \geq \frac{\chi_q^c + P_U \left( \mu_S^f \right) R_u}{\chi_u^c + P_U \left( \mu_S^f \right) R_u + \chi_q^c + P_Q \left( \mu_S^f \right) R_q} \equiv \hat{\Psi} \left( \mu_S^f \right).
\]

Since \( \Psi_i \left( \pi_i^*; \mu_S^f \right) \) is increasing in \( \pi_i^* \) for a fixed \( \mu_S^f \), we can thus conclude that \( \Psi_i \left( \pi_i^*; \mu_S^f \right) \geq \hat{\Psi} \left( \mu_S^f \right) \) if and only if \( \pi_i^* \geq \hat{\pi}_i^* \left( \mu_S^f \right) \).

**Proof of Lemma 5:**

*Proof.* Inspecting the formulas for the university students’ investment thresholds given in Appendix A.1, we can immediately see that under the condition \( \phi_c \leq (1 - \rho_u) / (1 - \rho_q) \), \( k_{SQ}^* \left( \mu_S^f \right) \geq k_{NQ}^* \left( \mu_N^f \right) \) and \( k_{SU}^* \left( \mu_S^f \right) \geq k_{NU}^* \left( \mu_N^f \right) \) for any value of \( \mu_S^f \in [0,1] \) and \( \mu_N^f \in [0,1] \). Moreover, the selective colleges’ optimal admission policy insures that the pool of students admitted to selective colleges is at least as “good” as that of the non-selective colleges in the senses that, first, they are more likely to be qualified; second the skill investment cost distribution will be better. The reason is simply that the selective colleges could follow a random admission policy and do as well as the non-selective colleges. But under the given condition, the students in selective colleges have more
incentive to further invest in skills regardless of their GS qualification; and moreover the education in selective universities are more productive. Thus in any equilibrium, \( \pi_S^* \) must be higher than \( \pi_N^* \).

It immediately follows from Lemma 1 that \( \mu_S^{t*} \geq \mu_N^{t*} \) in any equilibrium of the model.

\[\text{Proof of Lemma 6:}\]

\[\text{Proof.}\] Plugging the leading case equilibrium policies into the formulas of the investment thresholds given in Appendix A.1, we obtain:

\[k_{NQ}^* (0) = \phi_c (1 - \zeta_{QN}) (1 - \rho_q) \omega, \quad k_{NU}^* (0) = \phi_c \zeta_{UI} (1 - \rho_q) \omega.\]

In the leading case equilibrium, we can simplify Eq. (6) for \( k_h^* \) as

\[k_h = V_{SQ} (k_h; 1) - \rho_u V_{SU} (k_h; 1) - (1 - \rho_u) V_{NU} (k_h; 0).\]

By Lemma 3, we know that the above equation has a unique solution. We can verify that the unique solution satisfies \( k_h^* \geq k_{NQ}^* (0) \) if \( \phi_c \leq (1 - \rho_u \zeta_{UI} - (1 - \zeta_{UI}) (1 - \rho_q) ). \)

Similarly, \( k_l^* \) solves

\[k_l = \phi_l \{(1 - \rho_q) V_{SQ} (k_l; 1) + \rho_q V_{NQ} (k_l; 0) - V_{NU} (k_l; 0)\} \]

in the leading case equilibrium. One can verify that this unique solution satisfies \( k_l^* \leq k_{NU}^* (0) \) if and only if \( \phi_l \leq \phi_c \zeta_{UI} / [1 + (\rho_q - \zeta_{UI}) \phi_c] \).

\[\text{Proof of Lemma 7:}\]

\[\text{Proof.}\] We first show that \( \tilde{\pi} \) given in (13) is larger than \( \pi_N^* \). Recall from formula (11) that \( \pi_N^* = (Q_{N,l} + Q_{N,h}) / M_N. \) From Lemma 6 we know that \( k_l^* \leq k_{NQ}^* (0) \), and \( k_h^* > k_{NU}^* (0) \) under the stated conditions in the leading case equilibrium. After substituting these thresholds into the expressions of \( Q_{N,l}, Q_{N,h} \) and \( M_N \), we have \( Q_{N,h} = 0 \), and

\[Q_{N,l} = \phi_c \alpha \left\{ \phi_l G (k_l^*) \rho_q + [1 - G (k_l^*) + (1 - \phi_l) G (k_l^*)] G_U (k_{NU}^*) \zeta_{UI} \right\},\]

\[M_N = (1 - \alpha) (1 - \rho_u) [1 - G (k_h^*)] + \alpha \left[ \phi_l G (k_l^*) \rho_q + 1 - G (k_l^*) + (1 - \phi_l) G (k_l^*) \right].\]
Therefore,

\[
\pi_N^c = \frac{Q_{NI} + Q_{Nh}}{M_N} < \frac{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] G_{IU} (k^*_NU) \zeta_{UI}}{\phi_l G (k^*_l) \rho_q + 1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)} < \frac{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] G_{IU} (k^*_SU) \zeta_{UI}}{\phi_l G (k^*_l) \rho_q + 1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)} < \frac{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] \rho_u G_{IU} (k^*_SU) \zeta_{UI}}{\phi_l G (k^*_l) \rho_q + 1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)} = \tilde{\pi},
\]

where the first inequality follows from the fact that \( \phi_c < 1 \); the second inequality follows from \( k^*_NU < k^*_SU \); the third inequality follows from simple algebra; and the last equality follows from the definition of \( \tilde{\pi} \) given by (13).

Now,

\[
\Xi \equiv \frac{\tilde{\pi} - \tilde{\pi} (1 - \rho_q)}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} - \frac{\pi_N^c \rho_q}{\pi_N^c \rho_q + (1 - \pi_N^c) \rho_u} > \frac{\tilde{\pi} - \tilde{\pi} (1 - \rho_q)}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} - \frac{\pi_N^c \rho_q}{\pi_N^c \rho_q + (1 - \pi_N^c) \rho_u} = \frac{\tilde{\pi} \rho_q}{\tilde{\pi} \rho_q + (1 - \tilde{\pi}) \rho_u} - \frac{\pi_N^c \rho_q}{\pi_N^c \rho_q + (1 - \pi_N^c) \rho_u} > 0,
\]

where the first inequality follows from the fact that \( \tilde{\pi} < \phi_c \tilde{\pi} < \tilde{\pi} \).

**Proof of Lemma 8:**

**Proof. (Step 1)** We first show that if \( \zeta_{UI} \) is sufficiently close to one, \( \tilde{\pi} \) is monotonically decreasing in \( \phi_l \). Recall from (13) that

\[
\tilde{\pi} = \frac{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] \rho_u G_{IU} (k^*_SU) \zeta_{UI}}{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] \rho_u},
\]

Using the expression for \( G_{IU} \) given by (A9), we can rewrite \( \tilde{\pi} \) as

\[
\tilde{\pi} = \frac{\phi_l G (k^*_l) \rho_q + \rho_u \zeta_{UI} [1 - G (k^*_l) + G (k^*_SU) - G (k^*_l)]}{\phi_l G (k^*_l) \rho_q + [1 - G (k^*_l) + (1 - \phi_l) G (k^*_l)] \rho_u} = \frac{\phi_l G (k^*_l) \rho_q + \rho_u \zeta_{UI} [G (k^*_SU) - \phi_l G (k^*_l)]}{\phi_l G (k^*_l) \rho_q + [1 - \phi_l G (k^*_l)] \rho_u}.
\]
Now notice that \( \tilde{\pi} \) depends on \( \phi_t \) only through \( \phi_t G (k_t^*) \); also recall that \( k_t^* \) is increasing in \( \phi_t \) and \( k_{SU}^* \) is independent of \( \phi_t \) as long as we are in the leading case equilibrium. For expositional simplicity, denote \( m = \phi_t G (k_t^*) \). Thus

\[
\tilde{\pi} = \frac{m \rho_q + \rho_u \zeta_{UI} [G (k_{SU}^*) - m]}{m \rho_q + (1 - m) \rho_u}.
\]

Note that

\[
\frac{\partial \tilde{\pi}}{\partial \phi_t} = \frac{\partial \tilde{\pi}}{\partial m} \frac{\partial m}{\partial \phi_t} = \frac{\partial m}{\partial \phi_t} \frac{\rho_q \rho_u - \zeta_{UI} \left[ \rho_q^2 + G (k_{SU}^*) \left( \rho_q \rho_u - \rho_u^2 \right) \right]}{\left[ m \rho_q + (1 - m) \rho_u \right]^2}.
\]

Since \( \partial m / \partial \phi_t > 0 \), a sufficiently condition for \( \partial \tilde{\pi} / \partial m < 0 \) is that

\[
\zeta_{UI} > \frac{\rho_q \rho_u}{\rho_u^2 + G (k_{SU}^*) \left( \rho_q \rho_u - \rho_u^2 \right)}.
\]

Note that the right hand side of the above inequality is always less than one because

\[
[1 - G (k_{SU}^*)] \left( \rho_q \rho_u - \rho_u^2 \right) < 0.
\]

**Step 2:** We show that both \( \partial \pi_{SU}^* / \partial m \) and \( \partial \tilde{\pi} / \partial m \) can both be made arbitrarily close to zero when \( \phi_c \) sufficiently close to zero. This follows simply from the fact that \( \pi_{SU}^* \) and \( \tilde{\pi} \) are multiplicative in \( \phi_c \), specifically, \( \pi_{SU}^* \) is given by (see the proof of Lemma 7)

\[
\pi_{SU}^* = \frac{\phi_c \alpha \left\{ \phi_t G (k_t^*) \rho_q + [G (k_{SU}^*) - \phi_t G (k_t^*)] \zeta_{UI} \right\}}{(1 - \alpha) \left[ 1 - G (k_h^*) \right] \left( 1 - \rho_u \right) + \alpha \left\{ \phi_t G (k_t^*) \rho_q + [1 - G (k_t^*) + (1 - \phi_t) G (k_t^*)] \right\}};
\]

\( \tilde{\pi} \) is given by [see expression (14)]

\[
\tilde{\pi} = \frac{\phi_c \left\{ \phi_t G (k_t^*) \rho_q + [G (k_{SU}^*) - \phi_t G (k_t^*)] \rho_u \zeta_{UI} \right\}}{\phi_t G (k_t^*) \rho_q + [1 - \phi_t G (k_t^*)] \rho_u},
\]

where we used the expression of \( G_{UI} (\cdot) \) given by (A9).

**Step 3:** We show that, if \( \zeta_{UI} \) is sufficiently close to 1 and \( \phi_c \) sufficiently close to 0, then \( \Xi \) is strictly decreasing in \( \phi_t \). By law of total derivatives,

\[
\frac{\partial \Xi}{\partial m} = \frac{\partial \Xi}{\partial \tilde{\pi}} \frac{\partial \tilde{\pi}}{\partial m} + \frac{\partial \Xi}{\partial \pi_{SU}^*} \frac{\partial \pi_{SU}^*}{\partial m} + \frac{\partial \Xi}{\partial \tilde{\pi}} \frac{\partial \tilde{\pi}}{\partial m}.
\]

Since \( \partial \pi_{SU}^* / \partial m \) and \( \partial \tilde{\pi} / \partial m \) are arbitrarily close to 0 when \( \phi_c \) is sufficiently close to 0, and \( \partial \tilde{\pi} / \partial m \) is strictly negative if \( \zeta_{UI} \) is sufficiently close to 1, it suffices to show that \( \partial \Xi / \partial \tilde{\pi} \) is strictly positive, which is the case because

\[
\frac{\partial \Xi}{\partial \tilde{\pi}} = \frac{\rho_u \left[ 1 - \tilde{\pi} \left( 1 - \rho_q \right) \right]}{\left[ \tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} \left( 1 - \rho_q \right) \right]^2} > 0.
\]
Proof of Proposition 1:

Proof. Recall from expression (19) that

\[ C_f - C_c = \Delta \left\{ \Xi \left( \chi_q^f + \chi_u^f \right) + \frac{\tilde{\pi}}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} \left( \chi_u^c + R_q \right) \right. \]

\[ \left. - \frac{(1 - \tilde{\pi}) \rho_u}{\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)} \left( \chi_u^c + R_u \right) \right\} . \]

Thus, \( C_f - C_c > 0 \) if \( \chi_q^f + \chi_u^f \) is sufficiently large because \( \Xi > 0 \). Moreover, from Lemma 8, we know that when \( \xi_{UI} \) is close to 1 and \( \phi_e \) close to 0, \( \Xi, \tilde{\pi}/(\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)) \) and \( -(1 - \tilde{\pi}) \rho_u/(\tilde{\pi} + (1 - \tilde{\pi}) \rho_u - \tilde{\pi} (1 - \rho_q)) \) are all strictly decreasing in \( \phi_t \). Thus all three terms are largest when \( \phi_t \) approaches 0. Let \( Q \) denote the minimal threshold value of \( \chi_q^f + \chi_u^f \) for \( C_f > C_c \) when \( \phi_t = 0 \). By monotonicity, for any \( \chi_q^f + \chi_u^f > Q \), we can find \( \phi_t > 0 \) such that \( C_f > C_c \) continues to hold as long as \( \phi_t < \phi_t \).

Proof of Proposition 2:

Proof. (Part 1) When the AA is implemented in the firm hiring stage in an economy in the leading case equilibrium, we know that the firms will start to hire non-selective university students with unclear signals. Thus, AA in the hiring stage increases \( \mu_N^f \) from zero to a positive value, whereas \( \mu_S^f \) remains at 1. Observe from the expressions of \( k_{SQ}^* \left( \mu_j^f \right) \) and \( k_{SU}^* \left( \mu_j^f \right) \) detailed in Appendix A.1, we know that \( k_{SQ}^* \) and \( k_{SU}^* \) remain unchanged; and \( k_{NQ}^* \) and \( k_{NU}^* \) will decrease in \( \mu_N^f \) unless \( \rho_q = \rho_u \).

Now we show that \( k_h^* \) will decrease as \( \mu_N^f \) increases. First note that the increase in \( \mu_N^f \) does not effect the slope of the segments of the piece-wise linear curve described by the right hand side of Eq. 6). It does the initial height of the curve as well as the threshold points. Under the admission policy that \( \mu_h^* = 1 \), the initial height of the curve is decreased since \( - (1 - \rho_u) \omega \left( \phi_e \xi_{UI} \rho_q + \phi_c (1 - \xi_{UI}) \rho_u \right) \) is negative. Moreover we already showed that \( k_{NU}^* \) and \( k_{NQ}^* \) decrease in \( \mu_N^f \). Thus \( k_h^* \) decreases as a result of the increase in \( \mu_N^f \). [See Figure 2 for an illustration.] Similarly, \( k_i^* \) also decreases in \( \mu_N^f \) since the initial height of its corresponding curve is affected by \( \mu_N \) proportionally to \( \omega \phi_e \left\{ \rho_q^* - [\rho_q \xi_{UI} + (1 - \xi_{UI}) \rho_u] \right\} \), which is negative.

(Part 2) When AA is implemented in the university admission stage in an economy in the leading case equilibrium, we know that the selective universities will admit students from low
quality GS with unclear signals. That is, $\mu^c_l$ will increase from 0. By inspecting the formulas for $k^*_jQ$ and $k^*_jU$ detailed in Appendix A.1, we immediately see that they are not affected by an increase in $\mu^c_l$. Similarly, the equation for $k^*_h$ is not affected by $\mu^c_l$ either.

Now we show that $k^*_l$ may be increased when $\mu^c_l$ increases. A sufficient condition for $k^*_l$ to increase is that the initial constant term in the right hand side of Eq. (6) increases with $\mu^c_l$. With some algebra, we can show that the initial constant changes with $\mu_l$ proportional to

$$\phi_l \left\{ \left( \rho_q - \rho_q (1 - \rho_q) \phi_c - \rho_u^2 \right) - \zeta_{UI} \left( \rho_u - \rho_u (1 - \rho_q) \phi_c - \rho_u^2 \right) \right\}.$$  

Thus it is positive if and only if

$$\zeta_{UI} < \frac{\rho_q - \rho_q (1 - \rho_q) \phi_c - \rho_u^2}{\rho_u - \rho_u (1 - \rho_q) \phi_c - \rho_u^2}.$$  

(B13)

Note that if $\rho_q = \rho_u$, the above inequality always holds since $\zeta_{UI} < 1$. If $\rho_q < \rho_u$, then, for any given $\zeta_{UI}$, we can find $\rho_q$ sufficiently close to $\rho_u$ such that $\zeta_{UI}$ is smaller the threshold specified in the right hand side of (B13)